

**AN ANALYSIS OF HOW VISUALISATION PROCESSES CAN  
BE USED BY TEACHERS PARTICIPATING IN AN  
INTERVENTION PROGRAMME TO TEACH FOR  
CONCEPTUAL UNDERSTANDING OF GEOMETRY**

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## ABSTRACT

Visualisation in general and visualisation processes in particular have received much attention in the mathematics education research literature. Literature suggests that the appropriate use of visualisation helps learners to develop their conceptual understanding and skills of geometry as it allows them to visually interpret and understand fundamental mathematical and geometrical concepts. It is claimed that visual tools play an important role in communicating mathematical ideas through diagrams, gestures, images, sketches or drawings. Learning mathematics through visualisation can be a powerful tool to explore mathematical problems and give meaning to mathematical concepts and relationships between them.

This interpretive case study focused on how selected teachers taught concepts in geometry through visualisation processes for conceptual understanding as a result of an intervention programme. The study was conducted at four high schools by four mathematics teachers in the Kavango East Region in Northern Namibia. The participants were involved in a three-week intervention programme and afterwards taught three lessons each on the topic of geometry. The data collection method of this research was: focus group and stimulus recall interviews, classroom observations and recorded videos. This research is located in constructivism. I used vertical and horizontal analysis strategies to analyse the data. My analytical instrument consisted of an observation schedule which I used in each lesson to identify how each of the visualisation processes was evident in each of the observed lessons.

This study revealed that the participant teachers used visualisation processes in most of their lessons and these processes were used accurately in line with the requirements of the grade 8 mathematics syllabi. The visualisation processes were used through designed visual materials, posters and through the use of geometrical objects such as chalkboard ruler, protractor and compass. The results from this study also confirmed that visualisation processes can be a powerful instructional tool for enhancing learners' conceptual understanding of geometry.

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## **DEDICATION**

I dedicate this thesis to my parents, Mr. Muhembo Andreas and Mrs. Ntumba Theresia, for being the pillars of my life journey. Words alone are not enough to express my sincere appreciation for all your guidance and support. This thesis is also dedicated to my late grandmother Ngugho Irma Shipeku who passed away while I was at Rhodes pursuing this Masters.

## **DECLARATION OF ORIGINALITY**

I, Gottfried Mbundu Muhembo, student number 15M8813, hereby declare that this thesis entitled “An analysis of how visualisation processes can be used by teachers participating in an intervention programme to teach for conceptual understanding of geometry” is my own work, and a product of my research. It has not been submitted in any form to another institution. Where I have drawn on ideas of people from other publications or other sources, I have fully acknowledged these in accordance with Rhodes University, Education Department reference guide



Gottfried Muhembo

24 November 2017

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## **CHAPTER ONE**

### **CONTEXT OF THE RESEARCH**

#### **1.1 INTRODUCTION**

The aim of this research study was to analyse how visualisation processes can be used by teachers participating in an intervention programme to teach for conceptual understanding in geometry. This first chapter of this thesis introduces my study in the context of the background of the study in which I discuss how the study came about and its relation to visualisation in mathematics education. I further outline the research goals and questions of the study. This chapter also summarizes the methodology employed and the significance of the study. The final part of this chapter outlines the structure of the thesis by providing a glimpse of what is discussed in each chapter of the thesis.

#### **1.2 BACKGROUND OF THE STUDY**

This study sought to gain deeper insight into how visualisation processes in teaching mathematics develop conceptual understanding of geometry. All around us, images, symbols, and icons provide and convey meaning, creating a visual world of information (Murphy, 2009, p.1). Therefore, it is important for mathematics teachers to teach mathematics in general and geometry in particular, in a way that harnesses and promotes visualisation as opposed to relying on memorization and rote-learning. This study argues that teachers should create a visual environment when teaching mathematics. My focus was specifically on finding out how teachers make use of visualisation processes to enhance conceptual understanding of geometry in grade 8 mathematics.

It has been observed that “although difficulties in geometric thinking and reasoning are recognized and addressed in global educational research, problems in geometric conceptualization still prevail” (Dongwi, 2013, p.40). My experience as a mathematics teacher for eight years agrees with Dongwi’s observation. In many schools in Namibia, learners find it difficult to understand concepts used in geometry, thus making it difficult for them to perform well in mathematics. These difficulties are attributed to the fact that most teachers of

mathematics do not employ different instructional approaches to help learners comprehend geometrical concepts.

In addressing the observations above, Murphy (2009) suggests that visualisation processes can be used to meaningfully scaffold the teaching of conceptual understanding of mathematical concepts. Mathematics teachers should reinforce the notion that the world today is surrounded with visual objects that constantly provoke learners' curiosity. It is thus important that teachers are aware of the pedagogical implications and opportunities this has for their teaching and learning situation. It is mentioned that "teachers typically understand that information often comes through words and numbers and readily acknowledge that students must be taught to read and work with numbers" (Murphy, 2009, p. 1). But in my experience, all too often many teachers do not pay much attention to the ways information can be conveyed and understood visually. In my own experience as a mathematics teacher I have observed on many occasions that learners struggle with geometry and that many teachers do not make use of visual apparatus to teach geometry. Instead they rely heavily on oral presentations which involve listing steps and formulas.

Zodik and Zaslavky (2007) argue that "geometric problems should inherently be accompanied by figures and diagrams that represent the specific geometrical idea" (p. 265). Imagine teaching the properties of a rectangle to someone who has never seen one, but has only come across its characteristics in words? It will be difficult for that learner to visualise these properties in relation to a shape that is unfamiliar. An appropriate way to assist the learner is to make visible a rectangle through a sketch or a diagram and provide the learner with the opportunity to analyse it and attach its properties to this visualisation. Hence, I argue that there is a need to integrate a visual approach in our daily teaching in order to enhance conceptual understanding.

The purpose of this study is inspired by my passion for good teaching. The potential held by the incorporation of visualisation processes in the teaching of geometry within a classroom, which fosters learning that aligns with constructivism is at the heart of this study. Specifically, I focused on analysing how visualisation processes were used by teachers participating in an intervention programme to teach for conceptual understanding of geometry. Therefore, my focus was to understand how the participant teachers interacted with visual materials and how the materials were used in their geometry lessons.

### **1.3 RESEARCH GOALS AND QUESTIONS**

This study was inspired by some of the questions and gaps identified by Presmeg (2014) and *The ZDM Mathematics Education special edition* (2014, pp 151-157) journal which inter alia asked the following questions:

- What is the role of visualisation in the development of mathematical knowledge?
- What visualisation strategies do learners employ that enable them to construct meaningful conceptual content?

The themes of visualisation and visualisation processes thus framed my own research questions that are at the heart of this study.

#### **Research Questions**

The research questions that frame this study are:

- What are selected teachers' perceptions of the roles visualisation processes can play in the teaching of geometry to develop conceptual understanding through an intervention programme?
- How can visualisation processes be used in teaching geometry to enhance conceptual understanding?

### **1.4 RESEARCH METHODOLOGY**

Creswell (2007) defines research methodology as “an underpinning philosophical framework and fundamental assumptions that frame any research studies” (p.4). Bertram and Christiansen (2015, p.13) added that the methodology part of a research explains the approach that the researcher used to gather the data, as well as how the data were analysed. This qualitative research study is located in the interpretive paradigm. Interpretive methodology is directed at understanding phenomena from an individual's perspective, investigating interaction among individuals as well as the historical and cultural contexts which people inhabit (Creswell, 2007, p. 8). The interpretive research perspective aims to capture and share the understanding that participants encounter of what they are teaching (Kilpatrick, 1998, p. 98). In this study my intention was to investigate and understand the experience of the participant teachers on the role and the use of visualisation processes in teaching geometry for conceptual understanding.

This research adopted a case study approach. Yin (2009) defines a case study as an empirical inquiry in which the focus is on a contemporary phenomenon within real-life contexts and boundaries. This case study involved an interaction with selected teachers observing and recording their lessons and exploring the use of visualisation processes in geometry teaching for conceptual understanding after they participated in an intervention programme. The unit of analysis of this research was the participating teachers' practices focusing on the use of visualisation processes.

The study was carried out at four different schools in the Rundu circuit with four participants that are all Grade 8 mathematics teachers. The study was an intervention programme that was carried out in four different phases: Awareness workshop, planning, intervention and interviews (stimulus recall and focus group interviews). Data for this research was collected through observations and interviews. I observed 12 lessons (three lessons per teacher). After every observation, I interviewed the individual teacher and later had an interview with three of the participants together where I created an opportunity for the teachers to share their experience of teaching geometry using visualisation processes. Data was transcribed and analysed considering the themes that framed my analytical framework.

### **1.5 SIGNIFICANCE OF THE STUDY**

In the context of the new Namibia education curriculum, this study has the potential to inform policy makers and researchers in mathematics education of meaningful information about integrating visualisation into the design of policy guides and recommendations. It is anticipated that participating teachers will be empowered with various strategies involved in teaching geometry for conceptual understanding through visualisation. Furthermore, this research can help the Ministry of Higher Education and Training to incorporate visualisation into programmes that involve teacher-training institutions in order to adequately prepare student teachers to face the visual world when they become teachers. Further my study has the potential to help authors of mathematics textbooks to include visualisation processes in their books.

### **1.6 LIMITATIONS**

This research study was a qualitative case study which only focused on one circuit and four schools. The teachers that participated in this study do not represent all mathematics teachers in

that circuit, thus making it difficult for the findings from only four schools to be generalized to the whole Namibian educational context.

Participant teachers had their own commitments and could not make much of their time available, especially for interviews, which made it difficult to get their overall experience. One teacher of the four participant teachers did not attend the focus group interview to share his own experience of the intervention programme.

None of the four schools have sufficient geometrical objects to be used by both teachers and learners for constructions in geometry.

Some of the participant teachers did not present their lessons in full (45 minutes). Some lessons only lasted for 20-25 minutes, which made it difficult to analyse data collected from these lessons.

## **1.7 STRUCTURE OF THE THESIS**

This thesis consists of 5 chapters:

### **1.7.1 Chapter 1 (Context of the study)**

In Chapter one I introduce the study and outline the context, methodological aspect, significance and limitation of the study.

### **1.7.2 Chapter 2 (Literature review)**

In this chapter I present the literature review related to my study with a special focus on visualisation processes and conceptual understanding. I synthesize different readings that are relevant to visualisation processes. I review the concept of visualisation and visualisation processes. Furthermore, I discuss the aspect of geometry and the curriculum and the last part of this chapter presents the conceptual and theoretical framework of this study.

### **1.7.3 Chapter 3 (Methodology)**

This chapter presents the methodology that was employed in this study. I first outline the research goals and questions. The second part of this chapter discusses the research design

strategies for the study, and the third part presents the data collection tools and analytical framework, while the last part of this chapter discusses the issues pertaining to ethics.

#### **1.7.4 Chapter 4 (Data analysis and discussions)**

This chapter presents and discusses the findings of my research study. In this chapter I give a general description of the 12 lessons taught by the participant teachers. The chapter further discusses each teacher's lesson presentation with a special focus on visualisation processes. The chapter also gives an account of the similarities and differences in the way the teachers presented their lessons. In the last part of this chapter, I share some idiosyncrasies and participant teachers' perceptions about the use of visualisation processes during the intervention programme.

#### **1.7.5 Chapter 5 (Conclusion and recommendations)**

This is the concluding chapter where I consolidate the study findings with reference to the original research questions and within the contexts of the conceptual and methodological frameworks. This chapter presents a summary of the dominant themes, draws conclusions from the findings and makes recommendations and suggestions for future research. The chapter ends with my personal reflections of my research journey.

## **CHAPTER TWO**

### **LITERATURE REVIEW**

#### **2.1 INTRODUCTION**

The aim of this chapter is to review the literature that is relevant to this study. I begin this chapter by defining and discussing the history of visualisation and providing a broad discussion about visualisation processes and geometry. This chapter further examines concepts and theories that underpin the study. The main focus of the study is to investigate how visualisation processes are used by selected mathematics teacher to teach Grade 8 geometry for conceptual understanding. This chapter reviews the importance and significance of visualisation and geometry teaching, the curriculum aspect of visualisation in relation to geometry, and provides a review on critiques of visualisation.

#### **2.2 VISUALISATION**

The concept of visualisation has been widely used by many scholars in education research. Zazkis, Dubinsky and Dauterman (1996) looked at visualisation in terms of internal and external phenomenon and define visualisation as:

An act in which an individual establishes a strong connection between internal construct and something to which access is gained through the senses. Such connections can be made in either of the two directions. An act of visualisation may consist of any mental constructions of objects or processes that an individual associates with objects or events perceived by her or him as external. Alternatively, an act of visualisation may consist of the construction on some external medium such as paper, chalkboard or computer screen, of objects or events that the individual identifies with object(s) or process(es) in her or his mind (p.441)

Ho (2010) and Zimmerman and Cunningham (1991) note that visualisation does not equate to just forming a mental image, but rather it is specifically about visualising a concept or a problem rather than an idea.

### **2.2.1 Historical perspective of visualisation**

Visualisation has been studied for a little more than a century (Phillips, Norris & Macnab (2010)). Michelson (2017) noted that during the 1960s, methods of teaching and education were under scrutiny, with critics often noting how standard modes of learning did not take into account the fundamental social changes brought about by the use of images during the information age. Author Bamford discusses that human beings rely heavily on images to create meaningful understanding and interpretations of mathematical ideas (Bamford, 2003). He argued that at a very early age, children become deliberate producers of visual imagery and understand that graphic forms can be used to communicate. Bamford maintains that “pictures exist all around us, they surround us, the economy relies heavily on visual representations and a sense of design, style and feel. Understanding pictures is a vital life-enriching necessity” (Bamford ,2003, p.2).

L. Auburn and S. Ausburn (1978) added that we live in an era of visual culture, which influences our attitudes, beliefs, values and life-style. The use of images has changed the way we interact with our environment especially in the 21<sup>st</sup> century. Benson (1997) concurs and states that the presence of visual elements in today’s teaching and learning is increasing as the integration of images and visual representations with text in textbooks, instructional manuals, classroom presentations and computer interfaces broadens. Kress and Van Leeuwen (1996) assert that in the early years of schooling, children are constantly encouraged to produce images and illustrate their written work; as a result, images continue to play a very important role in their lives outside school. Due to increasing demand, visualisation today has gained momentum and is dominating in the educational spectrum, and many researchers of visualisation continue to advocate for visualisation to be a continuous part of schooling.

### **2.2.2 Defining visualisation**

Guzman (2002) states that visualisation is a “way of acting with explicit attention to the possible concrete representation of the object one is manipulating in order to have a more efficient approach to the abstract relationship one is handling”. Arcavi (2003) gave an equally comprehensive definition of visualisation as follows:

Visualisation is the ability, the process and the product of creation, interpretation, use of and reflection upon pictures, images, diagrams, in our minds on paper or with technological tools, with the purpose of depicting and communicating information, thinking about and developing previously known ideas and advancing understanding. (p. 217)

Arcavi (2003) views visualisation as a cognitive process and a product that is integral to this process. He sees technology as an important component of the visualisation process. He further emphasises that visualisation assists learners to engage with concepts and make mathematical meanings on a level that is not only symbolic and abstract. Fischbein (1987 as cited in Arcavi, 2003), stresses that “visualisation can accompany a symbolic development since visual imagery; by virtue of its concreteness can be an essential factor for creating the feeling of self-evidence and immediacy”. Arcavi argues that visualisation is no longer related to the illustrative purpose only, but is also being recognised as a key component of reasoning (deeply engaging with the conceptual and not merely the perceptual), problem-solving and even proving” (p.235). Duval (1999) concurs with Arcavi and defines visualisation as a cognitive activity that is intrinsically semiotic, i.e. neither mental nor physical. Presmeg (1992) discusses a number of visual images from concrete to abstract and highlights the importance of learners developing abstract patterns and dynamic imagery.

### **2.2.3 Visualisation in mathematics education**

Visualisation is increasingly being recognised as playing an important role in mathematics, especially when teachers and learners are interacting with geometrical concepts. The nature and role of visualisation and imagery in the teaching and learning of mathematics is complex (Jones, 1998). Much has been written about the value of visualisation and imagery in terms of the potential to enhance a global and intuitive view and understanding of various areas of mathematics (Bishop 1989, Fischbein 1987, Zimmermann and Cunningham 1991). Bishop (1989) maintains that when a person creates a spatial arrangement there is a visual image in the person’s mind, guiding this creation. He further states that mathematics is a subject which is concerned with objectivity and representing abstractions from reality and many of these representations appear to be visual (ibid).

Makina and Wessels (2009) state that “mathematics, as a human and cultural creation dealing with objects and entities quite different from physical phenomena, relies heavily on visualisation in its different forms and at different levels” (p.58). Zimmerman and Cunningham (1991) look at visualisation in mathematics as “the process of producing or using geometrical or graphical representations of mathematics concepts, principles or problems, whether hand-drawn or computer generated” (p.1). The authors further stress that “mathematical visualisation is the process of forming images (mentally, or with pencil and paper, or with the aid of technology) and using such images effectively for mathematical discovery and understanding” (p.3). On the other hand, Cobb, Yackel and Wood (1992) regard visualisation as a dualism created between mathematics in learners’ minds and mathematics in their environment, present in what they call ‘the representational view of mind’, which they find to be prevalent in mathematics education today.

In his analysis of learning with visualisation, Van de Walle (2010) defines visualisation as:

Geometry done with the mind’s eye. It involves being able to create mental images of shapes and then turn them around mentally, thinking about how they look from a different perspective, and predicting the results of various transformations. It includes mental coordination of two or three dimensions predicting the unfolding of a box (or net) or understanding a two-dimensional drawing or a three-dimensional shape (p.429).

#### **2.2.4 Types of visualisation**

Guzman (2002) states that “our human visualisation, even the apparently superficial phenomenon that we call ‘vision’, in its more psychological sense is not a process that merely involves the optical process of our eyes. It is much more complex since it entails in a quite important form, the activity of our brain. Guzman (2002) further mentioned that mathematical visualisation is not a univocal term, but depends on the level of communication. What goes on in the mind of one person may not necessarily be the same in the mind of another.

In an attempt to classify the types of visualisation, Guzman (2002) states that:

Mathematical concepts, ideas, and methods, have a great richness of visual relationships that are intuitively representable in a variety of ways. The use of them is clearly very beneficial from the point of view of their presentation to others, their manipulation when solving problems and in doing research. (p.2)

Against this background Guzman discussed four different types of visualisation which I outline below:

### **Isomorphic visualisation**

In this type of visualisation the object may have an “exact” correspondence with how we view its representations. This means that in principle, “it would be possible to establish a set of rules to translate the elements of our visual representations and the mathematical relationships of the objects they represent” (Guzman 2002, p.4). The importance of isomorphic visualisation manifests in most of our teachings. Guzman (2002) further argued that the manipulation of the objects that we perceive with our senses or with our imagination is normally easier and more direct than the handling of abstract objects. For example, in terms of visualisation, if the teacher is teaching about area of squares and rectangles, it is appropriate to bring to class examples of shapes that show area e.g. a cardboard box, and allow learners to work with these objects and discover what an area is before introducing the formula for calculating area of squares and rectangles to them.

### **Homomorphic visualisation**

In this type of visualisation, some of the elements have a certain mutual relation that imitates well the relationship between the abstract objects and provides us with support to guide our imaginations while proving mathematical processes (Guzman, 2002). An example of homomorphic visualisation will be angles formed in parallel lines. There is a clear relationship between the angles which makes it easier to determine the value of each angle. Learners need to know the properties of angles formed within parallel lines. Figure 2.1 below shows angles formed in parallel lines.



*Figure 2.1 Example of angles formed within parallel lines*

In this diagram the value of angle A is equal to angles C, F and H, the value of angle B is equal to angles D, E and G. Understanding of properties of angles formed in parallel lines will make it easier for learners to state the value of each angle in Figure 2.1. Guzman (2002) asserts that this type of visualisation in many cases can become a personal and subjective process, perhaps often not easily communicated but the effort to pass it on to our learners can be rewarding.

### **Analogical visualisation**

Under analogical visualisation, “we mentally substitute the objects we are working with by others that relate between themselves in an analogous way and whose behavior is better known or perhaps easier to handle, because it has been already explored” (Guzman, 2002). Cheshire, Ball and Lewis (2000) added that learning via analogy usually involves finding a set of systematic correspondences between a better-known source analogy and a more novel target. Loc (2008) said that analogy is a tool helping students to construct knowledge because of the processes of formulating and testing hypotheses, and analogy representations assist teachers to predict errors of students in teaching mathematics (Loc, 2007). Thinking of division and fitting something into a box is visual analogy. Explaining addition as directed movements on a number line is another example of analogy. The reason why analogy is used in mathematics as a visual tool is to help learners to understand the meanings of new knowledge in an easier manner; to introduce a new formula without its proof and to motivate learners (Loc & Uyen, 2014).

### **Diagrammatic visualisation**

In this kind of visualisation our mental objects and their mutual relationships concerning the aspects which are of interest to us are merely represented by diagrams that constitute a useful

help in our thinking process (Guzman, 2002). An example of diagrammatic visualisation is the tree diagram used in probability. Instead of using abstract and verbal methods to explain the possibility of an outcome in tossing a coin for example, a tree diagram can be used for this purpose because it's more visual. Guzman maintains that "the success that is experienced by great teachers in mathematics is often due to the efforts they make to transmit to others and to share with them not only their results but also processes of achieving the results" (p.8).

### **2.2.5. The role of visualisation in teaching and learning mathematics**

Burns (2006) asserts that while learning is still largely text-based, children are exposed to visual thinking through words, numbers, pictures, images, patterns, signs and symbols from a very young age. Kilpatrick, Swafford and Findell (2001) also observe that children begin learning mathematics way before they enter elementary school. Lowe (2000) stressed that today's students live in an information environment saturated with visual images, and educational materials are no exception. Similarly, Riesland (2005) concurs and states that by educating learners to understand and communicate through visual modes, teachers empower students with necessary tools to thrive in increasingly media-varied environment.

Duval (1999) asserts that "mathematics education has been very sensitive to needed changes over the past fifty years; representation and visualisation are at the core of understanding in mathematics" (p.3). The primary role of visualisation in teaching mathematics is to assist learners to analyse mathematical concepts using images or representations. Makina (2010) affirms that visualisation is a very important cornerstone in "teaching for understanding" by creating a platform where learners are more engaged with visual images. She maintains that mathematical visualisation is the process of forming images or constructing mental representations, and using such images effectively for mathematical discovery and understanding (ibid).

The use of pictures, diagrams, maps and charts are visual strategies that make understanding easier (Williams, 1983). Visualisation encourages active participation through interaction and exploring. Bishop (1989) states that "a learning environment in which structured and manipulative materials predominates can help to encourage the creation of visualisation" (p.174). Equally, Guzman (2002) discusses that:

Visualisation is extraordinarily useful in the context of the initial processes of mathematics as well as in that of the teaching and learning of mathematics. All these makes clear the convenience of training our own visual ability and introducing it to those to whom we are trying to introduce mathematics (p.31).

In an effort to achieve conceptual understanding of different geometrical concepts, it is important to ensure the appropriate use of the visualisation processes. Zimmermann and Cunningham (1991) state that understanding can be achieved by the use of symbolic, numeric and visual representations of ideas connected together. Makina (2010) argues that visualisation helps teachers to make instructional decisions about how to teach the content and the nature of tasks, while at the same time aiding teachers with the facilitation of lessons and with the ability to engage learners in realistic situations. The author further stresses that with visualisation, the overall goal of instruction is to help learners construct mental representations that correctly or accurately mirror mathematical relationships in instructional representations locked outside the mind (ibid).

The use of visual material should be a central focus when teaching for conceptual understanding. Gellert and Steinbring (2014) mention that visualisation representations can be used as an epistemological tool to explore mathematical concepts and bring about new meaning. Equally, Söbbeke (2005) affirms that in mathematics classrooms, visual diagrams help learners to see mathematical concepts and ideas more clearly. In light of this, Murphy (2009) encouraged the use of visual material in mathematics teaching:

Visual learning strategies can make a profound difference to a student's depth of understanding about mathematics. It is a powerful teaching tool for those who are natural visual/spatial learners, and indeed for all students. In fact, by using visual learning strategies in the teaching of mathematics, we can—and we should—increase the learning potential of all students and develop their ability to communicate mathematical concepts in an increasingly visual world (p.8).

Teachers' instructional approaches to teaching mathematics concepts can enhance or hinder the way learners acquire concepts. Ainsworth (2006) wrote that “when learners can interact with appropriate representations, their performance is enhanced” (p.183). She further added that

recently attention has been focused on learning with multiple representations. Consequently, combinations of representations can play a number of roles in supporting learning (ibid). “Visual representation such as diagrams has continued to be used to good purpose across mathematics” (Giaquinto, 2007, p.402).

### **2.2.6 Spatial visualisation**

The focus on spatial skills as a component of geometry instruction is not something new in mathematics education. “A variety of methods are already used by teachers in an attempt to improve students’ abilities to visualise and mentally manipulate geometric figures” (Boakes, 2009, p.2). Battista (1990) states that the balance between visual-spatial and verbal-logical thought may determine “mathematical cast of mind that influences how an individual processes mathematical information” (p.47). Wheatley (1990) added that “spatial sense plays a major role in mathematical reasoning. The author further states that spatial sense is indispensable in giving meaning to our mathematical experience. Also mentioned by Del Grande (1990) is that spatial visualisation is the ability to visualise imaginary movement in 3D space or to manipulate objects in the imagination (e.g. folding paper, and perforating it, and cutting shapes. Spatial visualisation involves the “ability to mentally manipulate, rotate, twist, or invent a pictorially presented stimulus object” (McGee 1979, p. 893).

Silverman (2002, pp. 1-2) asserts that visual-spatial learners are learners who make a replica of their ideas in pictures rather than words. It is also stated that spatial visualisation is important for the mathematical learning of concrete operational thinkers, for their thought is greatly dependent on concrete and pictorial representations (Battista, Wheatley, & Talsona 1982).

Spatial sense consists of two important components of geometric knowledge, namely spatial visualisation, which is the ability to visually compare shapes that have changed position on the plane-transformation geometry (Bassarear, 2012), and spatial orientation which operates when a fixed object is viewed from different points or when the position of an object is acknowledged (Battista, 2007). Learners that have spatial visualisation and spatial orientation are likely to perform better in geometry because they view shapes from different angles and are likely to interpret and analyse objects appropriately in order to help them develop conceptual understanding of the topic they are dealing with.

### **2.2.7 Challenges of visualisation in teaching and learning mathematics**

Despite the recommendations from literature that encourage the use of visualisation in mathematics, there seems to be reluctance on the side of many teachers to use visualisation. In my observations, Namibian teachers are not motivated to incorporate visual materials in their lessons. Some teachers claim that using visual materials is time-consuming and usually hampers their progress in completing the syllabus. Also, learners do not find visualisation to be part of mathematics and as a result they do not understand how visuals are related to the content they learn in the classroom. In the light of this, Eisenberg and Dreyfus (1991) mentioned three reasons why learners are reluctant to visualize: “a cognitive one (visuals are more difficult), a sociological one (visuals are harder to teach) and a cultural one related to beliefs about the nature of mathematics (visuals are not mathematical)” (p.30).

Cognitive difficulty refers to whether a visual is easier or more difficult to work with. Visual thinking requires higher cognitive demand than algorithmic thinking (Mwiikeneni, 2016). Arcavi (2003) mentions that learners who have been taught mathematics analytically find visual presentations more difficult and risky, thus they prefer an analytic approach.

Sociological difficulties have to do with the issue of instructional approach. School mathematics is sequential and algorithmic, and many teachers believe that sequential analytic representations are more pedagogical and efficient (Arcavi, 2003). A learner may fail to obtain marks because he/she used diagrams to give answers to questions. In my experience many teachers prefer learners to memorize formulas and rules rather than allowing learners to visualise. Cultural difficulties have to do with values and beliefs that our societies hold about doing mathematics which can be passed on from one generation to the other. Some people believe mathematics cannot be taught with the use of images but rather is a formalized subject with rules.

Students are often challenged to decode visuals by focusing on the overall meaning of visuals and encode visuals by carefully making choices of the type of representations (Quinell, 2014). This makes it difficult for them to make solid interpretations of visual representations. The idea that mathematics should be communicated in a non-visual framework is commonly shared in the mathematics community (Guzman, 2002). Eisenberg and Dreyfus (1989) added that when

learners devalue (Presmeg, 1997) visualisation, it can result in them hesitating to use visuals even when they become teachers. Arcavi (2003) noted that:

What we see is not only determined by the amount of previous knowledge which directs our eyes, but in many cases, it is also determined by the context within which the observation is made. In different contexts, the “same” visual objects may have a different meaning even for experts” (p.237).

Monoyiou, Papageorgiou and Gagatsis (2007) state that “the use of visual representations in mathematical teaching and learning is a multidimensional and complicated process and should be conducted with great attention” (p. 8). Guzman in his study points out that “visualisation leads to errors” (2002, p. 12). In agreement Steenpaft and Steinbring (2014) caution that mathematical visual images can be ambiguous elements that do not necessarily convey the concept effectively to students, and can lead to misunderstanding.

### **2.3 VISUALISATION PROCESSES**

When teaching for conceptual understanding through the use of visualisation processes, teachers need to be aware of these processes and teach accordingly. They should make use of these processes strategically to develop mathematical concepts in geometry in their classes. Hanson, Silver and Strong (1988) argue that the use of visualisation processes is not taught in schools in any organized way. (Debes (1969) identified 35 of what he referred to as visual literacy skills that can be used to teach conceptually. For the purpose of this study I have focused on only five of these because they are fundamental to the teaching of geometry, and refer to them as visualisation processes: *Designing visuals*, *manipulating visuals*, *interpreting visuals*, *analyzing visuals* and *visual language*. Each of these processes is discussed below in detail:

#### **Designing visuals**

Debes (1969) states that visual designs are of fundamental importance for the production of effective visual instructional materials. Fleming and Levie (1993) suggest that designers of visual images should take into account the basic principles of perception, so that they can produce teaching media that can communicate messages in a more efficient and effective way. Teachers should be able to demonstrate the use of basic visual elements such as lines, shapes, color and dimension in their teaching of geometry.

## **Manipulating visuals**

Just as writing is essential in the context of textual literacy, the capacity to manipulate visuals such as diagrams and sketches in conjunction with meaning-making is a core component in employing visual processes. It is essential to note that learners should not only have the capacity to derive literal meaning from a text, but that they should also be able to reach an understanding of how texts are produced. Teachers need to be able to think critically about and manipulate visual information to help learners to make a solid interpretation of its meaning (Rakes, 1999, p. 17). Teachers should be able to manipulate different representations and concepts in order to assimilate them into learners' already existing knowledge to make sense of the geometrical idea.

## **Interpreting visuals**

Rakes (1999) maintained that the ability to read, interpret and construct graphic displays is of growing importance in an increasingly visual world, as students are exposed to more computer-based electronic texts, which often rely heavily on graphic aids. Avgerinou (2001) asserts that interpreting visuals includes applying critical thinking skills to visuals: identifying and evaluating the validity of information communicated in the visual message. Teachers need to properly interpret visuals themselves before they can expect learners to do so. Felten (2008) added that "when teachers train learners well and allow them to practice visual skills they can develop the ability to interpret and employ diverse relationships of different visual forms effectively" (p. 60).

## **Analyzing visuals**

Avgerinou (2001) said that:

There are four phases of people's active involvement in the process of image decoding: Description of the graphic elements composing the image, analysis of the way those elements have been arranged, interpretation of the message being communicated and aesthetic appreciation of the image (p.286).

In order for learners to gain a deep understanding of mathematical objects and visuals they need to analyze them by interpreting them in different ways that make sense to them. Learners can analyze a shape by listing its properties or by relating it to a similar object. Carter, Hipwell and

Quinnell (2014) mentioned that “[a]n approach that aims to classify and teach visuals according to their properties is more likely to assist students in making meaning and in transferring knowledge between visuals with similar properties”. During teaching, teachers need to analyze various visual elements of objects by appropriately arranging and communicating them to the learners in relation to which grade they are in.

### **Visual language**

Visuals are a language with its own vocabularies, grammar and syntax. Debes (1969) asserts that a visually literate person should be able to read and write visual language, i.e. s/he should be able to decode and encode visual messages appropriately. Avgerinou (2001) asserts that “[i]f we are to become critically autonomous towards the visual images we are constantly surrounded by, we need to become more aware of the structure and functions of visual language” (p.286). Equally, Rakes (1999) stressed that teachers need to help students to become adept encoders and decoders of visual language so that they are equipped to deal with the increasing amount of computer-based information. Teachers can use techniques which promote the production of visuals to demonstrate learning. Furthermore, teachers can combine visual and verbal elements and present a visual language that can elicit learners’ feelings, emotions and attitudes.

These five processes discussed above form the basis of my analytical instrument which is broken down into observable indicators outlined in detail in Chapter 3, Table 3.6.

## **2.4 THE CONCEPT OF GEOMETRY**

The focus of this study is geometry teaching through an intervention programme in four schools in northern Namibia. The purpose is to observe and report on teachers’ practice in the teaching of geometry for conceptual understanding. Because instruction is assumed to have an impact on students’ learning (Peterson, Fennema, & Carpenter, 1989), “one way to try to enhance achievement in geometry is to improve instructional practice. The common belief is that the more a teacher knows about a subject and the way students learn, the more effective that individual will be in nurturing mathematical understanding” (Swafford, Jones and Thornton, 1997). It is imperative to realise that teachers’ instructional practice should be interlinked to bring about learners’ active participation in the mathematics class. It is therefore very important that “ways are found not only to suggest changes in teachers’ practices, but also to provide

necessary support and assistance for such desired changes to manifest in the [mathematics] classroom” (Sanni, 2009, p. 39).

Zhang, Ding, Stegall and Mo (2012) state that “students who struggle with learning mathematics often have difficulties with geometry problem-solving, which require strong visual imagery skills” (p.167). The authors further state that geometry and spatial sense are fundamental components of mathematics learning (ibid).

School geometry is the study of those spatial objects, relationships and transformation that have been formalised (or mathematised) and the axiomatic mathematical system that has been constructed to represent them (Clement & Battista, 1992, p.420). The Oxford Study Mathematics Dictionary defines geometry as the “area of mathematics relating to a study of space and the relationship between points, line, curves and surfaces” (Tapson, 1999, p.52). Jones (2002) asserts that the “study of geometry contributes to helping students develop the skills of visualisation, critical thinking, intuition, perspective, problem-solving, conjecturing, deductive reasoning, logical argument and proof” (p.125). He further states that geometry is a wonderful area of mathematics to teach because it is full of interesting problems and surprising theorems. Bleeker and Goosen (2009) assert that “geometry can be an effective tool in guiding learners towards abstract reasoning which we need in the wake of globalization with its challenges and opportunities” (p.19). Presenting geometry in a way that stimulates curiosity and encourages explanations can enhance students’ learning and their attitudes towards mathematics. French (2004), as cited in Atebe and Schäfer (2008), asserts that students’ general mathematical competencies have been closely linked to their geometric understanding.

The Royal Society/JMC report discussed in Jones (2002) suggests the following aims of teaching geometry:

- To develop spatial awareness, geometrical intuition and the ability to visualise;
- To provide a breadth of geometrical experiences in 2 and 3 dimensions;
- To develop knowledge and understanding of and the ability to use geometrical properties and theorems;
- To encourage the development and use of conjecture, deductive reasoning and proof;

- To develop skills of applying geometry through modeling and problem-solving in real world contexts;
- To develop useful ICT skills in specifically geometrical contexts;
- To engender a positive attitude to mathematics; and
- To develop an awareness of the historical and cultural heritage of geometry in society, and of the contemporary applications of geometry (p.124).

#### **2.4.1 The Namibian curriculum and geometry**

Geometry is recognized as an important part of the mathematics curriculum from kindergarten through to grade12. It is through geometry that children begin to develop an understanding of “geometric shapes and structures and how to analyze their characteristics and relationships” (National Council of Teachers of Mathematics (NCTM), 2000, p. 41). A study carried out by NIED (National Institute for Educational Development) Mathematics Education Officers in 2009 on the performance of learners in Mathematics at Upper Primary in the Okahandja district of Namibia, revealed that more than half of the learners (53%) cannot distinguish between different kinds of triangles and quadrilaterals (Namibia. MoE, 2009, p. 10). The absence of using visualisation processes strategically in mathematics teaching could attribute to the difficulties alluded to above and contribute to learners not acquiring concepts appropriately.

The Namibia national broad curriculum for basic education states that “Mathematical skills, knowledge, concepts and processes enable learners to investigate, model and interpret numerical and spatial relationships and patterns that exists in the real world” (Namibia. Ministry of Basic Education Arts and Culture [MBEAC], 2010, p.12). The experiences that learners have in the real world make it easier for them to relate to new knowledge that they learn in the classroom. The use of visualisation processes can be a powerful tool for transforming learners’ experiences into new knowledge through the use of diagrams and images. Idris (2006, p. 94) states that geometry is a unifying theme throughout the entire mathematics curriculum and as such is a rich source of visualisation for arithmetical, algebraic and statistical concepts.

Mateya (2013) added that in the mathematics curriculum, “geometry” as a domain of mathematics is considered as a key element which should be taught proficiently (p.50). In relation to the Namibian curriculum it is also observed that “one component of mathematics

education that makes extensive use of diagrams is the teaching and learning of geometry” (Jones, 2013, p.37). Also, geometry relates well to practical and real-life experiences as shapes are all-pervading. Geometry appeals to our visual, aesthetic and intuitive senses (Schäfer, 2003). Thus, geometry should be an area of mathematics education which is stimulating, curious and fun. Since geometry inherently involves diagrams and pictures, it is important for teachers to use visualisation appropriately in order to teach geometry conceptually and effectively. French (2004) explains that geometry has a vital role in the wider mathematics curriculum as it harnesses learners’ spatial awareness, develops learners’ reasoning skills and stimulates/informs and challenges their concept/facility.

According to Huang and Witz (2011), seeking an effective curriculum and instruction that would facilitate children’s conceptual understanding is a crucial issue for mathematics education. They add that employing a teaching curriculum that connects 2-dimensional (2-D) geometry motions and area measurement numerical calculations can be crucial. Geometry and visualisation are complementary in developing children’s conceptual understanding of geometry (Huang & Witz, 2011, p.10).

Table 2.1 The Geometry Curriculum. Adapted from the Grades 8-9 mathematics syllabus (Namibia MoE, 2015).

<b>GENERAL OBJECTIVES</b> Grade 8 Learners will:	<b>GRADE 8 SPECIFIC OBJECTIVES</b> Grade 8 Learners should be able to:
<b>Topic 6: Geometry</b>	
<b>Constructions</b>	
<ul style="list-style-type: none"> <li>Know how to perform geometrical constructions using a straight edge, a compass and a protractor</li> </ul>	<ul style="list-style-type: none"> <li>Use and interpret the geometrical terms such as point, a line, a line segment, a ray and a plane (<b>In Grade 8 a line refers to a straight line</b>).</li> <li>Measure different lines and angles accurately</li> <li>Identify and construct different types of angles accurately (acute, obtuse, right, straight and reflex angles)</li> <li>Construct squares, rectangles and circles from given data</li> </ul>
<b>Angle properties</b>	
<ul style="list-style-type: none"> <li>Know and understand angle properties to solve problems</li> </ul>	<ul style="list-style-type: none"> <li>Identify pairs of angles as complementary or supplementary</li> <li>Calculate unknown angles by applying the</li> </ul>

	<p>following angle properties</p> <ul style="list-style-type: none"> <li>- Angles formed on a straight line</li> <li>- Angles at a point</li> <li>- Angles formed at intersecting lines</li> <li>- Angles formed within parallel lines intersected by a transversal</li> <li>- Interior angles of triangles</li> </ul>
<b>Symmetry and Transformation</b>	
<ul style="list-style-type: none"> <li>- Understand the reflection of shapes and symmetry</li> </ul>	<ul style="list-style-type: none"> <li>• Identify reflections from given diagrams</li> <li>• Draw and describe reflections of simple shapes limited to horizontal and vertical lines of reflection</li> <li>• Identify and draw lines of symmetry for any given shapes</li> </ul>

The topics and lesson objectives outlined in table 2.2 are the lessons that were taught during the intervention programme. The participant teachers specifically focused on these concepts and competencies. Even though symmetry and transformation is part of the grade 8 Namibian syllabus, the topic could not be taught by all the teachers because the intervention period was not enough to cover all the topics under geometry.

#### **2.4.2 Challenges of geometry teaching**

Nikolodakis (2009) indicates that learners in general find defining and recognizing geometric shapes and the use of deductive thinking in geometry problematic. Burger and Shaughnessy (1986) echo the same sentiments when they state that “students are not sufficiently grounded in basic geometry and thus are forced to memorize concepts as their only recourse” (p. 46). Furthermore, Atebe and Schäfer (2008) in their study in Nigeria and South Africa indicate that participating secondary school learners had a limited and arguably inadequate knowledge of basic geometric terminology (pp. 110-123). Dongwi (2013) added that “[i]t is a common phenomenon that secondary school students globally struggle with simple geometric problems and their thinking and reasoning are generally below average” (p.40). She went on to stress that mathematics teachers’ approach to geometric instruction determines to a large extent the mathematical thinking strategies and dispositions that our learners attain and develop (ibid).

Burger and Shaughnessy (1986) mentioned that high school geometry as it is taught in most high schools is taught at a deductive level while most learners are only capable of reasoning informally about geometric concepts upon entrance into geometry. Piaget (1975) as cited in Wirszup (1976) shares this sentiment; he maintains that geometry instruction begins too late and when it is eventually taught moves from a measurement (quantitative) position to the recognition of shape. Piaget further argued that this makes teachers ignore the qualitative phase of transforming spatial operations into logical ones (ibid). Other researchers have also reported on learners' geometry related difficulties in their various fields of study. Nikolodakis (2009) found that learners find it very difficult to successfully write simple geometry proofs.

In my own experience as a mathematics teacher I have observed on many occasions that learners struggle with geometry and that many teachers do not make use of visual apparatus to teach geometry. Instead they rely heavily on oral presentations which involves listing steps and formulas. Ding and Jones (2006) point out that effective instruction in geometry requires teachers to develop sound instructional strategies and knowledge of useful resources and activities. Effective mathematics teachers reflect on their connected mathematical knowledge bases, which include content knowledge, pedagogical content knowledge, conceptual knowledge and procedural knowledge (Luneta, 2013).

## **2.5 CONCEPTUAL UNDERSTANDING**

Wu (1999) debates that the desire to achieve understanding in a subject such as mathematics while minimizing the component of skills is a most human one. But teaching mathematics for understanding is an extremely complex process (Hiebert & Lefevre, 1986). This section discusses conceptual knowledge and how students acquire conceptual knowledge in geometry. This part also touches on procedural knowledge and how it has a link to conceptual knowledge in mathematics education.

### **2.5.1 Working definitions of conceptual understanding**

Kilpatrick et al. (2001) recognize that using visualisation in teaching is one of the backbones of teaching for conceptual understanding. They state:

Students demonstrate conceptual understanding in mathematics when they provide evidence that they can recognize, label, and generate examples of concepts; use and

interrelate models, diagrams, manipulative and varied representations of concepts; identify and apply principles; know and apply facts and definitions; compare, contrast, and integrate related concepts and principles; recognize, interpret, and apply the signs, symbols, and terms used to represent concepts (p.9).

Kilpatrick et al. (2001) define conceptual understanding as “an integrated and functional grasp of mathematical ideas” (p.118). They further state that learners with conceptual understanding know more than isolated facts and methods. Conceptual knowledge helps learners learn new ideas by connecting it to what they already know. For example, learners at a very young age already will know what a mountain looks like and thus can relate it to a shape such as the triangle. What they already know is the shape of a mountain and the triangle becomes a newly formalized concept. Hierbert and Lefevre (1986) share the same sentiment and describe conceptual knowledge “as a knowledge that is rich in relationships, it can be thought of as a connected web of knowledge, a network in which the linking relationships are as prominent as the discrete piece of information” (p. 3). Hierbert and Lefevre (1986) emphasize that in order to achieve conceptual knowledge, learners need to construct a relationship between different information. This includes being able to visually connect different mathematical ideas and concepts. This has implications for teaching. It is argued in this study that using visualisation processes in the teaching of mathematics could enhance and promote the conceptual understanding in learners as articulated above.

### **2.5.2 Procedural and conceptual understanding**

Even though this study emphasizes the teaching for conceptual understanding of geometry, it is important to recognize the distinction between procedural and conceptual knowledge and acknowledge that the two cannot be separated from each other. The debate around these two theories of understanding includes whether conceptual or procedural understanding should occur first and whether these two are linked (Long, 2005; Pimm, 1995). Although instruction in mathematics concepts and procedures are important components to quality core mathematics instruction, it may be more effective to target an intervention on one or the other (Kanive, Nelson, Burns & Ysseldyke, 2014). Silver (1996) mentioned that competence in domains such as mathematics rests on children developing and linking their knowledge of concepts and procedures. Furthermore, it is stated that interventions that target students’ conceptual

understanding have been shown to be effective in correcting students' misconceptions of fundamental mathematics principles and in establishing an understanding of underlying mathematics concepts for problem-solving (Jitendra, DiPipi, & PerronJones, 2002; Ketterlin-Geller, Chard, & Fien, 2008). Long (2005) motivates for the inter-relationship and correlation of the two as well as the integration of procedures with the understanding of concepts.

Kilpatrick et al. (2001) refer to the two as conceptual understanding and procedural fluency and identified them as part of five intertwined mathematical proficiency strands (conceptual understanding, procedural fluency, strategic competence, adaptive reasoning and productive disposition). According to Kilpatrick et al. (2001), conceptual understanding is the comprehension of mathematical concepts, and procedural knowledge is carrying out procedures and rules (p.5). Procedural knowledge implies knowing formal rules, definitions, algorithms and strategies for doing a task, and conceptual knowledge involves deep understanding of the underlying mathematical ideas and concepts.

Procedural fluency is the knowledge of rules, symbols, and sequence of steps required to complete mathematical problems (Zamarian, Lopez-Rolon, & Delazer, 2007), and is often demonstrated by students quickly retrieving correct answers or completing a mathematical problem. Rittle-Johnson, Siegler and Alibali (2001) define procedural knowledge as the ability to execute action sequences to solve problems. This type of knowledge is tied to specific problem types and therefore is not widely generalizable. In contrast to procedural knowledge, they define conceptual knowledge as "implicit or explicit understanding of the principles that govern a domain and of the interrelations between units of knowledge in a domain" (p.346).

Kilpatrick et al. (2001) further recognize the importance of intertwining procedural and conceptual proficiency in mathematics education. They state that procedural fluency supports conceptual understanding and vice versa. Furthermore, mathematics proficiency strands have been discussed in many mathematics education research. Kilpatrick et al. (2001) mention that "in recognizing that no term captures completely all aspects of expertise, competence, knowledge, and facility in mathematics, we have chosen mathematical proficiency to capture what we think it means for anyone to learn mathematics successfully" (p.5). Their framework consists of five mathematics proficiency strands that form part of the conceptual framework of my study and they briefly define each of the strands as follows:

- *Conceptual understanding*: comprehension of mathematical concepts, operations, and relations
- *Procedural fluency*: skill in carrying out procedures flexibly, accurately, efficiently, and appropriately
- *Strategic competence*: ability to formulate, represent, and solve mathematical problems
- *Adaptive reasoning*: capacity for logical thought, reflection, explanation, and justification.
- *Productive disposition*: habitual inclination to see mathematics as sensible, useful, and worthwhile, coupled with a belief in diligence and one's own efficacy.

Figure 2.3 below shows the description of the five different mathematics proficiency strands as discussed by Kilpatrick et al. (2001).

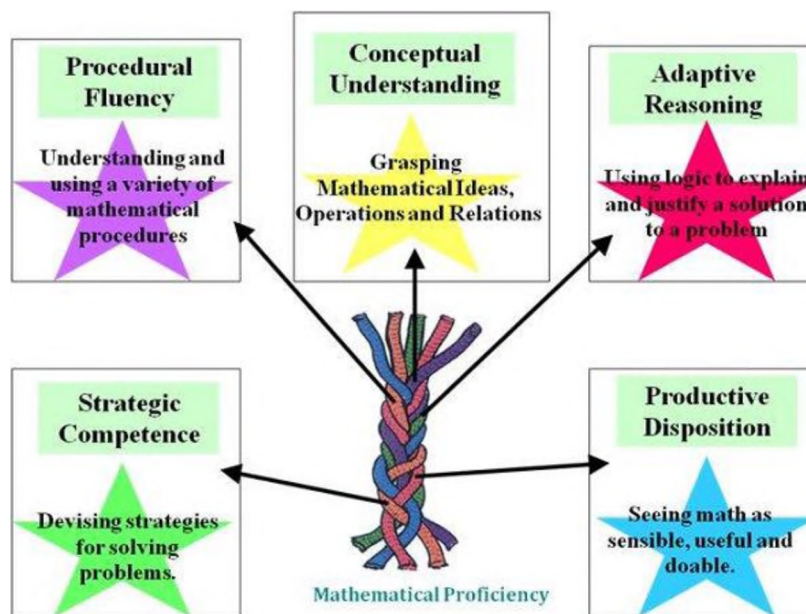


Figure 2.2: Kilpatrick et al. (2001) Mathematical proficiency model

### 2.5.3 Visualisation and conceptual knowledge

Pape and Tchoshanov (2001) discuss that the use of visual representations leads to improvement of learners' mathematical abilities and development of their advanced problem-solving and reasoning skills. Using visual representations facilitates learners' development of mathematical concepts (Presmeg, 1997). For example, when teachers use designed cardboards to teach about

angle properties, learners will have an opportunity to visualise the process and this helps them to easily grasp the concepts of angle properties. Learners must be given the opportunity to interact with one another and the teacher using visualisation processes. Through this interaction within problem-solving situations, knowledge of mathematical representation(s) and mathematical understanding emerges and develops (Pape & Tchoshanov, 2001 p.124).

## **2.6 THEORETICAL FRAMEWORK**

### **2.6.1 Constructivism**

Bonk and Cunningham (1998) observe that the traditional teacher-centered model in which knowledge is “transmitted” from teacher to learner, is rapidly being replaced by alternative models of instruction (e.g., learner-centered) in which the emphasis is on guiding and supporting students as they learn to construct their understanding. This traditional model may turn elementary-level lessons into an extremely theoretical, boring and non-effective process (Garcia & Pacheco 2013, p.25). In light of this argument, Ben-Ari (2001) states that the dominant theory of learning today is constructivism, which claims that knowledge is actively constructed by the student, not passively absorbed from textbooks and lectures. He further argues that “teaching techniques derived from the theory of constructivism are supposed to be more successful than traditional techniques, because they explicitly address the inevitable process of knowledge construction” (p.45). In the same vein Confrey (1990) writes that:

Constructivism can be described as essentially a theory about the limits of human knowledge, a belief that all knowledge is necessarily a product of our own cognitive acts. We can have no direct or unmediated knowledge of any external or objective reality. We construct our understanding through our experiences, and the character of our experience is influenced profoundly by our cognitive lenses (p.108).

### **2.6.2 Social constructivism and visualisation**

In order to design a sustainable 21st century learning environment, researchers need to make a commitment to conduct sustained research (Nardi & Schwarz, 2002), and this requires theoretical and methodological lenses congruent with the research (Wittrock, 1977). Therefore, this study is informed by Vygotsky’s (1962) social constructivism theory, which emphasizes the

learners' construction of knowledge rather than acquisition of knowledge from the teacher. He emphasizes that learners learn best when they are able to discover things. In this study it is argued that being able to visualise mathematical ideas is an inherent component in discovering ideas and concepts.

Yackel (2001) states that "Students construct their own meaning from the words or visual images they see or hear" (p.41). Thus, students need visual materials in order for them to construct meaningful knowledge. From a constructivist view, Pape and Tchoshanov (2001) highlight that the "necessary mapping between the concrete materials and the arithmetic algorithm (procedure) requires intensive social co-construction of meanings" (p. 123). Teachers and students need to co-construct their understanding of the steps of the mathematical operation while manipulating the materials (ibid.). Kilpatrick et al. (2001) adds that interpretation and use of visualisation objects in teaching is integral to the process of understanding mathematics.

Learners should not be given knowledge but rather be allowed to build their own theories and try them out instead of only consuming what they are told or instructed by the teacher. By facilitating a visual approach to learning mathematics, teachers encourage learners to imagine and visualise mathematical ideas and processes. Vygotsky (1962, p.83) further argues that a teacher who tries to give knowledge to students achieves nothing but an empty verbalism. Learners should thus be provided with learning opportunities such as visualisation activities that encourage discovery and exploration. Hannafin (1992) affirms that learning is not achieved by "mastery of formal knowledge as such, but rather by activities that progressively refine and quantify relationships among connected elements" (p. 54). Learners often come to school with experiences in the form of unstructured information. It is therefore the teacher's task to facilitate and create a learning and teaching environment that transforms this information into deep and meaningfully connected forms of knowledge. Ernest (1991) suggests that this is achieved in a mathematical environment that supports and encourages active construction of mathematical concepts rather than receiving it as a finished product from the teacher or text. I argue in this study that adopting a visual approach to teaching strongly encourages what Ernest is suggesting.

Moore (1997) asserts that students are not empty vessels to be filled with knowledge poured in by the teachers; they inevitably construct their own knowledge by combining their present experiences with their existing conceptions. Orton (1992) adds that in social constructivism,

learners are identified as active participants in constructing and reconstructing their own mathematical meaning. Students construct new knowledge particularly effectively when they are personally engaged in meaningful activities (Garcia & Pacheco, 2013). The intervention programme that is central to this study emphasises that meaningful activities are those that contain a strong element of visualisation. The learning programme that results from the intervention programme thus incorporates an interactive and activity-based approach to teaching mathematics with a rich infusion of visualisation processes and activities. Von Glasersfeld (1996) states that “knowledge is not passively received either by intuition or by communication, but is actively built up by students”. Students should be encouraged to interact with each other and the teacher to develop mathematical concepts together.

Using visualisation processes in mathematics teaching aligns well with social constructivism, because through visualisation learners have an opportunity to interact with different objects and develop different concepts. Giesen (2004) mentions that constructivism is a theory of learning which states that people create their own understanding and knowledge through experiencing things and reflecting on those experiences. Constructivism is thus central to this study because the intervention that is at the heart of this study foregrounds the use of a visual approach to teaching, which inherently involves active learning and engagement on a conceptual level with mathematical concepts. Learners actively engage in the intervention lessons through constructions and drawing of shapes. A visual environment allows learners to see what they are being taught by the teacher. The participating teachers create an opportunity for discussions whereby learners can share their experiences and observations in their groups—a key aspect in the constructivist approach. Through activities that are consistent with social constructivism, learners have the opportunity not only to learn mathematical skills and procedures, but also to explain and justify their own thinking and discuss their observations (Silver, 1996). An analytical instrument that is used during classroom observation is outlined in details on page 42, Table 3.6.

The purpose of this study is inspired by my passion for good teaching. The potential that is held by the incorporation of visualisation processes in the teaching of geometry within a classroom that fosters learning which aligns with constructivism is at the heart of this study. Specifically, I focused on how visualisation processes were used by teachers who participated in an intervention programme to teach for conceptual understanding of geometry.

## **2.7 CONCLUSION**

The purpose of this chapter was to present a synthesis of different literature related to visualisation in mathematics education. I first discussed the concepts of visualisation and visualisation processes and provided a contextual argument of how the concepts are being discussed by different mathematics researchers. The second part of this chapter is an outline of the concept of geometry and how geometry is related to visualisation and visualisation processes. The last part of this chapter is a discussion about the conceptual and theoretical frameworks that are at the heart of this study.

## CHAPTER THREE

### METHODOLOGY

#### 3.1 INTRODUCTION

This chapter presents the justification of the research methodology that is chosen for this research study. The objectives for undertaking this case study are to analyse how selected teachers teach geometry for conceptual understanding as a result of an intervention programme. Yin (2009) and Creswell (2007) regard educational research as essential, as it provides legitimate information, knowledge and principles to guide the decision-making, thinking and discussion processes in education. This chapter is divided into four parts: The first part is an outline of the research goals, questions and orientation, the second part of this chapter discusses the research design strategies for the study, the third part presents the data collection tools and analysis framework, while the last part of this chapter discusses issues pertaining to ethics.

#### 3.2 RESEARCH GOALS AND QUESTIONS

This study is inspired by some of the questions and gaps identified by Presmeg (2014) and *The ZDM Mathematics Education special edition* (2014, pp 151-157) journals which inter alia ask the following questions:

- What is the role of visualisation in the development of mathematical knowledge?
- What visualisation strategies do learners employ that enable them to construct meaningful conceptual content?

The themes of visualisation and visualisation processes thus framed my own research questions:

#### Research Questions

The research questions that frame this study are:

- What are selected teachers' perceptions of what roles visualisation processes can play in the teaching of geometry to develop conceptual understanding through an intervention programme?
- How can visualisation processes be used in teaching geometry to enhance conceptual understanding?

### **3.3 RESEARCH ORIENTATION**

This study employs a qualitative approach which focuses on how selected teachers teach visually for conceptual understanding in geometry. This approach provides me with the appropriate tools to extract rich and detailed data and evidence of visualisation processes used by the participant teachers during a carefully designed intervention programme. My study is an interpretive study with a central endeavor to understand the subjective world of human experience (Cohen et al., 2011), in this case the experience of teachers. Bertram and Christiansen (2014) discuss that:

Within the interpretive paradigm, researchers do not aim to predict what people will do, but rather to describe and understand how people make sense of their worlds, and how they make meaning of their particular actions. The purpose is to develop a greater understanding of how people make sense of contexts in which they live and work. (p.26).

The interpretive research perspective aims to capture and share the understanding that participants encounter of what and how they are teaching (Kilpatrick, 1998, p. 98). In this study my intention was to investigate and understand the experience of the participant teachers on the role and use of visualisation processes in teaching geometry for conceptual understanding.

### **3.4 METHODS (CASE STUDY)**

This research is a case study. Yin (2009) defines a case study as an empirical inquiry in which the focus is on a contemporary phenomenon within real-life contexts and boundaries. This case study involved an interaction with selected teachers observing and recording their lessons and exploring the use of visualisation processes in geometry teaching for conceptual understanding after they participated in an intervention programme.

The unit of analysis of this research is the participating teachers' practices focusing on the use of visualisation processes. Stake (1995) acknowledges that in a case study the researcher must avoid being drawn away from the focus of the study. Thus, it is important to keep focused on the unit of analysis in order not to be side-tracked from the central research questions of the research project. Since this research project is a case study, the findings only pertain to this particular case.

### **3.5 RESEARCH DESIGN**

This research was conducted in four different phases: *1. Awareness workshop, 2. Planning 3. Teaching Intervention Programme and 4. Interviews (focus group and stimulus response interviews)*

#### **3.5.1 Phase 1: Awareness (orientation) workshop**

During this phase I organized a workshop with invited grade 8 mathematics teachers in the Rundu circuit in northern Namibia. I did this in collaboration with the senior education advisor for Mathematics in that circuit, to run awareness workshops on how to harness the use of visualisation in the teaching of Mathematics. Due to the constraints on the national budget which made it difficult for the senior education officer to invite all the teachers of the circuit for a workshop, I decided to design a questionnaire which I sent out to all the schools with the aim of identifying the teachers to be invited to an awareness workshop. These questionnaires were collected and analysed and I selected four teachers from four different schools based on their responses to the questionnaires. The experience and qualifications of teachers was also considered in selecting the participants, see Table 3.5. I had discussed this idea with the advisor who was very amenable in collaborating with me. I introduced and discussed the concept of visualisation through a PowerPoint presentation and practical examples.

During this workshop, participating teachers were engaged in group discussions and activities about visualisation and visualisation processes. These included sharing their experiences working with visualisation and discussing the impact that it could have on developing learners' conceptual knowledge. I also discussed my research project with them and inspired them to participate in running an intervention programme that specifically foregrounds the use of visualisation in the teaching of geometry.

#### **3.5.2 Phase 2: Planning**

During this phase, I and the four participant teachers met again to plan how we were going to run the intervention programme. This involved a discussion on the integration and harnessing of visualisation in the teaching of twelve grade 8 geometry lessons (three lessons per teacher) over a period of four weeks. The teachers and I developed teaching programmes in the form of carefully designed lesson plans. The lessons included teaching strategies that make explicit use of visual artifacts and methods. The topics that were taught included geometry construction and

angle properties. I selected these topics because they are all covered under one chapter (geometry) of the new Namibian revised mathematics curriculum. Thus, this intervention programme coincided and synchronized with what the teachers are required to teach according to the national curriculum. The Tables below in Tables 3.1 to 3.3 show the planned schedule and provide a brief summary of the planned activities for each lesson from week 1 to week 3. The teachers were not limited to the visualisation activities in this summary; they were free to come up with other visual materials that align well with the following five visualisation processes: Designing visuals, manipulating visuals, visual language, analyzing and interpreting visuals. See appendix E for examples of the detailed lesson plans. A detailed discussion of how each of these lessons was presented can be found in Chapter 4 of this thesis.

*Table 3.1 Planned lesson schedule for Week 1*

<b>Teachers</b>	<b>Topic and details of the lesson</b>	<b>Visualisation activities</b>
<b>Teacher 1</b>	Geometry (construction) Use and interpret the geometrical terms such as a point, line, line segment, ray and plane	<ul style="list-style-type: none"> <li>• The use of a poster to demonstrate examples of a line, ray and line segment</li> <li>• Use of cardboard to design the materials to use in the classroom</li> <li>• Use of geometrical objects such as chalkboard ruler, protractor and compass</li> </ul>
<b>Teacher 2</b>	Geometry (construction) Use and interpret the geometrical terms such as a point, line, line segment, ray and a plane	<ul style="list-style-type: none"> <li>• Draw examples of a point, a ray, a line and a line segment on the poster and on the board</li> <li>• The teachers intend to use cardboards materials and display it on the board to explain the different concepts</li> </ul>
<b>Teacher 3</b>	Geometry (construction) Measuring different lines and angles accurately	<ul style="list-style-type: none"> <li>• Use of chalkboard protractor, ruler and compass</li> <li>• Use of a poster to show examples of lines and angles</li> <li>• Use of easily available material e.g. wires and ropes to explain measuring lines</li> </ul>
<b>Teacher 4</b>	Geometry (construction) Drawing and measuring lines and angles accurately	<ul style="list-style-type: none"> <li>• Use of chalkboard, protractor, ruler and compass</li> <li>• Refer to examples where angles are formed in your classroom</li> <li>• Use of a poster with examples.</li> </ul>

Table 3.2 Planned lesson schedule for Week 2

Teachers	Topic and details of lesson	Visualisation activities
<b>Teacher 1</b>	Geometry (Angle properties) Identify different types of angles (acute, obtuse, right, straight and reflex angles)	<ul style="list-style-type: none"> <li>• Show the different types of angles on the poster and on the board</li> <li>• Design visual materials from cardboards</li> <li>• Refer to materials in your classroom that form different angles</li> </ul>
<b>Teacher 2</b>	Geometry (Angle properties) Identify and construct different types of angles accurately (acute, obtuse, right, straight and reflex angles)	<ul style="list-style-type: none"> <li>• Use of geometrical objects such as ruler, protractor and compass</li> <li>• Design a poster with different types of angles</li> <li>• Allow learners to create their own visual materials</li> </ul>
<b>Teacher 3</b>	Geometry (Angle properties) Calculate unknown angles by applying the following angle properties, angles formed within parallel lines	<ul style="list-style-type: none"> <li>• Show examples of calculating angles on the board and on a poster</li> <li>• Use of straight objects of the same length to show examples of parallel lines</li> <li>• Allow learners to use properties of angles formed in parallel lines to find unknown angles on themselves</li> </ul>
<b>Teacher 4</b>	Geometry (Angle properties) Identify and construct different types of angles accurately (acute, obtuse, right, straight and reflex angles)	<ul style="list-style-type: none"> <li>• Construct different angles from cardboards</li> <li>• Allow learners to make their own types of angles</li> <li>• Use of a protractor to construct angles</li> </ul>

Table 3.3 Planned lesson schedule for Week 3

Teachers	Topic and details of lesson	Visualisation activities
Teacher 1	Geometry (Angle properties) Calculate unknown angles by applying the following angle properties: (a) angles formed on a straight line (b) angles at a point	<ul style="list-style-type: none"> <li>• Use of a designed poster</li> <li>• Define concepts using the visual materials and explain on the board</li> <li>• Use of different representations to teach the concepts</li> </ul>
Teacher 2	Geometry (angle properties) <ul style="list-style-type: none"> <li>- Identify pairs of angles as complementary and supplementary</li> <li>- Calculate unknown angles by applying the following angle properties (a) angles formed on straight line (b) angles at a point and (c) angles formed at intersecting lines</li> </ul>	<ul style="list-style-type: none"> <li>• Use of examples from the text book</li> <li>• Design a poster</li> <li>• Allow learners to measure the angles with the teacher's guidance using a chalkboard protractor</li> </ul>
Teacher 3	Geometry (Angle properties) Identify pairs of angles as complementary and supplementary	<ul style="list-style-type: none"> <li>• Draw examples on a poster</li> <li>• Use of electronic designed poster (if available)</li> <li>• Allow learners to come up with different types of supplementary and complementary angles</li> </ul>
Teacher 4	Geometry (Angle properties) Angles formed within parallel lines	<ul style="list-style-type: none"> <li>• Use of a poster</li> <li>• Use of a chalkboard ruler</li> <li>• Outline clearly on the board the properties of angles formed in parallel lines</li> </ul>

### 3.5.3 Phase 3: Teaching intervention programme

The intervention programme consisted of 12 lessons spread over four weeks. The aim of this intervention was to teach geometry using visualisation processes to develop learners' conceptual understanding of geometry constructions, angles, and transformation. All the lessons were video-recorded for the purpose of analysis. The aim was not to compare teachers but to see how visualisation processes were used by the teachers in the lessons that they taught. I visited each teacher once a week for the duration of three weeks. Before the teachers started teaching, I held a preparatory session with the individual teacher to find out what the teacher had prepared before going to the classroom. Table 3.4 is a timetable that shows the dates and the time that I observed the individual teachers.

*Table 3.4 Intervention programme schedule for Weeks 1 to 4*

Teachers	Dates	Time
<b>Week 1</b>		
Collection and analyzing of questionnaires, Awareness workshop		
<b>Week 2</b>		
<b>Teacher 3</b>	03 July 2017, Monday	08H20
<b>Teacher 1</b>	04 July 2017, Tuesday	09H20
<b>Teacher 4</b>	07 July 2017, Friday	11H55
<b>Week 3</b>		
<b>Teacher 3</b>	10 July 2017, Monday	08H20
<b>Teacher 1</b>	11 July 2017, Tuesday	10H30
<b>Teacher 2</b>	12 July 2017, Wednesday	07H00
<b>Teacher 4</b>	13 July 2017, Thursday	11H55
<b>Week 4</b>		
<b>Teacher 3</b>	17 July 2017, Monday	08H20
<b>Teacher 1</b>	18 July 2017, Tuesday	08H40
<b>Teacher 2</b>	19 July 2017, Wednesday	12H20
<b>Teacher 2</b>	20 July 2017, Thursday	10H20
<b>Teacher 4</b>	20 July 2017, Thursday	11H55

All the lessons were transcribed word for word as they were presented. Cohen et al. (2011) noted that “if the transcript is of the video tape, then this enables the researcher to comment on all of the non-verbal communication that was taking place in addition to the features noted from the audiotape” (p.427).

#### **3.5.4 Phase 4: Interviews (focus group and stimulus response interviews)**

During this phase of the research I conducted two kinds of interviews:

**1. Stimulus response interviews** (Cohen et al., 2011). These interviews entailed a conversation between each teacher and me, where we viewed the 3 recorded lessons of that particular teacher together. These interviews were a one-on-one discussion with the teacher to review each of their lessons and analyze the lesson based on the five visualisation processes of the analytical framework in Figure 3.5. Stimulus recall interviews in this study were carried out after every

lesson taught by the individual teacher. This also created an opportunity for the teachers not only to reflect on their teachings but also to provide them with opportunity to prepare well for the next lesson to be taught.

**2. Focus Group interviews** (Newby, 2010). These interviews were initially planned to be conducted at the end of every week but due to the fact that teachers were not available at the time we planned, the interviews were only conducted at the end of the intervention programme and three out of the four participant teachers participated. The questions for these interviews were directly related to the two research questions. The interviews were recorded and transcribed word for word including non-verbal actions.

### **3.6 RESEARCH PARTICIPANTS**

This study was conducted with four participant teachers from four different schools in Rundu circuit. Purposive sampling was used to select the four participants that were included in this research. Cohen et al. (2011) discuss that in purposive sampling researcher hand-picks the participants to be included in the sample on the basis of their typicality or possession of the particular characteristic being sought (p. 156). For the purpose of this study I selected the four teachers on the basis of their responses to the questionnaire in phase 1, see appendix H. The questions were deliberately designed to specifically get teachers' views about their teaching of geometry. I sent out 17 questionnaires and 10 were returned to me - from these ten I selected four to work with in this project.

The teachers that were identified showed willingness to use visualisation in their teaching and to participate in the intervention programme. Three of the four teachers that I selected were experienced teachers and all of them had more than five years' experience in teaching mathematics at the junior secondary level. These three teachers had qualifications that specialized in mathematics education. The fourth teacher was still a temporary teacher, but he had two years teaching experience of mathematics. Table 3.5 shows the participant teachers' qualifications and teaching experiences.

*Table 3.5 Qualifications and experience of participant teachers*

<b>Teachers</b>	<b>Qualifications</b>	<b>Teaching experience</b>
Teacher 1	Basic Education Teachers Diploma (BETD), (Math and Science Teachers Education Program (MASTEP), and International Computer Driving License (ICDL).	7 years
Teacher 2	BETD, Advanced diploma and Bed-Honours	7 years
Teacher 3	BETD and Advanced certificate in education	10 years
Teacher 4	Diploma in electrical engineering and diploma in Junior primary education	2years

### **3.7 RESEARCH TECHNIQUES**

The research techniques that were used to collect qualitative and interpretive data of this study were observation, stimulus recall and focus group interviews.

#### **3.7.1 Observation**

Observation in research provides the researcher with first-hand information. Observation means that the researcher sees for himself the context and site of the research study (Bertram & Christiansen, 2014). Cohen et al. (2011) state that the distinct feature of observation as a research process is that it gives the researcher an opportunity to collect live data in naturally occurring social settings (p. 456). During the intervention programme I video-recorded 12 lessons of the four participant teachers. These videos were recorded by me while the teachers were presenting their lessons. The video-recording focused entirely on the teaching process. I was stationed in an unobtrusive yet strategic position in the classroom that enabled me to capture the entire lesson event. Recording the lessons was important because I had an opportunity to go through the lessons afterwards by replaying the videos repeatedly.

Each video-recording was analyzed using the analytical instrument template discussed under the analytical framework in Figure 3.6. For the purpose of this thesis I initially intended to report only on 6 lessons. However, I reported on all the lessons because some of the lessons were very short as the teachers could not complete the lesson due to some urgent school activities that interrupted the lessons. However, there was sufficient evidence of visualisation processes for me to analyse.

### **3.7.2 Interviews**

#### *Stimulus Recall Interviews*

Slough (2001) observes that when teachers have the opportunity to reflect on their own teaching (such as in a stimulus recall environment), they find this useful in assessing and understanding their own teaching practice. Stimulus recall interviews in this study were conducted at the end of each lesson taught. The individual teachers viewed the videos alongside me. I asked the teachers to pause the video at any time they wished to reflect on their teaching with specific reference to using visualisation processes. I also stopped the videos at strategic intervals and asked open-ended questions such as “what were you trying to accomplish here? Why did you use this visual material at this point? How did the visual process encourage conceptual understanding here?” The open-ended questions strategy created an opportunity for participants to provide as much information as possible and accorded me an opportunity to pose follow-up questions. The themes discussed in the analytical instrument in Figure 3.6 formed the basis of these interviews. I was specifically posing questions that directly related to how the teachers used visualisation processes in the lessons.

#### *Focus Group Interviews*

The focus group interviews were conducted at the end of the three weeks of the intervention programme. The main aim of the focus group interview was to gather collective information about participant teachers’ experience of the interventions programme with a special focus on how visualisation processes developed conceptual understanding of geometry. The interviews were used to augment the data from the individual stimulus recall interviews and provided me with rich and deep insight into the practices of the teachers with specific reference to the research questions. One of the four participant teachers could not take part in these interviews because he had other commitments at the time.

### **3.8 ANALYTICAL FRAMEWORK**

The first step in the data analysis is to reduce data to enable the researcher to search for themes and identify patterns (Bertram & Christiansen, 2014, p. 119). Analyzing data helps to bring about understanding of the data. For example, “making sense of data in terms of the participants’ definitions of the situation, noting patterns, themes, categories and regularities” (Cohen et al., 2011, p. 537). The video recordings were analyzed using the analytical instrument illustrated in

the tables below in Figure 3.6. The teachers and I analysed the videos and teased out the roles of the visualisation processes used on the basis of this framework. The stimulus recall interview was the medium with which the lessons were analysed, which then shed light on the answers to the research question.

The focus group interviews were analysed based on the themes from the stimulus recall interviews, as these interviews specifically focused on the participating teachers' perception on how the use of visualisation processes enhance conceptual understanding of geometry. It was observed that the themes align with Debes' (1969) criterion used in the observable schedule.

The following visualisation processes drawn from the work of Debes (1969), as discussed in the context of this thesis, informed my observable schedule and analytical framework:

1. **Designing Visuals** - Ability to demonstrate the effective use of the visual elements of a mathematical concept (such as lines, shapes, angles and dimension)
2. **Manipulating visuals** - Making solid interpretations of visuals and using different visual representations
3. **Visual language** - Ability to read and write visual language, decode and code visuals (Produce and interpret visuals)
4. **Interpreting visuals** - Ability to use visuals in a variety of ways and apply them critically
5. **Analysing Visuals** - Ability to use visuals to illustrate and reinforce properties of different objects and shapes and arrange them to communicate effectively

I analysed these five visualisation processes with a special focus on the observable indicators as outline in the analytical instrument in Figure 3.6. I used this instrument during observations and my interest was to see how the teachers used the different visualisation processes in their lessons and rate them as rich, medium or no evidence depending on my observations. The instrument is divided into five sections and they were used simultaneously for each lesson.

Table 3.6. Analytical Instrument Tools used during observations

Teachers' activity in the classroom (Observable indicators)	Type of visualisation processes after Debes (1969)	Rich evidence	Medium Evidence	No evidence
Teacher use different representation during the lesson	<i>Manipulating visuals</i>			
Teacher demonstrated proper use of geometric concepts	<i>Manipulating visuals</i>			
Teacher used visuals in a way that allows learners to connect new concepts to what they already know	<i>Manipulating visuals</i>			
Teacher used visuals in an orderly and logical way to avoid confusing learners	<i>Manipulating visuals</i>			

Teachers' activity in the classroom (Observable indicators)	Type of visualisation processes after Debes (1969)	Rich evidence	Medium evidence	No evidence
Teacher brought to class visual materials	<i>Designing visuals</i>			
Visual materials brought to class are aligned with the subject content	<i>Designing visuals</i>			
Lesson was introduced with the use of visual materials	<i>Designing visuals</i>			
Teacher demonstrates the use of visual elements (lines, shapes, angles and dimensions)	<i>Designing visuals</i>			

Teachers' activity in the classroom (Observable indicators)	Type of visualisation processes after Debes (1969)	Rich evidence	Medium evidence	No evidence
Teacher used strategies that allow learners to produce visuals	<i>Visual language</i>			
Teacher used visuals in conjunction with his/her verbal explanations	<i>Visual language</i>			
Teacher responded to questions asked about the visuals by the learners	<i>Visual language</i>			
Teachers used visual language that was consistent with mathematical language and terminologies	<i>Visual language</i>			

Teachers' activity in the classroom (Observable indicators)	Type of visualisation processes after Debes (1969)	Rich evidence	Medium evidence	No evidence
Teacher properly identified and evaluated visual materials	<i>Interpreting visuals</i>			
Teacher asked questions that prompted critical thinking and critical reasoning among learners	<i>Interpreting visuals</i>			
Teacher concluded the lesson using visual materials	<i>Interpreting visuals</i>			

Teachers' activity in the classroom (Observable indicators)	Type of visualization processes after Debes (1969)	Rich evidence	Medium evidence	No evidence
Teacher communicated to learners using visual materials	<i>Analyzing visuals</i>			
Teacher used visual materials that deconstructed properties of shapes.	<i>Analyzing visuals</i>			
Teacher encouraged learners to work with the visuals in terms of the concepts that were taught.	<i>Analyzing visuals</i>			

### 3.9 VALIDITY

Maxwell (2009) asserts that validation is an important way of ruling out the possibility of misinterpreting the meaning of what participants say and do, and the perspective they have about what the research is about. Cohen, Manion and Morrison (2007) state that in qualitative research, validity should be addressed through the honesty, depth and scope of the data achieved. In this study I used triangulation (Gay, Mills & Airasian, 2012) to validate the data collected. For example, I used audio-video recordings, stimulus recall and focus group interviews. Using three methods at the same time helped me to identify common and different themes that emerged from this study. The video recordings helped me to easily conduct the stimulus recall interviews because questions for these interviews were drawn from the recorded video. The focus group interview was helpful in finding out the general experience and perceptions of the participant teachers, which enabled my first research question to be answered.

This study also adopted a strategy of member-checking to ensure internal validity and credibility. Merriam (2009) states that the process of member-checking involves a discussion of preliminary analysis and interpretations with the participant teachers before drawing any conclusions. Whenever I compiled teachers' responses to interview questions, I invited the individual teacher to verify whether what I recorded was true reflection of what they had responded. I also allowed the teachers to view the recorded videos with me after each lesson to ensure that I did not tamper with any information related to the videos. Member-checking helped me to identify my own biasness and misunderstanding. All four participating teachers were involved collectively to share their views and perceptions about the interventions.

### **3.10 ETHICS**

Issues of ethics are vital in the educational research process for a number of reasons. From a broad, collective point of view, ethics has to do with the application of moral principles to prevent harming or wronging others (Opie, 2004). In this section, I have highlighted some important aspects of ethics relating to my research study. Ethics in this study was aligned to Rhodes University ethical guidelines.

#### **3.10.1 Respect and dignity**

The aim of this research was communicated to the participants prior to carrying out the intervention programme. I invited all the participants to a central area where issues relating to this research were discussed; they were also invited to share their feelings regarding their involvements in this study. Everything pertaining to this study was explained clearly to the participants.

In order to ensure respect and dignity for the participants in this study, they were informed of their rights to withdraw from the study at any time if they found it uncomfortable to continue being part of it. I assured them that the data collected for the study would be kept confidential and anonymous. I also informed them that I would use pseudonyms (protection of participants, and site identities) in the final write-up of this thesis.

#### **3.10.2 Transparency and honesty**

Since my study was carried out in only one circuit of Kavango East in northern Namibia, I wrote a permission letter to the education director of Kavango East and the circuit inspector of Rundu circuit to request permission to carry out my study in their region. The director granted me permission in the form of a letter. See appendix B. A separate letter was written to the senior mathematics education officer in the region to request him to grant me an opportunity at his ministerial workshop to talk to the teachers and ask for four participants to be part of this research study. However, no such workshop took place during that period due to financial constraints. The senior education officer organized a workshop in his own free time and we called the teachers there to tell them about the study and what I was intending to do.

After selecting the four participant teachers, I wrote letters to their school principals asking to carry out my study in the different schools with the selected mathematics teacher. All school principals responded positively to my request and allowed me to carry out my study in their respective schools.

See appendix C. The four participant teachers were asked to complete a consent letter stating their willingness to take part in this study. They all completed these forms. Member-checking of the stimulus recall and focus group interviews was done in order to ensure that what was recorded was a true reflection of the teachers' responses.

### **3.10.3 Accountability and responsibility**

O'Leary (2004) states that an indicator of accountability in research is when the research process is transparent and open for any other researcher to trace the research methods, without compromising the confidentiality of the participants. In this study I was fully accountable and responsible for all data collected and kept them in safe documents both in my laptop and memory stick to avoid any loss or misplacement of the data. Since I am at the same professional level with the participant teachers of this research, the issue of position did not have any effect on my study. Three of these participants already knew me because we had studied together in the basic education teacher diploma. Therefore, my presence in their classroom did not cause any intimidation or discomfort that could possibly lead them to give responses that were contrived. All four participant teachers were positive in their responses and none of them tried to hide information or acted in a way of not wanting to share his/her experiences.

### **3.10.4 Integrity and academic professionalism**

In order to maintain integrity and professionalism in this study, I used a triangulation method to collect data, thereby ensuring that the data collected was authentic. This comprised of classroom observations, stimulus recall and focus group interviews. Data presented for this research study is a true reflection of the findings. I declare that the final thesis is my own work and where I have used other peoples' work I have acknowledged and referenced them according to the Rhodes University reference guidelines.

## **3.11 CONCLUSION**

In this chapter I first outlined the research goals and questions of this qualitative study. I further discussed the research orientation, methods, research design and research techniques that I employed in this study. This chapter also gave a discussion on the analytical framework that is at the center of this study, by providing a summary of the research instruments that were used to collect data. The chapter was concluded with a summary of issues pertaining to validity and ethics of this educational research.

## CHAPTER FOUR

### DATA ANALYSIS AND DISCUSSIONS

#### 4.1 INTRODUCTION

The purpose of this chapter is to present and discuss the findings of my research study. The main objective of the research was to analyse the use of visualisation processes by teachers who participated in an intervention programme to teach for conceptual understanding of geometry. This chapter begins with a general description of the 12 lessons (three lessons taught by each of the four teachers). The chapter further continues with the discussion of each teacher's lesson presentation, focusing on visualisation processes and themes that emerged from the lessons. The second part of this chapter analyses the similarities and differences among the four participants according to my analytical framework of: designing visuals, manipulating visuals, analysing visuals, interpreting visuals and visual language. In the last part of this chapter, I share some idiosyncrasies and participant teachers' perceptions about the use of visualisation processes during the intervention programme.

Data in this chapter is presented as authentically as possible. The participant's actual work, words and responses from the lesson observations and interviews has been used to support the data presented and they are coded as follows: for lesson observations (**T1L1**, T1 is for Teacher 1 and L1 stands for lesson 1, and I used Ls for learners' responses), for stimulus recall interviews (**T1SRI1**, T1 stands for teacher 1 and SRI1 stands for stimulus recall interview 1) and for focus group interviews (**T2FGI1**, T2 stand for teacher 2 and FGI1 stands for focus group interview 1).

#### 4.2 A BRIEF DISCRIPTION OF EACH LESSON

The analysed lessons were all on geometry in Grade 8. The main focus and content of the lesson centered on construction and angle properties. Each of the four teachers managed to teach three lessons.

**Lesson 1** (Construction); both T1 and T2 taught this lesson and the basic competency of this lesson was to use and interpret the geometrical terms such as a point, a line, a line segment, a ray and a plane. **Lesson 2** (Construction); this lesson continued the theme of construction. T1 and T2 continued with the lesson but this time it included identifying different types of angles (acute,

obtuse, right, straight and reflex angles). T3 and T4 focused on calculating unknown angles by applying the angle property of angles formed on parallel lines. **Lesson 3** (Angle properties); T1 and T2 focused on calculating unknown angles by applying the angle property of angles formed within parallel lines and angles at a point. T3 focused only on angles at a point and T4 focused on identifying the different types of angles as acute, obtuse, right, straight and reflex angles.

### **4.3 DESCRIPTION OF THE LESSONS WHERE VISUALISATION PROCESSES EMERGED**

The aim of the analysis below was to explore and identify how the teachers who participated in the intervention programme presented their lessons using visualisation processes (designing visuals, manipulating visuals, analysing visuals, interpreting visuals and visual language) to teach for conceptual understanding of geometry. The description presented in this chapter is in a vertical form and discusses what happened in each of the lessons taught by the four teachers. I first discuss lesson 1 to lesson 3 for each teacher according to the following structure:

1. Brief description of the lesson and how visualisation processes emerged.
2. Visualisation processes: here I present a bar chart which shows the extent of evidence observed against each visualisation processes criteria (see analytical framework tool in chapter 3 tables 3.6). I then discussed each of the five visualisation processes.
3. Stimulus recall interview: here I present a one-on-one stimulus recall interview that I conducted with each participant teacher after each lesson.
4. Focus group interview: here I present the focus group interview I had with the whole group after the entire intervention programme. Please note that teacher 1 did not participate in this interview and I thus do not report on his participation in this focus group interview.

The aim of this structure is to identify how visualisation processes are used in each lesson and to identify common and different teaching practices employed by the participating teachers.

The following coding criteria were used for each of the graphs (numeric data) drawn from the observation instruments:

**Rich evidence:** Here the teacher demonstrates an abundance and variety of the specific indicator. For example, under designing visuals, abundance would refer to the teacher bringing to the class four or more visual materials. It would also refer to the teacher engaging deeply with these visuals.

**Medium evidence:** Here the teacher only uses one or two visual materials and engages with them on a relatively superficial level of the specific indicators.

**No evidence:** Here the teacher does not use any visual materials and does not engage with any visual processes.

I have abbreviated the visualisation processes as follows: Designing visuals (**DV**), Manipulating visuals (**MV**), Visual language (**VL**), interpreting visuals (**IV**) and analysing visuals (**AV**).

#### 4.3.1 Lesson 1, teacher 1

##### 4.3.1.1 Brief description of lesson

This lesson was on construction and the objective of the lesson was for learners to know how to perform geometrical constructions using a straight edge, a compass and a protractor, and to use and interpret the geometrical terms such as a point, a line, a line segment, a ray and a plane.

##### 4.3.1.2 Visualisation processes

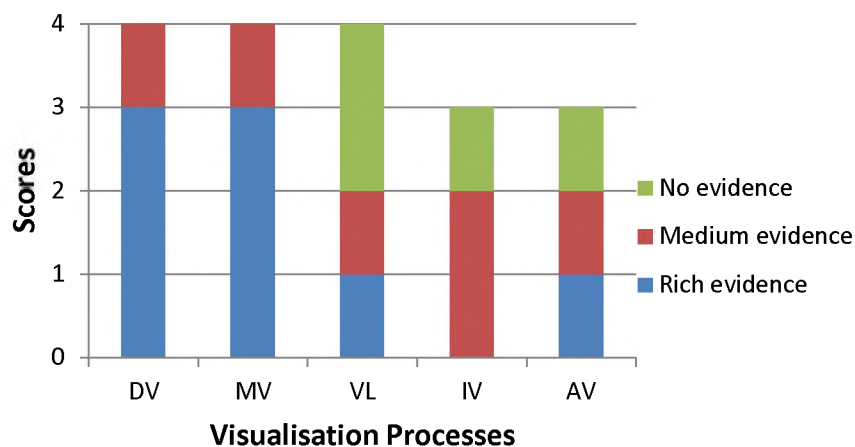


Figure 4.1 Teacher 1, lesson 1 Visualisation processes, numerical scores

Figure 4.1 above shows the summary of the scores of the visualisation processes in the teacher's lesson as per the coding criteria; this shows that there is evidence that teacher 1 used a number of visual representations under the following visual processes: designing visuals, manipulating visuals and interpreting visuals. However, little evidence was observed of analysing visuals and visual language.

The description below shows how each of these visualisation processes was used in lesson 1 for teacher 1:

**Designing visuals:** The teacher brought to class visual materials in form of a point, and an arrow as shown in Figure 4.2, that he had designed from card boards. He also brought to class a piece of a rope. The new lesson was introduced using a point.

**Manipulating visuals:** During this lesson, different representations e.g. demonstration on the board and verbally communicating to the learners while using the visual materials, were observed. In introducing the lesson, the teacher asked the learners to recall what they had learned about geometry in previous grades. The learners in return responded by naming the different types of angles (acute, obtuse, right, straight and reflex angles). The teacher listed each of these angles on the board for the learners to clearly see.

**Visual language:** The visual materials used in this lesson were in line with the teacher's verbal explanations. For example, the teacher used the rope to state the meaning of a line. Learners in this lesson did not ask the teacher any questions. However, the teacher posed a few questions that learners responded to.

**Interpreting visuals:** Questions asked by the teacher prompted critical thinking and reasoning because learners were asked questions based on the visual materials brought to class and they were further required to provide reasons for their answers.

**Analysing visuals:** Teacher 1 showed the learners an example of a point, which he had designed from cardboards. The teacher also used pieces of arrows that he had designed from box materials. He pasted it onto the board and used it to explain the concept of a line and a line segment. In addition to a line, the teacher used a piece of rope which he visually displayed on the

board. The teacher creatively linked each of the visuals together by going through each concept one by one. Figure 4.2 below shows how teacher 1 illustrated the visual materials on the board.

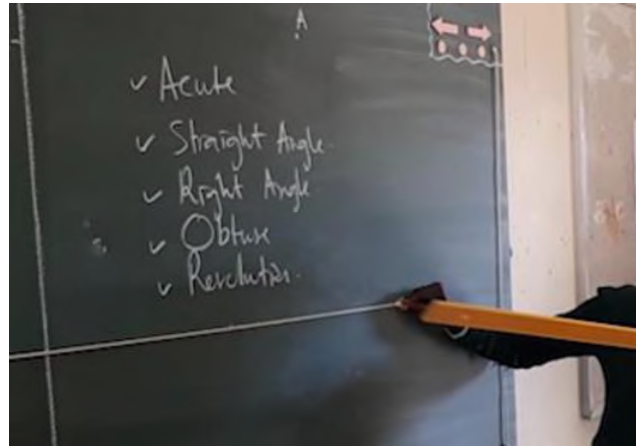


Figure 4.2 Teacher 1 displaying visual materials on the board

#### 4.3.1.3 Stimulus recall interview

For the teacher to reflect on his own teaching, I had a stimulus recall interview with teacher 1 where we watched the video together and analysed it together. I also wanted to understand why certain things were done the way they were done in the lesson. As a researcher, I was interested in knowing why the teacher introduced the lesson by first finding out what the learners already knew:

Researcher: Why do you think it was necessary for you to find out what learners already know about angles?

*TISR11: Alright, it's a good question, actually visualisation is a .... Concept.*

*So actually, looking at these learners when I first asked eee (ya) about geometry they were quiet (ya) so now first relating it, they are from grade 6, 7 relating what they know abit will now drive us through better.*

*I even tried asking them for example about geometry they still can't recall, and then latter I have to relate them to abit of angles which are still in geometry. Then you can see now learners*

are... ..It's very important to first look at what they already know okay? and then now you can see they are coming up with what geometry is about...angles and so....

Teacher 1 also revealed that he prefers writing down learners' responses as he asks them questions, because that helps them to see and visualise what they are saying and helps them to better remember the concepts. The rope as a visualisation tool was brought to the class because the teacher felt that most of the learners in his class are familiar with a rope because they have it at home, some use it as a clothesline and some use it for various games at home. Thus, bringing what they already know to the classroom and relating it to what one is teaching can improve understanding for the learners.

#### 4.3.2 Lesson 2, teacher 1

##### 4.3.2.1 Brief description of the lesson

Lesson 2 focused on construction with a specific focus on identifying the different types of angles as acute, obtuse, right, straight and reflex angle. As always, the teacher began the lesson by greeting the learners and welcoming them to the lesson.

##### 4.3.2.2 Visualisation processes

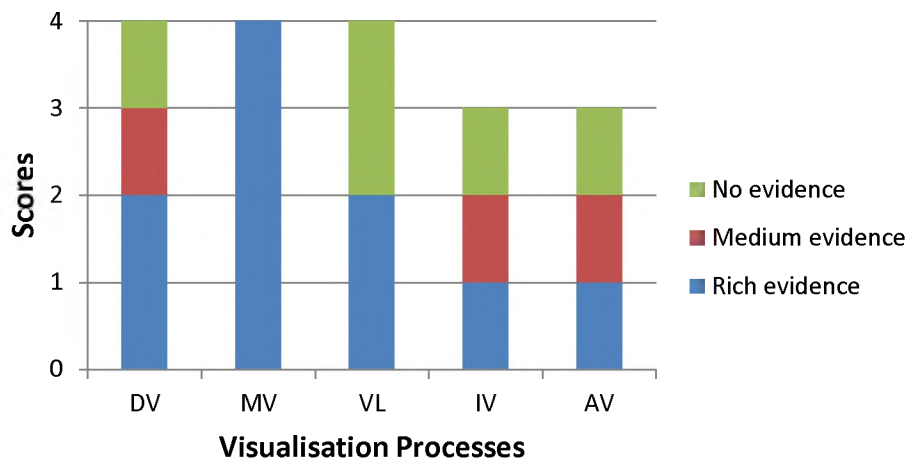


Figure 4.3 Teacher 1 Lesson 2 Visualisation processes, numerical scores

Figure 4.3 above shows the summary of the numerical scores of each of the five visualisation processes as presented in Lesson 2. The diagram indicates that there is evidence that visual material was used, especially when manipulating the visuals.

The description below is a discussion of how each of these visualisation processes were used in this lesson by teacher 1:

**Designing visuals:** The teacher brought a poster to class. He also brought an arrow which he used to demonstrate on the poster. The arrow was designed by the teacher to help him easily explain the types of angles. Teacher 1 demonstrated the size of each angle on the poster; he divided the poster into four quadrants ( $0^\circ$  to  $360^\circ$ ) as shown in Figure 4.4 below:

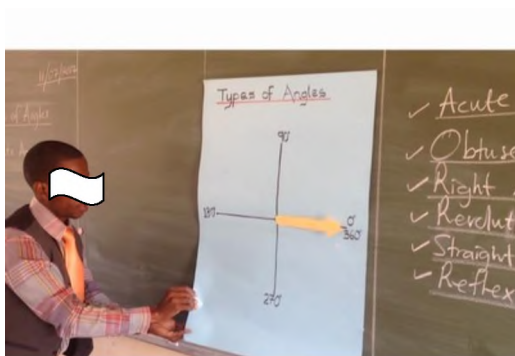


Figure 4.4 Teacher 1 demonstrating how each type of angle is formed on the poster using the arrow

**Manipulating visuals:** Teacher 1 used the chalkboard and the poster simultaneously throughout his lesson. While he demonstrated on the poster, he was also writing down on the chalkboard. He asked them to recall the types of angles that they had learned in previous grades.

*T1L2: Can we name the types of angles that we came across?*

*Ls: acute angle*

*T1L2: I will write them here before we ...*

*Ls: obtuse angle, right angle and revolution.*

As the learners were mentioning the angles, the teacher wrote down on the board as shown in the diagram below.



*Figure 4.5 Teacher 2 Demonstrating on the Board*

**Visual language:** The teacher demonstrated the use of mathematical language e.g. rotate, clockwise, and his verbal explanations aligned well with the visual materials that he used in the lesson.

**Interpreting visuals:** Teacher 1 then informed the learners that the angles mentioned by the learners would be looked at one by one from the smallest to the biggest angle. To teach each of the types of angles, the teacher used the arrow on the poster and moved it clockwise to show how angles increase in size from acute, obtuse, right, and reflex angles. He used the arrow to show that acute angles are less than  $90^\circ$ , right angles are equal to  $90^\circ$ , obtuse angles are greater than  $90^\circ$  but less than  $180^\circ$ , straight angles are equal to  $180^\circ$  and reflex angles are greater than  $180^\circ$  but less than  $360^\circ$ . The way the teacher moved the arrow clearly showed the angle sizes.

**Analysing visuals:** From time to time the teacher communicated to learners using the visual materials. However, the teacher did not encourage the learners to work with the visuals.

#### **4.3.2.3 Stimulus recall interview**

After the second lesson, we reflected on it through questioning and interacting with the teacher while viewing the recorded video. However, the teacher's responses were similar to the interview after his first lesson. In this interview the teacher still maintained that writing down the types of angles as learners are mentioning them helps learners to better recall them. He stated that the use of a poster in his lesson helped him to teach the types of angles according to their sizes, from the smallest to the biggest angle. The teacher stated that the poster which he used in his lesson helped the learners to see the formation of different angles, and using different colors

on the poster made it more attractive and captured learners’ attention. Teacher 1 added that the arrow that he used on the poster made it easier for him to demonstrate each type of angle in a limited period of time. He said the arrow is more visual and learners had an opportunity to see its direction and how the arrow formed different angles.

### 4.3.3 Lesson 3, teacher 1

#### 4.3.3.1 Brief description of the lesson

Teacher 1 in this lesson was teaching about calculating unknown angles by applying the following angle properties (a) angles formed within parallel lines and (b) angles at a point. Both these subtopics fall under the topic of angle properties. The main idea was to use the visual processes and tools to teach each of the concepts covered under this topic conceptually. Teacher 1 first greeted the learners and asked them to take out their mathematics books.

#### 4.3.3.2 Visualisation processes

Figure 4.6 below shows the summary of the numerical scores of each of the five visualisation process as per the coding criteria. The way the teacher used each of these processes is discussed thereafter.

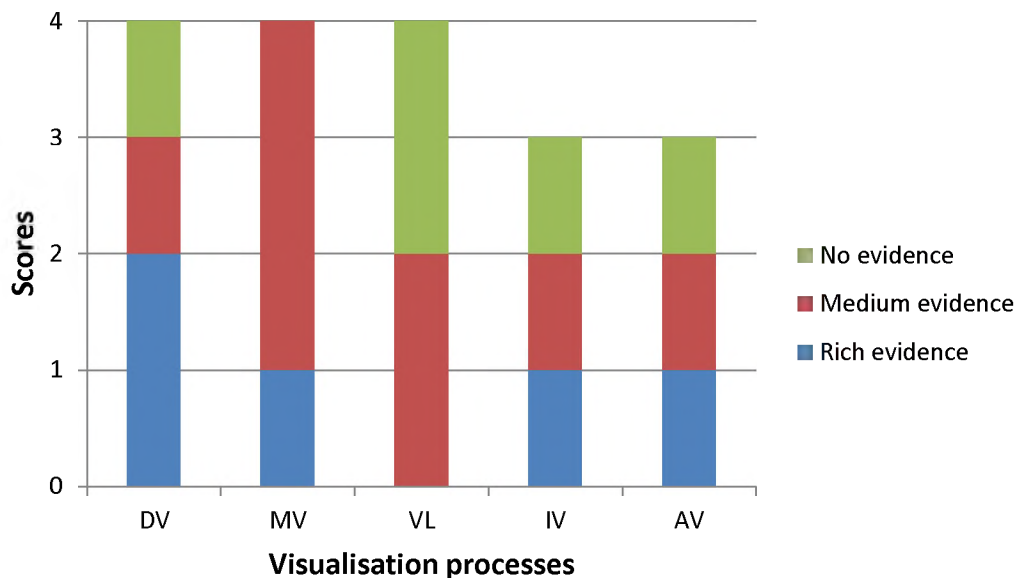
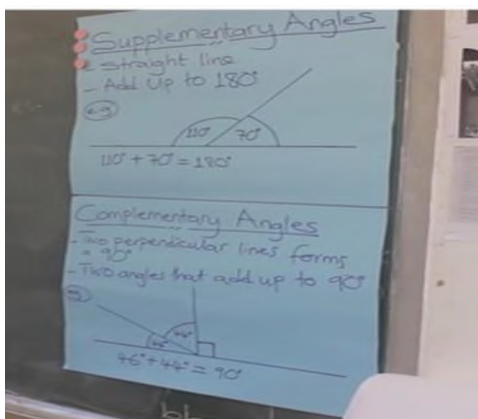


Figure 4.6 Teacher1, lesson 3 Visualisation processes, numerical scores

**Designing visuals:** In this lesson, the teacher used a poster as a teaching aid. The teacher also used a chalkboard ruler and protractor as visual objects. The materials brought to class were not used for the introduction of the lesson, but the teacher used visual elements such as drawing, lines and angles to lead learners to the topic to be discussed.

**Manipulating visuals:** The teacher started by looking at supplementary and complementary angles. He pasted a poster on the board and drew one example of a supplementary angle and another of a complementary angle. The illustration in Figure 4.7 below shows the example of the poster that was used to teach the two geometrical concepts of supplementary and complementary angles.



*Figure 4.7 Example of supplementary and complementary angles on a poster*

**Interpreting visuals:** Teacher 1 demonstrated that two supplementary angles add up to  $180^\circ$ . He used a chalkboard protractor to explain how two angles can add up to  $180^\circ$  and emphasized that any two angles that add up to  $180^\circ$  are referred to as supplementary angles. The teacher used an example of  $110^\circ + 70^\circ = 180^\circ$ . These two angles are also visually shown in Figure 4.7 above. Teacher 1 then moved on to complementary angles. He used an example of perpendicular lines that meet to form a  $90^\circ$  angle by drawing the lines on the board.

On the poster teacher 1 used an example of  $46^\circ + 44^\circ = 90^\circ$  to show that complementary angles are two angles that add together to form  $90^\circ$ . In order to supplement what he was demonstrating on the board, the teacher gave a few examples on the board and went through each of these, explaining each concept clearly. The angles on the board were clearly drawn using a chalkboard ruler. Teacher 1 moved on to teach the concepts of angles at a point. He showed the learners an

example of a revolution which he referred to as an angle at a point. Using a ruler, he demonstrated why angles at a point form a revolution as shown below.

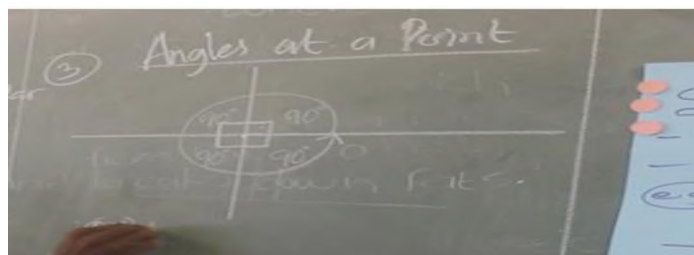


Figure 4.8 Example of angles at a point

The lesson was concluded with a recap of each of the concepts taught and later the learners were given an activity which was in line with the basic competencies and the visual processes that had been used in the classroom.

**Visual language:** The teacher emphasized the correct use of geometrical concepts and provided definitions of concepts on the chalkboard for learners to see them more visually and to conceptualize the concepts.

**Analysing visuals:** Teacher 1 encouraged his learners to always use a pencil and a ruler whenever they drew angles. The examples on the board were repeated frequently, hopefully to ensure that each learner was following what was being taught.

#### 4.3.3.3 Stimulus recall interview

In the last discussion I had with teacher 1, I interacted with him trying to find out why he did certain things that he did in the lesson. I first wanted to know why he introduced the concept of perpendicular lines if that was not the topic for the day. The teacher informed me that he brought in the concept knowing that learners were aware of the concept from grades 6 and 7. For him to talk about angles at a point, his explanation needed the aid of perpendicular lines. Teacher 1 stated that in order to introduce the concept of supplementary and complementary angles, he had to bring to the class a poster to help the learners to follow what he was explaining and later to show on the board how to calculate unknown angles formed on a straight line.

**TISR13:** *Just pause there. I only introduced them to supplementary and complementary angles using the visuals, so now I have to bring in the third one. Assuming that learners will now have understood*

*supplementary, complementary, now the other one's not on this visual, but it will now be drawn, quickly so their mind is now used that even if I draw, actually, this type of angle now will be understood.*

#### 4.3.4 Lesson 1, teacher 2

##### 4.3.4.1 Brief description of lesson

The following part shows how teacher 2 presented the same lesson of construction, focusing on using and interpreting geometrical terms such as a point, a line, a line segment, a ray and a plane. He used some powerful visualisation processes in this lesson.

##### 4.3.4.2 Visualisation processes

Figure 4.9 shows the summary of the scores as per the coding criteria. The diagram indicates that there is evidence that visual representations were used under the following visualisation processes: Designing, manipulating, interpreting and analysing visuals.

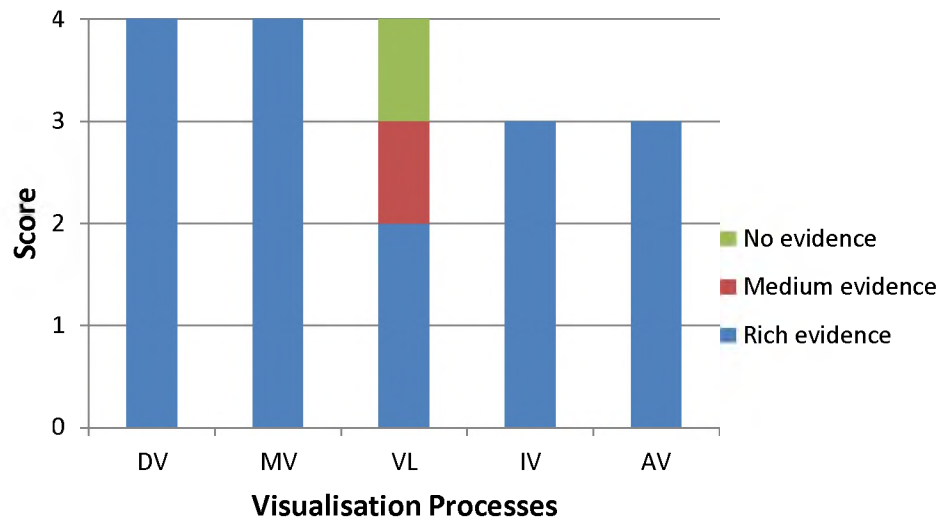


Figure 4.9 Teacher 2; lesson 1 Visualisation Processes, numerical scores

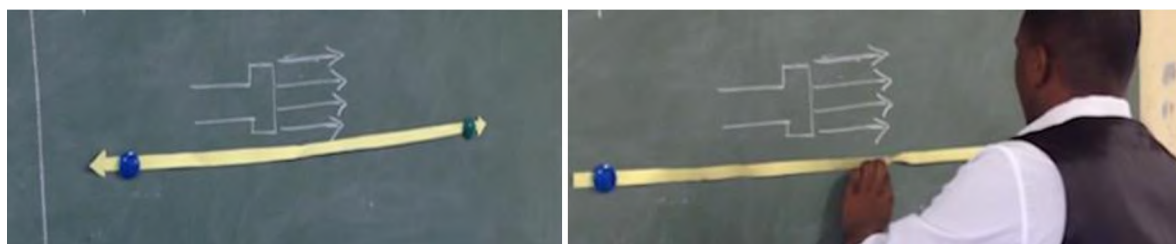
Visualisation processes in this lesson were used as follows:

**Designing visuals:** The teacher used the following visual materials in this lesson, a poster designed by the teacher, a line and a line segment made from cardboard materials, and a printed electronic poster. The teacher started the lesson by writing the topic *Geometry* on the board and pasted a poster with three different images on it. He then asked the learners to name each of the objects on the poster. He labeled the drawings 1, 2 and 3 and further asked the learners to state

the differences between them. One learner replied that drawing 1 has a beginning, drawing 2 has no beginning and drawing 3 has both beginning and end. Another learner responded that the first one has arrows on both sides, the second one has an arrow only on one side and the third has only points on both sides.

**Manipulating visuals:** The teacher explained to the learners that the first drawing is a line because according to the definition of a line, a line has no beginning and no end, the second drawing is a ray because it has a beginning but no end while the last one is a line segment because it has both a beginning and an end.

To further define the concept of a ray, the teacher used an example of a torch; he stated in the interview that he used that example because most learners are familiar with a torch, making it easier for them to grasp the concept. The teacher also brought to class materials designed in the form of a line as well as a line segment, as shown in Figure 4.10 below.



*Figure 4.10 Teacher 2, showing an example of a line and a line segment*

**Visual language:** The language used by the teacher aligned with the way he used the visual materials. For example, the teacher referred to the materials illustrated in Figure 4.10 as a line segment and a line. In this lesson, learners did not ask questions that required a response from the teacher.

**Interpreting visuals:** These visuals were displayed on the board and learners were asked to identify each of them following the examples from the poster. The learners correctly identified the visuals. Visual materials were continuously used by the teacher to ask questions that required the learners to reason critically. In teaching the concept of a plane, the teacher displayed a blank poster on the board and used it to demonstrate an example of a plane. Furthermore, towards the conclusion of the lesson, the teacher used a poster with different types of angles and informed

the learners that the next topic would be on the types of angles. The lesson was concluded by reviewing each concept that had been taught.

**Analysing visuals:** Whenever the teacher referred to an example, he tried to use the materials to communicate to the learners, by showing them the materials and asking them verbal questions to respond to.

#### **4.3.4.3 Stimulus recall interview**

This session was held after teacher 2 had taught his first lesson. The aim of this interview was for the teacher to reflect on his own teaching through a discussion with me, whilst observing and video-recording him. We viewed the video together and I wanted to understand why the teacher gave emphasis on the point in the introduction of the lesson. The teacher maintained that a point is a basic building block of geometry. Understating the properties of a point makes it easier for the learners to know concepts such as a line and a line segment. Teacher 2 stated that it was also important for him to find out in the introduction, what learners already know about geometry, because it helps him to establish what learners already know and what it is that may require explanation from him. During this interview, Teacher 2 further stated that it was always good for him to write down on the board what he was saying, as some learners could not hear properly; writing down what he was saying enabled those learners to see what the teacher was saying. Equally, it was important to use the poster and at the same time draw on the chalkboard, so as to re-emphasize the concepts. He said learners should be given time to see the visuals in different forms.

#### **4.3.4.4 Focus group interview**

Teacher 2 stated that when he first heard about visualisation in the awareness workshop, it fascinated him because visualisation is very important in geometry teaching.

*T2FGI1: You wouldn't teach geometry if there were no visuals, because geometry is not something that you can do abstractly; it must be done concretely, so that learners are able to see what you are saying as a teacher.*

He further stressed that his first lesson was very interesting because it was the first time that he had to look closely at the topic while considering the inclusion of visualisation in his lesson. On

the question about the type of visualisation processes that he employed in the lesson, teacher 2 said that he considered what learners see and think. He gave an example of the poster that he used in the classroom, stating that when he first displayed the poster on the board and asked questions, he wanted to know what the learners were seeing and what they thought it was.

Teacher 2 stated that the visuals that he used in the lesson were in line with the concepts he was teaching, because learners had an opportunity not only to hear a verbal explanation but were able to visualise objects that could help them to develop a conceptual understanding of geometry. Especially when teaching about lines, he advised that the teacher needed to bring to class an example of a line, visually enabling learners to see what a line is.

#### 4.3.5 Lesson 2, teacher2

##### 4.3.5.1 Brief description of lesson

This section analyses how Teacher 2 presented lesson 2, which was about identifying the types of angles as acute, obtuse, right, and straight and reflex angles. The teacher began the lesson by writing down the topic on the chalkboard. He called for learners' attention and requested them to look at him while he was teaching so that they did not lose focus.

##### 4.3.5.2 Visualisation Processes

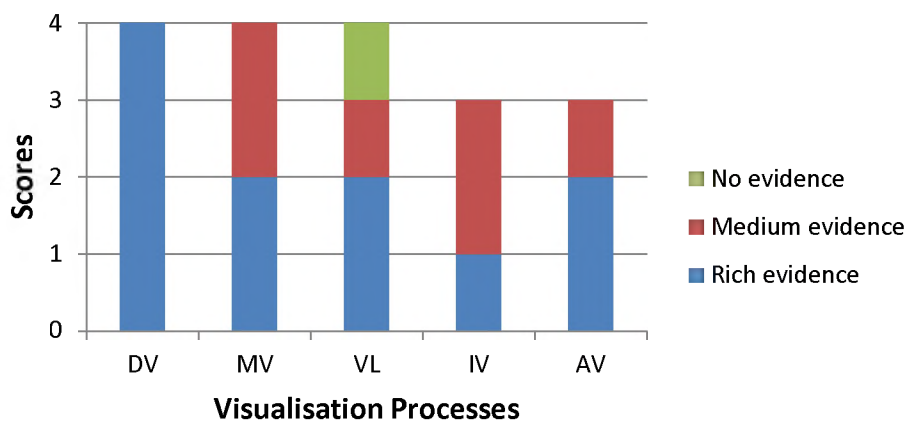


Figure 4.11 Teacher 2 Lesson 2 Visualisation Processes, numerical scores

Figure 4.11 indicates that under the observable indicators within designing visuals as a visualisation process, there is evidence that visual materials were used as compared to the other

four visualisation processes. It also shows that the teacher made use of manipulating visuals, visual language and analysing visuals. The figure depicts the scores for each of the five visualisation processes as per the coding criteria that I used.

The discussion below presents how each of the visualisation processes was used in this lesson:

**Designing visuals:** Teacher 2 recapped on the previous lesson by asking learners to define an angle.

*T2L2: Can someone define from yesterday's discussion, can someone define what an angle is, or if someone asks you what is an angle what will you say?*

*Ls: an angle is the amount of turn around a point*

Teacher 2 then drew an example of an angle on the board and lifted a protractor to show the learners that a protractor is used to measure angles. He emphasized geometrical terms such as a vertex and the arms of an angle.

**Manipulating visuals:** Using the protractor, the teacher measured the angle on the board. The angle measured  $126^\circ$ . He drew another angle on the board and this one measured  $50^\circ$ . As he was writing on the board one learner mentioned 50 degrees Celsius ( $^\circ\text{C}$ ). To ensure that there was no confusion between these two concepts, the teacher told the learner that degree Celsius is a unit used to measure temperature and degree is used to measure angles. Teacher 2 then told the learners that the focus of that particular lesson was on naming the different types of angles and not on constructing angles.

**Interpreting visuals:** As the lesson progressed, the teacher used the two angles drawn on the board to emphasize that angles are named according to their sizes. To supplement his explanation, he brought to class a poster which was divided into four quadrants with an arrow on it. He pasted the poster onto the board and used it to explain the difference in size between different types of angles. He moved the arrow clockwise to show an example of acute angles, right angles, obtuse angles, and reflex angles. When the teacher finished demonstrating using the visual material (poster), he explained again in more detail on the chalkboard, while at the same time writing down the sizes of each angle. The diagram below shows how the teacher demonstrated the types of angles on the poster using the arrow.

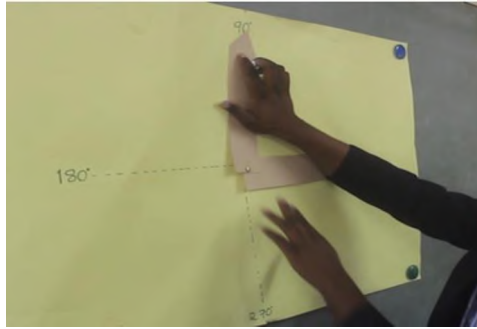


Figure 4.12 Teacher 2 demonstrating the types of angles on the poster

In this lesson, Teacher 2 used the visual materials to clarify the confusions that learners have between acute and reflex angles. He explained that acute angles are usually formed between the two arms of an angle, while reflex angles are formed at the vertex outside or behind the two arms of an angle.

**Visual language:** Teacher 2 used the visuals to pose verbal questions to the learners. He kept increasing the size of the angles while holding the materials and asked the learners to identify each angle according to its size. He repeated this process several times.

**Analysing visuals:** Teacher 2 concluded the lesson by reviewing each concept using the materials that he had prepared for this lesson, and asked the learners to come up with their own visual strips to demonstrate the types of angles.

#### 4.3.5.3 Stimulus recall interview

In trying to understand the teacher's teaching practice, I decided to have an interview with him after his lesson. We interacted together and reflected on his teaching. The following are some of the questions I asked him during the interview session.

**Researcher:** *How did the visual representation on the poster help learners to develop conceptual understanding?*

**T2SRI2:** *I believe it helps because when you just drawing an angle, let's say an acute angle and give an example of one, let's say  $50^\circ$ . That way some learners might not capture, but when you say I have a visual representation like this one, they understand like how angles are formed like*

*from 1 up to 89. You are showing from this with one visual aid, that way they will understand acute angle better than if you have to draw one or two acute angles.*

***Researcher:** I see, you want them to see that everything within this area is acute angles until at this point.*

***T2SRI2:** Yes, I believe that way is better when it's done than when you just draw one on the chalkboard.*

Teacher 2 was convinced that his learners captured the concepts he was teaching because he used a variety of teaching materials that learners were able to visualise and they responded promptly to the verbal questions that he asked them.

#### **4.3.5.4 Focus group interview**

In responding to the questions that I posed during the focus group interview, Teacher 2 recounted that preparing and teaching the second lesson was a little different from the first lesson, since in the second lesson he was aware that some of the learners did not properly capture the concepts of the previous lesson, and that made him plan the second lesson in a way that accommodated such learners. The teacher responded that in the second lesson, it was all about building up on the previous lesson and the visuals brought to class were an advantage in helping learners to learn the new concepts. On the question of how the visuals used are related to geometry teaching, the teacher responded that how we use and manipulate visuals is what matters. For example, if you as a teacher do not know how to use the protractor you are likely to confuse the learners. Teacher 2 further stated that it was important to first prepare well and know how to display your visuals in the classroom and to know how to manipulate and use language that learners could easily understand.

#### **4.3.6 Lesson 3, teacher 2**

##### **4.3.6.1 Brief description of lesson**

This was the last intervention lesson for Teacher 2; the basic competencies of the lesson were to identify pairs of angles as either supplementary or complementary angles, and to calculate unknown angles by applying the following angle properties: (a) angles formed on a straight line,

(b) angle at a point and (c) angles formed at intersecting lines. Teacher 2 first wrote the topic on the board and told the learners that the focus of the lesson would be on angles formed on straight lines.

#### 4.3.6.2 Visualisation processes

Figure 4.13 below shows the numeric scores of the individual visualisation processes as per the coding criteria. The diagram indicates the evidence that Teacher 2 in this lesson used visual material mostly under the *designing visuals* process, compared to the other visualisation processes.

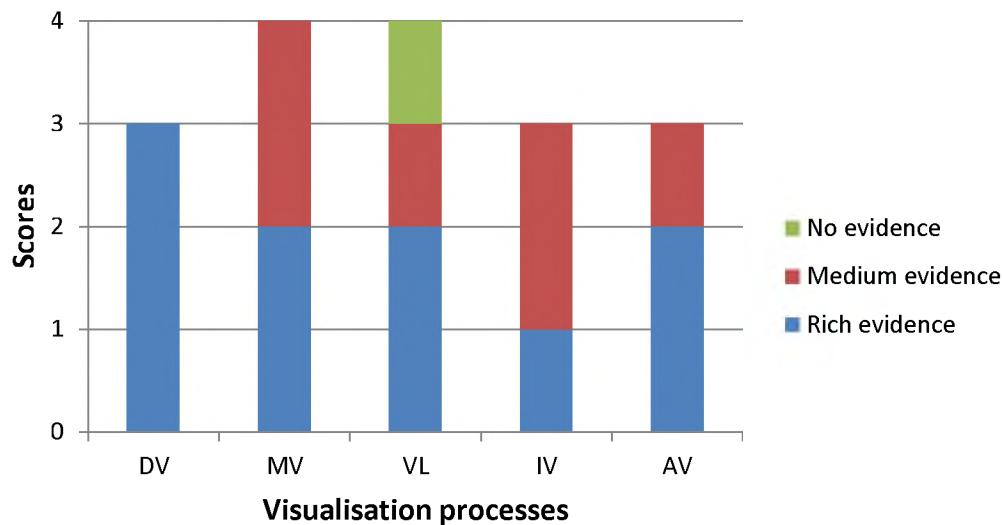


Figure 4.13 Teacher 2, lesson 3 visualisation processes, numerical scores

The discussion below outlines how each of these processes was used in lesson 3.

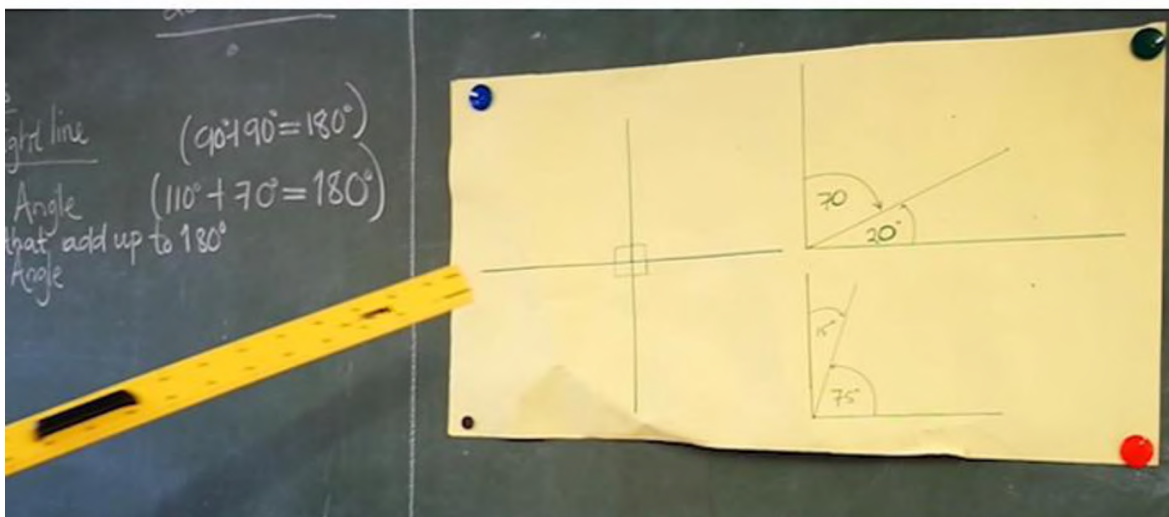
**Designing visuals:** In this lesson, the teacher used two different posters, a chalkboard ruler, and a protractor. The information displayed on the posters was in line with the basic competencies of that lesson as stipulated in the Mathematics grade 8 syllabus.

**Manipulating visuals:** To explain the two concepts of supplementary and complementary angles formed on straight lines, the teacher used a poster on which he drew two examples. He informed the learners that in example 1, one of the angles, measured with a protractor; measured  $70^\circ$  and therefore the remaining angle should equal  $110^\circ$ , to give a total of  $180^\circ$ .

**Visual language:** Teacher 2 used different representations in teaching the concepts. He used his voice, demonstration on the poster, and writing down on the board. He demonstrated good use of mathematical language e.g. when he drew an example of angles formed on a straight line, he used words like this is “not to scale” and explained to the learners that *not to scale* means that the angles were just randomly drawn without measuring with a protractor.

**Interpreting visuals:** The example on the poster was also shown on the board by the teacher, who emphasized that adding  $110^\circ$  and  $70^\circ$  to reach  $180^\circ$  would occur in the case of supplementary angles; supplementary angles could be defined as two adjacent angles that add up to  $180^\circ$ . The teacher also used the grade 8 mathematics text book to further define supplementary angles. Another example on the poster that the teacher used was  $90^\circ + 90^\circ$  to show that two right angles can also be supplementary angles that add up to  $180^\circ$ .

Teacher 2 instructed one learner to remove the first poster from the board and paste on the second poster. The second poster had a drawing divided into four quadrants, each forming  $90^\circ$ . He used the poster to introduce the concept of complementary angles. Figure 4.14 below shows an example of complementary angles.



*Figure 4.14 Teacher 2 showing examples of complementary angles on a poster*

The concept of perpendicular lines also emerged in this lesson, while Teacher 2 was explaining complementary angles. He explained what perpendicular lines are, while referring learners to

what they had already learned about types of lines. Using the poster above the teacher asked questions about the concept of complementary angles.

*T2L3: If there are two angles separated at  $90^\circ$ , for example  $20^\circ + 70^\circ$ , when you add them, what number will you get?*

*Ls: 90*

*T2L3: You are going to get?*

*Ls: 90*

*T2L3: thank you very much,  $70^\circ + 20^\circ$  gives us  $90^\circ$ .*

Teacher 2 then stated that when two angles add up to  $90^\circ$ , these angles are said to be complementary angles. The teacher wrote the definitions on the board and emphasized through examples on the board as well as the chalkboard, the difference between supplementary and straight angles, equally the difference between complementary and right angles. Teacher 2 further explained about angles at a point and angles formed at intersecting lines by giving examples on the board as illustrated in Figure 4.15 below.



*Figure 4.15 Teacher 2 showing an example of an angle at a point and angles formed at intersecting lines*

**Analysing visual:** The teacher concluded this lesson by going through each of the concepts discussed to ensure that the learners had grasped what was discussed. The teacher used and

communicated with a number of visuals in this lesson which makes it very interesting because the children had an opportunity to see as they interacted with the geometrical concepts.

#### **4.3.6.3 Stimulus recall interview**

During the stimulus recall interview that I had with Teacher 2, he clearly stated that in most cases learners get confused between straight angles and supplementary angles. They normally assume these two are the same.

***Researcher:** What geometrical concept were you bringing across when you differentiated between complementary, supplementary, right angle and straight-line angles?*

***T2SRI3:** There are two things that were in my mind, naming of angles, we name them that's a right angle, then there is a straight angle and then there is ah, all these angles and then there is also part of what we say angle properties, so these are two different concepts, but the difference is too thin. When you say complementary, two angles add up to 90 and a right angle is also 90°, so that confusion must be ironed out, that distinction although it's very little but it must be ironed out. So, the conceptual understanding is that when one says this, one does not mean the other one. When we say supplementary we do not mean straight angle. Straight angle is the whole name which is naming of angles but when we say supplementary, its angle properties, which means there are two angles which add up to 180°. There could be also angles that really are not looking straight but when you add them they give you 180°.*

Teacher 2 felt that in order to explain the distinction between types of angles and angle properties, he needed to use the poster and the chalkboard to clearly demonstrate the difference to the learners.

#### **4.3.6.4 Focus group interview**

Teacher 2 stated that the last lesson was good for him because the learners were already familiar with some concepts like an angle, a line and rotation, and they knew most of the objects used in geometry such as a chalkboard, a protractor and a ruler, and they knew how to use these objects. On the question of how different or similar his three lessons were in terms of visualisation, the teacher had this to say:

*T2FGI3: ..... already prepared. But then this third lesson, there were some visuals that I had to draw in front of the learners. So that is the difference, that the other one I brought, but this one I have to draw that, okay? Two lines are crossing, now this angle and this, they are vertical opposite and they are equal. So, for me the visuals were a bit different. As in the sense that, er, when I was talking about the first and second lesson, um, there were more of pre-prepared teaching aid. Which means I, the teaching aid come to the class and then we said and now, if I draw a one angle which is ninety, if I divide that angle somehow, between two, two parts, so I have to say this and that are together. To give me ninety, that is complementary. So that was now done all in front of the learners. So that was the difference in visual.*

I was also interested in knowing how the visuals that were used helped the learners to develop conceptual understanding of geometry. Teacher 2 maintained that the visuals helped the learners to first capture the concepts, apply the concepts and fully understand each concept that they were learning.

Teacher 2 stressed that apart from what was done directly during the intervention programme, there was a need to expose children to the real world. For example, when they just walk outside the classroom they should be able to see that there are lines that form shapes e.g. a stop sign. Alternatively, the teacher can bring a video with pre-prepared teaching materials and let learners watch something related to geometry. Equally we can expose the learners to the internet or teachers' sourcing materials from the internet can also be helpful. Teacher 2 concluded by saying that mathematics teachers should be encouraged to use visualisation in their teaching because there is no way learners will learn in an abstract fashion about parallel lines or angles without them seeing it in practice.

#### **4.3.7 Lesson 1, teacher 3**

##### **4.3.7.1 Brief description of lesson**

The main focus of this lesson was to measure different lines and angles correctly. The teacher first greeted the learners and started to write down some notes on the chalkboard, which the learners were copying into their exercise books.

##### **4.3.7.2 Visualisation processes**

Figure 4.16 below is the summary of the scores for each of the five visualisation processes for Teacher 3 lesson 1 as per the observable indicators. The diagram indicates that there was not

much evidence of visual processes being used in this lesson. However, there is medium evidence that shows that under the visual processes of manipulating and interpreting visuals, the teacher brought a limited number of visual materials to the classroom.

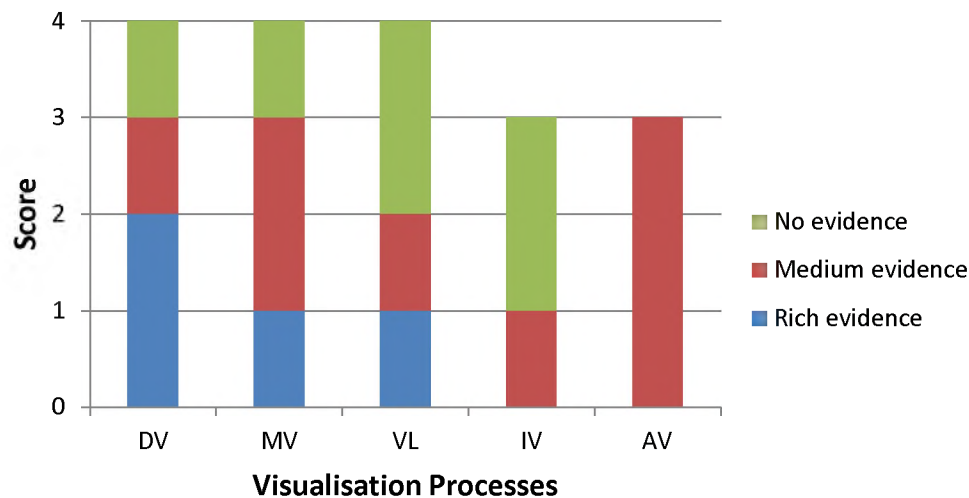


Figure 4.16 Teacher 3, lesson 1 visualisation processes, numerical scores

Below is a discussion of how each of the visualisation processes were used in this lesson

**Designing visuals:** He brought three visual objects: a chalkboard ruler, a protractor and a compass. Teacher 2 also made sure that each of his learners was in possession of geometrical materials used to draw lines and measure angles.

**Manipulating and interpreting visuals:** The teacher demonstrated how to draw a line using a compass. He did this on the board for all the learners to see clearly what he was doing. In order to measure different angles, the teacher used a chalkboard protractor to demonstrate all the steps. While drawing on the board, he also instructed the learners to draw the angles in their books. To join the arms of an angle, the teacher used a ruler on the board.

#### 4.3.7.3 Stimulus recall interview

After Teacher 3 taught his first lesson we came together to analyse his lesson, looking at how he used the visuals in the lesson. Teacher 3 mentioned in one of his responses to me that it was

important for him to first show the learners the objects that they were going to use in the lesson, hence he ensured that every learner was in possession of a mathematical set which contained the materials needed to draw lines and measure angles. He further said that after establishing that learners had the materials, he gave them notes which would be helpful to them at a later stage, especially when they were going to do an activity. He maintained that the use of visual objects such as a ruler, compass and protractor, which he used in his lesson, was motivated by the curriculum requirements that require teachers to use those materials when drawing and measuring angles.

#### **4.3.7.4 Focus group interview**

Teacher 3 did not have a comprehensive understanding of the concept of visualisation. He understood it as using a teaching aid in the class. However, he stated that using materials to teach the learners was much better than just teaching them verbally. The teacher indicated that his first lesson was interesting because many learners had an opportunity to know how a compass and a protractor looked. He stated that teaching a topic like geometry requires the teacher to use the objects to help learners to understand what he is teaching.

#### **4.3.8 Lesson 2, teacher 3**

##### **4.3.8.1 Brief description of lesson**

Teacher 3 in this lesson looked at angle properties focusing on calculating unknown angles by applying the following angle property: angles formed within parallel lines. This section analyses how Teacher 3 presented this lesson. After writing down the topic on the board he gave learners some notes on the board which took him about 8 minutes to finish. He instructed his learners to close their books and listen to his explanations.

##### **4.3.8.2 Visualisation processes**

The diagram in Figure 4.17 below shows the scores of each visualisation processes as outlined in the observation instrument. The diagram indicates that insufficient visual materials were used in this lesson.

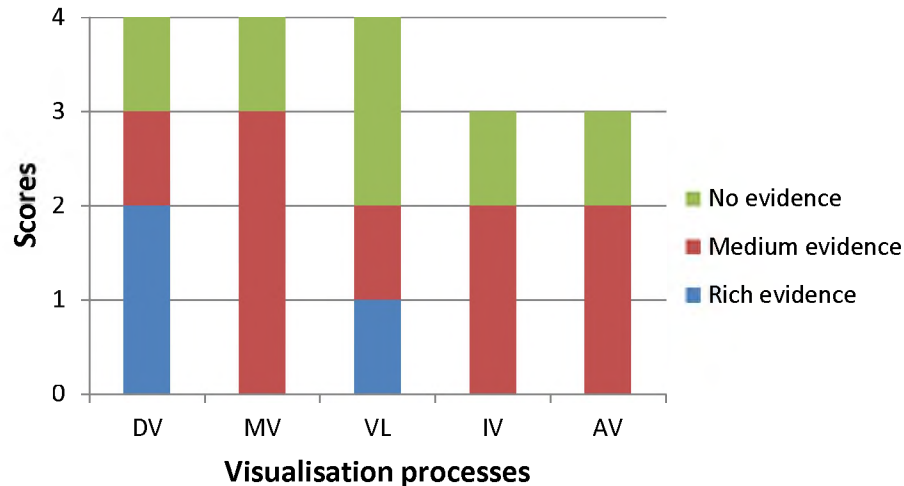


Figure 4.17 Teacher 3, lesson 2 visualisation processes, numerical scores

Below is a description of how the teacher demonstrated the use of the visualisation processes.

**Designing visuals:** Teacher 3 informed the learners that in this lesson they were going to look at alternate, corresponding and co-interior angles. He drew each of those angles on the board and stated their properties. Teacher 3 brought to class a piece of wire which he used as a visualisation tool to demonstrate examples of different angles formed within parallel lines.

**Manipulating visuals:** The piece of wire was used to show how angles in parallel lines form alternate angles, and to show examples of corresponding and co-interior angles. The teacher created a Z (alternate angle), an F (corresponding angle) and a U (co-interior angle). Learners had an opportunity to visualise this process because the wire was displayed on the board. Teacher 3 continued to explain about vertically opposite angles and again used the piece of wire to demonstrate that vertically opposite angles are equal.

**Analysing visuals:** Concepts taught in this lesson were presented one by one in a logical manner. The prepared visual material were used by the teacher to pose some verbal questions as he constantly lifted it up to show the learners and ask them questions. Activity given to learners was in line with the visual material brought to class.

#### **4.3.8.3 Stimulus recall interview**

The main aim of this interview was for me to understand fully how the teacher used the visuals in his class and to allow the teacher to reflect on his own teaching. The teacher stated that he prefers giving some examples on the board before explaining anything because he believes that examples help learners to be able to do the activity at the end of the lesson correctly. Teacher 3 said that it was important for him to bring to the class a piece of wire which he used to explain about angles formed on parallel lines; because the wire is visible and flexible the learners can use it to form both alternate, corresponding and co-interior angles.

#### **4.3.8.4 Focus group interview**

The teacher indicated that in the first lesson when he taught about constructing angles, learners were struggling. Most of them did not even know how to hold a chalkboard ruler or how to use their protractors but in the second lesson I could see they were at least able to work with the materials. The teacher stated that the wire which he used was a good visual object and it really helped him to explain more easily the three concepts to the learners. He stressed that he also used his hands, crossing them to demonstrate how angles are formed. Teacher 3 asserted that preparation was very important before teaching the learners.

#### **4.3.9 Lesson 3, teacher 3**

##### **4.3.9.1 Brief description of lesson**

The following section is an analysis of the third lesson which was also the last intervention lesson for Teacher 3. The teacher was initially going to focus on angles at a point, which is the topic that he wrote on the board. However, the notes that the teacher gave were about complementary and supplementary angles.

##### **4.3.9.2 Visualisation processes**

Figure 4.18 below shows a summary of the scores of each visualisation process as used in this lesson. The scores were rated according to the coding criteria.

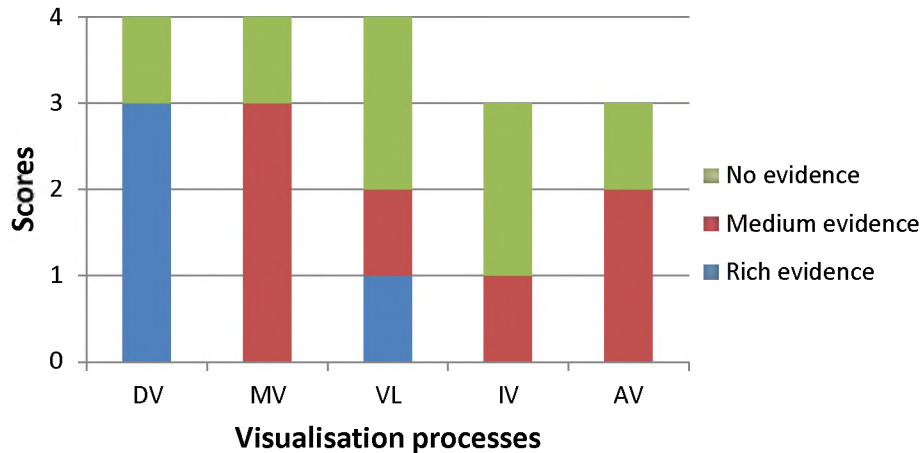


Figure 4.18 Teacher 3, lesson 3 visualisation Processes, numerical scores

Below is a description of how the teacher demonstrated the use of the visualisation processes.

**Designing visuals:** The teacher explained on the board the difference between complementary and supplementary angles. The properties of each type of angle were clearly drawn on the board. Teacher 3 used a piece of wire as a visualisation tool to teach the concepts. He designed the wire in such a way that it could form a complementary angle or a supplementary angle.

**Manipulating visuals:** The teacher gave examples of how to calculate the unknown angle in a pair of complementary angles, e.g.  $x + 55^\circ = 90^\circ$ , therefore  $x = 35^\circ$ . And in supplementary angles, he used the following example:  $60^\circ + a = 180^\circ$ , therefore  $a = 120^\circ$ , because two angles on a straight line form a pair of supplementary angles which add up to  $180^\circ$ .

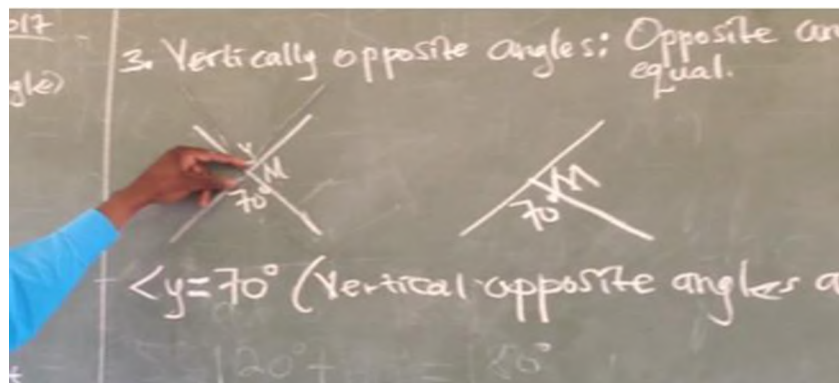


Figure 4.19 Teacher 3 demonstrating how to make vertically opposite angles using a piece of wire

Teacher 3 also touched on vertically opposite angles by drawing an example on the board and defining vertically opposite angles as equal. Figure 4.19 shows vertically opposite angles as drawn by the teacher on the board.

**Interpreting visuals:** Teacher 3 demonstrated on the board using a wire as illustrated in Figure 4.19 above, to show that vertically opposite angles are equal; at the same time he wrote the properties of vertically opposite angles on the board for learners to copy down. Teacher 3 also used his hands to teach the concept. He crossed his two hands and told the learners that an example of vertically opposite angles can be related to the way he crossed his hands to form opposite angles. He repeated teaching this concept with a number of examples and posing verbal questions that kept the learners engaged during the entire lesson.

**Visual language:** The visual material that the teacher used in this lesson aligns well with what the teacher was teaching, and the basic competencies as outlined in the mathematics syllabus.

#### 4.3.9.3 Stimulus recall interview

Teacher 3 indicated that he used the visual materials for his learners to see exactly what he was doing. According to the teacher, a piece of wire was a perfect material to use because it was flexible and he could make any type of angle from it.

#### 4.3.9.4 Focus group interview

Teacher 3 compared the three lessons that he had taught and pointed out some similarities and differences in the lessons:

*T3FGI3: to me, the only big difference was, first lesson I was just teaching them how to use the ruler and compass and so on. In the second lesson also, I was teaching them the three type of angles that are formed within parallel line. And I was using different materials. When it came to week three, I taught them about angles that are formed at a point. To talk about complementary angles, supplementary, I used the wire, different wires, and come to vertical. But can tell you that the learner, they were following. Even when I gave them activity, they were able to give the value of any given letter.*

Teacher 3 stated that the visualisation in his classroom helped the learners to see the reality of what he was teaching. Learners were able to answer questions properly and they were able to work with the materials. He added that learners needed to be given an opportunity through

projects to come up with their own materials rather than just teaching them and giving them topic tasks. Teacher 3 admitted that the intervention programme helped him to grow in his own teaching because previously he had not considered visual materials to be important. He then made a commitment that whenever he teaches geometry, he would ensure that visuals were used at all times.

#### 4.3.10 Lesson 1, teacher 4

##### 4.3.10.1 Brief description of lesson

This lesson was on construction with a specific focus on drawing and measuring lines and angles accurately. The teacher started the lesson by finding out what learners already know about lines and angles.

##### 4.3.10.2 Visualisation processes

Figure 4.20 below shows the scores of each visualisation process as outlined in the observation instrument. The diagram indicates that not many visual materials were used in this lesson.

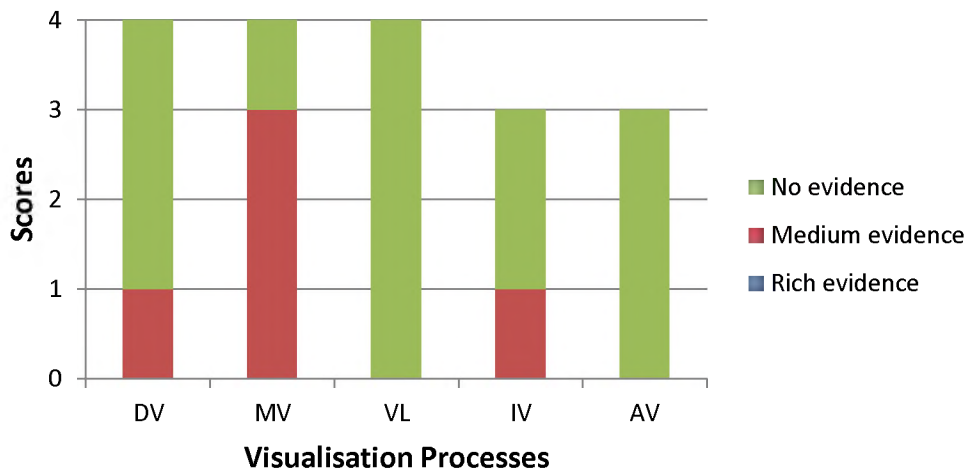


Figure 4.20 Teacher 4, lesson 1 Visualisation processes, numerical scores

**Designing visuals:** Other than the chalkboard ruler, the teacher did not bring to class any other visual materials that could be used to teach the concept of lines and angles.

The teacher started the lesson by writing down the topic on the board. She asked learners to recall and mention the types of lines that they had learned in grade 7. The learners mentioned the

following lines: straight lines, horizontal lines, and vertical lines. The teacher then informed the learners that in grade 8 the focus was not only on naming angles and lines but also on drawing and measuring lines.

**Manipulating and interpreting visuals:** She brought to class a chalkboard ruler which she used to draw and measure the lines. This was in contrast to teacher 3, who used a pair of compasses instead of a ruler to draw lines. After drawing the line on the board, she asked one learner to come to the board and measure the line using a ruler. The teacher then gave an activity to the learners; the activity was strictly on measuring and drawing lines. After teaching about lines, she drew an example of an angle on the board and asked a learner to come to the board and measure it. The first learner did not measure it correctly, while the second learner measured correctly but could not read the value of the angle correctly on the chalkboard protractor. After the learners' attempts to measure and read angles correctly, the teacher took over and explained to the learners how to measure angles and how to read the value of an angle on a protractor.

There was no evidence of the use of **visual language** and **analysing visuals** in this lesson; however, the teacher presented the concepts in an orderly manner that allowed learners to conceptualize the concepts logically.

#### **4.3.10.3 Stimulus recall interview**

After teaching her first intervention lesson, we decided to reflect as planned on the lesson that she had taught, viewing the video together. Teacher 4 stated that she first asked learners what they already knew about lines and angles because she was interested in building on their existing knowledge from previous grades. She added that sometimes it was important to allow learners to realize their own mistakes and eventually help them to correct those mistakes and she did this by inviting learners to the board and asking an individual learner to work out an example on the board. Teacher 4 emphasized the use of correct units when measuring lines. She stated that the main focus of the lesson was for learners to be able to measure lines that are already drawn. It is therefore important for learners to have rulers and use them to measure different lines. The teacher added that using the body expression to demonstrate how an angle is formed is crucial because students are able to visualise how the angle is formed. She maintained that the visual objects that she used made her lesson more practical.

#### **4.3.10.4 Focus group interview**

Teacher 4 stated that when the visualisation concept was first introduced to them she understood it as a way in which teachers allowed learners to see through the lessons. She said learners should be able to experience that which they learn not only through hearing. The teacher indicated that the first lesson was very exciting for the learners because they were able to follow her demonstration working with the objects despite the fact that it was a little bit uncomfortable because she had to spend the whole period in front of the chalkboard, something she is not used to. Teacher 4 added that using visualisation in the classroom was very interesting because learners were able to experience the activities instead of just memorizing the concepts. She said the use of visual materials made it easier for learners to understand the concepts because they were able to see what was going on and they could relate the visuals to the concepts being taught. Teacher 4 maintained that there is no way you can teach geometry theoretically, therefore geometry needs to be taught using visual materials.

#### **4.3.11 Lesson 2, teacher 4**

##### **4.3.11.1 Brief description of lesson**

The following section is an analysis of Teacher 4 teaching her second lesson, which focused on identifying different angles such as acute, obtuse, right, straight and reflex angles. The teacher introduced the lesson by asking learners to recall what they had discussed in the previous lesson, to which one learner responded that the topic was on measuring lines and angles. She again asked the learners to mention the name of the instrument that they used to measure angles. The learners responded correctly by saying that it was a protractor. The teacher informed the learners that the measuring angles was only the first part of angles, and that in the current lesson they were going to look at types of angles. The teacher first wanted to find out what learners already know from grade 7 about types of angles.

*T4L1: What are the types of angles that you have learned in grade 7?*

*Ls: Obtuse angle, acute angle, straight angle, right angle, revolution angle, reflex angle.*

*T4L1: Did we omit any angle, any which we did not mention? So those are the types of angles that we need to go and study today or which we are going to look at today.*

### 4.3.11.2 Visualisation processes

The diagram in Figure 4.21 below shows the scores of each visualisation processes as outlined in the analytical instrument.

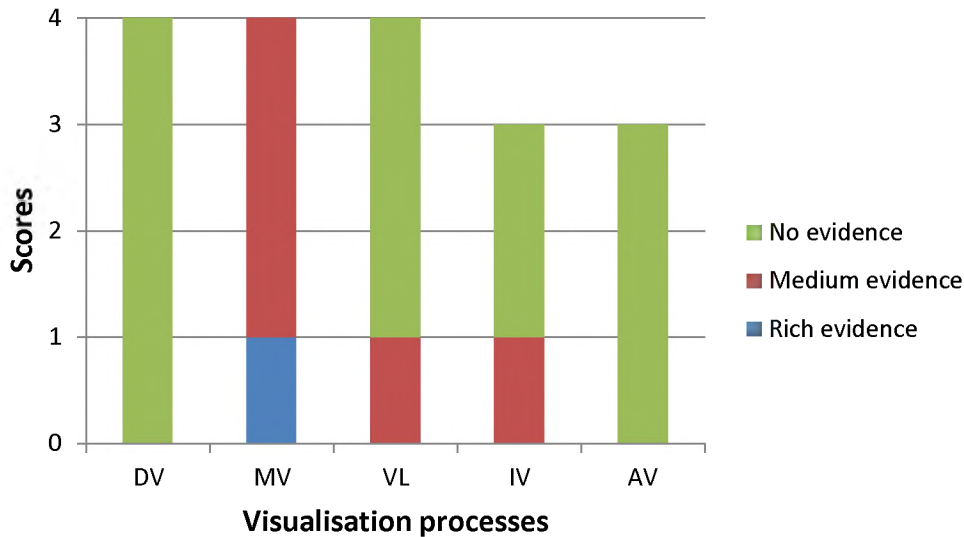


Figure 4.21 Teacher 4 Lesson 2 Visualisation processes, numerical scores

**Designing visuals:** Teacher 4 then wrote down the topic on the board and three types of angles viz. acute, obtuse and right angle. Teacher 4 asked the learners to define what an angle is. The learners indicated at first that they were struggling to answer the question. She then used her body expression to demonstrate that an angle is the amount of turn or rotation made by something. Learners were asked to use parts of their bodies to demonstrate examples of angles.

**Manipulating visuals:** The teacher used a various examples from within the classroom that form angles, e.g. the corner of the classroom building, the window frames and the door. She used her arm to show the difference between angles. She did this by stretching her arm to indicate that the more she stretches her arm the bigger the space between her arms is getting. Examples of acute, obtuse, right and straight angle were clearly shown using the arm.

**Visual language:** The teacher communicated the concepts in a way that did not confuse learners, for example the use of her body to teach concepts allowed the learners have an idea of what was expected from them.

**Analysing and interpreting visuals:** Teacher 4 then defined an acute angle as any angle that is between  $1^\circ$  and  $89^\circ$ , right angles as angles that are equal to  $90^\circ$ , and obtuse angles as being between  $91^\circ$  and  $179^\circ$ . The teacher spent sufficient time explaining each of the concepts by asking continuous questions and going through each of the examples one by one.

Teacher 4 then gave an individual activity based on the content that she had taught in that lesson. This lesson may not have had much visually designed material, however the teacher did use adequate examples of visual approach in her teaching using her body expression and some of the materials available in the class; this was crucial in developing the learners' conceptual understanding of geometry.

#### **4.3.11.3 Stimulus recall interview**

Similar to what I did in her first lesson, Teacher 4 and I met and reflected on her lesson for the second week. In our interactions the teacher highlighted that she normally asks learners first what they already know about the topic and as they are giving the answers she writes them on the board. This is because she wants all learners to take notes including those that cannot recall what they had learned in previous grades.

She added that she created an environment where learners could speak freely about what they already know concerning angles. Teacher 4 indicated that she used her body expression to demonstrate the formation of angles because she knows that turning around and body expression are something that learners are used to and they do it all the time without realising how important it is. The teacher felt that using the materials which are confined within the classroom walls e.g. doors, window frames, etc. was equally crucial in helping learners work with available materials to learn different geometrical concepts about angles.

#### **4.3.11.4 Focus group interview**

Teacher 4 stated that time was the biggest challenge for her since she could not finish all that she had prepared within a limited period of time. She stressed that using her body expression to demonstrate formation of angles helped her learners to easily acquire the concepts. She added that visualisation, especially in geometry should not just be focused on in the classroom.

**T4FGI2:** *We should go or relate it to the outside environment. For an example if I'm teaching a ruler or I'm teaching a kid about a ruler, in the classroom measuring lines, measuring angles. Maybe it's best also to exploit with the outside environment in a sense that where can learners see angles outside the classroom? Not only the classroom. Giving the projects, for instance, where they are able to go outside and relate to what they're learning in the classroom with the outside environment. It also helps a lot. It should not be something that is within the classroom and be left there. So, for instance you can give them a project go and paste from newspapers from where, go find where can angles be formed in the outside environment? Or buildings go and ask people who are constructing houses where do they use for instance angles. What, what type of material do they use? It also brings a lot in their mind.*

The following part is an analysis of how each of the four teachers taught their third lesson. Not all of them taught the same basic competencies but they all showed a good connection from their previous lessons. In this analysis I have discussed them one by one with a special focus on areas of their lessons where visualisation processes emerged.

#### 4.3.12 lesson 3, teacher 4

##### 4.3.12.1 Brief description of lesson

This lesson was on angles formed within parallel lines. The lesson is a continuation of the previous day's lesson which could not be completed. The teacher welcomed the learners to the lesson by greeting them first.

##### 4.3.12.2 Visualisation processes

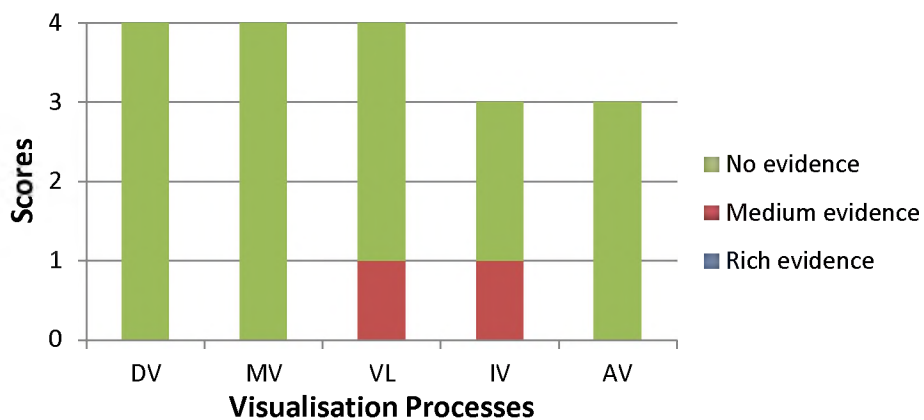


Figure 4.22 Teacher 4, Visualisation processes, numerical scores

Figure 4.22 above shows the scores of each visualisation process as outlined in the observation instrument.

**Designing visuals:** Teacher 4 started the lesson by writing down the topic on the board, which was angles formed in parallel lines. She defined parallel lines as lines that do not meet even though they have the same length and face in the same direction. **Manipulating visuals:** Teacher 4 also used an example of the roof of her class to supplement her explanation of parallel lines. She then drew two examples of parallel lines on the board. In explanation of angles that formed on a straight line, she wrote down a capital letter Z on the board and explained to the learners that angles that form a capital Z are called alternate angles and are equal. The teacher gave a number of examples of alternate angles and co-interior angles on the board. She interacted with the learners by asking them questions to which they responded.

This lesson lasted less than 30 minutes because it was a continuation of the previous lesson. There was not sufficient evidence of the use of **visual language**, **Interpreting** and **analysing visuals** in this lesson. However, she explained the concepts in an easy and logical way in order to develop learners' conceptual understanding of the concepts of alternate and co-interior angles.

#### **4.3.12.3 Stimulus recall interview**

Since this lesson was not a complete lesson, the teacher indicated to me that she had already used most of the visuals in her previous lesson and that she was confident that the learners understood the concepts that she was teaching them.

#### **4.3.12.4 Focus group interview**

Teacher 4 shared her experience throughout the intervention programme, with a specific focus on the three lessons. I wanted to know how different or similar were the three lessons that she had taught:

***T4FGI3:** to me the difference was that the last week, learners, you could see that the pace of doing the work was faster than the first two weeks. In the sense that I remember when even the first day when I introduced the angles formed in parallel lines. It was a little, you can see that the understanding is faster. Because they already knew what parallel lines are from the beginning and then they already understand what an angle is. Even when you bring the term such as the alternate angles, the opposite angles, angles*

*that are formed on parallel lines, they were already able to understand. It was easy for them to adapt it because they were already into the process that in the process of angles. They already understood a bit about angles. So, the lesson was a little bit faster. You could see the difference between the first two and the last one.*

Teacher 4 stressed that it was important for her to first make sure that the learners know the materials used in geometry and how to use those materials. The teacher emphasized that learners must be given activities that could enhance their creativity in geometry. For example, learners can be put into groups and given a project to construct their own types of angles using materials that are easily accessible to them. Teacher 4 admitted that the intervention programme was interesting for her because at first, she didn't understand the concept of visualisation but involvement in this programme had helped her to better understand the concept.

#### **4.4 SIMILARITIES AND DIFFERENCES IN THE WAY VISUALISATION PROCESSES WERE USED BY THE PARTICIPANT TEACHERS**

In this section, I present a summary of the analysis, focusing on the similarities and differences that I had observed among the four teachers that participated in the intervention programme. This summary aligns with the two research questions of this study:

- What are selected teachers' perceptions of the roles visualisation processes can play in the teaching of geometry to develop conceptual understanding as a result of an intervention programme?
- How can visualisation processes be used in teaching geometry to enhance conceptual understanding?

##### **4.4.1 Visualisation processes**

I will discuss this analysis looking at how each of the four visualisation processes unfolded during the intervention programme.

###### **4.4.1.1 Designing visuals**

Under this visual process, teachers were expected to bring to class visual materials that aligned with the subject content, introduce the lesson using visual materials and demonstrate the use of visual elements such as lines and angles. Debes (1969) argued that visual designs are

fundamentally important for the production of effective visual instructional materials. Teacher 1 and Teacher 2 brought into the classroom visual materials that were well connected to the content as prescribed in the Mathematics syllabus for grade 8. The two teachers demonstrated an extensive use of visual materials. The materials brought to class were designed by the teachers themselves. The teachers introduced both of their lessons while interacting with the visuals. They first considered finding out learners' prior knowledge by engaging learners in discussion through verbal questioning and activities.

Teachers 3 and 4 did not engage extensively with the use of visuals especially in their last lessons. They mostly used a chalkboard protractor and chalkboard ruler as their visual objects. However, Teacher 3 brought to class some materials, although they were not sufficient to teach all the concepts. All the four participant teachers demonstrated a high level of competency working with the materials that they brought to the classroom.

#### **4.4.1.2 Manipulating visuals**

Rakes (1999) asserts that teachers need to be able to think critically about and manipulate visual information to help learners make a solid interpretation of its meaning. Rakes (1999) further urged that teachers to manipulate different representations and concepts in order to assimilate them into learners' existing knowledge and thus make sense of the geometrical idea. Under this visualisation process (manipulating visuals), participant teachers were expected to use different representations in the lesson, demonstrate proper use of geometric concepts, create an opportunity for learners to connect new concepts to what they already know, and use visuals in an orderly and logical way to avoid confusing learners.

All four teachers used a variety of representations to present their lessons. Teachers 1 and 2 mainly used posters, the chalkboard and a number of self-made visual materials. They delivered the new concepts by first establishing what learners already know about the topic. They both started their lessons by asking verbal questions about the topics, to which the learners responded well. Teacher 3 on the other hand used the chalkboard more frequently. He had a strategy of first giving the learners notes before explaining the concepts and always giving the learners an activity at the end of each lesson.

Teacher 4 used different representations in her lessons. What was unique about Teacher 4 was the way she used her body expression most of the time to explain different concepts that she was teaching, for example when she demonstrated how an angle is formed. The teacher also made reference to materials found in the boundary of her classroom e.g. the door, tiles, window frame and the roof as examples to support some of her explanations. The four participant teachers presented all their lessons in a logical way. There was no sign that the learners were confused, or were not following properly because of the teachers mixing the materials. Each visual material brought to class was used for its intended purpose. Teacher 4 for instance taught a concept of lines and after teaching she gave learners an activity based on the concept of a line before she proceeded to the next concept, which were angles.

#### **4.4.1.3 Visual language**

Debes (1969) states that a visually literate person should be able to read and write visual language, i.e. he or she should be able to decode and encode visual messages appropriately. In order to fulfill the expectation of the following visual process (Visual language) teachers were expected to use techniques which promote the production of visuals to demonstrate learning. Participant teachers were also expected to use visuals in conjunction with the verbal explanation, respond to questions asked by the learners, and use visual language that is consistent with mathematical language and terminologies.

All participant teachers used language and terminologies appropriate to the level of grade 8 learners. There was no use of confusing words. In some lessons, teachers used mathematical language e.g. “not to scale”. The visuals used were in line with the language used. All teachers demonstrated an acceptable competency in reading and writing visual language in their lessons. I observed that none of the learners in the classrooms of the four teachers asked their teachers any questions that the teachers could respond to. Teachers were the only ones that asked questions both verbally and written on the board as a classroom exercise.

#### **4.4.1.4 Interpreting visuals**

Avgerinou (2001) asserts that interpreting visuals includes applying critical thinking skills to visuals. Felten (2008) adds that “when teachers train learners well and allow them to practice visual skills, they can develop the ability to interpret and employ diverse relationships of

different visual forms effectively” (p.60). Under interpreting visuals, teachers should be able to properly identify and evaluate visual materials, ask questions that can prompt critical thinking and critical reasoning among learners, and conclude the lessons with the use of visual materials. All teachers asked questions that required the learners to think and reason critically. Teacher 3 for instance asked learners to give reasons to support their answers when he taught about angles formed on parallel lines. Teachers 2 and 4 continuously asked questions throughout their lessons, creating a conversation with the learners. Teacher 2 used the visuals that he had brought to the class to conclude his second lesson. However, the other three teachers concluded both their lessons by only going through each concept one by one without using the visual materials.

#### **4.4.1.5 Analysing visuals**

In order for teachers to ensure that learners gain a deep understanding of mathematical objects, they need to analyze them by interpreting them in different ways that make sense to them. Under this visual process (analysing visuals), participant teachers were expected to communicate to learners using the visual materials, use visuals that could deconstruct properties of shapes, and encourage learners to work with the visuals. Teachers 1 and 2 managed to communicate with the learners while incorporating visuals into their lessons. For example, the teachers asked questions while holding the visual materials or pasting them on the board. Teachers 3 and 4 did not engage the learners deeply with the use of visual materials; they taught more in verbal communication than visual communication.

#### **4.4.2 General observation**

All participant teachers were focused on teaching the concepts and creating an opportunity where learners were free to share what they already know. The teachers used learners’ prior knowledge to introduce the learners to new concepts. All participant teachers taught three lessons each, although some of the lessons were not completed. This indicates that teachers were able to teach all the basic competencies that they had prepared in line with the new mathematics syllabus for grade 8. The majority of topics taught by the teachers were constructions and angle properties.

### **4.4.3 Idiosyncrasies**

The intervention programme of this research study focused mainly on finding out how participant teachers used the visualisation processes in their classrooms to teach for conceptual understanding of geometry. However, as teachers presented their lessons there emerged from the presentations something which is unique and I feel it is worth reporting on it in this analysis.

The first thing that attracted my attention is the way Teacher 4 used different gestures in her lessons. The way she used her arms, legs, head and elbows to demonstrate types of angles was amazing. Learners' attention was captured, and I could tell from their excitement that they were grasping the concepts very well. Teacher 4 may not have brought tangible visual materials into her classroom, but the way she related her examples to the materials found within the boundary of her classroom was notable. Another important observation which I found impressive was the way Teachers 1 and 2 made reference to the outside environment when they presented their lessons. When teaching about a ray, these teachers used an example of a torch to define the concept of a ray. They were convinced that learners had an idea of how a torch looks and how it works and that they could easily relate to it.

### **4.4.4 Synthesis of the visualisation objects used during the intervention programme**

The four participant teachers used a variety of visual materials throughout the lessons that each of them taught. These materials included: chalkboard ruler, protractor, chalkboard compass, mathematical sets for learners, posters, an arrow, a point, a line, a line segment (designed from cardboard materials), wires, a reed, and a rope. The teachers also used materials available in the classrooms such as: window and door frames, the roof, tiles and the door. Teachers also used their body expressions to define certain concepts.

## **4.5 TEACHERS' PERCEPTIONS ABOUT THE USE OF VISUALISATION DURING THE INTERVENTION PROGRAMME**

The findings of my study revealed that respondents held a common opinion that the use of visual processes in their teaching had a positive contribution to the way they taught geometry throughout the intervention programme. They referred to how each of the materials played an important role in developing learners' conceptual understanding of geometry. The teachers stated

that the use of the right material in a right way helped them to work well with the learners. The participant teachers also stated that reflecting on their own teaching practice assisted them to plan well while considering how learners acquire concepts. The opportunity for self-reflection through stimulated recall interviews helped them to have a better understanding of their own teaching while working on improving what they did not do well and maintaining that which went well.

The teachers also singled out the willingness of learners to co-operate during lessons, which made it possible for them to communicate well with their teachers and fellow learners. Furthermore, all teachers had a general understanding of the concept of visualisation, which made it possible for them to select the right visual materials to use in their specific classroom. Visual materials that were used by the teachers were environmental friendly and had no harmful effects on either the teachers or the learners. The videos that were recorded during lessons were appealing and clear, thus making it easier for me to go through them and develop questions for the interview based on the visualisation processes that emerged. Recording the videos also made it easier for the completion of the observation instruments.

All four teachers demonstrated a good understanding of the topic of geometry. This was cardinal because geometry is a mathematical domain at the heart of this study. The teachers also indicated that the opportunity that was created for them to meet first and discuss in a group prior to the intervention programme was very good because it gave them a platform from which to prepare their lessons together and learn from one another. During the awareness workshop, teachers had an opportunity to select appropriate visual materials that were suitable for grade 8 learners.

However, teachers equally indicated that they faced difficulties during the intervention programme. All participant teachers mentioned that on numerous occasions they ran out of time. They indicated that in most cases they were not able to deliver the lesson as planned due to time constraints. One of the four teachers also mentioned the issue of an overcrowded classroom, which made it difficult for him to communicate to all learners using the visual materials.

#### **4.6 CONCLUSION**

This chapter outlined the findings and presentation of data, with reference to the two research questions of this study. The first part of this chapter outlined and presented how each of the 12

lessons was conducted by the four participant teachers, and the last part of this chapter analysed the similarities and differences in the lesson presentations, and the views of the participant teachers regarding the use of visualisation processes in teaching geometry for conceptual understanding. The data that emerged from the lesson observations was based on the five visualisation processes (as per analytical framework) and the data that emerged from the stimulated recall and focus group interviews was based on teachers' self-reflections, experiences and perceptions.

Interestingly, the participant teachers acknowledged that the use of visual materials to teach for conceptual understanding was useful in their classrooms because learners had an opportunity to interact with the visuals and in the process, develop their geometrical concepts. In addition, the teachers advised that the use of visual materials to teach geometry should be the central focus of every mathematics teacher, because geometry by its nature requires learners to visualise what they are learning.

## CHAPTER FIVE

### CONCLUSION AND RECOMMENDATIONS

#### 5.1 INTRODUCTION

The objectives of this study were to analyse the use of visualisation processes by teachers who participated in an intervention programme to teach for conceptual understanding of geometry. In this chapter I present a summary of the findings, outline the significance of the study and provide recommendations to scholars of mathematics education and policy-makers. This chapter also presents the limitations of the study, suggestions for further research and my personal reflections as a result of this study.

#### 5.2 SUMMARY OF FINDINGS

During the data analysis process, a number of themes with special focus on visualisation processes emerged. Below are summaries of my findings relative to my two research questions:

##### Question 1

**What are selected teachers' perceptions of the roles visualisation processes can play in the teaching of geometry to develop conceptual understanding as through intervention programme?**

- The teachers stated that the use of visualisation processes made a positive contribution to the way geometry is taught to the grade 8 learners;
- The teachers indicated that geometry brings to the classroom an experience of the real world and presents a practical environment for learning mathematics;
- They stated that the use of visual materials helps to develop the way learners acquire geometrical concepts by providing them with an opportunity to physically interact with the materials and make concrete meaning of different objects;
- The teachers mentioned that the use of geometrical objects such as a chalkboard ruler, protractor and compass have a significant effect on the way students learn. They indicated that when learners are able to visualise how each geometrical object is used for construction, it helps them to capture and retain the concepts more easily;

- The teachers also indicated that visual materials help learners to think critically and assist them in problem solving;
- Teachers were of the opinion that learners should be given an opportunity to design their own visual materials from readily available resources in their surroundings;
- The demonstration of one of the teachers using posters and other designed materials helped learners to visualise and conceptualise the mathematical ideas;
- The teachers had a common view that when students were given an opportunity to interact with objects by touching, seeing or manipulating them, they could easily understand what they were being taught because they took ownership of their own learning instead of being given information by the teacher.

## **Question 2**

### **How can visualisation processes be used in teaching geometry to enhance conceptual understanding?**

- Visuals were used when introducing the lessons; teachers asked learners to recall what they already knew about the topic under discussion, while using visuals;
- Visuals were sometimes used simultaneously with verbal explanations, when the teacher used the materials while at the same time providing definitions and examples written down on the chalkboard;
- Visuals were sometimes used to pose verbal questions that students answered in reference to the materials;
- Visual materials were also used to provide real-life examples;
- Visuals were also used to develop conceptual understanding of geometry;
- Sometimes the materials were used at the end of the lesson to consolidate what had been taught and to review the main concepts of the lesson.

The teachers also indicated that using visual materials in their classrooms had some challenges. They said that the schools did not have sufficient geometrical objects to be used by both teachers and learners. The teachers said that using visual materials sometimes consumed a lot of time and as a result hampered their progress towards completing the syllabus. They also stated that

overcrowding of classrooms is a serious challenge, which makes it very difficult for them to work with materials.

### **5.3 SIGNIFICANCE OF THE STUDY**

In the context of the new Namibia education curriculum, this study has the potential to inform policy-makers and researchers of mathematics education with meaningful information and recommendations on integrating visualisation into the design of policy guides. This study helped me as a mathematics teacher to expand my own knowledge about geometry and about how to use effective instructional approaches to geometry. The study also contributed to my knowledge about visualisation in mathematics education.

It is anticipated that participating teachers will be empowered with various strategies involved in teaching geometry for conceptual understanding through visualisation. Furthermore, this research could help the Ministry of Higher Education and Training to incorporate visualisation into programs that involve teacher training institutions in order to prepare student teachers to embrace the visual world when they become teachers. My study has the potential to encourage authors of mathematics textbooks to include visualisation processes in their books.

### **RECOMMENDATIONS**

I am aware that the findings of this study cannot be generalized to the entire Namibia educational system; however, the results of this study could be used by teachers and educational personnel to improve the quality of mathematics education in our schools. I would therefore like to make the following recommendations to mathematics teachers, subject advisors and curriculum designers, textbook authors and lecturers at teacher training institutes:

- The circuits could identify key teachers who have proven to have successfully worked with visuals appropriately; these teachers could then be given further training and sent to schools to provide workshops on how to successfully incorporate the use of visualisation processes in the teaching of geometry for conceptual understanding;
- Mathematics teachers could encourage learners to design geometrical objects themselves, while guiding them and giving them instructions on how to use the available materials;

- Continuous professional development to equip teachers with necessary subject knowledge is fundamental to ensure that the teachers are at all times presenting appropriate information to learners in line with the curriculum;
- Subject advisors could call up regular ministerial workshops with mathematics teachers for the purpose of refreshing teachers' content knowledge and instructional approaches that encourage the use of visualisation;
- Teacher-training institutions could ensure that student teachers are well prepared in using visualisation to teach geometry. Student teachers could be well grounded and exposed to the use of visualisation processes especially during their practical training;
- Curriculum designers could consider incorporating and making visualisation compulsory in the teaching of mathematics. They could design the curriculum in such a way that it provides ample time for teachers to use visuals in their teaching time;
- It would also be appropriate for the authors of mathematics textbooks to include visualisation in their books by creating room for more diagrams and geometrical images that are interesting and easy for learners to analyse and make concrete meaning from.

## **LIMITATIONS**

This study had a small sample size. The findings of this study can thus not be generalized because the study only focused on four teachers and four schools. The study also only focused on one mathematical area which is geometry and only on some of the chapters in the grade 8 mathematics syllabus. The study only lasted four weeks; the time to collect sufficient data was thus limited. Ideally, I could have spent more time working with the teachers before going to observe them in their classrooms. Three of the classrooms where I observed the teachers were overcrowded, thus making it difficult for me to only capture the teachers' presentations without capturing learners.

Initially, I planned to have a workshop with all grade 8 mathematics teachers in the Rundu circuit to identify the four participant teachers. However, due to financial constraints I could not have the workshop as planned; instead I had to rapidly design a questionnaire to help me identify the teachers. Another problem that I encountered was finding time to interview the teachers after the lessons. Teachers had other commitments which made it difficult for me to carry out the

interviews as planned. Focus group interviews for example, were planned for the end of every week, but I could only have the interviews once at the end of the intervention programme, and one of the participant teachers did not partake in these interviews.

## **SUGGESTIONS FOR FURTHER RESEARCH**

Doing research in the area of visualisation in mathematics was very important for me as a researcher. However, since the study was a small-scale study, more research could be done in the same area. I suggest the following areas of research:

- Conduct research with more research sites and a bigger sample size for the findings to be generalized to a wider context;
- Conduct research which focuses on how students develop conceptual understanding without focusing solely on geometry;
- Conduct research which analyses geometry in the context of teacher' effective use of diagrams and images;
- Conduct research to find out how visualisation processes can be used to teach for conceptual understanding across all domains of mathematics.

## **PERSONAL REFLECTIONS**

It was with pleasure that I decided to take this journey. The opportunity to carry out this study is something I welcomed and was excited about. Doing this research gave me an opportunity to engage in a variety of reading and writing. The level of my writing has improved when compared with my first draft proposal. This journey taught me that in order to carry out educational research, one needs to devote much of one's time and energy to reading so as to understand the context in which you wish to do research.

Doing this research put me in a position where I was able to see the challenges and opportunities that teachers have in their daily interactions with the learners. Being an observer gave me a chance to look at mathematic teaching with a critical eye. Talking to teachers to hear their experiences is something I was grateful for, as I came to understand mathematics teaching from different perspectives. Finally, this research study prepared me well for future research on a doctoral level because it will be possible to expand the experience that I gained from this study. I

am also optimistic that I will be able to assist novice researchers to successfully complete their various research studies. I am grateful for this opportunity and the knowledge that I gained from this study will be shared with other educational stakeholders for the sake of the Namibian children.

## **CONCLUSION**

Using visualisation processes to teach geometry can be very effective in developing conceptual understanding. It is up to teachers to ensure that visuals are correctly incorporated in their daily teaching of geometry. The use of visualisation processes promotes an environment where teachers and learners interact with the objects and gain meaningful understanding of geometrical concepts.

This chapter, wherein I presented a summary of the findings, significance of the study, recommendations, suggestions for future research and an outline of my personal reflections of this study concluded my research study.

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## APPENDICES

### Appendix A: Ethical clearance



**RHODES UNIVERSITY**

Grahamstown • 6140 • South Africa

EDUCATION FACULTY • PO Box 94, Grahamstown, 6140  
Tel: (046) 603 8385 / (046) 603 8393 • Fax: (046) 622 8028 • e-mail: [d.wilmot@ru.ac.za](mailto:d.wilmot@ru.ac.za)

#### PROPOSAL AND ETHICAL CLEARANCE APPROVAL

**Ethical clearance number 2017.04.02.06**

The minute of the EHDC meeting of 1 June 2017 reflect the following:

**2017.04.02 CLASS B RESTRICTED MATTERS  
MASTER OF EDUCATION RESEARCH PROPOSALS**

*To consider the following research proposal for the degree of Master of Education in the Faculty of Education:*

***Gottfried Mbundu Muhembo (15M8813)***

*Topic: An analysis of how visualization processes can be used by teachers participating in an intervention programme to teach for conceptual understanding of geometry.*

*Supervisors: Prof Marc Schäfer*

*Decision: Approved*

This letter confirms the approval of the above proposal was noted at the meeting of the Faculty of Education Higher Degrees' Committee on 13 July 2017.

The proposal demonstrates an awareness of ethical responsibilities and a commitment to ethical research processes. The approval of the proposal by the committee thus constitutes ethical clearance.

Sincerely

Prof Marc Schäfer  
Chair of the EHDC, Rhodes University  
31 July 2017

**Appendix B: Approval letter from the Director of education**



**REPUBLIC OF NAMIBIA  
KAVANGO EAST REGIONAL COUNCIL**

**DIRECTORATE OF EDUCATION, ARTS AND CULTURE**

Telephone Number : 066 – 258 9000 / 258 9212  
Fax Number : 066 – 255 404 / 267 070  
Enquiries : **F. Kapapero**  
:

Private Bag 2134  
RUNDU  
NAMIBIA

Date : 01 June 2017

**TO WHOM IT MAY CONCERNED**

**SUBJECT: PERMISSION TO CONDUCT RESEARCH IN RUNDU  
CIRCUIT**

1. Kindly be informed that permission is hereby granted to Mr. Gottfried Muhembo, to conduct interviews at any Educational Institution in Rundu Circuit for Research purposes.
2. Your cooperation and support in this regard is highly appreciated and valued.

Yours sincerely,


**F. Kapapero**  
**DIRECTOR: KAVANGO REGIONAL COUNCIL**  
**KAVANGO EAST REGION**



01.06.2017  
Date

All official correspondence must be addressed to the Chief Regional Officer

## Appendix C: Approval letters from school principals

	[Redacted]	
	TEL: (066) 255414	P.O.BOX 1851 RUNDU .na
	Fax2email: 088645766	
	Inquiries: Principal	NAMIBIA

05 June 2017

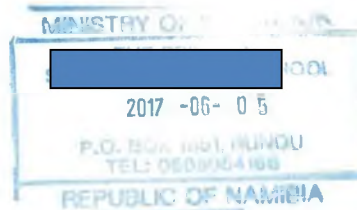
### TO WHOM IT MAY CONCERN

This letter acknowledges that I have received a request from Muhembo Gottfried M. a student at Rhodes University to conduct research study entitled "An analysis of how visualization processes can be used by teachers participating in an intervention program to teach for conceptual understanding of geometry" and I approve of this research to be conducted at our school.

I support the effort and will provide any assistance necessary for the successful implementation of this study. If you have question, please do not hesitate to call. I can be reached at +264810448989.

Sincerely

  
Kasera WK.  
PRINCIPAL





Enquiries: Mr. R.K Hamunyera  
Email: [REDACTED]

P.O. box 2304  
Tel/Fax: (066)256888

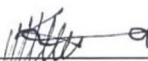
**To : Mr. Muhembo Gottfried M.**  
**RHODES UNIVERSITY**

**Re : Permission to do research at the School**

**Dear Sir**

I am writing this letter to inform you that you have been given the permission you requested to do your research at the School.

Yours Faithfully.

  
\_\_\_\_\_  
**RK HAMUNYERA**  
**SCHOOL PRINCIPAL**





30 June 2017

Mr. Gottfried Muhembo  
Rhodes University  
Grahamslow  
South Africa

**PERMISSION TO CONDUCT RESEARCH**

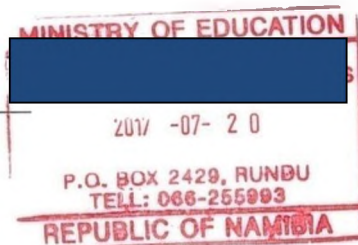
With reference to your letter date **15 May 2017**, bearing the above subject, permission is hereby granted for you to conduct your research at the above institution.

For any related arrangements, please do not hesitate but to contact us at the given address.

Thank you

Yours

  
Ms. Lucian E  
PRINCIPAL



## Appendix D: Participant teachers consent forms

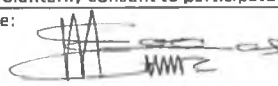
**INFORMED CONSENT FORM**

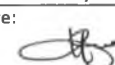
*An analysis of how visualisation processes can be used by teachers participating in an intervention programme to teach for conceptual understanding of geometry*

<b>Research Project Title:</b>	<i>An analysis of how visualisation processes can be used by teachers participating in an intervention programme to teach for conceptual understanding of geometry</i>
<b>Principal Investigator(s):</b>	<i>MR. G. W. Muhembo</i>

<b>Participation Information</b>
<ul style="list-style-type: none"> <li>I understand the purpose of the research study and my involvement in it</li> <li>I understand the risks and benefits of participating in this research study</li> <li>I understand that I may withdraw from the research study at any stage without any penalty</li> <li>I understand that participation in this research study is done on a voluntary basis</li> <li>I understand that while information gained during the study may be published, I will remain anonymous and no reference will be made to me by name or student number</li> <li>I understand that (other data collection requirements particular to this research, e.g. test results, personal information, video recording) may be used</li> <li>I understand and agree that the interviews will be recorded electronically</li> <li>I understand that I will be given the opportunity to read and comment on the transcribed interview notes</li> <li>I confirm that I am not participating in this study for financial gain</li> </ul>

<b>Information Explanation</b>
The above information was explained to me by: Mr. Gottfried Muhembo
The above information was explained to me in English and I am in command of this language: <i>yes.</i>

I, <span style="background-color: blue; color: black;">[REDACTED]</span> hereby voluntarily consent to participate in the above-mentioned research.	
Signature: 	Date: <i>29/06/2017</i>

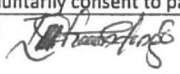
<b>Investigator Declaration</b>	
I, Gottfried Muhembo, declare that I have explained all the participant information to the participant and have truthfully answered all questions asked me by the participant.	
Signature: 	Date: <i>29/06/2017</i>


### INFORMED CONSENT FORM

Research Project Title:	An analysis of how visualization processes can be used by teachers participating in an in-service programme to teach for conceptual understanding of geometry
Principal Investigator(s):	MR. G. M. Muhembo

<b>Participation Information</b>
<ul style="list-style-type: none"><li>• I understand the purpose of the research study and my involvement in it</li><li>• I understand the risks and benefits of participating in this research study</li><li>• I understand that I may withdraw from the research study at any stage without any penalty</li><li>• I understand that participation in this research study is done on a voluntary basis</li><li>• I understand that while information gained during the study may be published, I will remain anonymous and no reference will be made to me by name or student number</li><li>• I understand that (other data collection requirements particular to this research, e.g. test results, personal information, video recording) may be used</li><li>• I understand and agree that the interviews will be recorded electronically</li><li>• I understand that I will be given the opportunity to read and comment on the transcribed interview notes</li><li>• I confirm that I am not participating in this study for financial gain</li></ul>

<b>Information Explanation</b>
The above information was explained to me by: Mr. Gottfried Muhembo
YES
The above information was explained to me in English and I am in command of this language:
YES

<b>Voluntary</b>	
I, [redacted]	
hereby voluntarily consent to participate in the above-mentioned research.	
Signature: 	Date: 29/06/2017

<b>Investigator Declaration</b>	
I, Gottfried Muhembo, declare that I have explained all the participant information to the participant and have truthfully answered all questions asked me by the participant.	
Signature: 	Date: 29/06/2017

### INFORMED CONSENT FORM

An analysis of how visualization processes can be used  
by teachers participating in an intervention programme  
to teach for conceptual understanding of geometry

Research Project Title:	
Principal Investigator(s):	MR. G. M. Muhembo

<b>Participation Information</b>
<ul style="list-style-type: none"><li>• I understand the purpose of the research study and my involvement in it</li><li>• I understand the risks and benefits of participating in this research study</li><li>• I understand that I may withdraw from the research study at any stage without any penalty</li><li>• I understand that participation in this research study is done on a voluntary basis</li><li>• I understand that while information gained during the study may be published, I will remain anonymous and no reference will be made to me by name or student number</li><li>• I understand that (other data collection requirements particular to this research, e.g. test results, personal information, video recording) may be used</li><li>• I understand and agree that the interviews will be recorded electronically</li><li>• I understand that I will be given the opportunity to read and comment on the transcribed interview notes</li><li>• I confirm that I am not participating in this study for financial gain</li></ul>

<b>Information Explanation</b>
The above information was explained to me by: Mr. Gottfried Muhembo
The above information was explained to me in English and I am in command of this language: YES

<b>V</b>	
I, [REDACTED]	
hereby voluntarily consent to participate in the above-mentioned research.	
Signature: [Signature]	Date: 29/06/2017

<b>Investigator Declaration</b>	
I, Gottfried Muhembo, declare that I have explained all the participant information to the participant and have truthfully answered all questions asked me by the participant.	
Signature: [Signature]	Date: 29 / 06 / 2017



**INFORMED CONSENT FORM**


An analysis of how Visualisation processes can be used by  
teacher participating in an intensive programme to teach  
for conceptual understanding of geometry

Research Project Title:	An analysis of how Visualisation processes can be used by teacher participating in an intensive programme to teach for conceptual understanding of geometry
Principal Investigator(s):	MR. G. M. Muhembo

<b>Participation Information</b>
<ul style="list-style-type: none"><li>• I understand the purpose of the research study and my involvement in it</li><li>• I understand the risks and benefits of participating in this research study</li><li>• I understand that I may withdraw from the research study at any stage without any penalty</li><li>• I understand that participation in this research study is done on a voluntary basis</li><li>• I understand that while information gained during the study may be published, I will remain anonymous and no reference will be made to me by name or student number</li><li>• I understand that (other data collection requirements particular to this research, e.g. test results, personal information, video recording) may be used</li><li>• I understand and agree that the interviews will be recorded electronically</li><li>• I understand that I will be given the opportunity to read and comment on the transcribed interview notes</li><li>• I confirm that I am not participating in this study for financial gain</li></ul>

<b>Information Explanation</b>
The above information was explained to me by: Mr. Gottfried Muhembo YES
The above information was explained to me in English and I am in command of this language: YES

<b>Voluntary Consent</b>	
	
hereby voluntarily consent to participate in the above-mentioned research.	
Signature: 	Date: 29/06/2017

<b>Investigator Declaration</b>	
I, Gottfried Muhembo, declare that I have explained all the participant information to the participant and have truthfully answered all questions ask me by the participant.	
Signature: 	Date: 29/06/2017

# Appendix E: Lesson Preparations

## Teacher 1, Lesson 1

10/07/2017  
8A(B, C, D)

LESSON PREPARATION FORM

GRADE 8 (A, B, C, D) SUBJECT Mathematics DATE 04 July 2017

TOPIC Geometry (Constructions)

TEACHING/LEARNING RESOURCES Chalk, Chalkboard, board ruler, rope

LEARNING OBJECTIVE Learners will know how to perform geometrical constructions using a straight edge, a compass and a protractor.

BASIC COMPETENCIES Learners should be able to use and interpret the geometrical terms such as a point, a line, a line segment, a ray and a plane.

PRESENTATION OF THE LESSON

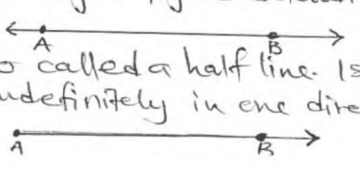
MONITORING OF HOME WORK Recaps on previous lesson.

INTRODUCTION Geometry (Geometrical constructions of angles, lines, two dimensional shapes). Explain.

PRESENTATION OF SUBJECT CONTENT AND LEARNING TASKS

TEACHER'S ACTIVITY	LEARNER'S ACTIVITY
Geometrical terms	✓ Take out note books
a) A point ✓ A straight figure extending indefinitely in both directions. ✓ An exact location.	✓ Pay attention in class
b) A line ✓ A straight figure extending indefinitely in both directions.	✓ Take notes when told to do so.

- a) A point :- Is an exact location represented by a dot.
- b) A line :- <sup>eg.</sup> Is a straight figure extending indefinitely in both directions.
- c) A ray :- Also called a half line. Is a straight line that extends indefinitely in one direction, from a certain point.



Teacher 1, Lesson 2

LESSON PREPARATION FORM

Rev 12/07/2017 (SA, SC)

NAME: [Redacted] DATE: 11/07/2017  
 GRADE: 8 A, B, C, D SUBJECT: Mathematics  
 TOPIC: Geometry -  
 TEACHING/LEARNING RESOURCES: Chalk, board, Poster,  
 LEARNING OBJECTIVE: Learners will know how to perform  
 geometrical constructions using straight edge, a compass  
 and protractor.  
 BASIC COMPETENCIES: Learners should be able to identify  
 different types of angles.

PRESENTATION OF THE LESSON

MONITORING OF HOME WORK: Recaps on previous lesson.  
 INTRODUCTION: Types of angles.

PRESENTATION OF SUBJECT CONTENT AND LEARNING TASKS:

TEACHER ACTIVITY	LEARNER ACTIVITY
Types of Angles	
- Acute Angle - less than $90^\circ$	- Take out notebooks.
- Right Angle - $= 90^\circ$	- Pay attention in lesson.
- Obtuse - Greater than $90^\circ$ but less than $180^\circ$	
- Straight - $= 180^\circ$	- Take notes when told to do so.
- Reflex - Greater than $180^\circ$	
- Revolution - $= 360^\circ$	
Ask learners to give the names of angle types.	-

Teacher 1, lesson 3

LESSON PREPARATION FORM

NAME: [REDACTED] DATE: 18/07/2017  
 GRADE: 8 (A.B.C.D) SUBJECT: Mathematics  
 TOPIC: Geometry: (Angle Properties)  
 TEACHING/LEARNING RESOURCES: Poster, Chalk, Ruler, Chalkboard.  
 LEARNING OBJECTIVE: Learners will know and understand angle properties to solve problems.  
 BASIC COMPETENCIES: Learners should be able to calculate unknown angles by applying angle properties.

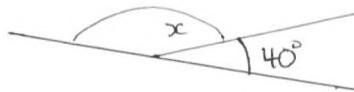
PRESENTATION OF THE LESSON

MONITORING OF HOME WORK: Recaps on previous lesson.  
 INTRODUCTION: Angle properties - Explain what will be looked at to the learners.

PRESENTATION OF SUBJECT CONTENT AND LEARNING TASKS:

TEACHER ACTIVITY	LEARNER ACTIVITY
✓ Angle properties	- Take out their note books
- Present the poster prepared to the learner showing steps to calculate Angle.	- Listen in the class
✓ Give more examples on the chalkboard.	
✓ Calculate and explain.	- Take note when informed to do so.

Eg.



$$x + 40^\circ = 180^\circ \text{ (Angle on straight line)}$$

$$x = 180^\circ - 40^\circ$$

$$x = 140^\circ$$

# Teacher 2, lesson 1

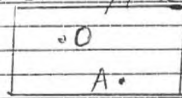
LESSON PREPARATION  
 GRADE: 89 SUBJECT: MATHEMATICS DATE: 02/07/17  
 THEME: Geometry: (Constructors)  
 TEACHING AIDS AND RESOURCE TO BE USED: poster of cards  
 LEARNING OBJECTIVE, OF THE LESSON: Learner will know how to perform geometrical constructions using a straight edge, a compass and a protractor  
 BASIC COMPETENCIES: Learners should be able to use and identify the geometrical terms such as point, a line, a line segment, a ray and a plane.

## PRESENTATION OF THE LESSON

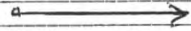
1. Monitoring of the homework done: N/A
2. Short introduction:

## 3. PRESENTATION OF THE LESSON CONTENTS

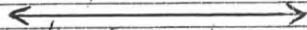
1. A Point  
 - a point is the basic building block of geometry  
 - It is a position on a plane.



2. A ray  
 - A ray starts from a point and may go to infinity  
 - It has starting point but has no end point.



3. A line  
 - A set of points without end in both directions



4. A line segment  
 - A line segment has two endpoints



5. A plane -> is a flat, infinite surface. has no edges or thickness  
 • A • B • C



4. CONCLUSION/CONSOLIDATION Use the cards from the posters to demonstrate back the point, line, line segments & plane to show it visually.

## 5. ASSESSMENTS/HOMEWORK/EXERCISE

6. OPPORTUNITY TO DEVELOP LEARNERS' LEARNERS ENGLISH READING AND WRITING SKILLS IN THE SUBJECT  
 Reading activities  
 Writing activities

PRINCIPAL/HOD

DATE

Teacher 2, lesson 2

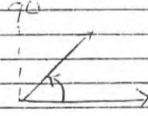
LESSON PREPARATION  
GRADE: 8G SUBJECT: MATHEMATICS DATE: 19/07/17  
THEME: GEOMETRY (ANGLES)  
TEACHING AIDS AND RESOURCE TO BE USED:  
LEARNING OBJECTIVE OF THE LESSON: Learner will know how to perform geometrical constructions using a straight edge, a compass and protractor  
BASIC COMPETENCIES: Learners should be able to identify and construct different types of angles accurately (acute, obtuse, right, straight & reflex angle).

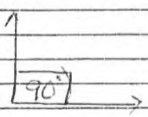
PRESENTATION OF THE LESSON

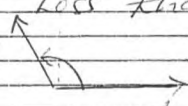
1. Monitoring of the homework done: \_\_\_\_\_
2. Short introduction: \_\_\_\_\_

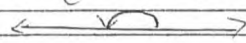
3. PRESENTATION OF THE LESSON CONTENTS

Types of angle

1. Acute angle  $\Rightarrow$  Less than  $90^\circ$  

2. Right angle  $\Rightarrow$  Equals  $90^\circ$  

3. Obtuse angle  $\Rightarrow$  Greater than  $90^\circ$  but less than  $180^\circ$  

4. Straight angle  $\Rightarrow$  Equals  $180^\circ$  

5. ~~Greater~~ Reflex angle  $\Rightarrow$  Greater than  $180^\circ$  but less than  $360^\circ$

6. Revolution Angle  $\Rightarrow$  Equals  $360^\circ$

4. CONCLUSION/CONSOLIDATION \_\_\_\_\_
5. ASSESSMENTS/HOMEWORK/EXERCISE \_\_\_\_\_
6. OPPORTUNITY TO DEVELOP LEARNERS' LEARNERS ENGLISH READING AND WRITING SKILLS IN THE SUBJECT  
Reading activities \_\_\_\_\_  
Writing activities \_\_\_\_\_

PRINCIPAL/HOD \_\_\_\_\_

DATE \_\_\_\_\_

Teacher 2, lesson 3

LESSON PREPARATION  
GRADE: 8G SUBJECT: Mathematics DATE: 20/07/17  
THEME: Geometry (Angles properties)  
TEACHING AIDS AND RESOURCE TO BE USED: \_\_\_\_\_  
LEARNING OBJECTIVE OF THE LESSON: Learner will Know and understand angle properties  
Solve problems  
BASIC COMPETENCIES: Learners should be able to Identify pairs of angle as  
Complementary or Supplementary, Calculate unknown angles by  
applying the following angle properties - angles formed on straight line.  
PRESENTATION OF THE LESSON

1. Monitoring of the homework done: \_\_\_\_\_
2. Short introduction: \_\_\_\_\_

3. PRESENTATION OF THE LESSON CONTENTS

ANGLE PROPERTIES  
Angle on straight line

(a) Supplementary = Two Angles add up to  $180^\circ$

(b) Complementary Angle = Two Angles that add up to  $90^\circ$   
When two perpendicular lines meet the form Angle  
up to  $90^\circ$

(c) Angle at a point  
= The angle around the point is  $360^\circ$

(d) Angle formed at intersecting lines  
= Vertically opposite angle are Equal.

4. CONCLUSION/CONSOLIDATION

5. ASSESSMENTS/HOMEWORK/EXERCISE

6. OPPORTUNITY TO DEVELOP LEARNERS' LEARNERS ENGLISH READING AND WRITING SKILLS IN THE SUBJECT

Reading  
activities  
Writing  
activities

PRINCIPAL/HOD

DATE

Teacher 3, lesson 1

LESSON 1

7cm = L

∴ x = 7cm

Topic 6: Geometry

General objectives

- learners must know how to perform geometrical constructions using a straight edge, a compass and a protractor

Specific objectives

- learners should be able to use measure different lines and angles accurately

AA

Measuring lines and angles

A line: Use pair of compasses or dividers

\* A line should not be measured directly with a ruler, but by using a pair of compasses or dividers on the line and then reading that distance with a compass.

An angle: Use a protractor

: There are two scales on a protractor

⇒ Work with the scale that starts from zero.

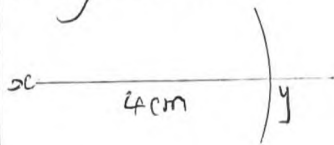
\* read the degrees: 20°

Acute angle. < 90°

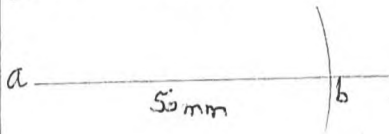
Obtuse angle. > 90°

E.g. Draw a line segment

(a)  $\overline{xy} = 4\text{cm}$

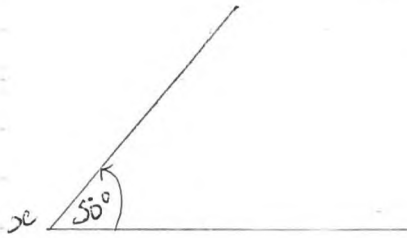


(b)  $\overline{ab} = 50\text{mm}$

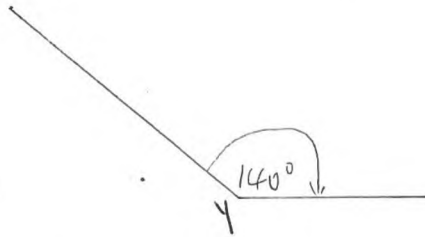


2. Measure the angles

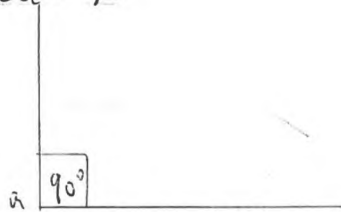
a)  $\angle x = 50^\circ$



b)  $\angle y = 140^\circ$



c)  $\angle a = 90^\circ$



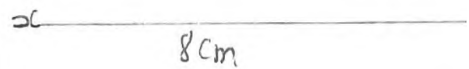
b)  $\angle M = 320^\circ$

$\therefore 360^\circ - 320^\circ = 40^\circ$

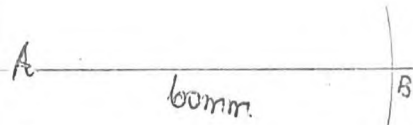
Classwork

1. Draw a line segment

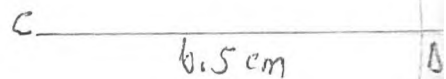
a)  $\overline{xy} = 8 \text{ cm}$



b)  $\overline{AB} = 60 \text{ mm}$

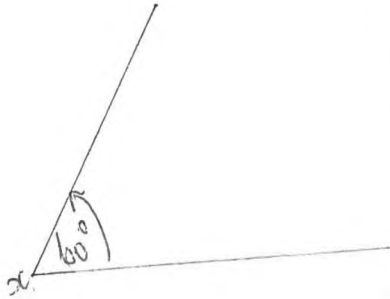


c)  $\overline{CD} = 6.5 \text{ cm}$

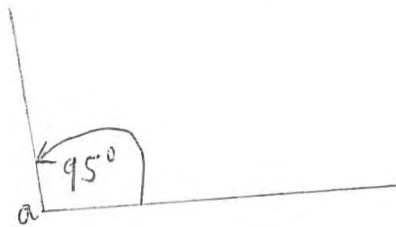


2. Measure the angles

(a)  $\angle c = 60^\circ$

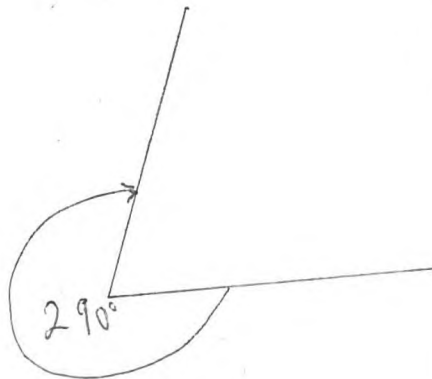


b)  $\angle a = 95^\circ$



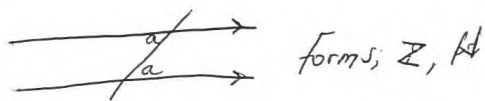
(c)  $\angle b = 290^\circ$

$\therefore 360^\circ - 290^\circ = 70^\circ$

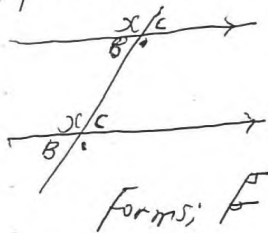


Angles formed within parallel lines ✓

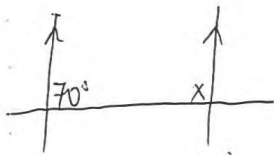
1. Alternate angles: Equal in parallel lines



2. Corresponding angles: Equal in parallel lines.

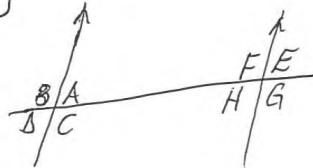


3. Co-interior angles: Supplementary in parallel lines  
 \* Add up to  $180^\circ$   
 → Formed forms:  $b d$



$$\begin{aligned} x + 70^\circ &= 180^\circ \\ x &= 180^\circ - 70^\circ \\ x &= 110^\circ \end{aligned}$$

Referring to the figure, write down the following pairs of angles



(a) Both pairs of alternate angles

$\angle C$  and  $\angle F$ ,  $\angle A$  and  $\angle H$

(b) All the pairs of corresponding angles

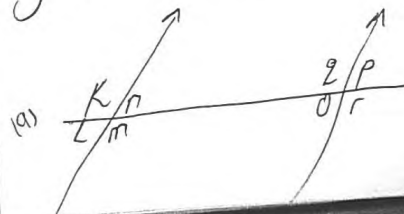
$\angle A$  and  $\angle E$ ,  $\angle C$  and  $\angle G$ ,  $\angle B$  and  $\angle H$   
 $\angle D$  and  $\angle F$

(c) Both pairs of co-interior angles

$\angle A$  and  $\angle F$ ,  $\angle C$  and  $\angle H$

Classwork

Referring to the figures, write down the following pairs of angles



(i) Both pairs of alternate angles

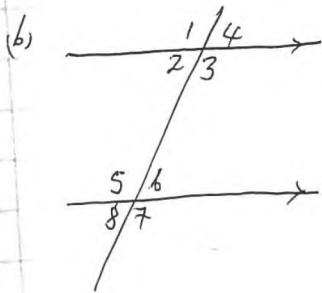
$\angle m$  and  $\angle q$ ,  $\angle n$  and  $\angle o$

(ii) All the pairs of corresponding angles

$\angle l$  and  $\angle o$ ,  $\angle m$  and  $\angle r$ ,  $\angle k$  and  $\angle q$ ,  $\angle n$  and  $\angle p$

(iii) Both pairs of co-interior angles

$\angle n$  and  $\angle q$ ,  $\angle m$  and  $\angle o$



(i) Both pairs of alternate angles  
 $\angle 3$  and  $\angle 5$ ,  $\angle 2$  and  $\angle 6$

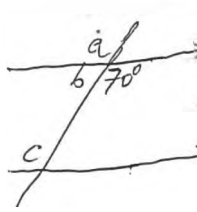
(ii) All pairs of corresponding angles

$\angle 1$  and  $\angle 5$ ,  $\angle 2$  and  $\angle 6$ ,  $\angle 3$  and  $\angle 7$ ,  $\angle 4$  and  $\angle 8$

(iii) Both pairs of co-interior angles  
 $\angle 2$  and  $\angle 5$ ,  $\angle 3$  and  $\angle 6$

Calculate angles within parallel lines

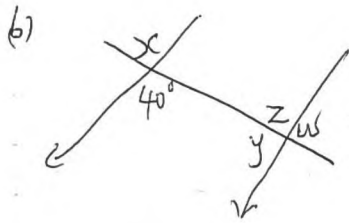
\*Remember: Alternate angle  $\alpha$   
Corresponding angle  $\beta$   
Co-interior angle  $\gamma$

E.g.  Find the angles marked with letters, give a reason in each case.

$a = 70^\circ$  (Vertical opposite angles are equal)

$c = 70^\circ$  (Alternate angles are equal in parallel lines)

$b + 70^\circ = 180^\circ$  (Supplementary angle / straight angle add up to  $180^\circ$ )



$\angle x = 40^\circ$  (Vertical opposite angles are equal)

$\angle z = 40^\circ$  (Alternate angles are equal in parallel lines)

~~$\angle x + y = 180^\circ$~~

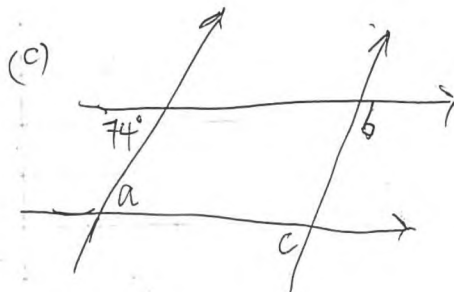
$\angle y + 40^\circ = 180^\circ$  (Supplementary angles  
Co-interior angles add up to  $180^\circ$ )

$$y + 40^\circ = 180^\circ$$

$$y = 180^\circ - 40^\circ$$

$$\angle y = 140^\circ$$

$\angle w = 140^\circ$  (Vertical opposite angles are equal)



$\angle a = 74^\circ$  (Alternate angles are equal in parallel lines)

$\angle c = 74^\circ$  (Alternate angles are equal in parallel lines)

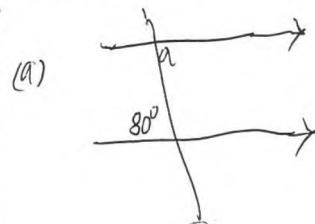
$$b + 74^\circ = 180^\circ$$

$$b = 180^\circ - 74^\circ$$

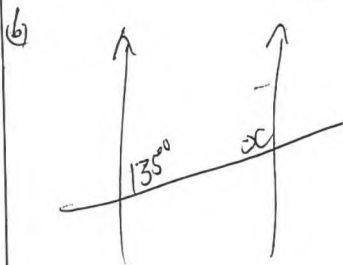
$$\angle b = 106^\circ$$

Classwork

Find the angles marked with letters and give a reason in each case.



$\angle a = 80^\circ$  (Alternate angles are equal in parallel lines)

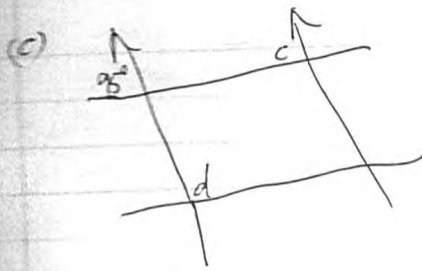


$\angle x + 135^\circ = 180^\circ$  (Co-interior angles add up to  $180^\circ$ )

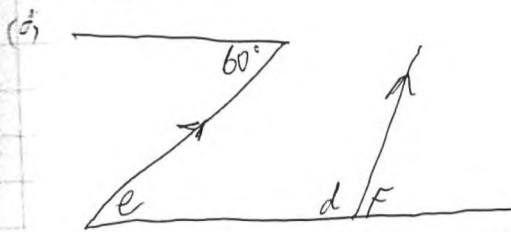
$$\angle x = 180^\circ - 135^\circ$$

$$\angle x = 45^\circ$$

Teacher 3, lesson 3



$\angle c = 95^\circ$  (Corresponding angles are equal in parallel lines)  
 $\angle c + \angle d = 180^\circ$  (Co-interior angles add up to  $180^\circ$ )  
 $\angle d = 180^\circ - 95^\circ$   
 $\angle d = 85^\circ$



$\angle e = 60^\circ$  (Alternate angles are equal in parallel lines)  
 $\angle f = 60^\circ$  (Corresponding angles are equal in parallel lines)  
 $\angle e + \angle d = 180^\circ$  (Co-interior angles add up to  $180^\circ$ )  
 $\angle d + 60^\circ = 180^\circ$   
 $\angle d = 180^\circ - 60^\circ$   
 $\angle d = 120^\circ$

Angles at a point

1. Complementary angles: Add up to  $90^\circ$  (Right angle)



$$a + 60^\circ = 90^\circ$$

$$\angle a = 90^\circ - 60^\circ$$

$$\angle a = 30^\circ$$

2. Supplementary angles: Add up to  $180^\circ$  (Straight angle)

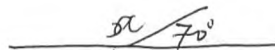


$$x + x + x + 60^\circ = 180^\circ$$

$$3x = 180^\circ - 60^\circ$$

$$3x = 120^\circ$$

$$x = \frac{120^\circ}{3}$$

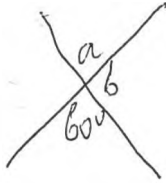


$$a + 70^\circ = 180^\circ$$

$$\angle a = 180^\circ - 70^\circ$$

$$\angle a = 110^\circ$$

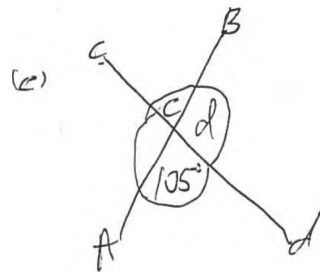
3. Vertically opposite angles: b) Opposite angles are equal.



$\angle a = 60^\circ$  (Vertical opposite angles)

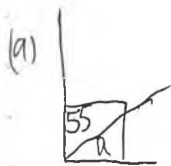
$b + 60^\circ = 180^\circ$  (Supplementary angle add up to  $180^\circ$ )

$$b = 180^\circ - 60^\circ$$
$$\angle b = 120^\circ$$



Classwork

1. Find the angles marked with letters.





Mathematic Lesson Plan

DATE 07/07/2017 GRADE 8H PERIOD 7 THEME Geometry

TOPIC Constructions

By the end of the lesson learners should be able to:

Measure lines and angles accurately.

Introduction:

Learners' pre-knowledge Test learner's knowledge on what a line is, how and where are lines formed. In addition, find what an angle is and how are angles formed.

Examples:

Possible answers

1. ~~Measure~~ Measure the given lines below. Use a ruler for (i) 10cm  
Your measurements (ii) 8cm

2. Measure the given angles below, use a protractor to measure the angles below. (iii) 19cm

Class room Topic task: Individual/ pair/ group

Possible Answers

do the activity on the hand-  
outs given:

1. Measure the lines  
given (see attached handout)

(i) (ii) (iii) (iv)

2. Measure the angles  
given below

(i) (ii) (iii) (iv)

a)  $4.7 \text{ cm} = 47 \text{ mm}$

b)  $5.6 \text{ cm} = 56 \text{ mm}$

c)  $8 \text{ cm} = 80 \text{ mm}$

d)  $1.8 \text{ cm} = 18 \text{ mm}$

2. (i)  $140^\circ$

(ii)  $40^\circ$

(iii)  $90^\circ$  (iv)  $160^\circ$

Assessment: Give feedback on the Chalkboard

while learners self mark their work  
Monitor the pair work as learners are working.

Conclusion:

- Emphasise on rules of using a protractor when using measuring angles.
- The middle point on the protractor serves as a starting point or vertex of the angle.

Home work: Individual/ pair/ group

1. Use a ruler to draw the lines with the length given in cm.
2. Use a protractor to draw the angles given below

HOD. 

Date 07/07/2017



Mathematic Lesson Plan

DATE 13.07.17 GRADE 8H PERIOD 4 THEME Geometry

TOPIC Types of angles

By the end of the lesson learners should be able to:


Name angles according to their degree.

Introduction:

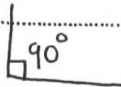
Learners' pre-knowledge Test on learners' pre-knowledge on types of angles based on the previous grade.

Examples:

1. Acute angles 

2. Obtuse angle 

3. Right angle



Possible answers

1. Between  $1^\circ$  to  $89^\circ$

2. Between  $91^\circ$  to  $179^\circ$

3. Exactly  $90^\circ$

Class room Topic task: Individual/pair/group

Measure the angles below  
and write their names  
according to their degrees  
(see attached hand-out)

Possible Answers

.....  
.....  
.....

Assessment: Collect learners handout for  
marking and give feedback afterwards

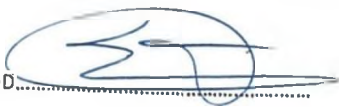
Conclusion:

Names of angles are given according  
to their sizes. A Right angle is  $= 90^\circ$  and  
makes a corner.

Home work: Individual/pair/group

Collect Pictures from media (news Papers,  
Magazine ect) where angles are formed.

HOD



Date

13/07/2017

DATE 20/07/17

TOPIC Angle

By the end of the lesson

I can

lines

Introduction:

Learners' pre-

Examples:





Mathematic Lesson Plan

DATE 20/07/2017 GRADE 8H PERIOD 7 THEME Geometry

TOPIC Angles formed in parallel lines

By the end of the lesson learners should be able to:

I identify angles formed in parallel lines.

Introduction:

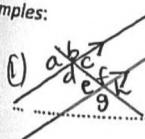
Learners' pre- knowledge Ask learners on what parallel

lines are

[Parallel lines are lines that have the same distance and they do not meet].

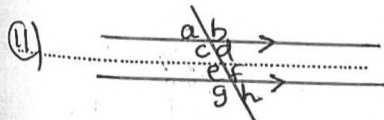
Possible answers

Examples:

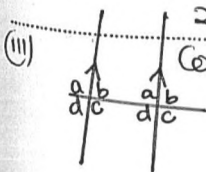


1. Identify alternate angles

1(i) d & f  
c & e



ii) d & e  
c & f

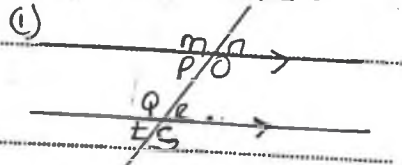


2. Identify Co-interior angles

2. (iii) a & b  
c & d

Class room Topic task: Individual/pair/group

1. Identify alternate angles from the lines below by writing their letters



possible Answers

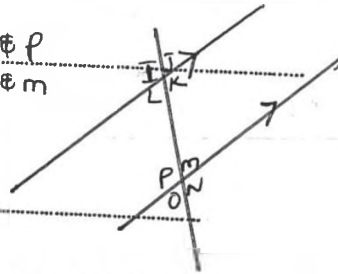
① P & R

O & Q

2. Identify Co-interior angles

Assessment:

2. L & P  
K & M



Learner's cross mark their work, while feedback is in progress.

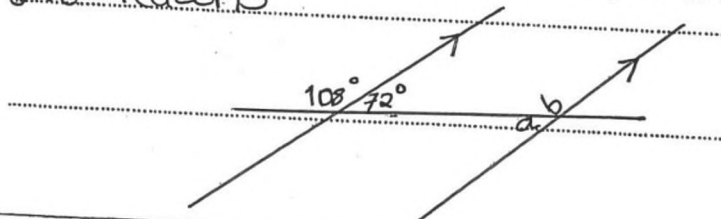
Conclusion:

Alternate angles form a capital Z letter, Or N letter and they are equal.

Co-interior angles form a U letter and they add up to  $180^\circ$  and  $a + b = 180^\circ$

Home work: Individual/pair/group

Calculate the angles marked by a letter & give reasons




HOD

Date


20/07/2017

**Appendix F: Permission letter to the Director of education**

PS - Please draft a letter to grant permission to Mr. Mateya



01.06.17



**RHODES UNIVERSITY**  
*Where leaders learn*

**PERMISSION LETTER TO DIRECTOR OF EDUCATION**

Mr. Gottfried Muhembo  
Rhodes University  
Grahamstown  
South Africa

15<sup>th</sup> May 2017

Mr. F. Kapapelo  
Director of Education  
Kavango East  
Rundu

**REQUEST FOR PERMISSION TO CONDUCT RESEARCH IN THE RUNDU CIRCUIT**

To whom it may concern

My name is Gottfried Muhembo, and I am full-time masters of education (MEd) student at Rhodes University (RU) in Grahamstown, South Africa. The research I wish to conduct for my masters full thesis requires me to observe Mathematics teaching in grade 8 and interview selected mathematics teachers. The title of my research project is: **An analysis of how visual literacy skills can be used to teach for conceptual understanding of geometry by selected grade 8 mathematics teachers.**

This research will be conducted under the supervision of Prof. Marc Schäfer, the SARCH Chair in mathematics education at Rhodes University.

I wish to collaborate with Mr. M. Mateya, a senior education officer for mathematics to conduct an awareness workshop on visualisation in mathematics and select four mathematics teachers from four different schools in the Rundu circuit as the participants of this research. I am therefore writing this letter to seek formal consent to approach, observe and interview the teachers and work with Mr. Mateya. For this reason I request your permission to carry out this research in your region in Rundu circuit as outlined in my attached research proposal between the 05<sup>th</sup> June 2017 to the 07<sup>th</sup> July 2017.

## **Appendix H: Focus group interviews**

### **Focus Group interview questions**

#### **Week 1**

1. What was your overall reaction to the concept of visualization?
2. For the lesson that you each taught in the previous week, how did you find it in general.
3. What visual processes did you employ and how do you think the learners find it.
4. How was the visual processes related to the geometric concept that you taught in your lesson.
5. How relevant did you find the use of visual processes to your grade 8 learners?

#### **Week 2**

1. Any notable difference in your preparation and teaching of the second lesson as compared to the first lesson?
2. How did the visual processes helped in developing the conceptual understanding of geometry of your students?
3. How can you relate visual processes to geometry teaching, in other words how can teachers use visual skills to teach geometry?
4. What role did the visual processes (visual language, manipulating visuals, analyzing visuals, interpreting visuals and designing visuals) that we discussed earlier played in developing conceptual understanding?

#### **Week 3**

1. In general, how different was your two lessons with the thirds lesson.
2. In terms of visualisation how different or similar was the three lessons.
3. Based on the grade 8 lesson you presented, what role do you think the visual processes played to enhance understanding of the three topics that you taught?
4. What do you think can be done differently to enhance conceptual understanding of geometry using the visualisation processes?
5. Do you think mathematics teachers should be encouraged to use visualisation in their teaching? Give reasons.
6. What is your overall experience of the intervention programme?
7. Any final thoughts?

## Appendix I: Questionnaire

### OVERVIEW

Literature indicates that learners in general find defining and recognizing geometrical shapes and the use of deductive thinking in geometry problematic. Furthermore, research shows that most students are not sufficiently grounded in basic geometry and thus are forced to memorize concepts as their only recourse. Therefore, the aim of this questionnaire is to find out teachers experiences and views concerning the teaching of geometry for grade 8. You are assured that your responses to this questions will be kept confidential and please feel free to answer the questions.

### PERSONAL INFORMATION

(a) Please indicate the school at which you are currently teaching and your mobile number.

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.....

(b) Gender (Please tick.): Female: ..... Male: .....

(c) Please indicate your age category (Please tick.): Below 25 yrs: ....., Between 25 and 30 yrs: ....., Between 30 and 40 yrs:..... above 40 yrs: .....

(d) What grades do you currently teach (Please tick.)

Mathematics Grade 8: .....

Mathematics Grade 9: .....

Mathematics Grade 10: .....

Mathematics Grade 11: .....

Mathematics Grade 12: .....

(e) Please record your educational qualifications below.

Academic qualifications (e.g. Grade 12 certificate, B. Sc, etc.):

.....  
.....

Professional qualifications (e.g. BETD, Bed Honours, Master of Education, etc.):

.....  
.....

## GEOMETRY QUESTIONNAIRE

1. In general, how do you find the teaching of geometry?

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2. What teaching strategy do you normally use when teaching geometry?

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3. How do you ensure that the concepts in geometry are clearly presented to your learners?

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4. What are the differences between the old curriculum and the newly implemented curriculum for grade 8 geometry?

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5. How often do you use teaching materials when teaching geometry?

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6. What teaching materials do you find effective and appropriate for grade 8 geometry, and how do you use these materials?

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7. What challenges do you encounter when teaching geometry in Grade 8?

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8. Is there any other information that you want to share regarding the teaching of geometry?

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