CASE STUDY: USING VISUAL REPRESENTATIONS TO ENHANCE CONCEPTUAL KNOWLEDGE OF DIVISION IN MATHEMATICS.

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ABSTRACT

Literature emphasizes how important it is that procedural and conceptual knowledge of mathematics should be learned in integration. Yet, generally, the learning and teaching in mathematics classrooms relies heavily on isolated procedures. This study aims to improve teaching and learning of partitive and quotitive division, moving away from isolated procedural knowledge to that of procedures with their underlying concepts through the use of manipulatives, visual representation and questioning. Learning and teaching lessons were designed to teach partitive and quotitive division both procedurally and conceptually.

The study explored the roles these manipulatives, visual representations and questioning played toward the conceptual learning of partitive and quotitive division. It was found that manipulatives and iconic visualization enhanced learning, and this could be achieved through scaffolding using a questioning approach. It was concluded that manipulatives and iconic visualization need to be properly planned and used, and integrated with questioning to achieve success in the learning of procedural and conceptual knowledge.

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ACRONYMS

CALD	Cambridge Advanced Learners' Dictionary
HLT	Hypothetical Learning Trajectory
MBEC	Ministry of Basic Education and Culture
Med	Masters' of Education
MIB	Ministry of Information and Broadcasting
M.o.E	Ministry of Education
(Day, Less, Par)	Day, Lesson, Paragraph
ZPD	Zone of proximal development

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CHAPTER 1

INTRODUCTION

1.1 Introduction

This chapter introduces my study, which aims to investigate the influence of manipulatives, visualizations and questioning in the learning of procedural and conceptual knowledge of division of whole numbers in a grade 4 class. This chapter consists of four sections. The next section provides the context of the study, the ground on which the investigation was built. This is followed by section three which briefly states the research goals and the research questions to which the study aims to respond. The last section gives the layout of the whole thesis.

1.2 Context of the study

In Namibia everyone has a right to education. This access is ensured by section 1 and 2 of article 20 of the constitution of the republic of Namibia, which declares free and compulsory primary education (Ministry of Information and Broadcasting (MIB), 1990). This education landscape is well defined by the four values that underpin the Namibian education policy. These values are: Access, Equity, Quality and Democracy (Ministry of Basic Education and Culture (MBEC), 1993). This study aims to contribute toward quality. Quality education as per education policy does not refer only to good performance in examinations (MBEC, 1993), as there are times when learners perform well in examinations yet are useless in the society (MBEC, 1993; Brown, Collins & Duguid, 1989). Quality education entails the knowledge of integrated skills and understanding. It is a cultivation or development of the ability to apply skills, information or knowledge accumulated through learning to unfamiliar situations or to solve real–life problems (MBEC, 1993; Brown et al, 1989). This learning is what Lampert (1992) and Brown et al (1989) called authentic learning, whereby classroom instruction allows learners to practice mathematics skills in a community of practice (the way mathematicians do). In order for classroom learning to be authentic in this way, skills and understanding must be integrated and learning has to be contextualized.

Reflection on my teaching indicates how school learning is often otherwise, first it relies heavily on procedural knowledge. When teaching a topic such as division, isolated procedures (skills) of how to divide are generally provided to learners. In most cases, these procedures are de- contextualized and isolated from their underlying concepts. In my experience, this teaching approach does not lead to effective learning because learners end up either using skills to perform well in examinations but are un-able to use these skills to solve real-life problems, or worse, learners would not even be able to use these skills to perform well in examinations.

Secondly, based on the history of the mathematics performance of the same candidates (samples) over the previous three grades; quality education had not been achieved to the satisfaction at all (see appendix 8.1). Over the period of three grades these learners had been achieving below 50% in mathematics and the highest performance reached by most was either 50% or below that.

Literature reveals that notations, procedures, symbols or conventions learned in isolation lead to difficulties in learning (Kilpatrick, Swafford & Findell, 2001; Davis, 1986, Hiebert & Lefevre, 1986).

This study looks at the possibility of improving my teaching of isolated procedural knowledge of division, by integrating conceptual knowledge. The study opts to explore concrete manipulatives and visual representations as recommended by a number of researchers (Zimmermann & Cunningham, 1991; Arcavi, 2003; Mudaly, 2010; Sinclair, Mamolo & Whiteley, 2011; Zazkis, Dubinsky & Dautermann, 1996). These authors strongly hold the view that such teaching enhances the learning of mathematical concepts.

1.3 Research Rationale

The aim of this study is to explore the influence of manipulatives and visualizations in the learning of procedural and conceptual knowledge of division of whole number in the class of grade 4. It also investigates the role of questioning in the learning process. To achieve this aim, an intervention program of fifteen lessons was developed and implemented in seven days. This intervention program was purposefully designed to use manipulatives and visual representations to build conceptual understanding of division of whole numbers in grade 4.

This study responds to the following research questions:

1) What learning emerged from the implementation of the intervention programme, which incorporates the use of manipulatives, visualization and questioning in the mathematic learning of the grade 4 class?

2) How did the learning of sharing (partitive) and quotitive (measuring out) division in that intervention program related to conceptual and procedural knowledge?

3) How did manipulatives and iconic visualizations support this learning, and what roles did they play in the learning of sharing (partitive) and quotitive (measuring out) division of the grade 4 class?

4) How were the questions used for scaffolding, and what role does questioning play in the learning of sharing (partitive) and quotitive (measuring) division of the grade 4 class?

5) What challenges were encountered when manipulative and visual representations were used to integrate division procedures with their underlying concepts?

1.4 Overview of the thesis

This section describes the layout of the thesis. The work comprises of six chapters.

The present chapter introduces the whole thesis. It also provides a brief description of the context of the study and the rationale for conducting it

Chapter two presents a review of the relevant literature. The review focuses on the following conceptual frameworks: division, conceptual and procedural knowledge, manipulatives, visualization and questioning. First, the main concepts are defined. Second, explanations are provided of how these concepts respond to the study's research questions. This chapter also touches on the literature that informed the intervention program from its developmental stage to its implementation stage, as well as the theoretical lenses that influenced the teaching and learning of this intervention program.

Chapter three describes the chosen research methodology. It also explains the rationale for the choices made. Chapter four presents an analysis of the findings, analyzing how manipulatives and visual representations underpinning the intervention program, influenced the integration of procedural and conceptual knowledge of division.

Chapter five discusses the findings by drawing a parallel between the findings and the literature. The discussion also evaluates the findings of how manipulative and iconic visualizations were found to enhance the learning of division, and the challenges encountered with a visual teaching approach. The final chapter contains the conclusion of the study.

Chapter 2

Literature Review

2.1 Introduction

In this chapter, literature relevant to the use of manipulatives and iconic visualization in learning is reviewed. The analysis is presented into two phases. The conceptual framework underpinning the lessons of this study's intervention program and the theoretical framework, the theoretical lenses that inform, influence and guide the teaching and learning practice of this intervention program's lessons.

The review begins with the phase of conceptual framework. The conceptual framework of this study looks at four aspects: (1) the mathematical subject content 'division'; (2) the theoretical concept of 'procedural' and 'conceptual' knowledge; (3) the pedagogical aspect '’manipulatives' and 'visual representations'; and (4) Questioning.

The review ends with the phase of the theoretical framework. The learning and teaching of this study was developed and implemented in guidance, influenced and informed by social constructivism with emphasis on socio-cultural theory of Vygotsky, in particular, the aspect of ’mediating learning', with important sub-aspects such as ZPD (zone of proximal development), scaffolding, instructions, visual representations and role of questioning..

2.2 CONCEPTUAL FRAMEWORK

2.2.1 Division

2.2.1.1 Introduction

In mathematics, division can be defined as the calculation of how many times a number goes into another (Cambridge Advanced Learners' Dictionary (CALD), 2008). Even though division is one of the required basic operations domains, it can be a difficult operation for children to understand because it builds on prior knowledge of three other operations (addition, subtraction and multiplication) (Anghileri & Beishuizen, 1998).

2.2.1.2 Kinds of division

Conceptually, there are two kinds of division: partitive and quotitive division (Kilpatrick, et al., 2001; Simon, 1993). Partitive division also termed partition or sharing division, is an equal sharing whereby an object or collection of objects is divided into a number of equal fragments or sub-collections (Kilpatrick, et al., 2001; Simon, 1993). In the case of whole numbers, the dividend must be larger than the divisor and the quotient must be smaller than the dividend (operand) (Simon, 1993). Quotitive division is also an equal division which is sometimes termed measurement division. It is a division whereby a unit quantity has to be measured out of a dividend (Kilpatrick, et al., 2001; Simon, 1993). In this case the dividend also has to be a larger number than the divisor. If a quotient is the whole number, the model can be seen as a repeated subtraction (Simon, 1993).

The mathematical representation for both forms (partition and measurement division) is the same i.e. they would all be 20 ÷5, but the difference lies in their meaning they carry and the process of division (Kilpatrick et al, 2001). Simon (1993) differentiates partitive and quotitive division in terms of intensive quantity (the number of items per groups). In partitive division the intensive quantity is the one generated, whereas in quotitive division the intensive quantity is the divisor (p.236).

> ***"Partitive: # items / # groups = # items per group and Quotitive: # items / # items per group = # group" (p.236).***

If this could be clarified in the form of a division problem, (20 ÷ 5) in **partitive division** the question would ask: if twenty apples are to be shared by five people, how many apples each person would get? In **quotitive division** the question would ask: if twenty apples are to be divided amongst people of whom each person would get five apples, how many people would get the apples? One can see that in all the problems above all problems have twenty apples to be shared or divided. The difference only came in the approach of division which is in partitive ***people to share are known*** hence one would only share the apples to the five children ***one by one until all the apples are equally shared.*** In quotitive division the ***number of***

people the apples are to be divided among them is not known hence one would be dividing by ***measuring out the size of five apples at a time give them to one person and keep measuring that size of apples out give to another until all apples get finished.***

2.2.1.3 Types of division problems

Division and multiplication are intimately related, forming different aspects of multiplicative thinking. Multiplicative thinking tasks also correspond to different categories. Kouba (1989); Mulligan & Mitchelmore (1997) explain how the quantities used or apparent in a problem and the relationship between them yield to the three major types of problems. Although Kouba and Mulligan used different names, the examples of problems they give indicate that they are both talking the same thing. The three major types of problems provided by Kouba (1989) are: first, the scalar problem e.g. 3 times as many candies. Mulligan & Mitchelmore (1997) called this category 'the multiplicative comparisons problems'. Second is the category called cross–product problems i.e. possible skirt, blouse combination (e.g. with 3 skirt one can use 2 blouse, how many blouses are needed for 9 skirts). This category of problems, Mulligan & Mitchelmore (1997) called it the Cartesian product. Third is the category called common equivalent set problems i.e. 3 candies per box for 4 boxes, which Mulligan & Mitchelmore (1997) also named equivalent groups problems. Mulligan & Mitchelmore (1997) added one more category of division problems called 'rectangular arrays' i.e. 3 rows of 4 children.

Kouba (1989) further explains that, apart from the apparent quantities in the problem, the unknown quantities also yield to the three major types of problems. These three types of problems are: first, multiplication (whereby the product is unknown). Second, measurement division (whereby the number of sets is unknown) and the third is partitive division (whereby the number of elements in each set is unknown).

This study focused on only the two types of division problems, the measurement (quotitive) and partitive (sharing), specifically on the category of common equivalent sets/groups problems. These are the division problems whereby number of elements per set is given while the number of sets is unknown or number of sets is apparent in the problem while the number of elements per set is unknown. According to Mulligan & Mitchelmore (1997) equivalents group problems can be of partition (sharing) or quotation (measurement). Below is the example of equivalent groups' problems as provided by Kouba (1989:150).

> "Measurement (Quotitive): there are 100 oranges in a sack. If you pack/fill 10 oranges in smaller packs, how many smaller packs will you fill with oranges?
>
> Partitive (sharing): for 100 oranges determine how many oranges each smaller pack will have, if you are to fill 10 of the smaller packs of oranges" (p. 150).

2.2.1.4 Intuitive models of division

Intuitive models are the solution strategies that children possibly or probably use when approaching division problems. Intuitive models are internal mental structures that correspond to a class of calculation strategies (Mulligan & Mitchelmore, 1997). They are the illustrations which represent abstract operations on whole numbers (Vest, 1971). Students use a range of solution strategies when solving division problems and these strategies are inferred to as various intuitive models of division (Mulligan & Mitchelmore, 1997).

The main intuitive model likely to be used for measurement division problems is 'repeated subtraction', while for partitive division problems is 'sharing' (Kouba, 1989; Mulligan & Mitchelmore, 1997). However, Kouba (1989) states that repeated subtraction strategy for measurement division problems and sharing strategy for partitive division problems, alone are not adequate. Children appear to employ other more solution strategies (intuitive models) when approaching division problems. Some identified intuitive models for measurement division are such as (1) repeated subtraction, a strategy whereby the quantity unit of the size of the groups is repeatedly taken away from the divided until the divided is finished i.e. 24 ÷ 6, a child could repeatedly take 6 away from 24 until reaches 0 and the possible answers for the process would (24 – 6 = 18), (18 – 6 = 12), (12 – 6 = 6), (6 – 6 = 0), (Mulligan & Mitchelmore 1997; Kouba, 1989; Vest, 1971). (2) Repeated addition or repeated building up, is a strategy of counting in multiples to determine the answer i.e. 24 ÷ 6 a child would repeatedly adds 6 or count in multiples of 6 until the divided number is reached to determine the answer the process would goes as follow: (6 = 6), (6+6 = 12), (6+6+6 = 18), and

(6+6+6+6 = 24). Repeated addition is simply an inverse of repeated subtraction (Mulligan & Mitchelmore, 1997; Kouba, 1989; Vest, 1971).

Intuitive models identified for partitive division are such as: (1) sharing by dealing i.e. 24 divided into 4 sets/ groups, a child would divide 24 by taking one by one to each 4 groups until 24 is finished (Kouba, 1989).

In addition to the intuitive models discussed above, Mulligan & Mitchelmore (1997) added two more intuitive models. These two models are: (1) multiplicative operation intuitive model. These are the strategies that use multiplication as the operation to solve measurement and partitive division problems. In these strategies, a child could guess and check solutions by multiplication or search for multiples of the divisor that would be equal to the dividend. (2) is direct counting, a strategy whereby one count out a quantity equal to a dividend, making tentative groups and sharing a quantity from one group to another until the dividend is equally shared, or dealing out the quantity of the dividend successively by counting in ones, twos, threes and so on to the specified number of groups until the dividend is exhausted. For the quotient, a possible strategy is measuring out groups of specified sizes until the dividend is exhausted and counting the number of groups to determine the answer (Anghileri & Beishuizen, 1998; Mulligan & Mitchelmore, 1997).

Anghileri & Beishuizen (1998) further elaborated how counting strategies can be made manageable and efficient especially when counting involves a large number (dividend) and counting seems impractical. Counting strategies involving large numbers (dividend) could be made manageable and efficient by chunking. Chunking is when a large number is made into chunks by either repeated addition/ subtraction or by being decomposed in any other form i.e. 96 ÷ 4, 96 could be chunked as (40 + 40 + 16) or (80 + 16) or in any other form to make division easier.

Preferences for choices of the use of intuitive model by children are determined and influenced by quantities and units (e.g. size of groups/ sets, number of sets/ groups or any other particular multiples involved) and the presence of superficial verbal cues (Mulligan & Mitchelmore, 1997). In every problem of sharing or quotitive division, the dividend must be divided in equal sized groups (or quantity units) and the effective identification of those quantity units leads to successful solutions (Mulligan & Mitchelmore, 1997).

2.2.2 CONCEPTUAL AND PROCEDURAL KNOWLEDGE

2.2.2.1 Introduction

In a broader sense, mathematics knowledge is knowledge as both *of* and *about* mathematics. Knowledge *about* mathematics pertains to understanding the nature of the mathematics discipline, whereas knowledge *of* mathematics refers to conceptual (concepts) and procedural (procedures) knowledge of mathematical subjects (Simon, 1993:234). The present study contributes toward the aspect of knowledge *of* mathematics, the conceptual and procedural knowledge of division of whole numbers.

Conceptual and procedural knowledge are the two identified and essential kinds of knowledge pointing to the understanding of concepts and skills that learners need to acquire in mathematics (Long, 2005; Rittle- Johnson, Siegler & Alibali, 2001). Competence in a domain such as division in mathematics rests on children developing and linking their knowledge of concepts and procedures (Rittle –Johnson & Alibali, 1999; Rittle- Johnson, et al., 2001). Thus being competent in division involves knowing concepts, symbols and procedures and how these are related (Hiebert & LeFevre, 1986).

The distinction between the two types of knowledge could be marked down by the two terms “understanding” for conceptual and “skills” for procedural (Hiebert & LeFevre, 1986). The two types of knowledge lie on continuum and cannot always be separated. The two ends of continuum represent their distinctions (Rittle- Johnson & Alibali, 1999; Rittle- Johnson, et al., 2001). Hiebert & LeFevre (1986) point out the core of each type of knowledge is easy to describe but the outside edges are hard to pin down. Conceptual and procedural knowledge, even though distinct, are related in a complex way (Long, 2005; Rittle- Johnson & Alibali, 1999). Hiebert & LeFevre (1986) explained that they are linked in a mutually beneficial ways, while Wu (1999:1) used the terms ***“completely intertwined”*** (P.1). Wu (1999:) further explains that in learning there is no line separating conceptual understanding and problem solving or basic skills; nor can one acquire the former without the latter, because skills and understanding are completely intertwined. Conceptual advances are invariably built on the bedrock of technique and they go hand in hand.

2.2.2.2 Definitions of conceptual and procedural knowledge of mathematics

Conceptual knowledge: is an implicit or explicit understanding of mathematical principles that govern the domain (Rittle- Johnson & Alibali, 1999; Rittle- Johnson, et al., 2001). This grasp or understanding of mathematical ideas is functional in a linked or integrated way (Kilpatrick et al, 2001; Rittle- Johnson & Star, 2007). Thus, this knowledge is rich in relationships, a connected web (network) of knowledge of which the linking relationships are as prominent as the discrete pieces of information (Hiebert & LeFevre, 1986; Long, 2005; Simon, 1993). Units of conceptual knowledge cannot be isolated pieces of knowledge. They only become part of conceptual knowledge if the holder recognizes its relationships to other pieces of information (Hiebert & LeFevre, 1986). The conceptual knowledge is flexible (transferrable), not tied to specific problem types and therefore can be generalized although it may not be verbalized (Hiebert & LeFevre, 1986; Kilpatrick, et al., 2001; Rittle- Johnson & Star, 2007; Rittle- Johnson, et al., 2001). In most cases, conceptual knowledge is assessed using either novel tasks or unfamiliar procedures whereby children need to rely on their knowledge of relevant concepts to generate methods for solving the problem as they do not already know a procedure (Rittle- Johnson, et al., 2001; Hiebert & LeFevre, 1986).

Procedural knowledge: is made up of two distinctive parts: a) the formal language or symbol representation system of mathematics, and b) the algorithms, rules and procedures for completing mathematical tasks (Hiebert & LeFevre, 1986; Long, 2005; Simon, 1993). The first part of the formal language or symbol representation system of mathematics refers to the familiarity with symbols used to represent mathematics ideas and awareness of the syntactic rules for writing symbols in an acceptable form. This knowledge of symbols and syntax of mathematics applies only to the awareness of surface features in isolation to the meaning of these features (Hiebert & LeFevre, 1986). The second distinctive part is that of the use of algorithms, rules and procedures for completing or solving mathematical tasks. This refers to step by step instructions prescribed on how to complete a task (Hiebert & LeFevre, 1986) and also the ability to execute action sequences to solve problems (Hiebert & LeFevre, 1986;

Rittle- Johnson & Star, 2007; Rittle- Johnson & Alibali, 1999; Rittle- Johnson, et al., 2001). Steps of actions for solving problems in procedural knowledge are executed in a predetermined linear sequence (Hiebert & LeFevre, 1986). Procedural knowledge may or may not be acquired meaningfully, in that it may be learned in isolation to its underlying concepts (Hiebert & LeFevre, 1986). Thus, procedural knowledge can be tied to only a specific problem type and therefore may not be widely generalized (Rittle- Johnson et al, 2001). Procedural knowledge may be assessed using routine tasks or standard arithmetic computation methods, whereby children are likely to use previously learned step-by-step solution methods for solving those types of problem (Hiebert & LeFevre, 1986; Rittle- Johnson, et al., 2001).

2.2.2.3 How to develop conceptual and procedural knowledge

The development of conceptual and procedural knowledge is achieved in four ways: First, through the provision of conceptual and procedural instructions (Hiebert & LeFevre, 1986; Rittle- Johnson & Alibali, 1999). Second, development is also achieved through encouragement of students to generate conceptual explanations, explaining their solution process (Kilpatrick, et al., 2001; Rittle- Johnson & Star, 2007). Third, is by construction of relationships between pieces of information. This linking process occurs between two pieces of information already stored in the memory, or between existing pieces of knowledge and the newly learned knowledge. Fourth, is by creating relationships between existing knowledge and new information entering the system (Hiebert & LeFevre, 1986; Long, 2005).

There are two level of constructing relationships: the primary and the reflective level. The primary level refers to relationships constructed at the same level of abstraction. It is a less abstract level whereby relationships are tied to a domain or within a specific domain. For example when learners explore the difference between sharing and quotitive division problems; this is a primary level because it is within division domain itself. The reflective level is a high level of abstraction and less tied to a specific domain. At this level, relationships are constructed by recognizing similar core features in pieces of information that are superficially different. The common features of different domains are pulled out and tie them together as a general idea (Hiebert & LeFevre, 1986; Long, 2005). For example when learners explored sharing and quotitive division problems in relation to other operation domains (addition, subtraction and multiplication); this is beyond division domain. To relate this knowledge, learners lose the common features (which are the dividend, divisor and quotient) in the division domain and move them across other domains. When the knowledge is learned beyond or across the domain itself it is learned at the reflective level.

Lastly, learning should be incorporated into real-life situation and contextualized (Davis, 1986; Lampert, 1992). Even more, knowledge should be learned in community of practice. That is, students should learn to do mathematics the way mathematicians do (Brown, et al., 1989; Lampert, 1992).

2.2.2.4 The importance of acquiring conceptual and procedural knowledge in a connected way.

Mathematics competencies rest upon acquisition of both types of knowledge in a connected way. When concepts and procedures are not connected, students may have a good intuitive feel for mathematics but be unable to solve problems, or they may generate correct answers without understanding what they are doing (Hiebert & Lefevre, 1986).

Conceptual and procedural knowledge need to be connected for the following reasons: firstly, connecting conceptual knowledge to formal symbol systems of mathematics gives symbols meaning (Hiebert & Lefevre, 1986). Secondly, connecting conceptual knowledge to procedural knowledge contributes to memory, the storage and retrieval of procedures and to their effective uses (Hiebert & Lefevre, 1986). When procedures are connected to their underpinning concepts, they become meaningful to the users and when stored can be successfully retrieved because, these procedures are stored as parts of networks of information, glued to a semantic relationship (Hiebert & Lefevre, 1986).

Thirdly, when procedures are linked to their underlying concepts this promotes flexibility or transfer of procedural use (generality), which reduce the number of procedures required to be learned (Hiebert & Lefevre, 1986, Rittle – Johnson, et al., 2001). Conceptual knowledge releases procedures from surface context in which they were learned and encourages their uses on other structurally similar problems. This transfer of procedures also reduces number of procedures to be learned and remembered by eliminating the need to learn different procedures for each tasks (Hiebert & Lefevre, 1986; Kilpatrick, et al., 2001). According to Long (2005) students whose conceptual knowledge is not grounded are able to apply procedures to similar problems, but if the context of the problem changes, they have to re-learn the procedure.

Children with greater conceptual knowledge would be more likely to generate meaningful explanations for why the correct answer is correct and how it was generated (Rittle – Johnson, et al., 2001). Thus learners should be made to generate conceptual explanations and describe solution processes verbally or in writing (Rittle – Johnson & Star, 2007).

2.2.3 VISUALIZATION

2.2.3.1 Introduction

Visualization is not a recent invention; diagrams are as old as mathematics itself (Zimmermann & Cunningham, 1991). People have used images for recording and communication since the cave-painting era (Arcavi, 2003; Kirrane, 1992). Even now, large parts of our communication in daily life depend on visual imagery (Mudaly, 2010). On the other hand, mathematics is a science of patterns and patterns are often accessible to visual metaphors. Even though not all patterns can be visualized, it is natural to try to find the most effective way to visualize these patterns and to learn how to use visualization creatively as a tool for understanding (Zimmermann & Cunningham, 1991). The history of mathematical concepts development points toward the importance of visualization (Woolner, 2004).

As stated in the previous chapter, this study aims to transform my teaching of division, enhancing the procedural approach by integrating conceptual understanding of division. Visualization was used as a tool and supplement but not as a replacement for verbal or other approaches to learning. Thus this study integrated visualization with a verbal approach; both modes were used through out the study.

Battista, et al., (1982), states that visualization is highly important to mathematics learning in primary grades, putting emphasis on concrete and pictorial representation. Thus, the visualization types used in this study were mainly that of real objects (stones) and drawing of pictures and icons.

2.2.3.2 Definitions that revolve around visualizations

Researchers have defined visualization in various ways. Below are some of the definitions of visualization that satisfy the term as used in this study. Mudaly (2010) used Arcavi's (2003, p. 217) definition:

> "Visualization is the ability, the process and product of creation, interpretation, use of and reflection upon pictures, images, diagrams, in our minds, on paper or with technological tools with the purpose of depicting and communicating information, thinking about and developing previously unknown ideas and advancing understanding" (p. 217).

Zazkis, Dubinsky & Dautermann (1996) defined visualization as an act in which an individual makes a strong connection between internal constructions to an external object accessed via senses. This act of constructions consists of mental constructions of objects or processes which the individual associates with objects or events perceived externally or the external constructions of objects on paper or chalkboard of object or events which the individual identifies with objects or processes in her/his mind.

In other words, visualization is the ability to represent, transform, generalize, communicate, document and reflect on visual information and the process of mental construction and transformation of visual images (Sinclair, Mamolo & Whiteley, and 2011). Zimmerman & Cunningham (1991, p.3) have given a simple definition:

> "Visualization is the ability to draw an appropriate diagram to represent mathematical ideas (concepts) or problems and to use the diagram to achieve the understanding and as an aid in problem solving" (p. 3).

According to these words visualization is not understanding but rather a means toward understanding. Using visualization in mathematics is more than just seeing a diagram. Mathematics visualization goes to an extent of visualizing a concept or problem. Visualizing a diagram simply means to form a mental image of that diagram but to visualize a problem or concept means to understand that problem in terms of a diagram or visual image. Thus a mathematical visualization process, which could be internal or external, is a process whereby, images are formed, mentally or with pencil and paper, and used effectively for mathematical discovery and understanding.

Visualization gives depth and meaning to understanding, serves as a reliable guide to problem solving and inspires creative discoveries (Zimmerman & Cunningham, 1991). People use diagrams or other visual representations not only as a vehicle to convey mathematical ideas or to explain but also for conviction (Hanna & Sidoli, 2007). Hanna & Sidoli (2007) explain that a proof is anything that is both convincing and explanatory, thus any visual representation that satisfies the two criteria is a proof.

2.2.3.3 What is involved in the visualization processes?

Visualization entails visualized and the visual literacy (Mudaly, 2010). It also involves visual thinking (reasoning) and analytic thinking (Mudaly, 2010; Zazkis, et al., 1996).

The visualized is anything which can be seen physically or mentally, e.g. the mental picture that comes to mind at the mention of parabolas (Mudaly, 2010). The visualized are visual images. Visual images are schemas depicting visual information. This includes the imagery that depicts shape, pattern or any form of picture in the individual's mind or externally. These extend to the imagery of verbal or number forms in an individual's mind or externally (Presmeg, 1989).

Stokes (2002) states that visual literacy is an ability to turn information of all types into pictures, graphics or forms that help communicate the information. It is visualization combined with logical thought (Mudaly, 2010): the ability to interpret images as well as to generate images for communicating ideas and concepts (Mudaly, 2010). Stokes (2002) expanded the definition as follows: visual literacy is the ability to translate, interpret, create, read and understand ideas presented as pictorial or graphic images as well as to turn ideas of all types into pictures or graphic or any images that help communicate these ideas. He further added that visual literacy is like a specific language, because its use and interpretation of images require one to be able to decode messages the images bear, in order to have meaning.

Visual thinking is defined as reasoning that consists of mental transformations of objects either constructed in the mind or externally (Zazkis, et al., 1996). Analytic thinking, which is the act of analysis, is a manipulation of objects or processes with or without the aid of symbols. To analyze is to break the compound into component parts in order to discern them, find relationships amongst those parts and synthesize them into various other wholes (Zazkis, et al., 1996). Analyses include mind-to-mind construction that involves reflecting abstractions of interiorizations, co-ordination, reversal and encapsulations. It is what most people refer to as logical analysis or reasoning.

Zazkis, et al., (1996) differentiated between analysis and visualization as follows: if mental manipulation is based on the meaning of the symbols and their syntactic rules then the act is analysis, but if the mental manipulation is based on the nature and configuration of symbols, the act is visualization.

2.2.3.4 The role of visualization in learning of mathematical procedures and concepts.

Viewed pictures often create clearer images in our minds because of the understanding of symbols or procedures attached to what we see (images). At the same time whilst seeing with the mind's eye, the mind is also capable of creating new information or transforming old knowledge (Mudaly, 2010). Visualization aids in connecting facts that might not necessarily be understood through verbal means. However, words and sentences (verbal) are also essentials in recording of the information acquired through visual means (Mudaly, 2010).

Mathematical visualization is also more than just visualizing diagrams. It goes to the extent of envisioning abstract concepts or problems (Arcavi, 2003; Kilpatrick, et al., 2001; Zimmermann & Cunningham, 1991). Visualization plays many roles toward the learning of mathematics procedures and concepts.

First, visualization, i.e. pictures, may be used to describe spatial relationships (Kirrane, 1992). This is because diagrams and mental images allow the individuals to free up more mental space for new imaging and for construction of relationships (Mudaly, 2010). According to Arcavi (2003) the visual display of information enables the individual to see the story and envision some cause –effect relationships.

Second, visualization is good at allowing pattern-matching approaches to problem solving (Kirrane, 1992). This is because visualization depicts clear sub-structures of a whole problem that leads to the identification or establishment of patterns (Arcavi, 2003). Sinclair, et al., (2011) further add that spatial, visual and kinesthetic approaches to learning mathematics help students make connections between various representations of a procedures underlying concepts.

Third, visualization is good at showing the structures of data (Kirrane, 1992). Visualization organizes data at hand in meaningful structures and can be an important tool to guide analytical development of a solution. Thus, visualization can be analytical process itself, which includes the solution which is general and formal (Arcavi, 2003). Visualization can be used as a way of working with pre-established products and also as an object of analysis (Arcavi, 2003).

Fourth, visualization links students' informal knowledge (old ideas) and their experiences to the new mathematical abstractions (Kilpatrick, et al., 2001). The provision of the correct physical stimuli (concrete or drawing) is essential for the connection of old ideas to new or foreign ones. These stimuli act as a tool to negotiate new ideas (Mudaly, 2010). Thus we see what we know because the mental cues after visualizing an image, direct our mind to some old information that we already know and an association is formed (Mudaly, 2010).

Fifth, during visualization processes, three things occur: collection of mathematical ideas, construction of meanings and transformation of knowledge (Mudaly, 2010; Kilpatrick, et al., 2001). This is because: (1) Visualization helps learners to discover their own misconceptions or errors and rectify them (Arcavi, 2003; Mudaly, 2010). (2) Visual conviction (strong feeling of being certain) leads to deeper understanding as it create opportunities for learners to explore, analyze and test their conjectures (hypothesis) (Arcavi, 2003; Mudaly, 2010; Tall, 1991). (3) Visualization help with justification, enabling learners to engage in critical reasoning by forming specific conjectures, and to work through it visually to the solution, thus enabling learners to see and explain logic behind their work (Arcavi, 2003; Mudaly, 2010). (4) Visualization can also help when describing objects or solution processes as well as with identification of objects (Kirrane, 1992).

Sixth, through visualization, knowledge is created with strong evidence and conviction in the minds of learners (Mudaly, 2010). This is because of iteration between physical/ mental images and analysis that enables learners to create new information. Seeing the diagrams and the analysis of the results, leads to achievement of high level of conviction in the learners' minds (Mudaly, 2010). In addition to that, visualization adds meaning and conviction to symbolic proof (procedural syntactic approaches), because it enables learners to engage with concept and meaning, which can be easily bypassed by symbolic solutions of the problem (Arcavi, 2003).

Seventh, visualized knowledge is vivid, enabling future remembrance (Arcavi, 2003). Visualization can be an effective and efficient means of condensing information, which makes it easy for memory storage and increases its future accessibility (Sinclair, et al., 2011).

Eighth, visualization is a good means for getting learners' attention (Kirrane, 1992). According to Arcavi (2003) diagrams and models are a central theme which develops and stabilizes interactions between people and things.

Lastly, visual representations can constitute proofs in and of themselves, because through visualization one does not only transform data, alter images and manipulate objects, one also examines features of mathematical objects (Hanna & Sidoli, 2007; Tall, 1991). Thus pictures can indeed form essential components of a proof as they are a means of understanding and when internalized can play a crucial role in logical demonstration, which can convey information sometimes as a whole proof. In that way, visualization enhances the ability of students to notice details, to conjecture, to reflect on and interpret relationships as well as to offer tentative explanations and proofs (Hanna & Sidoli, 2007). Even though a proof is the last stage of the process, visualization allows an exploratory stage of mathematical thinking, benefiting the building up of an overall picture of relationships which leads to solutions and /or proofs (Tall, 1991).

2.2.3.5 When and how visualization does play roles in the learning of mathematics procedures and concepts

Visual thinking can be an alternative and powerful resource for students doing mathematics (Nemirovsky & Noble, 1997), because some information is better presented visually rather than verbally (Stokes, 2002). However, many authors emphasize that the effectiveness of visual representation depends on its use in integration with other modes such as analytical, verbal, numerical and symbolical, and not when it is used in isolation (Nemirovsky & Noble, 1997; Stokes, 2002; Zazkis, et al., 1996; Arcavi, 2003; Mudaly, 2010; Zimmermann & Cunningham, 1991; Hanna & Sidoli, 2007; Tall, 1991). According to Zazkis, et al., (1996) visual and analytical thinking need to be integrated in order to construct rich understanding of mathematical concepts. This integration of verbal and visual elaboration strategies also facilitates mental connections in learning (Stokes, 2002). To achieve the greatest understanding visualization cannot be isolated from the rest of mathematics. Visual thinking must be linked to other modes of thinking and other forms of representation. One must learn how ideas can be represented symbolically, numerically and graphically and how to move back and forth among those modes (Zimmermann & Cunningham, 1991). When symbolic expression, with a good dose of symbolical sense, is integrated with visualization, it can also be used to inspire, guide and support visual reasoning (Arcavi, 2003).

The visualization process is iterative, working back and forth between visualization and analytical thinking. A learner visualizes an image (a picture or diagram) at first and then starts analyzing that image. After the mental process of analytical thinking the learner makes new meanings of the image and when going back to visualize the same image again, the learner will start re-assessing that picture/ diagram and new information will be added by the time the learner goes back to the visual image, until greater sense is made of what s/he sees (Mudaly, 2010; Zazkis, et al., 1996).

In addition to the integration of visualization with other modes of thinking, there are several other approaches to visualization which can make it effective, such as:

First, children develop visual-analytical thinking skills if presented with a training problem prior to their actual attempt at solving real visual problems (Mudaly, 2010). Visual literacy is a language the learner needs to learn. It includes being alert to visual messages and critically reading or viewing images as the language of messages (Stokes, 2002). In this training or practice learners should be helped to read or decode visuals through practicing decoding techniques. Decoding involves interpreting and creating meanings from visual stimuli (Stokes, 2002). Learners should also practice visual writing or encoding as a tool for communication for students to develop their visual abilities through use (Stokes, 2002).

Second, the visual representation must be very well structured. A visual representation which is well structured can carry a wealth of information very efficiently.

Third, visual representation has to be appropriate and relevant to the students' age and level. The design of the visual in learning should be related to the students' prior knowledge for it to be effective. One should also consider that young learners prefer simpler visuals and older learners prefer complex visuals. However, some argue that simpler visuals are usually more effective regardless of age group (Stokes, 2002).

Fourth, for visual representation to be effective, the use of visualization in learning must be well and carefully planned (Stokes, 2002).

Bakker & Hoffman (2005) describe the steps needed to ensure that diagrammatic reasoning takes place.

First: learners should construct of diagrams by means of representational system; such construction is motivated by the need to represent relations that students consider significant in a problem.

Second: learners should experiment with diagrams that are executed within a representational system and are rule-based. The rules define the possible transformations and actions but also the constraints of operations on diagrams (Bakker & Hoffman, 2005).

Third is observation: students have to observe the results of experimenting, and reflect on them (Bakker & Hoffman, 2005).

2.2.3.6 When visualization does not play roles in the learning of mathematical procedures and concepts

Visual representation may not be effective in the following circumstances:

First: harm is caused when students have weak visualization skills on that mathematical topic. This leads to a lack of meaning in the formalities of mathematics analysis (Tall, 1991). This is because the incorporation of visualization into mathematical activity brings its own set of cognitive demands (Noss, Healy & Hoyles, 1997).

Second: sometimes concepts that expert mathematicians regard as intuitive are not intuitive to students. So many students may have difficulties in reading diagrams and recognizing the transformations implied in them (Noss, et al., 1997).

Third: at times, students do not tend to make links between their visualization and analytical thought (Noss, et al., 1997).

Fourth, visualization that triggers excitement and offers mainly entertainments in presentation rather than encouraging thoughtful analysis of underlying meaning may interfere with learning (Stokes, 2002).

2.2.3.7 Challenges of using visual representations

Researchers have pointed out the following challenges students may encounter in learning to use visual representations.

The first is the difficulties that stem from over-use in textbooks and instructions of standard figures in standard positions (Clements, 1982). An image of a standard figure may include inflexible thinking which prevents recognition of concepts in a non-standard diagram (Presmeg, 1986). In the same manner, non-standard figures also do not always aid reasoning and problem solving, as pupils have difficulties in identifying concepts embedded in these diagrams (Clements, 1982). According to Sinclair (2003) a difficulty facing students is that mathematical pictures and diagrams contain a great deal of specific content represented in a concise but non sequential format. This requires students to apply their own organization to the information and to draw conclusions about how items are connected.

Second, we see what we know, so students do not necessarily see in diagrams what expert such as teachers, researchers or mathematicians do (Arcavi, 2003). In comparison to students, experts (teachers, researchers or mathematicians) know what to look for in a diagram, what to generalize from particular figure and so are able to employ a particular case or image to stand for a more general observation (Noss, et al., 1997). To students, images or diagrams may tie thought to irrelevant details (Arcavi, 2003; Clements, 1982; Presmeg, 1986), providing a stimulus that makes it difficult for a student to formulate necessary abstractions (Clements, 1982). As Tall (1991) warns, a picture may suggest false theorems; or distractions from the main elements in the original problem. The difficulty in perceiving the diagram or image loaded with intervening conceptual structures, are caused by two things: (1) our perception, which is shaped by what we know, our previous knowledge and experience in relation to that diagram and, (2) our perception, which is also shaped by the context within which that observation is made (Arcavi, 2003).

Third: people vary in their ability to form and use images. For example, they differ in ability to generate images, to find part of a whole image or to hold on image in mind (Kirrane, 1992).

The forth is that, sometimes students do not tend to make attempts at linking their visualization and analytical thought (Noss, et al., 1997).

The fifth challenge is that an uncontrollable image (an image formed in the learner's mind in an inflexible perception) may prevent the opening up of more fruitful avenues of thought especially if the image is vivid (Presmeg, 1986).

Arcavi (2003) has provided the three main categories in which these challenges/ difficulties are classified: (1) cultural, (2) cognitive and (3) sociological, see also (Sinclair, et al., 2011). The cultural difficulties are caused by the belief and values held about mathematics. This could stem from legitimate and accepted ways of doing mathematics, which a teacher and learners hold, which may prevent them do mathematics in a different way.

The cognitive difficulties include the level of cognitive demand, from easy to difficult. Visualization acts upon conceptually rich images rather than on routine, procedural ways of doing mathematics. Using a visual approach may implicitly imply that there are no procedurally “safe routine” to rely on. Consciously or unconsciously such situations may be rejected by students in a ground of being too slippery, risky or inaccurate (Arcavi, 2003).

Other cognitive difficulties arise from the need to attain flexible and competent translation back and forth between visual and analytical representations of the same situations, which is at the core of understanding much of mathematics (Arcavi, 2003). Learning to understand and comprehend with the use of visualization may be difficult, as it can be saturated, context dependant and non sequential (Arcavi, 2003).

Sociological difficulties are caused by the diversity, whereby the classroom may include students from various cultural backgrounds. Some students may come from visually rich culture. In contrast, visualizers may be under- represented amongst high mathematical achievers (Arcavi, 2003).

2.2.3.8 Ways to overcome challenges encountered in learning when using visual representations.

Various researchers have highlighted ways to overcome learning challenges caused by visual representations.

Clement (1982) states that just as learners are required to “paraphrase” sentences to ensure that they have processed them as propositions, with visualizations, a teacher might also require students to “paraphrase” diagrams by asking them to draw their own diagrams from memory or possibly to render a given diagram from a different viewing angle.

Mudaly (2010) states that students, prior to attempting to solve real problems, can be trained by being given an analytical reasoning/ visual reasoning skills tasks. He further states that difficult mathematical questions can be broken into focused simpler sub-problem to make learning and content more meaningful to learners (p 74). In addition, Mudaly (2010) suggests that learners can be given a chance to construct their hypotheses or conjectures, which then they can test visually in order to correct their misconceptions or proofs.

From a different perspective, Sinclair, et al., (2011) states the notion of incorporating verbal instructions and prompting and questioning and scaffolding into visual representation design or interpretation tasks. According to Sinclair, et al., (2011), verbal prompts and questions are the keys to supporting mathematical understanding during visual investigations. The prompts and questions help students to notice, to interpret and extend their thinking (Sinclair, et al., 2011). However, these prompts, questions and instructions should be made in correspondence to directing attention to possible mathematical relationships and should be open in nature (Sinclair, et al., 2011).

Sinclair (2011) states that questions are meant to help students notice and interpret; details should be provided with visual stimuli. Statements that prompt actions should be given with the necessary options, so that learners can take the required steps, and questions that invite explorations should be open options to support multiple paths. Finally, Sinclair (2011) states that scaffolding should be provided to students on tasks that they are unfamiliar with, exploration and prompts need to be more specific and focused to help direct the participants.

2.3 THEORETICAL FRAME WORK

The view of learning and teaching in this study is guided and informed by Bruner's theory of cognitive development and learning, and Vygotsky's theory of social constructivism.

2.3.1 Bruner's theory of cognitive development and learning: the stages of intellectual development.

The learning in this study passed through three intellectual stages as it used three different representations as a tool to mediate sharing (partitive) and quotitive (measurement out) division learning. The three main intellectual stages theorized by Bruner are: enactive, iconic and symbolic (McInerney & McInerney, 2006; Moyer, 2001). In an enactive stage a child learns by acting upon objects; doing learning with objects as a tool to mediate learning. After the enactive stage a child progresses to the iconic stage. In this stage a child's objects or symbols are represented by images. The child no longer needs to manipulate an object but rather learns through modeling, demonstration and pictures. At this stage the child can operate mentally with pictures. The symbolic stage is the last stage of progression that children enter. This is when children develop a capacity to think abstractly with symbols (McInerney & McInerney, 2006). According to Bruner, learning instructions should be sequenced for learning to occur best. Children should first experience it (enactive stage), then react to it concretely via icon formation, and finally symbolize it. A teacher gets best results at any age of learners if concrete, pictorial and symbolic presentations of the material are combined. As long as learners are given the opportunity to operate on the material at the appropriate levels of abstractions, the teaching will be successful (McInerney & McInerney, 2006).

2.3.2 Vygotsky's theory of social constructivism

The learning of this study was also influenced by Vygotsky's theory of social constructivism. This theory is also referred to as cultural-historical theory or socio-cultural theory (McInerney & McInerney, 2006). This theory emerged from Piaget's theories of cognitive development as it borrowed the idea that a learner learn actively, and learning is a result of interaction with the problem context where learners construct their own knowledge (Hall, 2007). However, Vygotsky's theory had gone beyond this to the extent of acknowledging the role of the social environment and its historical – cultural meaning in learning (Hall, 2007).

2.3.2.1 Zone of proximal development (ZPD)

The zone of proximal development is a gap/ region/ distance between the children's actual development level: the competence of what a child currently can do without adult assistance, and the level of potential development: what a child can accomplish with an adult assistance (Berk & Winsler, 1995; Winsler, 2003; Jacobs, 2001; Not, 1992; McInerney & McInerney, 2006; Tudge & Winterhoff, 1993; Verenikia, 2008; Moore, 2000).

In this zone a child's new capacities are first developed through collaborations and interactions with adults such as parents, teachers or more competent peers. To place learning in the ZPD, an appropriate level of difficulties needs to be established. This level must be challenging but not too difficult. Then the adult (teacher) provides a guided problem/ practice to the child, with a clear sense of the goal or outcome of the child's performance. This assistance is gradually withdrawn as the child gains the capacity to do the task independently (Winsler, 2003; Moore, 2000; Verenikia, 2008; Tudge & Winterhoff, 1993; Not, 1992; Jacobs, 2001; Berk & Winsler, 1995; McInerney & McInerney, 2006; Glassman, 2001; Kozulin, Gindis, Geyev & Miller, 2003; Hall, 2007).

2.3.2.2 Mediating learning and scaffolding in ZPD

According to Vygotsky, learning is mediated by the use of tools or social factors which sometimes are called cultural artefacts. These tools can be either physical or psychological, such as symbols, systems, signs, languages, manipulatives, iconic visual representations, pens, computers, calculators and so on. These tools are human products developed in various ways by different cultures over a historical time. These tools play a double role in learning.

First, they provide means with which to act upon the world. So learners do something with the tool that extends the learner's capacity in a particular way.

Second, the tool acts as cognitive scaffold that facilitates extension of knowledge into related areas (Hall, 2007; Glassman, 2001; McInerney & McInerney, 2006; Tudge & Winterhoff, 1993).

Learning is then mediated by a teacher or a more competent experienced peer through modeling of use of mediating tools and scaffolding learners to practice. Modeling involves explanations, coaching, diagnosis and direction while scaffolding provides structures or prompts in the learning environment (Hall, 2007:9).

In this study, *manipulatives* and *iconic visual representation* were tools used to mediate learning of sharing (partitive) and quotitive (measurement out) division. A questioning approach, which also involves language, was used to scaffold learning.

2.3.2.3 Scaffolding

Scaffolding is a metaphorical or umbrella term, used to describe and explain a certain kind of support and its role, which the learners receive in their interactions with parents, teachers or other mentors, as learners move toward new skills, concepts or levels of understanding (Winsler, 2003; Not, 1992; Verenikia, 2008; Murray, 2009). The term scaffolding was originally introduced by Wood, Bruner and Ross by the year 1976, in an attempt to relate or operationalize the concept of teaching and learning in ZPD (Not, 1992; Verenikia, 2008).

Scaffolding has three important features: (1) the essential dialogue of a discourse in which knowledge is co-constructed. (2) The activity within which knowledge is embedded. (3) The role of mediating tools used for learning (Verenikia, 2008). Scaffolding in the ZPD can be given through questioning or modeling participation, negotiation and engagement in activity, together with the teacher monitoring and maximizing the child's participation in the activity, by carefully modulating the type of assistance given and withdrawing the amount of adult assistance provided to keep the child in the ZPD (Murray, 2009; Winsler, 2003).

2.3.2.4 Questioning and its role in learning

In this study, a questioning approach was used as a means to scaffold learning. Questions were used to stimulate learners' thinking as thinking is believed to be driven by questions (Elder & Paul, 1998). Questioning has a crucial advantage in that the knowledge gained through it is transferrable. It teaches students to think for themselves and also provide an instant feedback on their understanding (Petty, 1993). Through questioning one can explore what learners already know, what they are thinking as well as their misconceptions. This is achievable through asking open-ended questions and question that involve the justification of ideas (Naylor, Keogh & Goldsworthy, 2004). Questions can be used also to control students' learning because they focus students' attention on specific features of the concepts being explored in the class. It gives students opportunity to communicate their reasoning process. Through questioning learners explain their thinking and one can discover their depth of misunderstanding or the nature of their misconceptions (Manouchehri & Lapp, 2003; Sinclair, 2003).

There are three important issues considered when questioning: form, content and purpose of the question. Form refers to whether the question is closed or open-ended. Content refers to types of information the question intend to obtain from students' thinking. Purpose refers to aim in relation to how you want students to be engaged in learning, if the aim of the question is for students to explore or investigate, and this should be apparent in the question (Manouchehri & Lapp, 2003).

2.3.2.4.1 How to do questioning

One should ask simple questions and avoid vagueness. Questions should be focused and specific. When asking, learners should be given plenty of time to think (Petty, 1993). A teacher should respond to all learners' questions with a further question, one that calls on respondents to think in fuller or in deeper ways. Question students' thought (statement) with the assumption that there is an agenda behind the thought that the teacher want to know and fully understand. Question students' thought to explore the following: (1) the information on which the statement was based, (2) inferences from which the students draw their conclusions, (3) the concept that defined and shaped the learner's statement, (4) whatever is taken for granted, (5) the implication and consequences implied by students or (6) to understand the learners' point of view (Elder & Paul, 1998).

2.3.2.4.2 Disadvantages of using questioning

Questioning is a time consuming activity and the problem with it in a whole class discussion is that it only allows few student to participate, as it would be difficult to involve all students in the group (Petty, 1993; Naylor, et al., 2004).

2.4 The intervention program

This intervention program design was guided by the Hypothetical Learning Trajectory (HLT) as explained by Simon & Tzur (2004). The HLT consist of three important aspects that need to be considered when planning a mathematical task: (1) The goals for students' learning; (2) The mathematical tasks that will promote learning and (3) The hypothesis about the process of students' learning (Sinclair, et al., 2011; Simon, 1995). According to Simon (1995) the planning of the mathematical tasks should allow students to have freedom to response to a situation on the basis of their past knowledge. At the same time Lampert (1992) and Brown, et al., (1989) proposed learning of mathematics to move back and forth between the practice of teaching in school and that of doing mathematics in a community of practice. Students need much more than abstract concepts and self-contained examples. They need to be exposed to the use of a domain's conceptual tools in authentic activity, with a teacher acting as a practitioner and using these tools in wrestling with problems of the world (Brown, et al., 1989).

Authentic teaching involves the withdrawal of the teacher or textbook authority in learning. Learning should be a messy conversation of questions and answers. When a learner responds to questions or makes a statement it should be treated as an assertion or a half-way established conjecture. Instead of a teacher judging the learners' statement (conjecture) to be either correct or incorrect, the teacher should put that statement to test for any mathematical truth in it, by giving an opportunity for learners to explain their thinking or justify their assertions. The legitimacy of such assertion would then be made based on the assumptions and reasoning supplied by the learner to justify it (Lampert, 1992).

CHAPTER 3

RESEARCH METHODOLOGY

3.1 Introduction

Gough (2001, p.4) defines research methodology as follows:

> "Research methodology refers to a theory of producing knowledge through research and provides a rationale for the way a researcher proceedings. More than one technique can be used and reasons for using those techniques in relation to the kind of knowledge or understanding the researcher are seeking" (p. 4).

This chapter discusses the research methodology opted for in this study and the rationale behind the choices made. The goals and rationale of the study are briefly presented in the next section. The study is informed by the interpretive paradigm and a qualitative approach was used to present the study's data analysis. The study is a case study, a case of visual representations with its unity of analysis the teaching and learning using manipulatives, visual representations and questioning on the intervention program. In this chapter, the samples, techniques and methods of data collection are also discussed. Attention is given to issues relating to validity and ethics. Ethical responsibilities of each member of the research community are also explained in this chapter. The chapter ends with discussion of how the data were analyzed, followed by the limitations experienced, and finally the conclusions.

3.2 RESEARCH GOAL

The goal of this study is to explore the influence of manipulatives, visual representations and questioning in the learning of integrated procedural and conceptual knowledge of sharing (partitive) and quotitive (measuring out) division in mathematics teaching at grade 4 level. For this goal to be achieved, a visual intervention program of fifteen lessons was designed. A questioning approach was used to scaffold learners to enable them reach the learning goals.

The research goal was split into sub-goals or research questions as follow:

1) What learning emerged from the implementation of the intervention program, which incorporate the use of manipulatives and iconic visualization in the mathematic learning of the grade 4 class?

2) How did the learning of sharing (partitive) and quotitive (measuring out) division learned in that intervention program relate to conceptual and procedural knowledge?

3) How were manipulatives and iconic visualization used to support this learning and what roles did they play in the learning of sharing and measuring out division of the grade 4 class?

4) How were the questions used for scaffolding and what roles does questioning play in the learning of sharing (partitive) and quotitive (measuring) division of the grade 4 class?

5) What challenges were encountered when manipulatives, visual representations and questioning were used to integrate division procedures with their underlying concepts?

3.3 RESEARCH ORIENTATION

3.3.1 The interpretive paradigm

The approach of this study was the interpretive paradigm.

Connole (1998, p.16) defines the interpretive paradigm as follows:

> "Interpretive paradigm is a paradigm modeled on historical, literacy and existential studies in which the subjective understandings of subjects are significant" (p.16).

Literature explains the interpretive paradigm, as a paradigm concerned for and focused on individuals' actions. Its central aim is to understand and interpret the subjective world of human experience (Cohen, Manion & Morrison, 2007), by extracting, exploring or discovering meaning that arises from the logic and facts of human actions (Connole, 1998). This study focused on the participants' actions and their experience with manipulatives, visualization and questioning in the learning of sharing and measurement division. The central aim of this study was to extract, explore and discover meanings and interpretations arising from the participants' experience with the use of manipulatives, visual representations and questioning to do sharing and measurement division activities.

One can only extract the meanings and interpret human experience once having ascertained the intentions of actors in the shared experience (Cohen, et al., 2007). In this study the intentions of the participants' actions were ascertained/ determined by the learning goals of these research activities. This was also made apparent in each particular task (shared experienced) of each lesson. For example, when the learning goal of the tasks was to use visual objects such as stones as a way of division; the participants used stones to divide in those activities. Thus the use of stones as means of division became the participants' intention for doing such activity. However, even though the intentions of the activities were uniformly ascertained, the realities emerged out of individuals' experiences differ, which was the focus of this study's analysis.

The only reality emerging from this paradigm is that constructed by the individuals involved in the research situation (Cresswell, 1994). Hence, this study's exploration focused on how individual participants experienced with manipulatives, visual representation and questioning used to do sharing and quotitive division in the study. In one given situation there could be multiple realities that required multiple methods of understanding them (Cresswell, 1994; Connole, 1998). In this study, multiple realities on one given task also existed for different participants because there were cases whereby learners (participants) got different results and held different perspectives out of the same task. Thus different methods such as observations, interview and document analysis were used to extract or discover the individuals' realities in order to present the relevant interpretation.

3.3.2 Qualitative Research

The process of data collection through the use of techniques/ methods of data collections (observations, interview and document analysis) and the presentation of the findings (data analysis) of this study adopted the qualitative research approach.

Cresswell (1994, pp.1-2) defined qualitative study as follows:

> "a qualitative study is an inquiry process of understanding a social or human problem based on building a complex, holistic picture, formed with words, reporting detailed views of informants and conducted in a natural setting" (pp.1-2).

In a qualitative study the researcher attempts to study human actions from the perspective of social actors with a primary goal of understanding and describing these actors' actions and experience (Babbie & Mouton, 2001). This study is qualitative. The participants' actions and their experience with manipulatives, visual representations and questioning were explored not only to find meanings, but also to explore how participants' meanings and their interpretation of division learning was built. The reporting of this study's findings reflects the holistic picture of the research process including a detailed description of the participants' actions.

3.4 METHODS

3.4.1 Case study

This is a case study. Cohen, et al., (2007) explain that a case study studies actions within a specific (or single) unit of analysis. Creswell (1994, p.12) added as follows:

"A case study is exploring a single entity or phenomenon (the case) bounded by time and activity (a program, event, process, institution or social group) and collect detailed information by using a variety of data collection procedures during a sustained period of time" (p.12).

The case of this study is the use of manipulatives, visual representation and questioning in the learning of sharing and measurement division. The unit of analysis is how manipulatives, visual representations and questioning enhanced the conceptual learning of sharing and measurement division in the intervention program.

A case study is bounded by time and activity (Cresswell, 1994). This case study is bounded in three dimensions: (1) the pedagogical boundary (visualization). (2) The conceptual/ subject content boundary (division). (3) Level/ grade (curriculum).

The pedagogical boundary: This study focuses on manipulatives, visual representation and questioning as tools for supporting learning of sharing and measurement division. An intervention program of fifteen lessons was designed, and its implementation enabled the data collection. In that intervention program, the following three tools were used: (1) concrete materials (stones), (2) drawing of diagrams and dots, and (3) questioning. The activities of the intervention program covered the two kinds of division sharing (partition) and measuring out (quotitive). For each kind of the division activities tools, manipulatives, visual representation and questioning were used for solving division problems in the following ways:

- In the first stage participants used stones (manipulatives) as a procedure to do division, learners divided using stones.
- In the second stage participants divided using diagrams of real objects. These were diagram of real objects such as apples or sticks.
- In the last stage participants did the division using icons (dots).

Conceptual/ subject content boundary: This study only focused on the two kinds of division, partitive (sharing) and measuring out (quotitive).

Partitive is an equal sharing whereby an object or collection of objects (dividend) is divided into a number of equal fragments/ sub- collections (Kilpatrick, et al., 2001; Simon, 1993). In this division the intensive quantity (number of items per group) is the quotient.

Quotitive division is where one has to determine how many times a unit quantity may be measured out of a dividend (Kilpatrick, et al., 2001; Simon, 1993). In quotitive division the intensive quantity (the number of items per group) is a divisor.

The other sub-boundary that lies within the conceptual division are the type of problems (tasks) used in the intervention. This study only focused on the equivalent groups types of problems of measurements (quotitive) and of partitive (sharing) division. These are the groups of division problems whereby the dividend is divided by either allocating the dividend equally in groups one by one or by measuring the unity of elements per group from the dividend until the dividend is used up. The final boundary under the conceptual frame is the type of number used. This study only used the division of whole numbers.

The level/ grade boundary (curriculum): The division content taught in this study is informed by the Lower Primary Namibian curriculum (Grades 1 – 4), particularly that of grade 4 (Ministry of Education (M.o.E), 2005). The division tasks that underpinned the intervention program of this study were taken from three Grade 4 textbooks used in Namibia (Westhuizen, 2005; Kruger & Augustyn, 2007; Van Rhyn, Nott & Westhuizen, 1999). Tasks were then later visually modified and adapted to satisfy the learning goals of the research activities in the intervention program.

3.5 SAMPLING

3.5.1 Orientation to sampling

Gay (1987, p.101) defines sampling as follows:

"Sampling is the process of selecting a number of individuals for a study in such a way that individuals represent the larger group referred to as a population" (p. 101).

The sampling of this study was guided by the following factors:

(1) The rationale of this research;

(2)The research method;

(3) Other key issues such as practicality and limitation factors.

The steps in sampling stipulated by (Gay, 1987) were followed:

(1) Identification of the research targeted population;

(2) Determination of the required sample size;

(3) Selection of the sample.

The sampling process is described in the next section below.

3.5.2 The Sample of the study

The sample consists of one class group of 14 Grade 4 learners at Delta Primary School (pseudonym) in the northern part of Namibia. The participants comprises of 10 boys and 4 girls. The group is a mixed ability group. The school has only one Grade 4 class. Its actual enrollment is 17 learners, but only 14 learners participated in this study. All 17 learners had equal opportunity to participate but 3 were not available as the intervention program took place during the school vacation.

The researcher also participated as the teacher teaching the participants. She was thus a participant – observer. She was involved in the study as both teacher and observer.

3.5.3 Criterion used and rationale behind sampling

The criteria that guided the sampling of this study are:

(1) The research goal;

(2) Accessibility and availability of the sample;

(3) Sample size and its representativeness;

(4) Technique of sample: (convenience sampling).

Research goal: Literature states that sampling should be based on what the research is trying to accomplish (Shank & Brown, 2007). The research goal is to improve the teaching and learning of division using manipulatives, visualization and questioning as tools to support learning. Manipulatives, visual representation and questioning thus underpin the intervention program. The research goal was to explore the role manipulatives, visual representation and questioning may play towards Grade 4 learners' conceptual understanding of the integration of division concepts and procedures. Grade 4 was chosen because at this level learners have yet engaged much with sharing and measuring out division. Thus investigating the implementation of manipulatives, visual representations and questioning at this level give a fresh picture of its effect in relation to conceptual knowledge development.

Accessibility and availability of sample: The grade 4 class group at Delta Primary School was selected as the sample of this study out of other suitable possible samples because of its accessibility and availability (Cohen, et al., 2007). Delta Primary School is the school where the researcher teaches Grade 4. It was thus easy to access parents for agreement purposes. Learners were also easily accessible and available for data collection during the school vacation as they live closer to the school.

Sample size and its representativeness: Cohen, et al., (2007) states there is no a clear-cut answer as to correct sample size. Sample size depends on the purpose of the study and the nature of the population under scrutiny (p. 101).

The representativeness to the population in this study is not meant for generalization to the group of this research. The purpose of this study and its results were not comparable to other possible samples in the targeted population. However, generalization of the result of this study could be extended to individuals finding themselves in a similar situation.

The class of 14 learners was reasonable in that the study only needed a class group/s which is manageable in relation to the quantity of data that needed to be collected and analyzed.

Convenience sampling: There are many techniques of sampling such as random sampling, stratified sampling, cluster sampling, systematic sampling, and convenience sampling (Gay, 1987; Cohen, et al., 2007). This study used convenience sampling, sometimes called accidental or opportunity sampling. It involves choosing the nearest individuals to serve as respondents or those who happen to be available and accessible at the time (Cohen, et al., 2007).

Being a part –time student one is expected to fulfill the MEd course requirements, one of which is to complete a course in a given time and in my case while continuing to fulfill my professional duties. The implementation period of this intervention program was the school vacation, the time agreed upon by all research stakeholders for the collection of data.

3.6 TECHNIQUES FOR DATA COLLECTION

For triangulation purposes, as explained by Cohen, et al., (2007), this study used the following methods of data collections: participant observation with video recording, document analysis and the focus group interviews with audio recording. Data collection started by administering the pre (diagnostic) task and ended with the same task as the post test.

3.6.1 Developmental phase of the intervention program.

3.6.1.1 The design

The intervention program was designed as a teaching and learning program of 15 lessons. The program had three main guiding components: (1) the learning goals, (2) the learning activity and (3) the hypothesis about learning process (Simon & Tzur, 2004). The learning content of the program was sharing (partitive) and quotitive (measurement) division (Kilpatrick, et al., 2001; Simon, 1993). The learning process was embedded within the division activities. The division problems or activities planned and done in the program were that of sharing and quotitive equivalent problems (Mulligan & Mitchelmore, 1997; Kouba, 1989). The hypothesis about the learning process led to the learning be planned in the hierarchical intellectual stages theorized by Bruner (McInerney & McInerney, 2006). Each kind of division was learned through the use of stones (enactive) through to the use of diagrams and dots (iconic) In every intellectual stage learning was interchangeable between enactive or iconic and symbolic (McInerney & McInerney, 2006).

3.6.1.2 The learning nature

The learning in this program left room for learners to construct their own knowledge. The role of the teacher was just to create an environment that facilitated learning, to ensure that interactions between individual learners and the activity or individual learners and the teacher or other were maintained and that learners were scaffolded to reach their learning goals. Learners were given the opportunity to explore division with real objects such as stones and also with diagrams. No teacher authority in terms of passing judgment on learners' statements was exercised (Lampert, 1992). Instead the teacher used questioning to help learners clarify their reasoning, or to push them closer to their potential level of development (Petty, 1993;

Manouchehri & Lapp, 2003; Naylor, et al., 20004; Elder & Paul, 1998). Where misconceptions arose, rather than rectifying these, the teacher referred learners to the division action process with stones, diagrams or dots, with the aim of pushing learners to uncover their own misconceptions and then rectify these themselves.

3.6.2 Diagnostic test-task.

The implementation of the intervention program was run in seven days. Two lessons were administered each day of the implementation of the intervention program. The first lesson of day one was the diagnostic task. The purpose of this diagnostic task was to determine the participants' prior knowledge about division. It consisted of:

(1) Oral questions that formed part of the interview questions. The questions were intended to clarify through asking participants to describe the division procedures they had used, to explain their understanding of division, and provided justification for their actions.

(2) Calculation questions where participants were asked to calculate division without any prior instruction. These questions were used to get a picture of the participants' prior knowledge on division calculation. The written work for these calculations formed a part of the data on the diagnostic task findings.

(3) Finally questions were asked on the way participants used manipulatives, visual representations (stones), as well as questioning used for scaffolding.

The lesson was video- and audio- recorded. The video- recorded data was used to help for the post- observation, while the audio- recorded data was used in drafting of lessons transcripts.

3.6.3 Observation and video recording

Observation was the main tool used for data collection in this study.

Good & Scates (1954, p.647) states that:

> "Observation seeks to ascertain the overt or covert behaviors of a person (and what it may reveal) by watching them as they express themselves in a variety of a

situation, selected to typify the conditions of normal living or to represent some special set of factors” (p. 647).

The observation focused on everything that happened in the lessons, with specific attention to the four following aspects: (1) details pertaining to the form of visualization used in the process of division; (2) how these forms of visual representations were used in this process; (3) what kind of knowledge was developed; and, finally (4) whether and how, visual representations played a role in the knowledge developed.

Participant observation: This study was a participatory observational research.

Babbie & Mouton (2001, p.293) explained that:

> “A participatory observational study is a study whereby a researcher is simultaneously a member of the group s/he is studying and a researcher doing a study” (p. 293).

This intervention program tried to ensure that the learning did not only include using manipulatives, visual representations and questioning, but led also to conceptual development. In other words, it tried to ensure that division (partitive and quotitive) was learned both procedurally and conceptually. This intervention program therefore needed a teacher capable of teaching in a way that ensured that the learning goals were met (i.e. the researcher herself).

Challenges of observation: Literature indicates that participant-researchers are faced with challenges in taking field notes (recording) and observing everyone else from a researcher’s point of view, while at the same time being one of the members of the group and (Babbie & Mouton, 2001; Gay, 1987). A further challenge was of taking notes of /recording more than one actions at a time when there was more than one interesting action going on at a time (Gay, 1987). These two challenges were solved through the use of video and audio recording devices.

Another challenge experienced was the length of the lessons. The study site (school) does not have electricity, and lessons took longer than the video recorder’ battery lasted. Video recording was therefore not used to record the entire lesson: only some parts were recorded, especially the actions of participants that involved the use of manipulatives and visual representations form. However, lessons were audio-recorded in their entirety. Recorded data was later transcribed for analysis.

3.6.4 Document analysis

Document analysis began by reviewing the mathematics schedules of the examination records over the three previous grades, 1, 2 and 3 in all the consecutive terms of each year April, August and December. These results are presented in the bar graphs as appendix 8.1 on the appendices chapter. The tables are presented in the sequential order of grades, which started with the results of grade one year 2009 to that of grade three year 2011. The reason of reviewing the past performance of the learners is to see how learners had been receiving quality education. Based on the (Namibian Curriculum for Basic Education, (NCBE, 2010) lower primary learners are not assessed through examinations. The end term results recorded are the accumulation of their recorded everyday to day continuous assessment, which is a record of these learners' everyday learning process. On the school schedules these examination statistic are recorded in the symbols. Based on the lower primary policy each activities of learners are to be out of 5 point scales, which have the following ranges: 1-1.4 = E, 1.5 – 2.4 = D, 2.5 -3.4 = C, 3.5 – 4.4 = B and 4.5 – 5 = A (National Policy Guide,). Looking at the tables very well in the appendices chapter appendix 8.1, one would notice that some learners' results of some terms or years are not shown. These are the learners who either had repeated some years in between or those who were admitted at school in years other than grade one.

Document analysis in this study also looked at the participants' activity worksheets. The worksheet tasks contained questions to test participants' knowledge of formal language or symbolic mathematical representation of the division ideas done practically with stones and diagrams. These questions required the translation of the ideas behind the division's actions into written mathematical representation. This mean after learners had done the division with stones or dots or with drawing; learners were then asked to translate the division action into symbolic number sentences. These questions were integrated into the lessons. They were not administered in isolation. The results of the earners' performance on numerical data are provided as Appendix 8.2.

According to Cohen et al (2007, p.414):

> "In tests, researcher has at their disposal a powerful method of data collection, an impressive array of tests for gathering data of numerical rather than verbal kind" (p. 414).

Cohen et al. (2007) further emphasized that tests should be designed with/ for a purpose. In this study, the tasks were designed so that the participants could record their answers on division on paper after they had used any given visual representation form to do the division; to draw diagrams or icons and used those drawings in the paper as a way of division; to transform knowledge from the practice of division with visual representations forms (stones, drawing or icons) to the procedural knowledge of division symbols (number sentence) and then to transform division knowledge to other operations such as addition, subtraction and multiplication and vice versa.

In as much as data from the document analysis informed the researcher on how some of the learning goals of the intervention program were met, it responded also to the research questions pertaining to the role of visualization in the achievement of these learning goals.

3.6.5 Interviewing with audio recording.

Any other method of data collection utilized in this study was interviews.

Babbie & Mouton (2001, p.289) explain the qualitative interview as follows:

> "Qualitative interview is an interaction between the interviewer and the respondents in which the interviewer has a general plan of inquiry but not specific sets of questions that must be asked in particular words and in a particular order. This interview, is essentially a conversation in which the interviewer establishes a general direction for the conversation and pursues specific topics raised by the respondents; ideally the respondent does most of the talking" (p. 289).

Good & Scates (1954, p.637) described the interview by saying that:

> "It is used for research purpose of gathering data directly from others in face-to-face contacts. Through interview, certain types of information can be secured only by direct contacts with people: for example data on the opinions, understanding and beliefs of people" (p. 637).

The interview questions for this study were prepared and integrated within the intervention program. They were intended to gather the data the researcher could only get through normal conversation. The purpose of these questions was to extract meanings and interpretations of the participants' actions from the participants' point of view. These meanings and interpretation were explored in relation to participants' procedural and conceptual knowledge of division, and the role visual representations did or did not play toward the participants' division knowledge development. Specifically, the interview questions sought clarity on the following aspects about the participants' division knowledge development: (1) the definitions of division; (2) their descriptions of the division process; (3) their Justifications for division methods and results; (4) their explanations for their actions; (5) the understanding shown through the examples they provided.

Focus group interview:

Babbie & Mouton (2001, pp.291-292) described:

> "A focus group interview is an approach whereby a researcher interviews a group of respondents and asks the respondents' individually in turns ensuring that each gets a chance to speak so that the interviewer ends up with all the individuals' responses of all members in the group"(pp.291-292).

The focus interview questions were integrated within the lessons of the intervention program for fear that if they were done after the lessons children might have forgotten certain things by then and the data would be compromised. The focus group questions were a follow-up to the written tasks and other lesson activities. They were asked immediately after the participants had just completed certain tasks. For example, after the participants had finished the division with stones, the researcher would ask them to describe the process of division and to justify their answers.

<u>Challenges of interviewing:</u> Good and Scates, (1954) highlighted factors that may cause limitations for the interview such as participants' experience and their ability to express themselves clearly. In this study, lessons comprised many tasks of different forms such as actions, written works and oral questions to be answered, which led to lessons taking up a lot of time. The average time for a lesson in this study was about 2 hours, and every day participants did two lessons (about 4 hours a day). It was difficult to ensure that for each member in the group got an opportunity to respond to interview questions. Also participants were inexperienced about such activities and struggled to express themselves. This prevented them from giving detailed answers to the interview questions asked.

3.6.6 Post-test task.

The post-test task was administered on day seven (lessons 14 and 15). In lesson 14 the test tested if learners could differentiate the two kinds of division. Learners were given the two divisions represented symbolically, which are: (35 ÷ 7) and (24 ÷ 3) and, were asked to divide the divisions in two different ways. Learners were also asked to describe the two different ways they can divide the two divisions. The task also tested the relationship of division knowledge to other domains such as addition, subtraction and multiplication. The second post-test was administered in lesson 15 and was the reiteration of the initial diagnostic test, to see if there any change in learning had occurred between the diagnostic test and the post-test.

3.7 VALIDITY ISSUES

3.7.1 Ethics

Research requires obtaining consent and cooperation from participants taking part or involved in any way in the study and even from the significant others in the study institutions or organization providing the research facility (Cohen, et al., 2007; Gay, 1987).

The study is the original work and has never been researched. The identities of people and the school involved in this study are stated by pseudonyms names for anonymity purposes. This study was based on the foundation of 'informed consent' (Cohen, et al., 2007) of all the concerned research stakeholders: the regional director of the region where the school (study site) belongs; the inspector of the circuit; the school board chairperson. Since the researcher is also the school principal and could therefore not have given herself permission; parents of the participant were asked to give the permission for the participation of their children.

The requisitions for permission from each of the three authorities were prepared in writing (Gay, 1987). In that requisition communiqué, the authorities were provided with the following information: (1) the purpose of the study; (2) the research title; (3) the researcher' responsibilities toward the participants in regarding to their right, anonymity and confidentiality were also stipulated; (4) the researcher's indication of the study site (the school) and the desired participants (grade 4 learners).

The study process began after the receipt in writing of permission was granted. The letters and permission in writing are provided as evidence in Appendices as (Appendix 8.1).

3.7.2 Triangulation in this study

Cohen et al (2007, p.141) defined triangulation as:

> "The use of two or more methods of data collection in the study of some aspect of human behaviors" (p. 141).

Triangulation techniques are intended to perceive the richness and complexity of human actions by studying them from different point of view. In this study, methodological triangulation approach was used (Cohen, et al., 2007). Three different methods for collecting data on the same case of manipulatives, visual representations and questioning were used: observation, document analysis and interview.

3.7.3 Validity of this study

A study is valid if it measures or explores what it supposed to measure (Cohen, et al., 2007; Gay, 1987). The validity of this study is based on the match between what this study intended to explore and its findings. This study aimed to investigate the roles that manipulatives, visual representation and questioning may play in the learning of integrated division procedures and concepts in mathematics at the level of grade 4. The findings also reported on the learning of sharing and measurement division and the influence of manipulatives, visualization and questioning in that learning. Thus for this, two criterions were opted to justify the validity of this study, which are: Descriptive validity and Content validity.

Descriptive validity: is the actual accuracy of the account, that the research report is not made up, selected or distorted (Cohen, et al., 2007). Evidence of the accuracy of this study is provided in the form of lessons transcripts in the appendices as (Appendix 8.3). Evidence is also available in the file.

Content validity: is achieved by the fact that a study tries to fairly and comprehensively cover the domain it purports to cover (Cohen, et al., 2007:137). This study is valid because the findings, analysis and discussions cover what was intended to be covered.

3.8 DATA ANALYSIS

The qualitative data analysis involves, organization, accounting for and explaining the data (Cohen, et al., 2007). Qualitative data is more open to ambiguity (Burton, Brundrett & Jones, 2008). To make sense of the data in terms of the participants' definitions of situations, the data need to be analyzed by noting patterns, organized in emerging themes and summarized into categories and regularities of these themes (Cohen, et al., 2007; Burton, Brundrett & Jones, 2008). These categories should be in relation to the research questions and goal and should be manageable (Lankshear & Knobel, 2004).

Burton et al (2008, pp.163-164) defined coding as:

> "A shorthand to identify data and allocate them to a system of categories and sub-categories" (pp. 163-164).

Coding of data and data organization that emerged in relation to conceptual frameworks need to be justified by literature (Burton, et al., 2008) some types of codes emerges from the collected data are: the pre-determined codes. These are the codes that are pre-determined by the literature or by the research title, and there are also those codes naturally emerging from data undetermined. The categories for analyzing derives from those predetermined and the undetermined one as follows: research goal, research questions, literature, participants, conceptual or theoretical frame works and researcher's experience (Burton, et al., 2008).

The data of this study was gathered using observation, interview and document analysis from the implementation of a teaching and learning of the intervention program of 15 lessons.

First, data from the lessons transcripts and from document analysis were organized into categories according to emerging themes. The coding of themes or categories of data was guided by the: research goal; research questions, and by literature on the relevant conceptual frameworks.

This is how the data of this study was categorized and coded. The data was categorized into four main themes: the procedural understanding of division, the conceptual understanding of division, challenge experienced and use of visualization.

Procedural understanding data: all the tasks that learners did in the lessons, which required ***division procedures*** (such as calculations) and the ***translation of the division actions (done with stones, drawings or dots) into mathematical symbolic number sentence*** yielded to the procedural understanding data. The data which portray the procedural understanding in this study is presented as a numerical data of bar graphs as Appendix 8.2. In the lesson transcripts (Appendix 8.5), the part of the conversation which yielded to procedural understanding was that of two sub- categories: the division procedures and the translation of division action into symbolic number sentences. These two sub-categories are code in different colours in the transcripts. The division procedures in the transcript are written in font with orange color. The conversation pertained to translation of division ideas into number sentences is shaded in a grey color.

Conceptual understanding data: the data that depict learners' conceptual understanding of division was extracted from the lessons transcripts (Appendix 8.5) and was portrayed in six different sub-categories. The six sub-categories and their coding are as follow:

- ***Definition of division;*** is the part of conversation in the transcript written in a font with a ***purple*** color.

- ***Describing the division procedures;*** is the part of conversation in the transcript written in a font with a ***green*** color.
- ***Division examples;*** is the part of conversation in the transcript written in a font with a ***dark red*** color.
- ***Justification of division actions or answers;*** is the part of conversation in the transcript written in a font with a ***blue*** color.
- ***Relation of division to other operation knowledge;*** is the part of conversation in the transcript shaded with a ***yellow*** color.
- ***Differentiating two divisions (sharing and quotitive);*** is the part of conversation in the transcript shaded with a ***dark yellow*** color.

Other theme emerged from the data are: ***the challenges experienced*** in the lessons and the ***use of visualization for specific learning support***. The two themes are also coded with different colors in the transcripts. Parts of the transcripts shaded in ***pink*** color are for the data pertained to ***challenges***. Parts of transcripts shaded in ***turquoise*** color are for the data pertained to the ***use of visualization*** for specific learning support.

After the data was categorized and coded, the data was linked by matching data of the same pattern, contrasting and comparing. The last step was explanation and theorization generation in relation to research questions and goal.

3.9 LIMITATIONS

The process of the data collection for this study faced some limitations.

Limitation of time: Activities of different forms were integrated within the lessons. The integration of tools in one lesson especially the questioning led to the lessons' taking up too much time (about 2 hours a lesson and 4 hours a day). By the time lessons approached the end, participants used to be tired.

Limitation was of lack of resources: Lack of electricity at the study site (the school) led to a researcher recording only some of the activities or parts of the lessons instead of recording the whole lesson.

Limitation caused by the integration of tools in the lessons: The interview questions and the document analysis activities were integrated within lessons. This led to the difficult in allowing every participants opportunity to participate.

Limitation of being a participant observer: The researcher hold two position during the process of data collection, which were: (1) as a teacher teaching the participants and (2) as the observer. It was not easy to be teaching and at the same time concentrating on observation as a researcher is supposed to do.

3.10 CONCLUSION

This chapter presented the methodological frame work for this case study. The next chapter presents the findings from the data collected in this study.

CHAPTER 4

THE RESEARCH FINDINGS

4.1 Introduction.

This chapter details the analysis of this study. It begins with the discussion of the nature of the intervention program from the design to its learning style. This was followed by the description of how the transcripts was used and referenced on the findings. Then the analyses of the diagnostic tests follow. The analyses continue by describing the lessons according to themes of the tools used for division and the kinds of division learned on those lessons. Tools are manipulatives (stones), diagrams and dots, and kinds of division learned were both sharing (partitive) and quotitive (measurement) division. The post-test tasks were also analyzed. Finally, findings were then analyzed under the themes in relation and response to research goals and research questions. The chapter ends with a concluding section.

4.2 How the transcripts was used and referenced on the findings.

The lesson conversation' transcripts form up the major part of this data (findings). They are provided as appendix 8.5 on the Appendices. The transcripts are typed in different colors to code different categories or sub-categories of the data (as described in chapter 3 section 3.8). On the appendices the lesson conversations are typed in the day the lesson took place, the lessons are numbered and the dialogue are numbered in paragraphs. In this chapter or in the whole study at large, the transcripts are referenced by the day, lesson and paragraph number i.e Day…, Less.., Par…, (this mean, or refer to the day on which the lesson took place the lesson number and the paragraph referred to).

4.3 The nature of the intervention program.

4.3.1 The design

The intervention program was designed as a teaching and learning program of 15 lessons. The program is provided as Appendix 8.4 on the Appendices. The program had three main guiding components: the learning goals; the learning activities, and hypothesis about the learning process. The overall learning goals for this intervention program were for learners to learn sharing and quotitive division procedures in a conceptual manner and relate the knowledge of that division learned to other domains of operations, which are addition, subtraction and multiplication. The learning was guided or embedded within the division activities. The division activities planned and implemented in the program were that of sharing and quotitive problems. The hypothesis of the learning process of this program led the learning to be planned in the hierarchy of Bruner's stages. Each kind of division was learned from the use of stones (enactive) through to the use of diagrams and dots (iconic) and in every activity, learning was related between enactive or iconic and symbolic.

4.3.2 The learning nature

The learning in this program left room for learners to construct their own knowledge. The role of the teacher was to create an environment to enable learning, to ensure the interactions between individual learners and the tools was maintained, as well as to scaffold learners in order to reach the learning goals. In this learning program learners were given the opportunity to experiment with real objects such as stones and with diagrams when performing division. The teacher did not pass judgment on learners' statements. Instead, the teacher used questioning to develop learners' clarity of reasoning, or to push them to their potential development. When possible misconceptions were identified, the teacher would refer learners to the division action process with stones, diagrams or dots, in order for learners to adapt and improve their own responses.

4.4 Diagnostic test results.

4.4.1 The description of the diagnostic test.

The diagnostic test was the first activity of the intervention program, which was administered on Day one of the program (see Appendix 8.5, Day 1, Less 1). The goal of the test was to assess learners' prior knowledge of division. It tested the procedural knowledge of division through calculation of division tasks and through its inverse multiplication tasks. The inverse multiplication tasks were also given to test the relation between multiplication and division. The test also assessed conceptual knowledge of division through interview (oral) questions, which asked learners to describe their division process, to justify this process and to provide everyday life examples of division. The test ended with two division tasks, to be carried out with stones. These last tasks also tested if learners can translate division actions with stones into the symbolic number sentence as learners were asked to translate their division action with stones into the symbolic number sentence.

4.4.2 Analysis of the test results.

The diagnostic test showed that learners' conceptual understanding of division was generally implicit, showing that learners' initial conceptual understanding of division was poor and shallow. Procedural knowledge was better than conceptual knowledge (Appendix 8.5, Day 1, Less 1).

Procedural knowledge: In question one of the diagnostic test, learners did the division calculation without any teaching and learning instructions given. The average performance of the learners (the average of the learners marks out of 5 is calculated as a mean) for the division calculation was 54% (see Appendix 8.2, figure 8.2(a)).

In question two, learners did the multiplication calculation, which was the inverse of division tasks in question one (see Appendix 8.2, figure 8.2(b)). The teacher gave prior instructions for question two using dots to show the patterns in the number values to help the learners gain familiarity with the task. At the chalkboard, teacher in conversation with learners drew and counted dots in circles to portray the pattern of how many fours are in twelve. The final diagram showed three circles as representing three groups each with four dots representing the number fours in twelve which carried the meaning that there are three fours in twelve (See Appendix 8.5, Day 1, Less 1, Par 48 -54). In this question, the mean of learners' performance was 56% (see Appendix 8.2, figure 8.2 (b)).

In questions 3 and 5, learners took 12 stones and divided them equally into 4 groups (12 ÷ 4) and took again 18 stones and kept measuring out 9 stones (18 ÷ 9). Eighty-nine percent of learners used the stones well and did the correct division in questions 3 and 5. When learners were asked to translate the division they had done with stones into a symbolic number sentence in questions 4 and 6, only 54% of learners translated correctly from division actions to symbolic number sentence of division (see Appendix 8.2, figure 8.2 (c)).

Conceptual knowledge: the conceptual knowledge of division in the diagnostic test was tested in three principles of conceptual knowledge: knowledge transfer; conceptual understanding and knowledge relationship. The results indicated that learners had a shallow and poor conceptual knowledge of division in those three different principles.

First, the ***division understanding*** of learners was tested when learners were asked to define division. Only one learner out of fourteen attempted to define division, which he did by providing an everyday example of division. The teacher tried through questioning, to scaffold learners in order to help them show their understanding, but only few responded to questions and their responses were in short statements of yes or no (see Appendix 8.5, Day 1, Less 1, Par 8 - 36). Another aspect indicating the lack of conceptual knowledge was that the procedural knowledge of division tested (the calculations and translation of division ideas to symbolic number sentence) was done in isolation to their underlying concepts. Learners were able to calculate (as indicated in Appendix 8.2, figure 8.2(a)), but did not show the meanings of symbols in the number sentence or the underlying concept attached to the procedures or signs on those number sentences.

Learners were also asked to describe their division process and to justify their division answers as a way of showing their understanding of division (see Appendix 8.5, Day 1, Less 1, Par 133-158, 217-228, 247-258 & 261-268 for description and also Appendix 8.5, Day 1, Less 1, Par 231-242, 259-260 & 269-273 for justifications). When asked to describe their division process only three learners attempted to answer. These three learners did not independently describe the division process completely. The teacher had to scaffold them through questioning to enable them complete their description.

Learners were then asked to justify their answer or their option for choosing the division number sentence in relation to the division performed with stones. Only two learners out of fourteen attempted to justify, and those who did, did not justify well. One described, rather than justifying and the others gave incomplete justifications (Appendix 8.5, Day 1, Less 1, Par 231-242, 259-260 & 269-273).

Learners were also asked to show their understanding of division through the provision of examples in (Appendix 8.5, Day 1, Less 1, par 274 -328). Seven learners gave division examples with small numbers ranging between 2 and 13. Some of the examples given were impractical. Such as the example of thirteen sweets shared by three people (13 ÷ 3) and that of seven sweets shared by three people (7 ÷ 3). (This was referred to as impractical because learners were expected to opt for a number that can be equally shared without a reminder). Some provided a practical example with incorrect answers

Knowledge relationship: Learners were asked to tell the relationship between the division tasks they did in question one and its inverse in question two. None of the fourteen learners were able to relate the division to multiplication, even after extensive scaffolding was given through questioning (see Appendix 8.5, Day 1, Less 1, Par 199 -217). Learners were only able to distinguish between division and multiplication; and above 50% of learners also calculated the division and multiplication well (as indicated in Appendix 8.2, figure 8.2(a) and (b)), but their knowledge did not go beyond the calculation skills.

Knowledge transfer: The only knowledge transfer in the diagnostic test was between two forms: the enactive (doing division with stones) and the symbolic. The 89% of learners used the stones well and did the correct division in questions three and five. When learners were asked to translate the division with stones in questions four and six, 54% of learners translated correctly from division actions to symbolic number sentence of division (see Appendix 8.2, figure 8.2(c)). The translation from the division actions with stones to symbolic number sentence of division appeared to be procedural knowledge as there was no indication of learners showing understanding of the meanings the signs in the number sentence carry.

4.4.3 The use of manipulatives and visualization during the diagnostic test

The use of manipulative and visualization does appear to have improved children's learning of division during the test. Learners' performance increased from 54% in question one when they calculated without stones and visualization prior to instructions to 86% performance, when learners were given a chance to do division with stones in questions 3 and 5 (see Appendix 8.2, figure 8.2 (a), (b) and (c)) for comparison.

Manipulatives and visualization appeared to play three roles in learning of division in this mediated test.

First, in question two of the diagnostic test, visualization (drawing of dots) played a role of learning support as the teacher incorporated it within teaching instructions to help learners visualize the pattern of number four in twelve (Appendix 8.5, Day 1, Less 1, par 48 -54). In questions 3 and 5, it still played a role of supporting learning as stones were used as a procedure for division.

Second, when learners gave an impractical example of thirteen sweets divided by three people (13 ÷ 3) (this was referred to as impractical because learners were expected to opt for a number that can be equally shared without a reminder) or a correct example with wrong answers of eight bananas shared by four people of which each would get four (8 ÷4 = 4) they were requested to perform this division with stones which helped them realize their error and at the same time rectified their errors (see Appendix 8.5, Day 1, Less 1, Par 273 -278 & 313 - 326).

Third, it played a role of providing feedback to the teacher. When there was a misinterpretation in the drawing patterns that the teacher drew on the chalkboard, the teacher interpreted the drawing of three circles each with four dots as follows: "four dots are the fours and three circles is the number of times the fours are in twelve". In contrary, Hanna interpreted the drawing as follows: there are four fours in twelve. When the teacher called her on the chalk board to show what she is referring to, Hanna came and counted dots inside the circle. To Hannah, the four dots inside the circles represented both the number four and the number of fours in twelve. The teacher was able to see what Hanna was referring to, when Hanna demonstrated using the drawing (see the conversation in Appendix 8.5, Day 1, Less 1, Par 54 - 112).

4.4.4 Challenges experienced during the diagnostic test

During the diagnostic test, two challenges were experienced.

First, is the misinterpretation of the drawing of dots (Appendix 8.5, Day 1, Less 1, Par 48-54) when the teacher drew three circles each with four dots to show that number four (represented by four dots) is there three times (represented by three circles) in twelve. Hanna interpreted this as there are four fours in twelve and all the fours she was just referring to, related to the four dots inside the circle (Appendix 8.5, Day 1, Less 1, Par 54 - 114).

Second, it was the ineffective use of stones to do division, a learner when instructed to divide 12 stones into 4 groups he divided them into 3 groups of 4 instead (Appendix 8.5, Day 1, Less 1, Par 159-163).

4.5 Lessons descriptions.

The description of the lessons below tells the story of what happened in terms of learning activities and their learning goals.

4.5.1 The description of lessons two and three, the learning of sharing division with stones.

In lessons two and three (see Appendix 8.4), learners used stones to do sharing division. After every sharing division problem with stones, learners were asked to translate (the translation refers to the transferring of division actions done with stones to symbolic number sentence) the division ideas they had done in actions to symbolic number sentences of division. In lesson three, the translation of division ideas done with stones into a symbolic number sentence was extended to other domains such as addition, subtraction and multiplication. In lesson two, learners did five different tasks of sharing while in lesson three they did three different division tasks. The learning goal of this lesson was to perform sharing division and then relate the understanding of that sharing division to other domains of addition, subtraction and multiplication.

The division problems for lesson two were: in question one; learners took 18 stones and shared them into three groups (18 ÷ 3). In question three; learners used 24 stones and shared them into four groups (24 ÷ 4). In question 5; learners used 30 stones and shared them into five groups (30 ÷ 5). In question 7; learners used 80 stones and shared them into four groups (80 ÷ 4) and in question 9; learners used 45 stones and shared them into 5 groups (45 ÷ 5). After learners had done the division with stones they translated the division action with stones into a symbolic number sentence. In this question, 59%; of the learners' performed the translation of all questions correctly (see Appendix 8.2, figures 8.2 (d)).

The division problems for lesson three were: in question one; learners took 20 stones and shared them into 5 groups (20 ÷ 5). In question 5; learners used 15 stones and shared them into 5 groups (15 ÷ 5). The division was done in two ways: (1) learners did the division using the stones in questions one and five (a division action) which they translated into a symbolic number sentence. (2) In questions two and six, learners translated (transfer by rewriting it) the same division done in questions one and five into a symbolic number sentence of repeated subtraction and repeated addition respectively. This was done to relate the division patterns to subtraction and addition. For example if (20 ÷ 5 = 4) then it would be done in a repeated subtraction as follow: (20 – 5 = 15), (15 – 5 = 10), (10 – 5 = 5) and (5 – 5 = 0), with that process one would see the pattern that five would be subtracted four times from twenty or if it is four it would be repeatedly subtracted fives from twenty to reach zero. In repeated addition learners keep adding 3 until fifteen is reached as follow: (3 = 3), (3 + 3 = 6), (3+3+3 = 9), (3+3+3+3=12), (3+3+3+3+3= 15) and this pattern of five threes in fifteen was borrowed or related from the division (15 ÷ 5 = 3). Eighty-nine percent of learners translated the division actions into symbolic number sentences of repeated subtraction and addition process correctly. Fifty percent of learners translated the division action with stones, which they did in questions one and five into a symbolic number sentence of division in question three and seven (see Appendix 8.2, figures 8.2 (e)).

4.5.2 The description of lessons four to seven, the learning of sharing division with diagrams and dots.

In lessons four and five, learners did the sharing division by drawing diagrams of apples, sweets, sticks, beans and cups. For each of the two lessons, learners did three different division activities. This is how the division in diagrams was done, first a number of small circles for the dividend, were drawn into a big box. Then, learners drew large empty circles representing groups or people sharing. To do the division, they 'moved' diagrams of sweets, apples or nuts from the big box to the 'group' circles, by cancelling diagrams from the big box with the dividend and redrawing them one by one in the circles representing groups or people to share. The cancelling and redrawing was done until all the diagrams in the big box were cancelled. The division in diagrams was translated into the symbolic number sentence of division and further extended into symbolic number sentence of repeated addition, subtraction and multiplication in lesson five.

The division problems done in lesson four were as follows: in question one; learners drew 18 apples and 3 empty boxes. Learners shared the 18 apples equally into the 3 empty boxes by cancelling three apples at a time and equally distributed them into the empty boxes by redrawing them one by one into those empty circles. This was done until all the 18 apples were cancelled out (18 ÷ 3). In question three; learners drew 24 sweets shared equally between 4 children in a similar fashion (24 ÷ 4). In question five; learners drew 30 sticks and 5 empty boxes and shared the 30 sticks equally into the 5 empty boxes (30 ÷ 5) (see Appendix 8.4). On analysis 60% of the learners did the division in the drawing correctly (see Appendix 8.2, figure 8.2 (f)). The division action done in these questions were translated into the symbolic number sentences and 60% of learners translated the division action into a symbolic number sentence correctly (see Appendix 8.2, figure 8.2 (f)).

The division problems done in lesson five were as follows: in question one; learners drew the sharing of 20 apples equally into 5 empty boxes (20 ÷ 5). In question five; learners drew the sharing of 15 sweets equally between 5 children (15 ÷ 5). In question nine; learners drew the sharing of 50 sticks equally into 5 empty boxes (50 ÷ 5) (see Appendix 8.4). On analysis, 81% of the learners did the division in the drawing correctly. Document analysis also revealed that 67% also translated the division action for questions two, six and ten into symbolic number sentences correctly. Translating questions one, five and nine into symbolic number sentences were performed correctly by 69% of learners (see Appendix 8.2, figure 8.2 (g)).

The activities in Lessons six and seven are very similar to those in lessons four and five. The difference was only that in lesson six and seven, learners did the sharing division by drawing dots (see the activities in Appendix 8.4).

Document analysis provided the evidence that in lesson six, 64% of learners did the division in drawing correctly. The division action done in drawing in questions one, three, five and seven were translated into the symbolic number sentence of division in questions, two, four, six and eight, which document analysis also revealed that learners of 75% also translated the division action into symbolic number sentence correctly (see Appendix 8.2, figure 8.2 (h)).

Document analysis provided the evidence that in lesson seven, 90% of learners did the division in drawing correctly. The division action done in drawing in questions one, five and nine were translated into the symbolic number sentence of repeated subtraction, addition and multiplication respectively in questions two, six and ten. Document analysis also revealed that 86% of learners also translated the division action into symbolic number sentence correctly. The division action done in drawing in questions one, five and nine were translated into the symbolic number sentence of division respectively in questions three, seven and eleven, which document analysis also revealed that 50% of learners also translated the division action into symbolic number sentence correctly (see Appendix 8.2, figure 8.2 (i)).

4.5.3 The description of lesson eight to thirteen, the learning of measurement division with stones, diagrams and dots.

In lessons eight and nine, learners used stones to perform measurement division. After every measurement division problems with stones learners were asked to translate the division ideas they had done in actions into symbolic number sentences of division. In lesson nine the translation of division ideas done with stones into symbolic number sentences was extended to other domains such as repeated addition, subtraction and multiplication. The learning goal of this lesson was to do measurement division and relate the understanding of that measurement division to other domains of addition, subtraction and multiplication. The division of these tasks was done this way, stones in the number of the dividend were taken and the same numbers of stones were repeatedly measured out from the dividend and groups were made for them until the dividend was finished (see Appendix 8.4) for activities.

The division problems for lesson eight were: in question one; learners took 18 stones and measured out groups of three stones until 18 stones were finished (18 ÷ 3). In question three; learners started with 27 stones and measured out groups of 9 until left with none (27 ÷ 9). In question 5; learners measured out groups of 20 stones from 115 (115 ÷ 20) (Appendix 8.4).

Analysis indicated that 90% of learners had performed the division with stones and translated this division action into symbolic number sentences correctly (see Appendix 8.2, figure 8.2 (j)).

The division problems for lesson nine were also performed by measuring out stones. The questions were: question one (40 ÷ 10), question five (16 ÷ 4) and question nine (90 ÷ 20). In these questions, they translated the division action with stones into symbolic number sentences using division and then again using subtraction, addition and multiplication respectively (see Appendix 8.4) for the activities. Evaluating learner performance, 88% translated the division actions into symbolic number sentences of repeated subtraction, addition and multiplication correctly and 79% translated into symbolic number sentences of division correctly (see Appendix 8.2, figure 8.2 (k)).

In lesson ten and eleven, learners did the measurement division by drawing diagrams of apples, stones, sticks and sweets.

The division problems done in lesson ten were as follows: in question one; learners drew 28 sticks and kept measuring out 4 sticks by cancelling out 4 sticks at a time from the box with 28 stick and redrew them in groups until 28 sticks were finished or were cancelled out (28 ÷ 4). In question five; learners drew 35 apples and kept measuring out 7 apples (35 ÷ 7). In question five; learners drew 32 sweets and kept measuring out 4 sweets (32 ÷ 4) (see Appendix 8.4) for activities. Document analysis provided the evidence that 69% of learners did the division in drawing correctly. The division action done in drawing in questions one, three and five were translated into the symbolic number sentence of division in questions, two, four and six, which document analysis also revealed that 67% of learners translated the division action into symbolic number sentence correctly (see Appendix 8.2, figure 8.2 (l) and (m)).

The division problems done in lesson eleven were as follow: in question one; learners drew 48 stones and kept measuring out 8 stones from the 48 stones (48 ÷ 8). In question five; learners drew 8 apples and kept measuring out 4 apples from the 8 apples (8 ÷ 4). In question nine; learners drew 14 sweets and kept measuring out 2 sweets from the 14 sweets (14 ÷ 2) (see

Appendix 8.4) for activities. Document analysis provided the evidence that 95% of learners did the division in drawing correctly. The division action done in drawing in questions one, five and nine were also translated into a symbolic number sentence of repeated subtraction, repeated addition and multiplication respectively in questions two, six and ten. As indicated in (Appendix 8.2, figure 8.2 (n)), document analysis also revealed that 79% of learners translated the division action into symbolic number sentence of repeated subtraction, repeated addition and multiplication correctly. The division action done in drawing in questions one, five and nine were translated into the symbolic number sentence of division respectively in questions three, seven and eleven. Document analysis also revealed that 60% of learners translated the division action into symbolic number sentence correctly.

In lesson twelve and thirteen, learners did the measurement division by drawing dots. This division was done in the same way as in with diagrams only that this time it was done with dots.

The division problems done in lesson twelve were as follows: in question one; learners drew 30 dots and kept measuring out 6 dots from the 30 dots (30 ÷ 6). In question three; learners drew 60 dots and kept measuring out 15 dots from the 60 dots (60 ÷ 15). In question five; learners drew 15 dots and kept measuring out 5 dots from the 15 dots (15 ÷ 5) (see Appendix 8.4) for activities. As indicated in (Appendix 8.2, figure 8.2 (o)), document analysis provided the evidence that 76% of learners did the division in drawing correctly. The division action done in drawing in questions one, three and five were translated into the symbolic number sentence of division in questions, two, four and six, which document analysis also revealed that 74% of learners translated the division action into symbolic number sentence correctly.

The division problems done in lesson thirteen were as follows: in question one; learners drew 20 dots and kept measuring out 3 dots from the 20 dots (20 ÷ 3). In question five; learners drew 36 dots and kept measuring out 6 dots from the 36 dots (36 ÷ 6). In question nine; learners drew 28 dots and kept measuring out 4 dots from the 28 dots (28 ÷ 4) (see Appendix 8.4) for activities. As indicated in (Appendix 8.2, figure 8.2 (p)), document analysis provided the evidence that 83% of learners did the division in drawing correctly. The division action done in drawing in questions one, five and nine were translated into the symbolic number sentence of subtraction, addition and multiplication respectively in questions two, six and ten, which document analysis also revealed that 88% of learners translated the division action into symbolic number sentence correctly. The division action done in drawing in questions one, five and nine were translated into the symbolic number sentence of division respectively in questions three, seven and eleven, which document analysis also revealed that 79% of learners translated the division action into symbolic number sentence correctly.

4.6 Post-test task

Two post tests were administered in (Appendix 8.5, Day 7, Less 14 & 15). The first post test in lesson 14 tested if learners can differentiate the two kinds of division: sharing (partitive) and quotitive (measurement out) division that they had learned in the previous 12 lessons. It also tested the knowledge relationship from the domain of division to the other operation domains (addition, subtraction and multiplication). Lesson fifteen was a repetition of the diagnostic test, given to assess the learning over this period.

In lesson 14, learners were given two symbolic division questions, and asked to perform each using a different kind of division. As indicated in (Appendix 8.2, figure 8.2 (q)), in question 1 (a) learners used the action or drawing of their choice to perform a sharing division of (24 ÷ 3). Eleven learners out of fourteen used the drawing of dots (icons), while 3 learners used stones. Thirteen learners out of 14 (93%) performed this sharing correctly. In question one (b) learners still were given a choice of either using stones or diagrams or dots to do the same division of (24 ÷ 3), but in this case they were asked to perform a quotitive division. Eleven learners out of fourteen used the drawing of icons (dots) as a way of division while the same 3 learners through out the test used stones to do division Only 10 out of 14, or 71% got a correct answer.

As indicated still in (Appendix 8.2, figure 8.2 (q)), in question two (a) learners performed a sharing division of (35 ÷ 7). Again, eleven learners out of fourteen, used the drawing of dots (icons) as a way of division while the same 3 learners used stones. Here 9 learners out of 14, or 64% did the correct sharing and got a correct answer. In question two (b) learners were asked to perform the same symbolic calculation using measurement division. As in the other cases, eleven learners out of fourteen drew icons (dots) while the same 3 learners used stones. In this question, 12 out of 14 (86%) did a correct quotitive division and got a correct answer.

As indicated in (Appendix 8.2, figure 8.2 (r)), in question 3, learners were given the choice of both the type of division and the way the division was drawn or acted out to do the division of (18 ÷ 6). Thirteen learners out of 14 performed quotitive division and 11 learners chose the drawing of dots while the same 3 learners used stones. Thirteen learners out of fourteen (93%) did the correct division in drawing and stones. All learners translated the division actions into a symbolic number sentence of division (18 ÷ 6 = 3). Only 8 learners out of 14, (57%) translated the division actions into a subtraction symbolic number sentence. eight learners out of fourteen (79%), correctly translated to addition and multiplication symbolic number sentences. Ten learners out of fourteen (71%) differentiated the two kinds of division orally and 12 learners out of 14 (86%) related the division to other domains: addition, subtraction and multiplication orally.

4.6.1 Differentiating the two kinds of divisions

In (Appendix 8.5, Day 7, Less 14, Par 24 – 27, 34-49, 53-63, 69-88, 91-102, 104-111, 113-135 & 137-156) 10 learners out of 14 differentiated the two kinds of division using the two division questions they were given symbolically. For the division of (24 ÷ 3) learners described how this division can be divided in two ways. First it was when 3 in the number sentence represent the number of groups, so they made 3 groups and allocated the dots one by one until the 24 dots were finished. Second it was when 3 was made the elements of group, which was divided by measuring out 3 dots from the dividend until the dividend were finished. The division of (35 ÷ 7) was also described in the same manner. The number 7 in the number sentence was used as either a group number or elements per group in the division (see also Appendix 8.5, Day 7, Less 15, Par 21-24, 85-88, 95-100, 139-142, 172-175, 188-191, 227-230, 248-251 & 273-276).

4.6.2 Relating division knowledge to other domains of addition, subtraction and multiplication.

In (Appendix 8.5, Day 7, Less 14, Par 167 - 200) the teacher made a conversation with one learner, to help the learner move across between division and other operation domains. Teacher asked: **“in eighteen there are three six right?”** learner: **“yes”**. Teacher: **“meaning if you are to subtract six from eighteen how many sixes would you subtract from eighteen?”** Learner: **“three”**. Teacher: **“and at addition how many sixes needed to get eighteen?”** Learner: **“three six”**. Teacher: **“and if you are to multiply, it is going to be six times what to get eighteen?”** Learner: **“is three times”**. Then the teacher reversed the pattern instead of asking six in eighteen she asked about the relationship of three to eighteen, in the same manner.

In (Appendix 8.5, Day 7, Less 14, Par 167 - 357) the teacher asked ten different learners the same questions. These learners reasoned in the same way as that one on previous conversation. They used the pattern that there are three sixes in eighteen and for that fact even in repeated addition it would be three sixes that would be repeatedly added to get eighteen, In repeated subtraction it would be three sixes that would be repeatedly subtracted from eighteen to reach zero. And in multiplication it would be six times three to get eighteen. And they all based their reasoning on the fact that there are just three sixes or six threes in eighteen.

4.6.3 Lesson fifteen

The lesson began with **Definition:** In (Appendix 8.5, Day 7, Less, 15, Par 3-8, 25-28, 40-43, 60-63, 89-94, 121-127, 129-138, 164-171, 186-187, 223-226 & 242-247), learners were again asked to define division as it was asked in lesson one. Twelve learners out of 14 (86%), provided a definition of division. Eleven of the learners defined it through the provision of division examples, which were given in the form of everyday knowledge. All the 12 examples given were practical in the context of division of whole numbers. The examples were given in both ways. The learner provided an example, and used the same example to explain how one is to divide it in either ways as sharing or quotitive.

The lessson continued with the division and multiplication calculation in question 1 and 2 respectively. Based on document analysis (see Appendix 8.2, figure 8.2 (s) and (t)) most (13) learners used icons (dots and sticks drawings) or stones to do question 1 and 2, only one learner computed. In question 3, learners were asked to use stones to do the division of 12 stones arranged equally in 4 groups. Thirteen learners out of fourteen (93%); responded correctly In question 4, learners were asked to translate the division action they did in question 3 into a symbolic number sentence of division. Twelve learners out of fourteen(86%) did this correctly (see Appendix 8.2, figure 8.2 (u)).

In question 5, learners were asked again to use stones to do the division by measuring out nine stones from 18 stones. All learners used the stones to do the division and got the correct answer. In question 6, learners were asked to translate the division actions with stone in question 5 into symbolic number sentence of division. Twelve learners out of fourteen(86%) translated the measuring division action with stones into symbolic number sentence of division correctly (see Appendix 8.2, figure 8.2 (u)).

Describing the division process: Ten learners described their division process to the extent of even describing the two different kinds of division sharing division as well as the measurement division (see Appendix 8.5, Day 7, Less 14, Par 165 -168 & Less 15, Par 37-39, 101-102 & 111-112).

Justification:Most learners were also able to justify their responses using reasons such as: the groups shared equal portions and the dividend was completely shared (Appendix 8.5, Day 7, Less 15, par 15-18; 80 -84; 103- 110; 161-163, 176- 183; 218-222, 239-241; 252-262; 277-282).

Knowledge relationship: Five learners who related division and multiplication reasoned using the patterns in the division. Using the division of (18 ÷ 6 = 3) learners said the inverse multiplication of 18 is 6 times 3 which was related on the basis that in all the number sentences there are same numbers which are 18, 6 and 3 and if there are six threes or three sixes in eighteen then even in multiplication it would be just six times three or three times six (Appendix 8.5, Day 7, Less 15, Par 13- 14, 31-32, 65-76, 147 -158; 194-217 & 263- 270).

4.7 The relation of the knowledge learned to procedural and conceptual knowledge.

4.7.1 The formal language or mathematical symbol representation.

All division problems for sharing and quotitive divisions were done or solved using stones and diagrams. Learners then translated these division actions into symbolic number sentences. The symbolic number sentence is the formal mathematical representation of these division ideas. The use of this symbolic mathematical representation of may be seen as procedural in accordance to the syntactic rules of how these symbols are to be written and how these rules correspond to the mathematical procedures of division.

The symbolic mathematical representations of sharing and quotitive division in this study were also learned conceptually. This was the case because learners related the symbolic mathematical representation not just to the mathematical procedure of division, but also to actions with stones, diagrams and dots that had a meaning in terms of sharing and grouping. Also, learners understood the meaning that the signs in symbolic representation (numbers in the number sentence) carried in terms of these actions. In lesson 14 (Appendix 8.5, Day 7, Less 14) of the post test of the study learners showed their understanding of the meanings carried by the signs in the number sentence such as (35 ÷ 7): the 7 in that number sentence could imply either a number of groups (sets) or the number of elements per group (set). Learners showed this understanding in two different ways, (1) when given the symbolic number sentence; they performed their division in two different ways using drawing of diagrams and dots or stones (Appendix 8.2, figure 8.2 (q)). (2) Learners' understanding was also shown when they verbally described the two ways one can do such division. Learners described the two different kinds of division this way: (1) for 35 divided by 7, one makes 7 groups and starts sharing stones one by one equally to seven groups until all 35 stones were finished, or by measuring out 7 stones from 35 stones and making groups for them until 35 stones were finished. In the same manner (2), 24 stones divided by 3 was described: one would make three groups and start allocating 24 stones to those three groups equally one by one until the 24 stones were finished, or by measuring out three stones from 24 stones and making groups for them until 24 stones were finished (Appendix 8.5, Day 7, Less 14, Par 24-27, 34-49, 53-63, 69-88, 91-102, 104-111, 113-135 & 137-156). These descriptions given by learners indicated that learners can differentiate between the two kinds of divisions. The descriptions learners provided were based on the symbolic mathematical representation of the two number sentences, and so indicated that the learners knew the meaning carried by the signs and numbers in the number sentences. This knowledge of meaning that learners portrayed indicated that the formal language or mathematical symbol representations of the number sentence were learned conceptually.

4.7.2 Other principles of conceptual knowledge apparent in the study.

First, knowledge learned in this study was ***transferrable***. Learners transferred the sharing and quotitive ideas from one form to another (knowledge was learned flexibly). One type of transferability apparent in the learning of sharing and quotitive division was that of the enactive, iconic and symbolic (see Appendix 8.2), which shows results of how learners were able to do division in actions with stones or drawings. It also shows how learners were able to translate their division actions into symbolic number sentences. The division ideas of partitive and quotitive division in this learning program was learned with stones (enactive) in every first two lessons (see lessons two and three for sharing and eight and nine for measurement in Appendix 8.4), this same division ideas was transferred in the forms of diagram and dots in the next four lessons and in all six lessons the same ideas were exchangeable transferred between stone, diagrams or dots to symbolic mathematical representation see (Appendix 8.2 and 8.4 and 8.5 all Paragraph colored orange), which explains how division was done with stones, diagrams and dots. When learning is transferrable and learned flexibly, that means the idea learned was generalized across different forms or domains. Learners were also able to transfer the same sharing and quotitive idea from the classroom division form to everyday real-life examples, which they did by providing the everyday examples (Appendix 8.5, all paragraph colored dark-red).

Knowledge learned by the learners in this study was ***rich in relationships (see Appendix 8.5, all Paragraph shaded in yellow colored)***, which is the other back bone principle of conceptual knowledge. This knowledge was rich in relationships because there was a linkage of it to other domains. In this study, the division knowledge was learned in relation to domains within division and external to division. Relationships were constructed by means of identifying the common features (common denominator), in this study were dividend, divisor and quotient numbers in the number sentence. Sometimes we can also refer to them as the division sub-structure or pattern, which even though the same was related differently in this learning. The sub-structures or division patterns in the division number sentence were related within the division domains of partitive and quotitive division as well as across different domains from division to addition, subtraction and multiplication see how it was very apparent in (section 4.5).

This study provided evidence of how relations of division patterns were constructed into levels; within the division domain itself and across; from division to other domains such as addition, subtraction and multiplication. The relation constructed within, learners related the division patterns within the division domain (Appendix 8.5, Day 7, Less 14, Para 24-27, 34-49, 53-63, 69-88, 91-102, 104-111, 113-135 & 137-156 & Day 7, Less 15, Par 21-24, 85-88, 95-100, 139-142, 172-175, 188-191, 227-230, 248-251 & 273-276). This started from lesson two to seven when they learned the division patterns of sharing (partitive division). In lessons eight to thirteen learners also learned the division patterns of measurement (quotitive division). The common features of the patterns of the two divisions were identified by the three number values: the dividend, divisor and quotient. In lesson 14 during the first post test, learners had shown the construction of relationships of these common features between sharing and measurement division. Learners first understood the meaning carried by a divisors and quotient in each kinds of division. These common features (dividend, divisor and quotient) were then related interchangeably between partitive and quotitive divisions by determining between divisor and quotient as to which one would be used as elements per set or as number of set in the division.

In this study, relationships were also constructed across other domains, learners related the division patterns of the same common features the dividend, divisor and quotient across the different domains from division to addition, subtractions and multiplication. After learners understood how many times the divisor can be taken out of a dividend (the quotient number) then they related it to addition, subtraction and multiplication this way, when 35 ÷ 7 is equal to 5, then it means 7 can be repeatedly added 5 times to get 35 and 5 can be repeatedly added 7 times to get 35. In the same manner, the common feature was also carried across to subtraction. In 35 ÷ 7 = 5, means 7 can be repeatedly subtracted 5 times from 35 to reach zero or 5 can be repeatedly be subtracted 7 times from 35 to reach zero. In the same manner, the common feature was also transferred to multiplication as an inverse of the division, whereby 35 ÷ 7 = 5 was reversed as 7 × 5 = 35 or 5 ×7 =35 see how well presented in (section 4.5) and the evidence from transcripts (Appendix 8.5, Day 7, Less 14, Par 169-186, 189-213, 220-240, 245-258, 263-268, 272-282, 284-319, 329-344 & 350-357 & Day 7, Less 15, Par 13-14, 31-32, 65-76, 147-158, 194-217 & 263-270).

4.8 The use of manipulations and visualizations and the roles they played in the division learning.

In this section the analysis focused on the way in which manipulatives and visualization was used to support learning and the roles it played in that learning.

4.8.1 The use of manipulatives and visualization in learning support

In this study, manipulatives (actions with stones) and visualizations (diagrams and dots) were used as tools to mediate the learning of division. The manipulatives and visualization were used as a ***procedure for division and for teaching instructions***. All division problems whether for sharing or quotitive division were solved through the use of stones, diagrams or dots, making the manipulatives and iconic visualization provide external forms for the division procedures (see Appendix 8.5, all paragraphs colored in orange). Manipulatives and visualization were used as physical tools for learning. These tools play a double role in learning: first they provide means within which learners can perform or act out division using them. Performing division with these tools extended their learning capacity to reach the learning goals of sharing and quotitive division. Secondly these tools also acted as scaffolds that facilitated extension of knowledge into related areas, as manipulatives and visualization helped by highlighting the division patterns which led to the construction of knowledge relationships within the division domain as well as across other domains such as addition, subtraction and multiplication.

As a procedure for division: In this study, stones, diagrams or dots were used as tools that provided means for learners to act upon the world, as learners were performing division with stones, diagrams and dots to extend their division learning capacity conceptually.

For sharing (partitive) division; learners divided all the division problems by taking stones or drawing diagram or dots for the number of a dividend. They then indicated a number of group position corresponding to the divisor and shared by allocating stones equally to groups one by one until all the dividend stones were finished or the diagrams or dots from the dividend drawing had been cancelled.

For quotitive division, learners again started the division problems by taking stones or drawing diagrams or dots for the number of a dividend. They then repeatedly measured out a number of stones, diagrams or dots corresponding to the divisor and made groups for them until the dividend stones were finished or had been cancelled from the dividend drawing (Appendix 8.5, all Paragraph colored in orange).

For teaching instructions: learning in this program was mediated by the teacher who was more competent and experienced in the subject content as well in the use of the tool. In findings, it was shown how learning was mediated by the teacher through the use and demonstration of stones, diagrams and dots.

First, the teacher incorporated the stones and drawing of diagrams or dots in teaching instructions several times to demonstrate their use as a procedure for division (Appendix 8.5, Day 1, less 1, Par 48 -54). In partitive division, the teacher showed how one indicates the groups positions either on the table or by drawing circles, rectangle or in any form. Learners were then shown how to share stones, diagrams or dots equally one by one. With stones, one only needs to allocate them to group positions one by one equally until the dividend is completely shared. In diagrams or dots a learner after drawing circles that represent groups to share, would need to cancel diagrams or dots from the box and redraw them into the groups. The process is repeated until all the diagrams or dots in the big circle were cancelled out and redrawn equally into smaller circles. The division result was then determined by recounting the stones, diagrams or dots in the smaller groups. Learners were also shown how they would use the stones, diagrams or dot to do the measurement division.

Second, the teacher also used the drawing of dots in (Appendix 8.5, Day 1, Less 1, Par 48 - 54) to show the patterns of number in the division. To visualize these patterns, the teacher drew the number of times the divisor is in the dividend as it was done in lesson one Day one, the teacher took 12 the dividend to show how many times four is in twelve. The teacher drew three circles in a number equal to a group number and in each circle she drew four dots equal to the elements per set/ groups, which was the same value as number four. This demonstration was done in interaction with learners in the process of demonstration, after each circle was drawn with four dots (the number of elements per group); the teacher asked learners to count the dots and learners were made aware that the sum of all the dots in all the circles should reach twelve. The teacher kept drawing the circles each with four dots in the number of elements per groups and made learners to count all the dots until when the number of dots in all the circles reached or sums up to 12 (the dividend). The final drawing portrays three circles each with four dots inside which was interpreted as the four dots representing number four and the three circles representing how many times four was in twelve. In (Appendix 8.5, Day 3, Less 5, Par 104 - 113) the teacher used the diagrams of sweets and drew in the same manner as it was done in lesson one. The teacher drew groups of three sweets to show how many 3s can build up 15. The teacher did the same demonstration of building 15 with 3 using dots in (Appendix 8.5, Day 4, Less 7, Par 105 - 115).

Third, the teacher used stones, diagrams and dots to demonstrate how they can be used as a procedure for the repeated addition, subtraction or multiplication. The teacher used the stones, diagrams or dots in the number of dividend and explained to the learners that for repeated subtraction one would use these stones by repeatedly taking out a number of stones equal to number of groups or elements per set from the dividend group of stones, counted the remaining and wrote the step in the number sentence, the process was then repeated until the result reach zero. For example in (Appendix 8.5, Day 4, Less 7, Par 44 - 55) teacher used dots to show 4 repeatedly subtracted from 20. This was how it happened, in step one of (20 – 4 =

16) the stones in a number of four were measured out from 20 stones and the remaining was counted to get sixteen. In the second step of (16 – 4 = 12) the stones in a number of four were still measured out of 16 stones and counted the remaining one again. The process was repeated until when a step of the remaining zero was reached, which is (4 – 4 = 0).

4.8.2 The roles that manipulatives (stones) and visualization (drawing of diagrams and dots) played in the learning partitive and quotitive division.

Manipulatives and iconic visualization played many important roles in the learning of sharing (partitive) and quotitive (measurement) division. These roles were: (1) manipulatives and visualizations played a role in rectifying or correcting learners' misconception, misunderstanding or their wrong answers; (2) The teacher used it to depict patterns of numbers in the division which led to construction of knowledge relationship; (3) It provided a strong conviction, evidence and proofs in learning, and (4) It also provided transparency and feedback of learning.

Findings in this study show how manipulatives and iconic visualization were used to help learners ***rectify or correct their misconceptions, misunderstandings or their wrong answers***. When learners misunderstood a point or gave an impractical example or a wrong answer a teacher used stones or drawings and the learner was requested to do the division with stones or drawings. Through the process of actions most learners came to realize that they were wrong and corrected their mistakes without the teacher telling them. In (Appendix 8.5, Day 1, Less 1 Par 273 – 278; 313 – 326) two learners gave impractical examples of division of whole numbers, which were thirteen sweets shared into three groups and seven sweets shared by three people (13 ÷ 3; 7 ÷ 3). These learners were requested to do their division with stones, after which they realized their examples were not practical and corrected them. In (Appendix 8.5, Day 7, Less 13, Par 4-7; 8) learners measured out three dots from twenty dots. The division which was supposed to give 6 groups of 3 dots and 2 dots that remained as a remainder. Two learners, instead of getting 6 groups of 3 dots, made 7 groups of which the seventh group had the two dots, the remainders. They concluded that there were 7 groups of 3 dots in 20. Learners were referred to the drawing of dots and asked to re-examine if all the groups looked the same. After learners examined the drawing they rectified their misconception.

This study shows how manipulatives and visualization provide ***strong conviction, evidence and proofs*** in learning. This is because they were used as means for understanding. Learning in this study used manipulatives and iconic visualization as proofs of where they had gone wrong in the process of their learning. As revealed in lessons transcripts, a learner who had done division with stones or diagrams wrongly or gave an impractical example of (13 ÷ 3) in (Day 1, Lesson 1) need not hear from the teacher that their conjectures were wrong. In (Appendix 8.5, Day 2, Less 3, and Par 337 345) the teacher used stones to prove how 15 stones shared by 5 groups of 3 stones can be related to 3 stones in each of 5 groups to build 15. In (Appendix 8.5, Day 4, Less 7, Par 61 – 65; Day 5, Less 10, Par 65 – 68; Day 7, Less 13, Par 46) when learners divided wrongly, that was when they were told to measure out 6 dots from 30 dots and made groups from them. Instead a learner made groups without knowing how many he supposed to make, or when the learner just simply got a wrong answer through the process of division, or when a learner in the processes of a repeated subtraction of 4 repeatedly subtracted from 24, she ended up having a step of (12 – 4 = 7). In all the wrong scenarios given above, what happened in this study was learners were referred back to the division actions to check if their works are in order. When the teacher made learners repeat the division process, doing the division with the question in mind of whether their division was correct or not or where they had gone wrong, helped learners to do this consciously. In the end through the process they discovered that they were wrong, where they had gone wrong and then rectified their mistake. This way, manipulatives and visualization provided learners with evidence and they learned with conviction. These conviction leads to a deeper understanding because learners were given opportunities to explore, analyze and test their conjectures.

Additionally, manipulatives and visualization was used to **visualize the patterns or division structures for construction of knowledge relationship:** When the teacher wanted to visualize the patterns of division in a specific division such as how many fours were in twelve

(12 ÷ 4 =) in (Appendix 8.5, Day 1, Less 1, Par 48 -54), the teacher drew the four dots in circles, which made up three circles each with four dots. The vision of four dots in each three circles portrayed the division sub structures that revealed that there were three fours or four threes in twelve. The display of these patterns in this study, made apparent the patterns or common features that were used to construct relationships either in a primary level (within the division domain between sharing and quotitive division) or reflective level which was the construction of relationship across other domains such as addition, subtraction and multiplication. The division sub -structures shown visually in this study as that of day 1 lesson 1; day 3 less 5 and day 4 less 7 , helped learners to construct relationships this way, once a learner was able to see how many a number of a divisor was in the dividend the pattern could be easily transferred to other domains. For the patterns of four in twelve, learners after they found out that there were three fours in twelve and four threes in twelve (12 ÷ 4 = 3). In the primary level the common feature of 4 in the number sentence referred to either the number of groups/ sets, which is sharing (partitive) division or elements per groups or sets, which is quotitive (measurement) division. This was indicated in the post-test (Appendix 8.5, Day 7, Less 14; Par 214 -220 & Less 15, Par 33-34, 44-45, 64, 77-78, 113-120, 128, 143-146, 192-193, 231-234 & 283-284) but in this lesson learners used the patterns of (35 ÷ 7) and that of (24 ÷ 3).

In the reflective level the division patterns or division sub–structure shown in the three circles each with four dots was related as follows, there were only three fours or four threes in twelve, which was visible in the drawing, thus in repeated addition only three fours or four threes need to be repeatedly added to build twelve. In subtraction only three fours or four threes need to be repeatedly subtracted from twelve to reach zero and in multiplication one need to multiply three times four or four times three to get twelve. For all the domains the conviction was because there were only four threes or three fours in twelve. This was also indicated in the post test (Appendix 8.5, Day 7, Less 14; Less 15).

4.9 The use of questions and the roles they had played in the learning of division.

The research also indicated that, manipulatives (stones) and iconic visualization (diagrams and dots) alone would have not been enough to mediate learning in such a way that learning would be within the Zone of Proximal development (ZPD) of the learners. Thus, scaffolding was given in the form of questioning throughout the study (Appendix 8.5, Day 1, Less 1 to Day 7, Less 13) to help learners reach the learning goals. Below is one of the extracts out of many that show how the teacher used questioning in (Appendix 8.5, Day 3, Less 5, Par 36 - 49).

Nab: "I took twenty" Teacher: "which twenty?" Nab: "the twenty apples" Teacher: "so you have used the apples drawing to get answer, how did you used the drawing then?" Nab: "I took twenty and subtracted". Teacher: "subtracted, how? I want you to tell me all what you have done". Nab "I took four out". Teacher: "you took four out and do what to the remaining? Did you counted them or what? Nab: "yes". Teacher: "did you then keep subtracting four?" Nab: "yes, and then sixteen remained". Teacher: "when sixteen remained, what did you do? Nab: "I took four from sixteen and twelve remained".

So questioning was used to stimulate learners' thinking, in such a way learners were assisted to push themselves a step further to a new capacity of development. In the conversation above a teacher had nowhere told that learner what need to be in the description of his division process but rather used questions to seek clarity, for confirmation, and for detail, to make a learners give missing facts or explain their actions and reasoning, and for regulation, to help that learner move to the next step. This extract indicates how questioning provided direction, when a learner gave an incomplete description or a justification which was not clear or an impractical example. Without this direction, learners would have found it more difficult to work successfully on the example. Questioning was opted for because learners need to construct their own knowledge and the teacher only needed to create an environment that would make it easy for them to do so. The teacher withdrew her authority of passing judgments to a learner of saying you are wrong or right and she nowhere told the learner that they were wrong or right. Instead she posed questions to a learner, questions that helped a learner to think further and see through division patterns to rectify themselves, or questions that made learners explain their thinking, which helped the learner to identify misconceptions.

It was through questioning, the roots to the challenges in this study were discovered and through the same approach, the misunderstanding or errors were rectified.

The problem experienced with questioning approach (as discussed in the limitations of the methodology: chapter 3, section 3.9) was that this approach makes lessons take too much time and only allows a few learners to participate with the teacher. A normal lesson takes only 40 minutes but lessons of this study took about 2 hours, there were two lessons in each day and that took about 4 hours in a day. This is because only simple and specific questions, questions that drive an individual to be focused to what need to be thought were used. Even though learners learned as a group, individual learners learned at different pace. Thus questioning each individual with simple, specific and focused questions, resulted in an extended conversation, because for an individual learner to reach a point where the questioning began to open up the learning required a conversation involving 3 to 5 questions. One of the longest conversations took place between the teacher and Hanna in (Appendix 8.5, Day 1, Lesson 1, Par 54 – 115). This made the lesson take a long time with fewer learners than required participating.

4.10 Challenges experienced during the implementation of this intervention program.

There were four challenges experienced during the teaching and learning of this intervention program. These were: (1) difficulties to divide large numbers with manipulatives and visualization, (2) misconceptions, (3) misinterpretations, (4) wrong use or ineffective use of visualization.

4.10.1 Large numbers

Extracts: In (Appendix 8.5, Day 1, Less 2, Par 184 – 200) show evidence of how learners had difficulties in using manipulatives and visualization to work with large numbers. When learners were given 80 stones to divide into 4 groups and 45 stones into 5 groups, only a few learners were able to do the correct division with stones. When learners were doing these divisions they used a counting strategy and in most cases they counted in ones from one to the dividend number. To count stones in ones from one to eighty or to forty-five was a challenge to learners as the numbers were large.

4.10.2 Misconceptions

As indicated in (Appendix 8.5, Day 7, Less 13, Par 5 - 8), learners' knowledge of division came at a point of understanding quotitive and sharing division as the division of a dividend divided into groups of equal potions until all dividend stones or diagrams were equally divided. However, when they divided with stones and diagrams, they became used to allocating the dividend stones equally into those groups until '***all the dividend stones are completely finished'***, but that was only possible in the context of division of the whole numbers if the dividend was a multiple of the divisor (number of groups in sharing or number of elements per groups in measurement). This understanding was carried over as a misconception to numbers that were not multiple of a divisor, numbers that resulted in a remainder. When learners were asked to measure out groups of threes from twenty (20 ÷ 3), they measured out six groups of three dots and there was a remainder of two dots left, because they were not used to something being left after the division, some learners decided to make a group for the two dots remained which led to the wrong answer of 20 divided by 3 equal to 7 instead of 6. The fact that all dividends should be equally measured out until the entire dividend was finished blinded them to the extent that they bypassed the fact that groups must also share equal portions. This image of division with the ***dividend completely measured out*** by the groups becomes a distracting detail tied to the image in the context of division of whole numbers with remainders. The image of seeing the dividend completely shared into groups distracted them to think that a seventh group with two stones should be seen as the same as the other six groups. This nullified the effect of the activity, which was supposed to make them question the difference, which was the beginning of knowledge extension from the division of whole numbers to that of fractions.

4.10.3 Different interpretations of the same visualization

In (Appendix 8.5, Day 1, Less 1, Par 55 - 115) after the teacher had drawn the diagram portraying patterns of three circles with four dots in each circle showing how many 4s are in

12, she asked a learner to read the drawing and tell how many 4s are in twelve. The learner kept stating that there and four 4s. When the learner was asked to point at the drawing what she is referring to as 4 fours by counting and pointing at them, the learner came and counted the four dots in the circle. So to a teacher, the diagram of four dots in each of the three circles was interpreted as four dots representing number 4 and the three circles as representing the number of fours in twelve, which were threes. However, to this learner the four dots represent both number 4 and at the same time it also represents the number of fours in twelve, which according to her they were four. The teacher had the knowledge or understanding of sharing and quotitive division. So in sharing division, a teacher interpreted the drawing as follows, the number of the circles represent groups or a divisor, the number of dot inside a circle represent a quotient and the total sum of the dots in all the circle represent the number of the dividend. However to learners it was a different interpretation which was vice versa to that of a teacher. In this case the teacher's knowledge of what sharing division is had directed the reading of the diagram. To a learner who had little knowledge of division, this may not be enough to help him/her read the drawing effectively. The same scenario also occurred in (Appendix 8.5, Day 2, Less 3, Par 65- 86) when a learner and a teacher interpreted the drawing portraying four circles with five dots in each circles (20 ÷ 4) differently, to a teacher the dots inside the circles and the number of the circles with their sum of all the dots in all circles represent a quotient, divisor and dividend. To a learner the five dots inside the circle represent both the number five in twenty and at the same time the number of how many times the fives are in twenty, so according to him the fives are there five times. The circles were not seen as a relevant point of focus to him.

4.10.4 Wrong use of manipulatives and visualization

The other challenge emerged from this study, was that some learners used manipulatives and drawing ineffectively. These learners instead of dividing stones in the number of the dividend into a number of groups (divisor), these learners either took a lesser number for the dividend or divided into a different number of groups. Some examples are: (1) In (Appendix 8.5, Day 1, Less 2, Par 1 & 83 - 92;Day 2, Less 3, Par 391 – 395; Day 4, Less 7, Par 10 -16; Day 5, Less 10, Par 286) when a learner was instructed to share 50 stones into 5 groups she shared them into 4 groups instead. (2) In (Appendix 8.5, Day 2, Less 3, Par 302; Day 4, Less 7, Par 113 - 116) when a learner was also instructed to build 15 with three diagrams (by repeatedly drawing three diagrams until fifteen is reached)instead he built by repeatedly drawing in five diagrams. (3) In (Appendix 8.5, Day 1, Less 2, Par 99 – 109;Day 2, Less 3, Par 91 -97; Day 4, Less 7, Par 125 & 172- 185) When a learner who was supposed to divide 24 stones into 4 groups instead he took 20 stones. All the above manners of ineffectiveness led to wrong answers.

4.11 Conclusions.

In this chapter, the analysis of the findings was presented. This analysis began with the description of the lessons. The analysis was then extended to the summary of the conceptual learning that emerged from the study program. The roles of manipulatives, visualization and questioning in that learning and the challenges experienced in the program were important issues in the analysis.

CHAPTER FIVE

DISCUSSION OF THE FINDINGS

5.1 Introduction.

In this chapter, the findings in chapter 4 are discussed in relation to the literature review presented in chapter 2 of this book. The discussion pays attention to the learning in the program as well as how the use of manipulatives, questions and visualizations influenced this learning. Challenges experienced in the teaching and learning program are also discussed. The discussion is presented under themes related to the research questions of this study.

5.2 The meaningful understanding of division that learners developed from the intervention program.

As it was reviewed already in the literature chapter (chapter 2, section 2.2.1.2) the two kinds of division are mathematically represented the same way i.e. (24 ÷ 3). Their differences are only carried in the meaning of the signs as related to their processes of division (Kilpatrick, et al., 2001; Simon, 1993). In the study we were able to tell that learners do understand that even though the symbolic representations of division look the same, they carry two different meaning of two different kinds of division.

After learners had passed through the process of learning in this program, experimenting with the use of stones, diagrams and dots (see chapter 4, section 4.4), learners were able to develop this understanding of division. Learners showed an understanding of the meaning carried by the sign in the division symbolic representation (see chapter 4, section 4.5). In section 4.5 of chapter 4 (the post test) it was evident that learners used the symbolic representation of (24 ÷ 3) and that of (35 ÷ 7) in two different forms of divisions. First, learners demonstrated that understating through the division action using stones and the drawing of dots. Given the symbolic representations above, learners used 3 and 7 on those number sentences as either number of groups or elements per sets and did the division in two different forms. Dividing (24 ÷ 3), (1) learners made 3 groups and shared 24 stones /dots equally into those 3 groups.

(2) They used 3 as the element per set and divided 24 dots /stones by measuring out 3 dots /stones out until 24 stone /dots were finished. The division of (35 ÷ 7) was also done in the same way. Secondly, learners were also able to show that understanding when they described the two forms of division orally. Thirdly, during the post test, (chapter 4, section 4.5) when learners provided their everyday examples they also went to an extent of explaining how their division examples can be divided in either as a sharing or as a quotitive division.

5.3 How the subject knowledge learned from the intervention program related to procedural and conceptual knowledge.

The sections below (5.3.1 & 5.3.2) discuss how the procedural knowledge of division in this program was conceptually learned. This discussion focuses on two aspects of division knowledge: (1) the formal language or symbolic mathematical representation and (2) the procedures used for division. Literature in (chapter 2, section 2.2.2.2) points out that if these two are taught in isolation then the knowledge would be procedural but if they are taught meaningfully then the knowledge could be classified as conceptual knowledge.

5.3.1 The formal language or symbolic mathematic representation.

The descriptions of lessons in chapter 4, section 4.4, sub-section 4.4.1 to 4.4.3 describes how the division process had passed through the stages of either from stones to symbolic representation or from diagram to symbolic representation. After every division action done with stones or diagrams/ dots, learners were asked to translate the division action into the symbolic mathematical representation of a division number sentence. Based on the procedural knowledge definition provided by (Hiebert & LeFevre, 1986; Long, 2005; Simon, 1993) in chapter 2, section 2.2.2.2; formal language or symbolic mathematical representation can be a procedural knowledge at some point. When the symbolic mathematic representation is learned procedurally, its knowledge only extends to the awareness of surface features and familiarity of syntactic rules of how these symbols are to be written or be used to represent the mathematical procedures (Hiebert & LeFevre, 1986; Long, 2005; Simon, 1993).

In this program, learners had learned the mathematical symbolic representation conceptually. In this case, learners learned the symbolic mathematic representation beyond procedural knowledge because apart from representing mathematical ideas carried in the division actions with stones, diagrams and dots to a correct symbolic number sentence (correct in a sense that it obey the syntactic rules of symbolic formal language of mathematics), learners also understood the meaning that the signs in symbolic representation (numbers in the number sentence) carry. In chapter 4, section 4.5 learners had not only represented their division action into the mathematical symbolic representation of the division number sentence but also had shown an understanding of the meanings carried by the signs in those number sentences. Learners had shown what such numbers sentences (35 ÷ 7 = 5) means in relation to the dividend, divisor and quotient. Apart from the meaning of dividend, divisor and quotient, these signs on the number sentence such as the 7 on the number sentence of (35 ÷ 7 = 5) could also imply the number of groups or people sharing in the division (if the division is a sharing) or the elements per set (if the division is quotitive)

By the end of the program (see chapter 4, section 4.5); learners had shown in three different ways that they had developed the understanding of these meanings carried by signs on the number sentence, (1) through demonstration, learners were given the two different symbolic representation of division number sentence the (24 ÷ 3) and the (35 ÷ 7); which they divided into two different kinds of division using drawing of diagrams and dots or stones. Learners used 3 and 7 in those number sentence and used them as either as number of groups or as number of elements per sets. (2) Learners' understanding was also shown when learners were made to describe the two ways one can do such division. (3) When learners were given a chance to provide real life example learners went to an extent of explaining how their division example can be done as either sharing or quotitive. So one can see that when learners translated their division actions into symbolic representation of division; it was not just mere rote learning or a use of these signs in isolation to their underlying concepts; learners did understand the meaning these signs carried. This is what literature says (chapter 2, section 2.2.2.2), when the knowledge of mathematical symbolic representations is used with the understanding of the meaning carried by the signs, the knowledge becomes conceptual knowledge (Hiebert & LeFevre, 1986; Long, 2005; Simon, 1993).

5.3.2 The procedures (skills) or actions used to solve division problems.

Division procedures/ skills used to solve the sharing (partitive) and quotitive (measuring out) division could also be classified as procedural knowledge. It is a procedural knowledge if executed in the predetermined linear sequence and learned in isolation to their underlying concepts (Kilpatrick et al, 2001; Hiebert & LeFevre, 1986; Rittle – Johnson & Alibali, 1999; Ritlle – Johnson & Star, 2007; Rittle – Johnson et al, 2001) (see chapter 2, section 2.2.2.2).

In this study, learners did not only follow the instructions of step by steps procedures with manipulatives and iconic visualization to do division and get the answer. The visualization and manipulatives embedded within the division procedures helped to visualize and embody the patterns or sub-structures of the number values in the number sentences. The patterns or sub-structures of division refer to the patterns embedded in the dividend number values i.e. the divisor and quotient pattern (we saw how learners were shown how many 4 are in 12 by drawing four dots inside each of the three circles). These division patterns from the division action form the basis for learners' justification explanations and their reasoning and this also had shown that they had either an implicit or explicit understanding of the concepts attached to their division actions or procedures (see chapter 4, section 4.7.2; the use of manipulatives and visualization to visualize patterns or sub-structures of number values). This understanding was one of the principles that made knowledge learned in this study conceptual knowledge (Rittle – Johnson, et al., 2001; Rittle- Johnson & Alibali, 1999).

5.3.3 Other principles of conceptual knowledge apparent in the study.

First, knowledge learned in this study was ***transferrable***. Learners transferred the sharing and quotitive ideas from one form to another (knowledge was learned flexibly). That is, the same division knowledge was embedded within activities of different forms such as: physical activities (stones), diagrams /dots as well as the symbolic form (see chapter 4, section 4.4, sub-section 4.4.1 to 4.4.3). This type of transferability agrees with the intellectual stages theorized by Bruner: the enactive, iconic and symbolic (McInerney & McInerney, 2006) (chapter 2, section, 2.3.1). The division ideas of partitive and quotitive division in this learning program were learned with stones (enactive) in every first two lessons, and was then learned again with diagrams and dots in the next four lessons and in all six lessons the same ideas of either sharing or quotitive division were translatable between stone, diagrams or dots and symbolic mathematical representation (chapter 4, section, 4.4). Based on the definition given by different authors, when learning is conceptual, it is transferrable and flexible, that means the idea learned can be generalized across different forms or domains (Hiebert & LeFevre, 1986; Rittle – Johnson & Star, 2007; Rittle – Johnson, et al., 2001; Kilpatrick, et al., 2001) (chapter 2, section, 2.2.2.2). Learners were also able to transfer the same sharing and quotitive idea from the classroom division form to everyday real-life examples, which they did by providing the everyday examples (chapter 4, section, 4.5).

Knowledge acquired by the learners in this study was ***rich in relationship***, which is the other back bone principle of conceptual knowledge (Hiebert & LeFevre, 1986; Simon, 1993) (chapter 2, section 2.2.2.2). When knowledge is rich in relationship there is a linkage of it to other domains.

In (chapter 4, sections 4.4, sub-sections 4.4.1 to 4.4.3), internal relations of division knowledge were learned (that is between sub domains the sharing and quotitive division) and external relations were also learned (across other domains such as subtraction, addition and multiplication). Relationships of knowledge are constructed by means of identifying the common features which are then related within the domain or across other domains. The common feature are like a common denominator e.g. the patterns of the (dividend, divisor and quotient in the number sentences).When the relation is constructed within a domain the relationship is at a primary level and when the relation is constructed across other domains, it is at reflective level (Hiebert & LeFevre, 1986) (chapter 2, section, 2.2.2.3).

In this program (chapter 4, section 4.7.2) common features (division patterns or sub-structures) were made visible with the help of stones and drawings. These are the patterns of dividend, divisor and quotient in the number sentences. These patterns or sub-structures made sense to the learners after they were made apparent, together with the meaning carried by the signs in representing number sentences. Learners then used these division sub-structures to construct relationships at primary and reflective levels (see how learners related the dividend, divisor and quotient in Chapter 4, section 4.5, & 4.6, sub-section, 4.6.2).

At primary level: The relationships of those patterns were constructed within the division domain between partitive and quotitive division. In the division domain, learners used the pattern of dividend, divisor and quotient and related it to sharing and quotitive division. Patterns were related like this: if for example the number sentence is ($60 \div 15 = 4$), then learner would use 15 in the number sentence differently for the two different kinds of division. Number 15 on the number sentence above could imply groups' number in the division (sharing) while in the quotitive division it would imply the number of elements per groups. Learners first understood the meaning carried by a divisors and quotient which is interchangeable between an intensive quantity (number of elements per set) and the number of groups or sets. These common features were then related interchangeably between partitive and quotitive divisions. That relationship construction explained above is what Hiebert & LeFevre (1986) called a primary level relationship (chapter 2, section, 2.2.2.3). Hiebert & LeFevre (1986) and Long (2005) explained that when common features are related within a domain (within a division domain itself) then it is a primary level relationship.

In reflective level: as analyzed in chapter (chapter 4, section, 4.5) the same common features (dividend, divisor and quotient) were also related to different domains of addition, subtraction and multiplication. After learners understood how many times the divisor can be taken out from a dividend (the quotient number) then they related it to addition, subtraction and multiplication this way, when $35 \div 7$ is equal to 5, then it means 7 can be repeatedly added 5 times to get 35 and 5 can be repeatedly added 7 times to get 35. In the same manner, the common feature is also carried across to subtraction. In $35 \div 7 = 5$, means 7 can be repeatedly subtracted 5 times from 35 to get finished or 5 can be repeatedly be subtracted 7 times from 35 to get finished. In the same manner, the common feature was also transferred to multiplication as an inverse of the division, whereby $35 \div 7 = 5$ was reversed as $7 \times 5 = 35$. The discussion above of how common features of division was related to other domains agree with that explained by Hiebert & LeFevre (1986) and Long (2005) in (chapter 2, section, 2.2.2.3).

5.4 The use of manipulatives and visualization and their roles in the learning of sharing and quotitive division.

In this section the discussion focused on the two ways in which manipulatives and visualization was used to support learning and the roles these played in that learning.

5.4.1 The use of manipulatives and visualization in learning support.

According to Vygotsky, (chapter 2, section 2.3, sub-section, 2.3.2.2) learning is mediated (supported) by the use of tools or social factors which are sometimes called cultural artefacts. These tools can be either physical or psychological (Hall, 2007; Glassman, 2001; McInerney & McInerney, 2006, Tudge &Winterhoff, 1993).

In (chapter 4, section 4.4, sub-section, 4.4.1 to 4.4.3 & section, 4.7, sub-section, 4.7.1), evidence had shown how manipulatives and iconic visualization were used as tools to mediate learning of sharing and quotitive division. The manipulatives and visualization were used as a ***procedure for division and for teaching instructions***. All division problems whether for sharing or quotitive division were solved through in action by use of stones, diagrams or dots, making the manipulatives and iconic visualization the tools for division procedures. This fact agreed with two points about tools for learning that different authors (in chapter 2, section 2.3.2, sub-section, 2.3.2.2) stated when they said (1) a tool for learning can be physical or psychological. (See chapter 4, section 4.4, subsections, and 4.4.1 to 4.4.3, which analyzed stones, diagrams and dots being used as physical tools to mediate division learning.) (2) Still in (chapter 2, section, 2.3.2, sub-section, 2.3.2.2) it was stated that these tools play a double role in learning: first they provide means within which to act upon the world. So learners do something with the tool that extends the learner's capacity in a particular way.When learners were using stones, diagrams and dots to do division as analyzed in (chapter 4, section 4.4, sub-section, 4.4.1 to 4.4.3), it was done in such a way that learners' division capacity is extended in the focused sub-domains of only sharing and quotitive division. Second tools act as scaffolds that facilitate extension of knowledge into related areas (Hall, 2007; Glassman, 2001; McInerny & McInerney, 2006; Tudge & Winterhoff, 1993). Even in the analyses, (chapter 4, section 4.5 and 4.6, sub-section 4.6.2, see also section, 4.7, sub-section, 4.7.2), evidence point to how drawings and stones were able to facilitate the extension of division knowledge to other related domains such as addition, subtraction and multiplication

As a procedure for division: In (chapter 4, section 4.4, sub-section 4.4.1 to 4.4.3) of the analysis, it was shown that stones, diagrams or dots were used as tools that provided means of doing division through action with real-world objects. Learners were doing division with stones, diagrams and dots to extend their division learning capacity conceptually. This agreed with what literature (chapter 2, section 2.3, sub-section 2.3.2.2) explains about tools for learning (Hall, 2007; Glassman, 2001; McInerny & McInerney, 2006; Tudge & Winterhoff, 1993). The agreement of the theory comes in because stones, diagrams and dots were used for division as physical tools used as division procedures to extend learners' capacity of division learning in sharing and quotitive division.

For teaching instructions: Hall (2007) state that learning is mediated by the teacher or more competent and experienced peer through modeling, use of tools and scaffolding learners' practice. In chapter 4 (section4.7, sub-section 4.7.1), it was shown how learning was mediated by the teacher through the use and demonstration of stones, diagrams and dots. In this section, the teacher (an experienced adult) used stones, diagrams and dots for teaching and learning instructions as well as to demonstrate how these tools can be used to do division of sharing and measuring out.

First, the teacher incorporated the stones and drawing of diagrams or dots in teaching instruction several times to demonstrate their use as a procedure for division. In partitive division, the teacher showed how one indicates the groups' positions on the table by drawing circles, rectangles or in any form. Learners were then shown how to share stones, diagrams or dots equally one by one. Learners were also shown how they would use the stones, diagrams or dots to do the measurement division.

Second, the teacher also used the drawing of dots to show the patterns of number in the division. To visualize these patterns, the teacher drew the number of times the divisor is in the dividend as it was analyzed in chapter 4 (section 4.7, sub-section 4.7.1), the teacher took a dividend such as 12 and draw circles in a number equal to a group number and in each circles draw dots equal to the elements per set/ groups. This demonstration was done in collaboration or interactions with learners in the process of demonstration. The demonstration of how division should be done with stones or in drawing as well as the collaboration between the teacher and the learners through learning are the principles that agree with mediating learning as reviewed in (chapter 2, section 2.3.2, sub-section 2.3.2.2)

Third, in (chapter 4, section 4.7, sub-section 4.7.1), it was also shown how the teacher used stones, diagrams and dots to demonstrate how they can be relate to repeated addition, subtraction or multiplication.

The three points rose above show how the teacher mediated (supported) learners to ensure they developed the new capacity needed for learning division, this learning, the theory of Vygotsky refers to as learning within their ZPD. The ZPD in this study refers to a gap between learners' actual level of development of sharing and quotitive division that learners portray in lesson one of the diagnostic test (chapter 4, section 4.3) and the potential level of development which in this case was determined by the learning goals (see the learning goals embedded in the intervention program as appendix 8.2): what the learners were expected to achieve in this learning. This outcome was shown by the findings in the post test (chapter 4, section 4.5). This agrees with what different authors defined as ZPD theorized by Vygotsky (chapter 2, section 2.3.2, sub-section 2.3.2.1 to 2.3.2.2) (Berk & Winsler, 1995; Winsler, 2003; Jacobs, 2001; Not, 1992; McInerney & McInerney, 2006; Tudge & Winterhoff, 1993; Verenikia, 2008; Moore, 2000). By using stones, diagrams and dots to support learning (chapter 4, section 4.7, sub-section 4.7.1), the teacher was establishing the stepping stone to the level of difficulties within the learners' ZPD; this mediation was done through a conversation between teacher and learners that was established through the questioning approach and this is learning through collaboration or interactions between teacher and learners, as Vygotsky theorized (Winsler, 2003; Moore, 2000; Verenikia, 2008; Tudge & Winterhoff, 1993; Not, 1992; Jacobs, 2001; Berk & Winsler, 1995; McInerney & McInerney, 2006; Glassman, 2001; Kozulin, et al., 2003; Hall, 2007) – see chapter 2, section 2.3.2, sub-section 2.3.2.1 to 2.3.2.2.

5.4.2 The roles that manipulatives (stones) and visualization (drawing of diagrams and dots) play in the learning partitive and quotitive division.

Manipulatives and iconic visualization played many important roles in the learning of sharing (partitive) and quotitive (measurement) division. These roles include: (1) rectifying or correcting learners' misconception, misunderstanding or their wrong answers; (2) depicting patterns of numbers in the division which led to construction of knowledge relationships, and (3) providing strong conviction, evidence and proofs in learning. These roles are discussed in detail below.

Findings in (chapter 4, section 4.7, sub-section 4.7.2) had shown how manipulatives and iconic visualization were used to help learners ***rectify or correct their misconception, misunderstanding or their wrong answers***. When learners misunderstood a point or gave an impractical example or a wrong answer a teacher used stones or drawing and the learner was asked to do the division with stones or drawings through the process of actions most learners came to realize that they are wrong and corrected their mistakes without the teacher telling them. This agreed with what Arcavi (2003) and Mudaly (2010) in (chapter 2, section 2.2.3, sub-section 2.2.3.4) had stated when said visualization helps learners to discover their own misconception or errors and rectify them themselves.

Findings in chapter 4 (section 4.7, sub-section 4.7.2) had shown how manipulatives and visualization had first provided ***strong conviction, evidence and proofs*** in learning. Based on Hanna & Sidoli (2007) and Tall (1991) in (chapter 2, section 2.2.3, sub-section 2.2.3.4), visual representations can constitute proofs. This is because they are means for understanding. Visualizations enhance the ability of students to notice the details, to conjecture, to reflect on and interpret relationships and to offer tentative explanations (Hanna & Sidoli, 2007). Visualization allows the exploratory stage of mathematical thinking, which benefits the building up of an overall picture of relationships which lead to the solution and proofs (Tall, 1991). Learning in this study used manipulatives and iconic visualization as proofs in this ways, manipulatives and iconic visualization played a role in proofing to learners where they had gone wrong in the process of their learning. As presented in (chapter 4, section 4.7, sub-section 4.7.2), a learner who had done division with stones or diagrams wrongly or gave an impractical example need not hear it from the teacher that something was wrong in their process. For example, when a learner gave an example of thirteen sweets shared by three people (13 ÷ 3), or divided wrongly, then learners were referred back to the division actions with the question in mind of checking if their works were in order. It was when the teacher made learners repeat the division process, exploring or doing the division with the question in mind of whether their division works, or where they had gone wrong, that learners checked consciously and in the end through the process they discovered that they were wrong, and where they had gone wrong and then rectified their mistake. This way, learners were provided with evidence and learned with conviction. These conviction leads to a deeper understanding because it led learners to explore, analyze and test their conjectures, see (chapter 2, section 2.2.3, sub-section 2.2.3.4) (Mudaly, 2010; Arcavi, 2003).

The second role of **visualizing the patterns or division structures to construct knowledge relationship:** When the teacher wanted to visualize the patterns of division in a specific division such as how many fours are in twelve (12 ÷ 4 =) the teacher drew the four dots in circles, which made up three circles each with four dots. The vision of four dots in each of the three circles portrays the division sub-structures that reveal that there are three fours or four threes in twelve, see (chapter 4, section 4.7, sub-section 4.7.2). This agreed with what literature in (chapter 2, section 2.2.3, sub-section 2.2.3.4) said on the role of patterns, sub-structure and relationship construction. Pictures may be used to describe relationships (Kirrane, 1992). These patterns in this study were the patterns or common features that were used to construct relationships either in a primary level (within the division domain between sharing and quotitive division) or at a reflective level which was the construction of relationships across other domains such as addition, subtraction and multiplication (chapter 4, section 4.6, sub-section 4.6.2). The division sub-structures shown visually in this study as analyzed in chapter 4, helped learners to construct relationships this way, once a learner was able to see how many a number of a divisor is in the dividend the pattern was easily related to other domains. This knowledge relation, analyzed in (chapter 4, section 4.5 and section 4.6, sub-section 4.6.2), also agreed with what literature said in (chapter 2, section 2.2.3, sub-section 2.2.3.4) as it explains that sub-structures or patterns allow the individuals to free up more space for new imaging and for constructing relationships (Mudaly, 2010). Visualization display information enabling the individual to see the story or envisioning some-cause effect relationships (Arcavi, 2003). This is possible because visualization depicts clear sub-structures of a whole problem that leads to the identification or establishment of patterns (Arcavi, 2003). It is good at allowing pattern-matching approaches to problems solving (Kirrane, 1992). This visualization of sub-structures of the concept helps with the organization of a concept in a meaningful way that can be a tool for analytical development of a solution (Kirrane, 1992; Arcavi, 2003).

5.5 The use and roles of questioning approach in learning of sharing and quotitive division.

Findings also indicated that manipulatives (stones) and iconic visualization (diagrams and dots) alone would have not been enough to mediate learning in such a way that learners would reach their potential development in their ZPD. Thus in this learning program, see (chapter 4, section, 4.8) a lot of scaffolding was given in the form of questioning as reviewed in chapter 2 (section 2.3.2.3 & 2.3.2.4) (Winsler, 2003; Not, 1992; Verenikia, 2008; Murray, 2009). So questioning in this learning program (chapter 4, section 4.8) was used to stimulate learners' thinking, this way learners were assisted to push themselves a step further to a new capacity of development as reviewed in chapter 2 (section 2.3.2.4) (Elder & Paul, 1998). Questioning was chosen because learners need to construct their own knowledge and a teacher needs only to create that environment that would make it easy for them to do so (as theorized by Vygotsky, see chapter 2, section 2.3.2). The findings in chapter 4 indicated how the teacher responded when a learner gave an incomplete description or a justification which is not clear or an impractical example. The teacher withdrew her authority of passing judgments to a learner and saying you are wrong or right (Lampert, 1992), instead she posed questions to a learner, questions that: help learners to think further and see through division patterns to rectify errors themselves; questions that made learners explain their thinking, and which helped the teacher to find out the learner' misconceptions see the example of an extract given in (chapter 4, section 4.8), as discussed in the literature review (chapter 2, section 2.3.2,sub-section 2.3.2.4) (Naylor, et al., 2004; Petty, 1993; Manouchehri & Lapp, 2003; Sinclair, 2003). It was through questioning the roots to the challenges in learning were discovered and through the same approach, that the misunderstanding or errors were rectified.

The problem experienced with questioning approach was given in the limitations of the methodology chapter (chapter 3, section 3.9 & chapter 4, section 4.8). This approach makes lessons to take a long time and only allowed a few learners than needed, to participate in the questioning. This is because only simple and specific questions, question that drive an individual to focused to what need to be thought were used. Although learners learned as a group, individual learners learned at different pace. Thus asking each individual simple, specific and focused questions, took a longer conversation. Asking took a longer conversation; because for an individual learner to reach a point where the teacher was driving her/him, it required a conversation involving 3 to 5 questions (see one of the longest conversations between teacher and Hanna in day 1lesson 1, Transcripts, Appendix 8.3). This made the lesson take a long time with fewer learners than desired being involved as reviewed in (chapter 2, section 2.3.2, sub-section 2.3.2.4.2)(Petty, 1993; Naylor, et al., 2004).

5.6 Challenges experienced in the teaching and learning of this intervention program.

The section below discusses the challenges experienced during the teaching and learning of this intervention program. Challenges emerging from this teaching and learning program were: (1) difficulties in dividing large numbers with manipulatives and visualization; (2) misconceptions; (3) misinterpretations, and (4) wrong or ineffective use of visualization.

Large numbers: Findings in (chapter 4, section 4.9, sub-section 4.9.1) show how learners had difficulties in using manipulatives and visualization to divide large numbers. When learners were required to divide 80 stones into 4 groups and 45 stones into 5 groups only few learners were able to do the correct division. When learners were doing these divisions they used a counting strategy and in most cases they counted in ones from one to the dividend number. To count stones in ones from one to eighty or to forty-five was a challenge to learners as the numbers are large. In (chapter 2, section 2.2.1, sub-section 2.2.1.4) Anghileri & Beishuizen (1998) suggested that counting strategies of large numbers need to be made manageable and efficient. This can be made by chunking the large number (dividend) into chunks through repeated addition/ subtractions or by decomposing that number in any form. In this study, learners were not yet taught how to decompose or chunk numbers. So the challenge was left for further studies. Further studies need to respond to the gap between the stage of using manipulatives and iconic visualization and the next stage when visualization is of no longer of use. How can learners be helped to divide large numbers? Is it still by visualization or other strategies?

Misconceptions: as analyzed in (chapter 4, section 4.9, sub-section 4.9.2), learners' knowledge of division came at a point of understanding quotitive and sharing division as the division of a dividend into groups of equal potions. However, when they divided with stones and diagrams, they used to allocate the dividend stones equally into those groups until '***all the dividend stones are completely finished'***, but that is only possible in the context of whole numbers if the dividend is a multiple of the divisor (number of groups in sharing or number of elements per groups in measurement). This understanding was carried over as a misconception to numbers that are not multiple of a divisor, numbers that resulted in the remainder. This happened when learners were given to measure out groups of threes from twenty. Learners measured out six groups of threes and there was a remainder of two left, because they are not used to something left after the division some learners decided to make a group for the two remainders which led to the wrong answer. Based on literature (chapter 2, section 2.2.3, sub-section 2.2.3.7), learners in this case could have standardized the division and created an image that the dividend must be completely shared after the division and bypassed the fact that groups must also share equal portions (Clement, 1982; Presmeg, 1986; Arcavi, 2003). This image, or vision of division of the ***dividend completely shared*** by the groups become an irrelevant detail tied to the image in the context of division of whole numbers. This is agreed to by Tall (1991) when he stated that pictures may suggest false theorems, which may be the distractions from the main elements in the original problem stimulus and this makes it difficult for a student to formulate necessary abstractions (Clements, 1982). In this study, the image of seeing the dividend completely shared into groups distracted the learners by encouraging them to think that a seventh group with two stones was equal to the other six groups, and so not identify the remainder. The remainder would enable an extension of knowledge from the division of whole numbers to that of fractions.

Different interpretations of the same visualization: findings in (chapter 4, section 4.9, sub-section 4.9.3) also showed how a drawing was interpreted in different ways by the teacher and by some learners. When the drawings of three circles each with four dots and the other of four circles each with five dots were drawn on the chalkboard; the number of circles in the drawing, their dots inside the circles and the sum of all the dots in all circles was supposed to be interpreted in relation to a quotient, divisor and dividend. So in sharing division, a teacher interpreted the drawing as follows, the number of the circles represent groups or a divisor, the number of dots inside a circle represent a quotient and the total sum of the dots in all the circles represent the number of the dividend. However some learners made a different interpretation to that of the teacher. Literature on visualization (chapter 2, section 2.2.3, sub-section 2.2.3.7) suggests that we see what we know. So students may not necessarily see in a diagram what an expert such as a teacher, researcher or mathematicians sees (Arcavi, 2003) this is because expert knows what to look for in the particular figure and so is able to employ a particular case or images to stand for more general observations (Noss, et al., 1997). In the case of this learning program a teacher know what sharing division is and the knowledge had directed in reading of the diagram. To a learner s/he may have a little knowledge of what division is but not enough knowledge to help him/her read the drawing in relation to that knowledge. To these learners the diagram might have tied their thought to irrelevant details (Arcavi, 2003; Clements, 1982; Presmeg, 1986; Tall, 1991). The false image in the learners mind may persist, thereby preventing the opening up of the thought to the original patterns (Presmeg, 1986).

Wrong use of manipulatives and visualization: the other challenge emerged from the analysis in (chapter 4, section 4.9, sub-section 4.9.4), was; there were some learners who showed the pattern of using manipulatives and drawing ineffectively. Based on the findings, these learners instead of dividing stones in the number of the dividend into a number of groups (divisor), these learners either took a less number of the dividend or divided in less or more number of groups. Some of the examples are such as: (1) when a learner was instructed to divide 50 stones into 5 groups, but rather she divided them into 4 groups instead; (2) When a learner supposed to build 15 with three diagrams, but instead he built with fives, and (3) When a learner who supposed to divide 24 stones into 4 groups, but instead he took 20 stones. All the above sorts of ineffectiveness led to wrong answers. This challenge shows some linkage to literature (chapter 2, section 2, 2, 3) on visualization on three different angles such as: (1) it could be caused by the ability to form and use manipulatives and images, as people differ in their ability (Kirrane, 1992) (chapter 2, section 2.2.3, sub-section 2.2.3.5). (2) The challenge also points to the social difficulties related to the use of manipulatives and visualization. This difficulty is caused by the learners' background of acquaintance and how they oriented to the use of manipulatives and visualization (Arcavi, 2003) (chapter 2, section 2.2.3, sub-section 2.2.3.7). (3) The challenge could also be led by the lack of enough training prior to the actual visual activities on how to use manipulatives and visualization, as proposed by (Mudaly, 2010; Stokes, 2002). In this case the lack of enough training through demonstrations is raised based on the individual capacity as learners learn at different pace (chapter 2, section 2.2.3, sub-section 2.2.3.7).

5.7 Conclusion.

In this chapter, the findings were discussed in line with literature. The discussion focused on the learning emerged from the program. It also discussed how this learning was conceptually learned. The roles manipulatives, visualization and questioning played in that learning and how it was played was also discussed. The discussion ended with the challenges experienced in the learning program.

CHAPTER 6

CONCLUSION

6.1 Introduction.

The purpose of this study was to investigate the influence of manipulatives and iconic visualization in the learning of integrated procedural and conceptual knowledge of sharing (partitive) and quotitive (measuring out) division in a grade 4 class. This learning with manipulatives and iconic visualization was scaffolded using a questioning approach.

6.2 Summary of the findings.

This study led to the following five conclusions.

1. ***Grade 4 learners, when learning through the use of manipulatives and iconic visualizations developed a meaningful understanding of division and were able to differentiate sharing and measurement division.***

The mathematical operation of division corresponds to two different practical actions (Kilpatrick et. al., 2001; see also chapter 2, section 2.2, sub-section 2.2.1.2). Often in a grade 4 class, a symbolic representation such as ($24 \div 6 = 4$) would be given and learners would perform the calculation procedurally without paying attention to the fact that the representation could mean two different things. After passing through this learning program incorporating manipulatives and iconic visualization (chapter 4, section 4.5), learners were able to take any division represented symbolically and demonstrate with stones, diagrams or dots, the two different kinds of division. They were also able to describe these different kinds of division processes orally (chapter 5, section 5.2).

In this study (as discussed in chapter 5, section 5.4, sub-section 5.4.2), the use of manipulatives and iconic visualization made the two kinds of division apparent, helping learners to see the difference between the two kinds of division. Also, the practical actions in each process of division provided an underlying meaning for each value and the operations in that number sentence.

2. ***The procedural knowledge of sharing and quotitive division was learned conceptually.***

In this study, it was evident in (chapter 4, section 4.6, sub-section 4.6.1) that learners learned the procedural knowledge conceptually. Procedural knowledge was learned in two forms: (1) the formal language or symbolic mathematical representation and (2) the division procedures or actions used to solve division problems.

In (chapter 2, section 2.2, sub-section 2.2.2.2) it was reviewed that when the symbolic mathematic representation is learned procedurally, this knowledge only goes to the extent of awareness of surface features and familiarity with syntactic rules of how these symbols are to be written or be used to represent the mathematical ideas. In this learning program, learners learned conceptual understanding beyond procedural knowledge because apart from able to represent the mathematical ideas carried in the division actions with stones, diagrams and dots to a correct symbolic number sentence (correct in the sense that it obeys the syntactic rules of symbolic formal language of mathematics), it was shown in (chapter 4, section 4.6, sub-section 4.6.1 and chapter 5, section 5.3, sub-section 5.3.1 to 5.3.2) learners were also able to understand the meaning that the signs in the symbolic representation as numbers in a number sentence (Hiebert & LeFevre, 1986; Long, 2005; Simon, 1993). That is, these procedures for division were learned attached to their underlying concepts. Also, it was evident (see chapter 4, section 4.6, sub-section 4.6.2 & chapter 5, section 5.3, sub-section 5.3.3) that this knowledge was transferable and it was rich in relationships, agreeing with Kilpatrick, et al. (2001), Rittle–Johnson & Alibali (1999), Rittle–Johnson & Star (2007) and Rittle–Johnson, et al. (2001) as reviewed in (chapter 2, section 2.2, sub-section 2.2.2.2).

3. ***Third, manipulatives and iconic visualization played a number of useful roles in the procedural and conceptual learning of partitive (sharing) and quotitive (measuring out) division.***

The use of manipulatives and iconic visualization played roles in the following:

(a) Manipulatives and iconic visualization helped learners ***rectify or correct their misconceptions, misunderstandings or their wrong answers***. As analyzed and discussed in (chapter 4, section 4.7, sub-section 4.7.2 & chapter 5, section 5.4, sub-section 5.4.2), when learners misunderstood a point or gave an impractical example or a wrong answer, the teacher referred learners to do the division with stones or drawings. Through the process of these actions most learners came to realize that they were wrong and corrected their mistakes without the teacher telling them. This agreed with what Arcavi (2003) and Mudaly (2010) had stated (see chapter 2, section 2.2, sub-section 2.2.3.4).

(b) They provide ***strong conviction, evidence and proofs*** in learning. Manipulatives and iconic visualization provided a proof in two ways. (1) the drawings were used several times by the teacher to visualize division patterns or sub-structures of number values in the number sentences i.e. the drawing of 4 dots in each of the 3 circles to show how many times number is in 12 (chapter 4, section 4.7, sub-section 4.7.2 & chapter 5, section 5.4, sub-section 5.4.2). (2) Using manipulatives and visualization convinced learners of their learning, because learners were referred to them for correction of misconceptions, misunderstanding and rectifying their wrong answers (chapter 4, section 4.7, sub-section 4.7.2 & chapter 5, section 5.4, sub-section 5.4.2). The use of manipulatives and iconic visualization as proofs was also supported by Hanna & Sidoli (2007); Tall (1991); and as for evidence and conviction, by Mudaly (2010); Arcavi (2003) in (chapter 2, section 2.2, sub-section 2.2.3.4).

(c) They provided ***transparency or feedback*** about learning to the teacher. Learners' understanding of division was made visible through their work done with stones or in drawings. In this way portraying their representations. This provision of learners' external representations enabled the teacher to see where the learner had gone wrong and where they were correct, and also enabled the learner to easily review their externally represented work (as analyzed in chapter 4, section 4.7, sub-section 4.7.2).

(d) They provided an ***active physical procedure for division and for teaching instructions***. Manipulatives and iconic visualization were used as tools to support learning and teaching instructions. (1) These tools were both physical and psychological (chapter 4, section 4.4, sub-section 4.4.1 to 4.4.3; 4, section 4.7, sub-section 4.7.1 & chapter 5, section 5.4, sub-section 5.4.1). (2) These tools played a double role in learning. First they provided means to extend the learner's capacity (chapter 4, section 4.7, sub-section 4.7.1 & chapter 5, section 5.4, sub-section 5.4.1) to act upon real objects. Second they acted as a scaffold that facilitates extension of knowledge into related areas (chapter 4, section 4.7, sub-section 4.7.1 & chapter 5, section 5.4, sub-section 5.4.1) see also the review on literature in (chapter 2, section 2.3, sub-section 2.3.2.2) (Hall, 2007; Glassman, 2001; McInerny & McInerney, 2006; Tudge & Winterhoff, 1993).

4. ***Learning of division with the use of manipulatives and iconic visualization was enhanced when questioning was incorporated within teaching and learning.***

A questioning approach was used to push learners through to reach the learning goals for these lessons. As analyzed and discussed in (chapter 4, section 4.8 & chapter 5, section 5.5), questions helped learners to think more deeply than what they already know, by making them justify or explain their sharing or quotitive reasoning; describe the division process; and to complete their thought. These responses also provided feedback on learning to the teacher (Elder & Paul, 1998; Lampert, 1992; Naylor, et al., 2004; Petty, 1993; Manouchehri & Lapp, 2003; Sinclair, 20003).

5. ***Fifth, learning with manipulatives and iconic visualization may also lead to some learning difficulties that need to be anticipated beforehand.***

The following difficulties were identified during the learning and teaching of this study.

(a) Some learners wrongly used the manipulatives and iconic visualizations and as a result they got wrong answers. When the structure was not clear to learners, they did not use these tools effectively. Example are when learners were told to divide 18 stones into 6 groups but rather divided into 3 groups as analyzed and discussed in (chapter 4, section 4.9, sub-section 4.9.4 & chapter 5, section 5.6, sub-section 5.6.2).

(b) Visualization and manipulatives failed to help learners when it comes to the division of large whole numbers as analyzed and discussed in (chapter 4, section 4.9, sub-section 4.9.1 & chapter 5, section 5.6, sub-section 5.6.2).

(c) There was a problem of different interpretations by the teacher and the learner, of the same visual arrangement. The teacher interpreted the patterns in the drawing in a different or even opposite manner to how the learner interpreted the same drawing. This was analyzed and discussed in (chapter 4, section 4.9, sub-section 4.9.3 & chapter 5, section 5.6, sub-section 5.6.2).

(d) The understanding of manipulatives and iconic visualization in relation to division created a misconception, an over generalization from all groups sharing equal portions, to the division of whole numbers with a remainder as analyzed and discussed in (chapter 4, section 4.9, sub-section 4.9.2 & chapter 5, section 5.6, sub-section 5.6.2).

6.3 Limitations.

The study has four limitations, two of which affected the collection of data and the other two had a negative impact on quality learning (see chapter 3, section 3.9).

First, as discussed in chapter 3, the lessons were too long – they took about 4 hours a day and did not allow all learners to be fully involved. This was caused by the integration of tools for data collection in a lesson. All three, the activities that contributed to document analysis, oral questions (interview) and the observation were incorporated within a lesson. An additional method that contributed to the time length was questioning (Petty, 1993; Naylor, et al., 2004) (see chapter 2, section 2.3, sub-section 2.3.2.4.2; chapter 3, section 3.9; chapter 4, section 4.8 & chapter 5, section 5.5). This limitation had a negative impact on the quality of learning as it effected the concentration of the learners.

The second limitation that had the negative impact on the learning of learners was the number of learners chosen in the sample. Because of the size of the sample, it was difficult for the teacher to follow the track of each individual's learning. The difficulties came in because not all individuals had the opportunity to respond to every question, as discussed in (chapter 3, section 3.5, sub-section 3.5.2).

The next two limitations had negative impact on the process of data collection. The third limitation was the lack of resources at the study site. There is no electricity at the study site and the video recorder used to record the learning and teaching could only work for 4 hours when it is on power. Due to this shortcoming the video was only used to record some parts of the lessons see (chapter 3, section 3.9).

The last limitation was that of being a participant observer. Working as both the teacher and the observer collecting research data on the lesson; made observation difficult (chapter 3, section 3.6, sub-section 3.6.3 & section 3.9).

6.4 Areas for further research

Manipulatives and iconic visualization had played roles in the learning of sharing and quotitive division, however evidence from the findings shows that there were challenges, pointing to areas that need a further investigation to extend the field of visualization and conceptual knowledge. Two future potential developments were discovered. (1) Manipulatives and visualization when used in this study did not appear to enhance the division of large numbers (chapter 4, section 4.9, sub-section 4.9.1 & chapter 5, section 5.6, sub-section 5.6.2). To extend the theory further, studies are needed to explore how learning could be moved a step further from the division of small number using manipulatives and visualizations, to the division of large numbers.

(2) This study focused on the division of whole numbers, evidence in findings reveals that there was another challenge in the study that points to the need of another angle of further potential development. Learners over generalized the misconception that since in measurement division groups of equal portions had to be measured out of a dividend, learners who divided to obtain a whole number with a remainder, still made a group for that remainder (see chapter 4, section 4.9, sub-section 4.9.2 & chapter 5, section 5.6, sub-section 5.6.2). This misconception of division of whole number with a remainder pointed to the need for future potential developments, to explore the use of manipulatives and visualization in the division of fractions.

6.5 Impact of the study on my teaching experience

The process of this study, especially when I passed through the teaching and learning of the intervention program impacted on my teaching experience positively. My teaching experience was improved on the following different angles:

(a) the use of manipulatives and visualization can only enhance the conceptual learning of procedural knowledge if it is:

(1)Well planned and structured in relation and response to the learning goal and subject content to be learned. (2) Learners do understand how to decode and encode the messages on that manipulatives and visualization. (3) It is linked to other forms of representations such as language, symbols and questioning.

(b) Questioning can be a powerful tool for scaffolding learning. Through asking questions, learners are stimulated to think for themselves which is good training for critical thinking. This way a learner is given an opportunity to create his or her own knowledge as theorized by the constructivists such as Piaget, Bruner or Vygotsky. Questions, depending on their form, content or purpose can guide a learner to learn in a desired way.

(c) Conceptual learning requires a lot of patience from the teacher and it can be time consuming. A teacher has to follow the track of every individual learner's learning and sometimes help them individually even when the learning of the individual is taking place within a class group. Individual learners learn at a different pace of which some of the learner would require much time and a lot of scaffolding to reach their ZPD.

6.6 Recommendations.

This study was a case study and thus does not guarantee generalization; however the results of this study can be a huge contribution to a teaching practice aimed to reach conceptual knowledge of division of whole number through the use of manipulatives, questioning approach and visualizations. The recommendation is that teachers may use these results to improve their teaching practice. These results can also be useful for the extension of the theoretical concepts pertaining to the use of manipulative, visualization and questioning for the influence of conceptual learning of division.

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8. APPENDICES

8.1 Appendix one

LEARNERS' PERFORMANCE ON MATHEMATICS FOR THREE PREVIOUS GRADES 1, 2 AND 3

The three tables below; shows the learners (samples)' performance in mathematics over the previous three grades, one to three, which are shown in three consecutive terms of April, August and December. Each table; after the performance of each candidates are shown end with the average of the entire group' performance, which is also show in consecutive terms April, August and December.

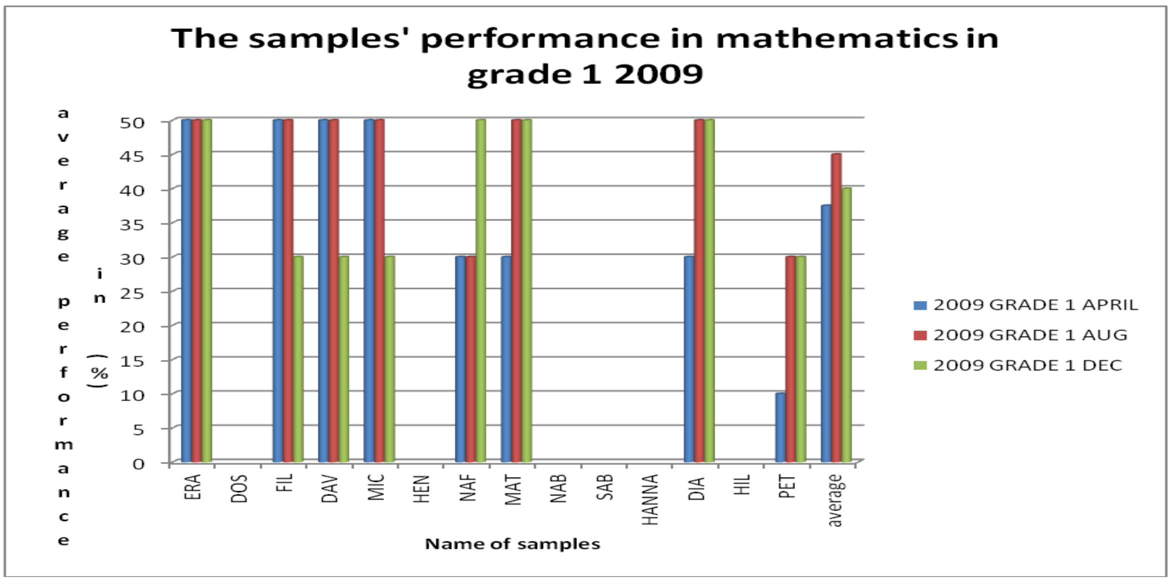


Figure 8.1 (a)

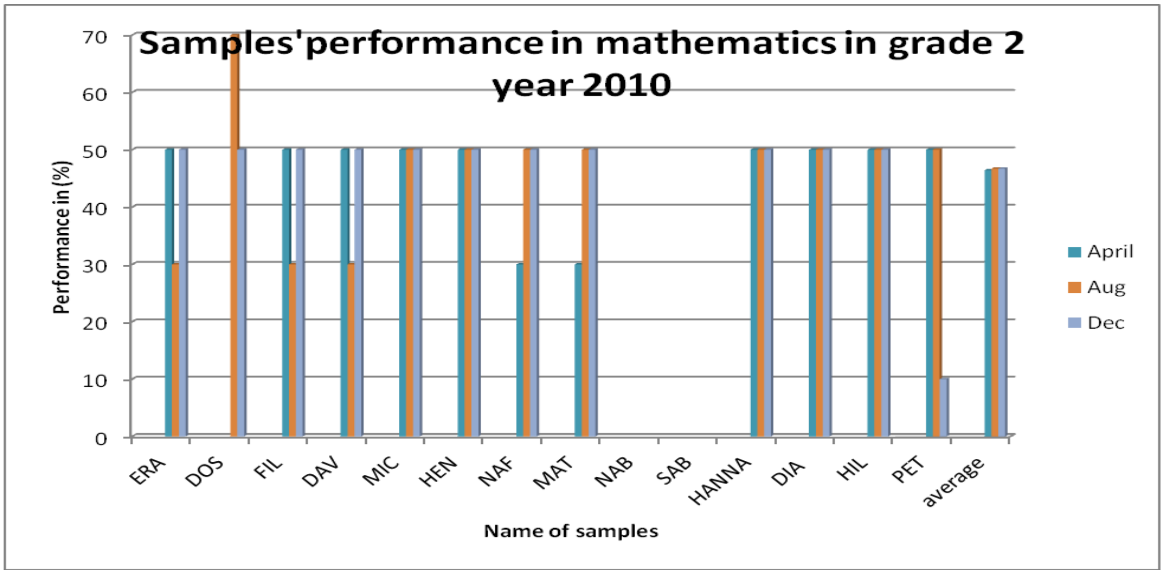


Figure 8.1 (b)

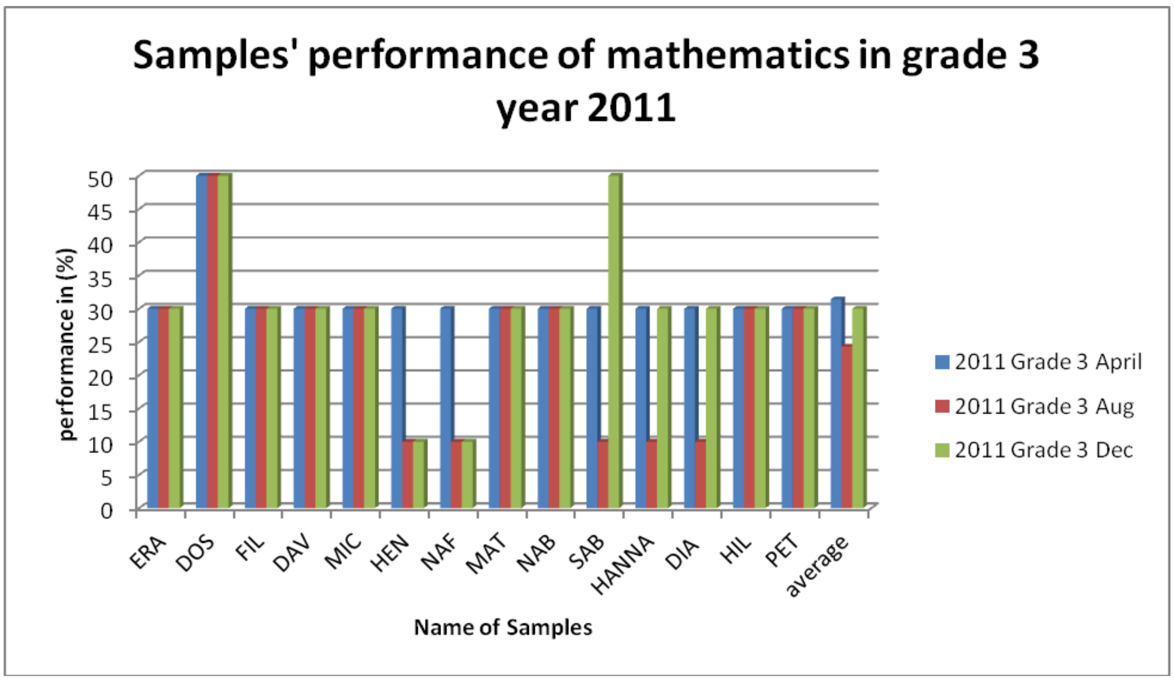


Figure 8.1 (c)

8.2 Appendix two

LEARNERS' PERFORMANCE ON THE PROGRAMME'S ACTIVITIES

The three figures 8.2 a, b and c shows the numeracy performance of learners in lesson one the pre- task lesson. The first figure 8.2 (a) shows the performance of learners for question one of the pre-task. In this question learners were given the division tasks to calculate without the division instructions. In question one, 3 learners got one mark, 3 learners got 2 marks, 4 learners got 3 marks, 3 learners got 4 marks and only one got all the five marks. The average is 8 learners out of 14, which are 57 % who performed above the 50% on question one.

Figure 8.2 (b) shows the performance of learners for question two in the pre-task, lesson one. The teacher gave the instructions using the dots to show the patterns in the number values prior to the task. Question two was multiplication questions the exact inverse of division task in question one. In question two, 3 learners got 1 mark, 3 learners got 2 marks, 3 learners got 3 marks, 4 learners got 4 marks and only 1 learner got all five marks. The overall average performance is 8 learners out of 14, which are 57%, who performed above 50% on the questions two.

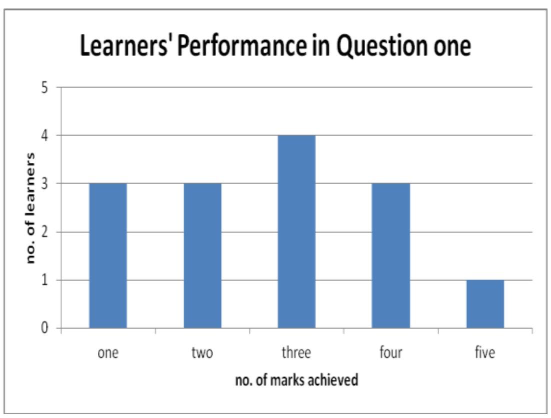


Figure 8.2 (a)

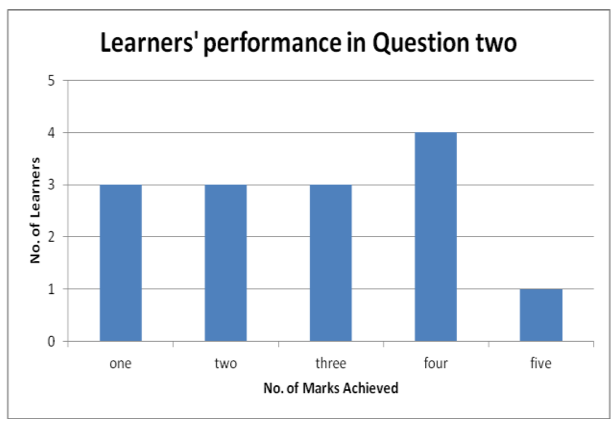


Figure 8.2 (b)

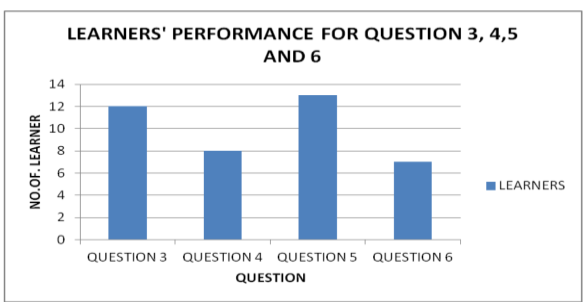


Figure 8.2 (c)

Figure 8.2 (c), shows the learners' performance in question 3, 4, 5 and 6. In question 3 learners used 12 stones and equally arranged them into 4 groups (12 ÷ 4), 12 learners out of 14, which are 86% divided using the stones and got the correct answer. This division action with stones in question 3 was translated into symbolic number sentence of division in question 4, whereby 8 learners out of 14, which are 57%; translated the division action of sharing with stones into symbolic number sentence of division correctly.

In question 5, learners took 18 stones and keep measuring out 9 stones (18 ÷ 9). 13 learners out of 14, which are 93% divided with stones correctly and got the correct answers. When this division of measurement out with stones in question 5 was translated into symbolic number sentence of division in question 6, only 7 learners out of 14, which are 50%; translated correctly the division actions into symbolic number sentences.

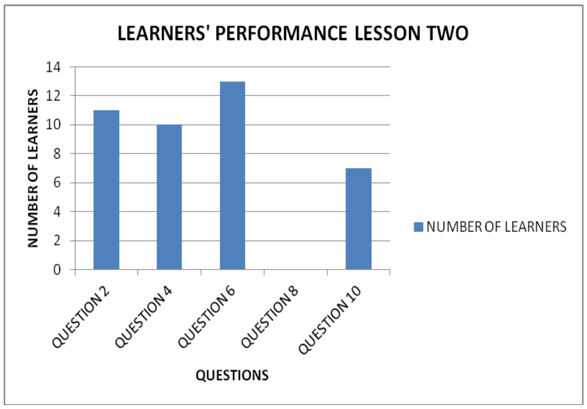


Figure 8.2 (d)

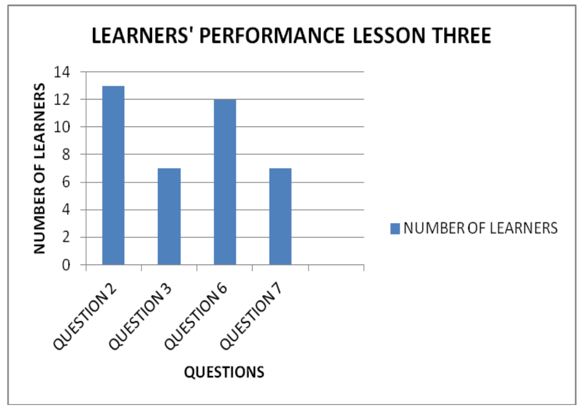


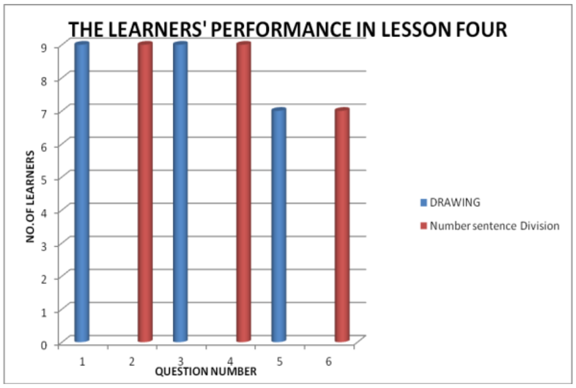
Figure 8.2 (e)

Figures 8.2 (d) and (e) show learners' performance in lesson two and three tasks. In the two lessons learners used the stones (manipulatives/ concrete objects) to do division. The stones were used in the value of dividends and were shared in a given number of groups. In lessons two learners did five different divisions, while in lesson three learners did three different divisions. In both lessons, the division actions with stones was translated to symbolic number sentence of division and in lesson three the translation into symbolic number sentence was also transferred to other operation domains, relating the division patterns to addition, subtractions and multiplication.

In figure 8.2 (d) shows learners' performance in question 1 to 10. In question 1, learners used 18 stones and divided them equally into 3 groups (18 ÷ 3). In question 2, learners translated the division actions' idea done in question 1 into symbolic number sentence of division. Eleven learners out of fourteen, which are 79% of learners, translated the division actions with stones to symbolic number sentence of division correctly. In question 3, learners used 24 stones and divided them equally into 4 groups (24 ÷ 4). The division actions of stones in question 3 were then translated to the symbolic number sentence of division in question 4. Ten learners out of fourteen, which are 71% of learners, translated the division actions with stones to symbolic number sentence of division correctly. In question 5, learners used 30 stones and divided them equally into 5 groups (30

÷ 5). The division actions of stones in question 5 was then translated to the symbolic number sentence of division in question 6, Thirteen learners out of fourteen, which are 93% of learners, translated the division actions with stones to symbolic number sentence of division correctly. In question 7, learners used 80 stones and divided them equally into 4 groups (80 ÷ 4). The division actions of stones in question 7 was then translated to the symbolic number sentence of division in question 8, four learners out of fourteen, attempted to do the translation but 0% of learners, translated the division actions with stones to symbolic number sentence of division correctly. In question 9, learners used 45 stones and divided them equally into 5 groups (45 ÷ 5). The division actions of stones in question 9 was then translated to the symbolic number sentence of division in question 10, four learners out of fourteen attempted to do the translation but only 1 learners which are 7% of learners, translated the division actions with stones to symbolic number sentence of division correctly.

Figure 8.2 (e) shows learners' performance of the task in lesson three. In question 1, learners took 20 stones and divided them equally into 5 groups (20 ÷ 5). The division action with stones in question 1 was translated into symbolic number sentence process of subtraction in question 2, relating the division patterns of 4 and 5 in 20 to subtraction. Thirteen learners out of fourteen, which are 93% of learners, related the division patterns into subtraction symbolic number sentence correctly, which is (20 ÷ 5 = 4) can be related to subtraction as (20 – 5 = 15); (15 – 5 = 10); (10 – 5 = 5); (5 – 5 = 0). The same division action with stones in question 1 was translated into symbolic number sentence of division in question 3, whereby 7 learners out of 14, which are 50%; translated the division action with stones into symbolic number sentence of division correctly. In question 5, learners used 15 stones and divided them equally into 5 groups (15 ÷ 5 = 3). Learners also used the same 15 stones and build 15 into 5 groups of 3 stones. The division action with stones in question 5 was translated into symbolic number sentence process of addition in question 6, relating the division patterns of 3 and 5 in 15 to addition. Twelve learners out of fourteen, which are 86% of learners, related the division patterns of (15 ÷ 5 = 3) into addition symbolic number sentence correctly, as follow (3 = 3); (3 + 3 = 6); (3 + 3 + 3 = 9); (3 + 3 + 3 + 3 = 12); (3 + 3 + 3 + 3 + 3 = 15). The same division action with stones in question 5 was translated into symbolic number sentence of division in question 7. Twelve learners out of fourteen, which are 86% of learners, translated the division action with stones into symbolic number sentence of division correctly.



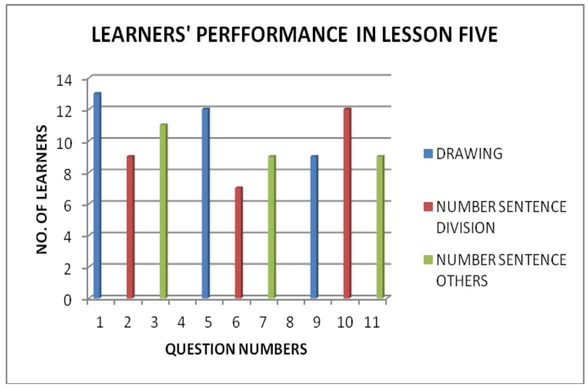


Figure 8.2 (f) **Figure 8.2 (g)**

Figures 8.2 (f) and (g) show learners' performance in the tasks of lessons four and five. In lesson four and five, learners used drawings of apples, sweets, sticks beans and cups, which they had used to do sharing/ partitive division activities. For each of the two lessons learners did three different division activities. The division in diagram was translated into the symbolic number sentence and further related division to symbolic number sentence of addition, subtraction and multiplication in lesson five.

As indicated in figure 8.2 (f), shows learners' performance for the tasks in lesson four. In question one, learners drew 18 apples and 3 empty boxes. Learners shared by redrawing the 18 apples equally into the 3 empty box cancelling three apples at a time and equally distribute them into the empty boxes until all the 18 apples are cancelled out and redrawn into 3 boxes (18 ÷ 3). Nine learners out of fourteen, which are 64% of learners, did the correct division in diagrams. The same division in diagrams in question one was translated into the symbolic number sentences of division in question two. Nine learners out of fourteen, which are 64% of learners, translated the division in diagrams into symbolic number sentence correctly.

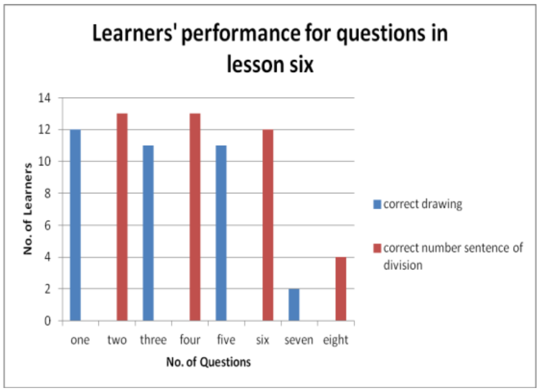
In question three, learners drew 24 sweets and 4 children. Learners shared the 24 sweets equally to 4 children (24 ÷ 4) by cancelling four drawing of sweets at a time and equally distribute them to four children until all the 24 sweets are cancelled out (24 ÷ 4) and redrawn under four children. Nine learners out of fourteen, which are 64% of learners, did the correct division in diagrams. The same division in diagrams in question three was translated into the symbolic number sentence of division in question four, whereby still 9 learners out of 14, which are 64% of learners, translated the division in diagrams into symbolic number sentence correctly.

In question five, learners drew 30 sticks and 5 empty boxes. Learners shared the 30 sticks equally into the 5 empty box (30 ÷ 5) by cancelling five sticks at a time and equally distribute into the empty boxes until all the 30 sticks are cancelled out (30 ÷ 5). Seven learners out of fourteen, which are 50% of learners, did the correct division in diagrams. The same division in diagrams in question five was translated into the symbolic number sentence of division in question six, whereby still 7 learners out of 14, which are 50% of learners; translated the division in diagrams into symbolic number sentence correctly.

Figure 8.2 (g) shows the learners' performance for the tasks in lesson five. In question one, learners drew 20 apples and circles to indicate 5 groups. They shared the 20 apples into 5 groups equally to get four apples in each of the 5 groups (20 ÷ 5 = 4). Thirteen learners out of fourteen, which are 93% of learners, did the division in diagrams correctly. The division in drawing in question one, was translated into a symbolic number sentence of repeated subtraction in question two, relating the division pattern of 4 and 5 in 20 to subtraction as follow (20 – 5 = 15), (15 – 5 = 10), (10 – 5 = 5), (5 – 5 = 0). Nine learners out of fourteen, which are 64% of learners, related the division patterns correct by translating the division pattern into repeated subtraction process. The same action of division in diagram in question one was again translated into the symbolic number sentence of division in question three. Eleven learners out of fourteen, which are 79% of learners, translated the division action in diagrams into the symbolic number sentence of division correctly.

In question five, learners drew 15 sweets in 5 groups, then they shared the 15 sweets into 5 groups equally to get 3 sweets in each of the 5 groups (15 ÷ 3 = 5). Twelve learners out of fourteen, which are 86% of learners, did the division in diagrams correctly. The division in drawing in question five, was translated into a symbolic number sentence of repeated addition in question six, relating the division pattern of 3 and 5 in 15 to addition as follow (3 = 3), (3 + 3 = 6), (3 + 3 + 3 = 9), (3 + 3 + 3 + 3 = 12) and (3 + 3 + 3 + 3+ 3 = 15). Seven learners out of fourteen, which are 50% of learners, related the division patterns correct by translating the division pattern into repeated addition process. The same action of division in diagram in question five was again translated into the symbolic number sentence of division in question seven. Nine learners out of fourteen, which are 64% of learners, translated the division action in diagrams into the symbolic number sentence of division correctly.

In question nine, learners drew 50 sticks and regrouped them into 5 groups of 10 bunches of sticks (50 ÷ 5 = 10), sharing 50 sticks equally into 5 groups of 10 sticks small bunches. Nine learners out of fourteen, which are 64% of learners, did the division in diagrams correctly. The division in drawing in question nine, was translated into a symbolic number sentence of multiplication in question ten, relating the division pattern of 5 and 10 in 50 to multiplication as follow (5 × 10 or 10 × 5 = 50). Twelve learners out of Fourteen, which are 86% of learners, related the division patterns correct by translating the division pattern into multiplication number sentence. The same action of division in diagram in question nine was again translated into the symbolic number sentence of division in question eleven. Nine learners out of fourteen, which are 64% of learners, translated the division action in diagrams into the symbolic number sentence of division correctly.



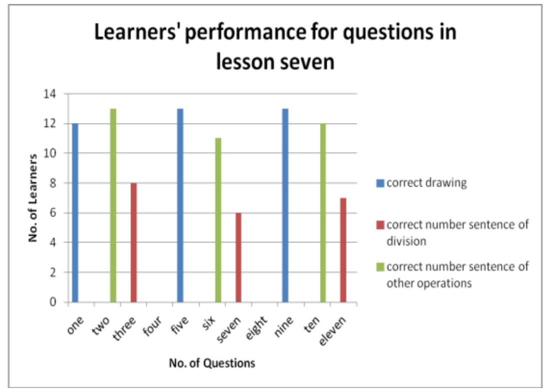


Figure 8.2 (h) Figure 8.2 (i)

Figures 8.2 (h) and (i) show learners' performance in the tasks of lessons six and seven. In lesson six and seven, learners used drawings of dots, which they had used to do sharing/ partitive division activities. In lesson six learners did four different division activities. The division in dots was translated into the symbolic number sentence and further related division to symbolic number sentence of addition, subtraction and multiplication in lesson seven.

As indicated, figure 8.2 (h), shows learners' performance for the tasks in lesson six. In question one; learners drew 18 dots and 3 empty circles to represent 18 apples shares into 3 boxes (18 ÷ 3). Learners shared by redrawing the 18 dots equally into the 3 empty box cancelling three dots at a time and equally distribute them into the empty boxes until all the 18 dots are cancelled out and redrawn into 3 boxes (18 ÷ 3). Twelve learners out of fourteen, which are 86% of learners, did the correct division in dots. The same division in dots in question one was translated into the symbolic number sentences of division in question two. Thirteen learners out of fourteen, which are 93% of learners, translated the division in diagrams into symbolic number sentence correctly.

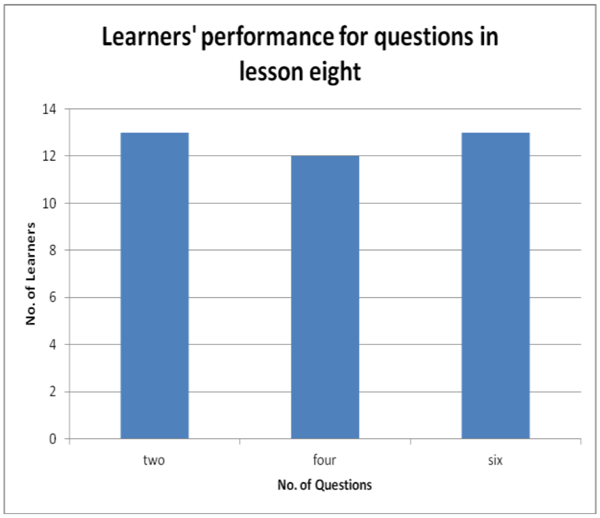
In question three, learners drew 24 dots and 4 rectangles. Learners shared the 24 dots equally into 4 rectangles, which represent 24 sweets shared by 4 children (24 ÷ 4). The division actions was done by cancelling four dots at a time and equally distribute them to four rectangles until all the 24 dots are cancelled out (24 ÷ 4). Eleven learners out of fourteen, which are 79% of learners, did the correct division in dots. The same division in dots in question three was translated into the symbolic number sentence of division in question four, whereby 13 learners out of 14, which are 93% of learners, translated the division in dots into symbolic number sentence correctly.

In question five, learners drew 30 dots and 5 triangles and equally distribute 30 dots into 5 triangles, which represent 30 sticks equally arranged into 5 groups (30 ÷ 5). Eleven learners out of fourteen, which are 79% of learners, did the correct division in dots. The same division in dots in question five was translated into the symbolic number sentence of division in question six, whereby 12 learners out of 14, which are 86% of learners; translated the division in dots into symbolic number sentence correctly.

Figure 8.2 (i) shows the learners' performance for the tasks in lesson seven. In question one, learners drew 20 dots and 5 circles representing the 20 appples shared equally by 5 people (20 ÷ 5 = 4). Twelve learners out of fourteen, which are 86% of learners, did the division in dots correctly. The division in dots in question one, was translated into a symbolic number sentence of repeated subtraction in question two, relating the division pattern of 4 and 5 in 20 to subtraction as follow (20 – 4 = 16), (16 – 4 = 12), (12 – 4 = 8), (8 – 4 = 4) (4 – 4 = 0). thirteen learners out of fourteen, which are 93% of learners, related the division patterns correct by translating the division pattern into repeated subtraction process. The same action of division in dots in question one was again translated into the symbolic number sentence of division in question three. Eight learners out of fourteen, which are 57% of learners, translated the division action in diagrams into the symbolic number sentence of division correctly.

In question five, learners drew 15 dots into 5 groups of three dots, then they shared the 15 dots into 5 groups equally to get 3 dots in each of the 5 groups (15 ÷ 3 = 5). Thirteen learners out of fourteen, which are 93% of learners, did the division in dots correctly. The division in dots in question five, was translated into a symbolic number sentence of repeated addition in question six, relating the division pattern of 3 and 5 in 15 to addition as follow (3 = 3), (3 + 3 = 6), (3 + 3 + 3 = 9), (3 + 3 + 3 + 3 = 12) and (3 + 3 + 3 + 3+ 3 = 15). Eleven learners out of fourteen, which are 79% of learners, related the division patterns correct by translating the division pattern into repeated addition process. The same action of division in dots in question five was again translated into the symbolic number sentence of division in question seven. six learners out of fourteen, which are 43% of learners, translated the division action in dots into the symbolic number sentence of division correctly.

In question nine, learners drew 50 dots and 5 rectangles and distributed the 50 into 5 rectangles representing 50 sticks regrouped equally into 5 groups of 10 bunches of sticks (50 ÷ 5 = 10). Thirteen learners out of fourteen, which are 93% of learners, did the division in dots correctly. The division in dots in question nine, was translated into a symbolic number sentence of multiplication in question ten, relating the division pattern of 5 and 10 in 50 to multiplication as follow (5 × 10 or 10 × 5 = 50). Twelve learners out of Fourteen, which are 86% of learners, related the division patterns correct by translating the division pattern into multiplication number sentence. The same action of division in dots in question nine was again translated into the symbolic number sentence of division in question eleven. Seven learners out of fourteen, which are 50% of learners, translated the division action in dots into the symbolic number sentence of division correctly.



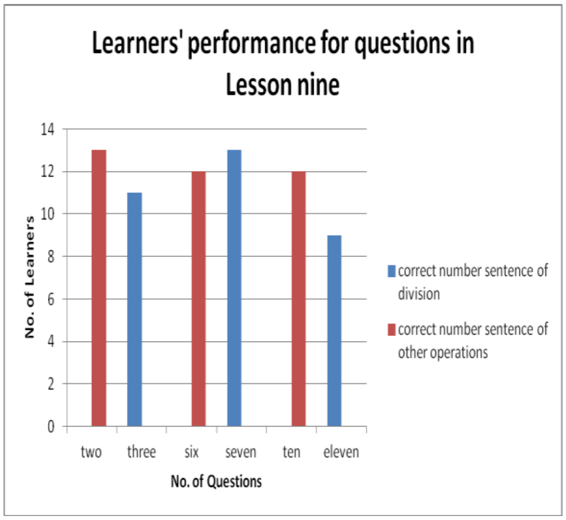


Figure 8.2 (j)

Figure 8.2 (k)

Figures 8.2 (j) and (k), show the learners' performance in tasks of lessons eight and nine. In these lessons, learners in groups surround the table and use stones to do quotitive division. In this division learners took stones in the number of dividend and keep measuring out the equal numbers of sets/ portion from the dividend until the dividend was finished. In each of lessons eight and nine learners did three different divisions. In both lessons, the division actions with stones was translated to symbolic number sentence of division and in lesson nine the translation into symbolic number sentence was also transferred to other operation domains, relating the division patterns to addition, subtractions and multiplication.

In figure 8.2 (j) shows learners' performance in question 1 to 6. In question 1, learners used 18 stones and divided by repeatedly measuring out groups of 3 stones from eighteen until eighteen stones get finished. Learners measured out six groups of three stones (18 ÷ 3 = 6). In question 2, learners translated the division actions' idea done in question 1 into symbolic number sentence of division. Thirteen learners out of fourteen, which are 93% of learners, translated the division actions with stones to symbolic number sentence of division correctly. In question 3, learners used 27 stones, which they divided by measuring out groups of 9 stones out of 27 stones. They measured out 3 groups of 9 stones from 27 stones (27 ÷ 9 = 3). The division actions of stones in question 3 were then translated to the symbolic number sentence of division in question 4. Twelve learners out of fourteen, which are 86% of learners, translated the division actions with stones to symbolic number sentence of division correctly. In question 5, learners used 115 stones, which they divided by measuring out groups of 20 stones from 115 until 115 stones get finished. They measured 5 groups of 20 stones and there was 15 stones reminder (115 ÷ 20 = 5 rem 15). The division actions of stones in question 5 was then translated to the symbolic number sentence of division in question 6, Thirteen learners out of fourteen, which are 93% of learners, translated the division actions with stones to symbolic number sentence of division correctly.

Figure 8.2 (k) shows learners' performance of the task in lesson nine. In question 1, learners used 40 stones and divided by repeatedly measuring out groups of 10 stones from 40 stones until 40 stones get finished. Learners measured out 4 groups of 10 stones (40 ÷ 10 = 4). The division action with stones in question 1 was translated into symbolic number sentence process of subtraction in question 2, relating the division patterns of 40 and 10 and 5 to subtraction. Thirteen learners out of fourteen, which are 93% of learners, related the division patterns into subtraction symbolic number sentence correctly, which is (40 ÷ 10 = 4) can be related to subtraction as (40 – 10 = 30); (30 – 10 = 20); (20 – 10 = 10); (10 – 10 = 0). The same division action with stones in question 1 was translated into symbolic number sentence of division in question 3, whereby 11 learners out of 14, which are 79%; translated the division action with stones into symbolic number sentence of division correctly. In question 5, learners used 16 stones and divided by repeatedly measuring out groups of 4 stones from 16 stones until 16 stones get finished. Learners measured out 4 groups of 4 stones (16 ÷ 4 = 4). The division action with stones in question 5 was translated into symbolic number sentence process of subtraction in question 6, relating the division patterns of 16 and 4 and 4 to repeated addition. Eleven learners out of fourteen, which are 79% of learners, related the division patterns into subtraction symbolic number sentence correctly, which is (16 ÷ 4 = 4) can be related to repeated addition as (4 = 4); (4 + 4 = 8); (4 + 4 + 4 = 12); (4 + 4 + 4 + 4 = 16). The same division action with stones in question 5 was translated into symbolic number sentence of division in question 7, whereby 13 learners out of 14, which are 93%; translated the division action with stones into symbolic number sentence of division correctly. In question 9, learners used 90 stones and divided by repeatedly measuring out groups of 20 stones from 90 stones until 90 stones get finished. Learners measured out 4 groups of 20 stones with 10 stones as a reminder (90 ÷ 20 = 4, rem 10). The division action with stones in question 9 was translated into symbolic number sentence process of multiplication in question 10, relating the division patterns of 90 and 20 and 4 to multiplication. Twelve learners out of fourteen, which are 86% of learners, related the division patterns into multiplication symbolic number sentence correctly, which is (90 ÷ 20 = 4) can be related to repeated addition as (20 × 4 + 10 = 90). The same division action with stones in question 9 was translated into symbolic number sentence of division in question 11, whereby 9 learners out of 14, which are 64%; translated the division action with stones into symbolic number sentence of division correctly.

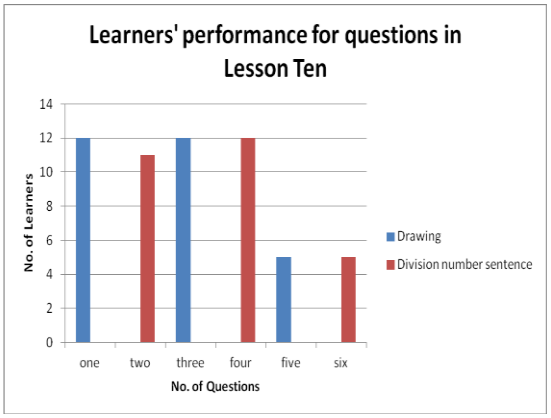


Figure 8.2 (l)

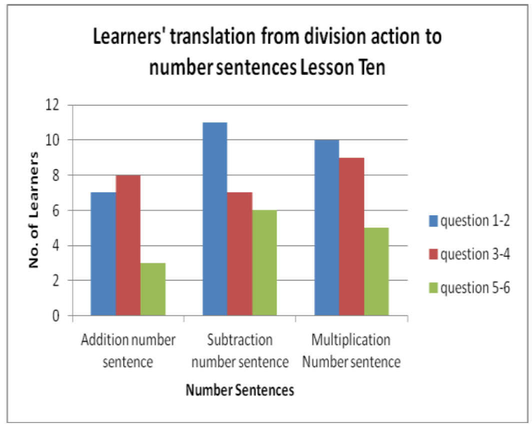


Figure 8.2 (m)

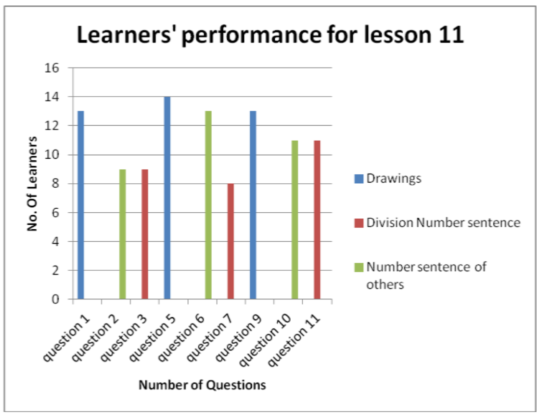


Figure 8.2 (n)

Figures 8.2 (l); (m) and (n) show learners' performance in the tasks of lessons ten and eleven. In lesson ten and eleven, learners used drawings of apples, sweets, sticks and stones, which they had used to do quotitive division activities. For each of the two lessons learners did three different division activities. The division in diagram was translated into the symbolic number sentence and further related division to symbolic number sentence of addition, subtraction and multiplication.

As indicated in figure 8.2 (l), shows learners' performance for the tasks in lesson ten. In question one, learners drew pictures of 28 drawn sticks. Learners divided by keep cancelling out 4 sticks from the 28 drawings and redraw them as a group (28 ÷ 4 = 7). Learners kept cancelling the 4 drawing of sticks making groups for them until 28 drawing of sticks got cancelled out. Twelve learners out of fourteen, which are 86% of learners, did the correct division in diagrams by cancelling 7 groups of 4 sticks each from 28 drawing of sticks. The same division in diagrams in question one was translated into the symbolic number sentences of division in question two. Eleven learners out of fourteen, which are 79% of learners, translated the division in diagrams into symbolic number sentence correctly. In question three, learners used the drawing of 35 drawn apples. Learners divided by keep cancelling out 7 apples from the 35 drawings and redraw them as a group (35 ÷ 7 = 5). Learners kept cancelling the 7 drawing of apples making groups for them until 35 drawing of apples got cancelled out. Twelve learners out of fourteen, which are 86% of learners, did the correct division in diagrams by cancelling 5 groups of 7 apples each from 35 drawing of apples. The same division in diagrams in question one was translated into the symbolic number sentences of division in question four. Twelve learners out of fourteen, which are 86% of learners, translated the division in diagrams into symbolic number sentence correctly. In question five, learners used the drawing of 32 drawn sweets. Learners divided by keep cancelling out 4 sweets from the 32 drawings and redraw them as a group (32 ÷ 4 = 8). Learners kept cancelling the 4 drawing of sweets making groups for them until 32 drawing of sweets got cancelled out. five learners out of fourteen, which are 36% of learners, did the correct division in diagrams by cancelling 8 groups of 4 sweets each from 32 drawing of sweets. The same division in diagrams in question one was translated into the symbolic number sentences of division in question five. Five learners out of fourteen, which are 36% of learners, translated the division in diagrams into symbolic number sentence correctly.

Figure 8.2 (m) still shows the performance of learners for lesson ten. For each question in lesson ten learners were asked to relate the division actions in drawings to addition, subtraction and multiplication. For questions 1 and 2, which has the division of (28 ÷ 4 = 8) only 7 learners, which are the 50% related this division to addition correctlly. Eleven learners out of fourteen, which is 79% of learners related the same division in question 1 and 2 into subtraction correctlly. Ten learners out of fourteen, which is 71% of learners related the same division in question 1 and 2 into multiplication correctlly. For questions 3 and 4, which has the division of (35 ÷ 7 = 5) only 8 learners, which are the 57% related this division to addition correctlly. Seven learners out of fourteen, which is 50% of learners related the same division in question 3 and 4 into subtraction correctlly. Nine learners out of fourteen, which is 64% of learners related the same division in question 3 and 4 into multiplication correctlly. For questions 5 and 6, which has the division of (32 ÷ 4 = 8) only 3 learners, which are the 21% related this division to addition correctlly. Five learners out of fourteen, which is 36% of learners related the same division in question 5 and 6 into subtraction correctlly. Five learners out of fourteen, which is 36% of learners related the same division in question 5 and 6 into multiplication correctlly.

Figure 8.2 (l) shows the learners' performance for the tasks in lesson Eleven. In question one, learners used the drawing of 48 stones that were already drawn. They divided by measuring out 8 drawing of stones from 48 by cancelling them out and make groups for them until the 48 drawn stones are cancelled out (48 ÷ 8 = 6). Thirteen learners out of fourteen, which are 93% of learners, did the division in diagrams correctly. The division in drawing in question one, was translated into a symbolic number sentence of repeated subtraction in question two, relating the division pattern of 8 and 6 in 48 to subtraction as follow (48 – 8 = 40), (40 – 8 = 32), (32 – 8 = 24), (24 – 8 = 16) (16 – 8 = 8) (8 – 8 = 0).

Nine learners out of fourteen, which are 64% of learners, related the division patterns correct by translating the division pattern into repeated subtraction process. The same action of division in diagram in question one was again translated into the symbolic number sentence of division in question three. Nine learners out of fourteen, which are 64% of learners, translated the division action in diagrams into the symbolic number sentence of division correctly. In question five, learners used the drawing of 8 apples, then they divided by measuring out 4 drawing of apples from 8, by cancelling them out and make groups for them (8 ÷ 4 = 2). All learners 14 out of 14, which are 100% of learners, did the division in diagrams correctly. The division in drawing in question five, was translated into a symbolic number sentence of repeated addition in question six, relating the division pattern of 4 and 2 in 8 to addition as follow (4 = 4), (4 + 4 = 8). thirteen learners out of fourteen, which are 93% of learners, related the division patterns correct by translating the division pattern into repeated addition process. The same action of division in diagram in question five was again translated into the symbolic number sentence of division in question seven. Eight learners out of fourteen, which are 57% of learners, translated the division action in diagrams into the symbolic number sentence of division correctly. In question nine, learners used the drawing 14 sweets and divided them by measuring out 2 sweets which they cancelled out from 32 and makes groups for them (14 ÷ 2 = 7). Thirteen learners out of fourteen, which are 93% of learners, did the division in diagrams correctly. The division in drawing in question nine, was related into a symbolic number sentence of multiplication in question ten, relating the division pattern of 2 and 7 in 14 to multiplication as follow (4 × 7 or 7 × 4 = 14). Eleven learners out of Fourteen, which are 79% of learners, related the division patterns correct by translating the division pattern into multiplication number sentence. The same action of division in diagram in question nine was again translated into the symbolic number sentence of division in question eleven. Eight learners out of fourteen, which are 57% of learners, translated the division action in diagrams into the symbolic number sentence of division correctly.

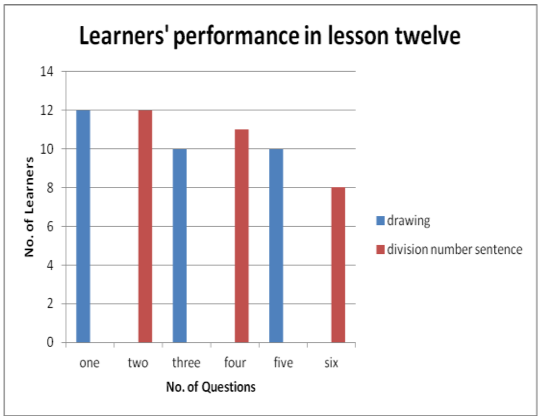


Figure 8.2 (o)

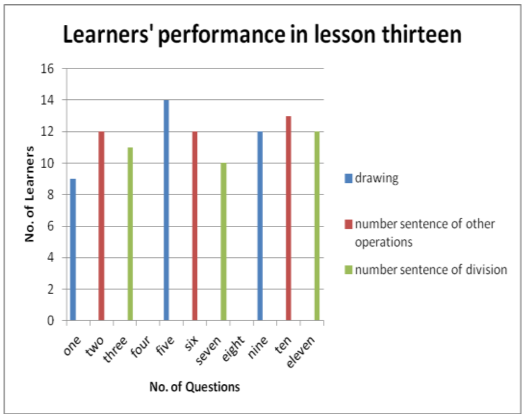


Figure 8.2 (p)

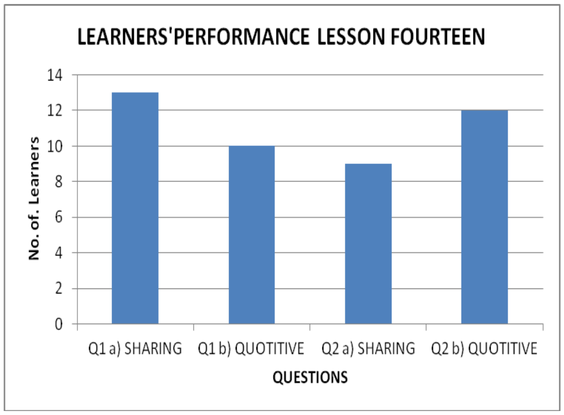
Figures 8.2 (o) and (p) show learners' performance in the tasks of lessons twelve and thirteen. In lesson twelve and thirteen, learners used drawings of dots, which they had used to do quotitive division activities. For each of the two lessons learners did three different division activities. The division in diagram was translated into the symbolic number sentence and further related division to symbolic number sentence of addition, subtraction and multiplication.

As indicated in figure 8.2 (o), shows learners' performance for the tasks in lesson twelve. In question one; learners drew 30 dots representing 30 pens. The idea is that learners are distributing 30 pens to people of which each would get 6 pens. Learners divided by keep cancelling out 6 dots from the 30 drawings of dots and redraw them as a group (30 ÷ 6 = 5). Learners kept cancelling the 6 drawing of dots making groups for them until 30 drawing of dots got cancelled out. Twelve learners out of fourteen, which are 86% of learners, did the correct division in drawing by cancelling 5 groups of 6 dots each from 30 drawing of dots. The same division in diagrams in question one was translated into the symbolic number sentences of division in question two. Twelve learners out of fourteen, which are 86% of learners, translated the division in diagrams into symbolic number sentence correctly. In question three, learners drew 60 dots representing 60 cookies. The idea is that learners are packing 60 cookies in packs of 15 cookies. Learners divided by keep cancelling out 15 dots from the 60 drawings of dots and redraw them as a group (60 ÷ 15 = 4). Learners kept cancelling the 15 drawing of dots making groups for them until 60 drawing of dots got cancelled out. Ten learners out of fourteen, which are 71% of learners, did the correct division in drawing by cancelling 4 groups of 15 dots each from 60 drawing of dots. The same division in diagrams in question three was translated into the symbolic number sentences of division in question four. Eleven learners out of fourteen, which are 79% of learners, translated the division in diagrams into symbolic number sentence correctly.

In question five; learners drew 15 dots representing 15sticks. The idea is that learners are distributing 15 sticks into groups (bunch) of each with 5 sticks. Learners divided by keep cancelling out 5 dots from the 15 drawings of dots and redraw them as a group (15 ÷ 5 = 3). Learners kept cancelling the 5 drawing of dots making groups for them until 15 drawing of dots got cancelled out. Ten learners out of fourteen, which are 71% of learners, did the correct division in drawing by cancelling 3 groups of 5 dots each from 15 drawing of dots. The same division in diagrams in question five was translated into the symbolic number sentences of division in question six. Eight learners out of fourteen, which are 57% of learners, translated the division in diagrams into symbolic number sentence correctly.

Figure 8.2 (p) shows the learners' performance for the tasks in lesson thirteen. In question one; learners drew 20 dots representing 20 apples. The idea is that learners are distributing 20 apples to people of whom each would get 3 apples. Learners divided by keep cancelling out 3 dots from the 20 drawings of dots and redraw them as a group (20 ÷ 3 = 6 reminder 2). Learners kept cancelling the 3 drawing of dots making groups for them until 20 drawing of dots got cancelled out. Nine learners out of fourteen, which are 64% of learners, did the correct division in drawing by cancelling 6 groups of 3 dots each from 20 drawing of dots. The same division in diagrams in question one was translated into the symbolic number sentences process of repeated subtraction in question two which related the patterns of 3 and 6 in 20 as follow: (20 – 3 = 17), (17 – 3 = 14), (14 – 3 = 11), (11 – 3 = 8) (8 – 3 = 5) (5 – 3 = 2). Twelve learners out of fourteen learners, which are 86% related the division done in dots into repeated subtraction correctlly. The same action of division in diagram in question one was again translated into the symbolic number sentence of division in question three. Eleven learners out of fourteen, which are 79% of learners, translated the division action in diagrams into the symbolic number sentence of division correctly.

In question five; learners drew 36 dots representing 36 sweets. The idea is that learners are distributing 36 sweets to children of whom each would get 6 sweets. Learners divided by keep cancelling out 6 dots from the 36 drawings of dots and redraw them as a group (35 ÷ 6 = 6). Learners kept cancelling the 6 drawing of dots making groups for them until 36 drawing of dots got cancelled out. All fourteen learners out of fourteen, which are 100% of learners, did the correct division in drawing by cancelling out 6 groups of 6 dots each from 36 drawing of dots. The same division in diagrams in question five was translated into the symbolic number sentences process of repeated addition in question six which related the patterns of 6 and 6 in 36 as follow: (6 = 6) (6 + 6 = 12) (6 + 6 + 6 = 18) (6 + 6 + 6 + 6 = 24) (6 + 6 + 6 + 6 + 6 = 30) (6 + 6 + 6 + 6 + 6 + 6 = 36). Twelve learners out of fourteen, which are 86% of learners, translated the division in diagrams into symbolic number sentence correctly. The same action of division in diagram in question five was again translated into the symbolic number sentence of division in question seven. Ten learners out of fourteen, which are 71% of learners, translated the division action in diagrams into the symbolic number sentence of division correctly. In question nine; learners drew 28 dots representing 28 sticks. The idea is that learners are distributing 28 sticks into groups (bunches) of 4 sticks each. Learners divided by keep cancelling out 4 dots from the 28 drawings of dots and redraw them as a group (28 ÷ 4 = 7). Learners kept cancelling the 4 drawing of dots making groups for them until 28 drawing of dots got cancelled out. Twelve learners out of fourteen, which are 86% of learners, did the correct division in drawing by cancelling out 7 groups of 4 dots each from 28 drawing of dots. The same division in diagrams in question nine was translated into the symbolic number sentences of multiplication in question ten which related the patterns of 4 and 7 in 28 as follow: (4 × 7 or 7 × 4 = 28). Thirteen learners out of fourteen, which are 93% of learners, translated the division in diagrams into symbolic number sentence correctly. The same action of division in diagram in question nine was again translated into the symbolic number sentence of division in question eleven. Twelve learners out of fourteen, which are 86% of learners, translated the division action in diagrams into the symbolic number sentence of division correctly.



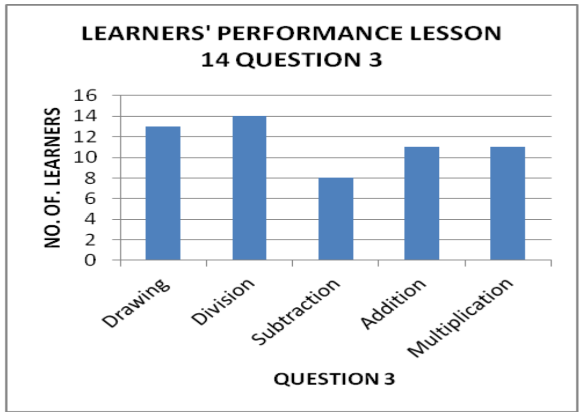


Figure 8.2 (q)

Figure 8.2 (r)

Figures 8.2 (q) and (r) show the learners performance for lesson fourteen. In figure 8.2 (q) shows the learners performance in question 1 and 2. The two questions had two division that are symbolically indetical in representation but different in term of how to divide. In both questions 1 and 2 the representation in (a)s are sharing division while their representation in (b)s are quotitive division. In question 1 (a) learners had option to use stones, drawing or dots to do the sharing of **24 stones** equally arranged into **3 groups.** Thirteen learners out of fourteen, which are 93% of learners did the sharing correctlly in drawing of dots. In question 1 (b) learners had option to use stones, drawing or dots to do the measuring out of **3 stones** from **24 stones.** Ten learners out of fourteen, which are 71% of learners did the quotitive division correctlly in drawing of dots. In question 2 (a) learners had option to use stones, drawing or dots to do the sharing of **35 stones** equally arranged into **7 groups.** Nine learners out of fourteen, which are 64% of learners did the sharing correctlly in drawing of dots. In question 2 (b) learners had option to use stones, drawing or dots to do the measuring out of **7 stones** from **35 stones.** Twelve learners out of fourteen, which are 86% of learners did the quotitive division correctlly in drawing of dots.

Figure 8.2 (r) shows learners performance on question 3 lesson fourteen. In this question learners were asked to opt for one betweeen stone, drawing of dots or any drawing to do division. learners were also at liberty to opt how to divide whether sharing or quotitive division. learners did the sharing division and opt to do division with the drawing of dots. thirteen learners out of fourteen, which are 93% opt to do division with drawing of dots, which they did correctly. All learners, which is 100% translated the same division in drawing correctly into symbolic division number sentence. Learners were also asked to relate the division to other operations whereby 8 learners out of 14, which is 57% related correctly into repeated subtraction. 11 learners out of 14, which is 79% related correctly into repeated Addition. 11 learners out of 14, which is 79% related correctly into inverse multiplication.

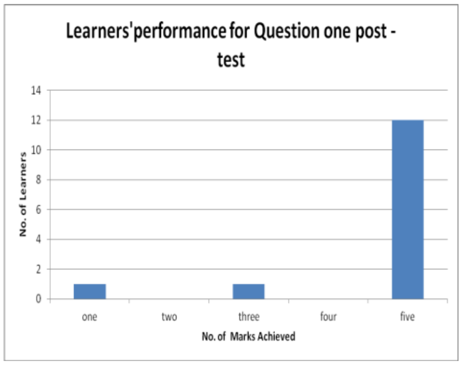


Figure 8.2 (s)

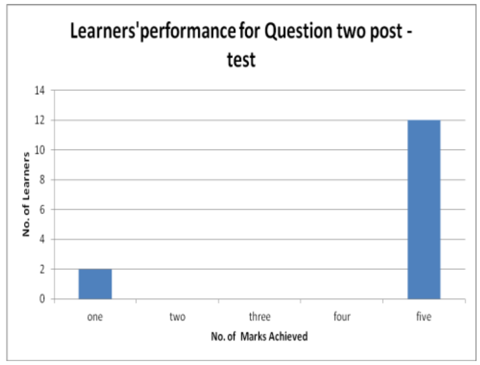


Figure 8.2 (t)

The three figures 8.2 s, t and u shows the numeracy performance of learners in lesson fifteen the post- task lesson. The first figure 8.2 (s) shows the performance of learners for question one of the post-task. In this question learners were given the division tasks to calculate without the division instructions. In question one, 1 learner got one mark, 1 learners got 3 marks and 12 learners got 5 marks. The average is 13 learners out of 14, which are 93 % who performed above the 50% on question one.

Figure 8.2 (t) shows the performance of learners for question two in the post-task, lesson one. The teacher gave the instructions using the dots to show the patterns in the number values prior to the task. Question two was multiplication questions the exact inverse of division task in question one. In question two, 2 learners got 1 mark and 12 learners got 5 marks. The overall average performance is 12 learners out of 14, which are 86%, who performed above 50% on the questions two.

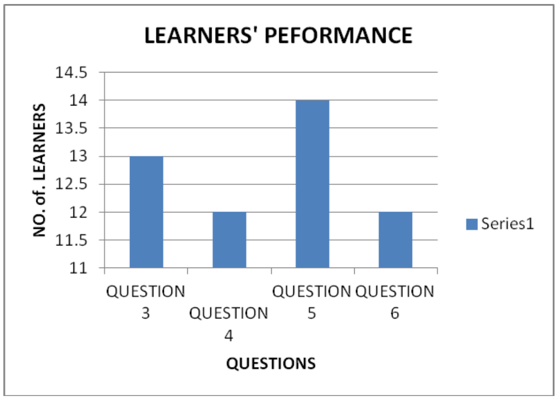


Figure 8.2 (u)

Figure 8.2 (u), shows the learners' performance in question 3, 4, 5 and 6. In question 3 learners used 12 stones and equally arranged them into 4 groups (12 ÷ 4), 13 learners out of 14, which are 93% divided using the stones and got the correct answer. This division action with stones in question 3 was translated into symbolic number sentence of division in question 4, whereby 12 learners out of 14, which are 86%; translated the division action of sharing with stones into symbolic number sentence of division correctly.

In question 5, learners took 18 stones and keep measuring out 9 stones (18 ÷ 9). 14 learners out of 14, which are 100% divided with stones correctly and got the correct answers. When this division of measurement out with stones in question 5 was translated into symbolic number sentence of division in question 6, only 12 learners out of 14, which are 86%; translated correctly the division actions into symbolic number sentences.

8.3 Appendix three

<u>ETHICS: LETTERS AND PERMISSIONS</u>

<u>Letter to the Director</u>

Dear Sir/ Madam

<u>Re: An appeal for a grant of the consent of a thesis</u>

I, Linea Beautty Joel, student number 609S6023, am a registered part time student at Rhodes University, Grahmstown (SA). I have been enrolled studying for a Master's degree specializing in Mathematics education since January 2011. I am hereby humbly appealing for a grant of consent from your office to carry out my thesis in your Region using XXXX school as a study site and the grade 4 learners as my study's participants (samples).

The aim of my thesis is to explore how visual representations, a teaching pedagogy may be used to enhance the integration of division procedures to their underlying concepts, in Mathematic teaching at a level of grade 4. To achieve this aim, the intervention programme of series of lessons, which is visually enriched is developed and need to be implemented over a period of a month (April to May).

Granting the consent will allow me to use XXXX school as my study site and a sample of grade 4 learners, which will enable the implementation of the intervention programme. In addition to that, it will also allow the use of video recording, field taking notes, the use of learners' sample of the tasks', learners' participation in interview and audio –taping of the interview, which will yield to data collection and data analysis.

The school, class –teacher(s), learners and parents will be asked for permission in regard to participation, video recording, audio-taping, field noting and the use of learners' results of the tasks. They will also ensured of the confidentiality, their right and democracy to participate and anonymity throughout and after the thesis process and will also be invited to proof read drafts of the thesis report to ensure the data record depict the accurate facts.

Attached is the consent form that needs your signature to signify your agreement/ permission on this regard matter.

Inquiries about this appeal can be gratefully forwarded to me at the contact number: XXXX

Yours Sincerely

Linea Beautty Joel

(The student)

LETTER TO THE INSPECTOR

Re: An appeal for a grant of the consent of a thesis

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The aim of my thesis is to explore how visual representations, a teaching pedagogy may be used to enhance the integration of division procedures to their underlying concepts, in Mathematics teaching at a level of grade 4. To achieve this aim, the intervention programme of series of lessons, which is visually enriched is developed and need to be implemented over a period of three months (February to April).

Granting the consent will allow me to use XXXX primary school as my study site and a sample of grade 4 learners, which will enable the implementation of the intervention programme. In addition, it will also allow the use of video recording, field taking notes, use of learners' sample of the tasks' results, learners' participation in interview and audio –taping of the interview, which will yield to data collection and analysis.

The school, class –teacher(s), learners and parents will be asked for permission in regard to participation, video recording, audio-taping, field noting and the use of learners' results of the tasks. They will also ensured of the confidentiality, their right and democracy to participate and anonymity throughout and after the thesis process and will also be invited to proof read drafts of the thesis report to ensure the data record depict the accurate facts.

Attached is the consent form that needs your signature to signify your agreement/ permission on this regard matter.

Inquiries about this appeal can be gratefully forwarded to me at the contact number: XXXX

Yours Sincerely

Linea Beautty Joel

(The student)

LETTER TO THE SCHOOL MANAGEMENT

Dear Sir/ Madam

Re: An appeal for a grant of the consent of a thesis

I, Linea Beautty Joel, student number 609S6023, am a registered part time student at Rhodes University, Grahmstown (SA). I have been enrolled studying for a Master's degree specializing in Mathematics education since January 2011. I am hereby humbly appealing for a grant of consent from the school management to carry out my thesis at XXXX school as a study site and the grade 4 learners as my study's participants (samples).

The aim of my thesis is to explore how visual representations, a teaching pedagogy may be used to enhance the integration of division procedures to their underlying concepts, in Mathematics teaching at a level of grade 4. To achieve this aim, the intervention programme of series of lessons, which is visually enriched is developed and need to be implemented over a period of three months (February to April).

Granting the consent will allow me to use XXXX school as my study site and a sample of grade 4 learners, which will enable the implementation of the intervention programme. In addition, it will also allow the use of video recording, field taking notes, use of learners' sample of the tasks' results, learners' participation in interview and audio –taping of the interview, which will yield to data collection and analysis. The school management is also humbly requested to communicate this with the class –teacher(s) teaching grade 4 learners, learners and parents about the carrying out of the thesis at the school.

Participants (class-teacher(s), learners and parents) will be asked for permission in regard to participation, video recording, audio-taping, field noting and the use of learners' results of the tasks. They will also ensured of the confidentiality, their right and democracy to participate and anonymity throughout and after the thesis process and will also be invited to proof read drafts of the thesis report to ensure the data record depict the accurate facts.

Attached is the consent form that needs the management member and school board member's signature to signify your agreement/ permission on this regard matter.

Inquiries about this appeal can be gratefully forwarded to me at the contact number: XXXX

Yours Sincerely

Linea Beautty Joel

(The student)

LETTER TO THE PARENTS

Dear parents

I Linea Beautty Joel am registered for a Master's degree in Mathematics education with education department at Rhodes University, Grahamstown (SA). To qualify for my Master's degree, I am required to write a thesis report. My thesis focuses on the area of teaching mathematics.

Based on experience, mathematics teachers get carried away by teaching mathematical procedures without linked to their underlying concepts, which does not makes sense to learners leading to poor performance of mathematics. My thesis aims to explore how visual representation a teaching pedagogy may be used to enhance the integration of division procedures to their underlying concepts, in mathematic teaching at a level of grade 4. To achieve this aim, an intervention programme with series of lessons, which are visually enriched is developed and need to be implemented in a grade 4 classroom lessons over the period of three weeks.

Granting me consent is part of the support I need from parents which will allow:

1) The participation of your child in the implementation of the intervention programme.
2) Video record the implementation of the intervention programme of which your child will be part of it.
3) Interview and audio – tape the child.
4) Takes notes of the child's participation during the implementation of the intervention programme.
5) Use the child's works (results) of the tasks as sample for data analysis and
6) Transcripts the video records and audio –tape records.

I ensure your that the participation of the child should be of their own will and they have a right to withdraw when see it fit. The data to be collected during the process of the thesis will be confidential and that anonymity underpins the ethic of the process that will be adhered to.

Attached is the consent form, which need your signature signifying that you had agreed to the above mentioned request.

All enquiries should be with pleasure forwarded to me at the contact number 0812739257.

Yours sincerely

Linea Beautty Joel

(The student)

OMBILIVE KAAVALI

Omuvali omusimanekwa

Ngame Linea Beautty Joel, omunasikola gwo Master's degree tandi ilongele omwaalu ko University ya Rhodes mo South Africa. Onda tegelelwa opo ndi ninge omapekapeko. Onda hogolola okuninga omapekapeko moshitopolwa shokulonga omwaalu.

Sho nda tala onakuziwa yokulonga kwandje, aalongi yomwaalu ohatu iyadha tatu longo uunona okuyalula omwaalu ndele eyalulo ndyono inali ikolelela komankankameno gomuule. Uunona ohawu longwa okuyalula omwaalu ihe ihawu yalula wushisho tawu ningi nenge wu vule okufatulula kutya omolwashike wuna okuyalulila komukalo ngoka. Elalakano lyomapekapeko gandje oku konakona omukalo gwokulonga omwaalu ngoka gwakwatelamo uumanya, omafano nokuthaneka, opo ndi gu konakone nkene tagu vulu okukwathela uunona okutseya okutopola neuveko lyomuule. Ekonakono lyomukalo nguka gwokulonga otali ka pula uule wootundi omulongo nane mule wethimbo lyiiwike itatu lwaampo.

Oku pandje epitikilo lyoye osho oshitopolwa she yambidhidho nda pumbwa oku ziilila kaavali, nota shika pitika ndje mwaambi tayi landula mpa:

1) Omunona gwoye opo akuthe ombinga mootundi dhomapekapeko ngaka.
2) Ndi thaneke omafano govideo gaashihe tashi ka ningwa mootundi ndhika moka woo omunona gwoye omotakala.
3) Okuningila omunona gwoye omapulapulo noku kwatela oonkundathana dhika mokakaseta.
4) Okundhindhilika pambapila iilonga yomunona mbyola ta ningi pethimbo lyomapekapeko opo shika ninge oshitopolwa shomushangwahokololo gwomapekapeko.
5) Ndika longithe iilonga yomunona mekonakono lyomapekapeko.
6) Okuka tula oonkundathana adhihe dhakwatelwa mokakaseta oshowo omafano govideo pambapila.

Oto yelithilwa woo kutya ekuthombinga olya manguluka,omunona okuna uuthemba okuzamo uuna ewete shapumbiwa. Kehe shimwe tashi kaningwa oshili oshiholekwa pokati komuningi gwomapekapeko, ngo taka konakona omapekapeko naakuthimbinga. Omadhina goshili itagaka longithwa momushangwahokololo gwomapekapeko.

Shaina okafoloma pevi hoka taka ulike kutya owa pitika omunona a kuthe ombinga nomuningi gwomapekapeko a ninge ashihe sha tumbulwa pombanda.

Omapulapulo ukitheni konomola yongodhi XXXX.

Neyelo Gweni

Linea Beautty Joel

(Omunasikola)

8.4 Appendix four

<u>INTERVETION PROGRAMME ACTIVITIES (*Joel Linea Beautty)*</u>

<u>Lesson one</u> **<u>Day one</u>**

<u>Diagnostic test</u>

<u>WORKSHEET ONE</u> **<u>ACTIVITY ONE</u>**

1. Calculate

a) $18 \div 3 =$

b) $24 \div 4 =$

c) $30 \div 5 =$

d) $80 \div 20 =$

e) $45 \div 9 =$

2. Look at these questions below and calculate

a) 18 is $3 \times \boxed{}$

b) 24 is $4 \times \boxed{}$

c) 30 is $5 \times \boxed{}$

d) 80 is $20 \times \boxed{}$

e) 45 is $9 \times \boxed{}$

3. Take 12 stones and arrange them equally in 4 groups. How many stones does each group have?
4. Write the number notation of the statement above?

5. Take 18 stones and measure out groups of 9. How many groups did you form?
6. Write the number notation of the statement above.

<u>Oral (interview) follow up questions for diagnostic test: Lesson one</u>

1. Tell (describe) what people do when dividing, i.e. if you see 18 ÷ 3 what do you do?

2. Can you tell what you did to get the answer in question 1?

3. Do you think multiplication is related to division? How do you think they are related?
4. Describe how you get the answer in question 3, what did you do to get the answer?
5. Justify: do you think that is the only correct answer you can get in question 3 and if yes, what makes you think so?
6. Can you give an example of the sharing situation as given in question 3?

7. Describe how you get the answer in question 5, what did you do to get the answer?
8. Justify: do you think that is the only correct answer you can get in question 5 and if yes, what makes you think so?
9. Can you give an example of the measurement out situation as given in question 5?

The purpose of the diagnostic test

The assumption of the study is that learners are used to the procedural version of division thus, the test aim to find out how efficient, accurate and effective can learner divide procedurally, the second aim is to find out the conceptual understanding of division they have that will be acquired through the follow up questions that learners would have to answer orally. The extent of their description will also show their adaptive reasoning. At the end of this diagnostic test the researcher would have the picture of the learners' prior knowledge of division.

Lesson two **Day One**

Partitive (Sharing) Question **Visualization aspect (concrete)**

Lesson objectives (sub-goal):

- Conceptually: learners should be able to identify that when sharing the divided and quantity units of sets/ group use to be provided. The dividend should be equally shared into the number of sets given. Sharing relates to making the desired number of groups, these groups must be equal. The result is the size of each group.
- Procedurally: learners should be able to (1) relate the division concepts to number representations (notations) i.e. 18 stone shared into 3 groups in number representation is $18 \div 3$ (2) to carry out procedures of sharing division which are: forming equal groups and dividing the stones into groups equally by counting.
- Adaptive reasoning: Learners would be able to describe how the sharing process, justify their solution based on basis of equal groups and size of groups formed, give examples of other sharing situations in a story or as number notation.

WORKSHEET TWO **ACTIVITY TWO**

1. Take 18 stones and share them equally into 3 boxes. How many stones does each group have?
2. Write the number sentence of the situation
3. Take 24 stones and share them equally between 4 learners, how many stones does each learner has?
4. Write the number sentence of the situation
5. Take 30 stones and pack them equally into 5 plastics bags, how many stones do each bag have?

6. Write the number sentence of the situation
7. Take 80 stones and arrange the equally into 4 groups, how many stones does each group have?
8. Write the number sentence of the situation
9. Take 45 stones and arrange them equally into 5 groups, how many stones do each group have?
10. Write the number sentence of the situation

<u>ORAL (INTERVIEW) FOLLOW UP QUESTIONS: LESSON TWO</u>

a) Why did you choose that number notation in question 1 (why didn't you choose addition, subtraction or multiplication rather?)

b) What answer did you get when you shared 18 stones equally into 3 boxes?

c) Can you tell me how you shared the 18 stones equally into 3 boxes or what you did to get the answer in question 1?

d) Justify: do you think you can get any other answer apart from the one you get in question 1? If not, what makes you so sure that that would be the only correct answer?

e) Can you come up with the other sharing situation you can think of as such in question 1?

a) Why did you choose that number notation in question 3 (why didn't you choose addition, subtraction or multiplication rather?)

b) What answer did you get when you shared 24 stones equally between 4 learners?

c) Can you tell me how you shared the stones or what you did to get the answer in question 3?

d) Justify: do you think you can get any other answer apart from the one you get in question 3? If not, what makes you so sure that that would be the only correct answer?

e) Can you come up with the other sharing situation you can think of as such in question 3?

a) Why did you choose that number notation for question 5 (why didn't you choose addition, subtraction or multiplication rather?)

b) What answer did you get when you packed 30 stones equally into 5 plastics bags?

c) Can you tell me how you shared 30 stones equally into 5 plastics or what you did to get the answer in question 5?

d) Justify: do you think you can get any other answer apart from the one you get in question 5? If not, what makes you so sure that that would be the only correct answer?

e) Can you come up with the other sharing situation you can think of as such in question 5?

a) Why did you choose that number notation in question 7 (why didn't you choose addition, subtraction or multiplication rather?)

b) What answer did you get when you arranged 80 stones equally into 4 groups?

c) Can you tell me how you shared the 80 stones equally into 4 groups or what you did to get the answer in question 7?

d) Justify: do you think you can get any other answer apart from the one you get in question 7? If not, what makes you so sure that that would be the only correct answer?

e) Can you come up with the other sharing situation you can think of as such in question 7?

a) Why did you choose that number notation in question 9 (why didn't you choose addition, subtraction or multiplication rather?)

b) What answer did you get when you arranged 45 stones equally into 5 groups?

c) Can you tell me how you shared the 45 stones equally into 5 groups or what you did to get the answer in question 9?

d) Justify: do you think you can get any other answer apart from the one you get in question 9? If not, what makes you so sure that that would be the only correct answer?

e) Can you come up with the other sharing situation you can think of as such in question 9?

Lesson three **Day two**

Intuitive models of division in partitive division: Visualization aspect (concrete)

Lesson objectives (sub-goal):

- Conceptually: learners should be able to explain how other concepts such as repeated subtraction, repeated addition and multiplication are related to partitive division.
- Procedurally: learners should be able to (1) relate the division concepts to number representations (notations) i.e. 18 stone shared into 3 groups in number representation

is 18 ÷ 3 (2) to carry out procedures of sharing division which are: forming equal groups and dividing the stones into groups equally by counting.

- Adaptive reasoning: Learners would be able to describe the subtraction, addition and multiplication process and relate it to sharing division, justify their solution based on basis of equal groups and size of groups formed, give examples of other sharing situations in a story or as number notation.

WORKSHEET THREE **ACTIVITY THREE**

1. Take 20 stones and repeatedly take away groups of 5 stones until 20 stones are finished. How many times was 5 stones taken away to finish 20 stones?
2. Can you rewrite the process in number notation? Begins as follow:

   20 – 5 =

   $\boxed{}$ - 5 =

3. Write a related number sentence of division to the process above.
4. If you cannot get it repeat the process this time by diving 20 stone into 4 groups of equal stones and see what you get after that write the number sentence of division that relate to that division process then.
5. Division can also be worked out by addition, take 3 stones keep adding 3 stones to them until 15 are reached. How many times did you add 3 stones together to reach 15? Follow up questions:
6. Can you rewrite the process in number notation? Begins as follow:

   3

   3 + 3 =

   $\boxed{}$ + 3 =

7. Write a related number sentence of division to the process above.

8. If you cannot get it repeat the process this time by diving 15 stone into 5 groups of equal stones and see what you get after that write the number sentence of division that relate to that division process then
9. Multiplication is an inverse of division, take 50 stones and regroup them into equal 5 groups of stones what number of stones times 5 gives 50 stones?
10. Can you rewrite the process in number notation? Begins as follow:

$$5 \times \boxed{} = 50$$

11. Write a related number sentence of division to the process above. Begins with 50 ÷ = $\boxed{}$
12. If you cannot get it repeat the process this time by diving 50 stone into 5 groups of equal stones and see what you get after that write the number sentence of division that relate to that division process then.

ORAL (INTERVIEW) FOLLOW UP QUESTIONS: LESSON THREE

a) Tell me why you think the process of subtraction above gave the same answer as that process of 20 stones arranged equally into 4 groups in question 1.

b) Can you tell me how you get the answer in the subtraction process question 2 and in division process question 1and how these processes may be related?

c) Justify: do you think you can get any other answer apart from the one you get in subtraction process or that of division in question 1 and 2? If not, what makes you so sure that that would be the only correct answer?

d) Can you come up with the same other subtraction situation you can think of as that of question 1 and 2 and relate it to the division?

a) Tell me why you think the process of addition in question 6 gave the same answer as that process of 15 stones arranged equally into 5 groups in question 5.

b) Can you tell me how you get the answer in the addition process question 6 and in division process in question 5 and how these process may be related?

c) Justify: do you think you can get any other answer apart from the one you get in addition process in question 6 or that of division in question 5? If not, what makes you so sure that that would be the only correct answer?

d) Can you come up with the same other addition situation you can think of as such of questions 5 and 6 and relate it to the division?

a) Tell me why you think the process of multiplication in question 10 gave the same answer as that process of 50 stones arranged equally into 5 groups in question 9.

b) Can you tell me how you get the answer in the multiplication process in question 10 and in division process in question 9 and how these process may be related?

c) Justify: do you think you can get any other answer apart from the one you get in multiplication process question 10 or that of division in question 9? If not, what makes you so sure that that would be the only correct answer?

d) Can you come up with the same other multiplication situation you can think of as such of question 9 and 10 and relate it to the division?

Lesson Four **Day Two**

Partitive (Sharing) Question **Visualization aspect (Drawing)**

Lesson objectives (sub-goal):

- Conceptually: learners should be able to identify that sharing relates to making the desired number of groups, these groups must be equal. The result is the size of each group.

- Procedurally: learners should be able to (1) relate the division concepts to number representations (notations) i.e. 18 stone shared into 3 groups in number representation is 18 ÷ 3 (2) to carry out procedures of sharing division which are: drawing equal groups and show in drawing the equal division of the stones into groups.
- Adaptive reasoning: Learners would be able to describe their sharing drawing process, justify their solution based on basis of equal groups and size of groups formed, and gives examples of other sharing situations in a story or as number notation.

WORKSHEET FOUR **ACTIVITY FOUR**

1. Draw to show 18 apples and shared equally into 3 boxes. How many apples does each box have?
2. Write the number sentence of the situation
3. Draw to show 24 sweets shared equally between 4 learners, how many sweets does each learner have?
4. Write the number sentence of the situation
5. Draw to show 30 sticks arranged equally into 5 groups of equal sticks, how many sticks do each group have?
6. Write the number sentence of the situation
7. Draw to show 80 beans filled equally into 4 bags, how many beans does each bag have?
8. Write the number sentence of the situation
9. Draw to show 45 cups arranged equally into 5 groups, how many cups do each group have?

10. Write the number sentence of the situation

ORAL (INTERVIEW) FOLLOW UP QUESTIONS: LESSON FOUR

a) Why did you choose that number notation in question 2 (why didn't you choose addition, subtraction or multiplication rather?)

b) What answer did you get when you shared 18 apples equally into 3 boxes?

c) Can you tell me how you shared the 18 apples equally into 3 boxes or what you did to get the answer in question 1?

d) Justify: do you think you can get any other answer apart from the one you get in question 1 and 2? If not, what makes you so sure that that would be the only correct answer?

e) Can you come up with the other sharing situation you can think of as such of question 1 and 2?

    a) Why did you choose that number notation in question 4 (why didn't you choose addition, subtraction or multiplication rather?)

    b) What answer did you get when you shared 24 sweets equally between 4 learners in question 3?

    c) Can you tell me how you shared the sweets in question 3 or what you did to get the answer?

    d) Justify: do you think you can get any other answer apart from the one you get in question 3 and 4? If not, what makes you so sure that that would be the only correct answer?

e) Can you come up with the other sharing situation you can think of as such of question 3 and 4?

a) Why did you choose that number notation in question 6 (why didn't you choose addition, subtraction or multiplication rather?)

b) What answer did you get when you grouped 30 sticks equally into 5 groups of equal sticks in question 5?

c) Can you tell me how you arranged the 30 sticks equally into 5 groups or what you did to get the answer in question 5?

d) Justify: do you think you can get any other answer apart from the one you get in question 5 and 6? If not, what makes you so sure that that would be the only correct answer?

e) Can you come up with the other sharing situation you can think of as such of question 5 and 6?

a) Why did you choose that number notation in question 8 (why didn't you choose addition, subtraction or multiplication rather?)

b) What answer did you get when you filled 80 beans equally into 4 bags in question 7?

c) Can you tell me how you filled the 80 beans equally into 4 bags or what you did to get the answer in question 7?

d) Justify: do you think you can get any other answer apart from the one you get in question 7 and 8? If not, what makes you so sure that that would be the only correct answer?

e) Can you come up with the other sharing situation you can think of as such of question 7 and 8?

a) Why did you choose that number notation in question 10 (why didn't you choose addition, subtraction or multiplication rather?)

b) What answer did you get when you arranged 45 cups equally into 5 groups in question 9?

c) Can you tell me how you shared the 45 cups equally into 5 groups or what you did to get the answer in question 9?

d) Justify: do you think you can get any other answer apart from the one you get in question 9 and 10? If not, what makes you so sure that that would be the only correct answer?

e) Can you come up with the other sharing situation you can think of as such in question 9 and 10?

Lesson Five **Day Three**

Intuitive models of division in partitive division: Visualization aspect (concrete)

Lesson objectives (sub-goal):

- Conceptually: learners should be able to explain how other concepts such as repeated subtraction, repeated addition and multiplication are related to partitive division.
- Procedurally: learners should be able to (1) relate the division concepts to number representations (notations) i.e. 18 stone shared into 3 groups in number representation

is 18 ÷ 3 (2) to carry out procedures of sharing division which are: forming equal groups and dividing the stones into groups equally by counting.

- Adaptive reasoning: Learners would be able to describe the subtraction, addition and multiplication drawing process and relate it to sharing division, justify their solution based on basis of equal groups and size of groups formed, give examples of other sharing situations in a story or as number notation.

WORKSHEET FIVE **ACTIVITY FIVE**

1. Draw to show 20 apples by repeatedly take away 5 groups of equal apples until 20 apples are finished. How many apples does each group have? And how many times did you repeatedly take away that number to finish 20 apples?
2. Can you rewrite the process in number notation? Begins as follow:

   21 – 4 =

   $\boxed{}$ - 4 =

3. Write a related number sentence of division to the process above.
4. If you cannot get it repeat the process this time by diving 20 apples into 5 groups of equal apples and see what you get after that write the number sentence of division that relate to that division process then.
5. Division can also be worked out by addition, draw to show 3 sweets added repeatedly until 15 sweets are reached. How many times did you add 3 sweets together to reach 15 sweets?
6. Can you rewrite the process in number notation? Begins as follow:

   3

   3 + 3 =

   $\boxed{}$+ 3 =

7. Write a related number sentence of division to the process above.

8. If you cannot get it repeat the process this time by diving 15 sweets into 5 groups of equal sweets and see what you get after that write the number sentence of division that relate to that division process then.
11. Multiplication is an inverse of division, draw to show 50 sticks and regroup them in drawing into equal 5 groups of sticks. What number of sticks times 5 gives 50 sticks?
12. Can you rewrite the process in number notation? Begins as follow:

$$5 \times \square = 50$$

13. Write a related number sentence of division to the process above. Begins with 50 ÷ = $\square$
14. If you cannot get it repeat the process this time by diving 50 sticks into 5 groups of equal sticks and see what you get after that write the number sentence of division that relate to that division process then.

ORAL (INTERVIEW) FOLLOW UP QUESTIONS: LESSON FIVE

a) Tell me why you think the process of subtraction in question 2 gave the same answer as that process of 20 apples arranged equally into 5 groups in question 1.

b) Can you tell me how you get the answer in the subtraction process in question 2 and in division process question 1 and how these processes may be related?

c) Justify: do you think you can get any other answer apart from the one you get in subtraction process question 2 or that of division in question 1? If not, what makes you so sure that that would be the only correct answer?

d) Can you come up with the same other subtraction situation you can think of as such in questions 1 and 2 and relate it to the division?

a) Tell me why you think the process of addition in question 6 gave the same answer as that process of 15 sweets arranged equally into 5 groups in question 5.

b) Can you tell me how you get the answer in the addition process in question 6 and in division process question 5 and how these process may be related?

c) Justify: do you think you can get any other answer apart from the one you get in addition process question 6 or that of division in question 5? If not, what makes you so sure that that would be the only correct answer?

d) Can you come up with the same other addition situation you can think of as such of questions 5 and 6 and relate it to the division?

a) Tell me why you think the process of multiplication in question 10 gave the same answer as that process of 50 sticks arranged equally into 5 groups in question 9.

b) Can you tell me how you get the answer in the multiplication process question 10 and in division process question 9 and how these process may be related?

c) Justify: do you think you can get any other answer apart from the one you get in multiplication process question 10 or that of division question 9? If not, what makes you so sure that that would be the only correct answer?

d) Can you come up with the same other multiplication situation you can think of as such of questions 9 and 10 and relate it to the division?

<u>Lesson Six</u> **<u>Day Three</u>**

<u>Partitive (Sharing) Question</u> **<u>Visualization aspect (Icons)</u>**

<u>Lesson objectives (sub-goal):</u>

- Conceptually: learners should be able to identify that when sharing the divided and quantity units of sets/ group use to be provided. The dividend should be equally shared into the number of sets given. Sharing relates to making the desired number of groups, these groups must be equal. The result is the size of each group.

- Procedurally: learners should be able to (1) relate the division concepts to number representations (notations) i.e. 18 stone shared into 3 groups in number representation is 18 ÷ 3 (2) to carry out procedures of sharing division which are: forming equal groups and dividing the stones into groups equally by counting.
- Adaptive reasoning: Learners would be able to describe how they used icon to represent the sharing drawing process, justify their solution based on basis of equal groups and size of groups formed, and gives examples of other sharing situations in a story or as number notation.

WORKSHEET SIX ACTIVITY SIX

1. Use dots to represent 18 apples and circle to represent boxes, draw to show 18 apples shared equally into 3 boxes. How many apples does each box have?
2. Write the number sentence of the situation
3. Use dots to represent 24 sweets and rectangle shapes to represent 4 learners, draw to show 24 sweets shared equally between 4 learners, how many sweets does each learner have?
4. Write the number sentence of the situation
5. Use dot to represent 30 sticks and triangle shapes to represent groups, Draw to show 30 sticks arranged equally into 5 groups of equal sticks, how many sticks do each group have?
6. Write the number sentence of the situation
7. Use dots to represent 80 beans and rectangle shapes to represent bags, draw to show 80 beans filled equally into 4 bags, how many beans does each bag have?

8. Write the number sentence of the situation

9. Use dots to represent 45 cups and circle shapes to represent groups, draw to show 45 cups arranged equally into 5 groups, how many cups do each group have?

10. Write the number sentence of the situation

ORAL (INTERVIEW) FOLLOW UP QUESTIONS: LESSON SIX

a) Why did you choose that number notation in question 2 (why didn't you choose addition, subtraction or multiplication rather?)

b) What answer did you get when you shared 18 apples equally into 3 boxes in question 1?

c) Can you tell me how you shared the 18 apples equally into 3 boxes in question 1or what you did to get the answer?

d) Justify: do you think you can get any other answer apart from the one you get in question 1 and 2? If not, what makes you so sure that that would be the only correct answer?

e) Can you come up with the other sharing situation you can think of as such of question 1 and 2?

    a) Why did you choose that number notation in question 4 (why didn't you choose addition, subtraction or multiplication rather?)

    b) What answer did you get when you shared 24 sweets equally between 4 learners in question 3?

    c) Can you tell me how you shared the 24 sweets equally between 4 learners or what you did to get the answer in question 3?

d) Justify: do you think you can get any other answer apart from the one you get in questions 3 and 4? If not, what makes you so sure that that would be the only correct answer?

e) Can you come up with the other sharing situation you can think of as such of question 3 and 4?

a) Why did you choose that number notation in question 6 (why didn't you choose addition, subtraction or multiplication rather?)

b) What answer did you get when you grouped 30 sticks equally into 5 groups of equal sticks in question 5?

c) Can you tell me how you arranged the 30 sticks equally into 5 groups or what you did to get the answer in question 5?

d) Justify: do you think you can get any other answer apart from the one you get in question 5 and 6? If not, what makes you so sure that that would be the only correct answer?

e) Can you come up with the other sharing situation you can think of as such of questions 5 and 6?

a) Why did you choose that number notation in question 8 (why didn't you choose addition, subtraction or multiplication rather?)

b) What answer did you get when you filled 80 beans equally into 4 bags in question 7?

c) Can you tell me how you filled the 80 beans equally into 4 bags or what you did to get the answer in question 7?

d) Justify: do you think you can get any other answer apart from the one you get in questions 7 and 8? If not, what makes you so sure that that would be the only correct answer?

e) Can you come up with the other sharing situation you can think of as such of questions 7 and 8?

a) Why did you choose that number notation in question 10 (why didn't you choose addition, subtraction or multiplication rather?)

b) What answer did you get when you arranged 45 cups equally into 5 groups in question 9?

c) Can you tell me how you shared the 45 cups equally into 5 groups or what you did to get the answer in question 9?

d) Justify: do you think you can get any other answer apart from the one you get in question 9 and 10? If not, what makes you so sure that that would be the only correct answer?

e) Can you come up with the other sharing situation you can think of as such of question 9 and 10?

Lesson Seven **Day Four**

Intuitive models of division in partitive division: Visualization aspect (Icons)

Lesson objectives (sub-goal):

- Conceptually: learners should be able to explain how other concepts such as repeated subtraction, repeated addition and multiplication are related to partitive division.

- Procedurally: learners should be able to (1) relate the division concepts to number representations (notations) i.e. 18 stone shared into 3 groups in number representation is 18 ÷ 3 (2) to carry out procedures of sharing division which are: forming equal groups and dividing the stones into groups equally by counting.

- Adaptive reasoning: Learners would be able to describe their use of icons representation of subtraction, addition and multiplication drawing process and relate it to sharing division, justify their solution based on basis of equal groups and size of groups formed, give examples of other sharing situations in a story or as number notation.

WORKSHEET SEVEN **ACTIVITY SEVEN**

1. Use dots to represent 20 apples and circle shapes to represent 5 groups, draw to show 20 apples by repeatedly take away 5 groups of equal apples until 20 apples are finished. How many apples does each group have? And how many times did you repeatedly take away that number to finish 20 apples?
2. Can you rewrite the process in number notation? Begins as follow:

   20 - 4 =

   $\boxed{}$ - 4 =
3. Write a related number sentence of division to the process above.
4. If you cannot get it repeat the process this time by diving 20 apples into 5 groups of equal apples and see what you get after that write the number sentence of division that relate to that division process then.
5. Division can also be worked out by addition, use dots to represent sweets, draw to show 3 sweets added repeatedly until 15 sweets are reached. How many times did you add 3 sweets together to reach 15 sweets?
6. Can you rewrite the process in number notation? Begins as follow:

   3

   3 + 3 =

$\square + 3 =$

7. Write a related number sentence of division to the process above.
8. If you cannot get it repeat the process this time by diving 15 sweets into 5 groups of equal sweets and see what you get after that write the number sentence of division that relate to that division process then.
9. Multiplication is an inverse of division, use dots to represent 50 sticks and rectangle shapes to represent groups, draw to show 50 sticks and regroup them in drawing into equal 5 groups of sticks. What number of sticks times 5 gives 50 sticks?
10. Can you rewrite the process in number notation? Begins as follow:

$$5 \times \square = 50$$

11. Write a related number sentence of division to the process above. Begins with $50 \div = \square$
12. If you cannot get it repeat the process this time by diving 50 sticks into 5 groups of equal sticks and see what you get after that write the number sentence of division that relate to that division process then.

ORAL (INTERVIEW) FOLLOW UP QUESTIONS: LESSON SEVEN

a) Tell me why you think the process of subtraction above gave the same answer as that process of 20 apples arranged equally into 5 groups.
b) Can you tell me how you get the answer in the subtraction process question 2 and in division process question 1 and how these process may be related?

c) Justify: do you think you can get any other answer apart from the one you get in subtraction process question 2 or that of division question 1 and 2? If not, what makes you so sure that that would be the only correct answer?

d) Can you come up with the same other subtraction situation you can think of as such of question 1 and 2 and relate it to the division?

a) Tell me why you think the process of addition in question 6 gave the same answer as that process of 15 sweets arranged equally into 5 groups in question 5.

b) Can you tell me how you get the answer in the addition process question 6 and in division process question 5 and how these process may be related?

c) Justify: do you think you can get any other answer apart from the one you get in addition process in question 6 or that of division in question 5? If not, what makes you so sure that that would be the only correct answer?

d) Can you come up with the same other addition situation you can think of as such of questions 5 and 6 and relate it to the division?

a) Tell me why you think the process of multiplication in question 10 gave the same answer as that process of 50 sticks arranged equally into 5 groups in question 9.

b) Can you tell me how you get the answer in the multiplication process in question 10 and in division process question 9 and how these process may be related?

c) Justify: do you think you can get any other answer apart from the one you get in multiplication process question 10 or that of division question 9? If not, what makes you so sure that that would be the only correct answer?

d) Can you come up with the same other multiplication situation you can think of as such of question 9 and 10 and relate it to the division?

Lesson Eight **Day Four**

Quotitive (Measurement) Question (concrete) **Visualization aspect**

Lesson objectives (sub-goal):

- Conceptually: learners should be able to identify that quotitive (measurement) relates measuring out the desired size of each group from the divided. The result is the number of groups.
- Procedurally: learners should be able to (1) relate the division concepts to number representations (notations) i.e. 18 stone shared into 3 groups in number representation is 18 ÷ 3 (2) to carry out procedures of quotitive (measurement out) division which are: forming groups of equal size and determine the groups by counting them.
- Adaptive reasoning: Learners would be able to describe the measurement process, justify their solution based on basis of groups of equal size and number of groups formed, and gives examples of other measurement (quotitive) division situations in a story or as number notation.

WORKSHEET EIGHT ACTIVITY EIGHT

1. Look at the 18 group of stones at the table A, groups of 3 stones are to be measured out, the first group of 3 stones is already measured out. Continue to measure out groups of 3 stones and find out how many groups can be measured out?
   Follow up questions:
2. Write the number sentence of this division situation
3. On table B, there are 27 stones and groups of 9 stones are to be measured out. The first group is already measured out for you. Continue to measure out groups of 9 stones and find out how many groups of 9 can be measured out?
   Follow up questions:
4. Write the number sentence of this division situation
5. On table C, there are 115 stones and groups of 20 stones are to be measured out. The first group of 20 is already measured out for you. Continue to measure out

more other groups of 20 and find out how many groups of 20 can be measured out?

Follow up questions:

6. Write the number sentence of this division situation

ORAL (INTERVIEW) FOLLOW UP QUESTIONS: LESSON EIGHT

a) Why did you choose that number notation in question 2 (why didn't you choose addition, subtraction or multiplication rather?)

b) What answer did you get when you divide 18 stones by measuring out groups of 3 stones in question 1?

c) Can you tell me how you measure out the groups of 3 from 18 or what you did to get the answer in question 1?

d) Justify: do you think you can get any other answer apart from the one you get in question 1? If not, what makes you so sure that that would be the only correct answer?

e) Can you come up with the other measurement out situation you can think of as such of question 1?

a) Why did you choose that number notation in question 4 (why didn't you choose addition, subtraction or multiplication rather?)

b) What answer did you get when you divide 27 stones by measuring out groups of 9 stones in question 3?

c) Can you tell me how you measure out the groups of 9 stones from 27 or what you did to get the answer in question 3?

d) Justify: do you think you can get any other answer apart from the one you get in question 3? If not, what makes you so sure that that would be the only correct answer?

e) Can you come up with the other measurement out situation you can think of as such of question 3?

a) Why did you choose that number notation in question 6 (why didn't you choose addition, subtraction or multiplication rather?)

b) What answer did you get when you divide 115 stones by measuring out groups of 20 stones in question 5?

c) Can you tell me how you measure out the groups of 20 stones from 115 or what you did to get the answer in question 5?

d) The last group has number of stones less than 20, what do we call that number?

e) Justify: do you think you can get any other answer apart from the one you get in question 5? If not, what makes you so sure that that would be the only correct answer?

f) Can you come up with the other measurement out situation you can think of as such of question 5?

Lesson Nine **Day Five**

Intuitive models of division in Quotitive (Measurement) Visualization aspect (concrete)

Lesson objectives (sub-goal):

- Conceptually: learners should be able to explain how other concepts such as repeated subtraction, repeated addition and multiplication are related to measurement out (quotitive) division.

- Procedurally: learners should be able to (1) relate the division concepts to number representations (notations) i.e. 18 stone shared into 3 groups in number representation is 18 ÷ 3 (2) to carry out procedures of quotitive (measurement out) division which are: forming groups of equal size and determine the groups by counting them.

- Adaptive reasoning: Learners would be able to describe the measurement process, justify their solution based on basis of groups of equal size and number of groups formed, and gives examples of other measurement (quotitive) division situations in a story or as number notation.

WORKSHEET NINE   ACTIVITY NINE

1. Look at the 40 stones at the table A, groups of 10 stones are to be measured out by repeatedly taking away 10 stones from the 40 stones until all the stones are finished, the first group of 10 stones is already measured out. Continue to measure out groups of 10 stones and find out how many groups can be measured out?

   Follow up questions:

2. Can you rewrite the number process sentences of this subtraction situation

   40 - 10 =

   $\boxed{}$ - 10 =

3. Write a related number sentence of division to the process above.
4. If you cannot get it repeat the process this time by measuring out groups of 10 stones out of 40 stones and see what you get after that write the number sentence of division that relate to that subtraction process then.

5. On table B, there are 16 stones arranged into groups of 4 stones measured out of 16 stones until they get finished. Start building 16 by adding the groups of 4 stones together. Find out how many groups of 4 you added to make 16 stones.

   Follow up questions:

6. Can you rewrite the process in number notation? Begins as follow:

   4

   4 + 4 =

   $\boxed{}$ + 4 =

7. Write a related number sentence of division to the process above.
8. If you cannot get it, repeat the process this time by diving 16 stones groups of 4 stones and see what you get after that write the number sentence of division that relate to that division process then.

1. On table C, there are 90 stones arranged in groups of multiples of 20 stones measured out. The groups of 20 are already displayed on the table for you. Count how many times groups of 20 gives 90.
2. Can you rewrite the process in number notation? Begins as follow:

   20 × $\boxed{}$ + $\boxed{}$ = 90

11. Write a related number sentence of division to the process above. Begins with 90 ÷ = $\boxed{}$
12. If you cannot get it repeat the process this time by diving 90 stones into groups of 20 and see what you get after that write the number sentence of division that relate to that division process then.

ORAL (INTERVIEW) FOLLOW UP QUESTIONS: LESSON NINE

a) Tell me why you think the process of subtraction in question 2 gave the same answer as that process of 40 stones measured into groups of 10 stones in question 1.

b) Can you tell me how you get the answer in the subtraction process question 2 and in division process question 1 and how these process may be related?

c) Justify: do you think you can get any other answer apart from the one you get in subtraction process question 2 or that of division in question 1? If not, what makes you so sure that that would be the only correct answer?

d) Can you come up with the same other subtraction situation you can think of as such of questions 1 and 2 and relate it to the division?

a) Tell me why you think the process of addition in question 6 gave the same answer as that process of 16 stones arranged groups of 4 stones in question 5.

b) Can you tell me how you get the answer in the addition process in question 6 and in division process in question 5 and how these process may be related?

c) Justify: do you think you can get any other answer apart from the one you get in addition process in question 6 or that of division in question 5? If not, what makes you so sure that that would be the only correct answer?

d) Can you come up with the same other addition situation you can think of as such of question 5 and 6 and relate it to the division?

a) Tell me why you think the process of multiplication in question 10 gave the same answer as that process of 90 stones arranged into groups of 20 in question 9.

b) Can you tell me how you get the answer in the multiplication process question 10 and in division process question 9 and how these process may be related?

c) Justify: do you think you can get any other answer apart from the one you get in multiplication process in question 10 or that of division in question 9? If not, what makes you so sure that that would be the only correct answer?

d) Can you come up with the same other multiplication situation you can think of as such of questions 9 and 10 and relate it to the division?

Lesson Ten **Day Five**

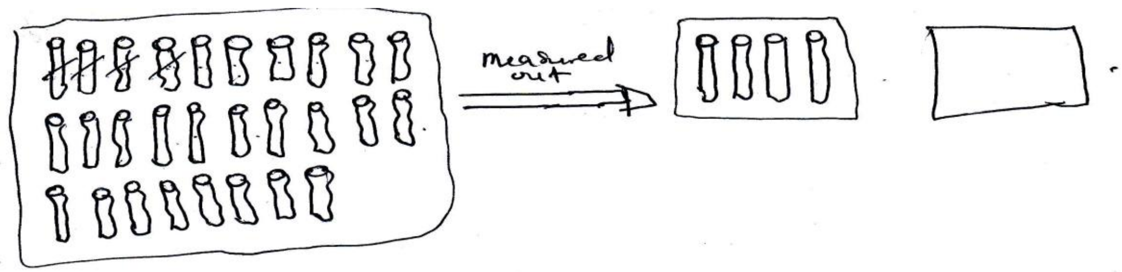
Quotitive (Measurement) **Visualization aspect (Drawing)**

Lesson objectives (sub-goal):

- Conceptually: learners should be able to explain how other concepts such as repeated subtraction, repeated addition and multiplication are related to measurement out (quotitive) division.
- Procedurally: learners should be able to (1) relate the division concepts to number representations (notations) i.e. 18 stone shared into 3 groups in number representation is 18 ÷ 3 (2) to carry out procedures of quotitive (measurement out) division which are: forming groups of equal size and determine the groups by counting them.
- Adaptive reasoning: Learners would be able to describe the measurement process, justify their solution based on basis of groups of equal size and number of groups formed, and gives examples of other measurement (quotitive) division situations in a story or as number notation.

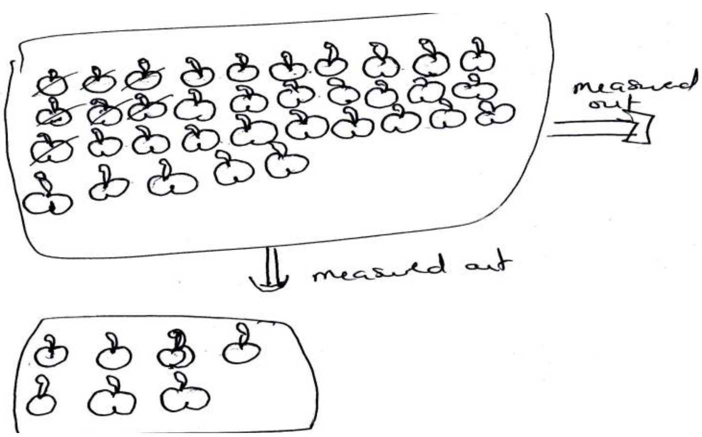
WORKSHEET TEN **ACTIVITY TEN**

1. The diagram below shows 28 sticks divided into group of 4 sticks. The first group is already measured out for you. Determine how many groups will be measured out.



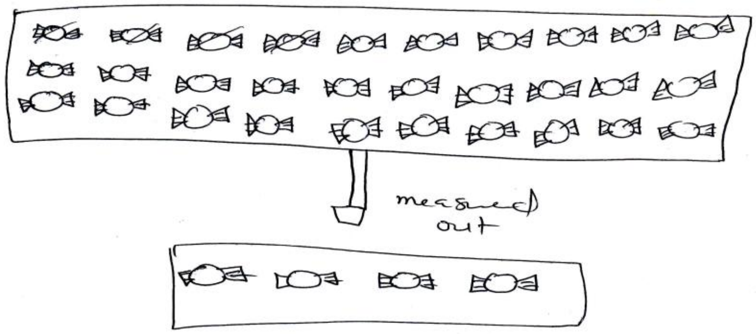
2. Write the number sentence of the situation

3. The diagram below shows 35 apples divided into groups of 7. The first group is already measured out for you. Determine how many groups will be measured out.



4. Write the number sentence of the situation

5. The diagram below shows 32 sweets divided into groups of 4 sweets. The first group is already measured out for you. Determine how many groups will be measured out.



6. Write the number sentence of the situation

ORAL (INTERVIEW) FOLLOW UP QUESTIONS: LESSON TEN

a) Why did you choose that number notation in question 2 (why didn't you choose addition, subtraction or multiplication rather?)

b) What answer did you get when you measure 4 sticks out of 28 sticks in question 1?

c) Can you tell me how you measure out 4 drawing of sticks from 28 drawing of sticks in question 1 or what you did to get the answer in question 1?

d) Justify: do you think you can get any other answer apart from the one you get as such of question 1 and 2? If not, what makes you so sure that that would be the only correct answer?

e) Can you come up with the same other measurement situation you can think of as such of question 1 and 2?

   a) Why did you choose that number notation in question 4 (why didn't you choose addition, subtraction or multiplication rather?)

b) What answer did you get when you measure the drawing of 7 groups of apples out of 35 apples in question 3?

c) Can you tell me how you measured the drawing of 7 groups of apples out 35 drawing of apples in question 3 or what you did to get the answer in question 3?

d) Justify: do you think you can get any other answer apart from the one you get in question 3 and 4? If not, what makes you so sure that that would be the only correct answer?

e) Can you come up with the same other measurement situation you can think of as such of question 3 and 4?

a) Why did you choose that number notation in question 6 (why didn't you choose addition, subtraction or multiplication rather?)

b) What answer did you get when you measured out 4 drawing of sweets out of 32 sweets in question 5?

c) Can you tell me how you measure out the drawing of 4 sweets out of 32 sweets in question 5 or what you did to get the answer in question 5?

d) Justify: do you think you can get any other answer apart from the one you get in question 5 and 6? If not, what makes you so sure that that would be the only correct answer?

e) Can you come up with the same other measuring situation you can think of as such of question 5 and 6?

Lesson Eleven **Day Six**

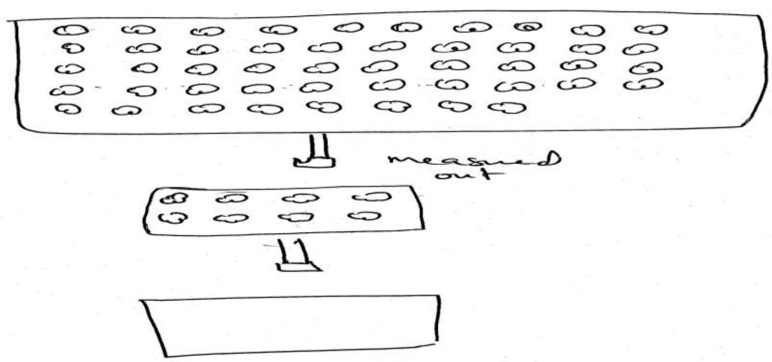
Intuitive models of division in Quotitive (Measurement) Visualization aspect (Drawing)

Lesson objectives (sub-goal):

- Conceptually: learners should be able to explain how other concepts such as repeated subtraction, repeated addition and multiplication are related to measurement out (quotitive) division.
- Procedurally: learners should be able to (1) relate the division concepts to number representations (notations) i.e. 18 stone shared into 3 groups in number representation is 18 ÷ 3 (2) to carry out procedures of quotitive (measurement out) division which are: forming groups of equal size and determine the groups by counting them.
- Adaptive reasoning: Learners would be able to describe the measurement process, justify their solution based on basis of groups of equal size and number of groups formed, and gives examples of other measurement (quotitive) division situations in a story or as number notation

WORKSHEET ELEVEN ACTIVITY ELEVEN

1. The diagram below show 48 stones, the first group of 8 is already measured out for you. Continue the drawing repeatedly taking away 8 to determine how many groups of 8 can be measured out.

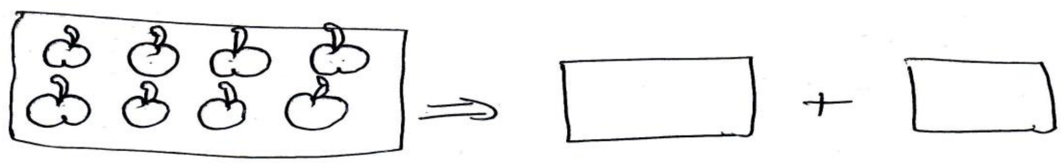


2. Can you rewrite the process in number notation? Begins as follow:

   40 – 8 =

   $\square$ - 8 =

3. Write a related number sentence of division to the process above.
4. If you cannot get it repeat the process this time by diving 48stones groups of 8in drawing and see what you get after that write the number sentence of division that relate to that division process then.
5. The diagram below shows 8 apples. Draw to start building other 8 apples by repeatedly adding groups of 4 apples together until the total of 8 apples is reached.

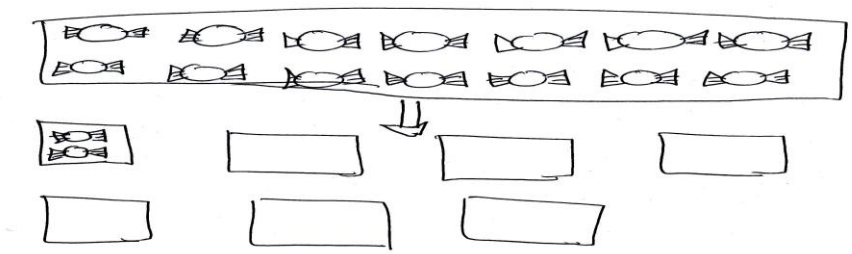


6. Can you rewrite the process in number notation? Begins as follow:

   4

   4 + $\square$=

7. Write a related number sentence of division to the process above.
8. If you cannot get it repeat the process this time by diving 8 apples into groups of 4 apples and see what you get after that write the number sentence of division that relate to that division process then.
9. The diagram below shows 14 sweets. Draw diagrams to show the measurement out of groups of 2 sweets, display them find out how many times 2 groups of sweets can make 14 sweets.



10. Can you rewrite the process in number notation? Begins as follow:

    $2 \times \boxed{} = 14$

11. Write a related number sentence of division to the process above.
12. If you cannot get it repeat the process this time by diving 14sweets into groups of 2 sweets in drawing and see what you get after that write the number sentence of division that relate to that division process then.

ORAL (INTERVIEW) FOLLOW UP QUESTIONS: LESSON ELEVEN

a) Tell me why you think the process of subtraction in question 2 gave the same answer as that process of division of drawing of groups of 8 stones measured out of 48 stones in question 1.

b) Can you tell me how you get the answer in the subtraction process in question 2 and in division process in question 1 and how these process may be related?

c) Justify: do you think you can get any other answer apart from the one you get in subtraction process in question 2 or that of division in question 1? If not, what makes you so sure that that would be the only correct answer?

d) Can you come up with the same other subtraction situation you can think of as such in question 1and 2 and relate it to the division?

a) Tell me why you think the process of addition in question 6 gave the same answer as that process of 8 apples groups of 4 in question 5.
b) Can you tell me how you get the answer in the addition process in question 6 and in division process in question 5 and how these process may be related?
c) Justify: do you think you can get any other answer apart from the one you get in addition process question 6 or that of division in question 5? If not, what makes you so sure that that would be the only correct answer?
d) Can you come up with the same other addition situation you can think of as such of question 5 and 6 and relate it to the division?

a) Tell me why you think the process of multiplication in question 10 gave the same answer as that process of 14 sweets in measurement of groups of 2 sweets in question 9.
b) Can you tell me how you get the answer in the multiplication process question 10 and in division process in question 9 and how these process may be related?
c) Justify: do you think you can get any other answer apart from the one you get in multiplication process question 10 or that of division in question 9? If not, what makes you so sure that that would be the only correct answer?
d) Can you come up with the same other multiplication situation you can think of as such of in question 9 and 10 and relate it to the division?

Lesson Twelve **Day Six**

Quotitive (Measurement) Visualization aspect (Icons)

Lesson objectives (sub-goal):

- Conceptually: learners should be able to explain how other concepts such as repeated subtraction, repeated addition and multiplication are related to measurement out (quotitive) division.
- Procedurally: learners should be able to (1) relate the division concepts to number representations (notations) i.e. 18 stone shared into 3 groups in number representation is 18 ÷ 3 (2) to carry out procedures of quotitive (measurement out) division which are: forming groups of equal size and determine the groups by counting them.
- Adaptive reasoning: Learners would be able to describe the measurement process, justify their solution based on basis of groups of equal size and number of groups formed, and gives examples of other measurement (quotitive) division situations in a story or as number notation.

WORKSHEET TWELVE ACTIVITY TWELVE

1. Use dots to represent 30 pens and circle shapes to represent people, draw to show 30 pens shared between people whose each get 6 pens. How many people get pens?
2. Write the number sentence of the situation
3. Use dots to represent 60 cookies and rectangle shapes to represent packs. Draw to show 60 cookies filled into packs of 15 equal cookies. How many packs have you filled?
4. Write the number sentence of the situation

5. Use dots to represent 15 sticks and 5 triangle shapes to represent groups, draw to show 15 sticks arranged equally into 5 groups. How many stick does each group have?

6. Write the number sentence of the situation

ORAL (INTERVIEW) FOLLOW UP QUESTIONS: LESSON TWELVE

a) Why did you choose that number notation in question 2 (why didn't you choose addition, subtraction or multiplication rather?)

b) What answer did you get when you gave 6 pens to each people in drawing of dots in question 1?

c) Can you tell me how you measure out 6 pens in drawing of dots from 30 or what you did to get the answer question 1?

d) Justify: do you think you can get any other answer apart from the one you get in question 1? If not, what makes you so sure that that would be the only correct answer?

e) Can you come up with the same other measurement situation you can think of as such of question 1?

   a) Why did you choose that number notation in question 4 (why didn't you choose addition, subtraction or multiplication rather?)

   b) What answer did you get when you filled 15 cookies into each pack from 60 (in the drawing of dots) in question 3?

   c) Can you tell me how you measure out drawing of 15 dots from 60 in question 3 or what you did to get the answer in question 3?

d) Justify: do you think you can get any other answer apart from the one you get in question 3? If not, what makes you so sure that that would be the only correct answer?

e) Can you come up with the same other measurement situation you can think of as such of question 3 and 4?

a) Why did you choose that number notation in question 6 (why didn't you choose addition, subtraction or multiplication rather?)

b) What answer did you get when you arranged the drawing of dots in groups 5 equal sticks of 15 sticks in question 5?

c) Can you tell me how you measure out in drawing of dots the groups of 5 in 15 in question 5 or what you did to get the answer?

d) Justify: do you think you can get any other answer apart from the one you get in question 5? If not, what makes you so sure that that would be the only correct answer?

e) Can you come up with the same other measurement situation you can think of as such of question 5?

Lesson Thirteen **Day Seven**

Intuitive models of division in Quotitive (Measurement) Visualization aspect (Icons)

Lesson objectives (sub-goal):

- Conceptually: learners should be able to explain how other concepts such as repeated subtraction, repeated addition and multiplication are related to measurement out (quotitive) division.

- Procedurally: learners should be able to (1) relate the division concepts to number representations (notations) i.e. 18 stone shared into 3 groups in number representation is 18 ÷ 3 (2) to carry out procedures of quotitive (measurement out) division which are: forming groups of equal size and determine the groups by counting them.

- Adaptive reasoning: Learners would be able to describe the measurement process, justify their solution based on basis of groups of equal size and number of groups formed, and gives examples of other measurement (quotitive) division situations in a story or as number notation.

<u>WORKSHEET THIRTEEN</u> <u>ACTIVITY THIRTEEN</u>

1. Use dots to represent 20 apples and circle shapes to represent groups. Draw to show the repeated subtraction (repeatedly take away) groups of 3 from 20 apples until 20 apples are finished. Find out how many groups of 3 you were able to make?

2. Can you rewrite the process in number notation? Begins as follow:

   20 - 3 =

   ☐- 8 =

3. Write a related number sentence of division to the process above.
4. If you cannot get it repeat the process this time by diving 20 apples groups of 3 in drawing and see what you get after that write the number sentence of division that relate to that division process then.

5. Use dots to represent 36 sweets and rectangle shape to represent groups. Draw to show groups of 6 dots repeatedly built (repeatedly added) to form 36 dots. Find out how many groups of 6 dots formed up 36 dots?

6. Can you rewrite the process in number notation? Begins as follow:

   6

$6 + 6 =$

$\boxed{} + 6 =$

7. Write a related number sentence of division to the process above.
8. If you cannot get it repeat the process this time by diving 36 sweets in groups of 6 in drawing and see what you get after that write the number sentence of division that relate to that division process then.

9. Use dots to represent 28 sticks and triangles to represent groups. Draw to show groups of 4, and find out how many times do groups of 4 makes up 28?

10. Can you rewrite the process in number notation? Begins as follow:

$4 \times \boxed{} =$

11. Write a related number sentence of division to the process above.
12. If you cannot get it repeat the process this time by diving 28sticks groups of 4in drawing and see what you get after that write the number sentence of division that relate to that division process then.

ORAL (INTERVIEW) FOLLOW UP QUESTIONS: LESSON THIRTEEN

a) Tell me why you think the process of subtraction in question 2 gave the same answer as that process of groups of 3 apples measured out from 20 apples in question 1.
b) Can you tell me how you get the answer in the subtraction process in question 2 and in division process question 1 and how these processes may be related?

c) Justify: do you think you can get any other answer apart from the one you get in subtraction process question 2 or that of division process in question 1? If not, what makes you so sure that that would be the only correct answer?

d) Can you come up with the same other subtraction situation you can think of as such of questions 1 and 2 and relate it to the division?

a) Tell me why you think the process of addition in question 6 gave the same answer as that process of groups of 6 sweets measured out from 36 sweets in question 5.
b) Can you tell me how you get the answer in the addition process question 6 and in division process question 5 and how these process may be related?

c) Justify: do you think you can get any other answer apart from the one you get in addition process question 6 or that of division process in question 5? If not, what makes you so sure that that would be the only correct answer?

d) Can you come up with the same other addition situation you can think of as such of questions 5 and 6 and relate it to the division?

a) Tell me why you think the process of multiplication in question 10 gave the same answer as that process of groups of 4 sticks measured out from 28 sticks in question 9.
b) Can you tell me how you get the answer in the multiplication process in question 10 and in division process question 9 and how these process may be related?

c) Justify: do you think you can get any other answer apart from the one you get in multiplication process in question 10 or that of division process in question 9? If not, what makes you so sure that that would be the only correct answer?

d) Can you come up with the same other multiplication situation you can think of as such of questions 9 and 10 and relate it to the division?

Lesson Fourteen **Day Seven**

Lesson Sub- goals

- To observe if learners had or can identify between partitive and quotitive division
- To see if learners can relate other concepts of addition, subtraction and multiplication to division

WORKSHEET FOURTEEN ACTIVITY FOURTEEN

1. Do activities a) and b), look at the process very carefully and tell how different you did the division.

   a) (24 ÷ 3) use drawing, dots or stones and divide **<u>24</u>** stones, dots or drawing arranged into **<u>3 groups</u>** of equal stones. How many stones does each group have?

   b) (24 ÷ 3) use drawings, dots or stones and divide **<u>24 stones</u>**, dots or drawings into **<u>groups of 3 stones</u>**

2. Do activities c) and d), look at the process very carefully and tell how different you did the division

   c) (35 ÷ 7) use drawing, dots or stones and divide **<u>35</u>** stones, dots or drawing arranged into **<u>7 groups</u>** of equal stones. How many stones does each group have?

   d) (35 ÷ 7) use drawings, dots or stones and divide **<u>35 stones</u>**, dots or drawings into **<u>groups of 7 stones</u>**

3. Use the visualization of your choice (stones, drawing or dots) and show how the following division is related to the following concepts

   18 ÷ 6 =

   Get it into the following approaches

   a) Repeated subtraction

   b) Repeated addition

   c) Multiplications

<u>ORAL (INTERVIEW) FOLLOW UP QUESTIONS: LESSON FOURTEEN</u>

1. Tell how different you did the division in a) and in b).

2. Can you relate that to addition, subtraction and multiplication?
3. tell how different you did the division you did the divisions in c) and in d)
4. Can you relate that division to addition, subtraction and multiplication?

Lesson Fifteen

Post test

THE PURPOSE

To test the learners' post knowledge if there is any change between the diagnostic test and the post test.

WORKSHEET FIFTEEN

ACTIVITY FIFTEEN

1. Calculate

a) $18 \div 3 =$
b) $24 \div 4 =$
c) $30 \div 5 =$
d) $80 \div 20 =$
e) $45 \div 9 =$

2. Look at these questions below and calculate

a) 18 is $3 \times \boxed{}$
b) 24 is $4 \times \boxed{}$
c) 30 is $5 \times \boxed{}$
d) 80 is $20 \times \boxed{}$
e) 45 is $9 \times \boxed{}$

3. Take 12 stones and arrange them equally in 4 groups. How many stones does each group have?
4. Write the number notation of the statement above.

5. Take 18 stones and measure out groups of 9. How many groups did you form?

6. Write the number notation of the statement above.

ORAL (INTERVIEW) FOLLOW UP QUESTIONS: LESSON FIFTEEN

a) Tell (describe) what people do when dividing, i.e. if you see 18 ÷ 3 what do you do?

b) Can you tell what you did to get the answer in question 1 and 2?

c) Do you think multiplication is related to division? How do you think they are related?

d) Describe how you get the answer in question 3, what did you do to get the answer in question 3?

e) Justify: do you think that is the only correct answer you can get in question 3 and if yes, what makes you think so?

f) Can you give an example of the sharing situation as given in question 3?

g) Describe how you get the answer in question 5, what did you do to get the answer in question 5?

h) Justify: do you think that is the only correct answer you can get in question 5 and if yes, what makes you think so?

i) Can you give an example of the measurement out situation as given in question 5?

8.5 Appendix five: Lesson Transcripts

DAY ONE **09 MAY 2012**

Lesson one

1.	Teacher:	good morning?
2.	Learners:	good morning teacher.
3.	Teacher:	how are you?
4.	Learners:	very well. How are you?
5.	Teacher:	I am very well.
6.	Teacher:	ok, we are going to have our holiday classes as we, as we've promised or as we've been waiting for. I hope you are ready. Isn't it?
7.	Teacher:	how many are we in the class? (One, two, three four ……..fourteen, teacher counting learners) we are fourteen, ok one of us is not going to be with us because he is sick. Dar is sick. He would not be able to be with us.
8.	Teacher:	all the classes that we would be busy with all these days, we would talk about division. Do you know what division is? Who know what division is? (Pause learners are just quiet) who can tell me what division is? Or when people are dividing, what do they do? (Pause) in your life can you remember anything you have divided or seen being divided? What does a person do when dividing? Let us talk or else we would not get out of here because we have a lot to do today. Sab! When a person divides what do they do? Or if they say divide what do you do?
9.	Sab:	is like when you have four things, then you divide with two and divide one portion here and another there.
10.	Teacher:	so Sab is saying; is like having four things and divide them with two and divide into two portions.
11.	Sab:	yes
12.	Teacher:	how do you divide them? Into un-equal portions or how?
13.	Sab:	in equal portions
14.	Teacher:	so if you are dividing you divide into equal portions right?
15.	Learners:	yes
16.	Teacher:	Can I divide ten sweets into two people?
17.	Learners:	yes
18.	Teacher:	how many each will get?
19.	One learner:	five
20.	Teacher:	so each one will have five sweets. Good! Today we will do division. Do you understand?
21.	Learners:	yes
22.	Teacher:	now tell me (the teacher started to write four different number sentence of four different operations at the chalk board), which number sentence on the chalkboard that show division here? Which number sentence on the chalkboard show division, is it A, B, C or D? Which number sentence on the chalkboard show division? We have A with this sign (pointing to the signs on the number sentences); we have B with that sign, C with that sign and D. which number sentence on the chalkboard that shows division? Which one of them at the chalkboard when you are given it you will divide?
23.	Hen:	is B
24.	Teacher:	is B dividing or subtracting?
25.	Hen:	subtracting
26.	Teacher:	I am asking the one that show division. Yes! (pointing to one learner)
27.	One learner:	is D
28.	Teacher:	so is D, if you are given this number sentence is showing division. What are you dividing? Who can explain that number sentence? What are you dividing? Who can give me an example using this number sentence in terms of how many things to be

divided and into how many portions? Era! If you are given this number sentence what does it mean? How many things and how should you divide them?

29. Era: you have twelve things to be shared by three people.

30. Teacher: you have twelve things to be shared by three people. If you have twelve things that are to be shared by three people, how many each will get? Yes! (giving a chance to one learner)

31. One learner: six

32. Teacher: will each get six? Is that so? How many each will get? Let me say, it is twelve sweets to be shared by three people, how many sweets each will get? If you are told to divide the sweets among them, how many sweets each will get? Yes! (pointing to another learner)

33. One learner: four

34. Teacher: what?

35. One learner: four

36. Teacher: yes is four ok,

37. Teacher: so I will give you the first activity. Everyone is expected to do this first activity. Can you see the paper on the table?

38. Learners: yes

39. Teacher: you can see activity number one on the paper?

40. Learners: yes

41. Teacher: Everyone look at your paper. Can you see where it is written calculate?

42. Learners: yes

43. Teacher: I want you to divide those numbers then as they are on the paper and give the answers. I will give you the worksheets to work on but the answer has to be filled on the question paper. Do you understand?

44. Learners: yes

45. Teacher: Write your names on those worksheets and question papers. Do you understand?

46. Learners: yes

47. Teacher: ok

48. Teacher: I want you to listen, so that when you will approach question two everyone will know what to do. Everyone listen to know what you are expected to do at question two. (teacher went to the chalkboard and write 12 is 4 times) At number two, there are number such as twelve is four times. Can you see something like that at number two? This on the chalkboard is just an example, I have not written the activity as it is on the question paper. But can you see something like that at question number two?

49. Learners: yes

50. Teacher: ok, this is just asking that for you to get this twelve, how many fours you need to have. (one, two, three, four, one, two, three, four, teacher counting dots that she makes in one circle each circle she put four dots inside)how many fours are these altogether?

51. Learners: eight

52. Teacher: eight, so we still need to add fours more to make twelve (one, two, three, four adding another circle with four dots inside) how many fours altogether now? How many dots altogether? Yes1 Era!

53. Era: twelve

54. Teacher: they are twelve, but how many fours are these? Or how many times are these fours? Yes! (pointing to Hanna)

55. Hanna: fours

56. Teacher: yes! (pointing to another learner)

57. One learner: three fours

58. Teacher: (talking to Hanna again) Hanna, how many dots are in this circle? (Pause) how many dots are inside this circle?

59. Hanna: four

60. Teacher: ok, four (in agreement)

61. Hanna: yes

62.	Teacher:	(pointing to the three circles as asking Hanna) this is four times one (pointing to one the circle with four dots)
63.	Hanna:	yes
64.	Teacher:	then how many fours are these? Or how many times are these fours? This is what I meant or asked in the first place Hanna?
65.	Hanna:	four
66.	Teacher:	no! This is four times one (pointing to one of the circle), this is also another four (pointing to the next circle with four dots) and this is another four (pointing to the last circle with four dots again). So how many fours are these? (Pointing to these circles each with four dots one by one) how many dots are inside this circles Hanna?
67.	Hanna:	four
68.	Teacher:	(pointing to the next circle) and here, how many dots are inside this circle?
69.	Hanna:	four
70.	Teacher:	there are four dots, so now I am asking. How many fours are these then? That is what I am asking (pause). Are you not able to count how many fours are here? Hanna!
71.	Hanna:	four
72.	Teacher:	count these fours for me and see how many fours are here (pause). Hanna! Count these fours for me aloud i.e. one, two so count the fours until you finish them. (pause) Hanna! Count these fours like, one, two, and three... Until you get the answer.
73.	Hanna:	counting (one, two, three)
74.	Teacher:	yes, so how many are these four when you counted them?
75.	Hanna:	four
76.	Teacher:	O! What?
77.	Hanna:	four
78.	Teacher:	o! Re-count them again aloud.
79.	Hanna:	(counting) one, two, three
80.	Teacher:	so how many fours are they?
81.	Hanna:	four
82.	Teacher:	four? Where is the fourth? Come to the chalkboard and count them pointing at them.
83.	Hanna:	one (pointing to the first circle)
84.	Teacher:	yes (in agreement)
85.	Hanna:	(starting to count again) one, two, three, four (but instead of counting circles she counted dots in the circle).
86.	Teacher:	can you see the number fours on top of each circle?
87.	Hanna:	yes
88.	Teacher:	ok, these fours on top of these circles like these refer to the values of dots inside the circles. Is that right?
89.	Hanna:	yes
90.	Teacher:	ok, how many times are these fours here? Count them for me.
91.	Hanna:	one, two, three, four (still counting dots)
92.	Teacher:	No! No! What you are counting is not in fours but in ones. How many dots are inside the circles?
93.	Hanna:	four
94.	Teacher:	ok, they are fours, in the same amount as the number 4 written on top isn't it?
95.	Hanna:	yes
96.	Teacher:	then count for me these fours; that are what I am asking.
97.	Hanna:	one, two, three
98.	Teacher:	yes, how many fours are these then?
99.	Hanna:	three
100.	Teacher:	they are three fours right?
101.	Hanna:	yes
102.	Teacher:	these are four times three that give twelve.
103.	Hanna:	yes

104.	Teacher:	one, two, three and four (counting the dots in the circle) this is the first four rights? One, two, three, four (counting in dots in the second circles) and this is the second fours right?
105.	Hanna:	yes
106.	Teacher:	one, two, three, four (counting the dots in third circle) and this is the third fourth right?
107.	Hanna:	yes
108.	Teacher:	and all these three fours together will make twelve right?
109.	Hanna:	yes
110.	Teacher:	you see it?
111.	Hanna:	yes
112.	Teacher:	(counting all the dots together in all three circles to show the total) one, two, three, four, five, six… twelve. So the fours time three. The four that are there times three gives twelve. Right? So that is then what you are going to do in activity one question two.
113.	Teacher:	(talking to the whole class) for number two in you question paper. There are numbers i.e. how many threes are needed to give you eighteen? Ok, you can make or draw threes in anyway, threes, threes (then the teacher makes dots for the threes) one, two, three (counting dots) this is just one three right?
114.	Learners:	yes
115.	Teacher:	one, two, three (making another three dots) that's another three. Right?
116.	Learner:	yes
117.	Teacher:	but if you add all these dots how many are they all together? Sab!
118.	Sab:	six
119.	Teacher:	six, so keep adding dots in threes until you reach eighteen and find out how many threes makes eighteen. Do you understand?
120.	Learners:	yes
121.	Teacher:	keep doing like that in all until you finish. Ok, those who were not done with number one continue with number one until you finish after that is when you can start with number two in the manner explained. Are we together?
122.	Learners:	yes
123.	Teacher:	ok
124.	Teacher:	now we will use the stones to divide. The next task says, take twelve stones and divide them into four groups. Find out that if you divide twelve stones into four groups of equal stones, how many stones would each group have? Do you understand?
125.	Learners:	yes
126.	Teacher:	how many stones did I say you take?
127.	Learner:	twelve
128.	Teacher:	twelve, and into how many groups will you share the stones?
129.	Learners:	four
130.	Teacher:	Four, so then you find out that if those stones are shared into those groups, how many stones will each group have? Do you understand?
131.	Learners:	yes
132.	Teacher:	ok, you can go and take stones.
133.	Teacher:	Sab! How did you do it? Did you took exactly twelve stones or how many did you take?
134.	Sab:	I took a handful of stones.
135.	Teacher:	yes! You took a handful of stones and what next did you do when you brought the stones at the table?
136.	Sab:	I counted twelve stones out of the stones I brought.
137.	Teacher:	and counted out twelve stones ok, how did you do it Fill? (Pause) how did you take your stones?
138.	Fill:	I took twelve stones.
139.	Teacher:	you took twelve stones from the box? Or how did you pick stones from the box?

140. Fill: I took a handful and counted out twelve.
141. Teacher: ok (murmuring you speak so low) Era! How did you do it?
142. Era: I took a handful from the box to the table and counted out twelve.
143. Teacher: how did you count?
144. Era: I counted out twelve.
145. Teacher: you counted out twelve because you need twelve rights?
146. Era: yes
147. Teacher: (talking to another learner) yes! Are you done?
148. Learner: yes
149. Teacher: how did you do it?
150. Learner: I took a handful of stones from the box and counted out twelve at the table.
151. Teacher: so you counted out twelve, (turning and talking to the whole class) now do you know the next step to continue? What did I say you will do?
152. Learners: you take twelve stones and divide them
153. Teacher: divide them into how many groups?
154. Learners: four
155. Teacher: how should be the stones shared in groups?
156. Learners: equal stones
157. Teacher: how should be the stones shared in the groups?
158. Learners: equally
159. Teacher: equally, so stones should be equally shared into those groups. Nab! How many groups you just made? (the teacher asked because she noticed Nab made a wrong number of groups)
160. Nab: three
161. Teacher: and how many groups did I say you should share the stones?
162. Learner: into four
163. Teacher: yes, I said into four groups, but not three. (Nab, stood and the teacher anticipate he is going to take more stones misunderstood that teacher is saying wrong number of stones rather than wrong number of groups). The number of stones you took is correct Nab, do not go take more stones but I say make these groups four and not three and find out how many stones each group would have if the groups are four. (Turning to another learner) how many groups did you make Dos?
164. Dos: groups are four
165. Teacher: they are four, and how many stones each group has?
166. Dos: three stones
167. Teacher: (speaking to the whole class) so write on your paper the number of stones each group has. Write that answer below question number three here. How many stones each group has? And write the answer. So first go back to your groups and find out how many stones each group has and write the answer. After writing the answer you go to the next question. The next question is saying take eighteen stones and keep taking out nine stones, nine stones until eighteen stones get finished. (Pause) are you done writing the answer?
168. Learners: yes
169. Teacher: Ok, if you done writing the answer you take those stones back to the box and take another eighteen stones from the box.
170. Teacher: are you done? Is everyone having eighteen stones?
171. Learners: yes
172. Teacher: have repeatedly taken out nine stones out of eighteen stones?
173. Some learners: yes
174. Teacher: put all the eighteen stones together and start taking nine stones repeatedly. Put them separately and count how many nines stones you took out from eighteen stones.
175. Teacher: make groups of stones, nine stones is one group each nine stones a group. (Pause) how many nine stones groups have you got? Write them write them beside question five, beside question five is where you will write the answer of how many nines you got out of eighteen. Did you write?

176.	Learners:	yes
177.	Teacher:	you write how many you took from eighteen?
178.	Learners:	yes
179.	Teacher:	ok, write beside question five, that as you count the nines (are one, two) you took out from eighteen. So write how many nines you took out from eighteen. If they are four then write four or whatever number they are. (Asking one of the learner) why are these stones scattered? I told you each nine stones you took out should be grouped as one group. Are these nine stones? Then make them a group like this is one group of nine and this is another group of nine like that. How many groups of nine are these? How many group of nine are there?
180.	Learner:	one, two
181.	Teacher:	have you written the answer then?
182.	Learner:	yes
183.	Teacher:	ok, can you see question four? Question four is asking you to write a number sentence. Have you noticed when you took those twelve stones and shared them into four groups?
184.	Learners:	yes
185.	Teacher:	do you think you can write that process in the division number sentence (pause) if you take eighteen stones and share them into three groups in the number sentence it would look as follow (showing the example on the chalkboard 18 ÷ 3 = 6) and you will get six. Right?
186.	Learners:	yes
187.	Teacher:	write a number sentence of question four for the twelve stones when you shared them into four groups, what did you get? So write twelve and divided and so on just as I have shown you eighteen stones divided by three equal to six. Write the number sentence in that manner at question number four. So the stones were twelve divided by what?
188.	Learners:	by four
189.	Teacher:	write like that at question four there. (Asking one learner) what did you got when you divided twelve by four? You wrote the answer there. Write the answer when you divided twelve stones into four groups, how many stones each group has? Good. (Talking to another learner) when you divided twelve stones, this is four but I can see you wrote two here. You did divide into four groups. Twelve divided into four groups and the answer you wrote here supposed to be the same as here, so write the correct number sentence.
190.	Teacher:	at question six you are also expected to write the number sentence of eighteen stones. You took eighteen stones and divide them by what? And how many stones did each group get? (Talking to one learner) ok, write here again. Look here! It is the same thing you did here. (Pointing to the same previous question) eighteen was shared into how many groups and how many stones each group got? Write that at question six. Good. And how many groups you got or how many nines did you took out of eighteen, write the answer.
191.	Teacher:	who can tell me, looking at question two, is that a division or a multiplication? (Pause) can you see the sign of operation at question two?
192.	Learners:	yes
193.	Teacher:	ok, look at it. Is that a division or multiplication? Hanna! Where is question two on your paper? (Hanna pointed at it) which sign is there? Is it division or is it multiplication? Hanna!
194.	Hanna:	multiplication
195.	Teacher:	Does everyone agree that it is multiplication?
196.	Learners:	yes
197.	Teacher:	Does everyone see it like that?
198.	Learners:	yes
199.	Teacher:	so I want you to think like this, do you see eighteen on question one (a)? Who can tell me what you can see at question one? Hil!

200.	Hil:	eighteen
201.	Teacher:	divided by what?
202.	Hil:	by three
203.	Teacher:	by three what answer did you get? What answer did you get? Nab, I mean Mic!
204.	Mic:	six
205.	Teacher:	you got answer six ok, that is question one (a), at question two (a) what is there? There is eighteen is, is what? Sab!
206.	Sab:	three
207.	Teacher:	three which is multiplied and there is a box there right?
208.	learners:	yes
209.	Teacher:	what did you fill in the box? Eras!
210.	Eras:	six
211.	Teacher:	you fill in six, now the question I want you to answer for me is this? Looking at division and multiplication did you notice anything similar or related? Look at it careful do you see anything similar or related? How multiplication and division does relate? Just look at question one a) you got answer six and even at question two a) you also got six, what does that mean? Yes Naf!
212.	Naf:	in question one a) there is eighteen divided by three equal to six and in question two a) there is eighteen is three times six
213.	Teacher:	ok, what does it tells you? What does it tell you? Does it tell you something? Look here all things are eighteen, in question one a) we divided eighteen and in question two a) eighteen became the answer. What does that tells you? (Pause) what it tells you about division and multiplication. You don't see anything? Eighteen divided by three you get six and six times three will give what? What does it give? Who can tell me something? Hil!
214.	Hil:	it gives six
215.	Teacher:	yes, what does that tells you in relation to division and multiplication? Can you get the division answer with the multiplication method? Do you think you can use multiplication to get the answer of eighteen divided by three? (Pause). You cannot do it? (Pause) can multiplication helps you to get the division answer?
216.	Learners:	yes
217.	Teacher:	how will you see it (pause) ok, if you cannot answer I will go ahead. When you divided in question one and two, how did you get the answer? Who can remember what you did to get the answer? You can look in your paper where you have written the answer (pause) Sab, for you to get this six, how did you have this six?
218.	Sab:	I took eighteen stones
219.	Teacher:	yes (in agreement)
220.	Sab:	and makes three circles representing groups
221.	Teacher:	yes (in agreement)
222.	Sab:	and count stones one, two, three (they way he show it means allocating stones one by one to three groups) and share the three stones two in each group
223.	Teacher:	two in each group?
224.	Sab:	I mean one stone in each group
225.	Teacher:	ok
226.	Sab:	then I continue allocating one by one
227.	Teacher:	Sab is saying, he kept taking three stones and allocates them one by one into groups until eighteen stones are done right?
228.	Sab:	yes
229.	Teacher:	did you all do it in that manner?
230.	Learners:	yes
231.	Teacher:	what makes you think that, that way you will get the correct answer? Or do you think there could be another way that can also gives you the same answer? I can see your answer, so I want you to respond to my question, Eras! Is there any other way you can use, that will still give you the correct answer? And what makes you think it will give you the correct answer? (Pause) is your answer correct? Era!

232.	Era:	yes
233.	Teacher:	what makes you think that, that way would give you a correct answer? (Pause) (Then the teacher turn to another learner) if someone tells you that your answer is wrong, how would you defend your answer that is correct? Sab! You would say you are correct because?
234.	Sab:	I will say I am correct because I took stones, I took eighteen stones and multiply with two
235.	Teacher:	o! Were you multiplying?
236.	Sab:	no
237.	Teacher:	what were you doing? You took eighteen stones and did what? (back to Era) Era, what were we doing?
238.	Era:	we were dividing
239.	Teacher:	yes we were dividing, continue
240.	Era:	dividing by three
241.	Teacher:	yes when divided by three
242.	Era:	then you keep counting out three, then make three circles and keep allocating stones to them.
243.	Teacher:	in the question of twelve stones you made four groups right?
244.	Learner:	yes
245.	Teacher:	then you start sharing the stones equally into those groups right?
246.	Learner:	yes
247.	Teacher:	can you tell me how you did it (turning to Dos) how did you do it?
248.	Dos:	first I divided into four groups
249.	Teacher:	ok, make four groups, one two, three and four, start taking stones and show me how you did it?
250.	Dos:	and count out stones and allocate three stones for each group
251.	Teacher:	how did you anticipate that there would be three stones in each group? How did you divide the stones for you to come to know that there would be three stones in each groups?
252.	Dos:	I divided to each group and give three stones to each groups
253.	Teacher:	let me say, first you make groups one, two, three and four and here are twelve stones. How did you divide stones into groups, that is the question I am asking?
254.	Dos:	I counted twelve stones and start dividing three stones into each four groups
255.	Teacher:	tell me how you divided then, did you allocate one stone per each group or how did you do it?
256.	Dos:	I started allocating stones one by one to each four group until twelve the stones get finished
257.	Teacher:	did you do that until twelve stones get finished?
258.	Dos:	yes
259.	Teacher:	ok, what makes you think that dividing in that way would gives a correct answer? (pause) (Turning to the whole class) ok in question number four, we took twelve stones. one, two, three, four, five, ……………….twelve (counting twelve stones) make four groups and start allocating stones to the groups (the teacher allocated stones one by one to each group until all four groups have three stones and twelve stones get finished) right?
260.	Learner:	yes
261.	Teacher:	now tell me how you did it, when we divided eighteen stones, when you were instructed to repeatedly take out nine stones because in that case I don't think you have been allocating stones one by one as in question four (turning to one learner) in the case of eighteen stones how did you do it? What did you do first?
262.	Learner:	I took eighteen stones
263.	Teacher:	you took eighteen stones. Then what did you do next?
264.	Learner:	and take out nine stones
265.	Teacher:	after taking out nine stones, what did you do next?
266.	Learner:	and take out nine stones again

267.	Teacher:	ok, meaning one can divide by counting or allocating one by one and you can also divide in a manner like taking out nines (meaning taking unit or elements per group from the dividend) or divide by allocating stones one by one to groups right?
268.	Learner:	yes
269.	Teacher:	what makes you think that you are correct, only two times nine would come out from eighteen? Is it possible to take out three nine stones out of eighteen?
270.	Learner:	no
271.	Teacher:	then what makes you think that only two nines would come out from eighteen and that you are correct?
272.	Learner:	because the stones get finished
273.	Teacher:	so the stones get finished and you conclude that, only two nines stones could be taken out from eighteen, because when you took out two nines stones from eighteen stones, eighteen stones were completely finished.
274.	Teacher:	I want someone to give me an example like that of twelve stones shared into four groups, which each got three stones. So give me an example using a different number. Who can give me such example and explain it in terms of what things are to be shared e.g. sweets and how to be shared, can you give such example? (Pause) Pet!
275.	Pet:	is like having thirteen sweets divided by three, four people
276.	Teacher:	how many sweets did you say?
277.	Pet:	thirteen sweets divided by four people
278.	Teacher:	how many people?
279.	Pet:	four people
280.	Teacher:	thirteen sweets shared by four people, can you please go and take thirteen stones share them into four groups and see what kind of answer you will get (turning to the class) give me example Sab!
281.	Sab:	I will take six sweets and share them to two people
282.	Teacher:	yes, how many sweets each one would have?
283.	Sab:	each would have three sweets
284.	Teacher:	so Sab is saying having six sweets and share them to two people each would get three. So I want everyone to give me your example. Fil, what example of sharing would you give?
285.	Fil:	is when you have ten sweets
286.	Teacher:	yes, and share them by how many people?
287.	Fil:	shared by four people
288.	Teacher:	four?
289.	Fil:	five people
290.	Teacher:	ok, five people how many sweets each one would have?
291.	Fil:	two
292.	Teacher:	two, ok (turning to another learner) give me your example Nab
293.	Nab:	(murmured but in the low voice)
294.	Teacher:	I do not hear you Nab speak louder
295.	Nab:	having four sweets
296.	Teacher:	ok when you have four sweets and share them how?
297.	Nab:	shared by two people
298.	Teacher:	how many sweets each one would get?
299.	Nab:	each one would have two sweets
300.	Teacher:	two, ok who else has an example? Hen
301.	Hen:	is like sharing six sweets amongst kids
302.	Teacher:	ok shared by how many kids?
303.	Hen:	shared by two kids
304.	Teacher:	how many sweets would each one have?
305.	Hen:	each one would get two, three
306.	Teacher:	each will have three, now Hil I do not want any example that has been already given by others because there are a lot of examples you can give.
307.	Hil:	having seven sweets

308.	Teacher:	how many people will share the seven sweets?
309.	Hil:	three people
310.	Teacher:	three?
311.	Hil:	yes
312.	Teacher:	do you think they would all share the sweets equally?
313.	Hil:	no
314.	Teacher:	Dos
315.	Dos:	when I have eight bananas
316.	Teacher:	yes
317.	Dos:	and share them to four
318.	Teacher:	yes
319.	Dos:	everyone would get four bananas
320.	Teacher:	how many bananas each one would get?
321.	Dos:	four
322.	Teacher:	four people? For eight bananas? How many bananas each one would get? (pause) people are four (one, two, three, four) and bananas are eight (pause) if you give one banana to each that means four are given out and four will remain, when you repeat to give two to each, will each get four for eight to get finished? (the teacher start to make drawing of eight dots she took four and allocate to four groups) Dos! I had already taken four, how many remained?
323.	Dos:	four
324.	Teacher:	one, two, three, four (allocating the remaining dots to groups again) now is there any left for you to give again?
325.	Dos:	no
326.	Teacher:	how many each would get then?
327.	Dos:	two
328.	Teacher:	each will just get two, ok

Lesson two

1.	Teacher:	take eighteen stones and share them into three groups and each group should have equal stones. Let there be only eighteen stones that you need at your table. Be able to tell how you divide and remember eighteen stones are to be divided into three groups equally. Divide the eighteen stones into three groups equally. Repeating, share them into three groups equally. Pet, how many groups have you made (counting Pet's groups) one, two, three, four, five, six. This is not three groups; I said groups should just be one, two, and three with the equal number of stones. That is the problem, Pet make three groups and keeps allocating stones to the only three groups.
2.	Teacher:	did you share stones into three groups? (Pause) did you share stones into three groups?
3.	Learners:	yes
4.	Teacher:	ok, what answer did you get when you shared the stones into three groups? How many stones does each group have? Yes!
5.	Learner:	six
6.	Teacher:	yes (turning to another learner)
7.	Learner:	six
8.	Other learners:	six, six, six, six (each learner mention their answer as they got their turn)
9.	Teacher:	so there are six stones in each group right?
10.	Learners:	yes
11.	Teacher:	if there are six stones in each groups, write for me a number sentence, write a number sentence of that, we said this is how one write a number sentence (showing to the number sentence on the chalkboard) write a number sentence containing eighteen divided by … and the answer you got. Write your number sentence below question two, below question two is where you will write your number sentence, you took

		eighteen stones and divided by what and get what? Ok, (talking to one learner) write your number sentence below question two. Good!
12.	Teacher:	have you written your number sentence?
13.	Learner:	yes
14.	Teacher:	what number did you divide, and how many stones were they?
15.	Learner:	two
16.	Teacher:	then why didn't you write eighteen divided by three and the answer you got? Ok, no problem leaves the answer you have written.
17.	Teacher:	ok, the question I now have is this, how did you divide these stones? who can tell me how you shared those stones, what did you do first and what did you do next for you to get the answer (stop the video because they are just talking now), who can tell me how the stones were shared, starting the explanation from the point when you went to the box to pick the stones, followed by what you did after you picked up the stones. Yes! (giving one learner a chance)
18.	Learner:	first I went to the box and take a handful of stones and count out eighteen stones
19.	Teacher:	yes
20.	Learner:	I counted three stones out of eighteen stones
21.	Teacher:	yes
22.	Learner:	I placed those three stones at the table distant from one another.
23.	Teacher:	ok
24.	Learner:	then I started allocating one stone by one beside each three stones.
25.	Teacher:	taking stones from where
26.	Learner:	from the eighteen stones
27.	Teacher:	ok
28.	Learner:	then I keep putting stones one by one beside each of three stones until the eighteen stones get finished and re count the stones at each group which are six.
29.	Teacher:	did you say until eighteen stones get finished
30.	Learner:	yes
31.	Teacher:	ok, how did you divide Mic?
32.	Mic:	I divided like this, I put a stone here, here and here and keep dividing, dividing (I guess he mean allocating stones) until stones become six.
33.	Teacher:	what makes you think that your answer is correct?
34.	Mic:	I counted them
35.	Teacher:	you counted the stones in each groups?
36.	Mic:	yes
37.	Teacher:	and you find out that there are six stones in each group?
38.	Mic:	Yes
39.	Teacher:	is it possible that you could get an answer that is not six?
40.	Mic:	no
41.	Teacher:	what makes you think that six is just the correct answer apart from counting those stones
42.	Mic:	because I divided
43.	Teacher:	yes we know you were dividing, but what if where you were taking this stones, you were able to take more stones from there?
44.	Mic:	yes I was able
45.	Teacher:	so you were able, when you took the sixth stones, where are your eighteen stones? I want you to start counting them and show me how you can get seventh stones because you said, it is possible, and no do it in the manner you did it before. Share those stones into three groups (the learner was busy re allocating) (after the learner is done) now how many stones do you have in your groups?
46.	Mic:	six
47.	Teacher:	six, so are you able to take more stones from the eighteens to make seventh in the group?
48.	Mic:	no
49.	Teacher:	why is not possible, that is the question I was asking?

50.	Mic:	there is nothing left
51.	Teacher:	because stones get completely finished from where you were taking right?
52.	Mic:	yes
53.	Teacher:	when you divided them equally into three groups each group got six stones right?
54.	Mic:	yes
55.	Teacher:	and where you took those stones the stones get finished right?
56.	Mic:	yes
57.	Teacher:	is that what makes you think that your answer is correct
58.	Mic:	yes
59.	Teacher:	who can give us an example of division as such, explaining how you could've divided a number of stones into a number of groups to get an answer. Dia
60.	Dia:	I can divide eight stones
61.	Teacher:	eight stones into how many groups?
62.	Dia:	three
63.	Teacher:	what?
64.	Dia:	three
65.	Teacher:	take eight stones and start divide them into three groups and see what you will get, count out only eight. Hurry up. Did you get eight stones?
66.	Dia:	yes
67.	Teacher:	ok, start dividing them into three groups and see if groups would equally share (only into two groups) no, no, no, no that you had made is only two groups, I said three groups divide eight into three groups of equal stones, the three you have just mention on your example (after she was done with the division) is it possible?
68.	Dia:	no
69.	Teacher:	is not possible right?
70.	Dia:	no
71.	Teacher:	ok, give me another example that is possible, which number is divided into what portion (turning to the whole class) so not every number divided by any number gives equal portions ok, (back to Dia) this eight stones can be divided into what equal groups?
72.	Dia:	three
73.	Teacher:	but you just mentioned three earlier which was not possible, mention or choose another number that is possible
74.	Dia:	two
75.	Teacher:	ok, start sharing these stones into two groups to see. Let all eight stones be allocated into two groups (after she is done) now how many stones does each group have?
76.	Dia:	(after counting) four
77.	Teacher:	who else can give me an example, Dav?
78.	Dav:	is like having ten stones
79.	Teacher:	yes, you can take ten stones divide them into how many groups?
80.	Dav:	two
81.	Teacher:	two and how many stones would each group have?
82.	Dav:	five stones
83.	Teacher:	yes each group would have five stones.
84.	Teacher:	ok, now we will continue with question three, take back all the stones from your table.
85.	Teacher:	you can shot the video now Mr. Sac (telling the video man) everyone go and take twenty four stones. Did you get twenty four stones?
86.	Learners:	yes
87.	Teacher:	divide them equally into four groups. So make four groups with equal stones. (A very long pause as learners are busy dividing) are you done?
88.	Learners:	yes
89.	Teacher:	Pet, how many groups have you just made
90.	Pet:	three
91.	Teacher:	well and how many did I say they should be?
92.	Pet:	four

93.	Teacher:	good! Why can't you do as you were told? (Turning to Naf) how many groups do you have Naf?
94.	Naf:	four
95.	Teacher:	four, I am coming (telling the other learners who were signifying that they are also done) you have made four groups rights! Ok, do you all have four groups?
96.	Learners:	yes
97.	Teacher:	ok, are you done?
98.	Learner:	yes
99.	Teacher:	Hanna, how many stones do you have in each group?
100.	Hanna:	four
101.	Teacher:	what?
102.	Hanna:	three
103.	Teacher:	when you counted the stones in each group are they three?
104.	Hanna:	(after counting) they are four
105.	Teacher:	(she went around asking learners the number of stones they have in each group and all learners who responded keep saying six, six, six. She did this to about twelve learners of which each of them has six as an answer, then she came back to Hanna) we have Hanna who got four stones in each of her groups (turning to Hanna) Hanna! Are you sure you took twenty four stones? (After Hanna recounted the stones) your stones are just twenty go and get four more stones to make them twenty four. Now make four groups and divide the twenty four stones in groups equally. (When discovered Hanna is doing it wrongly) re - start over in four groups, four groups, no! make those groups four (then teacher start assisting by showing her how to make groups and count them with her) this is one group, this is the second group, so make them four and start allocating equal stones to these groups (Hanna allocated but then stopped allocating in the middle of allocation before the allocation of stones is done) Hanna the stones are not finished continue allocating until the stones get finished. Allocate well, this stone was here and the other was there (after Hanna is done) now how many stones does each group has?
106.	Hanna:	four
107.	Teacher:	what
108.	Hanna:	four
109.	Teacher:	how did you get that answer because I did not see you counting them?
110.	Teacher:	(talking to one learner when she saw that he had written twenty -five) why writing twenty-five when we were dividing twenty- four stones? Were your stones twenty – five? (Talking to another learner when she saw that the learner had written four for an answer) when you divided twenty – four stones into four groups, did each groups got four stones? (Asking the whole class) did everyone write twenty – four divided by four right?
111.	Learners:	yes
112.	Teacher:	why divided sign and not plus, times or minus? Because you have written twenty – four divided by four equal to and then the answer. Why not chose plus, minus or times? Why Era!
113.	Era:	because we were dividing.
114.	Teacher:	ok, Era is saying is because we were dividing. Any other with the different opinion? Why not opted for minus, times or plus sign?
115.	Learner:	we were dividing.
116.	Teacher:	we were dividing ok, what do others have to say? Hil, why divided sign here and not plus or minus?
117.	Hil:	because we were dividing.
118.	Teacher:	so is because you were dividing those stones and were not subtracting or else, ok (to the whole class) now I want you to go and take thirty stones and divide them into five groups with equal stones.
119.	Teacher:	Hanna we said five groups do not start allocating second stones before determining or indicating your groups. No! Now you had make six groups destroy one group and

allocate the stones in the sixth group into five remaining groups. (After along break the teacher asked the whole class) are you done with the allocation?

120. Learners: yes

121. Teacher: ok, some are not done let us wait for them to finish (pause) ho many groups do you have Hen? (Pause) how many groups are these Hen?

122. Hen: four… five

123. Teacher: five, then let all the stones be allocated completely into these five groups and let there be space between the groups (to the whole class) are you done?

124. Learners: yes

125. Teacher: ok, if you are done tell me how many stones each group have?

126. Learners: (gave their answers as the teacher give them turn one by one) six, six, six, six, ….. until the seventh learner (then when the teacher reached the seventh learner she realized the learner only mentioned six but the groups has otherwise) six, are you sure you got six? Can you please count for me the stones in your groups? (After counting) how many stones are in your groups?

127. Learner: (after counting the stones) seven

128. Learners: (as the teacher continues giving them chances one by one until everyone got their turn) six, six, six, six, six, six …..

129. Teacher: so you got six stones, ok write below question six the number sentence that tells how many stones you divided with what and the answer you got. Write below question six, below question six below question six is here and that is were you are expected to write (showing one learner) you do not need to look at what others has done, everyone is doing their own things I do not know why Mic is peeping to Sab's (asking Dia) why did you put divided sign on your number sentence? Why did you write thirty divided by five? Dia!

130. Dia: (murmuring but too low to be heard) because are five groups

131. Teacher: you are speaking too low

132. Dia: because groups are five

133. Teacher: because the groups are five it makes you choose divided? Do the sign divided mean five groups? Why didn't you opt for minus, plus or times sign, why?

134. Dia: because we were dividing

135. Teacher: what?

136. Dia: because we were dividing.

137. Teacher: Dos why did you put division sign on your number sentence? Can you please expose you number sentence? Good! Why used division sign?

138. Dos: because we were dividing

139. Teacher: dividing how?

140. Dos: dividing into groups that are ………

141. Teacher: what is that you were dividing and how? That is what I am asking.

142. Dos: is stones

143. Teacher: yes I want to hear it all, like we divided this stones into this way.

144. Dos: we were dividing thirty stones into five groups

145. Teacher: how are the stones in the groups? (Pause) how are the stones in the groups?

146. Dos: six

147. Teacher: no you came to know that they are six after counting them, but how did you intended to share the stones into the groups? (Asking the whole class) did you know there would be six stones into each group.

148. Learners: no

149. Teacher: good! But when you were dividing what were your objectives you aimed to reach for each group? yes (giving a chance)

150. Learner: five

151. Teacher: not five but

152. Learner: equal stones

153. Teacher: your objective when dividing thirty stones was to divide them into five groups and each group would share stones equally right?

154.	Learners:	yes
155.	Teacher:	who can tell me how s/he divided or what s/he has done to get that answer from the beginning when you took stones from the box, what did you do? Yes! (giving a chance to a learner)
156.	Learner:	when I took thirty stones, I took thirty stones and start dividing into groups of six and counted groups and get the answer six.
157.	Teacher:	how did you know it is groups of six you should divide into when you were not aware how many stones each group would have? Tell me the process of how you did it. For i.e. I took this …….. from the box put them here and do this and that
158.	Learner:	I took stones from the box and put them on the table and divided into groups of six.
159.	Teacher:	who can explain what s/he has done clearly? Who can tell what s/he did to get these groups? Who can tell what s/he did to get these groups? Sab!
160.	Sab:	I took stones and take out five stones and place each one of the five stones separately to represent its groups. Then I kept allocating the remaining of thirty stones one by one to each of five stones until the stones get completely finished.
161.	Teacher:	well, you have explained it well. What makes you think, because you said if you divide thirty stones into five groups, you will get six stones in each groups, what makes you think that your answer is correct?
162.	Learner:	all stones are equal
163.	Teacher:	ok, all stones are equal, what else ensure you? Era!
164.	Era:	stones get finished.
165.	Teacher:	the stones you were dividing got finished, what else Naf!
166.	Naf:	because the stones I was adding got finished
167.	Teacher:	so is because the stones you were sharing (changing adding mentioned by the learner to sharing) got completely shared.
168.	Teacher:	Ok, (to the whole class) who can give me an example of division as such? Who can give me an example of division as such? Hil is like dividing what into what?
169.	Hil:	is like dividing (speaking very low)
170.	Teacher:	speak up loud
171.	Hil:	is like dividing ten sweets into ten kids and each will get five
172.	Teacher:	dividing ten sweets into ten sweets or what?
173.	Hil:	dividing ten sweets into five groups
174.	Teacher:	yes, into five groups and each one would get how many?
175.	Hil:	each would have five sweets
176.	Teacher:	are the sweets in groups still going to be five? Now take ten stones there and start dividing them into five groups see whether there will still be five stones in the groups (teacher and learner counting the stones) one, two, three, four five six …….., ten. Ok, now divide into five groups (one, two, three a learner doing it the wrong way) no, five groups that you've just said. First come up with five groups and do as it was done in the previous activity, how did you come up with groups? This are just two group you've made, make it five groups as you said Hil hurry up there are other things to make. Hil
177.	Hil:	yes
178.	Teacher:	you said ten stones right?
179.	Hil:	yes
180.	Teacher:	Divided into how many groups?
181.	Hil:	five
182.	Teacher:	well make five groups here then (when she started) why beginning a group with two stones? Don't we start a group with one stone? This is one, two, and three groups you've just made make them five as you said ten would be divided into five. Good! Keep allocating stones to these groups then until stones get finished (after she is done) are each group having five stones? (Pause) how many stones in each group?
183.	Hil:	two
184.	Teacher:	good! If you divided ten stones into five groups each group would have two stones.

185.	Teacher:	now I want you to take all the stones at the table back and come along with eighty stones and share them into four groups of equal stones.
186.	Teacher:	(after a long time) are you done with the division (pause) how many are done? Only one?
187.	Learners:	no
188.	Teacher:	ok, there are still people dividing lets wait for them (asking one learner) how many stones did you get at each group?
189.	Learner:	twenty- one
190.	Teacher:	are you sure you divided well? (The learner recounted) this group only have eighteen stones not enough even this group, you did not count well. Are you sure you took eighty stones?
191.	Learner:	yes
192.	Teacher:	these stones are not enough, they are not eighty. Recount them (turning to another learner) how many did you get?
193.	Learners:	(as they got chances to respond) twenty, eighteen, twenty, twenty-one, twenty-one, twenty –one, twenty, twenty –three, twenty
194.	Teacher:	now, take forty- five stones and divide them equally into five groups. Forty – five stones divided into five groups of equal stones.
195.	Teacher:	stones should be forty- five and divide them into five groups, how many did you get?
196.	Learners:	(one by one as each got the chance to respond) eight, eight, nine, eight, nine, nine.
197.	Teacher:	you did not took forty – five stones (turning to Hanna) Hanna! How many stones per each group? And you even did not divide into five groups as I told you. You have just made four groups, ok Era because you are done you can go home, and who else is done with your groups? Pet are done?
198.	Pet:	yes
199.	Teacher:	ok, you can go home also tomorrow come early people don't be late like today ok.
200.	Teacher:	(still went around the class tasking learners the number of stones each group have)
201.	Learners:	eight, nine, eight, nine, nine, six, nine…..
202.	Teacher:	everyone who is done can go home; if I gave you a pen leave it behind. Era, where did you put my pen? Sab! How many did you get?
203.	Sab:	nine
204.	Teacher:	you
205.	Learner:	nine
206.	Teacher:	Naf, are you done now?
207.	Naf:	yes
208.	Teacher:	how many did you get?
209.	Naf:	nine
210.	Teacher:	ok

DAY TWO **10 MAY 2012**

LESSON THREE

1.	Teacher:	good morning
2.	Learners:	good morning teacher
3.	Teacher:	how are you?
4.	Learners:	we are fine and how are you teacher
5.	Teacher:	you are fine? Ok, we are going to do reflection before we continue. Do you know what reflection is? (Pause) yes! Do you know what reflection is? (Learners gave a no sign) you don't know, ok, reflection means a remembrance of what happened yesterday, are we together?
6.	Learners:	yes
7.	Teacher:	so that when we do the activity today you will be able to connect it to what happened yesterday? Do you understand?

8.	Learners:	yes
9.	Teacher:	so if you can remember well, yesterday, what did we do yesterday? What did we do yesterday? Sab!
10.	Sab:	division
11.	Teacher:	division, yes we were doing division. So yesterday we were doing division, what were we using when dividing? What were we dividing with? Yes Fil!
12.	Fil:	stones
13.	Teacher:	stones, yes we were using stones. So we were using stones. Have you noticed/ realized what a person does when dividing?
14.	Learners:	yes
15.	Teacher:	if you know let me ask you this question, if you are told to divide fifteen by three what does that mean? It means you have how many things divided by what? Yesterday when you were dividing what did you do?
16.	Learner:	you have fifteen divided amongst three people
17.	Teacher:	you have fifteen things divided amongst three people, you see, or what else can you have? Who else can give me an example? There are many things we can divide on earth right? Yesterday we were using stones to divide, at home you share apples, at the party people share cool drinks, so there are many things people share on earth isn't it? Give me another example, anybody, anybody can give the example seeing that number on the chalkboard mean what? Era!
18.	Era:	you have fifteen sweets to be shared by three people
19.	Teacher:	you have fifteen sweets to be shared by three people, now tell me if you have fifteen sweets to be shared by three people, how many sweets each person would get? How many sweets each person would get? If these people are three, Dos!
20.	Dos:	each would get five sweets
21.	Teacher:	tell me how can one of the five get the five sweets? How do you get this five? What do you do first, and next to get the answer? How did you do it yesterday? Fil!
22.	Fil:	we were dividing
23.	Teacher:	dividing how?
24.	Fil:	we were sharing sweets amongst kids (he spoke so low)
25.	Teacher:	yes speak louder
26.	Fil:	we shared sweets amongst kids each got five
27.	Teacher:	he is saying we shared sweets amongst kids and each would get five. Who can tell me how we were dividing? How do we divide? Anybody, anybody, yes not only Era yes Era
28.	Era:	first you take out three stones
29.	Teacher:	yes first you take out ……… is it stones or sweets
30.	Era:	three sweets
31.	Teacher:	first you took out three sweets and imagine three people are there right?
32.	Era:	yes
33.	Teacher:	one is here, the second there and this is the third person
34.	Era:	yes
35.	Teacher:	yes continue
36.	Era:	than you keep allocating sweets each one by one
37.	Teacher:	yes and then
38.	Era:	until all sweets are finished
39.	Teacher:	until which sweets is finished?
40.	Era:	the fifteen sweets
41.	Teacher:	until the fifteen sweets get finished. So do you now all have a clue how people divide?
42.	Learners:	yes
43.	Teacher:	in case you are given to divide anything can you tell how to divide?
44.	Learners:	yes
45.	Teacher:	thank you, today, the activity we are going to do, I want you to know how to use subtraction, addition and multiplication do you know what subtraction, addition and multiplication are?

46.	Learners:	yes
47.	Teacher:	I want us to know how to use subtraction, addition and multiplication as a way of division. Do you understand?
48.	Learners:	yes
49.	Teacher:	for example, can you see this fifteen? Can you see this fifteen?
50.	Learners:	yes
51.	Teacher:	ok, from this fifteen, you can repeatedly take away threes until you get the division answer, and then you will see how many times you repeatedly take away threes because you are sharing fifteen amongst three people. Do you understand?
52.	Learners:	yes
53.	Teacher:	because you are sharing this fifteen amongst three people, so you are sharing amongst three people. So you repeatedly take away threes until fifteen is
54.	Learners:	finished
55.	Teacher:	yes, until fifteen get finished or you can also repeatedly add threes and see how many threes you added threes together to get fifteen or see how many times three would be there to get fifteen. That is what we will do today. I want by the end of the day to see if you can realize, using stones, realizing that we can use subtraction, addition or multiplication to get the division answer. Do you understand?
56.	Learners:	yes
57.	Teacher:	ok, the first activity we would do is subtraction, it say you will take twenty stones and keep repeatedly taking away five stones until twenty is ……
58.	Learners:	finished
59.	Teacher:	then you will tell me how many times you repeatedly took away five stones from twenty for twenty to get finished. When you repeatedly take away five stones, five stones, five stones, how many times were five stones taken out from twenty for twenty stones to get finished. Do you understand?
60.	Learners:	yes
61.	Teacher:	ok, you can go and take you stones now.
62.	Teacher:	how many of you have finished? How many of you are done? Raise up your hands everyone who is done. (After learners rose up their hands) that's good, ok there are one, two, two of you are still working, lets wait for them to finish. Now how many stones do you have Hanna? How many stones do you have altogether?
63.	Hanna:	twenty
64.	Teacher:	can you count them for me, these one if not needed return them to the box or else they will confuse you.
65.	Teacher:	how many stones did you get Fil? I mean no, no, no, how many fives did you take out from twenty?
66.	Fil:	two
67.	Teacher:	what?
68.	Fil:	two
69.	Teacher:	how many are those fives you took out from twenty stones for it to be finished?
70.	Fil:	five
71.	Teacher:	can you count those fives Fil? (Fil counted stones in the group instead of counting the groups) no, it is not stones you are to count but groups, like this one is one group of fives (representing one five), the next become two fives, or are these not five stones? now here you have taken five once, here is twice, so now count to see how many fives you have taken out of twenty for twenty to get finished, that is what I am asking.
72.	Fil:	five
73.	Teacher:	count these groups and see if they are five, those groups are they five? (Fil, still counting stones in a groups) I am not saying count stones, but count groups
74.	Fil:	four
75.	Teacher:	how many?
76.	Fil:	they are four
77.	Teacher:	what?
78.	Fil:	they are four

79.	Teacher:	yes, first you took out this five stones right? Is that so?
80.	Fil:	yes
81.	Teacher:	and took this other five right?
82.	Fil:	yes
83.	Teacher:	then you took this other five and this other five, that makes them how many fives you took out (pause) what?
84.	Fil:	four
85.	Teacher:	what?
86.	Fil:	four
87.	Teacher:	(the teacher now move around the class collecting the answers from learners each according to their turn) how many fives you took out? (she asked each one of them)
88.	Learners:	four, four, four, four, four (until the fifth learner)
89.	Teacher:	(asking the sixth learner) are you done? (Pause) are you done?
90.	Learner:	yes
91.	Teacher:	are you sure you repeatedly took out fives?
92.	Learner:	yes
93.	Teacher:	and your stones were twenty?
94.	Learner:	yes (the child started to recount the twenty stones)
95.	Teacher:	how many are they?
96.	Learner:	eighteen
97.	Teacher:	no go and make them twenty as you were told (turning to other learners) how many fives
98.	Learners:	four, four, four, four, four, four, four (learners all said four until all of them get their turn)
99.	Teacher:	now how many groups do you have on the table? How many groups do you have?
100.	Learner:	four
101.	Teacher:	you have four groups right? You have four groups. So you have twenty divided by ….. yes
102.	Learner:	five
103.	Teacher:	either divided by five equal to …..
104.	Learner:	four groups
105.	Teacher:	four groups or do you have twenty divided by four equal to five? Or which one do you have? Did you divide into four groups or were the groups five? Yes
106.	Learner:	four
107.	Teacher:	and each group has five stones right?
108.	Learner:	yes
109.	Teacher:	so all things are more less the same, so let us leave it like that for now, the question I want to ask you are this. Yesterday when you were dividing you have been making groups as such right?
110.	Learner:	yes
111.	Teacher:	four groups of equal stones which are how many stones per group? Hil
112.	Hil:	five
113.	Teacher:	but today you did not make groups, what have you been taking away and how? What have you been taking away?
114.	Hil:	taking away fives
115.	Teacher:	what have you done today?
116.	Hil:	we have been taking away five stones
117.	Teacher:	do you think subtraction and division are related in anyway? And if yes how are they related? Nab
118.	Nab:	is related when subtracting
119.	Teacher:	subtracting how? Even when you subtract two and three or four and fives or how is that subtraction? Is when you subtract how? Pet
120.	Pet:	is when you divide
121.	Teacher:	yes, dividing how? When do subtraction relate to division? Is when you subtract how? Sab

122.	Sab:	is when you subtract equal portions
123.	Teacher:	is when you subtract equal portions because like in this case we kept subtracting how many? Hen
124.	Hen:	is fives
125.	Teacher:	is fives so we did not subtract different amount of portions but equal portions. So if you keep subtracting equal portions and the number to be subtracted get finished or you reach zero the answer of how many you will repeatedly taking away would be the same as the division answer. If you say twenty divided into groups of five stones you will get four groups of equal stones right?
126.	Learner:	yes
127.	Teacher:	with equal number of stones which is five, meaning in twenty stones one can repeatedly subtract five fours. Now put all the stones in group together and now we will go to question two can you see question two?
128.	Learners:	yes
129.	Teacher:	that's where we will proceed to, there is four in question two so we would use four, then if you subtract four what would be left out? Yes
130.	Learner:	sixteen
131.	Teacher:	there would be sixteen left, keep subtracting then, in sixteen minus four what would be left? Keep subtracting four until twenty get finished or until you reach zero. Can you see this sixteen in the answer?
132.	Learner:	yes
133.	Teacher:	keep writing like that (showing learners how to write in the number sentence process) and keep subtracting until you reach zero. Start with those twenty stones and remove four stones and see what would remain.
134.	Teacher:	(Asking Nab) what remained?
135.	Nab:	eleven
136.	Teacher:	sixteen minus four is not eleven, count well Nab. Nab use stones and not fingers. Nab count the stones and not fingers and see how many remained
137.	Nab:	twelve
138.	Teacher:	now after subtracting four from sixteen twelve would remain right?
139.	Learners:	yes
140.	Teacher:	now put twelve here and subtract four from twelve, what would remain, subtract four from twelve and see what would remain. Dos!
141.	Dos:	eight
142.	Teacher:	yes, eight would remain, continue the process until you reach zero. Subtract four from eight and see what would remain. Subtract four from eight and see what would remain. Hil!
143.	Hil:	one
144.	Teacher:	no is not one. Yes
145.	Learner:	four
146.	Teacher:	if you subtract four from eight four will remain ok, four will remain. What will remain if you subtract four from four? If you subtract four from four what would remain? Dav!
147.	Dav:	two
148.	Teacher:	are you sure you have counted your stones well?
149.	Dav:	zero
150.	Teacher:	yes if you subtract four from four zero would remain. Can you subtract four from zero?
151.	Learner:	no
152.	Teacher:	ok, count for me how many fours you were able to subtract from twenty to get zero. Tell me how many times you repeatedly subtract fours to get zero, count them
153.	Learner:	five
154.	Teacher:	five, you can also count them here at the chalkboard, how many are they? Look at the list of the minus fours on the number sentence process, how many are they?
155.	Learner:	five

156.	Teacher:	they are five, meaning that twenty divided by four is …..
157.	Learners:	five
158.	Teacher:	five, while twenty divided by five become……
159.	Learners:	four
160.	Teacher:	four, you see it, but we got that answer through the use of subtraction method. You get that right?
161.	Learners:	yes
162.	Teacher:	(I can see Mic counted stones first and see how many fours he subtracted from twenty, some counted the fours at the chalkboard.) have you written the whole process in your paper?
163.	Learner:	yes
164.	Teacher:	you have written it right?
165.	Learners:	yes
166.	Teacher:	if the space is not enough and the subtraction number sentence process does not fit on the question paper write the process on the blank paper provided. Label it as question 2 and keep subtracting five from twenty. So write twenty and keep subtracting fives, there are four fives in twenty. What you do is take twenty stones and take away five stones recount the remaining stones and see the answer. Take away five stones again from the answer keep doing that until you reach zero. Everyone have twenty stones before you and start counting and when subtract the five stones keep them isolated (asking one learner) how many stones are left?
167.	Learner:	fifteen
168.	Teacher:	then write it on your paper, I said you should write the whole process on your paper and indicate it as question number 2. Twenty minus five, minus, minus and not divided look at the chalkboard. The five stones you subtracted did you put them separately?
169.	Learner:	yes
170.	Teacher:	where is the five you subtracted
171.	Learner:	is here
172.	Teacher:	then put it separately, now write fifteen here minus five and see what would be the answer and keep subtracting five stones until you reach zero (asking Hil) are you sure you have twenty stones Hil?
173.	Hil:	yes
174.	Teacher:	then where are the five stones you have subtracted?
175.	Hil:	they are here
176.	Teacher:	ok, count the remaining stones (after Hil had counted) and write the answer here. Where did you get the fours that you have written Hil? Now that you have taken five away, write the fifteen here and subtract five again. Count to find out the remaining again. Put or the five stones subtracted separately. Write the answer of the remaining stones, good continue subtracting five until you reach zero.
177.	Teacher:	(asking another learner) why have you started to write on your paper beginning from the middle of the paper. We will have a lot of things to do today and they might not fit on the paper. Label it question two here on top of the paper, then twenty minus fives and this answer is what you will begin with in the next step, then count your stones and subtract five stones and see what would remain. All the five stones you would subtract make them different groups so that you would be able to see how many fives you have taken from twenty.
178.	Teacher:	(asking another learner) are these fives my dear? That is why you always get wrong answers as a result of wrong counting look! I am not saying rub your written works, the written work is ok, good, good now you are at the step with ten as an answer. If they are not ten it means you have wrongly counted the stones when you subtracted five stones from ten stones. Write your answer here, what remained when you subtracted five stones from ten stones, all stones were ten, what remained when you subtracted five stones? This is minus and not divided, because you were subtracting ok, how many fives did you subtract? Naf!

179. Naf: four
180. Teacher: how many fives?
181. Naf: four
182. Teacher: you had taken out four fives, are your twenty stones finished? Dav!
183. Dav: yes
184. Teacher: how many fives did you subtract?
185. Dav: four
186. Teacher: yes, are you done counting Dos? Do you have twenty stones?
187. Dos: yes
188. Teacher: where are they written? (after seeing) why didn't you start with twenty?, because we started subtracting from twenty, at first you had twenty stones, twenty where you subtracted five to get this fifteen so your writing has jumped one step. Write that step on top here, minus five. What is the remaining? Good! How fives you subtracted from twenty to reach zero? You see with stones you are not able to it because you did put the groups of fives you removed out of twenty stones together. So you rather use your written works on paper. How will you see it on your written works?
189. Dos: they are four
190. Teacher: they are four. Are you done? (Asking another learner) how many fives were you able to subtract? Nab, why have you done the step for ten but you did not write ten here? The step of ten stones minus five where is it? When you subtracted five what was the remaining? Where are your groups of stones?
191. Nab: they are here
192. Teacher: how many are they?
193. Nab: they are four
194. Teacher: see now, but in your written works the fives are not four as in groups of stones, what is ten minus five?, yes take five away what will be the remaining?
195. Nab: five
196. Teacher: write your answer here then and write again five minus five to reach zero.
197. Teacher: who can tell me how we divided stones in four groups of equal stones? yes
198. Learner: I took twenty stones and make four groups
199. Teacher: from there, what do you do next?
200. Learner: I kept allocating stones one by one to each group.
201. Teacher: allocating stones one by one to each group until what?
202. Learner: until stones get finished
203. Teacher: allocating stones one by one at each group (one, two, three, four, one, two, three, four, the teacher was counting the dots that she was making in groups at the chalkboard the dots she allocated to four drawn groups in circles. After she is done she asked) do you have groups as such?
204. Learner: yes
205. Teacher: four groups with equal stones right?
206. Learner: yes
207. Teacher: how many stones does each group have? Yes Mic!
208. Mic: five stones
209. Teacher: five, now the question I have is this, do you think subtraction and division are related? Do you think you can get the division answer using the subtraction, look at the way we were subtracting can you see it there?
210. Learners: yes
211. Teacher: can you see how many five we have subtracted to reach zero?
212. Learner: yes
213. Teacher: and the five you subtracted are five just the same as that of the division with stones whereby you also made four groups of five stones right?
214. Learner: yes
215. Teacher: do you think they are related? Ok, let me change the question, why do you think you got the same answer as four fives and four groups of five stones? Here at subtraction you have subtracted four fives subtracted and when divided with stones you also got

		four groups each with five stones. Why may be did you get the same answer at both methods of subtraction and division? (Pause) hello! At first you were subtracting right?
216.	Learners:	yes
217.	Teacher:	you see when you were subtracting, out of twenty stones you repeatedly take out five stones until twenty got finished and how many fives did you take out?
218.	Learners:	four
219.	Teacher:	even when you divided with stones you got out four groups of five stones from twenty stones. So the question is here, why in both methods the subtraction and division methods you got the similar answers? That is the question I am asking. (Pause) you cannot see what makes the answers to be similar? When you divided how many stones each group has? How many stones each group has? Look on your table. Yes!
220.	Learner:	five
221.	Teacher:	five, when you subtracted what number had you been repeatedly subtracting? Dav!
222.	Dav:	five
223.	Teacher:	now, answer my question then, why may be you got similar answer at both processes? We have fives and four at both methods, why similar answers? Sab!
224.	Sab:	because all groups contains fives
225.	Teacher:	yes because all groups you took out has five stones, which are equal portions right?
226.	Sab:	yes
227.	Teacher:	now Sab is saying at both sides even when subtracting and when dividing with stones in all contains the same amount which is five. When you divided with stones groups have how many stones?
228.	Hil:	five
229.	Teacher:	and when you subtracted, what number you were repeatedly subtracting?
230.	Hil:	five
231.	Teacher:	your portions have the same amount that is why you got similar answers.
232.	Teacher:	now I want to ask you, do you think the answer you get is correct? If you divide twenty stones by five stones would you get four groups? And if you share twenty sweets to five people would each get four sweets? Or if you share twenty sweets to four people would each one of them get five sweets? Do you think your answer is correct?
233.	Learner:	yes
234.	Teacher:	why is your answer correct?
235.	Learner:	because twenty have only four fives
236.	Teacher:	ok, because there are four fives in twenty. So if you subtract four fives from twenty what will remain?
237.	Learner:	zero
238.	Teacher:	the remaining would be zero, so nothing would remain right?
239.	Learner:	yes
240.	Teacher:	ok, can you give me an example as such, subtracting in the same manner as such, so give me a number and explain it for i.e. you have …… things shared by these number of people, when you keep subtracting, or repeatedly taking that number for ….. Times you will reach zero. Who can give me an example as that we were doing now? Who can give me an example as that we were doing now? When we started, I gave you an example of fifteen divided by three and I said if you keep subtracting three repeatedly, fifteen would get finished. Yes Dos!
241.	Dos:	if I have fifty grapes
242.	Teacher:	yes
243.	Dos:	sharing them to five people
244.	Teacher:	yes
245.	Dos:	everyone would have ten grapes
246.	Teacher:	ok, if there are fifty grapes shared by five people every one would have ten grapes. So how many times you can take fives from fifty? (Pause) how many times you can

		subtract five from fifty for fifty to get finished? How many grapes did you say each person would get?
247.	Dos:	ten
248.	Teacher:	so, how many fives can you take out of fifty for fifty grapes to get finished?
249.	Dos:	ten
250.	Teacher:	yes you will need to take out ten fives for fifty grapes to get finished. I saw someone with a hand up with another example, Pet!
251.	Pet:	I have four sweets to be shared by two people
252.	Teacher:	how many people
253.	Pet:	four shared by two people each one would get two
254.	Teacher:	how many two would you subtract from four sweets for the sweets to get finished? Because I want the example in the subtraction form. She said she had four sweets shared by two people and each would have two sweets, the I say explain it in the subtraction way e.g. in four sweets subtract two and get the answer and subtract two from the answer until you reach zero how many two can you subtract from four for four to get finished?
255.	Pet:	one
256.	Teacher:	only one two, if you subtract two from four what would remain?
257.	Pet:	two
258.	Teacher:	and we said we only finish subtracting when we reach what?
259.	Pet:	zero
260.	Teacher:	then when you see that there is still two remaining then you are not done because two is not zero so four minus two remains two and you put that two here and two minus two? What would remain?
261.	Pet:	zero
262.	Teacher:	zero will remain, so now you are done right?
263.	Pet:	yes
264.	Teacher:	How many twos did you subtract from four then? How many twos you had subtracted from four?
265.	Pet:	two
266.	Teacher:	that is why even at division the answer is also two. Can you see it? That the subtraction method relate to division. If you repeatedly subtract the same number of portions until you reach zero for subtraction you count how many times you subtracted that number. So four divided by two is equal to what? Four divided by two is equal to? Yes! Everyone can you see at the chalkboard? Four divided by two is equal to?
267.	Learners:	two
268.	Teacher:	how many two can you take out from four to get zero? Naf!
269.	Naf:	two
270.	Teacher:	can you see how they are related now?
271.	Learner:	yes we can see it
272.	Teacher:	Hen, you said you can see it. What can you see (pause) can I give you another example to explain? Ok, if I say fifteen divided by three is equal to five. What does that mean?
273.	Hen:	murmuring (in a lower voice)
274.	Teacher:	it means what? What should you do with threes from fifteen to get zero? (Pause) who can explain it better? We said that, can you all look here, do you see that
275.	Learner:	yes
276.	Teacher:	twenty divided by five is equal to
277.	Learner:	four
278.	Teacher:	that means how many times you can take five out of twenty? How many times can you take away five s from twenty (telling learners with their hands up eager to give the answer) just tell me the answer and say it loud.
279.	Learner:	four times
280.	Teacher:	you can take the fives four times, that is why twenty divided by five is four right?

281.	Learner:	yes
282.	Teacher:	that is the same thing I was asking, that if fifteen is divided by three is equal to five. That means how many times can threes come out of fifteen? If you are subtracting, how many times would you subtract threes away from fifteen? Sab!
283.	Sab:	three times
284.	Teacher:	Dos!
285.	Dos:	four times
286.	Teacher:	Dav
287.	Dav:	five times
288.	Teacher:	is five times, is always similar to division answer, so you will subtract three, once, twice, thrice, fourth and fifth to get zero. If you subtract three here what would you get? Twelve minus three it would be nine. Nine minus three it would also be six. Six minus three it would be three and three minus three is zero. Can you now see the relation between subtraction and division?
289.	Learners:	yes
290.	Teacher:	you can see it right?
291.	Learners:	yes
292.	Teacher:	so if I told you to divide a number you can also do it by subtracting isn't it?
293.	Learners:	yes
294.	Teacher:	you think you can do it?
295.	Learners:	yes
296.	Teacher:	ok, we will see about that as the lesson goes ahead. Now we can go to question three. Question three is asking that "write this subtraction in the number sentence of division. How many things have you been dividing by what and what was the answer? Write the number sentence below question three. Write that number sentence of division stating that you have twenty stones divided by …… and so write it in the division form and not in subtraction form anymore. (Asking one learner) o! Child when you divided twenty divided five, did you get fifteen groups? (some learners were seen going back to their groups with stones count them to remind themselves and write) (Teacher telling one learner) it is not a minus, but divided sign. (As the teacher moved around she saw one learner peeping at the other learner's work) let everyone do your own things as you understand it you do not need to peep at other's work. I can see you have written it differently and I like it. It is all correct anyway.
297.	Teacher:	question five say, we can also get division answer using addition. This we do by repeatedly adding the same number, and then you count how many times you added that number to get the answer. Do you understand?
298.	Learners:	yes
299.	Teacher:	now we will continue with number five, that tell us that we can get the division answer using addition. You will repeatedly adding the same number over and over again until you reach the number you want to reach and count how many times you have repeatedly added that number. Start like in subtraction when we kept subtracting a same number repeatedly and counted how many times you subtracted the number for it to get finished. Now we will be building a number. Take all the stones to the box and only have the fifteen stones on your table. Do you have fifteen stones?
300.	Learners:	yes
301.	Teacher:	put them a little bit aside, put them aside a bit, just put them aside a bit, the instruction is saying (telling one learner) don't have more than fifteen stones at the table, take some of the stones back to the box leave only fifteen stones to avoid confusion. What we want to see is how many threes are in fifteen, because when you are dividing you want to find out how many threes are in fifteen right? Now we will not do it with a subtraction, but we will do it like this, you take three stones and put them separately so that you would be able to see that there are only three stones in each group and take another group of three stones put the three stones again as a group and after every each group of three stones you add, calculate the sum of all groups altogether. Then you take again another group of three stones and see what all groups will sum up, then

		you continue add the groups of three stones until the sum of fifteen is reached. When you reach the sum of fifteen, count how many threes were added to make fifteen. How many threes added together makes up fifteen? Do you think you can do it?
302.	Learners:	yes
303.	Teacher:	let me see, you do it then, find for me how many threes can be added to make fifteen (talking to one learner) threes, this are just fives you have made, see how many threes, how many threes added together can give you fifteen.
304.	Learner:	they are five
305.	Teacher:	they are five, but where are they because I cannot see them on you table (turning to the other learner) you have them right? Ok, there are five threes, ok, I want you to arrange your groups of three stones in order, and your groups of three stones should be arranged in order. (Talking to Naf) in order Naf, in order Naf, now I want you to give answers starting from one group of three stones. How many stones are these? Write that as number 6 on the other worksheet (paper) keep writing steps for threes starting from step one with one three, step two with two threes and keep getting the sum of threes in each steps until you reach the step with the sum of fifteen. (Pause) until you reach …..
306.	Learners:	fifteen
307.	Teacher:	are we together?
308.	Learners:	yes
309.	Teacher:	I told you because the work cannot fit on the question paper, do you number six on the other separate work sheet. (Talking to one learner) first you only need to start with this only one three, then in step two you add another one three and see how many stones altogether would be. Then in step three you add another groups of three stones and again see how many stones would be (sum). First you only have one three (represented by one group of three stones) do you see that?
310.	Learners:	yes
311.	Teacher:	three appearing once gives how many stones? Hil!
312.	Hil:	three stones
313.	Teacher:	good! Now put the two three stones together, when threes are there two times, what is the sum of their stones? Dav!
314.	Dav:	six
315.	Teacher:	six, then write six, now make the groups of three stones threes, when they are three groups, what would be the sum of their stones? Naf!
316.	Naf:	nine
317.	Teacher:	nine, the sum would be nine, now make the groups of three stones four, when the groups become four, what would be their sum?
318.	Learner:	twelve
319.	Teacher:	twelve now make the threes five, when the threes become five, what sum do you got?
320.	Learner:	fifteen
321.	Teacher:	now is clear that how many threes are in fifteen? When dividing you want to see how many threes are in fifteen rights? How many threes are in this fifteen? How many threes did you add together to get fifteen? Dav!
322.	Dav:	four threes
323.	Hil:	five
324.	Teacher:	they are five, so you add them five to give you fifteen. (Turning to Dav) you got twelve here when you summed up four threes and not fifteen.
325.	Teacher:	so let me ask you, can you see how addition and division are related? When can you get the similar answer between addition and division? When can you get the similar answer of addition to that of division is when you do what? Why the addition answer and that of division are the same?
326.	Era:	is because we were subtracting.
327.	Teacher:	we were not subtracting but adding. It was in the previous activity we were repeatedly subtracting the same number until the number gets finished. Yes
328.	Learner:	it is because we had been subtracting the same number.

329. Teacher: we did not subtract now we were adding or do you mean when we repeatedly add the same number? People can you see at the chalkboard very well what operation sign is this? Hil!
330. Hil: is a plus sign
331. Teacher: ok, that is why I asked that why addition also give an answer similar to that of division? What is that we had been doing that made answers to be similar? Naf!
332. Naf: is because we had been adding.
333. Teacher: adding what? Have we been adding different numbers, like five plus six plus seven or what? So what makes the answer to be similar to that of division?
334. Learner: because we repeatedly added the same number.
335. Teacher: we had been repeatedly adding the same number. If you look at all the five groups on your desk all groups have three stones or do groups have different number of stones? I.e. one with four or five and other with six? Ok how are the numbers you have been adding together? Era!
336. Era: same equal numbers
337. Teacher: is the same, equal, now let me say put these stones together again as one group and divide them into fives groups of equal stones and see how many stones each group would have. Put all the stones together and share them into five groups of equal stones and see how many stones are in the groups. (Talking to one learner) groups should not be too closer, like this group is too closer to the stones you are dividing. Let there be spaces between groups. How many stones are in each group?
338. Learner: three stones
339. Teacher: and how many groups are these?
340. Learner: five groups
341. Teacher: can you now see how division and addition are related?
342. Learner: because the stones are three
343. Teacher: three at what?
344. Learner: at each groups
345. Teacher: there are three stones at each groups, ok
346. Teacher: in addition how did you get the answer? What have you been doing to get the answer? Who can explain it well? Can you remember how we were adding those stones here right?
347. Learner: yes
348. Teacher: start with what we took first and all the steps that followed
349. Learner: yes
350. Teacher: who can tell us what had happened? How have you been taking stones? How many stones you took first? Yes!
351. Learner: three
352. Teacher: what did you do next then or what you have been doing over and over.
353. Learner: we have been adding them together
354. Teacher: adding them together, adding what together? That is what I want to hear.
355. Learner: adding threes together
356. Teacher: so you repeatedly add…
357. Learner: add threes
358. Teacher: you repeatedly add threes together until what?
359. Learner: until I reach fifteen
360. Teacher: so you repeatedly add threes together until you reach fifteen stones. The question I have is here, why do we keep getting the similar answer in addition as that of division? In division you divided fifteen stones in five groups and each group has how many stones? Hil!
361. Hil: three stones
362. Teacher: and in addition you said you repeatedly add what number have you been adding together?
363. Hil: threes

364.	Teacher:	why do you think you got the similar answers in both approaches, in division you have fifteen divided by threes equal to five, why in both approaches (addition and division) you got five the same answer. Naf!
365.	Naf:	is because all stones are equal
366.	Teacher:	is because all stones in all groups are equal, is that right?
367.	Learners:	yes
368.	Teacher:	when adding how many stones have you been repeatedly adding? Yes!
369.	Learner:	three stones
370.	Teacher:	even when you divided, how many stones were in each group?
371.	Learner:	three stones
372.	Teacher:	so in both methods we used the same number that is why you have similar answers that mean you can get a division answer using addition isn't it?
373.	Learners:	yes
374.	Teacher:	can you see how you can do it?
375.	Learners:	yes
376.	Teacher:	ok, do you think you think you can come up with an example as such? Who can give me the example and explain it in the same manner, i.e. you have how many things divided by this number, which is the same as this number repeatedly added times this number? Who can give me an example? Who can give me an example as such?
377.	Teacher:	(pause) ok, while you are thinking of an example, look at question seven, everyone go to question seven and write for me a division number sentence of what we were dividing. Fifteen divided by three gives you what? Write what we were doing, we were doing fifteen divided by three gives you what? Write it under question seven. Are you done? If you are struggling to get it go back to the stones and re-do the division of stones to get the answer. Divide by three or by five and see what you will get and then write your number sentence based on your observation. Are you done?
378.	Learners:	yes
379.	Teacher:	if you are done, take those stones back to the box and take other fifty stones from the box. Take those stones back to the box, handle them very well so they don't drop and scatter around the floor and take other fifty stones from the box.
380.	Teacher:	make five groups of stones out of fifty stones and all the groups should share the stones equally. All groups should share the stones equally. Remember all groups must have equal stones. Do you understand?
381.	Learners:	yes
382.	Teacher:	did you make five groups?
383.	Learners:	yes
384.	Teacher:	arrange the groups in order (row) ensure there are spaces between the groups. These groups you have, how many stones does each group have? Yes!
385.	Learner:	ten
386.	Teacher:	(asking another learner) how many stones each group have?
387.	Learner:	ten
388.	Learner:	(another learner) ten
389.	Teacher:	why your stones are not grouped? Why are they scattered? Count the stones in your groups to tell me how many stones you have in each group.
390.	Teacher:	ok, everyone got their five groups right?
391.	Learner:	yes
392.	Teacher:	(asking one learner) how many stones did you get in each group?
393.	Learner:	twelve
394.	Teacher:	what?
395.	Learner:	twelve
396.	Teacher:	are you sure you have counted well? One, two, three, four, (counting groups) these groups you have made are just four and not five. We said make five groups that is why you got a wrong answer. (Turning to another learner) how many stones in your groups?
397.	Learner:	ten

398.	Teacher:	(teacher went around asking the same question and all learners keep saying ten, ten, ten, until she reached the twelfth learner)
399.	Learner:	ten
400.	Teacher:	(asking the twelfth learner) ten how did you get ten when you have not counted? Hen, don't be answering what you just heard from others. Count the stones in your groups (after counting) how many stones?
401.	Hen:	ten
402.	Teacher:	(after realizing that Hen, is lying) count them aloud, aloud, aloud, saying one, two aloud.
403.	Hen:	one, two, three, four, five
404.	Teacher:	(when Hen, finished counting and stopped at five) how many stones?
405.	Hen:	ten
406.	Teacher:	count again aloud
407.	Hen:	one, two, three, four, five
408.	Teacher:	yes, how many is that?
409.	Hen:	five
410.	Teacher:	yes, but now you counted groups, now count the stones in the groups
411.	Hen:	one, two, three, four ………….. nine
412.	Teacher:	how many stones do you have in a group?
413.	Hen:	nine
414.	Teacher:	what?
415.	Hen:	nine
416.	Teacher:	nine (teacher turned to other learners and continues asking the same question)
417.	Learners:	(per their turn) ten, ten, ten, ten,
418.	Teacher:	how many stones in the group?
419.	Learner:	I am not done
420.	Teacher:	how many do you have in each group?
421.	Pet:	ten
422.	Teacher:	are you sure you do have ten stones in each group? How many groups did you made then? (Pause) can you please count your groups?
423.	Pet:	(after counting) they are ten
424.	Teacher:	but I said make five groups Pet, now make five groups and not ten. Leave these five groups the rest stones of the other five groups' re- allocate them to these five groups and see how many stones each group would have. Divide, no, no, no allocate all these stones to these five groups until the stones get finished. Put the stones there. Why did you left this one behind? Allocate them also. Why do you want to remove that one? That is how you people confuse yourself. Ok, now count the stones in each group (Pet counted and got ten stones in each group).
425.	Teacher:	so what I want us to discover is this. You have fifty stones that you will divide by five or into five groups, what did you said you got, never mind, how many tens did you get? How many tens did you get? Hanna!
426.	Hanna:	five
427.	Teacher:	you got five tens, one, two, three, four, five (the teacher counting the tens aloud) if it was multiplication what would you say? It is ten appearing there how many times?
428.	Learner:	five times
429.	Teacher:	five times, so the tens are there one, two, three, four, five times is that right?
430.	Learner:	yes
431.	Teacher:	and all these tens together will give you fifty right?
432.	Learner:	yes
433.	Teacher:	meaning the fives in fifty are there how many times? You will take fives how many times in fifty?
434.	Learner:	ten times
435.	Teacher:	or taking tens five times from fifty. Ok, the question I want to ask is this, do you think multiplication can help you get the division answer? Multiplication is implying that for you to have fifty, you need how many tens?

436.	Learners:	five
437.	Teacher:	hello! Hil, I am teaching and you are just busy pointing fingers at one another, not even listening. How will you even know what I am asking you here, coming to this classes mean one should be very, very paying attention? Why pointing fingers to Dav? Hil, why pointing fingers to Dav? Or do you want me to release you to go out this class?
438.	Teacher:	ok, how many tens would you take from fifty for fifty to get finished? Yes!
439.	Learner:	five
440.	Teacher:	the question I was asking is this, do you think you can get the division answer using multiplication? Can tell how many tens are in fifty? Hello! Can you tell how many tens are in fifty? How many tens are in fifty? Look at the groups on you desk. How many tens came out from fifty? Yes!
441.	Learner:	five
442.	Teacher:	meaning if you were to divide fifty by ten the answer would be what? What did you say about the number of tens in fifty just now? Era!
443.	Era:	five
444.	Teacher:	so, how many fives are in fifty? If tens are there five times, how many fives are there in fifty? Yes!
445.	Era:	ten
446.	Teacher:	can't you see the similarity, if you say ten times five is fifty can you see it?
447.	Learner:	yes
448.	Teacher:	ten times five is fifty you see that?
449.	Learner:	yes
450.	Teacher:	what are five times ten? What are five times ten? Yes!
451.	Learner:	is fifty
452.	Teacher:	is fifty, the answer will be similar. Numbers in multiplication does not matter which one come first because ten times five is fifty and even five times ten is also fifty. This is also related to division, if you divide fifty by five you will get? If you divide fifty by five what will you get? Hil!
453.	Hil:	ten
454.	Teacher:	is it related to multiplication or what? If you divide fifty by ten it gives you what? If you divide fifty by ten it gives you what?
455.	Learner:	five
456.	Teacher:	five, is that related to multiplication? How do division and multiplication relate to one another? (Pause) can you see at the chalkboard? Can you see what I am doing Dos? What did I say? At multiplication ten times five is what?
457.	Dos:	fifty
458.	Teacher:	if it is five times ten?
459.	Dos:	it gives hundreds
460.	Teacher:	what? Five times ten?
461.	Dos:	it gives fifty
462.	Teacher:	then I go to division fifty divided by five?
463.	Dos:	five
464.	Teacher:	so fifty divided by ten will give you?
465.	Dos:	five
466.	Teacher:	and fifty divided by five gives you?
467.	Dos:	ten
468.	Teacher:	look here at the chalkboard, can you see that this five and this ten on multiplication is the same as these on division (talking to the whole class) can you see the ten and five that give the fifty here?
469.	Learners:	yes
470.	Teacher:	when multiplying you use ten and five to get fifty. So even at division it gives these numbers. This fifty divided by five it gives ten and so fifty divided by this ten it gives you this five. Can you see it?
471.	Learners:	yes

472.	Teacher:	let me give you another example, six times two gives you what? Six times two is what? You can take the stones and make two groups of six stones and see what you will get. Yes!
473.	Learner:	twelve
474.	Teacher:	Are twelve. Good! And two times six? Make two stones six times and see what you will get. Yes!
475.	Learner:	is twelve
476.	Teacher:	can you see it here at multiplication?
477.	Learners:	yes
478.	Teacher:	you can see it at multiplication now I can bring the same number to division. What will be twelve divided by six equal to? Or how many six can you take from twelve?
479.	Learner:	two
480.	Teacher:	and two is the same, two that is there on multiplication side, on multiplication we were having twelve, six and two and even at division we have the same numbers twelve, six and two. Ok if I put twelve here divided by two what would be the answer? Yes!
481.	Learner:	six
482.	Teacher:	because there are how many twos in twelve?
483.	Learner:	there are six twos in twelve
484.	Teacher:	yes six twos. That means two there six times gives you what? Two times six gives you what?
485.	Learner:	twelve
486.	Teacher:	so why may be multiplication and division are related? (Pause) ok let me go to the next question. Have you seen that ten stones times five gives ….. (interrupted, Sab I want everyone to listen) ten stones there five times gives you fifty at multiplication tells us that there are five tens in fifty and ten fives in fifty. You have also seen on division that when you divided fifty stones into five groups, how many stones did you get in each group?
487.	Learner:	ten
488.	Teacher:	the question is here, why similar answers from multiplication and division? Yes!
489.	Learner:	because the stones were the same
490.	Teacher:	because the stones are the same, the stones that we divided were the same as the number of multiplication right?
491.	Learners:	yes
492.	Teacher:	what makes you think that your answer is correct?
493.	Learner:	because the stones get finished
494.	Teacher:	the stones get finished what else? What else tells you that you are correct? What if one group has ten stones the other has something else. So stones to be divided get finished and the groups are how?
495.	Learners:	the groups are enough
496.	Teacher:	enough in a sense that they are five as you were told to make them right? And all groups are how?
497.	Learner:	they all have equal number of stones
498.	Teacher:	all groups has equal number of stones
499.	Teacher:	I want someone to give me an example of multiplication the relationship between multiplication and division. Let someone give us the example of multiplication in relation to division. I have given you an example that if you have ten times five you'll get fifty in that you have three numbers ten, five and fifty and even at division you will have fifty divided by ten gives five or fifty divided by five gives ten. I also gave you the example of six, two and twelve, six times two is twelve and two times six is twelve, but when you divide is like a reversal of it isn't it?
500.	Learner:	yes
501.	Teacher:	twelve divided by six you will get two or when is divided by two you will get six. Who can give me an example as such, taking it from multiplication to division? (A very long pause) so no body is able to give an example? Ok, thank you.

LESSON FOUR

1.	Teacher:	we will draw in question one, draw eighteen apples. Do not draw too big apples or else your works will not fit on that paper and do not leave bigger spaces between the drawings, draw them closer. Eighteen apples (talking to one learner), like you, you have left a space on top of the paper why did you left it? Now continue your drawing going up some of the drawings, draw them on top of each other because you need a space for small boxes that you'll share or pack these apples. Do not draw too big apples or else they will finish up the space. Make sure you have drawn eighteen apples after drawing eighteen apples, share the apples into three boxes and ensure all boxes have equal apples. It is good that each one of you comes with a pencil from home then you do not have to wait for others to finish.
2.	Teacher:	(talking to the whole class) apples must be eighteen as you are told and remember to cross out with a line from the big box each apple transferred to the small box (shared) to avoid confusion. (talking to one learner) I told you to draw them as a group but not in a row, now where will you draw the three boxes? Use the rubber, that way you'll dirtify your paper.
3.	Teacher:	(talking to Hanna) how many apples are those Hanna? When the apples reach eighteen share them into three boxes, make three empty boxes and start putting apples inside the boxes. (talking to Dav) Dav! Why did you not put this box here? I told you already you need space for other activities. Your boxes supposed to be here all of them rub this one and put it here, because you'll have about other five things to draw on that paper, so do not waste the space. (Talking to the whole class) you see this one has done well by dividing her work with a line (showing the work of the one learner).
4.	Teacher:	are you done Hil?
5.	Hil:	yes
6.	Teacher:	how many apples did you put in each box?
7.	Hil:	six
8.	Teacher:	six
9.	Teacher:	ok, how many are done? Rise up your hand. (After few hands were up) so only one, two, three, four, we will wait until everyone is done.
10.	Teacher:	when you were sharing the apples in the boxes, what were you doing? Is it division, subtraction, addition or multiplication? Pet!
11.	Pet:	is subtraction
12.	Teacher:	so you were subtracting? Subtracting from the box or what? Yes!
13.	Learner:	it is division
14.	Teacher:	you were dividing, what were you dividing Pet?
15.	Pet:	box
16.	Teacher:	I cannot hear what you say you were dividing. I only hear the word box.
17.	Pet:	sweets
18.	Teacher:	which one? I want someone explaining things clear like this; I was sharing this like this Naf!
19.	Naf:	I was dividing apples taking them from the box sharing them between these three other boxes.
20.	Teacher:	ok, the apples you were sharing how many were they? (Pause) how many apples you first drew?
21.	Learner:	eighteen
22.	Teacher:	Shared into how many boxes?
23.	Learner:	three
24.	Teacher:	how many apples do each box shared?
25.	Learner:	six
26.	Teacher:	(talking to the whole class) can you write below question two the number sentence of what you were doing?

27.	Learners:	yes
28.	Teacher:	write it then, writing eighteen divided by whatever number you divided with which gave you an answer. Write it below question two. Below question two on your question paper. Write what you had divide with what the number and the answer you got. Have you written the number sentence below question two?
29.	Learners:	yes
30.	Teacher:	if you are done with question two go back to your worksheet, underline the work that is already done. After underlining write number three to label question three below the underlined works. Question three is saying, draw twenty – four sweets, draw twenty –four sweets and share them to four people. Ok share them between four people (showing on the chalkboard how learners should do it) I want to see your drawing drawn in this manner, a big box where to draw twenty – four sweets and four circles where to share the sweets, then allocate twenty – four sweets into the four circles until the sweets get finished. Do you understand?
31.	Learner:	yes
32.	Teacher:	so we do not know how many sweets would be in the groups yet.
33.	Teacher:	how many sweets do each person got?
34.	Learners:	six
35.	Teacher:	who else is done? (Pause) ok, if some of you are not done yet, let me ask Naf some questions while you are drawing. Naf, here you have written this number sentence of division, why division and not subtraction, addition or multiplication? What makes you think this operation suit what you were doing? (Pause) what were you doing?
36.	Naf:	I was drawing apples
37.	Teacher:	what I asked is this; the sign of operation you have used is that of division and not that of addition, subtraction or multiplication. What makes you think that this sign suit what you were doing? Why used division and not addition, subtraction or multiplication? Is because you were doing what?
38.	Naf:	is because I was subtracting
39.	Teacher:	now if you were subtracting why used the division sign then and not subtraction sign? Were you subtracting or were you dividing? This apples what were you doing with them into this groups?
40.	Naf:	I was taking them out from here and put them into the boxes
41.	Teacher:	put in the boxes how? Even the sweets what were you doing with the sweets and the kids?
42.	Naf:	I was sharing them to the kids
43.	Teacher:	good, the word share or divide is what I wanted to hear from you, because it agrees with the division sign you have used on the number sentence, so you put division sign because you were dividing apples into these boxes right?
44.	Naf:	yes
45.	Teacher:	and you were sharing sweets to the kids' right?
46.	Naf:	yes
47.	Teacher:	ok, let me ask you another question, how did you share or divide these apples? Did you put in one box first until the box is full or how did you share them?
48.	Naf:	I have been taking one by one and allocate and put into boxes until they become six.
49.	Teacher:	taking one by one from where? Where were you taking the apples when you put them into boxes?
50.	Naf:	from this box
51.	Teacher:	box with what?
52.	Naf:	apples
53.	Teacher:	how many apples in the boxes
54.	Naf:	twenty – eight
55.	Teacher:	is that twenty – eight? In that box there are twenty – eight apples? Is this number twenty – eight?
56.	Naf:	eighteen
57.	Teacher:	so you have been taking apples from the box with eighteen apples right?

58. Naf: yes
59. Teacher: sharing them into these boxes right?
60. Naf: yes
61. Teacher: how did you say you share them? You take …….
62. Naf: one by one until apples becomes six
63. Teacher: how did you know they would become six or do you mean you stopped when apples get finished?
64. Naf: until apples get finished
65. Teacher: ok, when you counted how many apples each box have?
66. Naf: six
67. Teacher: meaning if apples from the box were not finished, you would have continued allocating or put more apples into boxes right?
68. Naf: yes
69. Teacher: so is not because apples become six in the boxes that stopped you but what stopped you from continuing sharing or allocating is ….?
70. Naf: because where I was taking apples, apples get finished.
71. Teacher: ok, they got finished now it is clear. Was it the same when you shared sweets?
72. Naf: yes
73. Teacher: you have been taking sweets from here
74. Naf: yes
75. Teacher: sharing them to whom?
76. Naf: to kids
77. Teacher: these kids right?
78. Naf: yes
79. Teacher: until what?
80. Naf: until the sweets got finished from where I was taking.
81. Teacher: ok, until sweets got finished, how many sweets each child has?
82. Naf: six
83. Teacher: each child has six sweets. Ok, what makes you think that your answers are correct?
84. Naf: because where I was taking sweets which I shared to kids, the sweets got finished.
85. Teacher: where you took sweets to share to kids, sweets get finished and how are the sweets shared by the kids?
86. Naf: they have equal sweets.
87. Teacher: they have equal sweets; can you give me an example of what we had just done now? Examples of things you can share can you?
88. Naf: no
89. Teacher: you cannot give an example of something you can share amongst kids.
90. Naf: having four sweets
91. Teacher: ok, is like having four sweets shared by?
92. Naf: by two kids
93. Teacher: sharing them to two kids' right?
94. Naf: yes
95. Teacher: how many sweets each kid would have?
96. Naf: two
97. Teacher: so each kid would have two sweets right?
98. Naf: yes
99. Teacher: ok, now I want you to proceed to the next question that say draw thirty sticks and allocate these sticks into five groups of equal sticks. And find out how many sticks each group would have.
100. Naf: yes
101. Teacher: ok, first you draw thirty sticks, after you draw thirty sticks you make five groups, then keep allocating sticks to groups and remember to cancel the sticks you have allocated from the thirties. Then find out how many sticks each group has.
102. Teacher: who else is done?
103. Learner: yes

104. Teacher: let me have your worksheet. Ok, here you have written eighteen divided by three.
105. Learner: yes
106. Teacher: but on your work there are no answer, haven't you find out how many apples each box had?
107. Learner: I did
108. Teacher: then why didn't you write the answer? We normally write the number sentence like this i.e. 15 ÷ 3 = 5, but why your do not have an answer, that is what I am asking, or is it there?
109. Learner: yes
110. Teacher: ok, it is there only that your numbers are written into too small letters to see.
111. Learner: yes
112. Teacher: ok, now write on number four, the number sentence, stating what happened to the twenty -four sweets.
113. Teacher: so you are counting to get how many sweets each child got right?
114. Learner: yes
115. Teacher: so you will count the sweets of each child right?
116. Learner: yes
117. Teacher: how many sweets do each child got?
118. Learner: six
119. Teacher: now I will ask you questions. You will need to answer me. All your number sentences you had used division sign, what makes you think that what you were doing is division?
120. Learner: is because
121. Teacher: start with the apples question
122. Learner: is because I was packing them into empty boxes, putting apples into boxes one by one.
123. Teacher: so you were packing them into empty boxes, allocating apples one by one in the boxes?
124. Learner: yes
125. Teacher: after allocating apples one by one into empty boxes, how did the division ended up?
126. Learner: I finished because the apples got finished from the box
127. Teacher: ok, you finished dividing because all apples got finished in the box
128. Learner: yes
129. Teacher: are these apples also taken out, because you have not cancelled them?
130. Teacher: what makes you thinks that the answers that you got are correct? What if someone is to argue with you that the answer is not six but seven? What makes you think that no my answer must be correct?
131. Learner: is because what I was sharing got finished from the box.
132. Teacher: so is because thinks to be shared got finished from the box?
133. Learner: yes
134. Teacher: so meaning when the things to be shared get finished to you the answer would be correct, now what if after the shared things got finished, you realizes apples in the boxes are not equal, one box have five other box six and the other different? So what else makes you think that you are correct? Is because all boxes are…..?
135. Learner: is because the sweets shared are equal and the apples in the boxes are equal.
136. Teacher: can you give me an example of what you were doing right now?
137. Learner: yes
138. Teacher: what could it be
139. Learner: is like having ten maize cobs sharing them to two kids and each one of them would have five maize cobs.
140. Teacher: now I want you to do question five. It requires you to draw thirty sticks and after you draw thirty sticks start grouping them into five groups and ensure that each group share equal sticks taking the sticks from the thirty sticks. Who else is done? (five to six learners indicated by rising up their hands indicating that they are done)

141. Teacher: (talking to those who are done) write the number sentence at question four. Your number sentence must show what you were doing with twenty – four sweets. Write that number sentence below question four.
142. Teacher: (talking to Nab) Nab, you did not do as you were told. First I told you to draw twenty-four sweets and after you do that you share them to four children, now your twenty – four sweets are not here. Where were you taking the sweets you shared to kids?
143. Teacher: are you done
144. Learner: yes
145. Teacher: the question I have for you is this, when I peeped at many papers you have used division sign, like twenty – four divided by a number and then the answer. so why have you opted for division sign? What makes you use it? Era!
146. Era: is because we were dividing
147. Teacher: so you were dividing, dividing what?
148. Era: sweets and apples
149. Teacher: how
150. Era: for apples I was dividing them into boxes and sweets I was sharing them to children
151. Teacher: when you were dividing eighteen apples into three boxes, how many apples does each box get?
152. Learners: (per turns) six, six, six, six
153. Teacher: (talking to the fourth learner who also said six but her work shows otherwise) look at your work very well.
154. Learner: eight
155. Teacher: so you put eight in each box, who else said is done here? Dav, how many apples you put in each box?
156. Dav: six
157. Teacher: ok, who else is done how many did you put in each box?
158. Learner: four
159. Teacher: how many apples did you put in each box?
160. Learner: six
161. Teacher: are you done? How many apples did you put into each box?
162. Learner: four
163. Teacher: four ok, how many apples did you put in a box?
164. Learner: six
165. Teacher: six, now the question I want to ask you is this, how did you divide your sweets and apples? Who can tell me how s/he divided? How have you been dividing?
166. Learner: first I draw sweets in the box and draw four children and start giving children sweets one by one until all children get six sweets.
167. Teacher: did you stop dividing because each child get six sweets or is it because where you were taking the sweets got finished or what?
168. Learner: is because the sweets get finished
169. Teacher: meaning if there were still sweets left you would've been still sharing or giving sweets?
170. Learner: yes
171. Teacher: who can tell me why they think the answer they got is correct?
172. Learner: because what I was dividing got finished
173. Teacher: ok, the things to be divided got finished. What else, because you could have finished what you were dividing but things are not divided equally?
174. Learner: the boxes also shared equal things.
175. Teacher: the boxes shared equal things and that is what makes you think your answer is correct right?
176. Learner: yes
177. Teacher: ok, who can give me an example as such? Who can give me an example of things that you can divide in that manner Hil, is like dividing what?
178. Hil: is like dividing four sweets
179. Teacher: I do not want any example of four sweets anymore so is like dividing what into what?

180.	Learner:	is like dividing ten cool drinks into two people and every one would have five drinks.
181.	Teacher:	(talking to the whole class) I do not want any example of 4, 2, 5, and 10 anymore because everyone gives only an example of ten things divided by five or two. Now I want example of other numbers
182.	Teacher:	Nab, I want you to tell me, because you have not drawn like others. You had just drawn children that shared the sweets. Where have you been taking the sweets you shared to children? (Nab did not answer) are you done allocating sweets?
183.	Nab:	no
184.	Teacher:	do you know how many sweets each child would get?
185.	Nab:	no
186.	Teacher:	you don't know. How would you've come to know then?
187.	Teacher:	Dos, how many sweets you gave to each child?
188.	Dos:	six
189.	Teacher:	Dos, I have seen how you have done. You did not allocate sweets one by one to children until all sweets get finished but you gave sweets to one child at a time, to another child until you are done. Now how did you come to know that each child would have six sweets? What did you do to know that?
190.	Dos:	first I draw sweets and put them there and count them and share to children
191.	Teacher:	ok, sharing them to children, how did you know each would get six sweets? Because others have been sharing allocating one by one until all sweets get finished and then counted to find out how many sweets each child got, but you started giving each child six sweets, how did you know each child would get six sweets?
192.	Dos:	I counted sweets and find out they are twenty – four and started dividing to four children, I counted one, two, three, four, five and six and give to a child and repeated the same process until all children get six sweets.
193.	Teacher:	did you estimate that each would get six or what? Because you're counting ended at six sweets and not at five or seven.
194.	Teacher:	I can see that all other questions looks similar to the questions already done, after drawing thirty sticks and divided them, answer question six, which asks you to write a number sentence of what you were doing.
195.	Teacher:	Naf, how many sticks each group has?
196.	Naf:	eight
197.	Teacher:	the sticks you have drawn first were how many?
198.	Naf:	thirty
199.	Teacher:	thirty?
200.	Naf:	yes
201.	Teacher:	ok
202.	Teacher:	(asking another learner) have you drawn thirty sticks
203.	Learner:	yes
204.	Teacher:	how many sticks each group has?
205.	Learner:	six
206.	Teacher:	(asking Hil) how many sticks have you drawn?
207.	Hil:	thirty
208.	Teacher:	and how many sticks each group has?
209.	Hil:	six
210.	Teacher:	(asking Era) how many sticks did you draw?
211.	Era:	thirty
212.	Teacher:	when you divided, how many sticks each group has?
213.	Era:	six
214.	Teacher:	how many sticks did you draw Mic?
215.	Mic:	thirty
216.	Teacher:	thirty, how many sticks did you put in each group?
217.	Mic:	six
218.	Teacher:	six, are you sure your answer is correct?
219.	Mic:	yes

220. Teacher: what makes you think that your answer is correct?
221. Mic: because all sticks to be shared get finished.
222. Teacher: and how did you share the sticks in the groups?
223. Mic: I allocated one by one
224. Teacher: so you allocated one stick by one into each group, meaning how are the sticks allocated into each group?
225. Mic: equal sticks.

DAY THREE **11 MAY 2012**

LESSON FIVE

1. The lesson started with greetings and instructions of what is expected to be done in the lesson. Learners were told to write their names on all their question papers and worksheets
2. Teacher: draw twenty apples and draw five groups where you will share these apples into equal portions. So find out how many apples each groups will have.
3. Teacher: have you finished drawing?
4. Learners: yes
5. Teacher: can you write that into a number sentence.
6. Learners: yes
7. Teacher: ok, write the number sentence below question two
8. Teacher: Do you remember how last time we got the division answer by subtracting?
9. Learners: yes
10. Teacher: I want you to write, or label question two on the other paper you have drawn. How many apples did you draw first? Jaa!
11. Learner: twenty
12. Teacher: and each group has how many apples?
13. Learner: four
14. Teacher: and you how many did you get?
15. Learner: four
16. Teacher: and Fil how many apples each of your group have?
17. Fil: four
18. Teacher: Hanna how many apples each group has?
19. Hanna: six
20. Teacher: six, ok
21. Teacher: (she moved around asking learners the same question) how many apples each group have?
22. Learners: (learners responded each per their turn) four, four, four, four, four, four, four, four
23. Teacher: I want us to do as we did yesterday; you started with twenty apples isn't it?
24. Learners: yes
25. Teacher: then write the process on your work sheet paper, the blank paper. Start the process like this, writing like this, twenty apples minus four what would remain. Write the answer minus four again and see what will remain, keep subtracting four until you reach zero or zero remains. After the whole process count how many fours you have subtracted from twenty to reach zero. (She repeated the instruction again) can you tell me after the process that twenty divided by four is what?
26. Teacher: yesterday we said if four times five is twenty, so five times four is also twenty. In division is also like that. If twenty divided by five is four so twenty divided by four is also five. (Era, goes back to the apples drawing and subtract four, he counted the left over and wrote the answer. he took out four again and counted the left over and wrote the answer. he kept doing that until the apples got finished. Naf, Fil, Mic, and Sab also do the same thing. Hanna and Dav seems not so sure of what to do. Mat is done.) How did you have these answers Mat? What did you do first?
27. Mat: I took twenty and subtract four from it.

28. Teacher: which twenty you took? Did you go back to the drawing or you calculated in your mind or how did you do it?
29. Mat: I did the calculation in my mind
30. Teacher: so all these answer you got them calculating in your mind or did you use the drawing anywhere?
31. Mat: Some I used the drawing
32. Teacher: Ok, Dos how did you calculated? How did you get these answers?
33. Dos: I calculated in my mind first I calculated twenty minus four and there remained sixteen and then sixteen minus four there remained twelve and then twelve minus four there remained eight and eight minus four remained four and four minus four there would remain zero.
34. Teacher: so you calculated all these from you mind and have not used the drawing?
35. Dos: I have not used the drawing
36. Teacher: Nab, how did you calculated to get these answers? These answers how did you have them?
37. Nab: I took twenty
38. Teacher: which twenty
39. Nab: the twenty apples
40. Teacher: so you have used the apples drawing to get these answers? How did you used the drawing then?
41. Nab: I took twenty and subtracted
42. Teacher: subtracted how? I want you to tell me all what you have done.
43. Nab: I took four out
44. Teacher: took four out and do what to the remaining? Did you counted them or what?
45. Nab: yes
46. Teacher: did you then keep subtracting four or what?
47. Nab: yes and then sixteen remained
48. Teacher: when sixteen remained what did you do?
49. Nab: I took four from sixteen and twelve remained
50. Teacher: (asking another learner) how did you have the answers?
51. Learner: I got them when I took twenty drawing of apples and took four apples away and sixteen remained and I took four again from sixteen and twelve apples remained and I took away four apples from twelve again and eight apples remained and I took four apples away from eight and four apples remained and I took four apples from four again and zero apples remained.
52. Teacher: Hen, how did you have these answers?
53. Hen: I counted apples, I counted twenty of them and take and sixteen remained.
54. Teacher: and from there?
55. Hen: I counted apples again
56. Teacher: how many?
57. Hen: they are thirteen
58. Teacher: when you subtracted the first four there remained sixteen apples. Now why starting from thirteen and not from sixteen that remained. And here there remained twelve apples but you subtracted from fourteen instead of from twelve why?
59. Teacher: has anyone done writing the division and subtraction process on the paper?
60. Learners: yes
61. Teacher: this process is showing us how you subtracted fours from twenty until you reach zero
62. Learners: yes
63. Teacher: how many fours did you take away from twenty?
64. Learners: (a lot of learners raised up their hands and sounds as were eager to be given a chance) ah, ah, ah,
65. Teacher: Dav
66. Dav: five
67. Other Learners: (as per turn) five, five, five, five, five (it continued until reached eleventh)
68. Teacher: everybody took five fours from twenty right?
69. Learners: yes

70. Teacher: you subtracted until you reached zero right?
71. Learners: yes
72. Teacher: what I want to ask you is this, which means if you divide twenty by four, what will you get? Or let me change the questions this way. Do you think you can transform what we have done in subtraction to division; can you write it in a division form? Who can do it because you said from twenty you repeatedly take four fives times. It's like dividing twenty with four and gets what? Hil!
73. Hil: twenty divided by four and get five
74. Teacher: is divided by four and get five is that so?
75. Learners: yes
76. Teacher: does the process of subtraction related to twenty divided by four getting five is it? (after repeating the same question over and over again without getting an answer from learners she go back to the drawing)
77. Teacher: go back to the drawing of apples see if you can get four apples times five from the twenty apples, Mic what did you say?
78. Mic: I said I went back to the five groups of four apples; I arranged from twenty apples and counted the groups.
79. Teacher: ok, you counted that the four apples groups are there five times isn't it?
80. Teacher: now look at the chalkboard how the fours are repeatedly taken away from twenty, will this tells how many fours are in twenty or how many times one can take fours from twenty?
81. Learners: yes
82. Teacher: how many fours are in twenty? If there are twenty things, how many times you can take away four things?
83. Learner: is five times
84. Teacher: if someone says divide and the one who is subtracting, do you think you will all get the same answers? Sab!
85. Sab: yes
86. Teacher: why may be the answer is going to be the same?
87. Sab: because there are only five fours in twenty
88. Teacher: so is because there are only five fours in twenty, whether you divide into groups or repeatedly subtracting the four apples you will only take the four apples five times right?
89. Teacher: I want you to rewrite the process of subtraction into division way, transform the subtraction process, like the way you have been repeatedly subtract fours five times is the same as twenty divided by what and get what? Write this below question three.
90. Teacher: ok, now I want, yesterday we discussed that one can get the division answer by adding the same number repeatedly. Do you still remember that?
91. Learners: yes
92. Teacher: now at another worksheet, label number seven and draw for me the three sweets keep drawing three sweets in groups of threes until all the sweets you have drawn reach fifteen. After that you will then tell me how many three sweets you repeatedly added to get fifteen sweets. Let sweets be drawn in groups of three and keep adding three sweets until sweets sum up to fifteen.
93. Teacher: the question I have is how many threes did you needed to get fifteen?
94. Learners: (in turn) five, five, five, I am not done, I am not done, five, five, I am not done, I am no done, I am not done, five, I am not done, five
95. Teacher: so you used those threes to get fifteen?
96. Learners: yes
97. Teacher: now I want you to rewrite the process on the paper, first you took only three right? Only three sweets
98. Learner: yes
99. Teacher: then you added another group of three sweets and counted their sum which you find out it is not yet fifteen right?
100. Learner: yes
101. Teacher: now when you find out that the two threes did not reach fifteen, what did you do?
102. Learner: I added three more sweets

103. Teacher: then you keep adding until sweets reach fifteen; now I want you to write the process on the other paper how you have been adding threes until you reach fifteen, label the work question eight.

104. Teacher: (drawing the sweets at the chalkboard and showing the learners how to do the process at the chalkboard) first you started with the one only group of three sweets, which is one three (3). Then it became three times two when you added another group of three sweets (3 + 3) then they become three sweets times three when three more sweets was added (3 + 3 + 3), then they became three sweets times four when the three sweets were added (3 + 3 + 3 + 3), keep doing that until when the sum of the threes reach fifteen and write the sum of the threes at every step of the process.

105. Teacher: (asking the learners) when there were only three sweets it is one three isn't it? How many sweets did they become altogether when you added three more sweets Dav?

106. Dav: six

107. Teacher: how many sweets did they become when sweets become three times three

108. Learner: nine

109. Teacher: keep adding the threes and count until you reach fifteen; the row that has the sum of fifteen has how many threes?

110. Learner: five threes

111. Teacher: five, the row that makes up fifteen has five threes

112. Teacher: what does that mean, it mean how many threes are in fifteen? How many threes added together are in fifteen? Nab!

113. Nab: five

114. Teacher: ok, if you have fifteen, how many threes can you take out of fifteen? How many threes can you take out of fifteen?

115. Learner: five threes

116. Teacher: because fifteen was made up by how many threes added together? Hil!

117. Hil: five three

118. Teacher: meaning if they say divide fifteen by three, what answer would you get? If they say divide fifteen by three, what answer would you get? Sab!

119. Sab: five threes

120. Teacher: five threes, because there are how many threes in fifteen? Because there are how many threes in fifteen? Naf!

121. Naf: five threes

122. Teacher: five threes, now can you see how repeated addition can give you the division answer, can you see that addition process can also help you get the division answer right?

123. Teacher: now I want to ask, why the answers? Because you said when you repeatedly add threes, how many threes make up fifteen?

124. Learner: five

125. Teacher: ok, why the repeated addition answers gave the number of threes equal to or same as the division answer of fifteen divided by three

126. Learner: because all numbers are the same

127. Teacher: which same number are you referring to?

128. Learner: the threes

129. Teacher: ok, what else

130. Learner: in all the process we also used the same number fifteen.

131. Teacher: what else is common in that?

132. Learner: all groups has three sweets, which is all equal

133. Teacher: all groups in all process had equal and same number five of the sweet, because you were building fifteen in addition the same fifteen you divided and you were also building it with threes the same number three you divided by that is why the answers in both process is the same which is five.

134. Teacher: what makes you think that the answers you got are correct? What makes you think that the answers you got are correct? Hil!

135. Hil: is because when adding threes together sweets become enough.

136. Teacher: so sweets became enough as you only needed fifteen sweets right?

137.	Hil:	yes
138.	Teacher:	so you could no longer add another three because you had reached fifteen sweets?
139.	Hil:	yes
140.	Teacher:	and you intention was to build the fifteen sweets you were asked to?
141.	Hil:	yes
142.	Teacher:	and how many threes you had used when you built fifteen?
143.	Hil:	five threes
144.	Teacher:	who else has a different answer? What makes you think that your answer is correct?
145.	Learner:	is because all groups have equal number of sweets?
146.	Teacher:	all groups have equal number of sweets. Which groups?
147.	Learner:	groups of sweets
148.	Teacher:	and how many sweets per groups?
149.	Learner:	three sweets
150.	Teacher:	you could've added three more sweets to make eighteen sweets, so what makes you think your answer is correct?
151.	Learner:	is because I counted and the sweets are fifteen.
152.	Teacher:	because you counted fifteen sweets, does that mean you can no longer add?
153.	Learner:	yes
154.	Teacher:	why
155.	Learner:	because sweets get finished
156.	Teacher:	what do you mean they got finished, you were building? Jaa! (giving a chance to another learner)
157.	Learner:	because the sweets I was building get enough
158.	Teacher:	so sweets got enough, the exact number you were building.
159.	Teacher:	write this process in a division form, like fifteen divided by three and get what? Write that process in a division form, the process of addition building fifteen by threes. Write that below question nine on your question paper.
160.	Teacher:	yesterday we also talk about how multiplication can be used to get the division answer is that right?
161.	Learners:	yes
162.	Teacher:	use the space left or the next clean papers to draw fifty sticks and group them into five groups of equal sticks.
163.	Teacher:	as you are drawing keep thinking this, how many groups of how many sticks you take out from fifty sticks? And if you are dividing, how would you represent it? Write it into division number sentence.
164.	Teacher:	Hil, are you done?
165.	Hil:	yes
166.	Teacher:	into how many groups did you divide your fifty sticks?
167.	Hil:	five groups
168.	Teacher:	how many sticks each group has?
169.	Hil:	eight sticks
170.	Teacher:	each group has eight sticks? Mhh! Are you sure your sticks were fifty? Did you count them well?
171.	Hil:	yes
172.	Teacher:	ok, who else is done? If you are done rise up your hand.
173.	Teacher:	tell me if you are multiplying and you want to get fifty, how many groups do you have?
174.	Learner:	five
175.	Teacher:	so this is five, number five right? And how many sticks are in each group?
176.	Learner:	ten sticks
177.	Teacher:	ok, now if you are to multiply you will say five times what to get fifty?
178.	Learner:	five times ten
179.	Teacher:	ok, write that ten then in the column, does that mean if ten sticks are there five times will give you fifty sticks right?
180.	Learner:	yes

181.	Teacher:	ok, now I want you represent it into division number sentence. What would it look like, fifty divided by what to get what?
182.	Learner:	is fifty divided by five
183.	Teacher:	and get what?
184.	Learner:	get ten
185.	Teacher:	ok, write that process here then.
186.	Teacher:	how can you tell that these sticks are fifty? How did you come to know?
187.	Learner:	I counted
188.	Teacher:	how
189.	Learner:	I counted fifteen
190.	Teacher:	fifteen, which fifteen?
191.	Learner:	I counted ten, and then ten and then ten sticks.
192.	Teacher:	just continue
193.	Learner:	then ten again and ten again and from there I recounted them all and find out they are fifty sticks
194.	Teacher:	so you recounted all the sticks together and find out that ten sticks times five gives what?
195.	Learner:	the ten sticks times five will give me fifty sticks.
196.	Teacher:	what makes you think that your answer is correct and is not wrong?
197.	Learner:	is because the fifty sticks I was grouping got finished.
198.	Teacher:	continue explaining
199.	Learner:	and when I counted all sticks in each of five groups are equal and they are all ten
200.	Teacher:	can you give the example of multiplication as such we were doing and explain it in the terms such as, is like this number multiplied by this gives you this answer and in division you divide that number with this and get this answer.
201.	Teacher:	who else is done? Rise up your hand if you are done. Ok, how many sticks are in each group?
202.	Learner:	ten
203.	Teacher:	and in your group?
204.	Learner:	I am not done
205.	Teacher:	in your group?
206.	Learner:	Ten
207.	Learners:	(as per turn) ten, ten, ten, ten, eight, eight, I am not done, ten, ten
208.	Teacher:	looking at your question below it, there is $5 \times \ldots = 50$, so you say groups are five rights
209.	Learner:	yes
210.	Teacher:	how many sticks are in each group? So do it in multiplication, you have how many sticks in groups times five, which makes fifty sticks? Write that number in the box in the question paper, are you done?
211.	Learner:	yes
212.	Teacher:	if you were dividing, it means fifty divided by what gives what? Remember yesterday we used six and we said if you have six times two makes twelve, just like you have five times that number in the box on the question paper. Are you done?
213.	Learner:	yes
214.	Teacher:	so use this fifty sticks and write it in division form at the next page at question eleven, use the drawing of sticks and what you were doing to help you get the answer
215.	Teacher:	(asking Nab) did you make three groups on your drawing? (the teacher asked when she saw Nab had divided by three on his number sentence) (There were also some learners who did it well but their answer was wrong).
216.	Teacher:	what makes you think that the answer you got is correct? Dos!
217.	Dos:	yes
218.	Teacher:	what makes you think it is correct?
219.	Dos:	is because when I divided fifty sticks into five groups each groups had ten sticks.
220.	Teacher:	what if you could still have taken sticks where you were taking and make each group have eleven sticks?
221.	Dos:	when I kept allocating sticks one by one the fifty sticks got finished
222.	Teacher:	I want everyone to look here at the chalkboard and listen to me, you have five groups right?

223. Learner: yes
224. Teacher: and each groups has ten sticks right?
225. Learner: yes
226. Teacher: and all these five groups with ten sticks each which are five times ten give fifty rights?
227. Learner: yes
228. Teacher: and this is the multiplication 5 × 10 = 50, now my question is this, what makes you think that at division, if I divide fifty by five you will get ten as an answer? What led to this similarity that when you divided it was five groups each with ten sticks that came out of fifty and in multiplication it is also five times ten that makes fifty? Why the same answer? Why the same answer?
229. Learner: is because all things are the same
230. Teacher: what do you mean things are the same?
231. Learner: is because all number are the same
232. Teacher: which numbers are you referring to?
233. Learner: is the ten
234. Teacher: which is that ten you are talking? (Pause) when you divided, into how many groups did you divided?
235. Learner: five
236. Teacher: and when you multiplied into what multiples (or number of groups) did you multiplied the sticks?
237. Learner: five
238. Teacher: now why do you think the answer is the same, is because you have what in common?
239. Teacher: is because in all multiplication and division you arranged how many groups?
240. Learner: five
241. Teacher: and when divided the sticks you divided them into how many groups?
242. Learner: five
243. Teacher: and all the groups have how many sticks?
244. Learner: ten
245. Teacher: who can give me an example as such? When giving an example explain saying is like this number, times this number gives you this answer and reverse it to division just like we have done it at the chalkboard.

LESSON SIX

1. Teacher: we are going to do activity (lesson) six and we are going to use dots (the teacher reminded learners to remember that all activities are based on division and learners have to do them in relation to division. she also explained what the dots are or look like by making a dot on the chalkboard as well as how to use the dots)
2. Teacher: the first question say make eighteen dots representing eighteen apples, after drawing eighteen dots, start allocating the dots into three circles (the teacher draw a circle on the chalkboard to remind learner of how it look like) this is how a circle look like. After allocating dots into three circles one by one find out how many dots each circles would have.
3. Teacher: write on the question paper how many dots or apples (because we said dots represent apples) are in each circles? (Asking the learners) how many dots are in each circle?
4. Learners: (as per turn) six, six, six, six, six, six, six, six, six
5. Teacher: now if your answer is six write it on your question paper.
6. Teacher: now write the number sentence of what you have just done on question number two, that shows what you had divided by what and the answer
7. Teacher: in most of your question paper I can see most of you have written eighteen divided by ……, so you used division sign and not plus, minus or any other
8. Learner: is because we were subtracting

9.	Teacher:	so you were subtracting, but why didn't you use subtraction sign here you used division instead? (O! o! o! o! many learners rising up their hands eager to rectify the error) the subtraction you mentioned and the division sign you've used is a contradiction.
10.	Learner:	is because we were dividing
11.	Teacher:	dividing what?
12.	Learner:	is because we were dividing dots
13.	Teacher:	dividing dots how? Say clear things.
14.	Learner:	we were dividing eighteen dots into three circles
15.	Teacher:	so is that the reason why you used division sign, is it because you were dividing right?
16.	Learners:	yes
17.	Teacher:	tell me how you allocated those dots. Who can tell me the process? Mic how did you divided?
18.	Mic:	I kept cancelling three dots from eighteen and allocate the cancelled dots into three circles one by one until all the eighteen dots got finished.
19.	Teacher:	is there anyone who allocated dots differently, not allocating one by one?
20.	Learner:	no
21.	Teacher:	Nab, how did you do it?
22.	Nab:	I allocated putting in circle one by one
23.	Teacher:	taking the dots from where?
24.	Nab:	I was taking from here (pointing to where the eighteen dots are drawn)
25.	Teacher:	what makes you think that your answer is correct? What if these dots in circles supposed to be seven and not six instead or any other number which is not six?
26.	Learner:	is because the dots to be allocated got finished
27.	Teacher:	ok, the dots to be allocated got finished, what else?
28.	Learner:	is because all things are equal
29.	Teacher:	please specify what things being equal you are referring to?
30.	Learner:	the dots I allocated into circles are allocated equally
31.	Teacher:	ok, who can give me an example as such? An example of division in that manner. Sab!
32.	Sab:	is like having eight things divided into two circles each circles would have four things.
33.	Teacher:	who else can give me another example, Dav!
34.	Dav:	is like having twenty sweets shared by two kids of which each would have ten sweets.
35.	Teacher:	Dos!
36.	Dos:	is like having thirty sweets shared by two people of which each would have fifteen sweets.
37.	Teacher:	the next question still require you to draw dots representing twenty – four sweets, now you will share the sweets to four children that are going to be represented by four rectangles (the teacher draw rectangles on the chalkboard to remind learners of what rectangles look like) each rectangle represent a child.
38.	Teacher:	are you done sharing the twenty- four sweets amongst four kids?
39.	Learners:	(some) yes! (And some) no!
40.	Teacher:	So others are still dividing, to those who are done, write below question four, the number sentence of what you were doing, the number sentence representing what you were doing.
41.	Teacher:	at question three, it asks how many sweets each children have. Write the answer below question three. How many sweets each child would have? How many sweets each child would have? How many sweets each child would have?
42.	Teacher:	when you shared twenty – four sweets to four children, Hen, how many sweets each child got?
43.	Hen:	six sweets
44.	Teacher:	Hen how did you divided?
45.	Hen:	I divided putting one in each rectangle
46.	Teacher:	taking the dots from where?
47.	Hen:	from here (pointing to where twenty – four dots are drawn)
48.	Teacher:	ok, after allocated one dot to each rectangle did you ended up there or what happened next?
49.	Hen:	I continue taking dots from the twenty – four dots and allocated still one by one in the rectangles

50. Teacher: until what?
51. Hen: until twenty – four dots got finished
52. Teacher: ok, who did not get six sweets? Pet, how many did you get?
53. Pet: six sweets
54. Teacher: ok, now the next question I have is when I looked at most papers you have represented your actions as twenty – four divided by …., why used the division sign and not addition, subtraction or multiplication?
55. Learner: is because we were dividing
56. Teacher: dividing what?
57. Learner: is dots
58. Teacher: Naf, what were we dividing?
59. Naf: it is dots, allocating them to rectangles.
60. Teacher: how did you divided? I need someone else to tell me how you divided. Fil how did you divided?
61. Fil: I divided by allocating dots one by one on rectangles until the twenty – four dots get finished.
62. Teacher: now I want you to make the thirty dots representing sticks and allocate them into five groups of triangles, thirty dots and share them into one, two, three, four, five triangles (as she count she was drawing the triangles at the chalkboard).
63. Teacher: are you done dividing?
64. Learners: yes
65. Teacher: I want everyone to tell me how many dots each group has. Say it low so that other people would not hear you.
66. Learners: (as per turn) six, six, six, six, four, six, I am not done, six, six, six, six, six, six, six, six.
67. Teacher: now I what you to do the last activity and go home, the fifth question we will not do, but before you go to the next question write below question six a number sentence of what you were just doing.
68. Teacher: after question six, do question seven in which you will draw eighty dots and allocate them into four groups. Draw eighty dots and allocate them into four groups. Draw eighty dots and allocate them into four groups. Draw eighty dots and allocate them into four groups. Use four rectangles.
69. Learners: (learners were counting dots from one to eighty) one, two, three, four, five, six, seven, eight… eighty. (Learners were also heard counting the four dots they were taking from eighty dots and allocated to four rectangles) one, two, three, four, one, two, three, four…(they count dots as they were allocating them one by one to rectangles)
70. Teacher: write at question eight the number sentence of what you were doing.
71. Teacher: Era, why did you use the division sign?
72. Era: because we were dividing
73. Teacher: dividing what?
74. Era: dividing eighty dots into four rectangles.
75. Teacher: how many did you get in each group?
76. Learner: sixteen
77. Teacher: have you counted well
78. Learner: yes
79. Teacher: (talking to the whole class) before everyone go write for me an example on your question paper, an example of division.
80. Teacher: tell me your example
81. Learner: is like having eight things divided by two people everyone will have four things.
82. Teacher: ok, I do not want an example of eight anymore.
83. Learner: is like having ten biers shared by two people of which each one would have five biers.
84. Teacher: I do not want any example with ten, two and five anymore
85. Learner 1: is like having twenty things shared by two people of which each one would get ten things.
86. Teacher: ok, (giving a chance to another learner through signal)
87. Learner 2: is like having thirty things shared by three people of which everyone would have ten things.

88. Learner 3: is like having twenty things shared by ten people of which each one of them would have ten things.
89. Learner 4: is like having forty sweets shared by four people of whom each one of them would have ten sweets.
90. Learner 5: is like having four things shared by two people of whom each would get two things.
91. Teacher: give me another example I do not want four and two
92. Learner 5: is like having twenty things divided by two people of whom each one of them would get ten things.
93. Learner 6: is like having six sweets shared by three people of whom each would get two sweets.
94. Learner 7: is like having thirty sweets shared by the two people of which each one would have fifteen.
95. Learner 8: is like having forty sweets shared by two kids of whom each would get twenty sweets.
96. Learner 9: sharing hundred sweets to two people each will have fifty sweets.
97. Learner 10: having eighty sweets shared by two people of whom each would get forty sweets.

DAY FOUR 12 MAY 2012

LESSON SEVEN

1. Teacher: (greeted the learners and told learners that lessons are going to continue and they are going to divide in different manners, learners were told that they are going to use dots (icons) as a way of division and they will be expected to allocate dots into groups of equal dots.)
2. Teacher: in the first activity, I want you to make dots for me; do you remember what the dots are?
3. Learners: yes, they look like points
4. Teacher: ok, take your blank paper and label the task as number one and make twenty dots representing twenty apples, do not draw twenty apples but twenty dots that represent apples. After drawing twenty dots draw five circles that are representing five groups then start allocating the dot into five groups/ circles taking them from the twenty dots and cancel the dots allocated from the twenty dots. Allocate all dots until all twenty dots are finished and tell me how many dots each groups would have. Do you understand?
5. Learners: yes
6. Teacher: let me repeat just incase you did not hear properly. How many dots did I say you would draw?
7. Learners: twenty dots
8. Teacher: from the twenty dots keep taking out dots and allocate then into groups and the groups are five, after finishing allocating the dots, find out how many dots each group would have.
9. Teacher: remember you have to put the following in mind (1) how many things you were dividing? (2) Into how many groups? And (3) how many dots each group has? Because that is the idea to capture.
10. Teacher: Hanna! How groups did I say do?
11. Hanna: Five
12. Teacher: five, are you groups five? How many groups did you make?
13. Hanna: nine
14. Teacher: these groups of yours are they nine?
15. Hanna: they three
16. Teacher: three, why didn't you make five as you were told?
17. Teacher: (after almost everyone is done) I want you to tell me that after you allocated dots, how many dots each group have?
18. Learner: (as per their turn) four, four, four, four, four, four, four, four, four, four, four, four
19. Teacher: how many dots Hanna?

20.	Hanna:	four
21.	Teacher:	are these four? Count them for me.
22.	Hanna:	one, two, three (counting)
23.	Teacher:	(asking other learners) how many?
24.	Learners:	(per their turns) four, four, four, four
25.	Teacher:	ok, that means if you have twenty things divided/ shared by five people, how many each would have?
26.	Learners:	four
27.	Teacher:	each will have four things, good now I want you to label question two on your paper write twenty or representing twenty dots and keep subtracting four dots, and keep repeatedly taking away four dots until twenty dots get finished and see how many fours you will subtract to finish twenty dots. So start taking four dots from twenty and see what will remain, keep subtracting until you reach zero, then find out how many times you subtracted fours to finish to finish twenty. Do you understand?
28.	Teacher:	let me see your work. Dav! Look at the chalkboard, there is minus, minus, minus and not the division sign you have written there. So rubber the work you have written and do it right and label your work. Then start with twenty and subtract four. What would be the remaining? Keep subtracting four until twenty get finished. Do not write the answer without doing the calculation, use the dots for help and take four out to see what would remain, if you subtract four from twenty dots see how many dots would remain, that is better than writing the wrong answers without calculations.
29.	Dav:	(continued to do the wrong works)
30.	Teacher:	Dav! Dav! Can you see these twenty dots?
31.	Dav:	yes
32.	Teacher:	ok, take four and see what would remain or use whatever strategy (after Dav had counted) how many dots remained?
33.	Dav:	sixteen
34.	Teacher:	good! Then write sixteen and then take four from sixteen again and see what would be the answer, then keep subtracting four until you get zero.
35.	Teacher:	(asking another learner) did you use some fingers to calculate or you used the dots?
36.	Learner:	dots
37.	Teacher:	(talking to the whole class) you will tell me how many fours you subtracted from twenty for twenty to get finished?
38.	Teacher:	are you done how many fours did you repeatedly subtracted out from twenty for twenty to get finished? Mic!
39.	Mic:	five
40.	Teacher:	how many? (asking other learners)
41.	Learners:	five, five
42.	Teacher:	I want to see on your paper how you have recorded if it correlate with what you are saying
43.	Learner:	five
44.	Teacher:	what did you used when you calculated
45.	Learner:	dots
46.	Teacher:	where are the twelve dots then? Show me how you got twelve dots.
47.	Learner:	this is twelve dots
48.	Teacher:	ok, subtract four then to see what answer you will get.
49.	Learner:	(the learner start subtracting from dots that are more than twelve)
50.	Teacher:	no! Do not include these other dots, only use the twelve dots.
51.	Learner:	one, two, three, four (counting the dots to be subtracted)
52.	Teacher:	ok, now count the remaining dots to see how many dots remained.
53.	Learner:	one, two, three, four… eight.
54.	Teacher:	so is eight, is it the number you wrote here?
55.	Learner:	no
56.	Teacher:	(asking another learner) how many fours did you take to finish twenty?
57.	Learner:	five

58.	Teacher:	ok, five why have you written divided sign when we were subtracting?
59.	Teacher:	are you done? How many fours have you subtracted?
60.	Learners:	five, five, five
61.	Teacher:	if you take four from twelve is it seven that will remain? Take twelve dots from there and subtract four dots and see what will remain
62.	Learner:	(after she indicated the twelve dots)
63.	Teacher:	take four out then. (After the learner took four) so how many remained?
64.	Learner:	eight
65.	Teacher:	you see is it the one you have written? (Asking other learners) how many fours you subtracted?
66.	Learner:	five, five, five
67.	Teacher:	good! Everyone I asked said its five fours subtracted for twenty to get finished is that so?
68.	Learners:	yes
69.	Teacher:	(the teacher wrote the process at the chalkboard as follow) (20 – 4 = 16), (16 – 4 = 12), (12 – 4 = 8), (8 – 4 = 4), (4 – 4 = 0) (and counted the four being subtracted) one, two, three, four, five (she continue talking to learners) now I want to ask you this question, you repeatedly took out five fours isn't it?
70.	Learners:	yes
71.	Teacher:	so you divide twenty by four you'll get?
72.	Learners:	five
73.	Teacher:	when you divided with dots how many dots each group has?
74.	Learner:	four
75.	Teacher:	these dots were twenty and you allocated them into five groups of fours is that right?
76.	Learners:	yes
77.	Teacher:	what I want to ask you is this, why did we get the same numbers at both processes? The process of division with dots and that of subtraction, what is the common thing in them? from the twenty you repeatedly subtracted five fours and when you divided you also made five groups of four dots, why may be the same methods have similar answers? Era!
78.	Era:	because the numbers are equal
79.	Teacher:	which numbers
80.	Era:	fives
81.	Teacher:	so five is equal to which number? (after a long pause) I want you to tell me making comparisons, that this number at this process of dots is equal to which number at the subtraction process (the teacher kept repeating the same question, but the class is just quiet not answering, then later she changed the question)
82.	Teacher:	what is that similarity on both processes? Why do we have to get the same answers at both sides? (after a long pause, when learners could not respond she restart over again asking)
83.	Teacher:	when you regrouped the dots, how many groups did you grouped the small groups?
84.	Learners:	four
85.	Teacher:	are these groups four?
86.	Learner:	five
87.	Teacher:	five, which is similar to what numbers which you have been repeatedly subtracting?
88.	Learners:	fours
89.	Teacher:	it's the number of times you repeatedly subtract fours, so you have been subtracting fours until five times, what else is similar to the five fours we have been subtracting?
90.	Learner:	its because there are four dots in each of five groups (circles) and is similar to the five fours we had been subtracting
91.	Teacher:	(used the dots in the groups to clarify) these four dots in each groups, how many times were they removed from the twenty dots?
92.	Learners:	is five times
93.	Teacher:	which is the same as what in the subtraction process?
94.	Learner:	five fours

95.	Teacher:	now can you see these similarities?
96.	Learners:	yes
97.	Teacher:	tell me what similarities you see because you might just say yes when you might not really see it.
98.	Learner:	is because from twenty dots we removed four dots
99.	Teacher:	how many times were four dots removed from twenty dots
100.	Learner:	five times
101.	Teacher:	and for the subtraction, how many fours did you subtracted from twenty for twenty to get finished?
102.	Learner:	from the twenty I kept subtracting five fours also
103.	Teacher:	ok, that is why the answers are the same right? Who else can see the similarity?
104.	Teacher:	ok, if there is no one else to explain we can go ahead at question three on your paper, write this process as a number sentence of division.
105.	Teacher:	if you are done with that, I want you to label question four on the other worksheet and start making three dots per group, keep drawing three dots per group until you reach fifteen dots (then the teacher started to make three dots at the chalkboard to demonstrate to the learners) one, two, thee, (she counted as she makes the groups of three dots when she finished making the first group of three dots she asked the learners) how many dots are these?
106.	Learners:	three
107.	Teacher:	one, two three (the teacher added another group of three dots and asked the learners) how many dots altogether now?
108.	Learners:	six
109.	Teacher:	one, two, three (the teacher counting as she adds the third group of three more dots and she asked) how many dots altogether now?
110.	Learners:	nine
111.	Teacher:	one, two, three, (teacher counting as she adds more other group of three dots and asked the learners) how many dots are they now altogether?
112.	Learners:	twelve
113.	Teacher:	ok, keep adding the groups of three dots until dots altogether reach fifteen
114.	Teacher:	Nab! What are you doing? (turning to the whole class and ask) I told you to make three dots until you reach what?
115.	Learners:	fifteen
116.	Teacher:	Nab, I said make three dots and not three groups
117.	Teacher:	(asking another learner) how many dots are these? Did you make them in groups of three dots as you were told, Dav do it as you were shown at the chalkboard.
118.	Teacher:	(asking the whole class) are you done with your dots?
119.	Learners:	yes
120.	Teacher:	if you are done tell me how many three dots did you added to make fifteen? How many three dots were needed to make fifteen dots?
121.	Learner:	five, five
122.	Teacher:	I want when you give me the answer, I am also checking your work because some people have a way of imitating what others are saying. (Asking one learner) did you say five?
123.	Learner:	yes
124.	Other learners:	five, five, five, five
125.	Teacher:	which five Hanna? If you add all your dots together will they give you fifteen? Because I can see you make five dots in each group, so you have five dots in each group and that is five times five I can see. They must be three dots in each group (asking another learner) how many did you got?
126.	Learners:	five, five, five, five, five, five, five, five
127.	Teacher:	ok, now I want you to write it this way. Label the work number six. So at first you just started with just one three one circle with three dots inside, how many dots are in that first circle?
128.	Learners:	three

129.	Teacher:	is three so that means is three times one (3), when you added the circle with three dots (3 + 3) how many dots altogether were they?
130.	Learners:	six
131.	Teacher:	ok, keep adding three dots one more at a time until you reach?
132.	Learners:	fifteen
133.	Teacher:	until you reach fifteen, then see how many threes would give fifteen. See how the process is and write in the first row there is one three (3) in the second row there are two threes (3 + 3) in the third row there are three threes (3 + 3 + 3), so keep adding threes until you reach the row of threes that sum up to fifteen and tell how many threes makes fifteen.
134.	Teacher:	write the process at number seven, in a division number sentence, showing the repeated addition in a division form of fifteen divided by what ever number you were working with and the answer you got.
135.	Teacher:	(the teacher saw one learner who had written the addition process in division form as 3 ÷ 5 and asked the learner) does that mean the process meant three divided by five, was it three things divided into five?
136.	Teacher:	ok, I think everyone is done writing the number sentence at number seven is that so?
137.	Learners:	yes
138.	Teacher:	the question I want to ask is this , let us go back to the question we did with twenty dots, we re allocated the twenty dots into five groups of four dots, so which mean twenty divided by five you got four right? And twenty divided by four you get five. Even when you repeatedly subtract four from twenty, you took out five fours from twenty is that right?
139.	Learners:	yes
140.	Teacher:	the question I want to ask is this, how did you have the answer in subtraction process? What have you been doing? How did you came to know that five four can be taken out from twenty?
141.	Learner:	I made twenty dot and take out fours.
142.	Teacher:	so you repeatedly taking away fours out of twenty dots until what?
143.	Learner:	until twenty dots get finished
144.	Teacher:	what show you that now the twenty is finished.
145.	Learner:	is because I reached zero
146.	Teacher:	who can tell me the process of how we did it with division. How did you have the answer when you divided?
147.	Learner:	I make twenty dots and make five circles (groups), and then I cancelled five dots from twenty dots and share these five dots equally one by one to the five circles. I repeated the process until twenty dots get finished.
148.	Teacher:	what makes you think that your answer is correct, that your answers must be just correct, what makes you think that?
149.	Learner:	Is because the dots got finished from where I was taking them when allocating to groups.
150.	Teacher:	the dots got finished, what else apart from the dots being finished?
151.	Learner:	when I counted the dots in the circles they were all equal.
152.	Teacher:	who can give me an example as such? Who can give me an example and elaborate in division as well as in subtraction process?
153.	Learner:	is like having forty sweets shared by four people each would have ten.
154.	Teacher:	that is also the same as forty being repeatedly subtracted of how many (after a long pause). You said is forty (then she wrote on the chalkboard) divided by four each get ten (40 ÷ 4 = 40) this is the same as how much being repeatedly taken away for how many times for forty to get finished?
155.	Learner:	is like repeatedly taking five fours from forty.
156.	Teacher:	why five fours when in the number sentence there is ten? If you take five fours from forty will forty get finished?
157.	Learner:	is when you repeatedly take away ten fours from forty.
158.	Teacher:	or repeatedly taking away how many tens in forty to get finished?

159.	Learner:	four tens
160.	Teacher:	ok, now, lets go to the question of three dots repeatedly added until fifteen is reach, you find out that the answer is the same as fifteen divided by three and get what? Is like having fifteen divided by three and get what? Check what you have written on question seven, Naf, five threes added to give fifteen is the same as fifteen divided by what?
161.	Naf:	fifteen divided by five gives three
162.	Teacher:	is divided by five to get three, so if fifteen is divided by three one get three, Like there is three dots in each of five groups when you divided right?
163.	Learner:	yes
164.	Teacher:	ok, if we have fifteen dots, make fifteen dots in a box. Make fifteen dots in a box and divide them into five groups and see how many dots each group would have.
165.	Teacher:	(talking to one learner) where your fifteen dots are where you took the dots that you shared to five groups. I said first make fifteen dots and then later you allocate them to five groups of three.
166.	Teacher:	(after a long time) are you done?
167.	Learners:	yes
168.	Teacher:	ok, from fifteen dots, how many groups you make? How many groups were you able to make?
169.	Learners:	(as per their turn) five, five, five, five, five, five, five, five, five
170.	Teacher:	(talking to Hen) five, where are your five groups? (Pause) where are they? (Pause) you see Hen you have a tendency of imitating others without knowing that when you answer I always check your work to verify (asking another learner) how many groups are they?
171.	Learners:	(as per their turn) five, five, four
172.	Teacher:	four, where are they? Count them.
173.	Learner:	(counting) one, two, three, four (a learner counting but instead she was counting dots)
174.	Teacher:	I am asking the groups and not the dots. How many groups are they?
175.	Learner:	five
176.	Teacher:	if they are five with how many dots inside?
177.	Learner:	four dots
178.	Teacher:	are you sure your initially dots were fifteen?
179.	Learner:	yes
180.	Teacher:	ok, one, two, three, four, five, six, ………..….., fifteen (the teacher recounting the dots aloud to confirm)
181.	Teacher:	(using the learner's dots to re – do the division with the learner) first your cancelled the five dots from fifteen and allocated them to five groups like this, which makes all groups to have how many now?
182.	Learner:	two
183.	Teacher:	ok, and the last five dots, how many dots now are in the group?
184.	Learner:	three
185.	Teacher:	well, where did you get the fourth dots then? (Continuing asking others) how many groups?
186.	Learners:	(as per their turn) five, five, five
187.	Teacher:	do you remember how many threes were added to get fifteen? How many threes added to get fifteen?
188.	Learner:	five
189.	Teacher:	and how many groups you make when divided fifteen dots?
190.	Learner:	five
191.	Teacher:	ok, now the question I have is, why do you think you got the same answer in both process of addition and division? Why may be you get five at both sides? Why? (After the teacher repeated the answer several times with no answer from learners, the teacher changed the question) how many dots are inside the group?
192.	Learner:	three dots
193.	Teacher:	and what number you repeatedly add to get fifteen?

194.	Learner:	threes
195.	Teacher:	why then you added five threes and even when you divided you got five groups of threes dots? Why may be the answers are the same at both sides?
196.	Learner:	is because we added five threes and even in the five groups when divided there are three dots.
197.	Teacher:	so meaning, you were working with the same numbers at both sides, which is what? The number you had been adding is what?
198.	Learner:	is three
199.	Teacher:	so you had been adding the same number the three, three, three, three, three and even when you divided there is what in each group?
200.	Learner:	three
201.	Teacher:	so you have to get the same answer because if fifteen is divided by three you will get? So you have to get the same answer because if fifteen is divided by three you will get?
202.	Learner:	five
203.	Teacher:	another question I have is what makes you think that your answer must be just correct? What makes you think that your answer is correct? What makes you think that fifteen divided by three the answer is five and is correct?
204.	Learner:	is because all groups had equal dots
205.	Teacher:	what other facts makes you confident that you must be correct?
206.	Learner:	is because the dots I was allocating get finished.
207.	Teacher:	ok, the dots to be allocated get finished, what else (pause) where you able to get any numbers of threes from fifteen? The sixth threes or seventh or so?
208.	Learner:	is because when I counted the dots are just threes in each group and the groups are five
209.	Teacher:	meaning that there are only how many threes in fifteen?
210.	Learner:	five threes
211.	Teacher:	who can give me such example? Who can give me such example? Dos!
212.	Dos:	is like having hundred sweets divided amongst two people of who each would get fifteen.
213.	Teacher:	did you say each would get fifteen?
214.	Dos:	fifty
215.	Teacher:	ok, is like hundred sweets shared by two people of whom each would get fifty sweets, or how else can we put that example. Is like repeatedly adding what number for how many to get hundred?
216.	Dos:	is five
217.	Teacher:	ok, how many fives must you add to get hundred
218.	Dos:	ten fives
219.	Teacher:	will ten fives not give fifty?
220.	Dos:	it is twenty fives
221.	Teacher:	what else will you be adding to get hundred?
222.	Dos:	is fifties
223.	Teacher:	how many fifties?
224.	Dos:	two fifties
225.	Teacher:	we also discussed the multiplication is another way one can get division answer. I want you to be doing this for me (showing at the chalkboard and draw boxes) make for me fifty dots in the big box and make other five small boxes (groups) and share the fifty dots into the small five boxes (groups). Did you hear me well? How many dots did I say do?
226.	Learners:	fifty
227.	Teacher:	and into how many boxes did I say share them?
228.	Learners:	into five boxes
229.	Teacher:	good, first you make fifty dots in one box and then make five empty boxes and share the fifty dots into those five boxes (groups).
230.	Teacher:	are you done?
231.	Learners:	yes

232.	Teacher:	how many dots each group has? Hanna!
233.	Hanna:	six
234.	Teacher:	where are they I cannot see them in your work (the teacher continue with other learners) how many dots each groups has?
235.	Learners:	(as per their turn) ten, ten, ten
236.	Teacher:	did you count them?
237.	Learner:	ten, nine
238.	Teacher:	nine (turning to other learners) how many?
239.	Learners:	(as per their turn) ten, ten, ten, ten, ten, ten,
240.	Teacher:	meaning how many groups you have?
241.	Learner:	five
242.	Teacher:	and how many dots in each group?
243.	Learners:	ten
244.	Teacher:	so you have how many tens?
245.	Learners:	five tens
246.	Teacher:	yes tens are five, if you were to write it in the multiplication, you say you have ten times five which is equal to
247.	Learner:	fifty
248.	Teacher:	fifty or five there ten times became fifty, if you divide fifty by ten what will it be the answer?
249.	Learners:	five
250.	Teacher:	and if fifty is divided by five what will be the answer?
251.	Learners:	ten
252.	Teacher:	what makes you think that your answers must be just correct, that if you divide fifty by five you will get ten, what makes you think that your answer is correct?
253.	Learner:	is because in fifty there are only five tens
254.	Teacher:	there are five tens in fifty, if you take five tens from fifty will there be a reminder or fifty would be finished?
255.	Learner:	Fifty would be finished.
256.	Teacher:	so if you take ten fives from fifty, fifty would be finished right?
257.	Learners:	yes
258.	Teacher:	on your question paper, there is five times in a box give fifty, indicate the number in the box ($5 \times \ldots = 50$) and write or fill it in your question paper.
259.	Teacher:	when you finish go to question eleven and write the multiplication process into division, begin with fifty divided by what and get what?
260.	Teacher:	ok, I want someone to give me an example, explain it in multiplication and division form. Naf,
261.	Naf:	is like having thirty biers divided by three people each would have ten biers
262.	Teacher:	who can give this in multiplication
263.	Learner:	is like having three biers times ten that will makes thirty
264.	Teacher:	ok, Sab!
265.	Sab:	is like having hundred things shared by two people and each would get fifty.
266.	Teacher:	reverse it to multiplication
267.	Sab:	is two times fifty give you hundred
268.	Teacher:	or how else will you say it?
269.	Sab:	is fifty there two times gives you hundred
270.	Teacher:	ok, another example? Eras!
271.	Eras:	is like having twenty things to be shared by two people each will get ten things.
272.	Teacher:	ok, how is that in multiplication?
273.	Eras:	is two things there ten times gives you twenty
274.	Teacher:	Dos
275.	Dos:	if I have three - hundred to be shared by three people each would have hundred.
276.	Teacher:	say it in multiplication.
277.	Dos:	three times hundred is equal to three – hundred
278.	Teacher:	Nab

279. Nab: I have eighty sweets to be shared by two kids each will have forty sweets.
280. Teacher: Say it in multiplication.
281. Nab: two

LESSON EIGHT

1. Teacher: (teacher put stones at three tables and learners were called to do an activity in groups at each of the three tables. All groups had turns, when the group came to the table, what they do first is they count the stones and find out how many they are. Learners were counting aloud each members of the group were saying the counting of number names aloud together as a song. Each table represent the table had a different number of stones which learners in a group did their division).
2. Teacher: (teacher was giving instruction for table one) the stones needed are eighteen and from eighteen stones make groups of three stones taking them out from eighteen stones.
3. Teacher: ok, are you done taking out groups of three stones?
4. Group 1: yes
5. Teacher: how many groups of threes have you taken out from eighteen stones?
6. Group 1: six
7. Teacher: ok, write at your number two, what you have done transfer it into the division number sentence (this learners were to do it individually, each one of them would write on their own paper)
8. Teacher: why did you use division sign on your number sentence?
9. Learner: is because we were dividing.
10. Teacher: dividing what? You have not given the complete answer I want to hear the whole story that we were dividing this into what and so on.
11. Sab: we were dividing eighteen stones by repeatedly taking out threes stones until eighteen stones got finished.
12. Teacher: how did you divide, because I remember last time you taking stones and allocate them one by one to group?
13. Sab: we have been repeatedly taking out three stones until the eighteen stones got finished.
14. Teacher: you have been taking three stones until eighteen stones got finished? Ok, how many groups did you say make?
15. Sab: six
16. Teacher: what makes you think that eighteen when grouped in groups of threes one would measure out six groups and you are correct?
17. Learner: is because we took eighteen stones and divided them by three.
18. Teacher: what makes you think that you must just correct?
19. Learner: because the eighteen stones got finished.
20. Teacher: the eighteen stones where you were taking three stones got finished?
21. Learner: yes
22. Teacher: by the time it got finished, how many groups of threes did you removed?
23. Learner: six
24. Teacher: ok, can you give me an example as such? I am asking the whole group here, to give me an example (after a long pause) no, example? Ok,
25. Teacher: go to the second table and count for me the stones on that table (talking to other groups) I need another group to come at the first table.
26. Teacher: count the stones at the table and see how many are they (telling the group on the first table) in eighteen stones keep taking out the group of three stones and tell me in the end how many groups you were able to make
27. Teacher: (telling the group on the second table) from the twenty – seven stones keep taking out groups of nine stones and tell me in the end how many groups of nine you were able to make?
28. Teacher: (after learners are done) ok, are those groups of nine?
29. Group 1: yes

30.	Teacher:	now I want you to write this as a number sentence indicating what you were dividing with what and what answer you got?
31.	Teacher:	how many groups were you able to make?
32.	Group 1:	three
33.	Teacher:	each group has how many stones?
34.	Group 1:	nine
35.	Teacher:	and how many stones were they altogether were they?
36.	Group 1:	twenty – seven
37.	Teacher:	ok, write that as a number sentence then below question four.
38.	Teacher:	(talking to group two) how many groups were you able to make?
39.	Group 2:	six
40.	Teacher:	and how many stones each group has? Hen!
41.	Hen:	three
42.	Teacher:	all the stones you were working with here are how many?
43.	Group 2:	eighteen
44.	Teacher:	they are eighteen ok; now I want you to tell me that (turning to group 2) write below question two the number sentence of what you were dividing, with whatever number you were dividing and the answer.
45.	Teacher:	are you done writing the number sentence?
46.	Learners:	yes
47.	Teacher:	tell me how you did you choose this division sign on your number sentence.
48.	Learner:	is because we were dividing.
49.	Teacher:	you were dividing, dividing what?
50.	Learner:	stones
51.	Teacher:	you are not explaining well (giving a chance to another learner) ok,
52.	Learner:	is because were dividing twenty – seven stones.
53.	Teacher:	you were dividing twenty – seven stones, how?
54.	Learner:	we have been taking out groups of nine stones until the twenty – seven stones got finished.
55.	Teacher:	how many times did you take out groups of nine stones from twenty – seven?
56.	Group 1:	three, three, three
57.	Teacher:	describe how you had divided and get the answer.
58.	Learner:	from the twenty – seven stones we have been taking out groups of nine stones repeatedly until the twenty – seven stones get finished.
59.	Teacher:	what makes you think that your answer must be just correct?
60.	Learner:	is because all groups measured out have equal number of stones.
61.	Teacher:	what else ensuring you that your answer is correct because what if number of groups supposed to be more than that?
62.	Learner:	the stones where I was taking got finished.
63.	Teacher:	ok, is because the stones got finished, can you give ma an example as such?
64.	Nab:	is like having ten things divided by two people of whom each would get fifteen things.
65.	Teacher:	having ten things, but I said I don't want ten as an example anymore gives me another number.
66.	Nab:	is like having thirty things divided by two people of whom each would get fifteen things.
67.	Teacher:	ok, use your example in a way we had just finished explain it like this, saying it's like repeatedly taking out what in thirty.
68.	Nab:	is like repeatedly taking out groups of five from thirty.
69.	Teacher:	where do five come from again when your example only has thirty, two and fifteen, because you said thirty divided by two equal to fifteen. So put that example in the way we were just dividing stones.
70.	Nab:	is like repeatedly taking out fifteen from thirty.
71.	Teacher:	how many times?
72.	Nab:	two times

73. Teacher: will thirty be finished?
74. Nab: yes
75. Teacher: ok, (telling group one) you can go to the third table and count the stones on the table.
76. Teacher: (talking to group two) tell me why you have opted for that sign on your number sentence.
77. Learner: is because we were dividing.
78. Teacher: dividing what?
79. Learner: stones
80. Teacher: yes it is stones, but I want you to tell me the whole and clear story of that division.
81. Learners: we were dividing three stones
82. Teacher: three stones how? Just tell me the whole process of what happened, were all these stones three?
83. Learner: we were dividing eighteen stones.
84. Teacher: ok, dividing them how?
85. Learner: in three groups
86. Teacher: what?
87. Learner: dividing into three groups.
88. Teacher: are these groups three?
89. Learner: they are six
90. Teacher: Are six groups and each group has how many stones?
91. Learner: three stones
92. Teacher: so how did the division happen?
93. Learner: we have been taking threes at a time.
94. Teacher: until what? Say the full explanation.
95. Learner: we have been taking three at a time until eighteen stones got finished.
96. Teacher: how many times did you remove the groups of three?
97. Learner: six
98. Teacher: six, ok, now what makes you think that eighteen divided by three give six or when divided by six give three, what makes you think that those answers are correct
99. Learner: is because all stones are equal
100. Teacher: which stones are equal?
101. Learner: the stones in groups
102. Teacher: is that the only thing that makes you confident that the answers are correct? What if stones are still equal but fours (pause) Fil!
103. Fil: I had been counting until the stones get finished. (I guess he mean he had been measuring out until the stones get finished)
104. Teacher: when stones where you were taking got finished, then you stopped because no more stones to take or what?
105. Fil: no
106. Teacher: what do you mean that stones get finished then? What do you mean by stones get finished? (silence) Fil, stones were eighteen and you repeatedly took out threes until eighteen stones got finished, you could not take three more stones from the eighteen because they got finished, is that what you mean?
107. Fil: yes
108. Teacher: good! How many threes were you able to take out?
109. Fil: six
110. Teacher: ok, six, will you be able to give me an example of division as such?
111. Learner: it is like having sixty things shared by two people each would have thirty things.
112. Teacher: if it is in the way we have been dividing how would you say it? (pause, the learner could not say it but rather keep saying three, three, three)
113. Teacher: (asking group 1) are you done counting?
114. Group 1: yes
115. Teacher: how many stones are they?
116. Group 1: hundred and fifteen

117.	Teacher:	keep taking out twenty stones in groups until the stones get finished. Find out how many groups you can take (she faced to group 2) and you how many stones are those?
118.	Group 2:	twenty- seven
119.	Teacher:	ok, keep taking out groups of nine stones and find out how many groups you can take. (Facing to group 3) how many stones are those?
120.	Group 3:	eighteen
121.	Teacher:	keep taking out groups of three until eighteen stones got finished and find out how many groups you can take out from eighteen
122.	Teacher:	(after learners are done dividing) now it is time for questions, I will ask group 1 first because they are the one doing the last activity. How many stones altogether that you were dividing?
123.	Learner:	hundred and fifteen
124.	Teacher:	taking out groups of how many stones?
125.	Learner:	Twenty stones
126.	Teacher:	how many times did you take the group of twenty stones?
127.	Learner:	six times
128.	Teacher:	six times, so are all those six groups having twenty stones?
129.	Learner:	no
130.	Teacher:	so how many times you took groups of twenty stones?
131.	Learners:	five
132.	Teacher:	are there stones left?
133.	Learner:	yes
134.	Teacher:	how many stones left?
135.	Learner:	fifteen
136.	Teacher:	ok, they are fifteen; I want you to write that as a number sentence of what you were doing. Ok the question I want to ask is why have you used division sign?
137.	Learner:	is because we were dividing hundred and fifteen stones, taking out groups of twenty stones until the hundred and fifteen stones get finished.
138.	Teacher:	until hundred and fifteen stones get finished. Did you finish the stones?
139.	Learner:	no
140.	Teacher:	how many were left?
141.	Learner:	fifteen stones
142.	Teacher:	how do we call the stones that were left?
143.	Learner:	they are stones
144.	Teacher:	how do we call numbers that are left when dividing?
145.	Learner:	is reminder
146.	Teacher:	reminder, ok when divided hundred and fifteen stones by groups of twenty stones, how many groups did you get?
147.	Learner:	they are five groups
148.	Teacher:	how did you divide you say? How did you divide you say?
149.	Learner:	I kept taking out groups of twenty stones until there is only fifteen stones left.
150.	Teacher:	do you think your answer is correct? (Pause) and if you are correct, what makes you think that you are correct? (Pause) is your answer of hundred and fifteen stones divided into five groups of twenty stones and the reminder of fifteen correct?
151.	Learner:	no
152.	Teacher:	is it not five times? Are you able to take a sixth group of twenty stones or what?
153.	Learner:	no
154.	Teacher:	what does your no mean? Ok, let me repeat my question, do you think you are correct by getting an answer of five groups of twenty stones out of hundred and fifteen and fifteen stones as a reminder?
155.	Learner:	yes
156.	Teacher:	if you think you are correct, what makes you think you are correct?
157.	Learner:	is because when I took out groups of twenty repeatedly it came out five times and only fifteen was left.
158.	Teacher:	that fifteen is what? Or what makes you think you are done?

159.	Learner:	fifteen is not the same as or equal to twenty that I supposed to be taking out.
160.	Teacher:	and that means you are done because fifteen is not enough to take out as you needed to take out twenty is that so?
161.	Learner:	yes
162.	Teacher:	ok, can you give me an example of division as such.
163.	Learner:	is like having forty things shared by four people of whom each would get ten things.
164.	Teacher:	ok, would that have a reminder?
165.	Learner:	no
166.	Teacher:	give me an example with a reminder?
167.	Learner:	is like thirty things being shared by two people of whom each would get ten and ten would remain as reminder.
168.	Teacher:	ok (asking group 3) how many stones were you dividing?
169.	Group 3:	eighteen stones
170.	Teacher:	eighteen stones. Ok, from eighteen stones how many group of three stones did you took?
171.	Learner:	six
172.	Teacher:	six, now write below question two the division process and number sentence.
173.	Teacher:	(asking one learner who had used multiplication sign) so you were multiplying and not dividing?
174.	Teacher:	why did you opt for the division sign?
175.	Learner:	is because were dividing.
176.	Teacher:	dividing what?
177.	Learner:	eighteen stones
178.	Teacher:	how?
179.	Learner:	in three groups
180.	Teacher:	are the groups three?
181.	Learner:	no
182.	Teacher:	Dav! How did you divide?
183.	Dav:	we divided them into six groups
184.	Teacher:	where you from the beginning know that the groups would be six?
185.	Dav:	no
186.	Teacher:	then tell me how you divided.
187.	Dos:	we took eighteen stones, and then from eighteen we took out six stones.
188.	Teacher:	do not lie to me Dos; you were not taking out six stones from eighteen stones. What or how many stones have you been taking out?
189.	Dos:	we have been taking out stones one by one.
190.	Teacher:	have you been taking stones one by one?
191.	Learner:	is by threes
192.	Teacher:	ok, you have been taking threes stones from eighteen until?
193.	Learner:	until eighteen stones get finished
194.	Teacher:	good! That is what you have been doing, how many times did you took out threes?
195.	Learner:	six times
196.	Teacher:	what makes you think that eighteen divided by three is six and is correct or when it is divided by six are three and is correct?
197.	Dos:	(after a long break) is because the stones are equal
198.	Teacher:	the fact that stones are equal is not strong enough because you could've taken another number of equal stones from eighteen. What makes you think that your answer is correct? (after a long pause) What makes you think that your answer is correct? What makes you think that your answer is correct?
199.	Learner:	is because I divide
200.	Teacher:	because you divided is incomplete. I want a full explanation based on what you had done (let me leave you for a while, while you are thinking) (facing to group 2)
201.	Teacher:	how many stones were these?
202.	Group 2:	twenty – seven
203.	Teacher:	how many did you divided

204.	Learner:	we have been taking nine stones out of twenty – seven
205.	Teacher:	How many times did you take out nine stones?
206.	Learner:	three times
207.	Teacher:	ok, is three times, does that mean you were only able to take out three nines from twenty – seven
208.	Learner:	yes
209.	Teacher:	ok, write below question four a number sentence of what you were doing.
210.	Teacher:	the next question I want to ask you is this; do you think you are correct? If twenty-seven is divided by nine will you get three? If you keep taking groups of nine will you get three groups out of twenty – seven? And if so do you think your answer is correct? If so what makes you think you are correct and not wrong? Hen!
211.	Hen:	is because we, we have been dividing, dividing, mhhh groups
212.	Teacher:	Fil, what makes you think that you are just correct?
213.	Fil:	we have been taking groups of nine stones from twenty – seven stones until twenty – seven stones get finished
214.	Teacher:	so when you have been taking group of nine stone you finished twenty- seven stones after you three groups of nine stones were taken right?
215.	Fil:	yes
216.	Teacher:	ok, who can give me an example as such we were doing?
217.	Learner:	is like having thirty things shared by two people of whom each would get fifteen.
218.	Teacher:	ok, now I want you to count for me the stones on table three
219.	Teacher:	(back to group 3) did you get the reason why you think you are correct?
220.	Dos:	is because we took eighteen stones and we have been taking three stones out of eighteen by the time we reach the sixth group of three stones eighteen stones got finished.
221.	Teacher:	can anyone give me an example as such? Nab! Give me an example of division. Hil!
222.	Hil:	is like having thirty things divided amongst three people of whom each would get ten things.
223.	Teacher:	explain it in the way you have just dividing now.
224.	Hil:	you will be taking tens
225.	Dos:	is like having two Oros shared by two people of whom each would get five Oros
226.	Teacher:	How five Oros when they were just two? You said two Oro divided by two people, how came each would get five?
227.	Dos:	it is ten Oros divided by two people of whom each would get five Oros.
228.	Teacher:	ok, (talking to group 3) go to the second table and count the number of stones on the table.
229.	Teacher:	(asking group 2) how many groups of twenty did you removed from hundred and fifteen. Fil!
230.	Fil:	six groups
231.	Teacher:	do all six groups having twenty stones? Dia!
232.	Dia:	five groups
233.	Teacher:	is there any left over (then the teacher began counting the groups) one, two, three, four, five, six, because I see six groups here. How many stones are in the sixth group?
234.	Learner:	the sixth has fifteen stones
235.	Teacher:	Can you write below question six the number sentence of what you were doing, remember all stones were how many altogether?
236.	Learners:	hundred and five
237.	Teacher:	is it hundred and five or hundred and fifteen? Is it hundred zero five or hundred one five?
238.	Learners:	hundred and fifteen
239.	Teacher:	ok, you were dividing with what? Or what number of stones had you been taking out of hundred and fifteen?
240.	Teacher:	(asking one learner) is it fifteen or hundred and fifteen and is it minus sign here or divided, what were we dividing with, because here you have written minus, when you divided, how many stones each group have?

241.	Teacher:	(asking the whole group) how do we call the stones that are left over? How do we call the stones that are left over? (pause) you don't know?
242.	Learners:	yes
243.	Teacher:	Can you tell me how you have been dividing? Can you tell me how you have been dividing? How did you divide?
244.	Learner:	we have been taking out twenty stones
245.	Teacher:	ok, until what? Tell the whole process until the end.
246.	Learner:	until the stones get finished.
247.	Teacher:	did the stones get finished?
248.	Learner:	yes, until the stones get finished and fifteen stones remained.
249.	Teacher:	what makes you think that your answer must be correct? what makes you think that hundred and fifteen had only five groups of twenty stones
250.	Learner:	because five groups of stones are equal
251.	Teacher:	I asked that what makes you think that from hundred and fifteen only five twenty stones could be taken out?
252.	Learner:	is because we were dividing using five hundred stones
253.	Teacher:	are those stones five hundred? Say the number well, ok, what makes you think that only five of twenty stones can be taken out? What makes you think that you are right?
254.	Learner:	is because when we were dividing hundred and fifteen by taking out twenty stones and only fifteen stones remained.
255.	Teacher:	is it because when you divided and only fifteen was left, the how many times did you took out groups of twenty stones, you are not saying it clearly, is because the twenty stones were only taken out how many times?
256.	Learner:	five
257.	Teacher:	were you able to take them six times?
258.	Learner:	no
259.	Teacher:	why is it not possible?
260.	Learner:	because it will not be enough
261.	Teacher:	ok, who can give me an example as such, which one would divide and get the reminder? (Pause when no answer was given) ok, collect your papers and have a nice day.
262.	Teacher:	(talking to group 3) have you counted the stones? How many were they? Dos!
263.	Dos:	twenty – seven
264.	Teacher:	how many nines were you able to take out?
265.	Dos:	three times
266.	Teacher:	meaning you could only take three groups of nine stones and twenty – seven stones would be finished?
267.	Learners:	yes
268.	Teacher:	can you write below question four, the number sentence of what you were doing?
269.	Teacher:	(asking one learner) is it minus or divided? Which one were you doing? Subtracting three stones or dividing? I want you to take twenty – seven stones and show me now by subtracting three to see if you will get what you have written there (27 – 3 = 9) instead of (27 ÷ 3 = 9)
270.	Teacher:	I want you to tell me that, what makes you think that your answer must be correct? That if you take three nines the twenty – seven stones would be finished or twenty – seven stones divided by three is equal to nine or when it is divided by nine is equal to three?
271.	Dos:	is because when I took twenty – seven stones and divide them into three groups
272.	Teacher:	are you sure you were dividing into three groups, say exactly what you were doing because you were not dividing into three groups but you were taking out nine stones from the twenty – seven stones.
273.	Dos:	is because when I took out three nines, I counted them and they were three and after the three nines were taken out twenty – seven stones got finished.
274.	Teacher:	ok, how did you divided you say? What were you doing when dividing?
275.	Learner:	we took out nines

276.	Teacher:	did you just take out nine? I want a complete story of everything happened.
277.	Learner:	we had been taking groups of nine stones until twenty – seven stones got finished.
278.	Teacher:	you were taking until twenty – seven stones get finished, ok who can give me an example as such?
279.	Teacher:	you don't have an example
280.	Learner:	no
281.	Teacher:	you don't have?
282.	Dos:	is when having eighty apples divided by forty people of whom each would get two apples.
283.	Teacher:	ok do it in that way as we have been dividing just now.
284.	Dos:	is like taking two apples four times from eighty.
285.	Teacher:	do you mean four because two times four is just eight and not eighty
286.	Dos:	is taking two apples forty times for eighty apples to get finished.
287.	Teacher:	ok, go to the next table and count how many stones are on the table, repeatedly take out twenty stones from hundred and fifteen and find out how many groups of twenty stones you can make.
288.	Teacher:	(after learners finish counting) how many groups of twenty stones one can get from hundred and fifteen? How many groups of twenty stones you got?
289.	Learner:	five times
290.	Teacher:	are all these five groups really having twenty stones?
291.	Learners:	yes
292.	Teacher:	how many stones were they altogether?
293.	Dos:	one hundred and fifteen
294.	Learner:	five groups
295.	Teacher:	if you take out five groups of twenty stones from hundred and fifteen will it get finished?
296.	Learner:	no
297.	Teacher:	were there a left over?
298.	Learner:	yes
299.	Teacher:	how many?
300.	Learner:	fifteen
301.	Teacher:	is fifteen, now I want you to write below question six the number sentence of what you were doing
302.	Teacher:	do you all get that from hundred and fifteen if keep subtracting twenty stones you will get five groups, is that right?
303.	Learner:	yes
304.	Teacher:	now tell me how you divided. How did you divide? Dos!
305.	Dos:	I took hundred and fifteen stones (pause)
306.	Teacher:	ok, and what did you do next for these groups to appear? What have you been doing?
307.	Dos:	we take (pause)
308.	Teacher:	did someone else do the division for you?
309.	Dos:	no
310.	Teacher:	you did it yourself right?
311.	Dos:	yes
312.	Teacher:	but why you cannot tell me how you did it? Fil, Dav, Nab, anyone please tell me how you did the division.
313.	Learner:	we divided by taking out groups of twenty until hundred and fifteen stones get finished.
314.	Teacher:	you have been taking groups of twenty stones right?
315.	Learner:	yes
316.	Teacher:	when you removed five groups of twenty stones did the hundred and fifteen stones get finished? Did they get finished?
317.	Learner:	no
318.	Teacher:	what happened?
319.	Learner:	there were some stones remained.

320.	Teacher:	how many remained?
321.	Learner:	fifteen stones
322.	Teacher:	fifteen stones, ok, tell me what makes you think that you are correct, that if you repeatedly take out five groups of twenty stones from hundred and fifteen it would get finished? Dos!
323.	Dos:	I must be correct because when I took the groups of twenty stones, the hundred and fifteen stones get finished.
324.	Teacher:	did they all get finished Dos?
325.	Dos:	fifteen stones was left
326.	Teacher:	why leave fifteen stones? (Pause) how many numbers of stones were you taking out?
327.	Dos:	twenty stones
328.	Teacher:	why did you stop then when fifteen stones remained?
329.	Dos:	is because hundred
330.	Teacher:	what number of stones did you needed to take out of hundred?
331.	Dos:	hundred and fifteen
332.	Teacher:	no, hundred and fifteen is the number of stones where you were taking from. I asked which number of stones did you needed to take from hundred and fifteen.
333.	Dos:	is twenty
334.	Teacher:	ok, why stopped when reached fifteen stones then?
335.	Dos:	is because there are only five twenties in hundred.
336.	Teacher:	there are only five twenties in hundred, but this is hundred and fifteen?
337.	Teacher:	why did this fifteen remained makes you think that you will end up here and won't be able to continue? Why did you not even count the group of fifteen stones as one of the five groups of twenty stones?
338.	Dos:	is because is half
339.	Teacher:	because is half, do you know what half is? Is fifteen half of twenty?
340.	Dos:	no
341.	Teacher:	ok, you did not count the group of fifteen stones why? Do not tell me half because it is not suiting the situation and the answer is just on my question. How many stones you needed to take out from hundred and fifteen?
342.	Dos:	is twenty
343.	Teacher:	so then you did not count the group of fifteen because fifteen is not a……., or is not what?
344.	Dos:	is not enough
345.	Teacher:	yes it is not enough, it was just easy, you would have said that I could not include the group of fifteen stones because is not enough so your answer must be correct because only five groups of twenty stones were able to come out.
346.	Teacher:	give me an example that would have a reminder.
347.	Dos:	is like having fifteen sweets to be divided by two people
348.	Teacher:	how many sweets each would get?
349.	Dos:	five
350.	Teacher:	ok, the reminder?
351.	Dos:	is five sweets
352.	Teacher:	example?
353.	Learner:	is like having twenty – three sweets divided by two people and each will have ten sweets.
354.	Teacher:	and the reminder?
355.	Learner:	is five
356.	Teacher:	How many sweets did you say in the first place?
357.	Learner:	twenty – five
358.	Learner 2:	having twenty sweets divided by two people each will get five sweets
359.	Teacher:	is that so?
360.	Learner:	no, I have thirty sweets divided by two kids and each would get ten sweets
361.	Teacher:	is that so? Dav!

362. Dav: is like having twenty sweets divided by one child the child will get ten sweets and the reminder will be ten.

DAY FIVE **13 MAY 2012**

LESSON NINE

1. Teacher: (the teacher greeted learners and humbly asked them not to absent on Thursday the ascension day) then after that they did the reflection on the past day)
2. Teacher: all those days we did activities about what?
3. Sab: division
4. Teacher: we said one can get a division answer through different ways, let me give an example if you have twenty – four divided by six, how and how and ho and how can you divide that and get the answer?
5. Learner: you can take twenty four stones from the stones
6. Teacher: yes, you can take twenty – four stones
7. Learner: and make six groups
8. Teacher: and divide into six groups
9. Learner: then you keep allocating one stones to each group from the twenty – four until the twenty – four stones get finished.
10. Teacher: what other way can we use apart from using the stones (pause) we said we can use addition, subtraction and multiplication also isn't it? Who can tell me how we can get an answer through subtraction? Who can tell me how to use subtraction to get the division answer of twenty – four divided by six? Who can tell me how to use subtraction to get the division answer of twenty – four divided by six? Era! (pause) Sab!
11. Sab: you will do twenty – four dots
12. Teacher: no I want you to tell me how to do it through subtraction.
13. Sab: you will repeatedly taking away six from twenty – four until they get finished
14. Teacher: until what get finished? The six?
15. Sab: until the number get finished
16. Teacher: which number are you taking six away from?
17. Sab: from twenty – four
18. Teacher: then which number?
19. Sab: you repeatedly take six away from twenty – four until twenty four get finished
20. Teacher: good! until you reach zero
21. Teacher: we also have the way of addition how do you do it to get the division answer? (I want someone to take a chalk and do the process of subtraction on chalkboard) (Pause) can someone takes a chalk to do the process of subtraction on the chalkboard? Can someone take a chalk to do the process of subtraction on the chalkboard? (pause) Dos! Do you want to tell me that you cannot do it?
22. Dos: I can
23. Teacher: then why not rising up your hand because I know amongst you there are people who can know. Who else can come and do for addition on the other side of the chalkboard?
24. Teacher: (talking to Dos when realized he is getting wrong differences) count well Dos, count well, we have learned different way of counting even the use of sticks. (asking Dos) are those icons representing twenty – four
25. Dos: yes
26. Teacher: then take out four and see what the answer would be, then keep subtracting four from the answers until you reach zero.
27. Teacher: (asking the learner who did addition) Sab! Are done?
28. Sab: yes

29.	Teacher:	Now how many six are there in twenty – four or how many six needed to be added to have twenty – four (pause) did these one gave you twenty – four?
30.	Sab:	no
31.	Teacher:	then check on the rows the number of sixes that can give you twenty – four.
32.	Sab:	they are four
33.	Teacher:	so that means what would be the answer if one divides twenty – four by six? What would be the answer if one divides twenty – four by six?
34.	Sab:	is four
35.	Teacher:	what?
36.	Sab:	four
37.	Teacher:	can you see how addition can give you the answer to division?
38.	Sab:	yes
39.	Teacher:	Dos, how many sixes did you subtracted from twenty – four for twenty – four to get finished?
40.	Dos:	four
41.	Teacher:	meaning twenty – four divided by six is?
42.	Dos:	four
43.	Teacher:	(the teacher split 14 learners into two groups of 7 members each, the first group of seven learners come to the table)
44.	Teacher:	(talking to the group) count the stones on the table for me and tell me how many they are.
45.	Learners:	(counting together as a song and aloud) one, two, three, four, five, six, seven……………., forty.
46.	Teacher:	the instruction is saying, keep taking away groups of ten stones until forty is finished and find out how many tens are there. Each tens you remove from forty keep it in isolation as a group.
47.	Learners:	(counting out the tens) one, two, three, four, five… ten. One, two, three, four, five… ten. One, two, three, four, five… ten. One, two, three, four, five… ten.
48.	Teacher:	are you done?
49.	Learners:	yes
50.	Teacher:	how many times did you remove tens?
51.	Learners:	four times
52.	Teacher:	is that so?
53.	Learners:	yes
54.	Teacher:	you removed ten four times right?
55.	Learners:	yes
56.	Teacher:	now I want you to put back the stones in groups of tens together (after learners are done) have you put all the stones together?
57.	Learners:	yes
58.	Teacher:	now do question two. Do you remember what Dos was doing at the chalkboard, subtracting until he reach zero? Do the same process on your papers. Write forty minus ten and take forty stones and subtract ten stones to see what answer would be, and then write the answer. So keep removing (subtracting) from the answer like that until you reach zero.
59.	Teacher:	(assisting a learner who seems to be confused and do not know how to write the process well) look start with forty minus ten, what was the answer when you removed one ten? Ok, then thirty again writes thirty minus ten the answer is what? Continue like that until you reach zero.
60.	Teacher:	Hil! Look how Dos had done it at the chalkboard, so you also start with forty minus ten and write the answer, and the answer minus ten and write the answer and that answer again minus ten then the answer until you reach zero as an answer (she referred her to use the stones to help her get the answers) were these stones not forty from the beginning, so write forty because we started with forty, when ten stones were removed from forty what was the answer?

61.	Teacher:	(when realized that Hanna is copping the example from the chalkboard exactly as it is) Hanna! Do not copy the one written on the chalkboard, because is not what you just did now, you did for forty stones just now, but I see you writing sixteen minus instead of forty minus. If you are finding it difficult come back to the stones, take ten out and see what remains and write the answer. Do that until you reach zero.
62.	Teacher:	tell me when you divided forty by ten, what answer did you got? How many tens are in forty? That time when you repeatedly took away groups of ten stones from forty, how many tens did you take from forty for forty to get finished?
63.	Learners:	(as per their turn) four, four, four, four, four, four, four
64.	Teacher:	ok, when you started taking the tens out (subtracting) the process on your paper, how many tens did you subtracted?
65.	Learners:	they are four tens
66.	Teacher:	Fil
67.	Fil:	I took out four tens
68.	Teacher:	ok, now the question I have is, tell me the process of how you divided using the stones for to get that answer. What did you do and how did you started when divided them into groups? That's the question I am asking.
69.	Learner:	I have been taking out ten stones repeatedly
70.	Teacher:	ok, you had been taking out tens stones repeatedly until all the forty stones get finished right, how did you do it in subtraction process? Tell the process you went through to get the answer. Nab!
71.	Nab:	we have been taking (subtracting) tens then write the answer.
72.	Teacher:	I cannot understand, explain or tell the whole subtraction process clearly, what you did first next and the answer.
73.	Nab:	first we took ten stones from forty stones and counted the remaining stones and write the answer
74.	Teacher:	good! What else did you do because I want you to tell the story until the end?
75.	Nab:	we then take again ten stones and the counted the remaining and write the answer and take ten stones and counted the remaining and write the answer and take ten stones and counted the remaining and write the answer
76.	Teacher:	until what?
77.	Nab:	until stones get finished
78.	Teacher:	until stones get finished, ok you said when you divided with stones you got four groups of ten stones right?
79.	Learner:	yes
80.	Teacher:	and even when you repeatedly subtract, you subtracted tens four times is that so? So you got the same answer of four tens is that so?
81.	Learner:	yes
82.	Teacher:	what makes you think that your answer is correct? What makes you think that your answer is correct?
83.	Learner:	is because groups has equal number of stones
84.	Teacher:	apart from the equal number of stones in groups, what else is there? Naf!
85.	Naf:	even the stones I was adding got finished.
86.	Teacher:	what?
87.	Naf:	even the stones I was adding
88.	Teacher:	adding or subtracting?
89.	Naf:	the stones where I was subtracting got finished
90.	Teacher:	so is because where you were taking stone get finished and the stones are equal in the groups, how many stones did you take?
91.	Naf:	ten
92.	Teacher:	so you have been taking out tens?
93.	Teacher:	who can give me an example as such, in division and put it in the subtraction form and explain it like how one would repeatedly take a number from that number until it get finished. Who can give me an example?
94.	Pet:	is like having sixty things shared by two people each would have thirty.

95.	Teacher:	put that example in subtraction form, is like subtracting what number from sixty?
96.	Pet:	Taking tens from sixty.
97.	Teacher:	taking what from sixty?
98.	Pet:	fives
99.	Teacher:	so let me have your paper, you said (pointing to what is written on the paper) is like having sixty divided by two people of whom each would get thirty, put it into subtraction form.
100.	Pet:	I like having sixty things and take out tens.
101.	Teacher:	What?
102.	Pet:	taking out the tens
103.	Teacher:	taking what?
104.	Pet:	the tens
105.	Teacher:	no, I don't want you to use a number that is not on your example. Your example has sixty, thirty and two and those are the numbers I want to hear on your subtraction process.
106.	Pet:	I will be repeatedly taking out twos
107.	Teacher:	how many times would you be repeatedly taking out the twos for sixty to get finished?
108.	Teacher:	(now talking to the whole class) I want everybody now to look at the chalkboard; we said twenty – four divided by six will be what?
109.	Learner:	four
110.	Teacher:	when we came to subtract how many six did you came to subtract from twenty – four for twenty – four to get finished?
111.	Learner:	four
112.	Teacher:	ok, four sixes. If you want to subtract fours, how many fours would you subtract? Or if I say twenty – four divided by four what would you get? Hil!
113.	Hil:	six
114.	Teacher:	six, can't you see that we are just working with the same numbers twenty- four, six and four? Can't you see?
115.	Learners:	yes
116.	Teacher:	meaning even if you take them to multiplication version, how will you use them? How will you use these same three numbers of twenty- four, six and four? That day we said, if you have two times three you'll get six and if you have three times two you'll get what?
117.	Nab:	six
118.	Teacher:	if you take it to the division version you will say six divided by three that would give you what? Dav!
119.	Dav:	two
120.	Teacher:	six divided by two will give you what? Yes Dos!
121.	Dos:	three
122.	Teacher:	then use that in the case of twenty – four, for you to get twenty – four which numbers would you multiply? For you to get twenty – four which numbers would you multiply? Dos!
123.	Dos:	four and six
124.	Teacher:	you only have four and six, yes four times six or you can also say six times what?
125.	Dos:	six times four
126.	Dav:	six times four
127.	Teacher:	yes and it still gives you twenty – four, can't you still see that relation, that even when you subtract you will still use these numbers and so you need to subtract four sixes or six fours for twenty – four to get finished. So let me go back to Pet. How many twos would you repeatedly subtract from sixty for sixty to get finished?
128.	Pet:	thirty times
129.	Teacher:	you see another example?
130.	Sab:	is like having forty things
131.	Teacher:	ok, divided by how many people?
132.	Sab:	by four people

133.	Teacher:	how many things each one would get?
134.	Sab:	each one would get ten things.
135.	Teacher:	so ok, is forty things divided into/by four people of whom each would have ten things. I want you to use the subtraction methods. How many times would you subtract fours from forty for forty to get finished? (after a long pause) Sab this is the same thing we had done on the chalkboard, the same things Pet had just done now, we said in twenty – four you subtract four six times or six four times for twenty – four to get finished, even on your paper you have done it. How many fours can you subtract from forty for forty to get finished?
136.	Sab:	they are ten
137.	Teacher:	yes, ten is that difficult? If you start subtracting fours from forty you will subtract ten of them for forty to get finished. Another example from those who did not give their examples before (telling the other groups) let the group come and count these stones.
138.	Teacher:	after counting the stones, tell me if they are sixteen, then start making groups of four stones and find out how many fours (groups) would make up sixteen.
139.	Teacher:	how many fours are there into sixteen?
140.	Learner:	four
141.	Teacher:	so you find out that there are four fours in sixteen. How did you get the answer? What did you do with those stones to get the four fours?
142.	Dav:	from sixteen stones we have been taking the groups of four stones.
143.	Teacher:	you have been taking groups of fours from sixteen until stones get finished right?
144.	Dav:	yes
145.	Teacher:	take your blank paper, label question six and write the process of addition starting from one four and the answer, then make the two fours and their sum, then make the four three and their sum and make them four fours until you reach sixteen. Find out how many fours makes sixteen?
146.	Teacher:	(talking to group two) how many tens you took out from forty?
147.	Group 2:	four
148.	Teacher:	how did you have four tens? What did you do to get what you got?
149.	Learner:	we have been subtracting
150.	Teacher:	subtracting what? Your answer is not complete.
151.	Learner:	we had been subtracting the groups of ten stones
152.	Teacher:	subtracting the groups of ten stones until what? Yes!
153.	Learner:	until all stones get finished
154.	Teacher:	ok, until all stones get finished. How many groups did you get?
155.	Learners:	four times
156.	Teacher:	four times ok, now I want you to put the groups of stones together and do the next process of subtraction starting by forty minus ten, write the process on the worksheet, use the forty stones, remove ten and count the remaining one and write the answer of that forty minus ten, remove ten again, count the remaining one and write the answer keep doing that until you reach the difference of zero, just as it is done on the chalkboard. Can you see this example?
157.	Learners:	yes
158.	Teacher:	ok (talking to Hanna in group one when realized she is doing it wrongly) Hanna, first you started just with one four (which is represented by one group of four stones) one four, that is how addition process should start with one four (the teacher used the group of four stones to show) how many stones are in this group?
159.	Hanna:	four
160.	Teacher:	ok, they are four stones, then you make them two fours (two groups of four stones) can you see how these fours are there two times?
161.	Hanna:	yes
162.	Teacher:	how many stones are they altogether? When you get the answer write it here and make three fours, keep adding until you reach sixteen. (Turning to other learners) are you done?
163.	Learners:	yes

164.	Teacher:	(talking to one learner who has done it wrongly) there supposed not to be eight written here, but two fours, you now began with one four, then two fours, then three fours and so on. Fil! Look at how it is done at the chalkboard, there is one six, then two sixes, three sixes and four sixes and kept added, rub your everything because it is wrong and re-write the new one start with one four (she took four stones) and see how many stones are these (she added another group of four more stones next to the first one) and write two fours and count all the stones of all two groups. How many stones are they altogether? (When the learner finished with the whole process the teacher asked) did you reach sixteen?
165.	Fil:	yes
166.	Teacher:	for you to reach sixteen, how many fours did you added?
167.	Fil:	they are four
168.	Teacher:	that is a point I wanted to discover because with the way you were writing, 4, 8, 12, 16 you wouldn't have not been able to see the number of fours in those numbers
169.	Teacher:	(telling all learners) write a number sentence of what you were doing in a division form below question three on your question paper.
170.	Teacher:	what are we really doing in this class? (she asked the whole class when she realized some are doing it wrongly and some are confusing operation sign) what are we doing and that is what I asked in the morning, we said our lessons has to do with what?
171.	Learner:	division
172.	Teacher:	division, right?
173.	Learners:	yes
174.	Teacher:	yes, but I see some people had written minus on their number sentence (then the teacher asked Fil, when she realized he had written 40 ÷ 10 = 3) so when you divided forty by ten, was the answer three? When you used stones, how many groups of tens stones you made out of forty stones or when you subtracted, how many times did you subtracted ten from forty to reach zero?
175.	Teacher:	are you done?
176.	Learners:	yes
177.	Teacher:	you are all done? Ok, Fil, Naf, Mic and Pet, go back to a table with forty stones re-do the groups and see if what you have written on your answer is what you will still get with stones. (teacher referred them to the stones because they all have written 40 ÷ 10 = 30)
178.	Teacher:	(after they are done) did it give the answer you have written there?
179.	Learners:	no
180.	Teacher:	ok, write the answers as it out to be then.
181.	Teacher:	ok, the question I wanted to ask you is this, when you divided sixteen stones, how did you do that?
182.	Learner:	I have been taking out fours
183.	Teacher:	taking out fours until what?
184.	Learners:	until sixteen stones get finished?
185.	Teacher:	on the addition process, what did you do?
186.	Learner:	we had been adding fours until we reach sixteen
187.	Teacher:	so you had been adding fours until you reach sixteen, ok, when you divided how many fours did you get out from sixteen?
188.	Learner:	four
189.	Teacher:	ok, they are four, so you got the same answer on both processes. Why may be the same answer on both processes? Why may be the same answer on both processes? Why may be the same answer on both processes? Why may be the same answer on both processes? Sab!
190.	Sab:	is because we used sixteen
191.	Teacher:	is because we used sixteen and what about sixteen?
192.	Sab:	sixteen has only four fours
193.	Teacher:	ok, what makes you think that your answers are just correct?
194.	Sab:	is because all groups has equal stones

195.	Teacher:	all groups that you had been taking out from sixteen are equal, what other reason could have makes you think that you are correct?
196.	Sab:	is because the sixteen stones get finished.
197.	Teacher:	is because when you divided by the groups, four groups finish up sixteen is that so?
198.	Sab:	yes
199.	Teacher:	can you give me an example as such? Fil!
200.	Fil:	is like having forty sweets divided by two people of whom each would have ten sweets.
201.	Teacher:	if you have forty sweets divided by two people would each get ten?
202.	Fil:	yes
203.	Teacher:	o! Go and take forty stones from the box, divide them into two groups and see if each would have ten stones. Who else can give me another example? Mic!
204.	Mic:	is like having thirty things divided by three people of whom each would get ten.
205.	Teacher:	each would have ten, if you want to add like we were doing, which number would you repeatedly add to reach thirty?
206.	Mic;	you will be adding threes
207.	Teacher:	how many threes
208.	Mic:	ten threes
209.	Teacher:	or adding how many tens? If threes would be ten how many tens would they be?
210.	Mic:	they would be three
211.	Teacher:	for you to reach thirty, tens would be repeatedly added three times. Who else can give me an example? (after a long pause) if there is no more an example
212.	Teacher:	write that you were doing as a number sentence below question seven
213.	Teacher:	if you are done with the question seven go to the next table and count the stones at that table to find out how many are they.
214.	Teacher:	(talking to group 2) are you done writing the subtraction process on the paper?
215.	Learners:	yes
216.	Teacher:	why did some of you use the division sign instead of minus sign as you were doing it, as this process at the chalkboard? Even on your question paper you have it is there as forty minus ten, so rub those divided signs and replace them with minus. All the works are correct except the minus signs
217.	Teacher:	are you done? If you are done I want to ask you this. First you had forty stones which you divided into groups of tens stones is that right?
218.	Learner:	yes
219.	Teacher:	how did you divided? Who can tell me how you divided?
220.	Learner:	we took forty stones
221.	Teacher:	ok
222.	Learner:	and take out ten stones
223.	Teacher:	ok (after a long pause) did you just take out one ten or what? Say all things well until the end of the story.
224.	Learner:	we took four tens
225.	Teacher:	did you just take four tens at once or what? I want to hear everything in every step.
226.	Learner:	we been taking out tens repeatedly
227.	Teacher:	good! Until what?
228.	Learner:	until the forty stones get finished
229.	Teacher:	until forty stones get finished, how many tens did you take out from forty? How many times did you take ten out of forty?
230.	Learner:	four times
231.	Teacher:	four times ok, when you subtracted how you did do the whole process? What have you been doing? Who can tell me how you have been doing it? Dav!
232.	Dav:	we put all the stones of groups together and counted the stones out
233.	Teacher:	ok
234.	Dav:	we counted the remaining stones and write the answer
235.	Teacher:	from there

236.	Dav:	from the answer we took out ten stones again and counted the remaining one and write the answer
237.	Teacher:	did the process finish then?
238.	Dav:	no
239.	Teacher:	tell until when the story reach the end
240.	Dav:	then I took out ten from the remaining one and counted the remaining and write the answer
241.	Teacher:	until what how did you finish?
242.	Dav:	until zero is reached
243.	Hil:	we finish when there was a remaining of ten, when we subtracted ten and zero remained
244.	Teacher:	now the other question I have for you is this, when you divided stones in groups, you made four groups of ten stones, even when you subtracted you only subtracted four tens is that so?
245.	Learner:	yes
246.	Teacher:	why may be did you got the same answer? Even when you divided it was four groups of ten stones, and when you subtracted it were also four tens, why the same answer?
247.	Dos:	is because we take out ten stones and counted the remaining one
248.	Teacher:	my question is, when you divided with stones you make four groups of ten stones, even when you subtracted it was four tens you subtracted, why may be the same answer?
249.	Dos:	is because the forty got finished
250.	Teacher:	I don't think that is the answer, my question is why in forty when subtracting you took four tens and when divided into groups you made four groups of ten stones? dav!
251.	Dav:	it is because there are only four tens in forty
252.	Teacher:	yes, there are only four tens in forty or was it possible for you to take the fifth ten?
253.	Learners:	no
254.	Teacher:	yes, is because we used the same number which is forty, when we divided and when we subtracted and what are the things you were dividing with?
255.	Dav:	equal things
256.	Teacher:	yes we used the same number when dividing/ subtracting which is ten thus, the answer is the same
257.	Teacher:	do you think your answers are correct? If, so what makes you think that your answer are correct? What if it was possible to get five tens out of forty? (After several repetition and a long break of silence) it is ok, if you cannot reason why you think your answers are correct.
258.	Teacher:	who can give me an example as such?
259.	Dos:	is like having eighty things to be shared by forty people of whom each would get ten things
260.	Teacher:	ten?
261.	Dos:	no! each would have twenty
262.	Teacher:	ok, can you do it in the subtraction form? Is like taking what from eighty for it to get finished? Take the paper and write these numbers then. What did you say?
263.	Dos:	eighty money shared by forty people
264.	Teacher:	how much each would get?
265.	Dos:	twenty money
266.	Teacher:	are you sure?
267.	Dos:	ten money
268.	Teacher:	are you sure did you say people are forty? How many times will you remove forty from here?
269.	Dos:	two
270.	Teacher:	good! That is why I asked if you are sure. Now the subtraction, how many twos would you be subtracting from eighty for eighty to get finished? If you keep taking two from eighty? How many times two can be taken out?
271.	Dos:	it is four times

272.	Teacher:	there is no four on your example, look on your paper there is only eighty, forty and two
273.	Dos:	it is forty times
274.	Teacher:	if you want to be subtracting forty from eighty, how many times will you be taking forty?
275.	Dos:	two times
276.	Teacher:	two times ok, who else can give me the example? Hil!
277.	Hil:	I will have ten sweets shared by two people
278.	Teacher:	each would have?
279.	Hil:	five sweets
280.	Teacher:	if you have ten sweets and keep subtracting five, how many times would you subtract five from ten for ten to get finished?
281.	Hil:	two times
282.	Teacher:	and if you keep subtracting two from ten, how many times will you subtract it for ten to get finished?
283.	Hil:	five times
284.	Teacher:	ok, now give me another example, because I told you never to give me ten, five and two anymore
285.	Hil:	I would have four sweets
286.	Teacher:	say a bigger number than ten (after a long pause)
287.	Dia:	I will have thirty sweets shared by two people each would have fifteen.
288.	Teacher:	in the subtraction form, if keep subtracting twos, how many twos would you subtract from thirty for thirty to get finished? How many twos would you subtract from thirty for thirty to get finished? (after a long pause) who can help her, to tell how many times one can take two from thirty for thirty to get finished?
289.	Hil:	is for fifteen times
290.	Teacher:	ok, Dia! How many times would you take fifteen from thirty for thirty to get finished?
291.	Dia:	is two times
292.	Teacher:	I want you to write a number sentence below question seven that talk about what you were doing with forty stones.
293.	Teacher:	I want you to count these stones and find out how many….
294.	Teacher:	(asking group 1) are you done counting?
295.	Learners:	yes
296.	Teacher:	how many stones did you counted?
297.	Learners:	ninety stones
298.	Teacher:	you got ninety stones right?
299.	Learners:	yes
300.	Teacher:	keep taking the group of twenty stones until ninety stones get finished.
301.	Teacher:	(asking group 2) how many stones did you counted? How many stones did you counted?
302.	Group 2:	seventeen
303.	Teacher:	if they are seventeen take one stone away from them because they only need to be sixteen.
304.	Teacher:	(she turned again to group 1 and asked) how many your stones were they?
305.	Learners:	ninety
306.	Teacher:	did you remove the groups of twenty stones? Ok, I will come back.
307.	Teacher:	(talking to group 2) you said the stones are sixteen right?
308.	Learners:	yes
309.	Teacher:	keep taking groups of four stones and find out how many groups you can make from sixteen.
310.	Teacher:	are you done dividing sixteen in group of four stones
311.	Group 2:	yes
312.	Teacher:	(talking to group 1) are you done dividing ninety into groups of twenty stones?
313.	Group 1:	yes
314.	Teacher:	ok, how many groups of twenty stones did you got?

315. Learners: four times
316. Teacher: when you removed four groups of twenty stones, did the stones get finished?
317. Learners: no
318. Teacher: how many remained?
319. Learners: ten stones
320. Teacher: ten stones?
321. Learners: yes
322. Teacher: fill in the multiplication question to indicate how many times you removed groups of twenty stones and the remainder. So indicate how many times is twenty and plus what so that it can give you ninety. (talking to Fil) Fil! You are not done at all. You even have not indicated the remainder. Were group of twenty stones tens or four?
323. Teacher: now the question I want to ask you is this. Write below question 11, the number sentence on division of the ninety stones and twenty groups. (Talking to one learner) you have just written nineteen and not ninety (helping another learner) so ninety, if divided in the groups of twenty stones, how many groups did you have? And how many stones each group has?
324. Teacher: you have divided sixteen stones into groups of four groups?
325. Learners: yes
326. Teacher: how many groups did you get?
327. Learners: four
328. Teacher: four ok, I want you to write the addition process at question six. You start with one four, take one groups of four stones and count the stones to see how many stones are in one (one group of four). Write the process on the worksheet paper and label it number six. You start with one four, count stones of one group of four stones and write the answer. on the next row write two fours added and take two groups of four stones each and count all the stones together to see the sum of four plus four. Continue with three fours until you reach twelve (asking one learners) how many fours are these?
329. Learner: three fours
330. Teacher: why did you from one four jumped to three fours? Where are the two fours? The normal procedure is you start with one four, followed by two fours and then three fours and it goes on until you reach the sum of sixteen.
331. Teacher: who is at the row of four fours? (Showing one learner) for the four fours, you take four groups of four stones and count their stones altogether to get their sum. Did you get the answer already? (she faced to the other kids) if you see your fours do not sum up to sixteen add another more four stones, if they were three fours make them four fours and count their sum to see if you can get sixteen.
332. Teacher: ok, have you find out how many fours that gives or makes sixteen? (turning to Hen) Hen! Are you done?
333. Hen: yes
334. Teacher: let me see your work, you have written three fours that had given you fourteen. Hen! Takes the three groups of fours, see if they will give this fourteen. (After counting) you see! You were not counting at all you just wrote your thinks. Now, because you have not reached sixteen write four fours and make four groups of four stones again. Count the stones of all four groups and find out their sum.
335. Teacher: (asking all group members) how many fours gave you sixteen?
336. Learners: four
337. Teacher: when you divided with stones, how many groups of stones did you make?
338. Learners: four
339. Teacher: four groups, with four stones each right?
340. Learners: yes
341. Teacher: why do you think you got the same answer because when you added the fours they were four fours that gave you sixteen and when you divided you got four groups of fours stones. Why may be you got the same answer? Why may be Dav!
342. Dav: is because there are only four fours in sixteen

343. Teacher: yes! There are only four fours in sixteen, whether you are adding them or dividing in groups, only four fours are in sixteen. So even when you added to reach sixteen you added how many fours?
344. Learners: four
345. Teacher: when you used sixteen stones, how many groups of four stones you made?
346. Learners: four
347. Teacher: so that is it, you are working with sixteen on both addition and division and in sixteen there are on four fours.
348. Teacher: the next question I want to ask is this. Write the number sentence of what you were doing. If you are to write division number sentence, what would you say is divided by what to get what? So write that below question seven.
349. Teacher: the next question I want to ask is that, do you think your answer is correct? That if you take sixteen and keep taking groups of four stones, you would only take them four of them? Is that answer you have written there (because is reflecting sixteen divided by four groups of four stones each) correct?
350. Learners: yes
351. Teacher: what makes you think your answer is correct? (after a long pause) Dia! What makes you think that in this sixteen stones if I kept subtracting four stones I would only be able to remove them four times?
352. Dia: is because on sixteen there are only four groups.
353. Teacher: Hil!
354. Hil: is because if I divide sixteen by four, I will get four.
355. Teacher: ok, tell me your answer based on what you did with stones, what if I would have taken five or six or seven groups of four stones? Nab! Would you be able to take the fifth group of four stones? Or what makes you think you are correct? (Pause) yes!
356. Dos: is because there are only four fours in sixteen.
357. Teacher: what makes you think that they are not five? Hil!
358. Hil: is because the stones I was dividing or where I was taking stones, stones get finished.
359. Teacher: is because where you were taking the groups of fours when you took out or reached at the fourth groups stones get finished. Is that so?
360. Learners: yes
361. Teacher: or were you able to take the fifth group?
362. Learners: no
363. Teacher: the stones get finished and that is what makes you think you are correct right?
364. Learners: yes
365. Teacher: who can give me an example as such? Who can give me an example as such? Ok, I will leave you for a while and go to another group, while you are thinking of the examples (after she came back) did you got an example? Tell me an example.
366. Hil: when having twenty things divided by two kids each would get ten things.
367. Teacher: let me write it on your paper, so that I may use your answer to ask another question. You said having twenty things right?
368. Hil: yes
369. Teacher: shared by how many people?
370. Hil: are ten
371. Teacher: and each will have
372. Hil: ten sweets
373. Teacher: no! Repeat again. How many sweets you said?
374. Hil: twenty sweets
375. Teacher: divided by how many people?
376. Hil: two people
377. Teacher: and each person would get how many sweets?
378. Hil: ten sweets?
379. Teacher: I want you to put this in an addition approach. How many times would you add twos to get twenty?
380. Hil: ten

381. Teacher: ok, if you are adding ten, ten how many do you need to add to get twenty?
382. Hil: two
383. Teacher: (talking to the whole class) what I want you to note is, everyone look here, because we were doing division do you understand?
384. Learners: yes
385. Teacher: what we are doing is division, when you have twenty divided by two, what will it be the answer? Pet!
386. Pet: ten
387. Teacher: ten thank you, is that division right?
388. Learners: yes
389. Teacher: ok we have division, addition, subtraction and multiplication in these lessons. I want you to link or relate these to one another (the teacher drew four columns for each operations and wrote the headings of division, addition, subtraction and multiplication on top of each column) if we have this twenty divided by two, each would have how many? (Asking as she wrote the number sentence at the chalkboard on the column of the division).
390. Learner: ten
391. Teacher: (moving to a column of multiplication and asked) how will you get this answer in multiplication? Mic! Look here, which number we say you multiply to give another number? Era!
392. Era: it is two ten times
393. Teacher: two ten times will give what?
394. Learners: twenty
395. Teacher: twenty, meaning if you that two times ten are twenty even when dividing you will know ten is just there two times in twenty. Is that so?
396. Learners: yes
397. Teacher: and you can also see that twenty divided by ten is..
398. Learners: two
399. Teacher: yes because it is an inverse, and ten times two is?
400. Learners: twenty
401. Teacher: good, now go to the column of addition, which number would you add repeatedly to get twenty which also is in the number sentence of the division? Dos!
402. Dos: is twos
403. Teacher: is twos (the teacher started to repeatedly add the twos) one two (2 = 2), two twos (2 + 2 = 4), three twos (2+2+2 = 6), four twos (2+2+2+2 = 8), five twos (2+2+2+2+2 = 10), until the twos reaches?
404. Learners: ten
405. Teacher: ten, if they reach ten the value would be twenty. So let us move to the column for subtraction, there we used two here we will use tens. How many tens needed to make twenty? How many tens needed to make twenty?
406. Learners: two times
407. Teacher: two times, when you have one ten, the value is just ten. Is that so?
408. Learners: yes
409. Teacher: but when tens are two, the value would be?
410. Learners: twenty
411. Teacher: ok, let's go to the subtraction column. How many twos would you repeatedly subtract from twenty for twenty to get finished? How many twos would you repeatedly subtract from twenty for twenty to get finished?
412. Learners: Ten times
413. Teacher: is ten times, can you see we had been just using the three numbers in all the twenty, ten and two and this kind of relation you will see that that is what we will be doing all along in this activity
414. Learners: yes
415. Teacher: I want you to understand by the end of the day, that in this twenty you will be subtracting ten twos. One two three four five six seven eight nine and ten twos

(counting the twos) until ten times for twenty to be finished. (Or from twenty to reach zero) or you would be subtracting how many tens?

416. Learners: two

417. Teacher: good is two times for you to finish twenty or reach zero. Now in all the numbers you will give the examples and be able to relate it to divisions, addition subtraction and multiplication. Do you understand?

418. Learners: yes

419. Teacher: I want you also to be able to tell me the reason why? Why for instance if I subtract two times ten from twenty the twenty would get finished? This is because there are only how many twos in twenty? Hanna!

420. Hanna: is ten

421. Teacher: they are ten. So there is no way you will take out eleven twos from twenty while there is only ten. That is what I want you to understand. Where did you end up in your group? Is here ok, ok going to the next table and count the stones while giving instructions to the other.

422. Teacher: (asking group 2) how many stones did you get at your table?

423. Teacher: how many stones did you counted?

424. Learners: ninety

425. Teacher: ninety now keep taking away groups of twenty stones away from ninety and see how many groups of twenty stones you can take from them.

426. Teacher: (after a time) I want you to tell me how many twenties you were able to take out from ninety.

427. Learners: four twenties

428. Teacher: when you took four twenties, did ninety got finished?

429. Learners: no

430. Teacher: no, how many remained?

431. Learners: ten stones

432. Teacher: ten stones, how do we call those ten stones? How do we call things left after dividing?

433. Learner: is a reminder

434. Teacher: is a reminder, so in ninety there are how many twenties?

435. Learners: four

436. Teacher: and ten stones remained as a?

437. Learners: reminder

438. Teacher: write for me on page 28 the number of how many twenty plus what that gives ninety. How many times were those twenties and how many stones extra that makes ninety?

439. Teacher: write it again as a division in a number sentence (asking one learner) do we write ten as zero one (01) or as one zero (10) (asking another learner again) when you divided with twenty stones, how many times you removed groups of twenty stones? Mat! So you have written it differently but any way you are all correct.

440. Teacher: now you said in ninety there are only four groups of twenties stones right?

441. Learners: yes

442. Teacher: what makes you think that your answer is correct?

443. Dav: is because there are only four twenties in ninety.

444. Teacher: why do you say there is only four twenties and not five or six?

445. Hil: is because the stones we were dividing got finished

446. Teacher: were they finished? Did ninety stone got finished? Were there no ten stones left? Why?

447. Dos: is because the ninety stones that were there mhuu--- mhuu---

448. Teacher: when you subtracted four twenties, what happened that makes you think you are correct? Yes!

449. Hil: is because there were only ten left.

450. Teacher: and ten is not what? You have not clearly explained.

451. Hil: is not enough

452. Teacher: yes is not enough for twenty to be taken out again.

LESSON 10

1.	Teacher:	what I want you to do is this. On your question paper there are sticks (she started drawing sticks on the chalkboard) example one, two, three, four… ten (counting the ten sticks she is drawing on the chalkboard) is like having those sticks. Can you see them?
2.	Learners:	yes
3.	Teacher:	this is just an example I gave. Now I will be taking two sticks from the ten sticks by cancelling them out and each two sticks I will took out I will make groups for them. Each two sticks I took out from the ten sticks will be cancelled out from the tens. Then see how many groups of two sticks I would make out from ten sticks (she started taking two sticks and making groups for them and cancel them from the other side after she is done with the drawing she asked). How many groups of two sticks I made out from ten sticks?
4.	Learners:	five
5.	Teacher:	there are sticks drawn on your paper. Can you see them?
6.	Learners:	yes
7.	Teacher:	count the sticks for me and find out how many?
8.	Learners:	(after counting) twenty- eight
9.	Teacher:	they are twenty-eight, from twenty-eight sticks there are sticks that are already cancelled. Can you see them?
10.	Learners:	yes
11.	Teacher:	How many are cancelled?
12.	Learners:	four sticks
13.	Teacher:	redraw the sticks on the blank paper, start taking out four sticks by cancelling them out from the twenty-eight sticks and redraw them as a group. Now see how many groups you can make. Make sure the sticks you redraw in groups are cancelled out from twenty- eight so that you will not end up recounting them in the end and confuse yourself. How many sticks did we say you will be taking out from twenty-eight sticks?
14.	Learners:	four sticks
15.	Teacher:	four sticks good! After you have removed them for a group cancel them out from here.
16.	Teacher:	did you take groups of four sticks from twenty-eight?
17.	Learners:	yes
18.	Teacher:	how many groups were you able to take out from twenty-eight? Naf!
19.	Naf:	seven
20.	Teacher:	they are seven, so in twenty-eight, there are how many fours to be taken out? Yes!
21.	Learner:	seven
22.	Teacher:	seven, now I will change the question, because the sticks all together were how many?
23.	Learners:	twenty-eight
24.	Teacher:	take a blank paper and label question one, make four columns and write on top of each column the heading one Addition, the other, subtraction, the other division and the last multiplication. What I want you to do (you at this group I want you to do this) how many groups were they?
25.	Teacher:	the sticks all of them together, the one you have been cancelling and removing groups from. Hanna!
26.	Hanna:	eighteen
27.	Teacher:	is not eighteen but
28.	Learners:	twenty-eight
29.	Teacher:	so that is twenty-eight divided by what and got what? So write that number sentence in the column of the division and from there you move to the column for multiplication and write this as an inverse of that number times that number after multiplication move to the column of addition and indicate that twenty- eight is which

		number repeatedly added for whatever number. When you are done with the addition you move to the column for subtraction and write it as whatever number which is repeatedly subtracted out from twenty-eight for whatever number for twenty-eight to get finished.
30.	Teacher:	(she went to group 2 and explained to them about the sticks and showed them the example of ten sticks with groups of two sticks being cancelled out from them and told them to count the twenty-eight sticks and start removing groups of four sticks)
31.	Teacher:	(talking to group one) are you done counting? (Asking one child) on the addition column, what number would you be repeatedly adding? What number times what number will you be repeatedly adding to get twenty-eight? (Talking to another child again) seven plus four is eleven and not twenty-eight unless you want to say four sevens or seven fours. Look here! (Pointing to the example of twenty divided by ten equal to two) the tens in addition are two because there are only two tens in twenty and even in division we only see the numbers twenty, ten and two. Even in twenty-eight how many sevens are there? So you start with one seven (7 = 7), two sevens (7+7 = 14), (7+7+7 = 21), and then you continues depending on how many sevens are in twenty-eight. Naf! or how many fours?
32.	Naf:	seven fours
33.	Teacher:	if fours are seven then how many seven are in twenty-eight?
34.	Naf:	four sevens
35.	Teacher:	good! Even when you will be repeatedly adding and take sevens how many sevens will you repeatedly adds to get twenty-eight?
36.	Naf:	four
37.	Teacher:	good! And that is what I want to see. One seven first (7 = 7), two sevens next (7+7=14), followed by three sevens (7+7+7 = 21), then lastly the four sevens (7+7+7+7 = 28) and if you are using fours let them be seven times because there are four sevens and seven fours is that so?
38.	Teacher:	what are you doing Fil? We only have twenty-eight either divided by four or by seven. These two rub them. I cannot see what you have written (asking another learner) is this twenty-eight divided seven to get four?
39.	Learners:	yes
40.	Teacher:	do it again as twenty-eight divided by four, what will it give? How many fours can you took out from the drawing of twenty-eight sticks. If you are confused, go back to the drawings, use your drawing and count the fours. How many fours are they?
41.	Learners:	(murmurings) seven
42.	Teacher:	mhh! Why are you rewriting if it is already there here? Because you will just dirtify your paper. You already have this and this which are already correct. So go ahead to multiplication. Now rub all these are wrong. How do we do it in multiplication? Look at the example on the chalkboard when twenty was divided by ten and it gave two but how did we present that at multiplication? Which number was written first as factors (the learner started copying the exact example on the chalkboard) where did you get this ten if the numbers you are dealing with now has no ten, there at the chalkboard ten is used because in that division there were ten. I did not say copy the example as it is. I said use it as an example and do yours this one here of twenty –eight, seven and fours. Why writing this thing at addition when it supposed to be division. Look at the example at the chalkboard. You will see their different signs how they looks like, plus for addition and a sign like this (×) in multiplication a line across with dots (÷) for division and a line without dots for subtraction. So does this as it was done in the example, here in the division column you just need to tell me that twenty-eight is divided by what number to get whatever number? That is what you supposed to write at division. You can write it down here. Good! How will you write it on the multiplication for you to get twenty-eight? Which numbers would you multiply?
43.	Learners:	(in a lower voice) seven times four

44.	Teacher:	ok, write it then. This one rub it, because you are just working with these numbers here and there times what? (A learner said something in a lower voice) the numbers are just these one on division; these are the numbers you are working with. You only manipulate them accordingly. At addition which number would you repeatedly add? And how many times would you add that number? What does that number sentence in division tells you?
45.	Learners:	is fours
46.	Teacher:	how many fours?
47.	Learners:	seven fours
48.	Teacher:	or how many sevens?
49.	Learners:	four
50.	Teacher:	good! Which one is short or easy to write? Is it the addition of four sevens or the addition of seven fours? Write whichever is convenient to you.
51.	Learner:	is four
52.	Teacher:	so to you the fours are convenient all seven fours? And not the four sevens?
53.	Learner:	is sevens
54.	Teacher:	ok write then. Starting with one seven, to two, three and then four sevens to get twenty –eight. (Correcting a learner) here it is two sevens that would follow now and we put plus in the middle of sevens and the two seven row would be followed by three sevens and lastly the four sevens row because you said they need to be four to make twenty- eight. There are plus sign in between of sevens look at the example on the chalkboard, because we said they are added together. This division sign you put here rub it (explaining the signs) a line across with dot is division, a line across without dots is for subtraction here you put plus sign and a cross on multiplication column. You do not need to add twenty-eight itself, but look at the number that are in twenty – eight and repeatedly add them to get twenty-eight.
55.	Learner:	there are seven fours
56.	Teacher:	and there are also how many sevens?
57.	Learners:	four sevens
58.	Teacher:	ok, which one do you want to write? If it is fours, I want to see only fours added together starting from one four, two fours; three fours until they are seven added together (facing to another learner) you have not written your answers. Write your answers. Ok at subtraction you will repeatedly take away sevens until you reach zero. (To another) in subtraction it is twenty – eight we are subtracting from and we do not start with four minus but twenty-eight. Rub this thing! Even the addition of sevens is now wrong why write sevens as an answer is it not supposed to be twenty-eight? And even the order supposed to be one seven, two sevens, three sevens and four sevens like those sevens there three times gives what? Make seven dots three times and see what it will give.
59.	Learner:	twelve
60.	Teacher:	ok, count the dots see if you will get twelve (when a learner wrote twenty as an answer) it is not twenty at all. Do the counting again
61.	Teacher:	are you done?
62.	Learner:	yes
63.	Teacher:	how many fours did you take out from twenty-eight? How many fours did you take out?
64.	Learners:	seven
65.	Teacher:	how many fours did you take out?
66.	Learners:	seven
67.	Learner:	(other learners one by one) seven, seven, six
68.	Teacher:	are you sure? Let us count (using the stones and count) one, two, three, four you put it here, repeated until six times and asked what did you do with these remaining seventh fours?
69.	Learners:	(continuing to give their answers one by one) seven, seven,

70.	Teacher:	divide your whole paper into four equal columns. Write division in the first column, multiplication in the second column, addition in the next column and the subtraction in the last fourth column. (After a while) are you done?
71.	Learners:	yes
72.	Teacher:	now everybody look here. So that I give another example. Can you remember when you used the stones when you divided eighteen stones and got six?
73.	Learners:	yes
74.	Teacher:	in division you got ($18 \div 3 = 6$) is that so?
75.	Learners:	yes
76.	Teacher:	and we said even when you reverse the division as eighteen divided by six it will give three which are the same numbers as ($18 \div 6 = 3$) in the multiplication it will be still the same numbers which are ($3 \times 6 = 18$). What would be the answer of eighteen divided by six?
77.	Learners:	three
78.	Teacher:	even your sticks were twenty –eight divided by that number to get that number or reversed it and divide twenty –eight by this number to get that (referring to four and seven) are we together?
79.	Learners:	yes
80.	Teacher:	but this eighteen was just an example your number was what?
81.	Learners:	twenty-eight
82.	Teacher:	you see! So you will divide it with what?
83.	Learners:	with four
84.	Teacher:	these numbers eighteen, three and six were the numbers we used on the columns as an example. In addition you said how many threes were in eighteen?
85.	Learners:	six
86.	Teacher:	six, that is why we would be adding six threes (teacher used the same example to repeated subtraction and did it with learners)
87.	Teacher:	ok how many fours did you take out?
88.	Learners:	seven
89.	Teacher:	and how many sevens did you take from twenty-eight?
90.	Learners:	four
91.	Teacher:	they are four ok it show that you are done. Each one who is done continue with the next thing. Write the number sentence at question two.
92.	Teacher:	how many fours did you take out when you counted them Naf?
93.	Naf:	nine
94.	Teacher:	how can you take nine? Here you are even wrong. If they say subtract four from twenty-eight what will remain? Use your sticks and keep taking four and count the remaining one. Keep subtracting four again and count the remaining one and write the answers until you reach zero. Rub this because you did not count well.
95.	Learner:	I am done
96.	Teacher:	if you are done count these apples and see how many are they (talking to one learner) your subtraction process is very well but your three in fourteen minus seven that is not the answer. Go back to your sticks and see what would be the answer if you subtract seven sticks from the fourteen sticks (talking to another learner) when you have just moved to the next question are you done with addition.
97.	Learner:	yes
98.	Teacher:	how many fours have you added?
99.	Learners:	seven
100.	Teacher:	one, two, and three, four (counting the fours) I cannot see the seven fours added on your works but just four. Ok and here at subtraction, how many fours did you take out? I just do not know how you are counting. What is twenty-eight minus four?
101.	Learner:	twenty-four
102.	Teacher:	and now why did you not write twenty-four then? Keep subtracting until you reach zero (talking to another learner) let me see how you have done. Why do I see five on

		your answers when we do not have five in the multiple of four? See twenty-eight minus four is not twenty-seven at all. Count well.
103.	Teacher:	did you count the apples?
104.	Learners:	yes
105.	Teacher:	how many are they?
106.	Learners:	thirty –five
107.	Teacher:	now five are already removed out from thirty-five. So you keep taking five apples from thirty – five and makes groups for them
108.	Teacher:	you see Pet! You are progressing well. So when you took four from twenty-eight twenty four remained. So keep taking four, use the drawing to help you and continue until you reach zero. So go back to the drawing and remove four from twenty- four only. See what will remain and continue to do that until you reach zero and then find out how many fours were subtracted (after she is done) how many fours did you subtracted?
109.	Pet:	seven
110.	Teacher:	good you see you did it well. Now go to the next question. Count those apples to see what you will get but before that write a number sentence of what you were doing.
111.	Teacher:	are you done Hil? Why do you have a lot of things at division? Here at the division column, look at the example at the chalkboard. We only write the number of things we were dividing and the number of groups and the size of groups and between those numbers are only a divided sign and equal to sign. Why all these? How many things do you even have here? Let me see your things when you were dividing here. How many were you taking out?
112.	Hil:	fours
113.	Teacher:	how many fours you took out?
114.	Hil:	six
115.	Teacher:	are these six?
116.	Hil:	seven
117.	Teacher:	seven, then why did you write six here? This six make you have the wrong answers. Twenty- eight is divided by four. How many times did you removed that four here at the drawing? How many fours did you take from twenty-eight? (teacher helped Hil to relate the drawing to the number sentence)
118.	Teacher:	how many apples?
119.	Learners:	thirty – five
120.	Teacher:	keep taking five apples. No! No! No! I mean keep taking seven apples from thirty – five and make groups for them and tell me how many groups of sevens apples you were able to make
121.	Teacher:	are you done?
122.	Learners:	yes
123.	Teacher:	ok, how many groups of seven were you able to take from there (asking all the learners) have you written at number two the number sentence of what you were dividing Pet? Have you also?
124.	Pet:	no
125.	Teacher:	write the number sentence at question two, Sab! I see you also not have done it. Write on your question two (back to another)
126.	Teacher:	when you subtracted sevens from fourteen what did you get?
127.	Learner:	(after counting on the drawing) seven
128.	Teacher:	but I cannot see seven you have just written six. Write the correct number, if you are done go to the next question and write the number sentence of what you were dividing.
129.	Teacher:	how many groups of seven apples did you make out of thirty-five
130.	Learners:	five
131.	Teacher:	ok, write the division number sentence here of what you where dividing and the answer
132.	Learner:	I am done

133.	Teacher:	you are done? Have you also done the number sentence of what you were dividing?
134.	Teacher:	(after sometimes) you are done writing the number sentence right?
135.	Learner:	yes
136.	Teacher:	so how many apples were they altogether?
137.	Learner:	thirty-five
138.	Teacher:	thirty-five divided by what?
139.	Learner:	by seven
140.	Teacher:	ok, by seven and you got what you got right? Make four columns again on the other worksheets (asking one learner) how many sevens did you removed from the thirty –five? If you cannot remember go back to the drawing and see how many groups you took from thirty –five? How many?
141.	Learner:	five
142.	Teacher:	good draw columns then Pet! Are you also done?
143.	Pet:	yes
144.	Teacher:	but you have not written the number sentence on question four. How many sevens you have removed from thirty-five? The problem with you is that some of the groups you have just removed six apples and some groups are removed but not cancelled from the drawing of thirty-five. How many apples did you removed? (the learner started counting apples one by one instead of counting groups) I am asking groups of sevens apples. How many did you take out and not apples one by one.
145.	Learner:	five
146.	Teacher:	good, write the number sentence then of what you divided (after a learner had finished with the division column she kept quiet as if she was done)
147.	Learner:	I am done
148.	Teacher:	no you are not done. You have to relate this to addition, subtraction and multiplication again. All these columns should be filled
149.	Teacher:	(asking one learner about the first activity that used three and six instead of four and seven) how many group of fours did we removed from the drawing?
150.	Learner:	seven
151.	Teacher:	now why writing six if they were seven and why writing threes if we took groups of fours? I want to see you doing these corrections the time is going
152.	Teacher:	(talking to another learner) you have not used the signs correctly. See here it supposed to be plus sign. Here it supposed to be division sign and minus here.
153.	Teacher:	(talking to another learner) are you done?
154.	Learner:	yes
155.	Teacher:	how many fours have you added?
156.	Hil:	(after counting) six
157.	Teacher:	that mean you are not done because you have not reached twenty- eight. How many fours did we remove from the drawing?
158.	Hil:	seven
159.	Teacher:	meaning you need to add one more four. If you add one more four how many will they be?
160.	Hil:	seven
161.	Teacher:	seven, so do the other row now of seven fours and then add them together to get their sum.
162.	Dos:	I am done
163.	Learner:	(another learner) I am done
164.	Teacher:	you are done how many apples are those?
165.	Learners:	thirty-five
166.	Teacher:	ok, keep removing groups of seven apples, make groups for them and cancel them from the thirty-five apples drawing
167.	Hen:	I am done
168.	Teacher:	you are done? Which sign is this one? Is it minus or division sign?
169.	Hen:	minus

170.	Teacher:	ok, is minus. Hen, what were you doing with the drawing? Were you subtracting the groups of four or dividing apples into groups of four?
171.	Hen:	dividing
172.	Teacher:	but why have you not used the division sign then?
173.	Teacher:	(talking to another learner) if you are done go to your worksheet with four columns and do as we did in the last question for the all operations. So do it with thirty-five, seven and five, in addition how many fives do you think you will repeatedly add?
174.	Learner:	seven
175.	Teacher:	good! Can you see this thing you have done? Thirty –five apples divided into groups of seven apples to get five groups? You see that?
176.	Learners:	yes
177.	Teacher:	then write it as a number sentence and in multiplication it will be what times what to give what?
178.	Teacher:	ok, what does that mean? How many fives are in thirty –five? How many diagrams you have drawn? How many fives are in thirty –five? O! What were you removing and for how many times? What were you removing?
179.	Learner:	sevens
180.	Teacher:	how many sevens and that is what I am asking
181.	Learner:	five
182.	Teacher:	you removed five sevens right?
183.	Learner:	yes
184.	Teacher:	ok, start to add five sevens then. Remember you start with one, then two, then three including their sum until you reach thirty-five.
185.	Teacher:	how did you do it? Because you supposed to take seven apples at once and make a group for them and cancel them from the drawing box of thirty-five apples but that is not what you did. Like that how many groups did you make?
186.	Nab:	five
187.	Teacher:	you see they are now one, two, three, four, five, six (I guess a learner decided to make five groups because she heard others saying five forgetting that there was already a first group made from the thirty-five drawing of apples and by making five makes it six groups all together) can you see these apples?
188.	Nab:	yes
189.	Teacher:	what you are going to do is this. Can you see the apples already cancelled?
190.	Nab:	yes
191.	Teacher:	these are the first group of seven apples already drawn from thirty-five drawing of apples. What you will do is continue counting seven apples from the drawing of thirty –five in a box cancel them out from the box and make groups for them. keep doing that until all apples are cancelled out.
192.	Teacher:	(talking to one learner) how many sevens are these?
193.	Learner:	four
194.	Teacher:	then why writing their sum as fourteen when the four sevens have the sum of twenty-eight (to another learner) how many fours are these that gave you twenty-eight?
195.	Learner:	seven
196.	Teacher:	but I am just counting six and six gives twenty-four and not twenty-eight. So make them seven (she also turn to another learner and start helping her how to do the repeated subtraction)
197.	Teacher:	are you done? How many groups did you removed?
198.	Learner:	four
199.	Teacher:	no! count all the groups with seven apples that were removed from the big box
200.	Learner:	five
201.	Teacher:	good! Write the number sentence of what you were dividing. Era!
202.	Teacher:	(talking to Era) how many fives you added together?
203.	Era:	seven
204.	Teacher:	seven. But they have given you fifty? O! If you cannot get it go back to the drawing here you have been adding well. Here and here you are also correct but here you are

not correct count five fives make drawing for them if need be and see how many are they together. (After the learner had drawn the five circles each with five dots) count all the dots then to see what will be the sum.

205.	Teacher:	(talking to another learner) how many fours are these? (After counting) they are seven ok, but there is no plus sign and equal to sign. So how many fours did you took?
206.	Pet:	seven
207.	Teacher:	(when realized that the learner was just saying things out of blue) count them before saying the answers. You see where you are wrong you subtracted three here. We only subtract the same number until you reach zero. You only subtract fours, fours, fours and you can use your drawing to subtract and get the answer. When you take four from your drawing count the remaining and write the answer. Go back to the remaining and subtract four again, keep doing that until all is finished.
208.	Teacher:	(talking to another learner) how many groups of apples did you take?
209.	Dos:	four
210.	Teacher:	no! Count all the groups well that were removed from the big box because even this one that you already found done also come from the big box and is part of other groups.
211.	Dos:	(counting) one, two, three, four, five
212.	Teacher:	how many?
213.	Dos:	five
214.	Teacher:	good! if they are five write for me a number sentence of what you were dividing (talking to another learner) how many groups did you make?
215.	Learner:	five
216.	Teacher:	this drawing is it also a group or is it rubbed off?
217.	Learner:	it is there
218.	Teacher:	so you now have (counting) one, two, three, four, five, six because even this one is a group that you find already done and these apples comes from this big box also (after one group has been destroyed) good! How many groups do you have now?
219.	Learner:	five
220.	Teacher:	now write the number sentence of what you were dividing based on the number of apples altogether, number of groups and apples in a group (when the teacher saw that the learner has written the wrong dividend) how many apples were these all together?
221.	Learner:	thirty-five
222.	Teacher:	then why writing this thirty-four? (After a while) what will be an answer when seven apples are being subtracted from thirty –five? Because I do not think this twenty – seven will be the correct answer go back to the drawing. These apples cancelled already are seven see how many remained.
223.	Teacher:	ok here you have it correctly. You come to be wrong at this point. If you subtract seven from thirty-five what will remain? Go back to the drawing if you remove this seven apples what will be the remaining number of apples?
224.	Learner:	one, two, three, four, five, six, seven, ………….., twenty-eight (counting the remaining number of apples)
225.	Teacher:	oo! Yes is twenty-eight you are right. All your answers are correct. Now go to the last question.
226.	Teacher:	how many are those sweets?
227.	Learner:	thirty –two
228.	Teacher:	and how many sweets are already taken?
229.	Learner:	three
230.	Teacher:	no
231.	Learner:	four
232.	Teacher:	four, ok now count to check if those sweets are really thirty-two. Then if they are thirty-two keep subtracting groups of four sweets and make those groups here.
233.	Teacher:	who said is done here? How many sevens are these?
234.	Learner:	five
235.	Teacher:	did you include those that are already cancelled?

236.	Learner:	yes
237.	Teacher:	so they are only thirty. One, two, three, four, five, six, seven… thirty (counting the sweets in the drawing) ok add two drawing of sweets more. How many sweets are they now altogether?
238.	Learner:	thirty-two
239.	Teacher:	keep taking groups of four sweets then, this group already done for you is also included.
240.	Teacher:	are you also done?
241.	Learner:	yes
242.	Teacher:	(verifying) one seven, two seven, here you supposed to put three sevens. Rub all your wrong things and re-write the (the learner had written a correct sums of sevens but had not shown how many sevens are they that made up that sum) use the drawing to see the number of sevens that made up that sum.
243.	Teacher:	how many sevens did you take out from thirty-five?
244.	Learner:	four
245.	Teacher:	(talking to another learner) you do not have the correct subtraction answers. All these answers you wrote are wrong you have not counted well. Go back to the drawing, take seven from thirty-five and see what remains. Take again seven from the remaining and see what remain
246.	Teacher:	(talking to another learner) what were you doing? Did you add two sweets more?
247.	Learner:	yes
248.	Teacher:	can you see these four sweets?
249.	Learner:	yes
250.	Teacher:	they are already removed and made group for you. So you continue taking groups of four sweets and cancel them out.
251.	Teacher:	how many sweets did you take out?
252.	Learner:	seven
253.	Teacher:	seven, ok you are done. So now
254.	Teacher:	(talking to another learner) what did I say you do?
255.	Learner:	you said I should re-do this thing.
256.	Teacher:	have you done it correctly now? And if you have done it correctly why still in four subtracted from twenty-four giving the answer twenty- four still? If it is twenty-four minus four but the answer is still twenty-four. Did you use the drawing when calculated?
257.	Learner:	yes
258.	Teacher:	ok, first you subtracted four from twenty-eight and there was twenty-four remained. Is that so?
259.	Learner:	yes
260.	Teacher:	ok, if you take again four from twenty-four. What will be the remaining? You did not count well re-count again and keep taking the four from the remaining until the drawings get finished.
261.	Teacher:	(talking to another learner) I want you to count these apples for me (talking to another learner again) draw four columns and label the work as number three. How did you divide thirty-five? Ok, is with seven and you got five which is correct.
262.	Teacher:	are you done with question two?
263.	Dos:	yes
264.	Teacher:	how many groups did you take from here?
265.	Dos:	five
266.	Teacher:	have you answered the questions from here?
267.	Dos:	yes
268.	Teacher:	so according to your answer you took out four sevens?
269.	Dos:	they are five
270.	Teacher:	but why did you wrote four here? (After the learner had corrected his mistakes) good! now draw four columns

271.	Teacher:	(asking another learner) how many sevens did you take out? (Pause) count for me the groups you made (after a learner had counted) in clued this one because it also came from the drawings.
272.	Learner:	six
273.	Teacher:	it means if you had taken out six groups somewhere somehow you did not count well. You did not count well at all at all, because can you see this group here?
274.	Learner:	yes
275.	Teacher:	it also came from these apples. Do you want to tell me that the remaining apples would make six groups of seven? Rub all your works and re-do it well.
276.	Teacher:	(talking to the whole class) some people got it wrong because they like imitating others. When they hear other people say six and he will start to do what he heard from others or what he peeped on other people's papers, which is not good. Just do you works as I have instructed you to do.
277.	Teacher:	how many apples are these when you counted them?
278.	Learner:	thirty –five
279.	Teacher:	thirty – five these seven apples are already taken out. So you continue taking groups of seven apples from the drawing but do not repeat the one already cancelled out. Do cancel the apples you have removed from the drawing.
280.	Teacher:	(talking to another learner) if you have done the drawing of four columns, write the number sentence in those columns for division, subtraction, addition and multiplication.
281.	Teacher:	(talking to another learner) twenty – eight divided by seven does it gives seven again? How many groups are these?
282.	Learner:	seven
283.	Teacher:	and what sticks are inside the groups?
284.	Learner:	four
285.	Teacher:	then why didn't I see the numbers four, seven and twenty – eight on your division number sentence?
286.	Teacher:	(talking to Fil who was instructed to divide thirty-five apples into the groups of seven apples but rather made four groups and started allocating apples one by one to groups) rub you things and do what you are told. What I told you is count out seven apples and makes groups for them and cancels them out from the drawing. I did not tell you to allocate them one by one into groups. How did you even know the groups will be four when the number of groups is unknown?
287.	Learner:	I am done
288.	Teacher:	how many fours did you removed?
289.	Learner:	(starting to count) one, two, three, four, ……., nine
290.	Teacher:	are they supposed to be nine? Or may be you have not counted well (asking Sab) how many fours did you take out?
291.	Sab:	eight
292.	Teacher:	where did you get one four more from? (She goes back to the drawing and started to count with Sab) one, two, three, and four this is one four right? Is this a group?
293.	Sab:	yes
294.	Teacher:	(continuing to count the groups) one, two, three, four. And this is another group of four right? How many group are they now?
295.	Sab:	two four
296.	Teacher:	(continuing to indicate the fours from the drawing) one, two, three, four and this is the?
297.	Sab:	is the third
298.	Teacher:	they are now three groups (then she continue to count the remaining) one, two, three, four. Now how many fours are these?
299.	Sab:	four fours
300.	Teacher:	how many now?
301.	Sab:	five
302.	Teacher:	and now?

303.	Sab:	six
304.	Teacher:	now how many are they?
305.	Sab:	seven
306.	Teacher:	you see where the problem comes from? I told you to add two sweets to the drawing so the sweets can be thirty- two. Now where did you find these two other again? Can you just re-do it. Because they are not eight.
307.	Teacher:	(asking one learner) how many fours did you removed out?
308.	Learner:	they are seven
309.	Teacher:	(asking another learner) how many sevens did you add?
310.	Learner:	five sevens
311.	Teacher:	five sevens give thirty-five but then where did you took fours? If in this number sentence there is only five, seven and thirty-five. Where did you take these fours you repeatedly subtracted because you supposed only to be taking sevens or ……?
312.	Hil:	fives
313.	Teacher:	yes! How many fives?
314.	Hil:	seven
315.	Teacher:	or how many sevens?
316.	Hil:	five
317.	Teacher:	ok, this one rubs it and the dividend was not even twenty-eight but thirty-five. That means you copied the work of the first question to the second question because now we are not working with twenty-eight but with thirty-five.
318.	Teacher:	did you say you are done?
319.	Learner:	yes
320.	Teacher:	then write the number sentence of what you were dividing. Five times seven is what? Here you also did not write well. You have written the things you took there; also what you supposed to write here is a number sentence of what you were dividing here. Good! Now that you are done here go and draw four columns label the work as number five. These columns should be still labeled as we did for other first. One for division, one for addition, one for subtraction and the other for multiplication.
321.	Learner:	I am done
322.	Teacher:	how many groups did you got?
323.	Learner:	eight
324.	Teacher:	right! Write here then the number sentence of what you were dividing (after a long pause) have you written the number sentence of what you were dividing?
325.	Learner:	yes
326.	Teacher:	draw your four columns also, label them number five and do as you did in the last question, one for division, subtraction, addition and multiplication
327.	Learner:	I am done
328.	Teacher:	ok, where did we end up with you? What did I say do? I said divide the apples right?
329.	Learner:	yes
330.	Teacher:	did you have the drawing?
331.	Learner:	yes I am done
332.	Teacher:	ok, now write the number sentence of what you were dividing. (The teacher was saying what the learner is writing as she is writing the number sentence) so you have thirty-five divided by o! Did you have four or five? (Asking when she saw a learner writing four on the number sentence) were your groups four or five?
333.	Teacher:	(talking to Hil) ok, what did I say do?
334.	Hil:	you said I should remove groups of fours.
335.	Teacher:	ok, remove groups of fours, how many did you take out?
336.	Hil:	twelve
337.	Teacher:	if they are above ten you are wrong rub that answer. Do you notice what made you wrong? Hil! We have already discussed this. How many fours are in twenty-eight?
338.	Hil:	seven
339.	Teacher:	meaning there are how many fours in twenty-eight?
340.	Hil:	seven fours

341.	Teacher:	but why do you have a number of fours above ten? Can't you see you are wrong here? Because fours, if you repeatedly subtract them from twenty- eight they should be sevens and if you repeatedly subtract sevens from twenty-eight they should be fours. So you have not counted well. When you see your numbers are not corresponding to the numbers on your division number sentence there is something wrong.
342.	Teacher:	(talking to another learner again) are you done?
343.	Learner:	yes
344.	Teacher:	thirty-five divided by five is seven right?
345.	Learner:	yes
346.	Teacher:	and seven times five is thirty-five right?
347.	Learner:	yes
348.	Teacher:	on addition do you want to use fives or is it seven?
349.	Learner:	is fives
350.	Teacher:	how many fives? (pause) the thing is just here on the previous work of the previous column
351.	Learner:	three
352.	Teacher:	three, where is three in the division and multiplication number sentence? How many fives needed to be repeatedly added for you to get thirty-five? That is what I am asking. Because here the numbers are only five, seven and thirty-five. So how many fives would you repeatedly add to get thirty-five?
353.	Learner:	seven
354.	Teacher:	good! Let me see how you have added them starting from the first row with one five, the next row with two fives, the next with three fives, fours until you reach the row with seven fives.
355.	Teacher:	(asking another learner) where are you? So you have not yet written the division number sentence when I have told you the way it should be written (pointing to the example of the other one worked already) just as it is written here, so you write thirty-five divided by that number to get the answer. So you were dividing thirty-five with what? Was it seven or five? And what did you get? (After the learner had done it) good! How do you write that on multiplication? Look at the example on the other side of question one column?
356.	Learner:	I am done
357.	Teacher:	ok, you are even farther. Twenty-eight divided by seven is equal to four. Why is there seven and six on your work? Correct it.
358.	Dav:	I am done
359.	Teacher:	how are your things? You are not done. How many things were these?
360.	Dav:	twenty-eight
361.	Teacher:	but you did not write twenty-eight. How did you do it here? Did you add, subtract, divide or multiply? What did you do here?
362.	Dav:	I divided
363.	Teacher:	ok, why don't I see twenty-eight divided by whatever number you used and the answer you got? Or is it this one here?
364.	Dav:	yes
365.	Teacher:	on multiplication column is here right? And on addition how many fours did you add?
366.	Dav:	seven
367.	Teacher:	to give you twenty-eight, ok (reading the sums) four, eight, twelve, sixteen, twenty, twenty-four, twenty-eight, good! And here how did you subtracted? I cannot even see the subtraction but rather the addition because this sign you have used is an addition sign. You are not done. Is it sevens or fours you want to subtract? (After a pause) how many sevens are in twenty-eight? Go back to your drawing. How many sevens are in twenty eight?
368.	Dav:	four
369.	Teacher:	and there are how many fours?
370.	Dav:	seven

371.	Teacher:	seven, do you want to be subtracting fours or is it sevens from twenty-eight. That is what I am asking.
372.	Dav:	fours
373.	Teacher:	what?
374.	Dav:	fours
375.	Teacher:	ok, start taking fours out from twenty-eight then and see what would remain. You can use the drawing, if you take four from these and count see what will remain. When you get the answer then subtract four again. Keep doing that until you reach zero.
376.	Teacher:	what did I say do? (Asking another learner) you only missed here. This number on your number sentence supposed to be seven. You supposed to change that six to seven. So that seven times four gives twenty-eight. Now what you did just ruined everything. You are not done.
377.	Pet:	I am done
378.	Teacher:	who said is done here?
379.	Pet:	it's me
380.	Teacher:	where did you end up? How many sevens did you take out from twenty-eight?
381.	Pet:	five
382.	Teacher:	five sevens, very good now you can continue with the last question. The last question is saying these sweets are thirty –two but two of the sweets are missing so I am going to add two sweets for you so that they will become thirty-two. Can you see them?
383.	Pet:	yes
384.	Teacher:	ok, now make groups here. The first group of four sweets is already taken out from the drawing. Can you see how these sweets are cancelled out from the drawing and this group out side the box?
385.	Pet:	yes
386.	Teacher:	ok, continues taking out groups of fours from the drawing and cancel them out from the box until all sweets are finished. Who else is done?
387.	Teacher:	ok where are you now? Is four times seven equal to seven again? Ok, now Dia I want to ask you. How many sevens are in twenty-eight? Is this six or seven?
388.	Dia:	six
389.	Teacher:	the six that I told you to change long time ago. Are these groups six or are they seven? Who else is done?
390.	Teacher:	(asking a learner) how many sevens are in thirty-five? When you did your groups with your drawing. How many sevens did you take out from thirty-five?
391.	Learner:	five
392.	Teacher:	five ok, and how many fives are in thirty-five? If sevens are five, how many fives are they? We said these numbers are vice versa. So if seven is in thirty-five five times. So five is there how many times?
393.	Learner:	seven
394.	Teacher:	five is there seven times and seven is there how many times? How many sevens are in thirty-five?
395.	Learner:	five times
396.	Teacher:	five, so how many sevens are there you said? (Pause) is the same thing I asked you before.
397.	Learner:	five
398.	Teacher:	five and fives are there how many times?
399.	Learner:	seven times
400.	Teacher:	now at addition you only choose one number. Which one do you want to repeatedly add? Is it five or seven?
401.	Learner:	is fives
402.	Teacher:	ok how many fives will you add? (after a pause)
403.	Learner:	sevens
404.	Teacher:	sevens ok, starting with one five, two five, three until seven fives all with their sums.
405.	Teacher:	(talking to another learner) which number are you done with? Ok, the question five. How many sevens you repeatedly add (checking) is fives

406. Learner: yes
407. Teacher: ok and how many seven did you repeatedly subtract from thirty –five?
408. Learner: (after counting) five
409. Teacher: is five times but it seems you copy somewhere, because this is twenty-one but why is it not here? Your things are not in order because normally one write the answer and it is the answer you will subtract seven again rub your work and re-write in a clear manner. Just rub from the beginning.
410. Teacher: (asking another learner) where are you? Are five times seven equal to five when it supposed to be thirty-five? Is this number five or six even?
411. Learner: five
412. Teacher: write clear numbers, numbers are three in that numbers sentence thirty-five, seven and five. So do not re-write numbers. Or what do you wanted to do here? Good! How many does four sevens gives? Look at your drawing and make the sevens in four groups to see what it gives it will not give the eighteen that you have written here. See also what five sevens will give you.
413. Teacher: (talking to another learner) ok, this number was thirty-two. So you supposed to start with thirty-two minus eight and your eights supposed……, why do you have a sixteen in the row? Only eight supposed to be there it is the sum. You supposed to give their answers, rub from here and then makes two eights, three eights and so-on. Even on subtraction you supposed to start with thirty-two minus eight and keep subtracting eight from the differences until you reach zero.
414. Teacher: who said is done here? How did you worked? How many fives did you added?
415. Dos: sevens
416. Teacher: and how many five did you subtracted
417. Dos: seven
418. Teacher: can you see these sweets? They are just thirty because of a typing error. I will make them thirty-two. Can you see the four cancelled sweets?
419. Dos: yes
420. Teacher: they are here in a group. you will then continue to cancel fours and make groups for them until all the sweets are cancelled and puts into groups
421. Teacher: (asking another learner) what did I say do? When you subtracted four from thirty-two there remained twenty-eight, twenty – eight minus four is twenty-four, twenty-four minus four is twenty minus four is sixteen minus four is twelve minus four is eight minus four is four minus four is zero. How many fours did you subtracted?
422. Learner: eight
423. Teacher: can't you see that now you did it well. Good! Now draw a line to underline your work. So you can go to the next. You have not answered the next question that asked the number sentence of what you were dividing.
424. Teacher: (talking to a learner) can you see these apples?
425. Learner: yes
426. Teacher: count to see how many they are. Good! How many fours have you added here? How many fours have you added here?
427. Learner: seven
428. Teacher: seven, can you count them for me?
429. Learner: one, two, three, four, five, …………, seven
430. Teacher: underline here
431. Teacher: (to another learner) how many apples did you counted?
432. Learner: thirty-five
433. Teacher: thirty-five, from thirty-five there are already seven apples taken from the drawing. Can you see it?
434. Learner: yes
435. Teacher: ok, then continue taking the groups of seven apples from the drawing and cancel the apples already taken until all apples are taken out.
436. Teacher: write here the number sentence of how many things you were dividing by what number and the answer you got.

437.	Teacher:	good! Let me ask you before you go. Here you have used the division sign (division number sentence to relate to the drawing) why?
438.	Sab:	is because I was dividing.
439.	Teacher:	dividing what?
440.	Sab:	I was dividing twenty-eight sticks into groups of fours.
441.	Teacher:	how many groups of four did you take out from twenty eight?
442.	Sab:	(he first went to the drawing to count and confirm the answer) seven
443.	Teacher:	ok, how did you divide? How did you divide?
444.	Sab:	I have been taking the groups with four sticks each until all sticks are taken out.
445.	Teacher:	did you divided in the same manner for all questions today?
446.	Sab:	yes
447.	Teacher:	what makes you think that your answers are correct and you are not wrong? Are all your answers correct?
448.	Sab:	yes
449.	Teacher:	what makes you so sure that all your answers are correct even for question one, three or five?
450.	Sab:	is because the sticks when I counted them in all groups are equal and from where I was taking sticks got finished.
451.	Teacher:	ok, can you give me an example, in fact give me three examples for the three questions
452.	Sab:	is like having twenty things shared by two kids of whom each would have ten things.
453.	Teacher:	give me another example but it must not have number such as two, ten and five
454.	Sab:	is having thirty things shared by three people of whom each would get ten things.
455.	Teacher:	ok, another
456.	Sab:	is having forty things to be shared by two people of whom each would have twenty things.
457.	Teacher:	(asking another learner) if you subtract eight from thirty-two, is the answer going to be twenty-three? And how about this fifteen? Go back to the drawing and re-count your things. Keep subtracting eight from the difference, re-count the remaining one and see what your answer would be.
458.	Teacher:	(asking another learner) who said is done here?
459.	Hil:	I am done
460.	Teacher:	ok, how many groups did you take from thirty-five apples
461.	Hil:	five groups
462.	Teacher:	(counting to confirm) one, two, three, four, five, are all these groups having seven apples?
463.	Hil:	yes
464.	Teacher:	ok, so write for me a number sentence of what you were dividing and after you finish label your works as question three and divide your paper into four columns (pause) are you done?
465.	Hil:	yes
466.	Teacher:	how many sevens did you add?
467.	Hil:	five
468.	Teacher:	how many sevens did you subtracted
469.	Hil:	five
470.	Teacher:	(confirming) is this seven and four or is it fourteen? Is this seven and four or is it fourteen?
471.	Hil:	is seven and four
472.	Teacher:	(laughing) did you counted or what is wrong with you? These things are thirty-five is it so?
473.	Hil:	yes
474.	Teacher:	and you subtracted these seven things. How many remained?
475.	Hil:	(counting) one, two, three, four, five, six, seven… twenty –eight.
476.	Teacher:	twenty-eight rights? Take seven apples again from twenty-eight and see what will remain.

477.	Hil:	one, two, (she started counting in the groups of twenty –eight but not following them in a row only at random as she count she was jumping here and there)
478.	Teacher:	no that way you will just confuse yourself. Count them in order as they are drawn or in a columns
479.	Hil:	(continuing) five, six, seven… twenty –one.
480.	Teacher:	twenty-one, the one you have written here rights?
481.	Hil:	yes
482.	Teacher:	ok, take seven from this twenty-one again the remaining and count the remaining again
483.	Hil:	one, two, three, four,…………., fourteen
484.	Teacher:	so it is fourteen and not seventy four as you said rights?
485.	Hil:	yes
486.	Teacher:	and that is what I was asking you if the number you have written is seven four or fourteen. When you counted now did you count seven four or fourteen?
487.	Hil:	is fourteen
488.	Teacher:	ok, rub that seventy four and write fourteen then
489.	Teacher:	(asking one learner) how many groups did you take out?
490.	Learner:	seven
491.	Teacher:	no count them all even this one comes from these sweets.
492.	Learner:	(after counting them) eight
493.	Teacher:	eight ok, now write a number sentence of what you were dividing at question six.
494.	Teacher:	are you done? (Asking another learner) ok, if you are done let me add two sweets here. Now these sweets here altogether are thirty-two. Can you see them?
495.	Learner:	yes
496.	Teacher:	from these thirty-two sweets four sweets are already cancelled out and made a group for them out side the big box. Can you see them?
497.	Learner:	yes
498.	Teacher:	now continue taking four sweets out from the big box cancel them out and make group for them
499.	Teacher:	now start subtracting eight sweets from these thirty –two and see what will remain
500.	Learner:	one, two, three, four
501.	Teacher:	(talking to Dav) you have not counted well because four from twenty-eight is not going to give you seven. When you subtract four from twenty- eight there will be a lot that would remain. So go back to the drawing take four out and count the remaining one.
502.	Teacher:	Era! Are you done?
503.	Era:	yes
504.	Teacher:	ok, I want to ask you questions before you go. Era! Why have you opted for a division sign at question two, four and six? Why?
505.	Era:	is because we were dividing.
506.	Teacher:	choose one of the question to explain what and how you were dividing.
507.	Era:	like at number six we were dividing thirty-two sweets into eight groups.
508.	Teacher:	how did you know before hands that groups would be eight? Tell me what you had done step by step?
509.	Era:	I had been taking groups of fours until all the sweets are cancelled or got finished.
510.	Teacher:	until sweets got finished. When you divided into groups of four how many groups did you got?
511.	Era:	eight
512.	Teacher:	what makes you sure that all of your answers are correct?
513.	Era:	is because all groups has equal portions
514.	Teacher:	what else?
515.	Era:	and that the things I was dividing got finished.
516.	Teacher:	ok, how did you say you had been dividing? Use question three to explain.
517.	Era:	I have been taking out apples in group of five.
518.	Teacher:	are those apples five in your groups? Can you count them?

519.	Era:	I have been taking out groups of seven.
520.	Teacher:	how many apples were they altogether?
521.	Era:	they were thirty-five
522.	Teacher:	so you have been taking out groups of seven apples until what?
523.	Era:	until all apples get finished
524.	Teacher:	can you please give me an example as such?
525.	Era:	is like having thirty sweets shared by two people of whom each would get fifteen sweets or having twenty sweets shared by two people each would have ten sweets.
526.	Learner:	I am done
527.	Teacher:	what are you done with? Were you dividing groups?
528.	Learner:	yes
529.	Teacher:	how many groups did you take out?
530.	Learner:	seven
531.	Teacher:	no I don't think they are seven is this group not part of them? Count all the groups that come out from here.
532.	Learner:	(after counting) eight
533.	Teacher:	eight ok, write the number sentence of what you were dividing.
534.	Teacher:	(asking the learner) how many are fives?
535.	Learner:	seven
536.	Teacher:	seven ok, (the teacher started to check the answers for addition) five, ten, fifteen, twenty (and stop at the fifth row where it is written twenty-four) this is not twenty – four just count well, just because you know there are five sevens does not mean you should have the wrong sums. No, no, no this is not correct and this one. If you are finding it difficult to get the sum use the drawing and here you even forgot to put one of the five in the middle you only put the plus sign. You can also draw dots or sticks in fives to get the sums. Calculate your sum well they are not correct.
537.	Teacher:	(asking one learner) where did you take these fours? And it is subtraction but you put plus signs. You supposed to have thirty-five minus five. Yes minus five like that.
538.	Teacher:	(asking one learner when realized he had represented numbers wrongly) count these groups here.
539.	Learner:	one, two, three, four, (but instead of counting groups he was counting sticks in the group)
540.	Teacher:	I said count the groups and not the sticks the sticks you have done them well. They are four groups. Are these one? (pointing to groups)
541.	Learner:	one, two, three, four, five, six, seven
542.	Teacher:	then why did you write five on your number sentence? is that four which is there five times or seven times?
543.	Teacher:	(asking the whole class) the question I wanted to ask is why did you opt for the division sign in question two and question four?
544.	Dos:	is because we were dividing
545.	Teacher:	Dav! What were you doing here?
546.	Dav:	I was dividing
547.	Teacher:	dividing what Dav?
548.	Dav:	we were dividing twenty-eight sticks into groups. We took four sticks in each group
549.	Teacher:	your story is not complete it has to tell until the end of what you were doing.
550.	Dav:	I counted four sticks and put them into one group and counted another four sticks and make a group for them, I kept doing that until all the twenty- eight sticks got finished.
551.	Teacher:	until they got finished?
552.	Dav:	yes
553.	Teacher:	ok, after the division how many groups did you make?
554.	Dav:	seven
555.	Teacher:	seven ok, Dav! What makes you think your answers are correct?
556.	Dav:	is because the sticks I was dividing got finished

557.	Teacher:	so the fact that they got finished makes you think that you are correct? What if they are finished as you said but the groups you have divided them one five another group with eight (un equal distribution) would you still be correct?
558.	Dav:	no
559.	Teacher:	if not, that mean your answer is incomplete, so what makes you think you are correct (after a long pause) ok, if you cannot answer me. Give me an example of what could be divided in this manner.
560.	Dav:	is like having twenty sweets
561.	Teacher:	no I do not want twenty anymore that number is used many a times today.
562.	Dav:	is like having hundred sweets shared by two people each of whom each would have fifty sweets
563.	Teacher:	I want another one to tell me how you have been dividing. How did you divide Hil?
564.	Hil:	we have been ……….
565.	Teacher:	use number three to explain
566.	Hil:	we took thirty-five apples and divided them into five circles (meaning groups)
567.	Teacher:	how did you know the groups were five when they were not determined? Did you all divided like that? (Asking the whole class) because we have not divided as Hil is telling it. Who else can tell how you have divided? Because we did not make five groups (circles) first but what is that you have been doing? Naf!
568.	Naf:	we had been taking seven apples at a time make groups for them and cancel them at the big box, we kept taking seven apples making groups for them and cancel them from the big box until the apples in a big box get finished
569.	Teacher:	what makes you think that your answers must be just correct? Dos!
570.	Dos:	is because the thirty – five apples got finished and the apples in groups are all equal they are all seven in each group.
571.	Teacher:	(talking to the whole class) every one must give me an example and it should not be the one already given
572.	Dos:	is like having hundred sweets shared by ……
573.	Teacher:	people to share shouldn’t be two anymore because that is the same example Dav has just given now.
574.	Dos:	shared by fifty people of whom each would have
575.	Teacher:	two, these are the numbers already mentioned give another example
576.	Dos:	each would have fifteen
577.	Teacher:	no when people are fifty, fifty is just there two times in hundred.
578.	Dos:	each would have two sweets
579.	Teacher:	yes that is why I say that is the example Dav gave me. Another example? Pet!
580.	Pet:	having forty things shared by two people of whom each would have twenty
581.	Teacher:	ok, tomorrow I do not want forty, twenty and two anymore. Mat!
582.	Mat:	is like having thirty things shared by three people of whom each would have ten things
583.	Teacher:	yes
584.	Learner:	having fifty sweets shared by five people each would get ten
585.	Teacher:	Dos!
586.	Dos:	having hundred sweets shared by ten people each would have ten sweets
587.	Teacher:	good!
588.	Learner:	having thirty sweets shared by two people each would have fifteen
589.	Teacher:	I remember you giving me the same example so give me another one without thirty, two and fifteen
590.	Learner:	having forty sweets shared by two people each…..
591.	Teacher:	no that is also mentioned; forty divided by two each would have twenty. Did you not hear that? It was mentioned by Pet.
592.	Hen:	having thirty sweets shared by ten people of whom each would have ten sweets.
593.	Teacher:	Hil!
594.	Hil:	having fifty sweets shared by two each would have ten sweets

595.	Teacher:	no is not correct keep thinking I will come back to you. Even you Hen keep thinking your answer is not correct. I would come back to you.
596.	Learner:	I have three things shared by two people each would have (pause) five
597.	Teacher:	how many things did you say will be shared?
598.	Learner:	each would have…..
599.	Teacher:	no! No! No! I asked, how many things did you say will be shared?
600.	Learner:	three
601.	Teacher:	three as one, two, three (showing three stones)
602.	Learner:	yes
603.	Teacher:	and it would be shared by how many people you said?
604.	Learner:	two people
605.	Teacher:	how many each would get?
606.	Learner:	one
607.	Teacher:	unless you say so. Will there be a reminder or not?
608.	Learner:	one thing will remain
609.	Teacher:	yes
610.	Fil:	having fifty sweets shared by two people……..
611.	Teacher:	no! That is someone else's example and I am even waiting for her answer. Have you got your answers? (asking Hen and Hil)
612.	Hen:	yes
613.	Teacher:	ok how many each would get?
614.	Hen:	each would have twenty things
615.	Teacher:	you said you have thirty things to be shared by ten people is that so?
616.	Hen:	yes
617.	Teacher:	ok, how many each would get then?
618.	Hen:	each would have twenty things
619.	Hil:	having fifty sweets shared by five people each would have ten
620.	Teacher:	ok
621.	Learner:	having fifty things shared by three people and each would have ten
622.	Teacher:	no go and think again
623.	Learner:	is like having sixty sweets shared by three people each would have twenty
624.	Teacher:	yes
625.	Learner:	is like having eight sweets shared by two people and each would get three
626.	Teacher:	what?
627.	Learner:	each would get five
628.	Teacher:	o! Even eight and two you cannot get the correct answer. Use even your fingers and see what eight divided by two gives.
629.	Hil:	having ten sweets shared by five people each would have two
630.	Teacher:	ok, but I said ten; five and two are no longer to be used as an example. Yes!
631.	Hen:	having four sweets shared by two kids and each would have two.
632.	Teacher:	ok, just go home anyway. Yes!
633.	Learner:	each would have ten
634.	Teacher:	what was there to be divided?
635.	Learner:	each would have twenty
636.	Teacher:	no start like how many things you have to be shared by who before you tell what each would have or else we do not know what you were dividing.
637.	Learner:	I have twenty things to be shared by two people and each will get ten
638.	Teacher:	good go home
639.	Hil:	I will have six things shared by two people and each would get three things
640.	Teacher:	that is the grade 1 example, numbers bellow ten are not needed but anyway go home
641.	Learner:	is like having twenty things shared by two people and each would have ten things
642.	Teacher:	that was already mentioned by another but anyway go home.

DAY SIX 14 MAY 2012

LESSON ELEVEN

1.	Teacher:	(After the teacher greets the participants and reminded them to come for the lesson the next day) yesterday most of you went with unfinished work and I understand that it might be caused by the tables of columns that we added in the lesson, which were not part of it and I think some it could be they are still trying to understand what is going on. We are going to continue and the expectation is that those who do not understand today or tomorrow may understand it. I will still repeat the aim of these lessons, which is by the end we are expected to understand division and its relationship to other topics.
2.	Teacher:	(asking learners) what else could be related to division? Like yesterday we were relating division to what and what? Sab!
3.	Sab:	to addition
4.	Teacher:	to addition or what else? Fil!
5.	Fil:	subtraction
6.	Teacher:	to subtraction and also what?
7.	Sab:	multiplication
8.	Teacher:	yes multiplication, I want you to understand that if you are given a value i.e. four there twelve times? Who can tell me what four times twelve is? Who can tell me what four times twelve is? (pause) (The teacher changed the question) ok, ten times four is what? (Pause) the tens there four times are? Ten, ten, ten, ten
9.	Learner:	forty
10.	Teacher:	ok, and two times four is what? Because twelve is ten plus two (10 + 2) so twelve four times is like four tens (10 + 10 + 10 + 10) plus four twos (2 + 2 + 2 + 2). What are two times four? What are two times four? Two times four is what? Yes! Fil
11.	Fil:	one
12.	Teacher:	Sab! If the two are there four times?
13.	Sab:	four
14.	Teacher:	two there four times? What is the answer? yes!
15.	Learner:	eight
16.	Teacher:	yes eight, then what would be the whole answer of twelve times four? If you add the forty and eight together? Yes!
17.	Learner:	forty –eight
18.	Teacher:	yes it is forty – eight. It is forty – eight. Ok that is what we were doing yesterday. So you know that four there twelve times is forty – eight is that so?
19.	Learner:	yes
20.	Teacher:	so four there twelve times is forty-eight (writing it on the chalkboard) if you start adding twelve four, what will you get? Four there added twelve times, what would you get? Hil!
21.	Hil:	is forty –eight
22.	Teacher:	forty –eight divided by twelve gives what? Forty –eight divided by twelve gives what? Dav!
23.	Dav:	four
24.	Teacher:	so forty –eight divided by four will give what answer? Forty –eight divided by four will give what answer? (A! A! A! A! A! A! A! Learners shouted as they raised up their hands eager to be give a chance to say the answer) Naf!
25.	Naf:	is twelve
26.	Teacher:	Are twelve ok, how many twelve are in forty-eight? How many twelve are in forty-eight? Yes!
27.	Learner:	four
28.	Teacher:	yes they are just four. So if you were to add you will start with this one twelve. What will be the next step? Era!

29.	Era:	two twelve
30.	Teacher:	is two twelve. The next step is what? Fil!
31.	Fil:	is three twelve
32.	Teacher:	yes, it will be three twelve and what is the next step?
33.	Learner:	are the four twelve
34.	Teacher:	yes four twelve because there are only four twelve in forty –eight. But what do we put between the twelve?
35.	Learner:	plus sign
36.	Teacher:	plus sign. When it is one twelve the sum is just twelve is that right? But what will be the sum of two twelve? If you wish you can add two tens (10 + 10) plus two twos (2 + 2). Sab! What will be the sum?
37.	Sab:	twenty – two
38.	Teacher:	what?
39.	Sab:	twenty-two
40.	Teacher:	ok, twenty-two, how many three twelve gives? Here you just add the three tens (10 + 10 + 10) plus the three twos (2 + 2 + 2) Dav!
41.	Dav:	thirty
42.	Teacher:	thirty – what?
43.	Dav:	three
44.	Teacher:	you are right that it will be thirty if you add three tens but three twos will not give three. But what will three twos be? yes
45.	Learner:	thirty- five
46.	Teacher:	is not thirty –five but?
47.	Dos:	thirty – six
48.	Teacher:	thirty – six even the first answer was wrong because two twos give what? Dos!
49.	Dos:	twenty
50.	Teacher:	yes two tens gives twenty but two twos give what?
51.	Dos:	twenty –four
52.	Teacher:	twenty – four. Well who can give me the answer of twelve there four times. The four twelve give what? The four twelve give what?
53.	Learner:	forty – eight
54.	Teacher:	is forty – eight. Do you know that these are the things you also get when you repeatedly subtract? Now let us take forty-eight, how many twelve can you take from forty – eight? How many twelve needed to be subtracted from forty –eight for forty – eight to be finished?
55.	Learner:	four times
56.	Teacher:	yes four times. Just as it is in division we got four as an answer, when subtracting is like using the addition sum from bottom up-ward, in forty –eight there are four twelve, if you take one twelve how many would remain?
57.	Learners:	three twelve
58.	Teacher:	the same as these twelve on the sum, which would give what as an answer?
59.	Learner:	thirty –six
60.	Teacher:	is thirty –six, now if I take out twelve again, how many twelve would remain?
61.	Learner:	two twelve
62.	Teacher:	yes and this is like taking the twelve from thirty – six and thirty – six has three twelve so if you take or subtract one twelve from them two twelve would remain. Two twelve is equal to what? Naf!
63.	Naf:	twenty – four
64.	Teacher:	you see is that difficult now? In twenty –four which only have two twelve you will again subtract twelve. What would remain?
65.	Learner:	twelve
66.	Teacher:	is twelve. Now if I take away that only twelve remained, what would be the answer?
67.	Learner:	zero
68.	Teacher:	is zero. Now how many twelve did I remove from forty – eight?
69.	Learner:	four

70.	Teacher:	four twelve were taken out from forty – eight and forty – eight got finished. And that is what I meant when I said you should understand the relationship of division to other operations such as addition, subtraction and multiplication, because if I ask twenty-four divided by twelve is what? As long as you know how many twelve are there in twenty- four, then you know the answer.
71.	Teacher:	(the teacher distributed the question papers and worksheets. She also gave the instructions for learners to write their names on the answer sheets and the question papers)
72.	Teacher:	can you see your question paper? The first question asks, what does the diagram show? Who can tell us what the diagram is showing? Fil!
73.	Fil:	is showing the dots
74.	Teacher:	ok, it is showing dots representing the stones. How many are they? Sab!
75.	Sab:	they are forty – eight
76.	Teacher:	the question says is that, forty – eight dots, eight dots are already taken out from the forty –eight dots and made a group. But I want you to take a pen and cancel them out because they are not cancelled out from the big box.
77.	Teacher:	are you done cancelling the eight dots?
78.	Learners:	yes
79.	Teacher:	the eight dots you have cancelled out are the one already taken out for you and put already in this group. Can you see this group?
80.	Learner:	yes
81.	Teacher:	now keep taking eight dots from the big box and make groups for each eight dots you take from here and make sure you cancel them out from the big box. Keep doing that until all the dots are done cancelled out. Do you get it?
82.	Learner:	yes
83.	Teacher:	ok only eight dots are in each group and cancel each dot you took out from the big box and make group for them until all the dots in the big box get finished.
84.	Teacher:	if you are done take your worksheet and label question two and keep repeatedly subtracting eight from forty – eight. See how many eight can be repeatedly subtracted from forty – eight for forty eight to get finished. Use the dots to help you too. Start with forty – eight because the dots are forty – eight and keep subtracting eight dots from the forty – eight dots until you reach zero and see/ find out how many eights would be subtracted from forty –eight for forty – eight to get finished.
85.	Learners:	(learners used the forty-eight dots drawn on the big box count out eight dots hold them and count the remaining for an answer and wrote the whole difference number sentence of 48 – 8 = 40. They again go back to the remaining dots the 40 dots count out eight hold them and count the remaining dots and wrote the whole difference number sentence of 40 – 8 = 32. Learners kept subtracting eight from the differences until the forty – eight dots got finished. Some learners make circles of eight and count the remaining one.)
86.	Teacher:	(telling one learner who seem to have confused the minus sign with division sign) I said minus, because we are taking away and not dividing and you can also use the dots and keep subtracting /taking away eight dots until dots get finished and tell how many eights you were able to take out from forty- eight.
87.	Teacher:	(asking another learner) do you know which eight dots are already taken away?
88.	Learner:	yes
89.	Teacher:	ok subtract again right dots from the remaining forty and see what the difference would be.
90.	Teacher:	are you done?
91.	Era:	yes
92.	Teacher:	how many eight did you take out?
93.	Era:	six
94.	Teacher:	they are six right? When you were removing the groups of eights at division, how many groups of eight dots did you made out?
95.	Era:	six

96.	Teacher:	six, ok why may be you got the same answer at both division and repeated subtraction you got six eights?
97.	Era:	is because all things were of equal amount. The same amount which is twenty four.
98.	Teacher:	there is no twenty – four in this question or what amount do you wanted to say
99.	Era:	is because forty-eight is the same in both cases of division and of repeated subtraction
100.	Teacher:	ok in both cases you were working with the same amount which is forty – eight and what could be anther fact which could have led to the same answer?
101.	Era:	there is equal amount in both forty – eight and in both methods I was taking the same amount of eight dots.
102.	Teacher:	and that is the reason you got the same answer rights?
103.	Era:	yes
104.	Teacher:	ok who else is done?
105.	Learner:	I am done
106.	Teacher:	how many eight did took out from forty – eight?
107.	Learner:	five
108.	Teacher:	ok, you took only five eights from forty- eight (after she investigated how the learner worked and found out the mistake she made. The mistake the learner made was at the step when she subtracted eight from twenty – four) tell me, if we subtract eight from twenty- four is the answer going to be four? Take the twenty – four dots and remove eight dots and see what would be the difference.
109.	Learner:	(counting the dots) one, two, three, four, five, six, seven, …….., twenty, twenty-one, ……., twenty –four.
110.	Teacher:	ok now take eight from that twenty –four and see what the difference
111.	Learner:	one, two, three, four, five, …………, eight (counting eight from twenty - four)
112.	Teacher:	ok now count the remaining one and see what the remaining one
113.	Learner:	(counting) one, two, three, four, five, six, seven (she stopped)
114.	Teacher:	why do you stop at seven, when the twenty – four dots were up to here? Count well again if this is twenty –four, take eight and count the difference
115.	Learner:	(counting again) one, two, three, four, five… twenty –four.
116.	Teacher:	ok then count out eight from the difference again
117.	Learner:	(counting eight from twenty -four) one, two, three, four, five… eight.
118.	Teacher:	ok count to see the difference then
119.	Learner:	one, two, three, four, five, six… sixteen.
120.	Teacher:	so the difference is sixteen but you have not written sixteen here rub this wrong answer and write the correct one.
121.	Teacher:	(asking other learners) how many eights did you subtracted?
122.	Learner:	(counting) one, two, three, four, five, six, they are six
123.	Teacher:	six ok (after she checked the learner's number sentences of which some have the wrong differences) is this supposed to be seventeen? And here when you got forty – three did you counted well? (Referring a child to the forty – eight dots) where is the first eight dots you subtracted? (After the learner had shown them) ok now count the difference one and see if they are what you have written.
124.	Learner:	one, two, three, four, five, …………….., twenty, …………., twenty-eight, ………….., forty
125.	Teacher:	is it forty that you have written here? Because I see you have just written forty – three. So rub the forty – three and write the correct answer. even here count four eights and remove them and see if the remaining will be seventeen.
126.	Learner:	(counting the remaining dots after she had made an indication of four eights out) one, two, three.
127.	Teacher:	no! now you are re-counting even the dots of eight you've already subtracted. You only supposed to count the remaining one.
128.	Learner:	(counting again but this time the correct remaining one) one, two, three, four, ……, sixteen.
129.	Teacher:	how many did you counted?
130.	Learner:	sixteen

131. Teacher: but here you wrote seventeen. Count your things very well.
132. Teacher: who else is done?
133. Sab: I am done
134. Teacher: how many eights did you repeatedly subtracted out?
135. Sab: six
136. Teacher: and how many groups did you made with dots when you divided?
137. Sab: six
138. Teacher: now the question I want to ask is this. Why may be you got the same answer as six at both methods of division and that of repeatedly subtraction?
139. Sab: is because we were dividing forty – eight dots and even here we were subtracting from forty – eight.
140. Teacher: apart from using the same number of forty – eight at both sides what else could be the fact why you got the same answer?
141. Sab: is because there are six eights on forty –eight,
142. Teacher: is because there are only six eights in forty – eight, ok how did you have the answer?
143. Sab: I took forty – eight dots and subtracted eight and I counted the remaining dots and that gives me forty. I subtracted eight dots again from the forty the difference and counted the difference which is thirty – two and wrote the answer.
144. Teacher: meaning you kept repeatedly subtracting eight from forty – eight until forty – eight got finished? Is that so?
145. Sab: yes
146. Teacher: how did you do it at division with dots?
147. Sab: I have been repeatedly taking eight dots from the forty-eight until all the dots get finished.
148. Teacher: now what do you think is common in both processes?
149. Sab: the common thing is that in forty - eight there are just six eights
150. Teacher: ok is because there are just six eights in forty – eight. Even if you divided or through repeated subtraction you will only be able to take six eight right?
151. Sab: yes
152. Teacher: what makes you think that your answers are just correct and not wrong? They are correct right?
153. Sab: yes
154. Teacher: forty-eight divided by eight gave you six rights? And in forty – eight you only repeatedly subtracted six eights is that so?
155. Sab: yes
156. Teacher: then what makes you think that your answers are just correct and that you are not wrong at all?
157. Sab: is because when I took out eight dots from the big box, dots get finished.
158. Teacher: ok, is because six eights dots removed from forty – eight dots makes forty –eight dots to get finished
159. Teacher: you can now write the number sentence of what you were just dividing below question three. Era! You also need to write the number sentence below question three.
160. Teacher: Naf! How many groups did you get?
161. Naf: seven
162. Teacher: seven! Are all of your groups having eight dots? ok count these one here (counting the dots in eights) one, two, three, four, five, six because here I see you have indicated six eights, where did you find the seventh group? or is this not the eight dots you put into groups?
163. Naf: they are
164. Teacher: ok (pointing to the different eight dots in order and to their groups out side the box again) this group is for the first eight dots and you put them in this group is that so?
165. Naf: yes
166. Teacher: we also have these eight dots for the second group and these eight dots for the third group (the teacher continue pointing to groups until the sixth and she asked) where did you get the eight dots for the seventh group? Correct the error.

167.	Teacher:	if you are done. How many apples are these in the next question?
168.	Learner:	eight
169.	Teacher:	they are eight rights?
170.	Learner:	yes
171.	Teacher:	the instruction is, in eight apples keep subtracting four apples until the eight apples get finished.
172.	Teacher:	(asking other learners) how many of you are done? Ok, (asking one of the learners) how many eights did you take out from the drawing of forty – eight when you did the repeated subtraction?
173.	Hil:	eight
174.	Teacher:	eight, and here at the division drawing how many eight did you take out?
175.	Hil:	two
176.	Teacher:	look at the drawing of groups. How many eights are they?
177.	Hil:	they are six
178.	Teacher:	then if here they are six, where did you find the other two eight more on the other side? Did you count well? Ok now I want you to make eight dots in circles, in circles, in circles, to see if the eights will be eight as you got here and if they are not eight find out where you go wrong here. How many eights did you take out?
179.	Hil:	six
180.	Teacher:	six, ok, whoever is done now come to my table with your papers so that I can ask you together. Dos! How many eights did you take out from forty – eight for it to get finished?
181.	Dos:	six
182.	Teacher:	six, and when you made groups of eights in division, how many groups of eight were you able to make out from forty- eight?
183.	Dos:	six
184.	Teacher:	six, ok why may be in both methods when you subtracted eights from forty-eight and when you divided forty –eight dots in groups of eights you got the same answer as “six” eights or six groups of eight dots? Why the same answer?
185.	Dos:	is because I divided making circles and allocated in circles and in a big circle I took eight dots which I cancelled out and put into one circle.
186.	Teacher:	I did not ask you to describe how you divided, but asked that when you divided forty – eight drawing of stones this one can you see them?
187.	Dos:	yes
188.	Teacher:	you have been taking out eight stones and make groups for them is that so?
189.	Dos:	yes
190.	Teacher:	and you made six groups of eight drawing of stones each that you removed from the big box until the stones in the big box have been allocated in groups, is that so?
191.	Dos:	yes
192.	Teacher:	ok, even when you repeatedly subtracted eights from forty –eight, you also had subtracted how many eights?
193.	Dos:	six
194.	Teacher:	six, so the question is why you did have to get six the same answer in both methods? Six here at division and six there at repeated subtraction?
195.	Dos:	is because all number are the same
196.	Teacher:	which numbers are the same?
197.	Dos:	the forty – eight
198.	Teacher:	so when we divided we it was forty – eight we divided and when we subtracted it was also from forty – eight. What else do you think had led to this sameness of this answer apart from forty –eight?
199.	Teacher:	(she faced to the other learners who are at the table) all of you who are at this table write below question three the number sentence of what you were dividing.
200.	Teacher:	(facing back to Dos) apart from forty – eight being the same number used at both methods of division and repeated subtraction, what else could have led to this sameness of the answer? How many stones were you allocating in each group here?

201. Dos: eight
202. Teacher: and how many were you subtracting (what number were you subtracting) from forty – eight?
203. Dos: is eight
204. Teacher: ok what is that then on both sides that is common and can still lead to the same answer on both methods?
205. Dos: is because I was subtracting
206. Teacher: subtracting what?
207. Dos: from forty – eight stones
208. Teacher: yes, you have already mentioned about the forty –eight, but what else about it that can make you? What else that you were doing here there that could have led to the same answer?
209. Dos: is because there are only six eights in forty – eight
210. Teacher: is because there are only six eights in forty – eight rights?
211. Dos: yes
212. Teacher: does that mean that is the reason why you got the same answer? is it because you were subtracting and taking out the same number which is eight rights?
213. Dos: yes
214. Teacher: ok, good, now you can go and do the next question. Question five the next question says one can build eight with the fours, repeatedly added together. Now in the box with eight apples keep taking out four apples and put them in the box. Each four apples you take out must be cancelled out of the eight apples box.
215. Teacher: come back all of you who were here. (Asking one learner) how many eights did you take out?
216. Learner: they are six
217. Teacher: (after she realized that the number sentence of one learner is not well written then she asked the group of learners at the table and said) can you see this drawing of stones?
218. Learners: yes
219. Teacher: when you were drawing, were you subtracting from them or were you dividing?
220. Learners: dividing
221. Teacher: ok, so I expected one to write their number sentence as forty-eight divided which is division sign and the quotient at the end right? But that is not what I see from some people's works (she asked one learner) what is this is it six?
222. Learner: yes
223. Teacher: but then your sign is minus, when we were not subtracting but dividing. There supposed to be two dots on the line sign if you want to use it as a division sign. The division line sign has two dots (÷) and not (-)
224. Teacher: (telling the whole group) come here all of you. Let me give you the instruction for the next question, question five, So that you will be questioned later. Can you all see these eight apples?
225. Fil: yes
226. Teacher: I want you to divide them for me into these two groups. Each group should have four apples. So everyone go and do the division of apples, while I am asking your colleagues. You will remain behind because I am asking you.
227. Teacher: (asking the learner) let me ask you, when you divided these stones, taking out the groups of eights. How many groups of eights were you able to take out?
228. Fil: six
229. Teacher: six, ok and here you also subtracted how many eights from forty – eight?
230. Fil: six
231. Teacher: six ok, now the question I want to ask you is this, when you divided you were able to take out six groups of eight even when you subtracted you were able to subtract away six eight from forty- eight. Why may be did you got the same answer, from both methods you got the same answer as six groups of eight or six eights subtracted?
232. Fil: is because I was taking the stones from forty – eight stones taking out eight stones and count out the one that is left.

233.	Teacher:	what do you mean you counted out what was left, I am asking what is that makes you have the same answer, that when you divided you got six groups of eights and when you subtracted you got six eights? Why may be you got the same answer as six?
234.	Fil:	is because I divided by taking out eights and counted the one that remained.
235.	Teacher:	ok, how did you divided here?
236.	Fil:	I kept taking eight stones and make groups for them and cancel them out from the big box. I kept taking out eight stones until the stones got finished in the big box.
237.	Teacher:	until they got finished, ok how did you do it at the subtraction method
238.	Fil:	here I used the forty- eight drawing of stones. I took out eight and there was stones remaining.
239.	Teacher:	after taking eight stones out what did you do next?
240.	Fil:	when I took eight out I counted the remaining one.
241.	Teacher:	you counted the remaining one and do what next?
242.	Fil:	and write the answer
243.	Teacher:	ok, when you have written your answer?
244.	Fil:	after I have written the answer I took out eight again and counted the remaining one and write the answer
245.	Teacher:	ok
246.	Fil:	and come back here at the big box and take out eight stones again and count the remaining and write the answer again, I kept doing that until I reach zero.
247.	Teacher:	ok, do you think your answers are correct?
248.	Fil:	yes
249.	Teacher:	why do you think your answers are correct?
250.	Fil:	is because I was taking out eight stones and write the answer
251.	Teacher:	ok, you mean you have been taking out eight and write the answer, what makes you think that your answer is correct?
252.	Fil:	is because I was taking out eight stones and write the answer.
253.	Teacher:	that is the question I am asking. When you write answer where did you got it? Because you said you took eight from forty – eight and write the answers from where? Is there anything else you did to get your answer?
254.	Fil:	no
255.	Teacher:	what?
256.	Fil:	I took the forty –eight drawing of stones and took out eight and there was a remaining and write the answer
257.	Teacher:	ok, what makes you think that you are correct at the division side?
258.	Fil:	is because I was cancelling out eight and put them in their groups and I cancelled eight stones and put them in their group
259.	Teacher:	ok, I want you to divide these eight apples in groups of four apples. Each four apples you took out must be cancelled from the other side (telling the whole class) I want one who is not asked to come here.
260.	Teacher:	(asking Naf) ok, how many eights did you took out from forty – eight you can use the subtraction and count the eights subtracted or the division and count the number of groups
261.	Naf:	(counting them) one, two, three, four, five, six. They are six
262.	Teacher:	and when you divided how many groups of eight you made out of those stones?
263.	Naf:	(counting again the groups this time) one, two, three, four, five, six. They are six.
264.	Teacher:	they are six, ok the next question that I have is this, when divided forty – eight stones you got six groups of eight and even when you repeatedly subtracted eights from forty – eight you get six eights from forty – eight. So from both methods it is six eights, why may be did you get the same answer?
265.	Naf:	is because of the eights, the eights
266.	Teacher:	what about the eights?
267.	Naf:	is because the eights, from forty – eight stones I took eight stones out.
268.	Teacher:	so did you only take eight from forty-eight stones or what do you mean? What do you want to say?

269.	Naf:	I took out eight stones from forty –eight and write the answer
270.	Teacher:	do you understand my question? What I meant or wanted to say is that, you can see this forty – eight you started with when you repeatedly subtracted?
271.	Naf:	yes
272.	Teacher:	and these drawing of stones how many are they?
273.	Naf:	they are forty- eight
274.	Teacher:	why is then in this forty-eight when you repeatedly took out groups of eight and when you reached the sixth group forty – eight get finished and even in the forty – eight when you repeatedly subtracted the eight when you repeatedly subtracted eights six times the forty – eight get finished. Why may be six eights on both sides? Why do you have to get the same answer?
275.	Naf:	is because the stones get finished
276.	Teacher:	Naf! I know these stones get finished but my question is why you have to get the same answer at both sides of methods. When you divided you got six groups of eight and when you repeatedly subtracted you get six eights from forty – eight. Why are the answers the same at both sides? (Pause) don't you know why? How many did you say the drawing of stones was?
277.	Naf:	forty – eight
278.	Teacher:	and what number did you repeatedly take eight out?
279.	Naf:	forty – eight
280.	Teacher:	so it was all along forty – eight at both sides on this side it was forty – eight drawing of stones minus groups of eight right?
281.	Naf:	yes
282.	Teacher:	and at the other side it was forty – eight that you repeatedly subtract eight right?
283.	Naf:	yes
284.	Teacher:	ok then my question is why may be you get the same answer at both sides here at division it is six groups of eight and even there at subtraction it was six eight that you repeatedly subtracted so it is all along six eights from both side. Why then at division it is six groups of eights and at subtraction it is also six eights subtracted? What do you think led to this sameness of the answer? That is what I am asking (pause) tell me so that we can continue to the next question.
285.	Naf:	I do not know why.
286.	Teacher:	you do not know ok how did you divided?
287.	Naf:	I have been taking eight stones from the drawing and put them in a circle
288.	Teacher:	ok, tell me how you did it at subtraction?
289.	Naf:	I have been subtracting eights, like this, I took eight from forty – eight and count the remaining, which was forty and subtracted eight from forty and thirty –two remained and I took eight from thirty – two and twenty – four remained
290.	Teacher:	how did you do it? Did you use the drawing of stones
291.	Naf:	yes
292.	Teacher:	how did you use the drawing of stones?
293.	Naf:	I have been taking the eights out
294.	Teacher:	how? How did you do it?
295.	Naf:	I cancel them out
296.	Teacher:	ok, you cancel eight out and counted the remaining is that so?
297.	Naf:	yes
298.	Teacher:	After counting the remaining you then write the difference right?
299.	Naf:	yes
300.	Teacher:	how did you divide here?
301.	Naf:	I have been counting out the eights dots cancel them and make groups for them and count out another eight make a group for them and cancel them out from the big box. I have been doing that until all stones get finished.
302.	Teacher:	ok, until they got finished, what do you think is common in this division and this subtraction
303.	Naf:	the common thing is that the answer are similar

304.	Teacher:	ok is because the answer are similar right? Which parts in them are or what numbers in them are also similar?
305.	Naf:	the six
306.	Teacher:	how are the six similar? Can you indentify the numbers that are similar at both sides?
307.	Naf:	is because I got six eights at both sides
308.	Teacher:	ok why do you think it led to the similar answer?
309.	Naf:	is because the things are equal.
310.	Teacher:	which things are equal?
311.	Naf:	are the eights
312.	Teacher:	you mean the eights you have been taking out are equal right? At division it has been groups of eight and at subtraction it was also eight being repeatedly subtracted right?
313.	Naf:	yes
314.	Teacher:	what else make the answer to be equal or similar? (after a long pause) what is that number you have been dividing which is the same as this number you were repeatedly subtracting from?
315.	Naf:	forty – eight
316.	Teacher:	ok, meaning apart from taking out eights which is the same from both side, what else is the same at both sides?
317.	Teacher:	do you think your answers are correct?
318.	Naf:	yes
319.	Teacher:	why do you think your answers are correct?
320.	Naf:	is because all think are equal
321.	Teacher:	which things are equal?
322.	Naf:	the stones in groups
323.	Teacher:	what else made you sure that you are correct? What if groups could be seven and not six? What makes you think that six groups are the correct one?
324.	Naf:	is because all groups have equal stones and all the stones I was dividing got finished
325.	Teacher:	ok, the stones you were dividing got finished rights? Ok, that is good.
326.	Teacher:	(talking to the whole class) all those who are done with the question of the apples can you see number six?
327.	Learners:	yes
328.	Teacher:	copy it on the other worksheet. Label the work as number six and start with only one four until you reach the number of fours that can give you eight. When you divided eight apples, how many fours were you able to take out? Sab!
329.	Sab:	two
330.	Teacher:	that means eight can be built with how many fours?
331.	Sab:	two fours
332.	Teacher:	then keep adding fours until you reach eight. Then again write at number seven the number sentence of what you were dividing. Nab! Are you listening to me? What did I say you should write at blank clean paper? You see you did not listen very well. I said do number six on a blank paper worksheet and label your work as number six. Who can repeat what I have said you do at number six? Era!
333.	Era:	take the paper and label it six
334.	Teacher:	label it six and do what?
335.	Era:	write four
336.	Teacher:	write four just one four? I said build with fours to make what? Hil!
337.	Hil:	eight
338.	Teacher:	eight, starting with one four until the fours makes eight and that you should write it at the other blank paper. Nab! And at question seven I said you write a number sentence of how you divided eight apples.
339.	Learner:	I am done
340.	Teacher:	ok, those who are done, continue with the next question, question nine, count for me how many two are these and fill the answer at the box of question ten, that how many twos you were able to take out
341.	Learner:	I am done

342.	Teacher:	if you are done write at/ under question eleven the number sentence of what you were doing.
343.	Teacher:	Mat, when you divided forty – eight drawing of stones, how many groups of eight stones were you able to make?
344.	Mat:	six groups
345.	Teacher:	each group has how many stones?
346.	Mat:	eight stones
347.	Teacher:	eight so you have been taking eight stones from forty – eight stones until the groups of eight becomes (counting) one, two, three, four, five, six groups from forty – eight stones
348.	Mat:	yes
349.	Teacher:	how many groups did you say you were able to make? how many eights did you take out to finish them.
350.	Mat:	six
351.	Teacher:	six ok the question I have for you is this, when you subtracted the eight from forty – eight, you subtracted six of them for forty – eight to get finished right?
352.	Mat:	yes
353.	Teacher:	even in the forty – eight stones you were able to take out six groups of eight stones. Why may be you take out six the similar answer from both sides? (Pause) why may be you get six here and at the other side too? Mat! How many stones were these you said?
354.	Mat:	forty –eight
355.	Teacher:	forty – eight, ok how many eights you were able to take out you said?
356.	Mat:	six
357.	Teacher:	and this side you were subtracting from number forty - eight right?
358.	Mat:	yes
359.	Teacher:	how many eight were you able to subtract from forty –eight for forty – eight to get finished?
360.	Mat:	six
361.	Teacher:	that is why I asked why did you get the similar answer six here and there?
362.	Mat:	is because I counted stones here and there I also used the stones
363.	Teacher:	does that mean you did all the works of division and subtraction using this stones?
364.	Mat:	yes
365.	Teacher:	and it was the same stones you used when subtracting right?
366.	Mat:	yes
367.	Teacher:	and that makes you get the same answer rights?
368.	Mat:	yes
369.	Teacher:	why did you use stones when subtracting? (Pause) subtracted what makes you use the drawing of stones? (Pause) ok, how did you divided or how did you come up with the groups?
370.	Mat:	I started counting out eight stones form the drawing, cancel them out and make groups for them I kept doing that until all the drawing of stones got finished
371.	Teacher:	ok how did you do it here at subtraction side? Because you said you have used the stones here too. How did you do it?
372.	Mat:	I used this forty – eight drawing of stones I kept removing eight stones and counted the remaining one and write the difference. I kept doing that until all stones get finished.
373.	Teacher:	do you think your answers are correct?
374.	Mat:	yes
375.	Teacher:	so only six eights are to be subtracted out of forty – eight rights?
376.	Mat:	yes
377.	Teacher:	even when you divide forty- eight stones only six groups of eight stones you could get right?
378.	Mat:	yes

379.	Teacher:	what makes you think that your answers are just correct and not wrong? (Pause) what if you would have taken seven groups of eight from forty – eight? (Pause) can you or are you able to take nine groups of eight from forty – eight?
380.	Mat:	no
381.	Teacher:	then what makes you so sure that six groups are the only correct one?
382.	Mat:	it is because the stones would not be enough for eight or seven or nine groups of eight stones to be made from forty – eight stones
383.	Teacher:	so the stones were only enough to make six groups right?
384.	Mat:	yes
385.	Teacher:	then they are only enough for what?
386.	Mat:	they are only enough for six groups of eight stones
387.	Teacher:	so they were only enough to make six groups of eight rights?
388.	Mat:	yes
389.	Teacher:	what happened when you took out six groups of eight stones? Were there some stones left or what? (pause)
390.	Mat:	there would some stone left
391.	Teacher:	ok, if there would be some stone left for them to make a seventh group, if there was some stones left (pause) my question is you were using this forty- eight drawing of stones, where you had been taking out groups of eight stones rights?
392.	Mat:	yes
393.	Teacher:	are you saying after you were done making six groups, that you took out from forty-eight stones, that there were any stones left?
394.	Mat:	no
395.	Teacher:	so the answer is there was nothing or no stones left
396.	Teacher:	(asking the whole class) how many apples did each group has?
397.	Learners:	four apples
398.	Teacher:	you put four apples in each group rights?
399.	Learners:	yes
400.	Teacher:	have you done with question six?
401.	Learners:	yes
402.	Teacher:	I can see Hil, has not done it please Hil do it and label it, so (asking the whole class) you have written the fours rights?
403.	Learners:	yes
404.	Teacher:	did you get the number of fours that can make up eight?
405.	Learners:	yes
406.	Teacher:	if you are done that means you have also written below / under question seven, the number sentence of what you were dividing?
407.	Learner:	yes
408.	Teacher:	it you have done it have you also divided sweets into groups?
409.	Learner:	yes
410.	Teacher:	have you cancelled the sweets you have removed out?
411.	Learner:	yes
412.	Teacher:	you got the number of groups you were able to make rights?
413.	Learner:	yes
414.	Teacher:	have you written the answer of groups in the box provided?
415.	Learner:	yes
416.	Teacher:	if you have done it, how many twos are there in fourteen?
417.	Learner:	seven twos
418.	Teacher:	ok, write it then as a answer on your paper and when you are done with it write the number sentence below question eleven
419.	Learner:	yes
420.	Teacher:	ok, when you are done with that keep your worksheet because I would ask you some questions about your work late.
421.	Teacher:	ok, Mic come, Mic I want to know how many stones were this altogether?
422.	Mic:	forty – eight

423.	Teacher:	forty – eight rights? Ok, how many stones have you been taking out (pointing to the drawing of stones) how many stones are these?
424.	Mic:	eight
425.	Teacher:	eight, ok so in forty – eight stones, you have been taking out eight stones. How many times did you take these groups of eight stones?
426.	Mic:	six
427.	Teacher:	six, ok now let us come to the subtraction method, how many eights did you subtracted from forty – eight for forty – eight to get finished?
428.	Mic:	they are six
429.	Teacher:	they are six rights? Does that mean from number forty –eight one can only take six eights for the forty – eight to get finished? And from forty – eight stones one would only be able to take six groups of eight stones? why do you think you got the same answer of six eights at both sides of methods
430.	Mic:	is because forty – eight get finished
431.	Teacher:	forty – eight get finished, but my question is why did you from subtraction side you subtracted six eights and even at division it was six groups of eight stones, why may be six eights at both sides?
432.	Mic:	is because I was using the same number when dividing and when subtracting.
433.	Teacher:	which number is that?
434.	Mic:	is forty – eight
435.	Teacher:	ok, apart from that number forty – eight, what else could have led to the same answer? Apart from that number forty – eight, what else could have led to the same answer?
436.	Mic:	is because the stones get finished
437.	Teacher:	so when you removed six groups of stones from forty –eight stones the forty –eight stones get finished. The question is what else may be similar on both sides of division and subtraction that could have led to the same answer? Because you said in both methods you have used forty – eight, thus it led to the similarities in their answers. So what else is similar at both sides that could have led to these similarities of their answer?
438.	Mic:	is because these numbers I was taking out in groups which is eight is also the same as the one I was subtracting from forty – eight.
439.	Teacher:	ok, so is because you were taking out eight from forty - eight which is also the same as eight we were repeatedly subtracting from forty – eight
440.	Teacher:	how did you divided at the division method?
441.	Mic:	I had been taking eight stones from the drawing of forty – eight stones making groups for them and cancel them out from forty –eight stones until all the forty – eight stones are cancelled out and are allocated into groups.
442.	Teacher:	how did you do it at subtraction method?
443.	Mic:	I subtracted eight from forty – eight and got forty as a difference
444.	Teacher:	ok, you removed eight from forty – eight which forty – eight? Or where is that forty – eight? Did you use the stones? Or which forty- eight are you referring to?
445.	Mic:	this one (pointing to the drawing of forty – eight stones)
446.	Teacher:	ok, how did you do it?
447.	Mic:	first I took forty – eight drawing of stones and subtracted eight
448.	Teacher:	and how did you do it with stones? Because you said you used the stones to get the answers?
449.	Mic:	I counted out eight drawn stones
450.	Teacher:	ok, you counted out eight and put them aside
451.	Mic:	I then counted the remaining one and they were forty and wrote the answer. I took eight again from forty and thirty – two remained and I took eight from thirty – two and twenty – four remained and I wrote the difference. I took eight again from twenty – four and sixteen remained and I wrote the difference. I took eight again from sixteen and eight remained and I wrote the difference. I took eight again from eight and zero remained and I wrote the difference.

452. Teacher: ok, so that is what you have done. Ok do you think your answers are correct? That there is only six eight in forty – eight?
453. Mic: yes
454. Teacher: what makes you think that your answers are correct and not wrong?
455. Mic: is because I counted until I reach zero
456. Teacher: can you see you work with apples? How many apples each group have?
457. Mic: four
458. Teacher: four each, now if you repeatedly add the fours, how much will it be?
459. Mic: eight
460. Teacher: now in division you got two groups of apples and when you repeatedly add how many fours gave you eight?
461. Mic: two fours
462. Teacher: now my question is, when you added, you added two fours to get eight, even when you divided it was two groups of fours that came out of the eight. Why do you think you got the same answer of two fours at both sides?
463. Mic: is because I was dividing apples?
464. Teacher: dividing apples ok, what makes the answers similar? The two groups of four at division and the two fours at repeated addition?
465. Mic: it is because it is the fours I added four and four and even here at division it was the fours I was taking out the four and four in two groups.
466. Teacher: meaning when you added the two four they gave you eight rights?
467. Mic: yes
468. Teacher: and even in eight it was two fours that made eight. Ok two fours give or make eight rights?
469. Mic: yes
470. Teacher: and this is why you got the similar answers rights?
471. Mic: yes
472. Teacher: how did you do it at addition?
473. Mic: I counted two fours
474. Teacher: how?
475. Mic: I used my fingers and counted the fours until I got eight.
476. Teacher: have you been counting adding the fours together or how did you counted?
477. Mic: I took four and four fingers and counted them together and get eight
478. Teacher: so you found out that there are only four fours in eight right?
479. Mic: no they are two
480. Teacher: so what makes you think that your answer is correct?
481. Mic: when I took out two four apples, the eight apples get finished.
482. Teacher: so the apples get finished, what else made you sure that your answer is correct?
483. Mic: is because all groups had equal number of apples
484. Teacher: Hil! How many stones were this altogether?
485. Hil: they were forty – eight
486. Teacher: in forty – eight you were taking out the groups of how many stones?
487. Hil: is groups of eight
488. Teacher: and how many groups of eights you took out?
489. Hil: six
490. Teacher: you were also subtracting, how many eights were you able to subtract out?
491. Hil: six
492. Teacher: the question I want to ask you is this, these stones were forty – eight and you took out six groups of eights and even when you subtracted eight from forty – eight you subtracted six eights why similar answer at both method?
493. Hil: is because I worked with forty – eight at both methods
494. Teacher: ok, is because you worked with the same number at both side, what else made you to have similar answers?
495. Hil: there is six and eight at both sides of the methods
496. Teacher: what do you mean that there is six and eight at both sides?

497. Hil: I had been taking out/ subtracting eight at both sides
498. Teacher: how did you divided here?
499. Hil: I cancelled eight stones and put them in the groups
500. Teacher: and here how did you added
501. Hil: I kept taking eight stones from forty – eight and count the remaining for an answer and record it
502. Teacher: are your answers correct?
503. Hil: yes
504. Teacher: what makes you sure that you are correct?
505. Hil: because the stones get finished
506. Teacher: on question two you divided eight into two groups of four and when you added it was how many fours?
507. Hil: two fours
508. Teacher: why may be you get similar answers?
509. Hil: is because we worked with similar numbers which is fours.
510. Teacher: how did you add?
511. Hil: I took one four and another four and added them together.

LESSON TWELVE

1. Teacher: what you are going to do is this, in the previous lesson we were using drawn pictures. Now we are going to use dots to divide, take a blank paper and make thirty dots, those thirty dots represent thirty pens. How many dots did I say draw?
2. Learner: thirty
3. Teacher: after you draw thirty dots, keep taking out six (groups of six dots) like that it is like taking out how many pens?
4. Learners: six pens
5. Teacher: each six dots you removed makes circles for them and that represent a person receiving six pens. Each circle is a person receiving six pens. Keep giving six pens (by removing six dots and put them in their circles) after that you will tell me the number of people that would be able to get six pens are we together?
6. Learner: yes
7. Teacher: so take dots, one two three four five six and put them in the circle and you continue one two three four five six so you take another six dots and make a circle for them, keep doing that until thirty dots get finished. Do you understand?
8. Learner: yes
9. Teacher: ok
10. Teacher: (after a long pause) are you done?
11. Learner: yes
12. Teacher: if you are done go to question two, did you get the number of people that can be given the pens?
13. Learner: yes
14. Teacher: each person got six pens rights?
15. Learner: yes
16. Teacher: So continue with question two and write a number sentence of what you were dividing write that under question two on the question paper.
17. Teacher: if you are done make another sixty dots and keep taking out fifteen dots and put them in the rectangle boxes, label the work number three. After you are done tell how many groups of fifteen you made.
18. Teacher: are you done with the sixty dots?
19. Learner: yes
20. Teacher: you have removed the fifteen dots?
21. Learner: yes

22.	Teacher:	have you written down question four, the number sentence of what you were dividing?
23.	Learner:	no
24.	Teacher:	ok do it.
25.	Teacher:	Nab! I want to ask you, at multiplication, fourteen divided by two gives you seven. Do you think this will help you get the division answer? What are the similarities that would help you in both multiplication and in division?
26.	Nab:	is because seven in fourteen is there two times.
27.	Teacher:	and how many twos are there in fourteen?
28.	Nab:	seven
29.	Teacher:	meaning if you are to divide fourteen with two you will get what?
30.	Nab:	seven
31.	Teacher:	and if you are to divide fourteen by seven what would you get?
32.	Nab:	two
33.	Teacher:	can you see the relation? If you are given a multiplication work, do you think you can reverse it to division? (pause)
34.	Teacher:	(talking to the whole class) if you are done draw fifteen dots and label the work as number five
35.	Teacher:	(back to Nab) ok, let us go to the first question. How many eights did you take out from forty – eight?
36.	Nab:	six
37.	Teacher:	and when you subtracted how many eights did you subtracted out of forty – eight?
38.	Nab:	six
39.	Teacher:	so from forty – eight you subtracted six eight and from forty – eight drawing of stones you also got out six groups of eight. At both side you have six eights. Why do you think you got the similar answer? (A long pause) hello! Why may be you did get the similar answers? (Long pause).
40.	Nab:	is because forty – eight divided by eight will give forty
41.	Teacher:	what?
42.	Nab:	forty
43.	Teacher:	in this forty – eight divided by eight you will still get forty?
44.	Nab:	no
45.	Teacher:	but that is what you are saying, that if forty- eight is divided by eight you will get forty, or do you mean forty – eight minus eight? Can you repeat what you want to say?
46.	Nab:	is because in forty – eight there are only mhhh … eights (pause)
47.	Teacher:	how many eights? Finish your answer.
48.	Nab;	six of them
49.	Teacher:	do you think your answers are correct?
50.	Nab:	yes
51.	Teacher:	what makes you think that you are correct? (pause) (When the teacher could not get the answer from him) ok, you can go and sit down.
52.	Teacher:	are you all done?
53.	Learners:	yes
54.	Teacher:	you have drawn thirty dots in question one right?
55.	Learners:	yes
56.	Teacher:	and you have divided them into groups of six rights?
57.	Learners:	yes
58.	Teacher:	have you written the number sentence under question two?
59.	Learners:	yes
60.	Teacher:	have you done question three of sixty dots are you done with them?
61.	Learners:	yes
62.	Teacher:	you have been taking the fifteen dots until sixty get finished right?
63.	Learner:	yes
64.	Teacher:	have you written the number sentence of what you were dividing at question four?

65.	Learners:	yes
66.	Teacher:	are you done with question five?
67.	Learner:	yes
68.	Teacher:	how many dots you had made?
69.	Learners:	fifteen dots
70.	Teacher:	ok, divide fifteen into groups of threes, when you finish write rights?
71.	Teacher:	Sab! Looking at your work you have used division on your number sentence, why opted for a division number sentence?
72.	Sab:	is because we were dividing thirty into six circles.
73.	Teacher:	were circles six?
74.	Sab:	circles of six
75.	Teacher:	so you mean in all questions you were dividing that is why you have opted for division sign on your number sentence right? What answer did you get on the first question? How many people got six pens out of thirty?
76.	Sab:	five people
77.	Teacher:	and in question three how many packed of fifteen cakes you packed out from sixty?
78.	Sab:	four packets
79.	Teacher:	four packets, in question five how many sticks did you get in each groups? I mean how many groups did you make each with five sticks?
80.	Sab:	three groups
81.	Teacher:	how did you divided?
82.	Sab:	in the first question I made thirty dots and keep cancelling six dots and makes groups for them.
83.	Teacher:	here how many have you been counting out?
84.	Sab:	groups of fifteen dots
85.	Teacher:	ok, in question five, how many have you been counting out
86.	Sab:	is five
87.	Teacher:	ok, until all got finished rights?
88.	Sab:	yes
89.	Teacher:	what makes you think that your answers are correct?
90.	Sab:	is because all groups, the packets and people shared equal things.
91.	Teacher:	what else ensure you that your answers must be just correct?
92.	Sab:	is because the things I was dividing got finished after the division.
93.	Teacher:	write for me three examples of division here as a complete number sentence. Era!
94.	Teacher:	what makes you opt for the division sign when you translated your work to the symbolic number sentence?
95.	Era:	is because we were dividing.
96.	Teacher:	ok, you were dividing rights, ok how did you divided your works then?
97.	Era:	in question one, I have been taking out six, six, six until the thirty dots get finished.
98.	Teacher:	ok, in question three?
99.	Era:	I have been taking out fifteen, fifteen until sixty dots get finished.
100.	Teacher:	and in question five
101.	Era:	I have been taking out five dots, five dots until fifteen get finished
102.	Teacher:	in question one, how many people get the pens?
103.	Era:	five
104.	Teacher:	but your circles looks owe! They are six, or is this one cancelled?
105.	Era:	they are five. This one is cancelled.
106.	Teacher:	ok, rub it very well. How many packets of cakes did you packed out from sixty?
107.	Era:	four
108.	Teacher:	ok, do you think that your answers are correct?
109.	Era:	yes
110.	Teacher:	what makes you think that your answers are correct?
111.	Era:	is because all groups has equal portions
112.	Teacher:	what else ensuring you that your answers are correct?
113.	Era:	the things I was dividing get finished.

114.	Teacher:	they get finished right ok, go and write for me three examples here of division in a complete number sentence.
115.	Teacher:	you have used division signs when asked to translate your work to a symbolic number sentence why?
116.	Learner:	is because I was dividing.
117.	Teacher:	so is because in these works you were dividing rights? How many people got pens here?
118.	Learner:	five
119.	Teacher:	five, ok how many packets of cakes were you able to pack out from sixty? Each with fifteen cakes?
120.	Learner:	four
121.	Teacher:	four, how many groups were you able to groups your fifteen dots each with five dots?
122.	Learner:	three groups
123.	Teacher:	three, ok, how did you divide? How did you divide?
124.	Learner:	here, I cancelled out fifteens and makes groups for them I keep doing that until sixty dots get finished.
125.	Teacher:	so is that how you divided in all the questions, taking a number of dots and makes groups for them?
126.	Learner:	yes
127.	Teacher:	do you think your answers are correct?
128.	Learner:	yes
129.	Teacher:	what makes you think that your answers are correct?
130.	Learner:	is because I was dividing thirty I cancel and six and put in one group and cancelled out six again make a group for them until dots get finished
131.	Teacher:	you mean you have been counting out six dots, cancel them out and make group for them until the thirty dots get finished right? Have you done the same to all works?
132.	Learner:	yes
133.	Teacher:	ok, go and write for me the division examples here in a complete number sentence.
134.	Teacher:	Naf! You have used the division sign on your number sentence when asked to translate your work to a symbolic number sentence, why?
135.	Naf:	is because I was dividing.
136.	Teacher:	so is because you were dividing, what were you dividing?
137.	Naf:	its dots dividing them into circles
138.	Teacher:	ok, how many people were you able to give six pens each?
139.	Naf:	five people
140.	Teacher:	five, ok the cakes you were packing fifteens in each packet, how many packets were you able to pack out from sixty?
141.	Naf:	four packets
142.	Teacher:	ok, here you were dividing dots five in a group, how many groups of sticks were you able to make?
143.	Naf:	three
144.	Teacher:	three, ok, are your answers correct?
145.	Naf:	yes
146.	Teacher:	what makes you think that your answers are correct?
147.	Naf:	is because the dots I was dividing get finished.
148.	Teacher:	ok, where you were taking the dots when you were dividing, the dots get finished rights? What else makes you sure that your answers are just correct?
149.	Naf:	is because the groups had been equally allocated or have equal portion.
150.	Teacher:	ok, is because all groups have equal distribution. Ok go to the table and write three examples of division in a complete number sentence. Mic! Do not come anymore some else is already here
151.	Teacher:	the question I have for you is, I see you have used the division sign in your work when asked to translate your work to symbolic number sentence why?
152.	Nab:	is because I was dividing

153. Teacher: and the answer you get how many people were you able to give six pens? How many people were you able to give six pens each? How many people you gave six pens?
154. Nab: five
155. Teacher: in sixty biscuits (cakes) you were to pack out packets of fifteen, how many packets were you able to pack?
156. Nab: four
157. Teacher for fifteen sticks (dots) you were supposed to group them in groups of five, how many groups were you able to make?
158. Nab: three
159. Teacher: ok, do you think your answers are correct?
160. Nab: yes
161. Teacher: what makes you think that they are correct?
162. Nab: is because when I was dividing fifteen five only goes there three times.
163. Teacher: go and write three examples of division here. Mic!
164. Teacher: Mic! Oo! Why did you use division signs here and here (the teacher when pointing to his sign come across one that is wrong) and here you are wrong go and correct your mistake here.
165. Teacher: (asking another learner) Hil! Come. How many things were you dividing here?
166. Hil: thirty
167. Teacher: thirty, but you have written six divided by six instead of thirty divided by six why? I cannot even see your sixty dots here Hil! You are not done go and finish.
168. Teacher: Mat! You have used the division signs when asked to translate your works into symbolic number sentence, what makes you think it was division?
169. Mat: is because we were dividing.
170. Teacher: ok, is because you were dividing, how many people were you able to give six pens? (after checking Mat's circles) Mat! Are these pens six? (Counting them) so you did not give six pens but only five to each person? How many people were you able to give five pens each then?
171. Mat: six people
172. Teacher: how many packets were you able to pack?
173. Mat: four packets
174. Teacher: ok (after observed that the learner grouped sticks into threes instead of fives) so you grouped your sticks in groups of threes rights?
175. Mat: yes
176. Teacher: ok how many groups were you able to make?
177. Mat: five
178. Teacher: ok, how did you divided?
179. Mat: I divided like this, I cancelled three sticks (dots) and group them in one group, cancel another three and group them in their group keep doing that until they got finished.
180. Teacher: have you divided all your works in the same manner?
181. Mat: yes
182. Teacher: in question three you have been cancelling fifteens and make groups for them right?
183. Mat: yes
184. Teacher: and in question one you have been cancelling five and make groups for them rights?
185. Mat: yes
186. Teacher: are you answers correct?
187. Mat: yes
188. Teacher: what makes you think that you are correct?
189. Mat: is because (pause)
190. Teacher: hello! What makes you think that you must be just correct? That if thirty is divided by five you'll just get six or (pause) what makes you think that your answers are just correct? Can I call one while you are still thinking right?
191. Teacher: Dos! Come here. What makes you think that division sign is the right sign for the works you were doing?
192. Dos: is because we were dividing.

193.	Teacher:	is because you were dividing, how many people were you able to give six pens?
194.	Dos:	six
195.	Teacher:	six! How many pens were you giving to one person (counting the pens in each circle) is it five pens?
196.	Dos:	yes
197.	Teacher:	ok, how many packets were you able to pack out from sixty?
198.	Dos:	they are fifteen
199.	Teacher:	how many packets you packed out I asked?
200.	Dos:	six
201.	Teacher:	(pointing to the groups) are these packets six?
202.	Dos:	no they are four
203.	Teacher:	how many groups of sticks were you able to make?
204.	Dos:	three
205.	Teacher:	three, ok how did you divided?
206.	Dos:	I took thirty pens (dots) and share them to six people.
207.	Teacher:	six people! How did you know people would be six?
208.	Dos:	I have been taking five dots and give them to one person, take five again to another and continue until all pens (dots) get finished.
209.	Teacher:	have you been dividing in that manner to all of your works? Like in question three you have been taking fifteen and pack them in one packet or what?
210.	Dos:	yes
211.	Teacher:	all the sixty dots got finished rights?
212.	Dos:	yes
213.	Teacher:	are you answers correct?
214.	Dos:	yes
215.	Teacher:	what makes you think that your answers are correct?
216.	Dos:	is because the sixty I was dividing got finished and all packets got equal cakes.
217.	Teacher:	ok, is because sixty dots got finished and all packets got equal distribution of cakes rights? Write three examples of division in a complete number sentence.
218.	Teacher:	(back to Mic) have you thought of why you should be correct? (pause)
219.	Teacher:	(back to Mat) why do you think your answers were correct?
220.	Mat:	it is because it is not possible to take out five groups from fifteen.
221.	Teacher:	how not possible, when you have just made five groups out of fifteen or do you mean it is not possible to make more than five groups?
222.	Mat:	is because fifteen sticks are not enough
223.	Teacher:	enough for what? (pause a long pause) Mat! These sticks were how many?
224.	Mat:	fifteen
225.	Teacher:	how many groups of three were you able to make?
226.	Mat:	they are five
227.	Teacher:	then why do you think you are correct, is this what you have just done correct?
228.	Mat:	yes
229.	Teacher:	why then? Why do you think your answers are correct? (pause)
230.	Teacher:	ok, Mic! Come while Mat is still thinking (after seeing that Mic's works is not well done) Mic! What were we dividing is it not thirty, sixty and fifteen? Go and write your things well because they are not clear.
231.	Teacher:	(asking Hil) where are your sixty dots you were dividing?
232.	Hil:	they are here (pointing to groups of dots that are already done)
233.	Teacher:	yes, but before you makes groups you supposed first to makes sixty dots in one box that will be divided and it was then from the sixty dots you supposed to take groups from. Where did you take the dots in your groups then?
234.	Teacher:	(asking another learner) what makes you think the division sign you have opted fit the works you were doing?
235.	Learner:	because we were dividing.
236.	Teacher:	ok, because you were dividing, taking from the big box to smaller circles right?
237.	Learner:	yes

238.	Teacher:	and here what were you dividing?
239.	Learner:	is here
240.	Teacher:	how many sticks were these?
241.	Learner:	fifteen
242.	Teacher:	and how many groups were you able to make?
243.	Learner:	three
244.	Teacher:	and each group has how many dots?
245.	Learner:	five
246.	Teacher:	so it is fifteen divided by three to get five. Is this what you have written on your number sentence?
247.	Learner:	yes
248.	Teacher:	but I see you have written four here
249.	Teacher:	(telling all learners) each of you should write three division examples in a complete number sentence at the bottom of your question paper, ok
250.	Teacher:	Dav! You brought your paper. Why have you opted for the division sign when asked to translate your work to a symbolic number sentence?
251.	Dav:	is because we were dividing.
252.	Teacher:	dividing what?
253.	Dav:	I was dividing pens taking them from here and giving them to people
254.	Teacher:	how many people were you able to give six pens?
255.	Dav:	five
256.	Teacher:	the biscuits, you also divided them from sixty taking out packets of fifteen rights? So how many packets were you able to pack out from sixty?
257.	Dav:	five
258.	Teacher:	are these packets five? (asking as she pointing to the learner's works)
259.	Dav:	no they are four.
260.	Teacher:	ok, what makes you think that your answers are correct? (Pause) what makes you think that your answers are correct?
261.	Dav:	is because the things I was dividing get finished
262.	Teacher:	ok, is because the things you were dividing get finished, what else? Apart from the fact that things to be divided got finished, what else makes you sure that your answers are correct?
263.	Dav:	is because all groups had shared equal portions
264.	Teacher:	ok, Hil! Why have you used division sign?
265.	Hil:	is because we were dividing.
266.	Teacher:	do you think your answers are correct?
267.	Hil:	yes
268.	Teacher:	why do you think your answers are correct then?
269.	Hil:	Is because all groups got equal distribution
270.	Teacher:	ok, it is because of equal distribution rights?
271.	Teacher:	Mic! Why did you opt for division sign?
272.	Mic:	is because we were dividing.
273.	Teacher:	how many people did you give six pens each?
274.	Mic:	five people
275.	Teacher:	how many packets you got from sixty?
276.	Mic:	four packets
277.	Teacher:	what makes you thinks that your answers are correct?
278.	Mic:	is because the things I was dividing were completely divided.
279.	Teacher:	what else?
280.	Mic:	all groups had equal distribution
281.	Teacher:	ok, how did you divided?
282.	Mic:	I have been counting out one, two, three, four, five and six and put them in one group, I kept doing that until all things are equally distributed.
283.	Teacher:	have you been dividing in that manner in all the tasks?
284.	Mic:	yes

285.	Teacher:	Hen! Why have you opted for a division (sign) here?
286.	Hen:	because I was dividing
287.	Teacher:	dividing what?
288.	Hen:	I was dividing the dots into these small groups.
289.	Teacher:	how did you divided?
290.	Hen:	I divided putting here (he was saying and pointing to the small groups)
291.	Teacher:	taking from where?
292.	Hen:	taking the dots from here? (pointing to the big box)
293.	Teacher:	how were you taking dots
294.	Hen:	cancelling one
295.	Teacher:	one? Or what did you say? I cannot hear you.
296.	Hen:	cancelling fours
297.	Teacher:	oo! You mean you have been taking four dots rights?
298.	Hen:	yes
299.	Teacher:	and made groups for them right?
300.	Hen:	yes
301.	Teacher:	ok, what else did you do?
302.	Hen:	I cancel again and share to these
303.	Teacher:	these what?
304.	Hen:	the small groups
305.	Teacher:	ok, do you think your answers are correct?
306.	Hen:	yes
307.	Teacher:	you are correct? (Pause) do you think your divisions were correct?
308.	Hen:	yes
309.	Teacher:	what makes you so sure that your answers are correct?
310.	Hen:	is because I was dividing into groups
311.	Teacher:	is because you were dividing dots into groups' right?

DAY SEVEN **15 MAY 2012**

LESSON THIRTEEN

1.	Teacher:	(after greeting learners she gave learners question papers and worksheet)
2.	Teacher:	can you see question one on the question paper?
3.	Learners:	yes
4.	Teacher:	it says make twenty dots that would represent twenty apples. After you had drawn twenty dots (apples) keep taking out threes and make groups for them until all dots get finished or until you are unable to take out three any longer. These twenty dots should be drawn on the clear blank paper. Write your name on all your papers (the teacher repeated the instructions again)
5.	Teacher:	use the same worksheet, label the next task as number two and do a subtraction method. Start with twenty, keep repeatedly subtracting three until twenty get finished or three can no longer be subtracted.
6.	Teacher:	(asking one learner) are you sure your dots were twenty?
7.	Era:	yes
8.	Teacher:	and you were able to repeatedly subtract seven three? Era! Count your dots well, because there are no seven threes in twenty dots.
9.	Teacher:	hil! Are you sure that after repeatedly subtract threes from twenty zero was the remaining final answer? You see if you count this fourteen and subtract three, see if the difference would be nine that you have written here? I want to see you counting fourteen dots and subtract three from them (after Hil has done counting) ok, take three away from them and see what the difference would be.
10.	Hil:	is eleven

11.	Teacher:	what?
12.	Hil:	eleven
13.	Teacher:	is it eleven that you have written here? I see here you have written nine instead of eleven that is why you answer in the rest of steps are not correct because of this nine.
14.	Teacher:	(talking to all learners) if you are done, write a number sentence of what you were doing below question three.
15.	Teacher:	(talking to one learner) if you are done with the division, are you done? How many times were you able to repeatedly subtract three from twenty?
16.	Learner:	Five times
17.	Teacher:	(when realized the learner had some facts he had not considered) look at your work that you had done first before you answer. Do not just answer without knowing what you are saying.
18.	Learner:	(after she studied her work) they are six
19.	Teacher:	they are six ok now I want you to label number two for question two here, then write the process of subtraction, start with twenty minus three and get the difference, keep subtracting three from the differences until you reach zero or when three can no longer be subtracted.
20.	Teacher:	Dav! I don't think your last row is correct because one cannot subtract three from two remove this last row and on your number sentence this sign you used is minus and not division or do you wanted to use minus? What did you do were you subtracting or dividing?
21.	Dav:	dividing
22.	Teacher:	then put the two dots parallel to the sign line to make it division sign
23.	Teacher:	write under question three the number sentence of what you were doing. Naf! what were you doing is it division or subtraction
24.	Naf:	dividing
25.	Teacher:	but the why have you used minus sign the lines supposed to have dots if it is division.
26.	Teacher:	(asking a learner when realized that one of her row is wrong) what would be the answer after subtracting three from eleven? Take three from eleven and see what the difference would be?
27.	Learner:	eight
28.	Teacher:	you see I cannot see that eight on your answer and that wrong answer had made the rest of your answers to be wrong
29.	Teacher:	(telling the whole class) if you are done. Write number five and label it question five. Make thirty – six dots, keep subtracting six dots and make groups for them keep taking six dots and making groups for them until thirty – six dots get finished.
30.	Teacher:	(talking to one learner) are you done?
31.	Learner:	yes
32.	Teacher:	you cannot subtract three from two to get zero. Rub that thing is wrong or do you thing you can get three things from two things?
33.	Teacher:	then well this is not true here at your drawing, how many threes or groups of threes were you able to remove out of twenty?
34.	Learner:	six
35.	Teacher:	and here at the subtraction, how many three are these? (the learner counted the threes including the three of two minus three) no this you do not need to count it because it is not possible, so it can not be part of it. So how many?
36.	Learner:	they are six
37.	Teacher:	good! this should be rubbed away (asking the whole class) if you repeatedly subtract threes from twenty will you stop at zero or will it be a number less than three? if you repeatedly subtract threes from twenty will you stop at zero or will it be a number less than three? Yes!
38.	Sab:	you will not stop at zero
39.	Teacher:	you will not stop at zero but where would you stop at?
40.	Sab:	at two
41.	Teacher:	is at two, if you reach two you stop because you cannot subtract three from two

42.	Teacher:	are you done?
43.	Learner:	yes
44.	Teacher:	if you are done write the number sentence below question seven
45.	Teacher:	go again to the worksheet and do the addition process of six, keep adding six until you reach thirty – six, start with one six, two six, three six, until the number of six that can make thirty – six
46.	Teacher:	(talking to one learner who is doing the division wrongly) no, you cannot make groups first, how did you know how many groups needed? I said you cancel six dots out and make a group for them, then cancel another six and make a group for them and keep doing that until all the dots are finished, stop making those empty groups because you do not know yet the number of groups they will be.
47.	Teacher:	(talking to Naf) Naf! We do not add like that we started with one six in a row, two six in the second next row with their sum, here you supposed to have three six and their sum in the third row. And we keep doing that until the row that would have the sum of thirty – six, and then we would know how many six can make thirty – six. You can use the dots in groups to help you get the correct sums. For one six you use six dots only and for two six you count the dots of two groups with six dots each. (teacher used the drawing and show to the learner how to use the dots to get the sum)
48.	Teacher:	(telling learners that are done) if you are done make twenty – eight dots and label it question nine, keep taking out four dots and make groups for them.
49.	Teacher:	(asking Pet) where are you now? Is it at twenty – eight dots?
50.	Pet:	yes
51.	Teacher:	are you done with the other works?
52.	Pet:	yes
53.	Teacher:	o! Child, one, two, three, four, five (counting the six) now I want you to add for me the five six dots and see what would be their sum, just count up to the fifth group. Do not include this one because this is the sixth one. The thirty – six as a sum you put there is wrong because you have other row of six to be written down
54.	Teacher:	(talking to another child) where are you? Oo! You are at three six rights add them together to get their sum. So then keep adding all six together until when you reach thirty – six you will need to stop.
55.	Teacher:	(talking to another learner again) label number nine and make twenty – eight dots and then after, keep taking out fours dots, four dots, until dots get finished and see how many fours can finish twenty – eight.
56.	Teacher:	(talking to another learner) where are you? Ok, you are still adding the six; ok how many six are these?
57.	Learner:	they are three
58.	Teacher:	ok still make them four, until you reach thirty – six, if thirty – six is not reached then you are not yet done
59.	Teacher:	(telling another learner to do twenty – eight dots and to keep removing the fours until twenty – eight dots get finished)
60.	Teacher:	(the teacher tells another learner to do the repeated additions of six)
61.	Teacher:	where are you Dia? Have you added the six?
62.	Dia:	yes
63.	Teacher:	ok, have you written the number sentence of what you were doing under question seven or have you done it?
64.	Dia:	yes
65.	Teacher:	(talking to another learner) have you made groups of six from the big box?
66.	Learner:	yes
67.	Teacher:	if you had already done it then start the process of repeated addition, adding the six until you reach thirty – six. You can use the dots and count them when adding the six together. When it is one six, you take the only one group and see how many dots of one group of six will be. Two six will be given by the sum of two groups of six counted together, you keep doing that until when you reach the number of six that sum up to thirty – six.

68.	Teacher:	Dav! Have you seen how you can calculate? Now you are in a row of three six, what have you used to get the sum?
69.	Dav:	(pointing to the drawing of sticks) I used this.
70.	Teacher:	where else can you get the sum or what else can you use to get the sum? How many dots are in each of these groups?
71.	Dav:	six
72.	Teacher:	ok, you can also you the dots in the groups as six to get the sum (pointing to the circles) when it is one six it is the number of dots in one group in one circle, two six is the dots of the two circles altogether, three six is the dots of three circles like that.
73.	Teacher:	(asking one learner) how many fours have you taken out?
74.	Learner:	(counting) one, two, three, four, five, six, seven. They are seven
75.	Teacher:	seven fours, now I want you to write here that four times that number gives what? What number would it be four times that number? You can count the dots of all the fours to see what number it will be.
76.	Teacher:	(asking the whole class) who else is done? Ok, how many fours were you able to take out?
77.	Learner:	seven
78.	Teacher:	ok write it here then that is four times what giving what? (When the teacher finds out that the learner is re-counting the answer using the sticks) child the works is already here in dots. These are seven groups of fours. How many dots are they altogether for all the seven groups of fours?
79.	Learner:	twenty – eight
80.	Teacher:	is that not the answer of fours times seven?
81.	Learner:	(another learner) I am done
82.	Teacher:	you are done with this right?
83.	Learner:	yes
84.	Teacher:	label question nine here and make twenty – eight dots, after you are done with the dots, keep taking out four dots and make groups for them until all the dots are distributed into the groups
85.	Teacher:	(asking another learner) are you done?
86.	Learner:	yes
87.	Teacher:	how many groups of four dots did you take out?
88.	Learner:	seven
89.	Teacher:	complete number ten then, the number of dots and fill in the number how many dots were they altogether, in groups and the answer (when she realized that the child is not doing it well) child how many dots are in side each of these circle?
90.	Learner:	four
91.	Teacher:	and how many fours/ circles are these?
92.	Learner:	seven
93.	Teacher:	then let me see the four and seven, and how many altogether are all these fours?
94.	Learner:	twenty – eight
95.	Teacher:	well, let me see the equal to sign and the answer of four times seven
96.	Teacher:	Era, Sab, Fil, if you are done go to the next page and do question eleven, write the number sentence of what you were dividing.
97.	Teacher:	(talking to one learner) where are you? Is it at question six? Is this six or is it thirty – six? Are you stealing from someone or what is it? Because I can see you have written the sums but there are no the six that were added to make that sum. Now rub all your works because I do not know where you took these answers. I need to see the number of six that make up these sums. How did you get these sums? How did you get these sums? Or may be you are correct let me hear from you how you got these sums before you rub it.
98.	Learner:	I counted thirty – six
99.	Teacher:	from where? (pause) ok, now rub your work because you have not done it the way it supposed to be done we want to see in your work the number of six that gave you thirty –six, I want you to start adding six, starting from one six, then followed by two

six, three six, four six then it goes on until the number of six can sum up to thirty – six. Look here how others have done it (showing the learner to other's work)

		six, three six, four six then it goes on until the number of six can sum up to thirty – six. Look here how others have done it (showing the learner to other's work)
100.	Teacher:	(talking to another learner) now do question nine here, the twenty – eight dots and keep taking groups of fours dots until the twenty – eight dots get finished
101.	Teacher:	(asking another learner) why are you counting with sticks if you already have the six dots in groups? You can at least use these dots and again why are you subtracting when you supposed to be adding the six together? Rub this work and re-do it well, then label it question six, and start with one six and use the group of six dots. when it is two six, you make two groups of six dots count all the dots together to see the sum. You keep adding six and even groups until you reach thirty –six.
102.	Learner:	I am done
103.	Teacher:	ok you are done, up to where? How many fours did you take out of twenty –eight
104.	Learner:	seven fours
105.	Teacher:	seven ok, now answer question ten by filling here in the box the number of fours you were able to make out from twenty – eight
106.	Teacher:	(asking another learner) how many fours did you make out from twenty – eight?
107.	Learner:	seven
108.	Teacher:	then answer question ten that need to be filled, the number of fours you got and write their product of fours times seven after you are done continue to question eleven and write a number sentence of what you were dividing.
109.	Teacher:	(talking to another learner) are you done? Ho many fours did you take out?
110.	Learner:	seven fours
111.	Teacher:	ok then continue with question ten, fill the number of fours you get out from and the product of four times seven (after seeing that the child looks confused) yes when the fours are there seven times, how many dots are these? (Pause) how many dots are these altogether that you were dividing into groups?
112.	Learner:	twenty – eight
113.	Teacher:	ok, twenty – eight are they not the same dots that made up the seven groups of fours dots?
114.	Learner:	I am done
115.	Teacher:	you are done, ok how many fours did you get out from twenty – eight?
116.	Learner:	seven fours
117.	Teacher:	ok, write that number at question ten and the product of seven fours (when she see the learner is counting again) why counting again when these groups of fours came from twenty – eight dots?
118.	Teacher:	(talking to another learner) when you are done continue to question eleven and write the number sentence of what you were dividing.
119.	Teacher:	(talking to another learner) do question nine, label it nine and make twenty – eight dots, keep taking groups of four dots until twenty – eight dots get finished and find out how many groups of four you were able to make out from twenty – eight.
120.	Teacher:	(talking to another learner) do question nine and make twenty – eight dots, keep taking out groups of four dots until twenty – eight get finished. (the teacher repeated the same instructions to another learner again)
121.	Teacher:	Sab! You told me that twenty things divided into groups of threes will makes six groups of three rights?
122.	Sab:	yes
123.	Teacher:	even when you have been subtracting, how many threes have you repeatedly subtracted out?
124.	Sab:	six
125.	Teacher:	why may be you got the same answers?
126.	Sab:	is because there are only six threes in twenty
127.	Teacher:	do you think your answers are correct? Do you think your answers are correct?
128.	Sab:	is because there is only six threes in twenty
129.	Teacher:	give me an example of division and put it into subtraction form
130.	Sab:	is like having fifty things to be shared by five people each would have ten things

131. Teacher: now tell me how many fives are to be subtracted from fifty for fifty to get finished?
132. Sab: it will be ten fives
133. Teacher: and how many tens?
134. Sab: fives
135. Teacher: let us use it at division, how many fives needed to be added to get fifty?
136. Sab: ten fives
137. Teacher: or adding how many tens?
138. Sab: five
139. Teacher: ok, if you are multiplying, how do you multiply using these numbers to get fifty?
140. Sab: is five times ten
141. Teacher: Fil! When you divided with dots how many groups of three were you able to take out from twenty?
142. Fil: six threes
143. Teacher: and when you subtracted threes from twenty, how many threes were you able to take out from twenty?
144. Fil: six threes
145. Teacher: and when you subtracted threes from twenty, how many threes were you able to repeatedly subtract from twenty?
146. Fil: six threes
147. Teacher: why may be do you think you got the similar answers from both methods?
148. Fil: is because I was calculating
149. Teacher: the question I have is this, from division of groups you get six groups of threes from twenty and even at subtraction you also subtracted six threes
150. Fil: is because I took out six threes
151. Teacher: from where?
152. Fil: is because from the twenty I took out …… (pause) (Fil can do the calculations correctly but seem to have a problem explaining why he can also describe the process but cannot relate the idea across)
153. Teacher: Era! When you divided into groups how many groups of three did you make out from twenty?
154. Era: six
155. Teacher: and when you subtracted, how many threes did you repeatedly subtract from twenty to get finished?
156. Era: six threes
157. Teacher: why may be you have subtracted six threes the same as six groups of threes in division?
158. Era: the same twenty value I divided is the same twenty I was also subtracting from
159. Teacher: what else?
160. Era: is because the dots to be distributed get finished
161. Teacher: is because the dots get finished and get finished after you just take out six threes, were you able to take out seven threes?
162. Era: no
163. Teacher: why?
164. Era: it is because there are only six threes in twenty
165. Teacher: good! Do you think your answers are correct?
166. Era: yes
167. Teacher: why do you think you are correct?
168. Era: is because the things I was distributing got finished
169. Teacher: can you give me the example of division?
170. Era: is like having fifty things divided by two people and each would have ten things.
171. Teacher: I don't think so
172. Era: twenty things
173. Teacher: no twenty times two is forty
174. Era: thirty things

175.	Teacher:	if you take forty from fifty what would remain? If you take forty from fifty what would remain? If the tens are five in fifty like this (she was pointing to five fingers each representing ten) and they are ten, ten, ten, ten and ten and from these five tens you take away four tens, what would remain? (pause) ok go and take fifty stones and start dividing them into two groups
176.	Teacher:	when you divided twenty dots how many groups of three were you able to make out?
177.	Dos:	six
178.	Teacher:	when you subtracted threes from twenty, how many threes were you able to repeatedly subtract out?
179.	Dos:	six threes
180.	Teacher:	why may be you got similar answers which are six threes here at repeated subtraction and six groups of threes at division?
181.	Dos:	is because I was using threes
182.	Teacher:	do you mean you were using threes at both sides?
183.	Dos:	yes
184.	Teacher:	what else?
185.	Dos:	all side have the same values or numbers
186.	Teacher:	ok, give me an example of division
187.	Dos:	is like having forty things to be distributed by four people and each would be having ten things
188.	Teacher:	how many fours are in forty?
189.	Dos:	ten fours
190.	Teacher:	and how many tens are there?
191.	Dos:	four
192.	Teacher:	if you are to repeatedly add fours how many of them would sum up to forty?
193.	Dos:	they would be ten
194.	Teacher:	ok, while the tens would be how many?
195.	Dos:	they will be four
196.	Teacher:	if you are to multiply using the same numbers to get forty, what would you do?
197.	Dos:	I would multiply four by ten
198.	Teacher:	or what?
199.	Dos:	or ten by four
200.	Teacher:	do you think your answers are correct?
201.	Dos:	yes
202.	Teacher:	why do you think your answers are correct?
203.	Dos:	is because after division what I was dividing got finished
204.	Teacher:	what else?
205.	Dos:	the groups had equal distribution
206.	Teacher:	Era! I hope you now have the correct answer from stones. How many each would get if two share fifty things?
207.	Era:	yes
208.	Teacher:	how many each would get?
209.	Era:	twenty – five
210.	Teacher:	twenty – five, good, do you think your answers are correct?
211.	Era:	yes
212.	Teacher:	what makes you think that your answers are correct?
213.	Era:	is because all groups had equal distribution
214.	Teacher:	is because of the equal distribution, what else?
215.	Era:	what I was distributing got finished
216.	Teacher:	how many twos are in fifty?
217.	Era:	twenty – five
218.	Teacher:	and how many twenty – five are there?
219.	Era:	two
220.	Teacher:	if I start adding twos, how many two need to be repeatedly added to make fifty?
221.	Era:	twenty – five twos

222.	Teacher:	and if I want to get fifty, how many twenty –fives need to be repeatedly added to make it?
223.	Era:	two
224.	Teacher:	if you are to multiply what will you multiply and with what to get fifty?
225.	Era:	it would be two times twenty –five
226.	Teacher:	or twenty – five times what?
227.	Era:	times two
228.	Teacher:	Nab! When you divided twenty dots, how many groups of threes you were able to make?
229.	Nab:	four
230.	Teacher:	You can go home Nab
231.	Teacher:	Mat! How many groups of three dots you take out from twenty dots?
232.	Mat:	six
233.	Teacher:	six ok, when you repeatedly subtracted threes from twenty, how many threes were repeatedly subtracted from twenty?
234.	Mat:	Six threes
235.	Teacher:	six threes, ok, why may be you got similar answers of six threes at both sides? (Pause) why may be you got similar answers of six threes at both sides?
236.	Teacher:	ok let me ask you for repeated addition, if may be this one you cannot get it. Well these dots are thirty –six rights?
237.	Mat:	yes
238.	Teacher:	and you have been taking out six, six, six six right?
239.	Mat:	yes
240.	Teacher:	how many six did you take out from thirty – six?
241.	Mat:	five
242.	Teacher:	how many six did you repeatedly added to make thirty –six?
243.	Mat:	seven
244.	Teacher:	seven! In this thirty – six! How many six are in…
245.	Mat:	five
246.	Teacher:	are your dots really thirty – six?
247.	Mat:	yes
248.	Teacher:	(the teacher started to re-count the dots to find out if they are really thirty –six. She also started to re-do the groups of six with the learner for confirmation and also to find out which dots were left out and later discovered there were six dot s left out not allocated to any group)
249.	Teacher:	now that we have done it together you have the clue of how many six were made out from thirty – six right?
250.	Mat:	they are six
251.	Teacher:	ok, there are six six in thirty – six right?
252.	Mat:	yes
253.	Teacher:	if you are to repeatedly add six, how many are they needed to make thirty – six?
254.	Mat:	they would be six
255.	Teacher:	six, ok why may be they need to be six to make thirty – six or why do you think you got six groups of six dots and even when repeatedly subtract you got six six out from thirty – six? (pause) why didn't you got seven or eight six
256.	Mat:	no
257.	Teacher:	why
258.	Mat:	because the thirty –six I was distribution got finished, so I can't get out seven or eight.
259.	Teacher:	that mean this is because there are only how many six in thirty – six?
260.	Mat:	they are six
261.	Teacher:	so I will still ask the same question, why may be you got the similar answers the six groups of six dots and the six six at repeated addition?
262.	Mat:	is because there are only six sixes in thirty – six.
263.	Teacher:	(asking another learner) are these dots twenty – eight?
264.	Learner:	yes

265.	Teacher:	and you have been making groups of fours, four, four?
266.	Learner:	yes
267.	Teacher:	how many fours did you made out?
268.	Learner:	(after she counted the groups) they are seven
269.	Teacher:	they are seven ok, when you divided twenty – eight dots, you made out seven groups of four. So four there seven times gives what?
270.	Learner:	twenty – eight
271.	Teacher:	so we have the same numbers at division and multiplication that are twenty – eight, seven and four, why do you think there is such similarity?
272.	Learner:	is because there are only seven fours in twenty – eight
273.	Teacher:	give me an example
274.	Learner:	is like having hundred sweets to be distributed equally amongst twenty people of whom each would get ten sweets.
275.	Teacher:	no if people were ten then each would get ten but because they are twenty each would get?
276.	Learner:	five
277.	Teacher:	five thank you, meaning how many twenties are in hundred?
278.	Learner:	five
279.	Teacher:	and how many fives are in hundred?
280.	Learner:	twenty
281.	Teacher:	good! If I start subtracting twenties from hundred, how many of them would I be able to subtract from hundred?
282.	Learner:	five times
283.	Teacher:	and if I start subtracting fives from hundred how many of them needed to finish hundred?
284.	Learner:	it will be twenty times
285.	Teacher:	if I start repeatedly adding twenties how many do I need to add to make hundred?
286.	Learner:	they will be five
287.	Teacher:	five, but if I start repeatedly adding fives, how many are they needed to make hundred?
288.	Learner:	they will be twenty
289.	Teacher:	good! If you want to get hundred what numbers would you multiply to get hundred?
290.	Learner:	twenty times five
291.	Teacher:	or five times what? Or five times what?
292.	Learner:	times twenty
293.	Teacher:	do you think your answers are correct?
294.	Learner:	yes
295.	Teacher:	why?
296.	Learner:	because in thirty – six there are only six six.
297.	Teacher:	and at division what makes you think that you are correct, that only six groups of six would come out?
298.	Learner:	is because in thirty – six there are only six six
299.	Teacher:	were you able to take another group of six more than that?
300.	Learner:	no
301.	Teacher:	what made you think that you are done?
302.	Learner:	is because the thirty –six dots got finished and all groups had equal distribution.
303.	Teacher:	how many threes did you take out from twenty dots?
304.	Naf:	six
305.	Teacher:	and when you subtracted how many threes were able to repeatedly subtract from twenty for twenty to get finished?
306.	Naf:	six
307.	Teacher:	why may be did you get similar answer of six threes at both sides of division and of the repeated subtraction?
308.	Naf:	is because all numbers are the same.
309.	Teacher:	which numbers are the same?

310. Naf: is dots
311. Teacher: the dots are equal to what number?
312. Naf: they are equal to twenty
313. Teacher: the twenty you were subtracting threes rights?
314. Naf: yes
315. Teacher: do you think that your answers are correct?
316. Naf: yes
317. Teacher: what makes you think that your answers are correct?
318. Naf: is because all groups has equal distribution
319. Teacher: what else?
320. Naf: is because the things I was distributing got finished
321. Teacher: which things?
322. Naf: is the dots
323. Teacher: which dots get finished that you are referring to?
324. Naf: the one I was distributing
325. Teacher: give me an example
326. Naf: is like dividing thirty by three people of whom each would get twenty.
327. Teacher: mhuumhhh!
328. Naf: ten
329. Teacher: meaning how many threes would you take out from thirty for thirty to get finished? How many threes would you take out from thirty for thirty to get finished?
330. Naf: they will be ten
331. Teacher: and how many tens will you be able to repeatedly subtract from thirty for thirty to get finished?
332. Naf: three of them
333. Teacher: ok if you keep adding threes, how many of them would you need to make thirty?
334. Naf: ten
335. Teacher: and if I keep adding tens how many of them would be needed to make thirty?
336. Naf: three
337. Teacher: if I am to multiply to get thirty, how many times three would give thirty?
338. Naf: is times ten
339. Teacher: or ten times what?
340. Naf: Times three

LESSON FOURTEEN

1. Teacher: (the teacher distributed worksheet papers and question papers. Learners were given instruction to do the divisions in two ways as sharing and as quotitive. Learners were also given option to use any material of their choice which can be either stones or drawings to do the division)
2. Teacher: in question 1 A, let those using stones go and take twenty – four stones and those using drawing of dots take their worksheet and draw twenty – four dots. in question 1 A, let those using stones go and take twenty – four stones and those using drawing of dots take their worksheet and draw twenty – four dots. Label the work as number one A. Label the work as number one A. Label the work as number one A.
3. Learners: (shouting one by one) I am done. I am done. I am done.
4. Teacher: are you done making twenty – four dots?
5. Learners: yes
6. Teacher: are you done taking twenty – four stones?
7. Learners: yes
8. Teacher: I want you to notice the difference of the division in A and in B, and you will tell me this difference at the end of both divisions. In A it say make three groups, empty three

circles and in those three circles (representing three groups) keep allocating stones or dots equally until all the twenty – four dots or stones get finished.

9.	Teacher:	no Hanna! I said three groups like the way we were dividing in the first lessons of this program, like keep taking three stones and allocate them to three groups each one by one. Then you keep allocating as such until all the twenty – four dots or stones are equally shared. Those doing it with dots, make three empty circles keep allocating dots one by one in those three circles and cancel them out from the box of twenty – four dots.
10.	Teacher:	are you done?
11.	Learners:	yes
12.	Teacher:	no Hanna, I said three groups those you have made are four groups, so put all the stones back together and re-do the processes again. Make three groups and keep allocating stones equally to these groups (when the teacher noticed the group had unequal distribution at the end of division) are you sure you took twenty – four stones? Because the number of stones in groups does not show that. If stones were twenty – four, groups would have equal distribution of stones but I see some of your groups has five stones and some has six stones, so count the stones if they are twenty – four. If not twenty – four add stones to make them twenty – four.
13.	Teacher:	(talking to the whole class) all people dividing with stones, let them do their work any where even on the floor to create enough space.
14.	Teacher:	are you done?
15.	Learner:	yes
16.	Teacher:	ok, now you continue with question one B, you should still make twenty – four dots but this time do not make three groups but keep taking out three dots and make groups for them until all the twenty – four dots are finished. Those with stones take twenty – four stones still and this time keep taking out three stones at a time and make groups for them until twenty – four stones get finished.
17.	Teacher:	Pet! Did you get the number of groups you were able to get out from twenty – four?
18.	Pet:	yes
19.	Teacher:	Hanna! How many stones each group has? Hanna! How many stones each group has?
20.	Hanna:	eight stones
21.	Teacher:	ok, go and take another twenty – four stones again and from the twenty – four stones keep taking out three stones until the twenty – four stones get finished.
22.	Teacher:	now do question two; I hope you still remember how you did all the division in question one.
23.	Learner:	yes
24.	Teacher:	was there a difference on the two divisions or not?
25.	Learner:	there was a difference
26.	Teacher:	so there was a difference on how you divided rights?
27.	Learner:	yes
28.	Teacher:	ok, you will tell me about that difference later. On question two, make thirty – five dots and make seven empty groups (circles) and start allocating dots in those dots equally one by one until all the thirty – five dots are equally shared by seven groups that is question two C. those with stones, take thirty – five stones and make seven groups keep allocating stones in groups until all groups share thirty – five stones equally.
29.	Teacher:	are you done with thirty – five dots/ stones?
30.	Learners:	yes
31.	Teacher:	you have allocated them into groups rights?
32.	Learner:	yes
33.	Teacher:	then continue with question two D and make another thirty – five dots, again keep taking groups of seven dots until all the thirty – five dots get finished
34.	Teacher:	Pet! Let me ask you so that you can get a space for question two, Pet! When you divided in question one A and B did you notice any difference on the division methods?

35.	Pet:	yes
36.	Teacher:	what is the difference?
37.	Pet:	we were dividing
38.	Teacher:	that I know, I want to hear how you divided and the difference of that division.
39.	Pet:	in the first division, I made three groups first and keep allocating stones to groups equally one by one until all twenty – four stones get finished and in the second division I divided by taking out three groups from twenty – four until twenty – four stones are finished.
40.	Teacher:	in the second division have you been taking out three groups or do you mean groups of three?
41.	Pet:	yes, groups of three stones
42.	Teacher:	ok, finish the story; you have been taking groups of three stones until what?
43.	Pet:	until stones get finished
44.	Teacher:	at first you said you made three groups and start allocating stones one by one into groups until all twenty – four stones get finished, were groups having equal number of stones? Or how did you divided?
45.	Pet:	yes all groups shared equal stones
46.	Teacher:	ok, have you noticed the two different ways of division rights?
47.	Pet:	yes
48.	Teacher:	where is the difference you said?
49.	Pet:	is when you make groups first and that of repeatedly taking the same amount of number.
50.	Teacher:	ok put the twenty – four stones back in the box
51.	Teacher:	Nab! Come let me ask you, have you done dividing as I told you?
52.	Nab:	yes
53.	Teacher:	did you notice the difference in both divisions?
54.	Nab:	yes
55.	Teacher:	I want you to tell me the difference between how you divided in A and in B.
56.	Nab:	at first I divided stones until they get finished
57.	Teacher:	sharing stones until they get finished how?
58.	Nab:	dividing taking twenty – four stones and put into groups.
59.	Teacher:	do you mean you put all twenty – four stones into one group or what?
60.	Nab:	no into three groups
61.	Teacher:	then tell me how you share these twenty – four stones into these three groups?
62.	Nab:	I kept putting stones one by one in each group until all twenty – four stones get finished and in the second division in question one B, I have been taking stones one by one putting them into a group until…
63.	Teacher:	does that mean in both divisions you divided in a similar manner? Did you allocated one stone by one in all the division or you divided by taking three stones repeatedly in the second division? (pause)
64.	Teacher:	put the twenty – four stones back in the big box and come along with thirty – five stones Nab!
65.	Teacher:	those that are using the dots are you done?
66.	Learners:	yes
67.	Teacher:	have you been taking seven dots and make group for them?
68.	Learners:	yes
69.	Teacher:	have you seen the difference in how you were dividing?
70.	Learners:	yes
71.	Teacher:	where is the difference?
72.	Learner:	first I make three groups and put seven, seven in each group
73.	Teacher:	taking seven, seven here no, no, no tell me what you did first.
74.	Learner:	I make twenty – four dots.
75.	Teacher:	next
76.	Learner:	it was to make three groups (three empty circles representing groups)
77.	Teacher:	with what?

78.	Learner:	empty groups
79.	Teacher:	so now I want to hear how you put in or shared these dots in the groups
80.	Learner:	I have been taking three dots at a time and make group for them, taking three dots and make groups for them until all dots get finished
81.	Teacher:	Tell me the difference of division here in A and there in B.
82.	Learner:	you can divide by make groups first and allocate dots in the groups equally one by one
83.	Teacher:	and how else can you divide?
84.	Learner:	by taking out dots and make groups for them
85.	Teacher:	were they equal dots or unequal
86.	Learner:	is equal number of dots
87.	Teacher:	ok taking equal number of dots and make group for them until all the dots are equally distributed rights?
88.	Learner:	yes
89.	Teacher:	ok, Naf and Sab, I want you to answer these question for me. Are you done dividing in question one and two?
90.	Naf & Sab:	yes
91.	Teacher:	have you seen the difference on how to divide or not?
92.	Naf & Sab:	we have seen it
93.	Teacher:	have you done the division the same?
94.	Naf & Sab:	no
95.	Teacher:	what is the difference on division between that of A and B?
96.	Sab:	the difference is here, when we make twenty – four dots
97.	Teacher:	did you use twenty – four dots in A and B?
98.	Sab:	yes
99.	Teacher:	ok, now I want to hear where the difference is in both division, you may start like this, in both side we were dividing twenty – four but the difference is…..
100.	Sab:	the difference is in A we made three groups first and keep allocating the dots to three groups equally by cancelling three dots at a time and allocate them one by one to three groups until all the twenty – four dots get finished.
101.	Teacher:	ok, and in B how did you do it?
102.	Sab:	in B we have been dividing twenty – dots by taking out three dots at a time and makes groups for them, we kept taking three dots and make groups for them until all the twenty – four dots are finished.
103.	Teacher:	oo! Naf! where did you say is the difference?
104.	Naf:	the difference is here, in A we have been allocating dots one by one in groups until all groups get equal dots and the twenty – four dots are finished
105.	Teacher:	and in B how did you divided?
106.	Naf:	I have been dividing by taking out three dots at a time and make groups for them
107.	Teacher:	so you have been taking three dots repeatedly, make groups for them until the twenty – four dots got finished right?
108.	Teacher:	so what you say is a difference is that in one method you divide to groups by allocating equal portion one by one to each group
109.	Naf:	yes
110.	Teacher:	or dividing by taking out equal portion repeatedly until the dividends get finished right?
111.	Naf:	yes
112.	Teacher:	ok, Mic and Fil, are you done with the division? What difference on division you noticed? One of you should use question one while the other will use question two to tell me the difference you noticed in the division. have you seen the difference in A and B
113.	Mic:	yes
114.	Teacher:	what difference is that?
115.	Mic:	in question one A we have been cancelling out three dots and allocated them to groups
116.	Teacher:	shared them to the group how?

117.	Mic:	one by one to each group
118.	Teacher:	ok in B, how did you do it?
119.	Mic:	I have been dividing by cancelling out three dots and put them in one group, cancel another three and make them another group for them. I kept doing that until all twenty – four dots get finished.
120.	Teacher:	ok, what difference is there in question two C and D?
121.	Fil:	I divided seven things.
122.	Teacher:	you divided seven things, hoe did you divide seven or do you mean you kept taking seven or what do you mean? I do not understand say the whole story of what you did from the beginning.
123.	Fil:	first I made dots and cancel out seven and shared them one by one
124.	Teacher:	share them one by one to where?
125.	Fil:	in here (pointing to circles)
126.	Teacher:	ok, so first you made circles, how many circles are these you made?
127.	Fil:	seven
128.	Teacher:	ok, and then keep allocating one by one in these seven circles (groups) right? Ok in D how did you do it?
129.	Fil:	I made thirty-five dots
130.	Teacher:	ok and how did you divide then?
131.	Fil:	and take out eight dots
132.	Teacher:	so you have been taking out eight dots, eight dots?
133.	Fil:	yes
134.	Teacher:	did all thirty – five dots get finished?
135.	Fil:	yes
136.	Teacher:	ok, who else is done here?
137.	Teacher:	Mat, where is the difference in the two division of A and B? how did you divided in A and in B?
138.	Mat:	in A I divided like this, first I made twenty – four dots and make three empty groups and I started cancelling seven and put them in the one group, seven and put in another group
139.	Teacher:	so you have been cancelling seven?
140.	Mat:	yes
141.	Teacher:	are these dots in your groups seven?
142.	Mat:	(after counting) they are eight
143.	Teacher:	so you mean you have been taking out eight dots and put them in one groups and not seven?
144.	Mat:	yes
145.	Teacher:	how did you know there would be eight in each group? ok in B how did you divided?
146.	Mat:	I made twenty – four dots and seven groups and start allocating by taking one dot and put in a group take one dot and put in a second group until all the dots get finished
147.	Teacher:	(asking another learner) can you tell how you divided in A and in B? How did you do it in A?
148.	Learner:	I made twenty – four dots and later make three empty circles and from there I have been then, I cancelled three and allocated them one by one in each group. I kept cancelling three and allocating one by one in each group.
149.	Teacher:	ok in B how did you do it?
150.	Learner:	in B I have been dividing like this I first make twenty – four dots and I have been allocating one by one, one by one.
151.	Teacher:	is that how you divided
152.	Learner:	yes
153.	Teacher:	so you made empty circles first
154.	Learner:	yes
155.	Teacher:	how did you know the groups would be eight? How did you know the groups would be eight? (Pause) who can tell us how we divided in B? Because I don't think we have been allocating one by one in B. yes!

156. Dos: first we make twenty – four dots and keep cancelling out three dots and make a group for them until twenty – four dots get finished.
157. Teacher: I want all of you to tell me one thing, use stones, drawing or dots and divide eighteen by six, do it however you want to and write the answer when you finish.
158. Teacher: if you are done take a paper and draw four columns the one for division, for addition, for subtraction and for multiplication. Are you done with the drawing?
159. Learner: yes
160. Teacher: if you are done with the drawing …
161. Learner: yes we are done
162. Teacher: ok translate this division to the repeated subtraction beginning with eighteen minus or which number do you prefer?
163. Learner: minus six
164. Teacher: ok, then you start with eighteen minus six, keep subtracting six until you reach zero, then you go to the addition column and keep adding six until you reach eighteen and then you go to multiplication and write that six times what gives you eighteen. (some learners were using stones to relate division to addition, repeated subtraction and to multiplication)
165. Teacher: how did you have the answer?
166. Learner: I took eighteen things and kept taking six out until eighteen got finished and then after I am done I counted groups and find out they are three.
167. Teacher: after you find out that there are three six in eighteen, what do you do at subtraction? In eighteen there three six rights?
168. Learner: yes
169. Teacher: meaning if you are to repeatedly subtract six from eighteen, how many six would you subtract from eighteen?
170. Learner: three
171. Teacher: and at addition how many six needed to be repeatedly added to get eighteen?
172. Learner: three six
173. Teacher: and if you are to multiply it is six times what to get eighteen?
174. Learner: six times three
175. Teacher: so can you see how there is any relationship?
176. Learner: yes
177. Teacher: if I give you a different number will you still do it?
178. Learner: yes
179. Teacher: ok let me use three instead of six, how many three are in eighteen?
180. Learner: six three
181. Teacher: if you were to repeatedly subtract threes from eighteen, how many would they be for eighteen to get finished?
182. Learner: six threes
183. Teacher: if you are to repeatedly add threes to get eighteen, how many threes would they be?
184. Learner: six threes
185. Teacher: if you are multiplying it will be three times what that would give eighteen?
186. Learner: is three times six
187. Teacher: fil! Can you see on the subtraction, addition and multiplication columns?
188. Fil: yes
189. Teacher: so now we will start from the division. Can you see your division you have eighteen divided by six equal to what?
190. Fil: three
191. Teacher: meaning in eighteen there are how many six?
192. Fil: three
193. Teacher: then this gives you the idea of what you will do in subtraction, how many times would you repeatedly subtract six from eighteen for eighteen to get finished?
194. Fil: three
195. Teacher: three, ok, if you want eighteen how many six can you repeatedly add? if you want eighteen how many six can you repeatedly add?

196. Fil: three six
197. Teacher: three six good! How many times can six be multiplied to get eighteen? How many times can six be multiplied to get eighteen? How many times can six be multiplied to get eighteen?
198. Fil: three times
199. Teacher: because there are how many six in eighteen?
200. Teacher: there are three six in eighteen
201. Teacher: good! That is it whether dividing, adding, subtracting or multiplying the fact remains there are how many six in eighteen?
202. Learner: three
203. Teacher: when you divided you have seen how many six in eighteen?
204. Learner: three
205. Teacher: this made you subtracted how many six in eighteen for eighteen to get finished?
206. Learner: three
207. Teacher: and when you repeatedly added the six how many of them were needed to make eighteen?
208. Learner: three
209. Teacher: and in multiplication it is six times what to give you eighteen?
210. Learner: times three
211. Teacher: so you can see how they are related right?
212. Teacher: Dav! When you divided eighteen with six what was your answer?
213. Dav: three
214. Teacher: but you have not done it on subtraction, don't you know how to do the subtraction? If you subtract six from eighteen what would be the remaining? Are your things eighteen?
215. Dav: yes
216. Teacher: take out six and see what would remain (pause) what is the remaining?
217. Dav: twelve
218. Teacher: it is twelve so is not what you have written here rights? Or is this answer correct?
219. Dav: no it is twelve
220. Teacher: ok, it is good that you have realized that your answer is wrong and that you had corrected your mistake, how many six you need to repeatedly add together to get eighteen?
221. Dav: three
222. Teacher: three, ok, what number times six gives you eighteen?
223. Dav: three times
224. Teacher: ok let us use three, how many threes are in eighteen?
225. Dav: six threes
226. Teacher: if you are to subtract threes from eighteen how many can you repeatedly subtract from eighteen for eighteen to get finished?
227. Dav: six threes
228. Teacher: they are six, if you are to repeatedly add threes together, how many are they needed to make eighteen?
229. Dav: six threes
230. Teacher: ok, what number times three gives eighteen?
231. Dav: three times six
232. Teacher: ok, you are getting it I love it.
233. Teacher: Naf! When you divided how many threes are in eighteen?
234. Naf: Six threes
235. Teacher: and how many six are there?
236. Naf: three six
237. Teacher: if you are to subtract how many six can be repeatedly subtracted from eighteen for eighteen to get finished?
238. Naf: three six
239. Teacher: three, because there are only how many six in eighteen?

240. Naf: three six
241. Teacher: is it possible to subtract four six from eighteen?
242. Naf: no
243. Teacher: why wouldn't you be able to take four six from eighteen?
244. Naf: because they are only there three times
245. Teacher: if you are to repeatedly add the six together how many six are needed to make eighteen?
246. Naf: three
247. Teacher: three, if I add them two will they still give eighteen?
248. Naf: no
249. Teacher: because they must be in what number to give eighteen?
250. Naf: three
251. Teacher: what number times six gives eighteen?
252. Naf: six times three
253. Teacher: how many three did you say are in eighteen?
254. Naf: six threes
255. Teacher: sab! How many six did you say are in eighteen?
256. Sab: three
257. Teacher: if I am to repeatedly subtract six from eighteen how many six need to be subtracted for eighteen to get finished?
258. Sab: three six
259. Teacher: three six, is it possible to subtract five six from eighteen?
260. Sab: no!
261. Teacher: why?
262. Sab: because they are only there three times
263. Teacher: ok, they are only there three times if I start adding six together, how many six will give eighteen?
264. Sab: three six
265. Teacher: if I am to multiply, it will be six times what that will make eighteen?
266. Sab: three times
267. Teacher: Mat! When you divided how many six were you able to make?
268. Mat: (after counting) three
269. Teacher: (when the teacher noticed the number sentence has the wrong answer) do you mean you made twelve six from eighteen? Or why equal to twelve here? Where did you get this twelve? Mat! Where did you get this twelve?
270. Mat: I calculated in eighteen minus six and get twelve
271. Teacher: so you got this answer through subtraction and not by division rights? But the sign you put there is for division and not the subtraction sign. Now if you divide eighteen in groups of six, six, how many group of six would you make?
272. Mat: three
273. Teacher: ok, and that is what you supposed to write here (good) how many six are in eighteen we said?
274. Mat: three
275. Teacher: three six, if you are to repeatedly subtract six from eighteen how many six are needed to finish eighteen?
276. Mat: three six
277. Teacher: did this eighteen get finished when you repeatedly subtract six three times?
278. Mat: yes
279. Teacher: ok, if you are to add six, six an six how many of the six needed to make eighteen?
280. Mat: three six
281. Teacher: if we are to multiply, how many six needed to eighteen?
282. Mat: is three times
283. Teacher: (when noticed that the answer the learner is mentioning is not the same as the answer he has written on multiplication number sentence) is it what you have written here, six times three equal to eighteen? (pause) re-write it here clearly

284.	Teacher:	Dos! How many six are in eighteen?
285.	Dos:	three six
286.	Teacher:	if you are to subtract six, six, six, six how many number of six can finish eighteen?
287.	Dos:	Three six
288.	Teacher:	ok, where have you done it here on your worksheet because I told you to do it but I can not see where your works has been done? Here you subtracted six from eighteen and you got twelve you supposed to continue subtracting six until you reach zero.
289.	Teacher:	ok, if you are to repeatedly six together, how many six are needed to make eighteen?
290.	Dos:	three six
291.	Teacher:	if you are to multiply six it would be times what to make eighteen?
292.	Dos:	three six
293.	Teacher:	ok, now I will change how many six you said are in eighteen?
294.	Dos:	three six
295.	Teacher:	and how many threes are in eighteen? (Pause) let me say there were three were there is six and it would be eighteen divided by three, what answer would you get?
296.	Dos:	six
297.	Teacher:	so there are how many threes in eighteen?
298.	Dos:	six threes
299.	Teacher:	if I am to subtract threes, threes, three, from eighteen how many threes needed to finish eighteen?
300.	Dos:	six
301.	Teacher:	I would be able to subtract six threes from eighteen for eighteen to get finished. If I start adding threes how many of them are needed to make eighteen?
302.	Dos:	six of them
303.	Teacher:	six threes ok, what number times three makes eighteen?
304.	Dos:	six times
305.	Teacher:	ok, six times
306.	Teacher:	you worked with stones rights?
307.	Nab:	yes
308.	Teacher:	How many stones did you used? Eighteen rights?
309.	Nab:	yes
310.	Teacher:	how did you divided? How many six did you make out from eighteen?
311.	Nab:	three
312.	Teacher:	and how many threes are in eighteen?
313.	Nab:	six
314.	Teacher:	if you started subtracting six, six, from eighteen, how many six would finish eighteen?
315.	Nab:	three six
316.	Teacher:	three, will three six finishes eighteen? Will eighteen get finished?
317.	Nab:	yes
318.	Teacher:	how many times would you subtract threes from eighteen?
319.	Nab:	six threes
320.	Teacher:	if you start adding six, six, how many of the six would make eighteen?
321.	Nab:	four (after he finish counting on his paper)
322.	Teacher:	what number times six gives eighteen?
323.	Nab:	three six
324.	Teacher:	ok, three six gives eighteen in addition you said four six did you also used the stones here
325.	Nab:	yes
326.	Teacher:	and you were adding six stones to six stones
327.	Nab:	yes
328.	Teacher:	if it was six stones you did add all along, why did you got nine here at the row of six plus six, go and take six stones and put six stone here and another six stones here, count all the stones altogether to see what the sum of two six are. Will it give nine?
329.	Teacher:	Dia! How many six are in eighteen?
330.	Dia:	three six

331.	Teacher:	and how many threes are in eighteen
332.	Dia:	six three
333.	Teacher:	if I am to subtract six from eighteen, how many six would you subtract to finish eighteen (pause) how many six did you say are in eighteen
334.	Dia:	three six
335.	Teacher:	three six ok, if I start subtracting six how many of the six would finish eighteen?
336.	Dia:	three
337.	Teacher:	ok, if I start subtracting threes from eighteen how many threes would you subtract to finish eighteen
338.	Dia:	six
339.	Teacher:	if I start adding six, six, how many of the six needed to build eighteen?
340.	Dia:	three six
341.	Teacher:	and if it is the threes I repeatedly add how many would give eighteen?
342.	Dia:	six
343.	Teacher:	what number times six gives eighteen
344.	Dia:	three times
345.	Teacher:	(asking Nab) are these groups having six stones?
346.	Nab:	yes
347.	Teacher:	how many stones altogether when you counted them?
348.	Nab:	twelve
349.	Teacher:	but you have not written twelve here on your paper, you see how many will you get if you make six groups of threes?
350.	Teacher:	this is your paper right? How many six were you able to make?
351.	Learner:	three six
352.	Teacher:	if I am to repeatedly add six to get eighteen how many six will I add?
353.	Learner:	three six
354.	Teacher:	and the threes would be how many?
355.	Learner:	six three
356.	Teacher:	what number times six gives eighteen?
357.	Learner:	three times

LESSON FIFTEEN

1.	Teacher:	(the teacher gives the question paper to the learner to do the activity)
2.	Teacher:	the next activity is the same activity you did when we started the program. Do the calculation, this time you are free to use stones, drawing or dots to do the calculations.
3.	Teacher:	when people dividing what do they do? Do you know what division is?
4.	Sab:	yes
5.	Teacher:	what is division?
6.	Dav:	is like taking eighteen things and share them to three people of whom each would get six
7.	Teacher:	ok how do you do it? What do you do when dividing?
8.	Sab:	I take eighteen dots and indicate three groups/ people and start allocating the dots from eighteen one by one.
9.	Teacher:	how did you calculate your works?
10.	Sab:	I used dots
11.	Teacher:	because in question one you were dividing and in question two there was multiplication, do you think they are related?
12.	Sab:	yes
13.	Teacher:	how are they related?
14.	Sab:	it is because here you divide eighteen by three and at multiplication is also three and eighteen
15.	Teacher:	do you think your answers are correct? Do you think that your answers are correct?
16.	Sab:	yes

17. Teacher: what makes you think that your answers are correct?
18. Sab: is because if you are to divide eighteen things into three people each would just get six things.
19. Teacher: give me one example of division
20. Sab: is like having thirty things divided by three people of whom each would get ten things.
21. Teacher: I want you to divide using your example in two ways (sharing and quotitive division)
22. Sab: first you make thirty dots and then make three groups and you keep giving one dot to each until all the thirty dots get finished and that are equally shared.
23. Teacher: ok, in another division, how do you do it?
24. Sab: you still make thirty dots and keep taking out ten dots at a time giving them to one person (or making group for them) and take another ten and give them to a second person or make group for them and take another ten and give them to the third person or make group for them.
25. Teacher: Fil! Do you now know what division is?
26. Fil: yes
27. Teacher: what do you do when dividing?
28. Fil: you give people things one by one (pause)
29. Teacher: can you see the relation between question one and two?
30. Fil: yes
31. Teacher: you can see the common things in both questions rights? Let us take for instance thirty divided by five you got six, how will this be at multiplication?
32. Fil: you multiply thirty there time five.
33. Teacher: how did you do your works
34. Fil: I used stones
35. Teacher: give me an example of division
36. Fil: is like having forty things shared by two people each of whom would get twenty things
37. Teacher: how do you divide it?
38. Fil: I will share by giving one things by one until all forty things get finished
39. Fil: or taking out twenty things and give them to a child and another twenty things and give them to another child.
40. Teacher: Era! Can you tell me what people do when dividing? What is division?
41. Era: is like having twenty things shared by two people of whom each would get ten things.
42. Teacher: the question I asked is when you divide those twenty things, how do you divide? Or what do you do when dividing this twenty? What do people do when dividing? If they say divide this twenty things to two people what do you do and how do you divide?
43. Era: you take things and share them one by one to people until all things got finished.
44. Teacher: how did you get these answers or what did you use?
45. Era: I made sticks
46. Teacher: do you think there is a relationship between question one and two?
47. Era: no
48. Teacher: eighteen divided by three gives what?
49. Era: six
50. Teacher: and there are how many threes in eighteen?
51. Era: six
52. Teacher: so it is three times what that will gives you eighteen?
53. Era: six
54. Teacher: is that what you have written here? Because I see you have just written three times four instead of six
55. Teacher: ok how many fours are in twenty – four?
56. Era: six
57. Teacher: what number times four gives twenty – four?
58. Era: six
59. Teacher: is that what you have written here? Your things are just not correct.

60.	Teacher:	Naf! Do you now know what division is? If they say divide do you know how to divide? What is division? When a person divides what do they do?
61.	Naf:	it is like having thirty things shared by two people of whom each would have twenty.
62.	Teacher:	each would get what?
63.	Naf:	twenty
64.	Teacher:	ok I want you to go and take thirty stones from the box, start sharing them to two people and see how many each would get?
65.	Teacher:	(asking another learner) the question I want to ask you is this, do you think division and multiplication are related?
66.	Naf:	yes
67.	Teacher:	what is this relation?
68.	Naf:	if I divide twenty
69.	Teacher:	no, let us take or use this eighteen, how many three are in eighteen?
70.	Naf:	six
71.	Teacher:	if we move to multiplication, it will be three times what to give you eighteen?
72.	Naf:	six
73.	Teacher:	so what is common there in both methods?
74.	Naf:	is eighteen
75.	Teacher:	so the eighteen you were dividing and the eighteen the product at multiplication are the same right?
76.	Naf:	yes
77.	Teacher:	what did you use when you divided?
78.	Naf:	stones
79.	Teacher:	do you think your answers are correct?
80.	Naf:	yes
81.	Teacher:	what makes you think that your answers are correct?
82.	Naf:	is because the groups shared equal portions
83.	Teacher:	what else?
84.	Naf:	it is also the things I was sharing get finished.
85.	Teacher:	you gave me an example of twenty things shared by two people, can you tell me how one can divide this in two way?
86.	Naf:	is when you share things one by one, giving one to a person at a time
87.	Teacher:	or dividing how?
88.	Naf:	or giving ten to one person at a time and ten to another
89.	Teacher:	(asking another learner) do you now know what division is?
90.	Learner:	yes
91.	Teacher:	what is it?
92.	Learner:	is like having ten things shared by two people of whom each would get five things.
93.	Teacher:	ten things shared by two of whom each would get five, give me another example because I do not want an example ten, five and two
94.	Learner:	is like having seventy things shared by seven people of whom each would get ten things
95.	Teacher:	good, now the question is let me say you have seventy things to be shared by seven people, how do you share those seventy things?
96.	Learner:	you can share by giving people each one by one until all the seventy things are equally shared and finished.
97.	Teacher:	or divide how?
98.	Learner:	or divide in fives, fives, fives
99.	Teacher:	why used five when your example does not have five, it have seventy, seven and ten
100.	Learner:	or taking out ten give them to one person, take out another ten and give them to another person, keep taking ten giving to people until all the seventy is distributed.
101.	Teacher:	how did you get these answers?
102.	Learner:	I calculated
103.	Teacher:	do you think that your answers are correct?
104.	Learner:	yes

105.	Teacher:	what makes you think that your answers are correct?
106.	Learner:	is because I calculated
107.	Teacher:	yes I know you calculated but even calculation can be wrong you know, what makes you think that your answers must be just correct?
108.	Learner:	I used dots
109.	Teacher:	how?
110.	Learner:	I took eighteen dots and keep cancelling three, three, until eighteen get finished and count how many threes I got out which were six.
111.	Teacher:	how did you get these answers?
112.	Learner:	I counted
113.	Teacher:	how? did these drawing helped you to get an answer?
114.	Learner:	yes
115.	Teacher:	how many threes are in eighteen?
116.	Learner:	six
117.	Teacher:	ok, how did the drawing helped you see that?
118.	Learner:	I make eighteen dots and keep taking out threes, threes, threes until the threes were six.
119.	Teacher:	does that mean it makes you see how many threes are in eighteen rights?
120.	Learner:	yes
121.	Teacher:	Dav! Can you tell me what division is?
122.	Dav:	yes
123.	Teacher:	what is division?
124.	Dav:	is like having twenty things shared by five people of whom each would get five things
125.	Teacher:	each would have five?
126.	Dav:	yes
127.	Teacher:	twenty things shared by five people and each would have five again? (pause)
128.	Teacher:	aah! Go and take twenty stones there and start sharing them into five groups and see how many each group would get?
129.	Teacher:	(asking another learner) can you now tell me what division is.
130.	Dos:	yes
131.	Teacher:	what is division? Or what do you do when dividing?
132.	Dos:	is like having eighty things shared by ten people of whom each would get twenty things.
133.	Teacher:	twenty is only if you are to share eighty by four people.
134.	Dos:	one
135.	Teacher:	o! If each one gets one that means things were just ten, how many tens are in eighty?
136.	Dos:	they are forty
137.	Teacher:	I want you to be taking out the tens and see what they will give you.
138.	Dos:	they are eights
139.	Teacher:	good, each would get eight things; you said is like having eighty things shared by ten people of whom each would get eight things. If you are to divide what do you do? If you are to divide what do you do? (Pause) close your eyes and imagine you are dividing eighty things to ten people (pause) what did you see?
140.	Dos:	you divide giving them one by one to each person at a time until all get finished.
141.	Teacher:	how else can you divide that?
142.	Dos:	or giving eight to one person, eight to another person, keep giving eight to people until all the eighty things are equally distributed
143.	Teacher:	what did you used when you got these answers?
144.	Dos:	I used the dots
145.	Teacher:	how did you use them?
146.	Dos:	I made twenty – four dots and share them to six and each got four dots
147.	Teacher:	can you see the answer at division side?
148.	Dos:	yes
149.	Teacher:	did they help you get the answer at multiplication side?
150.	Dos:	yes

151. Teacher: how did they help you? Take one example
152. Dos: is like taking eighteen
153. Teacher: what is in eighteen?
154. Dos: there are three six in eighteen
155. Teacher: and how many threes are in eighteen?
156. Dos: they are six
157. Teacher: did it help you at multiplication?
158. Dos: yes
159. Teacher: how did you get the answers here? Or what did you used?
160. Dos: I compute them, I did not use stones or dots
161. Teacher: do you think this answer is correct? This of twelve divided by three equal to four, is it correct?
162. Dos: no
163. Teacher: why do you think is not correct? go and take twelve stones divided them into four groups and find out how many stones each group would get (the learner counted and found out that the answer he doubted is true)
164. Teacher: Nab! Do you now know what division is
165. Nab: yes
166. Teacher: ok, when you divided what do you do?
167. Nab: is like dividing sixty sweets into three people of whom each would get thirty.
168. Teacher: thirty, I don't think so because thirty there three times is ninety and not sixty how many each would have?
169. Nab: each would get ten
170. Teacher: but ten there three times is thirty and not sixty. Each would get how much?
171. Nab: twenty
172. Teacher: good, now tell me if you have sixty things sharing them to three people how do you do it? (Pause) what do you do?
173. Nab: I took sixty and started sharing to three people one by one at a time until all the sixty things are equally shared up.
174. Teacher: ok, how else do / can you divide that sixty? How else do / can you divide that sixty? How else do / can you divide that sixty?
175. Nab: or taking out twenty and give it to one person, another twenty and give it to another person and keep taking twenty giving to people until the whole sixty is equally distributed amongst people
176. Teacher: do you think your answers are correct?
177. Nab: yes
178. Teacher: because of what?
179. Nab: of the fours
180. Teacher: ok how many fours are in twelve?
181. Nab: there are five fours
182. Teacher: five fours
183. Teacher: they are three
184. Teacher: when you subtracted three fours from twelve did twelve get finished or there was anything left?
185. Nab: twelve get finished
186. Teacher: (asking one learner) do you now know what division is?
187. Learner: is like having sixty things shared by two people of whom each would get thirty things.
188. Teacher: if they say share this sixty things to two people, how many will you give to each? Or how will you share to them?
189. Learner: I will take the sixty things and start taking thirty giving to one child and take another thirty again and give it to another child.
190. Teacher: ok, how else would you do that apart from giving out thirties
191. Learner: I would give them one by one until the sixty is equally shared
192. Teacher: how did you get these answers? Did you use stones, dots or what?
193. Learner: is dots

194. Teacher: dots, ok did the answer in the first question helped you to get answers in the second question
195. Learner: yes
196. Teacher: how did it helped you
197. Learner: is because I used dots in both
198. Teacher: how many threes are in eighteen
199. Learner: three
200. Teacher: these threes, how many are they in eighteen?
201. Learner: three
202. Teacher: three? Look at what I am pointing, can you see this eighteen?
203. Learner: yes
204. Teacher: ok, how many threes are in eighteen
205. Learner: six threes
206. Teacher: ok, in this thirty how many fives are there?
207. Learner: they are six
208. Teacher: six, ok we have thirty, five and six
209. Learner: yes
210. Teacher: that is why I asked, that why do we have this similarities at both division and multiplication? (Pause) ok, how many fives are in thirty?
211. Learner: six
212. Teacher: and what number times five gives thirty? If let me say you are adding fives, fives, how many should they be to get thirty? If let me say you are adding fives, fives, how many should they be to get thirty? (Pause) o! Child how many fives did you say are in thirty?
213. Learner: six
214. Teacher: ok, now I am asking, how many fives are needed to make thirty?
215. Learner: six
216. Teacher: that is why I asked did division help you get the answers at multiplication?
217. Learner: yes
218. Teacher: do you think these answers of yours are correct?
219. Learner: yes
220. Teacher: what makes you think that your answers are correct? (Pause) what makes you think that twelve divided by four is three and is correct?
221. Learner: is because it is not finished
222. Teacher: what do you mean that it is not finished? (Pause)
223. Teacher: (asking another learner) do you know now what division is?
224. Learner: yes
225. Teacher: if they say divide, what do you do? When a person is dividing what do they do?
226. Learner: is like having eighty things shared by two people of whom each would get forty things.
227. Teacher: if they say divide this eighty things to two children, how would you do it or what would you do?
228. Learner: I would first take out forty things and give them to one child and again take another forty things and give it to another child.
229. Teacher: or how else do you divide it?
230. Learner: sharing by giving one by one to each learner at a time until eighty things are equally shared and things to be shared are finished.
231. Teacher: when calculated these answers, what did you used?
232. Learner: is dots
233. Teacher: how did you get answers at question two for multiplication? (Pause)
234. Learner: I used the dots
235. Teacher: (asking one learner) I can see there are similarities on question one and two. How are they related to one another? (Pause) how did the answers in question one helped you get the answers in question two?
236. Learner: I used dots

237. Teacher: so you also used dots
238. Learner: yes
239. Teacher: ok, what makes you think that you answers are correct?
240. Learner: because we were dividing
241. Teacher: yes we know we were dividing (pause) what make you think that you are just correct? (Pause)
242. Teacher: Pet do you now know what division is?
243. Pet: yes
244. Teacher: when one is dividing what do they do?
245. Pet: is like having hundred shared by two people of whom each would get one?
246. Teacher: if things are hundred and people sharing they are just two each would get one?
247. Pet: fifty
248. Teacher: if you were to divide hundred things to two people how would you divide the things?
249. Pet: I will divide by giving each one by one until all the hundred things are shared equally amongst two people
250. Teacher: and how else can you divide?
251. Pet: by taking fifty out of hundred and give it to one child and another fifty and give it to another child.
252. Teacher: do you think your answers are correct?
253. Pet: yes
254. Teacher: what makes you think that your answers are correct?
255. Pet: is because we were dividing.
256. Teacher: yes, I know we were dividing but what makes you think that you answers are correct? (Pause) how did you get your answers? Or what did you used was it stones or dots or did you just compute?
257. Pet: I used stones
258. Teacher: stones ok, so you took twelve stones and divided into four groups, so what makes you think your answers are just correct?
259. Pet: because all groups shared equal number of stones
260. Teacher: what else? (Pause)
261. Teacher: (asking another learner) what did you used to get these answers, was it stones, dots or what?
262. Learner: I used stones
263. Teacher: did the answers in the first question helped you to get answers in the second question?
264. Learner: yes
265. Teacher: how?
266. Learner: is because we worked with the same number eighteen at both side of division and multiplication
267. Teacher: what else
268. Learner: and the three we used at division is the same as the three in multiplication.
269. Teacher: and how many threes are in eighteen?
270. Learner: six
271. Teacher: ok (asking Dav) if you have twenty things divided by five each would get what?
272. Dav: four
273. Teacher: ok, how do you divide those twenty things to five people?
274. Dav: I would be sharing things one by one at a time until twenty things get finished.
275. Teacher: how else can you divide these twenty things?
276. Dav: I would be taking four things at a time and give them to one person, take another four things and give them to the second person so I will keep doing that until all twenty are equally distributed.
277. Teacher: do you think that your answers are correct?
278. Dav: yes
279. Teacher: why do you think that your answers are correct?
280. Dav: is because there are only six threes in eighteen

281.	Teacher:	but this is not what you have written, you have written eighteen, nine and two. I am asking this two works of yours, do you think these two answers are correct? Because in this work there is no three and six but rather nine and two.
282.	Dav:	is because if eighteen is divided into nine groups each would just have two stones.
283.	Teacher:	what did you used when calculated here?
284.	Dav:	is stones and dots
285.	Teacher:	(asking Dos) what did I say you go and do?
286.	Dos:	to take twelve stones and divide them into four groups
287.	Teacher:	ok, each group has how many?
288.	Dos:	each had three stones
289.	Teacher:	just the same things you have written on your paper rights?
290.	Dos:	yes
291.	Teacher:	I knew it all along even when I sent you to get the stones to do the division, you had the correct answer but you doubted your answer.