AN ANALYSIS OF MATHEMATICAL CONNECTIONS IN THE PRESENTATIONS OF FRACTION CONCEPTS IN NAMIBIAN GRADE 7 MATHEMATICS TEXTBOOKS

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DECLARATION OF ORIGINALITY

I, Pumulo Priscah Sibeso (19S9598) hereby declare that this thesis is my own original work and has not, in its entirety or in part been submitted to an institution of higher learning for a degree assessment purpose. All ideas, citations, quotations, and other materials derived from other people are duly acknowledged using the APA referencing style as per Rhodes University's Education department guidelines.

Signature: Date: 04 January 2024

DEDICATION

In loving memory of my late father and sister, who provided unwavering support and faith in my education. My iron lady, Kakwibu Joice Sibeso for her incredible determination and love. I express my gratitude and thanks for the support received throughout my journey.

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ABSTRACT

The global importance of mathematics in education systems is also recognised by the Namibian education authorities, which consider mathematics essential for everyday life and the development of science, technology, and commerce (MoE, 2009b). The National Mathematics Subject Policy Guide highlights the role of mathematical skills in investigating, modelling and interpreting numerical and spatial relationships. Textbooks, the heart of my study, mediate the curriculum and facilitate teaching and learning (Nicol & Crespo, 2006; MoE, 2009b; Apple, 1992; Ben-Peretz, 1990; Goodlad, 1984; Schmidt et al., 1997a).

In Namibia, the National Institute for Educational Development (NIED) curriculum panels oversee the evaluation and approval of education textbooks. The textbook selection process is primarily based on four criteria: adherence to the subject syllabus, comprehensive coverage of prescribed content, language and editorial quality, and the design and user-friendliness of the textbook (MoE, 2009b; 2015b). While the Namibian mathematics textbooks and the curriculum promote the application of mathematics in everyday life and connections to other subjects and cross-curricular issues, the NIED textbook evaluation criteria do not explicitly address these aspects.

This Namibian interpretive case study examined the mathematical connections within the topic of Common Fractions Mixed Numbers (CFMN) in three Grade 7 mathematics textbooks. The analysis revealed varying connections in each textbook, focusing on real-life connections and connections to other mathematical content areas. The purposeful nature of connections remained consistent across all textbooks, with exemplifying connections being the most prevalent. While all three textbooks align with the curriculum requirements and adequately cover each aspect, there is a lack of interdisciplinary connections, potentially reinforcing the perception of mathematics as a separate and discrete subject.

Participant teachers highlighted the importance of connection-making in teaching fractions and relied heavily on textbooks, particularly the *Platinum Mathematics* (PM) textbook. The study emphasised the universal significance of mathematical connections and recommended ongoing professional development and collaborative efforts to address challenges within the education system. Four emergent themes were identified: the need for workshops, professional development, and teacher empowerment; regional and cluster initiatives; leadership and advocacy; and collaboration and communication. The study contributes to understanding the importance of connections in mathematics education and provides recommendations for enhancing teaching practices. It also acknowledges its limitations and gives suggestions for improvement and future research areas based on the data findings.

Keywords: textbooks, textbook evaluation, mathematical connections/connection-making, fractions.

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LIST OF ABBREVIATIONS AND/OR ACRONYMS

CFMN	Common Fractions and Mixed Numbers'
CMC	Connections to other mathematics content
C-P-A	The Concrete-Pictorial-Abstract
CPS	Connection to problem-solving
CS	Connections to other subjects
SLI	Study of Language and Information
EHRD	Education for Human Rights and Democracy
EMLB	Excellent Mathematics Learners Book
EP	Explicit Presence
FGI	Focus group interview
FR	Food recipes
IP	Implicit Presence
LE	Lived Experience
LI	Learners interests
LPST	Lesson Planning Support Team
MCA	Millennium Challenge Account
MoE	Ministry of Education
NCTM	National Council of Teachers of Mathematics
NIED	National Institute for Educational Development
NSS	Hong Kong New Secondary School
OLDM	Oxford Let's Do Mathematics
P-A	pictorial to abstract
PM	Platinum Mathematics
PQA	Program and Quality Assurance
RELC	Regional Language Centre
RLC	Real-life connections
SPP	Senior Primary Phase
TAC	Textbook Approval Committee
TIMSS	Trends in International Mathematics and Science Study
USA	United States of America
VC	Visualisation connections
VO	Visual Objects

CHAPTER 1 : SITUATING THE STUDY

1.1 INTRODUCTION

This chapter provides an introduction to, and an overview of the study. The study aimed to analyse the nature of mathematical connections inherent in the presentations of fraction concepts in Namibian Grade 7 mathematics textbooks. I start by briefly discussing the notion of mathematics education internationally and locally, specifically focusing on Namibia's National Curriculum for Basic Education and its associated policies concerning textbooks. I then articulate the research goals and significance of the study, research questions and research methodology. The limitations of the study and the structure of the thesis are then briefly summarised.

1.2 CONTEXT/BACKGROUND

Mathematics is a universal, compulsory, crucial subject and one of the most emphasised in most education systems worldwide. The Namibian education system is no different, and it considers mathematics an indispensable tool for everyday life and essential to the development of science, technology, and commerce (Ministry of Education [MoE], 2009b). The *National Mathematics Subject Policy Guide* (2009b) reminds teachers that "mathematical skills, knowledge, concepts and process enable the learner to investigate, model, and interpret numerical and spatial relationships and patterns in the world" (p. 2). Textbooks are central in the mediation of the curriculum in teaching and learning processes, to enable learners and teachers to access mathematical content.

Textbooks are a universal and central resource for teaching and learning mathematics, as in most school subjects (Nicol & Crespo, 2006; MoE, 2009b). However, many developing countries are challenged by an unequal distribution of school reading materials in general and textbooks in particular. Textbooks are often the only material for teaching and learning that teachers and learners draw on for subject-specific content (MoE, 2008). Covid-19 and the national lockdown exacerbated the reliance on textbooks. In contexts where online learning was impossible, learners were required to use textbooks to make sense of mathematics. Also, the national and school-based examinations are often based mainly on the ability to reproduce what appears in prescribed textbooks (MoE, 2009b). Thus, their quality assumes particular importance.

In 2007, a survey of schools for Grades 5-12 in Namibia revealed that the textbook-to-learner ratio for the core subjects of English, Mathematics and Science averaged almost one textbook to five learners (Ministry of Education-Millennium Challenge Account [MCA], 2009a). In those years, a shortage of textbooks and other instructional materials was prevalent in Namibian primary and secondary schools. In July 2008, the United States of America (USA) provided grant funding for public investment in Namibian education through the Millennium Challenge Account (MCA) for five years. The MCA aimed to improve the quality of general education, access to textbooks and management thereof (MoE & MCA, 2010). The project's objective was "to give all learners access to a textbook for each priority subject" (p. 3). The project's primary focus was on developing and implementing effective procurement systems and procedures, and textbook management and utilisation at the school level (MoE, 2010, p. 3). Prioritised subject coverage was limited to English, Mathematics and Science from Grades 5-12 (MoE, 2009b). The *National Mathematics Subject Policy Guide* (2009b) also stipulated that "the ideal situation is that every learner has a textbook for mathematics" (p. 7). At the end of the project in 2015, the MCA report indicated a significant improvement in access to textbooks and a 1:1 learner-to-textbook ratio in the core subjects.

In Namibia, "all textbooks in education require the evaluation and approval of the National Institute for Educational Development (NIED) curriculum panels" (MoE, 2009b, p. 4). Textbook evaluators mostly base their selection on four criteria: conformity to the subject syllabus, the appropriate coverage of the prescribed content, the language and editorial quality, and the design, presentation and ease of use of the textbook (MoE, 2015b).

Notably, the Namibian mathematics curriculum suggests that one of the aims of mathematics education is to enable learners to apply mathematics in everyday life. It should link and connect to other subjects, cover cross-curricular issues and link with associated mathematical content (MoE, 2015b). Although the curriculum encourages mathematical connections, the NIED textbook evaluation criteria do not explicitly articulate this. Therefore, there is a gap in the Namibian textbook evaluation criteria. My study will analyse the three Grade 7 mathematics textbooks recommended by NIED, focusing on connection-making in fractions. The study envisages that it may, among other things, contribute to improving the quality of textbook evaluations and the design of suitable evaluation criteria in the Namibian education system.

In 2015, the Ministry of Education, Arts and Culture revised the curriculum from the pre-primary to Grade 12 levels. The curriculum revision also changed the textbook catalogue format and appropriate selection procedures. Since the evaluation of the Senior Primary Phase (SPP) learning support materials in April 2015, only three titles per subject per grade appear in the catalogue (MoE,2010; MoE, 2019). The selected three textbooks in Grade 7 mathematics for my study are the *Excellent Mathematics Learners' Book* [EMLB] (Mokwadi, Van Wyk, Van Rooi & Sacco, 2015), *Oxford Let's Do Mathematics* [OLDM] (Potgieter & Pretorius, 2015) and *Platinum Mathematics* [PM] (Hambata, Roos & van der Westhuizen, 2015). My study will analyse how these three textbooks support, motivate and embrace the process of connection-making in the context of fractions.

Educational standards worldwide have widely recognised the importance of making connections in mathematics education. It is recommended that teachers enable learners to recognise and make connections among relatable mathematical ideas (Mhlolo, 2012). Similarly, Mwakapenda (2008) and Parker (2006) emphasise that making connections is an essential experience for learners in the mathematics classroom. The Namibian mathematics curriculum recognises the need for mathematics education to enable learners to apply mathematical concepts in everyday life and to establish connections between mathematics and other subjects and cross-curricular issues (MoE, 2015b). Research has shown that learners' ability to recognise connections directly correlates with their mathematical understanding (Silver et al., 2009).

Weinberg (2001) argued that learners should be explicitly taught about the various mathematical connections through instruction, highlighting the interrelatedness of mathematical ideas. This approach deepens the learners' understanding and helps them view mathematics as a cohesive whole, connected to other subjects and relevant to their interests and experiences. Making connections in mathematics is crucial for understanding and meaningfully applying mathematical concepts. These connections can be made between different topics and concepts within and across grade levels, as well as between mathematics and other subjects and mathematics and everyday life (Annenberg Foundation, 2017). It is thus incumbent on textbooks to facilitate making connections in their texts.

My analysis of connection-making of the three textbooks in the domain of fractions will particularly focus on the following areas:

1. Real-world connections: By making connections between mathematics and real-life contexts, learners can better understand and appreciate numbers. This skill allows learners to see the relevance and practicality of mathematical concepts by relating them

to their own experiences and interests (Karakoç & Alacaci, 2015). Real-world connections can also enhance learners' understanding of mathematics, motivate them to learn, and improve their attitudes towards the subject (Gainsburg, 2008).

2. Connections to other subjects: Learners need to see the connections between mathematics and other curriculum subjects. Mathematics has significant relationships with subjects like Geography and Home Economics. For example, ratio, graphs, and statistics are used in Geography to analyse data, while mathematics is applied in Home Economics for budgeting and making financial decisions. Collaborative discussions between teachers of different subjects can help identify and develop effective approaches to teaching standard procedures in mathematics (Connections in Math, n.d.).
3. Connections in problem-solving: When teachers provide realistic examples in problem-solving, it helps link mathematical concepts with real-world knowledge and applications (Muijs & Reynolds, 2017, p. 261). By asking learners questions that encourage them to demonstrate and share their knowledge, connections become a natural part of their problem-solving process (Armitage, 2019).
4. Visualisation connections: Visual connections in mathematics lessons allow learners to visualise how mathematics applies to their daily lives. This makes mathematics more tangible and less abstract, as it becomes an observable phenomenon rather than a set of rules confined to the classroom (Blum, 2002; Cuban, 1976; Özdemir & Üzel, 2011; Patton, 1997). Visualisation connections can help overcome the perception that mathematics is complex and inaccessible (Karakoç& Alacaci, 2015).
5. Connections to other mathematical content areas: Fractions should be linked to other mathematical content, including decimals, percentages and ratios. These connections help learners see how fractions relate to other mathematical concepts and facilitate a deeper understanding of the subject (Connections in Math, n.d.).

Fractions "are a compulsory key component of the mathematics school curriculum in Namibia, South Africa and other countries" (Chikiwa, 2018, p 325). Although fractions are key components of mathematics, it is worth noting that fractions are particularly challenging for teachers to teach and learners to learn (Ausiku, 2022; MoE, 2015b; Nambira et al., 2009). Chikiwa (2018) found out that educational research consistently highlights the difficulty learners face in grasping common fractions. Studies by the Department of Basic Education (DBE, 2015) and Gabriel et al. (2013) indicate that difficulties with fractions persist beyond intermediate and senior phases, extending into secondary school and beyond (Schumacher &

Malone, 2017). Curricula in Namibia and South Africa typically expect foundational work with fractions, mixed numbers, and percentages to occur in grades 8 and 9, with earlier grades focusing on basic operations (DBE, 2011; [Ministry of Education, Namibia], 2012). Hence the focus on fractions in grade 7 level.

Fractions play a fundamental role in the mathematics curriculum within schools. They have significant meanings and features that extend across various mathematical areas such as algebra, geometry, probability and trigonometry. According to Bruce et al. (2013), fractions contribute to a foundational understanding of mathematics and profoundly impact learning other mathematical concepts. The understanding of fractions begins early in primary education and progresses with increasing complexity as learners' advance through the grades. However, it is often observed that learners enter Grade 7 with a poor understanding of fraction concepts that should have been mastered earlier. Niemi (1996) emphasises the importance of understanding fractions, particularly for learners in Grade 7, as it prepares them for more advanced topics in high school.

The challenges extend beyond learners. Woodward (1998) points out that teachers themselves may grapple with fraction concepts. Therefore, finding practical instructional approaches that support both learners and teachers in developing a solid understanding of fractions is essential, allowing them to make meaningful connections with this critical mathematical concept.

While fractions are ever-present in daily life – from dividing a pizza to measuring ingredients for a recipe – research and our own teaching experience at my school highlight a persistent disconnect: learners often struggle to grasp formal fraction concepts despite encountering them in practical situations. This observation aligns with studies by (Bruce et al., 2012; DBE, 2015, MoE, 2012) which demonstrate that understanding common fractions remains a significant challenge in mathematics education. Several factors might contribute to this disconnect. One possibility is the shift from concrete, real-world experiences to abstract mathematical symbols and procedures. Learners may struggle to bridge the gap between the intuitive understanding of dividing something into parts and the formal representation of fractions using numerators and denominators. Additionally, the lack of explicit connections between everyday experiences and classroom instruction may hinder the transfer of knowledge.

This highlights the need for pedagogical approaches that bridge this gap. By incorporating real-world contexts and manipulatives into fraction instruction, teachers can connect the abstract with the concrete, fostering a deeper understanding of these fundamental mathematical concepts. Future research could explore the effectiveness of such approaches in closing the gap between lived experience and formal learning of fractions.

In line with these ideas, my study aims to examine how textbooks promote connection-making in the context of fractions and explore selected Grade 7 teachers' perceptions and perspectives about the textbooks in this regard.

1.3 RESEARCH GOALS AND SIGNIFICANCE OF THE STUDY

The aim of this study is twofold. Firstly, the aim is to analyse the mathematical connections embraced within the fraction concept in the three selected Grade 7 mathematics textbooks, in terms of their frequency and purpose. Secondly, the aim is to explore the participant teachers' perceptions and attitudes on how the textbooks present the connections in fraction concepts. According to Schmidt et al. (1999) and Valverde et al. (2002), textbooks inform teachers, parents and policymakers on how to turn the educational objectives expressed in curriculum guides at the national, state or local level, into classroom lessons and reality.

Hopefully, this study will contribute to improving the quality, design and procedures of textbook evaluation criteria in Namibia. This could be achieved by revising the criteria for including mathematical connections. These should be precise and clear. Additionally, the study seeks to provide insights into the use and nature of mathematical connections in textbooks, raise awareness among teachers about their significance and inspire them to help their learners interpret the connections effectively. By emphasising the importance of connections, the study also hopes to encourage authors to incorporate suitable and effective mathematical connections in their textbooks, thereby improving conceptual teaching and learning of mathematics. Furthermore, the study's findings may contribute to the development of the education policy in Namibia, providing empirical evidence that can be used to inform decisions related to 2textbook evaluation and the mathematics curriculum. Lastly, the study may help education policymakers recognise the importance of post-use assessment of textbooks.

1.4 RESEARCH QUESTIONS

The three questions below guide the study:

1. How do selected Namibian Grade 7 textbooks encourage connection-making in the context of fractions?
2. What similarities and differences are observed in the three textbooks in promoting connection-making in the fraction concepts?
3. What are selected teachers' perceptions and perspectives of the three textbooks concerning connection-making in fraction concepts?

1.5 RESEARCH METHODOLOGY

This section elaborates on the research orientation. The study took two forms: a document (textbook) analysis and a case study, which will be elaborated on here. I also discuss the research instruments and the analytical tool.

Research orientation

This document (including the textbook analysis and case study) is underpinned by the interpretivism paradigm using a mixed-method approach and utilising qualitative and quantitative methods. According to Bertram and Christiansen (2022), in an interpretive paradigm, the researchers "do not predict what people will do, but rather describe and understand how people make sense of their world and how they make meaning of their particular actions" (p. 30). The choice of this paradigm aligns with my case study to *inter alia* understand how participant teachers make sense of mathematical connections. This was done through questionnaires and interviews, respectively.

I purposively selected the sources of data in this study, namely textbooks and participant teachers. A purposive sampling strategy is appropriate for selecting textbooks and study participants (Bertram & Christiansen, 2014). The three Grade 7 Namibian mathematics textbooks, namely the *Excellent Mathematics Learners Book* (EMLB), *Oxford Let's Do Mathematics* (OLDM) and *Platinum Mathematics* (PM), are approved for use in the Senior Primary Phase in Namibian schools. I chose these textbooks because they are based on the revised national curriculum of 2015 in Namibia (Namibia. MoE, 2010). The four participant teachers were also purposively sampled in this study based on their responses to the questionnaire. They are all Grade 7 mathematics teachers from the Grootfontein circuit in the Otjozondjupa region of Namibia.

Document analysis

In the context of document analysis research, McCulloch (2011) defined a document as "a record of an event or a process" (p. 249). In my study, the documents are the three selected textbooks. According to McCulloch (2011), Rose (2001, 2007) and Stray (1994b), textbooks can be recognised as primary documents in document analysis research. Only the fraction sections of the three Grade 7 Namibian mathematics textbooks that the NIED curriculum panellists approved are analysed. The focus of the analysis of my documents is the phenomenon (nature) and purpose of connection-making in fraction concepts.

Case study

Case study research is a systematic and in-depth study of one particular case (situation) in its context, where the case may be a child, a clique, a school, a teacher or a community (Cohen et al., 2018; Rule & John, 2011; Stake, 1995; Creswell, 1994; Adelman et al., 1976). In addition, Cohen et al. (2018) stated that the researcher aims to capture the reality of the participants' lived experiences and thoughts about a particular situation. For this study, the case consists of four selected Grade 7 mathematics teachers from the Grootfontein circuit in Namibia. The unit of analysis is the teachers' perceptions and attitudes regarding connection-making in fraction concepts in the three approved mathematics textbooks.

Research instruments

In this study, I used the following research instruments to collect data:

- document (textbook) analysis;
- a questionnaire;
- semi-structured individual interviews; and
- focus group interviews

Analytical tools

This section elaborates on the analytical tools used to analyse the data collected. I elaborate on each tool separately according to how I used it.

- *Document analysis*

The analytical framework comprises the five connection-making processes stated in Section 1.2 above. The five connection-making processes are split into indicators.

- ❖ Real-life connections: careers, food, recipes, learners' interests and lived experiences.
- ❖ Connections to other subjects: Agriculture, English, Geography, and Business.
- ❖ Connections to other mathematical content areas: money and finance, geometry, measurement, percentage, decimals.
- ❖ Visualisation connections: tables, graphs, pictorial/diagrammatic images and drawing
- ❖ Connections to problem-solving: real-life problems and cross-curricular issues (HIV & AIDs, population issues, ICT, road safety and globalisation).

I categorised the data in the three prescribed mathematics textbooks according to these five processes of connection-making. I analysed each category according to its different indicators. I then analysed the three selected textbooks on two levels: level 1 and Level 2.

- Level 1 analyses the presence (frequency) of the connections promoted in the fraction section. The presence or frequency analysis was done in two categories, Explicit Presence (EP) and Implicit Presence (IP), on a scale of 1-3 (1 – weak, 2 – moderate and 3 – strong) (see Appendix 1). The Merriam-Webster Online Dictionary (2022) defines explicit as "fully revealed or expressed without vagueness, implication, or ambiguity: leaving no question as to meaning or intent" and implicit as "present but not consciously held or recognized".
- Level 2 focuses on the purpose of the connections using three criteria: exemplifying, explanatory and decorative, measuring them on a scale of 1-3 (1 – weak, 2 – moderate and 3 – strong) (see Appendix 2). Exemplifying connection-makings are the ones that are suitable as completed examples or a variety of instances related to the entities or concepts the written language refers to. Illustrations that assist the text and provide new aspects of information necessary to clarify the explanatory, conceptual or causal mechanisms under consideration fall into the category of explanatory connection-making. In this case, decorative images are not intended to play a role in learning but to make the page more beautiful or interesting, or give only a general aesthetic sense or feeling of the textual spirit (Fotakopoulou & Spiliotopoulou, 2008).
- *Questionnaire*

The purpose of the questionnaire was to comprehensively examine the participant teachers' perspectives on mathematical connections in the classroom and gain valuable insights into the current state of connection-making skills among the participating teachers. I determined the number of participant teachers who agreed with the statements about the mathematical connections the textbooks used, and how they integrated ICT into their teaching practices. By sequentially presenting the findings from each section, I aimed to paint a holistic picture of the diverse factors influencing the teaching and learning of mathematics, ultimately contributing to the ongoing dialogue on effective instructional strategies in the field. I also identified potential areas for improvement in this domain of mathematics education in Namibia. I used descriptive statistics to organise and quantitatively analyse some questionnaire data. According to Bertram and Christiansen (2022), descriptive statistics involves transforming the data set into a visual overview, such as a table or graph, to summarise the data. I will present my quantitative analysis as tables and figures, and descriptions in percentage form will be given after each presentation.

- *Interviews*

I employed two types of interviews. Firstly, the semi-structured individual interviews were conducted to probe the participant teachers' perspectives regarding mathematical connections and the teaching of fractions. The questions posed during the interviews were structured to address specific aspects of the teachers' perceptions, attitudes to teaching fractions and their general outlook on the prescribed textbooks. Semi-structured individual interviews allow flexibility in obtaining detailed responses. Using probing questions enabled me to comprehensively understand the participants' perspectives because the participants felt more comfortable expressing themselves freely in this less structured interview environment (Bertram & Christiansen, 2014; Cohen et al., 2011, 2018). Furthermore, an open-ended question encouraged participants to share any additional comments or insights beyond the specific queries.

Secondly, the focus group interview aimed to gather a collective perspective through interactive engagement and discussions with my participants (Myers, 2009). The inherent flexibility of focus group interviews enabled me to interact directly with my participants and foster meaningful conversations that yielded diverse ideas (Watts & Ebbutt, 1987; Hevner & Chatterjee, 2010; Wilson,1997). As a researcher, my role during the focus group interviews was to initiate questions that all participants could answer and respond to. The focus group interviews served as a valuable methodological approach to gathering a collective perspective on the role of mathematical connections in textbooks. The structured format, participant engagement, and open dialogue created a conducive environment for insightful discussions, contributing to the overall depth of the research findings. To foster an environment of open dialogue, the participants were assured that there were no right or wrong answers. They were encouraged to freely express their points of view and use any mathematics textbook or resource during the interview. Furthermore, an open-ended question urged the participants to share any additional comments or insights beyond the specific queries.

The interviews were audio recorded and transcribed. I employed inductive analysis to examine the participant teachers' responses to the interview questions. Fotakopoulou and Spiliotopoulou (2008) described three main categorising strategies for qualitative analysis, where categories, themes, and patterns emerge from the data. During the analysis of the interviews, I allowed themes to emerge from the raw data.

1.6 LIMITATIONS OF THE STUDY

This study focused on analysing and interpreting mathematical connections with a limited sample size of only three textbooks and four teacher participants from a single circuit in the Otjozondjupa region. It is important to note that the findings cannot be applied universally to all other textbooks and regions. Although the scope of this study is limited, it can serve as a valuable starting point and springboard for future research.

1.7 STRUCTURE OF THE THESIS

This section provides a brief overview of the remaining chapters of the thesis.

Chapter 2 critically reviews the relevant literature. This review focuses on the roles and importance of textbooks in the teaching and learning of mathematics. Additionally, an overview of the process used to select mathematics textbooks for schools in Namibia was provided. The chapter also includes a summary of relevant information regarding mathematical connections. Lastly, I discuss the literature on social constructivism as a theoretical framework supporting this study.

Chapter 3 delves into the methodology employed. It offers a comprehensive overview of the chosen methods and the reasons behind selecting them. Additionally, the chapter provides the rationale for conducting the study and details the research instruments employed. It also acknowledges the limitations encountered during the study. In the conclusion of the chapter, an examination of the data analysis process is conducted, followed by a discussion on ethical considerations and the study's validity.

Chapter 4 focuses on data analysis of the three textbooks. The chapter concludes by concisely summarising all the data from the three selected textbooks.

Chapter 5 focuses on data analysis of the questionnaires, semi-structured individual interviews and focus group interviews. Moreover, I conclude the chapter by discussing some emerging themes from the analysis.

Chapter 6 of my research study presents the conclusion and recommendations. The key issues of my research findings are concisely summarised. Recommendations arising from the study are noted. Finally, the concluding remarks and personal reflections on my journey are outlined.

1.8 CONCLUSION

This chapter has provided an overview of the study, including the background and context of mathematics education in Namibia. It discussed the importance of textbooks in the teaching and learning of mathematics and highlighted the need for mathematical connections in curriculum materials. The research goals, significance of the study, research questions, and methodology were also outlined. The chapter concluded by acknowledging the study's limitations and summarising the thesis's structure. The following chapter discusses the relevant literature and theoretical framework.

CHAPTER 2 : LITERATURE REVIEW

2.1 INTRODUCTION

In any research endeavour, establishing a solid foundation is vital. It is the pillar upon which our investigation stands, empowering us to navigate new territories and explore the unknown (O'Leary, 2014). According to Bertram and Christiansen (2014), a literature review is "a section that puts a research study into the context of previous research by showing how it fits into a particular field" (p. 13).

In this chapter, I discuss all the conceptual and theoretical frameworks that underpin my study and which are relevant to my research. I first discuss the conceptual frameworks and then the theoretical framework. I end with a summary and conclusion.

2.2 CONCEPTUAL FRAMEWORK

2.2.1 Textbooks

In this section, I discuss the literature in relation to textbooks. The layout of this section is as follows:

- Mathematics textbooks
- Qualities of a good textbook
- Textbooks in the classroom
- Textbook and the curriculum
- Textbook analysis and evaluation
- Textbook evaluation in Namibia
- Textbook evaluation, approval methodology, textbook mark sheets and scoring system of the Namibian education system

2.2.1.1 Mathematics textbooks

Mathematics textbooks have been used as teaching and learning materials since ancient times. Euclid's *The Elements* in Ancient Greece, written around 300 BC, is considered the most successful mathematics textbook in the West (Boyer & Merzbach, 2011, p. 90). Similarly, Shen et al. (1999, p. 1) stated that *The Nine Chapters on Mathematics Art* from ancient China, dating back from 200-100 BC served as a textbook not only in China but also in neighbouring countries and regions until 1600 AD.

However, the study of mathematics textbooks, or textbook research, has a relatively shorter history. Six decades ago, Cronbach (1955, p. 4) observed that although textbooks were widely used in classrooms, research that focused on textbooks was scattered, inconclusive and often trivial. Research on textbooks is a more recent development than mathematics textbooks' long existence (Fan et al., 2013).

Textbooks are teaching and learning resources that are used all over the world to facilitate the understanding of mathematics. Sheldon (1988) characterised textbooks as published books specially designed to enhance the learners' linguistic, mathematical and communicative abilities. These foundational educational resources were perceived as instrumental in shaping learners' core competencies. Building upon this definition, Ur (1996) emphasised the dual role of textbooks, highlighting their significance as learning instruments and as crucial tools supporting the teaching process. It seems this perspective recognises the multifaceted nature of textbooks in educational settings.

Apparently, textbooks represent one of the most critical educational inputs (Pingel, 2010). Texts generally reflect basic ideas about national culture and often become flashpoints of cultural struggle and controversy. So, this viewpoint situates textbooks at the intersection of education and cultural discourse. Van Steenbrugge et al. (2013) contributed to this discourse by defining textbooks as printed and published resources designed for use by both teachers and learners. These materials provide explanations and exercises for learners, serving as instructional guides for teachers. This definition encapsulates textbooks' fundamental purpose and structure in the teaching and learning process.

Triantafillou et al. (2013) also delved into the disciplinary embedding of school textbooks. Describing them as textual products produced by discipline specialists, the authors highlighted the purpose of these materials within specific educational communities to achieve institutional goals. This insight acknowledges the nuanced variations in knowledge structures, norms of inquiry, vocabularies, representations and standards of rhetorical intimacy inherent in school textbooks (Bhatia, 2010).

The history of mathematics textbooks and the definition of textbooks provide a rich background for understanding the qualities of a good textbook. In the next section, I discuss the essential characteristics that make a textbook effective and influential in promoting effective mathematics teaching and learning.

2.2.1.2 Qualities of a good textbook

This section discusses the characteristics of good mathematics textbooks. Good textbooks must generally consider the specifically intended functions and the context in which they will be utilised. Their designs are critical to the users (Nicol & Crespo, 2006). The following elements should be taken into consideration when designing quality mathematics textbooks:

1. They should align with the curriculum. The textbooks should be written according to the prescribed syllabus and every aspect of the syllabus should be adequately covered (Gu et al., 2004). Similarly, Haggarty and Pepin (2002, pp. 567–590) state "quality texts should align with the frameworks or syllabi they represent".
2. The approach that the textbooks adopt and promote needs to be stimulating and creative (Gu et al., 2004).
3. Textbooks need to draw on mathematical history and be driven by research on teaching and learning mathematics. Qualified, experienced, and competent practitioners, such as mathematics professionals or a committee of experts, should write them (Gu et al., 2004).
4. The textbooks should use the style and vocabulary suitable for the age group of learners for whom the book is written and provide for individual differences. They should meet the needs of learners of varying abilities, interests and attitudes. The textbook's language should be simple, easily understandable, and within the pupils' grasp (Gu et al., 2004).
5. The textbooks must have appropriate exercises that align with the relevant assessment policy. They should also satisfy the examination demands (Gu et al., 2004).
6. Textbook content should be organised in order of increasing difficulty (Gu et al., 2004).
7. The textbooks should contain difficult problems or exercises to challenge mathematically gifted learners (Gu et al., 2004).
8. The textbook should stimulate the initiative and originality of the learners, and cultural and social aspects of learners need to be considered (Gu et al., 2004).
9. The quality of visual objects, such as illustrations and graphics, is a critical element that must be considered (MoE, 2015b). Tyson-Bernestein (1989) rationalised that clarity and meaning should be critical in analysing the textbook graphics' quality, including illustrations, charts, maps and graphs. Photographs and pictures help clarify the text (p.

92). Mikk (2000) emphasises the need for exciting, imaginative textbooks by asserting that "students have many sources of information available; if their textbooks are dull, they are unwilling to study them. Interesting and enthusiastic textbooks develop curiosity and interest in the subject" (p. 17).

10. The terms and symbols used must be current and internationally accepted. All the terms, concepts and principles used in the text should be clearly and accurately stated and defined (Haggarty & Pepin 2002).
11. The diagrams used in the textbook should be easily recognisable, and geometric constructions should be in proportion to the measurements prescribed by the problem (Gu et al., 2004).
12. Searle (1989) suggested that "All textbooks for use in classrooms should be reviewed by experts and by teachers and should be field-tested in classrooms" (p. 37). He further contends that field testing aims to determine whether the books are suitable for the children who will use them – whether the language, the illustrations, and the difficulty level are appropriate.
13. The textbook should offer suggestions to improve study habits (Gu et al., 2004).

Textbook evaluation is one of the many qualities of a good textbook, as noted in Point 12 above. Points 9 and 11 state visualisation as one of the criteria for a good textbook, hence the importance of this study. Textbooks are developed for classroom and beyond-classroom use.

2.2.1.3 Textbooks in the classroom

The role and function of the mathematics textbook have been widely discussed in the Trends in International Mathematics and Science Study (TIMSS) since the late 1990s. Textbooks are standard features in many classrooms worldwide and have been identified as essential vehicles for promoting curricula (O'Keeffe & White, 2017). The TIMSS study that explicitly focused on the use of mathematics textbooks internationally was the 1999 TIMSS-R. The study highlighted the association between textbooks and classroom practice in mathematics (Valverde et al., 2002). The 1999 study, reported by Valverde et al. (2002), highlighted the association between textbooks and classroom practice in mathematics. Based on the abovementioned elaborations, more research should be conducted on mathematics textbooks. Mainly, comparative studies include aspects aimed at identifying key features of mathematics textbooks and their role in teaching and learning (Fan & Zhu, 2000).

In many mathematics classrooms, textbooks are an essential part of what is involved when doing school mathematics; they provide frameworks for what to teach, how to teach, and outline the teaching sequence to follow (Apple, 1992; Ben-Peretz, 1990; Goodlad, 1984; MoE, 2009b; Nicol & Crespo, 2006; Schmidt et al., 1997a). The power of textbooks lies in their ability to serve as resources that introduce readers to worlds that are not immediately apparent or cannot be experienced directly (Sosniak & Perlman, 1990). In particular, textbooks have the power to provide an "organised sequence of ideas and information" to structured teaching and learning, which guide readers' "understanding, thinking, and feeling" as well as "access to knowledge which is personally enriching and politically empowering" (Sosniak & Perlman, 1990, p. 440).

However, Valverde and Schmidt (1997b) warn that a significant failing of textbooks occurs when teachers try to cover every aspect of it, hindering or ignoring the application of other suitable methodologies and resources for teaching and learning where necessary. The school textbook is also used as curriculum material that mediates school activities, while its use is situated in the social and cultural context of institutional teaching and learning (Rezat, 2007).

2.2.1.4 Textbooks and the curriculum

Curriculum implementation needs an instrument that bridges its implemented and intended imperatives. Rahmawati et al. (2020) suggest that textbooks are bridging instruments which include various learning materials. They are arranged according to the goals of learning and should align with the current implemented curriculum. A growing body of research has focused on the relationship between curricula, textbooks and learning (Valverde et al., 2002). Textbooks are the visible curriculum in most classrooms. Many classrooms' intended and implemented curriculums rely heavily on textbooks and different textbooks offer learners additional learning opportunities (Fuson et al., 1988; Mayer et al., 1995; Mesa, 2004; Valverde et al., 2002) (see Figure 2.1). TIMSS (2002) argues for a strong link between the curriculum and the textbooks by referring to the textbook as a 'surrogate curriculum'. In his book on comparing textbooks for the TIMSS, Howson (1995) pointed out that textbooks were one step nearer classroom reality than a national curriculum. Schmidt et al. (1997a) further argued that "two worlds – that of official intentions and that of actual classroom activities – are tied together, in part, by textbooks" p. 53).

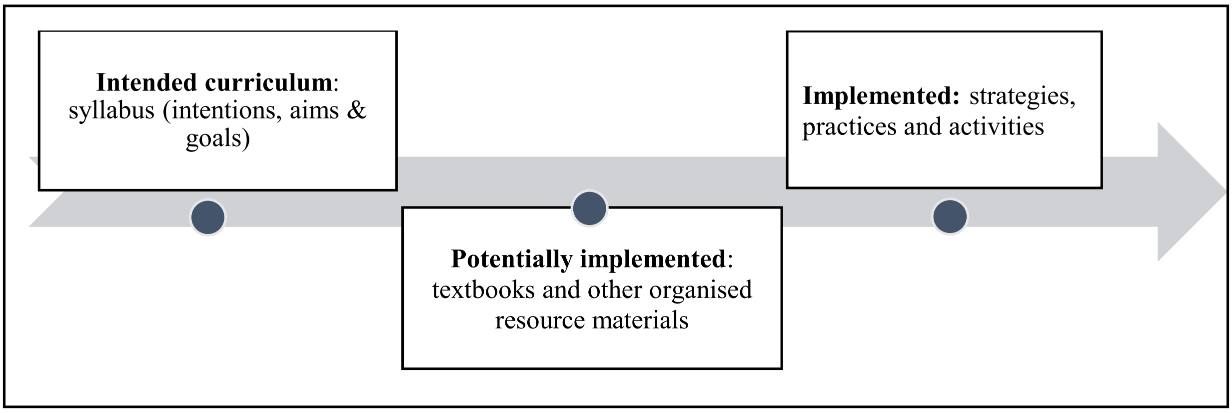


Figure 2.1: The place of textbooks
Source: Valverde et al. (2002, p. 13)

According to Valverde et al. (2002), textbooks play a crucial role at various levels of the curriculum as they bridge the abstract concepts of curriculum policies and the practical implementation by teachers and learners. So they act as mediators, facilitating the translation of the intentions embedded in curriculum policies into actionable tasks within the classroom (O'Sullivan, 2017; Valverde et al., 2002). Furthermore, school textbooks offer an essential resource that mathematics teachers rely on and provide detailed information in making decisions (Beaton et al., 1996; Fan & Zhu, 2000; MoE, 2016; Sun et al., 2023) such as:

- concerning the objectives and competencies to be achieved;
- about when it is best to convey content directly;
- about when it is best to let learners discover or explore information for themselves;
- concerning when learners need directed learning;
- on when the learners need reinforcement or enrichment learning;
- about when there is a particular progression of skills or knowledge that needs to be followed; or
- when the learners can be allowed to find their own way through a topic or area of content (MoE, 2016, p. 3).

Although textbooks seem vital for teaching and learning, not all are resourceful and user-friendly. O'Keeffe (2013) posits that "good textbooks need to consider content, value forming aspects, motivational elements, accessibility, illustrations, study guides – they must encourage a thirst for knowledge" (p. 4). Hence, it is essential to analyse critically and comprehensively what and how textbooks portray content knowledge. According to Verspoor (1989), textbooks play a significant role in enhancing the quality of education across all levels of educational progression. In support of this view, Fernig et al. (1989) observed that the absence of textbooks is perceived as detrimental to both the learning process and teaching practice.

2.2.1.5 Textbook analysis and evaluation

Evaluating textbooks is an essential aspect of education. Fan et al. (2013) elaborated that:

> Textbook analysis is a broad term primarily including (1) analysis of a single textbook or a series of textbooks, which often focuses on how a topic or topics are treated or how a particular idea or aspect of interest is reflected in the textbooks, and (2) analysis of different series of textbooks from the same country or, more frequently, different countries, often with a focus on identifying their similarities and differences. (pp. 636-637)

Sheldon (1988) suggests two reasons why we need to evaluate textbooks. Firstly, an evaluation can help teachers or programme developers select the appropriate textbooks for use in schools. Secondly, an evaluation of the merits and demerits of a textbook can familiarise teachers with its probable weaknesses and strengths (Sheldon, 1988). Mukundan et al. (2011) claimed that "this will enable teachers to make appropriate adaptations to the material in their future instruction" (p. 21). Cunningsworth (1995) and Ellis (1997) identified three types of textbook evaluations: pre-use, in-use and post-use evaluations. Mukundan et al. (2011) elaborated that

> Evaluation of textbooks for pre-use or predictive purposes helps teachers to select the most appropriate textbook for a given language classroom by considering its prospective performance. The second type of evaluation aids the teacher in exploring the weaknesses or strengths of the textbook while it is being used. Finally, post-use or retrospective evaluation helps the teacher reflect on the quality of the textbook after it has been used in a particular learning-teaching situation. (p. 21)

Wong (2011) cited several researchers in the area of materials evaluation (eg Ellis, 1997, 1998; Tomlinson, 2003) who stressed the importance of a post-use evaluation of a textbook since it can provide data regarding the actual effect of the materials on the users (Tomlinson, 2003; Tomlinson & Masuhara, 2004). Research studies identified significant differences in textbooks across the nations. The two large-scale studies that compared and contrasted the textbooks adopted in almost 40 countries concluded that textbooks vary "in a myriad of ways" (Schmidt et al., 1997b, p. 22) and that "they exhibit substantial differences in presenting and structuring pedagogical situations" (Valverde et al., 2002, p 17). However, Mukundan and Ahour (2010) state that empirical studies in post-use evaluation appear to be limited in the literature, with many biased towards pre-use evaluations. The National Institute of Educational Development (NIED) usually carries out the pre-use textbook evaluation mainly to ensure that it aligns with the curriculum in the Namibian education system (MoE, 2009b). This research study will, however, adopt a post-use evaluation/analysis process in its textbook analysis.

Authors and curriculum developers should consider various qualities of a good textbook. The quality of visual objects in textbooks, such as illustrations and graphics, are critical elements of a good textbook (MoE, 2015b). Similarly, Tyson-Bernestein (1989) suggests that clarity and meaning should be an essential criterion in analysing the quality of the textbook graphics, including illustrations, charts, maps and graphs, photographs and pictures because they help clarify the text. Neumann (1989) suggests that a significant criterion for evaluating a textbook includes the quality of the presentations and the attractiveness of illustrations. Tyson-Bernstein (1989) indicates that textbooks should motivate learners to read, stimulate class discussion, and help them think and problem solve. Further, the textbook should offer suggestions to improve study habits (Gu et al., 2004).

Curriculum developers and evaluators must consider a wide range of factors before deciding on the materials they develop or select for particular contexts. Richardson (2019) suggests that some of these factors should include the roles of the learner, teacher, instructional materials and syllabus. Additionally, Bell and Gower (1998) claim that to account for these roles effectively, the evaluator must gain an awareness of the learners' and teacher's needs and interests. Similarly, some scholars argue that evaluative criteria should be based on the teaching and learning context and the specific needs of the learners and teachers (Byrd, 2001; Sheldon, 1988). However, Mukundan et al. noted that "a review of the available checklists indicates that they have many identical evaluative criteria regardless of the fact that they had been developed in different parts of the world for different learning-teaching situations and purposes" (2011, p. 22).

Many well-established checklists, such as those of Cunningsworth and Kusel (1991) or Skierso (1991), examine similar dimensions like the physical attributes, aims, layout, methodology and organisation of textbooks. Other criteria that are present in most checklists include the way language skills (speaking, listening, etc), sub-skills (grammar, vocabulary, etc) and functions are presented in the textbook in the context of present socio-cultural settings (Cunningsworth, 1995; Harmer, 1991; Ur, 1996; Zabawa, 2001).

2.2.1.6 Textbook evaluation in Namibia

In Namibia, "all textbooks in education require the evaluation and approval of the Namibia National Institute for Educational Development (NIED) curriculum panels" (MoE, 2009b, p.

4). The Namibian curriculum and learning support materials review policy and review cycle includes a broad consultation with all stakeholders, including MoE officials and representatives of commercial publishing companies (MoE, 2015b). During 2011 and 2012, NIED outlined the main features of the Basic Education Curriculum Reform, which have now been accepted by the MoE and which constitute the basis upon which the number and type of subjects for the future curriculum are drawn up in the present policy (MoE, 2015b).

The reforms of the textbook evaluation and approval procedures aim to achieve greater control for Namibia's Ministry of Education, Arts and Culture over "procurement costs and physical content and presentational quality, and to introduce a 'competitive' evaluation to replace the previous 'publisher submission' approach" (MoE, 2015b, p. 1). A need was recognised to create an open, transparent and objective evaluation system controlled by a solid management system to guarantee a fair and objective evaluation process. The textbook review policy is implemented to avoid conflicts of interest between the evaluators and staff members of the ministry and others who might be involved in writing, editing or publishing learning materials submitted for evaluation and approval by private sector publishers.

The approval system that existed up to 2011, before the reform, had a number of problems, such as:

> the system did not require prices to be submitted for competitive evaluation and, therefore, did not provide the Ministry with any price control or negotiating position to achieve the best prices during procurement negotiations. There were also no required contractual obligations between publishers and the Ministry that ensure compliance with the minimum obligations that the Ministry of Education requires as a condition for approving a textbook for use in Namibian schools. Finally, there was no term or limit on the period of approval (MoE, 2015b, p. 1)

Also, importantly for this research, the textbook evaluation and approval procedures did not provide for in-use textbook evaluation data.

2.2.2 Textbook evaluation, approval methodology, textbook mark sheets and scoring system of the Namibian education system

In 2015, the Ministry of Education, Arts and Culture [MoEAC] revised the curriculum from the pre-primary to Grade 12 levels (MoEAC, 2015b). The curriculum revision also changed the official textbook catalogue format and appropriate selection procedures. Since evaluating the Senior Primary Phase (SPP) learning support materials in April 2015, only three titles per subject per grade appear in the catalogue. Although the official textbook catalogue is limited to a maximum of three core textbook titles per subject, categories such as supplementary materials, literature titles, dictionaries, glossaries and orthographies are omitted.

The Textbook Approval Committee (TAC) consists of six senior staff members of the Ministry of Education, Arts and Culture, which is the senior body within the MoE charged with an overview of the evaluation process, and meets after NIED has finalised the evaluation report. It comprises the Director of NIED, the Director of Programme and Quality Assurance (PQA), the NIED Deputy Director of Curriculum Research and Development, the PQA Deputy Director of Research Information and Publication, the Deputy Executive Director of Formal Education and the NIED Evaluation Administrator. Subject evaluation panels carry out the content evaluations. The evaluation panels use detailed criteria and scoring systems as the basis for their assessments of each textbook proposal. The panel comprises three to five members for each curriculum subject at the primary and secondary levels. The detailed criteria and scoring system apply to all content subjects in the Namibian curriculum. The Namibian curriculum and learning support materials review policy and review cycle of 2017 lists four criteria, and each criterion consists of different aspects for content subjects such as mathematics, on which textbook evaluators mostly base their selection (MoEAC, 2017):

A. **Conformity to the subject syllabus is assessed based on the following aspects:**
   - ✓ Aspect A is the coverage of required syllabus topics: How many of the topics required by the syllabus are missing in the textbook?
   - ✓ Aspect B is Coverage of Required Syllabus Competencies: How many of the competencies required by the syllabus are missing in the textbook?

B. **Content is assessed based on the following aspects**:
   - ✓ Aspect A is the sufficiency of the content: reviewers should analyse whether the content is very well adjusted to syllabus requirements and will provide sufficient work for the school year.
   - ✓ Aspect B is the accuracy and currency of the subject matter.
   - ✓ Aspect C is the sufficiency of the content.
   - ✓ Aspect D is the appropriateness of the content to the level of the learner.
   - ✓ Aspect E is the Organisation of the Subject Matter.

     Assessment of this aspect is based on the following statements:
       - ❖ The integration of skills throughout the textbook is sound and logical.
       - ❖ The sequencing of information within a unit is sound and logical.

- ❖ The information and skills are presented consistently throughout the textbook.
- ❖ Information or skills are organised into discrete units that are easy for learners and teachers to use.
- ❖ The different text types are well organised and signposted for learners and teachers.
- ❖ Learners are encouraged to use the text by themselves.

- ✓ Aspect F is the progression through the series.
- ✓ Aspect G is the promotion of cross-curricular issues: assessing whether text and illustrations consistently stress the following issues or groups of issues:
    - ❖ Gender issues
    - ❖ Environmental Learning
    - ❖ HIV and AIDS
    - ❖ Population Education
    - ❖ Education for Human Rights and Democracy (EHRD)
    - ❖ ICT
    - ❖ Road Safety
- ✓ Aspect H is the representation of Namibia's diversity (culture and environment)
- ✓ Aspect I is the representation of inclusive education. The text is assessed based on the following statements:
    - ❖ The text and illustration are of such a nature that they can be translated/interpreted/Braille for learners with visual impairments.
    - ❖ The text and illustration are of such a nature that they can be translated/interpreted/signed for learners with hearing impairments.
    - ❖ The material makes provision for learners with different educational needs to be used accordingly.
    - ❖ The design of the material encourages learners with special educational needs/learning difficulties to utilise it.
- ✓ Aspect J is the sufficiency and relevance of exercises, assignments/homework and activities.
- ✓ Aspect K is the availability of self-assessment exercises.
- ✓ Aspect L is the encouragement of skills development.
- ✓ Aspect M is the development of critical thinking and subject understanding.
- ✓ Aspect N is the thematic teaching / integration of other subjects.

C. **Language and editorial quality are assessed based on the following aspects:**
   - ✓ Aspect A is the accuracy, clarity and correctness of the language: in the case of English language teaching, standard British English should be used throughout the textbooks.
   - ✓ Aspect B is the appropriateness of the language for the level and background of the learner.
   - ✓ Aspect C is the editorial quality: the textbook is well-edited, and there are no examples of spelling errors or incorrect punctuation.

D. **Design, presentation and ease of use of the textbook are assessed as follows:**
   - ✓ Aspect A is the usefulness, relevance and accuracy of graphics and illustrations.
   - ✓ Aspect B is the quality and attractiveness of illustrations.
   - ✓ Aspect C is the ease of use. Assessment of the text is based on the following statements:
     - ❖ There is a good, informative content list.
     - ❖ The chapter headings, unit headings and sub-headings are clear and easy to understand.
     - ❖ Page numbering is clear and easy for learners to see and use.
     - ❖ Captions to illustrations are clear and informative.
     - ❖ Cross-referencing between text and illustrations is good and adds to the usefulness of both.
     - ❖ Key concepts are highlighted.
   - ✓ Aspect D is the balance between text and illustrations
   - ✓ Aspect E is the readability of typefaces and appropriateness of line spacing.

Research conducted by Nghifimule (2016) found out that although visualisation is not explicitly mentioned, it is partly incorporated in the last criterion, which looks at the usefulness, relevance and accuracy of graphics and illustrations and the quality and attractiveness of illustrations. However, Nghifimule (2016) suggested that the criterion insufficiently articulates the required characteristics of Visual Objects (VO). She argued that the criterion is insufficient to assess the appropriateness and quality of VOs in mathematics textbooks. Furthermore, this suggests a gap in the Namibian evaluation criteria for assessing VOs in mathematics. Hence, an analysis if visualisation will be incorporated in my study.

In her recommendations, Nghifimule (2016) acknowledged that textbooks are a universal and central element of teaching and learning mathematics. The study showed that VOs in mathematics textbooks are essential teaching and learning tools often used to enhance the learning of mathematical concepts (Steenpaß & Steinbring, 2014, p. 83). In the study, Nghifimule found out that mathematics textbooks are accessible to teachers and learners at most Namibian schools with a 1:1 learner-to-textbook ratio (MCA, 2015a). Thus, there is a need to take advantage of the availability of textbooks to get the best out of them, particularly in relation to the use of VOs evident in mathematics textbooks. Recommendations made by Nghifimule's study are summarised in Table 2.1 below. She suggested that these be implemented for the improvement of the quality of the textbooks in terms of VOs, in order to enhance the use of VOs among teachers and create awareness among authors to ensure that suitable and appropriate VOs are used. These recommendations apply to learners, teachers and teacher training institutions, authors, the Textbook Approval Committee (TAC) and further research.

Table 2.1: Summary of Nghifimule's recommendations

Learners	Teachers and Teacher training institutions	Authors	Textbook Approval Committee (TAC)	Further research
Learners require appropriate assistance and guidance from teachers and knowledgeable, competent peers to interpret VOs.	Teachers should be encouraged to teach and help learners to read and interpret diagrams in mathematics.	Authors should acquaint and familiarise themselves with computer technology to utilise and capitalise on the drawing capacity of the technologies at their disposal.	Formal platforms should be created whereby teachers and authors engage and collaboratively review the textbooks to determine whether the books are suitable for the children who will use them.	✓ Conduct similar research using all chapters. ✓ Investigate how learners interpret VOs evident in mathematics textbooks.
	Teachers should be encouraged to participate in field-test feedback sessions to give their inputs and contributions to authors to help improve the quality of the textbooks.	Authors need to do research and acquaint themselves with what learners know and what is trending to use familiar VOs to attract learners' interest in learning mathematics.	The MoE should subsidise the printing of textbooks to ensure that the quality of VOs is not compromised.	The use of mathematics textbooks of different grades at primary, junior and secondary levels.
	Teachers' induction and in-service training should include diagram interpretation, ie training on diagram literacy.	Authors should provide teachers with teachers' guides, where explanations for the different VOs are articulated.	The curriculum planners and textbooks evaluators need to revise the criteria for the evaluation of VOs used in mathematics textbooks and give explicit guidelines to authors.	General practices of how teachers help learners interpret VOs are evident in textbooks.

Source: Nghifimule, 2016, pp. 83-89

The criteria stated above partly incorporate connections under criterion B: Aspect F (progression through the series), Aspect G (promotion of cross-curricular issues), Aspect H (the representation of Namibia's diversity (culture and environment) and Aspect N (analysing thematic teaching/integration of other subjects). Although connection-making is recognised in the Namibian textbook evaluation criteria, the criteria do not state mathematical connections sufficiently and clearly. Hence, there is a gap in the Namibian evaluation criteria for assessing mathematical connections.

2.2.3 Connections

In this section, I elaborate on the literature surrounding connections and connection-making. The Merriam-Webster dictionary (2022) defines a connection as a causal or logical link, relation, or sequence between two or more ideas – for example, the relationship between mathematics and other subjects. Connection-making is the process of linking, creating or building a relationship between two or more things. In mathematics education, connection-making occurs when learners connect or create links between mathematical ideas, facts, procedures and relationships within mathematics and beyond mathematics (MoE, 2015b). The layout of this section is as follows:

- Mathematical connections
- What are connections?
  - Real-life Connections
  - Connections to other subjects
  - Connections to other mathematical content areas
  - Visualisation connections
  - Connections in problem-solving

2.2.3.1 Mathematical connections

Mathematics as a subject consists of interrelated concepts. This occurs not only between mathematical concepts but also relating to relationships to other disciplines and real life. The statement of The National Council of Teachers of Mathematics (NCTM) supports this: "Mathematics is not a collection of separate strands or standards, even though it is often partitioned and presented in this manner. Rather, mathematics is an integrated field of study. When learners connect mathematical ideas, their understanding is deeper and more lasting, and they come to view mathematics as a coherent whole" (NCTM, 2000). Kline (2015) stated "Mathematics is not an autonomous knowledge that can be perfect by itself, but was mainly to help people in understanding and mastering the problems of social, economic, and nature" (p. 38).

The NCTM published the Principles and Standards for School Mathematics for the USA in 2000 (NCTM, 2000). The Principles cover equity, curriculum, teaching, learning, assessment and technology. The Standards are explicitly related to numbers and operations, algebra, geometry, measurement, data analysis and probability, problem-solving, reasoning and proofing, communication, connections and representation. All these Standards were identified as critical for all levels of school mathematics. The connections standard specifies that instruction for all school levels should:

- ✓ Recognise and use connections among mathematical ideas: by emphasising mathematical connections, teachers can help learners build a disposition to use connections in solving mathematical problems rather than see mathematics as a set of disconnected, isolated concepts and skills.
- ✓ Understand how mathematical ideas interconnect and build on one another to produce a coherent whole. As learners progress through their school mathematics experience, their ability to see the same mathematical structure in seemingly different settings should increase. As learners develop a view of mathematics as a connected and integrated whole, they will have less of a tendency to view mathematical skills and concepts separately.
- ✓ Recognise and apply mathematics in contexts outside of mathematics. School mathematics experiences at all levels should include opportunities to learn about mathematics by working on problems arising in contexts outside of mathematics. These connections can be to other subject areas, disciplines and learners' daily lives.

The connections standard emphasises the importance of connections among mathematical topics and those between mathematics and other disciplines.

2.2.3.2 What are connections?

Making connections has proven to be an essential aspect of teaching and learning mathematics. Mhlolo (2012) postulates that "educational standards all over the world recommend that teachers enable pupils to recognise and to make connections among relatable mathematical ideas" (p. 176). In addition, Mwakapenda (2008) and Parker (2006) concur that making connections is among the essential experiences learners should gain in the mathematics classroom. Researchers broadly define mathematical connections as:

- a relationship between ideas or processes that one can use to link topics in mathematics;
- a process of making or recognising links between mathematical ideas;
- an association, a person might make between two or more mathematical ideas; and
- a causal or logical relationship of interdependence between two mathematical entities (Mhlolo et al., 2012, p. 2).

Empirical evidence shows that learners' ability to recognise connections directly links to their mathematical understanding (Silver et al., 2009). Barmby et al. (2009) claim that:

> To examine someone's understanding of a mathematical concept, it is important that we examine the connections that a person makes to that concept. Of course, we cannot see these internal connections directly; rather, we must observe the connections that a person can demonstrate and then infer understanding from these (p. 221).

In this regard, Weinberg (2001) advocates that learners should be made explicitly aware of these different possible mathematical connections through instruction that emphasises the interrelatedness of mathematical ideas. Furthermore, learners' understanding becomes more profound and more lasting, and learners come to view mathematics as (a) a coherent whole; (b) as connected with other subjects; and (c) as connected to their interests and experiences. The above resonates well with this study, although the focus and analysis will go beyond simple mathematical connections.

The Namibian mathematics curriculum suggests that one of the aims of mathematics education is to enable learners to apply mathematics in everyday life. In addition, it should also link to other subjects and within itself and cross-curricular issues (MoE, 2015b). I am interested in researching how textbooks promote connection-making in the context of fractions, as teachers find this topic difficult to teach (Ausiku, 2022; MoE, 2017; Nambira et al., 2009).

The study will analyse how these three textbooks support, motivate and embrace the process of connection-making in the context of fractions. Connections between mathematics topics and concepts within and across grade levels, mathematics and other subjects, and mathematics and everyday life make mathematics understandable and meaningful (Annenberg Foundation, 2017). Specifically, the analysis of connection-making will focus on the following:

2.2.3.3 *Real-life connections*

In my teaching experience, learners have repeatedly questioned the necessity of learning certain mathematics topics, such as fractions, in relation to their real-life contexts. Real-life connection-making is a skill that fosters an understanding and appreciation for numbers by focusing on the relationship between mathematical concepts and learners' experiences and interests (Karakoc & Alacaci, 2015). In particular, Kilpatrick et al. (2001) noted that "students' informal notions of partitioning, sharing, and measuring provide a starting point for developing the concept of rational number" (p. 232). Referring to learners, Kilpatrick et al. (2001) further noted that "their experience in sharing equal amounts can provide an entrance into the study of rational numbers" (p. 232). This was also supported by McLeod and Newmarch (2006) when they advised teachers to "use the knowledge and experience that all adult learners already have of fractions, particularly halves and quarters, and also the fractions that are part of everyday language" (p. 12). Real-life connections can bring benefits such as facilitating a better understanding of mathematical concepts, motivating learners and improving learners' attitudes towards mathematics (Gainsburg, 2008).

One of the most obvious ways fractions appear in daily life is through food and recipes. Precise measurements, often expressed in fractions, are crucial for successful baking or cooking. A recipe might call for $\frac{1}{2}$ cup of milk or $\frac{3}{4}$ teaspoon of baking powder. Dividing a cake or pizza into equal portions necessitates the understanding fractions. Time is another realm where fractions are ever-present. We use fractions of hours to represent minutes and seconds: 60 minutes in 1 hour, 60 seconds in 1 minute. Schedules like the school timetable often incorporate fractions, like how long the mathematics period lasts. Learners also use fractions to plan for their after school activities.

2.2.3.4 Connections to other subjects

Karakoç and Alacaci (2015) suggest that learners should not learn mathematics in isolation because it has significant relationships with other curriculum subjects – mathematics links to other curriculum subjects in numerous ways. For example, in Geography, learners use ratios to determine scale, graphs and statistics to analyse data; in Home Economics, learners use mathematics when budgeting and making appropriate money judgements. Teachers are encouraged to discuss with colleagues in other subject areas to identify and develop agreed-upon approaches to teaching standard procedures in mathematics (Connections in Math, n.d.).

In the realm of physical science, including physics, astronomy, and geology, mathematics forms the bedrock of analysis (Wilson, 2019). The study of motion and forces relies heavily on formulas involving distance, time, velocity, acceleration, and force. Furthermore, mathematics is essential for comprehending waves and energy transfer. Moving to the domain of chemistry, mathematics plays a vital role in understanding and manipulating chemical reactions (Zumdahl & DeCoste, 2020). The concept of stoichiometry, which deals with the balanced ratios of reactants and products in a chemical equation, utilizes mole ratios, molar masses, and unit conversions. These calculations ensure accurate predictions of reaction yields, a crucial aspect of laboratory experiments and industrial processes.

Biology also harnesses the power of mathematics to delve into the complexities of life. Population ecology, a field studying the abundance and distribution of organisms, utilizes differential equations and population genetics models to predict population growth, competition for resources, and the long-term viability of species (Begon, Townsend, & Harper, 2006). Mathematical analysis in genetics helps analyze inheritance patterns and predict the outcomes of genetic crosses, forming the backbone of modern breeding programs (Lewontin, 2000). Furthermore, in biochemistry, the study of enzymes and metabolic pathways heavily relies on mathematical modeling and the application of calculus to understand enzyme kinetics (Voet & Voet, 2013).

2.2.3.5 Connections to other mathematical content areas

The concept of fractions must be linked to other mathematical content, as shown in the figure below. Some obvious connections are decimals, percentages and ratios. Figure 2.2 below shows examples of mathematical content which integrates fractions.

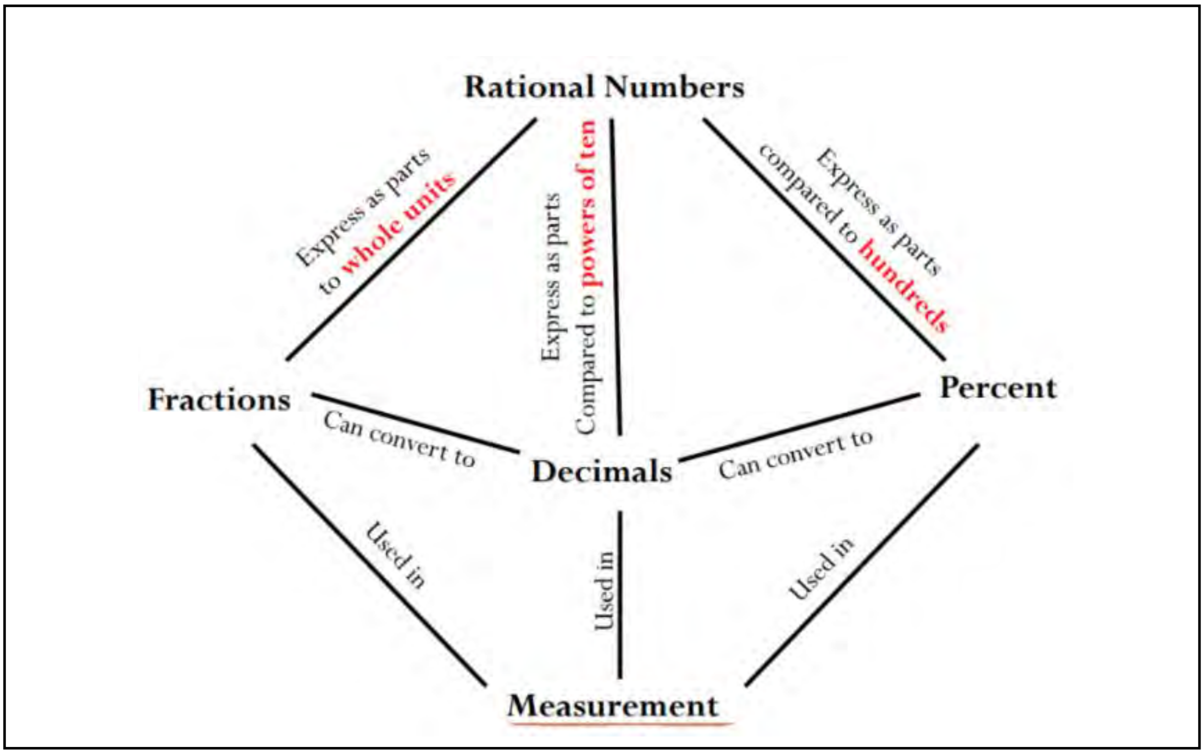


Figure 2.2: Examples of fractions within other mathematical content
Source: Connections in Math (n.d.)

2.2.3.6 Visualisation connections

Researchers suggest that when teachers use visual connections in their lessons, learners can see how mathematics fits into their daily lives by visualising mathematics (Blum, 2002; Cuban, 1976; Özdemir & Üzel, 2011; Patton, 1997). Mathematics becomes a visible phenomenon in their routines instead of abstract rules and concepts confined to the classroom. The abstract nature of mathematics is one of the reasons that may lead learners to think that mathematics is complex (Karakoç & Alacaci, 2015). Making visualisation connections in mathematics lessons is one way to mitigate this problem.

2.2.3.7 Connections in problem-solving

When solving mathematical problems, Muijs and Reynolds (2017) suggest that teachers use realistic examples leading to mathematical solutions. This connection-making process helps to link mathematical and real-world knowledge and applications (p. 261). Armitage (2019) suggests that asking learners questions that allow them to show and share what they know, connections become a natural part of their solutions.

2.2.4 Fractions

In this section, I elaborate the literature in relation to fractions. The layout of this section is as follows:

- Significance of learning fractions in schools
- Fraction constructs

2.2.4.1 Significance of learning fractions in schools

Fractions are an integral part of any school mathematics curriculum. They have rich meanings and features in mathematical areas such as algebra, geometry, probability and trigonometry. Bruce et al. (2013) asserted that "…… they are also deep, effecting foundational understandings that help or hinder the learning of other areas of mathematics" (p. 6). However, McLeod and Newmarch (2006, p. 12) observed that "all too often learners think of 'fractions' as being a discrete (and often difficult) topic that has no real connection with any other area of Maths". Bezuk and Cramer (1989) recognised the necessity to adequately develop concepts of fractions and their relationships within themselves or with other domains in mathematics. Thus, fractions should be integrated into different grades and subjects. Proficiency in fractions is essential for learners to be successful in mathematical problem-solving activities. Learners should feel confident and comfortable with computing fractions.

The foundations of engaging with and understanding fractions start early in primary education and continue to be critical components of more abstract concepts as one progresses through the grades. In my experience, learners often enter Grade 7 with a poor foundation in fraction concepts they should have learned earlier. Niemi (1996) says it is fundamental for any learner, especially in Grade 7, to understand fractions in order to cope with more advanced topics in the high school curriculum.

Difficulties in working with fractions can result in misconceptions about fractions and other related mathematical ideas. Barnett (2016) points out that "working with fractions is usually the first time in their schooling when learners give up on making sense of a concept and resort to simply following the procedure demonstrated by the teacher" (p. 12). Van de Walle et al. (2014) assert that a poor understanding of fractions often contributes to a misapplication of learners' whole number knowledge. Woodward (1998) notes that teachers often grapple with the concepts of fractions.

Learners' conceptual understanding of fractions rests on their ability to conceptually understand the meaning of fractions, reason about their size and make sense of operations with fractions. These are considered foundational knowledge of fractions (Bruce et al., 2013; Dyson et al., 2020; Van de Walle et al., 2014). Challenges encountered by learners in fractions attributed to poor fraction sense can adversely affect their understanding of other related mathematics concepts. Woodward (1998) defines fraction sense as:

> [A]n individual's ability to understand the meaning of fractions; to reason qualitatively about the absolute and relative size of fractions; and to make logical judgments about the reasonableness of calculations with fractions based on one's understanding of fractional numbers and the effect of operations on those numbers. (p. iii)

2.2.4.2 Fraction constructs

Many researchers have worked with different interpretations of fractions and their interconnectedness. Kieren (1976), for example, argued that an individual must have experience with multiple interpretations of fractions to understand fractions completely. Fractions consist of several sub-constructs. The four interrelated sub-constructs of ratio, operator, quotient and measure were first recognised (Freudenthal, 1983; Kieren, 1980; Vergnaud, 1983). Presently the part-whole is an essential aspect of fractions and the most commonly used. Kieren (1983) did not initially see the part-whole as a separate sub-construct because he identified the part-whole relationship as the foundation of rational number knowledge. Furthermore, he argued that the part-whole relationship encourages the development of the four sub-constructs of measure, quotient, operator, and ratio and is embedded in all of them. Behr et al. (1983) agreed with Kieren (1976) on the part-whole sub-construct as essential in understanding the other four sub-constructs. They, however, contested the idea that the part-whole could not form a separate sub-construct. Thus, they distinguished between part-whole, measure, quotient, ratio and operator as sub-constructs of fractions. Behr et al. (1983) suggested that the part-whole sub-construct coupled with partitioning, were the most fundamental constructs for rational number development; both were recognised as the basis for learning other sub-constructs.

Several studies revealed that the part-whole model might inhibit the development of the other sub-constructs, particularly the quotient sub-construct (Charalambous & Pitta-Pantazi, 2007; Pitkethly & Hunting, 1996). For example, when the fraction 'three-quarters' is only understood as three parts of a cake divided in quarters and not as a fair share of three wholes divided equally amongst four. Charalambous and Pitta-Pantazi (2005) concluded that of the five sub-constructs of fractions: "the part-whole interpretation of fractions should be considered as a necessary but not sufficient condition for developing and understanding of the remaining notions of fractions" (p. 239).

The findings from Charalambous and Pitta-Pantazi (2005), The Fractions Project (Steffe & Olive, 2010) and Wilkins and Norton (2018) encourage us to examine and explore how the different sub-constructs work together in practice, to provide a complete understanding of fractions. They maintain that focusing only on the part-whole sub-construct leads to a fragmented and partial understanding of fractions. Their work reveals constraints and limitations in the Behr et al. (1983) model of the sub-constructs of fractions. More complete understandings of fractions are achieved when teachers allow their learners to work with tasks that allow them to link the part-whole, measure, operator, ratio and quotient sub-constructs, and the unifying elements. This is very important for my research since I am interested in the mathematical connections in fractions presented in the textbooks. Analysing the selected textbooks will provide insight into what mathematical connections and fraction knowledge are facilitated for learners.

To meaningfully comprehend the concept of fractions, learners should understand their various fraction constructs (Bruce et al., 2013; Clarke, 2011; Van de Walle et al., 2014). According to Clarke (2011), the five fraction constructs which learners should master are part-whole, measures, operators, quotient, and ratio. See Table 2.2 below.

Table 2.2: The five fraction constructs adapted from Richardson (2019, pp. 21-22)

Fraction Construct	**Explanation**
Part-whole	It is a way of representing a part of a whole set of objects. It involves the partitioning of a shape/number of discrete objects into equal pieces (unitising) or determining how many parts would be in a whole set based on the part of the set (re-unitising).
Measure	It involves identifying a length and then using that length as a measurement piece to determine the length of an object (Van de Walle et al., 2013, p. 311). It is an appropriate construct to represent the magnitude of fractions.
Division/quotient	Division is the understanding of fractions that results from division/sharing.
Operator	Operator involves the multiple aspects of fractions, eg $\frac{2}{3}$ as finding two-thirds of the quantity.
Ratio	Ratio involves the comparison of part-part or part-whole. For example, the fraction $\frac{2}{3}$ could mean the proportion of those wearing jackets (part) to those not wearing jackets (another part), or it could represent those wearing jackets to the total number of learners in class (whole) (Van de Walle et al., 2014).

Bruce et al. (2013) attribute learners' poor understanding of fractions to traditional fraction instruction, which is predominantly part-whole oriented. Part-whole orientation deprives learners of the opportunity to see and extend their knowledge of fractions. Teaching based on the five fraction constructs enables learners to understand the nature of fractions and master their diverse, complicated operations (Zhang, 2018). Charalambous et al. (2010) found that "engaging students with multiple fraction constructs catalyses learning, not only because each fraction construct captures part of the broader notion of fractions, but also because moving from one construct to another reinforces understanding" (p. 142).

2.2.4.3 The part-whole sub-construct

The part-whole is an essential aspect of fractions and the most commonly used. Mamede et al. (2005) offer an understanding of this sub-construct. In the part-whole situation, the denominator describes the number of parts a whole has been divided into, while the numerator suggests the number of parts taken. Figure 2.3 below shows a diagram where the whole has been divided into four equal parts and three parts have been shaded, which can be represented by the fraction $\frac{3}{4}$.

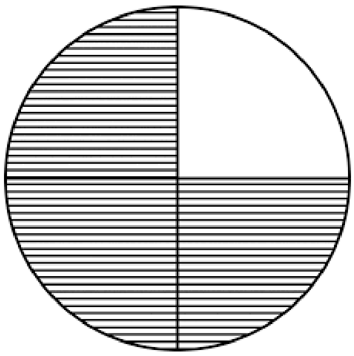


Figure 2.3: A part-whole representation of the fraction $\frac{3}{4}$

From the example above, the fraction represents the shaded part of a whole (circle) partitioned or divided into equal-sized pieces. The numerator refers to the number of shaded parts of the partitioned unit. In contrast, the denominator refers to the total number of equal-sized pieces (parts into which the unit is partitioned). Within any whole unit, the smaller the denominator, the bigger the partitions, resulting in fewer pieces. The bigger the denominator, the smaller the partitions, resulting in more pieces. When solving problems related to the part-whole sub-construct, it is important to establish how 'the whole' is considered and what is considered 'the part' of the whole. The resulting relationship between the dividend and the divisor is called the quotient. The example above illustrates that an object has been partitioned or divided into four equal parts, and three of those parts are taken. In cases where parts are taken from a whole, the numerator must be less than or equal to the denominator (Wong & Evans, 2008).

The part-whole sub-construct applies to continuous quantities and sets of discrete objects (Sowder et al., 1998), so whether the context involves part of a whole or part of a set, it is still considered a part-whole sub-construct. Regarding fraction instruction, using the correct model (for example, continuous quantities or discrete units to match the context) is important. If the context is related to the part of a set, discrete objects would be used, and if the context is related to the part of a whole, such as pizza or cake, then an area model would be appropriate (Post et al., 1982).

As noted, Behr et al. (1983) linked the part-whole sub-construct with the process of partitioning – one of the key unifying elements. The part-whole sub-construct involves the notion of partitioning or dividing a continuous quantity (including area, length and volume models) or a set of discrete objects into several equal parts (for example, four-sevenths as parts of a whole are interpreted as four of seven equal-sized parts (Lamon, 2020). Equal parts can be composed and recomposed to the initial whole (Lamon, 2005). Partitioning is a unifying element that connects the sub-constructs. The unit refers to the whole in the activities described below.

Activities used to develop the part-whole sub-construct of fractions include:

- ✓ partitioning of the unit, with this partitioning sometimes incorporated explicitly or given implicitly;
- ✓ identifying how many equal-sized parts make up a unit;
- ✓ unitising the unit, which is related to equivalent fractions, for example, mentally chunking the area into different-sized pieces and using this to name equivalent fractions (Lamon, 2020, p. 135); and
- ✓ providing units of various types, for example, discrete and continuous units to work with (Lamon, 2020).

Several ideas have developed from research, regarding the mastery of the part-whole sub-construct and the partitioning scheme (Charalambous & Pitta-Pantazi, 2005). Lamon (2020) suggests the following:

Learners must:

- understand that the parts into which the whole has been partitioned must be of equal size;
- be able to partition a discrete set or a continuous whole into equal parts;
- be able to identify whether the whole has been partitioned into equal parts;

- understand that the parts taken together must equal the whole regardless of the equivalent parts' size, shape, or arrangement;
- understand that the more parts the whole is partitioned into, the smaller the produced parts become; and
- develop unitising and reuniting abilities, which allow learners to reconstruct the whole based on its parts.

2.2.4.4 The measure sub-construct

The concept of the unit is central to the measure sub-construct. The measure sub-construct is used when working with number lines (finding points on a number line) (Behr et al., 1992). Iterating is an important aspect of the measure sub-construct (Van de Walle et al., 2014; Wilkins & Norton, 2018). The measure sub-construct can also facilitate the learners' understanding and development of adding and subtracting, fractions by helping them grasp that adding fractions with like denominators is adding iterations of the unit fraction that consists of the original fractions (Son & Lee, 2016). For example, if one recognises $\frac{3}{9} + \frac{4}{9}$ as three pieces of $\frac{1}{9}$ and four pieces of $\frac{1}{9}$, then combined, the answer is seven pieces of $\frac{1}{9}$ or $\frac{7}{9}$. Understanding this can eradicate the error of adding across numerators and denominators, ie, $\frac{3}{9} + \frac{4}{9} = \frac{7}{9}$ (Mack, 1995). Moreover, it can be extended to the addition and subtraction of unlike denominators (McNamara, 2015).

The measure sub-construct differs from other sub-constructs since a unit's number of equal parts varies according to how many times it is partitioned (Lamon, 2005; Van de Walle et al., 2014). Unlike the part-whole sub-construct that highlights how many parts the whole is divided into (which is referred to as the denominator of the fraction), the measure sub-construct highlights parts of the quantity needed out of the whole and is called the numerator of the fraction presentation (Van de Walle et al., 2014). Successively partitioning the unit in the measure sub-construct differs from the other interpretations in important ways. In the part-whole interpretation, comparing the number of equal parts taken from the whole requires a fixed number of equal parts in a unit. In contrast, the measure sub-construct allows the number of equal parts in the unit to vary and naming the fractional amount depends on how many times the partitioning process is carried out (Lamon, 2020; Van de Walle et al., 2014). When we take a part of a whole, we divide the whole into equal parts to establish a sub-unit (Naik & Subramaniam, 2008).

The sub-unit measures out the taken part. According to Lamon (2020), emphasis on the counting aspect at the cost of the measurement aspect is what has dominated the traditional teaching of fractions. Partitioning involving the formation of unit structures is underplayed in classrooms. The measure construct focuses on successively partitioning the unit since this allows us to measure with increasing precision (Lamon, 2020; Sowder et al., 1998). The measurements are called ‘points’, which can be modelled on a number line. Figure 2.4 illustrates how fractions can be presented on a number line.

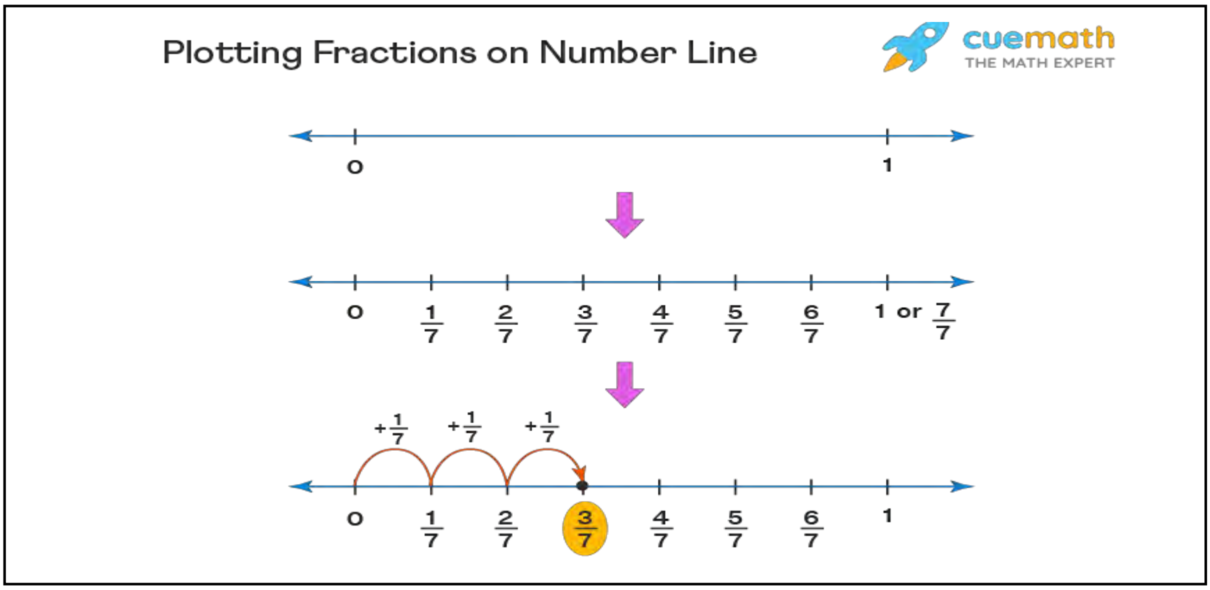


Figure 2.4: Examples of fraction representations on a number line

Source: Khurma, 2023, n.p.

The measure sub-construct also allows us to measure any amount by changing the unit of measure. For example, if litres in a problem do not work, we can partition them into millilitres. The measure sub-construct develops a strong notion of the unit and subintervals (Lamon, 2005; Wilkins & Norton, 2018). Class activities that include measure as a sub-construct would be successive partitioning of a number line and reading meters and gauges. Through successive partitioning, the measure sub-construct helps one learn about adding fractions (Behr et al., 1992) as equivalences emerge. Understanding equivalence aids learners in determining common denominators when adding and subtracting fractions. This can be achieved before introducing algorithms, resulting in a deeper understanding of fractions (Lamon, 2020).

The partitioning process helps learners understand the density of rational numbers, relative sizes, relative locations and fraction equivalence. According to Lamon (2020), “the density of rational numbers says that between any two fractions, there is an infinite number of fractions and that you can always get as close as you like to any point with a fraction" (pp. 173-174). Lamon suggests that for learners to master the measure sub-construct, they must:

- understand the concept of the density of the rational number, and
- be able to measure any distance from the origin using a given unit interval.

2.2.4.5 The division/quotient sub-construct

Unlike the part-whole sub-construct, the denominator in the quotient sub-construct describes the number of shares, and the numerator describes the number of items shared. Mamede et al. (2005) point out that a connection must be made between the meaning of the denominator (divisor) and the numerator (dividend) rather than treating these two quantities separately. Fractions as the result of division are called quotients. The quotient sub-constructs hold the idea that any fraction is the answer to a division sum. For example, $\frac{x}{y}$ is the 'fair share' of each person when a pizza, x, is shared amongst y people, where both x and y are whole numbers (Streefland, 2012). The fraction $\frac{x}{y}$ can also be considered as $x \div y$. This interpretation also comes from a partitioning situation. The numerator can be smaller than, bigger than or equal to the denominator in the quotient sub-constructs. Also, the answer (quotient) after equal sharing may be smaller than, bigger than or equal to the whole (unit). Therefore, two different measured spaces are considered, for instance, pizzas and people, and the outcome "refers to a numerical value and not the parts obtained by fair-sharing" (Charalambous & Pitta-Pantazi, 2007, p. 299).

Lamon (2020) suggests that for learners to master the quotient sub-construct, they:

- must understand and recognise that, unlike in the part-whole sub-construct, there are two different measure spaces for partitive division (eg, three CHOCOLATES shared among four FRIENDS);
- must understand that there is no restriction to the size of the fraction. It means that the numerator can be larger, equal to or smaller than the denominator, resulting in the answer being equal to, less or more than the unit (or whole);
- associate fractions with division and understand the role the dividend and divisor play; and
- must develop a thorough understanding of quotative and partitive division.

2.2.4.6 The operator sub-construct

The operator sub-construct describes the fraction as a function applied to a number, and rational numbers are "regarded as functions applied to some number, object, or set" (Behr et al., 1983; Marshall, 2012). Here, a fraction multiplies a quantity and either increases or decreases it. Behr et al. (1983, p. 96) referred to this as a 'stretcher/shrinker'. A function is applied to a quantity (number, object or set) when trying to find a fraction of it. A new value is formed when the operation shrinks (reduces) or stretches (enlarges) the quantity (number, object or set). This operation is a combination of multiplication and division.

The operator sub-construct uses a fraction to operate on a quantity or the result of a previous operation (Behr et al., 1983; Kerslake, 1986; Lamon, 2005; Post et al., 1982). It involves both shrinking and enlarging. For example, finding $\frac{3}{4}$ of a number, of a set of things, of a segment or of a geometrical picture, the operation would be to divide by four and then multiply by three or to multiply by three and divide by four. The function transforms the discrete set into another set with fewer or more elements. Calculating $\frac{3}{4}$ of 16 (sweets, centimetres, books, rands) can be done visually or numerically. Numerically, one would multiply 16 by three and then divide by 4: $(16 \times 3) \div 4 = 12$ (sweets, centimetres, books, rands). Figure 2.5 shows how one can calculate this alternatively through a visual representation.

Lamon (2020) suggests that for learners to master the operator sub-construct, they must:

- interpret a fractional multiplier in a variety of ways (eg $\frac{3}{4}$ should be interpreted either as $3 \times [\frac{1}{4}$ of a unit] or $\frac{1}{4} \times$ [3 times a unit] (Lamon, 2020);
- name a single fraction to describe a composite operation (when multiplication and division are performed, one on the result of the other) (Lamon, 2020); and
- identify the effects of an operator and relate inputs and outputs using a rule (eg, an input of 6 and an output of 13 results from a 13-for-6 operator (an operator that enlarges), thus the output is $\frac{13}{6}$ of the input.

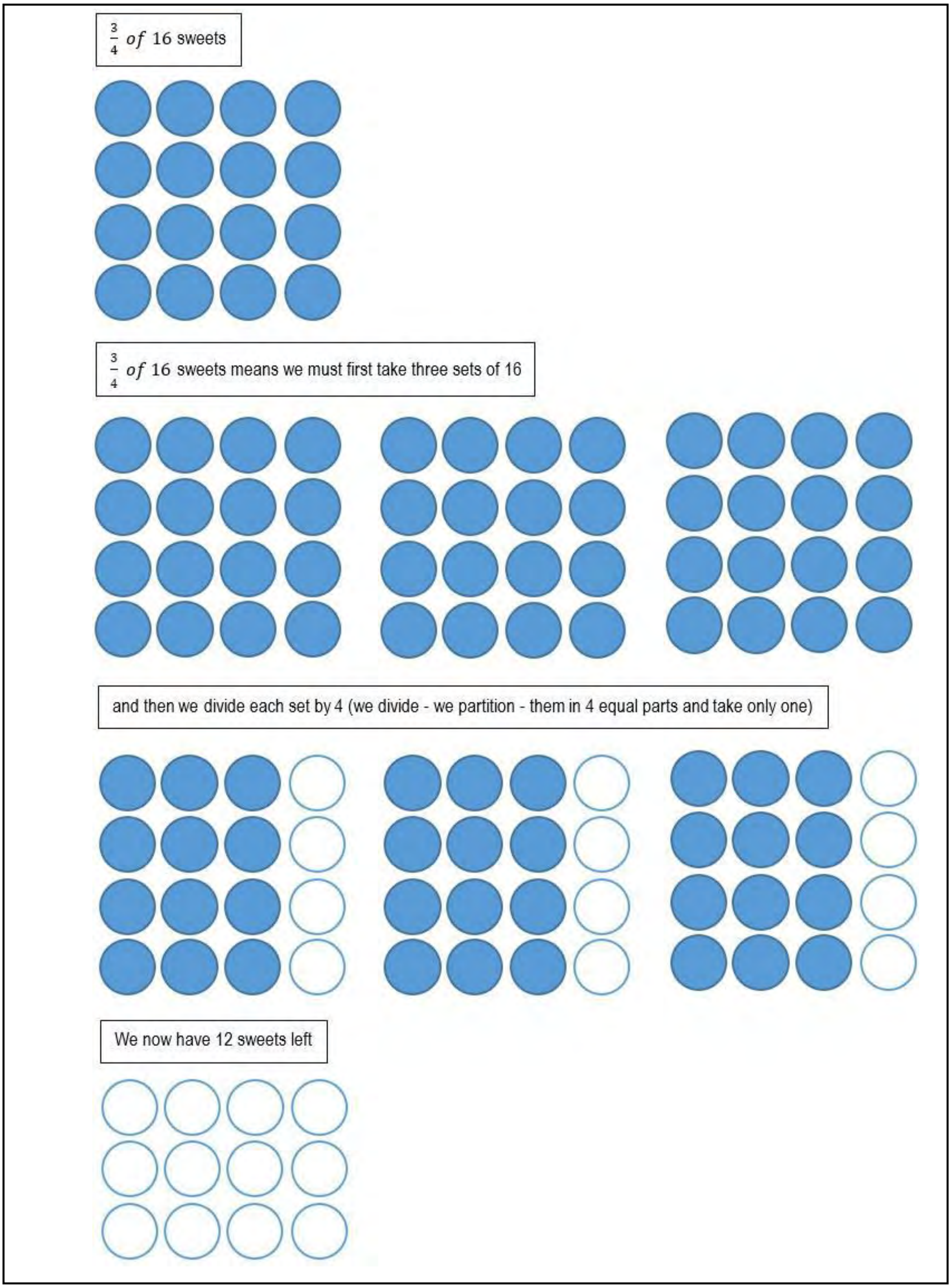


Figure 2.5: Fractions as operators applied to a set
Source: Charalambous & Pitta-Pantazi (2007, p. 312)

2.2.4.7 The ratio sub-construct

The ratio is the fifth and final sub-construct of fractions. The ratio sub-construct of fractions conveys the notion of comparing two quantities; therefore, it is considered a comparative index rather than a number (Carraher & Buckley, 1996). Hence, comparing two quantities in a given order is a ratio (Behr et al., 1983; Charalambous & Pitta-Pantazi, 2006; Lamon, 2020; Streefland, 1991). Fractions can be used in a ratio format. Thus, Behr et al. (1983) described fractions as ratios:

> Ratio is a relation that conveys the notion of relative magnitude. Therefore, it is more correctly considered as a comparative index rather than as a number. When two ratios are equal they are said to be in proportion to one another. A proportion is simply a statement equating two ratios. (p. 94)

The ratio sub-construct differs from the measure, part-whole, and operator sub-constructs in that the numerator and the denominator do not refer to the same quantity. Unlike the quotient and part-whole sub-constructs, the ratio sub-construct does not involve the idea of partitioning (Reys et al., 2014). Rate represents an extended ratio, "a ratio that applies not just to the situation at hand, but to a wide range of situations in which two quantities are related in the same way" (Lamon, 2020, p. 235). For example, N$3 per metre (m) is a rate that describes the relationship between cost in dollars and the number of metres in all the following instances: N$6 for 2m, N$24 for 8m, N$54 for 18m, and so on.

While the other four sub-constructs add, subtract, multiply and divide according to the same rules, ratios do not follow the same rules that can be used for fractions. For example, $\frac{3}{4}+\frac{2}{3}=\frac{3+2}{4+3}$ when $\frac{3}{4}$ and $\frac{2}{3}$ are ratios $=\frac{5}{7}$. When $\frac{3}{4}$ and $\frac{2}{3}$ is not a ratio, addition is done as follows: $\frac{3}{4}+\frac{2}{3}=\frac{17}{12}$ (Behr et al., 1992). Lamon (2005, p. 229) exemplifies this using a real-world context:

> "Yesterday Mary had 3 hits in 5 turns at bat. Today, she had two hits in six times at bat. How many hits did she have for a two-day total?
>
> Mary had 3:5 + 2:6 = 5:11 or 5 hits in 11 times at bat. If we were adding fractions, we could not write $\frac{3}{5}+\frac{2}{6}=\frac{5}{11}$ ".

Ratio and rate sub-constructs help develop a strong notion of equivalence and proportionality (Behr et al., 1983; Kieren, 1993). Lamon (2005) has argued that learners who studied ratio and rate as their primary sub-construct of rational numbers could switch between ratio and part-whole comparisons and experienced no difficulty with addition and subtraction of fractions (Lamon, 2005). Understanding rate/ratio fractions also helped the learners develop their own ways of reasoning about multiplication and division of fractions.

Lamon (2005) and Marshall (2012) suggest that to master the ratio sub-construct, learners must:

- construct the idea of relative amounts, ie, comparing measures of two parts of the same set (a part-part comparison) or the measures of any two different quantities (eg, cups of juice concentrate to cups of water);
- understand the relationship between two quantities and that they change together so that their relationship remains invariant; and
- realise that when two quantities are multiplied by the same non-zero number, the value of the ratio remains the same.

This discussion of the different sub-constructs of fractions was helpful in this study in offering insight into the range of sub-constructs necessary to develop an understanding of fractions for learners in Grades 7. The following section explains the importance of visualisation in mathematics education and fractions.

2.2.5 Visualisation

To visualise can mean to see something either in your mind or physically. In this section, I discuss the literature in relation to visualisation. The layout of this section is as follows:

- Visualisation
- Visualisation in fractions

Mathematics textbooks need to include visual objects. Presmeg (2006) noted that "a visual image is taken to be a mental construct depicting visual or spatial information, and a visualiser is a person who prefers to use visual methods when there is a choice" (p. 207). Fotakopoulou and Spiliotopoulou (2008) deem visible objects in textbooks essential for teaching and learning mathematics. Arcavi (2003) defines visualisation as:

> … the ability, the process and the product of creation, interpretation, use of and reflection upon pictures, images, diagrams in our minds, on paper or with technological tools, with the purpose of depicting and communicating information, thinking about and developing previously unknown ideas, and advancing understandings. (p. 217)

Arcavi's notion of visualisation resonates well with my study, as it focuses on connection-making, which is the product of creation. Zazkis et al. (1996) define visualisation as an act in which an individual establishes a strong connection between an internal construct and something to which access is gained through the senses. Thus, visualisation can be a powerful tool for teaching and learning mathematics (Arcavi, 2003; Fotakopoulou & Spiliotopoulou, 2008; Konyalioglu, 2005). Presmeg (2014) elucidates that visual teachers can connect mathematics content and other aspects such as the syllabus, subjects, prior knowledge, factors beyond the syllabus and most importantly, the real world. A study done by Reimer and Moyer (2005) found that virtual manipulatives helped learners learn more about fractions in class. Using multiple representations makes the fraction concept more concrete and understandable.

Although visual objects in textbooks are essential teaching and learning tools and resources, they have only recently attracted research attention (Fotakopoulou & Spiliotopoulou, 2008). Visualisation as a mathematical teaching strategy is not standard in Namibia, but some recent researchers (Ausiku, 2022; Chikiwa & Schafer, 2019b; Miranda & Alder, 2010; Nghifimule, 2016) have indicated that its application in the pedagogy of mathematics has been gaining momentum. Recent studies on visualisation processes in Namibian mathematics classrooms reflect that these processes not only arouse learners' interest in mathematics but also enhance their understanding of a variety of mathematical concepts (Ausiku, 2022; Chikiwa & Schafer, 2019b; Dongwi, 2018; Katenda, 2018; Muhembo, 2017; Nghifimule, 2016). However, researchers warn that visualisation could result in unexpected results if not correctly incorporated into the mathematics pedagogy (Ausiku, 2022; Huron et al., 2014).

Mathematics is regarded by many as "the model of a discipline based on rational thought, clear, concise language, and attention to the assumptions and decision-making techniques that are used to draw conclusions" (Makina, 2010, p. 4), thus a visual approach to working with fractions can facilitate a deep, comprehensive understanding of fractions. Through the incorporation of visuals in mathematics lessons, learners are encouraged to manipulate, reason and visualise fractions instead of responding to traditional teaching methods which emphasise the memorisation of rules and procedures. Zimmermann and Cunningham (1991) noted that "visualisation offers a method of seeing the unseen. It enriches the process of scientific discovery and fosters profound and unexpected insights" (p. 2).

4.2.1.1 Visualisation in fractions

Using visual objects is consistent with Bruner's model of representing mathematical ideas from enactive (concrete objects) to iconic (pictures or drawings) to symbolic (abstract symbols) (Loong, 2013). The Concrete-Pictorial-Abstract (C-P-A) sequence is the key instructional strategy for developing primary mathematics concepts in Singapore (MoE, 2007, 2012). This sequence is evident in the Namibian mathematics textbooks. Figure 2.6 shows two examples of this sequence in one of the three primary mathematics textbooks.

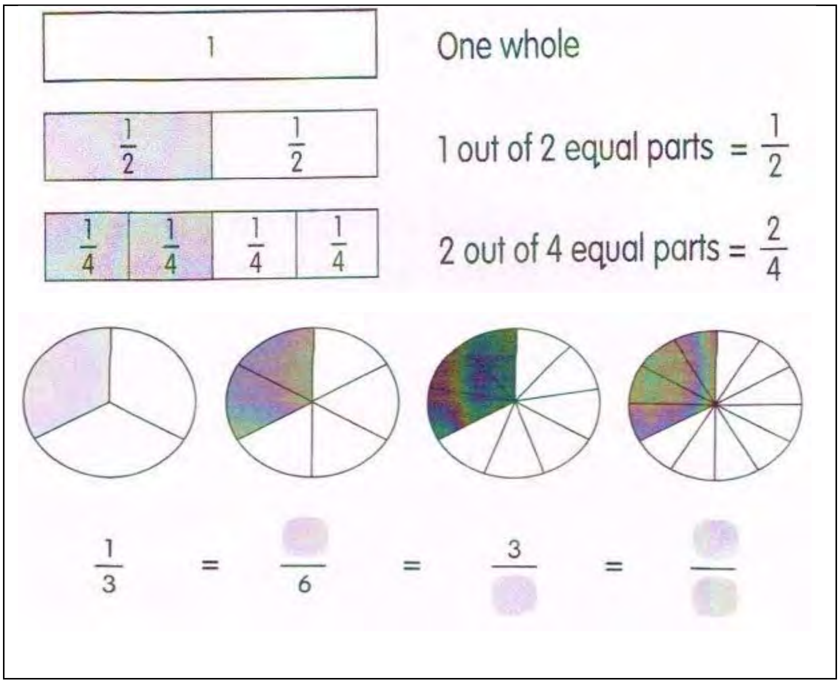


Figure 2.6: Examples of C-P-A sequence in one Singapore Primary Mathematics textbook
Source: Chang et al. (2017, p. 2)

In the textbook, pictorial representation (P) in the form of rectangular and circular models is used to introduce the concept of equivalent fractions. The parts of the area in the rectangular or circular models, representing respective fractions, depict the mathematical idea of equality. The learner learns that the parts of the area in the rectangular or circular models represent equal parts. With more exposure and practice with similar pictorial depictions, the learner eventually makes sense of the concept of equivalence between and among fractions. Soon, a reduced reliance on pictorial representations is observed while the operations with mathematical symbols or abstract (A) is observed. For example, when the equivalent fractions $\frac{a}{b} = \frac{c}{d}$ are understood, we can assume that there has been a successful transition from the pictorial to abstract (P-A). For example, Figure 2.7 below, taken from a Grade 7 Namibian textbook, gives a pictorial representation in the form of a rectangular model to compare common fractions.

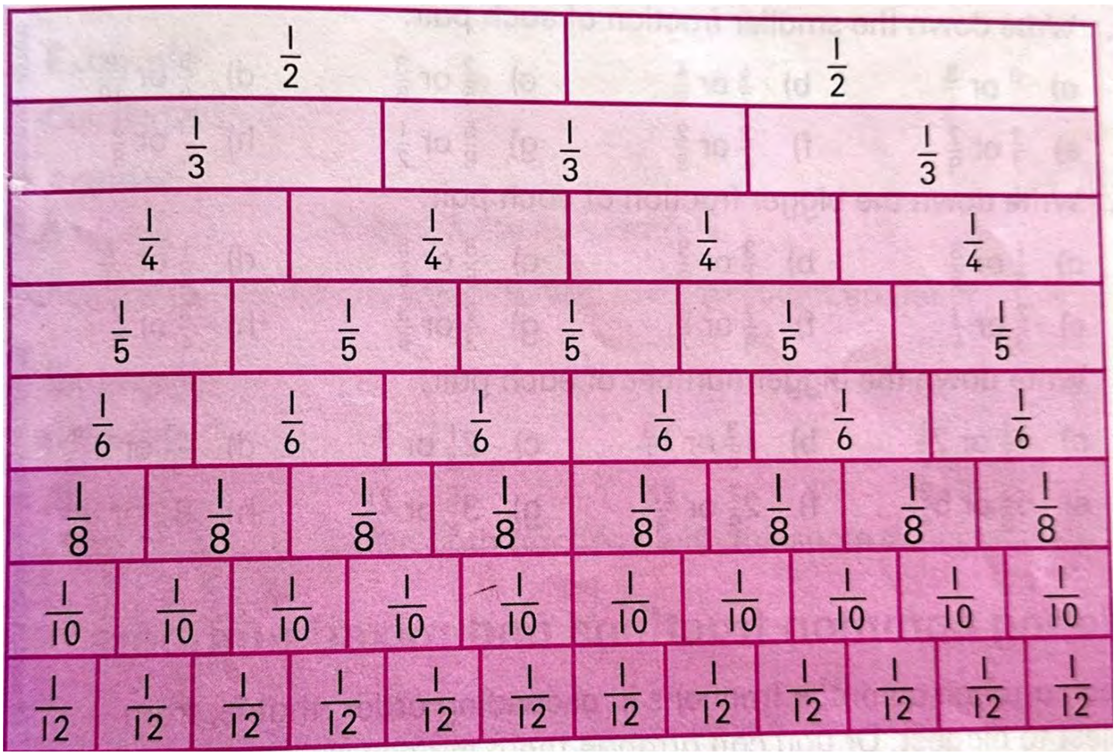


Figure 2.7: Example of representing fractions
Source: Hambata et al. (2015, p. 49)

A study by Reimer and Moyer (2005) found that virtual manipulatives helped learners in the class to learn fractions more effectively. Using multiple representations makes the fraction concept more concrete and understandable. However, Giaquinto (2015) warns us of the danger of over-generalisation when using images: for example, attempting to represent certain mathematical constructs that cannot be drawn diagrammatically. A visual approach to working with fractions can facilitate a deep, comprehensive understanding of fractions. Through the incorporation of visuals in mathematics lessons, learners are encouraged to manipulate, reason and visualise fractions instead of responding to traditional teaching methods, which emphasise the memorisation of rules and procedures. Zimmermann and Cunningham (1991) note that “visualisation offers a method of seeing the unseen. It enriches the process of scientific discovery and fosters profound and unexpected insights” (p. 2). The above explanations support my study because they illustrate the connection between visuals and fraction concepts. They also show how visualisation connections are essential in explaining the abstract concept of fractions.

2.3 THEORETICAL FRAMEWORK

2.3.1 Constructivism

The definition of constructivism is drawn from the developmental works of Vygotsky (1978), Piaget (1977), and the theoretical work of Bruner (1966). Maxwell (1992) describes constructivism as a "specific learning theory in which students' learning is based on experimentation, observation, and speculation about their own experiences" (p. 1). Caffarella and Merriam (1999) explain that the "constructivist stance maintains that learning is a process of constructing meaning; it is how people make sense of their experience" (p. 9).

Constructivism is grounded in the idea that all knowledge is constructed. The constructivist theory posits that the learner creates meaning from experiences by integrating prior knowledge with new knowledge (Eli et al., 2011). Through a constructivist lens, "mathematical knowledge is constructed, at least in part, through a process of reflective abstraction" (Noddings, 1990, p. 10). Learners' ability to construct and understand mathematical concepts, ideas, facts or procedures, thus involves making connections between old and new knowledge (Eli et al., 2011). Hiebert and Carpenter (1992) suggest that "many of those who study mathematics learning agree that understanding involves recognising relationships between pieces of information" (p. 67).

Making connections is integral to constructivism since learning for conceptual understanding involves building mental networks structured and connected like a spider's web, where "the junctures, or nodes, can be thought of as pieces of represented information, and threads between them as the connections or relationships" (Hiebert & Carpenter, 1992, p. 67). Eli et al. (2011) claim that "

> from a constructivist perspective, a mathematical connection can be thought of as a link (or bridge) in which prior or new knowledge is used to establish or strengthen an understanding of relationship(s) between or among mathematical ideas, concepts, strands, or representations within a mental network." (p. 299).

Gredler (2005) claims that the emergence of constructivism in mathematics education can be attributed to "dissatisfaction with information-processing theory, which suggested that students are acquiring isolated, decontextualised skills and are unable to apply them in real-world situations" (p. 89). Constructivism emphasises and promotes making connections in the teaching of mathematics. A mathematical connection is associated with making links to make sense of what is happening in the world. This view of mathematics enables learners to relate their knowledge of mathematics to the newly established knowledge. Learners have an informal knowledge of mathematics and make connections by forming relations between the constructed or built knowledge and the knowledge taught. The construction of knowledge is done in the mind, therefore, connections should be made between the mind and the concrete example at hand. Mathematics textbooks should thus be rich sources of concrete examples and real-life contextualisation.

According to Rezat (2007), the research in mathematics education has shifted its attention from only focusing on an individual learner's cognition and knowledge, to also considering contextual, socio-cultural and situated aspects of mathematics learning and knowing. In particular, the socio-cultural perspective now encompasses not only the classroom community's practices and culture, but also the artefacts that are utilized to mediate mathematical content and knowledge. Textbooks are such artefacts used in mathematics classrooms.

Sosniak and Perlman (1990) assert that the power of textbooks lies in their ability to serve as resources that introduce readers to worlds that are not immediately obvious or cannot be experienced directly. A textbook is a bridge between the known and the unknown world in teaching and learning mathematics. Çimer and Coşkun (2018) noted that "

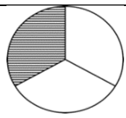
> according to the constructivist approach, learned information must be functional and should be transferable to practical applications … this requires textbooks to contain activities that help the students connect their learning to their lived reality." (p. 2010)

Hence, textbooks should offer opportunities for learners to learn more by themselves actively. Textbooks adopting a constructivist learning approach should help learners develop their learning skills, be active in their learning and motivate them to participate in their lessons (Dervişoğlu et al., 2004; Kabapınar, 2006; Kete & Acar, 2007; Kılıç, 2006; Küçükahmet, 2004; Esra & Hasenekoğlu, 2007). They should encourage learners to learn actively through thinking, investigating and analysing the provided information, creating new and original meanings and knowledge, and taking on responsibilities (Kabapınar, 2006; Özatalay, 2007; Pop-Pacurar & Ciascai, 2010). Textbooks have always been considered essential for teaching and learning mathematics worldwide. This notion is particularly true for fractions. In most cases, textbooks are the most readily available and accessible resource for teachers and learners in the classroom. Thus, textbooks are a necessary and critical means for knowledge construction, particularly in the concept of fractions, as these concepts are not always evident and apparent.

2.3.2 Constructivism, Bruner's model and fractions

Loong (2012) suggests that Bruner's mode of representing mathematical ideas is a practical example of the application of constructivism to fractions. To understand how learners', learn, Bruner (1990) developed three stages of representation: the enactive, iconic, and symbolic modes as mentioned above. Drews and Hansen (2007) noted that "these three modes of representing our experience are considered important to the development of children's understanding" (p. 20). See Table 2.3 below.

Table 2.3: Bruner's modes of representing mathematical ideas

Enactive	Iconic	Symbolic
Fractions as Part of a Set	Fractions as Part of a Whole	Fractions as Ratio of Two Numbers
		$\frac{1}{3}$

Source: Loong (2012, p. 40)

The enactive mode involves representing ideas through taking some form of action, such as manipulating objects. The iconic mode involves representing those ideas using pictures or images (Drews & Hansen, 2007). The symbolic mode, which is significant for this study, involves views expressed through language or symbols, where learners are expected to judge and think critically (Bruner, 1990). The modes in Table 2.3 align well with my study because they illustrate the transition of real-life objects (in this case, types of apples) to diagrammatical mode (visualisation in fractions) and finally, the symbolic (abstract mathematics taught in schools). A constructivist's point of view suggests that "Students construct their own meaning from the words or visual images they see or hear" (Yackel, 2011, p.41). Hence, learners and teachers need various visual and textual resources, such as books, technological devices and media, to mediate connection-making and to construct knowledge. Figure 2.8, obtained from one of the analysed textbooks for this research, shows the division of a whole number by a fraction; 8 apples can be divided by $\frac{1}{2}$ which is equal to 16 equal halves ($8 \div \frac{1}{2} = 16$).

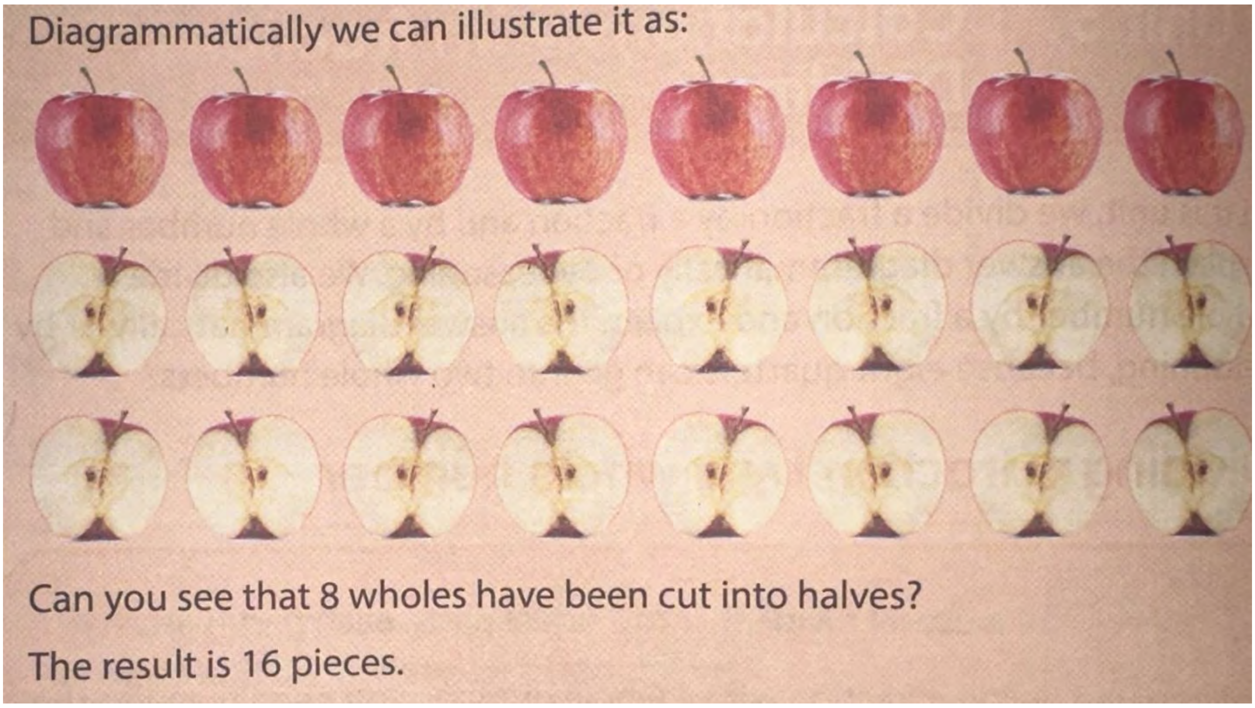


Figure 2.8: Reasoning with fractions diagrammatically
Source: Source: Mokwadi et al. (2015, p. 72)

Siebert and Gaskin (2006) suggest the application of partitioning and iterating when working with fraction concepts. They explain that partitioning is "creating smaller, equal-sized amounts from a larger amount". At the same time, iterating involves "making copies of a smaller amount and combining them to create a larger amount" (Siebert & Gaskin, 2006, p. 395). Learners can use images as tools to construct, create and act on fractions, and in this way, they create their own meaning of fraction concepts. See Figure 2.8 below.

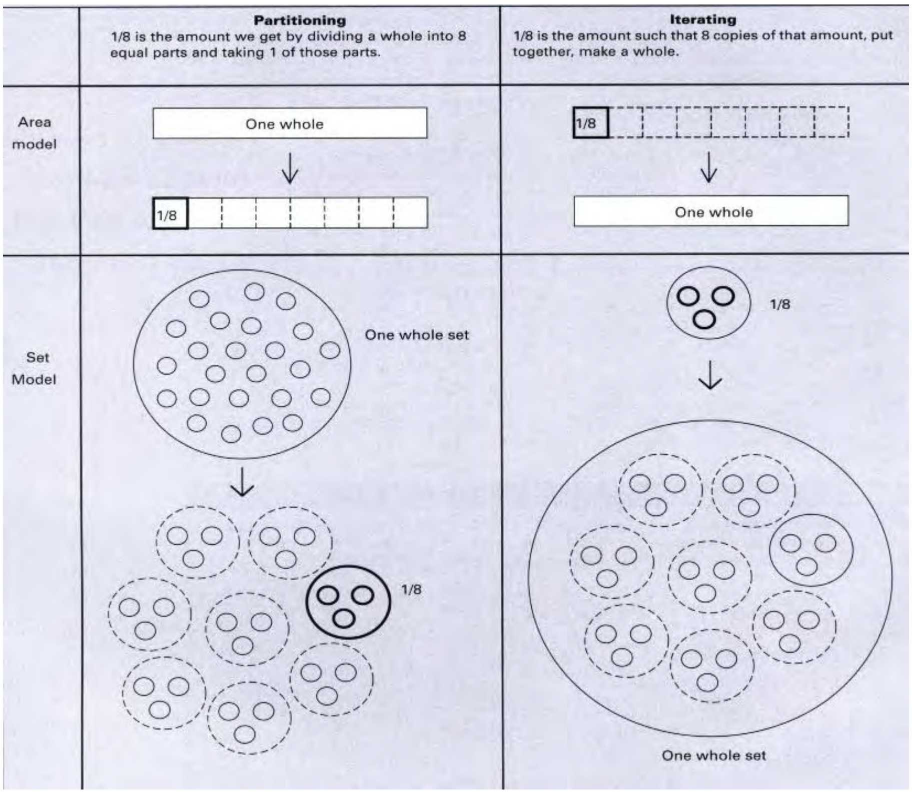


Figure 2.9: The fraction $\frac{1}{8}$ from partitioning and iterating perspectives using both area and set models
Source: Siebert and Gaskin (2006, p. 396)

The actions of partitioning and iterating enable learners to understand by doing, while the knowledge of partitioning and iterating supports learners in understanding fractional relationships and operating on them (Siebert & Gaskin, 2006). Constructivism supports my study because it embraces connection-making in fraction concepts and the sense of creation.

2.4 CHAPTER SUMMARY AND CONCLUSION

In this literature review, the links between representations and tasks were discussed. Different fraction sub-constructs and unifying elements, and an understanding of the interconnectedness of the sub-constructs supports learners' knowledge of fractions. Learners often receive limited interpretations and representations from the curriculum, with the part-whole and area models the most commonly used sub-construct and representation respectively, – hence, the need to carry out an in-use textbook analysis at the heart of this study. Research has noted how teachers struggle to find useful, appropriate and varied representations and models for teaching fractions, that reflect their multifaceted nature (Lamon, 1999; Post et al., 1993). They often therefore use representations or models comprising regularly shaped objects (circles or squares) divided into equal parts or use the number line (De Bock et al., 2007). But regularly shaped objects and number lines deal only with continuous wholes rather than discrete units.

Almost every learner asks, “Are fractions important, and why?” The answer is straightforward: “Yes!” Niemi (1996) says it is fundamental for any learner, especially in Grade 7, to understand fractions in order to cope with more advanced topics in the high school curriculum. Siegler et al. concluded “Secondary school pupils’ mathematics performance could be substantially improved if children gained a better understanding of fractions” (2012, p. 21). Fields such as architecture, medicine, chemistry, engineering, or technology involve precise and accurate calculations. Hence, a good understanding of the different meanings of fractions, including computations with fractions, is critical.

Learners experience fractions informally, for example, sharing sweets, dealing cards for a card game or baking/cooking (using recipes). Even though learners are confronted with fractions daily, they only experience abstract thinking when introduced to fractions in primary school. Basic arithmetic like addition, subtraction, multiplication and division of whole numbers is the main focus in lower grades to develop basic mathematics skills. In higher grades, fractions are found in topics such as algebra, geometry, probability and trigonometry. Fractions appear widely within the school curriculum, ranging from primary to secondary school. Thus, learners must have a good understanding of the meanings of fractions if they wish to achieve well in higher grades. The only way in which learners will gain a better understanding of fractions is if they are exposed to them through instruction and if they are taught in a way that is interrelated to other aspects of mathematics and real life.

Connection-making is an essential aspect of teaching and learning mathematics. Mhlolo (2012) postulates that “educational standards all over the world recommend that teachers enable pupils to recognise and to make connections among relatable mathematical ideas” (p. 176). The literature on fractions clearly shows how fraction constructs should be taught with clear connections within themselves and other mathematical aspects. Making connections is among the essential experiences learners should gain in the mathematics classroom. Empirical evidence shows that learners’ ability to recognise connections directly links to their mathematical understanding (Silver et al., 2009). An individual’s knowledge of a mathematical concept can be examined by the connections that they can make to that concept with other concepts, because we cannot see the internal connections directly (Barmby et al., 2009). Weinberg (2001) advocates that learners should be made explicitly aware of these different possible mathematical connections through instruction that emphasises the interrelatedness of mathematical ideas.

Mathematical instruction can be complemented by the use of resources in the classroom. Textbooks are such resources and are used worldwide. They are crucial to teaching and learning general school subjects and mathematics. The intended and enacted curriculum in many classrooms relies heavily on textbooks. In mathematics classrooms, textbooks are an essential part of what is involved in doing school mathematics; they provide frameworks for what to teach, how to teach, and outline the teaching sequence to follow (Apple, 1992; Ben-Peretz, 1990; Goodlad, 1984; Nicol & Crespo, 2006; MoE, 2009b; Schmidt et al., 1997b). Most teachers rely heavily on textbooks as their source of information for teaching and learning.

Several researchers in materials evaluation stress the importance of a post-use evaluation of a textbook since it can provide data regarding the actual effect of the materials on the users (Tomlinson, 2003; Tomlinson & Masuhara, 2004). However, Mukundan and Ahour (2010) state that empirical studies in post-use evaluation appear to be limited in the literature, with many biased towards pre-use evaluations. In the Namibian education system, NIED usually carries out the pre-use textbook evaluation mainly to ensure that it aligns with the curriculum in the Namibian education system (MoE, 2009b). The research will use a post-use evaluation/analysis process in the selected mathematics textbook analysis. The following chapter discusses the methodology of my study.

CHAPTER 3 : METHODOLOGY

3.1 INTRODUCTION

The focus of this study is to analyse the nature of mathematical connections inherent in the presentations of fraction concepts in selected Namibian Grade 7 mathematics textbooks. This chapter outlines and reports on the research methodologies used in this research study. It also provides reasons as to why the specific methods were chosen. The structure of this chapter includes the following: research goals, research methodology, research orientation, sampling, data collection tools and data analysis. The issues pertaining to validity, ethics and the limitations of the study are also discussed towards the end of this chapter.

3.2 RESEARCH GOALS

This study aimed to analyse how the three prescribed Grade 7 mathematics textbooks embrace making connections in fractions concepts. Firstly, it explored the mathematical connections in the three mathematics textbooks for the Grade 7 Namibian classrooms – their presence, types and purpose in the mathematical content. Secondly, the study aimed to understand the perceptions and perspectives of selected mathematics teachers about mathematical connections/connection-making in the three selected textbooks. The three research questions that framed this study were:

1. *How do selected Namibian Grade 7 textbooks encourage connection-making in the context of fractions?*
2. *What similarities and differences are observed in the three textbooks in promoting connection-making in the fraction concepts?*
3. *What are selected teachers' perceptions and perspectives of the three textbooks concerning connection-making in fraction concepts?*

Hopefully, this study will improve the quality, design, and more comprehensive procedures of textbook evaluation criteria in Namibia. Regarding education policy and practice, the study's empirical findings might inform the Namibian education policy on decisions regarding textbook evaluation and the mathematics curriculum. It might also enlighten textbook practitioners to recognise the importance of post-use assessment of textbooks.

3.3 RESEARCH ORIENTATION

This study was underpinned by the interpretivist paradigm using a mixed-method approach, with qualitative and quantitative methods. According to Bertram and Christiansen (2022), in an interpretive paradigm, the researchers "do not predict what people will do, but rather describe and understand how people make sense of their world and how they make meaning of their particular actions" (p. 30). Semi-structured individual and focus group interviews were conducted to gain insight into the participants' perspectives. The goal was to understand the phenomenon from their understanding and capture the multiple realities present. These interviews allowed for a deep exploration and understanding of the participants' views on the topic. I used qualitative analytical techniques to analyse the data collected from the interviews to derive meaningful insights and allowed themes to emerge from the raw data.

The study also examined the nature of mathematical connections in Namibian Grade 7 mathematics textbooks. I employed a mixed-methods approach to collect and analyse qualitative and quantitative data. According to Cohen et al. (2018), mixed methods research

> "typifies research undertaken by one or more researchers which combines various elements of both quantitative and qualitative approaches (eg with regard to perspectives, data collection and data analysis) to research, together with the nature of the inferences made from the research, the purposes of which are to give a richer and more reliable understanding (broader and deeper) of a phenomenon than a single approach would yield." (p. 32)

A questionnaire was used to collect quantitative data, measuring the participants' responses to specific statements. I then used descriptive statistical procedures to analyse this data. The study also utilised textbook analysis to collect quantitative data, focusing on quantifying and interpreting mathematical connections. Descriptive statistics were employed to analyse the collected data and gain insights into the presence and effectiveness of mathematical connections in the textbooks.

This study sought to understand the phenomenon by incorporating qualitative and quantitative methods. The utilisation of interviews, questionnaires and textbook analysis aimed to contribute to evaluating mathematics textbooks in the Namibian education system.

3.4 RESEARCH METHODS

The study took two forms: a document (textbook) analysis and a case study. In the context of document analysis research, McCulloch (2011) defined a document as "a record of an event or a process" (p. 249). In my study, the documents were three selected textbooks, which, according to McCulloch (2011), Rose (2001, 2007) and Stray (1994b), are recognised as the primary documents in document analysis research. Only the fraction sections of the three Grade 7 Namibian mathematics textbooks approved by the NIED curriculum panellists were analysed. The focus of the analysis of my documents was the phenomenon (nature) and purpose of connection-making in fraction concepts.

Case study research is a systematic and in-depth study of one particular case (situation) in its context, where the case may be a child, a clique, a school, a teacher or a community (Cohen et al., 2018; Rule & John, 2011; Stake, 1995; Creswell, 1994; Adelman et al., 1976). In addition, Cohen et al. (2018) stated that the researcher aims to capture the reality of the participants' lived experiences and thoughts about a particular situation. For this study, the case consisted of four selected Grade 7 mathematics teachers from the Grootfontein circuit in Namibia. The unit of analysis was the teachers' perceptions and attitudes of connection-making in fraction concepts regarding the three approved mathematics textbooks.

Several claims of strengths and weaknesses surrounding case study approach have been recorded over the years. Wellington (2015) suggested that they are "illustrative and illuminating, accessible and easily disseminated, holding the reader's attention and being vivid accounts which are 'strong on reality'" (p. 174). On the other hand, he noted that "they are not replicable and may not be representative, typical or generalizable" (p. 174). For my study, the findings thus pertain only to the participating teachers.

In contrast, Cohen et al. (2018) argued that case studies are "impressionistic, and self-reporting and may be biased. Further, they say that bias may be problematic if the case study relies on an individual's (selective) memory" (pp. 378-380). I discuss how I mitigated against biases for my case study in the ethics section below.

3.5 RESEARCH DESIGN

This study employed diverse data collection techniques and methods. Questionnaires were administered to a sample of 21 teachers. Four participants were then selected for semi-structured individual and focus group interviews from the questionnaire returns. Also, a textbook analysis was employed, as discussed above. The data generation process encompassed four distinct phases, elucidated on, below.

Phase One

During this phase, the primary objective was to develop the analytical tool (Appendices 1 and 2) and questionnaires (Appendix 3). The analytical tool was adapted from Fotakopoulou and Spiliotopoulou's (2008) model, as explained in the analysis section. In this phase, I tested the tool's reliability on one of the textbooks (*Excellent Mathematics Learners Book*) and identified any errors and limitations. I then made modifications and refinements to align the tool with Research Questions 1 and 2.

The questionnaire was developed and later improved through two iterations. The questionnaire was then distributed to all 23 schools in the Grootfontein circuit, specifically targeting Grade 7 mathematics teachers.

I also developed and finalised the individual and focus group interview schedules in this phase.

Phase Two

This phase aimed to address Research Questions 1 and 2. The three textbooks were analysed using my refined analytical tool. The analysis consisted of two distinct components, namely a vertical and a horizontal analysis.

Conducting vertical and horizontal analyses allows for a comprehensive understanding of the subject matter (Ricento, 2006). According to Charalambous et al. (2010), vertical textbook analysis is an in-depth examination of a specific textbook, focusing on its content, organisation, and pedagogical aspects. This type of analysis assesses how well the textbook aligns with the learning objectives, curriculum standards and target audience, and evaluates its effectiveness for teaching and learning purposes. In this study, the vertical analysis provided a detailed perspective by examining the connections featuring in the fraction concept, of each of the three selected textbooks. However, this analysis alone was insufficient to fully understand the implementation of mathematical connections in the fraction concept. Hence, I also conducted a horizontal analysis across the three selected textbooks in this study.

Horizontal textbook analysis, on the other hand, involves comparing multiple textbooks that cover the same subject area or grade level. It aims to identify similarities and differences among the textbooks in terms of content, teaching approaches, learning activities, assessment methods and overall quality (Charalambous et al., 2010; Mesa, 2007; Remillard, 2005; Rezat, 2006; Wijaya et al., 2015). By conducting a horizontal analysis, teachers and curriculum developers can identify the strengths and weaknesses of different textbooks, make informed decisions about textbook adoptions, and ensure that the chosen textbooks meet the curriculum goals and the learner’s needs. In this study, the horizontal analysis provided a detailed perspective by specifically examining the connections embodied the fraction concept across the three selected textbooks.

Charalambous et al. (2010) argued that horizontal and vertical analyses are appropriate approaches and criteria for analysing mathematics textbooks. Applying vertical and horizontal analysis methods in the Namibian Grade 7 mathematics textbook analysis provided valuable insights into these educational resources’ content, structure and effectiveness, thus aiding teachers and policymakers in making informed decisions about textbook selection and curriculum design.

Phase Three

In this phase, I analysed the questionnaires that were collected. A total of 50 survey questionnaires were distributed to all Grade 7 mathematics teachers in my circuit through the circuit office. Initially, only ten responded, which was deemed insufficient for this study. In an effort to increase participation, ten more questionnaires were distributed during the circuit cluster meeting, and 30 were emailed to schools that agreed to participate. This round of distribution resulted in an additional 11 responses, bringing the total number of completed questionnaires used in the study to 21. Participation in completing the questionnaire was voluntary.

I then used descriptive statistics and inductive analysis to analyse the questionnaires. This analytical approach provided a holistic understanding of the teachers’ perceptions and perspectives concerning mathematical connections and how these connections are presented in the three selected textbooks in relation to the fraction concept. This analytical process enabled me to answer Research Questions 1 and 3. Based on the questionnaires’ responses, I purposely selected four mathematics teachers to partake in semi-structured individual and focus group interviews.

Phase Four

After analysing the three selected textbooks and the returned questionnaires, I conducted semi-structured individual and focus group interviews with my four participants. These interviews were audio recorded and transcribed using the online transcription tool in MS Office 365. The transcriptions were then analysed inductively. In analysing both types of interview, I systematically examined the questions chronologically (see Appendices 4 and 5). As defined by Bertram and Christiansen (2014, p. 204), inductive analysis involves a researcher exploring patterns and themes in the raw data collected.

Mayring (2004, p. 268) proposed that the act of summarising content serves the purpose of condensing the material into more manageable proportions while staying true to the essential content. He further suggested that the inductive formation of categories occurs through summarising content, whereby the categories are generated from the text material. The conclusion of the individual and focus group interview analyses therefore involved a discussion on emergent themes that were not initially foreseen in the original interview schedule. Engaging with the four selected participants answered Research Questions 1 and 3.

3.6 SELECTION OF PARTICIPANTS

This section will present how participants and textbooks were selected. Firstly, the selection of textbooks will be elaborated upon, followed by the selection of participants.

3.6.1 Selection of textbooks

The textbooks used in this study were purposely selected. Purposive sampling is a valuable method for qualitative research, providing depth and flexibility in selecting cases based on specific criteria. This approach allows researchers to gain in-depth insights into complex phenomena and tailor the sample to meet the study’s needs. (Teddlie & Yu, 2007).

A purposive sampling strategy is appropriate in selecting textbooks for a study (Bertram & Christiansen, 2014). For the sake of validity, in this study I made specific choices about the textbooks to be included in the analysis, so all three approved mathematics textbooks for Grade 7 in the Senior Primary Phase were specifically included in the sample. These textbooks were chosen because they are based on the Namibian Revised National Curriculum of 2015 (MoE, 2010). As previously mentioned, the textbooks under study were the *Excellent Mathematics Learners Book* (EMLB), *Oxford Let’s Do Mathematics* (OLDM), and *Platinum Mathematics*

(PM). All three textbooks were published in 2015, with EMLB and PM published in Namibia and OLDM published in the United Kingdom.

Given the scope and focus of this study, only the Common Fraction and Mixed Numbers (CFMN) sections from each of the three selected textbooks were analysed. The concepts of CFMN were chosen because, from my own teaching experience, the literature and the analysis of mathematics performance, these concepts are particularly challenging, poorly performed and poorly understood by teachers and learners in Namibia (Ausiku, 2022; MoE, 2017; Nambira et al., 2009).

Chikiwa (2018) found out that educational research consistently highlights the difficulty learners face in grasping common fractions. Studies by the Department of Basic Education (DBE, 2015) and Gabriel et al. (2013) indicate that difficulties with fractions persist beyond intermediate and senior phases, extending into secondary school and beyond (Schumacher & Malone, 2017). Curricula in Namibia and South Africa typically expect foundational work with common fractions, mixed numbers, and percentages to occur in grades 8 and 9, with earlier grades focusing on basic operations (DBE, 2011; [Ministry of Education, Namibia], 2012). Extensive research on fraction instruction (Bruce et al., 2012; Katenda, 2017; Siegler & Forgues, 2014) reveals a persistent challenge: learners often struggle with these concepts (DBE, 2015)

3.6.2 Selection of participants

The participants for this study were Grade 7 mathematics teachers from the Grootfontein circuit in the Otjozondjupa region. Otjozondjupa region consists of three circuits namely the Okahandja circuit, the Ontjiwarongo Circuit and the Grootfontein cicuit. The Grootfontein circuit consists of 22 primary schools. Each school consiss of 1 to 6 mathematics teachers. The questionnaire were sent to 14 schools and eight schools were not reachable. I sent out 55 questionnaires and only 21 questionnaire were returned. From the 21 returned questionnaires, I purposefully selected four teachers to participate in the semi-structured individual and focus group interviews. The selection of these four was based on their working experience, proximity to my school, the depth and nature of their questionnaire responses and the participants' expressed interest, as evidenced by their provision of contact details and acknowledgement of availability. The pseudonyms Ms M, Ms P, Mr M, and Mr K were assigned for anonymity when quoting specific answers from the questionnaire and interviews. All four selected teachers were involved in the focus group interview sessions.

3.7 METHODS OF COLLECTING DATA

The data collection for this study incorporated four distinct research techniques, namely document analysis, a questionnaire, semi-structured individual interviews, and a focused group interview. As advocated by Bertram and Christiansen (2014), the selection of data collection methods must adhere to a "fitness for purpose approach," ensuring alignment between the methods chosen and the desired type of data to be collected (p. 41). Consistent with Cohen et al. (2018), my study employed data triangulation, a method involving multiple data sources to validate findings. Triangulation is a valuable strategy, mitigating the limitations of relying solely on a single method or data source. Incorporating multiple approaches enables researchers to enhance the reliability and validity of their findings, fostering a more comprehensive and nuanced understanding of complex phenomena by considering various angles and dimensions.

However, it is imperative to underscore the importance of careful planning and execution when employing triangulation. Researchers must thoughtfully consider the specific research question and context to ensure the integrity of the study. Triangulation in this study aimed to attain a more thorough and well-rounded understanding of the research problem by minimising potential biases or limitations inherent in a single method or data source.

3.7.1 Document analysis

This method involved analysing the three selected textbooks, the EMLB, OLDM and PM. Bowen (2009) defined document analysis as "a systematic procedure for reviewing or evaluating documents-both printed and electronic (computer-based and internet-transmitted) material" (p. 27). A document is "a record of an event or a process" (p. 249), and a textbook is recognised as a primary document in document analysis research as its basis (McCulloch, 2011; Rose, 2001, 2007; Secord, 2000; Stray, 1994:2). Bowen (2009) noted that documents are stable; hence, the researcher's presence cannot change what is being studied. Document analysis allowed me to gain insights into the mathematical connections inherent in the CFMN section of the selected textbooks.

3.7.2 Questionnaire

In this study, a comprehensive examination of teacher perspectives on mathematical connections in the classroom was undertaken by administering a structured questionnaire. Krosnick (2018) defines a questionnaire as a systematically prepared form or document with questions designed to elicit research participants' responses to collect quantitative and qualitative data. The questionnaire, divided into three distinct sections, probed various aspects of the teachers' experiences and attitudes related to the topic.

The questionnaire began with Section A, which focused on collecting demographic information about the respondents. This section allowed me to understand the backgrounds and experiences of the participating teachers. By analysing their demographic details, I could determine any potential patterns or correlations that might emerge in relation to their responses in the subsequent sections.

Section B of the questionnaire explored connection-making in the fraction section of the mathematics textbook commonly used in schools. The aim was to investigate how well the textbooks facilitate the integration of mathematical connections for teachers. The responses obtained in this section provided valuable insights into the effectiveness of the textbook in promoting connection-making skills. Question 1 of section B, for example, aimed to determine the teacher participants' knowledge of articulating connections in the fraction concepts of the selected textbooks. By analysing the responses to this question, I gained insight into the teachers' perceptions and perspectives about mathematical connections in the textbooks they use in their teaching practice.

Finally, the questionnaire concluded with Section C, which sought to elicit the teachers' perceptions and attitudes towards mathematical connections in the classroom. This section aimed to gauge the level of awareness and importance the teachers attribute to integrating connection-making skills into their teaching practices. Examining the responses in this section enabled me to understand the teachers' beliefs and attitudes towards integrating mathematical connections.

By analysing the responses, I gained valuable insights into the current state of connection-making skills among the participating teachers. I also identified potential areas for improvement in this domain of mathematics education in Namibia. By sequentially presenting the findings from each section, I aimed to paint a holistic picture of the diverse factors influencing the teaching and learning of connection-making in fractions, ultimately contributing to the ongoing dialogue on effective instructional strategies in the field. Find the attached sample of the questionnaire in Appendix 3.

3.7.3 Semi-structured individual interview

In this study, semi-structured individual interviews were conducted to further probe teachers' perspectives regarding mathematical connections and the teaching of fractions. The semi-structured individual interview (Appendix 4) is valuable for engaging in a nuanced conversation between the researcher and the respondents (Cohen et al., 2018). Semi-structured individual interviews have a more open format than structured interviews, allowing for flexibility in obtaining detailed responses. The use of probes in semi-structured individual interviews enables researchers to gain a comprehensive understanding of participants' perspectives. Participants feel more comfortable expressing themselves freely in this less structured interview environment (Bertram & Christiansen, 2014; Cohen et al., 2011).

All interviews were meticulously recorded using audio recording tools and subsequently transcribed using the online transcription facility in MS Office 365. The transcribed data served as the foundation for the subsequent analysis. The questions posed during the interviews were structured to address specific aspects of teachers' perceptions, perspectives on teaching fractions, and their general outlook on the prescribed textbooks.

The interview questions were strategically designed to gain insights into teachers' perspectives on mathematical connections (Question 1), their approach to teaching fractions (Question 2) and their general perceptions of the three textbooks concerning connection-making (Question 3). Furthermore, an open-ended question (Question 4) encouraged participants to share any additional comments or insights beyond the specific queries.

3.7.4 Focus group interviews

This study also involved using focus group interviews to gather a collective perspective through interactive engagement and discussion with my participants (Myers, 2009). The rationale behind selecting focus group interviews lies in their inherent flexibility, enabling direct interaction with participants and fostering meaningful conversations that can yield diverse ideas (Watts & Ebbutt, 1987; Hevnver & Chatterjee, 2010; Wilson,1997). As a researcher, my role during the focus group interviews was to initiate questions that all participants could answer and respond to. These were also audio recorded and transcribed. Prior consent was sought from participants for the audio recording of these sessions (see Appendix 5). The focus group interviews were divided into three parts.

During Part 1 of the interview, I welcomed and shared greetings with all the participants.

In Part 2, I introduced the purpose of the focus group interviews. I explained that these interviews were being conducted to obtain a collective view of the mathematical connections in the textbooks used by the participants. I emphasised that as a researcher, my role was to initiate interview questions, clarify misunderstandings and actively participate. I also explained that I was not an expert and should be considered a learner, as I was a part of the group.

To foster an environment of open dialogue, the participants were assured that there were no right or wrong answers. They were encouraged to freely express their points of view and use any mathematics textbook or resource during the interview. Participants were allowed to share additional thoughts, questions, or clarifications before proceeding with the structured questions.

Part 3 of the interviews consisted of structured questions. These were designed to probe the participants' understanding and experiences related to mathematical connections. Question 3 sought the participants' understanding and experiences regarding mathematical connections. Question 4 explored the participants' elaboration on whether the recommended textbooks explicitly presented mathematical connections. Finally, Question 5 concluded with an invitation for any final questions or participant comments.

The focus group interviews served as a valuable methodological approach to gathering a collective perspective on the role of mathematical connections in textbooks. The structured format, participant engagement and open dialogue created a conducive environment for insightful discussions, contributing to the overall depth of the research findings.

3.8 DATA ANALYSIS

Qualitative and quantitative data analysis are two distinct approaches used to analyse and interpret data in research, as stated in Section 3.3 above. This study employed both qualitative and quantitative analysis. This section describes the analysis process of the textbook, questionnaire and interview analysis.

3.8.1 Textbook analysis

My textbook analysis employed a quantitative approach at two levels.

For Level 1, the analytical tool examined the frequency of connections within the CFMN topic. For example, real-life connections were initially categorised into indicators such as career, food recipes, learners' interests and lived experience connections (see Appendix 1). This analysis sought the frequency of mathematical connections embraced in the textbooks. Firstly, I recorded the frequency of the mathematical connection on a table and then presented them using the vertical bar graphs as figures.

For Level 2 of the document analysis, I identified the number of connections present in the three selected textbooks and their respective purposes. The same categories and indicators stated in Level 1 above also applied to the Level 2 analysis (see Appendix 2). I also presented examples from the textbooks for each purpose and gave descriptions after each presentation.

3.8.2 Questionnaire

The questionnaire sought to determine the number of participant teachers who agreed with the statements about mathematical connections, the textbooks used and ICT integration in their teaching practices. My quantitative analysis was presented as tables and figures, and descriptions were given after each presentation in percentage form.

3.8.3 Interviews

I employed inductive analysis to examine the participant teachers' responses to the interviews and transcribed them in my research analysis. Fotakopoulou and Spiliotopoulou (2008) described three main categorising strategies for qualitative analysis, where categories, themes and patterns emerge from the data. During the analysis of the interviews, four additional themes emerged. These themes included workshops, professional development and teacher empowerment; regional and cluster initiatives; leadership and advocacy; and collaboration and communication.

3.9 VALIDITY

Validity in research refers to the extent to which the findings and conclusions of a study accurately represent the phenomenon being studied. In qualitative data, validity might be addressed through the honesty, depth, richness and scope of the data achieved, the participants approached, the extent of triangulation and the disinterestedness or objectivity of the researcher (Winter, 2000). To enhance the study's validity, I conducted individual interviews with participants and followed up with a focus group discussion. This approach allowed for validating the collected data through triangulation, seeking convergence of evidence from multiple sources. Thus, I ensured consistency and accuracy by comparing the responses given in individual and group settings.

In qualitative data, respondents' subjectivity, opinions, attitudes and perspectives can contribute to a degree of bias. Hence, validity should be seen as a matter of degree rather than an absolute state (Gronlund, 1981). Nevertheless, Maxwell (1996) explains that "triangulation reduces the risk of chances, associations and systematic bias, and relies on information collected from a diverse range of individuals, teams and settings, using a variety of methods" (p. 93). In this study, I adopted multiple data collection tools such as a questionnaire, semi-structured individual interviews, focus group interviews, and document analysis to validate the data and ensure its reliability. Using a combination of methods ensured that I thoroughly understood the research topic and reduced potential biases associated with a single method.

Before implementing the data collection tools, I took steps to pilot the analytical instrument. The purpose of piloting is to test the ability and effectiveness of these tools in answering the research questions, and thereby assess their appropriateness. This process helped to identify potential issues and make necessary adjustments before conducting the study. Furthermore, I utilised audio-recording devices during interviews to ensure a complete and accurate record of the discussions. I relied on the recorded data for analysis, enhancing credibility and transparency.

Moreover, I clearly explained the aims and objectives of the study to all participants. This step ensured that participants clearly understood the study and its goals, enabling them to align their responses with the research objectives. By enhancing each participant's understanding, I minimised potential misunderstandings or biases that could influence the validity of the research.

Lastly, I collected data from diverse sources, such as document analysis, questionnaires and interviews involving the four participants. This approach increases the richness and depth of data, providing a more comprehensive and valid understanding of the research topic.

3.10 ETHICS

Ethics in research refers to the principles and guidelines that govern the conduct of research studies involving human subjects. It involves ensuring that the research is conducted with integrity, respect for participants' rights and well-being, and adherence to ethical standards. Key elements of ethics in research include informed consent, confidentiality, the minimisation of potential harm, transparency in reporting, and maintaining objectivity and honesty in data collection and analysis (Oliver, 2003, p. 17). Ethical considerations are crucial to protect the rights and welfare of participants and promote the responsible and ethical advancement of knowledge (Howe & Moses, 1999, pp. 22-23). Research institutions, professional organisations, and regulatory bodies provide ethical guidelines to guide researchers in ethically conducting research. The Rhodes University Ethical Standards Committee and the Education Department Higher Degrees Committee approved this research under application number 2022-5888-7345 (See Appendix 11). The specific ethical principles adhered to in this study are detailed below.

Respect and dignity

Participants were informed of their voluntary participation and their absolute freedom to withdraw from the study at any point. The participants' anonymity was rigorously maintained by using pseudonyms. Mutual respect was upheld throughout the study and all collected data between the researcher and the supervisor were, and will continue to, remain confidential. Consent for using an audio recorder during interviews was explicitly obtained from the participant teachers, and all planned research activities occurred after formal classes as per the agreement with the director, the inspector and the principals.

Transparency and honesty

Formal consents were diligently obtained from key stakeholders, including the Director of Education in the Otjozondjupa region, the Inspector of Education in the Grootfontein circuit, the school principals for the questionnaire participants, the principals of the schools of the four participants and the questionnaire participant teachers (see Appendix 6, Appendix 7, Appendix 8, Appendix 9 and Appendix 10. The study's nature, purpose and content were communicated to all participants. The purpose of the questionnaire was spelt out. The questionnaire was not intended to judge the overall quality of the textbooks, but rather to analyse how the textbooks use connections in the context of fractions.

Accountability and responsibility

The data collected was handled with the utmost sensitivity and stored securely. Despite my role as a mathematics teacher in the circuit, participants were assured that this position did not compromise their responses.

Integrity and academic professionalism

This study is characterised by academic integrity, with the work being an original creation using the researcher's own words. Proper acknowledgement and referencing were employed when incorporating the works or ideas of others. The collected data was presented as is, without manipulation, compromise, fabrication or misreporting, upholding the highest standards of academic professionalism.

3.11 LIMITATION AND CHALLENGES

Although this study implemented various methodologies and ethical considerations, it is important to critically acknowledge potential limiting factors that could affect the research outcomes and interpretations. By identifying and addressing these limitations, future research efforts can be guided and improvements in study design can be made.

Sample bias

One potential limiting factor is sampling bias. The participants in this study were predominantly from the Grootfontein circuit in the Otjozondjupa region, which may result in a lack of diversity in the sample. It is possible that the characteristics of the selected participants could influence the findings, thereby limiting the generalisability of the results. It is important to recognise and consider this limitation when interpreting the findings. To mitigate this, I employed triangulation to minimise potential biases in the study. Triangulation involves using multiple data collection methods to enhance the validity and trustworthiness of qualitative research data (Cohen et al., 2011; Yin, 2016). I employed various methods in this study, such as document analysis, questionnaires and semi-structured individual and focus group interviews for data collection.

Generalisability

Limited generalisability is another limitation to consider. This case study focused on three selected textbooks and examined only one region in one circuit in the Namibian education context. Therefore, the findings may not apply to other textbooks. Extrapolating the results to different settings or demographics should be done cautiously, considering the specific context in which the study was conducted.

Fundings

Lastly, funding limitations impacted the scope of the study, sample size and available resources. With the limited funding, the study may not have been able to collect sufficient data or may have had to make compromises in research design. These limitations can affect the findings' reliability and should be considered when interpreting the results.

3.12 CONCLUSION

In this chapter, an overview was provided regarding the purpose of the study, the methods of selecting participants and the specific group of individuals targeted for research. The instruments utilised for data collection were also outlined. Additionally, the procedures for collecting and analysing the data were explained, along with examining the ethical considerations. The chapter also acknowledged the limitations and obstacles encountered during the study. It was emphasised that this research study was conducted as a case study and document analysis within the framework of an interpretive paradigm. The chapter concluded by addressing the data analysis process and elaborating on issues of ethics and validity. The subsequent chapter will present and discuss the collected data.

CHAPTER 4 : DATA ANALYSIS – TEXTBOOKS

4.1 INTRODUCTION

This chapter analyses the three textbooks in question. I present a vertical and a horizontal analysis (see Chapter 3, Section 3.5) of the mathematical connections prevalent in the three selected Grade 7 mathematics textbooks used in the Namibian education system – the *Excellent Mathematics Learner's Book* (EMLB) (Mokwadi et al., 2015), *Oxford Let's Do Mathematics* (OLDM) (Potgieter & Pretorius, 2015), and *Platinum Mathematics* (PM) (Hambata et al., 2015). The empirical focus is on Common Fractions and Mixed Numbers' (CFMN), and the presentation aligns with the analytical tool discussed in the methodology chapter (see Section 3.8). A consolidated summary of findings from these data sources is subsequently provided.

The data presented in this chapter sought to answer the research questions:

1. How do selected Namibian Grade 7 textbooks encourage connection-making in the context of fractions?
2. What similarities and differences are observed in the three textbooks in promoting connection-making in the fraction concepts?

4.2 VERTICAL DATA PRESENTATION OF THE MATHEMATICAL CONNECTIONS PREVALENT IN THE THREE SELECTED TEXTBOOKS IN THE CFMN SECTION

This section presents the data analysed from each of the three selected textbooks as stated in the methodology chapter (see Section 3.8.1 and Section 3.8.2).

For this vertical analysis, each textbook is analysed as per my analytical framework (see Appendices 1 and 2). I analysed each of the three textbooks at two levels.

However, before the Level 1 and Level 2 analyses of each textbook, I first discuss each textbook's general structure, followed by an analysis of the section CFMN in that particular textbook.

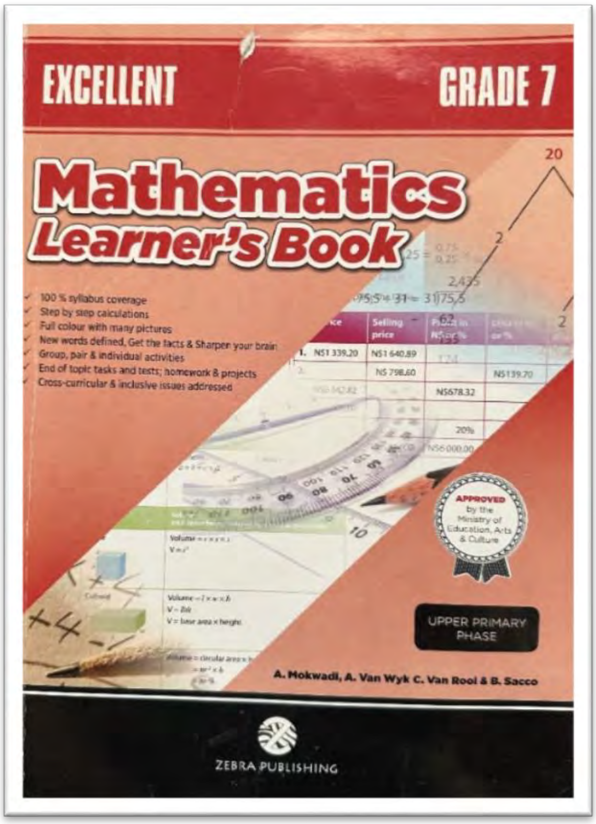
4.2.1 Analysis of the *Excellent Mathematics Learner's Book* (EMLB)

This section presents the general structure of the EMLB, Level 1 analysis and Level 2 analysis of the EMLB and the concluding remarks of the textbook analysis.

4.2.1.1 General Structure of the EMLB

This section describes the general structure of the EMLB, starting with the cover page and the back cover, and lists the features of the entire EMLB series and their function, as articulated in the textbook (Mokwadi et al., 2015). The textbook's front and back covers state the EMLB textbook's aim and list the entire textbook's main unique features (Mokwadi et al., 2015). See Figure 4.1 below.

Front cover



Back cover

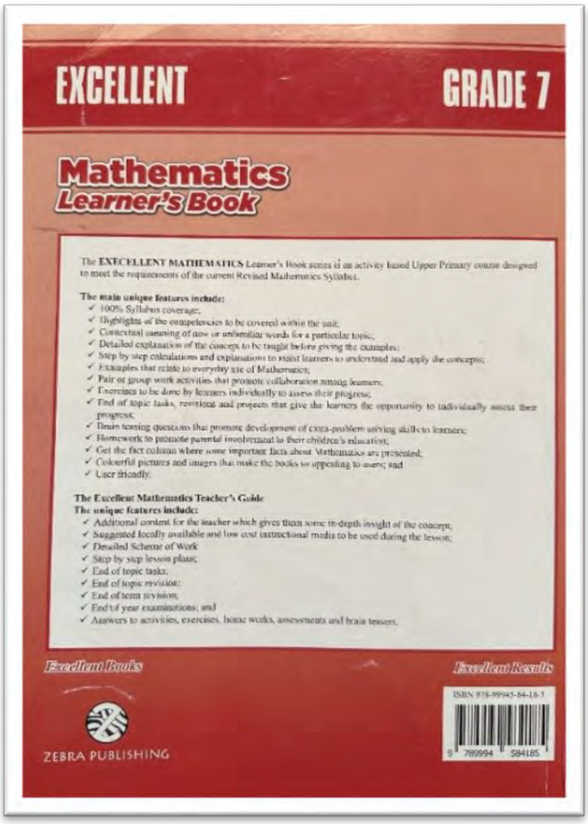


Figure 4.1: Front and back cover of the EMLB Textbook
Source: Mokwadi et al., 2015

This textbook consists of chapters on different mathematical topics, projects and investigations. The CFMN topic is located in Chapter 3 of the nine chapters of the EMLB textbook, comprising 16 of the 188 pages of the entire textbook. Each chapter in the textbook has the following features:

- ✓ The first page of each topic outlines the basic competencies (described in detail below) the learners need to achieve by the end of each topic. New words are stated and explained and pictures are presented (see Figure 4.2 below).

✓ There are several sections within the CFMN chapter. Each section in the chapter consists of the learning content, activities (intended to introduce learners to new mathematical ideas) and exercises (intended to practice mathematical procedures), described in more detail below.

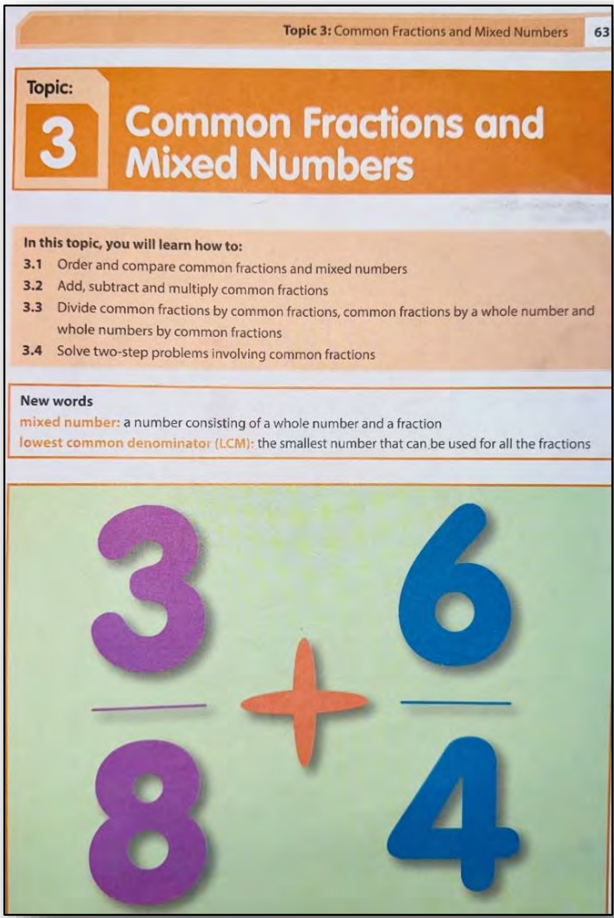


Figure 4.2: First page of the CFMN topic in the EMLB textbook
Source: Mokwadi et al., 2015, p. 63

As I understand the purpose of the textbook as articulated in this book, it is expected that concepts and skills development take place by working through the activities. The activities are designed to encourage learners to draw on familiar experiences and prior knowledge, explore new ideas, reflect on their learning and share ideas through writing, drawing, making and talking. Textbook tasks serve as a way to engage the learners in learning mathematics, shape their thinking or narrow or expand their understanding of the subject. Therefore, it is crucial for textbooks and other curricular materials to offer meaningful and diverse mathematical tasks (Glasnovic Gracin, 2018; Henningsen & Stein, 1997; Johnston-Wilder & Mason, 2004). The basic competencies stated at the beginning of each chapter highlight the essential mathematical concepts and skills expected to be understood and achieved after working through the activities. The exercises consolidate the concepts and skills developed through the activities. As such, they can be used both for practice and assessment purposes.

Basic competencies

The CFMN chapter consists of four basic competencies. These are:

- ✓ *Order and compare common fractions and mixed numbers*. For this competency, learners are expected to be able to compare common fractions and mixed numbers with denominators up to 10, using relationship signs (EMLB, 2015: MoE, 2015b). They are also expected to be able to arrange common fractions and mixed numbers with denominators up to 10 in ascending or descending order. This basic competency links well with my analytical tool because it points out that learners should be able to connect common fractions and mixed numbers. My analytical tool seeks *inter alia* to analyse the connections between the mathematical content.
- ✓ *Add, subtract and multiply common fractions*. Learners should acquire further knowledge on how to apply the four basic operations and learn how to use the order of operations. By the end of the topic, learners should be able to add and subtract common fractions and mixed numbers by finding common denominators, multiplying a fraction by a whole number and a fraction by a fraction and multiply mixed numbers (EMLB, 2015: MoE, 2015b).
- ✓ *Divide common fractions by common fractions, common fractions by a whole number and whole numbers by common fractions*. Learners are expected to be able to divide a whole number by a fraction, for example, $2 \div \frac{1}{4} = 8$, because eight quarters can go into two whole numbers. They should also be able to divide common fractions by a whole number and explain the answer diagrammatically or by reasoning, for example $\frac{1}{4} \div 2 = \frac{1}{8}$, Because if I divide $\frac{1}{4}$ into 2 equal parts, each part is $\frac{1}{8}$ (see Figure 4.3 below). This basic competency connects with my analytical tool by pointing out the visualisation connection. Learners are expected to show or illustrate their answers diagrammatically.
- ✓ *Solve two-step problems involving common fractions*. The learners are expected to be able to apply their knowledge of fractions to solve contextual word problems and express one quantity as a fraction of another quantity, for example, 5 kg of 25 kg as $\frac{5}{25} = \frac{1}{5}$ or 20 minutes of 1 hour as $\frac{20}{60} = \frac{1}{3}$ and calculate the fractional parts of a

quantity. Learners should also be able to determine suitable operations and strategies for fractions and calculations with fractions to solve three-step everyday-life problems involving fractions. This basic competency links with my analytical tool because it covers everyday life problems involving fractions. The analytical tool seeks to analyse the real-life connections presented in the EMLB textbook.

Figure 4.3: Reasoning with fractions diagrammatically
Source: Source: Mokwadi et al., 2015, p. 71

4.2.1.2 Level 1 analysis

In Level 1, I analysed the connections in mathematics knowledge prevalent in the CFMN topic of the EMLB. The analysis was applied to the learning content and activities because it is expected that concepts and skills development takes place by working through these activities. The activities are designed to encourage learners to draw on familiar experiences and prior knowledge, explore new ideas, reflect on their own learning and share ideas through writing, drawing, and making (Mokwadi et al., 2015; MoE, 2015b). Learners who use the textbook are expected to acquire mathematical knowledge through exploration and working cooperatively with other learners. The textbook implies that the teacher should explicitly transmit mathematical knowledge to the learners and the mathematical concepts and skills should be developed from whole class discussion by making use of the textbook (EMLB, 2015: Haggarty & Pepin, 2002).

Data presentation of the nature of connections in mathematics evident in the EMLB

Data showing the connections evident in the CFMN topic Level 1 analysis is presented in Table 4.1 and Figure 4.4 below. The learning content is analysed according to the analytical tool, as stated in the introduction of this chapter above.

Table 4.1: Analysis of EMLB

Connections in mathematics	Indicators	Explicit presence	Implicit presence	Total frequency
Real-life connections (RLC)	Learners' interests (LI)	4	0	4
	Lived experience (LE)	23	0	23
Connections to other Subjects (CS)	Agriculture	2	0	2
	Business Studies	2	0	2
Connections to other mathematics content areas (CMC)	Money & Finance	4	0	4
	Measurement	28	0	28
Visualisation connection	Pictorial/diagrammatically	5	0	5
Connection to problem-solving (CPS)	Real-life problems	27	0	27
Total		**95**	**0**	**95**

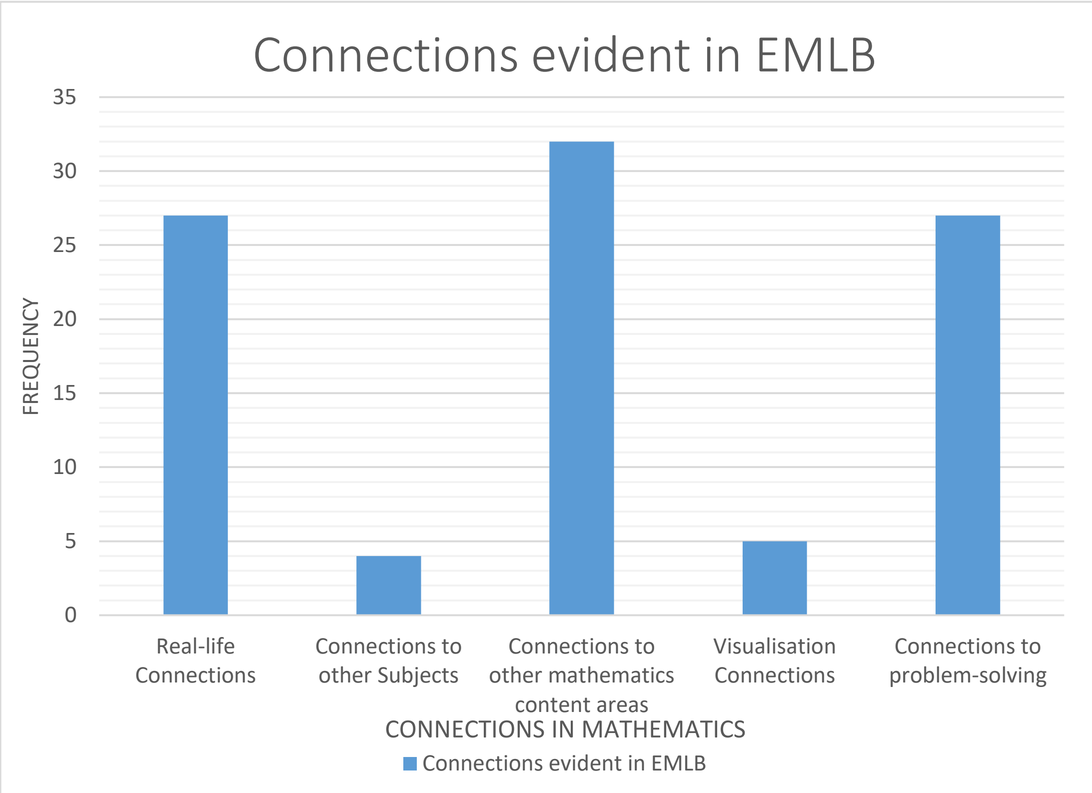


Figure 4.4: Connections evident in EMLB

Interestingly, Rebuschat et al. (2013) quote several researchers who explain implicit learning as an essential process of acquiring unconscious knowledge, further stating that it is a fundamental aspect of human cognition (Perruchet, 2008; Reber, 1993; Shanks, 2005). Many essential skills, including language comprehension and production, social interaction and intuitive decision-making, largely depend on implicit knowledge (Berry & Dienes, 1993; Dienes, 2012; Reber, 1993). In contrast, explicit learning is a mode of learning that usually occurs under conditions in which participants are instructed to look for patterns actively: that is, learning is intentional, a process that tends to result in conscious knowledge (Rebuschat et al., 2013).

Types of Connections

The connections are divided into the following five categories, with each category further divided into indicators:

Real-life connections

- ✓ Real-life connections are split into four indicators: food recipes (FR), learners' interests (LI), eg hobbies, lived experience (LE) and careers (C).

Connections to other subjects

- ✓ Four subjects are stated as indicators, and the rest are categorised as others to be analysed: Agriculture, English, Geography, Business Studies and others.

Connections to other mathematical content areas (CMC)

- ✓ Five areas of mathematical content are noted as indicators to be analysed: money & finance, geometry, measurement, percentages and decimals.

Visualisation connections (VC)

- ✓ Four indicators are listed under VC connections: tables, graphs, pictorial/diagrammatical images and drawing.

Connections to problem-solving (CPS)

- ✓ CPS has two indicators: real-life problems and cross-curricular issues (HIV & AIDs, population issues, ICT, road safety and globalisation).

In my Level 1 analysis, two real-life indicators are evident in the EMLB: LI (4) and LE (23). CS has two of its indicators evident: Agriculture has two connections and Business Studies has two as well. With CMC, two out of its five indicators are evident: money and finance has four connections presented, and measurement has 28 evident connections. The VC only has one indicator evident; the pictorial/diagrammatic presentation images have five evident connections. The connections to problem-solving have one of its two indicators evident: real-life problems have 27 connections, and cross-curricular issues were not presented. Figure 4.5 below illustrates the types of connections evident in the EMLB.

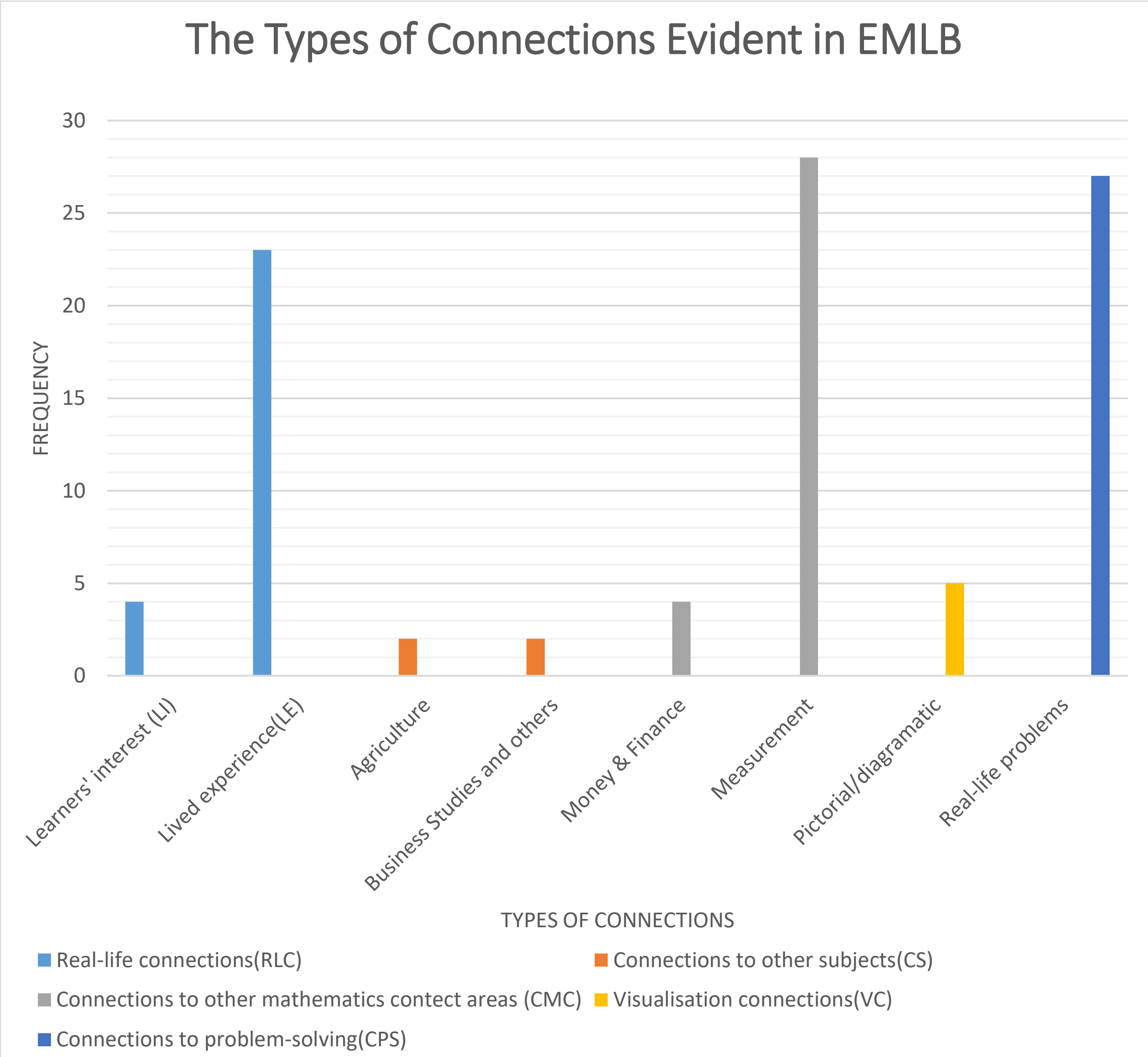


Figure 4.5: The types of connections evident in EMLB

Real-life connections

Connections to real life are divided into four indicators. A connection can be realistic, representing the concept true to an actual life situation, metaphoric, or as not having real existence. Figure 4.6 illustrates an example of a real-life connection presented in the EMLB. Cows are commonly domesticated animals in Namibia, especially in the Otjozondjupa region, and milk is one of the products that Namibian people get from their cows. The Otjozondjupa region is dominated by the Ovaherero people, who have reared cattle and led a nomadic lifestyle for decades (Online Guide to Namibia, n.d.; MyFundi n.d.). This real-life connection in the EMLB is an excellent example because learners can learn consciously about milk production, which they experience daily. Furthermore, this real-life connection also connects mathematics to the subject Agriculture, because it deals with animal farming and production. The example is contextualised and thus breaks the abstract notion of mathematics.

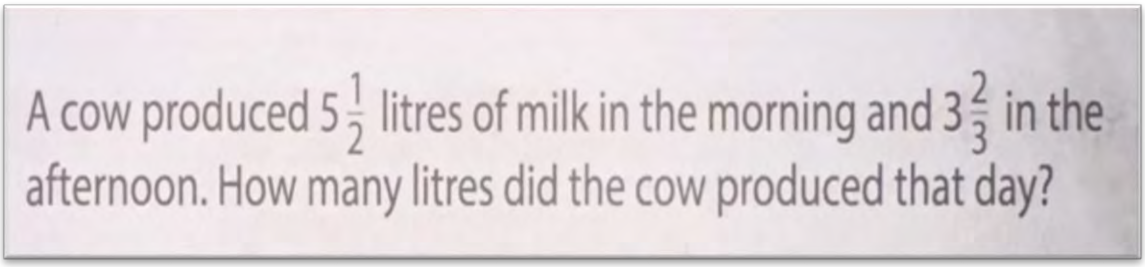
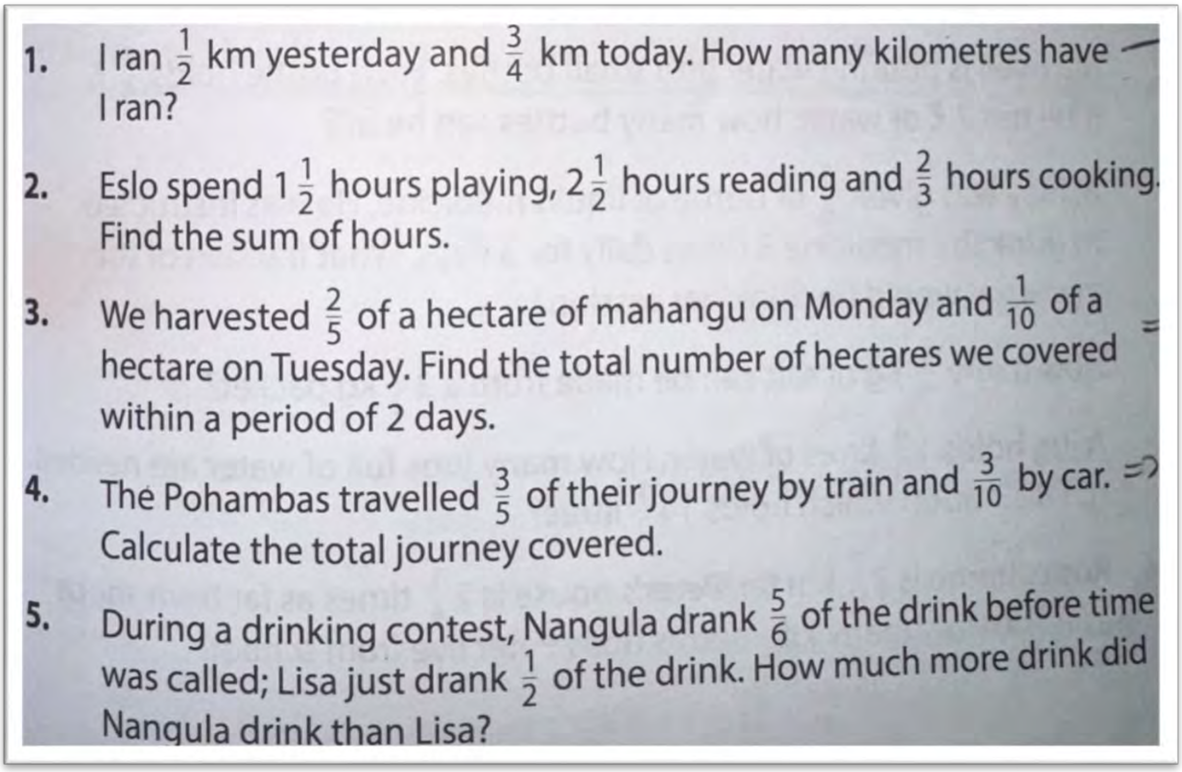

A cow produced $5\frac{1}{2}$ litres of milk in the morning and $3\frac{2}{3}$ in the afternoon. How many litres did the cow produced that day?

Figure 4.6: Example of a real-life connection and connections to problem-solving
Source: EMLB (Mokwadi et al., 2015, p. 69)

Connections to other subjects and other mathematical content

The relation to connections to other subjects has four subjects stated as indicators, and the rest are categorised as others to be analysed: Agriculture, English, Geography, Business Studies and others. The Namibian mathematics curriculum suggests that school mathematics should not be taught in isolation; instead, it should link to other subjects within itself and cross-curricular issues (MoE, 2015b). Figure 4.7 below shows examples of integrating fraction content into other subjects, other mathematics content and even real-life situations. Examples 1, 2, and 5 in Figure 4.7 integrate fractions with measurements in mathematics, covering length (distance), time and capacity. Example 3 in Figure 4.7 integrates fractions with agriculture and area.



1. I ran $\frac{1}{2}$ km yesterday and $\frac{3}{4}$ km today. How many kilometres have I ran?
2. Eslo spend $1\frac{1}{2}$ hours playing, $2\frac{1}{2}$ hours reading and $\frac{2}{3}$ hours cooking. Find the sum of hours.
3. We harvested $\frac{2}{5}$ of a hectare of mahangu on Monday and $\frac{1}{10}$ of a hectare on Tuesday. Find the total number of hectares we covered within a period of 2 days.
4. The Pohambas travelled $\frac{3}{5}$ of their journey by train and $\frac{3}{10}$ by car. Calculate the total journey covered.
5. During a drinking contest, Nangula drank $\frac{5}{6}$ of the drink before time was called; Lisa just drank $\frac{1}{2}$ of the drink. How much more drink did Nangula drink than Lisa?

Figure 4.7: Example of connections to other subjects and other mathematical content areas
Source: EMLB (Mokwadi et al., 2015, p. 75)

Connections to problem-solving

Figure 4.7 showing regarding problem-solving in fractions in the analytical tool 'Connections to problem-solving (CPS)' has two indicators: real-life problems and cross-curricular issues, which are split into five indicators (HIV & AIDs, population issues, ICT, road safety and globalisation). Example 4 in Figure 4.7 is a real-life problem-solving example: this will stimulate the learners' interest and engagement in the learning because Pohamba is a common name in Namibia, and the second president of Namibia is also Pohamba.

4.2.1.3 Level 2 analysis

The Level 2 analysis focused on the purpose of the connections using three criteria: exemplifying (Exa), explanatory (Exp) and decorative (D) (see Appendix 2). It also crosschecked the frequency of the connections' purpose. The Merriam-Webster online dictionary (2022) defines exemplifying as "to show or illustrate by example"; explanatory as "serving to explain"; and decorative as "serving to decorate". The Level 2 analysis aligns with visualisation in mathematics as a method of 'seeing the unseen' in images (Arcavi, 2003, p. 216), which is being recognised as a key component of reasoning. In agreement, Triantafillou et al. (2013, p. 1) acknowledge that "in mathematics, visualisation representations seem to undertake a more significant role acting as starting points or as the fundamental tools in reasoning”. The roles of visualisation are adapted and applied in connections in my study:

- ✓ Exemplifying – diagrams that exemplify the written text. They make the text and the mathematical idea more straightforward.
- ✓ Explanatory – captions provide an explanation or a classification of what is represented in the photographs.
- ✓ Decorative – there is no caption or reference from the main text to the photograph.

Purpose of the connections in the EMLB

The purpose of the connections is divided into three sub-categories adapted from the roles of visualisation objects:

- ✓ *exemplifying* is when the connection makes the learning content and the mathematical idea more transparent for the learners to grasp;
- ✓ *explanatory* refers to the connections with accompanying notes, which clarify the text and provide new information necessary to make the idea more transparent; and
- ✓ *decorative* is when the connection is made to look more attractive or ornamental – as a decoration. Often, there is no caption or reference from the main text.

The data for connections used for the above purposes evident in the EMLB, are presented in Table 4.2 and Figure 4.8 below.

Table 4.2: The number of connections evident in EMLB and their purposes

Connections in Mathematics	**Purpose (P)**			**Frequency**
	Exemplifying (Exa)	**Explanatory (Exp)**	**Decorative (D)**	
Real-life Connections (RLC)	25	2	0	27
Connections to other subjects (CS)	4	0	0	4
Connections to other mathematics content areas (CMC)	31	1	0	32
Visualisation connections (VC)	0	4	1	5
Connections to problem-solving (CPS)	27	0	0	27
Total	**87**	**7**	**1**	**95**

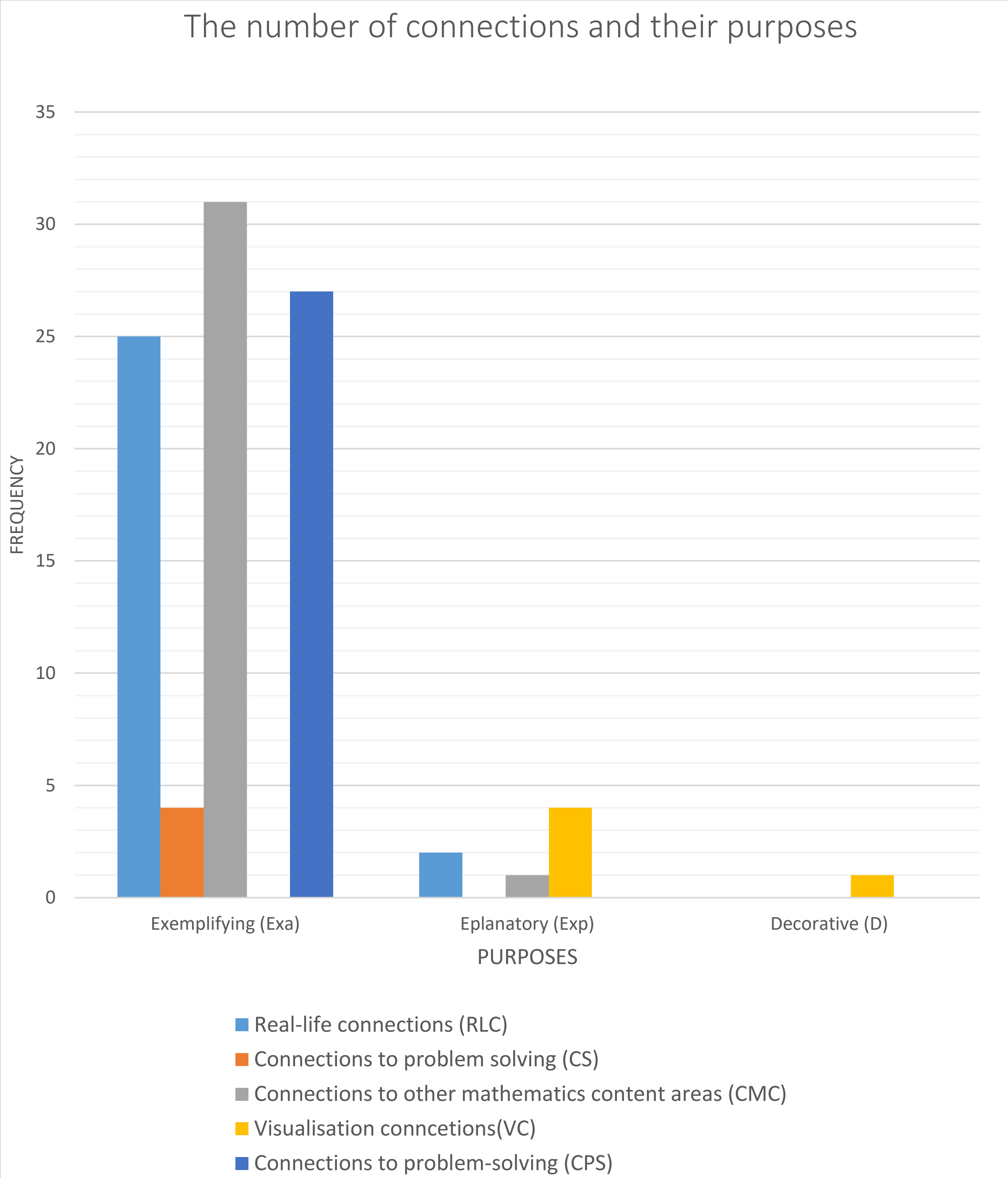
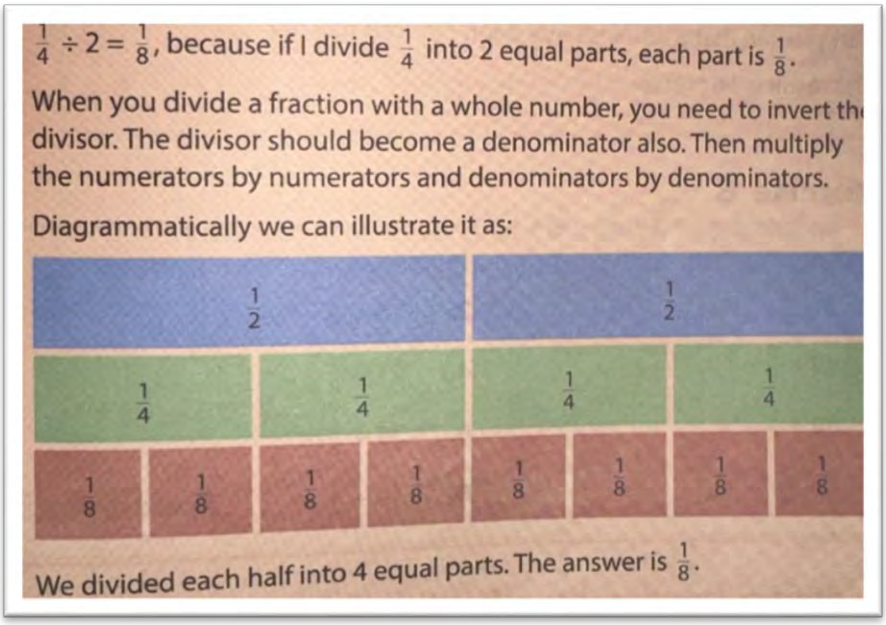


Figure 4.8: The number of connections evident in EMLB and their purposes

The exemplifying purpose

Figure 4.9 below shows an example of a visualisation connection (VC) serving an exemplifying purpose in the EMLB. In this example, a diagram is used to illustrate how a fraction can be presented or reasoned diagrammatically. Figure 4.9 explains how a learner can solve the fraction $\frac{1}{4} \div 2$. The summary above the diagram explains what the diagram illustrates, making the idea more transparent and easier for the learners to understand how a fraction can be divided into a whole number.



$\frac{1}{4} \div 2 = \frac{1}{8}$, because if I divide $\frac{1}{4}$ into 2 equal parts, each part is $\frac{1}{8}$.

When you divide a fraction with a whole number, you need to invert th divisor. The divisor should become a denominator also. Then multiply the numerators by numerators and denominators by denominators.

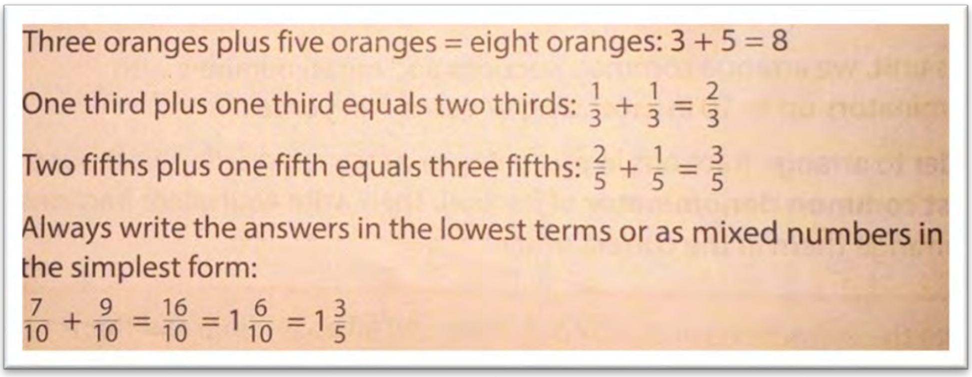
Diagrammatically we can illustrate it as:

We divided each half into 4 equal parts. The answer is $\frac{1}{8}$.

Figure 4.9: Example of a VC serving an exemplifying purpose
Source: EMLB (Mokwadi et al., 2015, p. 71)

The explanatory purpose

Figure 4.10 below illustrates addition in fractions. The figure begins by giving an example of the addition of whole numbers, followed by the addition of common fractions and finally, the answer is expressed in mixed number form. In this example, the connections with accompanying notes clarify the text and provide new information necessary to make the idea more transparent.



Three oranges plus five oranges = eight oranges: $3 + 5 = 8$

One third plus one third equals two thirds: $\frac{1}{3} + \frac{1}{3} = \frac{2}{3}$

Two fifths plus one fifth equals three fifths: $\frac{2}{5} + \frac{1}{5} = \frac{3}{5}$

Always write the answers in the lowest terms or as mixed numbers in the simplest form:

$\frac{7}{10} + \frac{9}{10} = \frac{16}{10} = 1\frac{6}{10} = 1\frac{3}{5}$

Figure 4.10: Example of a problem illustrating an explanatory purpose
Source: EMLB (Mokwadi et al., 2015, p. 66)

The decorative purpose

Figure 4.11 below illustrates the addition of fractions. This figure is placed on the first page of the CFMN chapter without any captions/instructions/solutions. The diagram is a visual object which does not serve any purpose. Hence, it is classified under decorative connections.

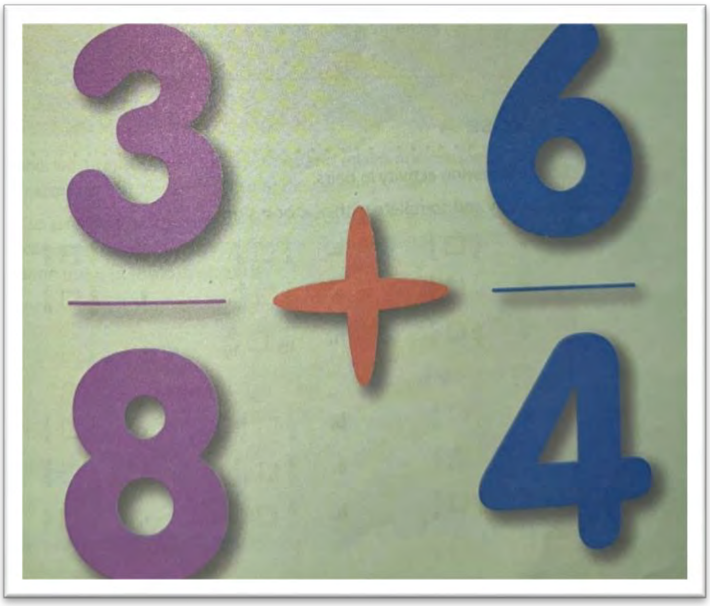


Figure 4.11: Example of a decorative purpose
Source: EMLB (Mokwadi et al., 2015, p. 63)

4.2.1.4 Concluding remarks regarding the EMLB

The Level 1 analysis analysed the connections in mathematics knowledge prevalent in the CFMN topic of the EMLB. The analysed data indicated the presence of the connections presented on the analytical tool applied, with Level 1 analysis 95 connections prevalent in the CFMN chapter. Of these, there were 33.7% connections to other mathematical content areas, followed by 28.4% real-life connections and real-life problems and 5.3% visualisation connections. Finally, 4.2% of the connections were to other subjects and another 28.4% to problem-solving. In the Level 2 analysis, the analytical tool analysed the purposes of the connections recorded in the Level 1 analysis. Data reflected that the connections prevalent in the CFMN chapter serve a purpose and are not vague. Ninety-two per cent of the CFMN learning content has an exemplifying purpose, followed by 7% having an explanatory purpose, and only 1% of the content serves a decorative purpose.

4.2.2 Analysis of the *Oxford Let's Do Mathematics* (OLDM) textbook

This section presents the general structure of the OLDM, Level 1 and Level 2 analyses of the OLDM and the concluding remarks on the textbook analysis.

4.2.2.1 General Structure of the OLDM

This section describes the general structure of the OLDM, starting with the cover page and the back cover, and lists the features of the entire OLDM series (Potgieter & Pretorius, 2015). The textbook's back cover states the OLDM course, approved series and aims. See Figure 4.12 below.

Front cover **Back cover**

Figure 4.12: Front and back cover of the OLDM Textbook
Source: Potgieter and Pretorius, 2015.

The back of the OLDM lists other approved series and subjects: OLDM Grades 4-7 and subjects like English Second Language, Natural Science and Health Education, Social Studies, Elementary Agriculture and Life Skills. The back cover also states the aims of the textbook. The OLDM claims that it:

- is written to the updated Senior Primary Mathematics syllabus and thoroughly develops the objectives and competencies in this syllabus.
- develops and reinforces core mathematical content and skills, including problem-solving and critical thinking.
- guides learners in working through problems using detailed modelled examples.
- includes many practical investigations for learners to master the required skills in real-world contexts.

- it has many illustrations to support teaching and explain the text visually.
- introduces character champions who increase learners' awareness of cross-cultural issues across the entire series.
- explains mathematics concepts and terms on pages they occur.
- provides an "assess yourself" page at the end of a topic for learners to reflect on their learning.
- gives learners further practice by having additional revision pages in every topic. (Potgieter & Pretorius, 2015)

The aim in bullet four above links strongly with my analytical tool because it addresses connections with everyday life problems involving fractions. I used my analytical tool to analyse the real-life connections presented in the OLDM textbook. Bullet five also connects with my analytical tool by emphasising the visualisation connections. Bullets two and three suggest that learners should be able to determine suitable operations and strategies to deal with fractions and compute with fractions to solve three-step problems.

The textbook consists of chapters on different mathematical topics, projects and investigations. The CFMN topic is located in Chapter 3 of the nine chapters of the OLDM textbook, comprising 18 pages of the 192 pages of the entire textbook. New words are stated and explained at the bottom of each page, and many pictures and diagrams are presented. Page five of the textbook has illustrated guidelines on how to use the textbook – see Figure 4.13 below.

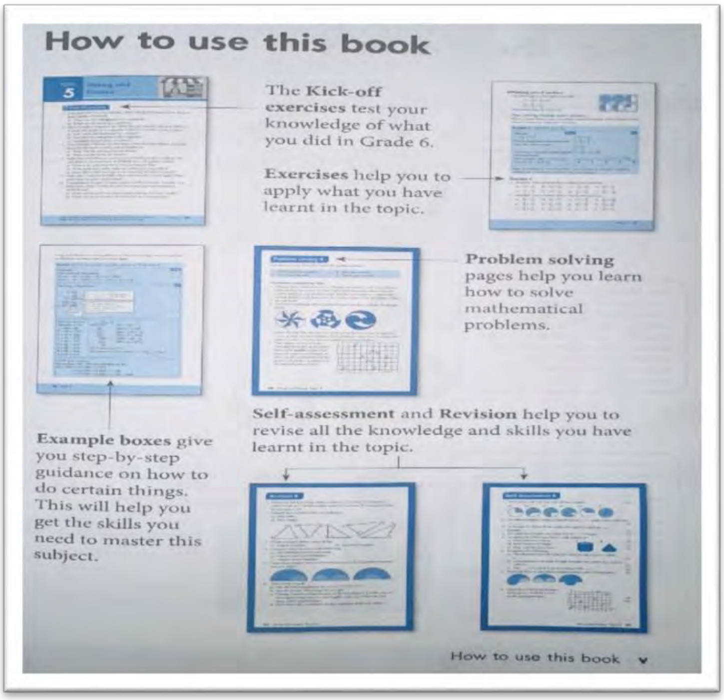


Figure 4.13: Guidelines on how to use the OLDM
Source: Potgieter and Pretorius, 2015, p. v

The textbook uses characters throughout, and on page six their profiles are elaborated on. See Figure 4.14 below.

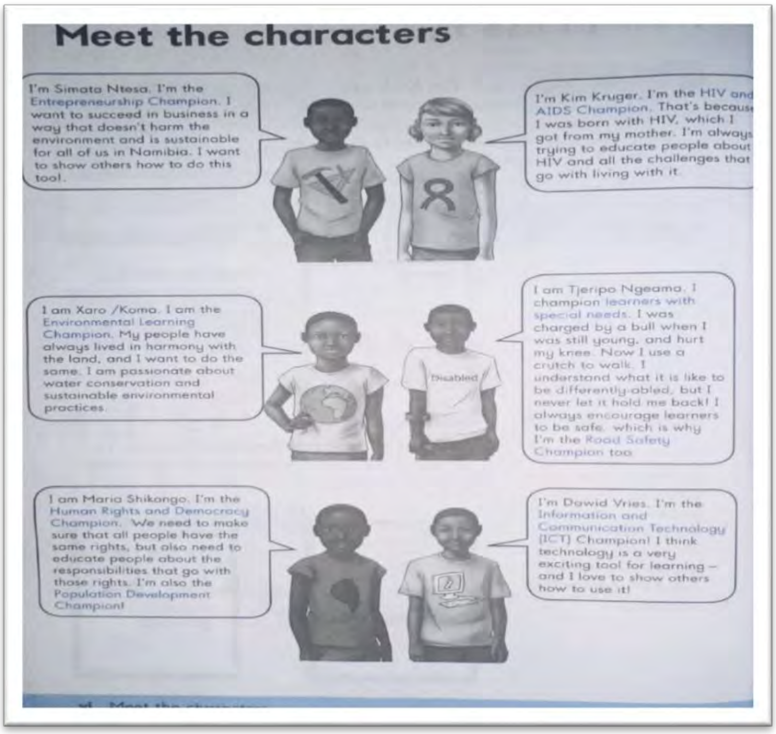


Figure 4.14: Profiles of the characters in the textbook
Source: Potgieter and Pretorius, 2015, p. vi

Characters depicted from diverse cultural backgrounds emphasise the narrative that mathematics is for all and not only for a select group of people. The characters elaborate on the cross-curricular issues that mathematics ought to consider. Some of the cross-curricular issues stated by the characters are noted in the analytical tool used in this study. These are ICT, HIV and AIDS, and environmental issues.

4.2.2.2 Level 1 analysis

In Level 1, I analysed the connections in mathematics knowledge prevalent in the CFMN topic of the OLDM. The analysis was applied to the learning content and activities because it is expected that concepts and skills development takes place by working through these activities. The activities are designed to develop and reinforce core mathematical content and skills – including problem-solving and critical thinking – guide learners in working through problems using detailed modelled examples and master the required skills in real-world contexts. Illustrations are provided to support teaching and explain the text visually (Potgieter & Pretorius, 2015; MoE, 2015b).

Data presentation of the nature of connections in mathematics evident in the OLDM

The data regarding the connections evident in CFMN from the Level 1 analysis is presented in Table 4.3 and Figure 4.15 below. The learning content is analysed according to the analytical tool, as stated in the introduction to this chapter above.

Table 4.3: Analysis of OLDM

Connections in mathematics	**Indicators**	**Explicit presence**	**Implicit presence**	**Total frequency**
Real-life connections (RLC)	Food recipes (FR)	3	0	3
	Lived experience (LE)	26	0	26
Connections to other Subjects (CS)	Agriculture	3	0	3
Connections to other mathematics content areas (CMC)	Money & Finance	5	0	5
	Measurement	35	0	35
Visualisation connections (VC)	Pictorial/diagrammatically	22		22
	Drawing	16	0	16
Connection to problem-solving (CPS)	Real-life problems	29	0	29
Total		**134**	**0**	**139**

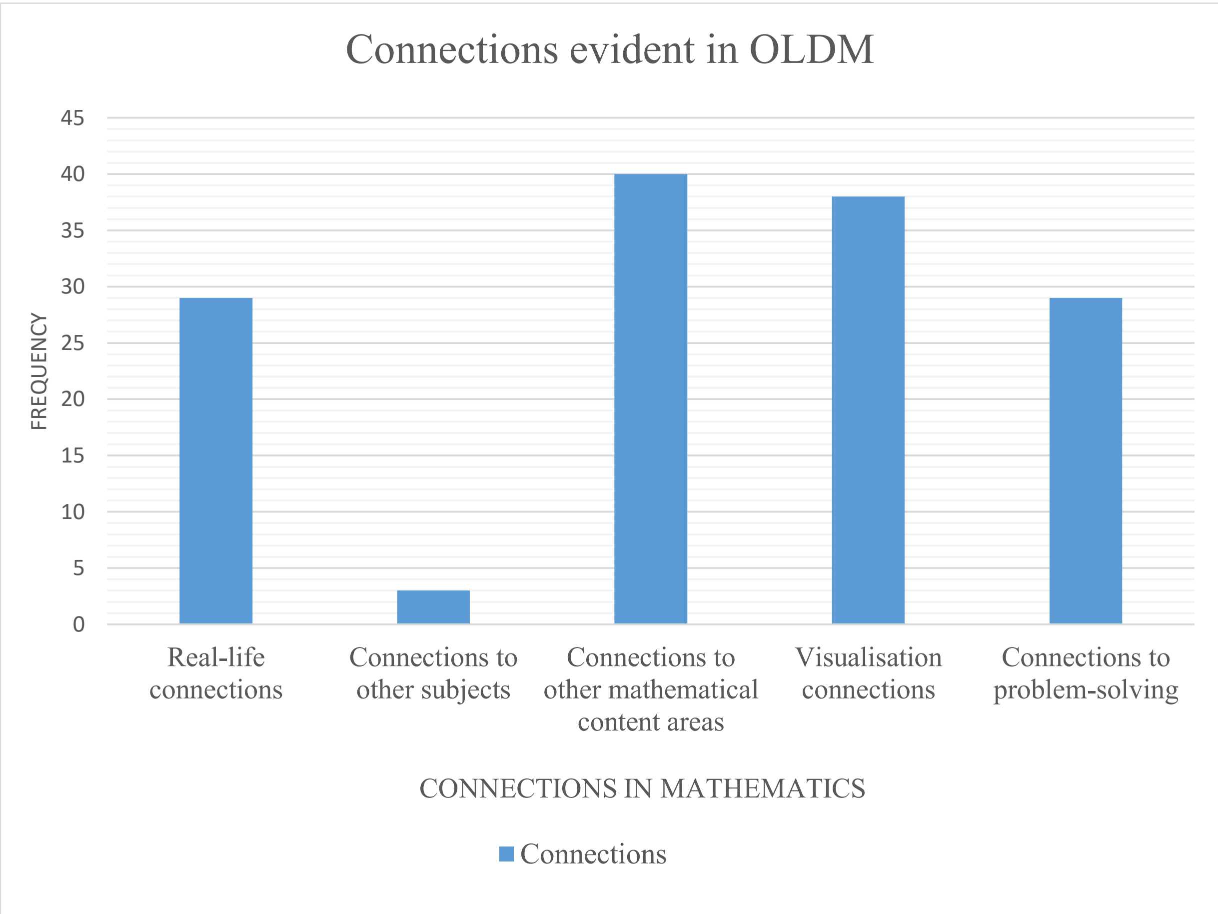


Figure 4.15: Connections evident in OLDM

Section 4.2.1.2 above describes implicit and explicit learning, and the types of connections analysed. The same criteria apply to the Level 1 analysis of the OLDM textbook as they did to the previous textbook.

In my Level 1 analysis, two real-life indicators were evident in the OLDM: food recipes (3) and lived experience (26). Connections to other subjects had only one of its indicators evident: agriculture, with 3 connections. For the section on connections to other mathematical content areas, 2 out of its 5 indicators were evident; money and finance had 5 connections presented, and measurement had 35 evident connections. Only two indicators were evident for visualisation connections; the pictorial/diagrammatic presentation had 22 evident connections and drawings showed 16 connections. Figure 4.16 below illustrates the types of connections evident in the OLDM.

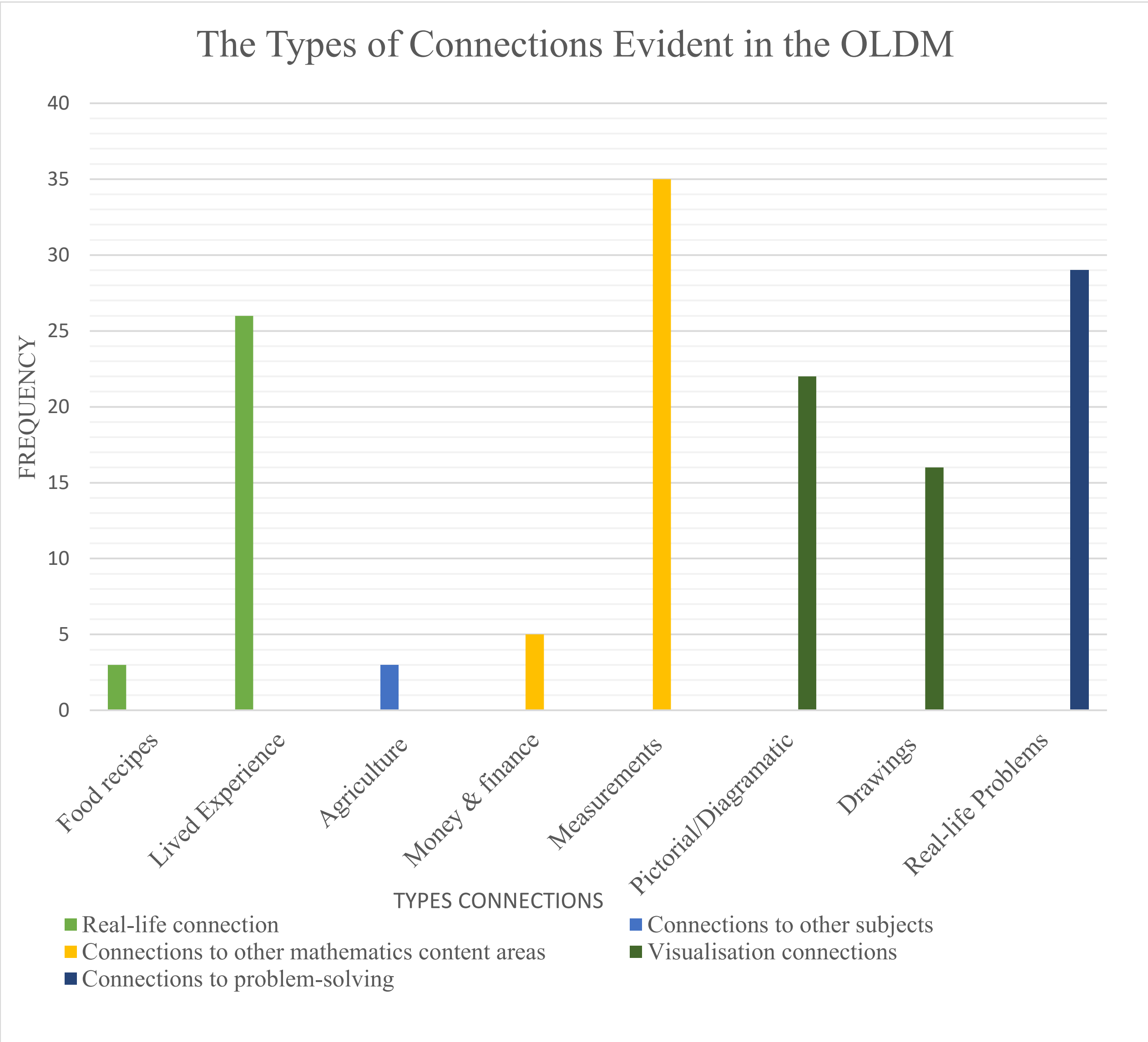
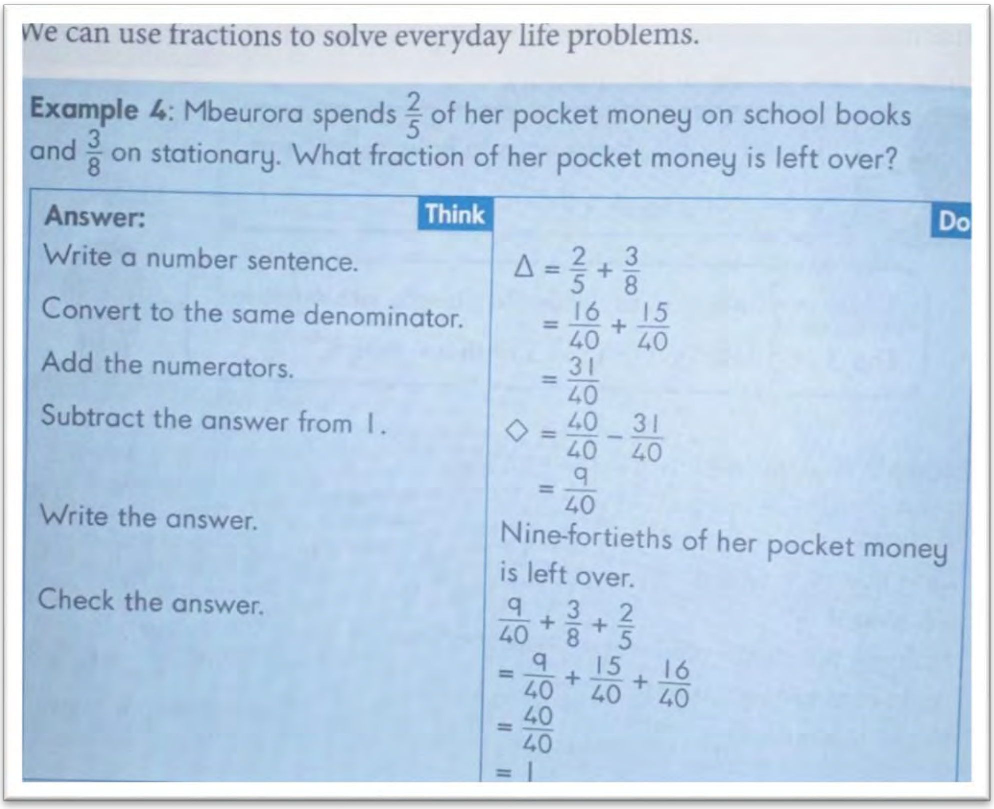


Figure 4.16: The types of connections evident in OLDM

Real-life connections

Connections to real life are divided into four indicators. A connection can be realistic, representing the concept true to an actual life situation, metaphoric, or not have a real existence. Figure 4.17 illustrates an example of a real-life connection presented in the OLDM. The caption of the example is self-explanatory, as it states, "We can use fractions to solve everyday life problems" (Potgieter & Pretorius, 2015). The example used uses a Herero name which is common in Namibia. Most of the learners using the textbook would be familiar with this name. Pocket money is something that learners encounter in their lives, and buying books and stationery is a norm for them.



We can use fractions to solve everyday life problems.

Example 4: Mbeurora spends $\frac{2}{5}$ of her pocket money on school books and $\frac{3}{8}$ on stationary. What fraction of her pocket money is left over?

Answer: Think	Do
Write a number sentence.	$\Delta = \frac{2}{5} + \frac{3}{8}$
Convert to the same denominator.	$= \frac{16}{40} + \frac{15}{40}$
Add the numerators.	$= \frac{31}{40}$
Subtract the answer from 1.	$\Diamond = \frac{40}{40} - \frac{31}{40}$ $= \frac{9}{40}$
Write the answer.	Nine-fortieths of her pocket money is left over.
Check the answer.	$\frac{9}{40} + \frac{3}{8} + \frac{2}{5}$ $= \frac{9}{40} + \frac{15}{40} + \frac{16}{40}$ $= \frac{40}{40}$ $= 1$

Figure 4.17: Example of real-life connections
Source: Potgieter and Pretorius, 2015, p. 64

Connections to other subjects

Four subjects stated as indicators in connections to other subjects and the rest are categorised as others: Agriculture, English, Geography, Business Studies and others. The Namibian mathematics curriculum suggests that school mathematics should not be taught in isolation; instead, it should link to other subjects within itself and cross-curricular issues (MoE, 2015b). Figure 4.18 below shows examples of integrating fraction content into the subject of agriculture. In the example in Figure 4.18, the harvesting of potatoes is used. Many learners are aware that agriculture deals with vegetable production and harvesting. Hence, learners could use their agricultural knowledge to help them solve the problem and contextualise it accordingly.

5. Linda harvests $4\frac{2}{5}$ kg of potatoes from one potato plant and $2\frac{9}{10}$ kg from another potato plant in her garden.
   a) How many kilograms of potatoes in total did Linda harvest? (2)
   b) How many more kilograms of potatoes did the first plant produce than the second plant? (2)

Figure 4.18: Example of connections to other subjects and other mathematical content areas
Source: Potgieter and Pretorius, 2015, p. 67

Connections to other mathematical content areas

Mathematical content areas should link and work with each other throughout the curriculum. In Figure 4.19 below, examples 1, 2, 4, 8, and 9 integrate fractions with mathematics measurements covering time, length (distance), money, mass and capacity. Most of the problems listed in Figure 4.19 are also real-life problems.

Exercise 13

1. Munya spends $\frac{2}{3}$ of 2 hours on homework and $\frac{1}{5}$ of 2 hours playing with his dog. How much time is left over to watch TV?
2. Karu's steps are $\frac{3}{5}$ m long. How far does she walk if she takes 8 steps?
3. Mary eats $\frac{1}{4}$ of a pizza. Jim eats $\frac{2}{3}$ of the rest of the pizza. What fraction of the whole pizza does Jim eat?
4. Moses earns N\$$4\frac{1}{2}$ if he works in the garden for one hour. How much will he earn if he works for $2\frac{1}{2}$ hours?
5. Eleni divides half of a cake into eighths of the cake. How many slices will she get?
6. Kukae divides $\frac{3}{4}$ of a loaf of bread equally between herself and 4 friends. What fraction of a loaf of bread does each one get?

7. Lifasi divides 3 equally large tarts into eighths. How many slices will there be in total?
8. How many $\frac{4}{5}$ kg tins can be filled from a 4 kg sack of flour?
9. Motor oil is sold in 2 ℓ cans or in 5 ℓ containers. Write the first quantity as a fraction of the second quantity.
10. A bucket can hold 10 ℓ. Heini pours 750 ml of water into the bucket. What fraction of the bucket has been filled?
11. Pendu gets N\$60 pocket money in one month. Tenu gets only N\$40. Write Pendu's pocket money as a fraction of Tenu's pocket money. Explain your answer.
12. A satellite takes $1\frac{1}{5}$ hours to revolve once around the earth. How many revolutions does it make in one day?
13. Gloria buys fruit juice in a 10 ℓ container. She empties it all into four 1 ℓ bottles and a number of $\frac{3}{4}$ ℓ bottles. How many of the $\frac{3}{4}$ ℓ bottles did she use?

Figure 4.19: Example of Connections to other mathematical content areas
Source: Potgieter and Pretorius, 2015, p. 65

Connections to problem-solving

The example in Figure 4.19 above can also be categorised as connections to problem-solving. My analytical tool CPS has two indicators: real-life problems and cross-curricular issues, which are split into five indicators (HIV & AIDs, population issues, ICT, road safety and globalisation). Figure 4.19 is a real-life problem-solving example. This is likely to stimulate the learners' interest and engagement in learning. For example, question one on Figure 4.19 states that "Munya spends $\frac{2}{4}$ of 2 hours on homework and $\frac{1}{5}$ of 2 hours on playing with his dog. How much time is left over to watch TV?". This example is relatable to the learners daily lives on how they spend their time after school..

4.2.2.3 Level 2 analysis

The Level 2 analysis focused on the purpose of the connections using three criteria: exemplifying (Exa), explanatory (Exp) and decorative (D) (see Appendix 2). See Section 4.2.1.3 for a detailed discussion of these three criteria.

Purpose of the connections in the OLDM

The data on connections used for the above purposes evident in the OLDM, are presented in Table 4.4 and Figure 4.20 below.

Table 4.4: The number of connections evident in OLDM and their purposes

Connections in Mathematics	**Purpose (P)**			**Frequency**
	Exemplifying (Exa)	**Explanatory (Exp)**	**Decorative (D)**	
Real-life Connections (RLC)	29	0	0	29
Connections to other subjects (CS)	3	0	0	3
Connections to other mathematics content areas (CMC)	40	0	0	40
Visualisation connections (VC)	27	6	5	38
Connections to problem-solving (CPS)	29	0	0	29
Total	**128**	**6**	**5**	**139**

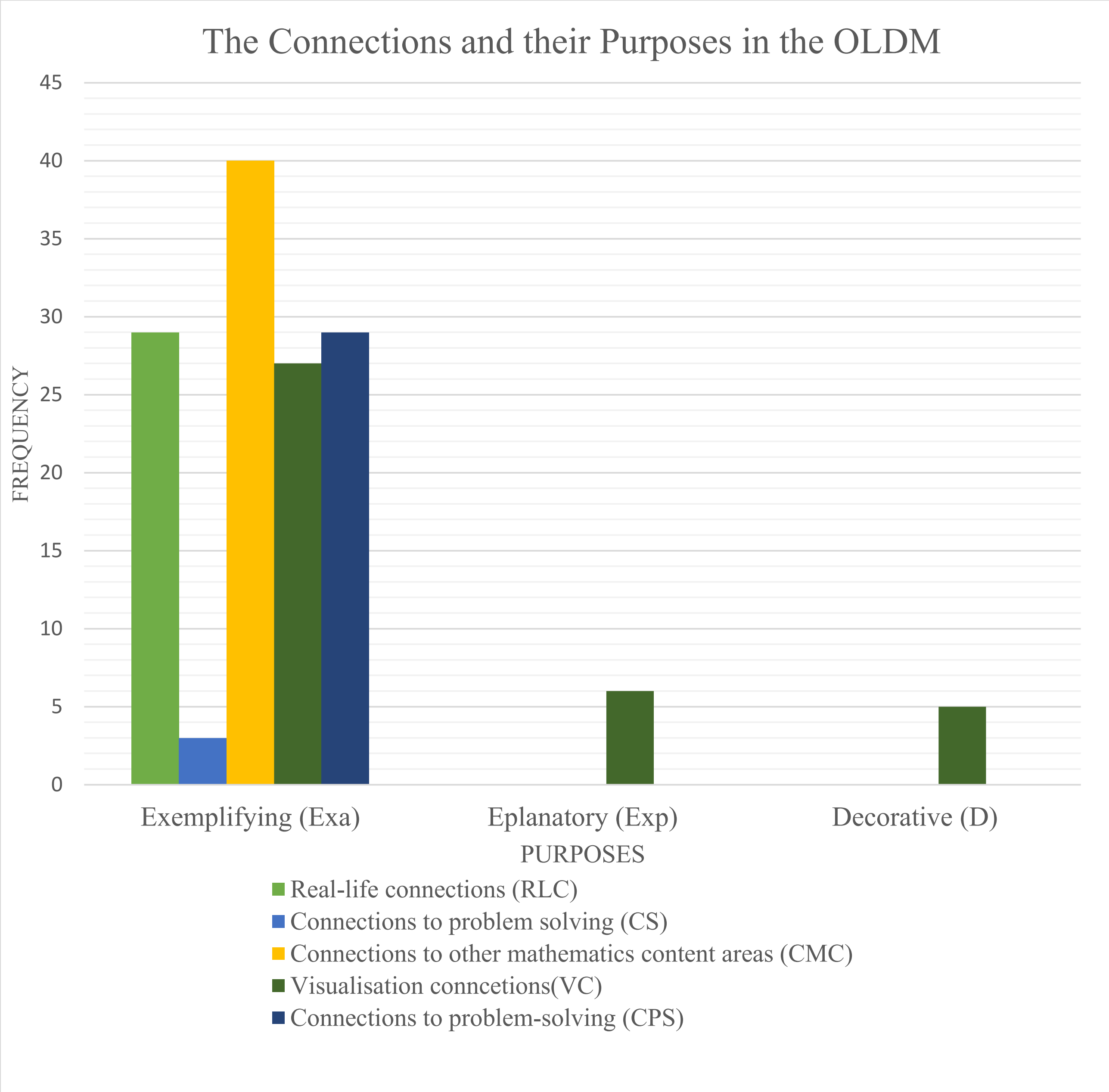


Figure 4.20: The number of connections evident in OLDM and their purposes

The exemplifying purpose

Figure 4.21 below shows an example of a visualisation connection serving an exemplifying purpose in the OLDM. In this example, drawings illustrate how a fraction can be presented or reasoned diagrammatically. Figure 4.21 explains how a learner can solve the fractions $\frac{1}{2} \div 2$; $\frac{1}{2} \div 4; \frac{1}{3} \div 2$ *and* $\frac{2}{3} \div 4$. Although no summary is provided, learners can interpret what the drawings represent, with their teacher's assistance.

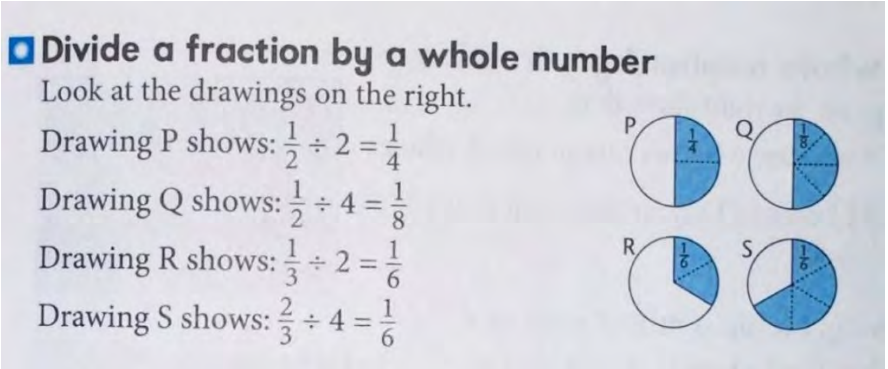


Figure 4.21: Example of a VC serving an exemplifying purpose
Source: Potgieter and Pretorius, 2015, p. 59

The explanatory purpose

Figure 4.22 below illustrates division in fractions. The figure begins by using the two characters having a conversation about the beginning of their school day. For the learners to be able to solve the problem, they have to express 45 minutes as a fraction of 60 minutes (1 hour). This shows the connection between whole numbers and fractions. In this example, the connections with accompanying notes clarify the text and provide new information necessary to make the idea more transparent.

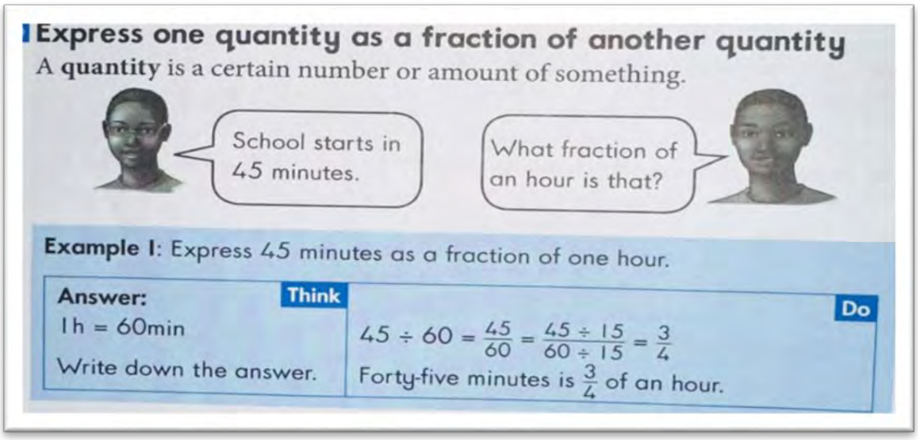


Figure 4.22: Example of a problem illustrating an explanatory purpose
Source: Potgieter and Pretorius, 2015, p. 62

The decorative purpose

Figure 4.23 below illustrates fabric-like rolled materials, presumably with different colours. This figure is placed on the first page of the CFMN chapter without any captions/instructions/solutions. The diagram is a visual object which does not serve any purpose. Hence, it is classified as having a decorative purpose.

Figure 4.23: Example of Decorative Connection Problem
Source: Potgieter and Pretorius, 2015, p. 51

4.2.2.4 Concluding remarks regarding the OLDM

Level 1 analysed the connections in mathematics knowledge prevalent in the CFMN topic of the OLDM. The data analysed using my analytical tool, indicates the presence of the connections. Level 1 analysis recorded 139 connections prevalent in the CFMN chapter. The connections to other mathematics content areas dominate the connections presented in the learning content of the CFMN chapter at 29%, followed by visualisation connections with 27%. Real-life connections and connections to problem-solving comprise 21% of all the connections presented in the CFMN chapter in the OLDM. The least presented connections are to other subjects, covering 2% of the chapter.

In the Level 2 analysis, the analytical tool analysed the purposes of the connections recorded in the Level 1 analysis. Data reflects that the connections prevalent in the CFMN chapter serve a purpose and are not vague. The exemplifying connections covered 92% of the CFMN learning content, followed by the explanatory purposes, which covered 4% of the content. The decorative purpose is also used, covering 4% of the learning content.

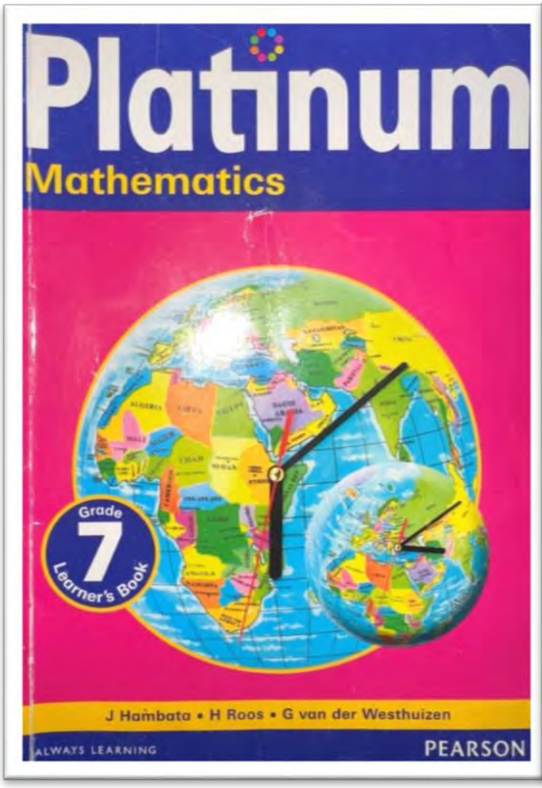
4.2.3 Analysis of the *Platinum Mathematics* (PM) textbook

This section presents the general structure of the PM, Level 1 analysis and Level 2 analysis of the PM and the concluding remarks regarding the textbook analysis.

4.2.3.1 General Structure of the PM

This section describes the general structure of the PM, starting with the cover page and the back cover, and lists the features of the entire PM series (Hambata et al., 2015). The textbook's back cover states the PM course, approved series and aims. See Figure 4.24 below.

Front cover **Back cover**



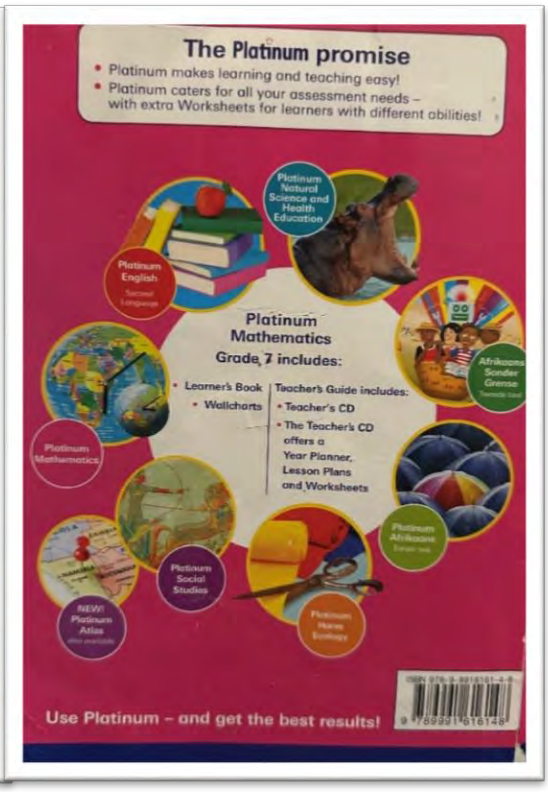


Figure 4.24: Front and back cover of the PM Textbook
Source: Hambata et al., 2015.

The back of the PM lists other approved series and subjects: PM Grades 4-7 and books such as *Platinum English Second Language*, *Platinum Natural Science and Health Education*, *Afrikaans Sonder Grense Tweede taal*, *Platinum Afrikaans Eerste taal*, *Platinum Home Ecology*, *Platinum Social Studies* and *New Platinum Atlas*. The back cover also states the aims of the textbook:

- Platinum makes learning and teaching easy!

- Platinum caters for all your assessment needs- with extra Worksheets for learners with different abilities! (Hambata et al., 2015)

The textbook consists of chapters on different mathematical topics, projects and investigations. The CFMN topic is located in Chapter 3 of the nine chapters of the PM textbook, comprising 19 pages of the 184 pages of the entire textbook. New words are stated and explained in the textbook's last three pages (Glossary), and an abundance of pictures and diagrams are presented.

4.2.3.2 Level 1 Analysis

In this section, I analyse the connections in mathematics knowledge prevalent in the CFMN topic of the PM. The analysis is applied to the learning content and activities because it is expected that concepts and skills development take place by working through these activities.

Data presentation of the nature of connections in mathematics evident in the PM

The data on the connections evident in the CFMN topic Level 1 analysis is presented in Table 4.5 and Figure 4.25 below. The learning content is analysed according to the analytical tool, as stated above in the introduction to this chapter.

Table 4.5: Analysis of PM

Connections in mathematics	Indicators	Explicit presence	Implicit presence	Total frequency
Real-life connections (RLC)	Lived experience (LE)	17	0	17
Connections to other subjects (CS)	Agriculture	7	0	7
	English	1	0	1
Connections to other mathematics content areas (CMC)	Money & Finance	4	0	4
	Measurement	17	0	17
Visualisation connections (VC)	Pictorial/diagrammatically	14	0	14
Connection to problem-solving (CPS)	Real-life problems	17	0	17
Total		**77**	**0**	**77**

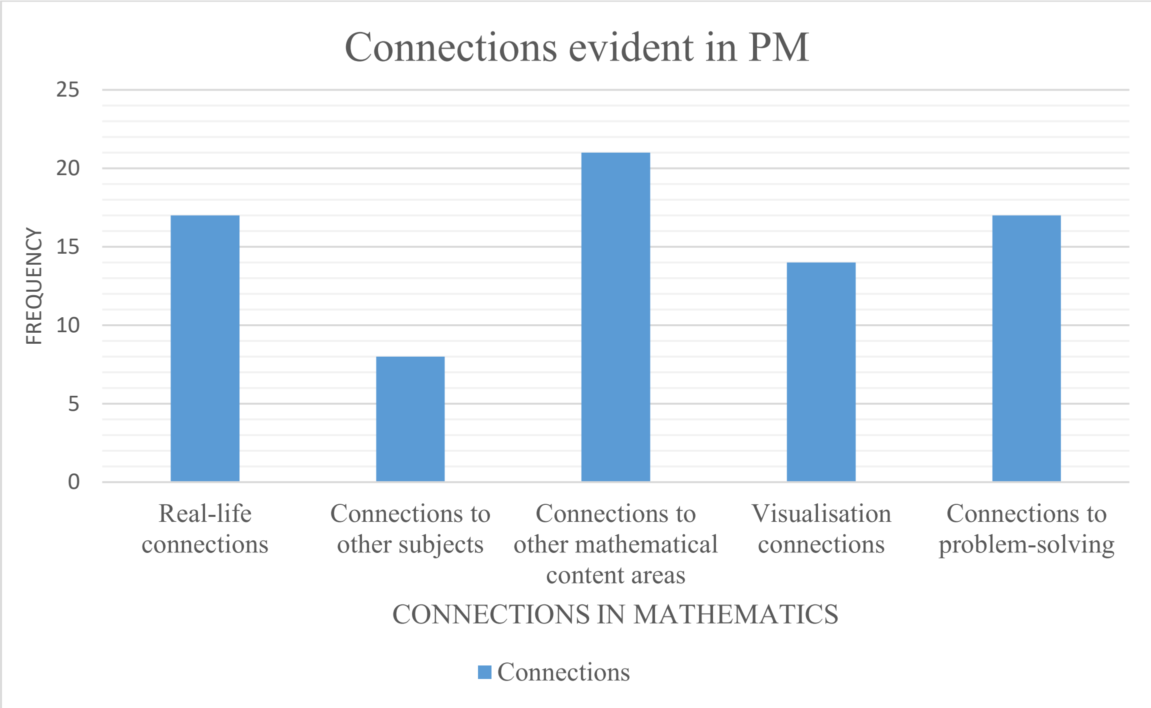


Figure 4.25: Connections evident in PM

Implicit and explicit mathematical knowledge are described in Section 4.2.1.2 above. The types of connections are also described in this section. The same criteria apply to the Level 1 analysis of the PM textbook as they did to the previous textbook.

One real-life indicator in my Level 1 analysis is evident in the PM: lived experience (17). Connections to other subjects have two indicators evident; agriculture has seven connections, and English has one connection. For connections to other mathematical content areas, two out of its five indicators are evident: money and finance have four connections and measurement has 17 evident connections. The visualisation connections only have one evident indicator; the pictorial/diagrammatic presentation has 14 evident connections. Figure 4.26 below illustrates the types of connections evident in the PM.

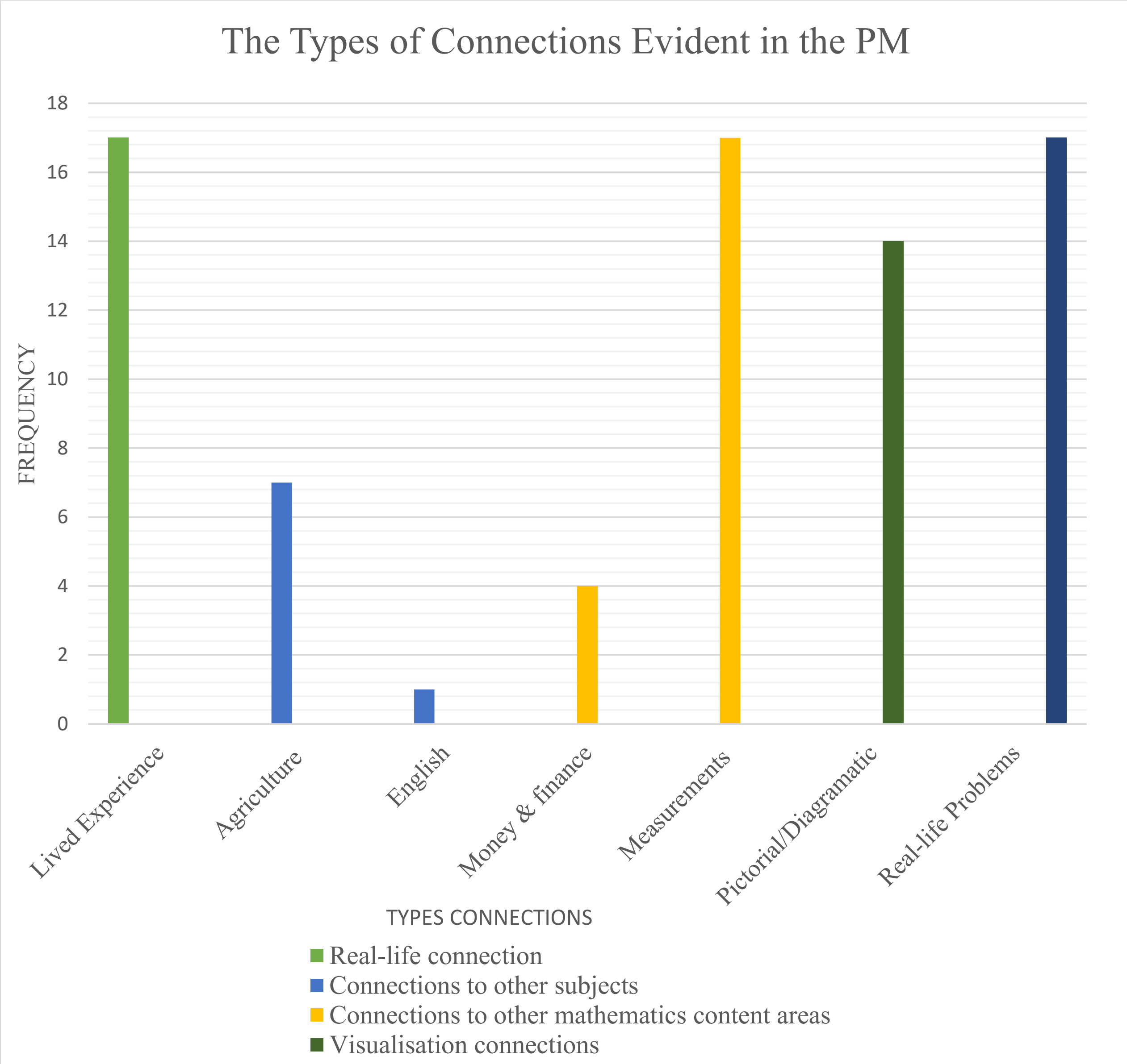
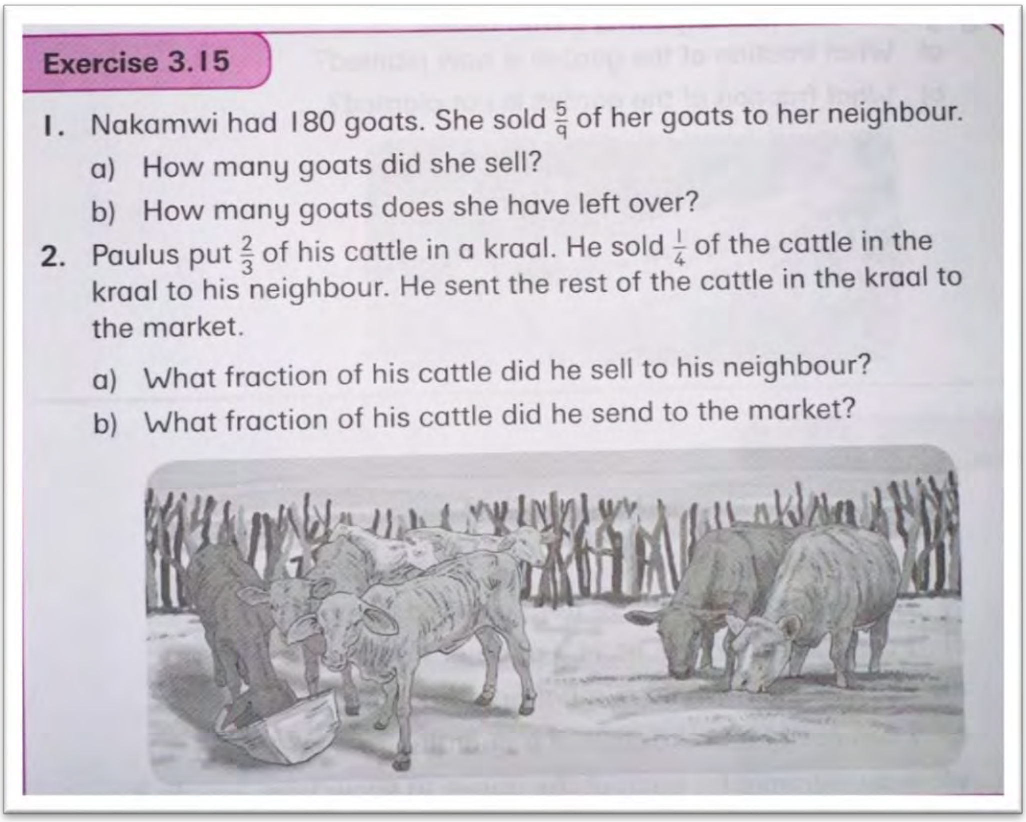


Figure 4.26: The types of connections evident in PM

Real-life Connections

Connections to real life is divided into four indicators. As mentioned previously, a connection can be realistic, representing the concept true to an actual life situation, metaphoric, or not have a real existence. Figure 4.27 illustrates an example of a real-life connection presented in the PM. The example uses goats and cattle to present the fraction concept. Cattle and goats are common domesticated animals in Namibia. Learners also learn about cattle and goats in the subject Agriculture.



Exercise 3.15

1. Nakamwi had 180 goats. She sold $\frac{5}{9}$ of her goats to her neighbour.
   a) How many goats did she sell?
   b) How many goats does she have left over?
2. Paulus put $\frac{2}{3}$ of his cattle in a kraal. He sold $\frac{1}{4}$ of the cattle in the kraal to his neighbour. He sent the rest of the cattle in the kraal to the market.
   a) What fraction of his cattle did he sell to his neighbour?
   b) What fraction of his cattle did he send to the market?

Figure 4.27: Example of a real-life connection

Source: Hambata et al., 2015, p. 65

Connections to other subjects There are four subjects stated as indicators in the section on connections to other subjects, and the rest are categorised as Business Studies and others: Agriculture, English, Geography and Business Studies and others. The Namibian mathematics curriculum suggests that school mathematics should not be taught in isolation; instead, it should link to other subjects within itself and cross-curricular issues (MoE, 2015b). Figure 4.27 above illustrates the integration of the subject Agriculture into the fraction concept. Figure 4.28 below shows examples of integrating fraction content into the subject of English. In the example in Figure 4.28, creative writing is used. Learners write essays in English, so the notion of a limited number of words or pages is familiar to them. Hence, learners could use their experiences in English to help them solve the problem and contextualise it accordingly.

Exercise 3.14

1. Helena, Sirudi and Juan handed in a creative writing task to their teacher. Sirudi wrote $2\frac{2}{5}$ pages, Juan wrote $1\frac{7}{8}$ pages and Helena wrote $3\frac{1}{10}$ pages.
   a) Calculate the total number of pages the teacher must mark.
   b) The teacher has marked $5\frac{1}{4}$ pages. How many are left to mark?

Figure 4.28: Example of connections to other subjects
Source: Hambata et al., 2015, p. 65

Connections to other mathematical content areas

Mathematical content areas should link and work with each other throughout the curriculum. In Figure 4.29 below, examples 8, 9 and 10 integrate fractions with mathematical measurements covering money and length (distance). Most of the problems listed in Figure 4.29 are also real-life problems.

8. Ben spends N$650 of his salary of N$5 200 on rent for a house. Calculate the fraction of his salary that he pays for rent.
9. Pumla's garden has an area of 8 m^2. She has planted 6 m^2 with vegetables. What fraction is left over?
10. Namibia covers 824 000 km^2 of land. If 335 000 km^2 is communal land, what fraction of the land is communal?

Figure 4.29: Example of connections to other mathematical content areas
Source: Hambata et al., 2015, p. 63

Connections to problem-solving

The examples in Figures 4.27, 4.28 and 4.29 can also be categorised under connections to problem-solving (CPS). It has two indicators: real-life problems and cross-curricular issues, which are split into five indicators (HIV & AIDs, population issues, ICT, road safety and globalisation). Figure 4.29 is a real-life problem-solving example which uses the country's name (Namibia). This familiarisation would stimulate the learners' interest and engagement in learning.

4.2.3.3 Level 2 analysis

The Level 2 analysis focuses on the purpose of the connections using three criteria: exemplifying (Exa), explanatory (Exp) and decorative (D) as described in see Appendix 2. See Section 4.2.1.3 for a detailed discussion of these three criteria.

Purpose of the connections in the PM

The data of connections used for the above purposes, evident in the PM, are presented in Table 4.6 and Figure 4.30 below.

Table 4.6: The number of connections evident in PM and their purposes

Connections in Mathematics	**Purpose (P)**			**Frequency**
	Exemplifying (Exa)	**Explanatory (Exp)**	**Decorative (D)**	
Real-life Connections (RLC)	17	0	0	17
Connections to other subjects (CS)	8	0	0	8
Connections to other mathematics content areas (CMC)	21	0	0	21
Visualisation connections (VC)	13	1	0	14
Connections to problem-solving (CPS)	17	0	0	17
Total	76	1	0	77

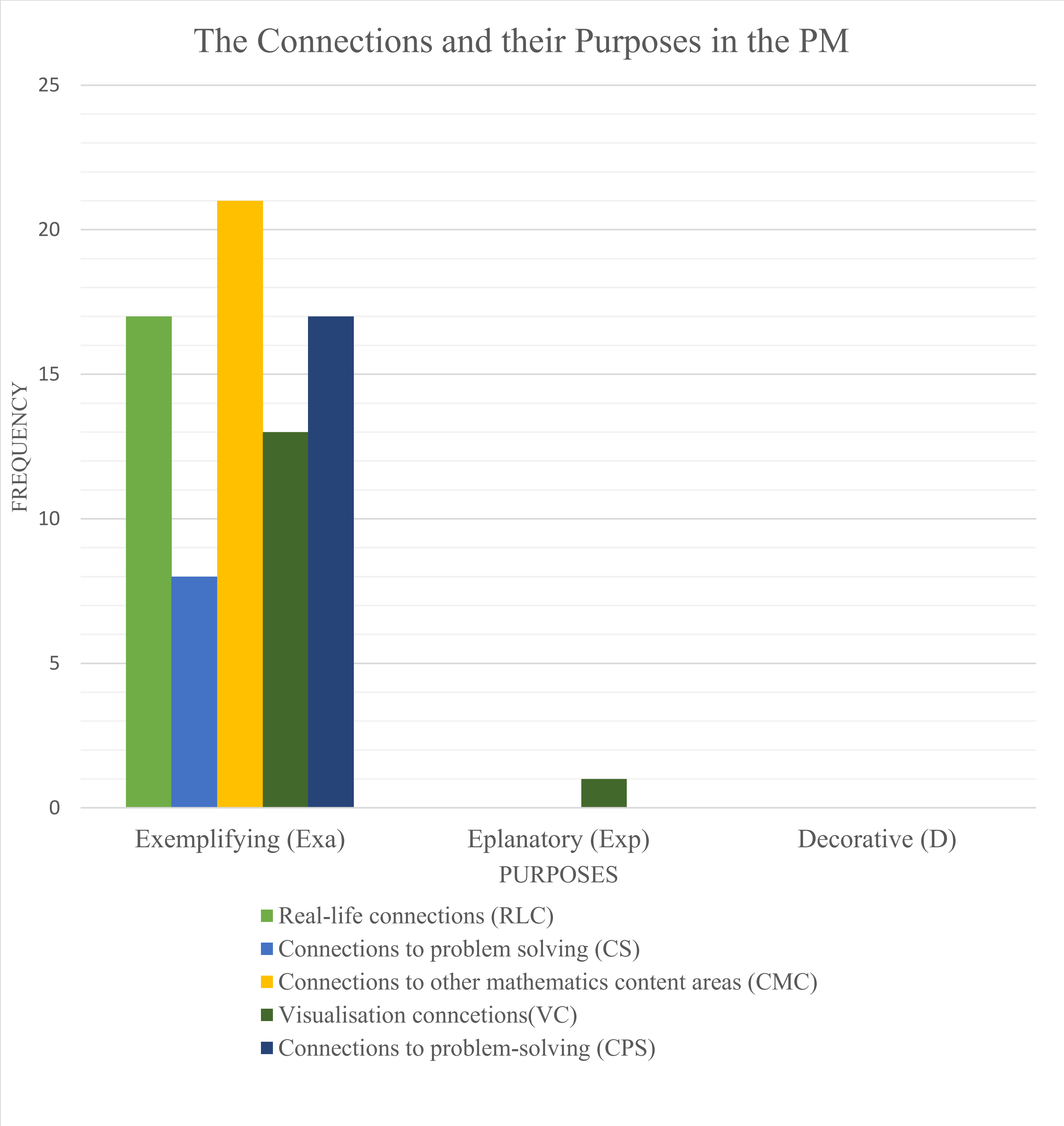


Figure 4.30: The number of connections evident in PM and their purposes

The exemplifying purpose

Figure 4.31 below shows an example of a visualisation connection serving an exemplifying purpose in the PM. In this example, diagrams illustrate how a fraction can be presented or reasoned through a diagram. Figure 4.31 explains how a learner can solve the fraction problem $5 \div \frac{2}{3}$. An explanation is provided under the diagrams.

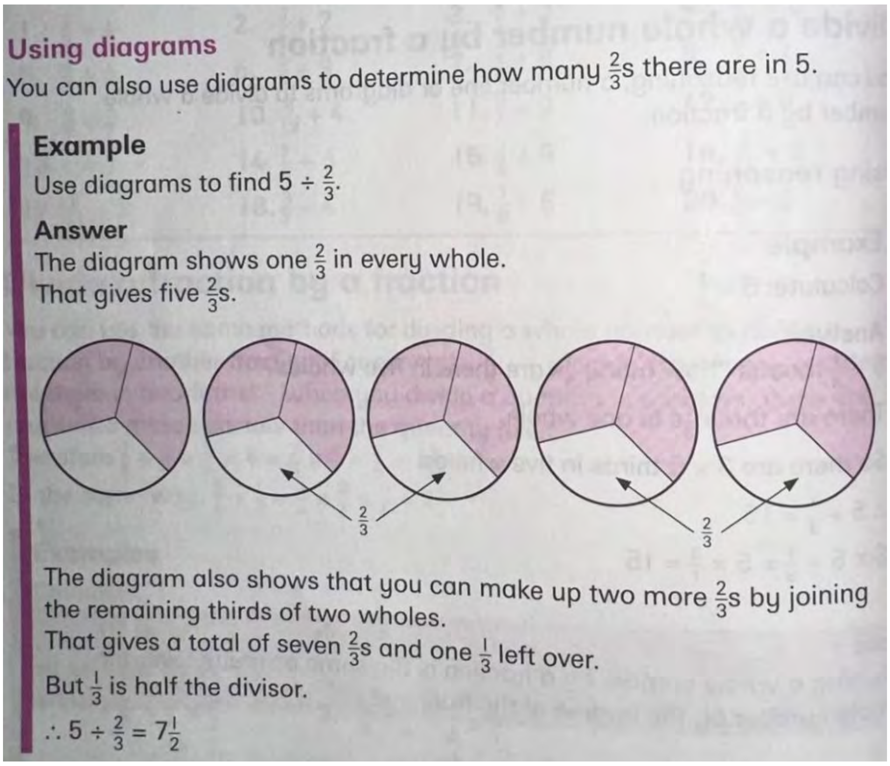


Figure 4.31: Example of a VC serving an exemplifying purpose
Source: Hambata et al., 2015. p. 60

The explanatory purpose

Figure 4.32 below illustrates a comparison of common fractions and mixed numbers. The visualisation connection (fraction wall) compares eight fractions in this example. Although no summary is provided, the diagram is accessible to interpret and is self-explanatory.

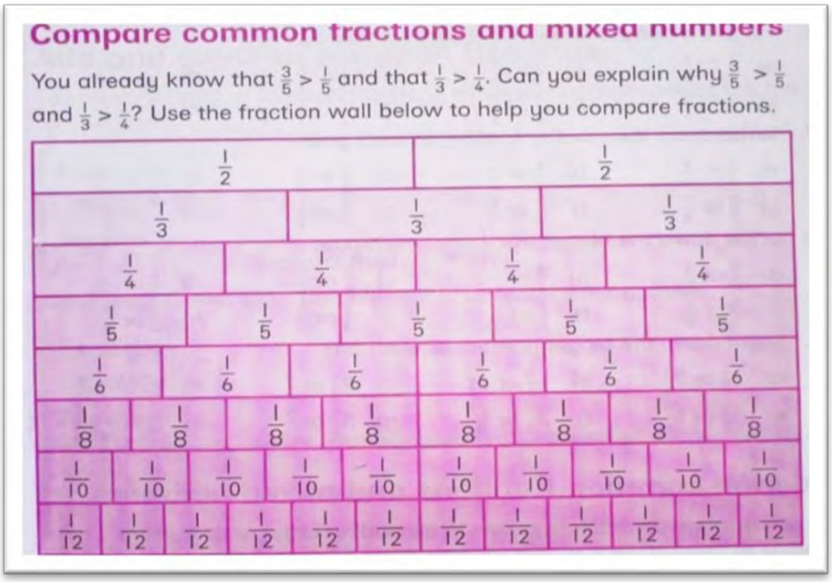


Figure 4.32: Example of a problem illustrating an explanatory purpose
Source: Hambata et al., 2015. p. 49

The decorative purpose

The PM does not have an example of the decorative connection purpose.

4.2.3.4 Concluding remarks regarding the PM

Level 1 analysed the connections in mathematics knowledge prevalent in the CFMN topic of the PM. The analysed data indicates the presence of many connections, with 77 connections recorded altogether. The connections to other mathematics content areas dominate by 27.3%, followed by real-life connections and connections to problem-solving with 22.1% for each. Visualisation connections cover 18.2%. The least presented connections are to other subjects, with only 10.4% of the total.

In the Level 2 analysis, I analysed the purposes of the connections recorded in the Level 1 analysis. Data reflects that the connections prevalent in the CFMN chapter serve a purpose and are not vague. Exemplifying purposes cover 99% of the CFMN learning content, followed by explanatory purposes, which cover 1%. The decorative purpose is not present in the PM.

4.3 HORIZONTAL DATA PRESENTATION OF THE MATHEMATICAL CONNECTIONS PREVALENT IN THE THREE SELECTED TEXTBOOKS

The central focus of this section is to horizontally analyse the three selected Grade 7 mathematics textbooks used by teachers and learners in the Namibian education system. As per the analytical framework (see Appendix 1 and Appendix 2), data collected and analysed were presented at two levels. Implicit and explicit learning, and the types of connections were discussed in Section 4.2.1.2. The same criteria were applied to the Level 1 analysis across the three selected textbooks.

In Level 2, I focused on the purpose of the connections using three criteria: exemplifying, explanatory and decorative (see Appendix 2). It also crosschecked the presence (frequency) of the connections' purposes. Please refer to Section 4.2.1.3 for a detailed discussion of these three criteria. The same criteria were applied to the Level 2 analysis across the three selected textbooks.

4.3.1 Level 1 analysis of the EMLB, OLDM and PM

In this section, I analysed the data collected on the connections in mathematics knowledge prevalent in the CFMN topic of the three selected textbooks. Please refer to sections 4.2.1.2, 4.2.2.2 and 4.2.3.2 of the analysis of the three textbooks for the elaboration on the analysed data.

Data presentation on the nature of connections in mathematics evident in the three selected textbooks

The data on the connections evident in the three selected textbooks' Level 1 analysis is presented in Table 4.7 and Figure 4.33 below. The learning content was analysed according to the analytical tool, as stated above in the introduction to this chapter. The description is provided below Figure 4.33 in percentages.

Table 4.7: Data presentation for the EMLB, OLDM and PM Level 1 analysis

Connections in mathematics	**EMLB**	**OLDM**	**PM**	**TOTAL**
Real-life connections (RLC)	27	29	17	73
Connections to other Subjects (CS)	4	3	8	15
Connections to other mathematics content areas (CMC)	32	40	21	93
Visualisation connection	5	38	14	57
Connection to problem-solving (CPS)	27	29	17	73
Total	**95**	**139**	**77**	**311**

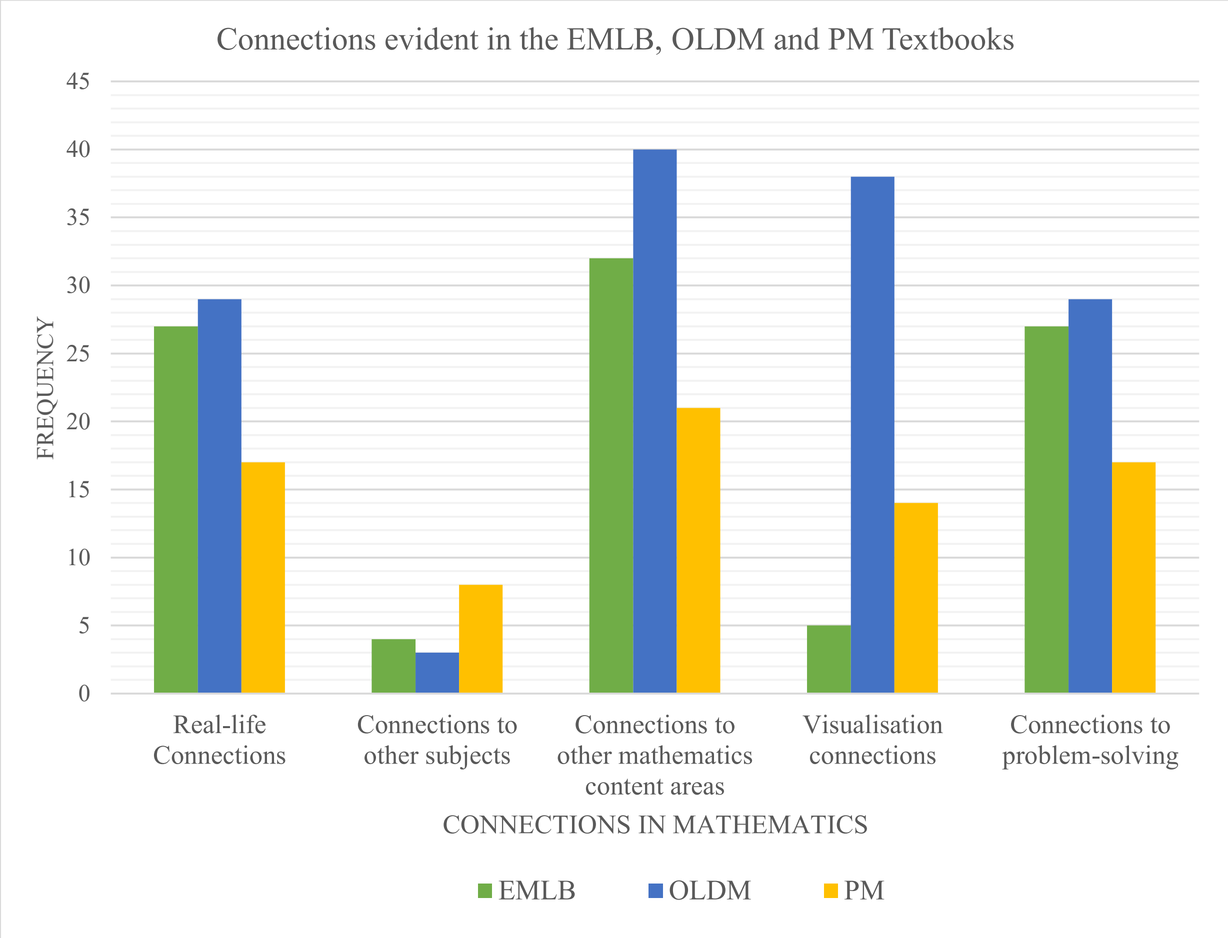


Figure 4.33: The connections evident in the EMLB, OLDM, and PM textbooks

The Level 1 analysis of the three selected textbooks indicates that all five connections stated as indicators on the analytical tool are presented across the CFMN chapters of the three textbooks. For the real-life connections:

- the OLDM tops the presentations with 40%,
- the EMLB presents 37% of the fraction content using real-life problems, and
- the PM only uses 23% of real-life connections.

The connections to other subjects are the least utilised connections across all connections presented in the three selected textbooks:

- the PM textbook uses the most connections to other subjects, with 53%, followed by
- the EMLB with 27%, and the
- OLDM with 20%.

The connections to other mathematical content areas are the most utilised across all three textbooks:

- the OLDM uses 43% connections,
- the EMLB uses 34%, and

- the PM only uses 23%.

Visualisation connections are the second-least used connections across the three selected textbooks:

- the OLDM uses the most, with 66.7% connections,
- the PM uses 24.6%, and
- the EMLB only uses 8.8%.

Connections to problem-solving share the same coverage as the real-life connections because only real-life problems are presented in the CFMN chapter across all three selected textbooks:

- EMLB uses 37%,
- OLDM uses 40%, and
- PM uses 23%.

The five connections mentioned above are divided into indicators. Section 4.2.1.2 gives a detailed discussion of the types of fractions and their indicators. For real-life connections, the lived experience indicator was the most common example across the three selected textbooks. The food recipe was only evident in the OLDM and not in the other two textbooks and the learners' interests were only evident in the EMLB. The agriculture indicator was the most common across the textbooks in its connections to other subjects. The EMLB connects to Business, and PM connects to the subject of English. The measurement indicator was the most common and most-used connection to other mathematical content presented across the three selected textbooks, followed by money and finance. All three textbooks used pictorial/diagrammatic images as visualisation connections. Only the OLDM used drawings. The real-life problems were the only indicator used across the three selected textbooks in connecting to problem-solving. The data collected on the types of connections evident in the three selected textbooks is presented in Figure 4.34 below.

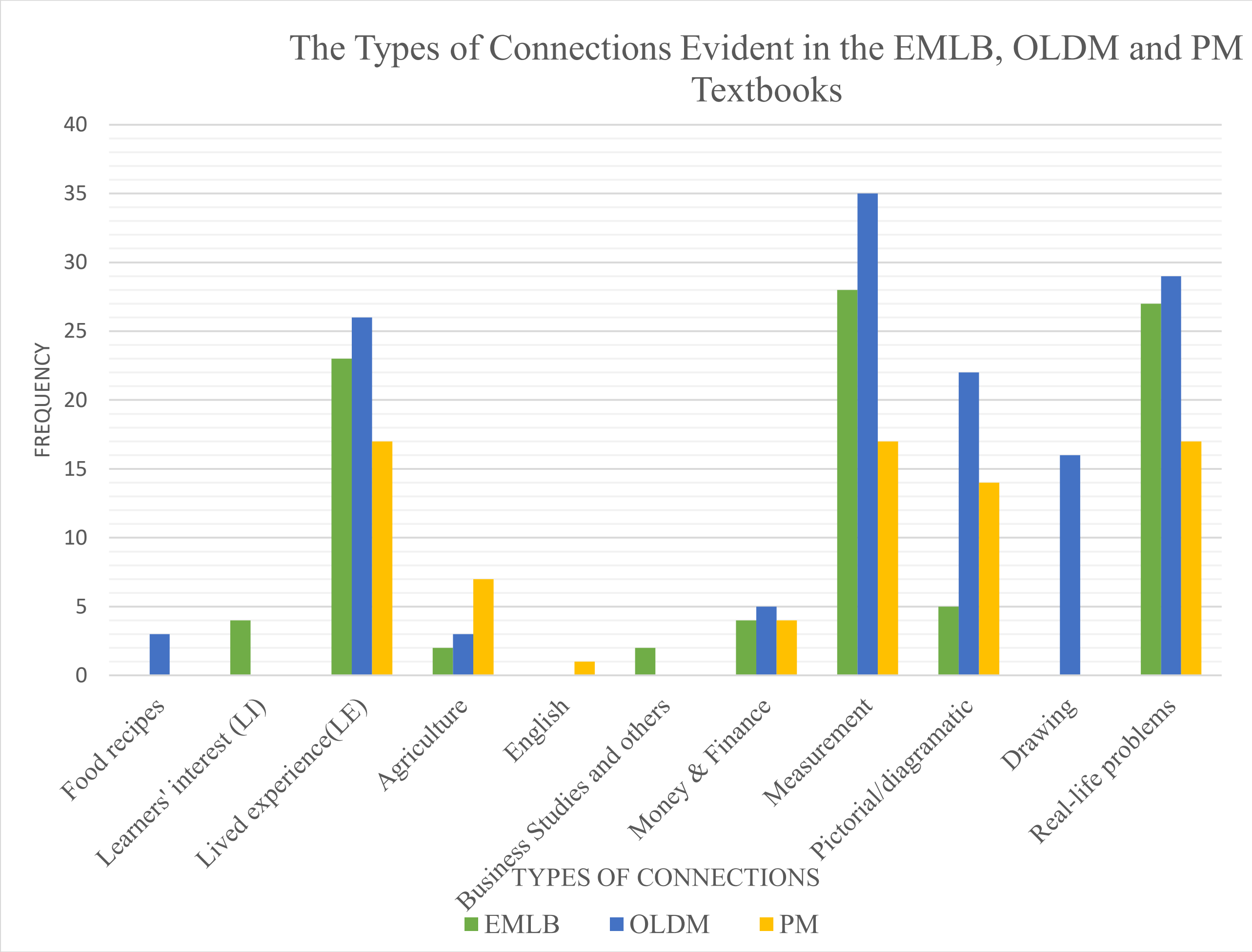


Figure 4.34: The types of connections evident in the EMLB, OLDM and PM textbooks

4.3.2 Level 2 Analysis

The Level 2 analysis focused on the purpose of the connections using three criteria: exemplifying (Exa), explanatory (Exp) and decorative (D) (see Appendix 2). The purposes are described in Sections 4.2.1.3, 4.2.2.3 and 4.2.3.3 of the analysis of the three textbooks for the elaboration on the analysed data. Section 4.2.1.3 gives a detailed description of these three criteria. The same criteria are applied to the data analysis for the three selected textbooks. The data on the connections and their purposes are presented in Table 4.8 and Figure 4.35 below. The description is provided below Figure 4.35 in percentages.

Table 4.8: The number of connections evident in the EMLB, OLDM, PM and their purposes

Connections in Mathematics		Purpose (P)			Frequency
		Exemplifying (Exa)	Explanatory (Exp)	Decorative (D)	
Real-life Connections (RLC)	EMLB	25	2	0	27
	OLDM	29	0	0	29
	PM	17	0	0	17
Connections to other subjects (CS)	EMLB	4	0	0	4
	OLDM	3	0	0	3
	PM	8	0	0	8
Connections to other mathematics content areas (CMC)	EMLB	31	1	0	32
	OLDM	40	0	0	40
	PM	21	0	0	21
Visualisation connections (VC)	EMLB	0	4	1	5
	OLDM	27	6	5	38
	PM	13	1	0	14
Connections to problem-solving (CPS)	EMLB	27	0	0	27
	OLDM	29	0	0	29
	PM	17	0	0	17
Total		291	14	6	311

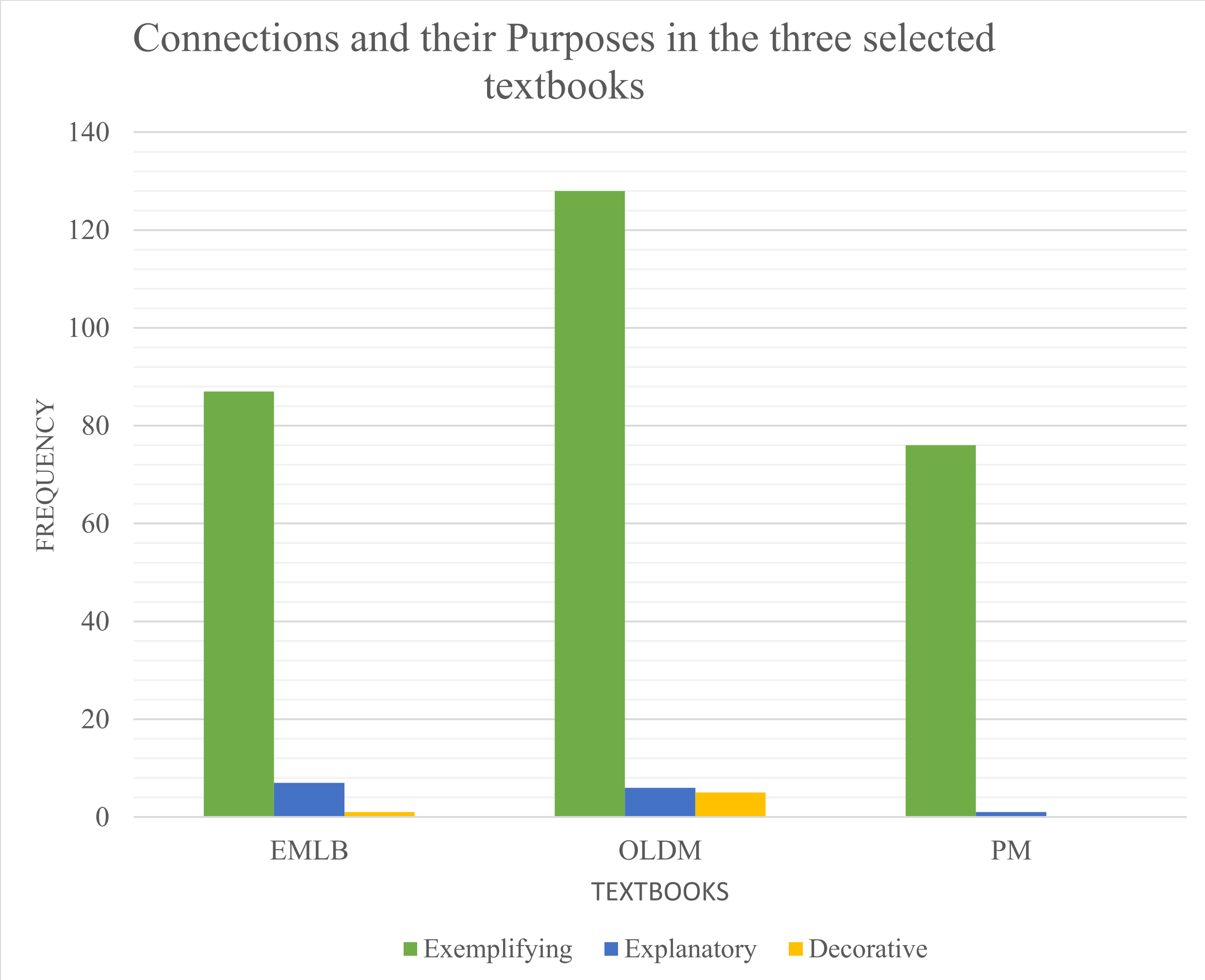


Figure 4.35: The number of connections evident in the three selected textbooks and their purposes

In the Level 2 analysis across the three selected textbooks, data reflects that the connections all served a purpose. The exemplifying purpose is the most used across all three textbooks:

- OLDM uses the most connections with 44%;
- the EMLB has 30%; and
- the PM only has 26%.

The explanatory purpose has 14 connections presented:

- the EMLB has 50%;
- OLDM has 43%; and
- the PM has 7%.

The decorative purpose is the least used purpose across all three selected textbooks, with only 6 connections:

- the OLDM has 83%;
- the EMLB has 17%; and

- the PM has no decorative connections.

4.3.3 Concluding remarks about the EMLB, OLDM and PM analysis

A textbook should bridge mathematics knowledge acquisition and mathematics curriculum attainment – see the literature review in Chapter 2, Section 2.2.1. In order to be resourceful and useful, textbooks should be of good quality. The three selected textbooks all have the following three qualities which meet the curriculum attainment. The following qualities are observed in the three selected textbooks (evident in the general structure of the three selected textbooks' presentation):

- the textbooks align with the curriculum;
- the textbooks are written according to the prescribed syllabus; and
- every aspect of the syllabus is adequately covered.

The analysis reflected that the EMLB goes the extra mile by listing, on the first page of each chapter, the basic competencies from the syllabus to be attained by the end of each chapter. The textbook guides learners on what to do and how to do it. On the other hand, in their chapters, the OLDM and PM do not state what the basic competencies are.

The real-life connections and connections to other mathematics content areas are strongly evident across the three selected textbooks. However, the connections to other subjects are not common across the textbooks. This could potentially be problematic as it creates a gap between mathematics and other subjects and, unfortunately, reinforces the notion of mathematics being a discrete and complex subject unconnected to other subjects.

Generally, it was observed that connections evident in the CFMN chapter of the three textbooks all had a purpose. The connections with an exemplifying purpose were the most prevalent across all three textbooks. The visualisation connection helped to clarify the text and assisted learners in imagining or actively representing the abstract fraction content into visual objects. The visualisation connections were mainly used for exemplifying purposes.

4.4 SUMMARY OF FINDINGS AND CONCLUSION

The study analysed three selected Grade 7 mathematics textbooks – EMLB, OLDM, and PM, examining connections within the CFMN topic. The Level 1 analysis for each textbook revealed varying prevalence and types of connections. The EMLB recorded 95 connections, focusing predominantly on real-life connections and connections to other mathematical content areas. The OLDM textbook had 139 connections, highlighting a dominance of connections to other mathematics content and visualisation connections. The PM textbook exhibited 77 connections, with a notable emphasis on connections to other mathematics content.

The Level 2 analysis probed the purposes of these connections. Across all textbooks, exemplifying connections were most prevalent, covering a significant portion of the CFMN learning content. The purposeful nature of connections was consistent, with the EMLB and OLDM featuring explanatory connections, while the PM had no decorative connections.

The evaluation of the selected textbooks against curriculum requirements indicates that all three align with the curriculum, follow the prescribed syllabus and adequately cover each aspect of the syllabus. Notably, the EMLB stood out by explicitly listing basic competencies at the beginning of each chapter, providing additional guidance for learners. Both the OLDM and PM lack this feature. Real-life connections and connections to other mathematics content were prominent in all textbooks, but connections to other subjects were less common. This lack of interdisciplinary connections could create a gap between mathematics and other subjects, reinforcing the notion that mathematics is a separate and difficult subject. The prevalence of purposeful connections in the CFMN chapters across all textbooks, particularly those serving an exemplifying purpose, indicated thoughtful design. In the next chapter, I analyse the questionnaire responses and the interviews.

CHAPTER 5 : DATA ANALYSIS – QUESTIONNAIRE AND THE INTERVIEWS

5.1 INTRODUCTION

This chapter presents the results of the administered questionnaires, followed by an analysis of the semi-structured individual and focus group interviews, involving the four selected participants. A consolidated summary of findings from these diverse data sources is subsequently provided.

The data presented in this chapter sought to answer the research questions:

- How do selected Namibian Grade 7 textbooks encourage connection-making in the context of fractions?
- What are selected teachers' perceptions and perspectives of the three textbooks concerning connection-making in fraction concepts?

Throughout the analysis of the interviews, various themes emerged, shaping the subsequent narrative. The chapter concludes with an in-depth exploration of these emergent themes, including the significance of mathematical connections, the necessity for teacher training in this domain, the role of workshops and professional development, initiatives at the regional and cluster levels, leadership and advocacy, and the importance of collaboration and communication. Although these themes are addressed individually, it is acknowledged that they are intricately interconnected.

5.2 QUESTIONNAIRE DATA PRESENTATION

The results of the responses from the questionnaires of the 21 teachers were analysed and are presented as follows:

- I started by presenting the responses to Section A of the questionnaire, which focused on the respondents' demographic details.
- This was followed by responses to Section B, which referred to the connection-making in the fraction section of the mathematics textbook the teachers use in schools.
- Finally, responses to Section C are presented as teachers' perceptions and attitudes on mathematical connections/making connections in the classroom.

5.2.1 Presentation of data from Section A of the questionnaire

Demographic data of the questionnaire respondents are presented in Table 5.1 below. The prescribed textbooks selected for the textbook analysis are highlighted in yellow for each participant using them. An elaboration of the demographic data statistics in Table 5.1 is provided below it.

Demographic data of the questionnaire participants is illustrated in Figure 5.1 below, starting with the gender, age and work experience of each teacher.

Table 5.1: Demographic data of the questionnaire participants

Participants	Gender	Age	Work experience as a teacher	Textbooks used in teaching fractions to Grade 7 learners	Subjects and grades taught
1. J.K.	F	$30 \leq x < 35$	6-10 years	• Excellent Mathematics Learner's Book • Let's Do Mathematics • Platinum Mathematics • Maths in My World • Maths For Life	• Mathematics 4-7 • Natural Science 4,5 & 7
2. C.K.	F	$35 \leq x < 40$	More than 10 years	• Excellent Mathematics Learner's Book • Platinum Mathematics	• Mathematics 7-9
3. K.S.	F	$20 \leq x < 30$	1-2 years	• Excellent Mathematics Learner's Book • Platinum Mathematics	• Mathematics 7 & 10 • Chemistry 10-11 • P/Science 8-9
4. SKM	F	$35 \leq x < 40$	6-10 years	• Platinum Mathematics • Champions	• Mathematics 7A-D • Natural Science 4D
5. EIN	F	$x \geq 50$	More than 10 years	• Platinum Mathematics	• Mathematics 5 & 7
6. N.P.	F	$40 \leq x < 50$	More than 10 years	• Excellent Mathematics Learner's Book • Let's Do Mathematics • Platinum Mathematics	• Mathematics Gr. 7 • Natural Science Gr.6
7. N	F	$20 \leq x < 30$	3-5 years	• Excellent Mathematics Learner's Book • Let's Do Mathematics • Platinum Mathematics	• Mathematics 4 • Elementary Agriculture 5 & 6
8. SO	F	$40 \leq x < 50$	More than 10 years	• Excellent Mathematics Learner's Book • Let's Do Mathematics • Platinum Mathematics	• Mathematics 7 A-D • Elementary Agriculture 5C-D
9. M.S.	M	$30 \leq x < 35$	6-10 years	• Excellent Mathematics Learner's Book • Let's Do Mathematics • Platinum Mathematics	• Mathematics 4 and 7
10. A.T.N.A.	F	$20 \leq x < 30$	Less than a year	• Excellent Mathematics Learner's Book	• Mathematics 7 • Natural Science 4, 5 & 7 • Elementary Agriculture 5-7
11. None	M	$35 \leq x < 40$	More than 10 years	• None indicated	• Mathematics 7, 8, 10 &11
12.SNA	M	$40 \leq x < 50$	More than 10 years	• Platinum Mathematics	• Mathematics 7 A, 6 A-D
13.MP	M	$30 \leq x < 35$	6-10 years	• Excellent Mathematics Learner's Book • Platinum Mathematics	• Natural Science 4 • Mathematics 4 & 7
14.P.N.	M	$20 \leq x < 30$	1-2 years	• Champions Mathematics	• Mathematics 6 & 7 • Natural Science 7 • Design and Technology 5
15.None	M	$30 \leq x < 35$	6-10 years	• Let's Do Mathematics	• Mathematics &

				• Platinum Mathematics	• Natural Science 4-7 • Social Studies 4-7
16.S.M.	M	$30 \leq x < 35$	3-5 Years	• Platinum Mathematics	• Mathematics 4-7 • Elementary Agriculture 5-7
17.None	M	$35 \leq x < 40$	More than ten years	• Excellent Mathematics Learner's Book	• Mathematics 4-7
18.Mr M	M	$x \geq 50$	More than ten years	• Let's Do Mathematics • Platinum Mathematics	• Mathematics G 6
19.JSD	M	$35 \leq x < 40$	6-10 years	• Let's Do Mathematics • Platinum Mathematics	• Mathematics 4 & 7
20.None	M	$40 \leq x < 50$	More than 10 years	• Excellent Mathematics Learner's Book • Platinum Mathematics	• Mathematics 7-9 • Physical Science 8-9
21.None	M	$30 \leq x < 35$	3-5 years	• Excellent Mathematics Learner's Book • Let's Do Mathematics • Platinum Mathematics	• Mathematics 4-7

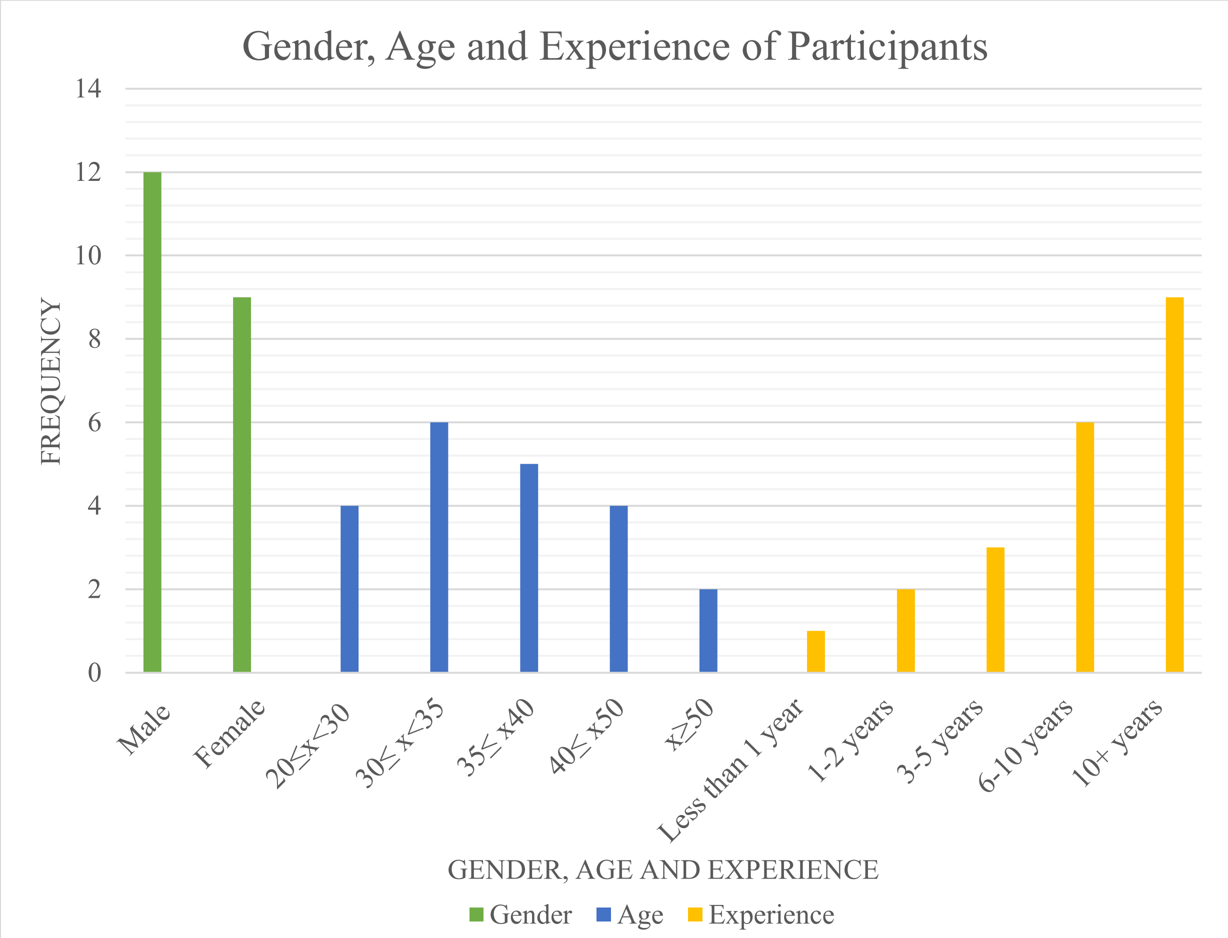


Figure 5.1: The gender, age and experience of the participants

Data reflects that 57% of the participants are male and 43 % are female. Five age groups were listed as indicators for the participants: $20 \leq x < 30;\ 30 \leq x < 35;\ 35 \leq x < 40;\ 40 \leq x < 50;\ \ x \geq 50$. Nineteen per cent of the participants are between 20 and 30 years of age, 28.6% are between the ages of 30 and 35, 23.8% are between the ages of 35 and 40, 19% participants are between the ages of 40 and 50 years and the last 9.5% are 50 years or older. The teachers' experience is divided into five indicators: 1 year or less, 1-2 years, 3-5 years, 6-10 years and 10 or more years of experience, with 4.8% of participants teaching for a year or less, 9.5% teaching for 1 or 2 years, 14.3% for three to five years, 28.6% for six to ten years, and 42.9% of the participants are mathematics teachers with ten or more years of experience.

Figure 5.2 below illustrates how many teachers use each of the three textbooks under analysis and others.

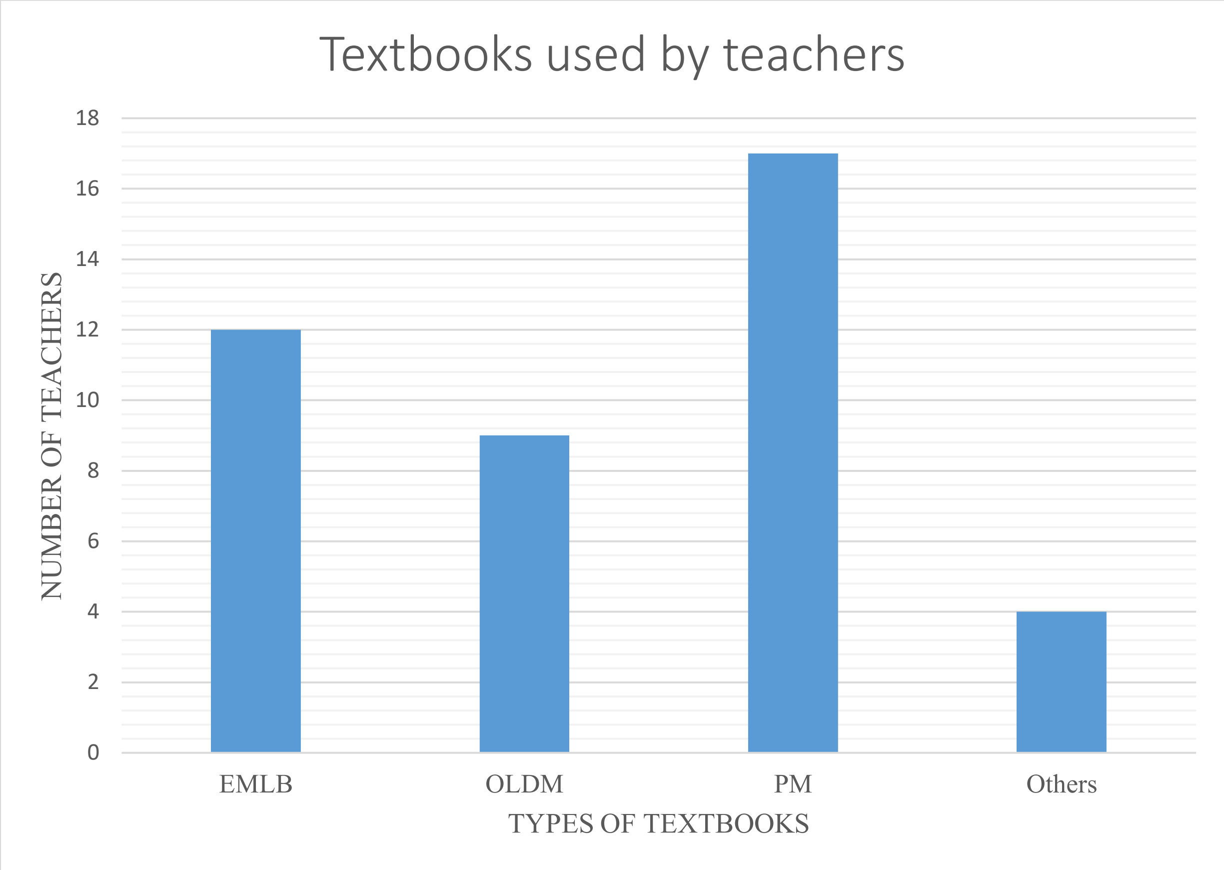


Figure 5.2: The textbooks used by teachers

Data collected reflects that PM is the most utilised textbook among Grade 7 mathematics teachers (40%), the EMLB is the second most used textbook by teachers (28%), and 21% of the data collected reflects that OLDM is the third most-used textbook. Other textbooks are used 9% of the time and 2% of the participants did not indicate which textbooks they used. The PM textbook may be the most used textbook because its is easily accessable in schools and the price is reasonable compared to the EMLB. The textbook catalogue reflects the cost of the prescribed textbooks (MoE, 2018, 2019)

In Figure 5.3 below, the subjects and grades taught by the respondents are presented.

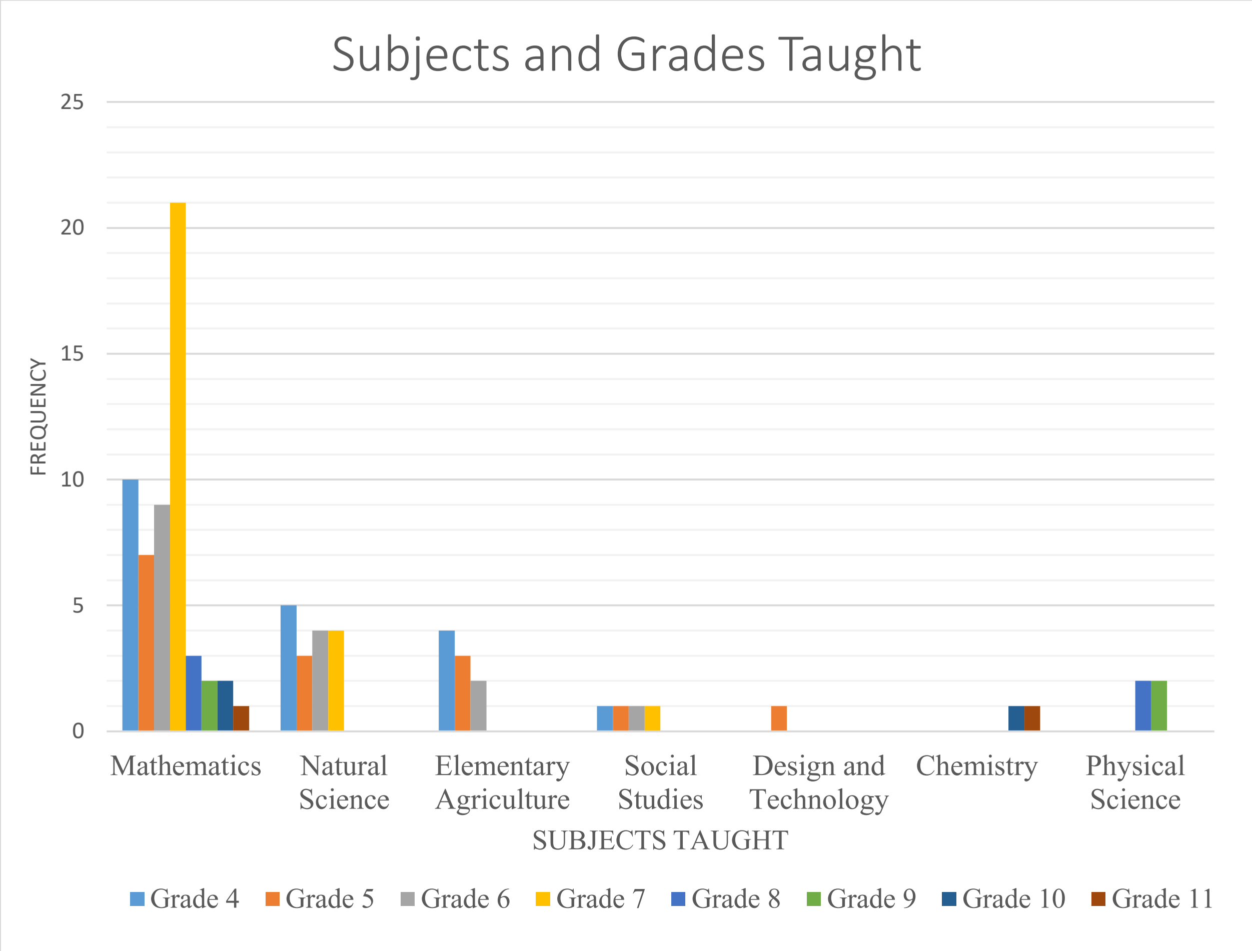


Figure 5.3: Grades and subjects taught by the participants

The data reflects that the participants also teach other subjects besides mathematics. They teach across the phases because some schools are combined, and the subject allocation applies to the entire school. The school is treated as one, so teachers teach across the phases. The Namibian education system is divided into four phases: Junior Primary (Grades 0-3), Senior Primary Phase (Grades 4-7), Junior Secondary Phase (Grades 8-9) and the Senior Secondary Phase (Grades 10-12) (MoEAC, 2015b).

5.2.2 Presentation of data from Section B of the questionnaire

Section B of the questionnaire sought to discover the teachers' perceptions of and perspectives on connection-making in the fraction section of their mathematics textbook. Section B consisted of six questions: data collected in Questions 1 and 2 are presented separately, and Questions 3 to 6 are analysed and presented together.

Question 1 analysis

Question 1 of Section A sought to discover the teachers' perspectives and perceptions on whether the textbooks they use explicitly articulate connections in the fraction concept. Participants responded with either a yes or no. Five connections in the analytical tool used in this study were listed as indicators: real-life connections, connections to other subjects, connections to other mathematical content areas, visualisation connections and connections to problem-solving. A summary of the data collected is presented in Figure 5.4 below.

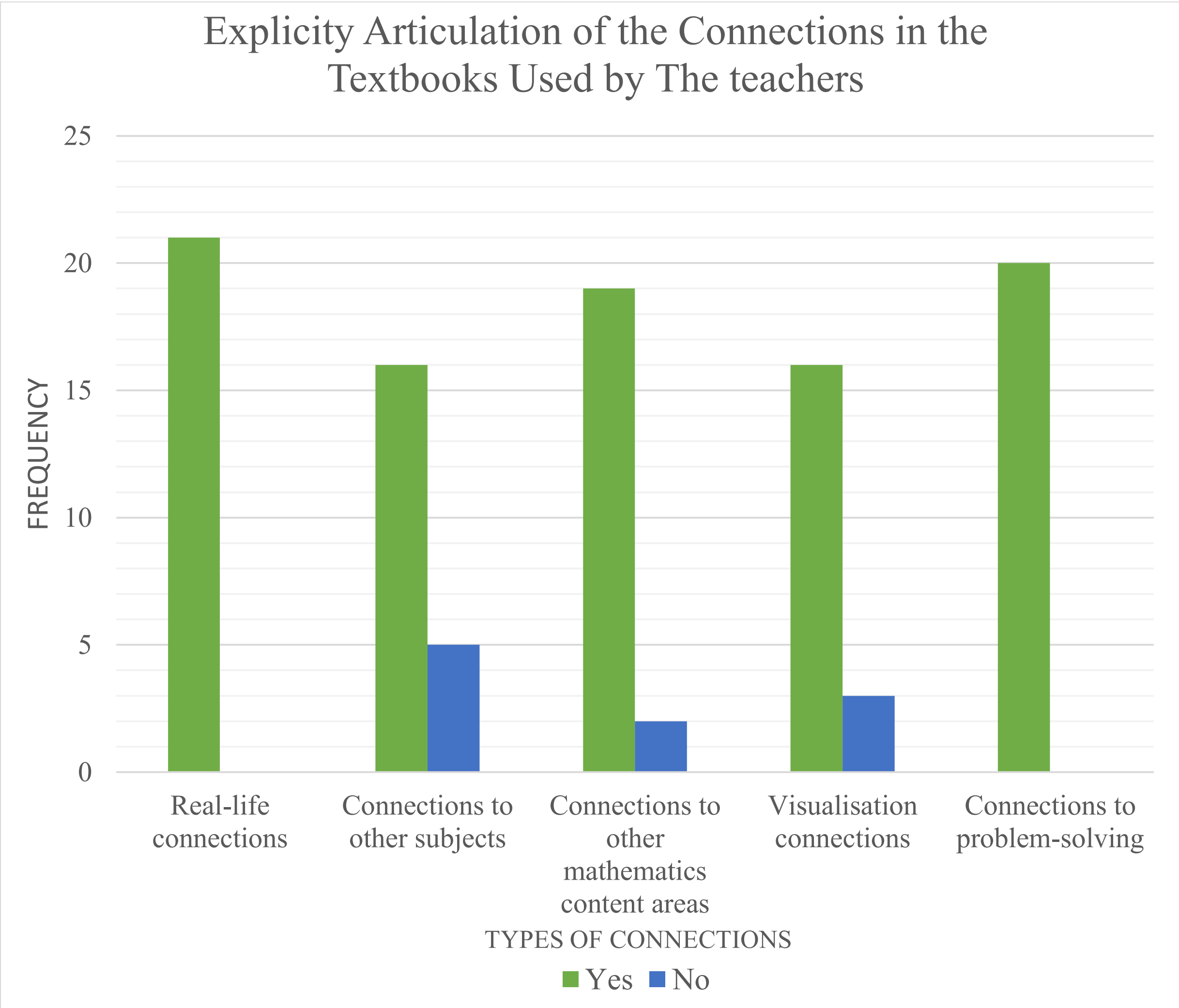


Figure 5.4: Example of connections explicitly articulated in the textbooks

Data collected reflects that 90% of the participants agreed that real-life connections, connections to other subjects, connections to other mathematics content, visualisation connections and connections to problem-solving are explicitly articulated in the fraction concepts chapter of the textbooks they use. Ten per cent of the respondents disagreed that the five connection indicators are articulated expressly in their textbooks.

Question 2 analysis

Question 2 of Section A sought to discover the teacher's perspectives and perceptions about the visual models prevalent in the textbook on the fraction concept. Participants responded with either a yes or no. Four visual models from the analytical tool used in this study were listed as indicators: tables, graphs, drawings and pictures, and the rest are categorised as 'others'. A summary of the data collected is presented in Figure 5.5 below.

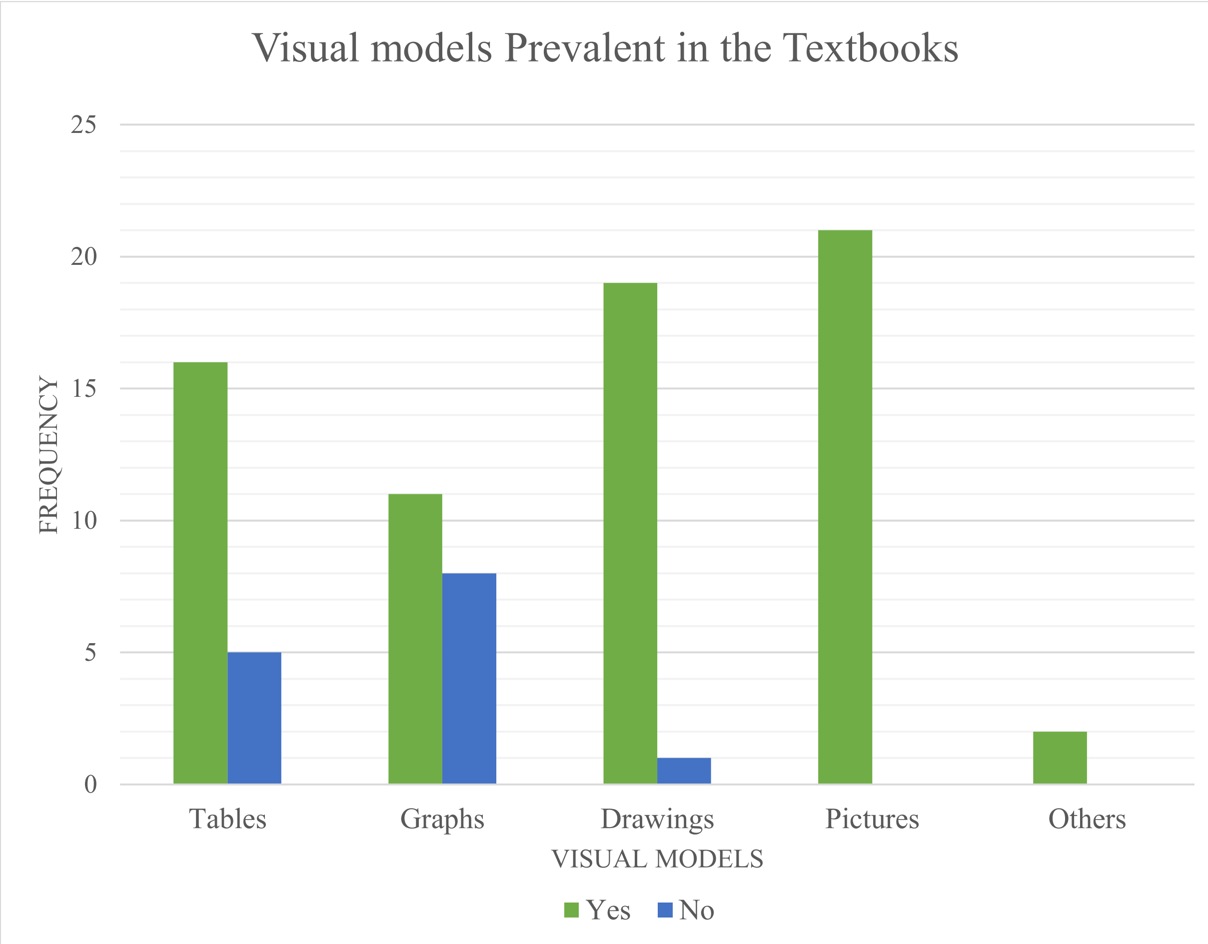


Figure 5.5: The visual models prevalent in the textbooks on the fraction concept

The data reflects that 82% of the participants agree that tables, graphs, drawings, pictures and other models are explicitly used in the fraction concepts chapter of the textbooks they use. Eighteen per cent of the respondents disagree that the four visual models are explicitly articulated in the same textbooks they use.

Questions 3, 4, 5 and 6 analysis

This section analyses Questions 3, 4, 5 and 6 of Section B of the questionnaire. The data is presented in Figure 5.6 below, and each question is elaborated on below the figure.

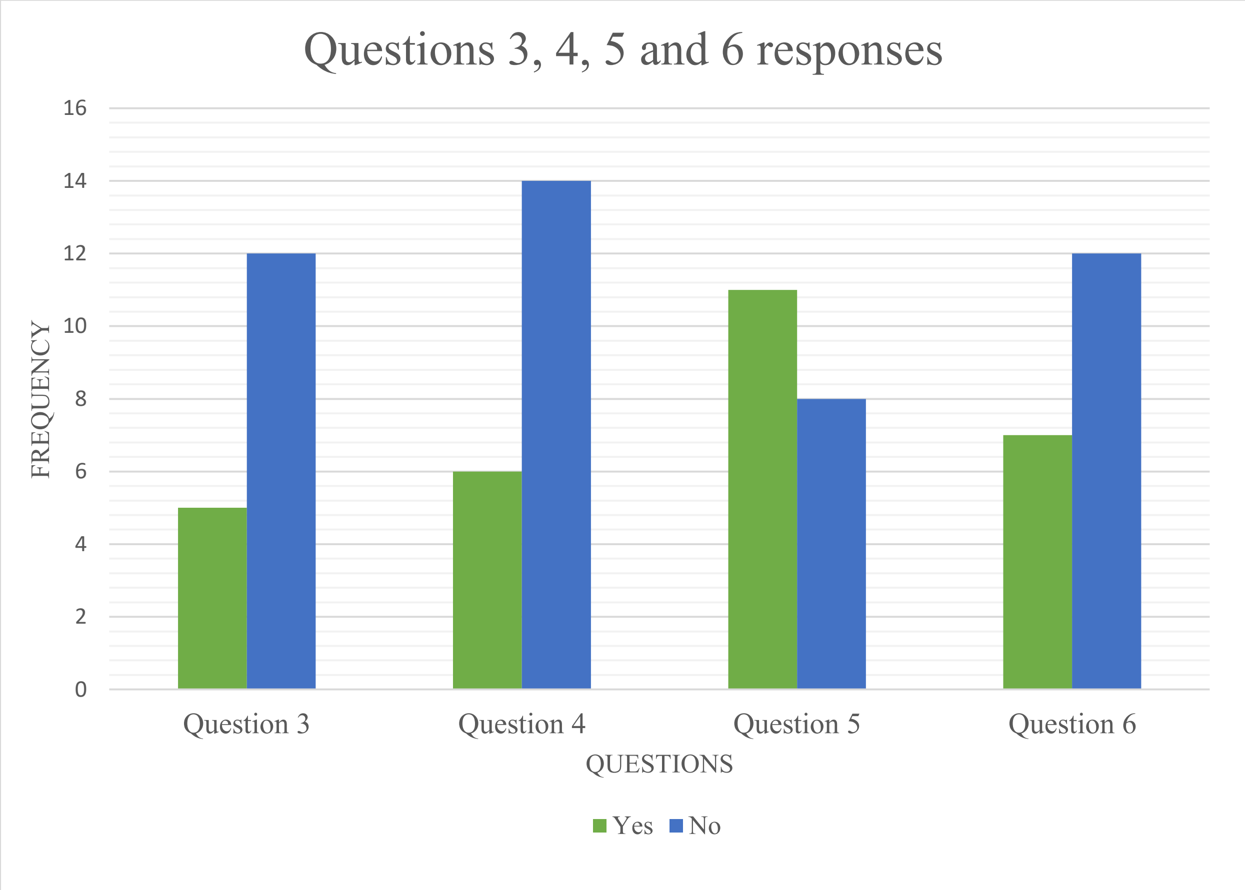


Figure 5.6: Responses to questions 3, 4, 5 and 6 of Section B of the questionnaire

Lesson Planning Support Team (LPST within the mathematics department)

Question 3 of Section B of the questionnaire sought to determine whether the participants' schools have a lesson planning support team (LPST) within the mathematics department. Participants responded by indicating either a yes or no response to Question 3. Twenty-nine per cent of the participants stated a yes, and 71% responded no to the question asked. Although some participants indicated that LPSTs are available at their respective schools, the need for such was indicated. For example, two respondents noted:

- *I need lesson planning support teams to be introduced at my school so that I get to learn from my colleagues how to teach fractions and other topics in mathematics.*
- *Our school need lesson planning support within our department as well as across other departments. Implementing this support team will enhance teaching and share important information in dealing with the failure of learners and helping out weak learners. Lack of necessary feedback after lessons is also a problem within our department.*

Lesson Planning Support Team (LPST across other departments)

Question 4 of Section B of the questionnaire sought to determine whether the participants' schools have a lesson planning support team (LPST) across other departments. Participants responded by indicating either a yes or no response to Question 4. Seventy per cent of the participants stated a no, and 30% showed a yes to the question asked. Although participants indicated that LPSTs in other departments are available at their respective schools, no further elaborations were provided in their responses.

Any other accessible resources

Question 5 of Section B of the questionnaire sought to determine whether teachers use other teaching resources besides the prescribed textbooks. The participants responded by indicating either a yes or no response to the question and spaces were provided to note down any other resources they use to teach the fraction concept. Fifty-eight per cent of the participants indicated a yes to the question, while 42% indicated a no. Participants referred to the following as other resources they use in their teaching: internet handouts, classroom pasted pictures, library textbooks, magazines, newspapers, mathematics puzzles, charts, posters, YouTube videos, examination reports, peer-teaching and past question papers. However, participants indicated a severe lack of teaching resources in their respective schools. For example, three respondents stated that:

- [They lack] *teaching resources in mathematics at the school currently: most learners are failing basic mathematics due to a lack of teaching and learning resources.*
- *The textbooks used are not detailed for learners to understand.*
- *Resources used for teaching mathematics is inadequate, making it difficult for teachers to relate mathematics teaching to real-life situation. Textbooks are also not enough; mostly, teachers use the chalkboard, and learners will not have enough time to copy the examples.*

ICT Integration in Fraction Concepts

Question 6 of Section B of the questionnaire sought to determine whether teachers integrated ICT into teaching the fraction concept. Participants responded by indicating either a yes or no response to the question, and spaces were provided for further elaboration and comments. Thirty-seven per cent of the participants answered yes to the question, while 63% indicated a no. Because 63% of the participants noted the absence of ICT integration in the concept of fractions, the need for integration was raised.

5.2.3 Presentation of data from Section C of the questionnaire

Section C of the questionnaire sought to discover the teachers' perceptions and attitudes on mathematical connections/making connections in the classroom. Section C consisted of six questions and the data from these was analysed together. The participants responded by marking one of the boxes of the five indicators: strongly agree, agree, uncertain, disagree and strongly disagree. A summary of the data is presented in Figure 5.7 below.

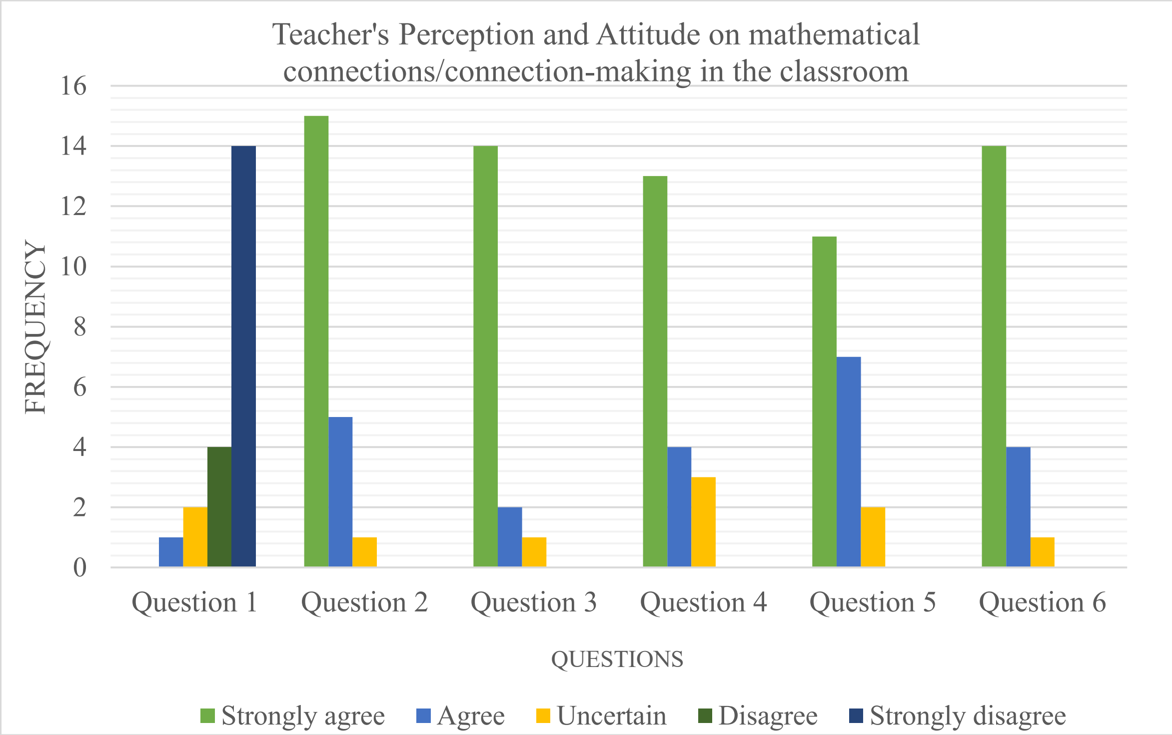


Figure 5.7: Responses to Section C of the Questionnaire

Question 1 of Section C of the Questionnaire

Question 1 of Section C of the questionnaire sought to discover the teacher's perceptions and attitudes about mathematical connections/making connections and their views about the statement 'making connections are disruptive when teaching fraction concepts.' The data reflects that 66.7% of the participants strongly disagree, 19 % disagree, 9.5% are uncertain and 4.8% agree. Two respondents commented:

- *connections are important because they integrate fractions and real-life, for example, the use of pizza models in fractions.*
- *Fractions broaden the learners understanding, motivation and critical thinking.*

In contrast, one participant agreed because they felt "*It's disruptive because while teaching you go back to the textbook or chart.*"

Question 2 of Section C of the questionnaire

Question 2 of Section C of the questionnaire sought to determine the teachers' perceptions and attitude on the effectiveness of making connections when teaching fraction concepts. The data reflects that 71.4% of the participants strongly agreed, 23.8 % agreed, and 4.8% were uncertain. Three participants further commented:

- *Yes, it is very effective because it is one strategy whereby a teacher connects the abstract to the concrete with physical and visual models.*
- *It allows learners to see the interrelatedness of fraction concepts and provides space for them to apply number concepts in other areas.*
- *Connection-making is effective because when teaching is correlated with the home and environment, the learner feels confident and ready to explore new things.*

Question 3 of Section C of the questionnaire

Question 3 of Section C of the questionnaire sought to determine the teachers' perceptions, attitudes and their views on the statement 'Making connections promotes learner-to-learner interaction'. The data reflects that 82.4% of the participants strongly agreed, 11.8% agreed, and 5.9% were uncertain. Three participants further commented:

- *It promotes collaborative learning because learners start discussing real-life examples of fractions they are exposed to daily at home and in other subjects.*
- *Learners can discuss and apply what they did in another subject to mathematical fraction concepts. Putting learners in groups will help them to have the confidence to do their work.*

Question 4 of Section C of the questionnaire

Question 4 of Section C of the questionnaire sought to determine the teachers' perceptions, attitudes and views on the statement 'Making connections helps improve learner performance in the fraction concepts'. The data reflects that 65% of the participants strongly agreed, 20% agreed, and 15% were uncertain. Two participants further commented:

- *Yes, it improves the learner's performance simply because learners will think outside the box and understand how fractions are used in real life.*
- *Concept development and understanding in fractions may deepen if only those involved in transmitting knowledge about concepts do it effectively so that the recipients can constructively do the same for others.*

Question 5 of Section C of the Questionnaire

Question 5 of Section C of the questionnaire sought to determine the teachers' perceptions, attitudes and their views of the statement 'The application of mathematical connections in teaching and learning can improve learners' critical thinking.' The data collected reflects that 55% of the participants strongly agree, 35% agree, and 10% are uncertain. Two participants further commented:

- *Connections enable learners to model and interpret numerical relationships and patterns that exist in real life.*
- *Critical thinking may emerge after understanding because one can evaluate, compute, analyse and use strategies that were never discovered by someone to simplify mathematics so math's connection can do.*

Question 6 of Section C of the Questionnaire

Question 6 of Section C of the questionnaire sought to determine the teachers' perceptions, attitudes and views on the statement 'Making connections engages learners' attention and motivates them'. The data reflects that 73.7% of the participants strongly agree, 21.1% agree, and 5.3% are uncertain. Two participants further commented:

- *Connections keep learners focused and curious and make them want to discover more about real-life situations they encounter daily.*
- *Attention and motivation depend on individual capabilities, which someone lacks while the other lacks. Those who lack motivation and engagement may be encouraged when making connections.*

5.2.4 Concluding remarks on the questionnaire analysis

All the participants noted that connection-making/mathematical connections are essential to the teaching and learning of the fraction concept to Grade 7 learners. Most of the teachers' responses stated that connection-making/mathematical connections make abstract concepts concrete and clarify mathematical ideas whose meanings are difficult to comprehend. Teachers noted that making connections benefits the learners in various ways:

- mathematical connections integrate mathematics into real life and broaden their understanding;
- making connections brings critical awareness of how mathematical concepts link with other subjects, for example, English, Natural Science, etc; and
- connection-making motivates learners and encourages them to be actively involved in learning.

Although most teachers noted the benefits of connection-making, some indicated they find them disruptive to the teaching and learning process because, time and again, they have to go back to the textbook or chart.

The data collected revealed that all the teachers who participated in the questionnaire rely heavily on textbooks as their resource for teaching and learning the fraction concept. Ninety-one per cent of the teachers indicated they use the NIED-prescribed textbooks and only 9% of the teachers use other textbooks. Although teachers use various textbooks, the PM is the textbook most used. Teachers even go the extra mile to use supplemental resources in addition to the prescribed textbooks. My analysis shows that 90% of the participants agreed that real-life connections, connections to other subjects, connections to other mathematics content areas, visualisation connections and connections to problem-solving are explicitly articulated in the fraction concepts chapter of the textbooks they use. Ten per cent of the respondents disagree that the five connection indicators are explicitly articulated in their textbooks.

Through the incorporation of visuals in mathematics lessons, learners are encouraged to manipulate, reason and visualise fractions instead of responding to traditional teaching methods, which emphasise the memorisation of rules and procedures. The data reflects that 82% of the participants agree that tables, graphs, drawings, pictures and other models are explicitly articulated in the fraction concepts chapter of the textbooks they use. Eighteen per cent of the respondents however, disagree that the four visual models are not explicitly articulated in the textbooks they use.

5.3 PRESENTATION OF THE DATA FROM THE SEMI-STRUCTURED INDIVIDUAL AND FOCUS GROUP INTERVIEWS

The audio-recorded semi-structured and focus group interviews with the four selected teachers were transcribed using the online transcription facility in MS Office 365. I analysed the transcribed results and present them as follows. Firstly, I present the responses to the semi-structured individual interview questions (see Appendix 4), where each part of the question is analysed individually. This is followed by presenting responses to the focus group interview questions (see Appendix 5). For both interviews, I analysed the questions in chronological order.

Secondly, I present a summary of the above responses to answer Research Questions 1 and 3, which are:

1. *How do selected Namibian Grade 7 textbooks encourage connection-making in the context of fractions?*
3. *What are selected teachers' perceptions and perspectives of the three textbooks concerning connection-making in fraction concepts?*

All four participants were cooperative throughout the interview and answered the questions confidently. The participants generally displayed good knowledge and experience of the subject matter. After the presentation of the interviews, the summary of responses per research question and the summary of findings will follow.

Finally, I discuss some themes that emerged from the interviews that were not anticipated in the original interview schedule and thereafter conclude the entire analysis chapter.

5.3.1 Semi-structured individual interviews

The process of data collection involved conducting individual interviews with the teachers. These interviews were audio recorded with the teachers' consent. To ensure accuracy, the interviews were transcribed exactly as they were spoken, without any information being omitted. The interviews were given a code, SSII, for semi-structured individual interview. Additionally, each teacher was assigned a specific code (pseudonym combined with interview code) to differentiate their responses. For example, Ms MSSII represents Ms M's semi-structured individual interview, while Mr KSSII represents Mr K's semi-structured individual interview. Once the interviews were transcribed, the qualitative data was organised by the questions asked during the interviews.

SSII Question 1: The teacher's perceptions concerning mathematical connections

All four participants indicated an awareness of mathematical connections from their own points of view, how they have used them to teach the fraction concepts and their own knowledge acquisition. For example Ms. M stated "That is, the is not the link between like cross curricular issues when we talk about. And mostly that includes the relationship between other subjects, such as geographical subjects, that inquires the data handling part, and so on". They pointed out that in education, teachers recognise these mathematical connections' vital role in building bridges between mathematics and other subjects or real-life situations. They believed these connections are instrumental in helping learners comprehend and appreciate the importance of mathematics. The four participants acknowledged that mathematical connections have the power to make mathematics more relatable and applicable to everyday life. By integrating mathematical topics into the curriculum, teachers find that learners establish meaningful connections within various subjects such as Geography, Science and English, and enhance their understanding of the mathematical concepts involved.

To illustrate the significance of mathematical connections, the participant teachers often provided examples of how mathematical concepts intertwine with real-life situations. They highlighted how counting, fractions and budgeting, among other constructs, are integral to practical scenarios, helping learners grasp mathematics' relevance and application in their daily lives. Regarding their teaching approach, the teachers emphasised the importance of demonstrating to learners how mathematics is relevant to their daily lives. By connecting to real-life experiences and situations, the teachers believed they could effectively engage learners and foster a deeper understanding of mathematical concepts. By connecting mathematics to other subjects and everyday life, the participant teachers said they aimed to help learners recognise the practical applications of mathematical concepts. They believed that by establishing these connections, learners can better appreciate the value and usefulness of mathematics in their lives.

The teachers who were interviewed had different perspectives on using mathematical connections when teaching fractions. Ms MSSII, for example, emphasised the importance of incorporating real-life connections, such as cutting fruit, to help learners understand the concept of fractions. She also mentioned the relevance of connecting fractions to other mathematical operations and problem-solving. Ms PSSII highlighted visualising techniques, such as using fruit and pizzas, to help learners grasp the idea of fractions as parts of a whole and the concept of sharing. Mr NSSII discussed how fractions are a part of everyday life, such as sharing food and money, and how problem-solving skills can be developed through fraction-related scenarios. Mr KSSII suggested using alternate representations, such as drawing and real-life situations, to teach fractions. He also mentioned the importance of connecting lessons to learners' prior knowledge, such as sharing sweets, to facilitate understanding. Despite the potential disruptions that can arise when using real-life materials, such as noise due to overexcitement, the teachers agreed that incorporating mathematical connections is essential for helping learners comprehend fractions effectively.

The four participants indicated a lack of training in mathematical connections. The responses from Ms MSSII, Ms PSSII, Mr NSSII, and Mr KSSII suggest that teachers must be trained to use mathematical connections. Ms MSSII acknowledged that teachers often use a traditional teaching approach, focusing solely on textbook content. Hence, she suggested that training would be beneficial. Similarly, Ms PSSII highlighted the changing world and curriculum, stressing the importance of training to keep up with new developments. Mr NSSII highlighted the need for teachers to familiarise themselves with the syllabus and make connections beyond basic subject content. He also mentioned challenges faced by schools in terms of a lack of materials. Finally, Mr KSSII agreed that teachers need training to overcome their fears and perceptions of the difficulty of teaching mathematics. He highlighted the importance of teachers being able to apply mathematical content to real-world examples to enhance learner understanding. Overall, the responses strongly support the idea that teachers should receive training in applying mathematical connections.

In summary, the four participants in this study all recognise the significance of mathematical connections in education. They believe these connections play a vital role in helping learners understand and appreciate the importance of mathematics. The participants stressed the need for teachers to integrate mathematical concepts into various subjects and real-life situations to make them more relatable and applicable to everyday life. They also highlighted the importance of providing examples and practical scenarios to help learners see the relevance and application of mathematics in their daily lives.

SSII Question 2: The teachers' perspectives concerning teaching fractions using connections

The advantages of using mathematical connections when teaching fractions are clear to the participant teachers. The participants said that connections make fractions more concrete, relatable and understandable for learners. By incorporating real-life scenarios, visual aids and problem-solving, teachers can engage learners and deepen their understanding of fractions. However, there may be some disadvantages to consider. For example, Ms MSSII mentioned that "using real-life objects like apples or pizzas could disrupt the lesson as learners become overly excited or distracted".

Additionally, Mr KSSII noted that the effectiveness of these connections may depend on the teacher's presentation and classroom management skills. Furthermore, he suggested that teachers should carefully plan and execute these strategies to ensure optimal learning outcomes. The four participants in this study indicated that with careful implementation, the benefits of mathematical connections in teaching fractions far outweigh the potential obstacles, making it a valuable approach for teachers to consider.

SSII question 3: The teachers' general perceptions of the three textbooks concerning making connections in fraction concepts

With regard to connection-making in the fraction concepts, the teachers interviewed had varying opinions on the textbooks recommended by NIED. Ms MSSII stated that she did not prefer any particular textbook and relied on multiple resources, including old textbooks and the internet, to gather teaching ideas. However, she mentioned that the most common textbook used by learners was PM. Ms PSSII preferred *Maths for Life* because it includes diagrams that help learners visualise concepts. She also favoured the EMLB textbook for its abundance of diagrams based on real-life objects, such as fruit, which allowed for practical application. Additionally, she appreciated the book's extensive examples and step-by-step explanations.

Mr NSSII, on the other hand, did not believe in relying solely on one textbook and mentioned having a collection of seven, some dating back to 1996. He felt that modern textbooks lacked content and explanations when compared to older ones. Although he did not mention specific titles, he preferred a blue book and another purple and red one. He emphasised the richness of content in these books, even if they contained topics not included in the syllabus. He also mentioned using the *Champion* textbook for its ability to address specific topics concisely, while turning to the *Diamond* textbook for in-depth knowledge enrichment. Lastly, Mr KSSII preferred the EMLB textbook. He believed it provided broader explanations and connected well with the syllabus requirements. He finds it more instrumental in addressing specific competencies than the other textbooks.

The participants' opinions on whether the textbooks they use embrace connection-making in the fraction concept and if the connections are clear enough for the learners to understand.

A few key points arose when analysing the teachers' opinions on the textbooks they use, regarding embracing connection-making in the fraction concept and the clarity of these connections for the learners. Ms MSSII acknowledged that the textbooks commonly used in the classroom embrace connection-making in the fraction concept by providing real-life examples for learners. However, she also noted that not all connections are mentioned. She believed the textbooks covered problem-solving aspects and highlighted subject-related issues, including fractions. Similarly, Ms PSSII agreed with the notion that the textbooks embraced connection-making. She explained that the textbooks start with more straightforward concepts and progressively build upon them. While she acknowledged that some elaboration might be necessary for learners to understand fully, she appreciated the structure provided by the textbooks.

On the other hand, Mr NSSII expressed a more critical view of the textbooks. He mentioned that none of the textbooks he has encountered fully address the importance of money and finance concepts, which are crucial for learners. He suspected that specific topics like geometry might be included for recreational purposes rather than substantial connection-making. However, he acknowledged that there are instances where real-life connections are made, such as when teaching budgeting. Mr KSSII affirmed that the textbooks utilise drawings as a representation to connect fractions and aid interpretation. For example, he stated that these drawings may include shaded or unshaded rectangles, helping learners visualise and understand the fraction concept. However, he cautioned that clear interpretations might not always be evident to the learners unless the teachers explained further and related the concepts to real-life situations.

SSII question 4: The participants' comments apart from the interview questions.

In their responses, the participants expressed their thoughts and comments about the interview process and their experiences with mathematics education. They highlighted several vital points apart from the interview questions. Ms MSSII mentioned that she learned about the connections and the interrelatedness of mathematics concepts in cross-curricular issues. She emphasised the importance of understanding these connections and how they can be applied. Ms PSSII expressed concern about the negative perception of mathematics among parents and the need for an awareness campaign to highlight its importance. She also hoped to gain knowledge from the interview process and acknowledged the importance of learning about connections in mathematics to enhance teaching practices.

Mr NSSII raised the integration issue regarding mathematics education being prioritised in the junior primary phase but often neglected in higher grades. He believed integration is crucial in helping learners grasp content better and connect to real-life situations. He also desired more hands-on training and demonstrations rather than verbal interviews. Mr KSSII expressed gratitude for my return visit and emphasised the importance of receiving feedback and recommendations based on the research findings. He hoped that the research would lead to improvements in mathematics education and the perception of the subject.

These comments highlight the participants' interest in learning and improving mathematics education, addressing the negative perceptions surrounding the subject and the need for practical training and recommendations to enhance teaching practices.

5.3.2 Focus group interview (FGI)

Data collection also involved conducting group interviews with the four participant teachers. These interviews were audio recorded with the teachers' consent. To ensure accuracy, the interviews were transcribed exactly as they were spoken, without any information being omitted. The interviews were given a code, FGI, which stands for focus group interview. Additionally, each teacher was assigned a specific code (pseudonym combined with interview code) to differentiate their responses. For example, Ms MFGI represents Ms M's focus group interview response, while Mr KFGI represents Mr K's focus group interview response. Once the interviews were transcribed, the qualitative data was organised by the questions asked during the interviews.

FGI Question 3: The teachers' understanding and experience concerning mathematical connections

The participants in the discussion highlighted the importance of mathematical connections in their understanding and experience of mathematics. They emphasised that mathematical connections are not limited to abstract concepts but extend to real-life situations and problem-solving. Ms MFGI focused on problem-solving as a critical aspect of mathematical connections. She explained that mathematical principles and techniques should be applied when solving classroom or real-life problems. For example, understanding angles and measurements is essential when building a house. Ms MFGI emphasised that everything in mathematics ultimately connects back to real-life situations.

Ms PFGI explicitly mentioned the connection between mathematics and time. She explained that time is a fundamental aspect of everyday life, and teaching learners about time is an essential mathematical connection. The concept becomes more relatable and applicable by relating mathematical concepts to something the learners use and experience daily. Mr NFGI shared his personal experience of realising the real-life connections in mathematics. He mentioned how he overlooked the connections until prompted to explore his syllabus. He mentioned geometry and highlighted that objects like tables, laptops, houses and beds are composed of geometric shapes and arrangements. Mr NFGI perceived these connections as enriching experiences, recognising the ubiquity of mathematical concepts in everyday life.

Mr KFGI summarised the perspectives shared by his colleagues, accentuating the importance of bringing life into mathematics education. He suggested that teaching mathematics separately from real-life applications may make it more challenging and less engaging for learners. Demonstrating the relevance and applicability of mathematics in daily life makes it more exciting and straightforward for learners to comprehend and appreciate.

The participants gave examples of mathematical connections from their practice.

In the discussions, the participants provided examples of mathematical connections in their practice. Ms MFGI mentioned using timetables in schools, which guide learners on when and where they should be. This demonstrates the connection between time and organisation, as schools could become chaotic without timetables. Ms PFGI shared an example of a learner who disliked mathematics but realised its importance when counting goats and measuring medication doses on a farm. This example illustrated how mathematics is necessary in real-life scenarios.

Mr NFGI called attention to the ubiquitous use of calendars and schedules, such as marking family members' birthdays and counting days until Christmas. He also mentioned the use of travel schedules, showcasing the practical applications of mathematics in planning and time management. Mr KFGI shared anecdotes about his grandmother, who used shadows to estimate time and used equal measures when sharing milk or cutting things into equal parts. This example highlighted the connections between time, measurement and fractions, emphasising that mathematics is applicable regardless of educational background.

Examples of mathematical connections that are practical and applicable when exploring fraction concepts and those which are not.

During the focus group interview, participants mentioned several mathematical connections that are practically used to explore the concept of fractions. Ms MFGI suggested using mathematics-to-mathematics connections, explicitly focusing on the four basic operations of adding, subtracting, multiplying and dividing fractions. She suggested that the operations are fundamental in understanding and working with fractions. On the other hand, Ms PFGI spoke about the importance of visual representations and concrete objects when teaching fractions. She stressed that diagrams and visual aids help learners comprehend concepts such as sharing and problem-solving. For example, learners can better understand fractions and their applications in real-life scenarios when they see concrete objects, like a loaf of bread cut into equal pieces for sharing.

Mr NFGI highlighted the relevance of fractions in real-life situations. He suggested that learners can observe fractional concepts in activities such as sharing equally from a common pot, measuring distances travelled and sharing meals. Furthermore, Mr NFGI said that recognising and understanding these real-life instances helps learners to connect abstract concepts like fractions to practical applications. Moreover, Mr NFGI also mentioned the role of fractions in other subjects. He talked about converting grades and percentages, indicating that fractions are used in subjects beyond mathematics. Fraction-based problem-solving was also mentioned, emphasising the importance of fractions in solving various real-life problems.

Lastly, Mr KFGI commented on the widespread use of fractions in different contexts. He mentioned how fractions are used in subjects such as calculating exam marks, where a fraction represents the percentage obtained. He also presented an example of sharing money among children, where fractions can be used to distribute money based on age. This demonstrated that fractions are not limited to mathematics but are present in various aspects of daily life.

Examples of connection-making ideas that the participants developed after participating in the questionnaire and the semi-structured interview.

The responses from Ms MFGI, Ms PFGI, Mr NFGI and Mr KFGI showed that that they have developed new connection-making ideas since participating in the questionnaire and semi-structured individual interviews. One common theme among the participants is the recognition of the limitations of traditional teaching methods. Ms MFGI acknowledged that the traditional approach of sticking to textbooks and syllabi without providing real-world experiences hinders the learners' understanding of the subject. She also pointed out time constraints as a contributing factor to this traditional approach. Ms PFGI echoed this sentiment by emphasising the need for teachers and learners to explore more and connect mathematics to the outside world. She suggested that creating awareness and shaping mathematics in a way that is interesting to others could lead to more people loving the subject.

Mr NFGI expanded on this idea by stating that although mathematics is present in everyday life through shapes and geometrical concepts, people still struggle to understand it. This highlighted the disconnect between using mathematics in practical applications and learners' difficulty in comprehending it. Mr KFGI supported this notion by suggesting that planning and carefully connecting lessons to real-life examples can enhance learners' understanding rather than spending time on topics that learners do not grasp. Moreover, an underlying concern expressed by the participants is the pressure and expectations placed on teachers by curriculum designers and leaders. Mr NFGI questioned why mathematics, prevalent in our surroundings, is considered problematic. Ms MFGI highlighted the limited guidance and support from subject advisors and suggested that building a solid foundation in lower primary levels is crucial for better comprehension in higher grades.

FGI Question 4: The teachers' general perceptions and perspectives of the three recommended Grade 7 textbooks' presentation of mathematical connections

In the discussion, Ms MFGI expressed dissatisfaction with the accessibility and content of the current textbook, PM. She believed that PM lacks straightforward content compared to other textbooks which the NIED textbook evaluation panel does not recommend. Although the teachers occasionally make copies of those additional books, they cannot use them consistently due to limited resources like broken copy machines. Ms MFGI spoke about the importance of textbooks being accessible to both teachers and learners. Ms PFGI agreed that the textbooks such as *Maths for Life*, EMLB, and *Diamond* explicitly present mathematical connections. She believed these three textbooks have more content and elaboration, making it easier for learners to understand, even without the teacher's explanation.

Mr NFGI highlighted the issue of learners' ability to understand the textbook content independently. He suggested that learners rely on teachers to explain the concepts because very few of them can comprehend a textbook's intended message. He mentioned a thicker textbook that had more explanations at the learners' level. Unfortunately, NIED does not recommend it, and Mr NFGI attributed this decision to business interests rather than pedagogical considerations. Mr KFGI mentioned the limitations in textbook selection due to government funding and approval processes. He believed that personal connections influence the selection committee to favour specific textbooks over others. However, he suggested that teachers have the flexibility to recommend textbooks that they find more suitable for their learners. They can inform parents of alternative options available in bookshops. Mr KFGI mentioned the importance of teachers having multiple textbooks to enhance their teaching resources.

Mathematical connections embraced by the textbooks identified by the participants

In the focus group interview, participants shared insightful perspectives on the mathematical connections embraced by textbooks. Ms MFGI spoke about the comprehensive coverage of mathematical connections in most textbooks, noting the intertwining of mathematics with various subjects. She highlighted the transition from data handling to a geographical context, demonstrating how mathematics extends its reach to other subjects. Ms MFGI also highlighted the prevalent role of the four basic operations and the integration of problem-solving throughout the curriculum. Ms PFGI contributed to the discussion by mentioning the interaction between mathematics and Arts as a subject. She discussed how teachers influence mathematical concepts in arts education, encouraging learners to apply mathematical principles in creative projects. Ms PFGI drew attention to the interconnectedness of subjects, asserting that arts, as a 'mother subject', influences other disciplines.

Mr NFGI further elaborated on the omnipresence of mathematical connections, expressing regret that they are often not explicitly mentioned. He cited practical examples, such as budgeting projects given to learners, as clear instances of mathematical applications. Mr NFGI showcased a notable project involving the creation of a soccer pitch, demonstrating how geometry played a crucial role in determining measurements. He argued that these connections exist but are often overlooked, and urged teachers to recognise and incorporate them into teaching. Lastly, Mr KFGI concisely reiterated the theme of mathematics extending to other subjects, summarising the overarching sentiment expressed by the participants.

The link between the mathematical connections to the learning content and the relevance to the Grade 7 level.

The responses provided by Ms MFGI, Ms PFGI, Mr NFGI, and Mr KFGI suggested that the mathematical connections in the textbooks are strongly linked to the learning content. The participants stated that the textbooks contain relevant examples and competencies that align with the syllabus. However, they noted that there are challenges when it comes to connecting the new topics in Grade 7 with the learners' prior knowledge. Due to the two-year gap caused by COVID-19, the participants said that learners have missed out on essential foundational concepts, such as geometry. This gap in knowledge hinders their ability to understand and apply higher-level content.

Additionally, the participants pointed out that the lack of communication and collaboration between lower primary and upper primary teachers worsens the issue. Ms MFGI highlighted the need for clear connection and knowledge sharing between teachers of different levels. She suggested that when teachers know what topics learners have covered in previous years, they can build upon their prior knowledge more effectively. This collaboration is essential to ensure a smooth transition and continuity in learning.

Furthermore, the responses acknowledged the importance of real-life connections in mathematics education. Mr KFGI emphasised the need for teachers to help learners interpret mathematical connections in real-life situations. He stated that by connecting abstract concepts such as shapes and algebra, to practical applications, learners are more likely to grasp the relevance of mathematics in their everyday lives. However, the responses also pointed out the challenges faced by teachers. Mr KFGI and Ms PFGI mentioned the need for teachers to understand the content to make meaningful connections with learners. It is essential for teachers to not only rely on textbooks, but also interpret and teach the content effectively. The participants suggested that this requires continuous professional development and support for teachers.

Suggestions and recommendations on the representation of mathematical connections in textbooks.

The focus group interview with Ms MFGI, Ms PFGI, Mr NFGI, and Mr KFGI provided valuable insights and recommendations on representing mathematical connections in textbooks. One critical piece of advice suggested by the participants was to ensure consistency and continuity in teaching from grade to grade. Ms MFGI suggested that learners stay with the same teacher throughout primary school to foster a cohesive understanding of concepts and terminology. She explained that this would help bridge the gap between prior knowledge and the introduction of new concepts. Furthermore, she stated that addressing misconceptions would contribute to a solid foundation for learners before they progress. Ms MFGI raised the issue of standardised tests and their value for identifying areas where learners lack knowledge and informing teaching strategies. However, Ms MFGI expressed concern about the phasing out of standardized test and the consequent lack of feedback. She suggested that standardised tests should continue to be utilised to improve teaching effectiveness.

Collaboration and communication among teachers are also crucial for ensuring a smooth progression of mathematical concepts, say the teachers. Ms MFGI and Ms PFGI advocated for greater cooperation among upper-primary and lower-primary teachers. They suggested that in the lower primary phase, teachers should specialise in certain areas of mathematics rather than be expected to teach all subjects. The teacher participants noted that having specialists for lower primary mathematics would improve the learners' overall understanding and performance.

The interviewees also highlighted the importance of understanding learners' prior knowledge and building upon it effectively. The connection between syllabus, textbooks and teachers' planning is crucial to achieving this. The textbooks should address all necessary competencies and be suitable for learners of different grades and abilities. Teachers should be able to interpret examples and provide real-life situations to make connections clear for learners. The teachers also recommend establishing a solid foundation in mathematics early, by introducing basic concepts in earlier grades and gradually building upon them. This would prevent learners from feeling overwhelmed by new topics and facilitate their understanding.

Moreover, the interviewees stressed the need for teachers to understand mathematics beyond memorising formulas and reciting answers. Furthermore, they suggested that teachers should be able to think critically and quickly and make connections between different topics. This would enable effective teaching and support learners' comprehension. Lastly, the participants emphasised that the representation of mathematical connections in textbooks should be unambiguous. Learners could then easily interpret diagrams, examples and formulas, enhancing their understanding and problem-solving skills.

FGI Question 5: Further questions and comments

The participants in the focus group interview had some final questions and comments before concluding the meeting. Ms MFGI requested a discussion summary to share with others to improve the current practices. She also highlighted the challenges teachers face in attending workshops in the afternoon when they are tired and might not be able to pay full attention. Ms PFGI suggested conducting a seminar on mathematical connections to create awareness and benefit other regional teachers. Mr NFGI emphasised the need to empower new teachers on the curriculum and subject content level, suggesting that workshops like this should be incorporated into cluster meetings. Mr KFGI commented on the firm group dynamic and recommended them for future endeavours. The participants desired to improve teaching practices and enhance mathematical connections in education. The various comments and questions emphasise the participants' commitment to ongoing professional development and dedication to addressing challenges within the education system.

5.3.3 Summary of responses per research question

This section focuses on the two research questions, namely Research Questions 1 and 3. The SSII and FGI responses are analysed together, and the participant teachers' pseudonyms are used as Ms M, Ms P, Mr N and Mr K. The findings are as follows:

Research Question 1: How do selected Namibian Grade 7 textbooks encourage connection-making in the context of fractions?

In response to Research Question 1, the teachers' responses reflected diverse notions on the presentation of mathematical connections in the recommended Grade 7 textbooks. Ms M criticised the lack of straightforward content, while Ms P found explicit mathematical connections in *Maths for Life*, EMLB, and *Diamond*. Mr N focused on learners' dependence on teacher explanations, and Mr K highlighted the limitations imposed by government funding and the importance of teachers having various textbooks.

The participants unanimously recognised the significance of mathematical connections in the textbooks. They stressed the role of connections in aiding learners' understanding and appreciation of mathematics. The participants advocated for integrating mathematical concepts into various subjects and real-life situations, making them more relatable and applicable to everyday life. They underscored the importance of providing examples and practical scenarios to demonstrate the relevance and application of mathematics to daily life.

Research Question 3: What are selected teachers' perceptions and perspectives of the three textbooks concerning connection-making in fraction concepts?

Regarding Research Question 3, the teachers varied in their preferences for textbooks. While Ms M favoured a mix of resources and Ms P preferred *Maths for Life* and EMLB, Mr N emphasised the richness of content in older books, and Mr K found the EMLB textbook instrumental in teaching mathematics to the grade 7 learners. Despite diverse opinions, all teachers acknowledged and used various textbooks to enhance their teaching of fraction concepts. The teachers expressed varying views on how much the textbooks embrace connection-making in the fraction concept. While Ms M and Ms P viewed the textbooks favourably, Mr N raised concerns about less relevant topics and Mr K stressed the importance of teacher-led explanations for better understanding. The participants collectively recommended prioritising training and resources for teachers to enhance their ability to integrate mathematical connections into teaching practices.

Throughout the interviews, the participants consistently highlighted the significance of real-life connections in aiding learners' comprehension of fractions. They emphasised the importance of connecting mathematical concepts to real-life situations, problem-solving and everyday experiences. The participants provided examples showcasing mathematics' relevance in various aspects of daily life, reinforcing the idea that mathematics is essential and applicable to everyone. Furthermore, the participants discussed various mathematical connections that are applicable and practical in exploring the concept of fractions. These included using mathematics-to-mathematics connections, visual aids, recognising fractions in real-life situations, using fractions in other subjects and applying fractions when problem-solving. However, there was no mention of connections that do not apply to fraction concepts.

In conclusion, the participants emphasised the universal nature of mathematical connections, spanning diverse subjects and practical applications. They recommended maintaining consistency and continuity in teaching, using standardised tests, addressing gaps in prior knowledge, improving collaboration among teachers and considering specialisation in junior primary mathematics to enhance mathematics education. The participants had additional questions and comments, expressing their commitment to ongoing professional development and dedication to addressing challenges within the education system.

5.4 SUMMARY OF FINDINGS

The teachers' responses highlighted the importance of connection-making in teaching and learning the fraction concept for Grade 7 learners. Connections were found to make abstract concepts concrete and clarify difficult mathematical ideas. Teachers noted that connection-making integrates mathematics into real-life situations, promotes critical awareness of interconnectedness with other subjects and motivates learners to actively engage in learning. However, some participant teachers found connection-making disruptive, as it required constant reference to the textbook or chart.

The data revealed that teachers rely heavily on textbooks for teaching the fraction concept, with 91% using the NIED-prescribed textbooks. The PM textbook was the most used among Grade 7 teachers. Most participants agreed that the five connection indicators (real-life connections, connections to other subjects, connections to other mathematics content, visualisation connections and connections to problem-solving) were explicitly articulated in the three selected textbooks. Similarly, the use of visuals, such as tables, graphs, drawings, pictures and models, was found to be explicitly articulated in the textbooks by 82% of participants.

The teachers expressed diverse opinions on the presentation of mathematical connections in Grade 7 textbooks, emphasising the importance of connections in aiding learners' understanding and appreciation of mathematics. Furthermore, the teachers varied in their textbook preferences but unanimously recognised the importance of various textbooks in enhancing their teaching of fraction concepts. They advocated for integrating mathematical concepts into various subjects and real-life situations to make them more relatable and applicable. They recommended training and resources to enhance teachers' ability to integrate connections effectively.

In conclusion, the participants stressed the universal significance of mathematical connections, advocating for their consistent integration into teaching practices. The study highlighted the importance of real-life connections and practical applications. The participant teachers also addressed the need for standardised tests, addressing gaps in prior knowledge, improving teacher collaboration and considering specialisation in junior primary mathematics. The participants expressed their commitment to ongoing professional development and dedication to addressing challenges within the education system, concluding with recommendations for ongoing professional development and collaborative efforts to address these challenges.

5.5 EMERGENT THEMES

In education research, the term "emerging themes" refers to areas of growing interest and investigation within the field (Lang, 2016). These themes highlight current trends and challenges in education, and often represent a shift in focus or approach compared to past research. Through the process of analysing the data from the interviews, four distinct themes emerged which were not anticipated. These are:

- Workshops, professional development and teacher empowerment
- Regional and cluster initiatives
- Leadership and advocacy
- Collaboration and Communication

Although they are discussed separately, it is noted that these are interrelated and intertwined.

5.5.1 Workshops, professional development, and teacher empowerment

The teachers articulated the necessity for workshops to augment their understanding of mathematical connections, addressing concerns related to timing and effectiveness. They underscored the importance of empowering teachers with enhanced subject content knowledge, particularly newcomers. Action research and initiatives were recommended to boost teacher expertise, especially in addressing gaps in mathematical understanding. They also called for a more collaborative and teacher-driven approach to curriculum changes, with teachers playing a pivotal role. Furthermore, they highlighted the importance of applying insights gained from the study in the classroom and promoting a culture of continuous learning among teachers.

5.5.2 Regional and cluster initiatives

Participants explored the potential of regional or cluster-level initiatives, proposing workshops on mathematical connections. They suggested that participant teachers take the lead in organising such workshops for the benefit of others in the region. The challenges faced by teachers, that are posed by curriculum designers and management expectations were highlighted, emphasising the importance of building a solid foundation at lower primary levels. They also emphasised the importance of building a solid foundation in mathematical connections at lower primary levels.

5.5.3 Leadership and advocacy

Concerns were raised by the participant teachers about the management's approach to teacher involvement in workshops and the practical implementation of mathematical connections. Participants called for better leadership and support from school management and curriculum designers to facilitate effective implementation and enable teachers to contribute meaningfully. Mr K recognised the potential of the participant group, suggesting they be recommended as a capable group that can significantly contribute to administering workshops on mathematical connections.

5.5.4 Collaboration and communication

Ms M emphasised the importance of summarising the interview discussions for better communication and advocated involving more stakeholders in the educational process. The participants highlighted the importance of collaboration and communication among teachers at different levels, particularly between upper-primary and lower-primary teachers. They stressed the need for better communication and cooperation to bridge the gap between the two phases and ensure a smoother transition for the learners.

5.6 CONCLUSION

In this chapter, I have presented and discussed the data collected from the questionnaires, semi-structured and focus group interviews. I concluded each section with a brief summary. The data obtained from the questionnaires and the two interviews reflected the views, perceptions, attitudes, experiences and perspectives of the participating Grade 7 mathematics teachers. The themes used to categorise and discuss the data aligned with the analytical tool, and some emerged from the interviews.

In the concluding chapter, I present a summary of the findings with specific reference to the research questions.

CHAPTER 6 : CONCLUSIONS AND RECOMMENDATIONS

6.1 INTRODUCTION

This chapter concludes my study, summarising my discussions and presenting the findings to my research questions. I also address the limitations of my study, provide recommendations and suggest future research areas based on the findings from my data. Finally, I conclude with remarks, reflecting on the journey and outcomes of the study.

6.2 SUMMARY OF THE RESEARCH FINDINGS

How do selected Namibian Grade 7 textbooks encourage connection-making in the context of fractions?

Textbooks play a crucial role in promoting mathematical connections. The Third International Mathematics and Science Study (TIMSS) (2002) emphasises the significance of a strong connection between the curriculum and textbooks, referring to the textbook as a 'surrogate curriculum'. The analysis of the three selected textbooks revealed that some textbooks provide opportunities for making connections, while others fall short in this aspect. The EMLB exhibited 95 connections, primarily emphasising real-life and mathematical content connections; the OLDM exhibited 139 connections, highlighting a dominance of links to mathematical content and visualisations; while the PM exhibited 77 connections, with a notable emphasis on connections to other mathematical content.

Evaluation against curriculum requirements confirmed strong alignment for all three textbooks, showcasing prevalent real-life and mathematical content connections. However, interdisciplinary connections were less common, potentially contributing to a perceived separation of mathematics from other subjects. The deliberate inclusion of purposeful connections in CFMN chapters across all textbooks, particularly those with an exemplifying purpose, indicated a well-thought-out design.

What similarities and differences are observed in the three textbooks in promoting connection-making in the fraction concepts?

A textbook should bridge mathematics knowledge acquisition and mathematics curriculum attainment. For resourcefulness and usefulness, textbooks should be of good quality (Apple, 1992; Ben-Peretz, 1990; Goodlad, 1984; MoE, 2015b; Gu et al., 2004; Nicol & Crespo, 2006; Schmidt et al., 1997a). The three selected textbooks all have the three qualities to meet the curriculum attainment. The following qualities were observed in the three selected textbooks (evident in the general structure of the three selected textbooks' presentation):

- the textbooks align with the curriculum;
- the textbooks are written according to the prescribed syllabus; and
- every aspect of the syllabus is adequately covered.

The real-life connections and connections to other mathematics content areas were strongly evident across the three selected textbooks. However, the connections to other subjects were not common across the textbooks. This could be problematic as it creates a gap between mathematics and other subjects and, unfortunately, reinforces the notion of mathematics being a discrete and complex subject unconnected to other subjects.

Generally, it was observed that connections evident in the CFMN chapter of the three textbooks all had a purpose. The connections with an exemplifying purpose were the most prevalent across all three textbooks. The visualisation connection helped to clarify the text and assisted learners in imagining or actively representing the abstract fraction content as visual objects. Researchers suggest that when teachers use visual connections in their lessons, learners can see how mathematics fits into their daily lives by visualising mathematics (Blum, 2002; Cuban, 1976; Özdemir & Üzel, 2011; Patton, 1997). The visualisation connections were mainly used for exemplifying purposes in the three selected textbooks.

The analysis reflected that the EMLB went the extra mile by listing the basic competencies from the syllabus to be obtained at the end of each chapter, on the first page of each chapter. The textbook guides learners on what to do and how to do it. However, the OLDM and PM do not state the basic competencies in their chapters.

What are selected teachers' perceptions and perspectives of the three textbooks concerning connection-making in fraction concepts?

The questionnaire responses reflected that 90% of the participant teachers believe that the prescribed textbooks explicitly articulate real-life connections, connections to other subjects, connections to other mathematics content, visualisation connections and connections to problem-solving in the fraction concepts chapter. However, 10% of the respondents disagree with this assessment.

The interview participants showed diverse opinions on the presentation of mathematical connections in Grade 7 textbooks, emphasising their crucial role in fostering the learners' understanding and appreciation of mathematics. Weinberg (2001) emphasised the importance of explicitly teaching learners about the various possible connections within mathematics through instruction that displays the interconnectedness of mathematical ideas. The interview data showed unanimous agreement among participants on explicitly articulating the five connection indicators (real-life connections, connections to other subjects, connections to other mathematics content, visualisation connections and connections to problem-solving) in the selected textbooks.

In conclusion, the analysis of the textbook, questionnaires and interviews all confirmed the importance of promoting mathematical connections in education. By fostering the learners' ability to make connections, teachers can enhance their understanding and application of mathematical concepts, ultimately leading to improved problem-solving abilities and lifelong mathematical proficiency.

6.3 LIMITATIONS OF THE STUDY

This section highlights the study's limitations, starting with generalisability, then sample bias and finally, funding. The three limitations were previously elaborated on in Chapter 3, Section 3.11.

Generalisability

The study's limited scope focused only on the 21 teacher participants for the questionnaire and four teacher interview participants within the Grootfontein circuit only. Despite its small scale, the sample from the participant teachers in this study shed light on the research questions and set the groundwork for further investigation with the remaining teachers.

Also, the study focused only on three textbooks in a specific Namibian region and circuit. Hence, the findings can only be applied to the three selected textbooks and the Grootfontein circuit. Caution is necessary when using the findings in different settings or demographics.

Limited funding impacted the study's scope, sample size and available resources.

Another limitation which emerged as I analysed the figures in the textbooks (but which did not form part of my analytical focus due to the scope of my study) was the issue of incorrect language use. Numerous diagrams such as Figures 4.6 and 4.7 had grammatical errors in their instructions. This is problematic and requires attention.

6.4 SIGNIFICANCE OF THE STUDY

This study aims to contribute to the improvement of the evaluation process of textbooks in Namibia by enhancing the criteria used for assessing mathematical connections. The focus was on providing clear and precise criteria that can be used to evaluate Grade 7 mathematics textbooks. The goal was to increase the teachers' awareness of the importance of mathematical connections and encourage them to help their learners recognise these connections effectively. Additionally, this study encourages authors to incorporate appropriate mathematical connections in their textbooks, enhancing the conceptual teaching and learning of mathematics.

6.5 RECOMMENDATIONS

Based on the research findings from the textbook analysis, questionnaires and interviews, in the context of connections in fractions, I make the following recommendations. These are categorised under teachers, authors and curriculum developers (textbook evaluators, Otjozondjupa region, Grootfontein circuit).

Recommendations regarding teachers

1. Encourage teachers to make the learners aware of the different connections they are making in their lessons. Helping learners recognise and understand these connections will contribute to their overall comprehension of mathematics.
2. Create a more conducive learning environment for learners to make connections in mathematics. Teachers should strive to make mathematics more accurate and relevant to the learners by using examples that make sense to them. Teachers should understand their learners' preferred learning styles and adjust their teaching accordingly.
3. Encourage all teachers to reflect on their teaching practices, particularly regarding the use of connections. This reflection will help the teachers identify what works and what does not, leading to continued professional development and improvement in teaching.
4. Recognise the potential of the teacher leaders and advocate for their involvement in improving mathematics education. These teacher leaders can be crucial in driving systemic changes and leading initiatives to enhance teaching and learning.

Recommendations regarding authors

1. Authors should ensure that the textbooks cover the entire curriculum thoroughly. Participant teachers in this study have expressed concerns about the depth of content, so it is crucial to include a comprehensive range of topics, providing the learners with a solid foundation in each mathematical concept.
2. Establishing an effective feedback mechanism is vital for ongoing improvement. Hence, I encourage the authors to seek feedback on textbooks and learning materials from teachers, learners, and parents. Their insights could help identify areas where the content is lacking or where there may be a need for improvement. This feedback loop would enable the authors and curriculum developers to continuously enhance the teaching materials and ensure the provision of high-quality content.
3. The authors should fully explore and incorporate mathematical connections in the textbooks. While the three selected textbooks show clear connections to real-life situations and other mathematical concepts, there is a lack of connections to other subjects. This could pose a challenge, as it may create a disconnect between mathematics and other fields of study, further enforcing the perception that mathematics is isolated and difficult to relate to other disciplines.

   I suggest including examples and exercises highlighting interdisciplinary connections to address this issue, demonstrating how mathematics intersects with other subjects such as science, art, technology and economics. This would help the learners to understand the relevance and application of mathematics in various contexts, fostering a more holistic and interconnected understanding of the subject. Additionally, incorporating cross-curricular projects and activities in the textbooks will encourage collaborative learning and show the learners how mathematics can be integrated into their overall educational experience.
4. In the textbook design, including the basic competencies and summaries of key mathematical ideas at the beginning of each chapter is helpful for both learners and teachers. These summaries help the learners understand the chapter's overall aim and expectations while providing a quick reference for critical mathematical concepts, formulas, and definitions. This ensures that the learners clearly understand what they need to learn and helps them prepare effectively for tests and examinations. It also serves as a helpful tool for revision, allowing learners to quickly refresh their knowledge on specific topics.
5. Authors need to ensure that correct grammatical language is used at all times.

Recommendations regarding curriculum developers (textbook evaluators, Otjozondjupa region, Grootfontein circuit)

1. Utilise experienced teachers as mentors to younger teachers, especially regarding explicitly using connections in teaching. Tap into the expertise of experienced teachers to guide and support novice teachers in incorporating connections into their teaching practices.
2. Organise meaningful workshops for the teachers to share their experiences and explore connections in teaching fractions. These workshops should focus on enhancing the teachers' understanding of mathematical connections and providing them with practical strategies.
3. Foster a culture of continuous learning by encouraging teachers to share their expertise and insights. This can be done through workshops, collaboration and incorporating new strategies into teaching practices.
4. To promote teacher collaboration and communication, the curriculum developers should consider organising regional or cluster-level initiatives, such as workshops on mathematical connections. These initiatives can provide valuable opportunities for the teachers to learn from each other and enhance their teaching practices.
5. Curriculum developers should ensure that the textbooks cover the entire curriculum thoroughly. Participant teachers in this study have expressed concerns about the depth of content, so it is crucial to include a comprehensive range of topics, providing the learners with a solid foundation in each mathematical concept.
6. By involving teachers in the curriculum and policy development process, their expertise and insights can help ensure that mathematical connections are adequately addressed and represented in the recommended textbooks. Teachers are familiar with the day-to-day realities of teaching mathematics and their learners' specific needs and abilities. Their input can help bridge the gap between policy development and classroom practice. Additionally, teachers can provide valuable feedback on the effectiveness and feasibility of proposed curriculum and policy changes. Their practical experience can highlight potential challenges or areas where adjustments may be necessary. Including the teachers in the development process can also help cultivate a sense of ownership and buy-in for the implemented changes.

   When the teachers feel that their voice and expertise are valued, they are more likely to actively engage with and implement the new curriculum and policies in their classrooms. Incorporating the teachers' input into curriculum and policy development can lead to more effective and relevant educational materials and practices that better support the learners' mathematics learning.

By implementing these recommendations, schools and teachers can create a more effective and engaging learning environment for teaching fractions, ultimately leading to improved mathematics education outcomes.

6.6 AREAS FOR FUTURE RESEARCH

The study was done on a small scale and in only one circuit in the region. Further research could be done in the form of an intervention programme, whereby different types of connections would be introduced to the teachers and the teachers would then practise and present them. An elaboration on the areas of future research is stated below:

Exploration of connections in teaching practice

- Conduct future research to delve deeper into possible connection-making in teaching practice
- Expand the sample size to obtain more comprehensive insights. This would provide a more far-reaching understanding of the impact of connections in teaching fractions and allow for comparisons across various contexts.
- Explore diverse geographical locations to uncover potential variations in connection-making practices and gain additional perspectives.

Learner perspectives and insights:

- Involve the learner perspectives as a component of a new research design, examining how their insights contribute to understanding how they perceive connections.
- Further research could involve analysing the learners' notebooks to determine the nature of learners' mathematical knowledge with regard to making connections.
- Complement the analysis with learner interviews and classroom observations to understand how learners engage with the textbooks.

Intervention programme for connection-making

- Implement future research in the form of an intervention programme, introducing various types of connections to teachers and assessing the impact on their practices and presentations.
- Investigate the relationship between the mathematical connections used in textbooks and the learners' access to mathematics.

- Examine how the mathematical connection in the textbooks influences the learners' comprehension and engagement with mathematical concepts.

Mathematics textbook content

- Explore the relationship between textbook content and learners' achievement, considering alternative methods such as analysing the learners' written productions, like notebooks, in addition to formal testing.
- Conduct similar studies in other mathematical domains.
- Research should be undertaken into appropriate and correct language use.

These areas for future research aim to build upon the current study's findings and address gaps in understanding the dynamics of teaching practices, learner perspectives, intervention programmes, textbook practices and the influence of mathematical connections on the learners' access to mathematics.

6.7 REFLECTIONS

My study of mathematics education has provided me with a deeper understanding of the subject. Through extensive research and analysis of literature and texts, I have realised the complexity of factors involved in teaching mathematics. This helped me recognise the importance of a solid orientation in mathematics education for effective teaching and learning.

During my research, I focused on fractions and recognised my own struggles in effectively teaching this topic. Despite my interest and proficiency in fractions, I acknowledged the need for further improvement in my own teaching approach. Reflecting on my experience, I found it a valuable experience to work with my research participants to gather comprehensive and relevant data. Interacting with them and listening to their perspectives was a practical learning experience.

As a first-time researcher, I found the entire research journey rewarding and enriching. I enthusiastically encourage fellow researchers to explore this burgeoning area of mathematics connections and contribute to expanding knowledge in this area. The research endeavour contributes to the academic landscape and offers a unique personal and professional growth opportunity.

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APPENDICES

Appendix 1: Analytical Table-Level 1

<u>Analytical table: Level 1</u>

Connections in Mathematics	Level 1 Frequency								
	Indicators		Explicit presence			Implicit Presence			Total frequency(Q)
			1	2	3	1	2	3	
Real-life connections (RLC)	Food recipes (FR)								
	Learners' interests (LI), e.g., hobbies								
	Lived experience (LE)								
	Careers (C)								
	Total								
Connections to other subjects (CS)	Agriculture								
	English								
	Geography								
	Business Studies and other								
	Total								
Connections to other mathematics content areas (CMC)	Money & Finance								
	Geometry								
	Measurement								
	Percentage								
	Decimals								
	Total								
Visualisation connections (VC)	Table								
	Graph								
	Pictorial/diagrammatic								
	Drawing								
	Total								
Connections to problem-solving (CPS)	Real-Life Problems								
	Cross-curricular issues	ICT							
		HIV & Aids							
		Population issues							
		Globalisation							
		Road safety							
	Total								

Appendix 2: Analytical Table-Level 2

Analytical Table: Level 2

Connections in Mathematics	Purpose (P)			
	Exemplifying (Exa)	Explanatory (Exp)	Decorative (D)	**Total**
Real-life connections (RLC)				
Connections to other subjects (CS)				
Connections to other mathematics content areas (CMC)				
Visualisation connections (VC)				
Connections to problem-solving (CPS)				
Total				

Appendix 3: Provisional Questionnaire

RHODES UNIVERSITY

Where leaders learn

Connections in fractions in Namibian mathematics textbooks.

Introduction

The purpose of this questionnaire is to elicit your perceptions and experiences of how the three prescribed Grade 7 mathematics textbooks in Namibia make use of connections in dealing with fractions. This questionnaire is one of my research instruments for my masters' research project which is entitled: *An analysis of how prescribed Grade 7 Namibian mathematics textbooks promote connection-making in the context of fractions."* The purpose of the study is to analyse how the textbooks make use of connections in the context of fractions and not to judge the overall quality of the textbooks.

Based on the responses to the questionnaire, I wish to then select 4 participants to be interviewed more in-depth about their perception of how the prescribed mathematics textbooks promote connection-making in fractions. Your name and cell phone number on the questionnaire will only be used for selection purposes and ease of contact for these 4 participants.

By taking part in this questionnaire, I consent that my responses may be used for the study above. I understand that my identity in this study will be protected by the code of ethics stipulated by Rhodes University. Having taken note of the above information, I freely volunteer to partake in completing this questionnaire.

Instructions

- Tick/cross in the appropriate box next to the answer of your choice.
- Kindly respond honestly to **ALL QUESTIONS** to the best of your ability.

DEFINITIONS:

1. **A connection is:** a causal or logical link, relation, or sequence between two or more ideas—for example, the relationship between mathematics and other subjects.
2. **Connection-making is the process of** linking, creating or building a relationship between two or more things. In mathematics education, connection-making occurs when learners connect or create links between mathematical ideas, facts, procedures, and relationships within mathematics and beyond mathematics.

Your name: ____ (*Kindly provide your name and cellphone number if you are interested in engaging in the interviews on the* *above mentioned research project. Your name and cellphone number on the questionnaire will **only** be used for the selection purposes of the 4 interviewees.)*

SECTION A: Demographic data

In the following question, please indicate your gender by ticking an **(x)** in the spaces provided.

Gender:

Male	
Female	

Age:

$20 \leq x < 30$	
$30 \leq x < 35$	
$35 \leq x < 40$	
$40 \leq x < 50$	
$x \geq 50$	

Working experience as an educator:

Less than a year	
1 - 2 years	
3 - 5 years	
6 - 10 years	
More than ten years	

The textbook you use in teaching fraction concepts to your grade 7 Learners

Textbook	
Excellent Mathematics Learner's Book	
Let's Do Mathematics	
Platinum Mathematics	
Others (specify):	

Subject (s) and grade (s) you are currently teaching:

____

____

____

____

SECTION B: Connection-making in the fraction section of the Mathematics textbook you use

<table>
<tr><th colspan="2">Respond by putting an (X) under the appropriate heading.</th><th>Yes</th><th>No</th></tr>
<tr><td rowspan="5">1. Are the following connections explicitly articulated in the fraction concepts in the book?</td><td>Real-life connections</td><td></td><td></td></tr>
<tr><td>Connections to other subjects</td><td></td><td></td></tr>
<tr><td>Connections in problem-solving</td><td></td><td></td></tr>
<tr><td>Visualisation connections</td><td></td><td></td></tr>
<tr><td>Connections to other mathematical content areas</td><td></td><td></td></tr>
<tr><td rowspan="5">2. Are the following visual models used when the textbook deals with fractions?</td><td>Tables</td><td></td><td></td></tr>
<tr><td>Graphs</td><td></td><td></td></tr>
<tr><td>Drawings</td><td></td><td></td></tr>
<tr><td>Pictures</td><td></td><td></td></tr>
<tr><td>Others:</td><td></td><td></td></tr>
<tr><td colspan="2">3. Does your school have lesson planning support teams (within the mathematics department) to promote making connections?</td><td></td><td></td></tr>
<tr><td colspan="2">4. Does your school have lesson planning support teams (across other departments) to promote making connections?</td><td></td><td></td></tr>
<tr><td colspan="2">5. Apart from the textbooks available, are there any accessible resources? If yes, note them down.
____
____
____
____
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______________________________</td><td></td><td></td></tr>
<tr><td colspan="2">6. Do teachers at your school integrate ICT in the teaching of fraction concepts?</td><td></td><td></td></tr>
</table>

Comment on any of the above responses:

____

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SECTION C: TEACHER'S PERCEPTIONS AND ATTITUDES ON MATHEMATICAL CONNECTIONS/MAKING CONNECTIONS in the classroom

1. Please respond by putting an **(X)** to indicate your level of agreement from strongly agree to disagree strongly.

Your perceptions and attitudes on connection-making adoption and use in the classroom	**Strongly agree 1**	**Agree 2**	**Uncertain 3**	**Disagree 4**	**Strongly disagree. 5**
1. Making connections are disruptive when teaching fraction concepts Elaborate: ______________________________ ______________________________ ______________________________					

2. Making connections is effective in teaching fraction concepts. Elaborate:					
3. Making connections promote learner to learner interaction Elaborate:					

4. Making connections help in improving learner performance in the fraction concepts Elaborate:					
5. The application of mathematical connections in teaching and learning can improve learners' critical thinking Elaborate:					
6. Making connections engages learners' attention and motivates them.					

Elaborate: ____ ____ ____ ____ ____ ____ ____ ____					

Do you have any additional comments you wish to make to your responses above?

____
____
____
____

Thank you very much for your participation.

Appendix 4: Semi-Structured Individual Interview Schedule

Research Question 3
What are the selected teachers' perceptions and perspectives of the three textbooks concerning connection-making in fraction concepts?

1. Teachers' perceptions with regard to Mathematical connections?
   1.1 From your point of view, what are mathematical connections?
   1.2 Can you describe them?
   1.3 Do you use them when you teach fractions? Please elaborate
   1.4 Have you tried using them? Please elaborate
   1.5 Were they helpful? Please elaborate
   1.6 Were you trained to use mathematical connections?
2. Teachers' perspective with regards to teaching fractions?
   2.1 What are the advantages of using connections when teaching fractions?
   2.2 What are the disadvantages or challenges of using connections when teaching fractions?
3. Teachers' general perceptions of the three textbooks concerning making connections in fraction concepts.
   3.1 With reference to making connections, do you prefer a particular textbook among those recommended by NIED when teaching fraction concepts? Please elaborate
      3.1.1 Which textbook(s) do you use most?
      3.1.2 Why?
   3.2 Do the textbooks you use in your opinion, embrace connection-making in the fraction concept?
      3.2.1 Are the connections in the textbooks clear for the learners' understanding?
4. Do you have any other comments apart from what I have asked?

Appendix 5: Focus group interview schedule

1. Welcome and share greetings with all the participants.
2. Introduce the purpose of the focus group interviews.

   These focus group interviews are conducted to obtain a collective view of the mathematical connections that textbooks you use promote. As a researcher, my role is to initiate interview questions, clarify misunderstandings, and participate. I am not an expert; please consider me a learner because I am part of you. As we discussed during the semi-structured interviews, I am requesting to record this meeting; please indicate by raising a hand. There are no wrong and correct answers. Feel free to give your point of view. This gives us all a second chance to remember, share, question and add what we left out in the questionnaire and semi-structured interviews. Feel free to use any mathematics textbook or any other resource during this interview. Are there any additions, subtractions, divisions or multiplications you would like to share before we begin?

3. What, in your understanding and experience are mathematical connections?

   3.1 Can you give examples from your own practice?

   3.2 Which ones are applicable and practical to use in exploring a fraction concept, and which ones are not?

   3.3 Have we developed new connection-making ideas since answering the questionnaire and semi-structured interviews?

   3.3.1 Can we share the new ideas?

4. Do you think that the recommended textbooks present mathematical connections explicitly? Please elaborate.

   4.1 What are the mathematical connections embraced by the textbooks that you can identify?

   4.1.1 Are they strongly or weakly linked to the learning content? Please elaborate.

   4.1.2 Are they relevant for the grade 7 level?

   4.2 Do you have any suggestions on the presentation of mathematical connections in the textbooks?

   4.2.1 What are your recommendations?

5. Do you have any further questions or comments before we end the meeting?

Thank you for participating in this study; let's put to use what we have learnt in this study in the classroom and share it with other colleagues too. Remember, mathematical connections are not just limited to fraction concepts! I will be delighted to hear from you all.

Appendix 6: Approved permission seeking letter from the school Principals for authorisation of the 4 participants

[redacted]

The Principal
Grootfontein Circuit Primary schools (Grades 4-7)
Grootfontein
Namibia

Dear Sir/Madam

Request for permission to conduct semi-structured individual and focus group interviews with teachers

An analysis of the nature of mathematical connections inherent in the presentations of fraction concepts in Namibian Grade 7 mathematics textbooks.

I, Pumulo P. Sibeso, a Masters of Education student registered at Rhodes University in South Africa (19S9598), humbly request permission from you to conduct the above interviews with your grade 7 mathematics teacher in your school. The research topic is: "*An analysis of the nature of mathematical connections inherent in the presentations of fraction concepts in Namibian Grade 7 mathematics textbooks*".

After carrying out a questionnaire with all the 150 Grade 7 mathematics teachers in the Grootfontein circuit and analysing them, the mathematics teacher of your school has been selected as one of the four participants to take part in one semi-structured individual and one focus group interview. The research will be conducted during weekends to avoid disrupting normal teaching and learning.

The participants will be involved voluntarily. They will thus not receive any incentives for being part of the study. They will, however, benefit from the research, which will add value to their teaching practice. Informed consent was obtained from the Director of Education in the Otjozondjupa region, the Inspector of Education of the Grootfontein circuit, and the teachers at the participating schools. The identity of all participants and their schools will not be revealed and will be treated with confidentiality.

The analysis of the collected data from this research will be written up as a MEd full thesis which will be archived at Rhodes University.

This research was approved by the Rhodes University Ethical Standards Committee and the Education Department Higher Degrees Committee. APPLICATION NUMBER: 2022-5888-7345. During the research, any concerns may be directed to the Rhodes University ethics committee,
Please feel free to contact me at (+264) 81 6545807, or my supervisor, Professor Marc Schäfer (PhD), the SARChI Chair in Mathematics Education at Rhodes University. His email address is

Thank you in advance for offering me this developmental opportunity.

Yours sincerely,

Pumulo P. Sibeso

Approved by:

[redacted]

Signature

MINISTRY OF EDUCATION
School Stamp
01 NOV 2023
OTJOZO... UPA REGION

[redacted]

The Principal
Grootfontein Circuit Primary schools (Grades 4-7)
Grootfontein
Namibia

Dear Sir/Madam

Request for permission to conduct semi-structured individual and focus group interviews with teachers

An analysis of the nature of mathematical connections inherent in the presentations of fraction concepts in Namibian Grade 7 mathematics textbooks.

I, Pumulo P. Sibeso, a Masters of Education student registered at Rhodes University in South Africa (19S9598), humbly request permission from you to conduct the above interviews with your grade 7 mathematics teacher in your school. The research topic is: "*An analysis of the nature of mathematical connections inherent in the presentations of fraction concepts in Namibian Grade 7 mathematics textbooks*".

After carrying out a questionnaire with all the 150 Grade 7 mathematics teachers in the Grootfontein circuit and analysing them, the mathematics teacher of your school has been selected as one of the four participants to take part in one semi-structured individual and one focus group interview. The research will be conducted during weekends to avoid disrupting normal teaching and learning.

The participants will be involved voluntarily. They will thus not receive any incentives for being part of the study. They will, however, benefit from the research, which will add value to their teaching practice. Informed consent was obtained from the Director of Education in the Otjozondjupa region, the Inspector of Education of the Grootfontein circuit, and the teachers at the participating schools. The identity of all participants and their schools will not be revealed and will be treated with confidentiality.

The analysis of the collected data from this research will be written up as a MEd full thesis which will be archived at Rhodes University.

This research was approved by the Rhodes University Ethical Standards Committee and the Education Department Higher Degrees Committee. APPLICATION NUMBER: 2022-5888-7345. During the research, any concerns may be directed to the Rhodes University ethics committee, ethics-committee@ru.ac.za
Please feel free to contact me at (+264) 81 6545807, pummie91@gmail.com) or my supervisor, Professor Marc Schäfer (PhD), the SARChI Chair in Mathematics Education at Rhodes University. His email address is m.schafer@ru.ac.za

Thank you in advance for offering me this developmental opportunity.

Yours sincerely,

Pumulo P. Sibeso

Approved by:

[redacted]

Signature

E EDUCATION
MARY SCHOOL
School Stamp 07

Appendix 7: Permission letter to conduct educational research from the Director

REPUBLIC OF NAMIBIA
OTJOZONDJUPA REGIONAL COUNCIL

DIRECTORATE OF EDUCATION, ARTS AND CULTURE

"Committed and Dedicated For Quality Education"

Tel no: 264 67 308000
Fax no: 264 67 304871
Enq: J. Sikeso
*Email:*jsikeso05@gmail.com

Private Bag 2618
Erf. 280, Sonweg Street
OTJIWARONGO
Namibia

18 January 2023

Ref No: 14/6/10

Ms. Pumulo P. Sibeso
081 654 5807

Dear Ms. Sibeso

SUBJECT: PERMISSION TO CONDUCT EDUCATIONAL RESEARCH

Your letter dated 16 January 2023, bears reference and is hereby acknowledged.

Regarding the above mentioned subject, the Directorate of Education, Arts and Culture is pleased to inform you that permission is granted as per your request to do an educational research in Grootfontein Circuit. The School Principals and Inspectors of Education will be notified of the approval by copy of this letter.

You are kindly requested to make sure that, the research process should by no means whatsoever disrupt teaching and learning.

Your cooperation in this regard will be highly appreciated.

Yours faithfully

OTJOZONDJUPA REGIONAL COUNCIL
DIRECTORATE OF EDUCATION
OFFICE OF DIRECTOR
2023 -01- 19
TEL: 067-308000 PRIVATE BAG 2618
FAX: 067-304871 OTJIWARONGO
OTJOZONDJUPA REGION

....
MS. J. MUTENDA
REGIONAL DIRECTOR

Appendix 8: Permission letter to collect data from the circuit Inspector

REPUBLIC OF NAMIBIA
OTJOZONDJUPA REGIONAL COUNCIL

DIRECTORATE OF EDUCATION, ARTS & CULTURE
SECTION: PROGRAMME AND QUALITY ASSURANCE

Tel: 264 67 242785
Cell no: 264 812185965
Enquiries: Chuma Kashuwa
Email: ckasaila@gmail.com

Dr. Nicky Iyambo Street
Private Bag 2019
Grootfontein
NAMIBIA

27 January 2023

TO : THE PRINCIPALS OF SCHOOLS
GROOTFONTEIN CIRCUIT
OTJOZONDJUPA REGION

SUBJECT : PERMISSION TO COLLECT RESEARCH DATA

This letter serves to permit **Ms. Pumulo P. Sibeso,** a teacher at Coblenz Combined School in Grootfontein Circuit of Otjozondjupa region, pursuing her Masters of Education, with the Institute of Rhodes University in South Africa (student no:19S9598), she has to conduct a case study with your grade 7 Mathematics teacher(s) at your school in the entire circuit. The research topic is **"An analysis of the nature of mathematical connections inherent in the presentations of fraction concepts in Namibian Grade 7 Mathematics textbooks"** and to collect data for her Dissertation as partial requirements to complete her studies.

On behalf of the Grootfontein Circuit and the Regional Directorate of Education, Arts and Culture, **Ms. Sibeso** is officially sanctioned to carry out her data collection project, for which the Directorate is cognizance of the fact that despite the project aimed at fulfilling her Masters in Education, the findings and recommendations to her studies might be to the advantage of the Grootfontein Circuit Schools in particular and the Otjozondjupa region in general.

You are therefore kindly implored to avail **Ms. Pumulo. P. Sibeso** your essential time and resources she would require in completing her project, in your respective schools.

Counting on your usual cooperation.

Yours Sincerely

....

CHUMA KASHUWA
ACTING INSPECTOR OF EDUCATION
GROOTFONTEIN CIRCUIT

OTJOZONDJUPA REGIONAL COUNCIL
DIRECTORATE EDUCATION
PRIVATE BAG 2019 GROOTFONTEIN
2023 -01- 27
TEL / FAX 067 242786
INSPECTOR OF EDUCATION
GROOTFONTEIN CIRCUIT

Appendix 9: Sample of approved letter from the school Principal for questionnaire data collection

[redacted]

The Principal
Grootfontein Circuit Primary schools (Grades 4-7)
Grootfontein
Namibia

Dear Sir/Madam

Request for permission to conduct an educational research project

An analysis of the nature of mathematical connections inherent in the presentations of fraction concepts in Namibian Grade 7 mathematics textbooks.

I, Pumulo P. Sibeso, a Masters of Education student registered at Rhodes University in South Africa (19S9598), humbly request permission to conduct a case study with your grade 7 mathematics teacher(s) in your school. The research topic is: "*An analysis of the nature of mathematical connections inherent in the presentations of fraction concepts in Namibian Grade 7 mathematics textbooks*". The study is planned to take place in two phases:

Phase 1: All the 150 Grade 7 mathematics teachers in the Grootfontein circuit will be given a questionnaire to complete. This questionnaire will elicit information about the teachers' perspectives and perceptions on how our prescribed Grade 7 mathematics textbooks promote connections in fractions.

Phase 2: Based on the questionnaire responses, I will purposefully select four participants to participate in semi-structured individual interviews. The group of four teachers will then also take part in a focus group interview. The research will be conducted during weekends to avoid disrupting everyday teaching and learning.

The participants will be involved voluntarily. They will thus not receive any incentives for being part of the study. They will, however, benefit from the research, which will add value to their teaching practice. Informed consent has been requested and granted from the Director of Education in the Otjozondjupa region and the Inspector of Education of the Grootfontein circuit; the teachers consent is yet to be requested after your authorisation. The identity of all participants and their schools will not be revealed and will be treated with confidentiality.

The analysis of the collected data from this research will be written up as a MEd full thesis which will be archived at Rhodes University.

This research is yet to be approved by the Rhodes University Ethical Standards Committee and the Education Department Higher Degrees Committee. APPLICATION NUMBER: 2022-5888-7290 2022. During the research, any concerns may be directed to the Rhodes University ethics committee, ethics-committee@ru.ac.za
Please feel free to contact me at (+264) 81 6545807, pummie91@gmail.com) or my supervisor, Professor Marc Schäfer (PhD), the SARChI Chair in Mathematics Education at Rhodes University. His email address is m.schafer@ru.ac.za

Thank you in advance for offering me this developmental opportunity.

Yours sincerely,

31 January 2023
Pumulo P. Sibeso

Approved by (Name of the Principal) [redacted] Signature

MINISTRY OF EDUCATION
Auttas Primary School
School Stamp
P.O. Box 1370, GROOTFONTEIN
OTJOZONDJUPA REGION

Appendix 10: Sample letter to the questionnaire participants

Appendix 10: Invitation letter to teachers (Participants)

[redacted]

[redacted]_School

Grootfontein

Dear Sir/Madam

Invitation of participation in an educational research project

An analysis of the nature of mathematical connections inherent in the presentations of fraction concepts in Namibian Grade 7 mathematics textbooks.

I, Pumulo P. Sibeso, a Masters of Education student registered at Rhodes University in South Africa (19S9598), hereby invite you to participate in a research study. The research topic is: "*An analysis of the nature of mathematical connections inherent in the presentations of fraction concepts in Namibian Grade 7 mathematics textbooks*". The purpose of the study is to **analyse** how the three prescribed mathematics textbooks promote connections in the context of fractions. The study is planned to take place in two phases:

Phase 1: All the 150 Grade 7 mathematics teachers in the Grootfontein circuit will be given a questionnaire to complete. This questionnaire will elicit information on your perspectives and perceptions on how our prescribed Grade 7 mathematics textbooks promote connections in fractions. By participating in the questionnaire you consent that your responses may be used for the study above. Your name on the questionnaire will only be used for the selection purposes of four interviewees who will be interviewed after the completion of the interviews. Your identity in this study will be protected by the code of ethics stipulated by Rhodes University.

Phase 2: Based on the questionnaire responses, I will purposefully select four participants to participate in semi-structured individual interviews. I will request consent to audio record the interviews. The group of four teachers will then also take part in a focus group interview. The focus-group interviews will be conducted using zoom meetings to cut costs for the participants and to observe Covid-19 protocols by not gathering. I will request consent to video record the zoom meeting. I will ensure that each participant is able to connect by zoom by assisting them via WhatsApp/phone calls. I will also adopt communication means such as email, phone calls, text messages, and WhatsApp chats to meet the participants' needs. The research will be conducted during weekends to avoid disrupting normal teaching and learning.

To avoid bias, I will strive to interact ethically, understandingly, and meaningfully with the co-researchers. Since English is used as a medium of instruction for delivering mathematics lessons in Namibian schools, English will be the primary language used in the study. However, during the focus group interviews, you may wish to express yourself in your home language, which will be translated into English.

I will keep strict confidentiality and privacy between you and my research supervisor. I do not intend to disclose your identity, change the study's results, or interfere with any of your concerns during the data presentation. I shall respect that desire if you wish to be identified by

your real name. You may withdraw from the study at any time. I will guard against exploiting my positionality by engaging reflexively in my actions throughout the research process.

You will be involved voluntarily and will thus not receive any incentives for being part of the study. You will, however, benefit from the research, which will add value to your teaching practice. Informed consent will also be requested from the Director of Education in the Otjozondjupa region, the Inspector of Education in the Grootfontein Circuit and the Principals at the participating schools. The identity of all participants and their schools will not be revealed and will be treated with confidentiality.

The analysis of the collected data from this research will be written up as a MEd full thesis which will be archived at Rhodes University.

This research is yet to be approved by the Rhodes University Ethical Standards Committee and the Education Department Higher Degrees Committee. APPLICATION NUMBER: 2022-5888-7290 2022. During the research, any concerns may be directed to the Rhodes University ethics committee, ethics-committee@ru.ac.za
Please feel free to contact me at (+264) 81 6545807, pummie91@gmail.com) or my supervisor, Professor Marc Schäfer (PhD), the SARChI Chair in Mathematics Education at Rhodes University. His email address is m.schafer@ru.ac.za

Thank you in advance for your participation.

Yours Sincerely

16 February 2022

Pumulo P. Sibeso

Declaration

I [redacted]___ (Full name and surname of participant) at this moment confirm that I understand the content of this letter and the nature of the research project. I agree to participate in this research project.

Signature of participant: ______ Date: 05 March 2023

Appendix 11: Ethics approval letter from the Rhodes University Research Ethics Committee

RHODES UNIVERSITY
Where leaders learn

Rhodes University, Education Faculty
Research Ethics Committee
PO Box 94, Makhanda, 6140, South Africa
Tel: +27 (0) 46 603 8393
Fax: +27 (0) 46 603 8028
email: e.rosenberg@ru.ac.za

https://www.ru.ac.za/researchgateway/ethics/

16 February 2023

PROF Marc Schafer

Education Department

M.Schafer@ru.ac.za

Dear PROF Marc Schafer and P. Sibeso

Re: An analysis of the nature of mathematical connections inherent in the presentations of fraction concepts in Namibian Grade 7 mathematics textbooks.

APPLICATION NUMBER: 2022-5888-7345

This letter confirms that your research ethics application has been reviewed and **APPROVED** by the Education Faculty Research Ethics Committee (EF-REC). Your permission letter(s) where applicable have been received and you are free to proceed with your study.

Approval is granted for 1 year. An annual progress report is required in order to renew approval for an additional period. You will receive an email notifying you when the progress report is due.

Should any substantive change(s) be made during the research process, that may have ethical implications, you should notify the Education Faculty REC Chair via email. This includes changes in investigators. The REC Chair will advise as to whether a new application is necessary.

Do keep this clearance letter secure and accessible throughout your study and after its completion. It will be needed when a thesis is examined and when publications are submitted to journals.

Please also submit a brief report to the REC Chair on the completion of the research. This can be done via email. The purpose of this report is to indicate whether the research was conducted successfully and whether any ethics-related matters arose that the committee should be aware of, in order to guide future studies.

Sincerely,

ERosenberg

Prof Eureta Rosenberg

Chair: Education Faculty Research Ethics Committee