AN INVESTIGATION OF THE ROLE OF PHYSICAL MANIPULATIVES IN THE TEACHING AND LEARNING OF MEASUREMENT IN GRADE 8: A CASE STUDY- USING SURFACE AREA AND VOLUME

A thesis submitted in partial fulfilment of the

requirements of the degree of

MASTER OF EDUCATION

(Mathematics Education)

Of

RHODES UNIVERSITY

by

SHAKESPEAR M.E.K CHIPHAMBO

NOVEMBER 2011

ABSTRACT

The purpose of this study is to investigate the role of physical manipulatives in the teaching and learning of measurement in Grade 8. The study focuses on how the use of physical manipulatives promotes learners' mathematical proficiency in relation to the five strands of Kilpatrick, Swafford and Findell (2001). The basis of the research is a case study in the interpretive paradigm involving 18 out of a cohort of 270 Grade 8 learners in the school where I teach.

The data was collected using a range of methods including: (i) baseline assessment tasks, first piloted using 7 Grade 8 learners and then given to the target group; (ii) an intervention programme with intervention tasks; (iii) a post-intervention task; (iv) observations during the intervention; and (v) individual interviews. The results of the baseline assessment and the post-intervention tasks were analysed both quantitatively and qualitatively.

My research findings indicate an overall improvement of the performance after learners engaged in using physical manipulatives. The average mark of the learners in the baseline assessment task was 23% and after the intervention programme the average mark was 31%. The responses from the learners interviewed showed that they were motivated and that the use of physical manipulatives assisted them in understanding the concepts of measurement, in particular surface area and volume. The results of my study thus reveal that the use of physical manipulatives in teaching and learning mathematics has a positive role to play in learners' understanding of surface area and volume at the Grade 8 level.

The findings of this case study support other research regarding the importance of using physical manipulatives in teaching and learning mathematics. They align with other findings that assert that manipulatives are essential mediating tools in the development of the conceptual and procedural understanding of mathematical concepts, clarifying and helping learners to visualize abstract mathematical concepts.

Key words: physical manipulatives; measurement; surface area; volume.

ACKNOWLEDGEMENTS

I am greatly indebted to the Lord God my Creator for His providence throughout my life in general and education life in particular. He has taken me by His hand and guided me to Masters Level, may His name be glorified.

In addition, I wish to give special recognition to the following individuals for the role they have played in my life of study:

My wife, Jennifer and my children for being so supportive with their encouraging words of saying "we are looking up to you do not give up studying.

My MEd supervisor Prof. Marc Schafer, for giving me the appropriate advices and information required in order to progress from one step to another during my years of study at Rhodes University.

Rob Kraft for his academic writing support and emphasis on the kind of information required for my thesis.

Kachoka family for its support ever since I came to South African. Their support paved the way for me to achieve the MEd programme.

Kwa-komani Comprehensive School principal, staff, school governing board and parents for granting me permission to conduct the research at their school.

And lastly, my HoD, Mr Matyedi who validated the baseline assessment as well as the post-intervention task for my research study.

DEDICATION

This thesis is dedicated to my immediate family for they sacrificed their time and resources in support of my studies.

DECLARATION OF ORIGINALITY

I, Shakespear M. E.K. Chiphambo declare that this thesis is my own work written in my own words. Where I have drawn on the words or ideas of others, these have been acknowledged using the references practices according to Departmental Guidelines.

Shakespear Chiphambo 30 November 2011

Signature:______________________________

TABLE OF CONTENTS

CHAPTER THREE: RESEARCH DESIGN AND METHODOLOGY

APPENDICES

LIST OF TABLES

Table Page

LIST OF FIGURES

ACRONYMS AND ABBREVIATIONS

1. BA	: Baseline Assessment
2. CITEd	:Center for Implementing Technology in Education
3. CRA	: Concrete Representational Abstract
4. DoE	: Department of Education
5. DSMATE	: Department of Science, Mathematics and Technology Education
6. FiMs	: Foebel-inspired Manipulatives
7. HoD	: Head of Department
8. MiMs	: Montessori-inspired Manipulatives
9. NCTM	: National Council of Teachers of Mathematics
10. PI	: Post-intervention
11. WCDoE	: Western Cape Department of Education

CHAPTER ONE: INTRODUCTION AND OVERVIEW

1.1 INTRODUCTION TO THE CHAPTER

In this chapter, I present the background of my research study, which aims to discuss the use of physical manipulatives in the teaching and learning of measurement. I also present the research problem and the rationale for my study. In addition, I discuss the context of my research study and its philosophical underpinnings. Thereafter I present a brief overview of the research methodology and its design. Lastly, I discuss some limitations of the study and provide an overview of the thesis.

1.2 BACKGROUND OF MY RESEARCH STUDY

Measurement is a critical topic in mathematics that "offers an opportunity for learning and applying other mathematics topics including number operations, geometric ideas, statistics, etcetera." (Clement & Bright, 2003, p. xi). It also enables learners to find out, to envision, locate and transform mathematical ideas (Department of Education [DoE], 2003). In order for learners to understand the concept of measurement effectively, the process of location, visualization and transformation needs to be attached to real life situations, for example, "measuring and comparing distances and time taken by learners from home to school, comparing distribution and allocation of land areas to population sizes and comparing the capacity of dams and the volume of water available through taps in a particular community" (South Africa. DoE, 2002). Although measurement is considered to be an umbrella topic in mathematics, research has shown that many learners find it an aspect of mathematics that is difficult to grasp and "do not [even] understand the attribute being measured or the units that are used for measurement" (Goos & Markar, 2008, p. 391).

Such evidence not only raises questions about the learning of measurement but also "raises questions about the effectiveness of the instruction that students are receiving in the content area" (Goos & Markar, 2008, p. 391). For example, students find it particularly difficult in finding the surface area and volume of a given object. Van de Walle (2004) argues that when the students are only taught the performance of the skills of a particular procedure they become reluctant to attach meaning to it. This problem poses many challenges to mathematics teachers. In order to respond to this challenge, (Penlington, 2009, unpaged) recommends that teachers should make use of "effective mathematical instruction that includes literal use of manipulative materials" In addition, in order to assist learners to differentiate between surface area and volume, "manipulatives can be used to clarify these misconceptions and build connections between mathematics concepts and representations, fostering more precise and richer understanding" of these concepts (Center for Implementing Technology Education Research Center, 2010, p. 1).

1.3 THE RESEARCH PROBLEM

Fundamental to the effective learning of measurement is conceptual understanding of concepts such as surface area and volume. The use of physical manipulatives lends itself well to the teaching of surface area and volume, and this research study aims to gain insight into the role of physical manipulatives in the teaching and learning of measurement in Grade 8 in the form of a case study using surface area and volume.

1.4 CONCEPTUAL FRAMEWORK

My research study is framed by an intervention programme that makes extensive use of physical manipulatives in the teaching and learning of measurement. I investigate how the use of physical manipulatives has an impact on the learning of measurement (specifically, surface area and volume) by using Kilpatrick, Swafford and Findell's (2001) model of mathematical proficiency. According to Kilpatrick, et al. (2001), mathematical proficiency comprises 5 interwoven strands:

1. Conceptual understanding which refers to the ability to represent mathematical ideas in different ways.
2. Procedural fluency which refers to the skills in performing procedures flexibly, accurately and efficiently.
3. Strategic competence which is the ability to represent mathematics numerically, symbolically, verbally and graphically.
4. Adaptive reasoning which refers to correct reasoning and justification of conclusion.
5. Productive disposition which refers to learners' passion, enthusiasm and recognition of the benefits and usefulness of mathematics. (p. 10)

This framework forms the basis of my analysis when examining the effect of my intervention on the mathematical proficiency of the learners with specific regard to learning about surface area and volume.

1.5 THE RESEARCH PROCESS

My research study is oriented in an interpretive paradigm which allowed me to explore how participants work with physical manipulatives in solving surface area and volume problems. I carried out my research at the school where I teach in the Queenstown District in the Eastern Cape of South Africa.

My research sample consisted of 18 Grade 8 learners and the research process was designed in 4 phases namely:-

Phase 1: Pilot and baseline assessment (Pre-test). A group of 7 Grade 8 learners was used to pilot the baseline assessment task. The task was subsequently administered to a group of 18 participants who were engaged in my research project.

Phase 2: Design of the intervention programme. The results of the baseline assessment task informed the final design of the intervention programme. Suitable intervention strategies were designed focussing on addressing the misconceptions that learners had in learning measurement, in particular surface area and volume of a cube and rectangular prism.

Phase 3: Administering of intervention tasks and observations. The intervention that was used had 4 tasks specifically focussing on informal and formal calculations of area, surface area and volume of prisms. An observation schedule with criteria aligned to the five strands of mathematical proficiency espoused by (Kilpatrick, et al. 2001) was used to evaluate learners' performance, the details of which are in chapter 3.

Phase 4: Post-intervention task similar to the pre-test was used to explore whether the intervention programme had had an impact on learners understanding of the surface area and volume in relation to a cube and a rectangular prism.

Thereafter, six learners were selected and I conducted semi-structured interviews with them in order to investigate how they used physical manipulatives in learning about surface area and volume.

Analysis of the data

The pilot task was analysed graphically. The baseline assessment and the post-intervention tasks were both analysed qualitatively and quantitatively. There was a comparison of how learners performed initially in the baseline assessment compared with the post-intervention.

For both tasks bar graphs were used to present the data for each of the items followed by a general discussion. In order to make the analysis of my observations as useful as possible, I used thick descriptions. The data from the semi-structured interviews data were categorised into themes (Davies, 2009).

Ethical issues

To ensure that my research study was undertaken in an ethical manner the following steps were taken (i) letters requesting consent to do the research at my work place were written and sent to the principal of the school and the school governing board, (ii) parents and guardians of the participants were also requested to sign the letter authorising their children's involvement in the research study, (iii) participants were made aware that their participation in the project was voluntary, (iv) learners were put at ease in my presence when they were doing the tasks, (v) participants were also informed that only their parents had the right to withdraw them from the study at anytime.

1.6 SIGNIFICANCE OF MY RESEARCH STUDY

The use of physical manipulatives can have a positive impact on the teaching and learning of measurement in all Grades of mathematics. This study could form a springboard to help all the learners in other grades to address their misconceptions in measurement, especially surface area and volume.

I hope that my research study will inspire other teachers to incorporate physical manipulatives in their teaching and learning of measurement.

1.7 LIMITATIONS

The limitations of my study were three-fold. Firstly, the sample size of my research was small so the results can thus not be generalised. Secondly, the research study was conducted within a limited time frame. Due to the busy programme at the school, I had to do my research after school hours when the learners were tired. Lastly, the number of physical manipulatives was limited as a result they could not be used by all the learners individually.

1.8 THESIS OVERVIEW

1.8.1 Chapter one

In chapter one, I present the background of the study which includes (i) my research problem and its rationale, (ii) the research context and its philosophical underpinnings, (iii) an overview of the research methodology and design, (iv) some limitations of the study and an overview of the thesis.

1.8.2 Chapter two

Chapter two reviews the literature relevant to my study of the role of physical manipulatives in the teaching and learning of measurement in Grade 8 mathematics.

1.8.3 Chapter three

In chapter three, I present a detailed description of the research design. This includes (i) the methods applied to collect data, (ii) selection of the participants, (iii) the sampling techniques applied, and (iv) a description of the instruments used in gathering data.

1.8.4 Chapter four

Chapter four presents the data analysis and discussion of the findings. The chapter specifically examines the role of physical manipulatives in learning measurement with understanding in the context of this study.

1.8.5 Chapter five

In chapter five, I present a summary of the main findings of my research study. I also discuss the limitations of the study, and offer some recommendations and ideas for further research.

CHAPTER TWO: LITERATURE REVIEW

2.1 INTRODUCTION

This chapter reviews the literature relevant to my study of the role of physical manipulatives in the teaching and learning of measurement in Grade 8 mathematics in particular my study aims to discover whether the use of physical manipulatives can be effective in the teaching and learning of measurement, specifically in finding the surface area and volume of cubes and rectangular prisms

Research done by Pestalozzi during the nineteenth century shows that the use of physical manipulatives in teaching and learning has a long history. Sowell (1989) noted that:

> Since ancient times, people of many different civilizations have used physical manipulatives to help them solve everyday mathematical problems, for example the Middle - East used counting boards; the Romans modified the counting board to create the world's first abacus. The Chinese and the Americans developed similar devices. The Mayans and the Aztecs both used corn kernels. The Incas used knotted string called quipu. The uses of manipulative materials were also included in the activity curricular of the 1930s. (p. 2)

The first true manipulatives were invented in the late 1800s. These took the form of maneuverable objects appealing to several different senses and were specifically designed for the teaching of mathematical concepts. Suydam (1984) reported that Friedrich Foebel in 1937 started the world's first kindergarten programme in 1937 to help his learners recognize and appreciate the geometric patterns found in nature. Since the early 1900s, Italian Maria Montessori advanced the idea further and designed many materials to help preschool learners discover and learn the basic ideas in mathematics and other subjects. Manipulatives are considered essential tools in teaching mathematics at the elementary school level; and the National Council of Teachers of Mathematics (NCTM) has always recommended the use of manipulatives in teaching mathematical concepts at all grade levels.

Some researchers commented that physical manipulatives are beneficial particularly for learning in young children, but are unnecessary for older children (Fennema, 1972; Wilkinson, 1974 & Friedman, 1978). On the other hand, Vance and Kieren (1971) discovered that regardless of their age learners learn mathematics well in laboratory settings where physical manipulatives are present. This led them to conclude that physical manipulatives enhance the teaching and learning of mathematics among the learners who are engaged in the activity.

Since my research findings will be analysed based on the five strands of Kilpatrick, et al.'s (2001) theory, I present the literature that support each of these strands on mathematical learning using physical manipulatives.

2.2 DEFINITION OF MANIPULATIVES.

According to Heddens (1986), Sowell (1989), Moyer (2001) and Van de Walle (2004), the term manipulative is applied to any object, picture or drawing that represents a mathematical concept or onto which the relationship for that concept can be imposed. Zuckerman, Arida and Resnick (2005) further defined manipulatives as physical objects that are specifically designed to foster learning in a teaching and learning environment.

Different researchers use different terms, for example, Van de Walle (2004) calls them concrete materials, Zuckerman, et al. (2005) refer collectively to these concrete materials as manipulatives. Kilpatrick, et al. (2001) used the following terms interchangeably: physical (concrete) materials or physical models or manipulatives. For the sake of consistency, in my study I call them physical manipulatives.

2.3 CLASSIFICATION OF MANIPULATIVES

Zuckerman, et al. (2005) broaden the perspective by classifying manipulatives into the following:

1. Foebel-inspired Manipulatives (FiMs)-these are designed materials used to foster modeling of real world structures, for example, using wooden blocks to build a structure that resembles a castle.
2. Montessori-inspired Manipulatives (MiMs)-they are designed to foster modeling of more abstract structures, for example, arranging Cuisenaire rods in different configuration that make numerical proportions. (p.1) MiMs can be both physical as well as digital. "Digital MiMs are computationally enhanced version of the physical objects.They involve physical construction, but the result of the construction is a simulation of generic structure rather than the real world example." (p. 2)

2.3.1 PHYSICAL MANIPULATIVES IN THE SCHOOL CURRICULUM

In this section, I present how the South African education system views physical manipulatives as essential tools for teaching and learning mathematics.

The Western Cape Department of Education [South Africa. WCDoE] (2000) views mathematics as:

> A human activity that involves: observing, representing, and investigating patterns and quantitative relationships in physical and social phenomenon and between mathematics objects themselves. Through this process new mathematical ideas and insights are developed. (p. 4)

The definition of mathematics embraces the notion that in order for the learners to develop new mathematical ideas and insights they must be engaged in activities that let them move from what Suydam (1984) referred to as:

> The concrete stage where the learners explore the concepts using [physical] manipulatives in a purposeful activity, through a representational stage where they visualise and communicate the concept at a pictorial level to the abstract where mathematical symbols (numerals, operations, etc.) are used to express the concept in symbolic language. (p. 3)

Suydam (1984) and Heddens (1986) describe that the transition is made possible when physical manipulatives from the learners' real world are used to represent mathematical ideas in a way that can clarify the ideas more simply than without them.

Secondly, the NCTM (2000) noted that the use of physical manipulatives is not only relevant to one specific mathematical topic and a particular level of learners. But they can be used in teaching mathematical concepts in all the grade levels and in different topics, for example topics such as categorizing, ordering, distinguishing patterns, recognizing geometric shapes and understanding relationships among them, exploring and relating spatial relationships, engaging in problem solving, learning about and investigating with transformations (NCTM, 2000 & WCDoE, 2000). In my study for example, learners worked out the volume of the boxes provided by filling them with dice. This showed the learners that the number of dice each of the boxes contains informally represents its volume.

In addition, Brooks and Brooks (1993) suggested that meaningful curriculum activities rely heavily on the use of physical manipulatives. The use of physical manipulatives to represent mathematical ideas is an effective practice for all learners regardless of their age or abilities (Bryant, Bryant, Kethley, Kim, Pool & Seo, 2008).

Kilpatrick, et al. (2001) recommend the school curriculum that promotes the use of physical manipulatives for the following reasons:

> (i) They provide a skill of linking informal knowledge and experiences to mathematical abstraction; (ii) they can be used as mathematical representations that help in clarifying ideas in ways that support reasoning and building understanding;

(iii) they support the development of efficient algorithms for basic operations; (iv) they enhance learners' conceptual understanding; (v) students are helped to correct their errors; and they enable learner to learner and teacher to learner conversation . (pp. 9, 31, 67, 353)

2.4 PHYSICAL MANIPULATIVES FOR CONCEPTUAL UNDERSTANDING

In this section, I show how physical manipulatives promote conceptual understanding in mathematics, "conceptual understanding refers to an integrated and functional grasp of mathematical ideas" (Kilpatrick, et al. 2001, p. 118). It is has been argued that learners with conceptual understanding know that mathematics is a subject of inter-related concepts which are not isolated; they have good retention of knowledge and can apply previous knowledge to new situations.

Mathematical concepts are explained by the use of representations. Representations are described as symbols or relationship of symbols, characters, or items that represent a certain mathematical idea (Goldin & Shteingold, 2001). This means physical manipulatives can work as "the mental tools view [that helps learners to learn mathematics]" (Chao, Stigler & Woodward, 2000, p. 285), for example:

> Stern's (1949) pattern board consists of a set of 10 wooden boards with fixed grooves into which blocks fit as a way of representing numbers from 1 to 10. This helps learners to understand the structure of numbers and acquire calculation skills. (p. 286)

Secondly, physical manipulatives can also be used as "the abstraction view" (Chao, et al. 2000, p. 285). Dienes (1964) defines the abstraction view as learners' capacity to theorize the mathematical relation of interest from a diversity of physical representations.

Clement (1999) also argues that [physical] manipulatives:

> Aid students in building, strengthening and connecting various representations of mathematical ideas that are meaningful to the learner, promote control and flexibility to the learner. And they are also consistent with cognitive and mathematics structures. (p. 50)

On the other hand, Greeno and Riley (1987) noted that physical manipulatives help learners to create and develop a mental representation of the necessary mathematical information that "bridges the gap between informal and formal mathematics" (Boggan, Harper & Whitemire, 2007, pp. 2 &3). For example, physical manipulatives must give meaning to the concept they represent or the one that needs to be clarified (Uttal, Scudder & Deloache, 1997). This implies that teachers must make sure that learners have to visualize the mathematical concept being addressed through the use of physical manipulatives.

It has been further argued that physical manipulatives also promote learners' conceptual development in mathematics (Kilpatrick, et al. 2001). For example, by using different types of physical manipulatives, learners have an opportunity of classifying, measuring, ordering, counting and using fractions (Wolfgang, Stannard & Jones, 2007). These opportunities are discussed by Pawat (1992) and Van de Walle (2004).

Prawat (1992) highlights the fact that learners' engagement through the use of physical manipulatives is considered not only necessary but also an essential condition for worthwhile learning which leads to conceptual development. Worthwhile learning entails learning that provides the logical contexts for students' abstract advancement (Cai, 2003). Chester, David and Reglin (1991) argue that using physical manipulatives help learners get connected to the real-world situations which eventually afford them the opportunity of worthwhile learning.

On the other hand, Van de Walle (2004) claims that the development of new concepts in learners takes place as learners use physical manipulatives to think and reflect on new ideas that emerge when they "[justify] their mathematical reasoning" (Heddens, 1997, p. 3). For example, "when learners have celery for a snack, they may develop an informal measurement system that helps them compare the length of celery pieces" (Maxim as cited in Stake, 1978, p.382). Gentner and Ratterman (1991) note that such extensive instruction and practice give learners opportunities to perceive and understand relationships between physical manipulatives and other forms of mathematical expressions.

Physical manipulatives should also be able to enhance learners' understanding of mathematical concepts. Understanding in this regard refers to "both constructing meaning and constructing systems of meaning" (South Africa.WCDoE, 2000, p. 3). Carpenter and Lehrer (1999) an the other hand, define understanding as the mental activity for the development of mathematical relationships and principles that are established when the learners get involved in the explanation of problems without being taught what to do in order to arrive at an answer.

Various findings world wide, consistently support the notion that the use of physical manipulatives in teaching and learning of mathematics is inextricable and cannot be divorced from mathematical content knowledge (Moyer, 2001; Clements & Bright, 2003). This implies that mathematics teaching must always be supported by the use of physical manipulatives to avoid learners being left mathematically handicapped.

Romberg and Kaput (1999) argue that the use of physical manipulatives in teaching and learning mathematics allows the exploration of problems as a way of learning mathematics for understanding. The assertion is that the use of physical manipulatives set a different view of how mathematics has to be taught and learnt as they "foster more precise and richer understanding [of mathematical concepts]" (Cited Research Center, 2010, p. 1).

Kilpatrick, et al. (2001) concluded that learners who use physical manipulatives in learning perceive mathematics as the subject of reasoning and intelligible which enable them to associate their informal facts and experiences to mathematical abstraction.

In an inquiry done by Suydam (1984) it was noted that physical manipulatives contribute to the development of well-grounded, interconnected understandings of mathematical concepts which can easily be reorganised when forgotten. Stein and Bovalino (2001) and Kilpatrick, et al. (2001) concurs with the findings saying that learners with such understanding can integrate and establish links between wide ranges of mathematical concepts. This type of understanding is called "integrated concrete knowledge, which is the knowledge built as we learn". (Clement, 1999, p. 48). An example of this is when, Sue McMillen's son, Jacob was asked to give the answer to ¾ + ¾. He solved the problem by thinking about the fractions in terms of money "75c plus 75c is $1.50, so ¾ + ¾ is 1½" (Clements & McMillen as cited in Clements, 1999, p. 48). Clements (1999) further argued that:

> For [learners] with this type of knowledge, physical objects, actions performed on them, and abstractions are all interrelated in a strong mental structure. Ideas such as '75,' '¾,' and 'rectangle' become real, tangible and strong as a concrete sidewalk. (p. 48)

Jacob's method of solving the problem shows that understanding and reasoning in mathematics can be achieved when the appropriate skills of solving mathematical problems acquired through the use of physical manipulatives is applied. Cass, Cates, Smith and Jackson (2003) suggest that it is important to identify the instructional strategies in order to facilitate knowledge acquisition, retention and generalization of mathematical skills.

With the use of physical manipulatives the instruction changes from the one that Romberg and Kaput (1999) refer to as:

> The three- segmented lesson which involves an initial segment where the previous day's work is corrected and next the teacher presents new material, often working one or two new problems followed by a few students working similar problems on the chalkboard. The final segment involves students working on an assignment for the following day. (p. 4).

More open ended activities that appeal to several senses such as, touching, visual, auditoria, etcetera contributes to learners' diminution of errors and increases in their scores on tests that require them to solve problems through investigation (Carrol & Porter, 1997; Clements, 1999; Sebesta & Martin, 2004).

Research points to the fact that physical manipulatives promote a child-centred lesson where the learning experience is confronted, rather than, for example, experiences like over emphasized area and volume formulae that confuse learners and inhibit learning measurement with understanding. Cited Research Center (2010) further argues that this confrontation fosters more precise and richer understanding about the concept being learnt because learners will be more engaged in "exploration of the formulae for area measurement" (Huang & Witz, 2010, p. 5). The implication here is that physical manipulatives need to be used in order for the learners to understand the concept of measurement.

According to Cain-Caston (1996) and Heuser (2000), physical manipulatives provide an environment in the mathematics classroom that is conducive to learning , especially when learners get engaged in reflection of their own mathematical experiences in learning mathematics. Uttal, et al. (1997) reveal that using physical manipulatives in reflecting on mathematical experiences helps learners to gain access to concepts, ideas and processes that remained inaccessible throughout the lesson. For that reason physical manipulatives are considered as objects to add support as mathematical lessons unfold (Papert, 1980).

Physical manipulatives are believed to help in "improving learners' retention both in the short and long term as long as they are used in a manner that reflects on their own experiences in learning mathematics" (Baggon, et al. 2007, p. 4). Furthermore, other research reveals that physical manipulatives are recommended as a means of improving learners' achievement for all levels which includes the range of the developmentally – slow learners to the able ones (Peterson, Mercer & O'Shea, 1998).

Uttal, et al. (1997) notes that the effective use of the physical manipulatives provides a way through the opaqueness of mathematical symbols and concepts, for example, by helping learners to develop new concepts, make connections between concepts and symbols and assess their understanding of the concepts being taught (Van de Walle, 2004).

On the other hand, physical manipulatives give the teacher an opportunity to "present learners with materials, situations and experiences that allow them to discover learning" (South Africa. WCDoE, 2000, p. 25). Unlike in the traditional method of teaching whereby rote learning is promoted and there is no learning of mathematical concepts behind the scenes (Hiebert & Wearne, 1992).

Other researchers also have discovered that when learners use physical manipulatives they master the technique of how to count (Clements, 1999). Secondly, their computational skills and retention of important mathematical specifics is supported (Carrol & Porter, 1997). In addition, learners learn to solve problems and their problem solving skills are enhanced (Carrol & Porter, 1997; Clements, 1999; Krach, 1998), therefore physical manipulatives help learners to gain a sound range of algebraic abilities (Chappel & Strutchens, 2001) which can enable them to present mathematical ideas in a variety of ways. Also learners' knowledge of fractions and ratios improves (Jordan, Miller &Mercer, 1998; Sebesta & Martin 2004). All these skills help learners to see the relationship that exists between mathematical concepts within the topic and between concepts in different topics.

Grouwns (1992), Cain-Caston (1996) and Heuser (2000) have also remarked that the use of physical manipulatives promotes mathematical learning and eliminates mathematical anxiety which they define as "more than a dislike towards mathematics" (Vinson, 2001, p. 89). Vinson (2001) adds that this anxiety is eliminated as learners develop their own understanding of mathematical concepts using physical manipulatives.

2.5 PHYSICAL MANIPULATIVES FOR PROCEDURAL FLUENCY

Kilpatrick, et al. (2001) defined procedural fluency:

> As the knowledge of procedures, knowledge of when and how to use them appropriately, estimation and use of skills in performing them flexibly, accurately, and efficiently. Procedural fluency supports mathematical methods that use physical manipulatives to explain certain mathematical concepts. (p. 121).

Van de Walle (2004) argues that, to support learners' procedural fluency, physical manipulatives should be used practically in a way that allows learners to discover mathematical concepts which lead to "conceptualization and understanding" (Heddens, 1997, p. 3). This tells us that physical manipulatives should be used as tools to teach mathematical concepts, so that they can "challenge and invite exploration, speculation and hard work [as learners manipulate them]" (NCTM, 2000, p. 19). Van de Walle (2004) comments that physical manipulatives help learners to explain ideas and clarify abstract concepts that they find difficult. This tells us that physical manipulatives should to be used as tools of facilitation to heighten learning and not to obscure the learning process if learners are to work with mathematical symbols flexibly.

2.6 PHYSICAL MANIPULATIVES FOR STRATEGIC COMPETENCE

"Strategic competence refers to the ability to formulate mathematical problems represent them and solve them. Learners should know a variety of solution strategies, and know which one to use for a specific problem" (Kilpatrick, et al. 2001, p. 124).

Moyer (2001), Girouard, Solovey, Hirshfield, Ecot, Shaer and Jacob (2007) suggest that physical manipulatives can serve as representations of mathematical ideas both explicitly and concretely because teaching entails using representations. For example, presenting the number of learners at a certain school per class graphically. The explicit representation also depends on how the teacher incorporates physical manipulatives into mathematical learning in order to help the learners move between representations to improve the growth and construction of various mathematical ideas. Romberg and Kaput (1999) argue that appropriate incorporation of physical manipulatives encourage learners to reflect explicitly on the characteristics of mathematical representations which are useful for solving problems.

Research reveals that learners "can only do mathematics if their minds have what is known as 'reflective intelligence': which is the ability of the mind to turn away from the physical world to itself" (Skemp, 1964). Lesh, Post and Behr (1987) assert that mathematical ideas have five different representations which are intertwined together, one cannot stand alone, which is shown in the diagram below.

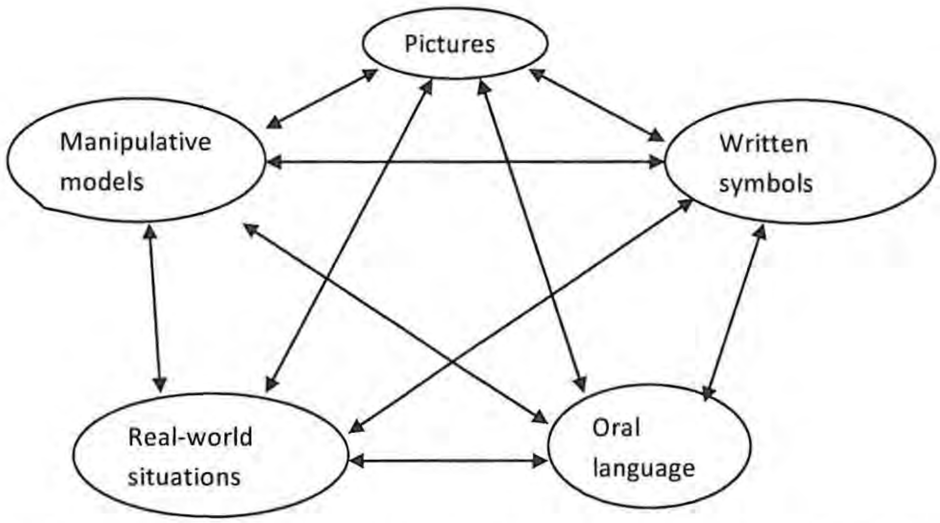


FIGURE 2.1 Five different representations of mathematical ideas (Adapted from Van de Walle, 2004, p. 30)

Van de Walle (2004) suggests that the movement between and among these representations improves the growth of mathematical concepts in learners which eventually enable them to use general principle to solve problems in real life. Lesh, et al. (1987) note that learners who are not able to move from one representation to another are more likely to have problems in solving mathematical problems.

In addition, these representations improve and broaden the development of children's mathematical concepts (Van de Walle, 2004) if they are selected to address the problems that learners have. (Thompson &Van de Walle, 1984; Wirtz, 1978).

Clements (1999) claims that learners need to make use physical manipulatives because they have what is referred to as "a sensory-concrete knowledge that needs to use sensory material to make sense of mathematical ideas." (p. 47)

Another study done by Dutton and Dutton (1991) suggests that teaching for understanding should follow Bruner's theory of the stages of cognition which emphasizes the use of concrete materials. The theory describes that learning starts with the use of semi-concrete or pictorial concepts and then symbolical problems. Kilpatrick, et al. (2001) add that physical manipulatives provide "models of learners' understanding of mathematical concepts in order for them to think mathematically to learn" (p.16).

The argument exists that the teaching and learning of mathematics using physical manipulatives is referred to as learning in action or learning by doing, (Seymour, 1996).

Vinson (2001) adds:

> The type of learning that engages learners in mathematical games, simulations, problem solving activities, discoveries and challenges. When learners are engaged in these types of activities they make meaning of mathematics content that has been learnt. (p. 91)

In support of the learning by doing concept, Piaget (1978) suggests that mathematical problems must be presented in the form of experiment, a real life situation or actual philosophical or theoretical dilemmas, and these problems need to be real, the ones that have been encountered in the history of human knowledge. This simply emphasizes the concept of "active learner, which suggests that learning is not the acceptance of knowledge which is out there, but it entails that learners engage with the world [to construct knowledge]" (South Africa.WCDoE, 2000, p. 29). Active learning is child-centred where learners' learning experiences get confronted, as they get engaged in the use of physical manipulatives. They are given opportunities to formulate their own questions and look for solutions to those questions.

Kilpatrick, et al. (2001) comment that in a child-centred lesson "[learners] solve word mathematical problems by 'acting out' the situation." For example, Marshall and Swan (2005) suggest that learners can use individual pattern block pieces to represent all angle sizes from 30 degrees to 360 degrees before they measure with protractors. The direct use of a protractor make learners view an angle as something simply measured by the protractor only. This won't give them a deeper meaning of an angle. Learners need to invent procedures that mirror the actions or relationships described in the given pattern. These relationships build up learners' competence in mathematics which includes knowing mathematical ideas, signs and actions and make meaning of how they are related to each other (Hiebert & Lefevre, 1986).

The use of physical manipulatives can facilitate a mathematical modelling process. Below is the process that has to be followed in order to model mathematical concepts.

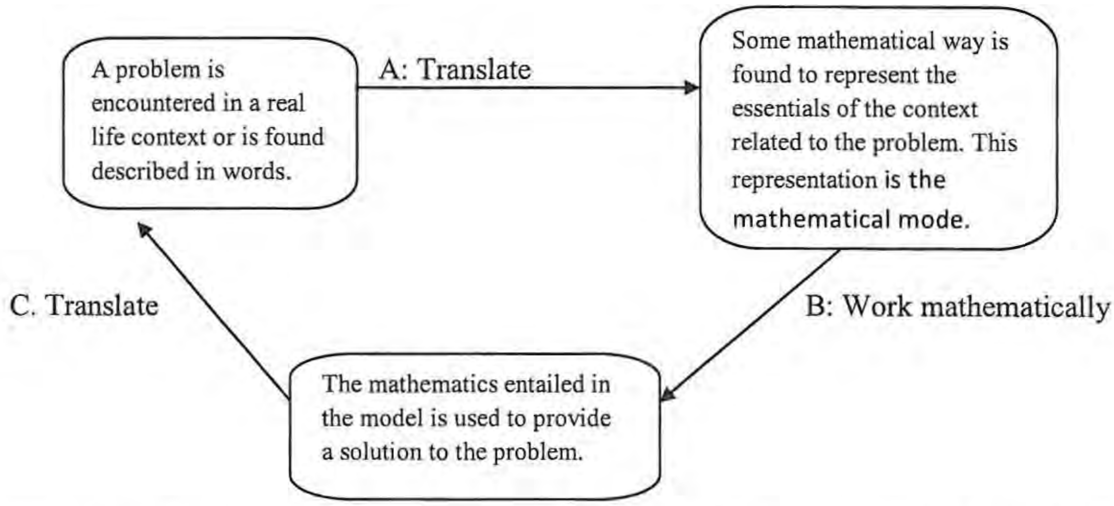


Figure 2.2. The systematic diagram suggesting the modeling process. (Adapted from Laridon, et al. 2006)

Laridon, et al. (2006) describe how the above systematic diagram works in modeling mathematical concepts as follows:-

> In translation stage A of the process, data is collected; facts related to the context are given or investigated and recorded. And then assumptions are made and stated. In stage B, mathematics known to the learners is used in setting up and solving the model. In translation stage C, learners interpret mathematical solutions in terms of the contexts, judge whether the solution makes sense when related to the contexts and they then accept or reject the solution. (p. 88)

Goos, Brown and Makar (2008) argued that physical manipulatives allow learners to practice measurement in real life settings where they are able to relate the given mathematical questions to the environment. According to Cai (2003) when learners model mathematical concepts using physical manipulatives they learn and understand important aspects of the concept or idea.

Laridon, et al. (2006) propose that, the approach for teaching and learning mathematical modeling should follow this process in order to give learners a chance to make assumptions that lead to simplification of real life ideas. This means that learners are equipped for problem solving in real life situations.

Since mathematically abstract concepts are often difficult to learn, physical manipulatives can be used to simplify these concepts. Zuckerman, et al. (2005) and Heddens (1997) suggest that the simplification of mathematical concepts using physical manipulatives is three-fold-

1. Sensory engagement- in this case learners use touch, vision and auditory senses as they relate real world situations to mathematics symbols;

2. Accessibility - dramatically improves the understanding of mathematics concepts of novices. Learners learn that there are many different ways to solve problems and that they can solve problems by application.

3. Group learning- provides a multi – hand interface, does not give the control to one person, facilitates natural group interaction and promotes group discussion, for example, to work cooperatively. Learners visualize mathematics concepts and are [afforded with] opportunity to present in front of the large group. (p. 3)

2.7 PHYSICAL MANIPULATIVES FOR ADAPTIVE REASONING

Kilpatrick, et al. (2001) define the term adaptive reasoning as:

> The capacity to think logically about the relationships among concepts and situations. This includes knowledge of how to justify the conclusions, intuitive and inductive reasoning based on pattern, analogy and metaphor. (p. 129)

Researchers note that physical manipulatives provide the learners with the tools for both thinking and communicating about mathematical concepts (Chao, et al., 2000). In such cases physical manipulatives provide learners with something "to talk about, explore with, think about and reason with" (Van de Walle, 2004, p. 30).

Marshall and Swan (2005) comment that physical manipulatives can also be catalysts for learning to promote learners' thinking abilities. For example, teachers' questioning technique change to "what if....... questions, that encourage further thinking about the relationships between the pieces in a hexagon" (Marshall & Swan, 2005, p. 146). Hiebert and Wearne (1993), and Thompson (1994) articulate that the use of physical manipulatives:

> Provide something for grounded conversations that end up in the construction of strong connections among ways of thinking about concrete situations and conventional mathematical language and notation. (p. 8)

Moyer (2001) argues that these tangible situations and conventional mathematical language and notation enable learners' abstract thinking to be closely coordinated with their concrete perceptions of the world. Thomas (1992) affirms that active manipulation of physical manipulatives provides learners with opportunities to develop a range of images that can be used in the mental manipulation of abstract concepts as they "hone their mathematical thinking skills" (Suydam, 1984, p. 3).

Thirdly, Van de Walle (2004) expounds that the use of physical manipulatives helps learners to understand and make meaning of their own learning. Thus learners are able to explain mathematical concepts to the teacher. These explanations can be in the form of proving or justifying solutions to a problem. They also enable learners to have grounded conversations as they interact with each other during the lesson; this eventually aids learners' understanding of mathematical concepts. According to Marshal and Swan (2005), physical manipulatives encourage learners to work towards a goal of mathematical achievement.

2.8 PHYSICAL MANIPULATIVES FOR PRODUCTIVE DISPOSITION

"Productive disposition refers to the tendency to see sense in mathematics, to perceive it as both useful and worthwhile, to believe that steady effort in learning mathematics pays off, and to see oneself as an effective learner and doer of mathematics"(Kilpatrick, et al. 2001, p. 125).

Learners can perceive mathematics as worthwhile if they can learn it by doing, they have to be hands-on; they can suggest what materials to use in order to find the solution to the given problem. South Africa. WCDoE (2000) claims that engagement and participation of the learners in their own learning using physical manipulatives is an imperative motivational force in successful learning. Thomas (1994) and Moyer (2001) expound that physical manipulatives provide learners with something to act upon as they reflect on their own actions in the process of learning mathematics.

Research on the use of physical manipulatives in the teaching and learning of mathematics reveals that those learners who use physical manipulatives during mathematics instruction, out perform those who do not, this proves that physical manipulatives enhance their curiosity and attitude towards mathematics (Driscoll, 1983; Greabell, 1978; Raphael & Wahlstrom, 1989; Sowell 1989; Suydam, 1989). Uttal, et al. (1997) also find that when physical manipulatives are used as symbols they strike a balance between helping learners by drawing in their curiosity towards mathematics learning and keeping them focused on the relevance of the object as a symbol.

2.9 TIPS ON THE USE OF PHYSICAL MANIPULATIVES

In this section, I present pointers on the use of physical manipulatives for the mediation of teaching and learning of mathematical concepts.

Suydam (1984) says that physical manipulatives must be well incorporated into the lesson to help learners to reason, solve problems, be creative and allow communication with other learners. For example, to develop place value concepts, "one stick should not be placed in the tens place. Instead a package of ten sticks bundled together with string should be placed in the tens place. Students need to conceptualize the idea of 'tenness'" (Heddens, 1986, p. 2).

Ball (1992), Fuson and Briars (1990), Resnick and Omanson (1987), Wearne and Hiebert (1988) suggest that extensive instruction and practice is required before physical manipulatives are employed in mathematical teaching and learning. For example, the teacher must plan the lesson that caters for learners with different abilities so a variety of physical manipulatives should be used.

South Africa. [WCDoE] (2000) and Van de Walle (2004) argue that teachers should not tell the learners how to use physical manipulatives, but let the learners discover the mathematical concepts for themselves. "This practice can help [learners] connect ideas and being able to integrate their knowledge to gain a deeper understanding of the required mathematical concept" (Suydam, 1984, p.3).

Steedly (2008) suggests that incorporating physical manipulatives into learning by using the teaching method known as 'Concrete Representational Abstract (CRA)' which is a 3 segmented instructional strategy, the teacher first uses concrete material to model the mathematical concept to be learned, then demonstrates the concept in representational terms, and finally in abstract or symbolic terms. For example, to calculate the volume of a cube, learners can fill in the cube with dice of the same size. The number of dice it holds is its volume (informal unit measure). Thereafter the learners investigate another method (not counting) to find the number of dice that particular cube holds, learners can discover that it has to be a: 'side x side x side'. The teacher then asks learners to find the volume of the cube measuring 5cm per side. This way the learners move from concrete to abstract gradually by building concepts after concepts from the simple to the complex.

2.10 WHAT IMPEDES THE USE OF PHYSICAL MANIPULATIVES IN TEACHING AND LEARNING MATHEMATICS

In this section, I present what the researchers say about the challenges that physical manipulatives can also bring into the teaching and learning of mathematics. And offer some solutions for teachers to avoid these problems.

Besides the fact that physical manipulatives support learning of mathematics, teachers should know they do not automatically provide mathematical meaning for the learners (Thompson, 1994). Moyer (2001) and Van de Walle (2004) note that it is a challenge for both teachers and learners to guide and perceive mathematical ideas in physical manipulatives. The challenge is to:

> 1. Interpret students' representations of their mathematical thinking, (2) reveal and represent connections among mathematical ideas and (3) develop appropriate concrete contexts for learning mathematics. (p.194)

Van de Walle (2004) further claims that misuse of physical manipulatives may lead learners to misunderstand the intended mathematical concept. For example, with that idea in mind Marshall and Swan (2005) suggested that when teachers instruct the learners what to do during teaching, no reflective thought may go into exploring the mathematical concepts

Hiebert and Wearne (1992) infer that if physical manipulatives are not well incorporated into the lesson they can promote rote learning which results in little or no learning of the mathematical concept behind the procedures. Van de Walle (2004) supports this notion discussed because "there is no physical example of mathematical concepts in the world, for example, the concept 'hundred' is a quantity relationship that a group of 100 items has a single item with a single of the same type." (p. 28).

From my experience, some teachers tend to rush learners and do not allow them time to understand and visualize the mathematical concept imposed onto physical manipulatives and as a result "learners' external actions would not always match the mental task intended by the teacher" (Clements, 1999, p. 47).

Research documents reveal that some teachers regard physical manipulatives as "tools for self-guided instruction, yet their relations with the mathematical concept needs to be guided and constrained by teachers' instructions [in order for the learners to clearly learn mathematics]" (Uttal, et al., 1997, p. 48). Fennema, Carpenter and Larman (1991) suggest that as a learning and knowledge are the products of intellectual processes such as planning, evaluating, recalling and strategizing teachers should be aware that some manipulatives may be of no use. Some have only a slight relationship to mathematical concepts (Moyer, 2001).

Swan, Marshall and White (2010) maintain that there are impediments to the use of physical manipulatives. Firstly, money, some of physical manipulatives are commercial and costly and it is not easy for some schools to get them due to financial constraints. Secondly, supervising classroom behaviour becomes a problem as many learners misuse the physical manipulatives in the classroom and see them as toys rather than materials to aid teaching and learning. Thirdly, the organisation of materials is an obstacle especially when it comes to the idea of borrowing and returning and sorting the material which result in missing pieces that need replacing.

Supporting this idea, Gilbert and Bush (1988) argue that the availability of the material is one of the factors that hinders the use of physical manipulatives when teaching mathematics. If they are not readily available the teacher just teaches without them. Secondly, due to teachers' workload, there may not be enough time for them to implement the use of physical manipulatives in the classroom. In large classes the use of physical manipulatives is deemed as time consuming. In my experience some teachers lack the knowledge of how to use physical manipulatives effectively and as a result they ignore them in the teaching and learning of mathematics.

2.11 LEARNERS' PROBLEMS IN LEARNING MEASUREMENT

In this section, I highlight some of the problems that most of the learners experience in learning measurement and provide suggestions of how to help overcome them.

Outhred, Mitchelmore, Mc Phail and Gould (2003) argue that learners do not understand the spatial attributes and the use of informal and formal units to measure and compare quantities. A lack of these basic skills, result in a misunderstanding of linear measurement which causes learners difficulty in measuring surface area and volume of prisms.

The argument exists that, developing measurement sense is not as simple as people think, but it is "more complex than learning the skills and procedures for determining a measure" (Clemence & Bright, 2003, p. 14). For example, to teach volume concept with understanding we need to consider "the structural problems of packing" (Clemence & Bright, 2003, p. 96). Teachers need to be aware of the value of listening and observing learners while they work. Clemence and Bright (2003) also caution teachers not to be too much abstract when teaching measurement (in all grades) because they leave learners with a procedural and conceptual deficit.

Van de Walle (2004) discovered that learners sometimes are unable to understand the conservation of units. For example, if the question says: find the area of a rectangle with sides measuring 7cm and 3cm. most of the learners solve this as follows: 7 x 3 = 21 or 7cm x 3cm = 21cm. They either ignore or do not understand that cm x cm = cm^2. Such misunderstandings "raise questions about the effectiveness of the instruction learners are receiving in the areas of [measurement]" (Goos, Brown and Makar , 2008, p. 391). In addition, Koshy, Ernest and Casey (2000) argue that such misconceptions might be due to carelessness and lack of subsequent knowledge on how to use standard units.

To support this, Outhred and Mitchelmore (1996) argue that learners who misunderstand area and measurement, experience problems in calculating the surface area of a given prisms. Most of these learners never engaged in "multiple measuring situations that encourage [them] to measure with non-standard and standard units." (Clements & Bright, 2003, p. 14). The research done by Carpenter, Coburn, Freys and Wilson (1975) reveals another problem where teachers do not give learners enough time to develop an understanding of area measurement as area. The argument is that teachers focus too much on formulae with a result negative effect on the learners' level of conceptual understanding of measurement. Also teachers put too much "emphasis on numerical operational skills [instead of] providing learners with more experiences with exploration of formulae, for example, exploration of formulae for area measurement" (Huang &Wirtz, 2010, p. 5).

The argument is that the traditional mathematical formulae result in a negative effect on learners' learning of measurement, for example, deficiency in correlation between 2-D geometry and measurement that is found in teaching material (Kordak & Balemenou, 2006). Based on these views Huang and Wirtz, (2010) say that if learners cannot understand 2-D geometry, they are more likely to find it difficult to solve measurement problems. Researchers suggest that the 2-D geometry and measurement should complement each other in order for the learners to develop the required abilities in measurement as they are closely related to each other (Fendel, 1987; Fuys; Geddes & Tischer, 1988).

Another problem that learners face, is that they are not given time for conversation as they learn measurement. Clements and Bright (2003) argue that there is a need for the learners to be given an opportunity to be in a "conversation and talk about the meaning measuring has for them." (p.14) This implies that learners' conversation should not be only about procedures, but that they must focus on how they come up with their procedures and on "big ideas about linear and two dimensional space" (Clemence & Bright, 2003, p. 14). Huang and Wirtz (2010) say that linear and 2-D space are considered big ideas when teachers include the understanding of the properties of basic shapes, congruence and geometry motions (flips, translations, rotation, etc) to serve as the foundation of constructing notions about measurement .

Research shows that most learners, especially those at high school have difficulty in understanding measurement because their teachers do not equip them at primary level with the basic skills or processes of measurement that are important later on, such as the capacity to: (i) contrast measurement (Barret, Jones, Thornton & Dickson, 2003; Grant & Kline 2003), (ii) select correct measuring apparatus (Clarke, Cheesemen, McDonough & Clarke, 2003), and (iii) measure from a fixed position (Lehrer, 2003).

In their research O'Keefe and Bobis (2008) discovered that the "language used" (p.394) by some teachers did not help learners to build up mathematical terminology as far as mathematics was concerned. This implies that when teaching measurement teachers must use the appropriate mathematical terms, for example, use terms like: a vertex not a corner, faces not sides and edges not sides.

2.12 CONCLUSION

To conclude, firstly, I discovered that the use of physical manipulatives in mathematics teaching and learning has a long history in education. For this reason the South African Department of Education promotes the use of physical manipulatives in order for the learners to acquire mathematical skills for the understanding of mathematical concepts. Secondly, the more technology is advancing the more types of physical manipulatives are designed; some are physical objects while others are computerized. Research highlights that mathematics teaching and learning can only be enhanced by the use of physical manipulatives which are well incorporated into the lesson.

Thirdly, the effective use of physical manipulatives is inextricably linked with good teaching practice. Clements (1999) said that good teaching practices entail the use of "physical manipulatives in the context of educational tasks to actively engage children's thinking with teachers' guidance." (p. 56)

In addition, the argument exists that mathematical ideas have five different representations namely: pictures, manipulative models, real-world situations, written symbols and oral language. Since these representations are intertwined, so teachers must make use of physical manipulatives in mathematics teaching in order to establish concrete mathematical ideas. Clements (1999) further suggests that teachers need to know that the main aim of using physical manipulatives is to build what is referred to as "Integrated-concrete ideas that help learners in the construction of meaningful ideas. For this reason physical manipulatives should be used before formal symbolic instruction such as teaching algorithms." (p. 46). Each element of the representations contributes to learners' understanding of mathematical concepts.

Not all manipulatives are appropriate for mathematics teaching, the following qualities summarized and described below by Clements (1999) show that:

> Good physical manipulatives are those that are meaningful to the learners; provide control and flexibility to the learner. Also they have characteristics that mirror, or are consistent with, cognitive and mathematics structures, and assist the learner in making connections between various pieces and types of knowledge. (p. 50)

In the next chapter, I present my research design and methodology.

CHAPTER THREE: RESEARCH DESIGN AND METHODOLOGY

3.1 INTRODUCTION

In this chapter, I present a detailed description of the research design I employed when conducting this research study. This includes: - (i) the methods used to collect data, (ii) selecting the participants, (iii) describing the sampling techniques and (iv) a description of instruments used in gathering data. In addition, I discuss how the baseline assessment was developed and validated in order to ensure that it was of an appropriate level and standard. Lastly, I describe the design of each of the instruments used in the study and explain why they were used.

3.2 THE CHOSEN PARADIGM

This research study is oriented in an interpretive paradigm. The interpretive paradigm offers an opportunity to engage fully in describing and understanding actions and procedures (Babbie & Mouton, 2001). In an interpretive orientation, the goal of the research is a narrative that goes deep enough to provide a scrutiny (Van Wynsberghe & Khan, 2007). The "meaning of human creations, words and experiences can only be ascertained in relation to the context in which they occur" (Terre Blanche & Durrheim, 1999, p. 125). By employing the interpretive paradigm, I was thus able to conduct in-depth research in a context that was highly localised and specifically related to experiences of Grade 8 learners. I observed learners engaged in various tasks of an intervention programme that utilised physical manipulatives to facilitate conceptual understanding of measurement.

3.3 RESEARCH METHODOLOGY–A CASE STUDY

This research is in the form a case study, defined by Leedy (1997) as a type of qualitative research in which data is collected directly from a specific group for the purpose of studying their interactions, attitudes or characteristics within their own environment (e.g. a school or classroom). The reason for using a case study is that it gives an opportunity to the researcher to study an aspect of a problem or only one phenomenon with in-depth scrutiny within a limited time frame (Bell, 1993, McMillan & Schumacher, 1993). Cohen and Manion (1992) add that another advantage of the case study is that it begins with the participants being engaged in real-life action and allows for the situation to improve should the need arise.

The unit of analysis for my case study was the experiences of a Grade 8 class that participated in an intervention programme that used physical manipulatives to learn about measurement. Van Wynsberghe and Khan (2007) suggest that "a case study is not exclusively about the case revealing itself as it is about the unit of analysis being discovered or constructed. [This means that] the unit of analysis must come into focus as the research progresses, not before" (p. 90).

3.4 RESEARCH SITE AND PARTICIPANTS

One definition of sampling suggests "having a choice about when and where to observe, whom to talk to or what information sources to focus on" (Maxwell, 2005, p. 235). My sampling was purposeful because of its "typical designation, which aimed to pick a small number of cases provide that would yield the most information about [measurement] phenomenon" (Teddlies & Yu, 2007, p. 7). Tashakkoi and Teddlies (2003) argue that small numbers of cases provides an opportunity for eliciting greater depth of information.

I carried out my research study at a high school in the Queenstown District. This is the same school where I am currently employed and I selected it because of its easy access. Another reason for selecting this school was because it is familiar to me so I could observe and interact with participants for an extended or intensive period of time. Lastly, I felt that my own work place would give me a greater understanding of the context due to my prior knowledge and insights I have of this school.

My sample of participants was made up of eighteen Grade 8 learners. From a cohort of 283 Grade 8 learners in my school, learners were asked to volunteer to participate in the research project. 18 learners volunteered. Grade 8 was chosen to participate in the research project for two reasons. Firstly, surface area and volume form an important aspect of measurement, specifically in surface area and volume of cubes and rectangular prisms. Secondly, Grade 8 represents the lowest grade of the selected school. If the study shows that physical manipulatives do have a positive role in teaching and learning, it will be an easy process to introduce them other grades to help clear their misconceptions relating to measurement, specifically in terms of surface area and volume of cubes and rectangular prisms.

3.5 RESEARCH PROCESSES AND DESIGN

The goal of my study is to investigate the role of physical manipulatives in the learning of measurement in selected Grade 8 learners. In particular, this study investigates how the use of physical manipulatives promotes learners' mathematical proficiency in relation to the five strands of Kilpatrick, et al.'s (2001) teaching for mathematical proficiency. I present and describe in detail the research processes and design below.

3.5.1 PHASE ONE–PILOT AND BASELINE ASSESSMENT (PRE-TEST)

The first phase of the research design was to pilot the baseline assessment task using seven learners who voluntarily offered to write the pilot task. After the pilot, no adjustments were made to the baseline assessment task. The task was subsequently administered to the group of 18 participants who were engaged in the research project. Below is the structure of the baseline task that was used to investigate the learners' prior knowledge of measurement.

According to the South African Department of Education [DoE] (2006), baseline assessment can take place at the beginning of an assessment task or grade or phase to establish what learners already know. In my research study it was designed for the following three purposes:

- firstly, to capture and explore learners' understanding of the concepts of surface area and volume,
- secondly, to reveal any misconception and misunderstanding learners might have regarding surface area and volume,
- lastly, I used the baseline assessment task to design a suitable intervention strategy that focused on addressing the misconceptions that learners have in learning measurement, in particular surface area and volume.

3.5.2 PHASE TWO–DESIGN OF THE INTERVENTION PROGRAMME

The intervention program that I designed was aimed at addressing not only the misconceptions learners demonstrated in the baseline assessment, but also to present the concepts of surface area and volume in an informal activity-based way so that learners were able to understand the difference between surface area and volume. In the intervention program a variety of physical manipulatives were used in order to engage learners in developing an understanding of surface area and volume. How the physical manipulatives were used is discussed in detail in section 3.6.3

3.5.3 PHASE THREE–ADMINISTERING OF INTERVENTION TASKS AND OBSERVATIONS

Since observations make it possible for a researcher to collect existing data from lived situations (Cohen, et al., 2000) I made extensive use of this technique. I observed learners working with physical manipulatives throughout the intervention program. Observations were used for data collection so that I could make rational decisions about participants' mathematical proficiency in terms of surface area and volume. Since "qualitative research emphasises process rather than outcomes" (Clarke & Ritchie, 2001, p. 309), the emphasis of the collected data was on "thick description" (Clarke & Ritchie, 2001, p. 277).

Clarke and Ritchie (2001) define thick description as an extensive narrative that captures the sense of behaviour as it occurs. The observation data of my research was generated whilst the participants were engaged in the intervention tasks. An observation schedule and the scripts used by the learners to write about each and every activity were used to collect data.

3.5.4 PHASE FOUR POST-INTERVENTION TASK (POST-TEST) AND INTERVIEWS

3.5.4.1 POST-INTERVENTION TASK

After the intervention program was completed, the participants were engaged in a post-intervention task which aimed at analysing the impact of the use of physical manipulatives in learning about surface area and volume. This task was very similar in structure to the baseline assessment, with only a slight difference in content.

3.5.4.2 SEMI – STRUCTURED INTERVIEWS

In order to investigate how learners felt about the use of physical manipulatives in learning about surface area and volume, I interviewed two participants individually from each of the following categories: (i) those who scored the highest score in the post-intervention task, (ii) those who scored an average mark, and (iii) those who scored poorly. Interviews are one of the qualitative data collection methods that are mostly employed in case studies (Gillham, 2000). Individual interviews were used because they afford access to the thoughts inside the person's head, making it feasible to understand what a person knows, likes or dislikes (Tuckman, 1972).

3.6 RESEARCH INSTRUMENTS AND CONTENT

For the purpose of what Cohen and Manion (1985) refer to as triangulation, I used different research instruments. According to Cohen, Manion and Morrison (2000), triangulation is described as the use of a range of methods of data collection for a research study. The reason for triangulation is to explore the research question from different angles (Flick, von Kardorff & Steinke, 2004). I now describe the research instruments employed for each phase of the research project.

3.6.1 PHASE ONE-PILOT AND BASELINE ASSESSMENT TASK (PRE-TEST)

The baseline assessment task is made up of two parts, A and B. Part A focuses on calculating the surface area and Part B focuses on calculating the volume of a cube and a rectangular prism.

Part A comprises five questions:

Question 1 required the learners to identify and name each of the objects presented in the form of a drawing. In question 2, learners were asked to write down the number of faces each object had. In question 3, learners had to identify the number of pairs of identical sides. In question 4, learners were required to calculate the volume of a cube and a rectangular prism (with given dimensions) without the aid of a diagram. Lastly, question 5 asked learners to work out the surface area of a rectangular prism with specific dimensions.

Part B comprises four questions:

Questions 6 and 7 required learners to find the volume of a cube and a rectangular prism. Question 8 asked for the calculation of the height of a rectangular prism given the volume and area of the base. Lastly question 9 required learners to calculate the length of a cube given its volume.

The baseline assessment task was presented to the participants as follows:-

Pilot and baseline assessment task

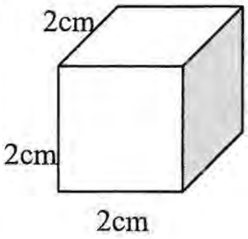
Instructions:

Answer all the questions

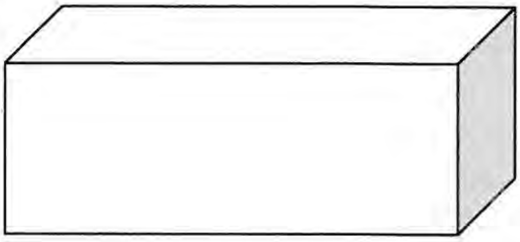
PART A

1. Name each of the objects drawn below.

(a)



(b)



2. How many faces does (i) object (a) have?

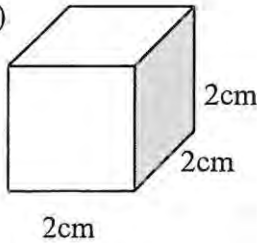
(ii) object (b) have?

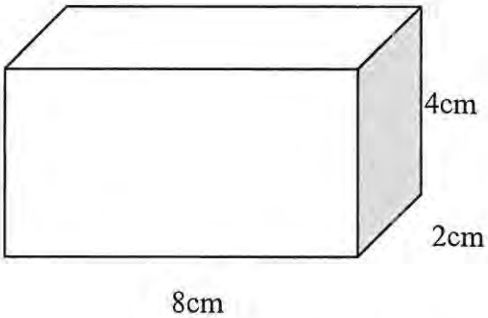
3. How many dimensions does object (b) have?

4. Calculate the **total surface area** of the object labelled (i) a

(ii) b

(a)

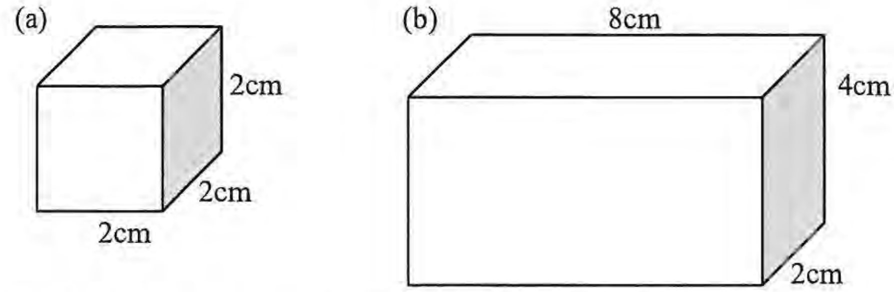




4. How much cloth is required to cover the outside part of a box with the following dimensions: length 12cm, breadth 8cm and height 6 cm?

PART B

6. Calculate the **volume** of each of the objects drawn below:



7. What is the volume of the box that has the following dimensions:

   length 12cm, breadth 8cm and height 6cm?

8. If the volume of a rectangular box is $240cm^3$ and the area of its base is $60cm^2$. What is the height?

9. If the volume of a cube is $64cm^3$. What is the dimension of each side?

3.6.2 PHASE TWO-DESIGN OF THE INTERVENTION PROGRAMME

The baseline assessment task helped me to design an intervention program that was aimed at addressing the misconceptions demonstrated by the participants in the baseline assessment. The baseline results showed that participants had difficulty in calculating surface area and volume of cubes and rectangular prisms. The results helped me to design four tasks which framed the intervention program. In order to address the identified problems and support the intervention programme learners were provided with the following resources: a multitude of dice, small tiles, grid papers, rulers, erasers, scissors, answer scripts, card for designing and constructing three-dimensional boxes.

3.6.3 PHASE THREE-THE CONTENT OF THE INTERVENTION PROGRAMME

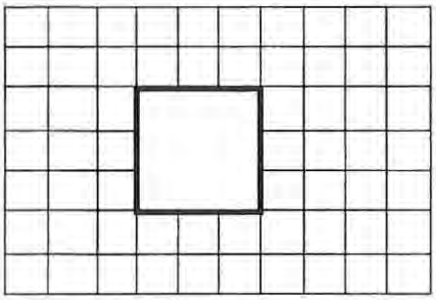
Task 1 of the intervention program contained eight different questions addressing the concepts of the area of a square and a rectangle. In order for the learners to be able to calculate the surface area of a cube and a rectangular prism, they need prior knowledge of how to find the area of a rectangle and a square. The task also aimed at supporting learners in understanding that the term "area" simply refers to the number of squares within the region covered or enclosed by a particular shape.

3.6.3.1 INTERVENTION PROGRAMME TASK 1

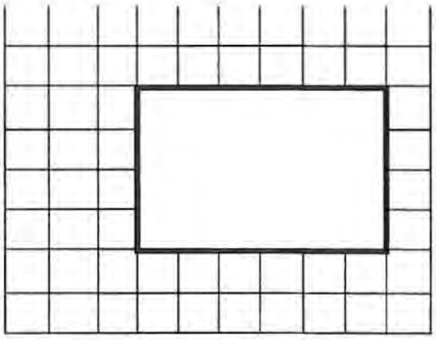
To begin with learners were engaged in activities such as identifying the number of squares within a certain region. Secondly, learners drew squares and rectangles on grid paper and determined each one's area in terms of the number of squares within the covered region. Lastly, learners were given small tiles in order to tile the region enclosed by a square and a rectangle. Task 1 is shown below.

1.1. Identify and name each of the shapes drawn on the grid paper below:

(a)



(b)



1.2. How many squares have occupied the region that is covered by:

(i) shape 'a'?

(ii) shape 'b'?

1.3. How many squares are covering each of the sides: the longer and shorter side of the shape labelled 'b'?

1.4.1. Use the grid paper provided to draw two diagrams, a square and a rectangle of any size you want.

1.4.2. (i) How many squares are there in the region covered by each of the diagrams you have just drawn?

(ii) Explain how you got the answers in Question 1.4.2. (i) above.

1.4.3. Investigate another way of finding the number of squares that cover the region occupied by each of the shapes drawn in Question 1.4.1.

1.5. Now use the squares (tiles) provided to answer the questions that follow:

1.5.1. Make up a square with:

(i) 10 tiles (ii) 25 tiles (iii) 16 tiles

1.5.2. Which number/s of tiles managed to give you a square shape?

1,5.3. What is the reason that the other tiles could not do this?

1.6. How many different rectangles can you make using:

(i) 10 squares? (ii) 7 squares? (iii) 12 squares?

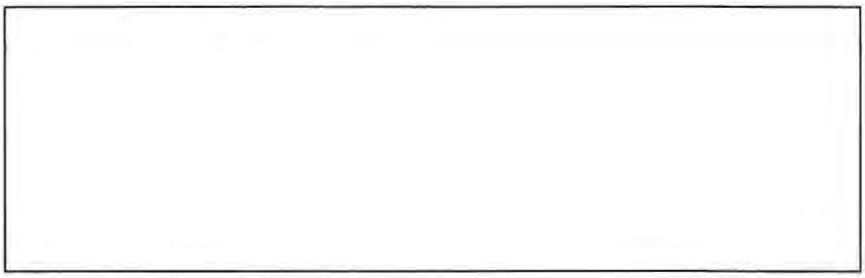
1.7. Now use the provided squares and triangles as your tiles to tile each of the shapes drawn below and then write down your findings. Use: **possible or impossible** as your answers

(i) Tile with squares (ii) Tile with triangles

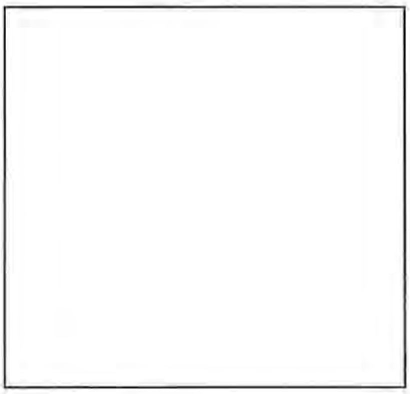
(iii) Now, tile the shapes below with both squares and triangles (mix them).

NB. Don't cut any tile use them as they are.

(a)



(b)



1.8. What are your findings in Question 1.7.? (Which tiles exactly cover the space occupied by each of the shapes without leaving any gaps?)

3.6.3.2 INTERVENTION PROGRAMME TASK 2

Task 2 consisted of five questions that required learners to draw the nets of a cube and a rectangular prism. The nets were drawn on the grid paper provided and were subsequently folded into the relevant cubes and rectangular prisms. This task also aimed at helping learners to calculate the surface area of the cube and the rectangular prism. Lastly, the task demanded that the learners should apply the concepts of surface area in a new situation. For more detailed structure, task 2 is shown below.

Use the provided resources to do the following activities:

2.1. Draw the net of:

(a) a cube measuring 4 squares per side.

(b) A rectangular prism with dimensions: 4 squares in length, 3 squares in breadth and 2 squares in height

2.2. (i) How many squares are covering each of the faces of a cube? (Label faces A – F)

(ii) What is the total number of squares that are covering the whole net of a cube?

2.3. (i) How many squares are on each of the sides of the rectangular prism net drawn in Question 2 (ii) above?

(ii) What is the total number of squares that are on the surface of the net of a rectangular prism?

NB. Learners must be informed that what they have just worked out in questions 2.2. and 2.3 is known as the surface area for each of the prisms in Question 2.2 above.

2.4. (i) How did you find the total number of squares on the:

(a) net of a cube?

(b) net of a rectangular prism?

(ii) Investigate another method that can be used to find the total surface area of:

(a) a cube

(b) a rectangular prism

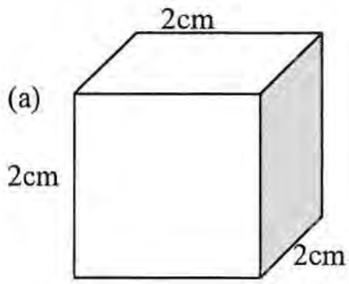
2.5. How much cloth is required to completely cover the outside surface of the rectangular prism measuring: 7 squares in length, 5 squares in breadth and 3 squares in height?

3.6.3.3 INTERVENTION PROGRAMME TASK 3

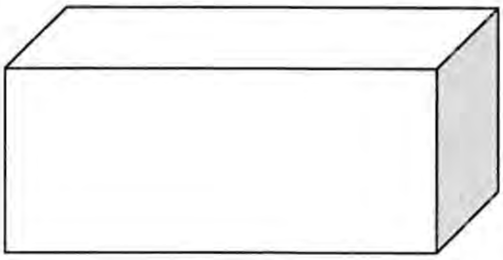
Task 3 focused on visualisation and the naming of solid shapes as well as counting the number of faces of the solid. It also required the learners to measure and calculate the surface area of a cube and a rectangular prism.

3.1. Carefully, study the diagrams below and then answer the questions that follow.

(a)



(b)



3.1.1. What are the names of the objects 'a' and 'b' drawn above?

3.1.2. How many faces does object (i) 'a' have?

(ii) 'b' have?

3.2. Use the resources provided to answer the questions that follow:

3.2.1. (i) Take the real object that is represented by the diagram 'a' in Question 3.1., above, identify and write down the number faces it has.

(ii) Measure and record its length, breadth and height. What did you find out?

(iii) Now, find the area of each and every face of the object.

(iv)Calculate the total surface area of the object.

3.2.2. Repeat the steps done in Question3.2.1. above, use diagram 'b'

3.3. (i) On the grid paper provided, draw the net of a cube and a rectangular prism and then fold them into prisms with the following dimensions:

(a) a cube with a side measuring 4 squares.

(b) a rectangular prism measuring, 7 squares in length, 5 squares in breadth and 3 squares in height.

(ii) Now, cut out each of the nets and fold them into boxes, with squares outside.

(a) What is the total number of squares on the surface of:

(i) a cube?

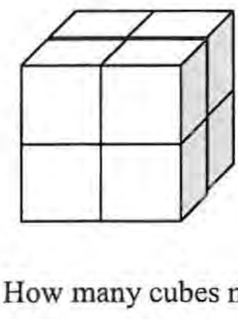
(ii) a rectangular prism?

NB. Your answers above: the first one is **the surface area of a cube** and the second one is the **surface area of a rectangular prism.**

3.6.3.4 INTERVENTION PROGRAMME TASK 4

The fourth task focused on the concept of volume of a cube and a rectangular prism. It comprised questions that engaged learners in the calculation of the volume of cubes and rectangular prisms by filling boxes with dice. In addition, learners were also asked to use the given dice to make different cubes and rectangular prism stacks. The main aim of this task was to establish the idea that the calculation of the volume of an object is all about finding the amount of space in a container, i.e. the number of dice that fill the solid. The details of Task 4 are shown below.

4.1. Study the diagram below and then answer the questions that follow:



4.1.1. How many cubes make up the stack drawn above?

4.1.2. Use the cubes provided to make a stack that is the same as the one drawn above.

4.1.3. (a) How many cubes did you use to make such a stack?

(b) Is your answer in Question 4.1.1, the same as the one in Question 4.1.3?

4.2.1. Make different stacks (each with all sides equal to each other) using the following number of dice or wooden blocks:

(i) 3

(ii) 9

(iii) 27

(iv) 36

4.2.2. Which number of dice makes up the required stack in Question 4.2.1?

4.2.3. (a) Construct 6 different rectangular prisms using 60 dice provided.

(b) Identify and write down the number of dice that are on the length, breadth and height of each of the stacks. Record your findings as follows:

Rectangular prism number	**Length**	**Breadth**	**Height**	**Total number of cubes**
1				
2				
3				
4				
5				
6				

4.3. Now use the dice and the boxes provided to do the following activities:

4.3.1. Find out how many cubes each of the boxes can hold, record your findings.

4.3.2. How did you find the number of cubes that exactly fit into: (a) a cube?

(b) a rectangular prism?

4.3.3. Now, investigate and write down another method that can help you to find the number of dice that fit into each of the given boxes.

3.6.4 OBSERVATION SCHEDULE

While the learners were engaged in different activities during the intervention programme my role was to give direction to each group where required and to observe how they used the physical manipulatives to answer the questions.

The observation schedule is comprised of 18 criteria, seven of which have been adapted from the National Survey of Science and Mathematics Education Observation Protocol (2000) with ratings ranging from 1 to 3 where 3 refers to as an extensive attempt, 2 to a moderate extent, and 1 to no attempt being made. Learners' engagement and performance during the intervention was evaluated based on the criteria in the observation schedule. These criteria are aligned to the five strands of mathematical proficiency advocated by Kilpatrick et al. (2001).

Table 3.1 Intervention programme's observation schedule

No.	DESCRIPTION	ALIGNMENT TO KILPATRICK'S STRANDS	RATINGS		
			3	2	1
1. *	The learner were able to present the given diagram using physical manipulatives	Conceptual understanding			
2. *	The learners were able to justify their solutions using physical manipulatives.	Adaptive reasoning			
3. *	The learners were able to follow basic instructions to do the task at hand.	Procedural fluency			
4. *	The learners respected each other's ideas, questions and contributions by attentively listening to each other.	Productive disposition			
5. *	The learners were able to use mathematics dictionaries in order to find the meanings of the terms surface area and volume.	Conceptual understanding			
6. *	The learners engaged with physical manipulatives (they were able to get the correct answers for all the questions).	Procedural fluency			
7. *	In their small groups each and every member was actively involved.	Procedural fluency			
8.	The learners' discussions of the given questions were guided by the physical manipulatives.	Conceptual understanding			
9.	The use of the programme allowed the learners to gain the skills i.e. communication skills, calculation skills.	Procedural fluency			
10.	The learners really used physical manipulatives in order to solve and understand the concepts of surface area and volume.	Strategic competence			
11.	The learners could move from the concrete stage through pictorial to the abstract stage of solving the surface area and volume of the prisms.	Strategic competence			
12.	The learners were actively engaged in doing the task at hand using physical manipulatives.	Procedural fluency			
13	The learners were motivated to do the task at hand (each and every learner was involved in doing the task).	Productive disposition			
14.	They showed an ability to understand the question presented diagrammatically (shown by solving the questions accurately)	Conceptual understanding			

KEY: 3.To a great extent. 2. Moderate. 1. No attempt has been made

*ADAPTED FROM: National Survey of Science & Mathematics Education Horizon Research, Inc. (2000). Inside the classroom: Observation and analytical protocol.

3.6.5 PHASE FOUR

3.6.5.1 POST-INTERVENTION TASK

The post intervention task was administered to the learners immediately after they had engaged in the intervention programme. The design of this task is similar to the baseline assessment. It also consisted of two parts which focused on the calculation of surface area of cubes, and rectangular prisms as well as the calculation of the volume of cubes and rectangular prisms. The main reason for setting this task was to explore whether the intervention programme had had an impact on learners' understanding of surface area and volume in relation to cubes and rectangular prisms. Below is the detailed description of the post-intervention task.

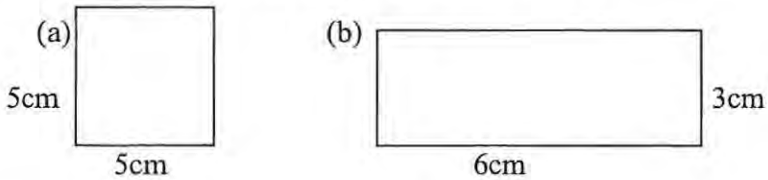
Post – intervention task

Instructions:

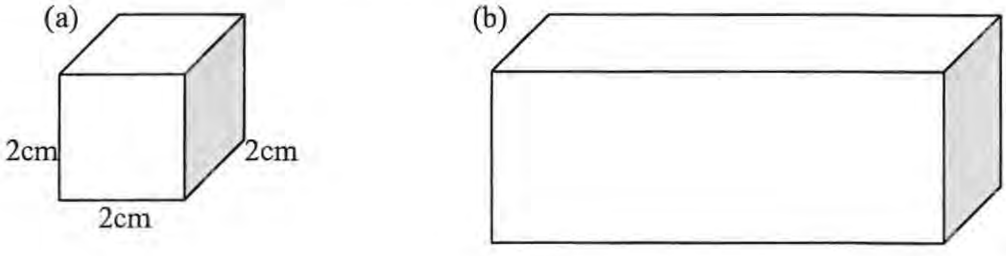
Answer all the questions

PART A

1. Calculate the area of the figures drawn below:



2. Name each of the objects drawn below.

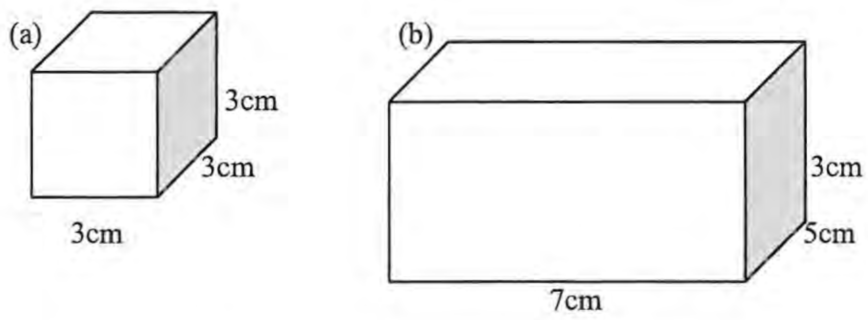


3. How many faces does (i) object (a) have?

(ii) object (b) have

4. Calculate the **total surface area** of the object labeled (i) a

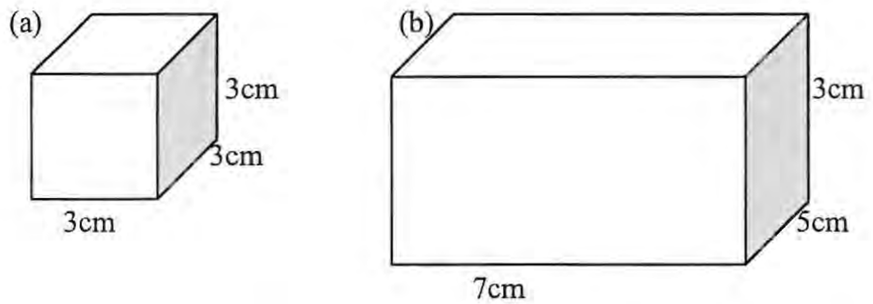
(ii) b



5. How much cloth is required to cover the outside part of a box with the following dimensions: length 10cm, breadth 8cm and height 5cm.

PART B

6. Calculate the **volume** of each of the objects drawn below:



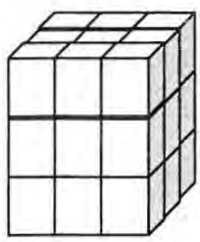
7. What is the volume of the box that has the following dimensions:

   length 12cm, breadth 8cm and height 6cm?

8. If the volume of a rectangular box is $60cm^3$ and the area of its base is $20cm^2$. What is the height?

9. If the volume of a cube is $27cm^3$. What is the dimension of each side?

10. What is the number of cubes used in the stack drawn below?



3.6.5.2 INTERVIEWS

Soon after the post-intervention task, six learners were chosen for the interviews. The selection of the interview participants was based on their performance in the post - intervention task which they wrote. I selected the two top performers, two medium performers and two low performers. These six were engaged in a semi-structured interview with open-ended questions. Bless and Higson-Smith (1995) argue that the open-ended nature of questions allows both the interviewee and the interviewer to discuss topics in detail. Also, it provides flexibility to probe for more details on the issues discussed. Below is the set of four questions that guided our conversations.

1. Do you have any comments about the use of physical manipulatives in teaching and learning of surface area and volume of prisms?

2. Do you like the use of the program [physical manipulatives] in learning about surface area and volume?

3. Did the program that you used help you to get a clear picture and explanation of how to find the surface area and volume of the given cube and the rectangular prism?

4. Apart from learning about surface area and volume of cubes and rectangular prisms, what else can you learn using these physical manipulatives?

3.7 ANALYSIS PROCEDURE

The collected data was analysed in the following sequence: piloted task, baseline assessment, intervention programme, observations, post-intervention task and individual interviews. The details of how the analysis was carried out are discussed in the paragraphs below.

3.7.1 ANALYSIS OF PILOT TASK.

The piloted data was analysed graphically and then in the form of a thick description based on how the participants responded to each question.

3.7.2 ANALYSIS OF THE BASELINE ASSESSMENT

The analysis of the baseline assessment task was done both qualitatively and quantitatively. Analysing data qualitatively means the "description of the data without using numbers" and quantitatively refers to "the statistical methods employed to present data in a visual way through the use of tables and graphs" (Department of Science, Mathematics & Technology Education [SMATE], 2007, p. 9). The results of the baseline and post-intervention tasks' questions that had the same content were presented in one bar graph. The learners' results were categorised into two those who improved and those who did not improve. Finally, a general discussion is presented for each graphical representation.

3.7.3 ANALYSIS OF OBSERVATIONS

Observations were analysed in the form of a thick description. The set of criteria used to analyse observations was implicitly based on Kilpatrick et al.'s (2001) five strands of mathematical proficiency:

1. Conceptual understanding which refers to the ability to represent mathematical ideas in different ways.
2. Procedural fluency which refers to the skills in performing procedures flexibly, accurately and efficiently.
3. Strategic competence which is the ability to represent mathematics numerically, symbolically, verbally and graphically.
4. Adaptive reasoning which refers to correct reasoning and justification of conclusion
5. Productive disposition which refers to learners' passion, enthusiasm and recognition of the benefits and usefulness of mathematics. (p. 10)

The ratings in the observation schedule allowed the data to be analyzed quantitatively. Graphs were used to illustrate the ratings.

3.7.4 ANALYSIS OF SEMI-STRUCTURED INTERVIEWS

The semi-structured interview data was analysed in themes. This was done by identifying similarities and differences in learners' responses to the interview questions. The interviews focused on how learners used manipulatives to solve the given mathematical tasks in the intervention programme, and attempted to elicit how the learners felt about the use of physical manipulatives. Finally, it helped me to explore and understand the feelings of the learners in the process of learning measurement using physical manipulatives.

Table 3.2 A summary of the research design and data collection process.

No.	INSTRUMENT	ACTION TAKEN IN ORDER TO COLLECT DATA	DATA COLLECTED
1	Baseline assessment	Piloting of the baseline assessment task using 7 participants	Qualitative results analyzed in the form of a descriptive script of how learners performed. Quantitative results analysed graphically, presenting how learners performed.
2	Baseline assessment	Administered to 18 participants	Qualitative results analysed in the form of a descriptive script of how learners performed. Quantitative results graphically, presented in a table.
3	Intervention programme	Administered to 18 participants	Qualitative results analysed in the form of descriptive script of how learners performed.
4	Observations	Use of observation schedule (observing learners using physical manipulatives during the intervention program)	-Analysed data in the form of a thick description that gives all the details on what happened during the intervention programme -
5	Post-intervention tasks	Administered to the participants	Qualitative results analysed in the form of a descriptive script of how learners performed. Quantitative results analysed graphically, presenting how learners performed.
6	Semi-structured interviews	6 participants engaged in individual interviewed.	Qualitative results in the form of interview transcripts.

3.8 CONTENT VALIDITY

To ensure the validity of the pre- and post-intervention tasks, I involved the head of the mathematics department at my work place. He analysed and validated the questions to see if they were appropriate in terms of content and grade level.

3.8.1 PILOTING OF BASELINE ASSESSMENT TASK

The baseline assessment task that I prepared for my participants was piloted using seven Grade 8 learners to ensure that the standard was consistent with Grade 8 mathematics, and that all questions were unambiguous.

3.9 ETHICAL ISSUES

In order to conduct this research study ethically I went through the following steps:

Firstly, a letter was written and sent to both the principal and the school governing board asking for consent to do the research in the school where I am working (appendix A). The school responded to my request as shown in appendix B. In order to avoid interference with the regular school programme, the research was carried out after school hours.

Secondly, as soon as the volunteers of the project were identified, letters and forms of consent were sent to the parents and guardians of the research participants requesting consent for their children to participate in the research project. For the details refer to appendices C and D. These letters were signed and brought back to me. In addition, the participants' guardians were informed that there were allowed to withdraw their children from the project at any time. This was done to avoid unnecessary absenteeism and dropout of the participants.

To ensure free participation the participants were made aware that their participation in the project was purely voluntary and that in the write-up of the data no names were to be mentioned. In addition, I informed the participants that participation in the study was likely to benefit them academically since measurement is a content area where many learners have misconceptions.

In order to democratize the research relationships I established what is referred to as "subject-subject relationship [with the learners] which is a freedom from professional control" (Clarke & Ritchie, 2001, pp. 319, 320). The learners were made to feel at ease with the entire research process, despite my presence. I was mindful of the fact that I was their teacher and that my presence could potentially make participants uncomfortable, and participants were encouraged to discuss whatever they felt was important in relation to the project regardless of my presence.

3.10 CONCLUSION

In this chapter, I discussed how the methodological framework was designed. The discussion included a description of the research processes and design, the chosen overall paradigm, research methods and instruments, content validity and ethical issues. In addition, I presented the instruments that were used in collecting the data, and that were used to answer my research question.

In the next chapter I present a discussion and analysis of the collected data.

CHAPTER 4: DATA ANALYSIS AND DISCUSSION

4.1 INTRODUCTION

The purpose of this study was to investigate the role of physical manipulatives in the teaching and learning of measurement in Grade 8. This case study focussed particularly on surface area and volume.

In this chapter, I present my data analysis and discussion of the findings. Data analysis refers to a rigorous and methodical exercise based on practical evidence gathered through interviews and observations. The data collected was analysed manually in order to acquire a clear understanding of the data (Danermark, Ekstrom, Jakabsen & Karrison, 2002).

The main sources of data in my study were a piloted task, baseline assessment tasks (pre-test), an intervention programme, an observation schedule, post –intervention task (post- test) and interview transcripts. Through the use of these instruments, I examined the role of physical manipulatives in learning measurement with understanding. I compared the results of the baseline assessment with those of the post –intervention task in order to identify the impact that the use of physical manipulatives has on the learners' understanding of surface area and volume concepts. In order to achieve this, I focussed on both the learners who improved and those who did not improve in the post-intervention task.

The first set of data that I analysed was the piloted task given to seven Grade 8 learners. After I piloted the baseline assessment task the results were analysed quantitatively focussing on how each and every learner attempted to answer all the questions. In order to maintain ethical issues and to ensure anonymity all the participating learners were assigned a code named A to G. The target group's codes were L1, L2, L3,...up to Ll8.

After the task was piloted, 18 participants were given the baseline assessment task (pre-test) which they wrote individually. The results of the baseline assessment task informed the preparation of the intervention programme. The baseline assessment task guided me in isolating the areas where learner had misconceptions. The learners engaged in the intervention programme for a total of 6 days. We spent one and half hours per day on the programme. The performance of the participants was explored with an observation schedule. The resulting analysis describes how learners were engaged in the use of physical manipulatives to address their misconceptions in measurement. For more information on how learners performed in the intervention programme, refer to appendix F

Once I completed the first part of the tests, I analyzed the data from the post-intervention task (post-test) was analysed. The learners' performance was illustrated graphically as well as qualitatively, by fully describing the data. Lastly, data collected from the semi-structured interviews transcripts were analysed qualitatively, for details refer to appendix G.

4.2 DATA PRESENTATION AND DISCUSSION

4.2.1 Objectives of piloting the task

The task was piloted to identify possible ambiguities in the wording of the questions and to test the appropriate depth of the content of the baseline assessment. A further reason for piloting was to obtain insight into how much prior knowledge on measurement the learners had from their year in Grade 7.

4.2.2 Results of the piloted task

In order to enhance the quality of the data analysis, I organised data from the pilot group in a graph which is presented in figure 4.1. For the details of the pilot task refer to chapter 3, section 3.6.1, pp. 43-45

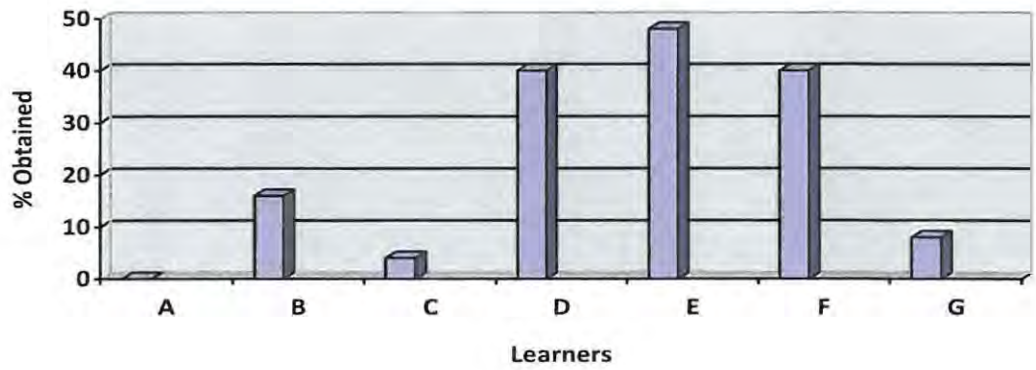


Figure 4.1 A graph showing the overall performance of the pilot group.

Most of the learners could fully answer one question and a part of the other. Learner A could not solve any of the questions correctly. Two learners B and C obtained 16% and 4%, respectively. A mark of 8% was obtained by learner G. Two learners (D & F) each obtained 40%. The score of 48 % was attained by learner E.

This data revealed that the task which the pilot group wrote was appropriately set at their level. The reason for the incorrect answers could be as a result of misconceptions due to faulty learning or carelessness or generalisation of the mathematical concepts. Some of the calculations showed that with the help of a better intervention programme these learners should be able to respond to most of the questions correctly.

Table 4.1 Question-per-question summary of learners' performance

LEARNER	QUESTION									OVERALL % OBTAINED
	1	2	3	4	5	6	7	8	9	
	%	%	%	%	%	%	%	%	%	
A	0	0	0	0	0	0	0	0	0	0
B	0	16	0	0	0	0	0	0	0	16
C	0	0	0	0	0	4	0	0	0	4
D	0	16	0	0	0	16	8	0	0	40
E	8	16	0	0	0	16	8	0	0	48
F	0	16	0	0	0	10	6	0	8	40
G	0	8	0	0	0	0	0	0	0	8

Table 4.1 shows the performance of the learners in the pilot task. It shows the marks obtained by individual learner per question.

In question 1 the general answer was that of naming a cube and a rectangular prism a square and a rectangle, respectively. In question 2 learners B, D, E, F and G correctly said that a cube and a rectangular prism each had 6 faces. Those learners who could not answer the question correctly said that each of these (a cube and a rectangular prism) had 1 or 2 or 3 or 4 faces.

In question 3, most of the learners said that a cube was a 2-dimensional solid shape. To find the surface area of a cube and a rectangular prim in question 4 almost 90% of the learners said that surface area = l x b x h, others got their answer in cm^2, while some in cm^3. For the details of learners' responses refer to appendix E.

Question 5 had the same responses as in question 4 with the exception of four learners, learner B said, l x b x h = 12cm 8cm 6cm =26 and learner C said: L x B x H =12 cm x 8cm x 6cm = $57cm^2$. The other group of two said that they needed 3 cloths.

Question 6 was solved in different ways as shown below:

Learner A: (a) V = b +l +h =2cm +2cm+2cm=$6cm^2$ (b) V=B+L+H=8cm+2cm+4cm=14cm

Learner B: (a) 6cm (b) 14cm

Learner C: (a) 2 +2+2+2=$8cm^2$ (b) 8+2+4+8+2+4=$25cm^3$

Learner F: (a) (2x2x2)cm =8cm (b) 4cm x 2cm x 8cm = 16cm

Learner G: L x B x H =12cm x8cm=96cm

Most of the learners thought that volume and perimeter are calculated by using the same procedure. Learner F conceptually knew how to find the volume, but was prevented from getting the correct answer by using the wrong procedure. Learner G did not respond to question 6 'a'.

In question 7; 57% of the learners decided to add the given dimensions in order to find the volume of a rectangular prism. Of all the learners only 43% managed to answer question 7 correctly.

57% of the learners responded to question 8 by saying $240cm^3$- $60cm^2$ =180cm. 43% of the remaining respondents got different answers. Of these 43%; 14% of them each got 160, 64 and 14400.

Question 9 was solved correctly by 14% of the learners; the remaining 86% did not solve it correctly.

General discussion

These results revealed the misconceptions that most of the learners have in their understanding of measurement. Most of them demonstrated that they could not differentiate area from volume of prisms. Also learners had problems in differentiating surface area from perimeter. These misconceptions informed me what to do in the designing of the intervention tasks.

4.3 OVERALL RESULTS OF BASELINE ASSESSMENT AND POST-INTERVENTION TASKS

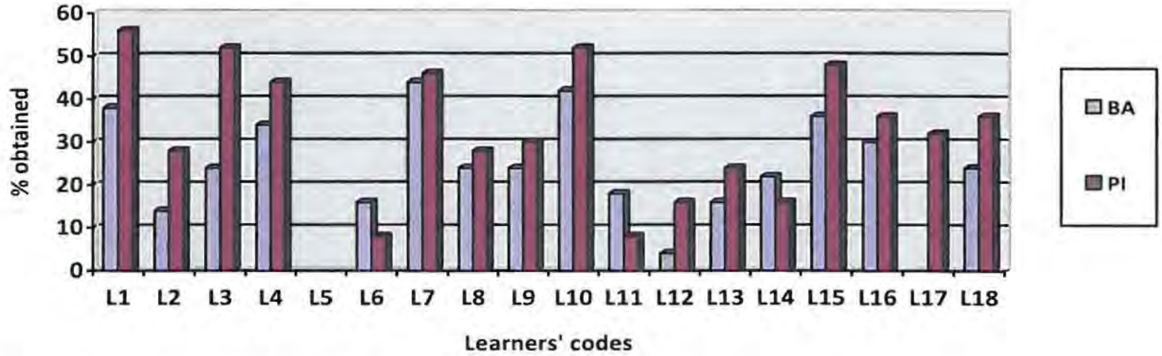


Figure 4.2: A graph showing learners' overall performance in both baseline assessment and post-intervention tasks, respectively.

Learners who improved : L1, L2, L3, L4, L7, L8, L9, L10, L12, L13, L15, L16, L17 and L18.

Learners who did not improve : L5, L6, L11 and L14.

As illustrated in graph 4.2; 78% of the learners improved in their performance in comparison with the marks they obtained in the baseline assessment task and 22% did not improve.

General discussion

From the results shown in graph 4.2 above we might say that the use of physical manipulatives in teaching and learning of measurement had a positive impact in learners' understanding of the concept of measurement. The reasons for this improvement may be that the use of physical manipulatives simplified and motivated learners to do mathematics. The reason why some learners could not improve might be that their level of understanding needed a better grounded prior knowledge of the use of physical manipulatives.

Another reason might be the one suggested by Sowell (1989) in chapter 2. Since these learners were from different primary schools, some had experience of how to use physical manipulatives, but for others it was the first time they had been confronted with the idea of using physical manipulatives in learning mathematics. My experience in teaching shows that one cannot assume that just by having physical manipulatives available, learning will necessarily take place. Teachers need to plan carefully when incorporating physical manipulatives into their teaching programmes. It takes lots of time and creativity to design appropriate activities that integrate into a mathematical lesson where physical manipulatives are used as tools to mediate learning. This often discourages teachers from using physical manipulatives and sometimes learners just play around with them without seeing any mathematical concept being clarified.

4.4 ITEM-PER-ITEM ANALYSIS

In this section, I present a table showing how each of the baseline assessment (BA) task questions correspond with the post-intervention (PI) task questions in content. Secondly, there will be a presentation of how learners performed per question in both baseline assessment and post-intervention tasks. Finally, I provide a general overview of the results per question.

Table 4.2 The correspondence of content of baseline assessment questions to post-intervention questions

Baseline assessment (BA) questions	Post-intervention (PI) task questions
Q. 1	Q. 2
Q. 2	Q. 3
Q. 3	
Q. 4	Q. 4
Q. 5	Q. 5
Q. 5	Q. 5
Q. 6	Q. 6
Q. 7	Q. 7
Q. 8	Q. 8
Q. 9	Q. 9
	Q. 10

4.4.1 BASELINE ASSESSMENT QUESTION 1 AND POST-INTERVENTION QUESTION 2

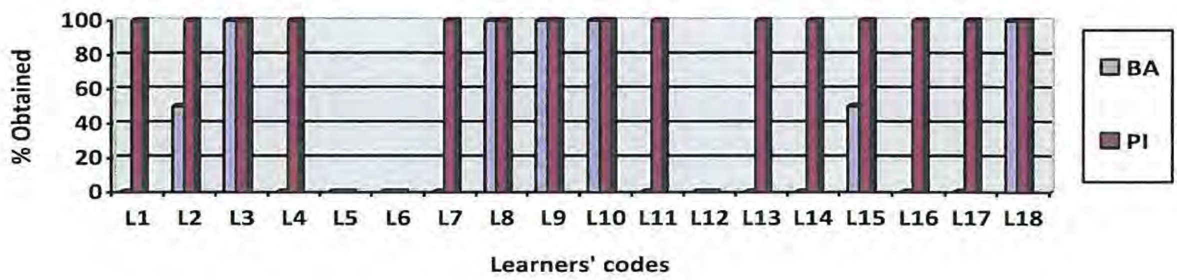


Figure 4.3: A graph showing learners' performance in Questions 1 and 2 of the baseline assessment and post–intervention task, respectively.

The graph drawn above shows that 83% of the learners improved and 17% did not improve in the post-intervention task as compared with the results obtained in the baseline assessment.

Learners who improved : L1, L2, L3, L4, L7, L8, L9, L10, L11, L13, L14, L15, L16, L17 and L18.

Learners who did not improve : L5, L6 and L12.

General discussion

From these statistics one can infer that the manipulatives have assisted the learners in improving their performance. Most of them could read and follow the instruction with ease. Those who did improve could not differentiate a square from a cube, and a rectangle from a rectangular prism. Their answers were the same as those of the fourth-grade learners in Ball's class who "believed that a rectangular prism was a rectangle because it was a box" (Ball, et al. 2005, p. 21). Lack of well grounded knowledge in mathematical terminology and the use of diagrams could possibly be further reasons for the failure to give the correct name to a cube and a rectangular prism.

4.4.2 BASELINE ASSESSMENT QUESTION 2 AND POST-INTERVENTION QUESTION 3

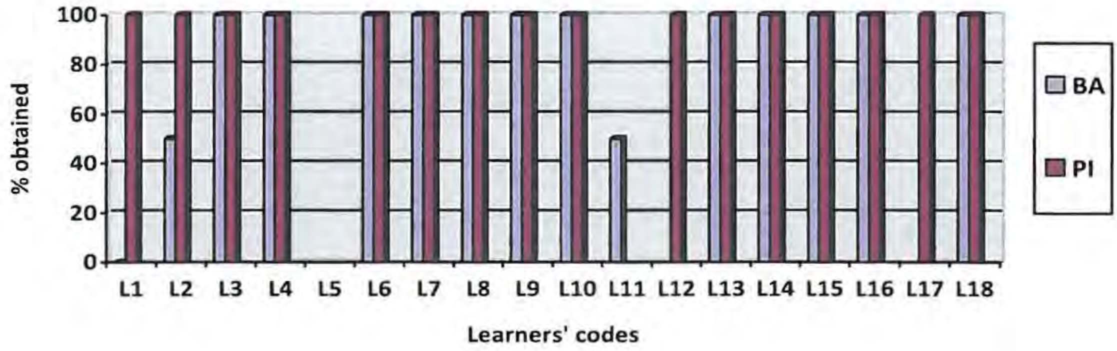


Figure 4.4: A graph showing learners' performance in Questions 2 and 3 of baseline assessment and post-intervention tasks, respectively.

89% of the learners have improved and 11% did not improve in the post intervention task. For the details of how those who did not improve responded to the questions refer to table 4.3.

Learners who improved : L1, L2, L3, L4, L6, L7, L8, L9, L10, L12, L13, L14, L15, L16, L17 and L18.

Learners who did not improve : L5 and L11

Table 4.3 Responses from learners who did not improve

Learner	Responses	
	Baseline assessment	**Post-intervention**
L5	2 (i) a cube has 3 faces (ii) a rectangular prism has 3faces	2(i) object 'a' has 4 faces (ii) Object 'b' has 3 faces
L11	2(i) the square has four faces (ii) the rectangle got six faces	3(i) They are 8 faces (ii) there are 8 faces

General discussion

The learners who did not improve show that they lack understanding of a 3-dimensional figure drawn on a flat paper. Also the term faces was not clear to these learners. L5 counted only the visible faces on the diagram. This response also highlights that this learner was not clear about what was asked. The misconceptions that still existed after this learner was engaged in an intervention task, shows that the use of physical manipulatives is not a guarantee that they simplify mathematical concepts for all learners.

4.4.3 AN OVERVIEW OF BASELINE ASSESSMENT QUESTION 3

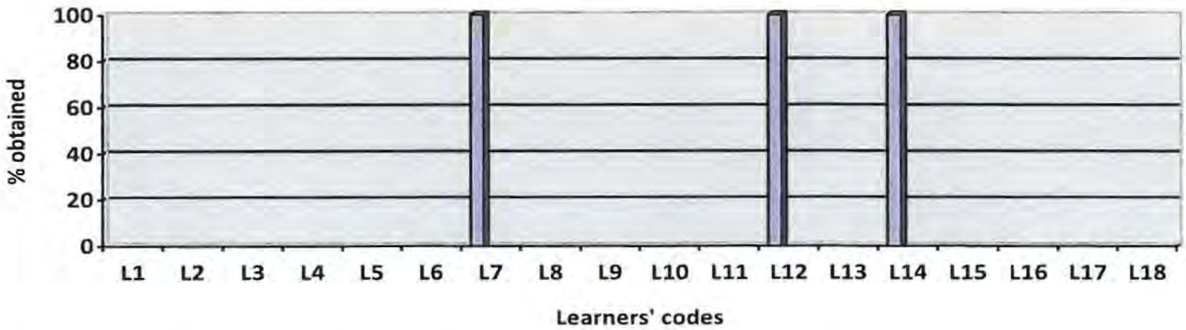


Figure 4.5: A graph showing learners' performance in Question 3 of the baseline assessment.

This question has no corresponding question in the post-intervention task. As shown in the graph above, 17% of the learners obtained a maximum of 100%. And 83% of learners could not answer the question correctly. Of these,

(i) 5,5% said that the sides of a cube are equal; the cube has one dimension, it has 4 dimensions and the last one just decided to ignore the question.
(ii) 11% of the learners said that the cube has 6 dimensions.
(iii) And lastly, 50% responded that the cube has two-dimensions.

General discussion

These results reveal that a 3-dimensional shape drawn on the paper appears to most of the learners as 2- dimensional. Another reason might be one of misinterpretation of the term dimension which has been confused with the term faces. Hence some of the learners said that it has 6 dimensions.

4.4.4 BASELINE ASSESSMENT QUESTION 4 AND POST-INTERVENTION TASK QUESTION 4

Question 4 comprises two parts, 4 'a' and 'b'. None of the participants could solve this question correctly.

Table 4.4 Learners' responses to baseline assessment question 4 'a' and 'b'

Learners	Responses	
	Question 4 'a'	**Question 4 'b'**
L1	They are six different sides object	
L2	L x B = 2 x 4cm = 8cm	1 + b=4 +2+8 = 14
L3; L7; L9; L14; L15 and L18	(2 x 2 x 2)cm = 8cm	L x B x H =(8 x 2 x 4)cm = 64cm
L4; L5; L12 and L17	The total surface area is 6cm	The surface area is 14 cm
L6; L11 and L13	The total is 20 cm	
L 10	Area= l xl = 2cm x2cm =4cm	l x b = 8cm x 2cm = 16cm
L16	2cmx2cmx2cm =8cm	4cm x 2cm x 8cm = 64cm

Instead of answering questions 4 'a' and 'b' separately 17% of the learners combined the two. They simply added together the dimensions of both a cube and a rectangular prism. It seems that this problem emanated from the misinterpretation of the word 'total surface area' in the question and they decided to add the given dimensions in both diagrams without taking into account what the question required them to do. 39% thought that total surface area of the cube and a rectangular prism calculated the same way as the volume. Because of the word 'area' at the end of 'total surface,' 6% thought that the question was asking for the area of the shapes, hence one learner said: Area = l x l and l x b, in question 4 'a' and 'b', respectively.

Table 4.5 Learners' responses to post-intervention task- question 4 'a'and 'b'

Learners	Responses	Learners	Responses
	Question 4 'a'		Question 4 'b'
L1; L3 and L4	3cm x 3cm x 3cm = $27cm^3$	L4	7cm x 5cm x3cm = $127cm^3$
L15 and L18	3cm x 3cm x 3cm=$27cm^2$	L3; L15 and L18	7cm x5cm x3cm =$105cm^2$
L7; L8; and L17	3cm x3cm x 3cm = 27cm	L8	7cm x5cm x3cm =105cm
L11; L12 and L13	3cm x3cm x 3cm = $33cm^2$	L2	$7cm^2$ x$5cm^2$ x$3cm^2$ =$105cm^2$
L2	$3cm^2$ x$3cm^2$ x $3cm^2$ =$18cm^2$	L11; L12 and L13	3cm x 5cm x 7cm = $151cm^2$
L10	3cm x12 = 36	L11	3cm x 5cm x7cm=150cm
L14	12cm	L17	3cm x5cm x7cm=165cm
L5	23cm	L1	7cm x5cm x3cm =$140cm^3$
L6	3cm x3cm x3cmx3cmx3cm = 20	L6	7cm x5cm x3cm =16
L9	261cm	L14	30cm
L16	3cm x 3cm x3cm=18cm	L9	3825cm
		L16	7cm +5cm +3cm =20cm
		L10	105 x 4 =420
		L5	No answer was given.

Findings in question 4 'a'

67% of the learners used the same procedure to solve question 4 'a'. Although their answers were different from each other, they employed the same reasoning in their understanding. They thought that total surface area of a cube and a rectangular prism are calculated the same as their volume. L10 thought that if 3cm is the measurement for an edge of a square then the square has 12 edges-this learner then decided to multiply 3cm by 12 edges. L14 added together the given dimensions of the cube and then multiplied the result by 2. This learner thought if there are three visible 2cm sides, then there are three-2cm sides on the opposite sides.

Findings in question 4 'b'

78% of the learners used the volume method to try and calculate the surface area of a rectangular prism; they multiplied the 3 given dimensions together. Despite the fact that their final answers differed from each other they used similar procedures to arrive at their answers. 22% of the learners solved the question differently. Of these 22%; 5, 5%, solved as follows: (7cm +5cm +3cm) x 2 = 30cm. Another 5, 5% multiplied the given dimensions together and then the result was multiplied by 4. This learner thought that there are four groups of the sides each made up of length, breadth and width. These findings are parallel with what Outhred and McPhail (2000); O'Keefe and Bobis (2008) found in their study.

Perhaps learners have these conceptual problems because of the way they were taught at primary level. Many teachers introduce the formulae before allowing learners to explore ways of solving the problem themselves.

4.4.5 BASELINE ASSESSMENT QUESTION 5 AND POST-INTERVENTION QUESTION 5

In question 5 none of the learners managed to solve the any of the questions correctly.

Table 4.6 Learners' responses to post-intervention task- question 5

	Question 5		Question 5
L1	(a) A= l x b x h =2cm x 2cm x2cm =8cm (b) A = l x b x h = 8cm x 2cm x4cm =64cm	L3, L7 and L9 L10 and L15 L11,L12,L13 L18	It is 400cm Surface area =l x b x h =10cm x 8cm x5cm =400cm^3 It is 400 You will need 400cm^2
L2	16cm	L1 L4	They will require a cloth that is 300 l x b x h =10cm x 8cm x 5cm = 300cm^3
L3, L9 and L18	(12cm x12cm) + (8cmx8cm) + (6cmx6cm) =144cm + 64cm + 36cm =244cm	L2	450 cloth can cover the box.
L11 and L13	No response was given	L5	The volume is 8cm
L7 and L14	52cm	L6	(a) 23cm (b) 15cm
L16	12cm x8cm x6cm=574cm =l x b x h cm	L8	L x b x h= 10cm x 8 cm x 5cm =330cm
L4, L6 and L8	26cm	L14	46cm
L10 and L15	It has to be 576cm	L16	L x b x h=10cmx8cmx5cm=81cm
L5 and L17	The cloth that is required must be 19cm	L17	The cloth must be 23
L12	Length 15cm, breadth 10cm and height 5cm		

L4; L6 and L8 added the three given dimensions together which is why they got 26cm. They thought that volume is calculated the same as perimeter. L11 and L13 did not give any answer to the question. From this we can draw two possible conclusions, both of them might: (a) have misconceptions and as a result they did not want to expose themselves; or (b) not know how to solve the question completely due to the lack of prior knowledge. L3, L9 and L18 seemed to have some idea of how to solve the question, but they applied an incorrect strategy. L10 and L15 thought that surface area of a cube and of a rectangular prism are calculated in the same manner that is why they simply multiplied the three dimensions together.

General discussion

Both baseline assessment and post-intervention tasks results of question 5 shows that these learners mostly memorise the algorithm of calculating volume and surface area of a cube and a rectangular prism without understanding their underlying concepts. Most learners simply randomly applied a formula without evidence of understanding the correct formula to use. Kilpatrick, et al. (2001) reminded us that "to present a problem accurately, [learners] must first understand the situation including its key features" (p. 124).

4.4.6 BASELINE ASSESSMENT QUESTION 6 AND POST-INTERVENTION QUESTION 6

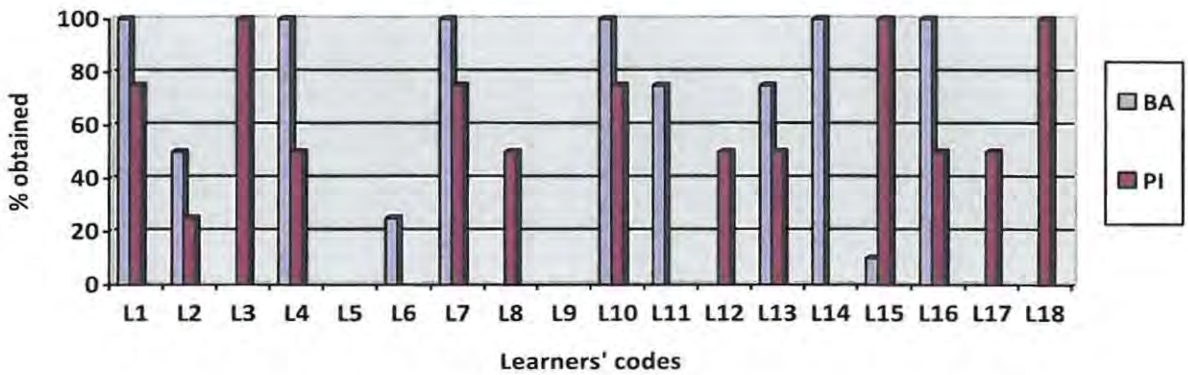


Figure 4.6: A graph showing learners' performance in Question 6 of the baseline assessment and post–intervention tasks.

Of the 18 learners, 33% improved in performance and 67% did not improve.

Learners who improved : L3, L8, L12, L15, L17 and L18

Learners who did not improve: L1, L2, L4, L5, L6, L7, L9, L10, L11, L13, L14 and L16

General discussion

Four learners (L5, L6, L12 & L17) thought that volume of prisms is calculated by adding together the given measurements. The most common problem however was that most learners could not multiply the three numbers correctly. For example, L12 and L13 said that 3cm x3cm x 3cm = 33cm. Kilpatrick, et al., (2001) also found this particular error to be an overwhelming problem in their study.

The majority of the learners could not distinguish between the unit measure of perimeter and area from the volume unit. For example, L17's final answer in the post-intervention task was worked out as follows: 3cm x3cm x 3cm =27cm^2 instead of cm^3.

L3, L9 and L18, have worked out the volume of the cube as follows: (2 x 2 x 2 x 2 x 2 x 2) cm =64cm. They possibly decided to multiply this because they considered the fact that a cube has 6 faces-that is why they multiplied six-twos together, each-2 representing one face.

4.4.7 BASELINE ASSESSMENT QUESTION 7AND POST-INTERVENTION TASKS QUESTION 7

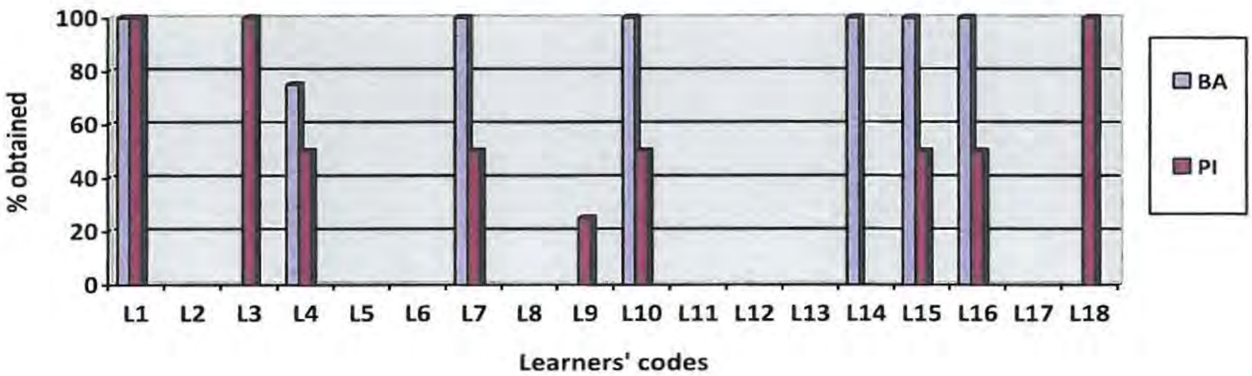


Figure 4.7: A graph showing learners' performance in Question 7 of the baseline assessment and post – intervention tasks.

Out of 18 learners 22 % improved and 78% did not improve.

Learners who improved : L1, L3, L9 and L18

Learners who did not improve: L2, L4, L5, L6, L7, L8, L10, L11, L12, L13, L14, L15, L16 and L17

General discussion

Learners' responses show some of the problems that learners experience in mathematics, for example, L7's responses:

```
  12cm
 x 8cm
 9 6
 x 6cm
5436
```

These calculations showed that L7 had no clear understanding of the concepts of place value and regrouping of the digits when dealing with multiplication of two digit numbers.

L2, L5, L8 and L17 could not get the correct answer; they thought that volume of a rectangular prism is calculated as follows: 12cm +8cm +6cm = 26cm. These calculations revealed learners' misconceptions in the difference between these two concepts, volume and perimeter.

Some learners, for example, L4 and L10 also could not get the correct answer after multiplying, they experienced the same problem as in the previous question. These problems show that mathematical concepts are intertwined. Learners who cannot understand properties of the four mathematical operations are more likely to have problems in most of the topics, like in measurement, probability, patterns, etc. learner needs to be "proficient" (Kilpatrick, et al. 2001, p. 116) in manipulating the four basic mathematical operations in order to solve mathematical problems correctly.

4.4.8 BASELINE ASSESSMENT QUESTION 8 AND POST-INTERVENTION TASK QUESTION 8

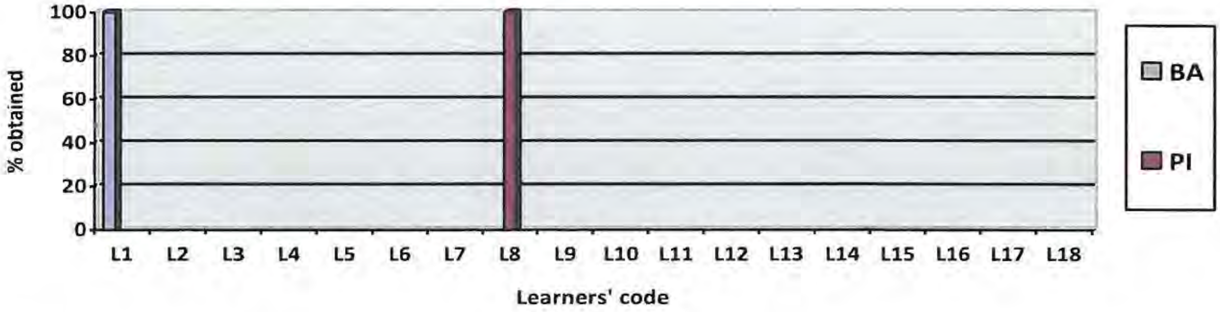


Figure 4.8 A graph showing learners' performance in Question 8 of the baseline assessment and post-intervention tasks.

Out of 18 learners, 6% improved and 94% did not improve.

Learners who improved : L8.

Learners who did not improve : L1, L2, L3, L4, L5, L6, L7, L9, L10, L11, L12, L13, L14, L15, L16, L17 and L18.

Graph 4.8 shows that most of the learners could not respond to baseline and post-intervention tasks correctly. Some of them decided not to answer the question.

General discussion

From the findings of my study, learners have shown that they have different misconceptions regarding the unit measure of volume. In question 8, some learners decided to subtract $60cm^2$ from $240cm^3$. Another group thought that to get the height they had to add together $60cm^2$ and $240cm^3$; the same method was also used by L18 in the post-intervention task. One of the learners got the answer, $300cm^5$. This learner thought that when you add cm^2 and cm^3 you get cm^5. L16 decided to multiply $240cm^3$ by $60cm^2$. Question 8 required a thinking approach in order to solve it. The results show that most of the learners have problems with questions that require them to apply knowledge and skills previously learnt to a new situation. It is also evident that most of the learners understand straight forward questions better than the ones that require the application of skills and knowledge.

4.4.9 BASELINE ASSESSMENT QUESTION 9 AND POST-INTERVENTION TASK QUESTION 9.

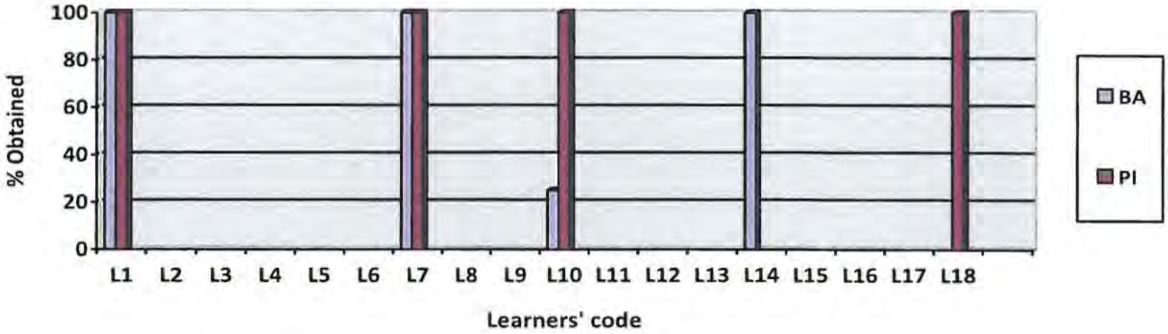


Figure 4.9: A graph showing learners' performance in Question 9 of the baseline assessment and post – intervention tasks.

Out of 18 learners 22% improved and 78 % did not improve.

Learners who improved : L1, L7, L10, and L18.

Learners who did not improve: L2, L3, L4, L5, L6, L8, L9, L11, L12, L13; L14, L15, L16 and L17.

General discussion

Some learners thought that since the cube has six faces they should allocate 2cm for each face, working on the assumption $(2cm)^6 = 64$. They did not know that in order to estimate the volume of a cube the focus is on three dimension (s x s x s) not faces.

One learner said the height was 16, the supporting idea here was that of a squares having 4 equal sides. The learner resolved that the volume of a cube: 16 x 4 = 64. L13 and 12's answer in BA and PI, respectively, showed that the given volume was for the 3 visible faces of the cube as they appeared on the paper. In order to get the volume of the whole cube each one of them decided to multiply the given volume by 2 since there is another set of 3faces that was not visible on the diagram.

4.4.10 LEARNERS' PERFORMANCE IN THE POST-INTERVENTION TASK QUESTION 10

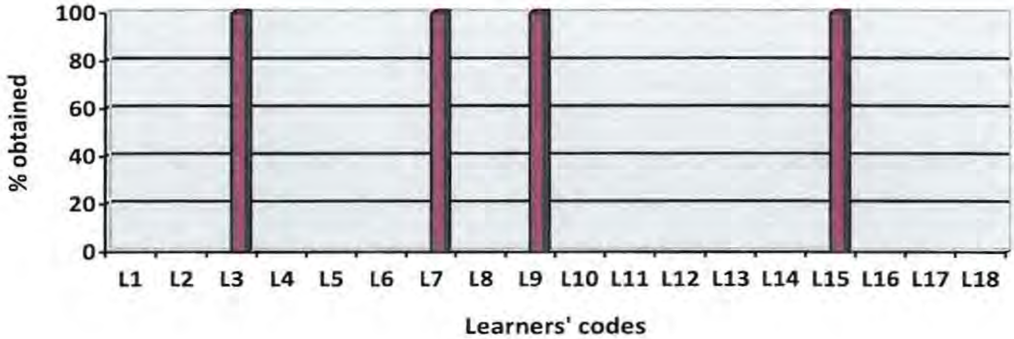


Figure 4.10: A graph showing learners' performance in Question 10 of the post- intervention tasks

Of 18 learners 22% answered the question correctly, they obtained 100% and 78% of them did not answer the question correctly.

General discussion

Most of the learners could not see that the diagram represented a 3-dimensional shape and so regarded it as a 2-dimensional shape. The reason for these misconceptions might be: (i) lack of prior knowledge in learning mathematics using physical manipulatives, (ii) learners' misinterpretation of mathematical representations (i.e. 3-dimensional shapes on a flat surface), (iii) insufficient time to engage with physical manipulatives might be another factor.

Most of the learners had a problem in identifying the number of dice that were in the stack. For example, some learners counted the squares seen on each of the faces of the stack.

4.5 INTERVENTION PROGRAMME'S FINDINGS

In this section, I present how learners engaged with physical manipulatives in the intervention programme. These findings are based on the learners' written responses and the information collected using an observation schedule.

During the intervention programme learners worked in four groups of different number of members as follows: - Group 1 (G.1) had four members, Group 2 (G.2.) – five members, Group 3 (G.3) and Group 4 (G.4) consisted of five members each. These groups were used throughout the intervention programme. Learners were asked to work in groups because of limited resources. I also wanted to investigate whether physical manipulatives could be mediating tools for group discussion. The presentation of the findings in this section is based on how learners worked in their individual groups using physical manipulatives.

4.5.1 INTERVENTION PROGRAMME TASK 1'S FINDINGS

G.1, G.2 and G.4 answered the first 6 questions (from question 1.1 to 1.3) in task 1 correctly. G.3 could not answer question 1.4.3 correctly.

Table 4.7 Questions which learners' groups could not answer correctly

Group	Question
G.1	1.6.1
G.3	1.4.3 1.5.2 1.6.1 (iii)
G.4	1.5.2 1.5.3

Out of 14 questions in intervention task 1, that is 57% of them were correctly done and 43% were correct.

4.5.2 INTERVENTION PROGRAMME TASK 2'S FINDINGS

Task 2 comprised 11 questions. Of the 11questions, 81% were not correctly done and 19% were done correctly. Of all the groups G.1 responded to all 11 questions correctly.

Table 4.8 Questions which groups could not answer correctly.

Group	Question
G.2	2.4 (i) and (ii) 2.5
G.3	2.3 2.4 (ii) 2.5
G.4	2.4 (ii) (b) 2.5

4.5.3 INTERVENTION PROGRAMME TASK 3'S FINDINGS.

Intervention programme task 3 comprised 19 questions. Out of those questions, 63% were answered correctly and 37% were incorrect. G.2 was the only group that managed to do all the questions correctly.

Table 4.9 Questions which groups could not answer correctly.

Group	Question
G.1	3.2.1 (iv) 3.2.2 (iii) 3.2.2 (iv)
G.3	3.2.1 (iii) 3.2.1 (iv) ; 3.3 (ii)
G.4	3.3 (ii) (b)

4.5.4 INTERVENTION PROGRAMME TASK 4'S FINDINGS

Intervention task 4 was comprised of 18 questions. Out of those questions, 67% were done correctly and 33% were incorrect.

Table 4.10 Questions which learners' groups could not answer correctly.

Group	Question
G.1	4.2.2 4.3.3
G.2	4.3.3
G.3	4.3.3
G.4	4.1.2 4.2.2

4.5.5 GENERAL DISCUSSSION

Learners' written work reflected that physical manipulatives helped them answer most of the questions correctly in each of the given activities. The reasons that some of the learners could not answer the questions correctly might be: (i) it was the first exposure to using these resources to do mathematics, (ii) questions were not clearly understood due to the language barrier, for example, G.2 and G.3 could not answer task 1question 1.5.2.

According to my observation schedule, 71% of the criteria revealed that learners worked with the manipulatives to a great extent. Only 29% of the criteria show that learners engaged with physical manipulatives moderately. Learners' performance when they were using physical manipulatives is described as follows:

(a) Learners who worked with the manipulatives to a great extent showed the following: (i) an ability to use physical manipulatives, (ii) were able to justify their solution using physical manipulatives, (iii) were able to work as a team, (iv) were able to use other resources to enhance the understanding of mathematical terms, (v) they were all actively involved, (vi) they were able to discuss the question using the physical manipulatives provided, (vii) they used physical manipulatives to differentiate surface area and volume of prisms, (viii) almost all the learners were motivated to do the task using physical manipulatives, and (ix) most of the learners could not solve the questions that required them to find the number of dice on the given cube stack.

(b) Learners who engaged themselves moderately in the intervention programme: (i) some could not follow basic instruction to do the task, in such cases I was always called to clarify the concept, (ii) others could not make the optimum use of physical manipulatives as a result they were not able respond to all the questions correctly, (iii) to a certain extent most of the learners could not multiply accurately, and (iv) also could not move from the concrete through to abstract questions (questions that required learners to solve a problem without physical manipulatives).

In addition, some learners could not identify square numbers from the given set of numbers. These results show that not all learners are proficient in number concepts. Thirdly, some participants did not know when and how to use procedures. For example, in task 2, G.2, instead of calculating the total surface area of a rectangular prism that has the following dimensions: length 7 squares, breadth 5 squares and height 3 squares, they added 7, 5 and 3 squares altogether to get 15 squares.

4.6 SEMI-STRUCTURED INTERVIEWS' FINDINGS

The interviews consisted of four semi-structured questions each one of which had one or two follow-up questions. In this section, I present the questions as sub-headings together with the learners' responses. A general discussion follows.

From the interviews three themes emerged namely: (1) similarities and differences in learners' responses to the interview questions, (2) Learners' understanding of the questions, (3) Learners' concerns regarding the use of physical manipulatives.

4.6.1 DO YOU HAVE ANY COMMENTS ABOUT THE USE OF PHYSICAL MANIPULATIVES IN TEACHING AND LEARNING OF SURFACE AREA AND VOLUME OF PRISMS?

Theme 1: Similarities and differences in learners' responses to the interview questions.

L. 2 and L. 5 said that *"the use of physical manipulatives has taught us easy methods of calculating the total surface are and volume of prisms. Secondly, the learning of how to find the total surface area and volume of prisms has been simplified"* (refer to appendix G)

L.3 and L.6 commented that mathematics concepts were better understood when physical manipulatives were used than when learning without them.

Theme 2: Learners' cognition of the questions.

L.4's response to question 1 of the interviews showed that the question was not well understood. This learner said that *"the dice were fitted into the boxes"* (refer to appendix G). And also other learners could not express themselves freely every time the question was asked.

4.6.2 DO YOU LIKE THE USE OF PHYSICAL MANIPULATIVES PROGRAMME IN LEARNING ABOUT SURFACE AREA AND VOLUME?

Theme 1: Similarities and differences in learners' responses to the interview questions.

In question 2- all six learners said that they liked the use of physical manipulatives in learning based on the reasons below.

Both L.1 and L.3 said that the programme was easy to use. L.3's second response said that, *"mathematical concepts were well understood"* (refer to appendix G). L.2 and L.5 said that they had the opportunity of using all sorts of tangible materials to solve mathematical problems. They also said that physical manipulatives motivated them because they were engaged in discussions and sharing ideas when doing the tasks.

L.4 liked the use of physical manipulatives because it was the first time they used them in learning surface area and volume. Secondly, this learner was interested to discover how dice can be used for teaching and learning mathematics. With a different response from all other learners, L.6 liked the use of physical manipulatives because it was part of learning.

4.6.3 DID THE PROGRAM THAT YOU USED HELP YOU TO GET A CLEAR PICTURE AND EXPLANATION OF HOW TO FIND THE SURFACE AREA AND VOLUME OF THE GIVEN CUBE AND THE RECTANGULAR PRISM?

Theme 1: Similarities and differences in learners' responses to the interview questions. 5 learners (L.1 – L.5) responded that the use of physical manipulatives gave them a clear picture and explanation of how to find the total surface area and volume of a cube and a rectangular prism. One learner (L.6) had a different view *"in some areas physical* *manipulatives did not give a clear picture of what to be done, especially the finding the number of squares outside the prisms"* (refer to appendix G).

In response to the follow-up question L.3 felt that physical manipulatives gave them an easy way of solving mathematics. L.4 said that physical manipulatives gave them something to use and refer to when they were solving mathematical questions. L.5 and L.6 discovered that physical manipulatives engaged both hands and mind; as a result *"they make you work in order to learn mathematics"* (refer to appendix G).

4.6.4 APART FROM LEARNING SURFACE AREA AND VOLUME OF CUBES AND RECTANGULAR PRISMS, WHAT ELSE CAN YOU LEARN USING THESE PHYSICAL MANIPULATIVES?

Theme 1: Similarities and differences in learners' responses to the interview questions.

L.1, L.2 and L.3 said that physical maniplatives can also be used to teach perimeter of shapes. L.3 suggested two other topics that could be taught, together with L.6, they said that capacity could be taught using physical manipulatives. L.5 said that *"I have no idea sir, since it is the first time to use these materials"* (refer to appendix G). Lastly, L.6, said that physical manipulatives can be used to teach data handling, but exactly how this would be done was not explained.

L.2 and L.3 responded to the follow-up question that the intervention programme simplified mathematical concepts and they were well understood. With a different view of all L.5 said that the programme promoted group work and they learnt from each other.

Theme 3: Learners' concern regarding the use of physical manipulative

Out of six interviewees, three (L.2, L.3 & L.6) raised their concern regarding the use of physical manipulatives in learning mathematical concepts. Of these three, two learners (L.2 & L.3) suggested that physical manipulatives must be used in teaching all topics in mathematics because they provide an accessible way of doing mathematical activities. L.6 advised that when using physical manipulatives optimum time is required in order to understand each and every step involved in learning mathematics.

4.6.5 GENERAL DISCUSSION OF THE INTERVIEWS FINDINGS

The identified themes reveal that the use of physical manipulatives has helped most of the learners to construct a clear understanding of mathematical concepts. Learners consider physical manipulatives to be important and helpful tools for teaching and learning mathematics. From the responses it is apparent that the use of physical manipulatives has helped them to learn and grasp mathematical concepts with greater ease. Mathematical language is a barrier to most of the learners which is why it is advisable that physical manipulatives should be used as much as possible in order to close the gap between learning and language.

4.7. CONCLUSION

In conclusion, I discovered that physical manipulatives can play an important role in learners' learning and understanding in mathematics. They motivate learners to concentrate on their task at hand, clarify and simplify concepts that are difficult to understand without the use of physical manipulatives and open up group discussion where all learners get actively involved in their own learning, and finally provide learners with something new to think and about reflect on as the lesson unfolds.

For the learners who did not improve, it is an indication that the mere use of physical manipulatives is not a remedy for all mathematical problems. There are many developmental gaps that need to be filled to ensure that all learners get the help they need to understand mathematics with the use of physical manipulatives.

In the next chapter, I present my final conclusion of the research project and make tentative recommendations on how this project can be taken further to support both learners and teachers in learning and teaching measurement. I also present the perceived limitations of my research project.

CHAPTER FIVE: CONCLUSIONS AND RECOMMENDATIONS

5.1 INTRODUCTION

In this chapter, I offer a summary of the main findings of my research study. Secondly, I place these findings in the context of the literature and discuss the study's significant to the field of mathematics education. I then highlight the limitations identified in the study. I conclude with recommendations for further research on the use of physical manipulatives in teaching measurement.

5.2 SUMMARY AND DISCUSSION OF THE MAIN FINDINGS

The findings of my research study reveal that physical manipulatives can have a positive impact in learners' understanding of concepts of measurement such as surface area and volume. The average mark obtained in the measurement tasks by the participants improved from 25% in the baseline assessment task to 33% in the post-intervention task, which indicates that after the use of physical manipulatives the performance of the learners improved. The introduction of the physical manipulatives may have had the following effects: (i) abstract concepts of mathematics were simplified as learners used physical manipulatives (ii) most of the learners were motivated as they enjoyed using physical manipulatives; (iii) the physical manipulatives mediated learning by allowing learner-to-learner and teacher-to-learner conversation as the instruction unfolded; and (iv) most of the learners were able to see and explore the difference between the concepts of surface area and volume concepts.

5.3 THE LINK BETWEEN THE LITERATURE AND THE RESEARCH FINDINGS

My research findings can be associated with the existing literature in the field as follows: (i) the results in the post-intervention task have shown that participants improved after engaging in the intervention programme that used manipulatives to mediate learning. In research studies conducted by, among others, Heddens (1986); Suydam (1984); Moyer (2001); Van de Walle (2004) and Kilpatrick, et al. (2001) the performance of learners also improved after they had been introduced to physical manipulatives.

(ii) Not all participants showed improvement after using physical manipulatives, indicating that the utilisation of physical manipulatives does not mean that learning will automatically take place. The use of manipulatives should be accompanied by meaningful teacher support. Other studies to have concluded similarly, that the use of physical manipulatives is not a guarantee for learning mathematics with understanding, include Fuson and Briars (1990), Hiebert and Carpenter (1992), and Resnick and Omanson (1987).

(iii) During the interviews some of the interviewees responded that they needed more time to apply physical manipulatives to exploring and understanding the underlying concepts. Sowell (1989), Sebesta and Martin (2004), and Kilpatrick, et al. (2001) also concluded that the length of time spent interacting with physical manipulatives affects the success of learners understanding mathematical concepts. They agree that long term use of physical manipulatives provides more benefit than does a short intervention.

(iv)The use of physical manipulatives facilitated meaningful visualisation of the surface area and volume concepts. It also engaged the learners in active participation as they explored and established for themselves that the area of a shape is about the number of unit squares in an enclosed region and that surface area is the number of squares on the surface of the given prism. This echoes the NCTM (1989) research findings, that physical manipulatives engaged learners in an active process of learning in which they created and discovered new mathematical concepts.

(v) Using physical manipulatives in groups facilitated peer interaction and cooperation. This is consistent with the official South African education policy which affirms that: "progressive education recognises the social aspect of learning, and uses conversation, interaction with others and the application of knowledge as an integral aspect of learning." (South Africa. WCDoE, 2000, p. 30).

(vi) In an interview, one of the participants said that "the use of physical manipulatives reminded me what we did at primary school" (refer to appendix F). The conclusions of research studies done by Cain-Castron (1996) and Heuser (2000) also indicate that to work with physical manipulatives gave their research participants opportunities to reflect on their past experiences.

(vii) Some of the participants could not get the correct answer after multiplication of single digit numbers, for example, they said that 3cm x3cm x 3cm = $33cm^2$. Kilpatrick, et al. (2001) encountered the same problem in their research study.

(viii) A number of participants found it difficult to understand what was required in the various tasks because they did not understand the mathematical terminology and language. South Africa. WCDoE (2000) points to a growing recognition that language and learning cannot be divorced. Language understanding plays a major role in learners' learning. Misunderstanding the language in a subject means misunderstanding the concepts being presented.

5.4 LIMITATIONS OF MY RESEARCH STUDY

Although my study showed that on average there was an improvement in the participants' performance, this result cannot be generalised to a broader context due to the smaller number of participants that I used. Secondly, the time taken to collect data was brief, and research indicates that when using physical manipulatives to increase mathematical proficiency adequate time is required (Kilpatrick, et al. 2001). The relatively short span of time in which I engaged with my participants was at times frustrating. I had to make use of times after school hours when many participants were tired and not willing to concentrate.

Lastly, I would have preferred to have sufficient physical manipulatives for each individual learner to explore the concepts of surface area and volume. Due to financial constraints this was not possible. Research suggests that "each student needs material to manipulate independently in order to understand the concept being addressed." (Heddens, 2010, p. 1). Nevertheless, the cooperative nature of the intervention had its own advantages: learners worked in small groups and interacted with their peers.

5.5 RECOMMENDATIONS FOR FURTHER STUDY

The use of physical manipulatives is but one way of encouraging learning about surface area and volume with conceptual understanding. Further research could explore other strategies, such as using video clips to teach these concepts.

The comparative utility of different physical manipulatives would be worth researching. Research could also focus on the nature of different intervention programmes that mediate the meaningful learning of measurement, making use of other measurement concepts.

Finally, a "research base needs to be established concerning the theoretical and practical issues on the selection criteria for manipulatives across a variety of domains and ability levels" (Chao, et al. 2000).

5.6 REFLECTION

Despite the many challenges that I have faced over the last two years of the coursework and research, I have learned many things about myself and my field of study. In particular, I have learned:

(i) to read extensively;

(ii) that teaching mathematics is not about following the textbook, but assessing the levels of your learners and preparing tasks based on their abilities;

(iii) that physical manipulatives enhance teaching and learning of mathematics;

(iv) that the research process requires lots of creativity, time, and resources in order to enable the collection of accurate and relevant data.

REFERENCES

Babbie, E., & Mouton, J. (2001). *The practice of social research.* Cape Town: Oxford University Press.

Ball, D. L. (1992). Magical hopes: Manipulatives and the reform of math education. *American Educator, 16*(2), 14-18.

Ball, D. L., Hill, H. C., & Bass, H. (2005). Knowing mathematics for teaching. Who knows mathematics well enough to teach third grade, and how can we decide? *American Educator, 30*(3), 14-17, 20-22, 43- 46.

Baroody, A. J. (1989). Manipulatives don't come with guarantees. *Arithmetic Teacher, 37*(2), 4-5.

Barret, J., Jones, G., Thorhton, C., & Dickson, S. (2003). Understanding children's developing strategies and concepts for length. In D.H. Clements & G. Bright (Eds.), *Learning and teaching measurement* (pp. 17-30). Reston: National Council of Teachers of Mathematics.

Bell, J. (1993). *Doing your research project. A guide for first time researchers in education and social sciences.* Milton Keynes: Open University Press.

Bless, C., & Higson-Smith, C. J. (1995). *Fundamentals of social research methods. An African perspective* (2nd ed.). Claremont: Juta.

Boggan, M., Harper, S., & Whitemire, A. (2007). Using manipulatives to teach elementary mathematics. *Journal of Instructional Pedagogies*, 1-8.

Brooks, J. G., & Brooks, M. G. (1993). *In search of understanding: The case for constructivist classrooms.* Alexandria, VA: Association of Supervision and Curriculum Development.

Brown, B. (2009). *Mathematics elective learning rational numbers.* BEd lecture notes, Rhodes University, Education Department, Grahamstown.

Bryant, B. R., Bryant, D. P., Kethly, S. A., Pool, C., & Seo, Y. J. (2008). Preventing mathematics difficulties in the primary grades: The critical features of instruction in textbooks as part of the equation. *Learning Disability Quarterly, 21*(1), 21-35.

Cai, J. (2003). What research tells us about teaching mathematics through problem solving. In F. Lester (Ed.). *Research and issues in teaching mathematics through problem solving* (pp. 241-254). Reston, VA: National Council of Teachers of Mathematics.

Cain-Caston, M. (1996). Manipulative queen. *Journalof instructional Psychology, 23*(4), 270-274.

Carrol, W. M., & Porter, D. (1997). Invented strategies can develop meaningful mathematical procedures. *Teaching Children Mathematics, 3*(7), 370-374.

Carpenter, T. P., Coburn, T. G., Reys, R, E., & Wilson, J, W. (1976). Addition and multiplication with fractions. *The Arithmetic Teacher, 23*(2), 137-142

Carpenter, P., & Lehrer, R. (1999). *Teaching and learning mathematics with understanding.* Mahwah, NJ: Lawrence Erbaum.

Cass, M., Cates, D., Smith, M., & Jackson, C. (2003). Effects of manipulative instructin on solving area and perimeter by students with learning disabilities. *Learning Disabilities Research and Practice, 18*(2), 112-120.

Chappel, M. F., & Structchens, M. E. (2001). Creating connections: Promoting algebraic thinking with concrete models. *Mathematic Teaching in the Middle School*. Reston, VA: National Council of Teachers of Mathematics.

Chao, S., Stigler, J., & Woodward, J. (2000). The effects of physical materials on kindergartners' learning of number concepts. *Cognition and Instruction, 18*(3), 285-316.

Chester, J., Davis, J., & Reglin. (1991). *Math manipulatives use and math achievement of third-grade students.* Retrieved from [ERIC 339591].

CITEd Research Center. *Learning mathematics with virtual manipulatives*. Retrieved June 18, 2010, from http://www.cited.org/index.aspx?page_id=151

Clarke, E., & Ritchie, M. (2001). *The practice of social research*. Cape Town: Oxford University Press.

Clarke, D., Cheesemen, J., McDonuogh, A., & Clarke, B. (2003). Assessing and developing measurement in young children. In D. H. Clements & G. Bright (Eds.), *Learning and teaching measurement* (pp. 67-80). Reston: National Council of Teachers of Mathematics. .

Clements, D. H., & McMillen, S. (1996). Rethinking 'concrete' manipulatives. *Teaching Children Mathematics, 2*(3), 270-279

Clements, D. H. (1999). 'Concrete' manipulatives, concrete ideas. *Contemporary Issues in Early Childhood, 1*(1), 45-60.

Clements, D. H., & Bright, G. (Eds.). (2003). *Learning and teaching measurement*. Reston, VA.: National Council of Teachers of Mathematics.

Cohen, L., & Manion, L. (1985). *Research methods in education* (4th ed.). London: Croom Helm.

Cohen, L., Manion, L., & Morrison, K. (2000). *Research methods in education* (5th ed.). London: Routledge.

Danermark, B., Ekstrom, M., Jakabsen, L., & Karrison, J. 2002). *Explaining society. Critical realism in the social sciences*. London: Routledge

Davies, S. M. (2009). *The potential of a stratified ontology for developing materials in community-based coastal marine environmental education process*. Unpublished PhD Thesis, Rhodes University, Grahamstown

Department of Science Mathematics and Technology (SMATE). (2007). *Research in Science and Mathematics Education*. Nelson Mandela Metropolitan University. Port Elizabeth: Goshawk Litho.

Dewey, J. (1938). *Experience and education*. New York: McMillan.

Dienes, Z. P. (1964). *Building up mathematics* (2nd ed.). London: Hutchinson Educational

Discoll, M, J. (1983). *Research within reach: Elementary school Mathematics and Reading*. Reston, VA: National Council of Teachers of Mathematics.

Dutton, W. H., & Dutton, A. (1991). *Mathematics children use and understand.* Mountain View, CA: Mayfield.

Fendel, D. M. (1987). *Understanding the structure of elementary school mathematics.* Boston, MA: Allyn and Bacon

Fennema, E. (1972). The relative effectiveness of a symbolic and concrete model in learning a selected mathematics principle. *Journal for Research in Mathematics Education, 3,* 173 -190.

Flick, U., von Kardorff, E., & Steinke, I. (2004). *A companion to qualitative research.* London: Sage Publications.

Friedman, M. (1978). The manipulative materials strategy: The latest pied piper? *Journal for research in Mathematics Education, 9,* 78-80.

Fuson, K. C., & Briars, D. J. (1990). Using base – ten blocks learning and teaching approach for first and second grade place – value and multi-digit addition and subtraction. *Journal for Research in Mathematics Education, 21,* 180-206.

Fuys, D., Geddes, D., &Tischler, R. (1988). The van Hiele model of thinking in geometry among adolescents, *Journal for Research in Mathematics Education Monograph, 3,* 1-196

Gentner, D., & Ratterman, M. J. (1991). Language and the career of similarity. In S. A. Gelman., & J.P. Byrnes (Eds.), *Perspective on thought and language: Interrelations in development,* (pp. 225-277). London: Cambridge University Press.

Gilbert, R., & Bush, W. (1988). Familiarity, availability, and use of manipulatives devices in mathematics at the primary level. *School Science and Mathematics, 88*(6), 459-469.

Gillham, W. (2000). *Case study research methods.* London: Continuum.

Girouard, A., Solovey, E, T., Hirshfield, L. M., Eccot, S., Shaer, O., & Jacob, J, K. (2007). *Smart blocks: A tangible mathematical manipulative: Learning through physical interaction.* Baton Rouge, LA: Medford.

Goldin, G., & Shteingold, N. (2001). Systems of representations and the development of mathematical concepts. In P. S. Moyer (Ed.), Are we having fun yet? How teachers use manipulatives to teach mathematics: *Educational Studies in Mathematics, 47*(2). Retrieved June 17, 2010, from http.//www.jstor.org/stable/3483327

Goos, M., Brown. R., & Markar, K. (Ed.). (2008), *Primary teachers' perceptions of their knowledge and understanding of measurement*. Australasia: Mega.

Grant, T, J., & Kline, K. (2003). Developing building bocks of measurement with young children. In D.H. Clements & G. Bright (Eds.), *Learning and teaching measurement* (pp. 46-56). Reston: National Council of Teachers of Mathematics.

Greabell, L. C. (1978). The effects of stimuli input on the acquisition of introductory geometry concepts by elementary school children. *School Science and Mathematics, 78*, 320-326.

Greeno, J. G., & Riley, M. S. (1987). Processes and development of understanding. In D. H. Clements, Concrete manipulatives, concrete ideas. *Contemporary Issues in Early Childhood, 1*(1), 45-60.

Grouwns, D. A. (Ed.). (1992). *Handbook of research on mathematics teaching and learning*. New York: Macmillan.

Hartson, R., & Boren, S. (1990). Experiential learning of mathematics: Using manipulatives. Martin, TN., & Charlesston, WV: *ERIC Clearinghouse on Rural Education and Small Schools.*

Heddens, W. J. (1986). Bridging the gap between the concrete and the abstract. *The Arithmetics Teacher, 33*, 14-17.

Heddens, W. J. (1997). *Improving mathematics teaching by using manipulatives*. Kent State University. Retrieved June 17, 2010,

from http://www.fedcuhnk.edu.hk/-fllee/mathfor/edumath/9706/13hedden.html

Heuser, D. (2000). Mathematics Class becomes learner centered. *Teaching Children Mathematics, 6*(5), 288-295.

Hiebert, J., & Lefevre, P. (1986). Conceptual and procedural knowledge in mathematics. An introductory analysis. In J. Hiebert (Ed.). *Conceptual and procedural knowledge: The Case for Mathematics*, (pp. 1-27), Erlbaum, Hillsdale, N. J.

Hiebert, J., & Carpenter, T. (1992). Learning and teaching with understanding. In D. A. Grouws (Ed.), *Handbook of Research on Mathematics teaching and Learning* (pp. 65-97). New York: MacMillan.

Hiebert, J., & Wearne, D. (1993). Instructional task, classroom discourse, and students learning in the second grade. American Educational Research Journal. In F. Lester (Ed.), *Research and issues in teaching mathematics through problem solving*. Reston, VA: National Council of Teachers of Mathematics.

Huang, H. E., & Witz, K. G. (2010). Learning and instructionDeveloping children's conceptual understanding of area measurement: *A curriculum and teaching experiment*. Retrieved July 15, 2010, from html:file//F:/ScienceDirect

Jordan, L., Miller, M., & Mercer, C. D. (1998). The effects of concrete to semi-concrete to abstract instruction in the acquisition and retention of fraction concepts and skills. Learning disabilities: *A Multidisciplinary Journal, 9*, 115-122.

Kilpatrick, J., Swafford, J., & Findell, B. (Eds.). (2001). *Adding it up: Helping children learn mathematics.* Washington, DC: National Academies Press.

Kordak, M., & Balomenou, A. (2006). Challenging children to view the concept of area in triangles in broad context. Exploiting the features of Cabri-II, *International Journal of Computers for Mathematical Learning 11*, 99-135.

Koshy, V., Ernest, P., & Casey, R. (Eds.) (2000). *Mathematics for primary Teachers*. London: Routledge.

Krach, M. (1998). Teaching fractions using manipulatives. *Ohio Council of Teachers of Mathematics, 37*, 16-23.

Laridon, P., Barnes, H., Jawurek, A., Kitto, A., Myburgh, M., & Pike, M, et al. (2006). *Classroom mathematics, grade 11 educator's guide*. Sandton: Heinemann.

Leedy, P.D. (1997). Practical research planning and design (6th ed.). In Department of Science Mathematics and Technology (SMATE). *Research in Science and Mathematics Education.* Nelson Mandela Metropolitan University. Port Elizabeth: Goshawk Litho.

Lehrer, R. (2003). Developing understanding of measurement. In J. Kilpatrick, W. G. Martin, & D. Schifer (Eds.), *A research companion to principles and standards for school mathematics* (pp. 179-192). Reston: National Council of Teachers of Mathematics.

Lesh, R., Behr, M., & Post, T. (1987). Rational number relations and proportions. In C. Janiver (Ed.), *Problems of representations in the teaching and learning of mathematics* (pp. 41-58). Hillsdale, NJ: Lawrence Erlbaum.

Lester, F. (Ed.). (2003). *Research and issues in teaching mathematics through problem solving.* Reston, VA: National Council of Teachers of Mathematics.

Marshall, L., & Swan, P. (2005). Developing mathematical thinking with assistance of manipulatives. *Reform, Revolution and Paradigm Shifts in Mathematics Education*, 144-147.

Maxim, G. W. (1989). Developing preschool mathematical concepts. *The Arithmetic Teacher, 37*(4), 36-42.

Maxwell, J.A. (2005). Qualitative research design: An Interactive Approach pp. 214-253. London: SAGE

Mc Millan, J. H., & Shumacher, S. (1993). *Introduction to qualitative research.* New York: Harper Collins.

Moyer, P. S. (2001). Are we having fun yet? How teachers use manipulatives to teach mathematics: *Educational Studies in Mathematics, 47*(2). Retrieved June 17, 2010, from http.//www.jstor.org/stable/3483327

National Council of Teachers of Mathematics. (1989). Retrieved July 10, 2001, from http:// standards.nctm.org.Previous/CurriEvstds/index.htm

National Council of Teachers of Mathematics. (2000). *Principles and standards for school mathematics.* Reston, VA: National Council of Teachers of Mathematics.

O'Keefe, M., & Bobis, J. (2008). Primary teachers' perception of their knowledge and understanding of measurement. In M. Goos, R. Brown., & K. Makar (Eds.), *Proceedings of the 31st Annual Conference of the Mathematics Education Research Group of Australasia* (pp. 391-397),University of Sydney.

Outhred, L., & Mitchelmore, M. (1996). Children's intuitive understanding of area measurement. In L. Puig & A. Gutirrez (Eds.), *Proceedings of the 20th conference of the International Group for the Psychology of Mathematics Education, 4* (pp. 91-98), Valencia, Spain: University of Valencia.

Outhred, L., Mitchelmore, M., McPhail, D., & Gould, P. (2003). Count me into measurement: A programme for the early elementary school. In D. H. Clements & G. Bright, (Eds.), *Learning and teaching measurement*: *2003 yearbook* (pp. 81-99). Reston,VA.: National Council of Teachers of Mathematics.

Papert, S. (1980). *Mindstorms:* Scranton, PA. Basic Books.

Penlington, T. (2009). The role of models in developing understanding. PGCE lecture notes. Rhodes University, Education Department. Grahamstown.

Piaget, J. (1978). To understand is to invent. In South Africa. Western Cape Department of Education. *Doing OBE part 1*. Cape Town: The Department.

Prawat, R. S. (1992). Teachers' beliefs about teaching and learning: A constructivist perspective. *American Journal of Education, 100*(3), 354-395.

Raphael, D., & Wahlstrom, M. (1989). The influence of instructional aids on mathematics achievement. *Journal for Research in Mathematics Education, 20*, 173-190.

Resnick, L. B., & Omanson, S. F. (1987). Learning to understand arithmetic. In D. H. Uttal K. V. Scudder., & J. H. Deloache, (1997). Manipulatives as symbols: A new perspective on the use of concrete objects to teach mathematics. *Journal of Applied Developmental Psychology, 18*, 37-54.

Romberg,T. A., & Kaput, J. J. (1999). *Mathematics worth teaching, mathematics worth understanding*. University of Massachussets: Dartmouth.

Sebesta, L. M., & Martin, S. R. M. (2004). Fractions building a foundation with concrete manipulatives. *Illinois School Journal, 83*(2), 3-23.

Seymour, D. (1996). Math is learning in action. Learning. In B. M. Vinson (Ed.), A comparison of preservice teachers' mathematics anxiety before and after a methods class emphasizing manipulatives. *Early Childhood Education Journal, 29*(2), 89-94.

Skemp, R. R. (1987). *The psychology of teaching mathematics* (Revised American Edition) . Hillside, NJ: Erlbaum.

South Africa. Department of Education. (2002). *National curriculum statement grades R-9 (General): Mathematics.* Pretoria: The Department.

South Africa. Department of Education. (2003). *National curriculum statement grades 10-12 (General): Mathematics.* Pretoria: The Department.

South Africa. Department of Education. (2006). *National curriculum statement Grade R-9. Orientation programme –Grade 8 and 9.* Pretoria: The Department.

South Africa. Western Cape Department of Education. (2000). *Doing OBE part 1.* Cape Town: The Department

Sowell, E. J. (1989). Effects of manipulative material in mathematics instruction. *Journal for Research in Mathematics Education, 20*(5), 498-505.

Stake, R. E. (2000). *Case study.* In N. K. Denzin & Y. S. Lincoln (Eds.), *Handbook of qualitative research* (pp.435-453). Thousand Oaks: Sage.

Steedly, K. (2008). Effective mathematics instruction. *National Dissemination Center for Children with Disablilities, 3*, 1-11.

Stein, M. K., & Bovalino, J.W. (2001). Manipulatives: One piece of the puzzle. *Mathematics Teaching in the Middle School, 6*(6), 356-359.

Suydam, M. (1984). Research report: manipulatives materials. *The Arithmetic Teacher, 31*(27)

Swan, P., Marshall, L., & White, G. (2010). Mathematics manipulatives: A panacea or pandora's box. EDU Conference.

Tashakkori, A., & Teddie, C. (2003). Issues and dilemmas in teaching research methods courses in social and behavioural sciences: US perspective. *International Journal of Social Research Methodology, 6*(1), 61-77.

Terre Blanche, M.T., & Durrheim, K. (1999). *Research in practice: Applied methods for* social sciences. Cape Town: University of Cape Town Press.

Thompson, C. S., & Van de Walle, J. (1984). Let's do it: The power of 10. *Arithmetic Teacher, 32*(7), 6-11.

Thompson, P. W. (1994). Notions, conventions and constraints: Contribution to effective uses of concrete materials in elementary mathematics. *Journal of Applied Developmental Psychology, 18*, 37-54

Tuckman, B.W. (1972). Conducting educational research. New York: Harcout Brace Jovanovich.

Uttal, D. H., Scudder., K. V., & Deloache, J. H. (1997). Manipulatives as symbols: A new perspective on the use of concrete objects to teach mathematics. *Journal of Applied Developmental Psychology, 18*, 37-54.

Vance, J., & Kieren, T. E. (1971). Laboratory settings in mathematics: What does research say to the teacher? *Arithmetic Teacher, 18*(8), 585-589.

Van de Walle, J. A. (2004). *Elementary and middle school mathematics.* Boston:

Pearson.

Van Wynsberghe, R., & Khan Samia (2007). *International Journal of Qualitative Methods, 6*(2), 80-94.

Vinson, B. M. (2001). A comparison of preservice teachers' mathematics anxiety before and after a methods class emphasizing manipulatives. *Early Childhood Education Journal, 29*(2), 89-94.

Wearne, D., & Hiebert, J. (1988). A cognitive approach meaningful mathematics instruction: Testing a local theory using decimal numbers. *Journal for Research in Mathematics Education, 19*, 371-389.

Wilkinson, J. (1974). In W. M. Fitzgerald & J. L. Higgin (Eds.), *Mathematics laboratories: implementation, research and evaluation* (pp. 35-60) Columbus: Eric Center for Science, Mathematics and Environmental Education.

Wirtz, R. (1978, April). An elementary mathematics curriculum for all children. Paper presented at the National Council of Supervisors of Mathematics meeting. San Diego, CA.

Wolfang, C., Stannard, L. L., &Jones, I. (2001). Block play performance among preschoolers as a predictor of later school achievement in mathematics. *Journal of Research Education in Childhood Education, 15*(2), 173-180

Yin, R. K. (1994). *Case study research: Design and methods.* London: Sage.

Zuckerman, O., Arida, S., & Resnick, M. (2005). *Extending tangible interfaces for education: Digital Montessori-Inspired Manipulatives.* Cambridge: MIT Media Laboratory.

APPENDIX A:

A LETTER OF CONSENT TO MY SCHOOL

15th November 2010

From: S.M. Chiphambo, 36 Prince Alfred Street, Queenstown, 5320.

To : The Principal and The School Governing Board, P.O. Box 1159, Queenstown, 5320.

Dear Sir/Madam

REQUESTING FOR TH E CONSENT TO DO A RESEARCH PROJECT

I am going to conduct the research project in mathematics education. I hereby request for your consent to engage Grade 8 learners in my project. The project will start from the end of January 2011 to mid- February 2011.

The project will be conducted as follows:

Days : Mondays, Tuesdays and Wednesday

Time : 1 hour 30 minutes after school

Duration : 2 to 3 weeks

The project is aimed to investigate the role of concrete materials in the teaching and learning of measurement in Grade 8. This project will benefit both the researcher as a teacher and the learner since mathematics is one of the crucial subjects.

Lastly, I will be glad if my request reaches your favourable consideration at anytime convenient to you.

Yours faithfully

SHAKESPEAR M. CHIPHAMBO (Student number 608c5101)

APPENDIX B

A LETTER OF RESPONSE FROM MY SCHOOL

KWA-KOMANI COMPREHENSIVE SCHOOL
QUEENSTOWN

1874 PELEM ROAD
MLUNGISI
QUEENSTOWN

P O BOX 1159
QUEENSTOWN
5320

TEL: 045 8381574 FAX: 045 8381574 E MAIL:

PRINCIPAL: Mr V.L. PAKADE

DATE : 29/11/2010

Dear Mr. Chiphambo

RE : Research project : yourself

On behalf of the SGB this serves to acknowledge receipt of your application for your research project which commences in 2011. I am therefore very much grateful to inform you that your application has been unanimously accepted by this committee without any reservations. Please kindly be aware that as long as you stick to your procedure outlined in your letter, we will give you our fullest support in every step of your way.

We wish you all the best for your current studies as well as your future plans.

Yours truly
Secretary ....

KWAKOMANI
1874 PELEM ROAD
MLUNGISI TOWNSHIP
29 NOV 2010
P. O. BOX 1159
QUEENSTOWN 5320
DEPARTMENT OF EDUCATION

APPENDIX C:

LETTER OF CONSENT TO PARENTS/ GUARDIANS

DATE.........................

From:

S.M. Chiphambo

To : Parents

Dear Parents

I am going to conduct the research project in mathematics education. I hereby request for the consent to engage your child....................................... as one of the participants, s/he has already volunteered to do so. The project will be conducted as follows:

Days : Mondays, Tuesdays and Wednesday

Time : 1 hour 30 minutes after school

Duration : 2 to 3 weeks

The project is aimed to investigate the role of concrete materials in the teaching and learning of measurement in Grade 8. This project will benefit both the researcher as a teacher and the learner since mathematics is one of the crucial subjects.

For the confirmation of allowing your child to be engaged in this research projects would you please complete the attached consent form and return it to me.

Thank for your cooperation in this regard.

Yours faithfully

SHAKESPEAR M. CHIPHAMBO (Student number 608c5101)

(Grade 8 Mathematics teacher)

APPENDIX D:

CONSENT FORM TO PARENTS/GUARDIANS

CONSENT FORM

I, ..., accept that my child,..(name of the child) be one of the participants in the research project conducted by Mr S.M. Chiphambo at..school as follows

Duration : 1 hour 30minutes

Days : Mondays, Tuesdays and Wednesdays

Time : After school for 2 to 3 weeks

As long as the project is in progress, I will not allow my child to quit the participation.

By signing this form, I solemnly accept the conditions of the project and I also declare that the information given above is true

Parent(s) signature :...

Contact number :..

For details of the research contact:

S.M. Chiphambo: 0760279032

Email: schiphambo@yahoo.com

APPENDIX E: PILOT TASK'S RESPONSES

LEARNER A

1. objects (a) is sque
objects b is rectangle

2 object(a) has ~~three~~ two face
object b ha three face

3. object(b) has 2 side

4 Area = S x S x S
= 2cm x 2cm x 2cm
= 8cm²

(b) Area = ~~S x S x S~~ L x b x h
= 8cm x 2cm x 4cm
= 64cm²

5. 12cm x 8cm x 6cm
= 586

6. Volume = B+h+h
2cm+2cm+2cm
= 6cm²

(b) volume = B+L+h
= 8cm+2cm+4cm
= 14cm

7 Volume = L+B+h
= 12cm+8cm+6cm
= 26cm²

8 240 - 60 = 180

9 Volume = S+S
= 32cm+32cm
= 64cm³

LEARNER B

1. a) ~~squer~~ ~~sqre~~ square.
   b) rectangle
2. ~~a) faces~~
   a) 6 faces.
   b) 6 faces.
3. a) two different side.
   b) two different sides.
4. a) 6 cm
   b) 14 cm
5. L x B x h
   12cm 8cm 6cm
   = 26
6. a) 6 cm
   b) 14 cm
7. [illegible] 26 cm
   b) [illegible]
8. 160
9. 32

31/01/1

31/01/11

LEARNER C

Part A

1.a Squar
b Rectangle

2 1 face

3 2 different sides

4a $L \times B \times H$
$= 2 \times 2 \times 2$
$= 6cm^2$

b $L \times B \times H$
$= 8 \times 2 \times 4$
$= 64cm^2$

5 $L \times B \times H$
$= 12cm \times 8cm \times 6cm$
$= 57cm^2$

Part B

6a) $2+2+2+2$
$= 8cm^3$

(1)

b) $8+2+4+8+2+4$
$= 25cm^3$

7 $12cm + 8cm + 6cm + 12cm + 8cm + 6cm$
$= 32cm^3$

9 $V - A = H$
$= 240$
$- 60$
$H = 180cm$

10 $64cm^2 \div 4$
$= 16$ dimension of each side

31/01/11

LEARNER D

31 January 2011

Maths Area

1(a) Square/cube ?
1(B) rectangle/rectangular prism ?
2.(a) 6 faces of object a. (2)
2.(B) 6 faces of object B. (2)
3.(B) They are 2 different sides of object (b) has.
4.(a) L X B
= 2cm x 2cm x 2cm
= 8 cm^2
4.(B) L X B
= 8cm x 2cm x 4cm
= 64 cm^3
5. They are 3 cloths.
6.

Volume

6.(a) L X B X h
= ~~12~~cm X ~~2~~cm X ~~2~~cm (2)
= 8 cm^3
6(B) L X B X h
= 8cm x 2cm x 4cm (2)
= 64 cm^3
7. The dimensions or the volume of the box is ~~336~~ 576 cm^3. (2)
8 The height is 180 cm^3.
9. The dimension of each side is 21 cm.

Total 21/?

LEARNER E

31 January 2011

Grade 8

Mathematics

Area

1. (a) Cube

(b) Rectangular prism

2. The are 6 faces of the cube. (2)

(b) The are 6 faces of the rectangular prism. (2)

3. The are 2 different sides that the rectangular prism has.

4. (a) L x b

= 2cm x 2cm x 2cm

= $8cm^3$

(b) L x b

= 8cm x 2cm x 4cm

= $64 cm^3$

5. The are 3 cloths.

Volume

6. (a) L x b x h

= 2cm x 2cm x 2cm (2)

= $8cm^3$

(b) L x b x h

= 8cm x 2cm x 4cm (2)

= $64 cm^3$

7. The volume of the box is ~~$336 cm^3$~~. $576 cm^3$ (2)

8. The height is 180 cm.

9. The dimension of each side is 21 cm

LEARNER F

Grade 8ⁿ 31/01/11

PART A

1.(a). Square
(b) Rectangle

2 Faces of object (a) are 6 six. (2)
Faces of object (b) are 6 six (2)
3 The are 2 the two difference sides.

2 X 2
= 4 X 2
= 8 cm

4 (a) L X B X H
2cm X 2 cm X 2 cm
= 8 cm

4 X 2 =
8 X 2
= 16 cm

12 X 8

(b) L X B X H
(4 X 2 X 8) cm
= 16 cm
5 L X B X H
(12 X 8 X 6) cm
= ~~26 cm~~ 132 cm

9. It 4 X 4 X 4 (2)

6(a) L X B X H (1)
(2 X 2 X 2) cm
= 8 cm

b) L X B X H (1)
4 cm X 2 cm X 8 cm
= 16 cm

7. 12 cm X 8 cm X 6 cm (1)
= 526 cm

8) 64

G

LEARNER G

PArt A

(a) Square
(b) Isocland

2
(a) 4 faces has 6 faces
(b) has 6 faces (2)

3. L×B×H
(4) (a)
L×B×H
(2×2×2) cm
=6cm
(b) L×B×H
(~~2×2~~ (4×2×8)cm
=16 cm

5 L×B×H
12×8
=96

6 L×B×H
(12cm × 8cm)
=96cm
~~L×L~~×1 L×B×H
(8) 64 16 cm

10. 64 + 32

1'4400

APPENDIX F:

AN OBSERVATION SCHEDULE

No.	DESCRIPTION	ALIGNMENT TO KILPATRICK'S STRANDS	RATINGS		
			3	2	1
1. *	The learner were able to present the given diagram using physical manipulatives	Conceptual understanding	√		
2. *	The learners were able to justify their solutions using physical manipulatives.	Adaptive reasoning	√		
3. *	The learners were able to follow basic instructions to do the task at hand.	Procedural fluency		√	
4. *	The learners respected each other's ideas, questions and contributions by attentively listening to each other.	Productive disposition	√		
5. *	The learners were able to use mathematics dictionaries in order to find the meanings of the terms surface area and volume.	Conceptual understanding	√		
6. *	The learners engaged with physical manipulatives (they were able to get the correct answers for all the questions).	Procedural fluency		√	
7. *	In their small groups each and every member was actively involved.	Procedural fluency	√		
8.	The learners' discussions of the given questions were guided by the physical manipulatives.	Conceptual understanding	√		
9.	The use of the programme allowed the learners to gain the skills i.e. communication skills, calculation skills.	Procedural fluency	√		
10.	The learners really used physical manipulatives in order to solve and understand the concepts of surface area and volume.	Strategic competence		√	
11.	The learners could move from concrete stage through pictorial to abstract stage of solving the surface area and volume of the prisms.	Strategic competence	√		

12.	The learners were actively engaged in doing the task at hand using physical manipulatives.	Procedural fluency	√		
13	The learners were motivated to do the task at hand (each and every learner was involved in doing the task).	Productive disposition	√		
14.	There is an ability to understand the question that is presented diagrammatically (shown by solving the questions accurately)	Conceptual understanding	√		

KEY: 3.To a great extent. 2. Moderate. 1. No attempt has been made

*ADAPTED FROM: National Survey of Science & Mathematics Education Horizon Research, Inc. (2000). In side the classroom: Observation and analytical Protocol.

APPENDIX G:

TRANSCRIBED INTERVIEW

KEY:

T: Teacher

L. 1: Learner 1

T	[I explained to the interviewee the aim of the interview and then I asked her the first question] After participating in this research project, what is your comment on the use of physical manipulatives in teaching and learning of surface area and volume of prisms?
L. 1	[She looked at me seemed a bit puzzled and then smiled] Mmm……I felt good.
T	[Follow up question] What do you mean by saying you felt good?
L. 1	[Smiled, but seemed to be uncomfortable and she could not respond to this question]
T	Please! Don't feel uncomfortable; feel free to give me any answer you have in your mind if I ask you a question.
L. 1	[At least, she felt relaxed and seemed ready to respond to questions] I mean that, I like the program which we have used to learn mathematics.
T	Why do you like the use of the program [physical manipulatives] in learning about surface area and volume?
L. 1	[Kept quiet for sometime, seemed to be out of words] It was easy to calculate the surface area and volume of the given cube and a rectangular prism.
T	[Follow up question] How easy was it to calculate the surface area and volume of the cube and a rectangular prism?
L. 1	[She looked at me and smiled, seemed to be thinking deeply] To find the area of a square and a rectangle that were drawn on the paper, I just counted the number of squares that covered each of the regions.
T	[Follow up question] How did you work out the surface area of a cube and a rectangular prism?
L. 1	I counted the number of squares covering one of the faces of a cube and multiplied it

	by 6 to find the total number of the squares on the surfaces of a cube.
T	[Follow up question] Why did you multiply by 6?
L. 1	[Laughed] Because all the faces of the cube are squares and they have the same number of squares covering each region.
T	Ok! That's interesting. What about the rectangular prism, how did you do it?
L. 1	I just counted the number of squares on each and every face of it and added them together.
T	[Follow up question] Did you try to investigate any other method of finding the number of squares [surface area] on the surface of the rectangular prism?
L. 1	No, the moment I got the answer, I just thought of moving on to another question.
T	Ok. Did the program that you have used help you to get the clear picture and explanation of how to find the surface area and volume of the given cube and rectangular prism?
L. 1	Yes, it did.
T	Ok. Now tell me how did you find the volume of the given cube and the rectangular prism?
L. 1	It was easy because I just filled in the dice into each of the prisms. And I found out that the number of the dices each one held that was its volume.
T	Ok. Apart from learning surface area and volume of cubes and rectangular prisms, what else can you learn using these physical manipulatives.
L. 1	Please! Can you repeat the question
T	[I paraphrased the question like this] What other topics in mathematics can be taught using physical manipulatives?
L. 1	Perimeter
T	How can it be taught? Please! Explain this.
L. 1	[she kept quiet, could not explain]
T	[when I discovered that the question was difficult for her due to language issue I decided to release her since all the key questions were answered] Ok. Thank you for your time, have a nice day. [She left]

KEY:

T: Teacher

L. 2: Learner 2

T	[I called the second learner for the interviews and welcomed him] you are most welcome. Please feel free to respond to any question I am going to ask you.
L.2	[Smiled and responded] Ok, sir, I will do so.
T	What is your comment regarding the use of manipulatives in teaching and learning about surface area and volume of prisms?
L.2	[Used gestures to show that repeat the question. Words came after some time and he said] Sir, I didn't get the question well, would you please repeat it or put it the other ways round.
T	[I paraphrased the question] Since you have used materials like dice, rulers and small boxes to find the surface area and volume of the prisms. How did you feel using these materials?
L.2	[Clearing his throat] I have learnt many things in this program.
T	[Follow-up question] You have learnt things like what?
L.2	I have learnt easy methods of calculating the total surface area and volume of prisms.
T	Do you like the use of these materials?
L.2	I like the way we have used all sorts of materials to solve mathematics problems
T	Why do you like the use of the program in learning about surface area and volume?
L.2	[Looked at the ceiling and smiled then answered] The teaching and learning materials you gave us really motivated me and I have been helped to do the work in an easy way. And also if we can use these things more often our minds can be opened.
T	[Follow up question] Would you please, give me an example of how easy it was to use manipulatives.
L.2	For example, to find the surface area of the prisms, I just counted the number

	of squares on each and every face of the cube and added them all together. This is how easy it was.
T	Did the program that you have used help you to get a clear picture and explanation of how to find the surface area and volume of the given cube and rectangular prism?
L.2	Yes, it did.
T	[Follow-up question] How? Would you please explain this?
L.2	For example, we were asked to draw the nets of a cube and a rectangular prism on the provided grid. These were later folded into boxes with squares on their surfaces. After that I had to find the surface area just by counting the total number of squares on the surface of each of the boxes.
T	What else?
L.2	To find the volume of each on the given prisms the dice help us a lot. We first find the definition of volume in the dictionary which defined it as the space occupied by the object. And then we filled each of the prisms with the dice in order to find how many dice each of the prisms could hold. This did not need us to think deeply because the materials you gave us made things easy.
T	Apart from learning surface area and volume of cubes and rectangular prisms, what else did you learn using these materials?
L.2	[Scratched his head for a while and then responded] I think perimeter
T	[Follow-up question] What else?
L.2	That's all what I know
T	Ok. Do you want to say something concerning this project?
L.2	I have learnt a lot from this project and my hope is that we continue having such projects. They can help us to understand mathematics better than using other methods.
T	Thank you for your time you may go, see you next time
L.2	[Left the room]

KEY:

T: Teacher

L. 3: Learner 3

T	[I welcomed learner number 3 and greeted her] Thank you for accepting to be one of the interviewees
L.3	[Smiled and responded] Ok, sir
T	What is your comment regarding the use of the physical manipulatives in teaching and learning about surface area and volume of prisms?
L.3	They have helped me a lot in understanding what you wanted us to learn.
T	[Follow-up question] How did the physical manipulatives help you in learning about surface area and volume of prisms?
L.3	I didn't know that dice and grid papers can be used in learning about surface area and volume of prisms.
T	[Follow up question] What else do you want to say?
L.3	I think it is quiet clear now in terms of the meaning of surface area and volume of prisms. And I like the use of this program.
T	Why do you like the use of the program in learning about surface area and volume of prisms?
L.3	I like it because it is easy to use. The concepts of surface area and volume of prisms are well understood now as compared to learning them without the use of anything.
T	Did the program that you have used help you to get a clear picture and explanation of how to find the surface area and volume of the given cube and rectangular prism?
L.3	Yes, it has really done the good job.
T	[Follow-up question] In what ways did the use of the program give you a clear picture?
L.3	[Laughed] We did some of these things in grade 7, but they were already out of my mind, now I have been reminded of what we did before.
T	What is your feeling concerning the way you were using the program?
L.3	My feeling is that we must use such resources often because they give us easy

	ways to solve mathematical problems.
T	[Follow up question] What easy ways of solving problems did you learn?
L.3	The first way is that the program has helped me to know that surface area is the total number of squares on the surface of the box. And also to find the volume of the box we filled it with dice, the number of dice it takes was its volume. This gave me the idea that volume is the space occupied by the object.
T	Apart from learning surface area and volume of prisms, what else can you learn using these physical manipulatives?
L.3	I think capacity and perimeter'
T	Ok, do you want to say anything concerning the project you were involved in?
L.3	[She nodded her head and kept quiet for a while and then answered]. I have discovered that these materials simplify mathematical concepts; it's easy to understand them.
T	Thank you for your time and participation in the project
L.3	Ok, sir [she left]

KEY:

T: Teacher

L. 4: Learner 4

T	[I called the learner by name,......and then I told him the reason why I called him to my classroom] I want to interview you based on the project you did recently.
L. 4	[Seemed amazed as if he was saying, interview, what for? He didn't say a word, but facial expression said something]
T	Feel free; please! Answer all the questions that I am going to ask you.
L. 4	Ok, sir.
T	What is your comment regarding the use of the manipulatives in teaching and learning about surface area and volume of prisms?
L. 4	[He did not answer as expected] The dice were fitted into the boxes.
T	What I am asking is that, do you like the way you have been learning about surface area and volume of prisms?
L. 4	Yes, I really like the way we were engaged in that project.
T	Why do you like the use of the program in learning about surface area and volume of prisms?
L. 4	[He looked at me and breathed deeply and then answered] It was the first time to use such materials. From that I have learnt many things.
T	[Follow up question] Things like what?
L. 4	Finding the total surface area and volume of a cube and a rectangular prism. And also that objects like dice can be used for teaching and learning of mathematics.
T	Did the program that you have used help you to get a clear picture and explanation of how to find the surface area and volume of the given cube and rectangular prism?
L. 4	Yes, it did.
T	[Follow up question] Explain, how did it give you a clear picture?
L. 4	Since we were doing the tasks in small groups, the materials you provide us with helped us to explain some ideas of surface area and volume of the prisms.
T	[Follow up question] Would you please be clear in your explanation. What

	were you doing in your groups?
L. 4	We used the given materials to find the surface area and volume of the prisms by counting the number of squares and filling the boxes with the dice.
T	What is your opinion concerning the way you were using the provided materials?
L. 4	I beg your pardon, sir. I can't get the question well.
T	[I paraphrased the question] What is your feeling about the use of material that I gave you?
L. 4	The program is good because it gives us something to use when we were to solve some mathematical questions.
T	Apart from learning surface area and volume of prisms, what else can you learn using these physical manipulatives?
L. 4	[He asked me] In which subject? In mathematics.
T	Yes, that is what we are talking about in this interview.
L. 4	[Seemed not clear, he finally said] Sir, I don't get you.
T	[I had to ask in another way] There are so many topics in mathematics. Which ones do you think we can use physical manipulatives to solve mathematics problems?
L. 4	I think, capacity
T	[Follow up question] What else?
L. 4	That is the only one I am sure of.
T	What can you say about the project you have just done?
L. 4	It is good.
T	[Follow up question] What do you mean by saying, it is good"
L. 4	It has taught us how to calculate the total surface area and volume of prisms in a very simple way.
T	Thank you for your time
L. 4	[The learner left the classroom]

KEY:

T: Teacher

L. 5: Learner 5

T	[I welcomed the fifth interviewee] Welcome .... and thank you for accepting to be interviewed.
L. 5	Its ok, sir.
T	What is your comment regarding the use of the manipulatives in teaching and learning about surface area and volume of prisms?
L. 5	It was ok.
T	[Follow up question] What do you mean by saying, it was ok?
L. 5	The program just simplified the learning of how to find the surface area and volume of the prisms.
T	Did you like the use of the provided physical manipulatives in learning about surface area and volume?
L. 5	Yes, I like the program very much.
T	[Follow up question] Why do you like the program?
L. 5	The program is so engaging that we were able to discuss and share our ideas when doing the tasks. The materials you provided helped us to work with what we could really see and discuss with.
T	Did the program that you have used help you to get a clear picture and explanation of how to find the surface area and volume of the given cube and rectangular prism?
L. 5	Yes, it has helped us to understand the questions and the concepts of surface area and volume very well
T	[Follow up question] How did it help you?
L. 5	[Kept quiet for a while, seemed to be thinking for an answer] I didn't know that the total surface area of a cube or a rectangular prism is all about the total number of squares on the surface of the cube or a rectangular prism. And also, volume, I have just learnt that, it is the space occupied by the object. The use and engagement with the materials has made the concepts clear to me.
T	What is your opinion concerning the way you were using the provided

	materials?
L. 5	I have discovered that they are really engaging. As I worked with them I could see that both mind and hands were used.
T	Apart from learning surface area and volume of prisms, what else can you learn using these physical manipulatives?
L. 5	I have no idea, sir, since it is the first time to use these materials.
T	Ok. Do you want to say anything about the project you were involved in?
L. 5	The project was ok, since we worked in groups we have also learnt from each other. Most importantly, I like the materials we used, for example boxes we see them of no use at home yet they are objects to use for mathematics learning.
T	Thank you for your time.

KEY:

T: Teacher

L. 6: Learner 6

T	Welcome to the interview, please feel free.
L. 6	Ok, sir, I will.
T	What is your comment regarding the use of the manipulatives in teaching and learning about surface area and volume of prisms?
L. 6	It's fine.
T	[Follow up question] what do you mean by saying it is fine?
L. 6	It is easy to understand mathematical concepts when real materials are used.
T	[Follow up] Do you like the program?
L. 6	Yes, I do
T	Why do you like the program?
L. 6	I like it because it is part of learning.
T	Did the program that you have used help you to get a clear picture and explanation of how to find the surface area and volume of the given cube and rectangular prism?
L. 6	It was not quite clear to me.
T	[Follow up question] Which part was not clear to you?
L. 6	The counting of units outside the surface of the box.
T	[Follow up question] What do you think need to be done in order for you to clearly understand the concept?
L. 6	I think if I can be engaged in such programmes for quite a long time, it will be clear. I think I didn't get it well because it is the first time to use such materials in learning, at primary we didn't do such things.
T	What is your opinion concerning the way you were using the provided materials?
L. 6	I think it is ok to be engaged in such activities they make you work in order to learn mathematics. If only we can be given enough time to engage ourselves in these activities, I think mathematics can be interesting and challenging
T	Apart from learning surface area and volume of prisms, what else can you learn using these physical manipulatives?

L. 6	I think, data handling
T	Ok, how can you use these materials for data handling?
L.6	I think counting of things.
T	Ok. Do you want to say anything about the project you were engaged in?
L.6	The project was ok. I think I have already said my concern about time.
T	I have noted your concern, thank you for coming for interview.