INVESTIGATING HOW THE USE OF VISUAL MODELS CAN ENHANCE THE TEACHING OF COMMON FRACTIONS FOR CONCEPTUAL UNDERSTANDING TO GRADE 8 LEARNERS.

A THESIS SUBMITTED IN FULFILMENT OF THE REQUIREMENTS FOR THE DEGREE OF

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ABSTRACT

The intention of this study was to explore how selected mathematics teachers used visual models to improve the teaching of common fractions for conceptual understanding to Grade 8 learners as a result of an intervention programme. This research study is part of the VIPROmaths project which seeks to research the effective use of visualisation processes in the mathematics classroom in South Africa, Namibia, Zambia, Switzerland and Germany.

This study which adopted a case study of teachers in Khomas Region, Namibia, is informed by constructivist learning theory. The study is situated within the interpretive paradigm and a multi-phase mixed method research approach was used. It focussed on analysing the use of visual models when teaching fractions namely: area model, number line model and a set model. The data were collected through survey questionnaires, observation and recall interview. The survey was conducted with the forty three mathematics teachers, from twenty secondary schools in Khomas region. The survey gave an overview of the nature and the use of visual models in schools. Three teachers purposively selected from the survey participated in the intervention program and were observed while teaching and interviewed after their teaching. Data were qualitatively and quantitatively analysed.

The findings of this study reveal that visualising fractions is one of the methods that can improve both teaching and learning by providing concrete evidence of otherwise abstract ideas and concepts. The teachers highlighted that models themselves guide learners through to the answer, as compared to working out solutions using symbols only. They further indicated that visual models improve learners' motivation, enhances understanding of fractions and encourages full participation of learners in the lesson. The study also found that use of visual models encouraged participation and it also boosted learners thinking capability. Teachers in this study preferred to use the area model as they found this model easier and more user-friendly in comparison with the number line and the set models. Teachers did not use the set model because of its complexity.

This study concludes that the use of visual models can help enhance the conceptual teaching and understanding of common fractions. It is hoped that the study contributes towards improving the quality teaching and learning of fractions in Namibia. Furthermore, it informs the teacher-training institutions in Namibia to integrate the use of visualisation in their training programmes to promote conceptual understanding of mathematics.

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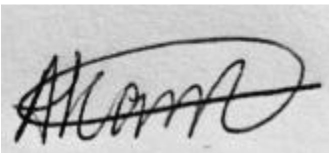
DEDICATIONS

I dedicate this humble work to my husband Rev Lukas Kaluwapa Katenda and my son Leuyanoshinge Latumbala Livingstone Katenda who have been my pillar through this journey and my spiritual, emotional and financial supporter, may God bless you always.

DECLARATION OF ORIGINALITY

I, Aune Kashikuka Katenda, Student number g13K6700, declare that this thesis entitled: ***Investigating how the use of visual models can enhance the teaching of common fractions for Conceptual understanding in Grade 8 learners,*** is my own work, written in my own words. Where I have drawn on words or ideas of others, these have been acknowledged according to Rhodes University Education Department referencing guidelines.



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CHAPTER 1

INTRODUCTION

1.1 INTRODUCTION

The aim of this research study was to investigate how visual models as visualisation tools can be used by teachers participating in an intervention programme to teach fractions for conceptual understanding to Grade 8 learners. In this chapter, I introduce my study by giving its background. Here I discuss how my study relates to the teaching of mathematics, giving more emphasis to the teaching of fractions in a Namibian context and its relation to visualisation in mathematics education. The chapter further outlines the research goals and research questions that guided this study. This chapter also summarizes the methodology employed, the theoretical framework and the significance of the study. The final part of this chapter outlines the structure of the thesis by providing a preview of what is discussed in each ensuing chapter.

1.2 BACKGROUND AND CONTEXT OF THE STUDY

The Namibian Ministry of Basic Education Arts and Culture's reformed curriculum advocates that all school graduates in Namibia are expected to be numerate and that the study of mathematics at junior secondary level should contribute to learners' ability to think logically, work systematically and accurately and solve real-world problems (Namibia. Ministry of Basic Education Arts and Culture, 2015). At this stage, more abstract mathematical concepts and reasoned arguments are introduced. The number concepts are formalised and include the whole range of real numbers, rational numbers and irrational numbers. At junior secondary level, learners use rational numbers to gain an understanding of common and decimal fractions. This study focuses specifically on enhancing the understanding of common fractions.

In Namibia, formal teaching of common fractions in schools starts in Grade 5 up until Grade 8 (Namibia. Ministry of Education, 2010). Table 1.1 below, presents the learning content of fractions in these four grades as indicated in the mathematics policy.

Table 1. 1: The learning content of common fractions in the Namibian mathematics syllabi for Grades 5-8 (Source: Namibia. Ministry of Education, 2015)

Grade 5	
Topic:	**Specific learning objectives:**
Fraction vocabulary	1. Use the correct terminology of fractions such as numerator and denominator; 2. Treat denominators of fractions as divisors e.g. a/b as a ÷ b whereby b represents the whole number of parts into which a whole is to be divided.
Comparing and ordering	3. Compare and order fractions with the same denominators or numerators e.g. $\frac{2}{5}$ and $\frac{4}{5}$ or $\frac{2}{3}$ and $\frac{2}{5}$; 4. Recognise equivalent fractions; 5. Compare and order fractions with different denominators including finding a common denominator, e.g. $\frac{1}{4}$, $\frac{1}{3}$ and $\frac{1}{5}$; 6. Locate fractions up to tenths as points on the number line.
Classification of Fractions	7. Identify proper fractions, improper fractions and mixed numbers; 8. Compare and order mixed numbers and improper factions in practical situations; 9. Convert between mixed numbers and improper fractions.
Fractional parts quantities	10. Calculate fractional parts of quantities.
Grade 6	
Equivalent fractions	11. Convert fractions to equivalents; 12. Write fractions in their simplest form.
Comparing and ordering	13. Compare and arrange fractions with the same and different denominators in a stated order by converting to a common denominator.
Addition and subtraction of fractions	14. Add and subtract fractions with the same and different denominators restricted to three terms/fractions; 15. Add and subtract mixed numbers.
Word problems	16. Solve two-step word problems involving addition (restricted to three terms) and subtraction with common fractions.
Grade 7	
The four basic operations and order of operations	17. Add and subtract common fractions including mixed numbers; 18. Multiply and divide fractions (including mixed numbers) by fractions; 19. Apply the correct order of operation BODMAS.
A quantity as a fraction of another quantity	20. Express one quantity as a fraction of another quantity;
Word problems	21. Solve three-step word problems involving common fractions.
Grade 8	
Common and decimal fractions	22. Convert common fractions to decimals by dividing the denominator into the numerator; 23. Convert terminating decimals to common fractions in simplest form; 24. Order and compare fractions; 25. Multiply and divide quantities by common fractions

The learning content of fractions in Grades 5 –8 as shown in Table 1.1 indicates that the teaching of fractions concentrates more on representing fractions symbolically, the interpretation of fractions and part-whole interpretation.

For example, in Grade 5 fractions are introduced to learners using symbolic fraction notation, whereby learners have to identify the numerator and denominator of the fraction notation (as shown in learning objective 1 in Table 1.1). The symbolic representation of fractions is also used for comparing and ordering fractions, classification of fractions and calculating fractional parts of quantities (as shown in learning objectives 3 – 5 and 7 – 10, in Table 1.1). The part-whole interpretation of fractions appears to be related to learning objectives 2 and 6, which regard the denominator of fractions as the number of parts into which a whole is divided, and to locate fractions up to tenths on the number line.

In Grade 6, the teaching of fractions is more of the part-whole explanation of fractions, which includes the use of similar models as the one used in Grade 5 to identify equivalent fractions (learning objective 11, in Table 1.1) and introduces the comparing and ordering of fractions with the same denominators only (learning objective 13, Table 1.1). However, the teaching of fractions in this grade particularly emphasises the use of procedures, such as how to find the lowest common denominator of two fractions and symbolic notation of how to teach equivalent fractions, comparing and ordering of fractions, addition and subtraction of fractions, as well as solving word problems (learning objectives 12 –16, Table 1.1).

The use of symbolic representation of fractions continues to dominate the teaching of fractions in Grade 7 and 8 as shown in the previous two grades. The content in these two grades is extended to multiplication, division of fractions and multiplication or division of quantities, but the emphasis is still on procedures and the use of symbols to solve problems. However, the teaching still emphasises the use of symbols and procedures for learning how to work with fractions (as shown in objectives 17 –25 in Table 1.1) above.

The use of symbols in fractions instruction, has in my view, resulted in a lack of understanding of fractions among learners. The general lack of understanding of fractions has been observed at all levels of education, from primary and secondary to tertiary levels (Bruce, Chang & Flynn, 2013). Clarke (2011, p. 36) asserts that "fractions are difficult to teach and difficult to learn." The results from research conducted in Namibia that looked at low performance in mathematics, has also indicated that fractions was one of the topics where learners perform poorly in all questions pertaining to fractions in examinations (Nambira, Kapenda, Tjipueja & Sichombe, 2009). It can be argued that the difficulties experienced in understanding fractions can be related to the way that fractions are taught in schools. The difficulties that learners encounter in learning fractions are further heightened by the existence of various fraction constructs; the unusual way in which fractions are written; the misapplication of whole number knowledge to fractions and the fact that little emphasis is placed on conceptual understanding of fractions (Hallett, 2010; van De Walle, 2014).

This research needs to be conducted in order to shed more light on how to improve the quality of the teaching and learning of fractions and create an awareness among teachers on the use of a different approach to teaching that can enhance the understanding of fractions. I am saying this because in teaching fractions, irrespective of which strategy or approach teachers choose to use, the focus should be on enhancing learners' conceptual understanding. I therefore argue that the incorporation of visualisation into the teaching of fractions is likely to address the difficulties associated with the understanding of fractions, because the use of visuals such as the fractional models, physical images of fractions and reflecting upon these images are fundamental processes to conceptualise fractions. As indicated by Presmeg (2006), that "visualisation is taken to include processes of constructing and transforming both visual mental imagery and all of the inscriptions of a spatial nature that may be implicated in doing mathematics" (p. 2), I thus contend that using visualisation processes in the teaching of fractions may provide more comprehensive understanding of fractions, since learners are able to work with fractions and reason about them conceptually as opposed to traditional teaching methods which place emphasis on the use of symbols and procedures.

1.3 RESEARCH GOALS

The aim of this study was to explore how selected teachers use fractional visual models to improve teaching common fractions for conceptual understanding to Grade 8 learners as the result of an intervention programme.

1.4 RESEARCH QUESTIONS

1. What is the nature of the different visual models used by Grade 8 mathematics teachers in the teaching of common fractions, prior to participating in an intervention programme?
2. How does the use of visual models enhance the conceptual understanding of fractions, if at all, to Grade 8 learners?
3. What are the selected Grade 8 mathematics teachers' experiences and perceptions in working with visual models to enhance the conceptual teaching of common fractions after participating in an intervention program?

1.5 THEORETICAL FRAMEWORK

This research study locates its theoretical underpinnings in constructivist learning theory. Constructivists assert that "knowledge is constructed" (Noddings, 1990, p. 7). Constructivism proposes that "we construct our knowledge of our world from our perceptions and experiences, which are mediated through our previous knowledge" (Simon, 1995, p. 115). Constructivism according to Ndlovu (2013), "focuses on the internal, cognitive or conceptual development of the learner's mind or discipline [mathematics] as a whole" (p. 4). In a constructivist perception of learning, people construct their own understanding and knowledge of the world through experiencing things and reflecting on those experiences (Gusta, 2011). In the same way, Cobb (1988) reiterates that the fundamental goal of mathematics is, or should be, to help learners build structures that are more complex, powerful, and abstract than those that they possess when instruction commences (p. 89).

It usually means teachers encouraging learners to use active techniques such as experiments and real-world problem solving to create more knowledge and then to reflect on and talk about what they are doing and how their understanding is changing (Gusta, 2011). "The constructivist teacher provides tools such as problem solving and inquiry based learning activities with which learners formulate and test their ideas, draw conclusions and infers, and pool and convey their knowledge in a collaborative learning environment" (p. 2).

1.6 RESEARCH METHODOLOGY

This research project is oriented in the interpretive paradigm. Interpretive approaches to research have the intention of understanding the world of human experience (Cohen & Manion, 1994), and suggest that reality is socially constructed (Mertens, 2005). This study essentially used two methods. The first consists of a survey of all mathematics teachers from twenty secondary schools in the Khomas region of Namibia; the second is a case study of a small cohort of teachers consisting of three Grade 8 mathematics teachers who took part in an intervention programme.

Since this study employs surveys, observations and interviews, a multi-phase mixed method approach was used. Mixed method research is a "research design (or methodology) in which the researcher collects, analyses, and mixes (integrates or connects) both qualitative and quantitative data in a single study or multiphase program of inquiry" (Creswell, 2014, p. 2).

The mixed method approach helped me to analyse my data from different perspectives. This means that data which was analysed qualitatively complemented the data analysed quantitatively. This adds to the validity and reliability of my study. A quantitative analysis of the survey using descriptive statistics provided me with a broad overview of the nature of visual models used in teaching common fractions in Grade 8 in the region concerned. Qualitative methods were used to analyse how the use of visual models enhances the conceptual understanding of fractions, if at all, in Grade 8 learners and the teacher's experiences and their perceptions of using the visual models after the intervention programme.

1.7 SIGNIFICANCE OF THE STUDY

The study contributes towards improving the quality of teaching of fractions in Grade 8. Firstly, this study investigated the nature of visual models that teachers in the Khomas region use to teach fractions. Secondly it looked at how the use of visual models may enhance the conceptual understanding of fractions in an intervention program. Finally, the study looked at understanding three selected mathematics teachers' experiences and perceptions about the use of visual models in their classrooms.

It is hoped that the study will contribute towards improving the quality of teaching and learning of fractions in Namibia. The study also hopes to make mathematics teachers aware of how to teach fractions using other methods, with the focus on improving learners' understanding of fractions. Lastly, the findings of this study may help create awareness in mathematics textbook writers and curriculum developers in Namibia to include the use of fractional models or the use of visualisations in the school syllabus and textbooks, in order to enhance the learners' conceptual understanding of mathematics.

1.8 STRUCTURE OF THE THESIS

Chapter One (Introduction): In Chapter one I introduce the study and present the context of the study, the research methodology, significance, the limitation of the study and finally the structure of the study.

Chapter Two (Literature review): presents the conceptual framework and literature that informs and shapes the analysis and interpretations of the findings of this study. Key concepts are fractions, fraction sub-constructs used in teaching fractions, visualisation, fraction visual models, conceptual understanding and constructivism theory.

Chapter Three (Methodology): describes the methodology and methods used to collect the data. It also describes the research orientation, research methods, research design, data collection method, and the sampling techniques used for this research. Furthermore, it discusses issues pertaining to data analysis, validity, and ethical considerations.

Chapter Four (Data presentation, Analysis and Discussions): presents the data from the three research instruments, namely survey questionnaires, observation (this includes different models used, and transcripts of the lesson videos) and stimulated recall interviews (the interview transcripts). In addition, it details the analysis of this data in order to generate the research findings for the three research questions outlined above. It further presents a discussion of the summary of the findings, linking the data analysis in Chapter 4 to the literature presented in Chapter 2.

Chapter Five (Conclusion and Recommendations): presents the key findings of the research findings, significant of the study, personal reflection and recommendations of this research study.

CHAPTER 2

LITERATURE REVIEW

2.1 INTRODUCTION

This chapter discusses the literature related to the teaching of fractions. The chapter particularly emphasises the teaching of common fractions using visual models as visualisation tools. The visual models to be discussed are the area model, the number line model and the set model. The chapter presents the teaching of fractions in the Namibian curriculum, it defines common fractions and other fraction constructs, and also describes different types of fractions. It also discusses the interpretation of fractions, the importance of teaching fractions and misconceptions associated with the teaching of fractions. The chapter then presents the concept of visualisation, introduces visualisation in mathematics teaching and the three visual models mentioned above. This chapter also looks at five strands of mathematical proficiency, but will specifically focus only on conceptual understanding. It further presents and discusses literature and texts on the teaching of fractions for conceptual understanding. Lastly, the chapter discusses the constructivism learning theory and constructivist views of teaching as its overarching theoretical framework.

2.2 TEACHING COMMON FRACTIONS IN THE NAMIBIAN CURRICULUM

Learners' understanding of fractions has proved to be a challenge over the years. The general lack of understanding of fractions has been observed at all levels of education, from primary and secondary to tertiary levels (Bruce et al., 2013). Clarke (2011, p. 36) asserts that "fractions are difficult to teach and difficult to learn." It can be argued that the difficulties experienced in understanding fractions can be related to the way that fractions are taught in schools. Bruce at el. (2013) further indicate that poor understanding of fractions can contribute to learners' lack of understanding of other mathematics concepts and their career choices. The difficulties that learners encounter in learning fractions are further heightened by the existence of various fraction constructs; the unusual way in which fractions are written; the misapplication of whole number knowledge to fractions and the fact that little emphasis is placed on conceptual understanding of fractions (Hallett, 2010; van De Walle, 2014).

In teaching fractions, irrespective of which strategy or approach teachers choose to use, the focus should be on enhancing learners' conceptual understanding of fractions. I argue that the incorporation of visualisation is likely to address these challenges as, by its very nature, the use of visualisation such as the visualising of fractions, manipulating them, constructing physical and mental images of fractions and reflecting upon these images are fundamental processes to conceptualise fractions. In agreement Presmeg (2006), define "visualisation as taken to include processes of constructing and transforming both visual mental imagery and all of the inscriptions of a spatial nature that may be implicated in doing mathematics" (p. 2). I thus argue that using visualisation processes in the teaching of fractions provides a more comprehensive understanding of fractions, since learners are able to work with fractions and reason about them conceptually as opposed to traditional teaching methods which place emphasis on rote learning and procedures.

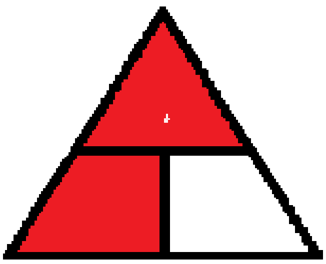
In the Namibian curriculum, visualisation is supported by the ideals of Learner Centred Education (LCE). Learner-centred education is an educational framework in which meaningful learning is promoted by helping learners take charge of their own learning (Namibia. Ministry of Education, 2015). This approach to teaching requires learners to be actively involved in their own learning through the construction of knowledge. It entails that learners learn best when they are actively involved in the learning process through a high degree of participation, contribution and use of manipulation (Namibia. Ministry of Education, 2015).

Learner-centred education requires that teachers use a variety of teaching materials such as manipulatives or visual models in order to help learners understand mathematics concepts, especially the concept of common fractions that many people think is difficult to teach and learn. Therefore, it is the duty of a teacher to find ways to make the topic of common fractions understandable and easier for learners to grasp. This study seeks to find other ways that teachers and learners can use to tackle problems associated with common fractions, as oppose to the algorithmic or symbolic ways of teaching that are currently being used in Namibia.

2.3 FRACTIONS

2.3.1 Definition of fraction

The term fractions' has been defined by many researchers. Hansen (2015) for example indicates that fractions can be used to refer to "any way of representing rational numbers, such as percentages, decimals and common fractions" (p. 9). On the other hand, Bruce et al. (2013) define a fraction as a "number which can tell us about the relationship between two quantities" (p. 8). These two quantities provide information about the parts, the units and the whole. My study is concerned with the development of common fractions. Common fractions refer to a representation of rational numbers in the form $\frac{a}{b}$ (Hansen, 2015). In this case, according to Hansen's (2015) definition of common fraction, a refers to the unit whereas b refers to the whole. In other words, a is the numerator that shows the part of the fraction being considered, while b is the denominator that shows how many times the whole is being divided as shown in Figure 2.1 below.



2/3 of the shaded triangle

Figure 2. 1: A representation of the fraction two-thirds.

The model shown in Figure 2.1 above illustrates that a full triangle is a unit or the whole. This unit is divided into three equal parts. The picture shows two coloured parts that represent two of the three equal parts. The numerator tells us how many of the parts in the unit are to be considered. The denominator tells us the total number of equal parts into which the unit is divided. In this example there are three equal parts in a triangle. The line between the numerator and denominator is known as the fraction bar. It is also called the division bar. This fraction is called two-thirds because two of the three parts in a triangle are considered.

Many researchers believe that learners would better understand the meaning of fractions if fraction is presented to them as in Figure 2.1 or if more emphasis was placed on understanding the different ways of describing a fraction or in the explanation fractional constructs. Researchers in this area, like Siebert & Gaskin (2006); Clarke, Roche, and Mitchell (2008); Lamon (2012) suggest the following possible constructs:

Part- Whole construct. Using the part-whole construct is an effective starting point for building meanings of fractions (Cramer, Monson, Whitney, Leavitt & Wyberg, 2010). Part-whole can be represented with a model by shading a region that represents a fraction as shown in Figure 2.1 above. In the part-whole interpretation, the denominator shows the number of equal parts of the whole and the numerator shows how many of those parts are included in the particular fraction. The part-whole construct is often represented with an area model, such as a shape partitioned into equal parts. The part-whole construct of fractions, when modelled with area diagrams, provides a useful tool for developing an initial understanding of fractions. For example, shading a part of a group of people such as $\frac{5}{6}$ of the class that went on the field trip. The area model could be useful in illustrating this as seen in Figure 2.2 below.

Figure 2. 2: Illustration of the fraction $\frac{5}{6}$ as a part-whole.

Measurement construct. Fractions as units of measure strongly connect with the concepts and processes of linear measurement. Scales are used to make linear measurements. When we measure an interval using a ruler, the unit of measure used is centimetres. Each centimetre represents a length that is one-hundredth of a whole metre. The measurement from the start of the interval to its endpoint tells us how many units the endpoint is from zero. If we wanted a more accurate measurement, we could subdivide the units (centimetres) into smaller equal parts (millimetres).

Instead of using a ruler, we could also use a number line. The distance between zero and one can be divided into equal lengths. For example, the distance can be divided into two parts to create a unit of measure called 'half'. Marking a point on the number line and labelling it as '$\frac{1}{2}$' means the point is a distance of one unit from zero. The unit can be divided into smaller parts that can either be six-twelfths, that still mean 'half' as shown one the number line in Figure 2.3 below.

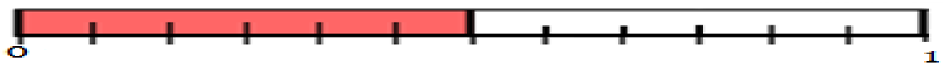


Figure 2. 3: An illustration of a measurement on a number line.

Operator construct. A fraction can be used to operate on a quantity. In other words, the fraction acts as a function.

For example, to find three-quarters of a certain number, several combinations of operations could occur. You could: divide by four then multiply by three; or multiply by three then divide by four. These situations indicate a fraction of a whole number, and learners may be able to use mental mathematics to determine the answer. However, Usiskin (2007) asserts that this construct is not emphasized enough in school curricula. Just knowing how to represent fractions does not necessarily mean learners will know how to operate with fractions, which occurs in various areas in mathematics (Johanning, 2008). The example below shows the fraction as an operator by finding what a quarter of eight is, in the first example, or in the second example that finds three quarters of eight, as shown by diamonds.

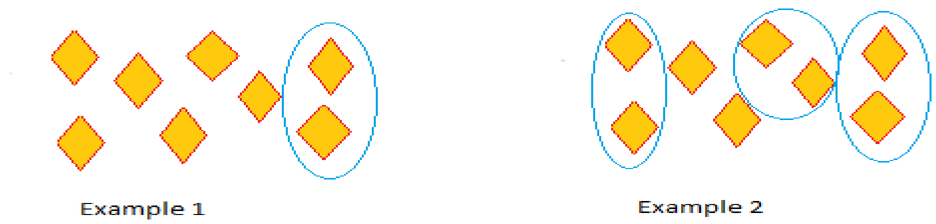


Figure 2. 4: An illustration of a fraction as an operator.

Ratio construct. The concept of ratio is yet another context in which fractions are used. Ratios are most commonly used to express the quantitative relationship between two groups, and can be modelled using discrete items. For example, the ratio of two to three can be represented as a group of two pentagons compared to a group of three circles: Another way of describing a ratio, is that it indicates the number of times one amount contains the other amount, or is contained within the other amount. We could say that the group of pentagons is two-thirds of the group of circles, or the group of circles is three halves of the group of pentagons as is shown in Figure 2.5 below.

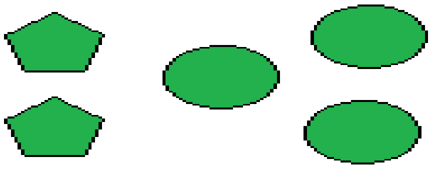


Figure 2. 5: Expressing the ratio 2:3 or as a fraction $\frac{2}{3}$.

In addition to the fraction constructs defined above, Way (2011) attempts to summarise some of the fundamental ideas of understanding fractions and suggests teaching tips as follows:

- Learners need to understand that particularly in the part-whole model, fractions refer to equal parts of a whole and they need to be able to name fractions both visually, verbally and symbolically (written). Although learners need to understand the information communicated by the numerator and denominator in describing a part- whole relationship, care needs to be taken when "reading" fractions. If the numerator and denominator are read as two separate numbers, such as 'two out of 3'or 'two over 3', this creates a barrier to perceiving the fraction as a number itself. It should therefore be read as 'two-thirds'.
- Learners need to understand the relationship between the number of parts and the relative size of the parts, through visualising the parts. This helps the learner to realise that the more parts a quantity is divided into, the smaller the pieces become. When connected to written fractions, this means 'the larger the denominator, the smaller the parts'. Learners might use this understanding as a strategy for comparing unit fractions (for example, to decide that $\frac{1}{4}$ is smaller than $\frac{1}{3}$), but they may need prompting to realise that this strategy is not so helpful when comparing non-unit fractions (for example, $\frac{3}{4}$ and $\frac{2}{3}$). However if they use visual models such as the one shown in Figure 2.6 below, they will be able to compare both unit and non-unit fractions without any difficulties.

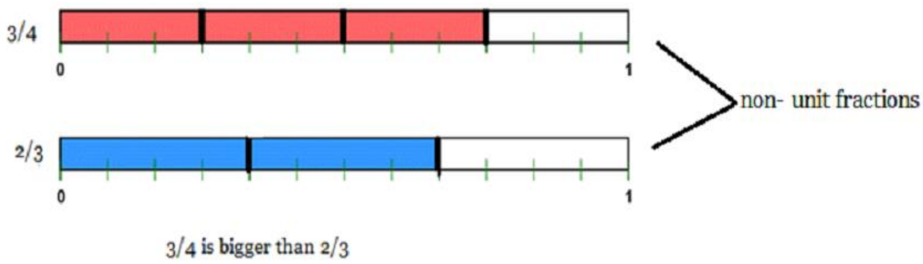


Figure 2. 6: A comparison of fractions on number line.

- Learners need to have a sense of the size of fractions in relation to one whole, including whether they are less than one (for example $\frac{1}{3}$), equal to one (e.g. $\frac{5}{5}$) or greater than one (for example, $\frac{5}{4}$). This often supports strategies around using benchmarks for estimation, particularly when ordering fractions. For example, the order $\frac{2}{4}$, $\frac{9}{6}$ and $\frac{1}{3}$ can be determined simply by deciding if each fraction is less than, greater than, or equal to one-half. However, this strategy will not be sufficient in all cases, so learners need a range of strategies to draw on, such as the use of a number line model that clearly shows the units and the fractions less or greater than one as see in Figure 2.7 below.

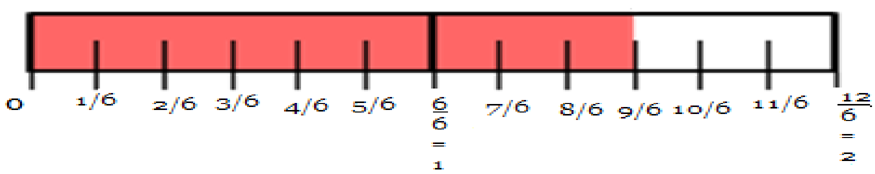


Figure 2. 7: Expressing a mixed number on a number line.

- Learners need to instantly recognise and name representations of commonly used fractions. Learners and teachers should consider strategies such as visual models such as the one in Figure 2.6 or 2.7 above that can be used to quickly recognise a fraction. Awareness of a range of models to present or name fractions will help learners develop stronger fraction understanding.

- Learners need to visualise, estimate and create representations of fractions by partitioning “wholes”, using a variety of models (for example, areas of various shapes, strips of paper or string, groups of objects). It is important to avoid always presenting learners with representations of fractions prepared in advance as this develops automatic responses rather than thinking and visualising. As learners already have difficulties creating their own representations, they need lots of practice in drawing shapes and using digital resources to build mental images that will allow them to visualise and estimate fractions.

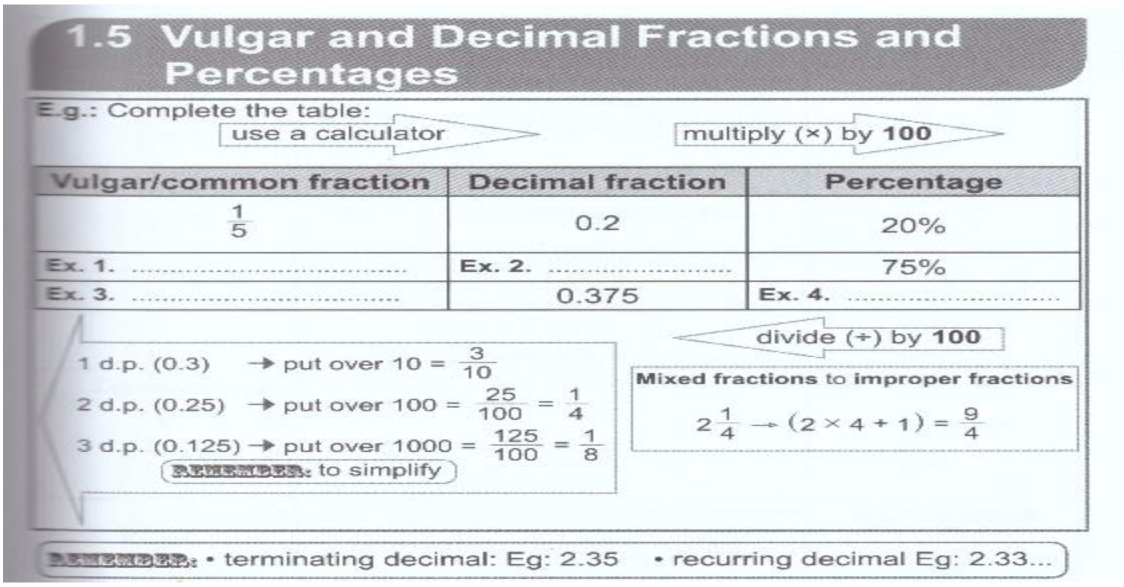
Learners who have developed these basic components of fraction sense will then have the capacity to apply their mental images of fractions to build more complex concepts and processes.

2.3.2 Types of fractions

In this study, common fractions are classified into three groups, namely proper fractions, improper fractions and mixed numbers. In the Namibian school mathematics textbooks, the three types of common fractions are defined as follows:

- Proper fraction is a fraction with a numerator smaller than the denominator e.g. $\frac{5}{11}$.
- Improper fraction is a fraction with a numerator bigger than the denominator e.g. $\frac{11}{6}$.
- Mixed number is a whole number together with a fraction e.g. $3\frac{3}{7}$ (Hambata, Roos, & van der Westhuizen, 2015, p. 49).

My observation of the Namibian school Grade 8 textbooks of mathematics by D'Emiljo (2009), suggests that the teaching of common fractions places emphasis on procedures how to convert common fractions to decimals and percentages or improper fractions into mixed numbers and vice versa, rather than visualising the conceptual relationship between improper fractions and mixed numbers. The textbooks use symbolic notation for identifying common fractions, decimals and decimal fractions, but they do not give conceptual interpretations of how to represent proper and improper fractions using manipulatives. The picture in Figure 2.8 below shows how fractions are typically taught in Namibia.



1.5 Vulgar and Decimal Fractions and Percentages

E.g.: Complete the table:

use a calculator → multiply (×) by **100** →

Vulgar/common fraction	Decimal fraction	Percentage
$\frac{1}{5}$	0.2	20%
Ex. 1.	Ex. 2.	75%
Ex. 3.	0.375	Ex. 4. ....

← divide (÷) by **100**

1 d.p. (0.3) → put over 10 = $\frac{3}{10}$

2 d.p. (0.25) → put over 100 = $\frac{25}{100} = \frac{1}{4}$

3 d.p. (0.125) → put over 1000 = $\frac{125}{100} = \frac{1}{8}$

REMEMBER: to simplify

Mixed fractions to improper fractions

$2\frac{1}{4} \rightarrow (2 \times 4 + 1) = \frac{9}{4}$

REMEMBER: • terminating decimal: Eg: 2.35 • recurring decimal Eg: 2.33...

Figure 2. 8: Symbolical teaching of common fractions in Namibia, (Source: Y=MX+C textbook (p.13).

This study thus seeks to establish conceptual interpretations of how to use visual models or manipulatives to represent or identify fractions, compare fractions and add and subtract fractions.

2.3.3 Interpretation of fractions

It is argued that learners' conceptual understanding of fractions has been a challenge over the years as indicated earlier on in this chapter; this trend has been observed at all levels of schooling (Bruce et al. 2013). Hence Nuffield (1991) indicates that it has been said that 'fractions' have been responsible for putting more people off mathematics than any other single topic. In fact, the very word 'fraction' has been known to make strong "men wince" (p. 32). This is possibly because of the way in which fractions are introduced to the learners, where certain learners still need more experience with the visual and practical aspect of creating simple fractions of shapes in order to gain a more secure understanding of what a fraction actually is. It is believed that learners need to have a concrete understanding of what the denominator and the numerator represent, through the use of visual manipulation. Nuffield (1991) asserts that "the headlong rush into computation with fractions, using such mumbo-jumbo as add the tops but not the bottoms' or 'turn it upside down and multiply', has often been attempted before the idea of a fraction or fractional notation has been fully understood" (*ibid.*).

It is argued that instead of learning a rule to remember how to solve a certain calculation, learners need to know why they are carrying out this rule. For example, when adding or subtracting fractions of a like denominator, the above-mentioned rule of ‘add the tops but not the bottoms’ will apply. However, if a learner does not understand why this is done they may also apply the same rule to the addition or subtraction of fractions with different denominators. They may remember the rule rather than the reason for using the rule. Learners need to understand this rule concerning common denominators. It will help them to understand how to change fractions so that they have a common denominator. Thus, if learners use visuals as shown below it may help them not to focus on rules and procedures, but rather use images and models to solve same or different denominator problems as shown in the model in Figure 2.9 below.

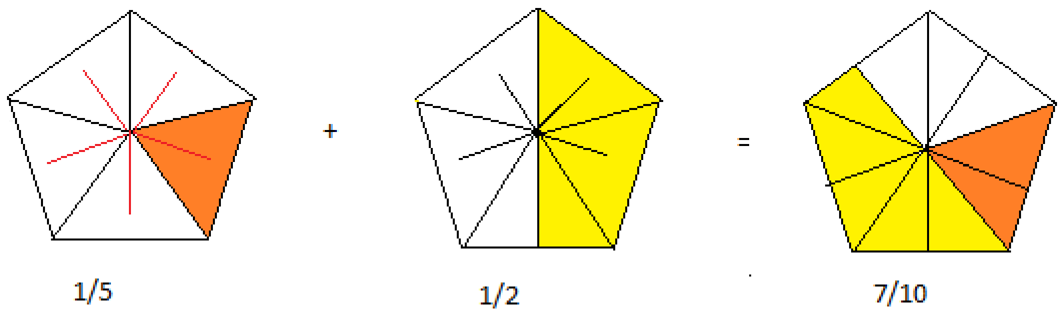


Figure 2. 9: Addition of fractions with different denominators using the area model.

The model in Figure 2.9 above shows that the addends $\frac{1}{5}$ and $\frac{1}{2}$ have unlike denominators. To work it out, first rename $\frac{1}{5}$ and $\frac{1}{2}$ to a like or common denominator (LCD) which is 10, by dividing each model visual into 10 parts as indicated with the short lines. Once each addend is written with like denominators, the numerators (coloured parts) can be added for the sum numerator.

Likewise, using the rule of ’whatever you do the bottom you must do to the top (and vice versa)’ or reducing a fraction to its simplest form would have no meaning if the learners do not understand the concept of equivalent fractions or know that fractions can be the same size but can be split into a different number of equal parts. It is necessary that learners have this visual knowledge of fractions in order to use and apply their knowledge across a range of different contexts.

2.3.4 The teaching and learning of fractions

Conceptual understanding of fractions is a challenging area of mathematics for learners to grasp (National Assessment of Educational Progress, 2005). This results in learners having difficulty retaining fraction concepts (Groff, 1996). Adults also struggle with fraction concepts (Lipkus, Samsa, & Rimer, 2001; Reyna & Brainerd, 2007), even when fractions are important to daily work-related tasks (Bruce & Ross, 2009). Fractions involve difficult-to-learn and difficult-to-teach concepts that present ongoing pedagogical challenges to the mathematics education community (Bruce et al., 2013). Some of the problems identified in the literature that make fractions so difficult are:

- The abstract way in which fractions are presented.
- Fractions do not form a normal part of the learners' environment.
- The tendency to introduce the algorithms for the operations on fractions before learners have understood the concept.
- The abstract definition of the operations on fractions.
- The formulation and practising of computational rules receiving too much attention whereas the fundamental concept of fraction is ill-developed (Lukhele, Murray & Olivier, 1999, p. 88).

Fractions are difficult to learn because they require deep conceptual knowledge of part-whole relationships (how much of an object or set is represented by the fraction symbol), measurement (fractions are made up of numbers that can be ordered on a number line) and ratios (Moss & Case, 1999; Hecht, Close & Santisi, 2003).

The challenges and misunderstandings learners face in understanding fractions (Hiebert & Wearne, 1988; NAEP, 2005; Gould, Outhred & Mitchelmore, 2006) persist into adult life and pose problems in such wide-ranging fields as medicine and health care, construction and computer programming. Therefore, helping learners to achieve a solid grounding in mathematics in general and fractions in particular has long-term high-stakes ramifications, suggesting that it is worth spending the time and effort to enhance student understanding to ensure their success in later mathematics, career and lives (Bruce et al., 2013).

The mathematics education community and researchers have much more work ahead to resolve these challenges presented by the learning and teaching of fractions. The implications are broad, but they are also deep, affecting foundational understandings that help or hinder the learning of other areas of mathematics. Behr, Harel, Post and Lesh (1993) for example, have insisted that "learning fractions is probably one of the most serious obstacles to the mathematical maturation of children" (Charalambous & Pitta-Pantazi, 2007, p. 293). The understanding of fractions is underpinned by larger mathematics cognitive processes including proportional reasoning (Moss & Case, 1999) and spatial reasoning (Mamolo, Sinclair & Whitely, 2011). In addition, Harvey (2011) suggests that in learning fractions learners need to be aware of two aspects or components of fractional parts: (1) the number of parts and (2) the equality of the parts (in size, not necessarily in shape). It is important to emphasize that the number of equal parts or fair shares that make up a whole determines the name of the fractional parts or shares. One of the best ways to introduce the concept of fractional parts is through sharing tasks.

Bruce et al. (2013) state that it is clear that a weak foundation in fractions can eventually cut learners off from higher mathematics and we must make strides through mathematics educational research and improve classroom practice to upgrade this situation. However, the problem is complex and requires a long-term commitment to gaining a greater understanding of how to support learners in building that solid foundation. It is therefore argued that it is important to teach fractions using manipulatives. A mathematics lesson that involves manipulatives allows students to develop reasoning skills, explore mathematical connections, and build number sense (Roddick & Silvas-Centeno, 2007). In other words, if mathematics teachers find other strategies that use manipulatives or visualisations they can overcome the difficulties involved in the teaching and learning of fractions.

2.3.5 Misconceptions linked to the teaching of fractions

There are many misconceptions connected to the teaching and learning of common fractions. Harvey (2011) indicates that learners learn better when they relate what is being taught to their previous knowledge. This means that when they encounter situations with fractions, they naturally use what they already know about whole numbers to solve the problems. Based on Harvey's (2011) research, there are a number of reasons why learners struggle with fractions. Examples of reasons for misconceptions are:

- There are many meanings of fractions such as discussed in 2.3.1 above.
- Fractions are written in a unique way.
- Learners overgeneralize their whole-number knowledge (McNamara & Shaughnessy, 2010).

It is important for a teacher to help learners see how fractions are alike and different from whole numbers. An explanation of common misapplications of whole-number knowledge to fractions follows, along with ways one can overcome the misapplication (Harvey, 2011).

Firstly, learners often think that the numerator and denominator are separate values and they have difficulty seeing them as a single value (Cramer et al., 2010). It is hard for them to see that $\frac{3}{4}$ is actually one number. One way to overcome this is by finding fraction values on a number line. This can be a fun warm-up activity each day, with learners placing particular values on a classroom number line or in their mathematics journals. Measure with various levels of precision (e.g., to the nearest metre), avoid the phrase "three out of four" (unless talking about ratios or probability) or "three over four" Instead, say "three fourths" (Siebert & Gaskin, 2006).

Secondly, learners often do not understand that $\frac{2}{3}$ means two equal-sized parts (although not necessarily equal-shaped objects). A teacher can overcome this misconception by asking learners to create their own representations of fractions using various manipulatives, representations and models, and on paper, cut/partition them into equal parts.

Thirdly, learners often think that a fraction such as $\frac{1}{5}$ is smaller than a fraction such as $\frac{1}{10}$ because 5 is less than 10. Conversely, learners may be told the reverse, i.e. the bigger the denominator, the smaller the fraction. Teaching such rules without providing the reason may lead learners to overgeneralize that $\frac{1}{5}$ is more than $\frac{1}{10}$. This misconception can be tackled by using visuals in the contexts that show parts of the whole. For example, the teacher can ask learners to draw a visual model such as the one on Figure 2.6. Through visualisation learners can see which fraction is bigger than the other one. Or the teacher can use the idea of fair shares: Is it fair if Mary gets one-fourth of the pizza and Laura gets one-eighth? Ask learners to explain why this is not fair and who gets the larger share.

Fourthly, learners often mistakenly use the operation "rules" for whole numbers to compute with fractions — for example $\frac{1}{2} + \frac{1}{2} = \frac{2}{4}$. To overcome this misconception the teacher can use visual models to add the fractions together such as the one explained on Figure 2.9 earlier. Emphasize estimation and focus on whether answers are reasonable or not.

Learners who make these errors do not understand fractions. Until they understand fractions meaningfully, they will continue to make errors by over-applying whole-number concepts

(Cramer et al., 2010; Siegler et al., 2010). The most effective way to help learners reach higher levels of understanding is to use visualisation that includes the use of multiple representations, multiple approaches, explanation and justification (Harvey 2011; Pantziara & Philippou, 2012).

2.4 VISUALISATION

2.4.1 Definition of visualisation

The term visualisation is defined in different ways by several mathematics researchers. According to Zimmermann and Cunningham (1991), "mathematical visualisation is the process of forming images (mentally, or with pencil and paper, or with the aid of technology), and using such images effectively for mathematical discovery and understanding" (p. 3). They further indicate that visualisation supplies depth and meaning to understanding, serving as a reliable guide to problem solving, and inspiring creative discoveries. In the same spirit, Lohse, Biolsi, Walker and Rueler (1994) define visualisation as "the study of mechanisms in computers and in humans, which allow them in concert to perceive, use, and communicate visual information" (p. 37).

Arcavi (2003) suggests a broad definition of visualisation for this study. According to him visualisation is the

> …ability, the process and the product of creation, interpretation, use of and reflection upon pictures, images, diagrams, in our minds, on papers or with technological tools, with the purpose of depicting and communicating information thinking about and developing previously unknown ideas and advancing understanding (p. 217).

In addition, Presmeg (2006) also defines "visualisation as taken to include the process of construction and transforming both visual and mental imagery and all of the inscription of spatial nature that may be implicated in doing mathematics" (p. 206). Furthermore, Rosen and Rolka (2006) define visualisation as "a powerful tool to explore mathematical problems and to give meaning to mathematical concepts and relationships between them" (p. 458). Moreover, Konyalioglu (2003) defines visualisation as a "bridge built between the world of experiments and the world of thinking and reasoning (p. 614). These definitions resonate well with the use of visual models in teaching fractions since they all focus on the process of constructing pictures, images or mental images to connect what is known to what is being learned and knowledge construction. Thus Zazkis, Dubinsky and Dautermann (1996), indicate that visualisation can be an act in which an individual establishes a strong connection between an internal construct and something to which access is gained through the senses.

In this study the definition of Arcavi (2003) above resonates strongly as it emphasises both the product and the process of visualisation as being important in advancing conceptual understanding of mathematics in general, and in my study fractions in particular, through using visual models.

2.4.2 Visualisation in mathematics teaching

In the teaching of mathematics, visualisation is not an end in itself but a means towards an end, which is to gain understanding (Fotakopoulou & Spiliotopoulou, 2008). Similarly, Konyalioglu (2003) states that by using a visualisation approach many mathematical concepts can become concrete and clear for learners to understand. The meaningful transformations of representations are at the core of understanding human information processing (Mckendree, Monaghan, Conlon, Lee, & Small, 2000).

Visualisation plays an important role as a powerful tool for learning mathematics and can be helpful when solving mathematical problems (Roseken & Rolka, 2006). The use of visualisations can help the learner to clearly distinguish between mathematical concepts. Researchers such as Harvey (2011) report that the use of visuals such as visual models used in the teaching of common fractions enhance comprehension and help learners to come up with possible solution opportunities. The use of visualisation can draw learners' attention by means of drawing and working with concrete models to come up with solutions (Jenks & Pecks, 1972; Konyalioglu, 2003; Arcavi, 2003; Konyalioglu, Asku, & Senel , 2011). Zimmermann and Cunningham (1991) indicate that visualisation supplies depth and meaning to understanding, serving as a reliable guide to problem solving, and inspiring creative discoveries.

Futhermore, Presmeg (1992) suggests that visualisation serves as a support to understanding and it also helps learners to visualise mathematical concepts. For instance, identifying a fraction using models can help a learner to identify the fraction that is presented by such model. Visualisation provides information that other modes of instruction do not, allowing the learner to develop deeper and richer concepts than being taught symbolically (Phillips, Norris & Macnab, 2010). It is also speculated that learning mathematics through visualisations can be a powerful tool to explore mathematical problems and give meaning to mathematical concepts and relationships (Rosken & Rolka, 2006).

Visualisation serves as an aid to finding the solution to problems (Phillips et al., 2010). Mesaroš (2012), agrees with other researchers mentioned above that the main aim of visualisation is to facilitate and support the learner's problem-solving process. Likewise, Piggott and Woodham (2011) indicate that visualisation has three purposes, namely to step into a problem, to model and to plan ahead.

- In the first purpose, visuals are used to help with understanding what a problem is about through creating a space to go into the detail of the situation and make it easy to understand before any generalisation can happen.
- In the second purpose, visualisation helps the teacher to model the processes being investigated and check for understanding.
- The last purpose of visualisation helps the learners to think and plan ahead.

Konyalioglu (2003), on the other hand, observes that some mathematicians and mathematics instructors are in favour of using visualisation in teaching mathematical concepts, while others oppose it. Konyalioglu (2003) further asserts that researchers supporting the visualisation method believe that it will enhance understanding, comprehension, self-confidence and creativity in mathematics education. Some maintain that visual thinking may be a strong alternative resource for learners by bringing new ways of thinking into mathematics and also underlining the importance of visualisation and visual reasoning within mathematics teaching.

Hac|salihog˘ lu (1998) refers to drawings as one of the visualisation forms, as a factor that can help improve learners' thinking ability. Jencks and Peck (1972) report that using visual models in problem solving facilitates learners' comprehension and creates solution-finding opportunities. They assert that visualisation in a form of drawing can also helps learning, particularly in the beginning phases in which the basic concepts are taught to learners. They also maintain that visualisation is crucial in enhancing problem-solving skills by playing an active role in ensuring long-term recall. Jencks and Peck (1972) argue that after working with concrete models/ drawings of fractional models, learners can establish links within the logic of problem solving, and develop distinctive formal rules — a process in which the teacher's function is simply to find a good model for the problem.

Although there is much research demonstrating the importance of visualisation in the teaching-learning process of mathematics, some researchers think that visualisation has never been brought to the forefront in this process. Researchers such as Eisenberg and Dreyfus (1991) suggest that teachers usually prefer the analytical process in teaching (even though they use visualisation in their own work), because they believe that visualisation cannot form a proof and it is not easy to establish and understand visual models. On the same note, Erog˘ lu (1992) adds that although the role of images in implicating and understanding relationships is undeniable, images can never be a part of a proof on their own and can only implicate the accuracy of a judgement. He further states that relying too much on visualisation may prevent mathematical thinking due to the limiting effect of a single-case scenario represented by an image. Similarly, Rival (1987) asserts that some researchers still have an old prejudice against visualisation in mathematics which concerns the reasons behind the preferences: assuming that mathematics should be exact, analytical, symbolic and algorithmic.

2.4.3 Visual models that can be used to teach fractions

This section presents a general discussion of the types of visual models that can be used for teaching fractions. It also presents and discusses the advantages and principles for the effective use of visual models to teach fractions.

In this study, the terms "visual model" represents two terms that are used interchangeably by different researchers. These terms are 'visual representations' and 'visual models'. Watanabe (2002) indicates that the two terms 'model' and 'representation' are not synonymous, and chooses not to use them interchangeably because 'representation' and 'model' have several different meanings but share some common meanings. He describes a model as anything that can be used to represent a mathematical idea such as;

> …a scale model of an object, a series of equations that mathematically model a physical phenomenon, a demonstration, something that illustrates or exemplifies a mathematical concept, concrete materials used in instruction, and so on (p. 457).

Visual representation according to Watanabe (2002) refers both to:

> ...process and to product—in other words, to the act of capturing a mathematical concept or relationship in some form and to the form itself. Moreover, the term applies to processes and products that are observable externally as well as to those that occur "internally," in the minds of people doing mathematics *(ibid.)*.

The two terms refer to the process of using symbolic, concrete and pictorial representations as well as words and relevant situations to explore concepts and communicate understanding.

In the context of this study, the terms 'visual model' or 'visual representation' refer to the teaching materials used to teach fraction concepts. The models that are particularly pertinent in this study are number lines models, area models and set models. Visual models according to Konyalioglu et al. (2012), are tools which are used to represent abstract concepts of mathematics in a constructive way, which leads to the mental interpretation of an image in the minds of learners. The National Council of Teachers of Mathematics (NCTM) (1989) indicates that visual models often make use of physical materials and other representations that help learners develop an understanding of fraction concepts.

A number of researchers have shown that it is important to represent mathematical ideas in multiple ways, including real contexts, physical models (manipulatives), pictures, verbalisations and written symbols (Lesh, Cramer, Doerr, Post, & Zawojewski, 2003). For fractions, this means learners need to explore a range of fraction content using different representations such as drawing shapes, cutting up objects, sharing sets of items, folding paper strips and string, naming fractions and locating fractions on number lines. Manipulating physical representations of fractions can support the development of strategies for processes such as comparing, finding equivalence and performing operations, but such activities also need to be firmly connected to symbolic representations (written fractions) and an understanding of fraction in particular. As indicated by Goldin and Shteingold (2001), Kamii, Kirkland and Lewis (2001), and Lamon (2001), understanding mathematical constructs is dependent upon the ability to connect and transfer meaning between different representations of the same ideas.

There is a range of models commonly used to support fraction instruction, such as sets of discrete objects, number lines (linear models) double number lines, and area models such as circles and rectangles (Harvey, 2011, p. 334). However, three common visual models are used in teaching fractions and they will be the focus of this study. They are:

- Area models (e.g. fraction circles, paper folding, geoboards)
- Number line models (e.g. fraction strips, number lines)
- Set models (e.g. counters and sets).

The use of visual models in mathematics to represent fractions has been recommended by many mathematics organisations, including the Standards for School Mathematics (2000), Texas Higher Education Coordinating Board (2009) and NCTM's Principles and National Mathematics Advisory Panel's Critical Foundation for Algebra (Council of Chief State School Officers, 2010). All these organisations have made calls for schools to train learners to use multiple representations of mathematical ideas fluently and flexibly (Gray, 2014). In addition, Koyalioglu et al. (2012) articulates that visual models help in enhancing problem solving skills by enhancing comprehension and also help learners to come up with a variety of possible solution opportunities. It is said that visual models can also play an important role in long term recall. Furthermore, Harvey (2011) suggests that teachers need to consider the effectiveness of models as a first priority, whenever they are to choose a model for fraction instruction. In the following section I briefly outline the models that are pertinent to my study.

2.4.3.1 Area models

Area models are key and useful to begin fraction explorations because they lend themselves to equal sharing and partitioning. Wong and Evans (2011) suggest that the pictorial (visual) representations of part-whole area models are described as the simplest representation, because the total number of equal parts in the area model matches and is associated with the fraction denominator, while the shaded parts are associated with the numerator, as discussed earlier in the chapter. Figure 2.10 below shows a fraction in the form of an area model. It shows an example of an area model representing the improper fraction $\frac{11}{6}$. To represent the improper fraction visually, the fraction has to be changed to mixed numbers where the full rectangle is a whole which is $\frac{6}{6}$ and the second rectangle represents the fraction $\frac{5}{6}$.

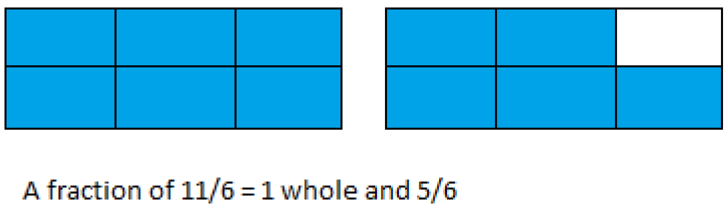


Figure 2. 10: An area model representing the fraction $\frac{11}{6}$.

Van de Walle et al. (2013) state that “a fraction is based on parts of an area” (p. 343). Circular fraction pieces are the most commonly used area models. One advantage of the area model is that it emphasises the part-whole concept of fractions and the meaning of the relative size of a part to the whole (Cramer, Wyberg & Leavitt, 2008). Area models can demonstrate how different shapes can be the whole. Grids or dot paper provide flexibility in selecting the size of the whole and the size of the parts. Many examples of area manipulatives are available, including circular and rectangular pieces, pattern blocks, geoboards, and tangrams. Most of these manipulatives can help learners develop concepts of partitioning.

Van de Walle et al. (2013) further suggests that it is always good and also easy to introduce fraction instruction using the area model, since it is well connected with equal sharing and partitioning. This can be achieved by ensuring that learners practice partitioning with area models. Partitioning is defined as the process of dividing a shape into equal-sized parts (Van de Walle et al., 2013). The same researchers further stipulate that whenever learners partition fractions using the area models, teachers should reinforce that fractional parts must be made of the same size and the number of equal-sized parts within the unit or whole, should determine the fractional amount (*ibid.*). An area model is a good visual for connecting the concept of equivalence to the standard algorithm for finding equivalent fractions (multiply both the top and bottom numbers by the same number to get an equivalent fraction) and further, area models can be used to work out all the operations, unlike some of the other models.

2.4.3.2 Number line models

Harvey (2011) describes the number line model in terms of the measure construct of fractions. He considers it to be very effective for helping learners to "co-ordinate information provided pictorially by the marked line together with the numbers which give information about scale" (p. 335). The number line model is also based on the fraction's distance from zero and allows for the numerical value of the fraction to be located relative to the unit of one. Figure 2.11 below shows the number line model.

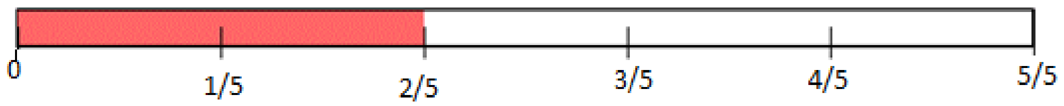


Figure 2. 11: The number line model representing the fraction two fifth.

The Figure 2.11 shows that the distance from zero to one is the unit or the whole in this particular situation. In this example there are five equal parts in the unit from zero to one and only two are coloured. This thus shows that the fraction represented by the number line is $\frac{2}{5}$.

In number line models, lengths or measurements are compared. Unlike in area models, number line models are said to be not widely used in primary and junior schools, yet they are the most challenging and essential tools and should be emphasised more in the teaching of fractions, for developing learners' conceptual understanding (Larson, 1987; van de Walle et al., 2013). Likewise with whole numbers, "the number line is used to compare the relative size of numbers" (Van de Walle et al., 2013, p. 345). Researchers on fractions (Petit, Laird, Marsden, 2010; Siegler et al., 2010) report that the number line helps learners to understand a fraction as a number rather than one number over another number and helps develop other fraction concepts. However, Mitchell and Horne (2011), indicate that the conventions for reading and drawing number lines to represent fractions, as described above by Harvey (2011), are sometimes confusing to the learners because they may use one or two of their part-whole interpretations to solve fraction tasks involving number lines as discussed below.

The first part-whole interpretation of the measure construct is the "part-whole segment of a line", often called the "measure sub-construct part-whole" (Mitchell & Horne, 2011, p. 53), in which a number line is interpreted as a segment of a rectangular object such as a paper strip. The second interpretation is to think of the length of a number line as a simple line that shows the whole and a fraction like $\frac{1}{3}$ would be thought of as a third of the way along a line, where the left-hand edge of the line is assumed to be at point zero (Mitchell & Horne, 2011). Nevertheless, learners need to think of a point on the number line representing $\frac{1}{3}$ as a number located between zero and one, and $\frac{7}{4}$ being located between one and two on the number line as a mixed number. Likewise, Pantziara and Philippou (2012) add that the ability to identify fractions as points on the number line can allow learners to think of fractions as single numbers.

2.4.3.3 Set models

In set models, the whole is understood to be a set of objects, and subsets of the whole make up the fractional parts. For example, three objects are one fourth of a set of twelve objects. The set of twelve in this example represents the unit, the whole or one. Harvey (2011) points out that the set model helps establish important connections with many real-world uses of fractions and with ratio concepts. The idea of referring to a collection of counters as a single entity makes set models difficult for some learners. For example, Figure 2.12 illustrates a set model for fractions explained in detail below.

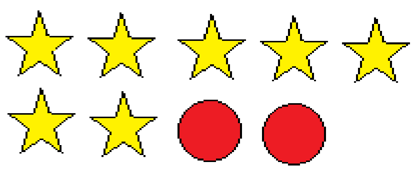


Figure 2. 12: An example of a set model.

The set model in the Figure 2.12 shows that nine is the denominator in the fraction i.e. the total number of shapes in the picture. If the question is asked what fraction of the shapes are stars, the answer would be $\frac{7}{9}$, because 7 of the 9 shapes are stars. However, the set model helps establish important connections with many real-world uses of fractions and with ratio and rate concepts. Counters in two colours are also an effective set manipulative. In another example, shapes can be flipped to change their colour to model various fractional parts of a whole set. Any countable objects (e.g. a box of crayons) can be a set model (with one box being the unit or whole). A common misconception that can be experienced with the use of set models is when learners focus on the size of a subset rather than the number of equal sets in the whole.

Table 2. 1 A summary of visual models used in this study (Source: Harvey, Pantiziara, & Philippou 2012).

Model	What defines the whole	What defines the part	What the fraction means
Area	The area of the defined region	Equal area	The part of the area covered as it relates to the whole unit
Linear or number line	The unit of distance or length	Equal distance/length	The location of a point in relation to zero and other values on the number line
Set	Whatever value is determined as one set	Equal number of objects	The quantity of objects in the subset as it relates to the defined whole

Teaching fractions using models appropriately can help learners clarify ideas that are often confused in a purely symbolic model (Harvey et al., 2012). Sometimes it is useful to do the same activity with two different representations and ask learners to make connections between the two. Different representations offer different opportunities to learn. For example, an area model helps learners visualise parts of the whole. A linear model shows that there is always another fraction to be found between any two numbers— an important concept that is underemphasized in the teaching of fractions symbolically. Some learners are able to make sense of one representation, but not another. Most importantly, learners need to experience fractions in real-world contexts that are meaningful to them (Cramer et al., 2010).

Gray (2014), suggests that the use of fraction models/visual models in the middle grades is considered key to learners' success to master or "conceptually anchor the algorithms used to work with fractions" (p. 7). He further states that fractional models/visual models help learners to make connections among fractional concepts, which eventually deepen their conceptual understanding. Cramer et al. (2008) also states that representations play a dynamic role in learning fractions as they allow students to understand mathematical concepts and relationships as well as to make sound mathematical arguments to convince oneself and others. According to Canterbury (2007), learners use fraction representations (visual models) in four different ways to solve fraction tasks as follows:

- To communicate and organise their mathematical thinking and reasoning;
- To obtain a visual representation of the task;
- To check the accuracy of their work;
- To have a clear picture of their task.

Similarly, research conducted by Fazio and Siegler (2011) revealed that visual models of fractions help develop conceptual understanding of computational procedures. Learners are often taught computational procedures without adequate explanation for why the procedures work. Yet research has shown a positive correlation between learners' conceptual understanding of fractions and their success in using procedures to solve problems. Thus it is advisable for teachers to focus on developing learner's conceptual understanding of a subject using visualisation rather than on algorithms and rote memorisation. The next section analyses a little more closely the notion of conceptual understanding.

The models presented in this section form a basis of the first analytical tool discussed in the data analysis section of the next chapter on page 49. The analytical tool will be used to analyse the data in Chapter four.

2.5 STRANDS OF MATHEMATICS PROFICIENCY

The strands of mathematical proficiency as defined by Kilpatrick, Swafford and Findel (2001) are "a composite, comprehensive view of successful mathematics learning" (p. 5). They indicate that the concept of 'mathematical proficiency' consists of five strands as follow:

- Conceptual understanding,
- Procedural fluency,
- Strategic competence,
- Adaptive reasoning,
- Productive disposition.

Although Kilpatrick et al. (2001) identified five strands for mathematics proficiency, my study will only focus on conceptual understanding, as this is the central element of proficiency that is at the heart of the study. The study specifically looks at how the use of visual models for teaching fractions may enhance Grade 8 learner's conceptual understanding of common fractions.

2.5.1 Conceptual understanding

Kilpatrick et al. (2001) define conceptual understanding as the "comprehension of mathematical concepts, operations and relations" (p. 116). Kilpatrick et al. (2001) further state that conceptual understanding involves the ability to integrate and connect mathematical ideas. These can be ideas about shapes and space, measures, patterns, functions, connections, proofs, fractions and so on. With conceptual understanding "learners gain confidence, which then provides a base from which they can move to another level of understanding" (pp. 118-119). Gabriel, Coche, Szucs, Carette, Rey, and Content (2012) define conceptual knowledge (or conceptual understanding) as "the explicit or implicit understanding of the principles ruling a domain and the interrelations between the different parts of knowledge in a domain" (p. 137). This includes "knowledge of central concepts and principles and their interrelations in a particular domain" (*ibid*.). Wiggins (2014) defines conceptual understanding as "the ability to justify, in a way appropriate to the learner's mathematical maturity, why a particular mathematical statement is true or where a mathematical rule comes from" (p. 1). This supports the notion that learners need to learn mathematical ideas with understanding.

Conceptual understanding in the context of mathematics is defined in different ways. According to Balka, Hull and Harbin (2012), conceptual understanding is a reflection of learners' abilities to reason in settings that demand the careful application of concept definitions, relations, or representations. Balka et al. (2012) further suggest that learners demonstrate conceptual understanding in mathematics in this way:

> They provide evidence that they can recognise, label, and generate examples of concepts; Use and interrelate models, diagrams, manipulatives, and varied representations of concepts; Identify and apply principles; Know and apply facts and definitions; compare, contrast, and integrate related concepts and principles; Recognise, interpret, and apply the signs, symbols, and terms used to represent concepts (p. 2).

Ainsworth (2006) asserts that conceptual understanding can also be developed through the use of multiple representations as a means to link different mathematical domains. This assertion has particular relevance to this study as multiple representations are often linked to visual representations. Kilpatrick et al. (2001) also suggest that learners with conceptual understanding know more than isolated facts and methods. They understand why mathematical ideas are important and the contexts in which they can be used. Learners with conceptual understanding are able to organise their knowledge into a coherent whole, which enables them to learn new ideas by connecting these ideas to what they already know, and they are then able to remember or retain these ideas.

In the teaching of fractions, which is the focus of this study, visual models of fractions as visualisation tools are used with the purpose of developing learners' conceptual understanding of fractions (Watanabe, Murata & Okamoto, 2012). Kilpatrick et al. (2001) also emphasise that one of the significant indicators of conceptual understanding is being able to represent mathematical situations in different ways and knowing how different representations can be useful for different purposes. It is through visualisation and the use of multiple representations that a solid conceptual understanding and the retention of mathematical concepts is developed (Cotrill, 1996; Cramer et al., 2010; Way, 2011). The effectiveness of the consistent use of representations is also supported by the findings of researchers Pirie and Kieran (1994), who found that learners hold on to the representations that they are initially exposed to as the grounding for their conceptual understanding.

2.5.2 Enhancing conceptual understanding of fractions through visualisation

The following are key indicators to teaching for conceptual understanding as adapted from Kilpatrick et al. (2001)

- *Building on prior knowledge*: This indicator emphasises the ability to see connections between the mathematics that the teacher is teaching and what the learners already know. The concept infers that learners need to learn mathematics with understanding and to construct new knowledge from their past experiences. This involves teachers adjusting the acquired knowledge to new situations, and helping learners to apply the acquired knowledge to solve new mathematical problems. For instance, as learners develop their understanding of the relative magnitude of unit fractions (for example, that $\frac{1}{2} > \frac{1}{3}$, given the same whole) and the variety of ways to compose and decompose fractions through these types of activities, they develop fraction number sense. Learners improve their understanding of relative magnitude by considering the impact on the size of the equal-partitioning as the denominator is changed. When learners use visual models, they can develop and test conjectures about these relationships. For example, if a certain area is divided into more regions, the regions become smaller.
- *The use of multiple representations or models*: This indicator refers to teachers' abilities to show different representations of the same mathematical concepts and the ability to provide evidence by clearly explaining and articulating mathematical concepts and ideas. The concept implies that teachers in their teaching should be able to manipulate representations, compare concepts, and apply facts and definitions to justify solutions

to mathematical problems using visual models. This is because the use of representations and different categories of visual models indicated in section 2.4.3 broaden and deepen the learners' understanding of fractions.

- *Emphasis on connections of ideas and concepts in mathematics:* This indicator refers to the teacher's ability to compare similarities and differences of a visual model and show learners how they connect with each other. They are able to integrate related mathematical concepts. The use of visual models can help learners clarify ideas that are often confused in purely symbolic teaching. Different representations offer different opportunities to learn. For example, an area model discussed in section 2.4.3.1 above helps learners visualise parts of the whole. A linear model discussed in section 2.4.3.2 above shows that there is always another fraction to be found between any two numbers (Harvey et al., 2012).
- *Connecting mathematics concepts to real world:* This indicator refers to the teacher's ability to connect and link mathematical knowledge to the outside world, and see the everyday significance of this knowledge. Learners need to experience fractions in real-world contexts that are meaningful to them (Cramer et al., 2010). For example, learners can cut different patterned blocks to help them develop a concept of partitioning and iterating, and to refer whatever they do to the particular whole that they already know.

The conceptual understanding indicators presented in this section form a basis of the second analytical tool discussed in the data analysis section of the next chapter on page 49. The analytical tool will be used to analyse the data in Chapter four.

Conceptual understanding and visualisation go hand in hand, as stated earlier by Balka et al. (2012). Learners demonstrate conceptual understanding by using interrelated models, diagrams, manipulatives, and varied representations of concepts. Through the use of manipulatives/representations which in the case of fractions are visual models, learners are able to clearly identify and represent fractions, unlike when fractions instruction is irrelevant. Moreover, I argue that visualisation and conceptual understanding strongly aligns with the theory of constructivism discussed in the next section. Constructivism *inter alia* places emphasis on the construction of knowledge and building on learner's prior knowledge through the use of manipulatives.

2.6 THEORETICAL FRAMEWORK

2.6.1 Constructivist learning theory

Constructivists assert that "knowledge is constructed" (Noddings, 1990, p.7). Constructivism proposes that "we construct our knowledge of our world from our perceptions and experiences, which are mediated through our previous knowledge" (Simon, 1995, p. 115). Constructivism according to Ndlovu (2013), "focuses on the internal, cognitive or conceptual development of the learner's mind or discipline [mathematics] as a whole" (p. 4). In a constructivist perception of learning, people construct their own understanding and knowledge of the world through experiencing things and reflecting on those experiences (Gusta, 2011). In the same way, Cobb (1988) reiterates that the fundamental goal of mathematics is or should be to help learners build structures that are more complex, powerful, and abstract than those that they possess when instruction commences (p. 89).

A number of authors indicate that a constructivist perspective maintains that individuals' views and understanding of the world around them are based on an ongoing lifelong process of building and constructing knowledge (Saunders, 1992; Kelly, 2000; Slater, Carpenter & Safko, 1996; Watkins et al., 2004). Constructivism also emphasizes the importance of a learner being actively involved in the learning process. In addition, Yackel (2001), indicates that "constructivist learners construct their own meaning from the words or visual images they see or hear" (p.41). Hence in the teaching of fractions learners need to be provided with various visual images, pictures and diagrams in order to construct their own knowledge or understanding of fractions and build on to what they already know.

2.6.2 Constructivists' views of teaching

According to Simon (1995), constructivism does not define a particular way of teaching nor does it determine the appropriateness or inappropriateness of teaching strategies *(ibid.)*. However, in a constructivist mathematics classroom it is acknowledged that "social interaction plays an important role in students' mathematical learning" (Cobb, Yackel & Wood, 1992, p. 5). In addition, Gusta (2011), suggests that constructivist teachers pose questions and problems and guide learners to help them find their own answers. They use many techniques in the teaching process. For example, they:

- Allow multiple interpretations and expressions of learning, such as the use of visual models.
- Prompt learners to formulate their own questions (inquiry).
- Encourage group work and the use of peers as resources (collaborative learning).

A constructivist classroom/teaching is further explained in Table 2.2 below that shows the distinction between the constructive classroom and a non-constructive classroom which Gusta (2011) called a symbolic tradition classroom.

Table 2. 2 Distinction between the traditional symbolic classroom and a constructivist classroom (Gusta, 2011).

Traditional classroom	Constructivism classroom
Curriculum begins with the parts of the whole. Emphasizes basic skills.	Curriculum emphasizes big concepts, beginning with the whole and expanding to include the parts.
Strict adherence to fixed curriculum is highly valued.	Pursuit of learner questions and interests is valued.
Materials are primarily textbooks and workbooks.	Materials include primary sources of material and manipulative materials.
Learning is based on repetition	Learning is interactive, building on what the learner already knows.
Teachers disseminate information to learners; learners are recipients of knowledge	Teachers have a dialogue with learners, helping learners construct their own knowledge.
Teacher's role is directive, rooted in authority.	Teacher's role is interactive, rooted in negotiation.
Assessment is through testing, correct answers	Assessment includes learner works, observations, and points of view, as well as tests. Process is as important as product.
Knowledge is seen as inert.	Knowledge is seen as dynamic, ever changing with our experiences.
Learners work primarily alone.	Learners work primarily in groups.

In a constructivist classroom a teacher encourages learners to use active techniques such as experiments; real-world problem solving to create more knowledge; and then to reflect on and talk about what they are doing and how their understanding is changing (Gusta, 2011).

> The constructivist teacher provides tools such as problem solving and inquiry based learning activities with which learners formulate and test their ideas, draw conclusions and infers, and pool and convey their knowledge in a collaborative learning environment (p. 2).

Gusta (2011) further suggests that in a constructivist classroom knowledge is constituted by the following tenets: construction of knowledge, active learning, reflection on learning, collaboration and inquiry-based learning. Gusta’s tenets are:

- Construction of knowledge. Learners are not blank slates upon which knowledge is etched. They come to learning situations with already formulated knowledge, ideas, and understandings. This previous knowledge is the raw material for the new knowledge they will create

- Active learning. Learners create new understanding for themselves. The teacher coaches, moderates, suggests, but allows the learners room to experiment, ask questions, and try things that don't work. Learning activities require the learners' full participation (like hands-on experiments). An important part of the learning process is that learners reflect on and talk about their activities. Learners also help set their own goals and means of assessment.
- Reflection on learning. Learners control their own learning process, and they lead the way by reflecting on their experiences. This process makes them experts in their own learning. The teacher helps create situations where the learners feel safe questioning and reflecting on their own processes, either privately or in group discussions. The teacher should also create activities that lead the learners to reflect on their prior knowledge and experiences. Talking about what was learned and how it was learned is really important.
- Collaboration. The constructivist classroom relies heavily on collaboration among learners. There are many reasons why collaboration contributes to learning. The main reason it is used so much in constructivism is that learners learn about learning not only from themselves, but also from their peers. When learners review and reflect on their learning processes together, they can pick up strategies and methods from one another.
- Inquiry-based learning. The main activity in a constructivist classroom is solving problems. Learners use inquiry methods to ask questions, investigate a topic, and use a variety of resources to find solutions and answers. As learners explore the topic, they draw conclusions, and, as exploration continues, they revisit those conclusions. Exploration of questions leads to more questions (pp. 3-4).

Hence, Cobb (1988) suggests that teachers should formulate teaching strategies that lead learners to make appropriate and meaningful constructions. These strategies may include visualisation, which is the use of drawings, diagrams and mental images.

Constructivism resonates very strongly with a visualisation and conceptual approach to learning and teaching because both place emphasise on providing representations/ideas based on developing learners' prior knowledge or experiences and enhancing understanding. In addition, conceptual understanding and constructivism suggest that the construction of knowledge is shaped by the social environment and teachers need to structure their teaching on the interactions between learners. Stein et al. (1994) point out, "in classrooms that are underpinned by constructivism, learners are active rather than passive; teachers are facilitators of learning rather than transmitters of knowledge" (p. 26). He further claims that classrooms that support a constructivist view of learning incorporate multiple representations [visual models], consider learners' experiences, focus on knowledge construction and create an environment that allows learners to engage with problems or activities.

Constructivism pays attention on how people learn. Taber (2011) suggests that mathematics knowledge results from people forming models in response to the questions and challenges that come from actively engaging math problems and environments - not from simply taking in information. One of the challenges in teaching is to create experiences that engage the learners' own explanations, evaluation, communication, and application of the mathematical models needed to make sense of these experiences. It is therefore important that teachers, as the facilitators of knowledge in the classroom, should help learners make connections between existing knowledge and learned knowledge. They can do so by guiding learners from what they already know to what they are about to learn. Taber (2011) also adds that from a constructivist position:

> …when teaching abstract concepts that cannot be directly shown or demonstrated to learners, the teacher needs to find ways to help learners make connections with knowledge that could be relevant: using models (visualisation), analogies and metaphors for example (p. 49).

These are the components of teaching that have the potential to enhance learner's conceptual understanding of a subject.

In this study, constructivism and visualisation are linked and interconnect at developing conceptual understanding for learners. Visualisation as indicated earlier in this chapter is the ability to construct images, diagrams, pictures to represent and communicate information to enhance conceptual understanding of a subject. Balka, Hull and Harbin (2012) also refer to conceptual understanding as a reflection of learners' abilities to reason in settings that demand the careful application of concept definitions, relations, or representations. In other words, conceptual understanding is gained by the use of models, diagrams, manipulatives (visualisation) to construct new knowledge and understanding (constructivism). This is well connected to theory of constructivism which stresses that mathematical knowledge results from learners forming models (visualisation) to construct knowledge from what is previously known. As learners construct diagrams, pictures, visual models and shapes, whether on paper or in their minds, conceptual understanding is enhanced. Arcavi (2003) confirms that visualisation is the ability, the process and the product of creation, interpretation, use of and reflection upon pictures, images and diagrams in our minds, on paper or with technological tools — with the purpose of showing and communicating information, thinking about and developing previously unknown ideas and advancing understanding. It is for this reason that this research is conducted: to investigate how the use of visual models as visualisation tools can enhance the understanding of common fractions in Grade 8 learners. The research process will be discussed in detail in the next chapter.

2.7 CONCLUSION

The learning of fractions is very complex. Researchers believe that fractions are difficult to teach and to learn, because fractions have multiple interpretations and representations. This study examines the use of visual models as a way of helping learners to understand fractions better. The use of visual models as visualisation tools are regarded as some of the best practices that can help learners to realise that fractions are indeed numbers.

The purpose of this chapter was to provide a contextual and theoretical background to the study. It provided literature focusing on fractions and visualisation with regard to using visual models (area, number lines, and set models), conceptual understanding and the constructivism theory of learning. The chapter also looked at the Namibian context with reference to the teaching content of common fractions. The chapter also emphasises the relationship between conceptual understanding, visualisation and constructivism and how they inform the teaching of common fractions.

In Chapter three I will present the research design decisions which arose from the theoretical frameworks I have presented in this chapter.

CHAPTER 3

RESEARCH METHODOLOGY

3.1 INTRODUCTION

This chapter presents the research methodology I used to collect data. The goal of this study was to explore how selected teachers used visual models to improve the teaching of common fractions for conceptual understanding to Grade 8 learners as a result of an intervention programme. The following research questions guided this study;

1. What is the nature of the different visual models used by Grade 8 mathematics teachers in the Khomas region of Namibia, in the teaching of common fractions prior to participating in an intervention programme?
2. How does the use of visual models enhance the conceptual understanding of fractions, if at all, in Grade 8 learners?
3. What are the selected Grade 8 mathematics teachers' experiences and perceptions in working with visual models to teach common fractions after participating in an intervention program?

In this chapter, I discuss the research site, sample and sampling methods, research orientation, research methodology, research design, data collection methods and tools, and data analysis. Finally, I discuss validity and reliability, and ethical considerations for this research project.

3.2 RESEARCH ORIENTATION

This study is oriented in an interpretive paradigm. Bertram and Christiansen (2014) describe a research paradigm as the representation of the worldviews that influence the approach the researcher chooses, to conduct his/her research in an acceptable way. Bertram and Christiansen (2014) further indicate that the research paradigm is crucial in every social science research, because it influences the type of research questions the researcher seeks to find answers to; the methods of collecting data; the choice of unit of analysis; and how the researcher analyses data and interprets the findings. This study relied on both qualitative and quantitative data collection methods. A multi-phase mixed method explanatory design as propounded by Creswell and Plano Clark (2007) was used to collect and analyse the data through an interpretive paradigm.

3.2.1 Interpretive paradigm

This study as stated earlier is oriented in the interpretive paradigm. According to Bertram and Christiansen (2014), the interpretive paradigm involves multiple interpretations of the research data. In educational research, the interpretive paradigm is used to understand "the meaning which informs human behaviour" (Bertram & Christiansen, 2014, p. 26). In addition, the interpretive paradigm tends to rely upon the participants' views of the situation being studied (Creswell, 2003).

An interpretive research orientation takes everyday experience and ordinary life as its subject matter, and asks how meaning is constructed and social interaction negotiated in social practices (Scott & Usher, 1999). An interpretive approach focuses on action (Cohen, Manion, & Morrison, 2007) for according to interpretive theorists, human action is inseparable from meaning. Experiences are classified and ordered through interpretive frames (Bassey, 1999; Scott & Usher, 1999). The central drive of research within the interpretive paradigm is to understand the subjective world of human experience (Cohen, et al., 2007).

This research approach therefore provides me with an appropriate platform to explore participating mathematics teachers' perception and experiences because it allows me to make sense of their classrooms and their understanding of visualisation from their own perspective. Furthermore the interpretative paradigm allows me to draw my findings from spoken and written responses of research participants, and are therefore authentic since they are the research participants' reports of their actual experience (Bertram & Christiansen, 2014).

3.2.2 Mixed method research

Since this study employs survey questionnaires, observations and interviews, a multi-phase mixed method approach is used. Mixed method research is a "research design (or methodology) in which the researcher collects, analyses, and mixes (integrates or connects) both qualitative and quantitative data in a single study or multiphase program of inquiry" (Creswell, 2014, p. 2). The mixed method involves collecting and analysing both qualitative and quantitative data.

> Quantitative research is a type of research in which the researcher studies a problem that calls for an explanation about variables; decides what to study; asks specific, narrow questions; collects quantifiable data from participants; analyses these numbers using statistics and graphs; and conducts the inquiry in an unbiased, objective manner.

> Qualitative research is a type of research in which the researcher studies a problem that calls for an exploration of a phenomenon; relies on the views of participants; asks broad, general questions; collects data consisting largely of words (or text) from participants; describes and analyses these words for themes; and conducts the inquiry in a subjective and reflexive manner (Creswell & Plano Clark, 2014, p. 53).

This study falls under an explanatory sequential mixed method approach whereby the quantitative data were collected and analysed first, and then the qualitative data was collected later (Creswell & Clark 2017). In explanatory sequential mixed methods, the quantitative data is analysed using statistical techniques and the qualitative data is analysed using thematic analysis, the two methods being integrated during the analysis. In my case, this was data from the survey that examined the nature of visual models used to teach fractions. The quantitative data was collected through a questionnaire. The qualitative data was collected through interaction with research participants, by observing the participants in their natural environment and interviewing them to find meanings and interpretations of the use of visual models in the teaching of common factions after the intervention (Maree, 2015).

The mixed method approach helped to analyse data from different perspectives. This means data analysed qualitatively, complements the data analysed quantitatively. This adds to the validity and reliability of this study. A quantitative analysis of the survey using descriptive statistics provided a broad overview of the nature of visual models used in teaching common fractions in Grade 8 in the region concerned, and the data from the questionnaires formulated the basis of the intervention programme. Qualitative methods were used to analyse teacher's experiences and perceptions of using the visual models after participating in the intervention programme. Quantitative analysis of the survey was utilised to support and expand qualitative findings from the interviews, thereby deepening the ultimate descriptions and narratives (Mackenzie & Knipe, 2006) in this study.

3.3 RESEARCH METHODOLOGY

This study essentially used two methods. The first consisted of a survey of forty three mathematics teachers in the Khomas region of Namibia, while the second is a case study of a small cohort of teachers consisting of three selected Grade 8 teachers who took part in an intervention programme.

3.3.1 Survey

The first method of his study is a survey. A survey can be defined as a “mean for gathering information about the nature, action, or opinions of a large group of people” (Pionsonnealt & Kraemer, 1993, p. 77). Surveys are inclusive, they require minimal administration and they are very easy for making generalizations (Bells, 1996). In addition, surveys can produce information about attitudes that are difficult to measure using observation techniques (McIntyre, 1999).

My survey questionnaire consisted of two sections: the first section was a general section with only five questions seeking the profile of the teacher. The second section consisted of ten questions which were used to identify the types of models the teachers are using to teach common fractions to the learners. Secondly, the survey was used to inform me (the researcher) about the types of models that need to be included in the intervention programme (see Appendix 1).

3.3.2 Case study

The second method of this study is a case study, which I regarded as appropriate for my research project because I was looking at a specific, real life phenomenon in a specific context, which is the use of visual models to teach common fractions (Rule & John, 2011). The case in this study consisted of three Grade 8 mathematics teachers from three different schools in the Khomas region of Namibia. The unit of analysis was twofold, namely the nature of visual models used by Grade 8 teachers, and the perceptions and experiences of teachers on the use of visual models to enhance the conceptual understanding of fractions in Grade 8 learners.

A case study has multiple definitions. Bertram and Christiansen (2014) define a case study as a “systematic and in-depth study of one particular case in context, where the case may be a person (such as a teacher, a learner, a principal or parent), a group of people (such as a family or a class of learners), a school, a community, or an organization” (p. 42). Bertram and Christiansen (2014) further describe a case study as a “style of research that is often used by researchers in the interpretative paradigm” (*ibid.*) with the aim of describing the nature of a particular situation. Creswell (2013) defines case study research as “a qualitative approach in which the investigator explores a real-life, contemporary bounded system (a case) … through detailed, in-depth data collection involving multiple sources of information (e.g. observations, audio-visual material, and documents and reports), and reports a case description and case themes" (p. 97). Hamilton and Corbett-Whittier (2012) also describe a case study as a "way of framing a particular bounded unit, providing guiding principles for the research design, process, quality and communication" (p. 10). They further explain that a case is a thing, a single entity, a unit which has boundaries. This can be a person, a group, an institution or an organisation. It focuses on collecting rich data by capturing the complexity of the case.

In my study a case study approach was suitable due to a number of reasons. Firstly, this research study was an in-depth study of a single case of three selected Grade 8 teachers participating in an intervention programme. The overall goal of the intervention programme was to explore ways to teach fractions using visual models with the purpose of enhancing learner's conceptual understanding of fractions.

The data was sourced from multiple sources such as surveys, observations and interviews. Creswell (2013) states that the use of multiple sources of data is good to develop an in-depth-understanding of the case. Further, the unit of analysis of this research is the spoken and written responses of the research participants. As stated by Bertram and Christiansen (2014), in a case study the researcher presents "the reality of the participants' lived experiences of, and thoughts about a particular situation" (p. 42).

3.4 RESEARCH DESIGN

Research design is defined as "the plan and procedures for research that span the decisions from broad assumptions to detailed methods of data collection and analysis" (Creswell 2009, p. 3). The research design of this research study involved five phases. The following is a description of the phases of data collection.

3.4.1 Phase 1: Design, piloting and dissemination of the survey questionnaire

The main aim of this phase was to develop the survey questionnaires, pilot and distribute the questionnaires to the teachers.

Pilot Study: During this phase, I carried out a pilot study of the survey questionnaire, with five mathematics teachers in my school. The teachers' responses enabled me to see if the questionnaire was reliable and effective to generate appropriate data. The teachers indicated to me that there were some questions that they could not understand because of some unfamiliar terms such as 'visualisation' and 'visual materials' that were used. The result of the pilot helped me to make necessary adjustments to the questionnaire so that it was suitable for the main study. After piloting, I included the meaning of 'visualisation' since the teachers could not understand what I was referring to when I asked about visualisation and visual materials in questions 3, 4, 5, 7, 8 and 10 (see Appendix 1 section B). The main aim of the survey was to find out what visual models (if any) the Grade 8 mathematics teachers in the Khomas Region used to teach fractions.

The survey: After the piloting, I disseminated the questionnaire myself to twenty secondary schools in the region. I explained the purpose of the questionnaire to the mathematics teachers and asked them to respond to it. I requested the Head of Department for mathematics and science of the schools to collect the completed questionnaires. However most of the teachers did not answer all the questions. Only twenty-five out of fifty questionnaires where completed. Since half of the questionnaires come back unanswered, I decided to ask the twenty-two mathematics teachers who are currently doing a BEd Honours degree concurrently to my studies, to fill in some more questionnaires. Not all of these BEd Honours students teach in the Khomas region, but since the whole country is using the same curriculum it was safe for me to assume that most of them taught common fractions in a similar way to those teachers from the Khomas region. From the BEd Honours students I collected eighteen completed questionnaires that brought the total number of questionnaires to forty-three.

3.4.2 Phase 2: Analysis of survey questionnaires and selection of participants

In this phase I analysed the survey questionnaires and selected the participants for the intervention. I analysed the survey questionnaires using descriptive statistics, which included frequency tables and graphs. The qualitative part of the survey questionnaire was analysed to inform my planning of the fraction models to be discussed in the workshop that was implemented in Phase 3. The analysis of the survey questionnaires gave me an overall picture of the nature of visual models that the teachers used and their views and understanding of the use of visual models in the teaching of common fractions. It also enabled me to select appropriate teachers to participate in the intended intervention program. The data analysed during this phase answered the first research question.

3.4.3 Phase 3: Awareness and planning workshop

In this phase, the selected participants were introduced formally to some of the approaches to teaching common fractions using visualisation (that is, using the area, number line, and set models). I ran two training workshops with the selected teachers that lasted for two hours each. Integral to the training program was to collectively study various models for teaching fractions, and creating an awareness of conceptual understanding in mathematics teaching. I, together with the participants, developed, debated and agreed on how teaching with the use of identified visual models could support the development of conceptual understanding of common fractions. The participating teachers were not mere recipients of my ideas but were active and took part in the development process of the models and their modifications and the debates that led to the finalisation of the models. However, due to time constraints, the participants opted to only teach using two models, namely the area model and number line model. During these workshops we also planned lessons that included problem solving or inquiry-based activities (see Appendix 4). During the workshops we adapted the two models — the area and the number line model to suit the needs of the participating teachers.

3.4.4 Phase 4: Implementation of the teaching program.

In this phase, each of the selected mathematics teachers were observed presenting their lessons on common fractions to their Grade 8 learners using the visual models prepared during the workshops in Phase 3. Each teacher presented four lessons. In order to capture their lessons, I video recorded each participant teaching four lessons. I also used the observation schedule (see Appendix 2) to capture the use of visual models used in each lesson. The data from the lesson observation is transcribed. This process took three months to complete. The analysis of these observations contributed to answering the second research question.

3.4.5 Phase 5: Stimulus Recall Interviews and Data analysis

During this phase, I conducted one-on-one stimulus recall interviews with each teacher, based on the video recording of their taught lessons. The interviews took place after I had completed an initial analysis of the video recording of each participant. The participants, with the aid of their video recorded lessons, reflected and elaborated on how they used the various models to teach common fractions. I also used these interviews to clarify issues and questions that had arisen in the questionnaire that they responded to in Phase 2 (see interview questions in Appendix 3). The data from the interviews were transcribed. The analysis of the Phases 4 and 5 contributed to answering the second and third research questions.

3.5 DATA COLLECTION METHODS

This study used three data gathering techniques, namely: survey questionnaires, observations and stimulus recall interviews. The rationale behind using these different techniques was to enhance validity and for the purpose of triangulation. According to Cohen, Manion and Marrison (2011), triangulation is "the use of two or more methods of data collection in the study of some aspect of human behaviour" (p. 203).

3.5.1 Questionnaire

The first data gathering method was a survey questionnaire. The questionnaire was distributed to fifty Grade 8 mathematics teachers at twenty different schools in the Khomas Region with the assistance of the school principals and heads of departments. I believed a survey questionnaire was an appropriate tool for this study as it enabled me to collect a variety of data from the fifth Grade 8 mathematics teachers in the region, however I only managed to collect forty three questionnaires back. In the questionnaire, I specifically sought information and responses about what visualisation tools (i.e. visual models/ representations) the teachers used to teach fractions (common fractions in particular). In the analysis of the teachers' responses I specifically looked at how these tools could be classified into the visual models as outlined in chapter two. I also sought teachers' general views and understandings of the use of fractional visual models in teaching common fractions (see Appendix 1).

3.5.2 Observation

The second method used for data collection was observation. Observation refers to "the systematic process of recording the behavioural patterns of participants, objects and occurrences without necessarily questioning or communicating with them" (Maree, 2015, pp. 83-84). During observations, I video recorded four lessons per participating teacher. I was specifically looking for data on how visual models were used in the lessons to enhance the learners' conceptual understanding of common fractions. These processes were identified and coded with the use of my analytical framework as discussed below. The observations of the twelve lessons were video recorded and transcribed. To observe the role visual models played, the analysis of the observations primarily focussed on the fractional concepts articulated in the analytical instruments (see Table 3.2 below). To observe the evidence of teaching constructively for conceptual understanding, the analytical template Table 3.3 below was used.

After the observation I initially analysed each video on my own and later collectively with the teachers during our interviews.

3.5.3 Stimulated recall interviews

Stimulated recall can be viewed as a subset of introspective research methods, which accesses participants' reflections on their experiences (Mackey & Gass, 2005), and for this study this is the teaching of fractions using visual models. These interviews were conducted with the individual teachers after I had finished video recording all the lessons. The purpose of the stimulus-recall interviews was to collect data from the participants about their perceptions and experiences from their teaching intervention. I also used this data to interrogate the learners' conceptual understanding of fractions, as well as the possible influence of the intervention.

3.6 RESEARCH SITE AND PARTICIPANTS

A purposive sampling technique was used to select the sample for this study. Purposive sampling is when the researcher "makes specific choices about which people, groups or objects to include in the sample" (Bertram & Christiansen, 2014, p. 60). In addition, Cohen et al. (2011) state that "in purposive sampling, often a feature of qualitative research, researchers hand-pick the cases to be included in the sample on the basis of their judgement of their typicality or possession of the particular characteristics being sought" (p. 156). The reason for using the purposive sampling is that it helped me to select teachers that fulfilled specific and unique criteria. The selection criterion for the participants for the intervention was based on the performance of the teachers' schools. My intention was to work with a range of teachers from schools performing at different levels: below average, at average, and above average in mathematics. The information on the schools' performance were obtained from statistics obtained from the regional office. The reason for using this performance-based selection was to seek insights into how teachers across the performance spectrum used visual models when teaching common fractions. Purposive sampling also gave me an opportunity to look specifically at the perception and experiences of the teachers that manifest from their teaching. Lastly, it allowed me to generate rich data since I was just focussing on a specific issue — that is the use of visual models to teach common fractions.

A sample of three Grade 8 mathematics teachers was selected from the questionnaire sent out to the mathematics teachers in the Khomas region to take part in the intervention program.

3.7 DATA ANALYSIS

The data analysis process collected both quantitative and qualitative data in an explanatory sequential mixed method approach whereby the quantitative data was collected and analysed first and then the qualitative data was collected later. The analysis process consisted of three stages as indicated in the Table 3.1 below.

Table 3. 1 Summary of the analysis process.

Stages	Date source	Analysis
Stage1	Survey questionnaire	• Analysed the questionnaires to find the nature of models used in the region • Analysed teachers view on how to teach fraction visually • Establish emerged themes
Stage 2	Observation	• Transcribed videos taken during teaching. • Analysed the lesson taught as per the observation schedules
Stage 3	Stimulus recall interviews	• Transcribed the audio recording • Analysed the interviews using the thematic analysis. • Established emerging themes.

The first stage analysed quantitative and qualitative data from the survey questionnaires. The quantitative data were analysed using descriptive statistics and presented in graphs. The qualitative data from the questionnaires was analysed according to the themes as they arose from different questions in the questionnaire. I used the following organisational themes shown below to classify the qualitative data drawn from the survey questionnaires.

- Teaching of fractions;
- Types of visual models teachers use;
- Teachers' views of how to teach fractions visually;
- Significance of using visual models when teaching fractions;
- The use of visual models to enhance conceptual understanding of common fractions.

The second stage analysed qualitative data from the classrooms observations. During the qualitative analysis, words, text or images are combined into categories of information that are presented to show the variety of ideas that were gathered during the data collection process

(Cresswell & Clark, 2007). This stage of analysis involved identifying and classifying the teachers' approaches to teaching common fractions using visual models (area model and number line model) according to the fractional teaching categories from the observation schedule, namely unit fractions, representation or identifying fractions, comparing fractions, addition and subtraction of fractions. The analytical templates shown in Table 3.2 and 3.3 below were used to analyse data at this stage. The same templates were used during the lesson observation to capture the use of visual models by teachers. The observation schedule in Appendix 2 Section A facilitated the process on the use of visual models to teach fractions. The analytical instrument in Appendix 2 Section B facilitated the second level of analysis which focuses on how (if at all) the use of visual models enhanced conceptual understanding.

Table 3. 2 Analytical template A.

Fractional teaching categories	Coding
Unit fractions	The teacher introduces fractions through unit fractions The teacher develops unit fractions though counting, composing and decomposing unit fractions using models The teacher shows equal-partition using models
Representation/identifying of fractions	The teacher uses models to introduce proper fractions, improper fractions and mixed numbers simultaneously
Comparing of fractions	The teacher uses models to compare different fractions: converting to a common denominator or using unit fractions.
Addition and subtraction of fractions	The teacher uses models to build on learners' intuitive understanding of the operation. The teacher uses models to solve problems (addition and subtraction)

Table 3. 3 Analytical template B

Conceptual understanding indicators	**Coding:** Approaches to enhance conceptual understandings through teaching. Explanation of each concept in relation to fractional model (area and number line)
Building on prior knowledge	The teacher uses fractional models that are familiar to what learners, to teach factions. The teacher makes use of what the learners already know, and draws from their past experiences.
Emphasis on connections of ideas and concepts in mathematics	The teacher uses fractional models to establish connections between different fraction concepts and establish relationships. The teacher uses fractional models to explore the relationships between concepts and how they are linked. Teacher encourages learners to explore connections between the concepts.
Connecting mathematics to real world	The teacher uses fractional models to connect mathematics to real world examples. The teacher uses everyday shapes and explains them in connection with the used fractional model.
The use of multiple representations or models	The teacher uses fractional models to represent fractions in multiple ways. The teacher uses fractional models to illustrate how to represent fractions, ordering, addition and subtraction of fractions in multiple ways and visually explain the concepts and explore the relationships.

The third stage analysed qualitative data from the stimulus-recall interviews. This stage of analysis involved identifying emerging themes from the interviews using thematic analysis. Braun and Clarke (2006) define thematic analysis as a "method for identifying, analysing, and reporting patterns (themes) within data" (p.6.). Thematic analysis is essentialist method for this study since it reports experiences, meanings and the reality of participants. In addition, thematic analysis examines the ways in which events (lessons), realities, meanings and experiences are the effects of a range of discourses operating within the society (Braun & Clarke, 2006). During this stage, the teachers' responses to the interview questions were transcribed and the themes that emerged from the different questions of the interviews are presented as follows

- Teaching of fractions
- Types of visual models teachers use
- Teachers' views on how to teach fractions visually
- Significance of using visual models when teaching fractions
- The use of visual models to enhance conceptual understanding of common fractions
- Challenges or limitations in the use of visual models

3.8 VALIDITY AND RELIABILITY

Validity is "based on the view that it is essential to demonstrate what a particular instrument measures, what it purports to measure, or that an account accurately represents those features that it is intended to describe, explain or theorise" (Cohen et al., 2011, p. 179). This research study maintained the internal validity of its findings through crystallisation. Crystallisation refers to the "practice of 'validating' results by using multiple methods of data collection and analysis" (Maree, 2015, p. 40). In this research study, three methods of data collection were used. Dempsey (2010) emphasises that the use of multiple data sources is crucial for ensuring the internal validity of the interpretive research. Maree (2015) also stresses that the reliability of the research findings can be enhanced by permitting people who have specific interests in the research to remark on or evaluate the research findings, interpretations and conclusions. This I did by giving back the transcribed lessons and interviews to the participants to check if the content of the transcription reflected what was taught and what we discussed in the interviews.

The research findings of this research are reliable, since I used two of the four triangulation types as defined by Denzil (1978) who asserts that triangulation is one of the key strategies for validating data. The two forms of triangulation that I used were: *Triangulation of data:* A combination of data drawn from different sources and at different times, in different places or from different people. *Methodological triangulation:* Triangulation within methods and between methods, the combination of different methods that link the qualitative and quantitative methods. My data where drawn from three teachers, from different schools. In addition, I also used different types of research instruments to collect my data and different methods as well.

In addition, to ensure validity of the survey questionnaire, observation schedule and the interview schedule. I piloted the questionnaire with five mathematics teachers at my school before sending it out to all the Grade 8 mathematics teachers in the region. I also asked the participants to pilot their lessons in the other class groups that were not observed, to ensure that the standard was consistent and that the models were not ambiguous. In the same was I conducted interviews with three mathematics teachers at my school to test the validity of the interview schedule. Lastly, to enhance credibility during the interviews I made use of an audio-recording device to gather a full record of the interviews. I clearly explained the aims and objectives of this study to all participants to enhance their understanding of the study.

3.9 ETHICAL CONSIDERATIONS

This research study respected the ethics for conducting research in a number of ways. Firstly, I obtained the written consent from the Regional Director of Education before conducting this research. Secondly, I obtained written consent from both the research participants and guardians of the participants, since the participating teachers were teaching learners in their classes and they were video recorded. Thirdly, I also explained the objectives of the study and possible contribution of the research to the research participants, staff and the school headmaster.

The participants were informed that they had a right to withdraw from the research at any time. Pseudonyms were used so that their names would remain anonymous. Teacher 1 is named Mr Mose, teacher 2 is named Ms Nalo and teacher 3 is named Mr Malele. The names of the participants and the school in the questionnaire were erased after the analysis of the questionnaires as they were merely meant to help me with the selection of participants for the intervention programme. Since my research collected some data by means of video recordings, participants were assured of their anonymity in my study. I also ensured that there was mutual respect between me and the participant. All the data collected would remain confidential between me and my supervisors and will be kept by the supervisor for five years. The consent letters to participants explicitly sought their approval to use the same data used for this study for publication in journals and conferences. In this research, lesson observations, workshops and interviews were scheduled in the afternoons to ensure that the research did not disturb the participants' daily activities.

To ensure transparency, the participants were involved in every phase of the project by giving them the opportunity to verify their responses in the stimulated-recall interview. On-going conversations between the participants and me ensured a sense of transparency and honesty to the participants. I was aware that my participants were my fellow teachers, I thus engaged with them with integrity at all times, keeping in mind all ethical standards of Rhodes University.

Integrity and professionalism were maintained at all times in this study. The entire thesis is my own work and I acknowledged and referenced other people's work according to the Rhodes University referencing guide for academic writing. The data collected are presented as they are without any manipulation or fabrication.

3.10 CONCLUSION

This part concludes the methodology chapter. This chapter described the methodology used in this study and a number of key ideas were discussed, namely the research site, sample and sampling method, the research orientation, the research design with five phases, the three methods used for data collection, the techniques and process for data analysis, issues pertaining to validity and reliability, and ethical considerations for this study.

CHAPTER 4

DATA PRESENTATION, ANALYSIS AND DISCUSSION

4.1 INTRODUCTION

This chapter presents, interprets and discusses the findings of the research project. The data presented in this chapter is drawn from three research instruments: survey questionnaires, lesson observations and the recall interviews. The chapter begins with the data presentation on the nature of the visual models that teachers used in their classrooms to teach fractions. These models are presented under the five categories that emerged from the questionnaire. Secondly, the research results from the classroom observations are presented, followed by the presentation of research results from the interviews with the teachers who participated in the intervention program. The summary of findings from the three sources of data is then presented. The chapter concludes with a discussion of each of these emerging themes, namely ***teaching of fractions; teaching fractions visually; the significance of using visual models in the teaching of fractions; and visual models as a tool for enhancing conceptual understanding of common fractions.*** Although these themes are discussed individually, it is acknowledged that they are interconnected.

4.2 PRESENTATION OF RESEARCH RESULTS FROM SURVEY QUESTIONNAIRES

The survey questionnaire was designed to provide the basis for the whole study and specifically to find an answer to the first research question which is "***What are the nature of the different visual models used by Grade 8 mathematics teachers in the Khomas region of Namibia in the teaching of common fractions prior to participating in an intervention programme?"*** It was also to find the teachers views on how they teach fractions in their classrooms. The teacher's responses helped to plan for the intervention that looked at how teachers can use visual models (area, number line and set models) to teach fractions with the purpose of enhancing learner's conceptual understanding of common fractions. The questionnaire was divided into sections A and B. Section A sought general information of the teachers who took part in the survey: information such as the teacher's age, qualifications and years of experience. This information is presented in Figures 4.1– 4.3 below.

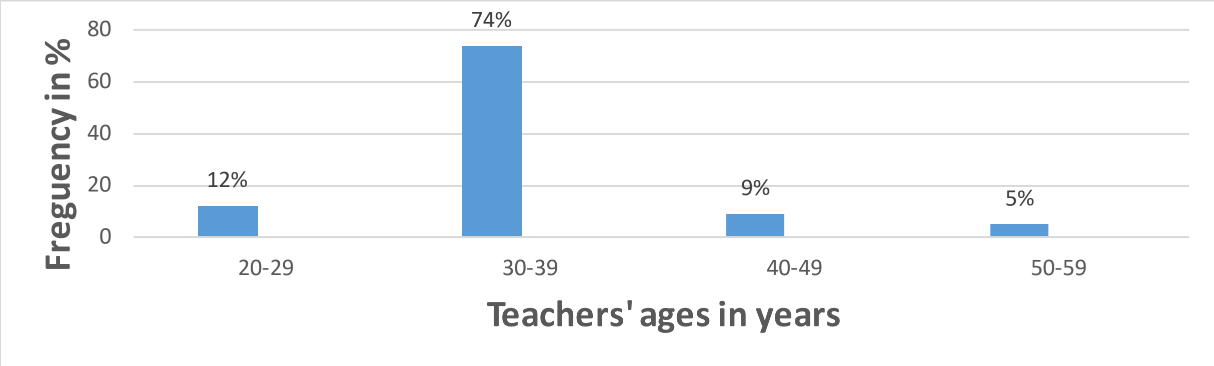


Figure 4. 1: Age distribution of the teachers surveyed

In this study, seventy-four per cent of the teachers who participated in the survey were between the ages of thirty to thirty-nine, the young teachers between the ages of twenty to twenty-nine are just twelve per cent, while the rest are teachers between forty and forty-nine and fifty and fifty-nine. This gave notion that the study involved older and experienced teachers and they would understand better matters related to visualisation in mathematics.

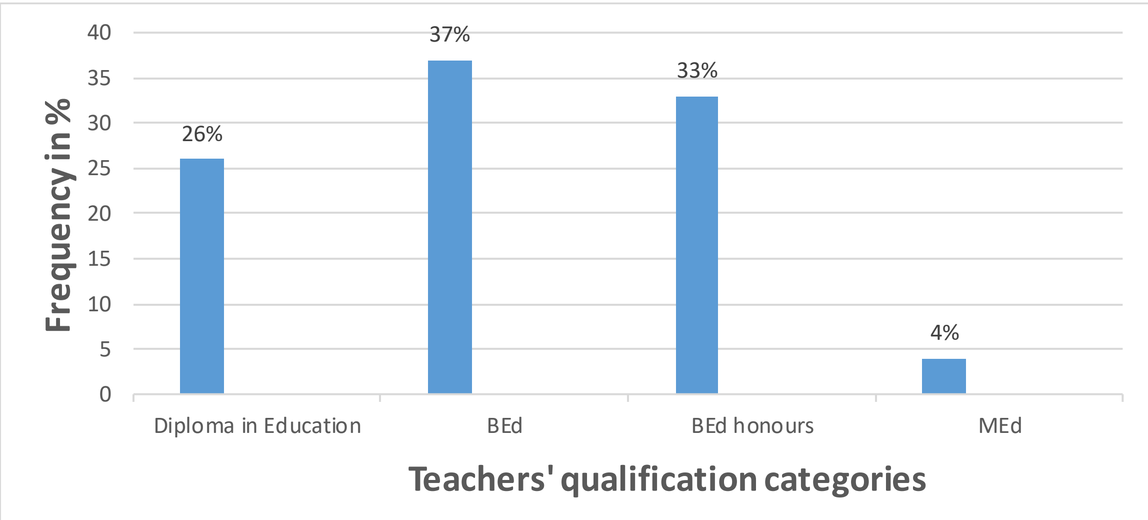


Figure 4. 2: Surveyed teachers' qualifications

The teachers who participated in the survey were qualified secondary school teachers with 37.2% of them having a BEd degree and 32.5% a BEd Honours degree. Thus, this study involved teachers who had undergone training to teach secondary school learners.

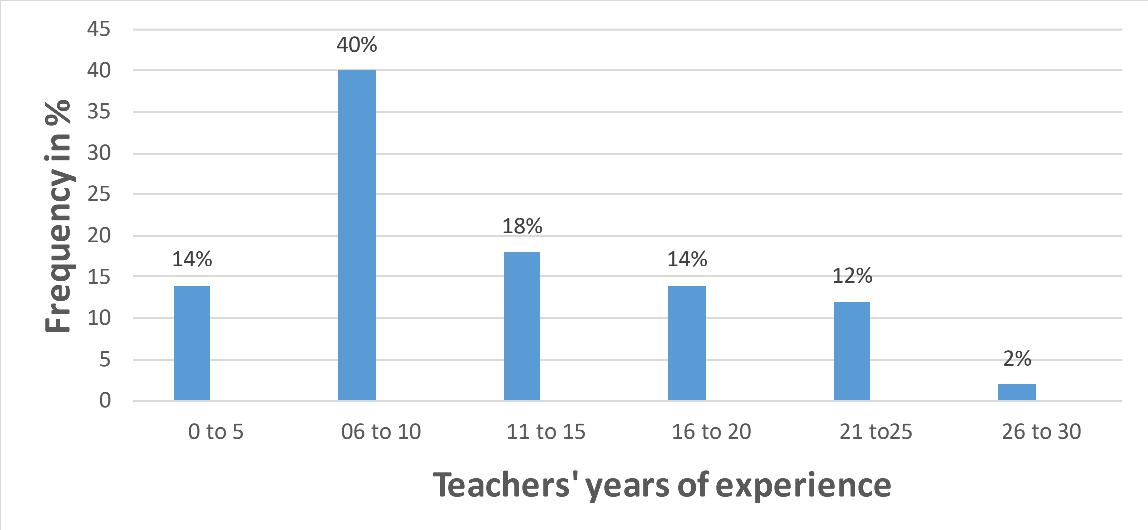


Figure 4. 3: Teachers' years of experience

In this survey 86% of teachers had six or more years of experience in teaching mathematics at secondary school level. In this study I consider such teachers to be very experienced. Thus it was expected that teachers with such a level of experience would be more familiar with the use of visualisation tools when teaching mathematics

Section B focused on teachers' views on how to teach common fractions using visual models. This part presents the views of different teachers to the questions from the questionnaires. This section had ten questions but they are clustered into five categories that are presented in this chapter (see Appendix 5). The teachers' responses are combined and presented in these categories. Individual teacher's responses are indicated with numbers since the questionnaires were numbered from T1–T43. The categories were: ***Teaching of fractions; the types of visual models teachers use; teachers' views of how to teach fractions visually; significance of using visual models when teaching fractions; and the use of visual models as tools for enhancing conceptual understanding of common fractions.***

4.2.1 Teaching of fractions

This category looked at how the teachers taught fractions in their classrooms. Teachers indicated that they use different materials to teach fractions and in different ways to make learners understand the concept of fractions better. The teachers indicated that they either use paper strips and drawings to show the numerator and the denominator, for example T27 indicated that she "*uses practical examples of sharing such as part of an orange, pizza, apple cut in pieces or bread to show unit fraction and relate fraction to a whole since a fraction is part of a whole*." Some indicated that they use manipulatives (*but no specific example of* *manipulative given*) and diagrams to shade out the fractions to simply to represent a fraction. Some teachers, such as T13, indicated that he "*draws circles on a hard paper and cut them in different fractions such as half* $\frac{1}{2}$, *thirds* $\frac{1}{3}$, *quarter* $\frac{1}{4}$ *and or draw diagrams of fractions on the chalk board or on papers prepared beforehand to indicate what a fraction represent."* Most teachers indicated that they only use symbols to teach fractions. Most of the answers given by the teachers are similar, however none of the teachers indicated specifically that they use visual models such as area, number line or set models.

4.2.2 Visual models used by teachers in the Khomas Region

In this survey, 88% of the teachers in the Khomas Region reported that they used some form of visual manipulatives when teaching fractions. Only 26% said they use none, as seen in Figure 4.4 below. In this region, 23% said they use diagrams to teach fractions. The other visual strategies had fewer teachers using them. This was because they said diagrams could be drawn quickly on the chalkboard or white board or on paper, thereby requiring relatively little time to prepare, rather than the other visual models. Information and Communication Technologies (ICT), which is expected to be gaining more use in this modern era, was used only by 2% (see Figure 4.4 below) of the Grade 8 teachers in the Khomas Region.

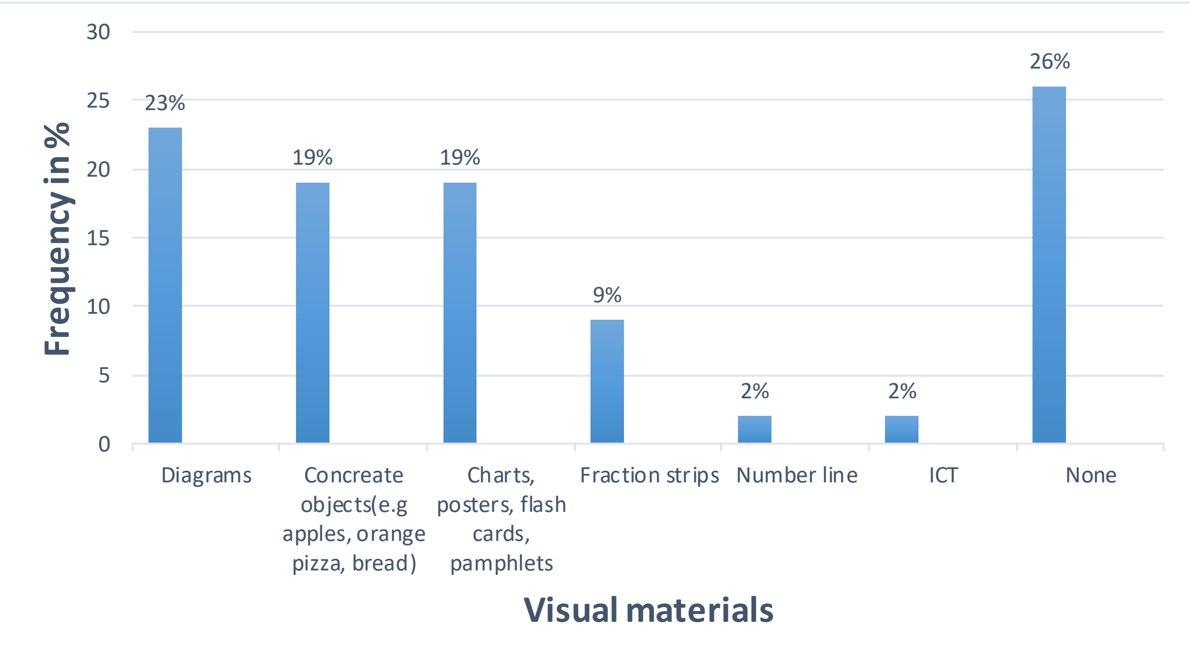


Figure 4. 4: Visual materials used to teach fractions

4.2.3 Teachers' views of how to teach fractions visually

This category sought teacher's views and ideas on how fractions can be taught using visuals. The significance of this section was to identify teachers' views on how they use visuals to teach fractions. The teachers stated that they present fractions to the learners by using visual materials such as number lines, diagrams, paper strips or pizzas cut in slices. Table 4.1 shows their responses.

Table 4. 1 Teachers' views on how to use visuals

Teacher	***Teachers' responses***
T24	Slice a pizza in six parts and if one learner get[s] one slice the remaining will be five-sixths that shows already a fraction and if the other part is put back it will show a whole and in the same time show addition of fraction.
T31	Cut an orange into eight pieces, and demonstrate using the pieces such that one piece of orange is removed there will be seven untouched, so the fraction of the piece that is removed is one-eighth that is one piece out of eight pieces.
T27	Draw columns where different fractions are divided into e.g. half, draw two columns and shade one and for two-thirds draw three columns and shade two.

Teachers gave an indication that fractions can be taught visually and in their responses as seen in Table 4.1 above, gave examples of how to teach or present fractions visually to the learners.

4.2.4 Significance of using visual models when teaching fractions

This section presents findings from teachers on the importance of teaching fractions using visual models. Teachers agreed that teaching using visuals is very important and it is needed. Their responses to the question were categorised into three themes as follows:

Table 4. 2 Significance of using visual models when teaching fractions

(i) To foster understanding

Teacher	***Teachers' responses***
T13	They help learners to recognise fractions and know that fraction is one number comprises of a numerator but not two numbers and a denominator and it help[s] learners to make sense of the fraction not just using symbols.
T33	Perception in children develop broadly when human senses are intensively involved and fraction concepts are a bit abstract because they represent quantities that are not whole. Common fractions are very different from whole numbers and learners tend to mix them, using visual aid will help learners to understand the meaning of fraction better.
T18	Pupils can best understand mathematical concepts when teaching progress from concrete to abstract.
T12	It help[s] learners to understand the concept of fraction much better and it help them to understand the concept of fair sharing.
	It help[s] visual learners and or those who are not good at using symbols to understand better.

(ii) *To aid connections*

Teacher	*Teachers' responses*
T12	They help learners to relate mathematics to every day's life, see realities and make comparisons or links.
T7	They helps learners to relate mathematics to real world situations, and it build[s] learner's knowledge from what is known to what will be learned.
T25	Visualisation is important in the sense that it helps learners to develop a broad concept of common fractions and help learners to relate fraction to other concepts.

(iii) *Knowledge construction*

Teacher	*Teachers' responses*
T35	Constructive instruction in mathematics requires that learners become involves in the learning; telling and showing are not enough we should involve them in their education by doing.
T42	Learners remember better when they see as opposed to when they just hear it. It help learners to grasp the concept easily and they hardly forget it because they will have the figure in mind.

Teachers viewed the use of models as providing access to knowledge, understanding and meaning during the learning process. Teachers also mentioned that visual models are significant for bridging the gap between theory and the concrete of fractions. They also indicated that visuals aid memory and retention of learnt material about fractions. In addition, they pointed out that the use of models encourages knowledge construction, since learners are involved in their own learning.

4.2.5 Visual models as tools for enhancing conceptual understanding of common fractions

The final category centred on teacher's views of how visual models can be used as a tool for enhancing conceptual understanding of fractions in Grade 8. To this question, most teachers agreed that visual models can be a tool for helping enhance the understanding of common fractions. Teachers' response are categorised into two themes as presented in table 4.3 below.

Table 4. 3 Visual models as tools for enhancing conceptual understanding of common fractions.

(i) To foster understanding

Teacher	***Teachers' responses***
T43	The use of visuals involves learners in their own learning that makes the learners understand because using visuals require learners to do the work as well so this will enhance their understanding since they can see, they do and touch. Seeing, touching clear out misconceptions and confusion and it print a vivid figure in the persons mind to enhance a broader and realistic understanding.
T30	Learners tend to contextualise the abstract part of fraction. Visuals help learners to consolidate their understanding of mathematics concepts.
T11	The demonstration with visual models whether on the chalk board, on paper or with concrete materials help learners to understand the concept better.

(i) Knowledge construction

Teacher	***Teachers' responses***
T33	It brings the figure that will permanently stay in learners mind and it will be always, because something that is done or observed take time to leave the memory of learners as it will keep reflecting in their mind.
T32	The practical make[s] it more understandable because it is more efficient than theory and it is related to real life situation. It help[s] learners to work out fractions without any difficulties and it help learners to make connections among concepts and relate them to reality,
T16	It is the best method that attach meaning to fraction, thus teaching without visual aids make the lesson meaningless.

Teachers acknowledge that the use of visual models can be a tool for enhancing understanding of common fractions, as they highlighted that visuals bring meaning to the lesson and they encourage learners to be involved in their learning. On the other hand, the teachers indicated that the use of visual models clear out misconceptions and confusions. Visuals also help to contextualise the abstract part of fractions since with visuals they see and touch whatever they are learning and this prints a permanent picture in their minds.

Finally, the last two questions of the questionnaires looked at how often and when teachers use visual models in their classrooms when they teach fractions. Teachers' responses to these questions were counted and presented in a bar chart below.

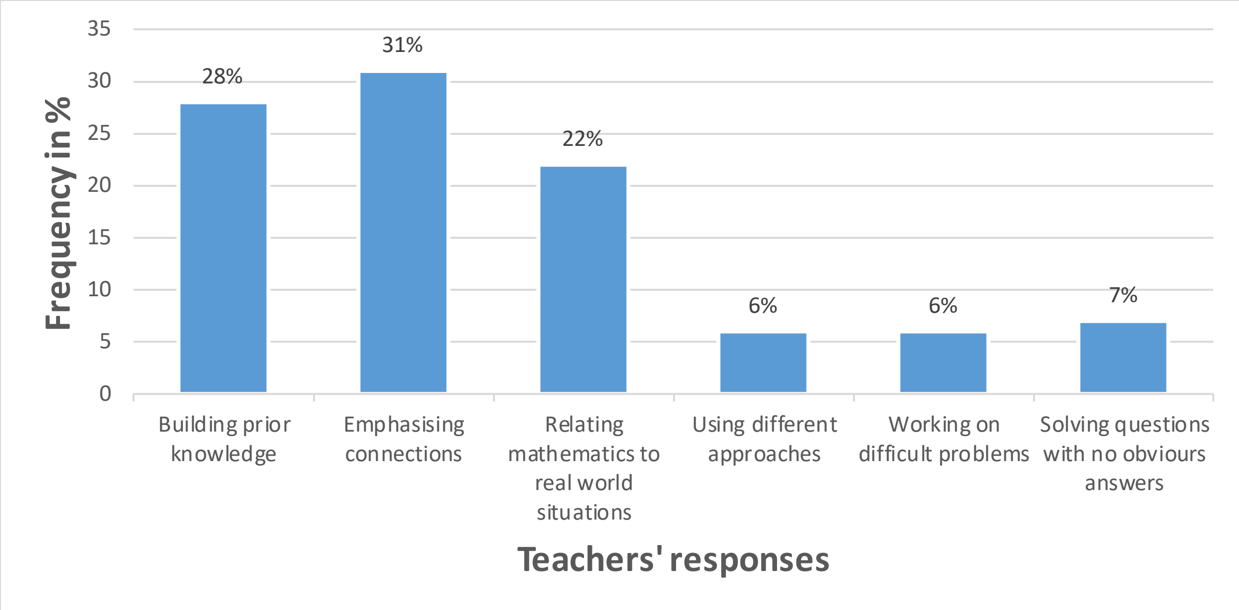


Figure 4. 5: Bar chart illustrating teachers' responses to the survey question

The teacher's responses shows that they predominantly used manipulatives to build learners' prior knowledge, when relating mathematics to real life situations and when they emphasise connections between concepts. They show high percentages in comparison to other approaches shown in the graph. This gives an indication that teachers do not often use visuals in their teaching.

4.2.6 Summary of the survey

It is noted from the teacher's responses that most teachers in this survey rarely used visual models to teach fractions. For those who used some manipulatives/visuals they were not used throughout the lesson. Although teachers did not use visual models such as area, number line or set models to teach fractions, their responses to the questionnaire supported the use of visualisation to teach common fractions. All teachers noted that visual models are central and imperative to the teaching and learning of common fractions. The majority of the teachers' responses stated clearly that visual models make abstract concepts concrete and clarify mathematical ideas whose meanings are difficult to comprehend. In addition, the teachers further asserted that visual models attract learners' attention and stimulate learners' interest in learning mathematics. They further argued that visual models also make mathematics fun and practical. Visual models enhance deep conceptual understanding. They highlighted that visual models help learners to grasp mathematical concepts without difficulty as they learn better by seeing visuals rather than symbols.

Teachers indicated that visual models play a significant role in mathematics as they aid learners to recall the concept discussed during the lesson. They added that the use of visual models involves learners in their own learning. This makes learners understand as using visuals requires learners to personally do the work as well. This, in turn, will enhance their understanding since they can see, do and touch. Seeing and touching clears up misconceptions and confusion, and it prints a vivid figure in the person's mind, to enhance a broader and more realistic understanding. It is noted that visual models also help learners to relate fractions to real life situations, thus enabling learners to link and connect the fractions learnt in class to real life situations. Although a few teachers indicated that they do not use visuals, they do however support the use of visuals as it enhances the conceptual understanding of fractions.

4.3 RESEARCH RESULTS FROM CLASSROOM OBSERVATIONS

This section presents the observations of lessons taught by three mathematics teachers from different schools in the Khomas Region. In their lessons, I focus on visual models that they used during their teaching of fractions to Grade 8 learners. The visual models that will be presented in these lessons are the area models and number line model only, and for all the teachers the models were drawn on the chalk board with no physical manipulatives being used. The set model was not used by any teacher during the observations due to its complexity, especially in the addition and subtraction of fractions. The teachers felt that the set model is too complicated and that it was going to confuse their learners. Therefore, the analysis of the lessons only looked at the two models: the area model and the number line model respectively. In addition the analysis of the lessons will mostly focus on the teachers since the aim of the study is to see how the teacher use that visual model to enhance learners understanding fractions.

The lesson observation answered the second research question:

How does the use of visual models enhance the conceptual understanding of fractions, if at all, in Grade 8 learners?

The lessons observed were analyzed according to the three teachers who participated in the intervention program. Teachers were observed four times as shown in the Table 4.4 below. Each lesson is presented separately by first giving the overview of the lesson then a discussion of each lesson according to the two observation schedules templates in Appendix 2. Table 4.4 shows the observed teachers' profiles.

Table 4. 4 Teachers' profiles

Teachers' names	Qualifications	Area of specialization	Years of experience	Number of lessons taught	Gender	Classroom observation coding	Interview coding
Teacher 1 Mr Mose	Bachelor of Education with Honours	Mathematics and Physical Science	5 years	4 lessons	Male	T_1V	T_1I
Teacher 2 Ms Nalo	Bachelor of Education degree	Mathematics and Physical Science	17 years	4 lessons	Female	T_2V	T_2I
Teacher 3 Mr Malele	Diploma in Education	Mathematics and Physical Science	10 years	4 lessons	Male	T_3V	T_3I

Table 4.4 above summarizes the profiles of teachers who participated in the intervention programs. It highlights their qualifications, the subjects they were teaching at the time of the study, their years of experience at the time of the study and the number of lessons that they taught/observed as part of this study. As can be seen from Table 4.4, all the three teachers who participated in the intervention were qualified to teach at junior secondary school level. They all had adequate teaching experience to better understand the pedagogical demands of the Namibian curriculum. It also presents the number of lessons that will be discussed in the next section. The last two columns in Table 4.4 show the coding that I used to conveniently identify the teachers during the classroom observations and recall interviews.

The analysis of the lessons discussed in this section looked at two components. The first component was the teaching of fractions using visual models, namely area and number line models. This component focuses specifically on how the teachers used the visual models as visualisation tool to teach: unit fractions; to represent or identify fractions; to compare fractions; to add fractions together and to subtract fractions; as per the analytical frame work discussed in Table 3.2 in Chapter 3.

The second component looked at how teaching using visual models enhance learner's conceptual understanding of fractions. This component specifically looked at how the teacher builds learners' prior knowledge, connecting fractions to other mathematical concepts and connecting fractions to real world situations, and how the teacher uses the different manipulatives or visual models to help learners understand the fractional concepts as per the analytical frame work discussed in Table 3.3 in Chapter 3.

4.3.1 TEACHER 1: MR MOSE'S LESSONS (T_1V)

4.3.1.1 Lesson 1

Lesson overview

Mr Mose taught the concept of common fractions using the area model. The models used were pre-drawn on the chalkboard before the lesson started. The focus of Lesson one was to use visual models to identify a fraction, compare fractions and to add fractions together.

Teaching using visual models as a visualisation tool

In this lesson, Mr Mose exclusively used the area model. He introduced the lesson by asking learners what a fraction represented, referring them to what they had learned in the previous grade. He asked them what the denominator, and the numerator is, and what they represent. He then explained the meaning of a whole and how the denominators and the numerators are related to the whole. He did this by showing them how to present a unit fraction using an area model as shown in Figure 4.6 below.

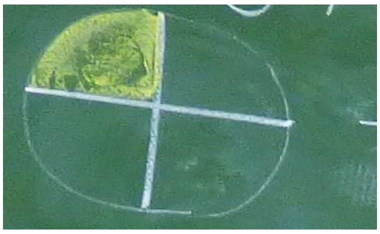


Figure 4. 6: Representation of a fraction (a quarter) on a model

Mr Mose then gave them other examples on how to present non-unit fractions that included proper fractions and mixed numbers as well. Mr Mose engaged learners in his lesson by asking them questions and by checking their prior knowledge. He engaged them in counting the fraction parts on a model.

The other topic covered in this lesson was how to compare fractions. He first articulated that in comparison they would look at which fraction was bigger and which one was smaller. He then checked learner's prior knowledge by asking them how one can know which fraction is bigger and which one is smaller. He then presented to them two fractions on an area model showing them how fractions can be compared as shown in Figure 4.7 below.

Figure 4. 7: Comparison of unit fractions

The last part of the lesson covered the addition of fractions. Mr Mose continued to refer learners to what they learned in the previous grade, by asking them what they needed to know before adding together fractions. He then started building their knowledge by adding fractions with the same denominators as in Figure 4.8 below.

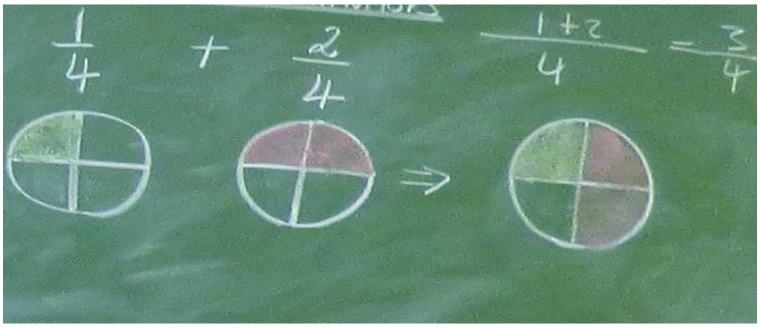


Figure 4. 8: Addition of fractions with the same denominators

Mr Mose then further gave more examples on adding fractions together as shown in the Figures 4.9 below. During his teaching, he kept on engaging learners in discussion and in finding the answers, as a way of building their knowledge of fractions.

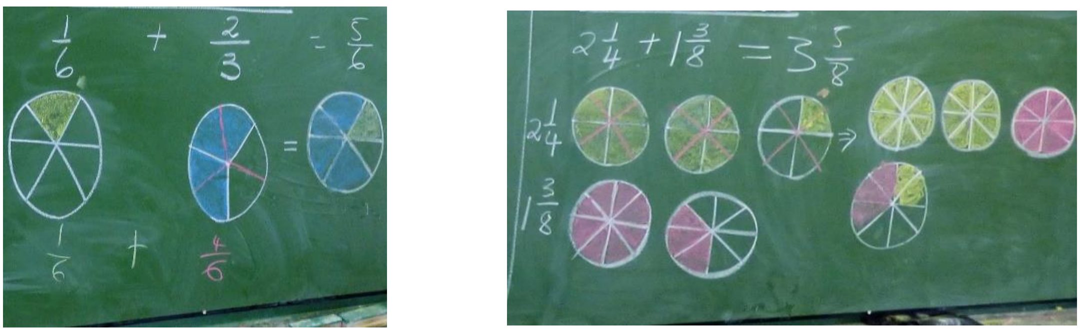


Figure 4. 9: Addition of fractions with different denominators

Teaching that enhances conceptual understanding of fractions

In this lesson, I noted three indicators of teaching for conceptual understanding.

a. Prior knowledge

The extract from the lesson below shows how the teacher referred to learners' prior knowledge by engaging them in discussions:

During the lesson introduction:

Mr Mose: A fractions is something like what? A number refer to what?
Ls: A numerator and a denominator
Mr Mose: A number that is up and a number that is down is that what you are telling me?
Ls: Yes Sir" [T_1V].

Comparison of fractions

Mr Mose: Ok good! Let's look at it in this way. Let's say this are pieces of pizza, now in one over two, you are two sharing the pieces and on the other fraction you are four sharing the pieces, in which group will get a big piece?
Ls: It's when you are two Sir
Mr Mose: Good yes you can see by using the drawing that the part that is divided by two is bigger than the one that is divided in four. So from the drawing we can tell which one is bigger and which one is smaller without even using a calculator or without any calculations. So the drawing will help you to see which one is big and which one is small [T_1V].

Addition of fractions

Mr Mose: what do we need to know when adding fractions?
L: We need to have same denominators.
Mr Mose: Yes we can only add together fractions if the denominators are the same. If the denominators are the same, we just add together the numerators. What can we do if the denominators are not the same?
L: We look for the lowest common denominator [T_1V].

The extracts above shows that Mr Mose asked questions which required learners to demonstrate their prior knowledge of fractions. Mr Mose's questions were aimed at reminding learners of what they already knew about fractions and to encourage them to use that knowledge as a foundation for what he was teaching. Mr Mose's questioning probed learners to provide evidence of what they had learned in relation to the visuals provided to them. In agreement, Balka et al. (2012) alluded to the fact that learners demonstrate conceptual understanding in mathematics by:

> Providing evidence that they can recognise, label, and generate examples of concepts; Use and interrelate models, diagrams, manipulatives, and varied representations of concepts; Identify and apply principles; Know and apply facts and definitions; compare, contrast, and integrate related concepts and principles; Recognise, interpret, and apply the signs, symbols, and terms used to represent concepts (p. 2).

b. Connecting mathematics to the real world

Mr Mose also attempted to connect fractions to real world situations when he was emphasizing their understanding of comparing fractions, by referring to fair sharing as seen in the extract and Figure 4.10 below.

> *Mr Mose: Very good! We clearly see by using drawings which fraction is bigger. So if we can put it in, practical example, if you are sharing the more you are smaller piece you get, the fewer you are the bigger you get. You do not need to subtract or do any sort of calculation here, we only look at the drawings and we can see which fraction is bigger and which one is smaller [T_1V].*

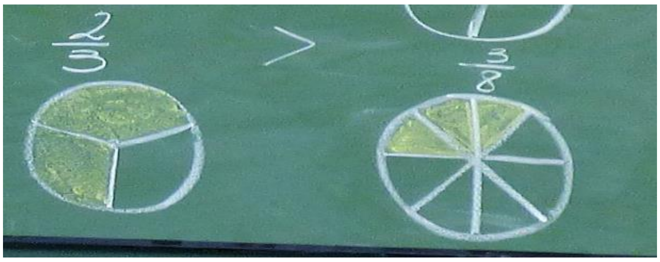


Figure 4. 10: Comparison of fractions using an area model

c. The use of multiple representations/models

Mr Mose assured learners that by visualising fractions, conceptual understanding will develop. Way (2011) indicates that learners need to understand the relationship between the number of parts and the relative size of the parts through visualising the parts. This will help the learner to realise that the more parts a quantity is divided into, the smaller the pieces become.

> *"Mr Mose: so with the drawings we can clearly see how the fractions Are represented, compared and added together without any calculation or the use of a calculator" [T_1V].*

See Figure 4.11 below that shows different representations (drawings) or visuals that were used in the lesson

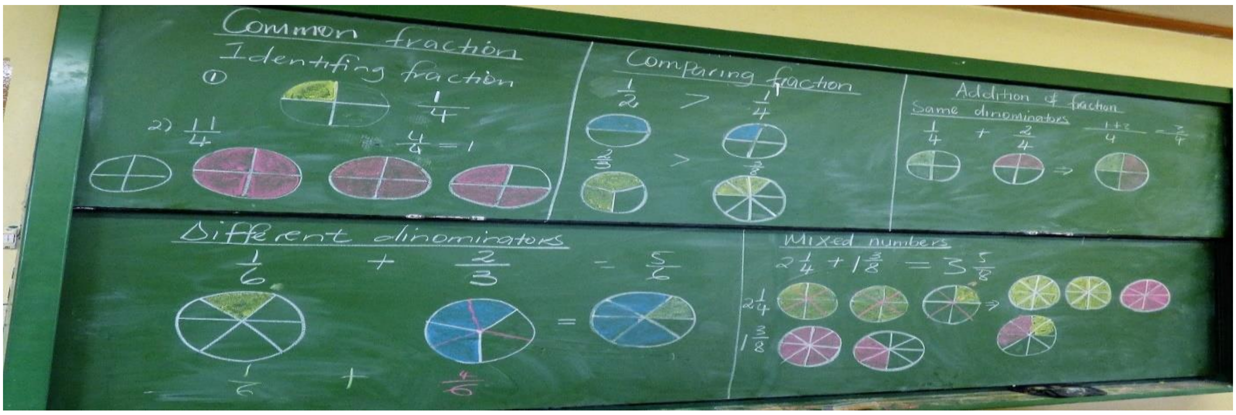


Figure 4. 11: Mr Mose's Representations

Mr Mose emphasised that the use of models is easy since learners can clearly see what the models represent and they can also easily compare fractions and add them together without using symbols, as seen in the picture of the whole lesson above.

4.3.1.2 Lesson 2

Lesson overview

In the second lesson, Mr Mose presented how to subtract fractions using the area model. Similarly, as with the first lesson, Mr Mose had pre-drawn models on the chalkboard, showing how to subtract different fractions using an area model.

Teaching using visual models as a visualisation tool

In this lesson, Mr Mose exclusively used the area model. In his introduction, Mr Mose referred learners to what they already learned in the first lesson. He also asked the learners what they knew about subtraction of fractions from the previous grade. He then gave them an example on how to subtract fractions with the same denominators, using models as shown in Figure 4.12 below.

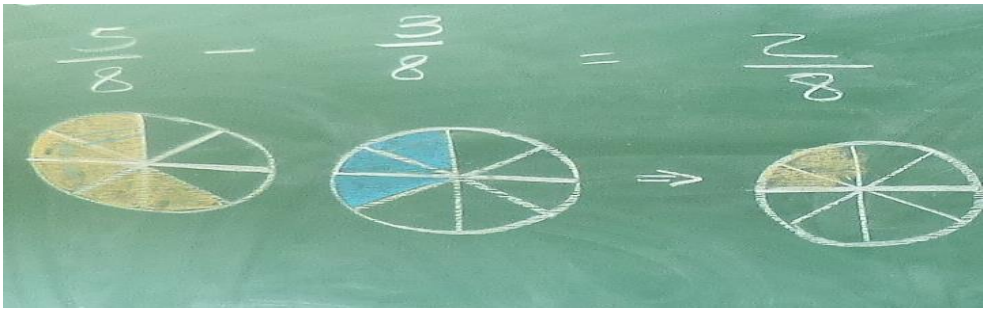


Figure 4. 12: Subtraction of fractions with the same denominators

Mr Mose further gave examples on how fractions with different denominators and mixed numbers can be subtracted. In the same example, he showed learners how to find the lowest common multiple of the denominators (LCM) and how to present the LCM on the model as seen in Figure 4.13 below.

Figure 4. 13: Subtraction of mixed numbers and finding the LCM

Teaching that enhances conceptual understanding of fractions

In this lesson, I noted two indicators of teaching for conceptual understanding.

a. Prior knowledge

Mr Mose referred learners to what they already knew by asking them questions related to the first lesson. The question he asked was:

> *If the denominators are not the same, we can just go ahead with subtraction?*

As shown in extract below:

> *Mr Mose: Yes we will look at how to subtract fractions using visuals. We do not need to use calculators. We will just use the drawings. First we will look at on how to subtract fractions when the denominators are the same. Remember the other day we said we can only add when the denominators are the same. Same apply to subtraction.* ***If the denominators are already the same, we can just go ahead with subtraction.*** *If they are not the same, we make them the same by…………..?*
> *Ls: (few learners answered) we have to find the lowest common multiple [T_1V].*

Prior knowledge was built into this lesson as the teacher kept on referring learners to what they had learned in the previous lesson. He reminded them of how to add fractions together, then helped them to relate addition to the subtraction that they were dealing with that day. This concurs with Gusta (2011), who articulates that learners are not blank slates upon which knowledge is etched. They come to learning situations with already formulated knowledge, ideas, and understandings. This previous knowledge is the raw material for the new knowledge they will create.

b. The use of multiple representations or models

Teaching for conceptual understanding also manifests itself in the use of multiple representations or visuals to show how to subtract fractions using models as shown in Figures 4.13 above and 4.14 below.

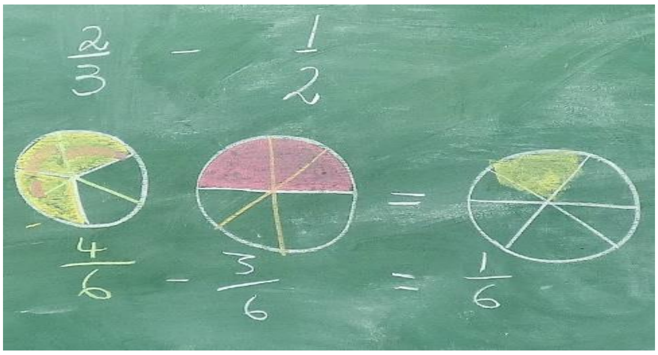


Figure 4. 14: Showing the subtraction of fractions with different denominators

The following explanation from the lesson, and as seen in Figure 4.13, shows that there is evidence of representations being used to show how fractions are subtracted using different manipulations that include finding/presenting the LCM on a model. Figure 4.14 further emphasises the presentation of the LCM on a model, as it shows how the two fractions with different denominators are presented on a model. On the other hand, his explanation on how to subtract fractions as shown below, shows teaching for understanding since it is based on the model used in class:

> *Mr Mose: Okay, that means I need to cut each of this fraction into six parts using a different colour by adding lines like how we were doing it with addition. Now the first fraction becomes four over six and the second one becomes three over six as well. They are on same denominator so we can subtract. Now we take out three parts from four parts by cancelling each parts and what remains on first fraction is our answer.*

Apart from Figure 4.14 above, Mr Mose also used other drawings in the lesson to illustrate subtraction of fractions using the area model.

4.3.1.3 Lesson 3

Lesson overview

In Lesson 3, Mr Mose taught fractions using a number line model. His lesson was presented on PowerPoint. The focus of Lesson 3 was on how to present a fraction on a number line, how to compare fractions using number lines, and how to add fractions using a number line. Similar to the other two lessons that Mr Mose taught before, this lesson had pre-prepared models already on the computer.

Teaching using visual models as a visualisation tool

In this lesson, Mr Mose solely used a number line model. Mr Mose introduced his lesson by referring learners to what they already learned in the previous lessons. He further related his lesson to the number line that learners learned in their previous grade, and to the area model that he had presented earlier. Then he showed how to identify a fraction on a number line, both proper and mixed numbers. He showed them as in Figure 4.15 below

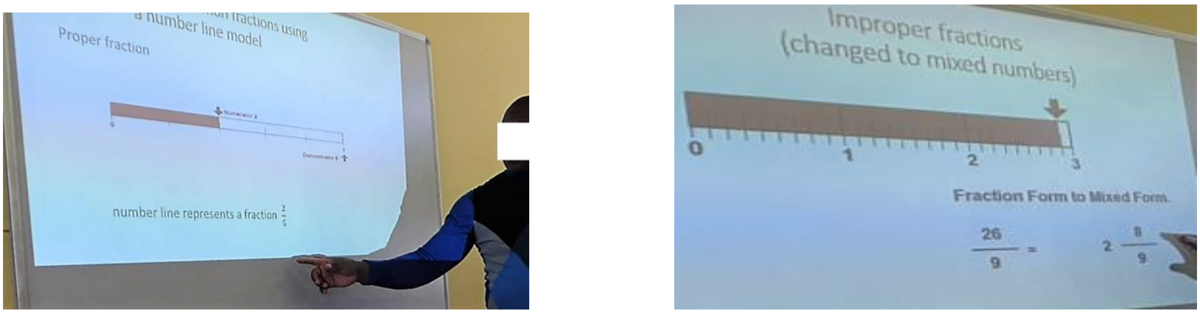


Figure 4. 15: Proper and improper fractions on a number line

He further explained how fractions could be compared on a number line, still referring to what learners had done on the area model. Lastly, he showed how to add fractions together using a number line. In his presentation of addition he also included how to find and present LCM on a number line (see Figure 4.16 below)

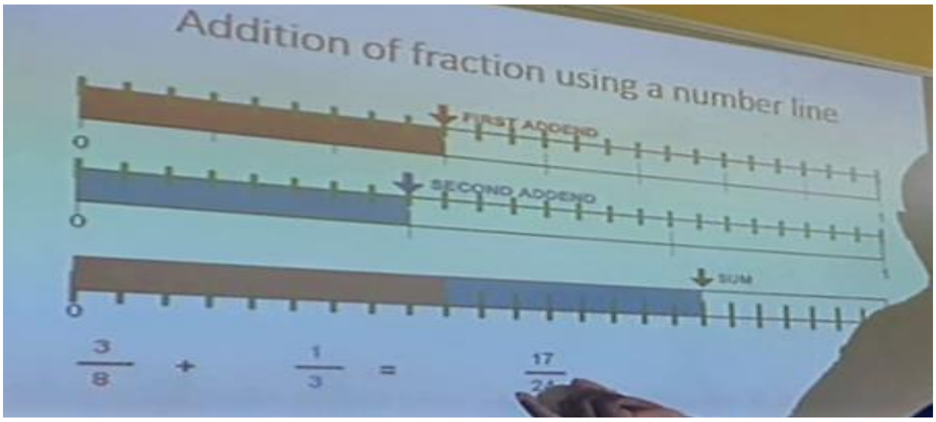


Figure 4. 16: Addition of $\frac{3}{8}$ and $\frac{1}{3}$ on a number line

Teaching that enhances conceptual understanding of fractions

In this lesson, I noted one indicator of teaching for conceptual understanding.

a. The use of multiple representations or models

Mr Mose used the models to make learners see and understand the notion of common fractions on a number line. His explanations were meaningful and involved learners as well. The example in Figure 4.17 below on comparison of fractions shows some indication that he taught for understanding.

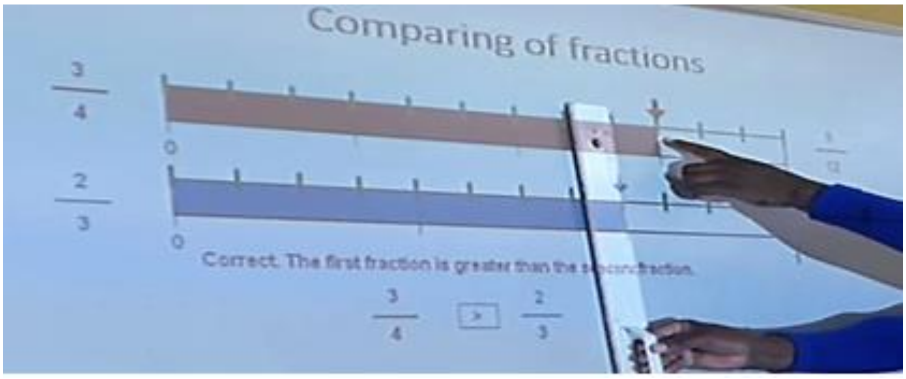


Figure 4. 17: Comparison of fractions on a number line.

In this lesson, different representations were used to show how fractions can be presented and compared on a number line. In Figure 4.17 above one can see how Mr Mose used the number line to show which fraction is smaller and which one is bigger visually and symbolically. In addition to this, he changed the fractions to the same denominator just to show the magnitude of the fractions he was presenting. This gives an impression of a teacher using different representations in one lesson to develop learners' understanding of what he is teaching.

4.3.1.4 Lesson 4

Lesson overview

In Lesson 4 Mr Mose taught how to subtract fractions using the number line model on a PowerPoint presentation, as in Lesson 3.

Teaching using visual models as visualisation tools

In this lesson, Mr Mose solely used a number line model. The lesson looked at how to use a number line model to subtract fractions. In his introduction, Mr Mose referred learners to what they had already learned in the previous lesson using a number line model. He then gave the learners the example of how to subtract mixed numbers. He first explained how to present the fractions on a number line relating to what they had already done in the previous lesson. He then explained the concept of subtractions and how to subtract the fractions on a number line. The first fraction was a fraction with same denominators. The second example he used were fractions with different denominators, and at this point he again showed learners how to present a LCM on the same number line. The example in Figure 4.18 below shows how to subtract fractions with different denominators and how to find the LCM.

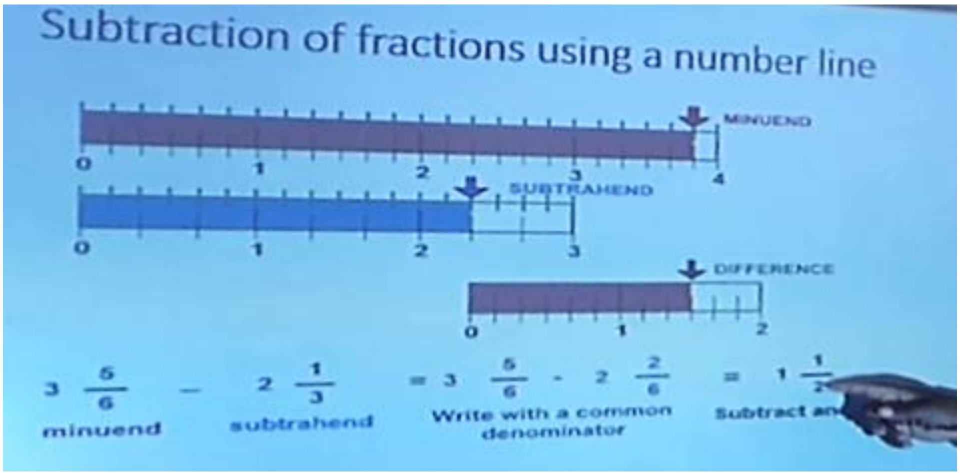


Figure 4. 18: Subtraction of fractions on a number line.

Teaching that enhances conceptual understanding of fractions

In this lesson, I noted one indicator of teaching for conceptual understanding.

a. The use of multiple representations or models

The extract below, Figure 4.18 above and 4.19 below from the lesson shows Mr Mose's teaching to enhance the learner's conceptual understanding of fractions. In this extract Mr Mose explains what happens when the two fractions are subtracted. The visuals that he used were clear and learners could relate to what he was explaining.

Mr Mose: Today we will do subtractions just like when we used area model but here we will use a number line. So we are going to look at how to subtract fractions using a number line. We have mixed number $4\frac{1}{8}$ minus $3\frac{7}{8}$, so we present our fraction in a number line, look at the first fraction presented and count for me the number of blocks between zero and one. How many are they?
Ls: 7, 9 and 4
Mr Mose: How many are they 4, I said count between zero and one. How many parts are there?
Ls: 8 Sir
Mr Mose: Yes they are eight because they represent the denominators. Now count how many whole are there?
Ls: 4 whole
Mr Mose: Yes you counts the full whole we have four whole and on the other intervals we only have one part coloured, meaning we have one over eight. On the other fraction we do the same. We have three whole and what?
Ls: Seven over eight
Mr Mose: We have three whole and seven over eight and that is what we are going to subtract?
Ls: Yes Sir
Mr Mose: So ***when we are subtract fractions such as $4\frac{1}{8}$ minus $3\frac{7}{8}$ using models we can cancel out the whole, so if we cancel out the three whole with the other three whole, and whatever we are left with is our answer****. What are we left with?*

Ls: One whole and one over eight
Mr Mose: yes we are left with a full whole and one over eight. We have to continue to subtract. What can we do?
L: sir I think we have to cancel out the small blocks and what remains is our answer [T_1V].

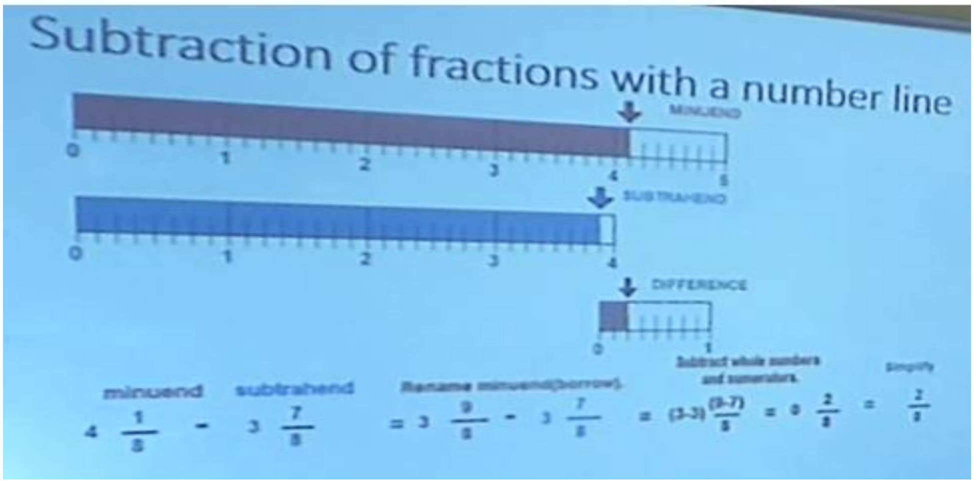


Figure 4. 19: Subtraction of mixed numbers on a number line.

In his teaching, Mr Mose kept on engaging learners in discussion, asking them questions that enhanced their understanding of fractions, and referred them to what they already knew. His questioning demonstrated that he used the learners' responses to ask more questions and to promote elaboration, positive thinking and knowledge construction (see the extract above). The way the models are used in this lesson can contribute to the enhancement of conceptual understanding of fractions in learners because they are clear, accurate and easy to read. Mr Mose's explanation of how fractions can be subtracted, helped learners to understand as well. In the extract he said

> *...when we are subtract fractions such as $4\frac{1}{8}$ minus $3\frac{7}{8}$ using models we can cancel out the whole, so if we cancel out the three whole with the other three whole, and whatever we are left with is our answer.*

In addition to the visual, the teacher presented the symbolical presentation of what the number line presents as seen in Figure 4.19. Cramer and Whitney (2010) and Way (2011) indicate that it is through visualisation and the use of multiple representations that a solid conceptual understanding and the retention of mathematical concepts is developed. The effectiveness of the consistent use of representations is also supported by the findings of researchers Pirie and Kieran (1994), who found that learners hold on to the representations that they are initially exposed to as the grounding for their conceptual understanding. In my view, Mr Mose used different representations, since he used models, explanations and symbols to illustrate how to subtract fractions using a number line model.

4.3.1.5 Consolidation of Mr Mose's Lessons

It was observed in Mr Mose's lessons that his teaching used the area and number line models. In his teaching, he kept on engaging through discussion and at the same time he kept on referring the learners to their prior knowledge. Mr Mose used area models drawn in the form of circles, which he had drawn prior to the lesson, not during the lesson. His explanations were well connected to the models he was using, however the learners kept on looking at the other models on the chalkboard as they were already there. For the number line model Mr Mose used a PowerPoint, and it was well presented because the drawings were accurate and clear for the learners to see. His explanation on the number line was constructive and clear and well connected to the model he was presenting. Mr Mose's teaching to enhance conceptual understanding was dominated by the use of multiple representations or models, as compared to how he built the learners' existing knowledge. The other two indicators, connecting mathematics to the real world and connecting fractions to other concepts in mathematics were not very visible in his lessons.

4.3.2 TEACHER 2: MS NALO'S LESSONS (T_2V)

4.3.2.1 Lesson 1

Lesson overview

In this lesson Ms Nalo examined how to teach fractions using the area model. She specifically looked at how to present a fraction visually on an area model and how to compare fractions using the area model. The models that Ms Nalo used, were drawn during the lesson.

Teaching using visual models as a visualisation tool

In this lesson Ms Nalo exclusively used the area model. Ms Nalo introduced her lesson by defining a fraction as part of a whole. She explained the concept of 'fraction' by giving a real life application of a pizza which is in a form of a whole. She then introduced the area model that it was in a form of a pizza. She explained how to present a fraction using an area model, still relating it to a pizza as she said:

> *If you have a pizza and it is divided in four parts and then one part is removed, what fractional part is removed?*

She further asked learners what types of fractions they had learned in Grade 7, and then she explained the types of fractions and gave examples of how each of them can be presented on a model as shown on Figure 4.20 below.

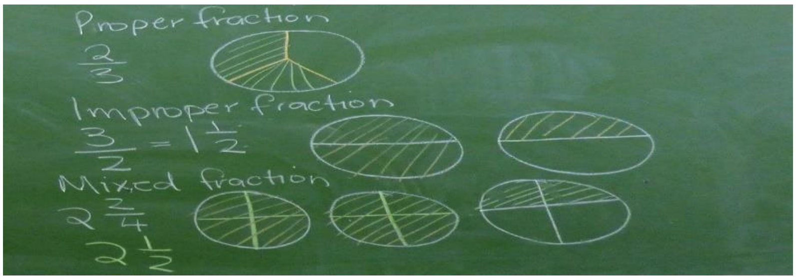


Figure 4. 20: Representation of fraction types using the area model

She explained to the learners how to present a fraction on a model. She said the part of the circle that is shaded represents the numerator and the number of times the models is divided represent the denominator as shown in Figure 4.20 above. She then moved to the comparison of fractions using the area model. Before she indicated to the learners how to compare fractions, she ask them how they could tell which fraction is bigger than the other —to seek their understanding. She then used the area model and showed them how to compare two fractions visually. At the end of the lesson she emphasised the comparison of fractions:

> *To compare the fractions we normally look at the denominators. The bigger denominator the smaller the fraction and the small the denominator the bigger the fraction.*

Teaching that enhances conceptual understanding of fractions

In this lesson, I noted two indicator of teaching for conceptual understanding.

a. Prior knowledge and connecting mathematics to the real world

In her introduction, Ms Nalo related much to what learners already knew, to make them understand the concept of fractions. She also looked at how they could visualise fractions and make them understand what was meant by a fraction as part of a whole by building the meaning of 'fraction' from the learners' prior knowledge.

Prior knowledge makes learning new knowledge easier since new information is connected to what learners already know. This connection forms a bridge between old and new information (Kilpatrick et al., 2001). From the lesson observation, the teacher used a model of a pizza to build a bridge between what is already known by learners to a new concept of a whole as she said:

> *...If for example we have pizza and its cut into four pieces, it's a whole cut into four pieces. Okay, let's say one of you takes a piece from the pizza, so this piece will be one piece, of a whole pizza right?*

Ls: Yes miss!
Ms Nalo: This piece it will one piece out of what?
Ls: Out of four

In her explanation in the lesson Ms Nalo explained the meaning of a fraction, referring, it to the real-world objects pizza. Since the Khomas region is an urban area, most learners understood the example used. In her explanation she helped learners to construct knowledge of a whole which was not familiar to most of them. Ms Nalo also helped learners to understand how a whole was related to a fraction, by connecting what she was teaching to the real world objects used in her explanation. This is an indication that the teacher was using a pizza which is real object, to scaffold learners from the known to the unknown.

4.3.2.2 Lesson 2

Lesson overview

In Lesson 2 Ms Nalo taught fractions using an area model. Her focus for this lesson was on addition of fractions using the area model.

Teaching fractions using visual models as a visualisation tool

In this lesson Ms Nalo exclusively used the area model. In her introduction to addition of fractions, she asked learners how to add fractions symbolically, just to remind them of how fractions are added together. She also asked the learners how fractions with different denominators are added together, just to test their knowledge on addition of fractions. Ms Nalo then introduced how fractions are added together using the area model. As she was showing the fractions on the models, she referred the learners to the first lesson and how the presented fractions on models. She then showed the learners how to add together fractions with the same denominators as shown in Figure 4.21 below.

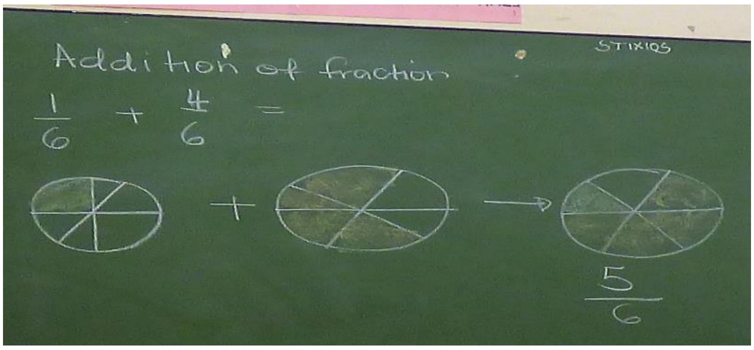


Figure 4. 21: Addition of fractions with the same denominators

She continued by showing learners how to add fractions with different denominators, how to look for the LCM and showed them on the model. Lastly, Ms Nalo showed how to add mixed fractions together, teaching them how to add whole numbers and then fractions.

Teaching that enhances conceptual understanding.

In this lesson, I noted one indicator of teaching for conceptual understanding.

a. Prior knowledge

The extract below shows how Ms Nalo taught to enhance the learners' understanding of fractions. The teacher's explanation of adding mixed fractions together shows how to present the LCM on a model.

Ms Nalo: Okay let's say we have $2\frac{1}{3}$ and $1\frac{1}{4}$ can we add that fractions straight away?
Ls: No, no miss
Ms Nalo: Hee, ***What can we do?***
Ls: We multiply two times three
Ms Nalo: Hee?
Ls: We make the denominators the same and then add them together [T_2V]

Her presentation in this extract engaged learners in a discussion that sought the learner's knowledge of addition of fractions. In the discussion the teacher used questions like "*What can we do?*" This gives an indication that the teacher was looking at what learners already know so that she could build onto it. Through the class discussions, learners created new understanding for themselves. The teacher coached, moderated and suggested but allowed the learners room to experiment, ask questions, and try things that don't work (Gusta, 2011). She used what learners know and visualised it to enhance their understanding of addition of fractions as seen in Figure 4.22, which shows the addition of mixed numbers and a demonstration of the LCM on a model.

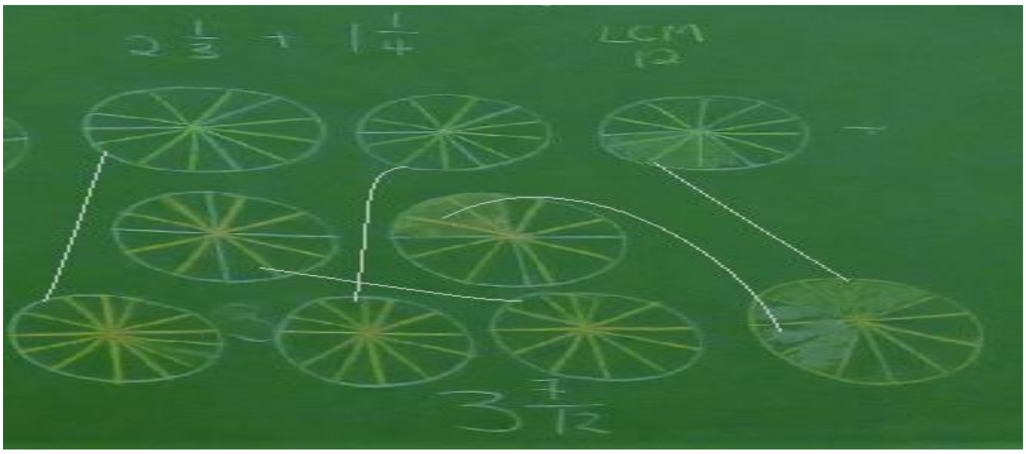


Figure 4. 22: Addition of mixed numbers with different denominators

4.3.2.3 Lesson 3

Lesson overview

In Lesson 3 Ms Nalo focused on the subtraction of fractions using the area model. Her models were drawn on the chalk board.

Teaching fractions using visual models as visualisation tools

In this lesson, Ms Nalo exclusively used the area model. She taught how to subtract fractions. In her introduction, she reminded learners of what was taught during the previous lesson — how to add fractions using area models. She then explained that when subtracting fractions, learners should first look at the denominators and if the denominators are the same they can just present their fractions on a model and then start to subtract by cancelling parts of the second fraction on a model from that of the first fraction. If the denominators are not the same they have to find the LCM. She then gave the learners the example of $\frac{7}{8}$ minus $\frac{1}{4}$, and worked it out as shown in Figure 4.23 below. In this example, Ms Nalo explained how to present the LCM in a model and how to subtract the shaded parts in the second fraction from the first fraction. She further explained how to subtract mixed fractions, also using models.

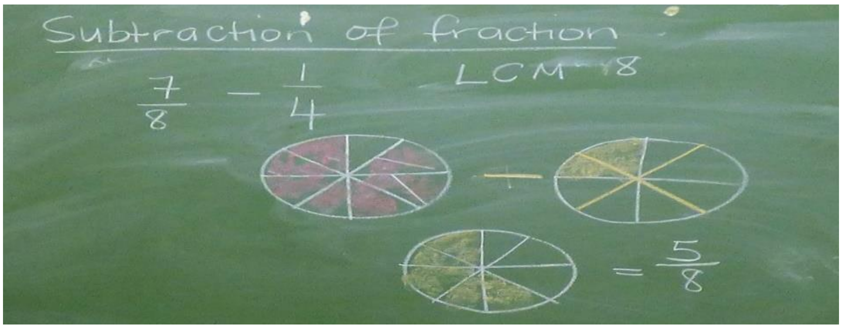


Figure 4. 23: Subtraction of fractions using area models

Teaching that enhances conceptual understanding of fractions

In this lesson, I noted one indicator of teaching for conceptual understanding.

a. The use of multiple representations or models

The extract from the video recorded lesson below shows teaching for conceptual understanding. The teacher explanation on how to find the LCM is related to the model in Figure 4.24. At the same time the teacher kept on engaging learners in the lesson by asking questions to monitor their attention and check for understanding.

Ms Nalo: To subtract one whole cancel another whole and then we are remaining with one and draw it as our answer. Then we move the one that are not full, look at the first one how many parts are shaded?
Ls: Two
Ms Nalo: How many are shaded in the second fraction?
Ls: One
Ms Nalo: Okay you are subtracting the shaded parts in the second fraction from the shaded parts in the first fraction. So if you cancel the one part with another, how many parts are remaining? [T_2V]

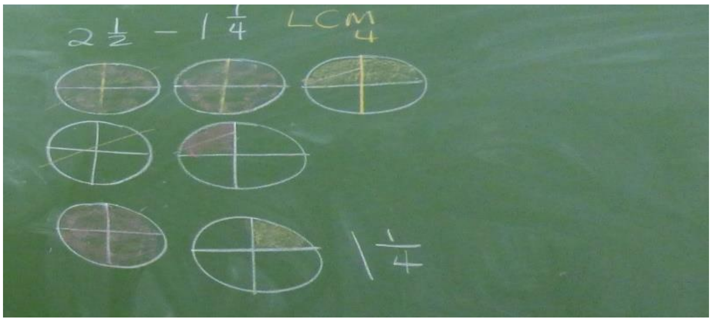


Figure 4. 24: Subtraction of fractions with different denominators.

As seen in the extract and in Figure 4.24 above, Ms Nalo used different representations (explanations, symbols and diagrams) in the sense that the learners were shown how fractions can be subtracted using different drawings/models and explanations. They were also shown how to use the models to find the LCM if the denominators are not the same. All the explanations given were based on the model presented to them on the chalkboard.

4.3.2.4 Lesson 4

Lesson overview

In Lesson 4, Ms Nalo demonstrated how to represent fraction, how to add fractions and how to subtract fractions using a number line model.

Teaching fractions using visual models as visualisation tools

In this lesson Ms Nalo exclusively used a number line model. She began her lesson by referring learners to what they had already done in the previous lessons. She started by saying that they had worked out fractions using area models, where they looked at how to represent fractions visually, compared fractions visually, and they added and subtracted fractions using visuals. For this lesson, she looked at how to work out fractions just as they did previously, however, this time around they used a number line model. She explained what a number line is, referring them to the previous grade. She then explained how to represent a fraction on a number line. She gave an example of 'a half' as she said:

> *If you have to represent a fraction of half for example, so you need to draw a number line in this case the fraction is positive, so the fraction will start from zero and remember the fraction is part of a whole, so on the other side will be one to make up a whole. Since this is a number line of one over two. How many times should we divide this number line?*
> *Ls: Two.*
> *Ms Nalo: You only, divide it into two parts, so we only divide our number line in the middle, so that it will have two parts. Now how many parts should we shade?*
> *Ls: Two, one, one………………….*
> *Ms Nalo: Nee? So, you only shade one part that represent a fraction of one over two, so it's one over two parts [T_2V].*

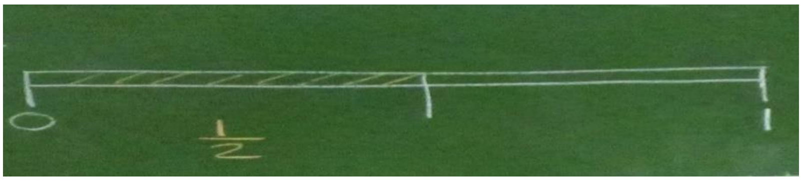


Figure 4. 25: Representation of a half ($\frac{1}{2}$) on a number line

Ms Nalo proceeded to show learners how to represent both proper and mixed numbers on a number line and she explained to them how to add and subtract fractions on a number line. She started with the addition of proper fractions with the same denominators, different denominators and with mixed numbers. She started with the example $\frac{2}{3} + \frac{1}{3}$ and showed learners how to add these two fractions together. She first explained that they have to represent the fractions on a number line and then add them together by combining the parts from the two number lines in one number line, as shown in Figure 4.26 below. However, they had to check if denominators are the same or not, just as they did with area model.

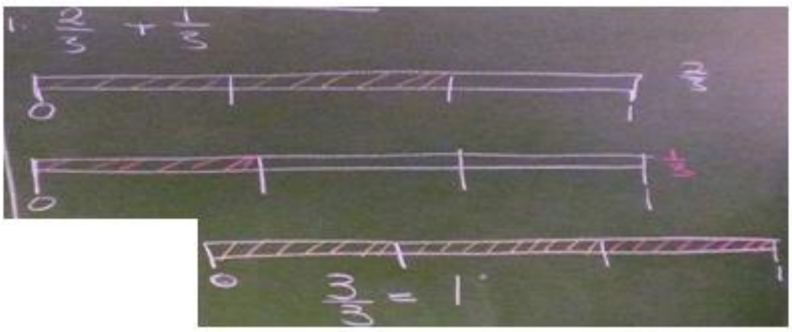


Figure 4. 26: Addition of $\frac{2}{3} + \frac{1}{3}$ on a number line

She further showed learners how to add fractions with different denominators and mixed numbers. She explained how to represent the LCM on a number line. In the same lesson Ms Nalo explained how to subtract fractions using a number line model. As with addition, she explained that to subtract the fractions using a number line, both fractions should first be represented on a number line. If they have different denominators they should look for the LCM. Ms Nalo explained that when subtracting, parts of the second fraction should cancel out parts of the first fraction (see example used in class in Figure 4.27 below).

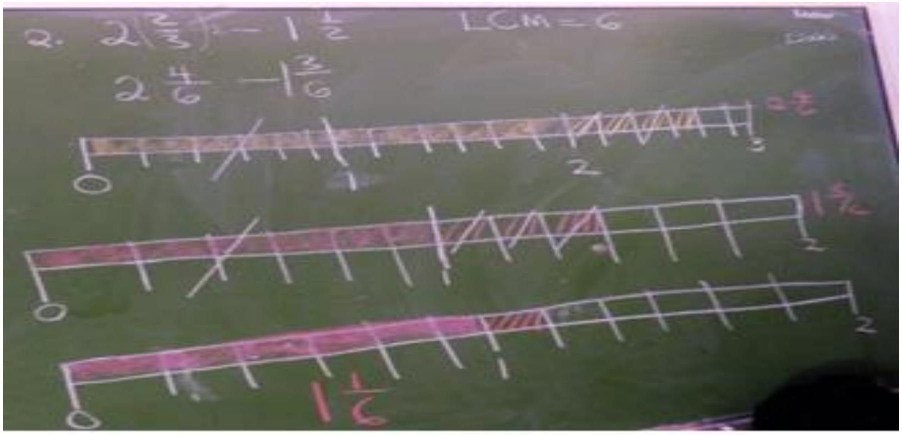


Figure 4. 27: Subtraction of fraction $2\frac{2}{3}$ - $1\frac{1}{2}$ on a number line.

Teaching that enhances conceptual understanding of fractions

In this lesson, I noted one indicator of teaching for conceptual understanding.

a. Prior knowledge

In her introduction of a fraction with a number line Ms Nalo explained how a fraction as part of a whole can be represented on a number line. Her explanation and demonstration of the model is shown in the extract and in Figure 4.28 below.

> *Ms Nalo: If you are representing this fraction ($\frac{4}{5}$) on a number line, then how many parts should we shade?*
> *Ls: Four.*
> *Ms Nalo: That is four over five parts, so that is how you represent a fraction on a number line.* ***Okay, let us say we have two whole shaded and how many parts of the fraction should we shade?*** *[T_2V].*

Figure 4. 28: Presentation of a fraction on a number line.

It is noted from the extract that Ms Nalo used drawings and questions that required learners to demonstrate understanding of how to represent a fraction on a number line, using both proper and mixed numbers. Her questioning probed for prior knowledge and required mathematical reasoning. The questions such as

…let us say we have two whole shaded, and how many parts of the fraction should we shade?

This question is asking learners to think of what they already know about representing fractions on a model. At the same time the question required a learner to think deeper and connect this type of model to the other model that they had already learned.

4.3.2.5 Consolidation of Ms Nalo's lessons

Ms Nalo had four lessons where she taught fractions using the area and number line models. In her lessons, Ms Nalo connected what she was teaching with the models that she was using. All her models were drawn on the chalkboard during the lesson. This was a very good idea since learners could see how the models are drawn and how they are used, as well as to find answers. She used circle shapes to present the area models, as well as the number line. During her presentation of the lessons, she kept on engaging learners in her lesson by creating a room for discussions. She kept on asking them questions to check if they understood what she was teaching. Teaching that enhances conceptual understanding was predominated by the use of representations and building learners' existing knowledge. She only tried to connect fractions to the real world in her introduction, but for the remaining lessons it was not used. Connecting the fraction concept to other concepts in mathematics did not happen in Ms Nalo's lessons.

4.3.3 TEACHER 3 MR MALELE'S LESSONS (T_3V)

4.3.3.1 Lesson 1

Lesson overview

In the first lesson, Mr Malele introduced learners to fractions by giving the meaning of fractions, the types of fractions and how to present a fraction on a number line.

Teaching fractions using visual models as visualisation tools

In this lesson, Mr Malele exclusively used a number line model. Mr Malele introduced the lesson by informing the learners that they would look at fractions using models. He defined a fraction as 'part of a whole', and he then explained what a 'whole' is and how a 'whole' is formed using different fractions. He then informed the learners that they would visualise fractions using a model called a number line model. He further discussed the three types of fractions: proper, improper and mixed numbers and gave examples of each. He explained how to change an improper fraction to mixed numbers because it is easier to represent a mixed number on a number line than an improper fraction. Lastly, he explained by showing on a model, how to represent a fraction on a number line — as shown in the extract below from the observed lesson.

> *Mr Malele: Let's say we have two over five as a proper fraction, you just draw a number line starting with zero to one that means it is one whole.*
> *Ls: Ooooooo…………*
> *Mr Malele: Now that one whole we divide it into five parts. We divide it now the numerator is the one we shade that means if I shade one part it is,* $\frac{1}{5}$ *if I shade two parts?*
> *Ls: 2/5*
> *Mr Malele: Three parts*
> *Ls: 3/5*
> *Mr Malele: Four parts*
> *Ls: 4/5*
> *Mr Malele: If its five parts that I shade it all and that is a whole. I shade the whole part which is equal to one. Okay back to our example you shade two parts out of five. So all proper fractions are presented like that [* T_3V *].*

In this lesson, Mr Malele taught learners how to visualise fractions using a number line. In the extract, his discussion with the learners was to engage them and see if they could visualise different fractions on a number line, as he asked them questions like "If I shade one part of five/two/three/four, what fraction will it give?" His intention was to build the learners' understanding of how fractions can be visualised.

Teaching that enhances conceptual understanding of fractions

In this lesson, I noted one indicator of teaching for conceptual understanding.

a. Prior knowledge

Mr Malele in his teaching made sure that he emphasised how fractions can be drawn and shaded on a number line. His way of asking learners and probing for understanding of what he was teaching aided teaching and learning. He posed questions that sought learner's prior knowledge of fractions. See the example of teaching to make learners understand the presentation of fractions on a number line.

> *Mr Malele: Next we look at how to present improper fraction, so this one we do not just present them like that we have to change them to…….?*
> *Ls: Improper, mixed (shout)*
> *Mr Malele: We change improper fractions to mixed number. Let's say we have a fraction* $\frac{5}{2}$ *which is an improper fraction, first thing you change it to mixed number meaning you will have two full whole and another whole which is not full. So we are saying that we have two full whole and the last one is not full.*

So we make our number line with three whole and each whole should be divided into two parts because of the denominator which is two [T_3V].

In the extract Mr Malele, asked learners questions that made them think of what they already knew. In this way his teaching was building on learners' prior knowledge, since learners indicated to him that improper fractions have to be converted to mixed numbers first before they are represented on a model. His teaching of how to present $\frac{5}{2}$ on a model was based on the learners' prior knowledge.

4.3.3.2 Lesson 2

Lesson overview

In Lesson two Mr Malele looked at how to compare fractions and how to add fractions together using a number line model.

Teaching using visual models (the number line) as a visualisation tool.

In this lesson, Mr Malele exclusively used a number line model. In his introduction of Lesson 2, Mr Malele reminded learners of what they had learnt in the previous lesson. He then informed the learners that in this lesson they would look at how to compare fractions and how to add fractions using a number line model. He asked them if they could still remember how to represent a fraction on a number line. He then said that to compare fractions they should use the symbols $<$, $>$ and $=$. He stated that they would use these symbols, but to see which fraction is big and which one is small they would use number lines and then apply the symbols. He explained that for easy comparing of fractions, they should always make the denominators the same by looking for the LCM. He indicated that this would help, since they would draw the number line, divide it into equal parts and then shade out the numerators so they could easily see which fraction is bigger and which one is smaller. See extract below for his explanation.

Mr Malele: For you to compare them before you draw them on a number line you look for lowest common multiple (LCM) of the denominators, so the LCM of these is 12. So you say $3\times\frac{3}{4}$ and $\frac{2}{3}\times 4$, now will have $\frac{9}{12}$ and $\frac{8}{12}$, now they are on the same denominators so you can compare them. So you can draw your number line with one whole. What is the denominator?
Ls: 12
Mr Malele: So you divide your whole into 12 parts, so the first fraction is $\frac{9}{12}$, how many parts are you shading?
Ls: 9
Mr Malele: 9 out of 12, yes this is the numerator that you are shading.
On the next one you shade how many parts?
Ls: 8

Mr Malele: Now we shade 8 parts in the second model, then you compare, do not forget your initial fractions so looking at the figure, which one is bigger? [T_3V]

Mr Malele then looked at how to add fractions using a number line. He explained that to add fractions together, learners should make sure that the denominator of the given fractions are the same, by finding the LCM as shown in the Figure 4.29 below.

Figure 4. 29: Addition of fractions with different denominators on a number line.

He further explained how to add mixed numbers together using a number line as shown in the figure below.

Figure 4. 30: Addition of mixed numbers on a number line.

Teaching that enhances conceptual understanding of fractions

In this lesson, I noted one indicator of teaching for conceptual understanding.

a. The use of multiple representations or models

In this lesson Mr Malele showed some evidence of teaching for conceptual understanding. This appears in Figures 4.29 and 4.30 above. His conversation with the learners as he was teaching gives an indication that the learners understood what was being taught. The discussion in the extract from the observed lesson below shows some evidence of teaching for conceptual understanding.

Mr Malele: Let us say we are given $\frac{3}{8}$ plus $\frac{1}{2}$ just an example, but we are using models. Are we together?
Ls: Yes Sir

Mr Malele: what do we look at first before we start adding fractions together?
L: we have to check if the denominators are the same
Mr Malele: ahaa! If they are not the same?
Ls: We look for the LCM.
Mr Malele: okay in this case they are not the same, so what is the LCM OF 8 and 2?
Ls: Two
Mr Malele: Not a factor, the LCM
Ls: Eight
Mr Malele: Eight, so the first fraction is already over eight. How do we do it?
L: We multiply both numbers by on the second fraction by four

On the other hand, his teaching was always based or connected to the model that he was presenting and he kept on referring learners to their prior knowledge so that they could build what they were being taught to what they already knew. His teaching on finding the LCM as seen in Figure 4.29, shows calculations and later represented in a model. This gives an impression that he used different representations (explanation, symbols and models) in the same lesson.

4.3.3.3 Lesson 3

Lesson overview

In Lesson 3, Mr Malele looked at how to subtract fractions using a number line model.

Teaching using visual models as a visualisation tool

In this lesson, Mr Malele exclusively used a number line model. In his introduction, Mr Malele reminded the learners of what they were taught in the previous lesson purely to attract their attention to the lesson. He also reminded the learners that to add or to subtract fractions they should make sure that the fractions that they are working with have the same denominators by finding the LCM. He then explained by using models, how to subtract fractions. He explained that to subtract a fraction on a number line you cancel the parts of the second fraction with the parts of the first fraction, and whatever remains not cancelled is the answer — as shown in Figure 4.31 below.

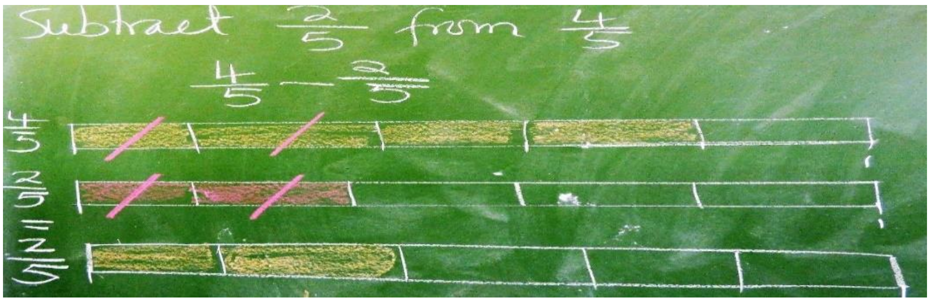


Figure 4. 31: Subtraction of fractions with same denominators on a number line

He emphasised that using models is easy and one need not use a calculator, but simply get the answers from the models. Mr Malele further explained how to subtract mixed numbers using the number line, with the example $3\frac{1}{2}$ -$1\frac{2}{3}$. This fractions has different denominators so he explained how to make the denominators the same by looking for the LCM, represent them on a number line and then subtracted them. See Figure 4.32 below as it was presented in the class.

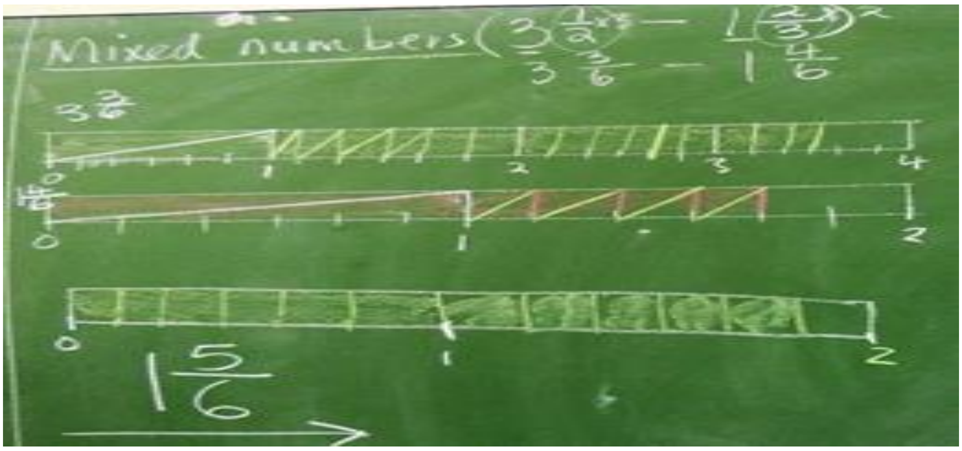


Figure 4. 32: Subtraction of mixed numbers on a number line.

Teaching that enhances conceptual understanding of fractions

In this lesson, I noted one indicator of teaching for conceptual understanding.

a. Prior knowledge

It was observed in Mr Malele's lesson that his teaching enhanced learners' understanding of fractions, because he made sure that he checked the learner's prior knowledge and then built on what they already knew. As he explained how to subtract $1\frac{2}{3}$ from $3\frac{1}{2}$, he competently explained how to find the LCM, how to present the LCM on a number line and how to subtract. See the extract below.

> *Mr Malele: Alright, let us look at mixed number, work out $3\frac{1}{2}$ minus s $1\frac{2}{3}$, look at your fractions there are whole and other whole that are not full. The denominators are also not the same,* ***so what do we do if the denominators are not the same?***
> *Ls: look for the LCM*
> *Mr Malele: yes we look for the LCM. Okay what is the LCM of 2 and 3?*
> *Ls: Six*
> *Mr Malele:* ***Six is the lowest so we multiply $3\frac{1}{2}\times$ 3-minus $1\frac{2}{3}\times 2$. So we will get $3\,\frac{3}{6}$ minus $1\frac{4}{6}$, then you draw your models so you draw your models nicely no need of calculator****. Now how many whole are we drawing in the first number line?*
> *Ls: four [T_3V]*

Mr Malele's engagement with learners in discussion and his questioning probed learners' existing knowledge of how to subtract fractions. A question like *what do we do if the* *denominators are not the same?* asked in the extract, required learners to think or reflect on what they had been taught or what they already knew, and then build on it. He also built learners' knowledge of how to make the denominators the same by showing them symbolically. He indicates in the extract that

> *Six is the lowest so we multiply* $\mathbf{3\frac{1}{2}} \times 3$ *minus* $\mathbf{1\frac{2}{3}} \times 2$. *So we will get* $\mathbf{3}\ \frac{3}{6}$ *minus* $\mathbf{1\frac{4}{6}}$ *then you draw your models so you draw your models nicely no need of calculator.*

This was later represented on a model as seen on Figure 4.33 above.

4.3.3.4 Lesson 4

Lesson overview

In Lesson 4 Mr Malele looked at how to work out fractions using an area model. He specifically looked at a representation of a fraction on an area model, and how to add and subtract fractions using an area model.

Teaching using visual models as a visualisation tool

In this lesson, Mr Malele solely used the area model. In his introduction, Mr Malele again reminded learners of what they had done in the previous lessons. He asked learners to explain how to present a fraction on a number line. He then asked learners if they could still remember the types of fractions that they did in the first lesson. He explained to them a bit by giving examples and reminded them that whenever they are working with models it is always good if they change the improper fraction to mixed numbers for easy representation on a model. He then introduced the idea of an area model, how it looks and how to represent a fraction on an area model — as shown in the extract below from the observed lesson.

> *Mr Malele:Area model we are just going to draw circles and we divide them, example to represent* $\frac{1}{2}$ *on an area model. We draw a circle and divide it into two parts and we shade how many parts?*
> *Ls: One.*
> *Mr Malele: Yes, one out of how many?*
> *Ls: Two.*
> *Mr Malele: Yes, one out of two that model represent one over two" [T_3V].*

He further explained how to represent both proper and mixed numbers on an area model and then looked at how to use an area model to add and subtract fractions. In addition and subtractions Mr Malele used different examples some of which were fractions with the same denominators and others with different denominators. For those with different denominators he showed learners how to make the denominators the same by finding the LCM. See examples given in the class in the Figures 4.33 and 4.34 below.

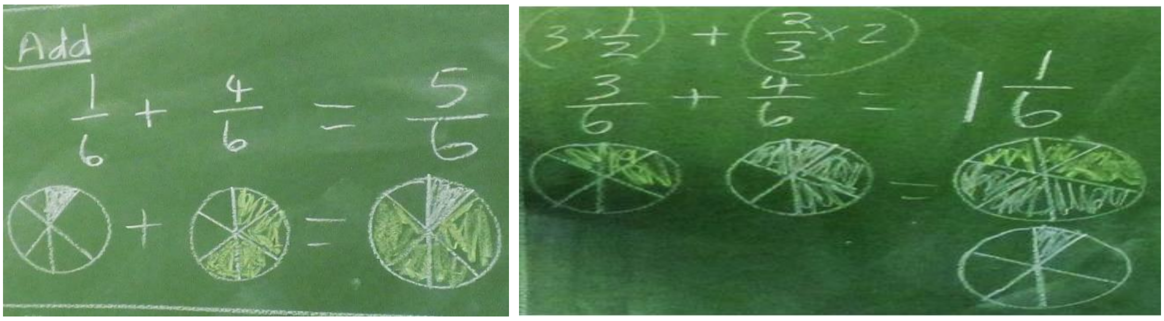


Figure 4. 33: Addition of fractions using the area model

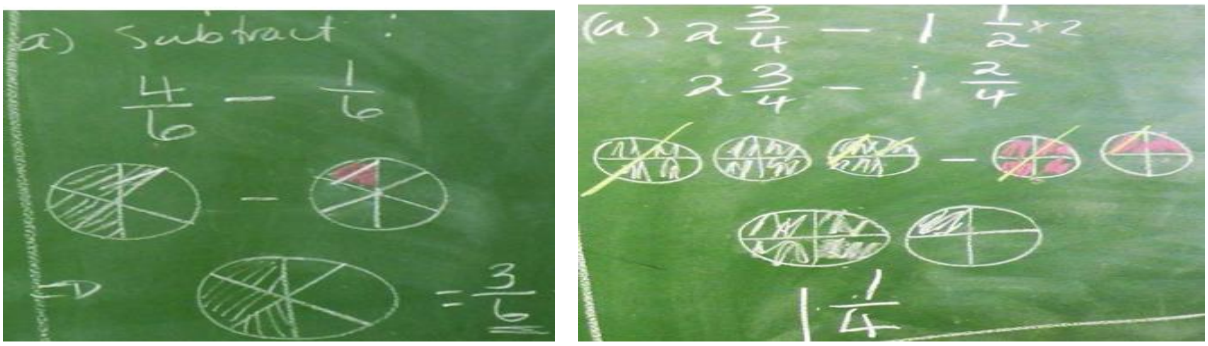


Figure 4. 34: Subtractions of fractions using the area model.

Teaching that enhances conceptual understanding of fractions.

In this lesson, I noted one indicator of teaching for conceptual understanding.

a. The use of multiple representations or models

In this lesson Mr Malele showed evidence of proper use of visuals and explanations that can enhance learners' understanding of fractions. The extract from the observed lesson and Figure 4.35 below from Lesson 4 shows how Mr Malele's explanations in the lesson were well connected to the visuals he used in the class.

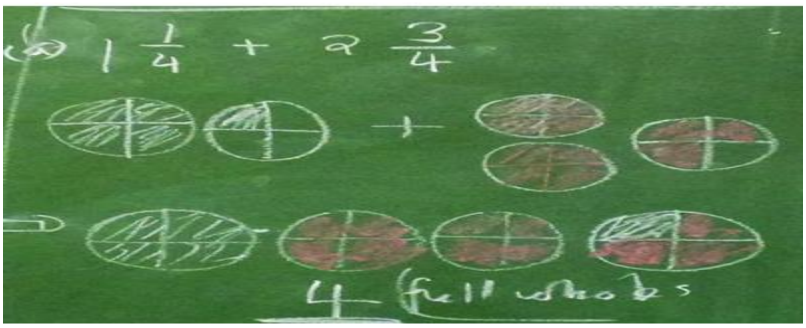


Figure 4. 35: Addition of mixed numbers on an area model

Mr Malele's discussion (as seen in the extract below) with the learners, shows that the learners were fully involved in the lesson as they answered the questions:

> *Mr Malele: Let us look at addition of mixed numbers, e.g. if we* have $1\frac{1}{4} + 2\frac{3}{4}$. On *the first fraction how many whole do we have?*
> *Ls: One.*
> *Mr Malele: Okay, we draw a full whole and divide it into four parts and shade it full, the next whole we shade one part. Its mixed number its one full and one over four plus, how many whole on the second fraction?*
> *Ls: Two whole.*
> *Mr Malele: Yes, two full whole, divided into four parts and fully shaded. And the one that is not full, how many parts to be shaded?*
> *Ls: Three.*
> *Mr Malele: Only three over four. To get the answer we add them together. So, your answer is four. [T_3V].*

It was observed that learners understood because most of them answered all the questions correctly. Lastly, the presentation of the models were clear and they were used in different ways, which helped most of the learners to understand the concept of fractions easily because they could clearly see what was being taught. As Harvey et al. (2012) indicate, the most effective way to help learners reach higher levels of understanding is to use visualisation which includes the use of multiple representations, multiple approaches, explanation and justification.

4.3.3.5 Consolidation of Mr Malele's lessons

Mr Malele taught four lessons where he used the number line model and the area model to teach common fractions to Grade 8 learners. The models that he used were all drawn on the chalkboard during the lessons. In his lessons he made sure that his explanations were related to the model that he was teaching. The models were astutely used to show learners how to find the answers. He also engaged learners in his lesson by asking them questions and involving them in finding solutions. He also tested the learner's prior knowledge at the beginning of all his lessons. It is noted that Mr Malele is the only one who emphasised changing improper fractions to mixed numbers for a better representation. In my observation, this was a good idea. Teaching for conceptual understanding for Mr Malele was also dominated by the use of multiple representations to show how to reach the answer. Other components were not used except for that of building learners' existing knowledge.

4.4 SUMMARY OF THE LESSON OBSERVATIONS

In the classroom observations, it was observed in the activities given to learners that the use of visual models helped most learners to develop an understanding of common fractions more successfully than how they were previously taught. It was noted that when the teachers were teaching, learners where paying more attention and they were fully engaged in the lesson. The discussions between the teachers and the learners were interesting and one could see that learners did understand what the teacher was resenting.

All teachers taught fractions using the area and the number line models. These were drawn on the chalkboard except for one teacher who used PowerPoint to present the addition and subtraction of fractions using the number line model. Two teachers preferred drawing the models while learners were present in class, and one preferred to draw before learners came to class. However, in my observation I noted that learners tend to understand better when the models were drawn during the lesson, because the learners could see what was being represented, unlike when they find models already drawn.

4.5 RESEARCH RESULTS FROM INTERVIEWS WITH THE PARTICIPANTS

The interviews were conducted with the three teachers who participated in the intervention program. The interviews took place after all the lessons were taught. The purpose of the interviews was to find an answer to the third research question:

What are the selected Grade 8 mathematics teachers' experiences and perceptions of working with visual models to teach common fractions, after participating in an intervention program?

The interview had six questions based on the taught lessons, and these questions were set to seek the teacher's perceptions and experiences of using visual models as visualisation tools, with the purpose of enhancing the learner's conceptual understanding of common fractions. From the interview questions, the following themes emerged: ***Teaching of fractions; type of visual models teachers use; teachers' views of how to teach fractions visually; the importance of using visual models when teaching fractions; the use of visual models to enhance conceptual understanding of common fractions; and challenges or limitations in the use of visual models.*** By reading and analyzing the responses of the three teachers, a sense of similarity and coherence is evident in both the teachers' remarks and responses to the questions asked. Throughout the interview process, all the teachers were cooperative and confident. Their responses are then combined and discussed as per the emergent themes as follows.

4.5.1 Interviews with the three teachers

4.5.1.1 Teaching of fractions

In the interviews, the teachers were asked what they now do differently compared to how they used to teach fractions prior to the intervention. All the teachers highlighted that they do not usually use visuals to teach fractions or even other topics in mathematics. However, after participating in an intervention, they noticed a major change in the learners' participation and understanding. This therefore gave an indication that they understood better then how they used to be taught. Mr Mose highlighted that

> *…the use of visuals improve learners' understanding of the topic. It boosts learner's interest in learning because when they are drawing, they feel like they are playing a game when they are drawing, and mind you learners enjoy drawing and doing practical's unlike with feeding them with lots of theories (T_1I) .*

In addition, Mr Malele indicated that

> *…the difference is that when you use models you are showing what you are explaining – you are putting it there and the learners can see it. They see that, ok, that is one over two and they can clearly see as they look on the chalkboard that one over two is there exactly represented. Actually that is a very big difference there. Using models is like practice because learners are seeing at the same times they are doing by drawing. Unlike with theories where learners only hearing but they cannot see what they are told represent. They only hear and they write but with visuals there is two three things involved, they hear, they see and they participate in their own learning and this by its own help in the understanding of what is being taught (T_3I).*

Ms Nalo on the other hand point out that

> *…when I was teaching fractions using models I saw that the learners are more participating unlike with the way we teach before this shows that they understand what is being taught. I think is because they can see how fractions are added or subtracted the other thing that I picked up is that the models stimulates learner's way of-of thinking" because their reasoning is quite different now. (T_2I).*

Teachers, in their experience of teaching using visual models, noted that visual models help in boosting learners' interest in learning, since models by nature are practical. When using models, learners are practically involved in their learning. On the other hand, teachers indicated that teaching using models encouraged participation and it also boosted learners' thinking capability as their reasoning has changed. The other aspect that all the teachers highlighted, is that the use of models shows an improvement in learners' understanding of fractions.

4.5.1.2 The teachers' preferred model

In this section I looked at the teachers' responses to the questions and how they make use of the visual models to help learners understand the topic of common fractions. All teachers taught fractions using the area model and the number line model. They pointed out that both models have the potential to make learners understand the topic of common fractions. However, they indicated that each model has its own challenges. From their points of view, they prefer area models, and they indicated that the area model is easy to draw and most learners understand the area models faster the number line model. Mr Mose indicated that:

> *…the area model is easy and convenient to use. With the area model, we were cutting the whole drawing – the way it was appearing – it's like we cut it more into smaller pieces so learners can see from the drawing, they can count that if it was like two over three and one over four, they all meet in twelve. So, when you cut in those twelve equal parts of a different colour inside that area model learners can see those parts that you draw in, as compared to the number line where you have to put the small lines on top. So I think the area model is the best and it is user friendly. Learners were able to draw and play around with it unlike with the number line" (T_1I).*

On the same note Mr Malele pointed out that:

> *….the two model that I have used, they are both helpful any way. But I have seen that using area models is quite easier. Let's look at the number line for instance. The number line when you draw it you have to divide the number line into continuous whole which is sometimes in my teachings made learners a bit confused. But when I have used the area model, where you just draw separate whole and you divide them into parts as given by the fractions that makes learners to understand it much better compared to the number line model. I have realized and also according to the feedback that learners gave me as we were looking at these things that area models is easy to use. But in my own capacity as a teacher, the area model is a bit easier because one, it's easier to draw and it's also easier to present (T_3I).*

The dominance of the area model in the observed classes was due to this model being easier to draw and use, and the ability to link the area model to prior knowledge. Ms Nalo argued that the number line model was not as easy to use as the area model when adding or subtracting fractions of different denominators. On the other hand, Mr Malele pointed out that a number line is confusing because of the continuous whole. All these factors lead to the dominant use of the area model I observed in this study.

4.5.1.3 Teachers' views of how to teach fractions visually

This question sought the teachers' views on how they taught fractions visually. All teachers were in support of teaching fractions visually because they could see that when mathematics is presented visually to the learners they tended to understand more successfully – unlike when they were merely taught theoretically with visual tools. Mr Mose indicated that

> *If I draw a circle, cut it into two parts and colour one part learners will see that it will form up a fraction one over two. Instead of just writing one over two without them knowing what one over two represent. (T_1I).*

Ms Nalo on the other hand supported the idea as she alleged that:

> *…..If we look at the area model the circles, when we were drawing the circle you can have one whole and one over two it's easier to see that you draw one circle a whole and then you draw another circle that you divide into two and then if you add one whole and one over three you again draw one full whole and then another one that you divide into three parts. So when you add these fractions together learners can clearly see that you are adding the whole and another whole together then they can see it's one and two then they became two whole and then they can see when you are adding one part from another whole*

fraction to the other, they can see how you are adding and become one and with models learners always get accurate answer. The only thing that they struggle with is to draw the parts equally but I think it's just a matter of time [T_2I].

The teachers' insight into using models is that it helps in showing exactly what a fraction represents. They noted that especially when working with operations (addition and subtraction), learners could really see what is being added or subtracted — unlike when teaching using symbols. Teachers have seen that working or visualising fractions is one of the methods that can help both teachers and learners to work easily, and in addition, visuals give accurate answers.

4.5.1.4 Significance of using visual models when teaching fractions

The three teachers acknowledged that it is vital to teach fractions using visual models or diagrams accompanied by explanations. The teachers further emphasized that models helped learners to understand what the fraction portrays and offers clear understanding of mathematical concepts. Mr Mose stated that:

….the model as we refer to them as visual, this means we want learners to learn by seeing. Learners learn better by looking at the drawing and they can see what is happening in class unlike with the old way were we use to use just numbers writing like two over three, then learners think that those are the two different numbers that you put on top and the other one down, they cannot really see what they mean" (T_1I).

Similarly Ms Nalo concurred with the statement by Mr Mose as she indicated that:

….for me, it is really important to use visual model because they help learners to visualise what is happening in class and it help them to recall what they were taught in class since when learners see the things, they don't forget easily. I'm saying this because when you are teaching without using visuals like how I use to teach before, learners just use to see numbers but they do not know really where those numbers come from, and sometimes they don't see how you put one number on top of the other and the one down. But when you are using the visual models, learners can see how a fraction is actually adding up to a whole, that is one full thing and if you divide into pieces they can easily see that there is a fraction being formed (T_2I).

Mr Malele indicated that:

"It's a quite helpful method of teaching because learners can see, they visualise what is happening in class unlike with theories were learners are just told numbers but they do not really see were those numbers are coming from (T_3I).

The teachers indicated that using visuals is learning by seeing and this pastes a permanent picture in the learners' minds, and helps them to recall too. Models help learners visualise concepts as they are presented in class.

4.5.1.5 Visual models as a tool for enhancing conceptual understanding of common fractions

The teachers revealed that visual models could be used as a tool to enhance learners' conceptual understanding of fractions. Models stimulate the learners' interest in learning mathematics and they help learners recall the mathematical concepts. In support, Mr Mose mentioned that

> *……I experienced that using visuals allows learners to be involved in their own learning because usually we used to struggle when we are teaching these fractions especially when we have to compare. But with visuals I have seen that when they look at the drawings it's very easy for them to compare especially when it came to comparison with the area model. The use of visual is very easy for our learners and it made them understand the topic of common fraction in no time. Using visual is one of the easiest method that helped our learners to get the correct answer very fast even the so called slow learners" (T_1I).*

In agreement Ms Nalo stated that

> *…..teaching fractions using this visual or visual models, it really made my work easier. All that I do now in my class, I can just draw my models on the chalkboard and teach them how to present fraction, how to add them together or subtract. So though they still need to learn how to find the lowest multiple the visual models really helps, me to teach fractions easily and my learners to understand how to work out fractions properly" (T_2I).*

The teachers feels that the use of models enhances learners' understanding of fractions, as it encourages learners to be involved in their own learning. They also indicated that models are easy to use and easily understandable — especially the area model — in comparison with symbolic teaching. They highlighted that models themselves guide learners through to the answer as compared to working out solutions using symbols.

4.5.1.6 Challenges or limitations in the use of visual models

The teachers acknowledged that there are numerous limitations associated with the use of visual models. All teachers indicated that although there are numerous challenges associated with the use of models, such as accuracy in drawing space to draw all the examples and materials, however, the major challenge is time. They stated that it took a lot of time to draw the models on the chalkboard and if they drew before the learners came to class, learners would not understand. In support, below is what the teachers said.

Mr Mose:

> *….the only challenge with this method is time, you have seen on the number line when I was using the power point it was already a bit fast. For the area model I tried to draw the models before they come to class but when they find them already drawn on the chalk board they do not concentrate much on where you are explaining, they will start to run through all the drawings and trying to find the answers even of the one that are not yet explained. So it was good if I draw while they are there but then it is time-consuming. So you might spend the whole period drawing and the moment you finish the drawing the lesson is already over (T_1I).*

Ms Nalo concurred as she said

Another challenge it's time. So sometime you just do one or two examples because you also need to monitor the learners by walk around and see how they are – how they are doing it. So, sometimes you just end up doing two or three examples and the time is already gone, though learners understand, it's a slow kind of process because for you to finish drawing three examples in the time we are given for a lesson is somehow not possible. Sometimes time goes without us completing what we have planned for the day" (T_2I).

Moreover, Mr Malele demonstrated that

….if we look at both like number line and area model you have to draw, you need enough space on the chalkboards and also you need to have time to draw. Even learners need time to draw. If you look at our classes we only have less forty five minutes per lesson. The time you draw on the chalkboard then the learners draw also in their books the forty minutes will already be over. The challenge is the time, so when you are presenting these lessons you need e much time" (T_3I).

Teachers indicated several challenges that they experienced, however the main challenge was that the drawing of models is time-consuming. Moreover, they suggest that if schools could be provided with technology or materials that are already equipped with the visuals it would help to teach mathematics using this method.

4.5.2 Summary of the interviews

The interviews were aimed at finding answers to the third research question that focused on teachers' experiences and perceptions of using visual models to teach fractions to Grade 8 learners. The teacher's responses to the interview questions indicated that the use of visual models is central to the teaching and learning of fractions. They indicated that the use of visual models enhanced the conceptual understanding of common fractions as learners could see what the teacher was representing and they themselves were also involved in their own learning by doing the drawings. All teachers asserted that the use of visual models made the lessons interesting and all learners paid attention. Additionally, the teachers echoed that the use of models was helpful as it developed a sense of fractions in the learners — unlike when fractions are taught using only symbols. They further indicated that the models helped even the so-called slow learners to catch up fast with the other learners.

They pointed out that the use of visual models helped them during teaching in terms of saving time and effort. To them, using visual models made teaching much easier and enhanced understanding of fraction concepts when they compared it with 'traditional' methods/symbolic method. The teachers indicated that models helped learners to construct their own knowledge because they were participating in their own learning.

They also indicated that although the method of using models is good and easy, there are challenges that come along with it. They asserted that the main challenge of using models is time. They found it time-consuming because the teacher has to draw the model and the learners have to draw as well, so by the time they finish the drawing, the lesson would already be over. They would end up only covering one or two examples in forty minutes. However, they also suggested that if visual models could be included in learners' textbooks it would assist both teachers and learners during the teaching and learning of fractions.

The other challenge that teachers noted is the accuracy of drawing, as the model has to be divided into equal parts. Finally, the teachers experienced challenges with presenting mixed numbers on a number line. They said it was a bit confusing because it is solely on one number line joined together, unlike the area model where each whole stands by itself.

4.6 SUMMARY OF FINDINGS AND EMERGING THEMES

The main findings from the three sources of data: the questionnaires, classroom observations and recall interviews are presented in this section.

It was noted from the questionnaires that most teachers do not use visual models to teach fractions. Some use visual materials but only when they are introducing the fractions to the learners. To teach they use only theory. Although they do not use visual models to teach fractions, they acknowledged that it is important to use visuals to teach fractions, as it enhances the learners' conceptual understanding of fractions. The teachers highlighted that the use of visuals involves learners in their own learning makes the learners understand, because using visuals requires learners to do the work as well. This, in turn, enhances their understanding as they can see, do and touch. Seeing and touching clears up misconceptions and confusion and it prints a rich picture in the person's mind, to enhance a broader, more realistic understanding. They further stated that visual models bring the figure that will permanently stay in learners' minds and that it would always be remembered, because something that is done or observed keeps reflecting in their minds and takes longer to leave the learners' memories. On the same note, the teachers asserted that visuals involves learners in their learning through practising what they are learning. Using practical teaching approaches makes learning more understandable because it is more efficient than theory and it relates the content to real-life situations. Visual models help learners to work out fractions with fewer difficulties and they help learners to make connections amongst concepts and relate them to reality.

Teacher responses in the survey and their inadequate use of visual models necessitated the need for an intervention program. Teachers were thus observed after the intervention program and the observations made are summarised here.

From the classroom observations, all three teachers used visual models to generate images during their lesson presentations, and used visuals models to develop the mathematical idea at hand. All teachers were able to connect and support their explanations of fractions through the models that they drew. This is in agreement with Arcavi's (2003) definition of visualisation as

> … the ability, the process and the product of creation, interpretation, use of and reflection upon figures, images diagrams, in our minds, on paper or with technological tools, with the purpose of depicting and communicating information, thinking about, developing previously unknown ideas, and advancing understanding (p. 217).

It was noted that when the teachers were teaching, learners where paying more attention and they were fully engaged in the lesson. The discussions between the teachers and the learners were interesting and one could see that learners did understand what the teacher was presenting.

Data from the interviews provides strong evidence to support the findings from the questionnaires and the classroom observations. The teachers indicated that the use of visual models helped learners to understand the topic of common fractions. They noted a major change in comparison to how they previously taught fractions. The teachers echoed that teaching using visual models makes the lesson interesting and learners were eager to attend the lessons because they understood what was being taught. They said that the learners where really engaged in the lessons and that they have since seen a major change in understanding. They mentioned that they could see that understanding in learners as they were answering questions in the activities as well as in the examinations. They further indicated that though the method is time-consuming it is the best method to teach fractions.

During data analysis, similarities were coded and categorized to bring about four major themes. The themes that emerged from the collected data are: ***teaching of fractions: teaching fractions visually; the significance of using visual models in the teaching of fractions; and visual models as a tool for enhancing conceptual understanding of common fractions.*** Although these themes are discussed separately, it is noted that they are interrelated.

4.6.1 Teaching of fractions

Learners' conceptual understanding of fractions has proved to be a challenge over the years (Bruce at al., 2013). The general lack of understanding of fractions has been observed at all levels of education, from primary, secondary to tertiary levels (Bruce et al., 2013). In agreement with the literature, the findings from teachers indicated that most of the learners have difficulties understanding fractions. It can be argued that the difficulties experienced in understanding fractions can be related to the way that fractions are taught in schools. In addition, Bruce at el. (2013) further indicate that poor understanding of fractions can contribute to learners' lack of understanding of other mathematics concepts and their career choices. However, Moyer (2014) points out that "students require appropriate assistance and guidance from teachers and knowledgeable peers as they select, interpret, and create visual models of mathematics" (p.3). This means that for learners to understand fractions they need manipulatives that help them to understand the topic better.

In addition, Way (2011) advocates that learners need to visualise, estimate and create representations of fractions by partitioning the 'whole', using a variety of models (for example areas of various shapes, strips of paper or string and groups of objects). The use of manipulatives or representations can help learners to understand fractions better. As learners already have difficulties creating their own representations, they need lots of practice in drawing shapes and using digital resources to build mental images that will allow them to visualise and estimate fractions. In support, Mr Mose indicated that

> *Using models is like allowing learners to practice because learners are seeing at the same times they are doing by drawing. Unlike with theories where learners only hearing but they cannot see what they are told represent (T_1I).*

Similarly, Makina (2010) affirms that visualisation is a very important cornerstone in "teaching for understanding" in mathematics because it helps the teacher to facilitate the lesson, as it creates a platform where learners are more engaged with visual images. Thus the most effective way to help learners reach higher levels of understanding is to use visualisation that includes the use of multiple representations, multiple approaches, explanations and justifications (Harvey, 2012; Pantziara & Philippou, 2012).

4.6.2 Teaching fractions visually

The use of visual objects are central elements to the effective teaching and learning of mathematics (Gellert & Steinbring, 2014). Thus, the roles of visualisation in mathematics teaching and learning cannot be over-emphasised. As defined by Presmeg (2006), "visualisation as taken to include the process of construction and transforming both visual and mental imagery and all of the inscription of spatial nature that may be implicated in doing mathematics" (p. 206), the teachers specified that visual models attract learners' attention and stimulate learners' interest in learning mathematics. They further argued that visual models also make mathematics fun, enjoyable and practical. In agreement the surveyed teachers stated that

> *The practical make it more understandable because it is more efficient than theory and it is related to real life situation. It help learners to work out fractions without any difficulties and it help learners to make connections among concepts and relate them to reality (T16).*

In addition, Konyalioglu (2003) asserts that researchers supporting the visualisation method believe that it will enhance understanding, comprehension, self-confidence and creativity in mathematics education. Some maintain that visual thinking may be a strong alternative resource for learners by bringing new ways of thinking in mathematics and also underlining the importance of visualisation and visual reasoning within mathematics teaching. On the same note, the teachers in the interview indicated that they noted that the use of visual models help to improve learners' participation in class and stimulate the learners ways of thinking. Zimmermann and Cunningham (1991) advocate that visualisation supplies depth and meaning to understanding, serving as a reliable guide to problem solving, and inspiring creative discoveries.

Sobbëke (2005) suggests that in mathematics classrooms, visual diagrams help the learners to better see mathematical concepts and ideas. In support, the findings from a questionnaire and the teachers who participated in the intervention suggested that visual models help to simplify and clarify the explanation, making it easier for the learners to grasp the abstract mathematical concepts easily. A figure gives a meaning better than the written text (Bosch, Chevallard & Gascón, 2006). Teachers in the survey questionnaires indicated that visuals help learners to consolidate their understanding of mathematics concepts.

4.6.3 Significance of using visual models in the teaching of fractions

The teachers acknowledged that is vital to use visual models as visualisation tools to teach common fractions. A research conducted by in Ho (2010) shows that visualisation is at the heart of mathematical problem solving and "it can be a powerful cognitive tool in problem solving" (p. 1). The teachers in the survey questionnaire indicated that learners learn better by looking at a drawing where they can see what is happening in class, unlike with the old way were we used to use just numbers — writing for instance two over three, then learners would mistakenly think that those are two different numbers and cannot really understand what they mean.

Researchers have shown that it is important to represent mathematical ideas in multiple ways, including real contexts, physical models (manipulatives), figures, verbalisations and written symbols (Lesh, Cramer, Doerr, Post, & Zawojewski, 2003). In the questionnaires, teachers indicated that visual models are important in such a way that it caters for all learners —

especially the slow learners. They solidify understanding, makes the lesson interesting to the learners and creates resourcefulness and enthusiasm in learners. They help learners to relate mathematics to real-world situations, and they build learner's knowledge from what is known to what will be learned. They help visual learners and those who are not competent at using symbols, to understand. It is the easy way to find the solution without any calculations involved.

In agreement, Koyalioglu et al. (2012) articulate that visual models helps in enhancing problem-solving skills by enhancing understanding and also help learners to come up with a variety of possible solution opportunities. Duval (1999) also suggests that "representation and visualisation are the core of understanding mathematics" (p. 3).

4.6.4 Visual models as a tool for enhancing conceptual understanding of common fractions

According to the findings, the teachers specified that the use of visual models is crucial to the teaching and learning of fractions and they enhances the conceptual understanding of common fractions.

In the questionnaires the teachers indicated that the use of visuals involves learners in their own learning and this makes the learners understand, because using visuals requires learners to do the work as well. This enhances their understanding, as they can see, do and touch. Seeing and touching clears up misconceptions and confusion and prints a vivid figure in the person's mind to enhance a broader and more realistic understanding. It produces a figure that will permanently stay in learners' minds and it will be remembered always, because something that is done or observed takes time to leave the memory of learners.

In support, Gray (2014) suggests that the use of fraction models/visual models in the middle grades is considered key to learners' success to master or "conceptually anchor the algorithms used to work with fractions" (p. 7). He further states that fractional models/visual models help learners to make connections among fractional concepts, which eventually deepen their conceptual understanding. Cramer et al. (2008) also state that representations play a dynamic role in learning fractions as they allow students to understand mathematical concepts and relationships as well as to make sound mathematical arguments to convince oneself and others.

Similarly, research conducted by Fazio and Siegler (2011) revealed that visual models of fractions help develop conceptual understanding of computational procedures. Thus, when teaching mathematics for understanding, the teacher must include both procedural and conceptual understanding (Wearne & Hiebert, 1988). Knowledge that has been learned with understanding provides a basis for generating new knowledge and for solving new and unfamiliar problems. When students have acquired conceptual understanding in an area of mathematics, they see the connection among concepts and procedures and can provide arguments to clarify why some facts are consequences of others.

4.7 CONCLUSION

In conclusion of this chapter, I have presented and discussed the data collected from survey questionnaires, classroom observations and interviews. Each section is discussed separately and concluded with a brief summary. The data stemming from the survey questionnaires and interviews was based on the views, perceptions and experiences of the participating mathematics teachers. The themes used to categorize and discuss data emerged from the survey questionnaires and interviews.

The teachers acknowledged that visual models are important and can be used as a tool for teaching and learning common fractions. Visual models can also be used as a tool for conceptual understanding to stimulate learners' interest in learning mathematics, to assist with reasoning, and to help them recall mathematical concepts. The teachers have also stated that the use of multiple representations enhance learners' conceptual understanding of a subject. In agreement, Ainsworth (2006) asserts that conceptual understanding can also be developed through the use of multiple representations as a means to link different mathematical domains. Some findings and conclusions, together with some recommendations will be broadly discussed in the next chapter.

CHAPTER 5

CONCLUSION AND RECOMMENDATIONS

5.1 INTRODUCTION

This chapter provides the conclusion and key findings of my study, which focused on the use of visual models to teach fractions to Grade 8 learners. The purpose of my study was to explore how selected mathematics teachers use visual models to improve teaching common fractions for conceptual understanding to Grade 8 learners as a result of an intervention programme. The chapter also presents the discussion of limitations and challenges of the study, the significance of the study, and recommendations and implications arising from the study. Finally, the chapter presents suggestions for future research, my personal reflections and the concluding remarks.

5.2 KEY FINDINGS

I present key findings according to the three instruments discussed in Chapter 3, namely the survey questionnaire, observation and the recall interview. The survey questionnaire was designed to address three research questions and to give an overview of the nature and use of visual models in schools. Observations were carried out during the intervention process with three teachers to answer the second research question. Finally, the recall interviews were conducted to provide answers to the last research question. In this study, a number of themes with specific focus on the use of visual models to teach fractions to Grade 8 learners emerged, and they are discussed in the previous chapter. However, the key findings of this study are discussed relative to my three research questions as follows:

Research Question 1

What is the nature of the different visual models used by Grade 8 mathematics teachers in the teaching of common fractions prior to participating in an intervention programme?

Forty-three teachers who participated in the survey listed some of the visuals that they use to teach fractions in their classrooms such as diagrams, concrete objects (apples, pizzas, oranges or sliced bread), charts, posters, flash cards, fractional strips and number lines. Some use technology, and the majority of the teachers indicated that they do not use any visual models to teach fractions. It is noted from the teachers' responses that they did not use the models such as area, number line and set model (see Chapter 2) to teach fractions. Although they did not use these specific visual models in their teaching of fractions, they stated that fraction-related visual models are needed to develop the following in learners:

(i) Fractional visual models foster understanding.

The majority of the teachers stated that visual models help learners to recognise fractions and know that a fraction is one number comprising of a numerator and a denominator — not two numbers. Some indicated that visuals help learners to understand the concept of fractions more satisfactorily. Visuals help them to understand the concept of fair sharing and they also help visual learners and or those who are not good at using symbols to understand fractions better.

Some teachers felt that the use of visuals involves learners in their own learning which makes the learners understand, because using visuals requires that learners to do the work as well. This enhances their understanding as they can see, do and touch. Seeing and touching eliminates misconceptions and confusion, and imprints a vivid picture in the person's mind that enhances a broader and more realistic understanding. The demonstration using seeing, touching and doing with visual models, whether on the chalk board, on paper or with concrete materials, helps learners understand the concept more easily. By the same token, some teachers indicated that learners' best understand mathematical concepts when teaching progresses from concrete to abstract, because learners tend to contextualise the abstract part of fraction more successfully.

(ii) Fractional visual models aid connection making

On aiding connections, the teachers mentioned that visualising fractions is vital in the sense that it helps learners to develop a broader concept of common fractions and to relate fractions to other concepts in mathematics such as ratio, percentages, ratio and algebra. Some indicated that visual models help learners to relate mathematics to everyday life, see realities and make comparisons or links. They further indicated that visual models assist learners to relate mathematics to real world situations, and they build learner's knowledge from what is known to what will be learned.

(iii) Fractional visual models help with knowledge construction

The surveyed teachers indicated that constructive instruction in mathematics requires that learners become involved in their learning. 'Chalk and talk' are not enough to teach concepts and learners should be involved in their learning, by doing. Learners remember more when they see, as opposed to when they just hear. Seeing helps learners to grasp the concept easily and they are less likely to forget it because they remember the visual image. Similarly, something that is done or observed stays in the memory of learners and because they continues to be built on as they continue learning.

In addition, teachers indicated that visuals by nature are practical, and make the concepts work more understandable because practical tasks are more efficient than theory and are related to real-life situations. It helps learners to work out fractions without any difficulties and construct their own conceptual knowledge and relate them to reality. Using visuals in teaching is the best method that attaches meaning to fractions, thus teaching without visual aids can make a lesson uninteresting and meaningless.

Research Question 2

How does the use of visual models enhance the conceptual understanding of fractions, if at all in Grade 8 learners?

During the classroom observations I noted that visual models were used when teachers were introducing the lessons. Teachers asked learners to recall what they already knew about fractions while using visuals models. This helped the learners to relate fractions to the model presented to them, to understand what fractions represent and how to identify a fraction in a model.

Visual models were sometimes used concurrently with verbal explanations. The teacher used the model while at the same time providing examples on how to work on what is written on the chalkboard: this helped learners to understand how fractions are either added or subtracted, as the teacher was using models while explaining how to add or subtract fractions, using a model. Visual materials were also used to connect fractions to real-life situations. Teachers used everyday examples to enhance understanding of what fractions represented. During the comparison of fractions, teachers used the concept of fair sharing to help learners understand which fraction is bigger and which one is smaller.

Visual models were sometimes used to help understand posed questions that students answered in reference to the model presented to them. This helped the teacher to assist learners during the answering of questions, by giving answers related to what was visually presented to them. Visual models were also used to check learners' understanding of the content taught in the lesson. In this case, the visual models were used in the learners' activities. This helped learners to answer questions in the activities as they were presented with the model, and at some point the learner had to present fractions on a model too

Research Question 3

What are the selected Grade 8 mathematics teachers' experiences and perceptions of working with visual models to enhance the conceptual teaching of common fractions after participating in an intervention program?

The teachers indicated that visuals assisted them to improve learners' ability to make sense of what a given fraction represented. The teachers' use of models helped to show learners exactly what a given fraction signified. They noted that especially when working with operations (addition and subtraction), learners could really see what was being added or subtracted — unlike when teaching using symbols. Teachers in this study noted that visualising fractions is one of the methods that can improve both teaching and learning by providing concrete evidence of otherwise abstract ideas and concepts. They highlighted that models themselves guide learners through to the answer, as compared to working out solutions using symbols only.

In addition, teachers indicated that visual models improve learners' understanding of the topic and it encourages full participation of learners in the lesson, since they are learning by doing. Teachers in their experience of using visual models noted that visual models help to boost learners' interest in learning. Teachers in this study experienced improvement in learner motivation due to the use of visual models. This was because of the practical element models afford to the teaching and learning of fractions and mathematics in general. Moreover, teachers indicated that teaching using models encouraged participation and it also boosted learners' thinking capability. Teachers highlighted that the use of models led to an improvement in learners' understanding of fractions. These teachers attributed the use of visual models to this improved enhancement of teaching and learning.

The teachers pointed out that visuals brought to their classrooms many advantages associated with learning by seeing. They provided their learners with the ability to recall images and pictures of previously learnt concepts.

The observed teachers found the area model to be easier and more user-friendly in comparison with the number line. Teachers felt that the number line was more difficult for them and learners to use. They conveyed that it was not easy to explain some fraction concepts and procedures, for example, showing the LCM when manipulating fractions. In addition, teachers could not use the set model because they indicated that using set models was a bit difficult for them and they feared that it could confuse their learners because of its complexity.

Lastly, they asserted that although the use of visual models is one of the best teaching approaches to enhance learners' understanding of fractions, they had some challenges. The main challenge that they all noted was that the drawing of models is time-consuming. However, they suggest that if schools could be provided with computers and projectors or other related technological tools this challenge would be alleviated. Alternatively, visual materials that are already prepared may also help to conceptually teach fractions and other mathematical domains.

5.3 SIGNIFICANCE OF THE STUDY

The study contributes towards improving the quality of teaching of fractions in Grade 8. Firstly, this study investigated the nature of visual models that teachers in the Khomas region use to teach fractions, and the findings indicated that most teachers do not use visual models to teach fractions to their learners. The findings in an intervention are aimed at making the teachers aware of how to teach fractions using other methods that can improve learners' understanding of fractions. This method will help mathematics teachers to be aware of the difficulties and misconceptions of fractions, and to plan their fraction instructions carefully to address these difficulties.

The other findings of this study suggest that the use of visual models helped learners to identify fractions, to develop a sense of the size of fractions in relation to each other, understand the meanings of fractions, find the LCM and apply basic operations with fractions. Therefore, in my view, these findings may create opportunities for more effective fraction instruction in Namibian teaching, and such instruction may enhance and improve learners' conceptual understanding of fractions.

Lastly, the findings of this study may help mathematics textbook writers and curriculum developers in Namibia to include the use of fractional models or the use of visualisation in the school syllabus and textbooks in order to enhance learner's conceptual understanding of mathematics.

5.4 LIMITATIONS AND CHALLENGES

The challenges that I encountered in this study included getting the survey questionnaires back from the mathematics teachers of the Khomas region. Some of the teachers complained that they did not get the questionnaires on time while some of them did not get them at all. Some further claimed that they gave the completed questionnaires back to their HODs, but I was unable to get them because they were misplaced. Furthermore, some questionnaires that I got back were partly completed, and in some cases, the answers were not elaborated upon.

The limitation to this study is the small sample size. The findings can thus not be generalized since the study was conducted only in three schools with only three teachers, and in one region. In addition, the study was conducted over a short period of time, focusing on a particular area of mathematics. Ideally, more time should have been spent on training the teachers and exposing them to the full potential of the use of fractional models to teach fractions using the four basic operations. The outcomes could have been more comprehensive if more than three teachers used all three fractional models for a longer period of time and in different grades. In, the three teachers only used the two models to teach fractions to Grade 8 learners and they all used them on the chalk board. If they were given enough time, they could have used different resources to teach fractions.

5.5 RECOMMENDATIONS

Even though the findings of this study cannot be generalized since it researched a small group of people all from the same region, the results can contribute to those who would like to use visualisation as teaching tools for developing learners' conceptual understanding of common fractions. Based on the results of this study, I would like to make the following recommendations: subject advisors and educational officers for mathematics should be encouraged to provide workshops for teachers on how to use visualisation as a tool, with special emphasis on aspects that develop learners' conceptual understanding of mathematics such as:

- connecting mathematics to prior knowledge
- connecting mathematics to the real world
- connecting mathematical ideas and concepts in mathematics
- the use of multiple representations or models
- connecting conceptual understanding with procedural fluency

Alternatively, teachers, especially those who participated in the intervention, can now explore more on the use of visual models. This has given them an opportunity to use them not only on the chalkboard but they could get other manipulatives such as 3-D shapes that are already designed to teach fractions. In addition, as the world is now more technologically advanced, teachers can also use technology to design clear and accurate models that they can use to teach in their classes instead of just using a chalkboard. Furthermore, the teacher-training institutions should integrate the use of visuals in their training to promote conceptual understanding. From my experience, schools are well equipped with different materials that teachers can use to come up with different models that they can use to teach their learners using different visual tools if they are trained. Teachers can then use those materials to come up with an area model that teachers who participated in the intervention said is easy to use and user- friendly.

5.6 SUGGESTIONS FOR FUTURE RESEARCH

As this study was carried out on only four components of fractions in Grade 8 (*identify/ represent fractions, comparing fractions, addition and subtraction of fractions*), it would be interesting to expand this study. The following are proposed possible areas for further research:

- Showing equivalence of fractions
- Using models to multiply and divide fractions
- Expand teaching fractions using visual models to other grades
- Use other models that can be used to teach fractions such as set model

5.7 PERSONAL REFLECTIONS

It was a pleasure to have walked this research journey. As a beginner researcher, I realize that research is a long journey that requires attention and an enormous amount of time. It is a journey where you are required to dedicate time to critically read, analyse and make sense of other people's writing. This has expanded my knowledge and understanding of visualisation in mathematics, common fractions and the constructivism learning theory.

This research study also allowed me to reflect personally on how I teach fractions to my learners. I will definitely support and encourage the use of visual models to teach fractions and try to find more visual tools that I can use to teach other topics in mathematics. The research has created a bond between me and other teachers in the region, and as a result we now discuss and share teaching materials.

5.8 CONCLUSION

In this chapter, I presented the key findings of this study, the significance of the study, the limitations, recommendations and suggestions for future research. My personal reflections concluded my research study. From this study, it is clear that visual models are essential resources for the teaching and learning of fractions. It is observed that teaching fractions using visual models helps learners to understand fractions more effectively.

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APPENDICES

Appendix 1: Mathematics teachers survey questionnaire

Section A

General information

Name……………………………………………………….....................................

School.……………………………………..

1. How old are you? *Tick one column only*

Under 25	
25-29	
30-39	
40-49	
50-59	

2. Are you female or male? *Tick one column only*

Male	
Female	

3. By the end of this school year, how many years will you have been teaching altogether? *Please round to the nearest whole number*...

4. (a) What teacher training qualification do you have? *Tick the appropriate column*

Grade 12	Certificate/ diploma in education	Undergraduate degree in education	Honour degree in education	Masters of education

5. Indicate with a tick which pedagogical practice characterise a good mathematics teacher.

Tick one column in each row	*Not important*	*Somewhat important*	*Very important*
Emphasises formulas and procedures			
Teachers in a sequential and logical manner			
Uses manipulatives or visual aids to teach			
Encourages learners to be creative, collaborative, reflective and inquiry based.			
Links mathematics to other subjects and to the real world situations			
Encourages learners to construct knowledge by visualising the mathematical concepts that are being taught.			

Section B

In this section, the questions refer to your mathematics teaching of common fractions. Please remember that this study's focus is on visualisation and the nature of fractional visual materials that teachers can use to teach common fractions to enhance the conceptual understanding of common fractions in grade 8 learners. *(Fractional visual materials refers to the manipulatives that a teacher can use to teach fractions).*

1. How do you teach common fractions in your classroom?
2. Have you ever experienced any difficulties or challenges making learners understand the topic of common fractions? If yes please elaborate.
3. In you teacher training did you cover the topic of visualisation and its role in teaching and learning of mathematics?
4. In your teaching, how would you illustrate the topic of common fraction in a visual manner?
5. Which visual materials do you use when teaching common fractions?
6. Where do you get these teaching materials and who prepares them for you?
7. Explain the importance of using visual materials in teaching of common fractions?
8. In your mathematics lessons, how often do you use visual materials to do the following:

Tick one column in each row	*Never*	*Some lessons*	*Most lessons*	*Every lesson*
Building on learners prior knowledge				
Emphasising connections of ideas and concepts in mathematics				
Connecting mathematics to the real word				
The use of multiple representations to solve problems				

9. In mathematics lessons, when do the use of fractional visual materials work best?

Tick one column in each row	*Never*	*Some lessons*	*Most lessons*	*Every lesson*
Explaining the reason behind the idea				
Representing and analysing relationships				
Working on a problem for which the is no immediate obvious method or solution				
To solve problems				

10. In your view, can the use of visual materials help learners to understand the topic of common fraction?

Appendix 2: Observation schedule

Name ……………………………………………………………………………………………….

Date……………………………………………………………………………………………....

Topic………………………………………………………………………………………………

Section A

This schedule is intended to capture the use of visual models in the classroom.

	Visual models			
Observable indicators	***Area model, number line model and set model*** The part of the area covered as it relates to the whole unit			
Coding	**1**	**2**	**3**	**4**
Unit fraction The teacher introduce fraction through unit fraction				
The teacher develops unit fractions though counting, composing and decomposing unit fractions using models				
The teacher shows equal- partition using models				
Representation/ identifying of fraction The teacher use models to introduces proper fraction, improper fraction and mixed numbers simultaneously				
Comparing of fraction The teacher use models to compare different fractions. Converting to a common denominator or Using Unit Fractions.				
Addition and subtraction of fractions The teacher uses models to build on learners' intuitive understanding of the operation.				
The teacher uses models to solve problems (addition and subtraction)				

The Likert scale below will be used to analyse the visual model used in the classroom

Codes	**Categories**	**Descriptions (**fractional visual models)
1	Strong evidence	Rich evidence of fractional visual models used (at least 3-4 different examples using models are given)
2	Medium evidence	Some fractional visual models used. (at least 2 or more examples using there models are given)
3	Weak evidence	Few fractional visual models used. (only1 or 2 examples given using models)
4	No evidence	No fractional visual model used.

Section B

This scale is intended to capture the learner's conceptual understanding of fraction through the use of models in the classroom.

Conceptual understanding indicators	Approaches to enhance conceptual understandings through teaching. Explanation of each concept in relation to fractional model (area, number line and set model), the observable indicators.	Coding			
		1	2	3	4
Building on prior knowledge	The teacher uses fractional model that are familiar to what learners know to teach faction. The teacher makes use of what the learners already knows and draws from their past experiences.				
Emphasising on connections of ideas and concepts in mathematics	The teacher uses fractional models to establish connections between different fraction concepts and establish relationships. The teacher uses fractional models to explore the relationship between concepts and how they are linked. Teacher encourages learners to explore connections between the concepts.				
Connecting mathematics to real world	The teacher uses fractional models to connect mathematics to real world examples. The teacher uses every day-shapes and explain it in connection with the used fractional model.				
The use of multiple representations or models	The teacher uses fractional models to represent fractions in multiple ways. The teacher uses fractional model to illustrate how to represent fractions, ordering, addition and subtraction of fraction in a multiple ways and visually explain the concepts and explore the relationships.				

Appendix 3: Interview schedule

1. During our lessons you have been teaching learners on how to work out fractions using fractional visual models. In your view and experience do you think it is important to use visual models when teaching fractions? Why, please elaborate.
2. In your opinion and experience, how did you make use of visual models to help learners to understand the topic of common fraction? Please provide me with some examples.
3. Looking at the models that you used in class, which model in particular do you think can help learners to understand the content you taught better?
4. What experiences did you gain working with visual models that may enhance the teaching of common fractions as a result of participating in the intervention program?
5. What challenge did you experience using these models? How can these challenges be overcome.
6. What do you see different as compared to the teaching of fractions you have been doing as you taught using visual models?

Appendix 4: Lesson preparation

Teaching common fraction using visual models

Lesson 1

Teaching content	**Learning objective;** **at the end of the lesson learners should be able to:**
1. Identifying fractions	• Name fractions from the visual model presented to them • Draw the visual model and present fractions both proper and mixed numbers on the model • Equal partitioning of the model
2. Comparing fractions	• Compare fractions with same or different denominators using models as well as mixed numbers

Lesson 2

3. Addition of fractions	• Add together proper fractions with same and different denominators using models • Look for lowest common multiple of the denominators • Add together mixed numbers using models

Lesson 3

4. Subtraction of fractions	• subtract proper fractions with same and different denominators using models • Look for lowest common multiple of the denominators • subtract mixed numbers using models

Appendix 5: Emerged themes from a questionnaire

The questions used in the survey questionnaire of the teachers were clustered according to the following themes.

Themes	Question No.
Teaching of fractions	Q 1
Type of visual models teachers use	Q 5 & Q 6
Teachers views of how to teach fraction visually	Q 3 & Q 4
Significance of using visual models when teaching fractions	Q 7
The use of visual models to enhance conceptual understanding of common fractions	Q 10

Appendix 6: Emerged themes from the interviews

Themes	Question No.
Teaching of fractions	Q 6
Type of visual models teachers use	Q 2
Teachers views of how to teach fraction visually	Q 2 & Q 3
Significance of using visual models when teaching fractions	Q 1
The use of visual models to enhance conceptual understanding of common fractions	Q 3 & Q 4
Challenges or limitations in the use of visual models	Q 5

Appendix 7: Ethical clearance

RHODES UNIVERSITY

Grahamstown • 6140 • South Africa

EDUCATION FACULTY • PO Box 94, Grahamstown, 6140
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PROPOSAL AND ETHICAL CLEARANCE APPROVAL

Ethical clearance number 2017.12.08.09

The minute of the EHDC meeting of 05 December 2017 reflect the following:

2017.12.8 CLASS B RESTRICTED MATTERS
MASTER OF EDUCATION RESEARCH PROPOSALS

To consider the following research proposal for the degree of Master of Education in the Faculty of Education:

Mr Aune Katenda (13K6700)

Topic: Investigating how the use of visual models can enhance the teaching of common fractions for conceptual understanding in grade 8 learners.

Supervisor: Professor M Schäfer
Co-Supervisor: Dr C Chikiwa

Decision: *Approved*

This letter confirms the approval of the above proposal at a meeting of the Faculty of Education Higher Degrees' Committee on the 5 December 2017.

The proposal demonstrates an awareness of ethical responsibilities and a commitment to ethical research processes. The approval of the proposal by the committee thus constitutes ethical clearance.

Sincerely

Ms Zisanda Sanda
Secretariat of the EHDC, Rhodes University
8th December 2017

Appendix 8: Approval letter from the Director of education Khomas Region

REPUBLIC OF NAMIBIA
KHOMAS REGIONAL COUNCIL
DIRECTORATE OF EDUCATION, ARTS AND CULTURE

Tel: [09 264 61] 293 9411
Fax: [09 264 61] 231 367/248 251

Private Bag 13236
WINDHOEK

File No: 12/3/9/1

Aune Kashikuka Katenda
P.O. Box 158
Windhoek

REQUEST FOR PERMISSION TO CONDUCT RESEARCH AT SCHOOLS IN KHOMAS REGION

Your letter dated 15 January 2018 on the above topic has reference.

Permission is hereby granted to you to do research for your Master's Degree Program Education with the topic of research title: "Investigating how the use of visual models can enhance the teaching of [redacted]

The following must be adhered to:

- Permission must be granted by the School Principal;
- Teaching and learning in the respective schools should not be disrupted;
- Teachers who will take part in the research should do so voluntarily;
- A copy of your thesis with the findings/recommendations must be provided to the Directorate of Education, Arts and Culture, Khomas Regional Council.

I trust this confirmation will assist.

Yours sincerely

PP

MINISTRY OF EDUCATION
AND CULTURE
PRIVATE BAG 13236 WINDHOEK
22 -01- 2018
DIRECTOR
KHOMAS REGION

Gerard N. Vries
Director of Education, Arts and Culture

Appendix 9: Letter requesting permission from the Principals

RHODES UNIVERSITY
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REQUEST FOR PERMISSION TO CONDUCT RESEARCH AT YOUR SCHOOL

Aune K Katenda

P.O. box 158

Windhoek

15 January 2018

The School principals

Khomas Regional council

Directorate of Education, Art and Culture

Private bag 13221

Windhoek

To whom it may concern

Re: Request for permission to conduct research at your school

My name is Aune Kashikuka Katenda, and I am a Master's degree student at Rhodes University (RU) in Grahamstown, South Africa. The research I wish to conduct for my Master's full thesis requires me to send questionnaires, observe and interview Mathematics teachers at high school level. The research study is entitled "Investigating how the use of visual models can enhance the teaching of common fractions for conceptual understanding in grade 8 learners". The aim of this research is to create an awareness amongst mathematics education researchers and teachers about the role of visualisation processes in the teaching and learning of mathematics. The participation of teachers in this study is important because it will enhance their understanding of fraction teaching using visual models and it will enhance your learner's conceptual understanding of fractions. This research will be conducted under the supervision of Professor Marc Schäfer and Dr Clemence Chikiwa.

This letter serves to seek formal consent to approach mathematics teachers, the learners and the parents of the learners who will be involved in this research. Further, I would be grateful if I may access appropriate documents at their discretion. For this reason I request your permission to visit schools premises to conduct my research as outlined in my research proposal. The programme will not disturb the school programme because it will take place after school

I have received ethical clearance from Rhodes University, a copy of which I attached to this request. As part of this I undertake to ensure that the name of the schools and all participants will be replaced with pseudonyms and that all the material I collect as part of the research will be accessible only to myself and my supervisors.

Upon completion of the study, I undertake to provide you and the teachers with access to the research findings. If you require any further information, please do not hesitate to contact me at 0812984707 or katendaa@gmail.com.

Thank you for your time and consideration in this matter.

Yours sincerely

Aune Kashikuka Katenda

Student number: 13k6700

Rhodes University

Appendix 10: Participating teachers consent forms

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INFORMED CONSENT FORM

Research Project Title:	Investigation how the use of visual models can enhance the teaching of common fractions for conceptual understanding in grade 8 learners.
Principal Investigator:	Aune Kashikuka Katenda

Participation Information

- I understand the purpose of the research study and my involvement in it
- I understand the risks and benefits of participating in this research study
- I understand that I may withdraw from the research study at any stage without any penalty
- I understand that participation in this research study is done on a voluntary basis
- I understand that while information gained during the study may be published, I will remain anonymous and no reference will be made to me by name or student number
- I understand that data collection requirements particular to this research, e.g. observation results, video recording and interviews will be used
- I understand and agree that the interviews will be recorded electronically
- I understand that I will be given the opportunity to read and comment on the transcribed interview notes
- I confirm that I am not participating in this study for financial gain
- I understand that my personal details will not be reviled

Information Explanation

The above information was explained to me by: Aune Katenda

The above information was explained to me in English and I am in command of this language

Voluntary Consent

I, hereb[redacted]e above-mentioned research.

Name: [redacted]	Date: 02/ 02/ 2018
Signature	

Investigator Declaration

I, Aune Kashikuka Katenda, declare that I have explained all the participant information to the participant and have truthfully answered all questions ask me by the participant.

Signature:	Date: 02/ 02 / 18

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INFORMED CONSENT FORM

Research Project Title:	Investigation how the use of visual models can enhance the teaching of common fractions for conceptual understanding in grade 8 learners.
Principal Investigator:	Aune Kashikuka Katenda

Participation Information
• I understand the purpose of the research study and my involvement in it • I understand the risks and benefits of participating in this research study • I understand that I may withdraw from the research study at any stage without any penalty • I understand that participation in this research study is done on a voluntary basis • I understand that while information gained during the study may be published, I will remain anonymous and no reference will be made to me by name or student number • I understand that data collection requirements particular to this research, e.g. observation results, video recording and interviews will be used • I understand and agree that the interviews will be recorded electronically • I understand that I will be given the opportunity to read and comment on the transcribed interview notes • I confirm that I am not participating in this study for financial gain • I understand that my personal details will not be reviled

Information Explanation
The above information was explained to me by: Aune Katenda
The above information was explained to me in English and I am in command of this language

Voluntary Consent	
I, hereby voluntarily consent to participate in the above-mentioned research.	
Name: [redacted] Signature	Date: 02/ 02 /2018

Investigator Declaration	
I, Aune Kashikuka Katenda, declare that I have explained all the participant information to the participant and have truthfully answered all questions ask me by the participant.	
Signature:	Date: 02/ 02 / 18

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INFORMED CONSENT FORM

Research Project Title:	Investigation how the use of visual models can enhance the teaching of common fractions for conceptual understanding in grade 8 learners.
Principal Investigator:	Aune Kashikuka Katenda

Participation Information
• I understand the purpose of the research study and my involvement in it • I understand the risks and benefits of participating in this research study • I understand that I may withdraw from the research study at any stage without any penalty • I understand that participation in this research study is done on a voluntary basis • I understand that while information gained during the study may be published, I will remain anonymous and no reference will be made to me by name or student number • I understand that data collection requirements particular to this research, e.g. observation results, video recording and interviews will be used • I understand and agree that the interviews will be recorded electronically • I understand that I will be given the opportunity to read and comment on the transcribed interview notes • I confirm that I am not participating in this study for financial gain • I understand that my personal details will not be reviled

Information Explanation
The above information was explained to me by: Aune Katenda
The above information was explained to me in English and I am in command of this language

Voluntary Consent	
I, hereby voluntarily consent to participate in the above-mentioned research.	
Name: [redacted] Signature	Date: 01/ 02/ 18

Investigator Declaration	
I, Aune Kashikuka Katenda, declare that I have explained all the participant information to the participant and have truthfully answered all questions ask me by the participant.	
Signature:	Date: 02/ 02/ 18

Appendix 11: A sample of approval from the parents/guardian

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REQUEST FOR PERMISSION FROM PARENTS

Aune K Katenda

P.O. box 158

Windhoek

15 January 2018

Dear Parents or guardian

Re: Request for permission for your child to take part in my research.

My name is Aune Kashikuka Katenda, and I am a Master's degree student at Rhodes University (RU) in Grahamstown, South Africa. The research I wish to conduct for my Master's full thesis requires me to send questionnaires, observe, interview and video record Mathematics teachers at a high school. This research will be conducted under the supervision of Professor Marc Schäfer and Dr Clemence Chikiwa.

This letter serves to seek formal consent from you that I may video-record your child's teacher when he/she teaches mathematics whilst your child participates in his/her class. I wish to video-record your child's teacher for a period of 2 – 3 months while he/she teaches fractions. The children of the class will not be part of the video recordings, but in the event of your child's face appearing in the video I will blur his/her face. Please sign the table below if you grant permission.

I, hereby voluntarily grant consent that you may video-record in my child's mathematics class for the above-mentioned research study.	
Parent or guardian name: Parent or guardian signature:	Date: 10 /02 / 2018

Thank you for your time and consideration in this matter.

Yours sincerely

Aune Kashikuka Katenda

Student number: 13k6700

Rhodes University