AN INVESTIGATION INTO AN AFTERSCHOOL INTERVENTION PROGRAMME AIMED AT IMPROVING LEARNERS' MENTAL COMPUTATION SKILLS

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ABSTRACT

This case study centres round an afterschool intervention programme aimed at enhancing mental computation skills of Grade 8 learners at a secondary school in the Erongo educational region of Namibia. Nine research participants took part in the study, and the mental computational strategies exhibited by these participants constitutes the unit of analysis. The study is anchored within an interpretive paradigm and is theoretically underpinned by constructivist epistemology. Kilpatrick, Swafford and Findell's (2001) model of mathematical proficiency provides the conceptual framework supporting the study. The research was carried out in four sequential phases – an initial pre-test, the intervention itself, a follow-up post-test, and a focus group discussion. The study highlights the underdevelopment of mental computation skills and the associated lack of appropriate mental computational strategies in secondary school learners. It is recommended that appropriate time within the school curriculum be allocated for the development of learners' mental computation skills.

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DEDICATION

I dedicate this thesis to my three lovely daughters Nomagugu, Nontombi and Nolwazi. Girls, I appreciate the support you rendered to me throughout this research journey.

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CHAPTER 1

INTRODUCTION OF THE STUDY

1.1 INTRODUCTION

The main focus of this study is to develop, implement and research an afterschool intervention programme aimed at enhancing mental computation skills of Grade 8 learners. This chapter introduces the study. The contextual background of the study is described, followed by a brief description of the research goals, research design and the research process. The chapter concludes with a brief overview of the structure of the thesis.

1.2 CONTEXT OF THE RESEARCH

My interest in improving learners' mental computation skills in mathematics stems from the following points of reference: (i) personal experience as a mathematics educator (thirteen years as a high school teacher, three years as a teacher educator, and six years as an education officer), (ii) literature which underscores the importance of mental computation skills, and (iii) emphasis on the role of mental computation skills in contemporary school curricula.

My personal experience has revealed that many secondary school learners are unable to perform even relatively simple mental computations such as basic multiplication and division and the addition and subtraction of two or more digit numbers. In addition, basic facts such as doubling and halving seem to be lacking in many learners. Given these observations, developing and researching an afterschool programme aimed at expanding and enriching learners' repertoire of mental strategies is clearly an enterprise worth pursuing.

Although the pocket calculator is an important 'tool' for helping learners carry out demanding computational tasks, it should not be seen as a substitute for carrying out calculations that can readily be performed mentally. While the use of calculators is embraced in the Secondary School curriculum in Namibia, and while there are important skills which can be developed through use of the calculator, the contextual backdrop to this research project is the underdevelopment of *mental* computational skills and the associated lack of appropriate mental strategies.

Maclellan (2001) characterizes mental calculation as a process whereby a numerical calculation can be carried out quickly and accurately without the assistance of any external devices, and in a manner that makes use of a specific strategy in a conscious manner (Maclellan, 2001). It is important to take note of the emphasis on the 'conscious' nature of the process and the fact that it involves the use of 'strategies' to arrive at an answer. The strategies used during mental computations thus need to be used consciously as well as flexibly. Heirdsfield (2002) highlights the importance of mental computation skills as being a vital component of learning as learners engage with numbers, take informed decisions about what procedures to follow, and create their own strategies. Furthermore, McIntosh, Nohda, Reys, and Reys (1995) note that "in addition to recognizing mental computation as an important skill, there is renewed interest in using it as a vehicle for promoting thinking, conjecturing, and generalizing" (p. 238).

Reys and Reys (1986) make the sobering remark that over 80% of real-world problem solving makes use of mental computation. Clearly there is a need for these skills to be nurtured and developed within the classroom. Indeed, as Reys and Reys (1986) point out, although there have been numerous calls in the past for specific changes to the Mathematics curriculum in the USA, "all seem to agree on one thing: mathematics programs must give significantly more attention to the development of skills in mental computation" (p. 4). Within the Namibian Mathematics curriculum, Grade 5 learners are expected to make use of different strategies to carry out mental arithmetic calculations involving the addition and subtraction of whole numbers from 0 to 10 000 (Namibia. Ministry of Education, 2006). In Grade 6 this is extended to all four basic operations (addition, subtraction, multiplication and division) for whole numbers up to 100 000, and up to 1000 000 in Grade 7 (Namibia. Ministry of Education, 2006). In Grade 8 and Grade 9 the basic competencies include recalling multiples of numbers less than 10, as well as recalling square numbers up to 20^2, cubic numbers up to 10^3, as well as powers of 2, 3 and 5 up to 2^{10}, 3^5 and 5^4 respectively (Namibia. Ministry of Education, 2007). From Grade 10 onwards, no explicit mention is made of mental computation.

Thus, although some mental computation is explored as part of the Namibian Mathematics curriculum, the question is to what extent this is meaningfully engaged with in the classroom, and to what extent such mental computational strategies are embraced by the learners. The nature of mental computation problems demonstrated by learners at secondary school level, as

personally experienced by this researcher, puts into question whether the competencies as envisaged by the Ministerial curriculum documents are really being met by these learners. It is against this backdrop that this study – developing and researching an afterschool programme aimed at enriching learners' repertoire of mental strategies – is framed.

1.3 RESEARCH GOAL AND QUESTIONS

The purpose of this study was to design, implement and research an afterschool intervention programme aimed at enhancing mental computation skills of Grade 8 learners at a secondary school in the Erongo education region of Namibia. The study is framed by the following guiding questions:

(a) What is the nature of the mental computation problems displayed by the participants?

(b) In what ways did the afterschool programme enhance the participants' mental computation skills?

(c) What is the nature of the participants' experiences in the afterschool programme?

1.4 RESEARCH DESIGN

This study engages with the lived experience of the research participants as they engage with the activities of an afterschool mathematics programme, and the study is anchored within an interpretive paradigm (Cohen, Manion & Morrison, 2011). Nine research participants took part in the afterschool programme, and the research took the form of a case study (Yin, 2009). The case under scrutiny was the group of nine students who took part in the research process, while the mental computation strategies exhibited by the participants constitute the unit of analysis. The research was carried out after normal school hours at a secondary school in the Erongo region of Namibia over a period of eight weeks.

1.5 RESEARCH PROCESS

The research was carried out in four sequential phases – an initial pre-test, the intervention itself, a follow-up post-test, and a focus group discussion. The pre-test was used to assess the mental computation skills of the nine participants. Data from the pre-test served as an initial benchmark with respect to mental computation ability and was also used to inform the activities of the intervention programme. The intervention programme itself took the form of an afterschool

mathematics club that met twice a week for a period of eight weeks. The programme was dedicated to intensive brainstorming sessions where the researcher explored learners' computation strategies, created a platform for learners to learn from one another's strategies, and exposed learners to a variety of different mental computation strategies. Upon completion of the intervention programme a post-test, similar in structure and content to the pre-test, was administered. The research process concluded with a focus group discussion involving all nine research participants. The purpose of the focus group discussion was to engage participants in a discussion around their experiences in the afterschool intervention programme.

1.6 OVERVIEW OF THE THESIS

This section provides a brief overview of the following chapters in this thesis.

Chapter 2 begins with a focus on the researcher's personal experience in the teaching field with reference to learners' mental computational skills. This is followed by a review of literature relating to the importance of mental computation as well as the different types of mental computation strategies. Constructivism, the theoretical framework underpinning the study, is then discussed. Finally, the conceptual framework of Kilpatrick, Swafford and Findell's (2001) model of mathematical proficiency is reviewed, with specific reference to the strands of conceptual understanding, procedural fluency and adaptive reasoning.

Chapter 3 provides an overview of the methods, techniques and procedures used to carry out the research. The chapter begins with a brief discussion of the research goals, orientation, methodology and design. The participant selection, research site and data collection techniques are then described. The chapter concludes with a discussion of the data analysis as well as validity and ethical considerations, and finally the challenges and limitations of the study.

Chapter 4 presents and discusses the results of the study. The chapter is structured in line with the four phases of the study, namely the pre-test, intervention, post-test and focus-group discussion. The chapter concludes with a discussion which draws the results from the different phases together.

Chapter 5 is the final chapter and provides a summary of the research findings in relation to the research questions as well as a brief discussion of the limitations and the significance of the study. The chapter concludes with recommendations and suggestions for further study.

CHAPTER 2

LITERATURE REVIEW

2.1 INTRODUCTION

The purpose of this chapter is to provide a conceptual backdrop to the study. The chapter begins by reviewing aspects of mental computation, its relevance and importance, and the rationale for researching the development of mental computation skills in school-going learners. The chapter also highlights a number of important aspects of constructivism which forms the theoretical framework underpinning the study. Finally, the conceptual framework of mathematical proficiency, specifically Kilpatrick, Swafford and Findell's (2001) model of mathematical proficiency, is delineated.

2.2 THE NATURE OF PROBLEMS LEARNERS ENCOUNTER IN MENTAL COMPUTATION

The researcher's personal experience has revealed the following mental computations as being problematic for many secondary school learners: (i) addition and subtraction of two or more digit numbers (ii) basic multiplication and division of numbers. Basic number facts like factors and multiples are lacking in many learners. Many children are unsure of basic multiplication tables. In addition, basic facts like halving and doubling, which are very useful when it comes to computation problems involving multiplication and division, also seem to be lacking in many learners. The following represent a few examples of the kinds of problems that learners struggle to carry out through mental computation:

1. $24 + 79$ 2. $97 - 43$ 3. $466 + 212$ 4. $921 - 619$ 5. 16×25 6. 36×25
7. $1200 \div 4$ 8. $6.5 \div 0.5$ 9. $90 \div \frac{1}{2}$ 10. $4 \times 3\frac{1}{2}$ 11. 5% *of* 60

The problems shown above become easy to solve once learners' mental computation skills are developed, i.e. once they have acquired the necessary strategies to handle mental computation problems. Some of these strategies will be discussed later in this chapter.

2.3 MENTAL COMPUTATION/CALCULATION (DEFINED)

In research literature the terms *mental calculation* and *mental computation* are often conflated, and in this study the two terms will be used interchangeably. Maclellan (2001) characterizes mental calculation as a process whereby a numerical calculation can be carried out quickly and accurately without the assistance of any external devices, and in a manner that makes use of a specific strategy in a conscious manner (Maclellan, 2001). There are a number of important aspects of mental computation that can be drawn from this definition. Firstly, the view of mental computation being a 'process' suggests that it can be seen as a step-by-step means of carrying out an operation in a clearly defined and logical manner. Furthermore, it is important to take note of the emphasis on the 'conscious' nature of the process and the fact that it involves the use of 'strategies' to arrive at an answer. It is not simply a process that can be accomplished without some deeper thinking being employed.

Plunkett (1979) characterizes the algorithms followed in a mental calculation process as being "fleeting, variable, flexible, active, holistic, and constructive" (p. 3), and that they require understanding. This characterization demonstrates how complex these mental computation strategies are. The strategies used during mental computations need to be used consciously as well as flexibly. Reys (1986) defines mental computation as "the process of producing an exact answer to a computational problem without any external computational aid" (p. 22). Sowder (as cited in Hartnett, 2007) defines mental computation as "the process of carrying out arithmetic calculations without the aid of external devices" (p. 346), while Buys (2008) describes mental computation as "moving quickly and flexibly through the world of numbers" (p. 122). A characteristic feature of all these definitions/characterizations of mental computation is the notion of the computation process being carried out in 'one's head' and not on a piece of paper, and that the process should be swift and accurate.

A good understanding of basic number facts and also a sound knowledge of number bonds, especially the number bonds up to 10, are a prerequisite to effective development of mental computation strategies. As Maclellan (2001) remarks, mental computation needs to be recognized for its complexity:

> …it is a richly connected web of mental computation … for which the child needs a knowledge of number relationships, a facility with basic facts, an understanding of

> arithmetical operations, the ability to make comparisons between numbers and possession of base-ten place value concepts. (p. 153)

Mental computation, unlike written algorithms, requires deeper knowledge of work with numbers than mere recalling of known procedures. Hartnett (2007) underscores that "the key difference is the need for some application of a deeper knowledge of how numbers work" (p. 345). The mental manipulation of multiple numbers at a given moment places a heavy demand on the working memory. Having well developed mental computational skills can ease this load. Heirdsfield (2000) distinguishes two aspects of memory which seem to be significant when it comes to mental computation, namely "load on working memory while calculating, and retrieval from long-term memory of facts and strategies" (p. 6). Well-developed mental computation skills should not exert much load on one's working memory.

Threlfall (2002), as a way of characterizing mental strategies, describes mental computation as a process of "constructing a sequence of transformations of a number problem to arrive at a solution as opposed to just knowing, simply counting or making a mental representation of a 'paper and pencil' method" (p. 30). This highlights the aspect of conscious engagement that for many authors characterizes mental computation. With the use of their own constructed strategies, learners can execute logically derived step-by-step procedures which have a likelihood of leading to the correct solution to a given problem. There is some deeper thinking involved in this process, unlike mere regurgitation of learnt facts or memory recall of standard algorithmic procedures or 'paper and pencil' methods. This is further highlighted by McIntosh (2005) who remarks that "…mental computation demands active thought about numbers and operations rather than unthinking recall of procedures" (p. 7).

Drawing from the literature cited above, for the purposes of this thesis the researcher characterizes mental computation as involving: a) carrying out numerical calculations in the 'head', without the aid of any external device; b) logical step-by-step procedures; c) use of strategies to arrive at a solution; d) a good understanding of numbers and their properties; e) a thorough knowledge of basic number facts.

2.4 IMPORTANCE OF MENTAL COMPUTATION

Although teaching is not the focus of this study, it is nonetheless important to note that teachers should consciously help their learners to develop their computation skills in the classroom. As Reys (1984) clearly states, "mental computation should be a visible part of an elementary mathematics program" (p. 550). In other words, every mathematics syllabus should embrace the development of mental computation skills, preferably right from the elementary grades. Reys (1984) further brings to our attention what he believes to be five widely accepted reasons for teaching mental computation, namely:

> (1) it is a prerequisite for successful development of all written algorithms; (2) it promotes greater understanding of the structure of numbers and their properties; (3) it promotes creative and independent thinking and encouraging students to create ingenious ways of handling numbers; (4) it contributes to the development of better problem-solving skills; and (5) it is a basis for developing computational estimation skills. (p. 549)

The points highlighted above suggest that any child who has a sound development of mental computation strategies is likely to demonstrate a well-developed and creative understanding of the structure of numbers and their properties, and is also likely to command a positive disposition when it comes to working with numbers and problem solving situations. Issues of flexibility in terms of which mental computation strategy to apply and at what instance to apply a particular strategy are embedded in such a mentally computationally literate child.

Varol and Farran (2007) also point out that "mental computation helps children understand how numbers work, how to make decisions about procedures, and how to create different strategies to solve math problems" (p. 89). The overarching goal of developing the mental computation skills of our learners is for them to actively put into use their basic number facts to become flexible at handling a variety of mental calculations. Calculators and other external computation aids should only be resorted to when absolutely necessary. Reys and Reys (1986) make the sobering remark that over 80% of real-world problem solving makes use of mental computation, and as such it makes sense that we develop our learners in such a way that they can face real-world challenges.

McIntosh et al. (1995) reveal some important reasons why mental computation is such a vital skill to develop in children. First and foremost, it is considered a universally valued skill. Mental computation skills are used in real world problem solving situations, and these skills are also

critical prerequisites for the process of estimation. Secondly, mental computation skills provide opportunities to engage in mathematical thinking, thereby strengthening number sense and other general processes related to problem solving. The mathematical skills we develop in our learners should empower them to face the real world problems they meet in life after school; and a sound development of computational skills does just that.

McIntosh et al. (1995) further elucidate that "in addition to recognizing mental computation as an important skill, there is renewed interest in using it as a *vehicle* for promoting thinking, conjecturing, and generalizing" (p. 238). Notions of critical thinking, conjecturing and generalizing lie at the heart of what mathematics is all about, and encouraging these mathematical processes through the use of mental computation is a powerful means of engaging with and developing important mathematical ideas.

Research has also shown that developing mental computation skills of children has rewarding outcomes as compared to relying only on written algorithms or even calculation with external devices. Maclellan (2001) posits that:

> …conventional written algorithms encourage children to carry out the different steps of a computation without actually thinking about them. Mental strategies, on the other hand, demand that the child be actively thoughtful to determine what the numbers mean and how these might be changed in appearance but not in value. (p. 147)

In his comparison of standard written algorithms with mental computation, Maclellan (2001) points out that in standard written algorithms students tend to carry out 'blindly' different steps until reaching the solution, often with limited thought being put into what they are engaged in. In contrast, in mental computation, by its very nature, students are compelled to apply some thinking to the actions they undertake, thus resulting in far greater active engagement on the part of the student.

Hartnett (2007) echoes similar sentiments when she posits that

> Mental computation strategies are different from written algorithms in that they require more than the application of a remembered procedure. The key difference is the need for some application of a deeper knowledge of how numbers work. (p. 345)

It is for these reasons that mental computation should be given a prominent place within the school curriculum. According to Varol and Farran (2007), there is a growing body of research

which affirms that "there is evidence that instruction on mental computation can lead to both increased understanding of number and flexibility in working with numbers" (p. 92). However, caution should be taken not to 'prescribe' mental computation strategies to children. Rather, children should be helped to construct their own mental computation strategies as much as possible. Literature reveals that learners who are good mental computers are often those who invent their own strategies and display less dependence on teacher-taught strategies. In addition, a great deal of value can be gleaned from students sharing and discussing their individual strategies, thereby verbalizing their thought processes. A conducive classroom environment should be created for children to construct their own strategies and to be able to share these with their peers.

Carpenter et al. (1998), in an endeavor to balance the idea of teaching strategies and that of encouraging learners to construct their own strategies, caution that if children were to be taught computation strategies directly, "there would be a danger that children would learn them as rote procedures in much the way that they learn standard algorithms" (p. 19).

The National Statement on Mathematics for Australian Schools (Australian Education Council, 1991, as cited in Heirdsfield, 2002) underscores the importance of mental computation in the curriculum:

> People who are competent in mental computation tend to use a range of personal methods which are adopted to suit the particular numbers and situation. Therefore, students should be encouraged to develop personal mental computation strategies, to experiment with and compare strategies used by others, and to choose from amongst their available strategies to suit their own strengths and the particular context. (p. 5)

Children should be given opportunities to develop their own strategies. This promotes greater understanding and flexibility in computation. Heirdsfield (2011) suggests that "when teaching mental computation, the emphasis should be on children developing their own strategies by exploring, discussing, and justifying their thinking and solutions" (p. 2).

Reys (1984) draws our attention to one benefit of mental computation instruction which relates to problem solving; he remarks that "several studies indicate that spending 10 minutes a day for several months on mental computation activities improves problem-solving performance as well as mental computation skills for both fast and slow learners" (p. 550). These are results based on empirical evidence and suggest that it is the consistency and regularity with which learners are

encouraged to engage in activities geared towards developing their mental computation skills which is important. The more frequently learners engage with tasks demanding use of mental computation, the more they develop these skills.

Students who are good at mental computation possess accurate and efficient mental strategies. They also possess an integrated understanding of number facts such that they are able to use equivalent representations of numbers to their advantage when computing mentally. Heirdfield (2002) underscores that "the value of mental computation is to promote understanding and flexibility in number and operations, that is, to develop number sense" (p. 5). Development of number sense in turn has a bearing on the development of computational strategies, hence the close link between the two.

Literature reveals how mental computation opens children's minds to working well with numbers, for example constructing their own mental strategies and making informed judgments about the procedures they pursue in solving problems. As Hope (as quoted in Tsao, 2004) remarks, "a highly skilled mental calculator could use various ingenious calculation methods, including distributing and factoring" (p. 76).

Reys and Reys (1986) remark that:

> Although calls for specific curricular changes are varied, all seem to agree on one thing: mathematics programs must give significantly more attention to the development of skills in mental computation and … much less attention to traditional written algorithms for computation. (p. 4)

Reys (1985) puts forward a number of benefits of teaching mental computation:

- Mental computation promotes an understanding of the base-ten number system as well as of basic number properties.
- Mental computation encourages thoughtful inspection of a problem prior to the application of an algorithm.
- Mental computation rewards flexibility in dealing with various forms of numbers.
- Mental computation nurtures the development of a keen number sense.
- Mental arithmetic utilizes visual thinking skills. (p. 46)

Threlfall (2002), summarizing some reviewed literature, gives four reasons for teaching mental calculation:

1. Most calculations in adult life are done mentally
2. Mental work develops insight into the number system ('number sense')
3. Mental work develops problem-solving skills
4. Mental work promotes success in later written calculations (pp. 29-30)

Different researchers have highlighted varied reasons for the importance of including mental computation in school curricula. For example, McIntosh (2004) reveals that "adults in their everyday lives use mental computation for over three quarters of all their calculations, whereas written calculation and calculator use are each involved in less than fifteen percent of all their calculations" (pp. 1-2). Karantzi (2010) remarks that mental computation forms an important component of the teaching and learning of mathematics since "it develops problem solving skills, provides opportunities for making calculated estimates and contributes to the understanding of the concept of number" (pp. 3-4), sentiments echoed by many educationalists and researchers.

2.5 MENTAL COMPUTATION STRATEGIES

Primarily, children need to start developing mental computation strategies right from pre-school. This can be handled through discussions based on the natural computation strategies they possess before coming to school. Strategies, based on number facts like count on, doubles, near doubles, bridging ten, tens facts, etc., can be discussed, mainly for single digit numbers, in the early years of their primary education. Beishuizen and Anghileri (1998) point out that "automatising basic number bonds like complements in 10 is an important prerequisite for flexible mental arithmetic" (p. 521). For example, 1 + 9 = 10, 3 + 7 = 10, 4 + 6 = 10, 5 + 5 = 10, or any other combination of numbers which add up to 10.

A great deal of research has been carried out in identifying the type of strategies children use when confronted with computation problems. A number of standard strategies go by different names in different parts of the world. For the sake of clarity, and to establish a general understanding of these different strategies, for the purposes of this thesis I will adopt specific names for these strategies drawing from the literature relating to mental computation strategies. Although by no means exhaustive, the following list provides a summary of some of the most commonly used mental computation strategies. The purpose of this list is to provide a clear description of each strategy:

Counting on & counting back

This is a strategy which involves counting on or counting back. It can be employed right from the addition and subtraction of single-digit numbers up to multiple-digit numbers. For example, in order to calculate 11 + 5 we could simply count on from 11, i.e. 12, 13, 14, 15, and 16 (counting on by 1). In order to calculate 19 – 6: one could count back from 19, i.e. 18, 17, 16, 15,

14, and 13 (counting back by 1). In instances where the larger number appears second, e.g. 5 + 11, pupils should have the flexibility and criticality to realise that counting 11 on from 5 would be equivalent to counting 5 on from 11. The counting on and counting back strategy can also be used to count in 2s (doubles), 5s, 10s or indeed any number. Learners need to be flexible in using this strategy depending on the nature of the computation problem presented.

Near doubles

This strategy is used to compute numbers that are close to doubles or where the creation of a double can facilitate the handling of a computation problem. For sums of single-digit numbers, e.g. 5 + 6, after noticing that 6 is one more than 5 we can swiftly arrive at the answer by doubling 5 and adding 1, 'double 5 + 1 = 11'. For two-digit numbers, e.g. 17 + 19, we see that 19 is two more than 17 and can thus calculate the sum as follows: 'double 17 + 2 = 36'. An analogous situation works for subtraction. For example, 93 - 45 can be seen as 'double 45 + 3 – 45 = 45 + 3 = 48'. 'Seeing' 93 as 'double 45 + 3' is a skill learners need to develop.

Partitioning

This strategy involves mentally splitting numbers into tens and units before adding or subtracting them. For example, in order to calculate 37 + 24, first split the 37 into 30 and 7 and the 24 into 20 and 4. The tens are added to give 30 + 20 = 50, and the units are added to give 7 + 4 = 11. Finally the 50 and the 11 are added to give 61. Similarly, in order to calculate 54 – 32, split 54 into 50 and 4, 32 into 30 and 2, then 50 – 30 = 20, 4 – 2 = 2, and finally 20 + 2 = 22. The idea can also be extended to include decimals. We often use this strategy when adding money. The money is split into dollars and cents, each part is worked out separately, and the two answers are then added together to arrive at the final answer, for example: N$33.50 + N$16.80 = N$(33 + 16) + N$(0.50 + 0.80) = N$49 + N$1.30 = N$50.30.

Aggregation

For the addition and subtraction of two-digit numbers, this strategy involves keeping one number intact and then partitioning the other number into tens and units. For example, to add 27 + 35, keep 27 intact, split 35 into 30 and 5, add 30 to 27 to get 57, and then add the remaining 5 to 57 to get 62. The aggregation strategy is very similar to the partitioning strategy, with the distinction that in the aggregation strategy only one of the numbers is split.

Stringing

This can be seen as a variation of the aggregation strategy where the splitting of the second number is less overt. In addition and subtraction problems involving multi-digit numbers, the first number is left intact while the second, rather than being split, is rather added to or subtracted from the first in parts. For example, to calculate 325 – 249, we would subtract the 249 in stages by first subtracting 200, then 40 and finally 9. Thus, 325 – 200 is 125, 125 – 40 is 85, and finally 85 – 9 is 76.

The stringing strategy can also be used, in a slightly modified form, in multiplication questions. For example, to calculate 48 x 6, the multiplication is carried out in two stages, firstly multiplying 48 by 2, and then multiplying the result by 3. The process of multiplying by 6 has thus been carried out in two stages by using a combination of smaller numbers (2 and 3) whose product is 6. A similar process also works in the case of division.

Bridging to 10 and compensating

This strategy involves bridging a number to a multiple of ten, adding/subtracting the second number to that multiple of ten, and then making an adjustment to compensate for the bridging. By way of example consider the sum 28 + 35. To begin with, bridge 28 to 30. The addition of 30 to 35 can be swiftly calculated to give 65. Finally, we need to adjust the 65 down by 2 in order to compensate (i.e. cancel out) the initial bridging of 28 to 30. Subtracting 2 from 65 thus gives the final answer of 63. A similar process works for subtraction. Consider the sum 67 – 32. Begin by bridging 32 to 30. Subtracting 30 from 67 gives 37 which then needs to be adjusted down by 2 (to compensate for the initial bridging) to give a final answer of 35. A degree of critical thinking is required here – note for instance that even though the initial bridging was 2 units *down*, the final compensation was *also* 2 units down since the bridging resulted in *less* being subtracted. Bridging to 10 clearly eases the calculation demands in computation problems of the type 86 – 39 since it is far easier to subtract 40 than to subtract 39.

Mental image of pen-and-paper algorithm

This strategy involves carrying out a mental computation problem by mimicking a pen-and-paper method in one's mind through a mental image – e.g. adding two 2-digit numbers by lining them up one below the other, adding the units, performing a carry-over if necessary, and finally adding the digits in the tens column.

Halving and doubling

This is a strategy used in multiplication problems where a computation problem is transformed into a less demanding one by halving one number whilst simultaneously doubling the other. Consider the following multiplication: 16×3. By halving 16 to 8 and simultaneously doubling 3 to 6 we have $16 \times 3 = 8 \times 6 = 48$. The process of halving and doubling can also be carried out more than once depending on the nature of the problem. For example: $28 \times 25 = 14 \times 50 = 7 \times 100 = 700$.

Doubling

This is a strategy which can transform seemingly difficult division problems into manageable computation problems. For example, in order to calculate $33 \div 1\frac{1}{2}$, we can simply double both the dividend as well as the divisor to arrive at the equivalent division problem $66 \div 3$ which can be calculated far more easily. As with the strategy 'halving and doubling', this strategy can also be carried out repeatedly.

Multiplicative thinking

This is a strategy which involves seeing repeated addition as multiplication, for example seeing 37 + 37 + 37 as 3 x 37.

Favourable factorisation

This strategy involves recognising favourable factors which can make multiplication or division of numbers much easier. For example, in order to calculate 36 x 25 one could see the 36 as having favourable factors 9 and 4 and use this to calculate the product as follows: 36 x 25 = 9 x 4 x 25 = 9 x 100 = 900.

Although children can be encouraged to develop mental computation strategies, it is important that they also become flexible in using these strategies. The richer the strategies developed, the more flexible learners become in tackling computational problems. For example, McIntosh et al. (1995) point out that a problem like 8×45 can be handled in any number of different ways through mental computation. One could for example double 45 to get 90, double it again to get 180, and double it a third time to get 360. Alternatively one could split the 45 into 40 and 5 and then proceed by multiplying 40 by 8 to get 320, 5 by 8 to get 40, and then adding the 320 and 40 to get 360. A third method could be to bridge 45 to 50, multiply 50 by 8 to get 400, and then to

compensate for the bridging by subtracting 8 multiples of 5 (i.e. 40) from 400 to get 360. A different approach could be to bridge 8 to 10, multiply 45 by 10 to get 450, and then compensate by subtracting two multiples of 45 (i.e. 90) from 450 to get 360.

2.6 THEORETICAL FRAMEWORK

This study is underpinned by a constructivist epistemology. Constructivism is a theory that relates to the conceptualization of knowledge, and how this knowledge is acquired. The basic tenet of constructivism views the acquisition of knowledge as a process of personal cognitive construction, undertaken by the learner, in a process of meaning making. This is in line with what Applefield, Huber and Moallem (2001) also say:

> Constructivism proposes that learner conceptions of knowledge are derived from a meaning-making search in which learners engage in a process of constructing individual interpretations of their experiences. (p. 37)

The learner is viewed not as a passive recipient of knowledge but rather as an active constructor of knowledge, knowledge which itself is constructed on the basis of already existing knowledge. Applefield et al. (2001) also posit that: "Dialogue is the catalyst for knowledge acquisition" (p. 38).

Palincsar (1998) highlights that from a social constructivist perspective, learning can be seen as "the appropriation of socially derived forms of knowledge that are not simply internalized over time but are also transformed in idiosyncratic ways in the appropriation process" (p. 365). Ndlovu (2013) makes an important point that a critical aspect of constructivist epistemology is for personal constructions of knowledge to be "communicated, justified and accepted by the group" (p. 6). An important component of the research process was thus for participants to share and discuss their own mental computational strategies, the idea being that each participant's prior knowledge would add to the general mental strategies of the group as a whole. Furthermore, it was hoped that the process of discussing and sharing strategies could potentially lead participants to construct 'new' mental computation strategies both individually and collectively. Within the context of the afterschool intervention programme, the other learners together with the researcher played the role of the social community, offering the setting for the construction of knowledge, the posing of questions, and the discussion of ideas. According to social constructivist principles, each participant has a role of 'meaning-making' of the constructed knowledge by being actively

involved in the construction process. Constructivism views the knowledge a child constructs as a tentative human construction based on previously acquired knowledge. Duit (1996) highlights the following three principles of constructivism:

- Knowledge is not passively received but is built up by the cognizing subject
- The function of cognition is adaptive and enables the learners to construct viable explanations of experiences
- The process of constructing meaning always is embedded within a social setting of which the individual is part. (p. 42)

It is these guiding principles that I adhered to in conducting this study. All participants were encouraged to be actively involved in coming up with viable mental computation strategies. The viability of the constructed computation strategies lies in the usefulness of this knowledge for the constructor. Within the context of the afterschool intervention programme, what participants constructed needed to make 'sense' not only to them but to their peers as well. With the careful guidance of myself as researcher, the participants were given full autonomy, encouraged to share their views as much as possible, and to discuss their findings or newly constructed ideas with their peers.

Constructivist epistemology is consistent with an inquiry approach. With this in mind, important aspects of the afterschool intervention programme incorporated hands-on activities, which solicit construction of mental strategies, and engagement in discourse which promotes cooperative learning. Probing questions were used which make clear connections between prior knowledge and 'new' ideas/strategies.

To summarize, in this study, the intervention programme was geared at soliciting participants' active engagement with respect to constructing their own mental computation strategies as well as sharing, discussing and justifying their strategies with the other participants. Constructivist epistemology thus not only informed the activities of the intervention programme, but also provided the researcher with an appropriate lens through which to engage with children's construction of knowledge.

2.7 CONCEPTUAL FRAMEWORK

McIntosh et al. (1995) remind us that "mental computation (or thinking) strategies [should] be born out of conceptual understanding and active problem solving rather than memorized rules or

standard procedures" (p. 238). The conceptual framework supporting this study is Kilpatrick, Swafford and Findell's (2001) model of mathematical proficiency, in which there are five interwoven strands, namely:

- *conceptual understanding* - comprehension of mathematical concepts, operations, and relations
- *procedural fluency* - skill in carrying out procedures flexibly, accurately, efficiently, and appropriately
- *strategic competence* - ability to formulate, represent, and solve mathematical problems
- *adaptive reasoning* - capacity for logical thought, reflection, explanation, and justification
- *productive disposition* - habitual inclination to see mathematics as sensible, useful, and worthwhile, coupled with a belief in diligence and one's own efficacy. (p. 116)

Although the five strands making up this model are "interwoven and interdependent in the development of proficiency in mathematics" (Kilpatrick et al., 2001, p. 116), for the purposes of this study the focus will specifically be on the strands of *conceptual understanding, procedural fluency* and *adaptive reasoning.* These three strands are further elaborated on below.

Conceptual understanding

Kilpatrick et al. (2001) view conceptual understanding as "an integrated and functional grasp of mathematical ideas" (p. 118). Knowledge learnt should be organized into a coherent whole and not left as isolated and unrelated facts. A student who has a well-developed conceptual understanding can be characterized as having a profound comprehension of mathematical concepts, the ability to carry out operations well, and should be able to establish relations amongst concepts. Such a student tends to make connections easily, and knowledge gained forms the basis for building on new knowledge. As Kilpatrick et al. (2001) point out, "A significant indicator of conceptual understanding is being able to represent mathematical situations in different ways and knowing how different representations can be useful for different purposes" (p. 119).

Furthermore, Kilpatrick et al. (2001) acknowledge that:

> When students have acquired conceptual understanding in an area of mathematics, they see the connections among concepts and procedures and can give arguments to explain why some facts are consequences of others. (p. 119)

By way of example, consider the calculation 5.41×4.23 for which a student arrives at an answer of 2288.43. A student with conceptual understanding will quickly realise that $5 \times 4 = 20$ hence make a connection that the answer to the product 5.41 x 4.23 should be slightly bigger than 20. This may lead to the realization that the error lies with the decimal point, and that the value of 22.8843 is probably the correct result. In this instance, conceptual understanding in relation to magnitude was critical to identifying the potential source of the error.

It is interesting to see the link between Kilpatrick et al.'s (2001) definition of conceptual understanding and how Hiebert and Lefevre (1986) view the same concept. Hiebert and Lefevre (1986) describe conceptual understanding as:

> Knowledge that is rich in relationships. It can be thought of as a connected web of knowledge, a network in which the linking relationships are as prominent as the discrete pieces of information. Relationships pervade the individual facts and propositions so that all pieces of information are linked to some network. (p. 3)

One important feature of this definition, which resonates very strongly with Kilpatrick et al.'s (2001) notion of conceptual understanding, is the richness of the connections which exist between concepts.

Star (2005) alludes to the fact that a close scrutiny at the definition of conceptual understanding highlights that it is "defined in terms of the *quality* of one's knowledge of concepts - particularly the richness of the connections inherent in such knowledge" (p. 407). For Rittle-Johnson and Alibali (1999), conceptual knowledge represents "explicit or implicit understanding of the principles that govern a domain and of the interrelations between pieces of knowledge in a domain" (p. 175).

A critical aspect in all of these characterizations of conceptual understanding relates not only to the quality of the individual pieces of knowledge, but to the depth that is generated through connecting and relating mathematical ideas to create a network of understanding.

Procedural fluency

Kilpatrick et al. (2001) view procedural fluency as "knowledge of procedures, knowledge of when and how to use them appropriately, and skill in performing them flexibly, accurately, and efficiently" (p. 121). Students who possess procedural fluency have the ability to perceive the necessary steps, algorithms or sequences to carry out a particular task leading to a correct

solution. For example, faced with the calculation $199 + 67$, students who have procedural fluency should immediately recognize that the 'round to 10 and compensate' strategy (or some variation thereof) would be an appropriate procedure to employ. Thus, add 1 to 199, perform the calculation 200 + 67 to get a sum of 267, and then finally subtract 1 from the total to compensate for the initial rounding to 10.

Hiebert and Lefevre (1986) define procedural fluency as follows:

> …familiarity with the individual symbols of the system and with the syntactic conventions for acceptable configurations of symbols. The second kind of procedural knowledge consists of rules or procedures for solving mathematical problems … [such as] chains of prescriptions for manipulating symbols. (pp. 7-8)

Command of mathematical symbols and the mastery of rules or procedures for solving mathematical problems thus constitute the essence of the notion of procedural fluency. In mental computation, a student coming across a problem like "Determine 55% of $600" should immediately show acquaintance with what the symbol '%' means, the convention of the word 'of' and what it means in mathematical language, and also knowledge of the dollar sign '$'. Lack of familiarity with such symbols will contribute to the student's failure to correctly compute an answer to the question. In addition, students need to have appropriate procedures and strategies at their disposal to perform the calculation efficiently once they have interpreted the symbols and syntactic conventions. One could for example split the calculation into two parts, first determining 50% of $600, then determining 5% of $600, and finally adding the two to get a final answer. 50% of $600 represents half of $600, i.e. $300. Since 1% of $600 is $6, 5% of $600 must be 5 x $6, i.e. $30. Thus 55% of $600 = $300 + $30 = $330. No procedural fluency can be achieved without the combined knowledge of symbols, rules and procedures.

Adaptive reasoning

According to Kilpatrick et al. (2001), adaptive reasoning refers to "the capacity to think logically about the relationships among concepts and situations" (p. 129). A student who possesses adaptive reasoning should be able to carefully weigh the available options to a solution route, critically look at the correctness and validity of a solution to a given problem, and finally justify the conclusion. As Kilpatrick et al. (2001) point out, one uses adaptive reasoning to "navigate through the many facts, procedures, concepts, and solution methods and to see that they all fit

together in some way, that they make sense" (p. 129). Adaptive reasoning goes beyond informal explanations and justification of solutions. As Kilpatrick et al. (2001) put it, it also includes "intuitive and inductive reasoning based on pattern, analogy, and metaphor" (p. 129). A student's ability to draw analogical correspondence while problem solving displays a powerful reasoning skill.

According to Kilpatrick et al. (2001), research suggests that reasoning ability is typically manifested in students when the following three conditions are met: "They have a sufficient knowledge base, the task is understandable and motivating, and the context is familiar and comfortable" (p.130). If the knowledge base of the student is weak, this in turn affects the ability to reason logically and arrive at correct and valid conclusions.

Adaptive reasoning is evident when a student shows the ability to justify his or her thought processes, and is able to provide sufficient reason for carrying out a task in a particular manner. There is always a need to probe students to make explicit their thought processes in solving problems, as this allows one insight into their proficiency in carrying out and solving a task. For example, in solving the problem $\frac{37+37+37}{3}$, a student who simply adds the three numbers in the numerator to get 111 and then divides this by 3 to get 37 is simply following a taught procedure 'blindly' without reasoning. A student who displays adaptive reasoning should be able to make the transition from additive to multiplicative thinking, i.e. 3 lots of 37 divided by 3 is 37, that is $\frac{3\times 37}{3}=37$.

Of importance in the discussion of mathematical proficiency is that these various strands are interwoven and interrelated, and it is thus crucial that they are developed together.

2.8 CONCLUSION

This chapter focused on the rationale for the study and outlined the nature of problems children encounter with regards to mental computation. Definitions and examples of specific mental computation strategies were then described. The chapter concluded with an outline of constructivism as the theoretical backdrop to the study as well as the notion of mathematical proficiency as the conceptual framework of the study.

CHAPTER 3

METHODOLOGY

3.1 INTRODUCTION

The purpose of this chapter is to provide a clear overview of the methods, techniques and procedures used to carry out the research. The chapter begins with a brief discussion of the research goal, orientation, methodology and design. The participant selection, research site and data collection techniques are then described. The chapter concludes with a discussion of the data analysis as well as validity/reliability and ethical considerations, and finally the challenges and limitations of the study.

3.2 RESEARCH GOAL AND QUESTIONS

The main purpose of this study was to investigate an afterschool intervention programme aimed at enhancing mental computation skills of Grade 8 learners at a secondary school in the Erongo educational region of Namibia. The study was framed by the following guiding questions:

- What is the nature of the mental computation problems displayed by the participants?
- In what ways did the afterschool programme enhance the participants' mental computation skills?
- What is the nature of the participants' experiences in the afterschool programme?

3.3 RESEARCH ORIENTATION

This study is anchored within the interpretive paradigm. According to Cohen, Manion and Morrison (2011) "the central endeavour in the context of the interpretive paradigm is to understand the subjective world of human experience" (p. 17). Since the study seeks to engage with the lived experience of the research participants as they engaged with the activities of an afterschool mathematics programme, the interpretive paradigm is most appropriate for this study. In line with the interpretive paradigm, the researcher engaged in discussion with the research participants, asking probing questions where necessary, in order to elicit rich data around each participant's mental computation strategies.

3.4 RESEARCH METHODOLOGY

This research takes the form of a case study (Yin, 2009). The case in this instance is the group of 9 students who took part in the research process, while the mental computation strategies exhibited by the participants constitute the unit of analysis.

Leedy and Ormrod (2010) highlight the importance of the case study methodology in that it promotes understanding of a specific situation under study and is useful for investigating focused instances of human interaction, in this case the interaction of the participants in the context of an afterschool programme focussing on mental computation strategies.

3.5 RESEARCH DESIGN

The research was carried out in four phases which in total extended over a 10 week period:

Phase 1: Pre-test

Phase 2: Intervention (afterschool programme)

Phase 3: Post-test

Phase 4: Focus group discussion.

- **Phase 1 – Pre-test**

The study began with the research participants individually answering a baseline pre-test assessment composed of 8 mental computation tasks (Table 3.1).

Table 3.1 Pre-test mental computation questions

Question Number	Question
1	Add 32 + 7 + 18 + 25
2	Three cell phones are priced N$3 600, N$4 000 and N$4 400. What is the total cost of these cell phones?
3	400 – 222
4	Subtract 173 from 998
5	36×25
6	What is 55% of N$320?
7	$55 \div 2\frac{1}{2}$
8	Take $\frac{2}{3}$ of 126 oranges

Each question was individually projected onto a whiteboard using PowerPoint and a data projector. The participant was then required to answer the question through the use of mental computation only. No external aids were permitted, and the question had to be answered orally. After answering the question, the researcher asked the participant to explain their thinking process in order to gain insight into the mental strategies employed by the participant in answering the question. Probing questions were asked where further explanation was deemed necessary. The next question was then projected onto the whiteboard and the process was repeated. The pre-test was important to gain baseline data of the mental computation strategies exhibited by the research participants.

- **Phase 2 – Intervention (afterschool programme)**

The intervention took the form of an afterschool mathematics programme. The participants met twice a week for a total of eight weeks with each session having duration of between two and three hours. The programme created a platform to expose learners to a variety of different mental computation strategies as well as for learners to learn from one another's mental strategies.

The results of the pre-test were important in terms of informing the afterschool programme, specifically with respect to the nature of the mathematical tasks. The weekly sessions were centred round a computer game called *Tux of Math Command*. The game aims at developing students' mental computation skills as well as speed. The game is played by a single player. Mental computation questions appear at the top of the screen and descend to the bottom. The aim of the game is to correctly answer each question before it hits the bottom of the screen. *Tux of Math Command* is pre-loaded with a variety of mental computation problems, ranging from operations with single digit numbers to two or more digit numbers. For the purposes of this study, both single digit and two-digit numbers were used. Table 3.2 summarises the types of questions posed by *Tux of Math Command*.

A single computer was used and each participant was allowed to play one or two rounds of the game. The computer screen was projected onto a whiteboard, so while each player played the game the other participants were able to watch and mentally calculate answers themselves. After playing one or two rounds each, the researcher and the participants engaged in discussions. It is in these discussions that 'new' mental computation strategies were explored.

Table 3.2 *Tux of Math Command* example questions

Question type	Example of typical question
Addition of single digit numbers	$6 + 7 =$
Sums to 10, 15 and 20	3 + 7 = ; 9 + 6 = ; 11 + 9 =
Addition and subtraction of two digit numbers	13 + 26 = ; 43 – 25 =
Multiples of 2 to 15	2 x 9 = ; 3 x 8 = ; 4 x 7 = ; 5 x 4 = 12 x 5 = ; 13 x 12 = ; 14 x 5 =
Division by 2, 3, 4 … 12	$12 \div 2 =$; $9 \div 3 =$; $28 \div 4 =$; $15 \div 5 =$ $24 \div 6 =$; $42 \div 7 =$; $72 \div 8 =$; $36 \div 9 =$
Adding and subtracting integers	–3 + 11 = ; 9 – (–5) = ; –7 – 6
Multiplication and division of integers	$25 \times -3 =$; $20 \div -4 =$

- **Phase 3 – Post-test**

A post-test comprising 8 mental computation problems similar to the pre-test was administered at the end of the research period (Table 3.3). The questions were answered using the same procedure as with the pre-test.

Table 3.3 Post-test mental computation questions

Question Number	Question
1	Add 17 + 15 + 3 + 26
2	Three cell phones are priced N$360, N$400 and N$440. What is the total cost of these cell phones?
3	600 – 333
4	Subtract 153 from 798
5	28×25
6	What is 55% of N$440?
7	$33 \div 1\frac{1}{2}$
8	Take $\frac{2}{5}$ of 225 oranges

- **Phase 4 – Focus group discussion**

A focus group discussion was held at the end of the eight-week programme. In the focus group discussion participants were asked to reflect on the afterschool intervention programme in terms of their experiences engaging with mental computation strategies.

3.6 DATA COLLECTION METHODS

This study made use of pre- and post-tests, an intervention programme in which field notes were taken, and a focus group discussion. Both the pre- and post-tests were administered orally in a manner that allowed for one-on-one engagement with the participants' solution strategies. Pre- and post-test discussions as well as the focus group discussion were audio-recorded and subsequently transcribed. Field notes were taken during the intervention itself.

Use of different data collection methods is useful in qualitative research because it helps the researcher to collect more data than would have been possible using a single data collection method. Kane and Brún (2001) support the use of multiple methods since such an approach provides richer and "'stronger' information than using a single technique" (p.108).

The pre-test was used to answer the first research question. The post-test, as well as field notes taken during the intervention itself, were used to answer the second research question, while the focus-group discussion was used to answer the third research question.

3.7 RESEARCH SITE AND PARTICIPANTS

The research was conducted at a secondary school in the Erongo educational region of Namibia. The school was purposefully chosen due to its proximity to the researcher's place of work. The nine research participants were also purposefully selected from a group of Grade 8 learners. Since this research was carried out after normal school hours, committed and hardworking learners were preferable for this study. The learners' former Grade 7 teacher was consulted to assist in identifying appropriate learners. This pragmatic choice of research participants represents an instance of purposeful sampling (Cohen et al., 2011).

3.8 DATA ANALYSIS

Both the pre-test and post-test were analysed in terms of the correctness of learner responses. In addition, each participant's verbal description of their thinking process allowed each of the eight pre- and post-test items to be coded for the solution strategy employed (counting on and counting

back, partitioning, near doubles, aggregation, stringing, favourable factorisation, multiplicative thinking, bridging to 10 and compensating, halving and doubling, and doubling).

Qualitative data from the focus group discussion was analysed in terms of emerging themes. These themes were developed and categorized over time through repeated engagement with the data. Audio recordings of the focus group discussions were first transcribed. Repeated engagement with these transcripts led to the identification of common ideas that were gradually refined into themes and ultimately coded. These themes included student empowerment, personal knowledge construction, necessity of prior knowledge, perceived value of the programme, and confidence.

3.9 SUMMARY OF THE DESIGN AND TOOLS

Table 3.4 provides a summary of the research process.

Table 3.4 Summary of the research process

Phase	Tool	Aim	Data	Analysis
1	Pre-test	To ascertain a baseline for participants' existing mental computation strategies	Responses to questions Verbal explanations of thinking processes Transcript of audio recordings	Analysis of solutions in terms of accuracy and strategy used
2	Intervention	To create a platform for the development of 'new' mental strategies	Field notes Verbal discussions of solution strategies	Emerging themes Analysis of solutions to intervention questions in terms of accuracy and strategy used
3	Post-test	To assess the development of mental computation strategies	Responses to questions Verbal explanations of thinking processes Transcript of audio recordings	Analysis of solutions in terms of accuracy and strategy used
4	Focus group discussion	To allow students to reflect on their experience	Audio transcripts	Emerging themes

3.10 VALIDITY

Validity refers to whether the data collected and analysed is trustworthy. Moore (1998, p. 163) describes validity as "the degree to which an evaluative device measures what it is supposed to measure". The pre- and post-tests were given to a mathematics colleague for validating the appropriateness of the assessment items. The focus group discussion questions were also circulated to appropriate colleagues for validation purposes.

3.11 ETHICAL CONSIDERATIONS

My position as a researcher was fully explained to all the pre-identified stakeholders. The stakeholders identified in this study were: i) the Education Officer for Mathematics and Science in the Erongo Education region; ii) the Principal of the school which the participants attend; iii) the parents/guardians of the research participants; and iv) the research participants themselves. The objective of the research was fully explained to all stakeholders before embarking on the research journey, and positive responses to letters of consent were received from all.

The participants' engagement in this study was on a voluntary basis, and all participants were informed of their right to withdraw from the process at any stage. According to Trochim (2010), "the principle of voluntary participation requires that people must not be coerced into participating" (p. 15). This was strictly adhered to in this study.

The issue of confidentiality was highlighted and the participants were informed that their identities would be kept anonymous throughout the research period and thereafter.

3.12 CHALLENGES AND LIMITATIONS

The main challenge was adhering to the agreed upon timetable. Although ten students agreed to take part in the study, one participant withdrew shortly after commencement of the programme. In terms of the limitations, this research took the form of a case study. As such, and given the small sample size, it is not possible to make generalisations from the research findings.

3.13 CONCLUSION

It was the purpose of this chapter to provide an overview of the data collection and analysis processes. The next chapter focuses on the analysis of the data and discusses the findings of the study.

CHAPTER 4

RESULTS, ANALYSIS AND DISCUSSION

4.1 INTRODUCTION

This chapter presents and discusses the results of the study. The chapter is structured in line with the four phases of the study, namely the pre-test, intervention, post-test and focus-group discussion. The chapter concludes with a discussion which draws the results from the different phases together.

4.2 PRE-TEST

Responses to the pre-test were analysed in terms of their accuracy as well as the specific mental computation strategy employed. These strategies have already been described in detail in Section 2.5 but for convenience are summarized in Table 4.1 along with their respective codes.

Table 4.1 Strategy coding

STRATEGY	CODE	DESCRIPTION	EXAMPLE
COUNTING ON	CO	This strategy involves counting on in 1s, 2s, 3s etc. in a step-by-step fashion.	Question: 45 + 6 Solution: 45, 46, 47, 48, 49, 50, **51**
COUNTING BACK	CB	This strategy involves counting back in 1s, 2s, 3s etc. in a step-by-step fashion.	Question: 37 – 4 Solution: 37, 36, 35, 34, **33**
NEAR DOUBLES	ND	Seeing one number as being almost the double of another.	Question: 30 + 31 Solution: 30 + 31 = 2 x 30 + 1 = **61**
PARTITIONING	PT	In addition and subtraction, this strategy involves splitting numbers into tens and units before adding or subtracting them. The idea can also be extended to multiplication and division.	Question: 38 + 45 Solution: (30 + 40) + (8 + 5) = 70 + 13 = **83** Question: 27 x 8 Solution: 20 x 8 + 7 x 8 = 160 + 56 = **216**
AGGREGATION	AG	For the addition and subtraction of two-digit numbers, this strategy involves keeping one number intact and then partitioning the other number into tens and units.	Question: 43 + 29 Solution: 43 + 20 + 9 = 63 + 9 = **72**

STRATEGY	CODE	DESCRIPTION	EXAMPLE
STRINGING	ST	In addition and subtraction problems involving multi-digit numbers, the first number is left intact while the second, rather than being split, is added to or subtracted from the first in parts. The basic idea can also be extended to multiplication and division where the process is carried out in stages.	Question: 325 – 249 Solution: 325 – 200 = 125; 125 – 40 = 85; 85 – 9 = **76** Question: 48 x 6 Solution: 48 x 2 x 3 = 96 x 3 = **288**
BRIDGING TO 10 AND COMPENSATING	BC	This strategy involves bridging a number to a multiple of ten, adding/subtracting the second number to/from that multiple of ten, and then making an adjustment to compensate for the bridging. The idea can also be extended to multiplication and division.	Question: 28 + 35 Solution: 30 + 35 = 65 65 – 2 = **63** Question: 3 x 19 Solution: 3 x 20 = 60 60 – 3 = **57**
MENTAL IMAGE OF PEN & PAPER ALGORITHM	MI	This strategy involves carrying out a mental computation problem by mimicking a pen-and-paper method in one's mind.	Question: 364 – 186 Solution: $\begin{array}{r} {}^{3}\not{3}^{2}\ \not{6}^{15}\not{4}^{14} \\ -\ 1\quad 8\quad 6 \\ \hline 1\quad 7\quad 8 \end{array}$
HALVING & DOUBLING	HD	In multiplication problems, this strategy transforms the task into a less demanding one by halving one number while simultaneously doubling the other.	Question: 16×3 Solution: $16 \times 3 = 8 \times 6$ = **48**
DOUBLING	DB	In division problems, this strategy transforms the task into an easier one by doubling both the dividend and divisor.	Question: $33 \div 1\frac{1}{2}$ Solution: $66 \div 3$ = **22**
MULTIPLICATIVE THINKING	MT	This strategy involves seeing repeated addition as multiplication.	Question: 40 + 40 + 40 Solution: 40 x 3 = **120**
FAVOURABLE FACTORISATION	FF	This strategy involves recognising favourable factors which can make multiplication or division of numbers much easier.	Question: 28 x 25 Solution: 7 x 4 x 25 = 7 x 100 = **700**

Table 4.2 summarises the eight pre-test questions that were presented individually to the research participants.

Table 4.2 Pre-test questions

Question	Answer
1. Add 32 + 7 + 18 + 25	82
2. Three cell phones are priced N$3 600, N$4 000 and N$4 400. What is the total cost of these cell phones?	12 000
3. 400 – 222	178
4. Subtract 173 from 998	825
5. 36×25	900
6. What is 55% of N$320?	176
7. $55 \div 2\frac{1}{2}$	22
8. Take $\frac{2}{3}$ of 126 oranges	84

Table 4.3 provides a summary of the analysis of the pre-test. The nine participants are each identified by a letter from A to I. The abbreviations 'Acc' and 'Stra' stand for accuracy and strategy respectively. The question numbers relate to the questions given in Table 4.2, and the strategy codes relate to those given in Table 4.1. Each question was analysed in terms of accuracy and strategy use. An incorrect answer was coded 0 and a correct answer was coded 1. The 'Group success rate' gives the total number of students who were successful in getting the correct answer for a given question, while the 'Individual success rate' gives the number of correct scores obtained by each student.

Table 4.3 Pre-test strategy analysis

Learner	Q1		Q2		Q3		Q4		Q5		Q6		Q7		Q8		Individual success rate
	Acc	Stra	Acc	Stra	Acc	Stra	Acc	Stra	Acc	Stra	Acc	Stra	Acc	Stra	Acc	Stra	
A	0	AG	1	MI	1	MI	1	MI	0	MI	0	MI	1	MI	0	MI	4
B	1	AG	1	AG	1	MI	1	MI	0	MI	0	MI	0	MI	1	MI	5
C	0	AG	1	AG	1	MI	1	MI	0	MI	0	MI	0	MI	1	MI	4
D	1	AG	1	AG	1	MI	0	MI	0	MI	0	MI	0	MI	0	MI	3
E	0	AG	1	AG	1	MI	1	MI	0	MI	0	MI	0	MI	0	MI	3
F	0	AG	0	AG	1	MI	1	MI	0	MI	0	MI	0	MI	1	MI	3
G	1	AG	1	MI	0	MI	1	MI	0	MI	0	MI	0	MI	1	DB	4
H	1	AG	1	AG	1	CO	1	MI	0	MI	1	MI	0	MI	0	MI	5
I	0	AG	1	AG	1	MI	1	MI	0	MI	0	MI	1	MI	1	MI	5
Group success rate	4		8		8		8		0		1		2		5		

Close scrutiny of Table 4.3 reveals that the highest score was 5 correct answers out of a possible 8 (5/8). This was obtained by three of the nine learners. Three learners scored 4/8 while a further three scored of 3/8, the lowest score obtained. Four strategies were employed by the participants in this pre-test activity, namely MI, AG, CO and DB (see Table 4.1 for coding summary).

The group success rate reveals that Q2, Q3 and Q4 were the best answered questions, with eight of the nine participants correctly answering these three questions. Q5 was the most poorly answered question with no one correctly answering it. The other four questions, namely Q6, Q7, Q1 and Q8 registered group success rate scores of 1/9, 2/9, 4/9 and 5/9 respectively. In general the problems involving multiplication and division (i.e. Q5, Q6, Q7 and Q8) were poorly done.

The mental image of pen and paper strategy (MI) was the most predominantly used strategy in this activity. Out of the 72 responses (9 participants each answering 8 questions), 54 used the MI strategy. This was followed by aggregation (AG) which was used in 16 of the 72 responses. The counting-on strategy (CO) was used only once – by learner H in Q3. The doubling strategy (DB) was also used only once – by learner G in Q8 – who approached the question by first determining $\frac{1}{3}$ of 126 to 42, and then doubling this answer to get 84. Only four of the twelve strategies described in Table 4.1 were used. The strategies of CB, ND, PT, ST, BC, HD, MT and FF were not used at all. Table 4.4 summarises the strategy distribution per question.

Table 4.4 Strategy distribution (per question)

STRATEGY	Q1	Q2	Q3	Q4	Q5	Q6	Q7	Q8	FREQUENCY
CO	0	0	1	0	0	0	0	0	1
CB	0	0	0	0	0	0	0	0	0
ND	0	0	0	0	0	0	0	0	0
PT	0	0	0	0	0	0	0	0	0
ST	0	0	0	0	0	0	0	0	0
AG	9	7	0	0	0	0	0	0	16
BC	0	0	0	0	0	0	0	0	0
MI	0	2	8	9	9	9	9	8	54
HD	0	0	0	0	0	0	0	0	0
DB	0	0	0	0	0	0	0	1	1
MT	0	0	0	0	0	0	0	0	0
FF	0	0	0	0	0	0	0	0	0

Table 4.5 shows the success rate of each strategy used. Under the 'Outcome' column, 'F' represents a fail (i.e. a question answered incorrectly) while 'S' represents a success (i.e. a question answered correctly). The 'frequency of success' is given per strategy and represents the percentage of the 72 total responses that were correctly answered using the specific strategy employed. The 'frequency of failure' was similarly calculated. The 'frequency of attempts' represents the percentage of the 72 total responses attempted using a particular strategy. The 'success ratio' was calculated by dividing the 'frequency of success' by the 'frequency of attempts'.

Table 4.5 Strategy frequency count (Pre-test)

Problem	Outcome	SOLUTION STRATEGIES												SOLUTION OUTCOMES		
		CO	CB	ND	PT	ST	AG	BC	MI	HD	DB	MT	FF	TOTAL	SUCCESS	% SUCCESS
1	F	0	0	0	0	0	5	0	0	0	0	0	0	9	4	44.4%
	S	0	0	0	0	0	4	0	0	0	0	0	0			
2	F	0	0	0	0	0	1	0	0	0	0	0	0	9	8	88.9%
	S	0	0	0	0	0	6	0	2	0	0	0	0			
3	F	0	0	0	0	0	0	0	1	0	0	0	0	9	8	88.9%
	S	1	0	0	0	0	0	0	7	0	0	0	0			
4	F	0	0	0	0	0	0	0	1	0	0	0	0	9	8	88.9%
	S	0	0	0	0	0	0	0	8	0	0	0	0			
5	F	0	0	0	0	0	0	0	9	0	0	0	0	9	0	0.0%
	S	0	0	0	0	0	0	0	0	0	0	0	0			
6	F	0	0	0	0	0	0	0	9	0	0	0	0	9	0	0.0%
	S	0	0	0	0	0	0	0	0	0	0	0	0			
7	F	0	0	0	0	0	0	0	7	0	0	0	0	9	2	22.2%
	S	0	0	0	0	0	0	0	2	0	0	0	0			
8	F	0	0	0	0	0	0	0	4	0	0	0	0	9	5	55.6%
	S	0	0	0	0	0	0	0	4	0	1	0	0			
FREQUENCY OF SUCCESS		1.4%	N/A	N/A	N/A	N/A	13.9%	N/A	31.9%	N/A	1.4%	N/A	N/A	48.61%	35	48.6%
FREQUENCY OF FAILURE		0.0%	N/A	N/A	N/A	N/A	8.3%	N/A	43.1%	N/A	0.0%	N/A	N/A	51.39%		
FREQUENCY OF ATTEMPTS		1.4%	0.0%	0.0%	0.0%	0.0%	22.2%	0.0%	75.0%	0.0%	1.4%	0.0%	0.0%	100.0%		
SUCCESS RATIO		100%	N/A	N/A	N/A	N/A	63%	N/A	43%	N/A	100%	N/A	N/A			

A number of points can be gleaned from Table 4.5. In Q1, four students managed to use the AG strategy with success while 5 students attempted to use the AG but without success. In Q2, seven students used the AG strategy, six successfully and one unsuccessfully. The other 2 students successfully used the MI strategy. In Q3, seven students successfully used the MI strategy. Only one student used the MI strategy unsuccessfully, while one student successfully used the counting-on strategy (CO). In Q4, all nine students used the MI strategy, eight successfully and one unsuccessfully. None of the students succeeded in answering Q5 or Q6 correctly, and all participants attempted answering these two questions using the MI strategy. These were the two

most challenging questions for the participants. In Q7, only two students successfully used the MI strategy. The other seven students attempted to employ the MI strategy but without success. In the last question, Q8, only four students succeeded in using the MI strategy correctly. Four students attempted to use this strategy but without success while one student successfully managed to use the doubling strategy (DB) correctly for this question.

Overall, only a very small number of different strategies were used, and of those four that were used, two clearly predominated – MI and AG. Although there were 54/72 attempts to use the MI strategy, only 23/72 were successful in using this strategy, with 31/72 attempts being unsuccessful. For the AG strategy, 10/72 attempts were successful while 6/72 attempts were not successful. Questions 2, 3 and 4 were well done with 8/9 students getting the solutions correct. Questions 5 and 6, which involved multiplication, were poorly done, with none of the students correctly answering either of these questions.

4.3 INTERVENTION (AFTERSCHOOL PROGRAMME)

In this section I focus on capturing the development of Student H to model the progress of a student in the intervention programme, drawing on field notes and transcripts that were captured towards the end of the intervention programme. The basic tenet of constructivism views the acquisition of knowledge as a process of personal cognitive construction, undertaken by the learner, in a process of meaning making. With this in mind, it is important to note that Student H actively participated and engaged in discussions over the course of the afterschool programme. For purposes of reference, in the pre-test Student H scored 5/8 and made use of the AG strategy twice, the MI strategy five times, and the CO strategy once.

As discussed in the literature review, many learners are unsure of basic multiplication tables. In addition, basic facts like halving and doubling, which are very useful when it comes to multiplication and division, also seem to be lacking in many learners. This resonates strongly with Student H who showed evidence in the pre-test of not having mastered many of these basic skills. However, as the intervention unfolded, Student H's basic numerical skills developed as she engaged with mental computation strategies. By way of example, in response to the calculation of 5 x 6 she remarks *"Sir, 5* x *6 = 30, basic fact"* (line 46). Interestingly, she goes on to say, *"I can also think of doubling 5 to get 10 and at the same time halve 6 to get 3. Then 10* x *3 = 30"* (line 47). Here we see not only evidence of knowledge related to basic multiplication

tables, but also the halving and doubling (HD) strategy, which was never exhibited in the pre-test, as well as flexibility in calculating the product in more than one way.

There is also evidence of Student H having developed the partitioning strategy (PT) for addition of two-digit numbers, a strategy which also did not man itself in the pre-test. For example, in response to the calculation 18 + 14 she gave the correct answer of 32. Her explanation of her mental reasoning clearly shows a partitioning strategy: *"Sir, I added the units first, 8 + 4 = 12, and then I was left with a 10 and a 10 which gives me 20. Then if I add 12 to 20 I get 32, which is my final answer"* (lines 11 – 12). The mental flexibility developed by this participant is demonstrated by her immediate engagement with two other ways of solving the same problem. The first of these was: *"... you could also subtract 2 from 14 and then add it to 18 to make it a multiple of 10 which is 20. Then from 14 you will be left with 12. Then 12 + 20 = 32."* (lines 14 – 15). In this approach, Student H utilizes a bridging to 10 and compensating strategy (BC). The second alternative approach, demonstrating the skill of "moving quickly and flexibly through the world of numbers" (Buys, 2008, p. 122) was as follows: *"... you could also add a 2 to 18 to make it a 20. Then 20 + 14 = 34. Then at the end remember to subtract a 2 from 34. 34 – 2 = 32"* (lines 17-18). Although this is still a BC strategy, it has been used in a slightly different way.

A near double strategy (ND) surfaced when Student H solved the following mental computation problem: 18 + 17. She gives an answer of 35 and explains her thinking process as follows: "*Sir, I split 18 into 17 + 1, then I have a double 17 plus 1, that is 18 + 17 = 17 + 1 + 17 = 2 (17) + 1 =34 + 1 = 35"* (lines 20 – 24). Demonstrating her flexibility in manipulating numbers and working with different strategies, she then goes on to say, *"Sir, I can also add 1 to 17 to make it an 18. Then 18 + 18 = 36 and then I subtract 1 from 36 to get 35"* (lines 25 – 26). The calculation of 12 + 17 further demonstrates Student H's flexibility in engaging with different strategies. Her answer to this problem was 29 and she explained her thinking process as follows: *"Sir, 12 – 3 = 9. If I add 3 to 17, which are friendly numbers, I get 20. Then 20 + 9 = 29"* (lines 31 – 32), a clear BC strategy. She further remarks that *"... I can also add 3 directly to 17 to get 20. 20 + 12 = 32. Then 32 – 3 = 29"* (line 33).

A multiplicative thinking strategy (MT) surfaces when solving 14 + 7: *"Sir, 14 is double 7. Then 14 + 7 = 2* x *7 + 7 = 3* x *7 = 21"* (lines 38 – 41). She is quick to also give an alternative route to

the solution using a BC strategy: *"Sir, I could also split 7 into 6 + 1, add 6 to 14, as they are friendly numbers, to get 20. Then 20 + 1 = 21"* (lines 42 – 43).

The halving and doubling strategy (HD) seems to be have been used correctly in determining the product of 4 x 9. Student H's response was *"Sir, I will halve 4 and double 9, to get 2* x *18 = 36"* (line 50). Knowledge of equivalent mathematical statements plays a crucial role as students engage in the transformation of mathematical expressions en route to a solution. This is particularly true if one looks at student H's engagement in the solution to 5 x 36 in which she also used an HD strategy: *"Sir, if you halve 36 you get 18 and if you double 5 you get 10. The product of 5* x *36 is the same as 10* x *18 = 180"* (lines 52 – 53). The products 5 x 36 and 10 x 18 are equivalent. The basis of the halving and doubling strategy is the transforming of a mathematical statement from one form to another which, although equivalent to the original statement, is easier to calculate. This is in line with what Threlfall (2002) posits when characterizing mental computation as a process of "constructing a sequence of transformations of a number problem to arrive at a solution as opposed to just knowing, simply counting or making a mental representation of a 'paper and pencil' method" (p. 30).

Evidence of further transformation steps is given in her response to the calculation 400 – 222. Student H remarks, *"Sir, I will split 400 into 300 + 90 + 10 and 222 into 200 + 20 + 2. Then 400 – 222 = 300 + 90 + 10 – 200 – 20 – 2 = 100 + 70 + 8 = 178"* (lines 65 – 68). The splitting of 100 from the 400 followed by the further splitting of 100 into 90 and 10 to give a 'hundreds, tens and units' structure to the original 400 shows a clearly developing sense of mental dexterity.

In the calculation of the product 36 x 25, Student H also made use of transformation, in this instance by using the favourable factorization (FF) strategy: *"I will break down 36 into factors so that 36* x *25 = 9* x *4* x *25 = 9* x *100 = 900"* (line70). This question is identical to Q5 in the pre-test. At pre-test level Student H attempted to answer this question, unsuccessfully, using the MI strategy (mental image of pen and paper algorithm). After a number of sessions during the intervention period she was able to answer the same question using a far more elegant strategy. Interestingly, by the end of the intervention session Student H hardly ever resorted to the mental image of a pen and paper strategy (MI) which was the predominant strategy she made use of in the pre-test.

Student H reveals that her mental computation strategies have been developed and refined over time due to her involvement in the afterschool intervention programme. In the focus group discussion she had the following to say about her experience of the programme:

> *I learnt a lot like finding the strategies without sir having to tell us, like finding them from our own heads. In class for example at school in multiplication of numbers, I have made use of halving and doubling to get to a solution in class at school, [also] the strategy of compensating. These strategies have made me get to the answers much quicker unlike some of my classmates who were not exposed to this programme. They still use calculators for simple problems which can be done mentally. I have become richer in strategies and get to answers much quicker.* (lines 68 – 74)

Student H remarks that she feels she has managed to construct new knowledge (mental strategies) without anyone 'spoon-feeding' her. She feels she has been empowered by the programme and can now do things she could not easily do before. She has become richer in terms of the mental strategies she is now able to draw on, and is able to perform calculations far more quickly than before this programme.

During the focus group discussion Student H expressed her feelings about how she has been empowered by the programme. In our discussions about the use of calculators in the classrooms, this is what she had to say:

> *I will always cross-check my answers from calculator by trying to do the problem mentally and then compare the answers… The mental strategies which we picked up in this program have definitely empowered us as we can use them anywhere to carry out computation problems.* (lines 204 – 208)

In relation to the importance of mental computation strategies, Student H makes the following remarks:

> *I think these strategies are important to use so that we can share with other learners in class. It might even be good to consider starting a maths club which might be the right place to share these strategies with others.* (lines 211 – 213)

In the pre-test, Student H scored 5/8 and made use of the AG strategy twice, the MI strategy five times, and the CO strategy once. The post-test results show a marked improvement. With regard to accuracy, she scored 8/8 in the post-test. In addition, she managed to employ a far greater variety of mental computation strategies in the post-test. At pre-test level, 5/8 questions were attempted using an MI strategy – a mental image of pen and paper algorithm. In the post-test the MI strategy was abandoned in favour of new strategies such as DB, PT, FF, MT and AG.

4.4 POST-TEST

Table 4.6 summarises the eight post-test questions that were presented individually to the research participants.

Table 4.6 Post-test questions

Question	Answer
1. Add 17 + 15 + 3 + 26	61
2. Three cell phones are priced N$360, N$400 and N$440. What is the total cost of these balls?	1 200
3. 600 – 333	267
4. Subtract 153 from 798.	645
5. 28×25	700
6. What is 55% of N$440?	242
7. $33 \div 1\frac{1}{2}$	22
8. Take $\frac{2}{5}$ of 225 oranges?	90

Table 4.7 provides a summary of the analysis of the post-test. As with Table 4.3 the abbreviations 'Acc' and 'Stra' stand for accuracy and strategy respectively. The question numbers relate to the questions given in Table 4.6, and the strategy codes relate to those given in Table 4.1. Each question was analysed in terms of accuracy and strategy use. An incorrect answer was coded 0 and a correct answer was coded 1. The 'Group success rate' gives the total number of students who were successful in getting the correct answer for a given question, while the 'Individual success rate' gives the number of correct scores obtained by each student.

Table 4.7 Post-test strategy analysis

Learner	Q1		Q2		Q3		Q4		Q5		Q6		Q7		Q8		Individual success rate
	Acc	Stra	Acc	Stra	Acc	Stra	Acc	Stra	Acc	Stra	Acc	Stra	Acc	Stra	Acc	Stra	
A	1	PT	1	MT	1	PT	1	BC	1	FF	0	PT	1	DB	1	DB	7
B	1	PT	1	MT	1	ST	1	PT	1	HD	0	PT	0	DB	1	DB	6
C	1	AG	1	AG	1	ST	1	PT	1	FF	0	PT	1	DB	1	DB	7
D	1	AG	1	MT	1	ST	1	PT	1	HD	0	PT	1	DB	0	PT	6
E	1	AG	1	AG	1	ST	1	PT	0	HD	0	PT	1	DB	1	DB	6
F	1	PT	1	AG	1	ST	0	PT	1	FF	1	PT	1	DB	0	DB	6
G	1	AG	1	AG	1	ST	1	ST	1	FF	1	PT	1	DB	1	DB	8
H	1	AG	1	MT	1	PT	1	PT	1	FF	1	PT	1	DB	1	DB	8
I	1	PT	1	AG	1	ST	1	PT	1	FF	0	PT	1	DB	1	DB	7
Group success rate	9		9		9		8		8		3		8		7		

Close scrutiny of Table 4.7 reveals a number of important points. Firstly all participants improved their individual success rates compared with the pre-test. Individual success rate of 8/8 was attained by two participants, Students G and H, while three students scored 7/8 and the remaining four students scored 6/8. Questions 1, 2 and 3 were the most successfully answered questions with a group success rate of 9/9 for each question, whilst Q6 was the most poorly answered with a group success rate of 3/9.

Table 4.8 summarises the post-test strategy distribution. Table 4.8 reveals that the strategies PT, DB and AG had highest frequency count of 23, 17 and 10 respectively. Importantly, a far greater variety of strategies were made use of in the post-test. In total, eight different strategies were used, the numbers in brackets showing the number of times that each strategy was used: PT (23), AG (10), MT (4), ST (8), BC (1), FF (6), HD (3) and DB (17). There is also an interesting shift in that participants who relied heavily on the MI strategy in the pre-test completely abandoned this in favour of other strategies in the post-test. Table 4.8 summarises the strategy distribution per question.

Table 4.8 Post-test strategy distribution (per question)

STRATEGY	Q1	Q2	Q3	Q4	Q5	Q6	Q7	Q8	FREQUENCY
CO	0	0	0	0	0	0	0	0	0
CB	0	0	0	0	0	0	0	0	0
ND	0	0	0	0	0	0	0	0	0
PT	4	0	2	7	0	9	0	1	23
ST	0	0	7	1	0	0	0	0	8
AG	5	5	0	0	0	0	0	0	10
BC	0	0	0	1	0	0	0	0	1
MI	0	0	0	0	0	0	0	0	0
HD	0	0	0	0	3	0	0	0	3
DB	0	0	0	0	0	0	9	8	17
MT	0	4	0	0	0	0	0	0	4
FF	0	0	0	0	6	0	0	0	6
TOTAL	9	9	9	9	9	9	9	9	

Table 4.9 shows the success rate of each strategy used. Under the 'Outcome' column, 'F' represents a fail (i.e. a question answered incorrectly) while 'S' represents a success (i.e. a question answered correctly). The 'frequency of success' is given per strategy and represents the

percentage of the 72 total responses that were correctly answered using the specific strategy employed. The 'frequency of failure' was similarly calculated. The 'frequency of attempts' represents the percentage of the 72 total responses attempted using a particular strategy. The 'success ratio' was calculated by dividing the 'frequency of success' by the 'frequency of attempts'.

Table 4.9 Strategy frequency count (Post-test)

Problem	Outcome	SOLUTION STRATEGIES												SOLUTION OUTCOMES		
		CO	CB	ND	PT	ST	AG	BC	MI	HD	DB	MT	FF	TOTAL	SUCCESS	% SUCCESS
1	F	0	0	0	0	0	0	0	0	0	0	0	0	9	9	100.0%
	S	0	0	0	4	0	5	0	0	0	0	0	0			
2	F	0	0	0	0	0	0	0	0	0	0	0	0	9	9	100.0%
	S	0	0	0	0	0	5	0	0	0	0	4	0			
3	F	0	0	0	0	0	0	0	0	0	0	0	0	9	9	100.0%
	S	0	0	0	2	7	0	0	0	0	0	0	0			
4	F	0	0	0	0	0	0	0	0	0	0	0	0	9	9	100.0%
	S	0	0	0	7	1	0	1	0	0	0	0	0			
5	F	0	0	0	0	0	0	0	0	1	0	0	0	9	8	88.9%
	S	0	0	0	0	0	0	0	0	2	0	0	6			
6	F	0	0	0	6	0	0	0	0	0	0	0	0	9	3	33.3%
	S	0	0	0	3	0	0	0	0	0	0	0	0			
7	F	0	0	0	0	0	0	0	0	0	1	0	0	9	8	88.9%
	S	0	0	0	0	0	0	0	0	0	8	0	0			
8	F	0	0	0	1	0	0	0	0	0	2	0	0	9	6	66.7%
	S	0	0	0	0	0	0	0	0	0	6	0	0			
FREQUENCY OF SUCCESS		N/A	N/A	N/A	22.2%	11.1%	13.9%	1.4%	N/A	2.8%	19.4%	5.6%	8.3%	84.72%	61	84.7%
FREQUENCY OF FAILURE		N/A	N/A	N/A	9.7%	0.0%	0.0%	0.0%	N/A	1.4%	4.2%	0.0%	0.0%	15.28%		
FREQUENCY OF ATTEMPTS		0.0%	0.0%	0.0%	31.9%	11.1%	13.9%	1.4%	0.0%	4.2%	23.6%	5.6%	8.3%	100.0%		
SUCCESS RATIO		N/A	N/A	N/A	70%	100%	100%	100%	N/A	67%	82%	100%	100%			

Strategies PT and DB had the highest frequency of attempts of 31.9% and 23.6% respectively, with corresponding success ratios of 70% and 82% respectively. Although the strategies AG, BC, ST, MT and FF had low frequency of attempts, they each enjoyed a success ratio of 100%, while the HD strategy had a success ratio of 67%. The strategies CO, CB, ND and MI did not feature in the post-test activity. Questions 1, 2, 3 and 4 (the problems involving addition and subtraction) were the best answered questions, with all nine participants correctly answering them. Question 6 was the most poorly answered question in the post-test, with only three participants answering it correctly. No participants were able to answer Q6 correctly in the pre-test.

Figure 4.1 and Figure 4.2 provide a comparison of the strategies employed, along with their success rates, in the pre-test and post-test. The success of the programme in terms of increasing participants' access to mental computation strategies is clearly illustrated.

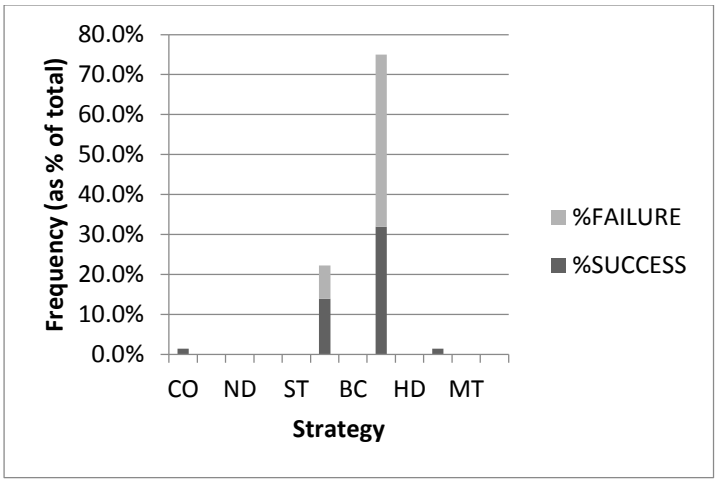

Figure 4.1 Pre-test strategy distribution

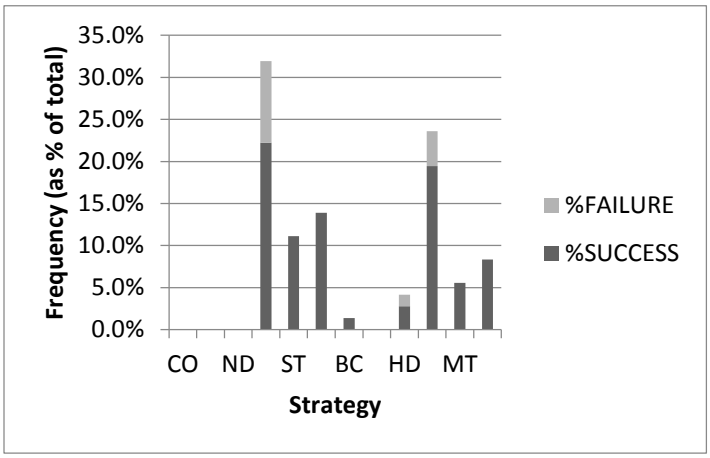

Figure 4.2 Post-test strategy distribution

4.5 FOCUS GROUP DISCUSSION

The following six questions guided the focus-group discussion:

- What has been your experience with the programme?
- What did you learn from the programme, if anything?
- What is the importance, in your own opinion, of mental calculations?
- How are you intending to make use of knowledge you may have acquired?
- How do you perceive the role of mental computation inside and outside the classroom?

- The current Grade 10 examination is calculator-based, i.e. you are allowed to use your calculator. What is your opinion about use of calculators in school?

The focus group discussion was analysed in terms of emerging themes. These themes were developed and categorized over time through repeated engagement with the data. These themes are: student empowerment, personal knowledge construction, necessity of prior knowledge, perceived value of the programme, and confidence.

4.5.1 STUDENT EMPOWERMENT

Most participants felt they had been empowered by the afterschool intervention programme. Student H for example feels she has been empowered by the programme and that this has benefitted her schoolwork:

> *…it has also been helping me in my school work because I have been able to get to solutions faster by doing calculations mentally*. (lines 29 – 30)

For Student B the empowerment was manifested in the liberation from being bound to the calculator:

> *I gained new knowledge. We were used to using calculators and also counting using fingers. Now I can do a lot without using a calculator.* (lines 49 – 51)

For Student I the empowerment was experienced in an increased sense of critical thinking:

> *…these mental computation strategies also help us to think logically when solving math problems. The step-by-step procedures we go through help us to develop some sort of logic reasoning.* (lines 225 – 227)

Student B felt sufficiently empowered by the programme to share her knowledge with her brother and classmates who had not taken part in the afterschool programme:

> *Sir I would like to share some of the strategies with my fellow students so that they can also start using them in class. I am also sharing this with my brother who is in a higher grade than me and he enjoys it*. (lines 100 – 102).

With reference to the use of calculators in the classroom, Student H made the following insightful remarks which illustrate the empowerment she has experienced through her participation in the programme:

> *Sir, there is nothing wrong with using calculators; it's just that they spoil you in terms of thinking, you do not think out of the box any more, you just run for a calculator. You actually do not know how to arrive at an answer, you wouldn't know yet because*

> *the calculator did it for you. You do not get steps or processes how to get to a solution.* (lines 125 – 128)

For Student H there were also further practical aspects to this empowerment:

> *…For example when I am outdoors, those strategies can help me calculate change at the shops when I am sent by my parents. It's easy to tell when somebody cheats on you with your change; it helps me to quickly and mentally check how much change I expect to get instead of rushing for a calculator in the shops.* (lines 143 – 149)

4.5.2 PERSONAL KNOWLEDGE CONSTRUCTION

Heirdsfield (2011) makes the important point that when engaging learners with mental computation tasks, "the emphasis should be on children developing their own strategies by exploring, discussing and justifying their thinking and solutions" (p. 2). Within the afterschool programme, participants had the opportunity to construct, interpret, and acquire mental computation strategies in communication with their peers. During the course of this programme, a lot of exploration, discussion and justification of strategies and students' thinking processes took place.

Applefield et al. (2001) posit that within a constructivist framework, "…learner conceptions of knowledge are derived from a meaning-making search in which learners engage in a process of constructing individual interpretations of their experiences" (p. 37). Throughout the intervention programme the participants were given the opportunity to do just this, to actively engage in the construction of mental computation strategies in an individual sense-making process. As Student H remarks:

> *Sir I learnt a lot from the experience we gained in the project. I learnt a lot like finding the strategies without sir having to tell us, like finding them from our own heads.* (lines 68 – 69)

The same student goes on to say:

> *Sir, at a stage where students are taught how to use traditional methods of computing, they should also be encouraged to explore other possible ways of solving the same problems and this way they may come up with these mental strategies. They might get used to discovering these strategies; teachers do not really need to teach them but rather make students discover these strategies. They just have to encourage children to find other ways of solving mathematics problems. There should be a lot of discussion amongst students and the teachers and new ideas develop in that situation*. (lines 250 – 256)

It is clear that this student feels she has gained knowledge through being exposed to an environment in which she was accorded the opportunity to construct her own mental computation strategies.

4.5.3 NECESSITY OF PRIOR KNOWLEDGE

According to constructivist principles, prior knowledge is an essential prerequisite for the construction of new knowledge. Students entered the intervention programme with prior knowledge in the form of pen-and-paper strategies. Although the emphasis of the intervention programme was on mental computation strategies, this prior knowledge formed an important backdrop to the process of developing alternative strategies. As Student A remarks:

> *I was used to these traditional methods; like working out a computation problem from right to left. Now I know how to get to an answer in a quicker way.* (lines 38 – 39)

Furthermore, students also acknowledged the importance of fundamental background knowledge to the process of developing increasingly sophisticated mental computation strategies. As Student B remarks:

> *As students we should practice going through times tables as these form the foundation of problems involving multiplication and division. Processes like factorization should be developed a lot*. (lines 257 – 259)

4.5.4 PERCEIVED VALUE OF THE PROGRAMME

There are clear benefits of establishing a programme of this nature. As Reys (1984) points out, "spending 10 minutes a day for several months on mental computation activities improves problem-solving performance as well as mental computation skills for both fast and slow learners" (p.550). Participants of the programme were quick to see its potential benefits, and their remarks in the focus group discussion support the continuation of such a programme in one or other form. Student F makes the following comment for example:

> *Sir I think this program should continue and should be extended to other learners as well so that we all develop skills of solving math problems mentally. I feel it's fair to include other learners as well so that they are also exposed to these strategies*. (lines 228 – 230)

Student H makes the following remark:

> *I really think there should be a programme of this nature right from primary school so that kids acquire these skills right from a tender age. By the time they get to secondary*

school level, they will have gained a lot of strategies which will in turn help them learn new things during their mathematics lessons. (lines 217 – 220)

During the course of the afterschool programme participants came to appreciate the power and usefulness of being able to engage with mental computation skills. Student G makes the following remark in relation to the use of calculators in the classroom:

Sir over reliance on calculators makes you lose confidence in solving math problems, the day a student does not have a calculator he/she might think it's impossible to solve a particular problem without a calculator. (lines 182 – 184)

Student F highlights that engaging with mental computation skills encourages awareness of mathematical processes:

Sir even in the examination we write, they do not ask for a correct answer to a problem they also ask for the method of arriving at that answer. A student who does not have mental computation skills will find it difficult to explain the procedures or methods of getting to an answer. The calculator does not show any method sir. (lines 185 – 188)

4.5.5 CONFIDENCE

An interesting theme running through the data relates to participants' development in terms of their personal confidence. It was clear from the responses of many participants that they found the afterschool programme a safe and supportive environment to explore their ability to express themselves in front of others. Student D comments that she found the experience liberating in terms of being able to express herself: *"Sir I learnt how to speak in a group, to express yourself without fear"* (line 42). Student F made a similar comment: *"I also learnt how to talk openly in front of a group of people…"* (lines 52 – 53). This sentiment is echoed by Student C: "*...I also learnt how to speak with confidence in a group"* (lines 66 – 67).

Student H felt that she was confident enough with the various mental strategies to explain them to her fellow classmates who hadn't been part of the intervention programme:

…a lot of kids in class have problems with multiplication of big numbers like two-digit and three-digit numbers. They have problems mostly with addition, subtraction, multiplication or dividing numbers. So I can actually help them with the strategies that I have acquired in this programme. I will have to explain through demonstration on the board; make them understand and learn new strategies too. They cannot be limited by the traditional methods we learn from our teachers since our primary school days. (lines 90 – 95)

Student I made the following remark in relation to the computer game that formed a large part of the intervention programme:

> *Sir in the computer games we played I learnt that it's not good to panic because if you panic you will fail to solve even simple problems. I trained myself to concentrate in order to score high, drive away fear. Panicking is a big challenge in playing games and also in learning new things. Games make you develop confidence in doing something.* (lines 292 – 295)

4.6 DISCUSSION

The theoretical framework which underpinned this study was constructivism, while Kilpatrick et al.'s (2001) model of mathematical proficiency, specifically the strands of conceptual understanding, procedural fluency and adaptive reasoning, constituted the conceptual framework which supported the study.

Duit (1996) highlights the following three principles of constructivism:

- Knowledge is not passively received but is built up by the cognizing subject
- The function of cognition is adaptive and enables the learners to construct viable explanations of experiences
- The process of constructing meaning always is embedded within a social setting of which the individual is part. (p. 42)

These three principles informed the structure of the afterschool programme in which the knowledge a child constructed was seen as a tentative construction based on previously acquired knowledge. Constructivist epistemology is consistent with an inquiry approach and, as such, important aspect of the afterschool intervention programme incorporated hands-on activities like computer games which solicit construction of mental strategies, and engagement in discourse which promotes cooperative learning. Dialogue and discussion constituted important ingredients of the programme which allowed the personally constructed knowledge of each research participant to be "communicated, justified and accepted by the group" (Ndlovu, 2013, p. 6). An important component of the intervention programme was thus for participants to share and discuss their own mental computational strategies, the idea being that each participant's prior knowledge would add to the general mental strategies of the group as a whole. The researcher along with the participants formed the social community offering the setting for the construction of computational strategies, the posing of questions, and the discussion of ideas.

All participants were encouraged to be actively involved in exploring viable mental computation strategies. The results of the pre-test revealed the use of only four mental computation strategies (MI, AG, CO and DB), with the MI strategy (mental image of pen and paper algorithm) being the dominant strategy. The intervention, post-test results, and the focus group discussion revealed evidence of newly acquired and constructed mental computation strategies (CO, CB, ND, PT, ST, AG, BC, DB, MT, FF and HD) in the various participants. The viability of these newly constructed mental computation strategies lay in the successful use and exploration thereof in responding to discussion questions during the intervention. In addition, all students' results improved at post-test level as compared to their achievement at pre-test level – Student H and Student G managing to score 8/8 in the post-test. The MI strategy, which dominated the pre-test solution strategies, was completely abandoned at post-test level – a clear indicator that the participants had internalized alternative and more elegant mental computation strategies. Furthermore, the strategies exhibited at the post-test level had individual success rates ranging from 75% to 100% as compared with a range of 37.5% to 62.5% at the pre-test level.

The students' participation in this afterschool intervention programme was a positive experience for all nine participants. Students felt empowered by the programme through their development of new mental computation strategies.

4.7 CONCLUSION

This chapter presented the results, analysis and discussions of the mental computation strategies developed by students during the afterschool intervention programme. The next chapter brings together the findings of the study in relation to the original research questions.

CHAPTER 5

FINDINGS AND CONCLUSION

5.1 INTRODUCTION

This chapter provides a summary of the main ideas of the study and presents the research findings in relation to the original research questions. It encompasses a brief overview of the research design and process, and summarises the findings and significance of the study. The chapter concludes with recommendations and suggestions for further study.

5.2 REVIEW OF THE OBJECTIVES

The main purpose of this study was to design, implement and investigate an afterschool intervention programme aimed at enhancing mental computation skills of Grade 8 learners at a secondary school in the Erongo educational region of Namibia. The study was framed by the following guiding questions:

(d) What is the nature of the mental computation problems displayed by the participants?
(e) In what ways did the afterschool programme enhance the participants' mental computation skills?
(f) What is the nature of the participants' experiences in the afterschool programme?

5.3 OVERVIEW OF THE CONTEXT

My interest in investigating learners' mental computation skills in mathematics stems from personal experience that many secondary school learners are unable to perform even relatively simple mental computation tasks such as basic multiplication and division and the addition and subtraction of two or more digit numbers. In addition, basic number facts such as doubling and halving seem to be lacking in many learners. While the use of calculators is embraced in the Secondary School curriculum in Namibia, and while there are important skills which can be developed through use of the calculator, the contextual backdrop to this research project is the underdevelopment of *mental* computational skills and the associated lack of appropriate mental strategies in secondary school learners.

Reys and Reys (1986) make the sobering remark that over 80% of real-world problem solving makes use of mental computation. This highlights the need for these skills to be nurtured and developed within the classroom. Maclellan (2001) characterizes mental computation as a process in which numerical calculations are carried out quickly and accurately through the conscious use of specific strategies and without the assistance of any external aids (Maclellan, 2001). Heirdsfield (2002) highlights the importance of mental computation skills as being a vital component of learning as learners engage with numbers, take informed decisions about what procedures to follow, and create their own strategies.

Within the Namibian Mathematics curriculum, Grade 5 learners are expected to make use of different strategies to carry out mental arithmetic calculations involving the addition and subtraction of whole numbers from 0 to 10 000 (Namibia. Ministry of Education, 2006). In Grade 6 this is extended to all four basic operations (addition, subtraction, multiplication and division) for whole numbers up to 100 000, and up to 1000 000 in Grade 7 (Namibia. Ministry of Education, 2006). In Grade 8 and Grade 9 the basic competencies include recalling multiples of numbers less than 10, as well as recalling square numbers up to 20^2, cubic numbers up to 10^3, as well as powers of 2, 3 and 5 up to 2^{10}, 3^5 and 5^4 respectively (Namibia. Ministry of Education, 2007). From Grade 10 onwards, no explicit mention is made of mental computation.

Thus, although some mental computation is explored as part of the Namibian Mathematics curriculum, the question is to what extent this is meaningfully engaged with in the classroom, and to what extent such mental computational strategies are embraced by learners. It is against this backdrop that this study – developing and researching an afterschool programme aimed at enriching learners' repertoire of mental computation strategies – is framed.

5.4 OVERVIEW OF THE RESEARCH PROCESS

This case study, involving nine Grade 8 learners, was anchored in the interpretive paradigm. The study was carried out in four sequential phases – an initial pre-test, the intervention, a follow-up post-test, and a focus group discussion. The pre-test was used to assess the mental computation skills of the research participants. Data from the pre-test served as an initial benchmark with respect to mental computation ability and was used to inform the activities of the intervention programme itself. The intervention took the form of an after school mathematics club that met twice a week for a period of eight weeks. The programme created a platform for learners to learn

from one another's strategies, and exposed learners to a variety of different mental computation strategies. Upon completion of the intervention programme a post-test, similar in structure and content to the pre-test, was administered. The research process concluded with a focus group discussion involving all nine research participants. The purpose of the focus group discussion was to engage participants in a discussion around their experiences in the afterschool intervention programme.

5.5 FINDINGS OF THE STUDY

The findings of the study are summarised below in relation to the three guiding research questions.

5.5.1 What is the nature of the mental computation problems displayed by the participants?

The pre-test comprised eight questions. Of the nine research participants, three were able to correctly answer five of the eight questions, three were able to correctly answer four of the questions, and three were only able to correctly answer three of the eight questions. Of the eight mental computation questions, the first four involved addition and subtraction (two questions of each) while the last four involved multiplication and division (two questions of each). The questions involving addition or subtraction were much better answered than those involving multiplication or division. Of those questions involving addition or subtraction, three were correctly answered by eight of the nine research participants. Interestingly however, Question 1 (32 + 7 + 18 + 25) was only answered correctly by four of the nine participants. Of those questions involving multiplication or division, none of the research participants was able to correctly answer Question 5 (36×25). Question 6 (what is 55% of N$320?) was correctly answered by only one participant, while Question 7 ($55 \div 2\frac{1}{2}$) was correctly answered by only two participants. The final question (take $\frac{2}{3}$ of 126 oranges) was correctly answered by five of the nine research participants.

In terms of the mental computation strategies displayed by the participants, there was a clear over-reliance on the mental image of pen and paper strategy – a strategy that simply mimics traditional pen and paper computational algorithms. Of the 72 total responses, 54 made use of this approach. In the absence of appropriate mental computation strategies to draw on, the

participants seem to have resorted to a mental analogue of standard pen and paper techniques. The only mental computation strategy evidenced in more than one learner was the aggregation strategy which involves keeping one number intact and partitioning the other number into tens and units. This strategy was used by all participants in Question 1 (32 + 7 + 18 + 25) and by seven of the nine participants in Question 2 (Three cell phones are priced N$3 600, N$4 000 and N$4 400. What is the total cost of these cell phones?).

5.5.2 In what ways did the afterschool programme enhance the participants' mental computation skills?

In terms of a comparison of the pre-test and post-test results, all participants improved with respect to their individual success rates. Of the nine research participants, two were able to answer all eight questions correctly, three were able to answer seven questions correctly, and four participants were able to answer six questions correctly. Furthermore, the strategies exhibited at the post-test level had individual success rates ranging from 75% to 100% as compared with a range of 37.5% to 62.5% at the pre-test level. This shows a marked improvement.

However, the most striking feature of the post-test results is that a far greater variety of strategies were made use of when compared with the pre-test. In total, eight different strategies were used, the numbers in brackets showing the number of times that each strategy was used: partitioning (23), aggregation (10), multiplicative thinking (4), stringing (8), bridging to 10 and compensating (1), favourable factorisation (6), halving & doubling (3) and doubling (17). There is also an interesting shift in that participants who relied heavily on the MI strategy in the pre-test completely abandoned this approach in favour of other strategies in the post-test – a clear indicator that the participants had internalized alternative and more elegant mental computation strategies.

5.5.3 What is the nature of the participants' experiences in the afterschool programme?

The themes emerging from the focus group discussion gave insight into the participants' experiences in the afterschool programme. These themes were: student empowerment, personal knowledge construction, necessity of prior knowledge, perceived value of the programme, and confidence.

Within the afterschool programme, all participants had the opportunity to construct, interpret, and acquire mental computation strategies in communication with their peers. The participants felt they had been empowered by the afterschool intervention programme through their development of new mental computation strategies. This empowerment took different forms for different participants. For one participant the empowerment was experienced in an increased sense of critical thinking, for another it was manifested in the liberation from being bound to the calculator. One participant felt sufficiently empowered by the programme to share her knowledge with her brother and classmates who had not taken part in the afterschool programme.

According to constructivist principles, prior knowledge is an essential prerequisite for the construction of new knowledge. Students entered the intervention programme with prior knowledge in the form of pen-and-paper strategies. Although the emphasis of the intervention programme was on mental computation strategies, this prior knowledge of the participants formed an important backdrop to the process of developing alternative strategies.

The participants were quick to appreciate the power and usefulness of being able to engage with mental computation skills, and all nine participants supported the continuation of such a programme in one form or another.

The students' participation in this afterschool intervention programme was a positive experience for all nine participants who, in addition to their engagement with alternative mental computation strategies, found the experience supportive in terms of developing their personal confidence.

5.6 LIMITATIONS OF THE STUDY

The main focus of this study was to develop, implement and research an afterschool intervention programme aimed at enhancing mental computation skills of Grade 8 learners at a secondary school in the Erongo educational region of Namibia. The greatest limitation of the study is the sample size. The findings of a case study carried out on nine purposefully selected Grade 8 learners from a particular school cannot be generalised.

5.7 SIGNIFICANCE OF THE STUDY

This study provides data that can assist policy makers at both ministerial level as well as institutions of higher learning with respect to programmes that could potentially enhance the development of mental computation strategies in school-going pupils. Given that the development of mental computation ability is likely to have significant bearing on improving learners' mathematical proficiency, and given the importance of the use of mental computation in everyday problem solving, attention to the development of mental computation ability within our schools is paramount. It is hoped that this study has shed light on the importance of this particular issue.

5.8 RECOMMENDATIONS AND SUGGESTIONS FOR FURTHER STUDY

Based on the findings of this study, the following recommendations are put forward:

- The Ministry of Education should be encouraged to allocate appropriate amounts of time within the curriculum for the development of mental computation skills. This should be done all the way through from primary school to high school level.
- Institutions of higher learning tasked with training teachers at primary as well as secondary school level should incorporate sufficient time to engage with strategies aimed at developing learners' mental computation ability.
- Emphasis should be placed on learners constructing their own mental computation strategies rather than being 'spoon-fed'.
- In-service training needs to be given to teachers who are already in the teaching field but who lack the necessary skills to help learners develop their mental computational ability.

In addition, the following suggestions are put forward as possible future research avenues to explore:

- Since this study focused on nine students from a single secondary school, there is need to extend this study to a larger sample size, incorporating students from other schools and other regions of Namibia.
- Extending this study to other countries in the SADC region could also be a worthwhile endeavour.

5.9 CONCLUSION

The importance of mental computational skill, both in terms of ability as well as flexibility, cannot be underestimated. Not only is mental computation an important skill with respect to mathematical proficiency, it is a crucial element in the day-to-day problem solving of all citizens of our country. It is hoped that this study has highlighted the need for more time and attention in school to be focused on developing the mental computational ability of our learners.

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Appendix A – Letter to Principal

P O Box 212
Arandis
Namibia

7 January 2014

The Principal
[redacted]
Erongo Education region
P O Box 97
Arandis
Namibia

[redacted]
2014 -01- 10
[redacted]

Dear Mr. R. Geiseb

MEd research proposal

I have successfully gone through the process of finalizing my research proposal for a Master of Education degree (Mathematics Education) through the Education Department of Rhodes University. Dr. Duncan Alistair Samson will be my supervisor.

The proposed research focuses on investigating an afterschool intervention programme aimed at improving learners' mental computation skills. The central goal of the research is to investigate the effectiveness of an afterschool intervention programme in enhancing learners' mental computation skills at secondary school level.

I intend to draw my research participants from a selected group of grade 8 learners. Looking at the research topic, these grade 8 learners are ideal to work with at this early stage of their course before they fully get exposed to calculator usage. Not only will these selected learners be ideally suited to the data collection protocol, but I believe they will find the experience a most worthwhile and fascinating extension activity.

The whole research exercise where the participants will be involved is envisaged to last no more than 5 weeks and our activities will be realized in the afternoons, outside normal academic hours at a time convenient to each individual research participant, twice a week. It is proposed that this research activity be conducted from an appropriate venue on the school campus e.g. the higher level Mathematics classroom – research participants will thus be in familiar surroundings.

I am aware of the various ethical considerations pertinent to research in the social sciences. Anonymity of both the school as well as the research participants will be assured at all times, as well as in the final thesis. Secondly, only those learners who agree to participate in the study through voluntary informed consent will form part of the research sample, and participants will have the freedom to withdraw from the study at any stage. In the case of audio recordings, consent will also be obtained from participants' parents or legal guardians.

This letter serves as a formal request that I be allowed to conduct the proposed research at [redacted] Secondary School. I undertake to make the final thesis available to the school, as well as the research participants, after its examination by Rhodes University, if requested.

If you are willing to grant permission for this research to be undertaken as outlined above, I would be much obliged if you could sign the consent form below.

Sincerely,

Appendix B – Information to parents

Rhodes University, Grahamstown, South Africa
Education Department

Mathematics Education M Ed Research Project

INFORMATION TO PARENTS/GUARDIANS OF RESEARCH PARTICIPANTS

Introduction
I am a part-time M Ed (Mathematics) student in the Education Department of Rhodes University. My research focus is on developing/enhancing students' mental computation skills through an afterschool intervention programme. The Principal of your child's school has granted permission for me to conduct the proposed research in his school, and your child has been selected as a potential research participant for the study. This letter provides important information regarding the study, and formally requests your consent for your child to take part in the study.

Description of involvement
The project is scheduled to start the third/fourth week of January 2014 and it is hoped to be complete within five weeks from the inception. The data collection process involves audio recording individual research participants while they are engaged in solving a number of mental computation problems. Research participants will be asked to "think aloud" while solving the given problems. In addition, field-notes might be taken by the researcher. This phase of the data collection process will be conducted outside normal academic hours at times convenient to the research participants. The research will be conducted from an appropriate venue on the school campus – research participants will thus be in familiar surroundings and will not need to leave campus. The time commitment per research participant is variable, but is unlikely to exceed 20 hours in total, spread over a period of five weeks.

Risks and benefits
There are no foreseeable risks involved in participating in the study. Your child's participation in the study will not only further his/her own understanding of and appreciation for this important section of the syllabus, but will ultimately contribute to the distillation of pedagogical strategies which will be able to be used in the classroom context to further other pupils' understanding of mental computation problems

Participants' rights
Your child's participation in this study is strictly optional, and at his/her and your own personal discretion. Should you agree for him/her to take part in the study he/she retains the right to withdraw, without explanation, at any point. Anonymity and confidentiality are guaranteed at all times, both during the research process itself and in the final written thesis.

Audio recordings
The audio recordings are necessary for detailed analysis of your child's strategies when solving the mental computation questions. It may be necessary for two to three other researchers to listen to portions of the recordings in order to provide input during this analysis process.

Consent
After you have read and understood this letter, and made any necessary clarifications about your child's involvement in the research process, please complete and sign the attached form.

Appendix C – Example of pupil consent form

RESEARCH PARTICIPANT CONSENT FORM

FULL NAME: [redacted]

HOUSE NUMBER: 2086 SEX (MALE/FEMALE): Female

DATE OF BIRTH: 05 September 2013

CELL NUMBER: none

DETAILS OF PARENTS OR LEGAL GUARDIANS:

NAME: [redacted]

EMAIL ADDRESS: [redacted]

PHONE NUMBER: [redacted]

I [redacted] hereby agree to take part in the research study to be conducted by Kwethemba Michael Moyo, a M Ed (Mathematics) research student in the Education Department of Rhodes University, Grahamstown. I understand what will be required of me in the role of research participant. Furthermore, I am aware that I retain the right to withdraw from the research project at any point without explanation.

With particular reference to the audio recordings, I hereby give consent to the following:

		*	
~~Yes~~	No		For 2 to 3 additional researchers to listen to portions of the audio recordings in order to assist with the analysis thereof.
~~Yes~~	No		For transcripts of some audio recordings to be included in the final written thesis.
~~Yes~~	No		For some audio recordings to be included in conference presentations.
~~Yes~~	No		For some audio recordings to be used in teaching seminars for the purposes of teacher development.

* Please TICK the appropriate box

Signed: [redacted] Witness: [redacted]

Date: 27 January 2013 Date: 27 January 2013