

VISUALISATION PROCESSES IN TEACHING PATTERNS AND THEIR GENERALISATION: PERCEPTIONS AND EXPERIENCES FROM SENIOR PHASE MATHEMATICS TEACHERS

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SIMON CHATIMA

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ABSTRACT

This study investigated how senior phase mathematics teachers used visualisation approaches to teach patterns and their generalisation as a result of an intervention programme. This project is an integral component of the VIPRO MATHS Project and one of a number of post-graduate research projects which has a particular focus on visualisation processes in mathematics education in the Southern African region. This case study of Senior Phase mathematics teachers in the Chris Hani West District (CHW) in the Eastern Cape Province of South Africa used a social constructivist theory. The study is grounded within an interpretive paradigm explanatory qualitative method design. It surveyed thirty-seven Senior Phase mathematics teachers from schools in the CHW district to get an idea of how teachers teach patterns and their generalisation, prior to an intervention process. After the survey, nine teachers were purposively selected to take part in an intervention programme. They were then observed (teaching) and interviewed to get their perceptions about what factors enable and/or constrain the teaching of patterns and their generalisation when visualisation approaches were used. The data from the survey questionnaire responses, the lesson observations and the interviews were analysed qualitatively.

The findings from the survey questionnaire responses revealed that teachers in CHW district value the use of visualisation approaches, although they used visuals to teach patterns in a way that lacked depth, skill and rigor prior to the intervention. Challenges such as difficulty in keeping abreast with the prescribed syllabus, the lack of resources, time constraints, and the limited knowledge of algebra learners have for successfully transferring pictorial representations to symbolic presentations, emerged from the survey data. This necessitated the need for an intervention which focused on the use of a variety of visual strategies and tools to teach patterns and their generalisation effectively. Data from lesson observations showed that all observed participant teachers used visuals to generate image- related questions and created platforms for lively classroom discussions. The teachers used the generated questions to build on learners' prior knowledge and to promote learner understanding of pattern dynamics that stem from pictorial representations. The data from the interviews revealed that teachers' perceptions had drastically changed after the intervention. This was because they were now equipped and empowered to effectively make use of visual strategies that enabled them to: build on learners' prior knowledge, to manipulate visuals for the purpose of prompting learning, develop cognitive perception in learners, cultivate learner innovation and originality and enhance problem solving techniques.

This study concludes that the use of visuals enhanced conceptual teaching of patterns and their generalisation in the observed participants. This has the potential to provide mathematics education

researchers and curriculum developers with a strong basis to include visualisation processes and strategies into the design of policy documents and to initiate further research efforts. The implications for the teachers are that learners need to be taught ways to create and interpret visual representations in order to understand the dynamics of transferring a pictorial pattern to its resultant symbolic representation expressing pattern regularity. It is also hoped that the results of this study will be utilised by mathematics subject advisers and teacher training institutions to improve the teaching of patterns and their generalisation.

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DEDICATION

I especially dedicate this thesis to my late mother, Getrude Gladys Sibanda. Thank you mother for teaching me to persevere until dreams come true and for instilling in me a spirit of hard work despite the prevalence of seemingly insurmountable challenges. I will live to fulfil your undying hopes of the prosperity of our family.

DECLARATION OF ORIGINALITY

I Simon Chatima student number (10C7589), hereby declare that this thesis entitled ***“Visualisation processes in teaching patterns and their generalisation: Perceptions and experiences from senior phase mathematics teachers”*** is my own work and a product of my research efforts. It has not been submitted in any form to another institution. Where I have used the ideas of others and from other publications or any other sources, I have fully acknowledged these in accordance with Rhodes University, Education Department reference guide.

Simon Chatima

Date: November, 2021

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ACRONYMS USED IN THE STUDY

APV	Applicability of Visuals
BPK	Building on Pre-Existing Knowledge
BUV	Benefits of Using Visuals
CHW	Chris Hani West District
CKC	Collective Knowledge Construction
CNS	Constraining Factor of Visualisation
COA	Counting Analysis
CUV	Challenges of Using Visuals
DRA	Difference Rate Analysis
ENB	Enabling Factor of Visualisation
EXA	Explicit Analysis
FLV	Flexibility of Visuals
FUV	Frequency of Use of Visuals
PBS	Problem Solving Techniques
REA	Recursive Analysis
SYI	Symbolic Inscription
TIMSS	Trends in International Mathematics and Science Study
TPE	Teacher Perception and Experience
TPU	Teaching Approaches Used
USV	Usability of Visuals
VIU	Visual Usage
VQG	Visual Question Generation
VTI	Visual Tool Inspection
VTT	Visual Tool Transformation
WOA	Whole Object Analysis

CHAPTER 1: CONTEXT TO THE STUDY

1.1 INTRODUCTION

The purpose of this study was to explore the teaching of patterns and their generalisation by selected Senior Phase mathematics teachers. The study endeavoured to uncover factors that enable and/or constrain the teaching of patterns and their generalisation when visualisation approaches are used. The emerging teachers' perceptions were explored, based on the teachers' experiences before and after an intervention programme. This chapter begins with the research context and background from which I discuss the genesis of this study and its relation to visualisation in mathematics education. This discussion is followed by an outline of the research goals and the research questions that guided this study. The chapter then provides a summative overview of the methodology that I followed, the data analysis, the theoretical framing and the significance of the study. The limitations of this study are also listed, and the chapter concludes with a brief outline of the structure of the thesis that provides indicators of what is discussed in each chapter.

1.2 BACKGROUND OF THE STUDY

The teaching of patterns and their generalisation in mathematics uses a variety of methods that may or may not incorporate visual approaches (Barbosa & Vale, 2015). The study of patterns in school mathematics has been the focus of some research studies in the past years. In the teaching of patterns, the transition from working with numbers to algebra when generalising patterns has its own challenges. While any mathematical concepts, ideas and methods "have a richness of visual relationships that are intuitively representative in a variety of ways" (Guzman, 1996, p. 2), teaching of patterns may benefit immensely from visual approaches. Some prescribed textbooks in South Africa's school system (*Jet Educational Services Resources, 2015; Platinum Mathematics Grade 9, 2013*) are full of pictures, which in my experience are rarely used as visual tools for enhancing learner understanding by teachers. Teachers prefer routine methods involving algorithms and simple narrative descriptions (Warren, 2005a). In many cases, given pictorial sequences are simply reduced to numeric sequences (Samson, 2012) without engaging with underlying mathematical concepts. It appears that this practice by mathematics teachers negates the prospects of meaningful utilisation of visualisation techniques in the mathematics classroom. The South African National Curriculum Statement (NCS) policy document for Grades 7-9 specifically emphasises the use of visualisation by suggesting that the aim of teaching mathematics is to "communicate effectively using visual,

symbolic and/or language skills in various modes” (Department of Education, 2011, p. 5). It was from this premise that I decided to investigate the perceptions of Senior Phase mathematics teachers about what enables and/or what constrains the teaching of patterns when visuals are used before and after an intervention.

In South Africa the teaching of mathematics, particularly patterns and algebra, has long been the concern of many individuals (curriculum developers, academic researchers, tertiary institutions, teachers, recruitment agencies, parents and learners). This list also includes teachers of mathematics (Budaloo, 2015) like myself. Despite the urgency for developing working strategies to teach mathematics concepts involving patterns and generalisation most interventionist efforts seem to be relatively unsuccessful. In a diagnostic report by Trends in International Mathematics and Science Study (TIMSS, 2016) on the 2015 Grade 9 National Mathematics Exam, the information in Table 1.1 was provided for exam items involving pictorial patterns.

Table 1.1: Table 1.1: Comparison of percentage learner performance in patterns and algebra

Part C (patterns & algebra)	Correct responses	Partially correct responses	Incorrect responses	Omitted
South Africa	8.4%	0.4%	79.6%	1.8%
International average (n=38)	19.3%	1.6%	49.9%	29.3%

(Source: TIMSS Diagnostic Report, 2016, p. 65)

The report portrays the inherent problems in the teaching and learning of patterns in South African schools as most learners failed to identify the rule or adequately explain a pattern even when the problem was presented in pictorial form. Thus, learners exhibited an unsound knowledge of number patterns and algebra.

In teaching mathematical concepts such as patterns and algebra (the heartbeat of mathematics), teachers need to show learners how to navigate and interpret pictorial forms by extracting essential information from set problems. They should then translate these from their visual reasoning into the language of mathematics. Similar reports like the one above (see Table

1.1) have been made over several years. Despite these diagnostic analyses and recommendations to teachers, results in mathematics have not shown a marked improvement over five years as shown by the National Senior Certificate DBE performance report (see Table 1.2).

Table 1.2: DBE Percentage candidate performance in Mathematics 2015-2019

Year	2015	2016	2017	2018	2019
% of learners attaining above 30%	49.1	51.1	51.9	58.0	54.6

Intervention initiatives continue to dominate the mathematics education space in South Africa and the wider Southern African region. This is done as a means of encouraging both mathematics teachers and learners to participate in mathematics development initiatives designed with the intention of presenting the diverse nature of mathematics (Budaloo, 2015). The initiatives are also a means of encouraging learners to appreciate the relevance of mathematics in solving problems that plague our societies.

The search for more effective ways of teaching mathematics is paramount for most mathematics researchers (Budaloo, 2015). There are many research projects which advocate the use of visual approaches in the teaching of mathematics. However, not all research has shown a positive correlation between visual strategies and mathematical understanding (Duncan, 2010; Budaloo (2015). The poor results in South African schools both at Senior Phase level and at Senior Certificate level, has created some concern, and the different stakeholders in education have felt that it was becoming difficult to attain success. As a result, researchers have to now constantly look for new ways to address this concern (Fleisch, 2008). This study focuses on finding alternate ways, such as using visual strategies to enhance the teaching of patterns, since much literature seems to suggest the unlimited benefits of using visual approaches in the teaching of mathematics (Malaty, 2006; Chiappini & Bottino, 2002).

There is a need to research effective pedagogical strategies that can enhance the successful teaching of mathematics in South African schools. Visualisation approaches are henceforth considered some of the ways that can develop effective strategies in teaching mathematics concepts. This study has focused on exploring the views of teachers on the use of visual approaches in teaching patterns and was guided by the following research goals and questions:

1.3 RESEARCH GOALS

The goals of this study were three-fold:

1. to investigate how widespread was the use of a visual approach to teaching patterns by Senior Phase mathematics teachers in CHW District, prior to an intervention programme.
2. to investigate how selected Senior Phase mathematics teachers apply visual approaches to teaching patterns and their generalisation after an intervention programme.
3. to investigate the experiences and perceptions of selected Senior Phase mathematics teachers in order to establish what factors enabled and/or hindered the teaching of patterns when visuals were used before and after an intervention.

1.4 RESEARCH QUESTIONS

1. How did Senior Phase mathematics teachers in the CHW district teach patterns and their generalisation prior to participating in an intervention programme?
2. How do selected Senior Phase mathematics teachers teach patterns and their generalisation after participating in an intervention programme?
3. What are the selected teachers' experiences and perceptions of teaching patterns and their generalisation prior to and after participating in an intervention programme?
4. What are the enabling and/or constraining factors of applying a visualisation approach to teaching patterns and their generalisation?

1.5 RESEARCH METHODOLOGY

This research study is framed within an interpretive paradigm and informed by social constructivist views. The main purpose of an interpretive study is to describe and interpret phenomena with an attempt to get shared meaning with others (Pollard, 2002). Interpretivists believe that reality is socially constructed (Maree & Pietersen, 2016), following that people studied in their social contexts or natural environments yield comprehensive understanding of the perceptions they have of their own activities. It was my role as a researcher in the interpretive paradigm to comprehend, articulate and clarify social reality through the eyes of the different participants (Cohen, Manion & Morrison, 2011). This study applied two methodological approaches. The first one was a survey conducted on a group of forty Senior Phase mathematics teachers in the CHW district, whilst the second one comprised of a case

study on nine selected teachers that participated in an intervention programme from which they were interviewed after an implementation process.

Data was collected through a survey questionnaire, lesson observations, stimulated recall interviews and a focus group discussion. The selected teachers utilised teaching programmes developed during the intervention programme. The intervention programme consisted of three workshops (which were conducted within the research sites) in which visual strategies were learnt and discussed in detail and visual teaching tools were developed. Lesson observations, conducted during the implementation phase, were video-recorded. Each teacher participant was scheduled to be observed three times, in three different cycles which included a pilot study, in his/her own classroom environment. My focus as a researcher was on how teachers made use of various visual strategies and techniques to promote visualisation when teaching patterns and their generalisation.

Thirty-seven Senior Phase mathematics teachers responded to a survey questionnaire that was administered in the sub-district of CHW (Komani CMC) to establish how widespread the use of visual approaches was when teaching patterns. There are about 160 Senior Phase mathematics teachers in CHW district according to a curriculum coverage report of 2019 (Zambodla, 2019). The survey was followed by an intervention programme that involved nine teachers, purposively selected. The nine teachers included three Grade 7 teachers, three Grade 8 teachers and three Grade 9 teachers. The intervention focused on how to effectively teach patterns using visual tools, followed by how to develop teaching programmes with ample visual tools and strategies to use when teaching patterns and their generalisation.

The teachers then implemented the visual strategies in their respective classes and were video recorded in the process. Each video-taped lesson was followed by a reflective one-on-one stimulated recall interview. Each participant was observed and video recorded at least twice, which included a pilot study observation. The interview process was done to establish individual participants' views and experiences on whether the use of visuals enhanced or impaired the teaching of patterns. At the end of the implementation process a focus group interview was conducted with the remaining eight participants. The focus group interview, which focused on the actual teaching process using visuals and on factors of visualisation which enabled and/or constrained the teaching of patterns, was also audio recorded and transcribed. The participants reflected on how visuals were used in teaching and gave their views about whether visualisation enhances and/or impedes the teaching of patterns. Thus, collective meaning of the investigated phenomenon was constructed.

1.6 DATA ANALYSIS

The process of data analysis started immediately, in concurrence with data collection from the initial survey. Data analysis was carried out in three stages. In Stage 1, data from the survey questionnaire responses of selected Senior Phase teachers in CHW district was analysed qualitatively and discussed according to emerging themes. The choice of procedures to use when analysing data from questionnaire responses was limited to the themes that were derived from the questionnaire items.

In Stage 2 of the analysis process, I qualitatively analysed the data from lesson observations. Analytical tools specially developed for the purpose of observing the implementation of the intervention programme were used to analyse how five visual strategies were used by participants in teaching patterns and their generalisation. In Stage 3, a narrative thematic analysis approach, defined as a method of identifying, analysing and reporting patterns within data collected from interviews, was used (Braun & Clark, 2006). Interviews were used as crucial tools to get less measurable data from direct sources (Canary, 2019). Analysing interviews thematically allowed me to provide relatable stories and perspectives.

Collective meanings and constructions from the interviews, which included focus group discussions, were analysed to get an understanding of what enables and/or constrains the teaching of patterns when visual approaches were used.

1.7 SIGNIFICANCE OF THE STUDY

This research study, in the context of the education curriculum in South Africa, has the potential to inform curriculum developers and mathematics researchers about including visualisation approaches into the design of policy guidelines. Besides contributing to my knowledge about visualisation in mathematics education, this project also helped me and other mathematics teachers to expand knowledge on how to effectively use instructional methods that involve visual strategies. This knowledge was acquired through information sharing with participants through collective knowledge construction in both the intervention and the interview process.

Selected Senior Phase mathematics teachers were potentially empowered with different approaches involving teaching patterns, using visualisation strategies. The findings of this study have the potential to assist researchers in mathematics education and curriculum developers, to integrate visual approaches in their developmental initiatives to ensure adequate preparation of teacher trainees and practising teachers, in order to enhance innovative ways of teaching for progress.

The findings of this study may assist authors of mathematics textbooks to include the use of visualisation in their textbooks. This will enhance effective teaching for the development of critical thinking and innovation among learners. Newly developed textbooks will empower teachers with visually innovative skills of handling mathematics concepts like patterns and their generalisation.

1.8 LIMITATIONS

It was not easy to get the completed questionnaires for collecting initial data for the study from survey participants (Senior Phase mathematics teachers). Covid-19 pandemic restrictions made it very difficult to distribute the questionnaires and get speedy responses from participants. There was a slight deviation from the original plan of having the questionnaires completed in an awareness workshop. Instead, the workshop was conducted online. Some questionnaires were completed online but I managed to have the majority of questionnaires completed manually. I had to seek the assistance of the Senior Phase mathematics subject advisor to distribute the questionnaires to various schools, even though it was not easy to get the completed questionnaires back. There were gaps in some of the questionnaires and some of the responses lacked adequate elaboration.

Implementation and observations took an unnecessarily longer time than was planned, since participating schools had their classes coming to school on a rotational basis due to Covid-19 pandemic restrictions. In some instances, there were clashes in the timetables of different schools, which further exacerbated my observation time frame. In all the participating schools lesson durations had been reduced from sixty-minute to thirty-minute lessons. Teachers had to deal with stringent time limitations to successfully deliver worthwhile lessons. I had to ask for ten-minute extensions of the lessons from the school administrators to at least observe adequately taught lessons. Teaching with visuals demands ample time to for effective presentation (Bozkurt, 2017 cited in Liaga, 2019).

The other challenge I encountered was that of interviewing teachers after they had taught their lessons. Teachers often came up with excuses and this had a negative impact on my interview plan. There were also several interruptions during the interview processes with some individual participants as some interviews had to be conducted from the comfort of participants' homes. However useful insights on the use of visualisation strategies in teaching patterns were provided by the study.

1.9 OPERATIONAL DEFINITIONS

Visualisation is “the ability, the process and product of creation, interpretation, use of and reflection upon pictures, images, diagrams, on paper or with technological tools, with the purpose of depicting and communicating information, thinking about and developing previously unknown ideas and advancing understanding” (Arcavi, 2003, p. 217). The study focused on internal and external visualisations of linear patterns that were presented in pictorial format.

Visual strategies refer to the use of visual tools (for example pictures, diagrams, tables, flow charts and manipulatives) that involve didactic techniques in teaching mathematics concepts. This study explored the use of such techniques in teaching patterns and their generalisation. Some of the visual strategies that selected teachers commonly used in this study included visual tool generation, visual tool inspection, image use and visual tool transformation.

Generalisation strategies are the fundamental activities involved in shaping mathematics ideas and applications, showing process in a broader context to help problem solving and cultivate knowledge frameworks of those processed products (Kashefi, Hashemi, & Abu, 2013; Tall, 2002). This study focuses on five such generalisation strategies. These are: recursive analysis, counting, whole-object analysis, difference rate analysis and explicit analysis.

Enabling factors are forces that facilitate or enhance individual or collective efforts to effectively work through tasks and processes for quality construction of knowledge and understanding. Enabling factors examined in this study were predisposed to include knowledge, beliefs, values, attitudes and self-efficacy that appeal to people’s motives for behaviour change in carrying out new and previously experienced activities (Atkin, 1996).

Constraining factors are items or factors that restrict or limit the execution of certain tasks that influence positive interest and perceptions. If one enabling factor is not taken into consideration and/or neglected, that aspect may become a constraining factor in itself (Rivera, 2011). Constraining factors in this study were taken to be those inhibiting aspects of visualisation that prevented the successful teaching of patterns when visual approaches were used.

1.10 THESIS OUTLINE

This thesis comprises of five chapters, the contents of which are outlined below.

1.10.1 Chapter 1 (Context of the study)

This chapter gives a brief introduction of the thesis, that includes the background to the study, research goals and research questions, a brief explanation of the methodology and analysis

process, the significance of the study and limitations thereof, as well as the operational definitions that were used in the study.

1.10.2 Chapter 2 (Literature review)

In this chapter I discuss the conceptual framework and literature that informed and shaped the analysis and interpretations of the findings of this study. The chapter begins by defining patterns and generalisation. This is followed by a thorough examination of the various generalisation strategies used in teaching patterns by visual to non-visual approaches. The chapter goes on to give a working definition of visualisation and its use in teaching patterns. I then proceed to examine what literature says about factors that enable and/or constrain the teaching of patterns when visualisation approaches are used. A brief discussion then follows of how the use of visualisation can develop conceptual and critical understanding in learners. This is followed by a detailed theoretical framework of how social constructivism can be used to anchor the teaching of patterns. The philosophical tenets of social constructivism and their related theoretical principles and pedagogical strategies are extensively discussed in this chapter. Lastly, the method by which data was analysed is briefly discussed.

1.10.3 Chapter 3 (Methodology)

This chapter provides a graphical account of how the research was designed and conducted. The chapter discusses the research orientation, the research methods and research design. It also discusses data collection processes and tools and the sampling techniques used in the study. It further discusses the techniques involved in data analysis. It wraps up by discussing in detail issues of validity and ethics.

1.10.4 Chapter 4 (Data presentation, Analysis and Discussion)

This chapter presents and discusses the outcomes of this research study. It starts by presenting and critically analysing data from the three research instruments (survey questionnaire, observations and interviews). These are analysed to produce findings from the four research questions stated in 1.4 above. The chapter concludes by summarising the research findings from the research process.

1.10.5 Chapter 5 (Conclusions and Recommendations)

This final chapter concludes the study by making a summary presentation of the dominant and outstanding themes. It draws conclusions from the findings and makes recommendations and

suggestions for future research studies. This chapter ends with my own personal reflections that emanated from what I experienced in the investigative process for this research study.

1.11 CONCLUSION

This chapter gives the reader an idea of the background of the study as well as introduces the intricate issues that were researched in this study. The methodology with which research questions were systematically answered were explained. The way data was analysed was also discussed. A diagnostic explanation of the limitations to the study was made manifest. A brief description of the operational definitions that were used in the research process were explained before giving a brief outline of what to expect in each of the five chapters that make up the thesis in this research endeavour.

CHAPTER 2: LITERATURE REVIEW

2.1 INTRODUCTION

Teaching of patterns and their generalisation using images and visual tools at Senior Phase and other levels of mathematics learning is a curriculum requirement in South Africa and in various parts of the world. However, at present the phenomenon of teaching pattern generalisation using images and/or technological tools and pictorials is not well understood neither by teachers nor students and is scantily researched in the Southern Africa region (Budram, 2009). This study undertakes to investigate how teachers can teach patterns and their generalisation using pictures, images and other related visual tools.

The objective of this study is to investigate the factors that enable and/or constrain the teaching of patterns when visualisation approaches are used. To shape the direction of this study, in this chapter I review literature that has explored strategies and techniques used in teaching patterns and generalisation in mathematics teaching. I explore the conceptual and theoretical understanding of what visualisation entails in the process of teaching patterns.

2.2 PATTERNS AND GENERALISATION

a) Patterns in mathematics teaching

A pattern is the rule between elements of a series of mathematical objects which are constructed (Guerrero & Rivera, 2002). Thus, it is referred to as a system of repetitious and orderly arrangements of objects or shapes (Olkun & Toluk-Ucar, 2006). Papic and Mulligan (2005) define a pattern as a spatial or numerical regularity. Mathematics has been characterised as 'the study of patterns' by a number of researchers (Hardy, 1992; Steen, 1988; Devlin, 1994 and Fried, 2010). In view of the above definitions of a pattern, this investigation intends to investigate the teaching of patterns on the basis that a pattern is a key concept for understanding mathematical knowledge and concepts. Pattern studies are the basis for the understanding of the system and logic of mathematics and observing of mathematical relationships (Burns, 2000 cited in Tansil & Ozdas, 2009).

Basically, there are five different levels involved in teaching and learning pattern skills. These are: 1) recognise a pattern, 2) describe a pattern, 3) copy a pattern, 4) extend a pattern and 5) create a pattern. Ginsburg (2018) cited in Mulligan et al. (2020), describes approaches to teaching patterns that can be adopted for the purpose of this investigation. One such approach

is to seize upon teachable moments that arise in the everyday environment, for instance visual constructions that children create in their day to day activities. Another approach is through guided instruction, in which the teacher uses manipulatives or other visual tools (pictures, shapes, tables and diagrams) to systematically develop ideas about a variety of patterns.

Regardless of the specific content involved in teaching patterns, teachers should emphasise thinking and communicating about patterns and the structure underlying them (ibid.). Visual patterns develop relational thinking and often students use them to make sense of patterns that would otherwise be inaccessible. Thus, when children are prompted to see for instance the first picture in the second picture and the second picture in the third picture, they are equipped to adopt a more functional perspective from a relational perspective (Pershan, 2015). This study seeks to investigate what impacts the teaching of patterns, particularly linear patterns, when taught from a pictorial perspective that moves into a functional perspective to develop generalisation skills of such patterns (Clarke & Willie, 2016).

Most teaching and learning of patterns involve exploring ideas in algebraic thinking. According to Annenberg (2017), the process of exploring patterns involves the following sequence of events: First the teacher explores how learners think visually about patterns by finding out what the learners see in a visual representation of a pattern. He/she then listens to how the learners describe the [different elements of] pattern in a narrative and how they predict and continue the pattern. At the end of this sequence, the teacher asks the learners to come up with a general rule to describe this pattern.

“Finding patterns is a subjective activity” (Annenberg, 2017, p. 3). This supports the idea that different people notice different things. What one person sees often differs from what another perceives. A teacher’s careful interpretative instruction can assist learners to see a pattern in a cognitively enlightened way. He/she can achieve this through an investigative inquiry approach. This involve questions such as: *How many elements can you see in the first picture? How does this relate to the second and third pictures? Can you predict the fourth and tenth pictures? Find a common rule that governs the number of elements for the entire sequence. Describe how you have formulated your common rule and interpret it to others* (Annenberg, 2017). Mathematical situations often pose an overabundance of information, visual cues, and possible patterns, making it impossible to process everything at once (Lobato, Hohensee & Rhodhamel, 2013). Pattern observing helps learners to overcome this problem by attending to only some of the input. According to Lobato et al. (2013), pattern noticing is defined as selecting, interpreting and working with particular mathematical features or regularities when multiple sources of

information compete for one's attention. Patterns are crucial in the teaching of mathematics in the following ways: 1) they help learners make predictions by recognising and creating a pattern based their own observations; 2) understanding patterns helps learners prepare for learning complex number concepts and mathematical operations; and 3) patterns allow learners to see relationships and develop generalisations. Thus, a pattern-based methodology approach challenges learner to use higher order skills and emphasises exploration, investigation, conjecture and generalisation (Vale & Cabrita, 2015), and this makes looking for a pattern a powerful problem-solving strategy.

Pattern tasks give learners the opportunity to observe and verbalise their own generalisations and translate them symbolically (English & Warren, 1998), thus developing algebraic thinking in learners. Learning through patterns is a powerful approach to algebraic thinking (Vale & Cabrita, 2015).

b) Generalisation in mathematics

Generalisation – which is the means of communication and the tool of thinking – is the basic tool for the development of mathematical knowledge and the centre of mathematical activities. It is the main goal of mathematical instruction (Jones, 1993). Generalisation is a basic structure of algebra and patterns are the basic structure of generalisation (Sherrocks & Threlfall, 1998 cited in Zakis & Lijedahl, 2002).

Generalisation can be defined from a mathematical point of view as “looking for a bigger picture” (Tall, 2011), “extending concepts to a bigger area to investigate about it” (Mason, Burton & Stacey, 2010). Mathematics teaching means nothing unless problem solving occurs in the form of generalisation (Polya, 1988). Using generalisation is necessary to have effective teaching and learning at all levels of mathematical learning (du Plessis, 2018).

From Tall's (2002) point of view, components of generalisation can appear in three ways: expansive, reconstruction and disjunctive. Expansive generalisation involves extending the existing pattern information without changing their previous ideas and structure. Reconstruction occurs when a learner reconstructs his/her cognitive area of knowledge. Disjunctive generalisation problems can be analysed in higher class of knowledge. Learners can solve new problems by adding aspects of disconnected pieces of information together (Mason et al., 2010 cited in du Plessis, 2018).

It can be concluded that generalising is a major factor in developing the algebraic thinking of children (Lesley & Freiman, 2004). Generalisation is the principle of algebra and the search of pattern is the first step in generalisation.

Three main generalisation strategies are defined in research studies. These are: 1) Recursive approach; 2) Explicit approach; and 3) Whole-object approach (Stacey, 1989; Vale & Cabrita, 2008). In the recursive approach, when making a generalisation the learner is taught to extend a given pattern sequence using the common difference, building on previous terms and based on observable features of the visual figure.

To use the explicit approach to generalise patterns learners are taught to discover a symbolic rule based on the context of the pattern problem. This allows the immediate calculation of any output value, given the correspondent input value. This requires a careful and explicit strategy that involves pattern noticing in the pattern figure context.

The whole-object approach of a visual nature involves considering a term of the sequence as a unit and using multiples of that unit. The ultimate generalisation includes a final adjustment based on the context of the pattern problem (Adapted from Barbosa & Vale, 2015).

These and other generalisation strategies, which includes counting and difference rate will be used in this investigation. The difference rate approach involves using the common difference as a multiplying factor where the ultimate generalisation proceeds to an adjustment of the resultant process (Warren, Cooper & Lamb, 2006). The counting approach entails drawing a pattern figure and counting its elements to establish a regular rule (Stacey, 1989). These strategies will be used to establish what works and what does not work in teaching patterns from a visual perspective.

c) Patterns and generalisations in teaching

Researchers over the years have been investigating the development of pattern generalisation ability, mainly focusing on learners and students' ability to generalise (Stacey, 1989; Presmeg, 2006; Arcavi, 2003). Warren (2005a) asserts that "The power of mathematics lies in relations and transformations which give rise to patterns and their generalisations." (p. 305). Mathematics as a subject is defined as the study of patterns (Hardy, 1992). The importance of understanding patterns (from a visual perspective) and being able to identify and work with patterns is crucial in the teaching of patterns and their generalisations (Mulligan & Mitchelmore, 2009).

Generalisation can be considered as a well-known fundamental and valuable skill of mathematics teaching and learning which has many applications in mathematics and appears in many forms (Mason, 2010, Kashefi et al., 2013). Generalisation is therefore a crucial mathematical concept that permeates all branches of mathematics and is highlighted in the teaching of the subject at all school levels and beyond (Dindyal, 2007). Mason (1996) describes generalisation as “the heartbeat of mathematics” (p. 65). Generalisation in patterns involves extending the range of reasoning or communication beyond the case or cases being considered, by explicitly identifying and exposing commonality across the given case or cases (Kaput, 1999). Dreyfus (1991) cited in Warren (2005) views generalisation as the recognition of some general characteristics of a given situation. Such generalisation results in the systematic expansion of an individual learner’s knowledge structure of patterns or any other mathematical domain under study. Ellis (2007, p. 197) defines generalisation as “a process which engages learners in at least one of the following activities: 1) identifying commonality across cases, 2) extending ones’ reasoning beyond the range in which it originated, or 3) deriving broader results from particular cases.” The generalisation of a pattern is vital in that it builds expressions beyond the perceptual field (Radford, 2006).

In many instances in this study, the following generalisation strategies will be used in tandem with the strategies mentioned above. These are: 1) Near-generalisation, 2) Distant generalisation, 3) Constructive generalisation and 4) Deconstructive generalisation. These are selected for their functionality in promoting symbolism in generalising tasks involving repeating, growth, linear, numeric and most importantly, figurative or visual patterns.

Near generalisation uses such strategies as counting and recursive analysis (Tanisil & Ozdas, 2009). The counting strategy involves drawing a figure and counting its elements. Such a strategy can be used in growing patterns that involve geometric shapes (Barbosa & Vale, 2015). In some instances, counting may involve the capability to grasp a commonality of some particulars of a visual pattern, then extend or generalise this commonality (Tanisil & Ozdas, 2009). On the other hand, a recursive solution involves finding a local rule and recognising and using elements and changes from previous information in the pattern to find the next one in the series (Lannin, 2006). The recursive approach in near generalisation also involves extending the sequence using a common difference, building on previous terms or features of the figure, or counting using tables in the discovery of subsequent terms (Cabrita et al., 2012).

Explicit strategies are determined in the use of deconstructive generalisation. This involves discovering a rule based on the context of the problem posed by a visual figure that allows the

immediate calculation of any output value given the corresponding input value (Barbosa & Vale, 2015).

A constructive generalisation occurs when the configurations used to depict a pattern are broken up into non-overlapping parts and a formal rule is established by adding up these parts. However, when different parts are overlapped to form the configurations and the rule is expressed by counting each part of the configuration separately followed by subtracting the overlapping parts, deconstructive generalisation is produced (Samson, 2012).

Whole-object analysis strategies are determined in using both constructive and deconstructive generalisation. In whole-object analysis a term of a sequence is considered as a unit and multiples of that unit are used. A final adjustment is made based on the context of the pattern problem. Tasks, in both constructive and deconstructive generalisation, are focused on pattern exploration premised on the idea that seeing a pattern is the first step towards identifying the disjointed elements which are present in a figure, to be assembled (Rivera & Becker, 2008).

Generalisation should lead mathematical communication to such a level where the focus is no longer on the individual cases or situations themselves, but rather on the patterns, procedures, structures, and the relationships across and among these cases (Kaput, 1999). Radford (1996) on examining number patterns, posits that generalisation is a procedure that draws a conclusion from a sequence of “observed facts”. While generalisation may seem to be an obvious component of patterns in school mathematics, there are some teaching and learning challenges that cannot be ignored. My study intends to focus on linear patterns, and how visualisation enables and/or constrains the generalisation process in mathematics teaching.

The structure of a pattern is interpretable in many ways. Visuality of patterns brings some advantages to learners when solving such patterns. Barbosa and Vale (2015) assert that seeing a pattern is a key first step in searching for regularity in a pattern. This requires the development of perceptual and cognitive agility in learners by the teacher.

Stacey (1989) identified three strategies commonly used by learners in generalising patterns: (a) **Counting** – where a learner draws a picture to represent the situation and then counts the desired attributes; (b) **Difference rate** – where learners build a larger unit by multiplying the unit’s position by the common difference, and (c) **Whole-object** – where learners use a known value as a unit to construct a larger unit using multiples of the unit. Since Stacey’s (1989) study, several other studies have been carried out to understand and categorise the strategies evidenced by learners of various levels, when they solve pattern problems in different contexts.

Barbosa and Vale (2015) collated these strategies, and they are presented in a table as shown below.

Table 2.1: Generalisation Strategies applied in visual patterns (Barbosa & Vale, 2015, p. 59-60)

Strategy	Nature	Description
Counting (C)	Visual	Drawing a figure and counting its elements.
Whole-object (no adjustments) (WO ₁)	Non-visual	Considering a term of a sequence as a unit and using multiples of that unit.
Whole-object with visual adjustment (WO ₂)	Visual	Considering a term of the sequence as a unit and using multiples of that unit. A final adjustment is made based on the context of the problem.
Whole-object with numeric adjustment (WO ₃)	Non-visual	Considering a term of the sequence as a unit and using multiples of that unit. A final adjustment is based on the numeric properties.
Recursive (R ₁)	Non-visual	Extending the sequence using the common difference, building on previous terms (numeric relations).
Recursive (R ₂)	Visual	Extending the sequence using the common difference, building on previous term (features of the figure).
Difference rate (no adjustment) (D ₁)	Non-visual	Using the common difference as a multiplying factor without proceeding to a final adjustment.
Difference rate (with adjustment) (D ₂)	Visual	Using the common difference as a multiplying factor and proceeding to an adjustment of the result.
Explicit (E ₁)	Non-visual	Discovering a numerical rule that allows the immediate calculation of any output value given the correspondent input value.
Explicit (E ₂)	Visual	Discovering a rule, based on the context of the problem, that allows the immediate calculation of any output value given the correspondent input value.
Guess and check (GC)	Non-visual	Guessing a rule by trying multiple input values to check its validity.

d) Visual generalisation strategies explained

Counting (C), Whole-object with adjustment (WO₂), Recursive (R₂), Difference Rate with adjustment (D₂) and Explicit analysis (E₂) are selected visual generalisation strategies which need to be discussed here in greater detail as they are expected to feature extensively in this study. This study focuses on figurative/visual patterns and how they can be taught successfully with minimum hindrances and substantial enablement.

Recursive analysis is a near generalisation strategy. It draws from the fact that seeing a pattern is a necessary first step in pattern generalisation (Lee & Freiman, 2006). According to Barbosa et al., (2015), traditionally learners and teachers translate visual information into symbolic or numerical information. They look for a sequence and use their inductive reasoning. Depending on the level of the learners, they can use a recursive approach which uses finite differences between consecutive term values as shown below.

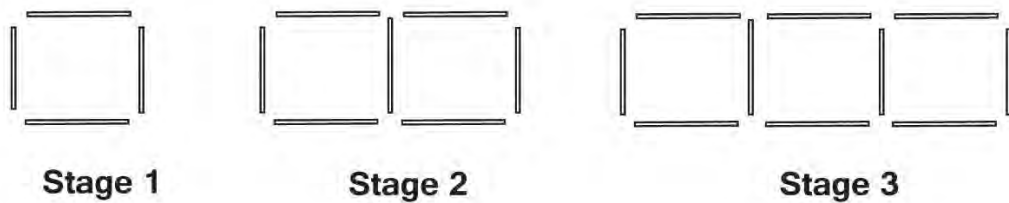


Figure 2.1: Matchstick rectangular pattern

Stage number	Number of matchsticks	How number of matchsticks was obtained
1	4	$3(1) + 1$
2	7	$3(2) + 1$
3	10	$3(3) + 1$
4	13	$3(4) + 1$
5	16	$3(5) + 1$
6	19	$3(6) + 1$
7	22	$3(7) + 1$

Figure 2.2: Numerical solution of the Matchstick pattern using finite differences

The final symbolic rule to this type of generalisation will be $3n + 1$. In other words, the number of matchsticks at each subsequent stage is found by adding the finite difference “three” to the number of matchsticks in the previous stage can be used for generalisation purposes.

Counting is a form of far generalisation that requires a sound arithmetic foundation (Vale & Cabrita, 2008) for the learning of symbolic algebra. Pattern and algebra should be taught in combination with number concepts. Numerical tasks of counting have their foundations in the recognition of patterns. Counting tasks can be a way to develop some skills that equip learners to translate visual patterns into numeric expressions, and later to reach the symbolic far generalisation of a given pattern (Barbosa, Vale & Cabrita, 2015). Thus, visual pattern tasks are considered a tool to enable comprehensible numerical expressions. The counting generalisation strategy can be illustrated by the triangle dot pattern shown below.

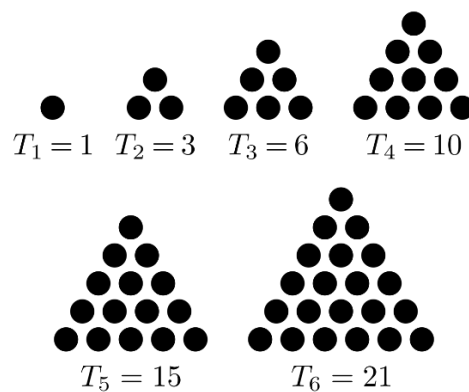


Figure 2.3: Triangle dot pattern

Counting from the top level going down, the number of dots increase consecutively by one dot at each subsequent level. Inversely a learner can count the number of dots at each base level as the stage number and count consecutively backwards as one goes up the levels until he/she reaches the apex with only one dot. For instance, at Stage 10, the counting goes as follows: $10 + 9 + 8 + 7 + 6 + 5 + 4 + 3 + 2 + 1 = 55$. This process requires a sound knowledge of arithmetic.

From a didactics perspective the teacher can prompt learners to reason from a visual perspective that half the number of each stage multiplied by that stage number plus one dot will give the total number of dots at that particular stage. Generalisation can thus be drawn from this conjecture and presented algebraically as $n/2(n + 1)$. This rule applies at all stages of this linear pattern as shown below for stage 10.

$$10/2(10 + 1)$$

$$5 (11)$$

55 dots

Whole-object analysis is a constructive approach to generalisation. The learner uses a portion of a visual figure as a unit to construct a larger unit using multiples of the unit. The learner may adjust for any over counting or undercounting where applicable. For instance, a rod can be made by joining cubes and placing stickers on the exposed faces on each cube. A rod with a length of ten cubes will thus have forty-two stickers. By visually analysing the trend, a rod length of fifteen cubes would take $42 + 5(4)$ stickers since each additional cube adds 5 stickers and one sticker must be removed when a new cube is added to the rod. This makes a total of four stickers added to each cube. The scenario needs to be adjusted a little when a rod length of twenty cubes is constructed. We take $42 + 9(4)$ stickers since 20 is a multiple of 10, where the logical rationale for using 9 instead of 10 can be understood from a visual perspective when the rod of cubes and stickers is constructed physically or diagrammatically on paper. (Adapted from Lannin, Barker & Townsend, 2006).

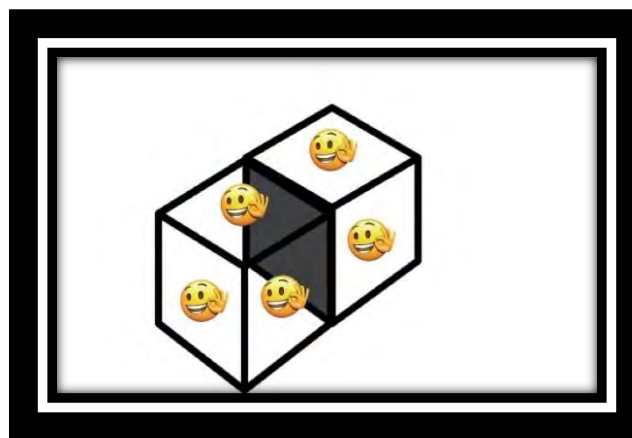


Figure 2.4: Diagrammatic representation of a rod made of 2 cubes and 10 stickers

As the order of the term becomes far, and learners start basing their reasoning on the common difference between terms, the generalisation strategy becomes **Difference rate analysis**. In an example given by Barbosa, Vale and Palhares (2009), an unknown number of task cards is hung on a notice board in a certain format, using a known number of pins (as shown below).

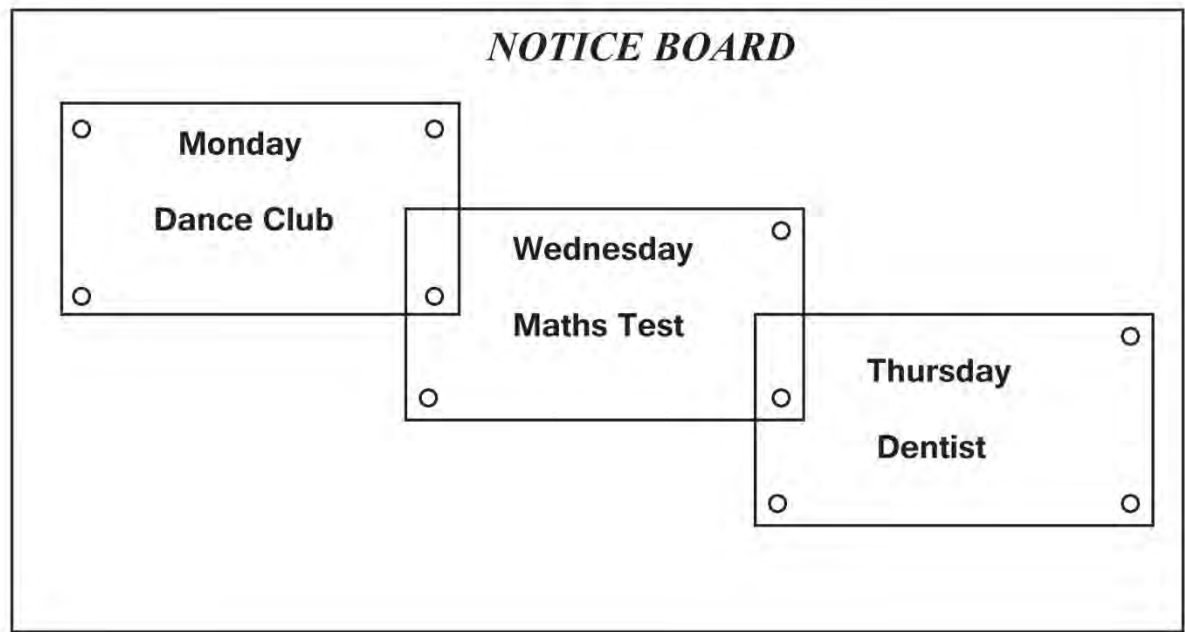


Figure 2.5: Task cards on a notice board

Learners were asked to find out and support their deductions of how many task cards Joshua could hang on his notice board if he was given a box containing 600 pins. The following is a solution by one learner who opted to use the difference rate approach to solve the problem.

$$3 \times n = 600$$

$$3 \times 200 = 600$$

As we need one more pin for the last card, we can hang 199 cards!

This is a typical example of the application of a difference rate (D_2) strategy where three is the common difference and ' n ' is the number of cards to be hanged. The strategy tactfully draws its conclusion from the context of the pattern figure with a final adjustment.

Explicit analysis (E_2) is a deconstructive generalisation strategy. According to Samson (2012), a learner given the visual figure below, went on to use a deconstructive approach of generalising the resultant visual pattern from it.

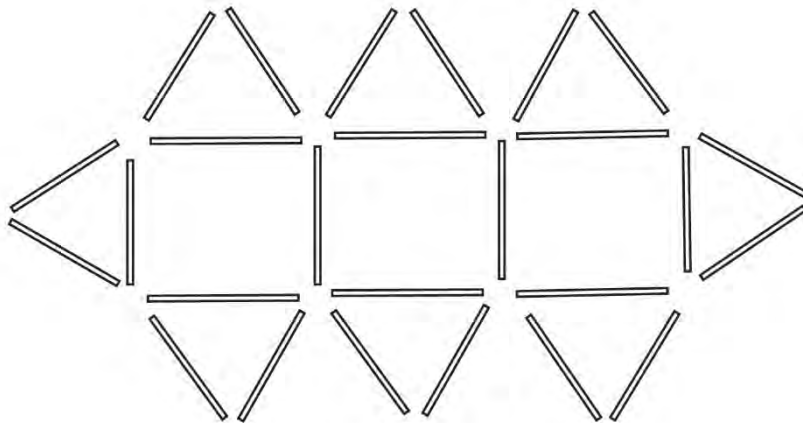


Figure 2.6: Matchstick visual pattern

The learner subdivided the whole figure into smaller units of triangles, V-shapes and single vertical sticks as shown below.

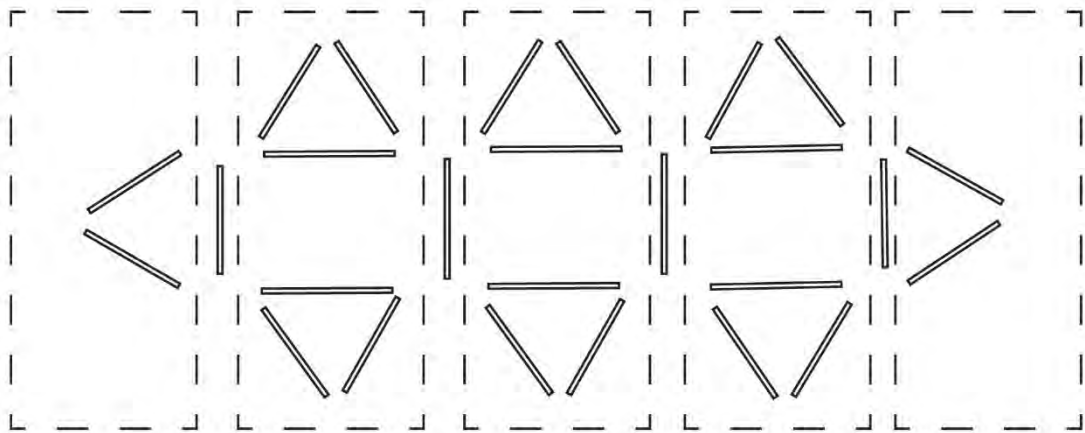


Figure 2.7: Deconstructed matchstick pattern

The triangles were not visualised separately, but in pairs situated vertically below and above one another. For the n th stage there will be n pairs of triangles, each pair requiring six matchsticks and thus $6n$ matchsticks in total. The n th term will also contain $n + 1$ vertical sticks. Finally, the V- shapes at each end, which are constants and thus independent of whichever term is under consideration, will always require four sticks. The total count thus comes to $6n + (n + 1) + 4$. This is an example of explicit generalisation which is deconstructive in nature.

2.2.1 Visualisation

a) Definition

The term visualise encompasses “making visible” and “making [a] mental image” as its core concepts. This implies that until we ‘visualise’ some things, that ‘something’ does not assume a visual form. That is to say that an image exists through a process of creation, perception, manipulation and logical interpretation (Manovich, 2010).

Many definitions of the term visualisation have been put forward by various scholars and researchers. Here I examine just a few of the definitions of visualisation. Arcavi (2003) defined visualisation as a process of creating, interpreting and reflecting upon pictures and images on paper with the purpose of depicting information and ideas. Other definitions of visualisation are more elaborate than “concept images” proposed by Tall and Vinner (1981). Some researchers defined visualisation as the use of spatial variables (position, size, shape, and more recently curvature of lines [as in *Geogebra*] and movement) to represent key differences in (numerical) data and reveal important patterns and relations (Manovich, 2010; Presmeg, 2006). Hebb (1982) cited in Presmeg (2006) defined visualisation as “the occurrence of mental activity corresponding to the perception of an object ...”

It can be summarily concluded that visualisation include kinds of imagery which depict shape, pattern or notion of imagery. Imagery which attains the vividness and clarity of a picture can conveniently be included in the definitive summary (Clements, 1982 cited in Presmeg, 2006). This summative idea allows for the possibility that verbal, pictorial, numerical and/or mathematical (algebraic) symbols may be arranged spatially to form the kind of imagery referred to by Paivio (1971) as “number forms”. This study will explore the use of visualisation in teaching patterns, based on the conceptual definitions of patterns and generalisation explained in sections 2.2 (a) and (b).

b) Visualisation in patterns

The potential to open meaningful spaces for group discourse and exploration when pattern generalisations are developed from pictorial forms cannot be underestimated (Kadunz, 2015). In some past studies the potential richness of pictorial contexts embedded in visualisation approaches have often been perceived to be degraded to simple rote exercises, where the emphasis is on finding the general solution of a given pattern. In many cases a given pictorial sequence is simply reduced to an equivalent numeric sequence (Samson, 2012).

Visualisation skills which frequently involve recognition of pattern and structure, are positively correlated with mathematical achievement (Arcavi, 2003; Thomas, 2000) and analogical reasoning (English, 2004). Mulligan (1995) reiterated that students who recognise the structure of mathematical patterns, presentations and processes acquire deep conceptual understanding. Teaching processes involving patterns reveal a number of mechanisms of visualisation that are subtle and complex. Visual reasoning and justifications as evident in emerging generalisation processes emanate from teaching processes that use visualisation approaches (Samson, 2012).

Visualisation processes in teaching patterns have considerable value as a precursor to formal algebra (English & Warren, 1998). Samson (2012) suggested that pictorial representations can readily be reduced to purely symbolic equivalents provided that the pictorial context has been meaningfully understood. Thus visualisation, as applied in teaching patterns, can be viewed as complex and working in two opposing ways. Firstly, to visualise elements in a visual tool in order to form a symbolic image of the concept; and secondly, a concept image may be narrowed by the visual elements. We cannot form an image of a concept and its examples without visualising its elements (Herskowitz, 1989).

According to findings by Papic and Mulligan (2007) visually inclined teaching generally produces visually alert learners who are able to deal with a variety of pattern problems in a highly esteemed cognitive way.

The use of visualisation in teaching mathematics, particularly patterns and their generalisation, is central to this study. Many researchers recognise the importance of the role that visualisation plays in teaching mathematics. Arcavi (2003) defines visualisation as

... the ability, the process and the product of creation, interpretation, use of and reflection upon pictures, diagrams, in our minds, on papers or with technological tools, with the purpose of depicting and communicating information thinking about and developing previously unknown ideas and advancing understanding (p. 217).

This definition encompasses visual thinking that involves creating mental images, carefully interpreting them, logically utilising them and effectively reflecting upon them. Shatri and Buza (2017) suggest that the abilities of teachers to use visual tools (symbols, pictures, tables and/or images) advances concept development and critical thinking. Using visualisation in teaching patterns is critically important in that both concept and skill develop side by side, and this is vital for mathematics teaching in general.

In mathematics as well as in other subjects, thinking is aided by visual representation of concepts (Budram, 2009). It is almost impossible to think without the aid of images. The centrality of visualisation as an important contributor to teaching and learning processes is evident in research. Ben-Chaim (1989) wrote that “Visualisation is a central component of many processes for making transitions from the concrete to the abstract modes of thinking” (p. 50). Visualisation refers to the cognitive functions in visual perception where pictures combine aspects of natural reasoning with more formal shapes to enhance cognitive understanding (Arnheim, 1991). Such aspects of visualisation have the potential to enhance the teaching of mathematics concepts.

According to Arcavi (2003), mathematics teachers the world over use visualisation as its aspects gain increasing prominence in mathematics and mathematics education. Visualisation plays a pivotal role in making the teaching of mathematics easy, interesting, fun and capable of effecting cognitive development (Winn, 2015; Bozkurt, 2017). However, this does not eliminate the need to critically analyse the use of visualisation techniques and meaningfully reflect on their application in teaching mathematical concepts.

Some researchers’ findings claim that visualisation is time consuming therefore a deterrent to teaching (Bresciani & Eppler, 2015; Naps et al., 2002). Kashefi (2015) however, argues that on the contrary, teaching with visuals generally proves to be less time consuming than normal algorithms when it comes to teaching patterns.

Some findings suggest that instead of positively affecting the thinking process, teaching through visualisation can negatively affect thinking by putting pressure on the cognitive process – thereby affecting learning (Kashefi, 2015). To add to the negative voice is the claim by Bozkurt (2017) cited in Liaga (2019) that visuals cannot provide prompts and feedback to learners. On the other hand, many research findings point out that cognitive development is greatly enhanced through visualisation. Some proponents of visualisation even maintain that critical thinking and meaningful reflection can only be enhanced when and if visualisation is used with greater care and insight (Lee, 2007; Makina, 2010).

Research suggests that learners learn abstract, new and novel concepts more easily when they are taught in both verbal and visual forms (Konyalioglu, 2011). It can be said that visual media makes concepts more accessible and helps with recall (Fadirani, Van Bilon & Schoeman., 2017). Visualisation, if properly taught and planned, can help learners, and connect with events that are culturally and contextually relevant. In this way teaching with visualisation empowers learners to be co-creators of knowledge rather than mere consumers (Fadirani et al, 2017).

Hence, through visualisation, learners can connect theories taught in class with real world events and concepts (Bransford, Vye & Steveens, 2006).

Traditionally and from an epistemological point of view, teaching mathematical concepts with images demonstrates how to make thinking visible. Thus, we have a long tradition of visualisation within mathematics education which is based on and supported by practical and theoretical practices (Kadunz, 2015; Hoffman, 2005).

The process of exploring patterns from a visualisation point of view, involves the following sequence of events. First the teacher explores how learners think visually about patterns by finding out what the learners see in a pattern. The teacher then listens to how the learners describe the pattern in a narrative and how they predict and continue the pattern. At the end of the sequence, the teacher asks the learners to come up with a conjecture and general rule to describe this pattern (Jurow, 2004; Annenberg, 2017). Teaching of patterns interlinks a tabular format, a pictorial format, a narrative format and an analytical algebraic format (Sullivan, 2011). The focus of teaching patterns and their generalisation, in this study, will be directed at making the pictorial format the centre of the teaching process through interactive teaching and logical construction of valid mathematical arguments. Drawing from these ideas it can be concluded that the use of visual approaches stands out as significant in teaching a variety of mathematics concepts.

2.2.2 Teacher's visual strategies in pattern generalisation

According to Barbosa and Vale (2015), teaching patterns basically involves four methodological approaches. That is, a teacher may approach the concept from a tabular format, a narrative format, a pictorial format and an analytical algebraic format as illustrated in Figures 2.8, 2.9, 2.10 and 2.11. All four approaches can be used in tandem, but the objective of this study is to prioritise the pictorial approach which entails visualisation.

TABLE FORMAT

INPUT	1	2	3	4	5	6	10
OUTPUT	6	10	14	18	22	26	?

Figure 2.8: Tabular format of a sequential pattern

NARRATIVE FORMAT

Adding a particular input value of the sequence to its subsequent consecutive input value and doubling the result gives the value of the output of that particular input.

Figure 2.9: The narrative format of the sequential pattern

PICTORIAL FORMAT

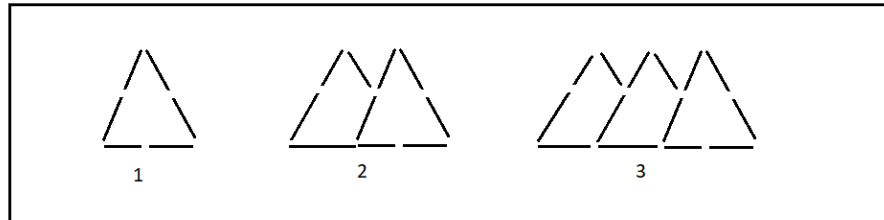


Figure 2.10: Pictorial format of the sequential pattern

ALGEBRAIC FORMAT

The algebraic format can be deduced from the premise that if we multiply any stage number of the pictorial format by 4 (the difference between output values) and add 2, we get the number of matchsticks at each stage. This translates to the algebraic expression: $4n + 2$. Or tripling the input value and adding 2 more than the input value. This transform to: $3n + (n + 2)$.

Figure 2.11: Algebraic format of the sequential pattern

Thinking about and teaching patterns involves communicating the regularity in words/pictures and/or tables in a mathematically concise form that others can understand (Annenberg, 2017). The picture, the narrative and any other representation of a pattern needs to be carefully communicated to the learner through constructive pedagogy (careful didactic inquiry). Thus, the proper use of shape, size, image and colour, together with culturally recognisable pictures in teaching patterns, result in effective teaching with visuals as suggested by Naidoo (2012). To the teacher, visualisation processes should involve construction and transforming both visual and mental imagery and the inscriptions implicated in doing mathematics when teaching patterns as suggested by Presmeg (2006). (Adapted from *Teachers' Resources: Annenberg Learner*, 2017).

2.2.3 Enablements and/or constraints in teaching patterns

Teaching patterns and their generalisation, using any approach, can pose some instructional difficulties or open up hidden possibilities (Virginia Department of Education, 2004). Research presents some factors that may enhance or constrain the teaching of patterns at school level when a visualisation approach is used. Such factors may arise from the following analytical points raised by Rivera (2011):

1) The extent to which teachers justify and construct plausible algebraic structures of pictorial and other linear patterns may enhance or constrain the teaching of patterns. This implies that the teacher's pedagogical instructions should build on the learners' intuitive understanding of structure, which can then be developed into the domain of algebra. Teachers need to foster the habit of searching for regularity between and within mathematical objects. This helps them to focus on relational attributes that are grounded in continued application of observed regularity (Radford, 2012). An omission of this instructional strategy may result in constrained effects of teaching patterns.

2) The extent teachers which to allow the development of sensory and cognitive perception to grow together affect the teaching of patterns. The relevance and promise of applications of perceptual learning (PL) to high level cognitive domains cannot be underestimated (Kellerman et al., 2009). Applying perceptual learning to rich cognitive instructional domains is crucial, exciting and rewarding – more so in teaching pictorial and linear patterns.

3) The extent to which teachers infer symbolic reasoning from visual representations has notable effects on teaching patterns. Several studies have compared how learners learn from schematic (symbolic reasoning) versus realistic (visual reasoning) representations of patterns and diagrams (Butcher, 2006; Joseph & Dwyer, 1984). Teaching patterns from a combination of abstract and concrete objects affect learners' problem-solving practice, near transfer (inference between symbolic and visual representations) and learning perceptions in quite a positive way (Moreno et al., 2011).

4) The extent to which teachers verbally describe features of a pictorial pattern (functional relationships) has similar ramifications on the enhancement or constraining of teaching patterns. Good instruction is not just about developing learners' facility with performing various procedures, such as finding regularity. Instruction should also help to develop a narrative understanding of patterns, ability to represent pattern structure and fluency in moving among multiple representations of patterns (Kalchman & Koedinger, 2001).

It can be concluded that if one enabling factor is not taken into consideration and/ or neglected, that aspect may become a constraint in itself (Rivera, 2011). The table below makes an attempt to categorise and summarise enabling and constraining factors to teaching patterns through visualisation approaches.

Table 2.2: Constraining and enabling factors of using visualisation adapted from Rahim (2009), Whitley (2004) and Gooding (2009).

Enabling factors to teaching patterns through visualisation	Constraining factors to teaching patterns through visualisation
1. Constant interaction with a variety of visual tools promotes conceptual understanding and deeper cognitive reasoning.	1. Compatibility issues with traditional curricula may arise when teaching approaches emphasise the use of pictorials.
2. Visuals can be used flexibly in a variety of pattern to deepen generalisation skills.	2. Teacher's over involvement in visual deconstruction and rule formulation may violate the constructivist teacher/learner contract.
3. Working with a variety of visuals has the potential of developing both visual and cognitive perception.	3. Classroom culture may limit learner autonomy and creativity in expressing personal visual perceptions.
4. Working with a variety of visual figures encourages the development of problem-solving techniques.	4. Group dynamics in a particular setting may make some learners feel intimidated to display personal creativity and visual perception. This may put unnecessary pressure on their cognition.
5. Encourages visual innovations and enhancement of individual originality.	5. Learners' limited knowledge of algebra may hinder progress in generalising from a pictorial.
6. Opens the door for learner exploration with deepening visual reasoning.	6. Time allotted to tasks and the nature of the task may hinder progress in developing conceptual understanding.

To get a clearer understanding of how these tabulated factors enable and/or constrain the teaching of patterns, the following elaborative explanations are quite necessary.

It is true that a variety of visual tools promote both visual and cognitive perception (Rahim, 2009). Some proponents argue that overuse of visual tools may unsettle the compatibility issues of such use with existing curricula (Gooding, 2009). A teacher who uses a greater variety of visual tools in teaching patterns may risk not completing or missing out on some crucial aspects of the prescribed syllabus content. However careful use of a variety of visual figures definitely promotes both cognitive perception and critical reasoning in learners.

A single visual tool can be used flexibly to develop a variety of general rules. This has the effect of deepening generalisation skills and enhancing algebraic understanding in learners (Rahim, 2009). Teachers who use a variety of generalisation skills give greater enablement for pattern understanding to their learners and thus develop both their symbolic and cognitive reasoning simultaneously. However, Whitley (2004) argues that the use of a single visual tool to develop a variety of generalisation skills gives the teacher a dominant role in the teaching and learning process, which infringes negatively on the constructivist teacher/learner contract envisaged by Vygotsky (1978). The learner need not be treated as a passive recipient of knowledge but as an active player in the construction of new knowledge (Dolittle & Hicks, 2012). The teacher's over involvement in visual deconstruction and rule formulation violates this constructivist position on the teaching and learning process – even for patterns and generalisations (Whitley, 2004).

When a teacher uses a variety of visuals to cater for individual differences in teaching patterns, this has the positive potential of developing in learners both visual and cognitive perception. This allows cognition and visual perception to grow together (Gooding, 2009). Seeing a pattern is the first step in developing the cognition necessary for the growth of perceptive and cognitive understanding of mathematical concepts. On the other hand, if classroom dynamics are such that other learners are not used to the expression of deeply personal creativity and individualised perceptions by others, this will have a negative impact (Rahim, 2009). Teaching and learning in a non-tolerant classroom culture can limit learner autonomy and creativity when a teacher uses a variety of visuals to cater for individual differences. Visual and cognitive perception does not necessarily develop in an environment where learners are afraid of being jeered at or looked down upon when they express their own perceptions of a given pattern.

However, the use of a variety of visual figures in teaching patterns encourages the development of problem-solving techniques (Whitley, 2004). The more learners get used to seeing a variety of visual figures, the more they develop schematic strategies to deal with each of them and the more they enhance their problem-solving abilities. On the contrary, according to Bozkurt (2017) cited in Liaga (2019) too, many visual figures will have the negative impact of putting unnecessary pressure on the learner's cognition. Too many visuals limit the learners' creative ability which can have the negative impact of minimising problem-solving abilities. Furthermore, an overdependence on a wider variety of visuals in a sizeable classroom creates group dynamics that results in some learners feeling intimidated to display personal creativity (Rahim, 2009).

A visualisation approach to teaching patterns has the potential to encourage learner innovation, thus enhancing individual originality (Gooding, 2009). Learners will develop visual innovations as they collectively or individually manipulate visual figures for generalisation purposes. However, learner innovation cannot be achieved by the use of visualisation approaches alone, in the absence of learners' intricate algebraic knowledge. Learners' limited knowledge of algebra may negatively affect progress in generalising from a pictorial pattern (Whitley, 2004).

Careful and calculated use of visualisation approaches in teaching patterns opens opportunities for learner exploration. As learners explore new possibilities in dealing with a variety of strategies, visual reasoning deepens, and this widens their cognitive and perspective abilities (Rahim, 2009). Vigorous exploration of pattern regularities when using a variety of visual figures takes a considerable amount of time and skill. Thus, time allotted to the task at hand may hinder progress in developing deep conceptual understanding of patterns and their generalisation. However, Kashefi (2015) argued that learners took less time to generalise a pattern when they used a visual approach than when they use any other approach.

2.2.4 Conceptual teaching and understanding of patterns

Teachers with conceptual understanding of teaching mathematics know more than isolated facts, methods and approaches. They understand why a mathematical idea is important and the kind of contexts in which it is useful (Bahr, 2008). In teaching patterns and their generalisation, mathematics instruction by teachers must ensure that conceptual knowledge is deeply embedded (Molina, 2014).

When learners truly master a pattern concept, they are able to show all the steps in detail and connect the processes and related concepts. Once they reach this level of understanding, the teacher can then expose them to more efficient ways of expressing or performing those processes. Thus, relying on algorithms and procedures in teaching patterns and focusing on shortcuts results in teaching efficiency, and not mathematical concept development (Molina, 2014; Chen, 2009). Molina (2014) suggests that shortcuts become the means to get to the solution with the unfortunate result of bypassing conceptual understanding. This kind of trade-off makes it difficult for learners to understand more complex topics as they advance in schooling.

To improve the teaching of patterns and maximise instructional time, the teacher should find and utilise connections among mathematical concepts and ideas. This will promote the idea of working smarter and not harder as posited by Chen, Reys and Reys (2009). Teaching and

learning is a complex interconnected web that develops as teacher and learner acquire experiences in which they reason inductively and deductively (Dreambox, 2017). Making connections from one skill to another helps the teacher and the learner to reach higher depths of knowledge and understanding.

According to Bahr (2008), developing patterning fluency requires a balance and connection between conceptual understanding and generalisation proficiency. One of the most robust findings of research is that conceptual understanding is an important component of proficiency, along with factual contextual knowledge and procedural facility (*Principles and Standards for School Mathematics*, 2000 cited in Bahr, 2008). It can be summarised that learners who master procedures of generalising patterns without understanding their visual contexts are often not sure when or how to use them to develop plausible generalisations.

Many teachers use patterns to promote generalisation as a pre-algebraic activity (Mason, 1996). If algebra is used as a tool for expressing generalities, conceptual exploration of patterns thus lays the foundation for algebraic reasoning and understanding (Kaput, 2007). The idea of using numerical rather than visual patterns can be problematic to reach generality, and consequently to get an algebraic expression or formula that can generate any term of a given sequence (Vale & Cabrita, 2015).

Conceptual understanding in teaching patterns entails the use of counting tasks to develop skills that provide learners with the abilities to translate visual patterns into numerical expressions, and later to reach the far generalisation of a given pattern (Vale & Cabrita, 2015). In this study I am interested in encouraging teachers to develop didactic strategies that enable future teachers and students to work with pattern tasks involving generalisation that is grounded in visual/figurative contexts. Patterns are an effective way to encourage learners to explore important ideas in the study of algebra as conjecture and generalisation (Yeats, 2004 cited in Vale & Cabrita, 2015).

Algebra as a pattern related concept is more than just manipulation of variables and formulas (Stacey, 1989). The teacher should strive to make learners experience generalisation tasks to get formula construction and remain flexible and creative for long periods in their search for solutions in solving pattern problems. If teachers are not in the habit of getting learners to work at expressing their own generalisation, then mathematical thinking is not taking place (English & Warren, 1998). Conceptual understanding is thus paramount to the teaching of patterns and their generalisations.

2.3 THEORETICAL FRAMEWORK: SOCIAL CONSTRUCTIVISM

Introduction

The theory of social constructivism has been used in qualitative research quite extensively, mainly for the following reasons: 1) social constructivism is an interpretive framework whereby individuals seek to understand their world and develop their own particular meanings that correspond to their lived experiences; and 2) a major focus of social constructivism is to uncover ways in which individuals and groups participate in the construction of their perceived social reality (Doolittle & Hicks, 2012). Social constructivism involves looking at the way social phenomena are developed, institutionalised, known and made into tradition by humans (Creswell, 2013). The rationale for using the theory of social constructivism in this study can therefore be justified along these lines. Social constructivism is defined as a sociological theory of knowledge according to which human development is socially situated and knowledge is constructed through interactions with others (Vygotsky, 1978). This definition and the preceding rationale for choosing social constructivism embraces the issues to be investigated in this study.

This study will be informed by the social constructivist theory as envisaged by Vygotsky (1962). Vygotsky's theory supports the idea that teaching is an act of social interaction where collective and creative input combine. Meaning making, drawn from learner involvement, proceeds from ensuing social processes (including the teacher's instructional ingenuity), that include applying strategies and developing concepts that work for particular situations, as suggested by Beghetto (2016).

Constructivism involves the active creation and modification of thoughts, ideas and understanding that develops because of experiences that occur within socio-cultural contexts (Doolittle & Hicks, 2012). This idea will be used extensively to theoretically shape the form of this research study which will be critically discussed in distinct sections and informed by research literature. These sections will include theoretical principles of learning and teaching, basic philosophical tenets of social constructivism and possible pedagogical strategies for teaching patterns and their generalisation.

a) Basic philosophical tenets of social constructivism

Doolittle and Hicks (2012) discuss three basic tenets of social constructivism that are linked to the six theoretical principles of social constructivism (to be discussed in 2.3b) below). These tenets are listed here:

1. Knowledge is not passively accumulated but is rather the result of active cognising by individual learners.
2. Cognition organises and makes sense of one's experiences and is not a process to render an accurate representation of an external reality.
3. Knowing has its roots in both biological/neurological construction and in social, cultural, and language-based interactions.

(Adapted from Doolittle & Hicks, 2003).

The first tenet conveys the fact that active cognising in learners in a classroom environment develops from the premise that, despite having the same learning experience, every learner bases his/her learning on the understandings and meanings personal to them. Teaching processes that assist learners to take personal responsibility to learn is an aspect of a social constructivist classroom (Vygotsky, 1962).

In view of the ideas expressed in the second tenet, individual cognition involves collaboration and shared understanding in problem-solving activities such as will be found in the teaching and learning of patterns. Adults and more knowledgeable peers assist learners in their development by guiding their participation in the relevant activities, thus conforming to Vygotsky's proposed zone of proximal development (ZPD) (Adam, 2017). The phenomenon of cognition as being extended beyond the individual makes it possible for information processed between the individuals, the visual tools and other learning subjects to assume meaning. Thus, the individual's abilities and achievements cannot be understood outside the connection of the social circle (classroom environment) and culture in which they are immersed (Hatchins, (2003) cited in Adam (2017).

The third tenet acknowledges the role of nature and nurture in social settings and how these influences the acquisition of knowledge in formal learning environments. Social constructivism emphasises the importance of social interaction as the way of learning rather than individual cognition. According to Vygotsky (1962), a calculated reliance on a social activity has the potential of tapping into the language, culture and context for teaching purposes. Socially available tools such as artefacts, language and culture, can provide a platform from which the teaching of mathematics domains such as patterns, can benefit. The construction of knowledge is thus fundamentally (collective) "reconstruction of structures performed in the external reality" (Doolittle & Hicks, 2012, p. 78), existing in the learners' immediate environment.

b) Basic theoretical principles of teaching and learning guided by social constructivism

Six principles of teaching and learning guided by the social constructivism thinking will be considered in this section. These are as follows: 1) the construction of knowledge and meaning making are individual and socially active processes; 2) knowledge construction involves social mediation within cultural contexts; 3) the construction of knowledge is fostered by authentic and real world environments; 4) construction of knowledge takes place within the framework of the learner's prior knowledge and experience; 5) construction of knowledge is integrated more deeply by engaging in multiple perspectives and representations of content, skills, and social realms; and 6) construction of knowledge is fostered by learners becoming self-regulated, self-mediated, and self-aware (Doolittle & Hicks, 2003).

Knowledge – according to Vygotsky (1978) – is constructed through social interactions shared from collective group experiences. Learning is thus an active, social process that involves learners constructing new ideas/concepts based on previously gained knowledge. In teaching patterns and their generalisations, personal ideas that result from organising the experiential world need to fit in with constraints imposed by physical and social reality. Not only should a learner be instructed to use his/her personal perception of a pattern but incorporate the views of others (Ernest, 1997).

In view of principle 2, the learning of pattern concepts occurs when one learns from others in a socio-cultural context. The use of culturally relevant visual manipulatives assists the learner to readily understand pattern concepts in a familiar context (Nodding, 2011). Thus, in order to construct knowledge that fosters authentic real-world scenarios, teachers teaching patterns and their generalisations using pictorials need to strive to make the learning environment real and authentic by designing tasks with familiar contexts (Vygotsky, 1962).

The fourth principle can be discussed in close association with ideas related to the works of Vygotsky (1978), Bandura (1977), and also nested in the theories of Dewey (1930), Piaget (1972) and Bruner (1978). These theories make reference to the idea that: “when learners come to class, they do not come without prior knowledge of whatever concept is to be taught”. It is therefore essential for teachers teaching patterns using pictorials to build their knowledge of what the learner is bringing to the learning environment in terms of experiential knowledge (Glaserfeld, 1999).

Multiple perspectives emerge when learners interact with a variety of pattern representation content, skills and social realms (Weinberger, 2011). The teachers' strategies, in teaching patterns and their generalisations, should thus incorporate a variety of techniques that may include some or all of the following strategies: counting, whole-object analysis, recursive

analysis, difference rate, explicit analysis as well as guess and check techniques (Barbosa & Vale, 2015). This idea is highlighted in the fifth principle of social constructivism.

The sixth principle emphasises learner autonomy in knowledge construction but does not neglect the role of the teacher in guiding learners to take charge of their individual knowledge construction. The zone of proximal development (ZPD) as envisaged by Vygotsky (1978) can be noted here where a learner becomes aware of what he/she can achieve without the help of others and what he/she can achieve with the help of more knowledgeable peers. However, this should happen under the teachers' instructional guidance and negotiated learning interventions (Doolittle & Hicks, 2003).

Teachers' perceptions and experiences about how visualisation impacts on teaching processes will be investigated, based on the premise that mathematics (including the teaching of patterns) is a theory of form and structure that arises within interaction and language dynamics in a social context, guided by these principles (Ernest, 1999).

c) Pedagogical strategies related to some philosophical tenets and theoretical principles of social constructivism.

Below is a table showing relational delineation of the philosophical tenets of social constructivism, their theoretical principles and possible pedagogical strategies that will be used to anchor this research study.

Table 2.3: Relational Delineation of the Philosophical Tenets, Theoretical Principles, and Pedagogical Strategies of Social Constructivism (Adapted from Doolittle & Hicks, 2003).

Tenet 1: knowledge is not passively accumulated, but rather is the result of active cognising by the individual. Tenet 2: cognition organises and makes sense of one's experience, and is not a process to render an accurate representation of an external reality. Tenet 3: knowledge has its roots in both biological/neurological construction and in social, cultural, and language-based interactions.	Principle 1: the construction of knowledge and the making of meaning are individually and socially active processes. Based on: Tenet 1 Principle 2: the construction of knowledge involves social mediation within cultural contexts. Based on: Tenets 1, 2 & 3 Principle 3: the construction of knowledge is fostered by authentic and real-world environments. Based on: Tenet 3 Principle 4: the construction of knowledge takes place within the framework of the learners' prior knowledge and experience. Based on: Tenets 1 & 2 Principle 5: the construction of knowledge is integrated more deeply by engaging in multiple perspectives and representations of content, skills, and social realms. Based on: Tenets 1, 2 & 3. Principle 6: the construction of knowledge is fostered by learners becoming self-regulated, self-mediated, and self-aware. Based on: Tenet 1	Strategy 1: teachers and learners should be prepared to implement visual technology as a tool for inquiry. Based on: Principles 1, 3 & 4. Strategy 2: teachers should use visual tools to create authenticity, which facilitates the process of learner inquiry and action. Based on: Principles 1, 2, 3 & 4. Strategy 3: teachers should use visual tools to foster local and global social interactions such that learners attain multiple perspectives on mathematical concepts and ideas. Based on: Principles 1, 2 & 5. Strategy 4: teachers should facilitate learner knowledge construction by using visual technology to build on learners' prior knowledge and interests. Based on: Principles 1, 4 & 6 Strategy 5: teachers should enhance the viability of learner knowledge by using visual technology to provide timely and meaningful feedback. Based on: Principles 1, 4 & 6. Strategy 6: teachers should cultivate learners' academic independence by using a variety of visual tools to foster autonomous,
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		creative, and critical thinking. Based on: Principles 1, 2, 3, 4, 5 & 6
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With the backdrop of this research literature and theoretical framing, for this study it is expected that the social constructivist teacher should uphold multiple perspectives from participants' respective experiences that ensure that their implementation cultivates relational connections (Kim, 2001). These should emerge as the teachers interact with teaching and learning tools (and their learners) to establish how visualisation impacts on the teaching of patterns and their generalisations. Visualisation processes, when carefully handled, have the potential of building bridges between the world of experiments and the world of thinking and reasoning (Konyalioglu et al., 2012). Thus, the social realities perceived by participants need to be examined in accordance with the pattern of thinking emerging from the different social discourses.

The data collected from this study process was analysed, premised on the fact that it this is a qualitative research study, anchored by a social constructivist theoretical framework. The thematic content narrative analysis was used to analyse data collected from survey questionnaire responses, observational analytic tools and interview transcripts.

Regardless of approach, a vast array of data sources should be available to the narrative analyst, including transcripts from interviews, focus groups, samples of conversations, published literature and media-based materials (Hodges et al., 2008). This research study strived to fulfil the expectations of such criteria, based on its multiple data sources. The data sources for this study included questionnaire responses, video transcripts, notes from analytical tools and interview transcripts. Narrative analysis focuses on the use of language within a social context (Salkind, 2010). Through social interaction and in using language, people produce versions of particular events – thereby creating their reality. Different people might therefore have different perceptions and versions of the same event.

In this study, selected teachers taught the same topic (patterns) using the same approach (visualisation), but there arose different constructions of how the approach impacted on the teaching process. The participants' versions of reality can be referred to from a narrative analyst's perspective as interpretative repertoires or metaphors (Porter, 1996). In this study,

narrative analysis was used to understand how participants construct meaning about what factors impede or enable the teaching of patterns when visual approaches are used.

According to Bamberg (2010), narratives provide portals into realms of knowledge about what has been experienced. The narrative approach involves inquiry directed at the narration of human experiences or inquiry that produces data in a narrative form (Buttino, 2015). This researcher used narrative analysis premised on the idea that emerging perspectives need to be examined critically to come up with a balanced understanding of the issues involved. In this study the participants' power to present experiential knowledge and the researcher's grounded experiences and emerging multiple perspectives intersected to bring about a rich narrative.

The concept of thematic analysis, grounded in narrative context, will be discussed in detail in Chapter Three, which deals with methodology and research design, to blend it conveniently with the theoretical framing of social constructivism discussed in 2.3 above.

2.4 CONCLUSION

The context in which this study was conducted needs to be understood from the perspectives of all parties involved (the researcher and the participants). Thus, literature is used to conclusively tie up the issues that emerged from the research process to support an analytical process that sufficiently examines all components of the investigated phenomenon.

The reader is hereby informed that the study does not necessarily seek to examine the ultimate truth regarding the subject of interest, but to make substantial contributions to ongoing research studies. The findings of this study can be used as a basis for further in-depth studies about what enables and/or what hinders the teaching of patterns and their generalisation when a visualisation approach is applied.

The way and method by which this investigation was undertaken is discussed in detail in the next chapter. The researcher strived to dissect with appropriateness the aspects of research discussed here and marry them coherently with literature.

CHAPTER 3: METHODOLOGY AND RESEARCH DESIGN

3.1 INTRODUCTION

In this chapter I present the description of the research process. I start by stating my research goals and the research questions that informed them. I proceed to define my research orientation, which was a qualitative approach grounded in an interpretive paradigm. I then proceed to dissect the research methods I employed in the study, which included a survey and a case study. Research participants were selected from a sample population of Senior Phase mathematics teachers. The study then proceeded with a research design that comprised of six phases. The first phase dealt with the process of distributing and piloting a prepared questionnaire and the subsequent selection of research participants. The second phase involved the analysis of data collected using the questionnaire. The next phase, Phase 3, dealt with how the intervention process was conducted and how teaching programmes which were to be used in the implementation process were compiled. After this phase, a pilot study of lesson observations was conducted as Phase 4 of the research design. This was followed by Phase 5, in which the actual lesson observations were conducted as selected teachers implemented the knowledge acquired in the intervention programme. In the last phase of the research design, Phase 6, I conducted two parts interviews: one-on-one stimulus recall interviews with each of the participant teachers and a focus group interview.

Study methodology then examined data collection instruments, which included a survey questionnaire, observations and interviews. The collected data was then critically analysed using thematic approaches. The analysed data was then validated accordingly, and issues of ethical concerns was discussed and explained with regard to how the research process unfolded.

Below is a diagrammatic overview showing the key aspects of the research methodology - see Figure 3.1 below.

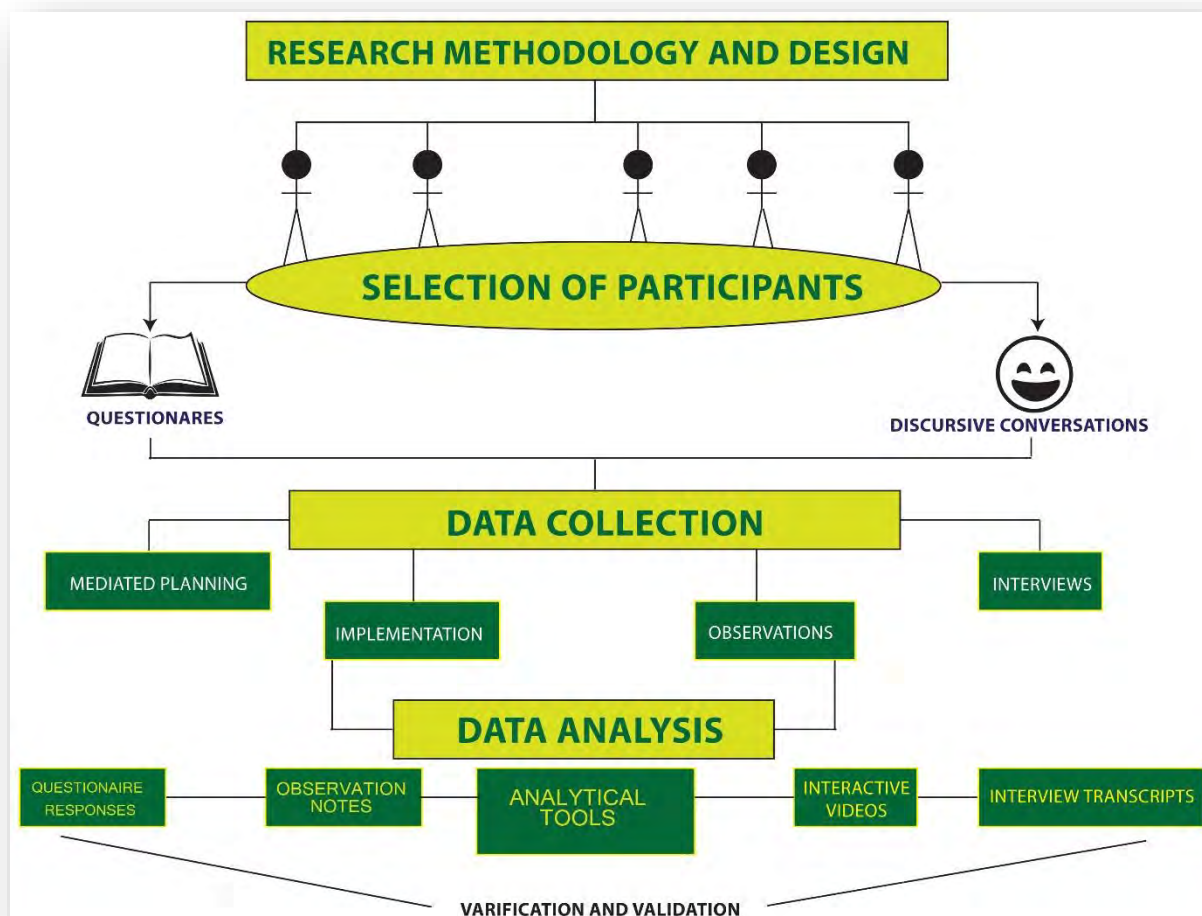


Figure 3.1 Diagrammatic representation of the key aspects of the research methodology

3.2 RESEARCH GOAL AND RESEARCH QUESTIONS

The main purpose for conducting this research study was guided by three main goals: Firstly, to investigate how widespread was the use of a visual approach to teaching patterns by Senior Phase mathematics teachers in the Chris Hani West District prior to an intervention. Secondly, to investigate how Senior Phase mathematics teachers in Chris Hani West District apply visual approaches to teaching patterns after an intervention programme. Thirdly, to investigate the experiences and perceptions of Senior Phase mathematics teachers in order to establish what factors enable and/or hinder the teaching of patterns before and after an intervention. To achieve these goals the research process was designed to answer the following research questions:

- 1) How do selected Senior Phase mathematics teachers in Chris Hani West District teach patterns prior to participating in an intervention programme?

- 2) How do selected teachers in Chris Hani West District teach patterns and their generalisation after participating in an intervention programme?
- 3) What are the selected teachers' experiences and perceptions of teaching patterns and their generalisation prior to and after participating in an intervention programme?
- 4) What are the enabling and/or constraining factors of applying a visualisation approach to teaching patterns and their generalisation?

The research methodology or strategy for this study was determined by the nature of the research question and the phenomena being investigated (Denzin & Lincoln, 2005). Thus, the research format used in the investigation was seen as a tool to answer the research questions. This study was aimed at exploring and understanding the views and collective meanings constructed by participants (comprising of nine Senior Phase mathematics teachers) about what factors enable and/or constrain the teaching of patterns when visual approaches are used. The study did not aim to provide the ultimate truth about this research topic but rather to investigate a particular way of looking at and deriving meaning on the phenomena under investigation.

3.3 RESEARCH ORIENTATION AND APPROACH

This research study is a case study using a qualitative approach, and framed within an interpretive paradigm, which is informed by social constructivist views.

3.3.1 Interpretive Paradigm

A research paradigm refers to a researcher's worldview which—implies the perspective, or thinking, or shared beliefs that inform the meaning or interpretation of research data (McKenzie & Knipe, 2006). My research study reflected on my beliefs about the phenomenon under investigation, and how I experienced it through the eyes and actions of my participants. I used it as the lens through which I looked at the world of pattern teaching. It was the conceptual lens through which I examined the methodological aspects of this research project to determine the research methods that I would use and how the data would be analysed (Lather, 1986).

My central endeavour of using an interpretive paradigm was to understand the subjective world of human experience (Guba & Lincoln, 1989). I had to make an effort to “get into the heads of the subjects being studied” so as to understand what they were thinking or the meanings they were making in the context of teaching patterns using visuals (Kivunja, 2017. p. 33). I made every effort to try and understand the viewpoint of the participants I observed rather than use my own viewpoint as an observer. I placed emphasis on understanding the individual

participants and their interpretations of the phenomenon under study, hence my key tenet to an interpretive paradigm was that reality is socially constructed (Bogdan & Biklen, 1998). This, I realised, was the reason why an interpretive paradigm is often referred to as the constructivist paradigm. In an interpretive paradigm, theory does not precede research but follows it so that it is grounded in the data generated by the research process (Corbin, 1990).

3.3.2 Qualitative research approach

First and foremost, qualitative research is holistic in nature. It looks at the bigger picture and begins with a search for understanding of the parts that make the whole as it looks at the relationships within a system (Denzin & Lincoln, 2005).

This research study combined processes that included survey questionnaire responses, interactive discourse from an intervention process, insights from observed lessons, individual participant views from an interview process and collective meaning making from a focus group discussion. Through a process of searching to understand the parts that made up the whole concept of teaching patterns and their generalisation from a visual perspective, this researcher was able to examine relationships within a system of perceptions and experiences.

Secondly, a qualitative approach to research demanded time-consuming analyses. It required ongoing analysis of the data collected. This researcher started off the process of analysis from the initial onset of data collection. Data analysis in this research study took long and weary hours as I took my time to systematically analyse teachers' views before and after the intervention programme. The analysis process started without delay from the survey questionnaire responses and proceeded to the lesson observations that took place after an intensive intervention programme. The process of analysis then entered into a connective phase as data from the interview process that ensued was collated and critically analysed. Analysis did not end there, it proceeded to examine teachers' collective views from data collected from a focus group interview process. Even then data analysis continued to its finalisation point as I engaged participants in a member checking processes.

A qualitative approach also requires the researcher to become the research instrument. This aspect of the qualitative research approach allowed me to incorporate space for the description of my own biases and ideological preferences. A qualitative approach is not complete without the researcher's informed contributions that shape and give credence to the phenomenon under investigation. This was the reason why I used my own biases to some extent in analysing data. I wanted to both influence and be affected by the research process (Bahraim, 2016). I also

wanted to become a credible qualitative researcher with a whole new way of thinking about what counts as evidence in an investigated phenomenon in qualitative data. The concept of reflexivity and of a researcher being a primary instrument through which research is conducted, helped me to understand the complexity inherent in qualitative research data (Xu & Storr, 2012).

Reflexivity pertains to the analytic attention of the researcher's role in qualitative research (Palaganas et al., 2017). Reflexivity is both a concept and a process (Dowling, 2006). As a concept, it refers to a certain level of consciousness; it entails self-awareness (Lambert, Jorneen & McSherly, 2010) which means being actively involved in the research process. As a researcher I realised that I had to be very much part of the social world that I was studying (Frank, 1997; Morse, 1991). Reflexivity as a process emerged as introspection on the role of subjectivity in the research process. It was a continuous process of reflecting on the participants' values (Parahoo, 2006) and recognising, examining and understanding how their interactive engagement, experiences, locations and assumptions affected their practice and perceptions (Hessen-Biber, 2007).

Reflexivity changed me as a researcher in many ways as I shared the changes brought about by me during the research process. It highlighted the journey of discovering how I, as a researcher shaped, and how I was shaped by the research process and outputs (Palaganas et al, 2017). The key to reflexivity was for me to make the relationships and influences between participants and me, the researcher, visibly explicit.

Last but not least, in this study the qualitative approach incorporated consent decisions and was responsive to ethical concerns. Participants for the research study were recruited on a voluntary basis and necessary documentation was used to confirm their consent to take part in the research process. Even though the selected participants signed a participatory contract, the document was just for formality purposes as participants could still opt out if need be at any given point during the research process. Participant anonymity was guaranteed [by the use of pseudonyms and/or codes in collected data].

3.4 RESEARCH METHODS

In this study, the research methods I used included a survey and a case study. In this section I explain how these two methods were applied to examine Senior Phase mathematics teachers' perceptions and experiences about the use of a visual approach to teaching patterns and their generalisation.

3.4.1 Survey

In qualitative research a survey is used to answer questions that have been raised; to solve problems that have been posed or observed; to establish baselines against which future experiences and comparisons can be made; to analyse trends across different experiences; and generally, to describe what exists, in what frequencies, and in what contexts (Isaac & Michael, 1997, cited in Glasow, 2005). This was the rationale against which I made my choice of a survey approach to research the phenomenon under study.

In this study, I opted for the survey method because of its flexible use as a research approach to investigate a wide range of topics. This researcher capitalised on employing the questionnaire as a survey tool for data collection. A survey is a traditional way of conducting research (Mathers, Fox & Hunn, 2007). I was convinced that a survey was going to be particularly useful for my own non-experimental descriptive design that was aimed at describing contextual reality. The survey approach is frequently used to collect information on attitudes and behaviour. This would fit conveniently well with my research intentions of getting collective meanings from participants' perceptions and experiences about what enables and/or constrains the teaching of patterns when a visual approach was used.

My survey was restricted to a representative sample of the potential group that I was interested in, for reasons of practicality and cost-effectiveness. My research survey was by nature, a cross-sectional survey, since it was carried out at just one point in time to provide a snapshot of how Senior Phase mathematics teachers in CHW District taught patterns and their generalisation. Despite this, the findings to this investigation were intended to be used in follow up activities with teachers in the district to cultivate a culture of using visualisation approaches in teaching mathematical concepts. The survey took an explorative form that set out to describe how the teaching of patterns was enabled and/or hindered by the use of visuals.

I took two important steps to set in motion my survey process. First, I developed a sample plan that I would use to select the samples from a survey population. My approach to this was to pilot a pre-prepared survey questionnaire on a few individual teachers picked at random from the geographical sites of my intended research. Having got an idea of how the teachers would respond to my survey questions, I adjusted my questions accordingly.

My second step was to co-opt the Senior Phase mathematics subject advisor to assist me with the contact details of potential respondents from my research population which included forty Senior Phase mathematics teachers from the CHW District. This assisted me in the distribution

and retrieval of the questionnaires. I then drafted criteria by which I would then select a sample of nine teachers from the population for the survey. Emails and WhatsApp messages were the media through which the questionnaire was to be administered in adherence to rules and regulations of a volatile Covid-19 environment at the time of the survey. Procedures were put in place for obtaining population estimates from the sample data and for estimating the reliability of those population estimates. I had to identify desired response rates from the various circuits in the district and my preferred level of accuracy of responses from the pilot survey. In the end I obtained thirty-seven questionnaire responses that were suitable for use in my research process, since some questionnaires were not fully completed or not returned at all.

3.4.2 Case Study research

Case study methodology provides tools for researchers to study complex phenomena within their contexts. Likewise, the phenomenon to be investigated in this study presented complex issues that required careful exploration in their actual contexts (Barxter & Jack, 2008).

An approach to case study research is usually based on a constructivist paradigm. This aspect of a case study views truth as a relative entity which is dependent on an individual's perspective. This paradigm recognises the importance of the subjective human creation of meaning but does not absolutely reject some notion of objectivity (Stakes, 1995; Yin, 2003). Constructivism is built upon the premise of a social construction of reality (Searl, 1995). This researcher made an informed choice of using case study research in this investigation. Since one of the advantages of this approach is the close collaboration between researcher and participant to construct meaning while enabling participants to tell their stories, this choice is appropriate.

Another aspect of a case study methodology in research is that it enables a researcher to examine the data within a specific context. In many cases a case study method selects a small geographical area or a very limited number of individuals as the subjects of a study. In their true essence, case studies explore and investigate contemporary real-life phenomena through detailed contextual analysis of a limited number of events or conditions, and their relationships (Zainal, 2007). It is against this background that this research study was conducted.

On the other hand, Yin (1984, p. 23) defines the case study research method

as an empirical inquiry that investigates a contemporary phenomenon within its real-life contexts: when the boundaries between phenomena and context are not clearly evident and in which multiple sources of evidence are used.

This resonated well with my research process in which evidence came from three different sources: questionnaires, observations and interviews.

3.5 SAMPLING AND PARTICIPANTS

The trend in qualitative research usually focuses on using relatively small samples (Lyell, 1998). Research participants are generally selected because they are able to provide rich descriptions of their experiences, thereby providing information that is rich and able to challenge and enrich the researcher's understanding of the phenomena of interest (Crabtree & Miller, 1992: Hutchinson & Wilson, 1991).

This researcher opted for purposive sampling for this particular research. Purposive sampling, also known as judgemental, selective, or subjective sampling is a form of non-probability sampling. In purposive sampling the researcher relied on his own judgement when choosing members of a population to participate in a study. Purposive sampling is often used in qualitative research for the identification and selection of information-rich cases related to the phenomenon of interest (Palinkas et al., 2015).

In this study two samples were used, first the sample of Senior Phase mathematics teachers who responded to the survey questionnaire. Then there was the sample of the nine selected teachers who took part in an intervention process and subsequently implemented the intervention knowledge. In the first sample I used random purposive sampling and I had to apply the principles of convenience purposive sampling for the second sample. I acknowledge my use of purely subjective tendencies to choose participants for the intervention, although I used a criterion of purpose to at least collect data that could be qualitatively generalised.

This researcher's choice of selecting participants from research sites that are in close proximity to him as well as selecting participants on the grounds of their availability was largely informed by convenience sampling techniques. However, effort was applied in selecting participants to establish a sample population that could be said to be statistically representative of the greater population under consideration. The sample of participants for this study more-or-less fitted in with a heterogeneous profiling tendency that could at least be qualitatively generalised.

In purposive sampling the researcher goes to the a community or group of people on purpose because he/she thinks that the individuals fit the profile of the people that the researcher needs to reach for the intended phenomenon of interest. Findings from purposive sampling do not necessarily have to be representative of the greater population of interest, but they are qualitatively generalizable.

The more prior information I have as the researcher about my particular group of people or community of interest, the better the sample for my purpose. In this study the researcher opted to use maximum variation purposive sampling, slightly tempered by convenience sampling techniques, since these sampling procedures were judged to blend in quite well with the purpose of this study. The purpose of this study was to investigate the views of Senior Phase mathematics teachers about the constraining and enabling factors of using visualisation to teach patterns. The teachers' perceptions were investigated from experiences at an individual and collective level. Thus, heterogeneous views were investigated from varying sources to provide a rich base of data and information for the phenomenon of interest. The diversity of views in this study emanates from the variety of contexts experienced by participants from the various research sites and from responses from survey questionnaire data. The way the two samples were selected is explained below in section 3.5.1 and section 3.5.2.

3.5.1 Senior Phase mathematics teachers in the district

The fact that I had to involve Senior Phase mathematics teachers since I was also a Senior Phase mathematics teacher in the same district suggested that I use a convenience purposive sampling method for my research population. After all I had in the past years interacted and shared practices with quite a number of this population group in my capacity as a lead mathematics teacher conducting district facilitated teacher development programmes.

However, choosing Senior Phase mathematics teachers in the CHW District as my research population from which I would select my research sample did not in any way present a conflict-of-interest issue. Since I was conducting a study on a separate platform with completely different intentions, I was convinced my research efforts did not in any way conflict with the teacher development programmes of the CHW District, but rather that it supplemented the district's teacher development efforts in a positive way. After all, I was only going to involve forty teachers out of a total of about 160 Senior Phase mathematics teachers in the district. This population sample was reasonably representative according to qualitative research requirements.

3.5.2 Participants for the intervention

The main idea when using purposive sampling is to reject the individuals who do not fit a particular profile when creating a sample. The type of purposeful sampling that was considered and selectively used in conjunction with convenience sampling to select research participants in this study, was maximum variation. According to Foley (2018), maximum variation purposive

sampling (which is also referred to as heterogeneous purposive sampling), is a technique used when the researcher is looking to examine a diverse range of cases for a particular phenomenon. This type of sampling technique allowed me to gain much insight from as many angles as possible.

Using principles of maximum variation purposive sampling, I narrowed my sample to nine participants. I did this by adhering to a set of criteria for my selection purposes. My selection criteria included the following: teachers to be selected had to be practising Senior Phase mathematics teachers currently teaching either Grade 7, 8 or 9; they had to have taken part in the CHW District teacher development programme I once facilitated in or any such related programme; they had to have trained to teach mathematics either at GET or FET level; and they should be currently teaching in the iNciba Circuit where the researcher is based as a Senior Phase mathematics teacher. Above all they should have shown interest by participating in the initial survey for teachers in the district.

3.6 RESEARCH DESIGN

In a qualitative study, research design should be a reflexive process operating through every stage of the research process (Hammersley & Atkinson, 1995). My research design took a logical progression of six distinct phases. In this section I give an explanation of how these phases panned out in my study.

Phase 1: Questionnaire piloting, distribution and selection of nine participants

After having made preliminary inquiries from a number of mathematics teachers in the district, I piloted my pre-prepared questionnaire on at least one teacher from every circuit in the CHW District. The Senior Phase mathematics subject advisor assisted me in identifying individuals with whom to pilot my survey questionnaire. After retrieving the pilot questionnaires, I then accordingly adjusted my questioning style in order to get precisely what I intended as suitable responses for my research study.

Partly due to the need to maintain the project at a cost effective level and partly due to compliance to Covid-19 regulations by minimising contact with survey participants, I had to use online methods for the distribution of questionnaires to potential participants. I was assisted by the subject advisor who gave me the contact details of the teachers and also added a message to the teachers encouraging them to be supportive of my research efforts. I had to also put a message to the teachers explaining the nature and purpose of my research initiatives. All in all I distributed sixty questionnaires by email and by WhatsApp.

From the thirty-seven fully completed and returned questionnaires, I embarked on the process of selecting nine participants: three Grade 7 teachers, three Grade 8 teachers and three Grade 9 teachers, using the criteria I mentioned in Section 3.5.2 above. I also selected teachers with survey responses that would conveniently answer my research question. Soon after selection was finalised, I briefed the participants and asked them to complete consent forms after deliberating all the issues concerning the study and its ethical considerations. For regulatory and sustenance purposes, the participants were also asked to sign a commitment contract as a formality.

Phase 2: Analysing the questionnaire data

The process of analysing data from the research started as soon as the completed questionnaires started trickling in. A thematic narrative approach which derived from the way the survey questions were phrased, was used to analyse the questionnaire responses.

Table 3.1 Predetermined and emerging themes used to analyse questionnaire responses

Predetermined themes		
Theme	Survey question derived from	Theme description
Theme 1	Question 1	Approaches used to teach patterns.
Theme 2	Question 2	Approaches that involved visuals.
Theme 3	Question 3	Frequency of use of visuals.
Theme 4	Question 4	Challenges of using visuals.
Theme 5	Question 5	Effectiveness of visuals used.
Emergent themes		
Theme 1	Question 5	Role of visuals in teaching patterns.
Theme 2	Question 5	Benefits (pay-off) of the use of visuals.

The data from the survey questionnaire was analysed in accordance with the themes derived from the way each question from the questionnaire was responded to by the participants.

Predetermined themes 1 to 5 were derived from each of the five point questions of the survey questionnaire see Table 3.1. Emergent themes were derived predominantly from question number 5 from the survey questionnaire also see Table 3.1.

Analysis was carried out through drawing up similarities and differences from teachers' responses. The aim was to examine how teachers taught patterns and their generalisation prior to an intervention programme. Effectively, findings from this aspect of the study automatically answered Research Question 1: *How do selected Senior Phase mathematics teachers in the Chris Hani West District teach patterns prior to participating in an intervention programme?* Some of the themes used to analyse teachers' perceptions were predetermined from the survey questions whilst some themes emerged from the way teachers responded to the survey questions.

Phase 3: Planning of teaching programme and workshop with teachers

The data that I collected from the survey compelled me to conduct an intervention programme in a constructive way. I decided to share my experiences with the participants and encouraged them to implement well-planned visual strategies and intended to examine the effectiveness of a visual approach to teach patterns and their generalisation.

Initially I intended to conduct a series of workshops with the participants at a central venue where we would engage in interactive discussions and planning of teaching programmes that emphasise the use of visuals to teach patterns. However, this was not feasible in a Covid-19 environment. Instead, I visited teachers in their respective sites and conducted the workshops there with limited numbers (three teachers to be precise, since the schools from which the participants came from had all the Senior Phase classes under one site).

I conducted three different workshops at each site. The first workshop involved the use of different visual strategies to teach patterns and incorporated the development of visual tools and the planning of a single lesson teaching programme. The second workshop involved the use of visual generalisation strategies and again the development of visual tools and a single lesson teaching programme. The third workshop session involved the identification of factors that enable and/or factors that constrain the teaching of patterns when a visual approach is used. The workshop took place in a discursive environment where teachers demonstrated some of the aspects of a visual approach in peer teaching activities, which were later critically discussed by participants. Each workshop session took two hours, and they were all conducted after

school. The researcher took the role of moderator and guide throughout the workshop sessions and provided an intricate knowledge base for teaching patterns from a visual approach.

Phase 4: Pilot study

Due to Covid-19 restrictions the managers of the research sites limited the number of visits I would be able to make to their respective schools. As a result, I could not conduct pilot study lesson observations with all the participants' at all three sites. I conducted two pilot study observations in research site A and one pilot study in each of the other two research sites. Thankfully this gave me an idea of the kind of data I would expect to collect from my main observations.

A pilot study or preliminary study is referred to as a small scale of a complete process or a pre-test for a particular research instrument such as an observation or an interview. In this study a pilot study was carried out in order to discover problems and barriers related to the phenomenon under investigation and assess the acceptability of the observation protocol. The pilot study was also conducted to test drive research equipment processes that would unfold (Hayat, 2014).

Phase 5: Implementation of intervention programme

Soon after the intervention programme, the participant teachers embarked on an implementation process in which they taught patterns in their respective schools. I observed a total of nineteen lessons from eight teachers, since one participant withdrew from the study citing personal commitments as the reason. The table below shows the summary of lesson observations made. Observations occurred in all the three research sites identified: School A, School B and School C. The participants were assigned codes pertaining to their respective sites and grade level taught. For instance, a Grade 7 teacher from school A would be referred to as TA7. Each participant was observed a minimum of two times and some teachers were observed three times, which included the pilot study observation.

Table 3.2 Breakdown of lesson observations conducted with participants

Code of observed teacher	Frequency
TA7	3
TA8	2
TA9	2
TB7	2
TB8	0
TB9	3
TC7	2
TC8	3
TC9	2
TOTAL	19

Eight out of the nine selected participants were observed teaching the topic patterns. Three of the participants were observed three times each which included a pilot study observation. Due to limited time and the difficulties of operating in a volatile pandemic environment, the other five participants were only observed two times each. One participant opted out and was thus not observed – see Table 3.2 above.

The implementation process and observation took longer than necessary because classes were taking place on rotational basis due to the pandemic. This delayed data collection to a considerable extent. This aspect made planning of observational schedules quite difficult as one grade would be in school on certain days of the week and teaching time was drastically reduced from one hour per lesson to thirty minutes per lesson. Teaching with visuals requires time and preparation but the situation compromised both time and teacher enthusiasm to adequately prepare for effective teaching. Even though the implementation process happened under dire conditions I managed to however collect ample data for analyses purposes.

The observation process involved three observation schedules. Some themes for the observations were informed by literature whilst some emerged from the implementation process. Table 3.3

portrays how observations were carried out according to seven pre-determined themes and three emergent themes.

Table 3.3 Observation themes used during implementation

Pre-determined themes			
Themes	Theme description	Visual strategy used	Codes
1	Near generalisation strategies incorporating question generation	-Counting analysis -Recursive analysis	VQG REA/COA
2	Far generalisation strategies incorporating visual tool inspection	-Whole-object analysis	VTI WOA
3	Constructive generalisation strategies incorporating visual usage and symbol inscription	-Difference rate analysis	VIU, SYI DRA
4	Deconstructive generalisation incorporating visual tool transformation	-Explicit analysis	VTT EXA
5	Applicability of visuals		APV
6	Flexibility of visuals		FLV
7	Usability of visuals		USV
Emergent themes			
1	Prevalence of problem-solving dynamics		PBS
2	Evidence of collective knowledge construction		CKC
3	Evidence of building on pre-existing knowledge		BPK

Videos of lesson proceedings were taken during the implementation process. The videos were then transcribed, and it was from this transcribed data that the observation themes were derived. See extracts of video transcripts in Section 4.3.

Phase 6: Interviews

The researcher conducted one-on-one stimulus recall interviews with each participant after each lesson. Another interview was conducted with the whole group of participants to get an idea of their perceptions about what enables and what hinders the teaching of patterns when a visual approach

was used. Special reference was made during the interviews as to what transpired in the lessons, from the videos taken during the implementation process. Each teacher was interviewed twice after teaching each lesson. Teacher TC7 was the only one who was interviewed once as she excused herself from the second interview session citing work commitments. The interviews were also transcribed, and the data used to draw up themes for analysis purposes. Table 3.4 gives an overview of the frequency with which interviews were conducted.

Table 3.4 Overview of stimulus one-on-one interviews conducted with participants

Teacher's code	Frequency
TA7	2
TA8	2
TA9	2
TB7	0
TB8	0
TB9	2
TC7	1
TC8	2
TC9	2
TOTAL	13

Pre- determined themes for the interview process were derived from transcribed data from the interviews and from literature. Emergent themes came from how the participants responded to questions in both one-on-one interviews and in the focus group interview.

Figure 3.2 shows the predetermined themes (first category) that came from the transcribed data and from literature.

CODE	MEANING
ENB1	Enabling factor 1: promotes of conceptual understanding.
ENB2	Enabling factor 2: deepens generalisation skills.

ENB3	Enabling factor 3: develops problem solving techniques.
ENB4	Enabling factor 4: develops cognitive perception.
ENB5	Enabling factor 5: encourages innovation and originality.
ENB6	Enabling factor 6: opens room for exploration.

Figure 3.2: Excerpt (A) on Data categorization for data collected from interviews

Figure 3.3 shows the predetermined themes (second category) that were also derived from transcribed data and from literature.

CODE	MEANING
CNS1	Constraining factor 1: compatibility issues with traditional curricula.
CNS2	Constraining factor 2: violates constructivist teacher/learner contract.
CNS3	Constraining factor 3: limits learner autonomy.
CNS4	Constraining factor 4: limits learners' display of creativity.
CNS5	Constraining factor 5: hinder conceptual development/tasks not completed.
CNS6	Constraining factor 6: hinder progress if learners lack knowledge of algebra.

Figure 3.3: Excerpt (B) on data categorisation for data collected from interviews

Figure 3.4 shows the emergent themes (third category) that were derived from data transcribed from the focus group interview.

Code	Meaning	Description
TPE1	Teacher perception and experience 1	Perceptions about applicability of visuals before and after the intervention.
TPE2	Teacher perception and experience 2	Insights about the flexibility of visuals and about what works and what does not work when visuals are used
TPE3	Teacher perception and experience 3	Beliefs about the usability and effectiveness of visuals.

Figure 3.4: Excerpt (C) on data categorisation for data collected from interviews

3.7 DATA COLLECTION INSTRUMENTS

Three research instruments were used to collect data in this research study. These were a survey questionnaire, observations and interviews. In this section I present how these research instruments were used to collect data.

Questionnaire

A qualitative questionnaire is a survey tool which uses open-ended questions to produce long-form written answers. Survey questionnaire questions aim to reveal opinions, experiences, narratives, perceptions or accounts. Survey questionnaires are a useful precursor to interviews or focus group discussions as they identify initial themes or issues to then explore further phenomena in the research process (Jensen, 2010). For the reasons stated in the brief explanation of survey questionnaire above, I found it fitting to use a questionnaire in my study as it blended well with the nature of my subject of interest.

The selection an appropriate method for data collection such as a survey questionnaire depends on a number of factors that include: access to potential respondents, subject matter and the motivation of the respondents. Questionnaire application in qualitative research can be relatively cheaper and quicker if the sample is large and widespread as in my study (Mathers et al., 2009).

Because my sample population was large and widespread (there are more than 160 Senior Phase mathematics teachers in the CHW District, situated in widely-spread schools, in an area of about 120 square kilometres) I opted to use a survey questionnaire. I used a questionnaire partly because it was cheaper and quicker to collect data in such an environment.

The research question I intended to be answered by questionnaire responses was: How do Senior Phase mathematics teachers in the CHW District teach patterns and their generalisation prior to participating in an intervention programme?

To ensure a high response, rate I accompanied the questionnaires with a covering letter stating the purpose and scope of my survey. I carefully and thoughtfully designed my questionnaire in such a way that it captured the audience and eliminated possible biases towards certain approaches. It was necessary for me to find a way of reducing the number of participants in the survey without compromising the findings in any way. I used random sampling to achieve this. I was aware that random sampling – if adequately and carefully conducted— would ensure internal validity (Mathers, Fox & Hunn, 2009). To obtain a random sample I had to define the population from which it was to be drawn. The population (also known as the sample frame) was made up of Senior Phase mathematics teachers who were currently practising and had had experiences of having taken part in teacher development initiatives run by the district or the Provincial Education Department.

I designed a questionnaire with open-ended questions. In this way, respondents were allowed to interpret the questions in their own way so that the responses I got were rich in detail but not so difficult to code. My questions generally flowed logically from one to the next. For me to achieve the

best response rate, questions flowed from the least sensitive to the most sensitive; from the factual and behavioural to the attitudinal and from the more general to the specific (Kabir, 2016).

In my design I used statements which could be interpreted in the same way by members of the population of interest. I also used statements where persons with different options would give different answers. I ensured that I used one aspect of the construct I was interested in per question item. I achieved this by using positive statements and I avoided negatives or double negatives. My questioning techniques were tailored in such a way that they did not make unnecessary assumptions about the respondents.

The actual questions of the questionnaire made use of clear and comprehensive wording, with correct spellings, correct grammar and punctuation. I ensured that questions avoided items that contained more than one question per item. The questions were not in any way biased or leading the respondents towards an answer.

I was targeting a sample population of forty Senior Phase mathematics teachers, so I distributed sixty questionnaires – only to get a response rate of thirty-seven respondents. This provided me with adequate data to analyse and set my investigation in motion with nine selected teachers from the survey as my research sample. From the data I obtained from the questionnaires I then saw the need to conduct an intervention programme with the nine selected teachers after which I observed them teaching patterns in their respective schools. I observed each of the participants and collected data.

- **Observations**

Observation is a method of data collection in which the researcher observes participants within a specific research field. Participant observation involves the observer being a member of the setting in which they are collecting data. Most observations seek to answer the question “what is going on here?” (Bryant, 2009, p. 2).

Observation means that the researcher is better able to understand and capture the context within which people interact. This first-hand experience with the setting allows the researcher to be open to discovery and inductiveness, rather than guessing what the context is like. Thus, the researcher may see things that routinely escape the awareness of the participant, using different methods. Observation provides a chance to learn things that people may be unwilling to discuss in interviews (Bryant, 2009). It was from this position that I chose to use observations in order to bring the best of a narrative from the point of view of an eye witness to the proceedings that shaped teachers experiences and perceptions.

The research question I intended to be answered by the implementation of the intervention as I observed the teachers teaching was: *How do selected Senior Phase mathematics teachers in the CHW District teach patterns after participating in an intervention programme?* Observations have to be tied to the research question(s), which should act as a guide to what is to be observed (Bryant, 2009). My observation followed a semi-structured format that involved the development of observation templates in the case of this study I used three prepared observational schedules or analytical tools. The first one focused on what visual strategies were used by teachers to teach patterns in general. The second observational tool focused on what visual strategies teachers used to teach generalisation of linear patterns. The third observation schedule focused on factors that enabled and/or constrained the teaching of patterns and their generalisation based on several criteria of applicability, flexibility and usability of visuals when a visualisation approach was applied.

Piloting data collection from an observational process is an important way of determining what is important to include or what can possibly be ignored. I piloted my observations in quite a limited way since circumstances at the time of my data collection process did not allow me the liberty of piloting all participants – see Table 3.1. However, I got an opportunity to test my equipment and got an idea of what to expect in the actual observational process. I video recorded all the observed lessons and took down important notes during the observations, which included three pilot studies.

My role as a researcher in the observations was quite crucial for authentic data collection. I took the position of a background (non-participant) observer but with all due ethical concerns taken care of. For instance, before the observations I ensured that I made the participants aware of the purpose of the observation and asked for their free consent. Each participant was assigned a pseudonym for anonymity purposes. I made notes during the observations concerning what the teachers were doing and what were they trying to accomplish, how exactly they did it, how they characterised what was going on and the assumptions they made. I also took into consideration how I, as the researcher, felt about collecting this unfolding data and how my views about what was happening would affect the data (reflexivity). If I happened not to be sure whether or not to record an event, I recorded it anyway since it is better to have too much data than to have not enough data.

I was going to analyse my observation data in accordance with its appropriateness to my overall analytical strategy. Thus, my observation work was solely deductive, in which data relied heavily on what transpired rather than on what I assumed to have occurred. In that regard my qualitative data was associated with terms such as authenticity, trustworthiness, reflexivity, particularity and subjectivity. The observation data's subjectivity took into account biases and awareness about injustices that emanated from the application of a visual approach in mathematics teaching.

Trustworthiness identified the nature and sources of inequalities posed by using a visual approach on learners of mixed abilities. Authenticity represented and respected the perspectives of the participants. Observation data was triangulated with other sources (questionnaire responses and interviews) to make visible the ways in which participant teachers and their respective learners, as well as the researcher, benefited from the power of collective interactions. Reflexivity was achieved as the researcher engaged participants respectfully and collaboratively. In the end, the analysis of data collected from observations took on a crystallisation rather than generalisation precedence as the researcher identified potential change-making strategies by building the capacity of the participants involved.

The observation data and findings contributed to theory and possible dialogue in a clear experiential and values context. The observation data thus raised such analytical questions as: *what do I see going on? What did I learn from the observation process? What views should I include to give an authentic story of what actually transpired?* Data from the videos taken during observation was then transcribed for analysis purposes.

- **Interviews**

According to Kvale (1996) a qualitative research interview seeks to describe meanings of the central themes in the life world of the research subjects. Thus, the main task in interviewing participants is to understand the meaning of what the interviewees say. Interviews seek to cover both the factual and the derived meaning level, though it is usually more difficult to interview on a meaning level.

Interviews are particularly useful for getting the story behind participants' experiences. The interviewer can then pursue in-depth information around the topic of interest from what proceeds from the interview interactions process (Kvale, 1996). My interview process was thus premised on these theoretical ideas.

In an interview process the role of the interviewer is to ensure that the participants could construct the meaning of their experiences without feeling that they are being judged.

Once the participants agreed to be observed and interviewed after an intervention and implementation process, an appointment was made with each participant at a time convenient to both the researcher and the individual participants. The interviews were scheduled to take place at the participants' workplaces or their homes, depending on what was suitable for each individual participant. The background of the research was explained to the participants as well as the ethical considerations relating to their participation. The researcher did not adopt an expert position and was

open and transparent with the participants. This allowed the participants to easily talk about their views and experiences (Edwards, 1990).

The researcher explained to the participants that the research study was prompted by the challenges that he faced in his practice as a Senior Phase mathematics teacher and as a lead teacher taking part in a teacher development programme. The researcher did not have to provide unnecessary details on these challenges faced. Transparency helped the researcher put the participants at ease and also helped with their disclosure of pertinent issues regarding the teaching of patterns using pictorials. In keeping with the idea of openness in the research approach of collecting data, an emphatic connection was created between the researcher and the participants through sharing common experiences (Matsumoto, 1996).

The only interview questions that were pre-formulated were the opening questions to both one-on-one interviews and the focus group interview: *“Can you please shed light on the types of visual strategies you used to teach patterns and why you preferred these as such?”* and *“What common views, if any, do you share about the ways you teach patterns and their generalisation before and after the intervention programme?”* The interviews were relatively unstructured with the researcher beginning with the same broad question for each participant and allowing subsequent questions to be guided by the ensuing conversation. All participants were interviewed by me and all interviews were audio recorded with prior consent from the participants.

The participants were allowed to speak freely on their own terms about the phenomena in question (Mouton & Marais, 1991). Through the use of unstructured interviews, the researcher was able to follow particularly interesting avenues of conversation that emerged in the interview discourse. The participants were treated as experts in their own right and allowed to make the most of the opportunity to tell their stories. In this way, the researcher was able to relate to participants in subjective ways on their own terms, rather than on the researcher’s terms (Edwards, 1990).

The researcher remained on high alert to sense any inclination by participants to screen or filter the information they provided. When the researcher sensed any such evidence of holding back information by participants, he did not risk entering into a therapeutic role with the participant in question.

The interview style was based on the following qualitative research interview methodology described by Neuman (2000, p. 32):

- Questions are tailored to the participant’s situation.
- Researcher shows interest in responses and encourages elaboration.

- Researcher makes the interview a friendly conversational exchange but with slightly more direction from the interviewer.
- Researcher interspaces the interview process with jokes, aside stories and diversions which are also recorded.
- Interviewer uses open-ended questions with frequent probing.
- Researcher and participant jointly control the pace and direction of the interview.
- Interviewer adjusts the participant's norms and language usage in a polite and diplomatic manner.

As described before in this section, the interviews were guided by participants' responses and the direction of the study was thus mutually guided by the researcher and the participants. It was easy for the researcher to establish rapport with the participants and to engage in a conversation with them about the subject of investigation.

In qualitative research the researcher plays a fundamental role as an instrument of data collection (Creswell, 1994) and data interpretation. Seroka (1999) cited in Leonard, (2005) states that qualitative research interviews are an interactive process that cannot be value-free. Therefore, the idea that interviews should be neutral is not possible. Parker (1994) argues that any of research is undertaken by subjective individuals and this subjectivity needs to be acknowledged. However, Parker (1994) also reiterates that when a researcher acknowledges this subjectivity, he/she should be able to account for what has led him/her to investigate the subject of interest. From conducting interviews in an unstructured format, I was able to gather substantial amounts of detailed information from the participants with minimum effort. The relaxed structure of my questioning techniques gave ample support for participants to be open, generous and honest with essential information for the study. The unstructured nature of the interview process allowed me to be flexible so that I could change focus at any given turn as the interview progressed. Above all, I was able to ask further probing questions to gain more in-depth information. Thus, I was able to collect participants' ideas, opinions, values and beliefs about the phenomenon under investigation. Such were the advantages I got from the use of unstructured interview approaches.

Limitations to the use of an unstructured approach to interviews manifested when participants went off topic when responding during relatively lengthy discussions on the topic under study. I had to battle with multiple and unnecessary diversions as a researcher and moderator of the interview process. In some instances, these diversions led the interviews to go on for longer than the allocated and mutually agreed times. The diversity of responses I got from the various participants was so wide that I found it extremely difficult to produce a coherent report of the findings of the research process.

On another note, limited data resulting from winding and repetitive views from participants made it difficult for me to accurately and adequately reflect upon the views of the wider population concerning the phenomenon under study (Kvale, 1983).

In a synchronous communication of time and space, face to face interviews take advantage of social cues such as voice, intonation and body language. In instances in which I used face to face to face interview processes, I got a lot of extra information that I added to the verbal answers of the participants in question. All of the participants chose to be interviewed face to face with one exception. One participant opted for both face to face and telephone interviews. However, in face to face interviews, social cues and their tendency of visibility led to disturbing interviewer effects as I tried to guide the interviewees in a specific direction with my behaviour. This had the negative effect of participants holding back essential information. This disadvantage was however diminished by my use of an interviewer protocol governing the level of my interactive involvement in the interview process through conscious awareness of the dynamics of face to face interview processes.

One of the advantages of face to face interviews prevalent during the course of this study was that there was no significant time delay between question and answer. The researcher and the participant could directly react to what the other said or did. I noticed that most of the answers of the interviewees were more spontaneous and without the need for extended reflection. This phenomenon of the face to face interview enabled me to minimise on time. Audio recording in face to face interviews reduced the danger of not taking any notes during the interview process – which is essentially vital. Note taking helps to check if all aspects of the questions on the unstructured list have been answered. Note taking saved the day when my recording device malfunctioned and also when I negligently forgot to push the “record” button. Notes became essentially useful when I got stuck in the drill of the largely time-consuming process of transcribing an audio. According to Bryman (2001) one hour of audio recorded information takes five to six hours to transcribe.

Face to face interviews assisted me a great deal to create a good interview ambience. They helped me in making more use of standardisation of the situation under investigation. The emerging views from interviewed participants assisted me with structural formulation of the standard by which I was going to analyse the research findings.

On the other hand, telephonic interviews gave me an extended access to my participant base. They enabled me to contact and work with the participant that had difficulties in scheduling a face to face interview with the researcher (for example mothers and people with business or personal commitments). There was a drastic reduction of social cues when telephonic interviews were

conducted with participant. This resulted in reduced and fewer possibilities to create a good interview ambience (Opdenakker, 2006).

3.8 DATA ANALYSIS

Data analysis in qualitative research involves a range of processes and procedures where the researcher moves from the qualitative data, which have been collected, into some form of explanation, understanding or interpretation of the people and situations under investigation. Qualitative data analysis is usually based on an interpretive philosophy. The idea is to examine the meaningful and symbolic content of qualitative data (Sunday, 2012).

The researcher used a thematic narrative approach to analyse the data collected from the research process. Thematic analysis, described from a constructivist perspective, refers to categories of knowledge forms created through interactions of individuals with the world they experience (Cotty, 2003 cited in Ahmed, 2008). There is no meaning outside of a person's perception. We construct our own realities, thus knowledge is contextual, contingent, and subject to interpretation (Douglas, 2017).

Constructivism informs the thematic analysis of this study by considering the explicit statements of the research participants to be their own construction of what teaching with visualisation means and how it impacts on the teaching of patterns and their generalisation. The researcher needed to make efforts to ensure that the authentic voice of the participants was heard. Thus, the ideas and knowledge forms collected in themes and given coded descriptions were seen to capture the empirical reality of the phenomenon under study. This made it more imperative for me to use thematic analysis as a method of analysing data from the interviews.

However, our views as researchers have an impact as we choose the words that constitute our codes. We define what we see as significant in the data and describe what we think is happening (Douglas, 2017).

I had to make use of my research questions to group data and then look for similarities and differences in teacher perceptions about what factors enable and/or constrain the teaching of patterns when visualisation approaches were used. I also used emergent frameworks from the survey, the observations and the interview processes to group the data and look for relationships.

The ideas that informed my choice of this analytic approach were based on the following principles guiding qualitative data analysis: first, people differ in their experiences and understandings of reality – the constructivist idea of the existence of multiple realities in every social interactive engagement; second, a social phenomenon cannot be understood outside of its own context (context-bound); third,

the understanding of human behaviour emerges slowly and non-linearly; and fourth, exceptional cases may yield insights into a problem or new ideas for further inquiry (Sunday, 2012).

In view of the approach and guiding principles chosen, this study used the thematic type of qualitative analysis. Thematic analysis is the process of identifying patterns within qualitative data (Maguire & Delahunt, 2017). From the perspective of teaching and learning, thematic analysis is a method rather than a methodology (Braun & Clarke, 2006). This means that unlike many qualitative methodologies, it is not tied to a particular epistemological or theoretical perspective. This makes it a very flexible and considerably advantageous method given the diversity of work involved in teaching and learning.

In my analysis I followed Braun and Clarke's (2006) six step framework. In my view this analytic approach offered a clear and usable framework for carrying out a thematic analysis on the subject of interest. The goal of a thematic analysis is to identify themes (patterns in the data that are important or interesting) and use these themes to address the research or say something about arising issues. This is much more than simply summarising the data; a good thematic analysis interprets and makes sense of the data.

Two levels of analytical discourse are involved in thematic analysis. At the semantic level of thematic analysis of the explicit or surface meanings of the data the analyst is not looking for anything beyond what a particular participant has said or what has been written. The latent level looks beyond what has been said and starts to identify or examine the underlying ideas, assumptions and conceptualisations that are theorised as shaping or informing the semantic content of the data.

In my analysis, I moved from one step to the next; however, the phases were not necessarily linear. I moved forward and backward many times when dealing with the complex data from the survey, the observations and the interviews.

Step 1: familiarising with the data

The first step in my qualitative analysis was reading and re-reading the questionnaire responses, the video transcripts and the interview transcripts. I made myself quite familiar with my entire body of data or data corpus (questionnaire responses, video transcripts from lesson observations and interview transcripts), before proceeding with the process of analysis. At this stage it was important that I took notes and recorded all early impressions.

Step 2: Generating initial codes

In this phase I started to organise my data in a meaningful and systematic way. Coding reduces lots of data into small chunks of meanings. The method of data generation and coding was determined by my perspectives and the research questions.

Glense (2006) cited in Feza (2015) defined coding as a progressive process of sorting and defining scraps of collated data that are applicable to research purpose. The coding process I used consisted of re-reading the transcripts and identifying recurring words, ideas, or patterns generated from the data. I had to re-read the narratives and highlight, within each narrative, prominent ideas and recurring words or messages. I then developed a corresponding code, a shorthand designation, to easily identify the recurring words/ideas for that passage and placed it in the margin. After completion of the coding of the first transcript, a master code list was constructed. As I proceeded with the next transcript, codes were pulled from the master code list if found to be applicable, or new codes were created and added to the master code list. My initial master code list contained five codes (recurring patterns/themes).

Codes were then placed into a logical category or a word or phrase describing some segment of my data that was explicit. Categories reflected the themes that had become apparent and represented the major findings of the study. For this study the codes were condensed into three major themes each with sub-categories. The themes and sub-categories were questionnaire response themes, observational themes and themes emerging from interviews and focus group discussions, each highlighting the following major themes: 1) how teachers taught patterns and their generalisation prior to an intervention; 2) how teachers taught patterns and their generalisation after an intervention; and 3) teachers perceptions and experiences about what enables and/or constrains the teaching of patterns when visuals are used.

Step 3: Searching for themes

As defined earlier, a theme is a pattern that captures something significant or interesting about the data and/or the research question (there are no hard and fast rules about what makes a theme). A theme is characterised by its significance. I examined all the codes I had assigned to data and found that some fitted together into a defining theme. I organised the codes into broader themes that seemed to say something specific about each research question. My themes were predominately descriptive as they described patterns in the data relevant to the research question. Table 3.5 shows all the preliminary themes that I identified, along with the codes that are associated with them.

Table 3.5 Categorisation of themes from collected data

Theme	Theme description	Code
Themes from the survey questionnaire responses		
1	Teaching approaches used to teach patterns and their generalisation	TPU
2	Frequency of use of visuals	FUV
3	Challenges of using visuals	CUV
4	Benefits of using visuals to teach patterns	BUV
Themes from lesson observations		
1	Use of near generalisation strategies that include counting and recursive analysis	COA/REA
2	Use of distant generalisation strategies that include whole-object analysis	WOA
3	Use of constructive generalisation strategies that include difference rate analysis	DRA
4	Use of deconstructive generalisation strategies that include explicit analysis	EXA
5	Applicability of visuals in teaching patterns	AVS
6	Flexibility of visuals in teaching patterns	FVS
7	Usability of visuals in teaching patterns	UVS
Themes from the interview process		
1	Enabling factors of visualisation -promotes conceptual understanding -deepens generalisation skills -develops problem solving techniques -develops cognitive perception -encourages innovation and originality -opens room for exploration	ENB1 ENB2 ENB3 ENB4 ENB5 ENB6
2	Constraining factors of visualisation -compatibility issues with traditional curriculum -violates constructivist teacher learner contract -limits learner autonomy -limits learners' display of creativity -hinders conceptual development	CNS1 CNS2 CNS3 CNS4 CNS5 CNS6

	-hinder progress when learners have limited knowledge of algebra	
3	Teachers' perceptions and experiences -perceptions about applicability of visuals to teaching patterns -insight into the flexibility of visuals to teach patterns -beliefs about the usability of visuals to teach patterns	TPE1 TPE2 TPE3

Step 4: Reviewing themes

During this phase I reviewed, modified and developed the preliminary themes that I identified in Step 3. It was useful to gather together all the data that was relevant to each theme. I read and scrutinised the data associated with each theme and considered whether the data really supported it. The following were the things that I had to think about: First, do the themes make sense? Second, does the data support the themes? Third, am I trying to fit too much or too little into a theme? Fourth, are there themes within themes? Fifth, are there other themes within the data (emerging themes)? Some of the adjusted changes are shown in Figures 3.2, 3.3 and 3.4 in Section 3.6 above.

Step 5: Defining themes

This is the phase in which I did the final refinement of the themes and subthemes. The main aim of this phase was to identify the essence of what each theme was about. What is the theme saying? How do sub-themes relate to the main themes and how do the themes relate to each other? Table 3.5 was used as a basis from which a final thematic map was derived in order to come up with the relationships between themes, based on the teachers' perceptions.

Data analysis is simply the process of making sense from the data. To do this, this researcher immersed himself in the data and consolidated the data focusing on those segments that provided insight into the research questions. The researcher then compared the segments and looked for patterns/themes embedded in the data. The researcher interpreted what was said by participants and made meanings or understandings, which became the findings of the study.

Step 6: Write-up

Usually, the endpoint of a research process is some kind of report. The last stage of my thematic analysis was the interpretation of the data or simply making meaning from the data. Interpretation consisted of studying the categories and their corresponding codes to determine whether there were

any overarching themes or theories that provided insight on the factors that enable and/or constrain the teaching of patterns when visualisation approaches are used.

Data from collated information in questionnaire responses, analytical tools used during observations of lessons, conversations from interactive discussions, video transcripts and transcribed texts from interviews were reflexively analysed at each stage of the data collection process.

The identity of the participants was removed from the transcripts to maintain their confidentiality. For instance, data from individual questionnaire respondents was coded from TR1 to TR37. Pseudonyms were assigned to participants in order to protect their identity, while providing information relating to their experiences. The recorded interviews were transcribed within twenty-four hours of being conducted. The intention of this rapid transcription process was to allow the researcher to become familiar with the data as quickly as possible.

Analysis was a continuous process in which the protocols of research were read over a period of time and each time a deeper level of analysis was reached. As a result, the data analysis and the literature review occurred in tandem. While the literature review guides the researcher in observing certain aspects in relation to the topic under investigation, the process of data analysis also informs the relevant literature that was undergoing review.

Although thematic analysis can be focused on body language and other forms of visibility as a means of discourse (Powers, 2001), this study mainly focuses on analysing the spoken words that were transcribed to create a written narrative. In addition, the participants' social and historical contexts and experiences were taken into consideration. During analysis the researcher constantly reflected on the questions asked during the survey and interviews and the responses provided by the participants. This was done in order to avoid making irrelevant analyses that would not meet the objectives of the study.

The analysis process was completed in three phases and each phase is outlined in the section below.

Phase 1

In Phase 1 of the analysis process, the researcher began by reading data to get an overall sense of the data and experience the different participants' frameworks. This phase of analysis was concerned with making an understanding of how teachers in the CHW District taught patterns prior to an intervention. This study's analysis serves to provide an independent reality check of predetermined theory change, which helped participants to assess, learn from and demonstrate the social impact of their teaching processes. Qualitative impact assessment protocol places project beneficiaries' voices (in this case Senior Phase mathematics teachers) at the centre of the evaluation and analysis process. This enables

them to share and feedback their experiences in an open, credible and respectful way (Remnant & Avard, 2016).

The researcher gathered evidence of the initial and subsequent impact of the investigation through casual statements collected directly and indirectly from the participants. Data from survey questionnaires was then triangulated with data collected from lesson observations and from interviews that ensued after the implementation process. Once the data was collected the researcher used robust thematic coding methods, systematically coding for drivers, outcomes and attribution.

Phase 2

The second phase of the analysis process involved the identification of common constructions or meanings (categorisation) that emerged from the lesson observations. The researcher made efforts to interpret what transpired in the observed lessons by drawing similarities and differences based on identified themes. Identified constructions were then deconstructed to gain a better understanding of each observed aspect as presented by different participants. During the deconstruction phase the researcher examined the texts for contradictions, similarities and ambiguities emerging from the constructions of the participants. In this phase the researcher identified the objectives of the study based on the language used by the participants. The similarities and differences were examined within and across grade levels. The data was analysed in accordance with the thematic categories raised during the coding process. The dominant aspects around the teaching of patterns and their generalisation were examined in this context.

Phase 3

Following the process of categorisation and the search for differences and similarities in how participants attached meaning to situations, the researcher then integrated the different constructions. In this stage of analysis, the researcher attempted to construct a holistic and comprehensive view of the emerging narratives or constructions that emerged from the interview process. The teachers' perceptions and experiences about what enables and/or hinders the teaching of patterns when visuals are used became explicitly manifest as I processed the interview transcripts.

When the need arose during the third phase of analysis, the researcher conducted additional interviews to make adequate sense of the emerging constructions. This process ensured the stability and credibility of the findings to this study. Qualitative data analysis was interactive and typically coincided with data collection (Gibbs, 2002), hence the possible inclusion of additional interviews during the data analysis process (the additional interviews were appended to the original transcripts).

3.9 VALIDITY

Criticism has often been made against qualitative research for lacking rigour and credibility associated with traditional quantitative research (Horsburgh, 2003). The emphasis in quantitative research is on accuracy of data and the extent to which data can be generalised. According to Denzin and Lincoln (2005), quantitative research concerns itself with the extent to which results are consistent over time (reliability) and whether the research truly measures that which it was intended to measure (validity).

On the other hand, qualitative research disputes the idea of generalisability of results and argues that meaning is historically situated and therefore no two people can experience the same “problem” in the same way (Denzin & Lincoln, 2005). Qualitative research acknowledges that there is always the prevalence of a multiplicity of information and therefore results can thus not be generalised across different contexts (Denzin & Lincoln, 2005). In qualitative research, reference is made to various techniques that can be used to evaluate and validate qualitative studies. There are various hallmarks of qualitative research that I used to ensure the validity of this study (Moss, 2016). One of the hallmarks is the construct of reflexivity which, as described in Section 3.3, clearly illustrates the researcher’s subjective involvement in the research process. The researcher acknowledges that his framework and context has an influence on the conception of the research. By referring to my personal journey, as a researcher, with regard to the topic of discussion, I provided the context and foundation upon which the research is grounded.

Validity in qualitative research can be seen in the extent to which the researcher provides sufficient detail to enable the reader to interpret the meaning and context of what is presented (Popey et al., 1998). Validation in this research study thus depends on the transparency with which my data collection and analysis procedures are presented. However, the usefulness of the term validity in qualitative studies is regarded with scepticism by some researchers (Wolcott, 1990, cited by Hangashi Jr et al, 2019; Smith, 1983). Kirk and Miller (1986) discuss the importance of truth in the quality of the results in qualitative studies and the search for objectivity in the construction of knowledge. However, threats such as limitations in the access to data, and the complexity and limitations of the human mind to gauge the quality of research results can be mitigated upon by reflexivity techniques.

There are various types of validities that apply to research. The most important types of validity for qualitative studies, according to Maxwell (1992), are descriptive, interpretive and theoretical validities. In descriptive validity the researcher does not embellish or distort the information. Situations and facts reported are those that are seen and heard. Interpretive validity refers to the researcher’s sensitivity and mental processes used to capture and interpret/construct the meaning of objects, events and behaviours of people engaged and involved in the studied phenomenon.

Theoretical validity refers to how much theoretical explanations that are developed through research analysis are consistent with the data collected. Every theory has the components: concepts and categories and describes how they relate to each other.

In the context of this study, validity can be replaced by the term trustworthiness (Onwuegbuzie & Johnson, 2006) or rigour (Golafshani, 2003), since without trust and rigour, research would be worthless or, even worse, it would be a fictional piece (Morse, Barret, Olson & Spiers, 2002). According to Lincoln and Guba (1985), the trustworthiness of a research study is the central aspect of the issues that are conventionally called validity and reliability.

One of the hallmarks of validity in qualitative research described by Horsburgh (2003) and Popey et al. (1998) is the interpretation of subjective meaning. This refers to the process of using the participants' accounts to analyse and interpret data. Good qualitative research should include a description of the context within which the research was conducted. This is done in order to provide the reader with an understanding of the context of the study.

Triangulation is used in this study to ensure a research account that is rich, robust, comprehensive and well developed (Thurmond, 2001). Triangulation involves using multiple data sources in the investigation to produce understanding. Theoretical validation and triangulation are particularly valid to this study. Theoretical triangulation refers to the use of multiple perspectives to examine and interpret the data. The benefit of theoretical triangulation is that it provides a broader and deeper understanding of the research findings (Thurmond, 2001).

Creswell (2014) defined qualitative validity and reliability as the steps a researcher takes in order to check for accuracy of the findings, by employing certain procedures. In this study I used several strategies to ensure validity and reliability of my investigation. The first strategy was chosen for its potential to promote credibility (internal validity): 'member checking', the sharing of interview transcripts, analytical thoughts, and/or drafts of the report with research participants to make sure I was representing them and their ideas accurately. The second strategy involved having an expert audit review of my data analysis by my supervisor, who served on my Master's degree committee. The third strategy involved my clarification of the bias that I as the researcher brought to the study. I openly and honestly discussed the biases (past experiences, assumptions and opinions) that would shape my interpretation of the research study. The fourth strategy meant that I provided detailed procedures used for data collection and analysis. The strategy I used to promote transferability (external validity) included providing rich, 'thick' descriptions. The term 'thick' describes a highly descriptive detailed presentation of the setting, and in particular the findings, with supporting evidence presented as

narrative quotes. The other strategy I employed was to use maximum variation sampling techniques during sample selection, thus allowing for a greater range of narrative application by my readership.

In this research study validity was determined by the extent to which the data obtained from the participants had been consistently checked to the point where data analysis processes become self-correcting (Gibbs, 2002; Schurink, 1998). This researcher validated the data to an extent where he was able to identify when to continue, stop or modify the research process (Morse, Barret, Olson & Spiers., 2002).

In this research process the researcher was on the lookout, throughout the whole process, for any evidence of the data being repetitive or irrelevant to the research questions. This helped me to determine when to stop or continue with the process of investigation. Qualitative research focuses on understanding the phenomenon under investigation (Babbie & Mouton, 2001). In so doing, analysis in this study, aimed at ensuring that the research findings captured what the participants viewed as the factors that impede or necessitate the teaching of patterns when visualisation approaches are used. This was captured verbatim from the participants' responses and analysed accordingly.

Validity in this regard was measured by the extent to which the analysis reflected what was said by the participants. This understanding is in line with the theoretical framework for this study (social constructivism), which does not aim to seek ultimate truth but is rather focused on identifying the constructions by the participants concerning the narratives under investigation (Vygotsky, 1978). Both qualitative research and social constructivist frameworks are concerned not so much with whether data is true, but with capturing and understanding what is said and experienced at a point in time (Babbie & Mouton, 2001). This is because both paradigms contest the notion of objectivity.

In qualitative research, stability is used as an indication of validity (Burns, 2006). Stability in qualitative research refers to the trustworthiness of data and is concerned with whether the observations are repeatable (Gibbs, 2002) during the data collection process. To ensure that the data was stable in this study, the researcher continuously reflected on the objectives of the study by ensuring that the participants were guided to remain within the domain of the study. The stability of the findings was concerned with the degree to which the findings responded to the focus of the inquiry and not the biases of the researcher.

Consistency in the answers received from different participants enabled me to compare the extent of stability in the findings. Stability was also measured by the extent of the level of my empathy and the extent to which I reflected on my understanding of issues to the participants (Gibbs, 2002). The social constructivist paradigm maintains that the researcher is a key instrument in the construction of

meaning. The researcher may have an overarching role in the construction of meaning but he certainly does not have the mandate to impose views in a dictatorial fashion. By reflecting on my understanding of what participants said, I was able to confirm or negate the meanings derived from the conversations in order to construct shared meanings and realities.

The context in which behaviour is understood and interpreted in qualitative research is crucial. The context included the researcher's viewpoints on the subject matter and how the research procedure affected and was impacted on by the researcher (Schurink, 1998). The trustworthiness and validity responsibility of a research process lies entirely with the researcher.

Reliability in research measures the extent to which the research findings will remain consistent across repeated investigations in different circumstances with different investigators and the extent to which such findings are generalisable (Gibbs, 2002). However, this study adopted a qualitative methodology with social constructivism as a theoretical framework. These two approaches both argue that there are multiple realities and perspectives and that people construct narratives differently, from similar contexts, at different times. As a result, it was not possible for data to remain consistent across repeated investigations with different participants. Constructivist discourse and qualitative research are not interested in making generalised statements about human behaviour (Shurink, 1998).

Qualitative research therefore refers to dependability rather than reliability. According to Lincoln and Guba (1985), the process of allowing for external audits is aimed at fostering the dependability of the data presented during the research process. Allowing an external person to evaluate whether or not the findings, interpretations and conclusions are supported by the data, allows me to ascertain the extent to which the presented data is dependable. This researcher allowed the analysis of the data and interview texts to be discussed with the project supervisors who then made their own interpretations of the information gathered and constructively questioned the analysis made. According to Thurmond (2001), if every person involved in the same data analysis comes to the same outcome then it is more likely that the findings are true and therefore dependable.

3.10 ETHICS

Several ethical considerations were taken into account to ensure that the study was conducted in an appropriate manner (Babbie & Mouton, 2001). To comply with ethical considerations in conducting research all participants were asked to provide verbal and written consent to be observed and interviewed – see Appendix 3. The participants willingly and voluntarily consented to participate in the research study after they were approached by the researcher (Lee, 2000: Neumen, 2000). The research purpose and processes were explained to the participants. Even though participants were

requested to sign a negotiated contract to minimise dropouts from the study – see Appendix 4. This was done in keeping with the suggestion by Silverman (2009) that highly formalised ways of securing consent should be avoided in favour of fostering relationships. The negotiated contract form was designed in such a way that ethical regard for participants was sustained.

The purpose of the research was explained to the participants and the participants were told that should they wish to withdraw at any point during the investigation they could do so. Permission to video record classroom observations and audio record interviews was obtained from the participants and all stakeholders' involved – Appendices 5a and 5b. It was further explained to the participants that their information would remain confidential and that the specific content of individual and group interviews would only be discussed with the researcher's supervisors. In the final report the identity of the participants was removed and pseudonyms used for participant identification. For instance, the Grade 7 teacher to be interviewed from the first research site was referred to as TA7, and the Grade 9 teacher to be interviewed from the third research site was referred to as TC9 and so on. This was done to secure anonymity among participants.

The researcher found it appropriate to emphasise the confidentiality of the information collected and to establish trust with the participants from the very early phases of the investigation. While sharing the purpose of the study with the participants, the researcher also shared his background and experiences as a Senior Phase mathematics teacher. This had the effect of encouraging the participants to willingly and freely share their experiences and deep-seated views.

At the end of the investigation the researcher debriefed the participants by talking critically about the process itself, and discussed with the participants the impact this investigative process and findings could have on the population of interest. The aim of the debriefing process was to ensure that the participants are not left emotionally harmed or traumatised from the drill of the research process.

The observable benefits of the study were manifest and became observable by the mood of the participants during and after research processes (observations and interviews) and the potential impact this would have on the research population. There will be information dissemination workshops that will be conducted by the researcher targeted on the research population soon after the findings have been established. The tone of the conversations regarding the research findings after the research process will manifest its impact (Motseme, 2002).

Any risks, either minor or major, that this research process posed on the research subjects were taken care of by the researcher. The researcher took drastic measures to minimise at all costs psychological or emotional harm to the participants. Since participants were operating in circumstances that were

technically different to their normal routine, they were compelled to work under unnecessary pressure, causing them discomfort and probably fatigue, and they needed emotional support from the researcher and from their peers. This was taken care of by openly talking about it beforehand and collective ways of overcoming these effects were negotiated in the initial stages of the research process. The fact that another human being was going to be present to observe a participant could trigger some form of embarrassment and uneasiness. The researcher had to discuss all such phenomena and find ways to overcome them as a collective group with the participants.

This researcher employed the bow tie analysis as a risk management technique. The bow tie analysis technique is one of the most particular techniques available in helping identify risk mitigations in qualitative research (Shuttleworth, 2017). The technique starts by looking at a risk event and then projects this in two directions. To the left, all potential causes of the event are listed and, to the right, all potential consequences of the event are listed. It is then possible for the researcher to identify and apply mitigations (or barriers) to each of the causes and consequences separately, effectively mitigating both the probability of risk occurrence and the subsequent impacts, should the risk still occur.

3.11 CONCLUSION

This chapter outlined how the research was conducted: the process used to select participants, the method used to collect data as well as the data collecting instruments used. The approach that was used in analysing the data from a survey questionnaire, the process and analytical tools used to gather data from lesson observations, the interview process and the way texts from transcribed interviews and observation videos were illustrated and categorised. The aim of this study was to understand the participants' constructions of reality, thereby illustrating ways in which phenomena under investigation are collectively and individually constructed by selected participants. Experiences and perceptions of selected Senior Phase mathematics teachers about what enables and/ or constrains the teaching of patterns using a visualisation approach were examined and analysed from a narrative context of predetermined and emergent themes in order to construct shared meanings about the investigated phenomenon. The findings from this elaborate research process are outlined in Chapter 5 of this thesis.

CHAPTER 4: DATA PRESENTATION, ANALYSIS AND DISCUSSION

4.1 INTRODUCTION

This research study was conducted to investigate how widespread was the use of a visual approach to teaching patterns and their generalisation by Senior Phase mathematics teachers in CHW District prior to an intervention. The study also sought to investigate how selected Senior Phase mathematics teachers applied visual approaches to teaching patterns and their generalisation after an intervention programme. Subsequently, the study aimed to investigate the experiences and perceptions of participant teachers in order to establish what factors enabled and/or constrained the teaching of patterns and their generalisation before and after the intervention when they used visual approaches. In this chapter I present, analyse and discuss the data collected. The chapter begins with a presentation and analysis of survey questionnaire data to investigate how teachers in the Chris Hani West District taught patterns and their generalisation prior to an intervention programme. This is followed by a presentation, analysis and discussion of data from lesson observations and from a series of interviews carried out after the implementation of the intervention. The chapter then proceeds to the integration of the data collected from questionnaires, lesson observations and the interviews. Data from the survey questionnaires was used to answer Research Question 1, whereas Research Question 2 was answered using data from the lesson observations and lastly, data from the interviews answered Research Questions 3 and 4. As all data forms unfold, teachers' perceptions and views about what factors enhance and/or hinder the teaching of patterns, prior to and after the intervention, are revealed and discussed accordingly.

4.2 SURVEY QUESTIONNAIRES

The data from survey questionnaire responses of the thirty-seven Senior Phase mathematics teachers ($n = 37$) from the CHW District were qualitatively analysed using the thematic approach. Both predetermined and emerging themes were used to analyse teachers' questionnaire responses prior to an intervention. My main interest at this stage was to gain an understanding of how Senior Phase mathematics teachers in the district taught patterns and their generalisation before an intervention. Each questionnaire respondent was allocated an identification in form of codes. The codes were arrayed ranging from Tr_1 to Tr_{37} as explained in Chapter 3, Section 3.1.

The data from questionnaires is presented and analysed in two parts. Firstly, the discussion of predetermined themes which were noted from literature and used to formulate the five questions of the survey questionnaire. Secondly is the discussion of themes that emerged from the way Senior Phase mathematics teachers responded to the questions in the questionnaire (see Figure 3.2). The

data collected at this survey stage and its unfolding implications to my study gave me the reason for carrying out an intervention process with a selected number of the surveyed teachers.

4.2.1 Analysis of questionnaire data – Predetermined themes

Approaches to the teaching of patterns

The first question from the questionnaire was to investigate approaches that teachers were using to teach patterns prior to an intervention programme. The responses from surveyed teachers indicated that they are using more than one approach to teach patterns and their generalisation. For instance, Tr₂₂'s response (see Table 4.1) shows that this teacher used more than one method to enhance learner understanding, although the teacher did not state the actual methods. Furthermore, Tr₂₄ reiterated that he used a variety of approaches (see Table 4.1).

Table 4.1 Teachers' various approaches to teaching patterns

Teacher	Response
Tr ₂₂	I teach patterns using approaches that make my learners understand quickly and with ease.
Tr ₂₄	My teaching of patterns involves several methods and techniques that assist learners to understand patterns in an easy way.
Tr ₈	I teach using different methods to cater for different mind sets and bring understanding to them.
Tr ₉	I make use of pictures and prediction methods.
Tr ₁₂	My teaching of patterns includes strategies and techniques that makes understanding easy and learning enjoyable.
Tr ₁₃	I use the environment around me to make learners identify patterns and proceed to numerical situations.
Tr ₁₆	I usually make learners count from pictures and manipulates such as matchsticks.

Teachers went on to give reasons why they chose the approaches they used. For instance, Tr₂₂ and Tr₂₄ used teaching approaches that were aimed at making it easy for learners to understand patterns and their generalisation. Tr₈ on the other hand used a variety of approaches with the aim of catering for different mind sets. The variety of teaching methods used by Tr₉ included pictures and what he termed 'predictions', which I presume was the product of analysing the pictures used in different pattern problems. In the same light, Tr₁₂ used a variety of strategies and techniques aimed at making the learning of patterns and their generalisation easy and enjoyable. Tr₁₃ had her own way of teaching

patterns. She used pattern forms in the learners' immediate environment. Learners would identify patterns in the classroom surroundings to connect to numerical situations.

The teachers in the CHW District used a variety of approaches to teach patterns (Table 4.2). My interest was to check whether some of these approaches involved visuals. I was interested in finding out the nature of the visuals, their form and these teachers' sources of these visuals. Question 2 in the questionnaire was meant to probe for such information. Some of the teachers' responses are presented in Table 4.2 below.

Table 4.2: Teachers' approaches that involved a variety of visual strategies and techniques

Teacher	Response
Tr ₁₅	The strategies I use include pictures, images, tables and diagrams and sometimes flow charts. This assist me a lot to get the concept of patterns across to learners.
Tr ₁₇	Yes I use pictures and sometimes manipulates.
Tr ₂₁	The visual techniques I use sometimes include pictures and tables.
Tr ₂	Yes I use pictures, images and diagrams. I take them from text books and past exam papers.
Tr ₃	Yes I do use images, tables and pictures because it helps learners to understand better.
Tr ₂₃	Sometimes I make use of pictures to make learners understand.
Tr ₁₄	Not really. I use pictures in rare instances.

From the responses in Table 4.2, teachers in CHW District indicated that that they made use of visual approaches in their teaching of patterns and their generalisation. They even went on to mention the kinds of visuals they use such as pictures, images, tables and diagrams (Table 4.2). Some of these teachers were clear as where they got these visuals from, for example Tr₂ who indicated that he takes them from books and past examination papers. Another teacher gave a reason why he uses visuals to teach patterns. Tr₃ said that he uses visuals because they help learners to understand better. This view was also supported by Tr₂₃. Although the use of visual approaches by teachers in the district can be confirmed in this data, what could not be established at this point, was the effectiveness of this widespread use and its acknowledgement by the teachers.

However, not all surveyed teachers acknowledged the use of visuals in teaching patterns and their generalisation. T₁₄ voiced very negative views about the use of visuals to teach patterns (Table 4.2).

Frequency of use of visual approaches

Teachers in this study were asked how often they used visuals when teaching patterns and their generalisation. Some of the teachers' responses are given in Table 4.3 below.

Table 4.3. Teachers' use of visuals

Teacher	Response
Tr ₁₁	I use pictures when explaining and when demonstrating to learners.
Tr ₁₂	At all times during my teaching of patterns I use pictures to make my explanations clear.
Tr ₂₂	Quite often with good progress.
Tr ₂₉	I use pictures most of the time when I want my lesson to be exciting and effective for teaching.
Tr ₃₁	I make use of pictures and other visuals oftentimes to get learners to understand the trend in given patterns.
Tr ₃₅	I usually ignore pictures but use them when I need to explain patterns to learners.
Tr ₂₃	Sometimes I use pictures, images, and diagrams and manipulates to explain with vividness the concept of patterns.

The data in Table 4.3 implies that teachers used visuals frequently to explain and demonstrate mathematics concepts. The teachers used words like 'at all times', 'quite often' 'most of the time', 'oftentimes' (see Table 4.3) to show how frequently they were using visuals. In some instances, teachers found the use of visuals quite unavoidable (see the response by Tr₃₅). It seems, before the intervention, that teachers had found the use of visuals to teach patterns an appealing approach hence they said they utilised visuals quite frequently in their teaching of patterns and their generalisation.

Even though the use of visuals by selected teachers was considerably frequent, the quantity of the visuals used did not reflect concisely on what I expected from a widespread use of visuals. For instance, I noted that teachers mentioned that they used visuals in the form of pictures and manipulatives without giving examples of the nature of these pictures and manipulates. Neither did they briefly state how they used these in conjunction with other visual accessories like tables and diagrams. Thus, the quantity and quality of the visuals used by teachers did not match the frequency with which they were used. This assumption is evidenced by the number of claims to using pictures and to the implied quality of visuals used by teachers in their responses. From what I observed, teachers teaching Grades 7 and

8 advocated that they used visuals quite frequently but with a lack of variety – see responses by T₁₂ and T₂₉. On the other hand, Grade 9 teachers used quality visuals which included pictures, tables, diagrams and manipulatives with quality application of these visuals – see responses from T₂₃ and T₃₁.

Challenges of using visual approaches

After investigating the frequency with which teachers used visuals to teach patterns, I proceeded to look at the challenges teachers faced when teaching patterns and their generalisation before the intervention. Teachers in the CHW District acknowledged facing a number of challenges when teaching patterns using visuals. This aspect was addressed by teachers' responses to Question 4 of the survey questionnaire.

Table 4.4. Teachers' challenges when using visuals

Teacher	Response
Tr ₁₃	The only challenge I face is in preparing enough visual tools for my lessons with limited resources.
Tr ₁₅	I struggle to get learners to give narrative descriptions of what they see in a pictorial pattern as I interacted with them.
Tr ₁₉	The worst challenge I face is getting learners to find the general rule from a pictorial context with minimum assistance since they have limited knowledge of algebra.
Tr ₂₉	My most common challenge comes when learners fail to transfer verbal explanations of visual representations into symbolic representations and the time it takes to do it.
Tr ₂₃	My challenge is in getting learners to work independently without compromising syllabus coverage.

From the data in Table 4.4, the nature of challenges teachers faced when teaching patterns using visuals included preparation due to limited resources, learners struggling to give narrative descriptions of pictorial contexts, learners taking longer times to translate verbal explanations of pictorial patterns to symbolic representations, learners' failure to work independently without compromising syllabus coverage. These challenges faced by the survey teachers posed daunting constraints on the teaching of patterns. However, the challenges could be overcome if measures were taken to train teachers on the correct and effective use of visualisation approaches.

Challenge Number 1 stated above can be attributed to the teachers' constant focus on the numerical aspects of a pattern even when it could be presented successfully in a visual context with visual tools that have been thoroughly prepared. This conditioned tendency by teachers to focus on numerical

aspects of a pattern often creates an obstacle to efficient planning and preparation of visual tools for adequate and comprehensive generalisation (Noss, Healy & Hoyles, 1997).

In the context of visual pattern teaching, if learners are encouraged to grasp figures discursively it is necessary to take into account their complexity through narrative descriptions of their pictorial contexts. This could have mitigated the teachers' Challenge Number 2 above, by allowing a discursive establishment of the skill of generalisation that considers the context of the pictorial figure, with less difficulty (Sasman, Liebenberg, Linchevski & Olivier, 1999). This could also have assisted teachers in making learners take shorter times to translate verbal explanations of pictorial patterns to symbolic representations. In a way, teachers had been engaging in improper application of direct proportion in exploring linear patterns by using strictly numerical reasoning; and thereby implying the meaningless manipulation of visuals on the pretext of adhering to traditional syllabi (Lannin, 2006).

Thus, coming from this background of sticking to traditional curriculum demands, teachers failed to allow learners how to explore different types of patterns from which they could apply visualisation and manipulate figures to facilitate generalisation – see response by T₂₃. I can conclude that teachers' challenges comprised of social factors, which resulted from interactions, which, if not well handled, have the potential of impacting negatively on the learners' thinking. These challenges also comprised of cognitive factors which depended on the mental structures the teacher had to develop in learners through a visual approach. Last but not least, the challenges faced by teachers involved the structure of the task— which had a bearing on the type of pictorial figure, the values attributed to its independent elements and even the ability of the learners to see these values and connections (Mason, 1986).

Effectiveness of visuals for teaching patterns

The use of visuals to teaching patterns and their generalisation was found to be generally effective by teachers responding to Question 5 of the survey questionnaire Responses to these effects are shown in Table 4.5.

Table 4.5. The effectiveness of the use of visuals

Teacher	Response
Tr ₂₃	Pictures are an effective way of teaching patterns.
Tr ₄	It makes learning and teaching fun.
Tr ₈	They [visuals] create a long-lasting memory in learners' minds.

Tr ₁₅	Teaching with visuals is a favourable approach since it makes the lessons exciting and learners understand better from seeing.
Tr ₇	They are [only] favourable for patterns that are hard for learners, which they are only able to understand with pictures and diagrams.

The responses in Table 4.5 confirm that teachers had some degree of appreciation of the effectiveness of using visuals to teach patterns even prior to the intervention programme. For example, the responses by Tr₂₃ and Tr₁₅ support this observation. However, there was some essence of not giving the effectiveness of visuals full credence in teaching patterns as evidenced by the response by Tr₇ who seemed to have given the effectiveness of visuals the benefit of the doubt. The teacher seems to prefer to use visuals only in certain circumstances where the pattern required such, and certainly not for all teaching situations. However, there is ample evidence in the data, as with Tr₄ and Tr₈, to afford credence to the effectiveness of using visuals to teach patterns and their generalisation.

In their responses to survey questions, teachers showed that they were quite aware of the effectiveness of using a visual approach to teaching patterns. However, the responses did not explicitly point out in which ways the use of visuals can be viewed as an effective approach to teaching patterns. The one thing that the teachers mentioned as an effective aspect of using a visual approach was the fact that it makes the learners excited – but did not elaborate to what subsequent effect or with what educational ramifications.

Hasan, Hamad and Iqbal (2015) pointed out that the excitement that learners experience when patterns or any other mathematical concepts are taught using visuals comes with the following benefits: first, visuals have the potential to make learners retain more pattern concepts; second, they help learners to learn more effectively when they are inspired through visuals; third, visuals help teachers develop image formulations that assist learners to see and hear properties from pictorial figures; fourth, they help teachers provide a basis for conceptual thinking in learners; fifth, they help teachers to instil increased narrative proficiency in learners; and sixth, visuals assist teachers to make learning permanent.

4.2.2 Emerging themes from the analysis of questionnaires

The role of visuals in teaching patterns

Some of the responses to the questionnaire related to the role played by visuals in the teaching of patterns, and these were noted. Here are some of the teachers' responses that talked to the aspect of the role of visuals on the teaching of patterns.

Table 4.6. Some teachers' insights into the effect of visuals on teaching

Teacher	Response
Tr ₃₁	Use of pictures ... clarifies the concept of patterns.
Tr ₁₃	Visuals help teaching to go smoothly and cause learners to like mathematics.
Tr ₄	It makes learning and teaching fun.
Tr ₁₅	Pictures make my teaching easy.
Tr ₁₆	Visuals bring mathematics concepts to reality.

The data in Table 4.6 gives the direct role of the use of visuals on teaching patterns. For instance, the responses from T₃₁, T₁₃ and T₁₅ speaks directly on the effect of visuals on teaching patterns in particular, whilst T₄ and T₁₆ does so in a somewhat indirect manner. Therefore, the teachers found the use of visuals to have a positive role on the teaching of patterns as evidenced by the responses in Table 4.6.

Teachers who participated in the awareness workshop that involved responding to a survey questionnaire expressed underlying insights into the future of applying a visual approach to teaching patterns. By advocating that visuals clarify the concept of patterns to learners, teachers acknowledged the idea of giving preferential importance to learners' opinions regarding visual tool understanding. By mentioning that visuals make teaching go smoothly and cause learners to like mathematics, the teachers seemed to suggest that if teachers share opinions about how students regard the usage of visuals it would enhance the teaching/learning processes.

To collectively enjoy the fun and ease of execution of a visual approach to teaching patterns that would bring mathematics concepts to reality, teachers need to take the following initiatives: arrange workshops and conferences for teachers to improve their skills of using visuals; lobby authorities like the local Department of Education to offer periodic training sessions for teachers already in the field regarding the use of visual teaching and learning resources; and last but not least, they should think about forming a forum for teachers in the district to meet periodically to assess the effectiveness of teaching approaches that include the use of visuals (Hasan, Hamad & Igbal, 2015).

Resultant pay-off of the use of visuals on learners

The second emerging theme from teachers' questionnaire responses was the influence or resultant pay-off of the use of visuals on their learners. Teachers' responses in the Table 4.7 below reveal how this phenomenon unfolded.

Table 4.7. Some teachers' responses showing how the use of visuals benefits learners

Teacher	Response
Tr ₁₇	Teaching with visuals develop cognitive understanding in learners.
Tr ₁₈	Learners grasp concepts more easily.
Tr ₂₀	Visuals assist learners to get an idea of how a pattern is unfolding.
Tr ₂₄	Visuals help learners to understand patterns the easy way.
Tr ₃₃	Visuals help me to develop my learners' perceptive understanding.

The responses by Tr₁₇, Tr₂₄ and Tr₃₃ talk of the positive pay-off of visuals on the learners' cognitive and perceptive understanding. On the other hand, Tr₁₈ and Tr₂₀ voiced how the use of visuals assisted learners in their acquisition of knowledge with ease. From the teachers' responses it appears that the use of visuals was an indispensable aspect of teaching and learning of patterns. Since visuals seem to demystify mathematics concepts like patterns and their generalisation, it became imperative to engage teachers in thorough teacher development processes. This would have further lasting impact on the learners who interact with these teachers.

Although teachers did not expressly state that teaching with visuals has a significant role in the teaching of patterns, they were evidently aware of the following aspects of visualisation: First, visuals stimulate thinking and improve the learning environment; second, the use of visuals substitutes a monotonous classroom drill characterised by traditional teaching approaches; third, visuals develop personalised learning in learners as they experience successful and pleasant learning in the classroom environment; fourth, the correct use of a visual approach is useful and relevant, especially when it relates to syllabus content; and fifth, visuals give the teacher insights into learner opinions, perceptions, experiences, failures and successes. These effects of teaching using visuals featured directly and indirectly in the teachers' responses to survey questions.

Reflections from both pre-determined and emergent themes

Considering the individual responses to the survey questionnaire, I saw the need for organising an intervention programme. This move would consider extensive and more focused efforts to workshop teachers on the effective application of a visual approach to teaching patterns. The responses to the questionnaire made a noteworthy impact on my research initiatives and set up the foundational basis for my investigation. I needed to find out what actually enables and/or hinders the teaching of

patterns when visualisation approaches are used, from the teachers' views and experiences, and in an investigative environment.

The data I collected from the questionnaire respondents gave me the impression that the use of visuals to teach patterns makes teaching effective. The use of visuals plays a crucial role in shaping the way pattern knowledge is acquired by learners. Visuals also have a lasting effect on the cognition of learners as they sharpen their perceptive understanding. However, teaching with visuals has its fair share of challenges, some of which include technical and operational difficulties. The frequency with which teachers employ the use of visuals also has far-reaching consequences on them as an effective and reliable approach to teaching patterns and their generalisations.

This data gave me the impression that if adequate and effective intervention is employed on these teachers, the use of visualisation approaches in teaching patterns could make beneficial contributions to the teaching of patterns and to mathematics education. Thus, I used the thematic responses and the emerging themes from the questionnaires as the basic context to engage a group of selected teachers from this survey in an intervention programme. In this intervention programme, I assisted teachers to develop teaching programmes that involved the use of visual strategies to teach patterns and their generalisation. After the intervention, the selected teachers taught pattern lessons in their respective classes which I observed and later analysed.

4.2.3 Summary of survey questionnaire responses

It became clear to me that there was varied usage of visuals in teaching patterns and their generalisation among Senior Phase mathematics teachers in CHW district prior to my intervention. The degree of usage varied significantly, amidst a plethora of challenges and teachers' experiential propensities. Underlying challenges such as a lack of preparation time, lack of resources, time constraints, adherence to the prescribed syllabi content and learners' background knowledge of algebra were noted as reasons why teachers were not using visualisation strategies to teach patterns and their generalisation more meaningfully. As such, based on the evidence of these underlying challenges and pattern teaching dynamics presented in Section 4.2, I concluded that the teachers' involvement in an intervention programme was vital. I then drafted an intervention programme based on the observations made in this section and what literature has said.

The next section focuses on what I observed during teaching after selected teachers were involved in the intervention programme.

4.3 LESSON OBSERVATIONS AND VISUAL PROCESSES

Selected teachers were engaged in an intervention programme and observed teaching their learners after attending the intervention programme. The taught lessons were video recorded, and the videos were transcribed. This process was carried out in order to answer Research Question Number 2: ***How do selected teachers in CHW District teach patterns after participating in an intervention programme?***

In this section I present data collected from lesson observations. The data was categorised in accordance with themes that emanated from literature and derived from prepared observational tools. Also included in the data are themes that emerged as lesson observations unfolded. Videos taken from lesson proceedings were transcribed and the data was collated with visual indicators outlined in the observation schedules (see Figure 3.2). Three observation schedules were utilised for the thematic analysis process in this section. Table 4.8 gives an overview of how analysis of lesson observations was conducted grade by grade and across grades.

4.3.1 Analysis of observed lessons

Use of near generalisation that incorporated visual question generation

The analytical presentation below gives succinct details of how patterns were taught using near generalisation, incorporating visual question generation, from lessons observed in Grades 7, 8 and 9. The nature of interactions that transpired in the observed lessons is depicted in the extracts from the video transcripts in this presentation (see Table 4.8). The following descriptive codes were used to indicate the occurrence of represented aspects:

COA– Counting analysis; VQG – Visual Question Generation; and REA – Recursive Analysis.

Table 4.8. Teachers' use of near generalisation

Manifestation of near generalisation and visual question generalisation		
Grade 7 teachers	Grade 8 teachers	Grade 9 teachers
12: L3: If you multiply the stage number by 3 you get number that is 1 less than the number of sticks at that stage. For instance at stage 2 if you say 3 multiply by 2 you get 6 which is 1 less the number of sticks at that stage which is 7. The same happens at stage 3. 3 multiply by 3 is 9 which is less 1 the total number of sticks at that stage which is 10. [REA] 13: Excellent. What then should we do to get the number of sticks at each given stage? 14: L4: We should multiply each stage number by 3 and add 1 for each given stage. [REA]	9: Can we use this common difference to deduce a rule for the pattern? Let us start by multiplying each stage number by the common difference. [VQG] 10: Can someone present that in algebraic form where n is the stage number. 11: L6: $6n - 2$. [REA]	1: (Teacher puts up a visual tool of overlapping stick triangles). In this pattern of overlapping stick triangles what regular trend do you notice? [VQG] 2: L1: The sticks at the base of the triangles increase by 1 stick at each stage. [COA] 3: L2: The number of sticks making up the upright body of the triangles increase by 3 sticks. [COA] 6: Good. What is the difference between increase in body sticks and increase in base sticks? 7L4: $3 - 1 = 2$ 8TA9: What then is the sum of that figure and the stage number? 9L5: $(n + 2)$ 10TA9: Now let us combine the two expressions together. What do we get? 11L6: $3n + (n + 2)$ [REA]

Manifestation of near generalisation and visual question generalisation		
Grade 7 teachers	Grade 8 teachers	Grade 9 teachers
13: What should we do with the box in the centre? [VQG] 14:L7: If we add the single box to the boxes in the arms we get the total number of boxes at each stage. [COA] 15: Excellent! Now let us analyse the pattern trend using our knowledge of algebra and the common difference. 16:L8: You mean from the idea that stage number multiply by 4 plus one box? [REA] 17: Yes my dear. 18:L9: it is easy $4n + 1$. [REA]		2: L1: There is one triangle in stage 1, two triangles in stage 2 and three triangles in stage 3. [COA] 3 : Are all the triangles fully visible at each stage of the pattern? [VQG] 4: L2: No, not exactly. Some parts of the triangles in stages 2 and 3 are partly obscured. 5: Quite correct. 6: What then will happen if we multiply the 4 sticks with each stage number (n) and add the 2 sticks of the upper top of each triangle? [VQG] 7: L6: We get $(4n + 2)$. [REA]
9: By counting the small squares at each stage of the displayed pattern can someone come up with a fitting numerical sequence for the pattern? [VQG] 10:L1: There are five small squares in stage 1, nine small squares in stage 2 and 13 small squares in	2: Someone count the number of bricks at each stage of the pattern? [COA] 3:L1: There are 4 bricks in stage 1, 8 bricks in stage 2, 12 bricks in stage 3 and 16 bricks in stage 4. [C]	3: How many sticks anchor any fully visible triangle at the base of each stage of the triangular pattern? [COA] 4:L2: Two sticks.

Manifestation of near generalisation and visual question generalisation		
Grade 7 teachers	Grade 8 teachers	Grade 9 teachers
stage 3. Therefore the numerical sequence is 5; 9; 13. [COA] 11: Good counting. Thank you. I would like you to assist each other in your respective groups to come up with a re-represented format of the same pictorial pattern that portray the way each stage is presented. Clue: the peripheral squares in relationship to the central square. We can do it come on. [VQG]	4: Can you describe by what ratio is the pattern growing from one stage to another? [VQG]	5: Can we formulate multiplicative expression that portray the number of the sides of a triangle and its respective stage number? 6:L3: Yes we can say $3 \times n$ or $3n$. [REA] 7: Excellent. Can we now formulate an addition expression that portray the stage number and the two base sticks that appear at each fully visible triangle? [VQG] 8: L4: We can say $n + 2$. 9: TC9: Let us now add together the two expressions and use the combined expression to apply to any given stage of the pattern. What do we get? [VQG]

From lessons taught by the Grade 7 teachers (see Table 4.8), visual question generation became visible on three separate occasions. The first occurrence appears in line 13 of the lesson taught by TB7. The other two occurrences are evident in line 9 and line 11 from the lesson taught by TC7. In these three extracts teachers asked meaningful questions based on the input image. The primordial problem tackling technique was visible in the lesson taught by TC7 – see line 11. Near generalisation strategy in the form of recursive analysis became visible in lines 15 and 17 for the lesson taught by TA7, and also in lines 15 to 17 for the lesson taught by TB7. The observable trend from the extracts was that teachers encouraged learners to extend sequences using a common difference, by utilising features of the given visual figure. Counting analysis as a near generalisation strategy was made use of by TC7 as evidenced in line 10. Overall, teachers teaching patterns in Grade 7 extensively utilised the visual question generation technique and the recursive analysis strategy, but somehow underutilised the counting analysis strategy. The underutilisation of counting analysis strategy was partly due the limited scope of the visual tools used by the teachers and partly due to the way teachers presented their questions to the learners. The teachers concerned could possibly have overlooked the power of counting elements of the figure used in contexts that prompt learners to manipulate visuals, as they work with pattern problems – see Table 4.8.

On the other hand, Grade 8 teachers made use of visual question generation in two separate instances, once by TA8 in line 4 and the other instance also in line 4 from the lesson taught by TC8. However, there was moderate use of the counting analysis strategy by TC8 – see line 2 and line 3. Recursive analysis strategy was scarcely used by observed teachers from Grade 8. Visual question generation was used by Grade 9 teachers at a slightly advanced stage as it included evidence of deep learning methods – see line 7 and line 9 from the lesson extract by TC9. Teachers in Grade 9 also made extensive use of both counting and recursive analysis strategies. For instance, counting appears in lines 2 and 3 from the extract by TA9, in line 2 from TB9's lesson extract and in line 3 from TC9's lesson extract (see Table 4.8). Whereas recursive analysis manifests in line 11 from the lesson taught by TA9, in line 6 from TB9's lesson extract and in lines 11 and 12 from TC9's lesson extract.

All aspects of the use of near generalisation that incorporated visual question generation were covered in all the lessons observed across grades. The level of depth and engagement slightly differed as I observed that visual question generation did not explicitly involve deeper learning in the first two lower grades but graduated slightly as I focused on Grade 9 teachers. Near generalisation involved mostly recursive analysis across the grades and teachers did not utilise counting analysis as I expected them to do.

According to Yamakwa and Mitamura (2016), visual tool question generation involves generating meaningful questions based on an input image. It is a multimodal problem tackling technique, involving image understanding and natural language generation, and especially when using deep learning methods. However, the aspect of deep learning did not explicitly manifest in most of the lessons taught by the selected teachers. Either they lacked execution skills, or they had poorly prepared visual tools. The aspect of visual question generation was examined together with near generalisation of patterns in a way that was reflective of adequate utilisation of knowledge gained from the intervention across grade levels. Near generalisation entails the finding of the subsequent term in a given series of generated patterns (Ozdas & Tanisli, 2009). This aspect (of using near generalisation) was well handled across grades and presented fewer problems for the teachers and their learners.

Use of far/distant generalisation that incorporated visual tool inspection

In this presentation the following descriptive codes were used to indicate the occurrence of represented aspects: WOA – Whole object analysis; VTI – visual tool inspection.

Table 4.9 below contains lesson extracts from video transcripts that depict the kind of interactions that revealed the aspects described above.

Table 4.9: Teachers' use of distant/far generalisation during instruction

Manifestation of distant generalisation and visual tool inspection		
Grade 7 teachers	Grade 8 teachers	Grade 9 teachers
7: What familiar thing have you noticed in the numeric pattern? 8: L5: The numbers seem to be square numbers with 2^2, 3^2, and 4^2 only that they do not start with 1^2. [VTI] 9: Exactly! So what would the pictorial pattern look like if we adjust by adding a stage before and another stage after the last stage? 10: L6: Then we will have the number of rhombuses being 1; 4; 9; 16 and 25. 11: Excellent! What then can we conclude to be the general rule of the pattern if we represent every stage number with the letter 'n'? [VTI] 12: L7: If we square each stage number we get the number of rhombuses at that stage. So the rule is n^2. [VTI]	1: Good. Now count the number of sticks in stages 1, 2 and 3 that I have drawn on the chalkboard. 2: L2: There are 6 in stage 1. 3: L3: 11 in stage 2. 4: L4: And 16 in stage 3. 5: TA8: What trend have you noticed in the entire pattern? [VTI] 6: L5: The pattern grows at each stage by 5 sticks more. 7: TA8: What happens if we multiply the 5 sticks and each stage number? 8: L6: We get a number that is 1 less than the number of sticks at that stage. [WO₂] 9: So how do we manipulate such an idea to get the rule that governs every stage of the pattern? [VTI] 10: T7: We can say 5 multiply by the stage number plus one stick. [WOA]	21: Thank you class. We have succeeded in finding a rule for the pattern. Is there any other way? [VTI] 22: L13: If we square each stage number and multiply by 4 and then add 3 we get the same solutions. 23: TA9: Let us try it for the whole of stage 3. 24: L14: $(3^2 \times 4) + 3 = 39$ [WOA] 25: L15: We can conclude that the rule $4n^2 + 3$ applies at all stages of the pattern. 26: Exactly we have succeeded in finding two options for the rule to the pattern.
2: L1: They are rhombuses.		5: Good presentations. Now let us use the information we have to come up with an all-

Manifestation of distant generalisation and visual tool inspection		
Grade 7 teachers	Grade 8 teachers	Grade 9 teachers
3: Exactly, and what shape make up each stage of the tiling pattern? [VTI] 4: L2: Stage 1 is a 2 by 2 square. 5: L3: Stage 2 is a 3 by 3 square. 6L4: Stage 3 is a 4 by 4 square. [VTI] 7: Now count the tile slabs at each stage and present the numbers in a numerical sequence for the entire pattern. 8: L5: Stage 1 = 4 tiles, stage 2 = 9 tiles and stage 3 = 16 tiles. [WOA]		encompassing rule that can be used at every stage of the pattern. [VTI] 6L4: the 6 sticks of the two triangles at each segment gives us the term $6n$. 7: L5: Each stick added at each segment give us the term $(n + 1)$. 8: L6: The 4 sticks of the constant V-shapes remain as a constant 4. [VTI] 9: Good reasoning. So now let us combine the terms into an algebraic expression. 10: L7: It gives us $6n + (n + 1) + 4$. [WOA]
13: L3: There is one set of 4 squares added to one separate square in stage 1, [VTI] 14: L4: I can see two sets of 4 squares added together with one separate square in stage 2. 15: L5: Also there are three sets of 4 squares added together and added to one separate square in stage 3.	13: Good. What is the best and easiest way to arrive at the number of small squares making up the central part of each stage of the picture pattern? Please use your knowledge of square numbers. [VTI] 14: L5: We square each stage number. Like $3^2=9$. 15: Brilliant what then should we do with the four squares at each corner of the central square?[VTI]	1: (Referring to the figure on display on the chalkboard). How many sides does a triangle have? 2: L1: A triangle has three sides. 3: How many sticks anchor any fully visible triangle at the base of each stage of the triangular pattern from a whole object perspective? [WOA]

Manifestation of distant generalisation and visual tool inspection		
Grade 7 teachers	Grade 8 teachers	Grade 9 teachers
16: What is the rule that we can derive from what has been said about the pattern? 17L6: I see. The rule is $4n + 1$. Where n represents the stage number. [VTI] 18: Quite correct. You can try drawing stages 4 and 5 of the original pattern and shape by growing the shape using multiples of the unit figure as an exercise. [WOA]	16: L6: Just add them to get the total number of squares. $3^2 + 4 = 13$ small square to apply to the whole figure. [WOA]	4: L2: Two sticks. 5: Can we formulate multiplicative expression that portray the number of the sides of a triangle and its respective stage number? 6: L3: Yes we can say $3 \times n$ or $3n$. [VTI]

The observed trend in the lessons taught by Grade 7 teachers was as follows: Visual tool inspection manifested extensively for all the three teachers as shown in Table 4.9. Examples are shown in lines 8, line 11 and line 12 for the lesson extract from TA7; in lines 3 - 6 for the lesson extract from teacher TB7 and in lines 13, 14, 15, 16 and 17 for the lesson extract from TC7. There was evidence of focused mental scanning of the qualities of the image – see line 14 from TC7’s lesson extract as shown in Table 4.9. The process of teaching included selectively attending to input and feedback from existing schemata as evidenced in lines 4-6 for the extract from TB7 – see Table 4.9. Far generalisation through the visual strategy of whole object analysis was carried out by at least two teachers in Grade 7. The evidence of this aspect can be found in line 7 and line 8 of the extract from TB7’s lesson, and also in line 18 of the extract from the lesson taught by TC7 as shown in Table 4.9.

Visual tool inspection is evident in line 4 and line 9 in the lesson taught by TA8. This same aspect of visual tool inspection appeared in a more elaborate manner in line 13 and line 15 in the lesson taught by TC8 as shown in Table 4.9. However, whole-object analysis was explicitly used by teachers in Grade 8. They considered a term of the sequence as a unit and used that unit with adjustments, based on the context of the visual problem at hand. This evidence can be found in lines 8 and line 10 from the extract from TA8’s lesson and also in line 16 from the extract from TC8’s lesson – see Table 4.9.

Visual image inspection was observed in lines 21, lines 5-8 and line 5 from lesson extracts from TA9, TB9 and TC9 respectively (see Table 4.9). Whole object analysis is evident in line 24, line 10 and line 3 of lesson extracts from TA9, TB9 and TC9 respectively as the teachers engaged their learners in far generalisation techniques.

Visual tool inspection and whole object analysis were taught in a more or less similar manner across the grades, however, there was evidence of a higher plane of visual inspection among Grade 9 teachers. The evidence of how visual tool inspection was systematically and skilfully examined can be found in lines 6-8 of the extract from TB9’s lesson (see Table 4.9). The meticulous way in which visual inspection was handled by TC9 in lines 5-6 shows a more informed and advanced strategy execution that has the potential to develop cognitive maturity in learners. Possibly the nature of the visual figure used, assisted the teachers to teach patterns effectively from a visual perspective.

Visual image inspection, also known as visual tool inspection, involves a focused mental scanning of the qualities of the image within working memory. This results when one examines an image in order to answer questions about it (classifying and scrutinising it). It also entails noticing similarities and differences in shape (connection), and selectively attending to input and feedback from existing schemata (Makina, 2010). The aspect of visual inspection was handled by teachers across grade levels with resounding success, pointing to their effective use of the knowledge gained in the intervention

programme. This aspect of visual tool inspection was examined together with far generalisation – which entails the finding of an arbitrary term in a given series of a pattern, where a learner may be encouraged to carry out generalisation by generating a ‘rule’ without finding all subsequent terms until the particular arbitrary term is established (Barbosa & Vale, 2015). Selected and observed teachers across grades did a sterling job in this respect, thanks to their thorough planning and use of appropriate visual tools.

Constructive generalisation with visual image use and symbolic inscription

The following codes were used to indicate the occurrence of the represented aspects:

VIU – Visual Image Use; SYI – Symbolic Inscription; DRA – Difference Rate Analysis.

Table 4.10 contains lesson extracts that depict interactions that involved visual image use, symbolic inscription and the use of difference rate analysis in teaching patterns.

Table 4.10 Teaching of patterns using constructive generalisation

Manifestation of constructive generalisation, image use and visual inspection.		
Grade 7 teachers	Grade 8 teachers	Grade 9 teachers
2:L1: There are four rhombuses in stage 1. 3: L2: There are 9 rhombuses in stage 2. 4: L3: There are 16 rhombuses in stage 3. 5: Can someone present the pattern in numeric terms? 6L4: 4; 9; 16. [VIU]	8: Now try multiplying each stage number by the common difference 3. What did you observe? [DRA] 9: L5: Stage 1 $\times 3 = 3$, Stage 2 $\times 3 = 6$ and Stage 3 $\times 3 = 9$. It gives the number of sticks at each stage! [VIU]/[DRA] 10: Quite correct. Can we now deduce the general rule to this pattern by representing the stage number with the letter n? 11: L6: Yes. We can say n^3 or $3n$. [SYI]	10: What can we say about the number of dots at each stage of side (flank) of the pair of wings of the butterfly at each stage of the butterfly pattern? 12: L7: There are 2 dots in stage 1, 4 dots in stage 2 and 6 dots in stage 3. 13: Does that give any recognisable numeric pattern? 14: L9: Yes. 2; 4; 6 [VIU]
14: If we are to extend the pattern by one more stage what will we have? 15: L9: We will end up with the sequence 1; 4; 9; 16; 25. [VIU] 16: Meaning that each stage number (n) is being squared. [SYI] 17: L10: Yah n^2 becomes the general rule of the tile pattern. [SYI]		4: Does this information remain the same if the figure were to be reconstructed to represent stages 1 and two? I think there is one way to find out. Let us have volunteers to draw stage 1 and stage 2 on the chalkboard assuming the original figure is stage 3 of the pattern. 5: Good presentations. Now let us use the information we have to come up with an all-encompassing rule that can be used at every stage of the pattern. [VIU]

Manifestation of constructive generalisation, image use and visual inspection.		
Grade 7 teachers	Grade 8 teachers	Grade 9 teachers
13: L3: There is one set of 4 squares added to one separate square in stage 1, [DRA] 14: L4: I can see two sets of 4 squares added together with one separate square in stage 2. 15: L5: Also there are three sets of 4 squares added together and added to one separate square in stage 3. [VIU] 16: What is the rule that we can derive from what has been said about the pattern? 17: L6: I see. The rule is $4n + 1$. Where n represents the stage number. [SYI]	5: L2: The pattern is growing by a difference rate of 4 at each stage. If we multiply stage 1 by 4 we get 4, multiplying stage 2 by 4 we get 8 and multiplying stage 3 by 4 we get 16 and so on. [VIU]/[DRA] 6: TC8: Good reasoning. Can you deduce from this conjecture how many bricks will make the 10th stage of this pattern if it is possible? 7: L3: Of course you need to multiply the stage number 10 by 4 and get 40 which will be the number of bricks at this stage. [DRA]	3: Can we formulate an equation using the number of pins attached to each separate card and the total number of pins given to the bursar? [VIU] 4: L2: Three multiply by the number of cards gives the total number of pins. [DRA] 5: Can we present that as an algebraic expression? 6: L2: Yes. $3 \times n = 58$. [SYI] 7: The equation can be simplified as $3 \times n = 57 + 1$. Can we then say conclusively that the bursar would hang 19 cards? [DRA]

Visual image use and symbolic inscription were examined concurrently because of their functional role in pattern teaching. Teachers expressed these two aspects in a variety of ways when tackling pattern teaching. Visual image use is displayed in line 5, line 15 and lines 13-15 from lesson extracts taught by TA7, TB7 and TC7 respectively – see Table 4.10. Whilst symbolic inscription was demonstrated in lines 16 and 17, and in line 17 from lesson extracts taught by TB7 and TC7 respectively. However, constructive generalisation – which involved difference rate analysis – was moderately used by Grade 7 teachers. This aspect was only demonstrated in one instance – see line 13 from the lesson extract taught by TC7 as shown in Table 4.10.

For the Grade 8 teachers, visual image use is shown in line 9 and line 5 respectively for TA8 and TC8, whereas symbolic inscription is displayed in line 11 and line 7 respectively for the two observed teachers. Difference rate analysis is demonstrated in line 5 and line 7 for TC8. The evidence that visual image use was carried out in Grade 9 can be seen in lines 12-14, lines 3 and 4 and line 3 respectively for TA9, TB9 and TC9, whilst difference rate analysis only appears in line 5 for TC9 (see Table 4.10).

Constructive generalisation was not tackled to my expectations by teachers across all grade levels. The use of a common difference as a multiplying factor and subsequently proceeding to an adjustment of the result was however tackled by TC9 quite effectively – see line 7. It was also in this same line and less explicitly so in line 17 for TC7 and line 11 for TA8, that I observed the symbolic inscription. In these instances, teachers shifted their focus from an analysis of viewing mathematical symbols as external supports for reasoning, to an activity of symbolising through reasoning with visual facts and inscriptions.

Image use occurs when an image is employed in the service of some mental operation. It is the mental involvement in refining a shape by using, changing and saving images for concept understanding (Kosslyn, 1994 cited in Makina, 2010). Symbolic inscription, on the other hand, refers to an analysis of pattern that has shifted its focus from viewing mathematical symbols as external supports for reasoning, to an activity of symbolising where learners participate in practices of symbolising through reasoning with tools and inscriptions (Cobb, 2002). These two aspects were examined together with constructive generalisation which entails finding stable relationships within and across the stages of a pattern, to enable learners to perform extensions accordingly in a mathematically consistent manner (Rivera, 2009). However, this expected phenomenon did not work out as I had anticipated. Probably, participant teachers found the aspects of constructive generalisation too demanding and potentially upsetting to their need to remain committed to adhering to current syllabus content.

Use of deconstructive generalisation that incorporated visual tool transformation

In this section I present aspects of deconstructive generalisation that involved the use of visual tool transformation techniques by the observed teachers. In this presentation and analysis, the following codes were used to indicate the occurrence of represented aspects as they manifested in the observed lessons.

VTT – Visual Tool Transformation; EXA – Explicit Analysis.

Table 4.11 How patterns were taught using deconstructive generalisation

Manifestation of deconstructive generalisation and visual tool inspection		
Grade 7 teachers	Grade 8 teachers	Grade 9 teachers
1: (Teacher draws a matchstick pentagonal hut pattern on the chalkboard comprising of three stages). Class can you make for me the fourth stage of this pattern using the matchsticks I have provided in your groups? 2: (Learners get busy as they assist each other to come up with the transformed stick figure. After 8 minutes the teacher stops the activity and proceed to ask questions). [VTT]	3: How many matchsticks make up the fourth stage of the pentagonal figure you have just constructed? Can someone describe how we can explicitly get the number of sticks deriving it from the nature of the figure? [VTT] 4: L1: The figure comprises of five sided pentagons. So if we multiply this figure by the stage number 4 and then add 1 we get the number of sticks at this stage. [EXA]	15: Now let us square these numbers and add the three dots of the butterfly body and see what happens. Can someone try it for stage 3? [VTT] 16: L11: $6^2 + 3 = 39$. This is exactly the number of dots at this stage. [EXA] 17: L12: I tried it with stage 2. $4^2 + 3 = 19$. The number of dots at stage 2.
1: The pattern I have displayed on the chalkboard forms a certain letter can someone tell me the letter? 2: L1: Letter X. 3: Good observance. Any other observed characteristics of the elements of the pattern figure? 4: L2: The centre of the shape remains constant with one fixed box.		20: How does this apply to the pattern we have? 21: L7: Applying the rule to stage 3 we will have $(4 \times 3) + 2$. Which gives us 14 the number of sticks in 22stage 3. [EXA] 23: Excellent. Try it with any stage of the pattern you will see that the rule perfectly fits. 24: ALL: (After carrying out the activity). Yes it does.

Manifestation of deconstructive generalisation and visual tool inspection		
Grade 7 teachers	Grade 8 teachers	Grade 9 teachers
5: L3: The four arms of the X grow by one box at each stage. 6: Brilliant! Is there any other observable trait of the pattern? [VTT]		
8: Ok class today we will focus on pictorial patterns as we explore pattern formations. Look at the pattern I have displayed on the chalkboard. Let us see what we can learn about it. [VTT] 9: By counting the small squares at each stage of the displayed pattern can someone come up with a fitting numerical sequence for the pattern? 10: L1: There are five small squares in stage 1, nine small squares in stage 2 and 13 small squares in stage 3. Therefore the numerical sequence is 5; 9; 13. [EXA] 11: Good counting. Thank you. I would like you to assist each other in your respective groups to come up with a re-represented format of the same pictorial pattern that portray the way each	4: Can you explain the different elements that make up the picture pattern? 5: L2: The centre is a 2 by 2 square and there is a square box at each corner of the 2 by 2 square. That give us $2^2 + 4$. 6: Quite correct. Assuming that the four squares at the corners remain unchanged can someone draw a 3 by 3 square pattern with similar elements? [VTT] 7: L3: (draws the picture on the chalkboard). 8: Can someone now draw the disjointed parts of stage 3 of the pattern drawn by the Last learner? [VTT]	7: The equation can be simplified as $3 \times n = 57 + 1$. Can we then say conclusively that the bursar would hang 19 cards? 8: L3: There were three cards and ten pins in the demonstration image. So since we need one extra pin for each card in the pattern image, we hung 3 cards. [VTT] 9: Yes we hung 3 cards. If we multiply 3 by 3 and add 1 we get the total number of pins which is 10. How many cards therefore required 58 pins?[EXA] 10: L4: I think he need 19 cards. Since $3 \times 19 = 57$ and if we add 1 pin we get the 58 pins. [EXA]

Manifestation of deconstructive generalisation and visual tool inspection		
Grade 7 teachers	Grade 8 teachers	Grade 9 teachers
stage is presented. Clue: the peripheral squares in relationship to the central square. We can do it come on.[VTT]		

In the lessons taught by selected teachers, visual tool transformation aspects manifested as follows: in line 2, line 4 and 5 as well as in line 8 and 11 when it was used by TA7, TB7 and TC7 in that respective order (see Table 4.11). Although the teachers in question used visual tool transformation, their application of this thematic aspect was not up to my expectations. A lot could have been done to show what happens when one changes or operates on an image. TC7 made an attempt to this effect as evidenced in line 8 of the extract from her lesson (see Table 4.11). Deconstructive generalisation, with respect to explicit analysis, was carried out in one instance in this grade level – see line 10 from the lesson extract for TC7 as shown in Table 4.1.

For Grade 8 teachers, the only aspect of visual tool transformation that extensively covered the technique, manifested in two distinct instances. The evidence is found in line 3, and lines 7-8 respectively for TA8 and TC8. Explicit analysis manifested for both TA8 and TC8 in line 4 and line 7 respectively. Lesson proceedings from TA9 and TC9 exhibited visible aspects of visual tool transformation shown in line 15 for TA9, and in line 8 for TC9. Whilst explicit analysis became evident in the lessons taught by all three selected Grade 9 teachers (TA9, TB9 and TC9) as found in lines 17 and 18, line 21 and lines 9 and 10 respectively, their styles and approach to explicit analysis differed significantly – see Table 4.11.

Visual tool transformation and explicit analysis were covered by the lesson proceedings that occurred across the grade levels, although in varying degrees of depth. Visual tool transformation was dealt with quite extensively across the grades whilst explicit analysis was used at a deeper level in both Grades 8 and 9, but less deep in Grade 7. This could be attributed to the nature of the visual tools used by the teachers, as well as the cognitive level of the learners involved in the different grade levels.

Visual tool transformation or image transformation occurs when a person changes or operates on an image – rotating, zooming, translating or scanning it. It also involves the movement of images of shapes into related shapes e.g., squares into hexagons (Kosslyn, 1994). This aspect was examined together with deconstructive generalisation, which involved employing different ways of counting and structuring discrete objects or parts of a pictorial pattern into an algebraically useful type (Rivera, 2009). Both visual tool transformation and deconstructive generalisation were aspects of teaching patterns and their generalisation that selected teachers handled with skill and tactful precision. This was quite reflective of the knowledge they had acquired from the intervention process.

Evidence of applicability, flexibility and usability of visuals

The next level of analysis involved examining how the teaching of patterns by selected teachers exhibited aspects of applicability, flexibility and usability. These aspects were examined to establish how a visual approach enabled and/ or constrained the teaching of patterns and their generalisation. Table 4.12, Table 4.13 and Table 4.14 contain extracts from the video transcripts that manifested the aspects to be analysed.

The following codes were used to show aspects represented from the observation process: AVS- application of visual strategies, FVS- flexibility of using visual strategies and UVS- usability of visual strategies.

Table 4.12 Applicability of visuals during teachers' instruction

Manifestation of the applicability of visual strategies		
Grade 7 teachers	Grade 8 teachers	Grade 9 teachers
13: Good conclusion. Now I would like you to draw stage 5 and then predict the number of rhombuses at stage 10. Is this applicable at all stages of the pictorial pattern? 14: L8: It worked with stage 5 and stage 6 also. [AVS]	12: Good. Now use this general rule to predict the number of sticks in stages 10 and 20. Also draw for me stages 4 and 5. [AVS]	11: Try applying the algebraic expression to each stage of the pattern. What do you see? 12: L7: I tried stage 3. $3 \times 3 + (3 + 2) = 14$. The number of sticks at that stage. 13: L8: The same happens at stage 1 and stage 2. [AVS]
17: L10: Yah n^2 becomes the general rule of the tile pattern. [SI] 18: Now use the rule to find out how many tile slabs will there be in stages 8 and 10. 19: L11: The rule is quite applicable to both stages. $8^2 = 64$ and $10^2 = 100$ slabs. [AVS]		21: L7: Applying the rule to stage 3 we will have $(4 \times 3) + 2$. Which gives us 14 the number of sticks in stage 3. [E₂] 23: Excellent. Try it with any stage of the pattern you will see that the rule perfectly fits. 24: ALL: (After carrying out the activity). Yes it does. [AVS]
16: What is the rule that we can derive from what has been said about the pattern? 17: L6: I see. The rule is $4n + 1$. Where n represents the stage number. [SI]	20: TC8: Quite right! Let's see if this works also with stages 1 and 4, If we consider the rule to be $n^2 + 4$. [SI] [AVS]	10: L5: I applied $3n + (n + 2)$ to stage one and I got 6 the total number of sticks in stage one.

Manifestation of the applicability of visual strategies		
Grade 7 teachers	Grade 8 teachers	Grade 9 teachers
18: Quite correct. You can try drawing stages 4 and 5 of the original pattern and shape by growing the shape as an exercise. Does the rule apply to these stages? [AVS]		11: L6: I applied the expression to stage two of the pattern and I got 10 the total number of sticks at that stage. [AVS]

The applicability of a visual approach to teaching patterns and their generalisation became evident in the way that the lessons taught by Grade 7 teachers were direct and practical – see line 14 for TA7 and lines 18 coincidentally for TB7 and TC7 respectively. Visual applicability was demonstrated by TA8 and TC8 by indirectly or directly prompting learners to apply what they had learnt – see lines 17 and line 20. Similarly, Grade 9 teachers employed a practical approach to their teaching of patterns. They prompted learners to apply the knowledge they had collectively constructed from the teaching of patterns through a visual approach – see lines 11, 12 and 13 for TA9, lines 21, 23 and 24 for TB9 and lines 10 and 11 for TC9. The trend of applying knowledge gained from interacting with visual tools to different circumstances, was prevalent across all three grade levels. There was evidence that teachers were teaching their learners to apply their pre-existing knowledge as well as newly acquired knowledge of pictorials to the learning of patterns, just as the teachers were applying knowledge from the intervention to the teaching of patterns – see Table 4.12.

Table 4.13 Flexibility of visuals during teachers' instruction

Manifestation of flexibility of visuals		
Grade 7 teachers	Grade 8 teachers	Grade 9 teachers
13: Good conclusion. Now I would like you to draw stage 5 and then predict the number of rhombuses at stage 10. Is this applicable at all stages of the pictorial pattern? 14: L8: It worked with stage 5 and stage 6 also. [FVS]	10: We can say 5 multiply by the stage number plus one stick. Or alternatively we can use the number of the sides of a pentagon and multiply it by the stage number and add 1. [FVS]	25: L15: We can conclude that the rule $4n^2 + 3$ applies at all stages of the pattern. 26: Exactly we have succeeded in finding two options for the rule to the pattern. [FVS]
14: If we are to extend the pattern by one more stage what will we have? 15: L9: We will end up with the sequence 1; 4; 9; 16; 25.[FVS]		10: L8: $6 \times 3 + (3 + 1) + 4 = 26$. This is the total number of sticks in the original figure which is stage 3 of the pattern. 11: L9: I have checked with stage 2 the rule perfectly applies. 12: L10: It also does with stage 1. [FVS]
5: L1: We can simply count the tiles physically or we can add the number of tiles in each row like $2 + 2$ in stage 1, $3 + 3 + 3$ in	10: TC8: Can someone now draw the disjointed parts of stage 3 of the pattern drawn by the Last learner? [FVS]	13: TC9: Exciting isn't it? We can conclude that the other general rule for this pattern is $3n + (n + 2)$. Can't we? 14: ALL: Yes we can. [FVS]

Manifestation of flexibility of visuals		
Grade 7 teachers	Grade 8 teachers	Grade 9 teachers
stage 2 and $4 + 4 + 4 + 4$ in stage 3.[FVS]	11: Excellent thinking. Let us try something different. Can you think of a way to find out which stage in the sequence will have 76 bricks? [FVS]	

In lessons taught by Grade 7 teachers, flexibility entails moving back and forth between a pictorial presentation to a numerical or symbolic one. This required the teachers' role in calculating perceptive understanding in learners in order for them to transfer visual ideas to symbolic ideas. This evidence can be seen in line 15 for TB7 and in line 5 for TC7. For teachers teaching Grade 8, the trend was such that learners were encouraged to use the visual tools in a flexible way so as to construct new knowledge based on the original pictorial figure and idea. Encouraging learners to think in alternative ways about a picture cultivates their cognitive perception. This evidence is found in line 10 for TA8 and for the most part, in lines 10, 11 and 12 for TC8. Teachers teaching Grade 9 patterns and their generalisation concentrated more on finding alternative rules for given pictorial patterns, deriving their rules from the context of the original pictorial tool – see line 26 for TA9 and line 13 for TC9. The evidence in lines 26 and 13 indicate how teachers used a visual approach flexibly – see Table 4.13. Thus, flexibility involved changing mathematical presentations, thinking in an alternative way about a picture and using alternative rules to describe a pictorial pattern.

Table 4.14 Usability of visuals during teachers' instruction

Manifestations of usability of visuals		
Grade 7 teachers	Grade 8 teachers	Grade 9 teachers
22: Brilliant! Can we therefore conclude that the general rule for this visual pattern is $3n+1$ where 3 is the infinite difference and n is the stage number? Can we use this rule in similar situations? 23: ALL LEARNERS EXCITEDLY): Yes! [UVS]	4: Can anyone give me the numerical pattern derived from the number of sticks at each stage of the pattern? 5: L3: 3; 6; 9 and so on. 6: What is the common difference between each stage? 7: L4: Three sticks. [UVS]	16: Excellent! We have succeeded in getting a rule for the pattern through our manipulations. [UVS]
13: What should we do with the box in the centre? [VQG] 14: L7: If we add the single box to the boxes in the arms we get the total number of boxes at each stage. [UVS]		10: Let us see if this information will make any sense if we try to manipulate it. The 4 sticks of the upright body of the visible triangle appear at every stage of the pattern. Likewise the 2 top sticks appear with every triangle at each stage of the pattern. What then will happen if we multiply the 4 sticks with each stage number (n) and add the 2 sticks of the upper top of each triangle? [UVS]
18: Quite correct. You can try drawing stages 4 and 5 of the original pattern and shape by growing the	11: Excellent thinking. Let us try something different. Can you think of a way to find out which 12stage in the sequence will have 76 bricks? [FVS]	10: L4: I think he need 19 cards. Since $3 \times 19 = 57$ and if we add 1 pin we get the 58 pins.

shape as an exercise. Does the rule apply to these stages? [UVS]	13: L4: We can do it this way: $4 \times n = 76$ therefore $n = 76 \div 4 = ?$ 14: L5: Thus it is the 19th stage. [UVS]	11: That's 100 percent correct. Thank you. [UVS]
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Usability for teachers teaching Grade 7 entailed using a discovered general rule for different situations of a visual problem – see line 22 for TA7, line 14 and line 18 for TB7 and TC7 respectively in Table 4.14. The evidence of a visual approach by the Grade 8 teachers was evidenced by the transferability of information from picture to symbolic presentation – see lines 4, 5, 6 and 7 for TA8 and lines 11, 12, 13 and 14 for TC8. On the other hand, usability of a visual approach for Grade 9 teachers depended on the ability of learners to manipulate elements of the pictorial tool or parts of the symbolic rule, to allow some necessary adjustments to the final rule – see the evidence in line 16 for TA9, line 10 for TB9 and also line 10 for TC9 as shown in Table 4.14. Generally, usability of a visual approach in this investigation yielded the following: a visual approach becomes usable if a discovered rule can be used for different situations of a pictorial problem; when information from a picture is transferable to a symbolic presentation; and when learners are able to manipulate elements of a pictorial tool and/or parts of their symbolic aspect to allow some adjustments to the final rule.

4.3.2 Presentation and analysis of emergent themes from lesson observations

From analysing themes arising from lesson observations that were informed by literature and the observation tools, I proceeded to the analysis of themes that emerged from the lesson proceedings themselves. These were problem-solving, collective knowledge construction and building on learners' pre-existing knowledge. Table 4.15, Table 4. 16 and Table 4.17 contain excerpts from the video transcripts showing the prevalence of these themes.

The following acronyms were used to indicate the prevalence of aspects of emergent themes: PBS – Problem Solving; CKC – Collective Knowledge Construction; and BPK – building on pre-existing knowledge.

Table 4.15 Excerpts from video transcripts with evidence of problem-solving

Manifestation of problem-solving techniques		
Grade 7 teachers	Grade 8 teachers	Grade 9 teachers
9: L2: Yes. Three sticks are added at each stage. 10: So what happens if we multiply the number of sticks added to each stage by that 11 particular stage number? [PBS]	8: Now try multiplying each stage number by the common difference 3. What did you observe? 9: L5: Stage 1 $\times 3 = 3$, Stage 2 $\times 3 = 6$ and Stage 3 $\times 3 = 9$. It gives the number of sticks at each stage! [PBS]	11: Try applying the algebraic expression to each stage of the pattern. What do you see? 12: L7: I tried stage 3. $3 \times 3 + (3 + 2) = 14$. The number of sticks at that stage. [PBS]
10: Someone to assist please. 11: L6: Stage 1 multiply by 4 is 4, stage 2 multiply by 4 is 8 and stage 3 multiply by 4 is 12. [PBS]		10: Let us see if this information will make any sense if we try to manipulate it. The 4 sticks of the upright body of the visible triangle appear at every stage of the pattern. Likewise, the 2 top sticks appear with every triangle at each stage of the pattern. What then will happen if we multiply the 4 sticks with each stage number (n) and add the 2 sticks of the upper top of each triangle? [PBS]
12: Let us use the visual figure drawn by Group C. Can we formulate an expression that involve the separate elements of the visual and their respective stage number? [PBS]	11: Excellent thinking. Let us try something different. Can you think of a way to find out which stage in the sequence will have 76 bricks?	5: Can we formulate multiplicative expression that portray the number of the sides of a triangle and its respective stage number? 6: L3: Yes we can say $3 \times n$ or $3n$. [PBS]

Manifestation of problem-solving techniques		
Grade 7 teachers	Grade 8 teachers	Grade 9 teachers
	13: L4: We can do it this way: $4 \times n = 76$ therefore $n = 76 \div 4 = 19$. Thus it is the 19th stage. [PBS]	

Evidence of the use of problem-solving techniques

Selected Grade 7 teachers employed some problem-solving mechanisms in their teaching of patterns and their generalisation. They encouraged learners to engage in collective activities on modelling mathematically and visually constructed ideas – see line 12 from the lesson extract for TC7 (Table 4.15). This phenomenon exhibited the effectiveness of using visual tools – see line 10 by TB7. Case studies have shown that learners can successfully use visual tools to visualise and model pattern problems to support them in testing and examining mathematical approaches (Saenz-Ludlow & Kadunz, 2016).

Classroom communication based on cooperative and elaborate mathematical activity presented a novel problem-solving task, generating creativity and deeper thinking – see lines 8 and 9 from the extract for TA8. Learners solving mathematical problems by representing in writing or graphical form, engage in abductive reasoning – where inductive and deductive reasoning are not enough to describe the newly formulated ideas. This was what seemingly transpired when a learner attempted to solve a pattern problem – see lines 11, 12 and 13 from the extract for TC8 (Table 4.15). By abduction, individuals generate hypotheses to explain surprising facts that involve symbolic representations derived from pictorial forms. Teachers focused on the learners' mathematical thinking and learning in order to support them in investigating their own interpretations of mathematical knowledge in patterns – see lines 10 and 11 for TB9 and lines 5 and 6 for TC9 as shown in Table 4.16.

Overall, the selected teachers encouraged collective effort among learners when solving pattern problems. As teachers effectively used visual tools, learners gained knowledge to visualise and model pattern problems. This supported learners to test and examine the visual approach employed by their teachers. Furthermore, problem solving activities assisted the learners to reason by generating hypotheses to explain unexpected facts. This gave teachers the opportunity to focus on learners' thinking and learning (Saenz-Ludlow & Kadunz, 2016).

Evidence of collective knowledge construction

Table 4.16 Excerpts from video transcripts with evidence of collective knowledge construction

Manifestation of collective knowledge construction		
Grade 7 teachers	Grade 8 teachers	Grade 9 teachers
12: L3: If we multiply the stage number by 3 we get a number that is 1 less than the number of sticks at that stage. For instance at stage 2 if you say 3 multiply by 2 you get 6 which is 1 less the number of sticks at that stage which is 7. The same happens at stage 3. 3 multiply by 3 is 9 which is less 1 the total number of sticks at that stage which is 10. [[CKC]	4: Can we use this common difference to deduce a rule for the pattern? Let us start by multiplying each stage number by the common difference. [PS] 5: L2: In stage three if we multiply 3 by 6 we get 18 but we have 16 sticks in this stage which is 2 less than what we get. 6: L3: The same happens at stages 1 and 2. 7: L4: Even at stage 5 it is the same.[CKC]	1: (Teacher puts up a visual tool of overlapping stick triangles). In this pattern of overlapping stick triangles what regular trend do you notice? 2: L1: The sticks at the base of the triangles increase by 1 stick at each stage. 3: L2: The number of sticks making up the upright body of the triangles increase by 3 sticks. [CKC]
13: What should we do with the box in the centre? 14: L7: If we add the single box to the boxes in the arms we get the total number of boxes at each stage.		5: Good observation. Now can we describe in detail the component elements at each stage of the pattern based on the structure of the picture? 6: L3: There are 4 matchstick elements to each of the upright body of each fully visible triangle. 7: L4: The matchstick elements at the base of each stage increase by one stick from one stage to the other. [CKC]

Manifestation of collective knowledge construction		
Grade 7 teachers	Grade 8 teachers	Grade 9 teachers
15: Excellent! Now let us analyse the pattern trend using our knowledge of algebra. 16: L8: You mean from the idea that stage number multiply by 4 plus one box? [CKC]		8: Quite correct.
1: Morning good learners. Today we have a visitor who wants to see how we learn patterns. By the way what is a pattern? 2: L1: Things that are arranged in an orderly sequence. 3: TC7: Can you give examples of patterns. 4: L2: Like the numbers 2; 4; 6; 8 5: L3: Like the tiles on the floor. 6: L4: Like the pictures on your dress. [CKC]	14: L5: We square each stage number. Like $3^2=9$. 15: Brilliant what then should we do with the four squares at each corner of the central square? 16: L6 We can formulate a separate rule for them. 17: L7: No ways. Just add them to get the total number of squares. $3^2 + 4 = 13$ small square to apply to the whole figure. [CKC]	7: The equation can be simplified as $3 \times n = 57 + 1$. Can we then say conclusively that the bursar would hang 19 cards? 8: L3: There were three cards and ten pins in the demonstration image. So since we need one extra pin for each card in the pattern image, we hung 3 cards.

Co-construction of mathematical knowledge is linked to successful communication that is mediated by visible signs and their rule-like transformations (Saenz-Ludlow & Kadunz, 2016). In this study co-construction of mathematical knowledge is referred to as collective knowledge construction (CKC). This phenomenon is exhibited in lines 1-4 of the lesson extract by TC7– see Table 4.16. The teacher engaged learners in a collective knowledge formulation activity where several contributions were made to construct a central idea about patterns. This collaborative phenomenon was also visible as the teacher’s and learners’ ideas complimented each other see– lines 13, 14, 15 and 16 from extract for TB7 (see Table 4.16).

Collective knowledge construction requires that involved parties come to a consensus, even if there are areas in which they do not see or think in the same way – see lines 14, 15, 16 and 17 from the extract for TC8. The teacher plays a significant role in directing and controlling collective knowledge construction – see how this panned out in the lesson taught by TA8 – lines 4-7 from the extract, as shown in Table 4.16.

Learners construct and co-construct their mathematical meaning in the midst of mathematical communications. If they are motivated and well mediated upon, such meanings progressively become refined to meet the objective meanings that are the focus of mathematical exchange (Saenz-Ludlow & Kadunz, 2016). Line 7 and line 8 from the lesson extract by TC9 exhibited such communicational and mediation dynamics by the teacher in question. The same phenomena of mediated communication were noticeable in lines 5, 6, 7 and 8 from the lesson extract by TB9– see Table 4.16.

My observations were supported by the fact that co-construction of knowledge occurs where there is good and clear mediation and communication. I saw learners collaborating freely in knowledge construction when visual tools used were presented and utilised effectively. Teachers motivated their learners through well-mediated mathematical meanings and in turn got progressively refined ideas from their learners. This occurred across grade levels but with varying degrees of use by the participating teachers.

Evidence of building on pre-existing knowledge

Table 4.17 Teachers building on prior knowledge of their learners

Manifestation of building on pre-existing knowledge		
Grade 7 teachers	Grade 8 teachers	Grade 9 teachers
5: Brilliant. The kind of patterns we are going to learn today are called pictorial patterns. By the 6way what other type of patterns did we learn last time? 6: L3: Numeric or number patterns. [BPK]	4: Can anyone give me the numerical pattern derived from the number of sticks at each stage of the pattern? 5: L3: 3; 6; 9 and so on. 6: What is the common difference between each stage? 7: L4: Three sticks. [BPK]	6: L3: The body element is constant with three dots at each stage. 7: L4: At stage 1 the wing span makes a square of 4 dots with one dot at each top and bottom flank. 8: L5: At stage 2 the wing span makes a square of 16 dots with 4 dots at each top and bottom flank. 9: L6: At stage 3 the wing span makes a square of 36 dots with 9 dots at each top and bottom flank.[BPK]
9: Does the sequence remind you of any numerical sequence you have come across before? 10: L6: It is 4; 9; 16		3: Are all the triangles fully visible at each stage of the pattern? 4: L2: No, not exactly. Some parts of the triangles in stages 2 and 3 are partly obscured. 5: Good observation. Now can we describe in detail the component elements at each stage of the pattern? [BPK]

Manifestation of building on pre-existing knowledge			
Grade 7 teachers		Grade 8 teachers	Grade 9 teachers
11: L7: These are square numbers. 2 squared, 3 squared and 4 squared. [BPK]			
9: By counting the small squares at each stage of the displayed pattern can someone come up with a fitting numerical sequence for the pattern? 10: L1: There are five small squares in stage 1, nine small squares in stage 2 and 13 small squares in stage 3. Therefore the numerical sequence is 5; 9; 13. [BPK]	3: L1: 8 squares. 4: Can you explain the different elements that make up the picture pattern based on your knowledge of a square? 5: L2: The centre is a 2 by 2 square and there is a square box at each corner of the 2 by 2 square. [BPK]	3: Can we formulate an equation using the number of pins attached to each separate card and the total number of pins given to the bursar? 4: L2: Three multiply by the number of cards gives the total number of pins. 5: TC9: Can we present that as an algebraic expression? 6: L2: Yes. $3 \times n = 58$. [BPK]	

The interpretation of new schemata for the purposes of inferring to pre-existing (real object) knowledge is a continual transformational process which does not happen at random. Neither does this transformational process occur as a result of one-and-only-one of the multitude of interpretations (Saenz-Ludlow & Kadunz, 2016). Saenz-Ludlow & Kadunz (2016) also argue that a learner's memory is, in fact, a reservoir of interpretations that keeps connecting the inferences of the interpretation of the original (real object) thoughts.

In this study I referred to the phenomenon explained above as the building up of new interpretations from the pre-existing knowledge of the learner, by the mathematics teacher (BPK) in short. In lines 5 and 6 for TA7, the teacher is trying to build upon the learner's pre-knowledge of square numbers. Similarly, TB7 makes an attempt to build on learners' previous knowledge of a sequence of square numbers – see lines 9, 10 and 11 (see Table 4.17).

In lines 4, 5, 6 and 7, TA8 makes an attempt to make learners connect their previous knowledge of number patterns to pictorial formats, with considerable success. TC8 in lines 4 and 7 makes a remarkable connection with learners' previous knowledge of square numbers and a learner quickly builds on this previous knowledge and interprets the figure in a whole new way as evidenced in Table 4.17.

Learners' pre-existing knowledge of geometric figures (particularly triangles) saved the day for the grade 9 teachers. In lines 6, 7, 8 and 9 I witnessed the teacher (TA9) making a connection by building up his interpretation of a triangular figure to bring learners to formulate new knowledge and interpretations of the figure. Likewise, TC9 prompted his learners to formulate new interpretations from their observations of a card pattern – see lines 4 and 5 in Table 4.17.

New interpretations of pattern phenomena were built upon existing schemata as the teachers selectively used transformational processes that were not random, but carefully initiated to relate to pre-existing knowledge of the learners. This phenomenon was observed across all three grade levels

4.3.3 Summary of lesson observations

In all the eighteen lessons I observed, I recorded the prevalence of the seven pre-determined themes and three emergent themes, but with varying degrees of application by participant teachers. The theme that was adequately and effectively dealt with was visual question generation. This theme aspect was used across grades and even overlapped into the other themes. However, visual tool transformation was scantily dealt with. In the few instances visual tool transformation was employed by teachers, they only applied it in principle, that is mental insinuations of transformative intentions

were made visible. Visual image use and symbolic inscription were also given fair treatment by teachers implementing the knowledge they acquired from the intervention process.

Indicators of the use of acquired visual strategies were also visible but these strategies were used either superficially or were, in some instances, completely amiss. In Grades 7 and 8 classes, the most prevalent visual strategies were counting and recursive analysis for lesson 1 and difference rate and recursive analysis in lesson 2. In Grade 9 classes, difference rate and whole object analysis dominated in both lesson 1 and lesson 2. Explicit analysis was adequately utilised by one teacher in Grade 8 and by two teachers in Grade 9.

The use of visuals in teaching patterns presented more enabling factors than constraining factors as evidenced by the widespread applicability of visual strategies applied by the participant teachers. Due to inadequate preparation of visual tools by teachers (refer to Tr. 13 in the questionnaire responses), the ones that were used were not sufficient to allow flexibility of use in solving emerging pattern problems. However, the tools that the teachers used were evidently usable to a considerable extent.

Problem solving and the building up of new knowledge on pre-existing knowledge were treated as emerging themes. These were effectively covered in the lessons at all grade levels. The other theme, which was treated as an emerging theme, was the co-construction of knowledge. This occurred between teachers and learners and among learners themselves. The dynamics associated with collective knowledge construction were visible across grade levels.

In general, visualisation was considerably enabling to the teaching of patterns and their generalisation as most of its aspects were implemented with notable success as evidenced in the lesson extracts taken from the observation process. Even so, there were several constraining factors that were highlighted by this observation process: for instance, over involvement of the teacher in the teaching and learning process and learners' limited knowledge of algebra h delayed the generalisation process to some extent – see extract from TA9. In my perspective visualisation was utilised by teachers in a generally enabling way and less constraining way. I took this view, although I was yet to verify my preliminary findings by engaging participant teachers in an interview process to either confirm or reconsider my observations. This view was to be best answered by my conducting of a series of interviews to get a true picture of the perceptions of the teachers concerning what enables and/or constrains the teaching of patterns when visual approaches are used. After class observations, I conducted one-on-one stimulus recall interviews and one focus group interview to ascertain my findings in this study.

4.4 TEACHERS' EXPERIENCES AND PERCEPTIONS ABOUT TEACHING WITH VISUALS

4.4.1 Experiences and perceptions

To answer Research Question 3: What are the selected teachers' experiences and perceptions of teaching patterns and their generalisation prior to and after participating in an intervention programme? I conducted a total of fourteen interviews, thirteen of which were stimulus recall one-on-one interviews and one was a focus group interview. The purpose of the interviews was to ascertain selected teachers' perceptions and experiences on whether the use of visuals enhanced and/or constrained the teaching of patterns and their generalisation. The interview data is presented in the following section under the theme: Experiences and perceptions.

In this section I present and analyse data from the focus group interview. The purpose is to examine teachers' perceptions and experiences about teaching patterns and their generalisation using a visual approach. To examine teachers' perceptions and experiences three sub-themes were used: *teachers' perceptions and experiences before and after the intervention; teachers' insights about what works and what does not work when a visual approach is employed to teach patterns; and teachers' beliefs about the effectiveness of a visual approach to teach patterns and their generalisation.*

4.4.1.1 Teachers' perceptions and experiences before and after the intervention

Participant teachers had collective views about the use of visuals to teach patterns before and after the intervention. I made this observation after interacting with the participant teachers in a focus group discussion I conducted soon after the implementation process. The extracts below show how teachers' perceptions converged in the construction of meaning regarding the use of visuals. The codes at the bottom of the extracts are described in Figure 3.6.

TC8: *Before the intervention I was of the opinion that teaching patterns using pictures is a waste of time and mere laziness on the part of the teacher.* [TPE1]

TA8: *Me too – I thought using pictures does not count much for teaching patterns. I thought it was child play that does not instil real cognition in learners.*

TA7: *I also had the view that teaching patterns using pictures fail to invoke critical thinking which I saw happening, on the contrary, when I made use of pictures to teach patterns and their generalisation.* [TPE1]

Although the participants were from different research sites and had different teaching backgrounds, they all had the same views about teaching patterns using visuals before and after the intervention. It can be concluded that before the intervention teachers were influenced by preconceived ideas about

teaching with visuals. However, after the intervention there was a remarkable shift of perception as teachers began to appreciate the use of visuals to teach patterns and their generalisation.

A notable shift of perception about the use of visuals to teach patterns then occurred. For instance, TC7 had the following impression to share during the one-on-one stimulus recall interview process when asked if she would recommend the use of visuals after the intervention:

TC7: Yes, I would encourage other teachers to use visuals in teaching because from what I experienced, visuals help learners connect their everyday experiences with abstract mathematics concepts. Visuals are also easy to use [contrary to my judgement before the intervention] and enhance learner understanding of mathematics. [TPE1]

Another sign that teachers' understanding of the use of visuals was drastically transformed after the intervention came from TA8. She had the following thoughts to share:

TA8: The experience I had after the intervention made me to have a more practical approach to teaching patterns that is positive and easy to execute. Before the intervention I struggled to strike a working rapport with my young learners. This changed completely, with the use of visuals I can teach patterns to learners of all age groups and achieve outstanding results. [TPE1]

This evidence gave me a comforting foundation for trusting the effectiveness of the use of visuals to teach patterns and their generalisation. I now had the mandate to scrutinise further what really worked and what did not work for the teachers when they employed the use of a visual approach to teaching patterns. This aspect would allow me to get a fair and balanced interpretation of the participants' realisations about the use of visuals.

4.4.1.2 Teachers' insights about what works and what does not work when visuals are used

Participants expressed achievements and setbacks during the implementation process. This gave me a notion of what actually worked and what did not work when visuals were used to teach patterns and their generalisation. One participant had some misgivings, as evidenced in the statements below:

TC8: I preferred visuals because they made my work easier and it was easier for the learners to grasp the concept of patterns. Also my learners understood better when they see from pictures rather when they reason from numeric presentations.

TC8: I had limited visuals in my lesson, not enough to cater for problems that emerged during lesson delivery. Learners became a bit agitated wanting to explore more from more challenging visuals which I could not provide during the duration of the lesson. [TPE2]

TC8: *Once a teacher takes time and skill to prepare visuals he/she can save time and will be able to teach patterns effectively and in manageable time space. [TPE2]*

On a similar note, TA9 voiced a belief that visuals work for the greater good if utilised properly. However, the teacher also lamented the lack of skill among learners to effectively transfer picture elements to corresponding symbolic rules and how difficult it could be to use visuals effectively when time constraints are not taken care of adequately. This is what the teacher had to say:

TA9: *The pictorials encouraged learners to construct meaningful knowledge about a pattern and this was seen working in their verbal explanations of particular patterns based on the context of the picture. The pictures also aroused interest in learners to explore more about the patterns. However things did not work as planned when it came to the task of formulating algebraic rules for the patterns. Learners did not readily transfer the picture elements to a corresponding symbolic presentation of the pictorial pattern. It took longer time to get learners to master this technique, as a result tasks were not completed in time. [TPE2]*

TA9: *It was also difficult for my learners to identify the functional rate in difference rate analysis. The visual tools were either complex or did not identify with what they normally see in everyday life. After careful guidance and linking of ideas learners grasped the concepts. [TPE2]*

However, during the member checking exercise, TA9 conceded that learners' limited knowledge of algebra, rather than the use of visuals impeded progress and resulted in rendering the use of visuals not to work as effectively as he expected. My next step was now focused on assessing the general beliefs of teachers about the effectiveness of using visuals to teach patterns and their generalisation.

4.4.1.3 Teachers' beliefs about the effectiveness of a visual approach to teach patterns

The general belief amongst participants about the effectiveness of using a visual approach to teach patterns and their generalisations can be found in the following statements:

TA9: *I think teaching using pictures is a better [and quite effective approach] than the use of concrete numbers. Learners love pictures and they arouse their interest to learn and to explore patterns. Numbers are too abstract and tend to confuse to the extent that they miss important concepts which they could easily access when a picture is used to teach patterns.* [TPE3]

TC9: *Contrary to the belief that when you teach patterns using visuals you risk not finishing the syllabus, I do disagree, because as you have seen in this lesson I covered quite a variety of patterns using visual approaches which could not have been possible had I employed the traditional method of teaching patterns. Visual approaches make it possible for a teacher to finish the topic well ahead of prescribed time and have ample time for revision purposes.* [TPE3]

Despite the popular belief by participants that the use of visuals is an effective approach to teaching patterns, one participant was rather doubtful about the efficacy of visuals. The teacher had the following to contribute:

The visual strategies were effective to some extent. Quite a number of my learners are slow learners and it took some time to get them to effectively manipulate visuals and/or read patterns from pictures. They also had difficulties to explain pattern trends and give verbal explanations. Furthermore, brighter learners got bored when visual aspects of pictorial pattern were at length used to explain a regularity they had readily observed from the onset. [TPE3]

Despite this single observation, the popular and general belief of participants that the use of visuals is an effective way of teaching patterns and their generalisation did not diminish.

From what I gathered from the teachers' impressions and insights about the use of visuals to teach patterns and their generalisation, perceptions before and after the intervention changed significantly from negativity to positivity. Insights about what works and what does not work when a visual approach is employed to teach patterns were inclined towards the view that visuals mostly work effectively when they are correctly used to teach patterns and their generalisation. Teachers' beliefs about the effectiveness of a visual approach to teach patterns and their generalisation were biased towards a positive attitude for the approach.

I also believe that this shift in belief by participant teachers (coupled with their general perceptions and insights) about the use of visuals to teach patterns, permanently shaped the teachers' pedagogical practices and hope that they will cultivate it and share it with their colleagues for development purposes. My belief is that this will go a long way towards moulding better instructional practices among teachers in the district and beyond.

4.5 ENABLING AND/OR CONSTRAINING FACTORS OF VISUALISATION

From the collective views of participant teachers, I gathered during the interview process, I came up with the following data that outlines the challenges and breakthroughs teachers encountered during the implementation process. This allowed me to conveniently answer Research Question number four: ***What are the enabling and constraining factors of applying a visualisation approach to teaching patterns and their generalisation.*** This aspect of the study is discussed in this section under the theme: *Enabling and constraining factors.*

Enablers of using a visualisation approach

From the responses I got from teachers participating in the one-on-one interviews after the implementation process, I gathered that teachers experienced a number of factors that enabled them to teach patterns and their generalisation quite effectively. The following subthemes were used as indicators to establish to what extent the interviewees found the use of visuals enhancing/enabling to the teaching of patterns: *promotion of conceptual understanding; deepening of generalisation skills; development of problem-solving techniques; development of cognitive perception; encouragement of innovation and originality; and facilitation of learner exploration.*

The data presented below is referenced according to the theme codes presented in Figure 3.4.

4.5.1 Promotion of conceptual understanding

Some aspects of how a visualisation approach enabled the teaching of patterns and their generalisation can be seen in the interview extract below. This occurred when TB9 answered a question which was indirectly aimed at probing whether the teacher found visual strategies enhancing of the teaching of patterns and at the same time promoting conceptual understanding.

The strategies [I used] were effective to some extent. Quite a number of my learners are slow learners and it took time to get them to effectively manipulate objects and/or read patterns from pictures. They could however explain regular trends and give verbal explanations. This prompted them to look for rhythm in the pictorial pattern. This even helped non-visual learners

to understand and appreciate the regularity they could see in a picture...one learner shouted in excitement that he had got it! This really stood out for me. [ENB1]

The fact that the teacher acknowledged that learners could explain regular trends in pictorial patterns, indicates that visuals had facilitated exploration for learners. The learners could also look for regularities (rhythm) in the pictorial pattern. There is no better way of seeing the development of explorative tendencies than this – see extract above. Furthermore, the use of visuals allowed learners to effectively manipulate pictures and read patterns successfully, even though it took them longer to do so – see evidence from the interview extract. This indicates that the use of visuals encouraged the development of conceptual understanding of patterns.

4.5.2 Deepening of generalisation skills

The use of visuals was acknowledged by TA7 as having the potential to enable the teaching of patterns in a variety of ways which included deepening learners' generalisation skills. The extract below from the interview transcript speaks in that regard.

Yes definitely since the pictures prompt them to connect to previous knowledge of arithmetic and algebra if these were well taught and integrated with their knowledge of what they can construct from visual presentations. Pre-knowledge is essential in learning any new concept in mathematics visual patterns included. [ENB2]

My interpretation of this statement is that since visuals prompt learners to use their knowledge of algebra (see extract), the teacher implied that this would improve their generalisation skills as they employ this knowledge in symbolic representations. The teacher even went to the extent of saying that this knowledge was essential for concept development in pattern learning.

The aspect of visuals enabling learners to deepen their generalisation skills was highlighted again in another instance: TA8 explained how visuals inspired strategies like difference rate and whole-object analysis, enabling him to make learners explore patterns in quite an enabling way that yielded much success. The extract below supports this observation.

Yes I made use of counting, difference rate and to some extent whole-object analysis. To my dismay learners were more able to explore pictorial patterns using counting analysis and just a few learners could readily generalise from a picture context using difference rate and whole-object analysis. [ENB2]

The fact that there were at least some learners who could generalise from the context of a picture, is evidence enough to confirm that visual teaching indeed enabled the process of teaching to improve learners' generalisation skills.

Hence, the use of visuals enabled both conceptual understanding and enhanced learners' generalisation skills. TA7 further reiterated the enabling power of the use of visuals to teach patterns in the extract below:

Visuals promote critical and positive thinking as they bridge the gap between abstract and concrete thought. The reasoning from pictures has a lengthy retention span in the minds of the learners and thus lay the basis for the development of deeper conceptual understanding.
[ENB1]

4.5.3 Develop problem-solving techniques

Not only did the use of visuals enable the teaching of patterns by enhancing conceptual understanding and deepening generalisation skills, it also encouraged the development of problem-solving techniques among learners. This aspect, together with one other aspect (encouragement of innovation and originality among learners) were voiced by TC7 in the extract below.

As you saw in the lesson from the video learners were displaying critical thinking in the way they queried about certain aspects of the visuals used and you could see them exhibiting problem solving techniques as they interact with a variety of visuals. For instance a learner asked how we arrived at the tenth term of a pattern of triangles without having to draw individual stages. In some instances learners came up with something original that I did not think about or predicted in my planning. [ENB3]

It appears the teacher's use of visuals to teach patterns influenced the learners to think in a way that was both critical and progressive. This spurred the learners on to engage in self-directed problem-solving activities as they interacted with the visual tools. Thus, the learners became visibly innovative and original in their dealing with pattern problems. This autonomous development happened unexpectedly for the teacher concerned, further confirming the power of visuals in enabling the teaching of patterns.

4.5.4 Develop cognitive perception

TA8 further highlighted the enabling effect of a visual approach to teaching patterns and their generalisation, by allowing cognitive perception to evolve simultaneously with problem solving techniques. The extract below gives evidence to that effect:

To me it was the level of excitement which manifested itself as learners found a general rule for a given pattern after being skilfully guided to it by their own pattern exploration and the teacher's mentorship. Even when they were giving examples in life of things that have regular patterns like tiles, bridge rails ceiling decorations. They were making connections between what they were taught and what they experience in everyday life. [ENB4]

The highlighted phenomenon here indicates that visuals encouraged learners to enhance their generalisation skills as they initiated deeper explorations of patterns. This assisted learners to enhance their cognitive perception, an aspect that is essential for the teaching and learning of patterns.

It was TA9's consciousness about the power of visuals to instil cognitive perception in learners that caught my attention as I went through the interview transcripts. The teacher had the following realisation and insight to share:

TA9: From what they saw in a picture pattern, learners could respond with confidence and were in a position to extend and analyse a pattern better than when it was presented in numerical or tabular format. Learners' descriptions of patterns became more robust and clear in a positive way when they interpreted patterns from pictures. [ENB4]

This was a clear indication of the enabling effect of visuals to teach pattern concepts. Learners could reason from what they saw in a picture and proceeded to give innovative explanations in a cognitively charged explanation.

4.5.5 Encourage innovation and originality

There is even an element of visuals potentially encouraging innovation and originality in learners from evidence in the extract that follows:

TB9: In more than just one way visual approaches assisted learners to work independently. This laid conducive ground for them to deal with generalisation of patterns in creative ways. Learners' explorative and innovative skills were sharpened by the use of visuals and I could see they were clamouring for more challenging tasks that involve pictorials.

From the tone in this extract it can be seen that the teacher had used visuals in quite an effective way, so as to invoke originality and innovation among his learners. Thus, the power of visuals was once again seen to be enabling the teaching of patterns and their generalisation.

4.5.6 Facilitate further exploration

TA7 had the following conception to add to the many factors that enable visuals to be effectively used to teach patterns and their generalisation:

TA7: There were such moments, really more special, when learners used knowingly or unknowingly far generalisation strategies to predict terms far spaced from the initial term of reference. For instance predicting the twentieth or fiftieth term of a pictorial pattern whereas it could be have been difficult to imagine its magnitude or show it in a physical form. The learners could predict through rigorous exploration aided by the picture and come up with the extended terms of the pictorial pattern. Such were the moments that brought me pleasure and satisfaction especially when learners could do it with minimum help from me. [ENB6]

It is apparent from this extract that visuals promoted learner exploration to the effect that it made the teachers' work easy and quite significant. Visuals, with little doubt, enabled the teaching of patterns and their generalisation in a way that is noteworthy and could thus be considered for use in a variety of mathematics concept teaching situations. The evidence from the interview transcript supports the fact that visuals enable the teaching of patterns and their generalisation in more than just one intriguing way.

A visual approach thus enabled the teaching of patterns in a variety of ways that included: promoting conceptual understanding of patterns; deepening of generalisation skills; developing problem solving skills; developing of cognitive perception; encouraging of innovation and originality and facilitating learners' deeper exploration. After examining these enabling factors, I shifted my attention to looking at what factors then hindered the teaching of patterns when a visual approach was employed.

Challenges/constraints of using a visual approach

I had to examine what teachers experienced as challenges or constraints when teaching patterns and their generalisation using visual approaches. The data I present in this section draws from the interview process I conducted with participant teachers after each phase of the implementation process. The challenges experienced by the participant teachers are hereby categorised as subthemes in accordance with how they were viewed as hindering the use of a visual approach to teach patterns and their generalisation: *raising of compatibility issues with traditional curricula; violation of the constructivist teacher/learner contract; limitation of learner autonomy and creativity; intimidation of learners resulting in limited creativity (group dynamics); taking longer to complete tasks; and hindrance of progress due to learners' limited knowledge of algebra.*

4.5.7 Compatibility issues with traditional curriculum

Participant teachers experienced several challenges that constrained their teaching of patterns using visual approaches. Here I present and analyse some of these challenges. In the extract below TC8 highlighted some of the challenges she faced, that could be associated with issues of compatibility with the prescribed traditional curriculum. The codes at the bottom of each extract are derived from Figure 3.4.

I had limited visuals in my lesson, not enough to cater for problems that emerged during lesson delivery. Learners became a bit agitated wanting to explore more from more challenging visuals which I could not provide during the duration of the lesson. I ended up not meeting the syllabus requirements as per policy document.[CNS1]

Due to the demands of having to prepare adequate visual tools in order to teach patterns effectively, the teacher could not cope up with learner demands. This raised compatibility issues with the existing curriculum. The fact that learners were agitated, further compounded the challenge since this would impact negatively on syllabus coverage for the teacher. This was a serious constraint of applying a visual approach to teaching patterns. However, this could have been avoided had the teacher taken time to prepare her visual tools thoroughly with close consultation with the policy document.

TA8 experienced similar challenges as presented in the extract below.

The challenges came as a result of my limited visual tools and asking more questions based on them. Once I had exhausted the few that I had prepared I had to keep on going back to further explore other alternatives in the ones I had already used. Learners were visibly bored by this repetitive process. This challenged me as a teacher and dampened learner interest. [CNS1]

I observed that the teacher experienced an acute compatibility issue which was of his own making. However, the teacher acknowledged his mistake and promised to rectify it in future. See the extract below to understand his position when answering a follow up question to his previous challenge:

In future I will prepare ample visual tools to cater for all planned visual strategies and enough to engage learners in group and individual activities. [CNS]

The teacher seemed to have realised that to keep abreast of current curriculum demands he has to plan accordingly.

4.5.8 Violation of the constructivist teacher/learner contract

It is a fact that if a teacher over involves himself or herself in the teaching and learning process of any mathematical concept, the constructivist teacher/learner contract is violated. Such was the challenge

faced by some participant teachers in their teaching of patterns and their generalisation using a visual approach. The following extract from the interview with TA8 highlights this aspect.

Actually my lesson was basically teacher centred. I admit I somewhat over involved myself in the teaching learning process. Understandably so I had to take a dominant role as a mentor and guide. Had I given learners autonomy to self-direct their learning I could not gotten anywhere since I would have to contend with divergent views and ideas that would demand time and skill to reconcile? That way I would have lost a lot of time. On the other hand the learning environment and time allocated to teaching and learning did not permit such liberties to listen to every voice and get most learners to participate in knowledge construction. [CNS2]

The teacher started off by admitting guilty of over involving himself in the lesson proceedings, but was however quick to defend his intentions of doing so. He went on to give plausible reasons for taking a dominant role – such as fear of running out of time when trying to get learners to construct meaningful knowledge about patterns.

4.5.9 Limitation of learner autonomy and creativity

The other challenge that participant teachers experienced, was the limitation of learner autonomy by their use of visuals to teach patterns. This phenomenon was highlighted in an interview with TB9. See his exact words in the extract below:

The challenges I faced were that instead of helping them, some of the visuals caused some form of distractions for some of my learners. The visuals were a source of digression and this paused challenges and counter production in my teaching process. Learners ended up doing playful things due to the learners' short concentration span. [CNS3]

Since visual tools and manipulatives became a source of distraction, learners could not use their autonomous power to use a visual approach to learning. This was a serious onslaught to learner autonomy since the approach negatively impacted on this inherent autonomy to learn. Instead of enabling the teaching of patterns, the use a visual approach ended up limiting learner creativity. However, the teacher could have capitalised on this digression to develop useful skills in learners by progressively guiding them through their play-learning, since children learn from playing.

In another instance, TC9 experienced a different phenomenon which had serious limitations on learner autonomy and creativity. See the extract below.

For the leaners it was quite challenging to move from a pictorial representation of a pattern to its equivalent symbolic representation. I was quite challenged to explain to them how this

could be done through exploring the picture. It was easy for learners to understand the picture but transferring its meaning to an algebraic rule was quite difficult for them. In future I will make them analyse the components of the picture and use the ideas to come up with verbal descriptions and then transform these into meaningful algebraic representations. [CNS3]

From the teacher's point of view it was the way in which the pictorials were presented and analysed that limited learner autonomy to construct meanings and symbolic meanings from them. The teacher also alluded to the fact that the learners' limited knowledge of algebra contributed to their lack of creativity.

4.5.10 Group dynamics

Group dynamics and classroom culture were some of the challenges experienced by teachers when they taught patterns using a visual approach. This manifested in the interview with TC9. If learners are either shy to express themselves or are afraid of how their classmates react to their contributions, this may also limit learner participation and hence creativity. See the extract below:

My learners had visible difficulties moving from a pictorial presentation to a symbolic representation. Probably they were shy to showcase their perceptive powers or they feared being booed by their peers. In future I will encourage my learners to give more elaborate narrative descriptions of a pictorial pattern and utilise those explicit narratives to formulate fitting symbolic rules. I will encourage them to be self-confident and to focus on the task at hand. [CNS4]

Indeed, if the situation in this class was as the teacher suggested, no real progressive participation and creativity would have been realised. In such a non-conducive environment especially with a visual approach (which demands collective and autonomous construction of knowledge), progress would not take place. The teacher has a role to play, as suggested by TC9, to take full control of the classroom culture by exercising authority.

4.5.11 Limited knowledge of algebra (tasks not completed)

The time it takes to complete any mathematical task has an impact on conceptual development in learners. This aspect was evident in the interview process with TA9. See the extract below:

Drawbacks manifested themselves when learners could not move on their own to formulate rules for patterns and also in my use of limited visual strategies due to limited time which resulted from learners' limited knowledge of basic algebra. [CNS5]

It was indeed the time factor that impacted on how long it took the teacher and his learners to complete tasks with sufficient conceptual development. The teacher seemed to suggest that the use of visuals, or rightfully so, the proper use of visuals, determines whether concepts are developed sufficiently. Otherwise, tasks were completed in time and the concept of patterns developed by some of the participant teachers. For instance, TC9. See the extract below.

Yes it definitely is because it fits in well with the syllabus content and can be done in precisely the prescribed time frame and does instil the required cognitive skills in learners. All my tasks were completed in time.[CNS5]

Last but not least, the challenge that participant teachers experienced which constrained the teaching of patterns and their generalisation when visuals were used was learners' limited knowledge of algebra. The extract below sums it all up.

The problem was adaptation to new situations and failure to read the picture in a constructive manner since they were used to finding rules to numerical formatted patterns. The learners' lack of knowledge of algebra became a hindering factor as I was the one to constantly guide them to come up with working rules for given pictorial patterns. [CNS5]

From the participant teacher's point of view, learners had a very limited knowledge of algebra. This affected the effective use of visuals, thus hindering progress for the teacher to teach pattern generalisation in an effective way. However, the teacher hopefully assisted learners to generalise without violating the teacher/learner contract mentioned earlier on.

4.6 HOW VISUALS ENABLED AND/OR CONSTRAINED THE TEACHING OF PATTERNS

In this section I present data predominantly collected using Observation Schedule C, to examine how teachers found visual strategies applicable, flexible and usable as they taught patterns and their generalisation. Although this aspect was dealt with in Section 4.3.1 (see themes 5, 6 and 7), I deemed it necessary to explicitly explain it in this section for emphasis purposes. (I felt that some aspects from Observation Tool C were not adequately addressed).

Applicability of visuals was examined with the following indicators in mind: constant interaction with a variety of visuals to promote understanding; evidence of problem-solving techniques; compatibility with traditional curriculum; and teachers' involvement in the learning process. **Flexibility** of the use of visuals was examined indicative of the involvement of a variety of problems to develop generalisation skills; evidence of exploration with deeper reasoning; learner autonomy and creativity; group dynamics that promoted learning progress and visual perception. At the same time **usability** of visuals was examined indicative of potentiality to develop cognitive perception; the encouragement

of learner innovation and originality; learners' effective use of algebraic knowledge to advance symbolic representations; and the time it took to complete tasks with adequate knowledge construction.

4.6.1 Teachers' views about the applicability of the use of visuals across the grades

4.6.1.1 Constant interaction with a variety of visuals

There is remarkable evidence that the teachers engaged in constant interaction with a variety of visuals to promote learner understanding. For instance, TA7, when I paused the question "What made it possible for you to successfully deliver the concept of patterns?" he emphatically stated, *"Using different approaches and a variety of pictorials and manipulates as teaching aids."* This was a clear indication that the teacher engaged learners in constant interactions with a variety of visuals to promote learner understanding. TC7 added another dimension to the applicability of visuals in teaching patterns and their generalisation. He had the following to say about this indicator: *"With visuals like pictures and diagrams learners can see and make sense of what they see and reason from concrete to abstract thought using their experiences with the real world. I use such variety of strategies because they enable learners to understand quicker and better."*

As with the Grade 7 teachers, Grade 8 teachers also conceded that engaging learners in constant interactions with a variety of visuals was an indicator of applicability, but at a limited scale. TA8 made the fair admission that *"Actually I concentrated more on visual strategy with one tool as a precursor to future pattern teaching and learning. This worked fairly well with my learners as they interacted quite successfully with various numerical and pictorial contexts at all stages of the lesson."* TC8 made a similar admission: *"I had limited visuals in my lessons, not enough to cater for problems that emerged during lesson delivery. Learners became a bit agitated wanting to explore more challenging visuals which I could not provide during the duration of the lesson."* I considered this to be a constraining factor to the use of visuals to teach patterns effectively.

For a visual approach to be applicable, there should be constant interaction with a variety of visuals in order to promote understanding of pattern concepts. *"The strategies I used with a variety of visuals ... made it easier for learners to understand [pattern concepts] and it made my work easier as a teacher to get the concept of patterns to learners with very little difficulty."* It is apparent in this statement by TA9 that a variety of visuals had the potential to promote learner understanding. TB9 added to this view that *"The visuals I used included pictures, tables and natural objects some of which I made myself. These visuals could be manipulated to explain patterns like matchsticks and cubes. I preferred using these variety of visuals to help learners understand the concept of patterns better."*

4.6.1.2 Evidence of problem-solving techniques

Even though there was little evidence that Grade 7 teachers employed problem solving techniques, TC7 pointed to an instance where problem solving was alluded to in one of her lessons. She said *“As you saw in the lesson from the video learners were displaying critical thinking in the way they queried about certain aspects of the visuals used and you could see them exhibiting some form of problem solving techniques as they interacted with a variety of visuals.”* Of course, what I observed in that particular lesson amounted to some form of problem-solving activity, although it did not have all its interpretive implications.

Another applicability factor arose in the evidence that learners developed problem-solving skills when visuals were used to teach patterns. This aspect was made prevalent by the contribution made by TC8. She said: *“Teaching patterns using pictures cause learners to develop skills of organisation as well as perfect their skills of critically analysing a pictorial pattern.”* There is no better way of acknowledging the development of problem-solving techniques than the scenario portrayed in this insightful contribution. The teacher further added that *“This helps learners to take charge of their learning and get a deeper understanding of pattern concepts.”*

A visual approach, or any other teaching approach, cannot be considered fully applicable if there is no indicative evidence of problem-solving techniques used. TC9 made an important contribution to this aspect in the following statement: *“I used to give my learners (before the intervention) an array of numeric patterns and ask them to extend to two or three stages [and figure out the rule] without intricate engagement with the pattern. With the use of visuals learning became more meaningful and quite involving, and learners enjoyed it as they engaged in rigorous problem-solving activities.”* This evidence of problem-solving techniques being used by teachers also evidenced in the following contribution by TA9: *“When learners were verbalising their pattern descriptions and analysing pictorials I could sense unconcealed excitement as they engaged in insightful explanations and constructions through problem solving techniques.”*

4.6.1.3 Compatibility with traditional curriculum

Teachers conceded that teaching patterns using visuals was quite a compatible approach with the traditional curriculum. TA7 shared his experiences and insights about this phenomenon in the following way: *“A teacher who dedicates himself or herself to thorough preparation of visual tools makes his/her teaching of pattern content much easier and is likely to flow with the syllabus content and finish the syllabus in a short space of time and use the extra available time for revision purposes.”*

Compatibility of the use of a visual approach with the traditional curriculum as an indicator of applicability was shown in the following contribution by TC8. She said *"I preferred visuals because they make my work easier [without neglecting the syllabus content] and it was easy for the learners to grasp the pattern concept. Learners understand better when they see from pictures rather than when they reason from abstract numeric representations."* TA8 made a forthright statement to this effect: *"Teaching using visuals is quite compatible with what is required of me to teach from the syllabus. It does not in any way disrupt my syllabus coverage, I do not know with the other mathematics teachers."*

What use would a visual approach to teaching patterns and their generalisation be, if its application was not compatible with traditional curriculum demands? The compatibility of a visual approach to the prescribed syllabus is an indication of its applicability to teaching and learning. TB7 said to this effect: *"To a greater extent this [visual approach] was very much in line with what I had planned for the lesson following curriculum guidelines to teaching patterns at Senior Phase level."* This was ample evidence of the compatibility of a visual approach to teaching patterns and their generalisation. TC9 added to this view in the following statement: *"Yes it definitely is [compatible] because it fits in well with the syllabus content and can be done in precisely the prescribed time frame."*

4.6.1.4 Teacher's involvement in the teaching process

Another indicator of the applicability of a visual approach to teaching patterns and their generalisation which was considered, was the teachers' involvement in the teaching process. I considered if there was in any way over involvement of the teacher in the teaching and learning process this would amount to a constraint, since it violates the teacher/learner constructivist contract. When I asked TC7 if learners could proceed on their own to generalise a pattern with less of her help after collectively exploring a visual pattern, the teacher had the following to say: *"Some brilliant learners can be able to do that with minimal assistance from me but the task of generalisation is generally not an easy task for average to below average learners, so I have to chip in to ease cognitive pressure on the learners."* To me this indicated that there was a bit of over- involvement of the teacher when employing the use of visuals.

"My learners could read and understand a pattern, extend a pattern successfully and describe a pattern without my help. I only assisted them when it came to analysing a pattern and coming up with a general rule through prompting. Otherwise they could do quite a lot with minimum assistance from me." This was a contribution made by TA8 when responding to whether his use of visuals was indicative of applicability in terms of teacher involvement in the teaching process. To a greater extent, his response was indicative of an enabling factor of visualisation in teaching patterns and their generalisation.

The level of teacher involvement and its intensity in the learning process when a visualisation approach is used to teach patterns and their generalisation can be indicative of its applicability. TC9 had the following to say about this aspect: *“I made sure that my own involvement was minimal since I wanted my learners to understand through pattern exploration. I had to make use of the very bright learners to assist less capable learners to get things done whilst I provided technical guidance.”* With this level of teacher involvement, the teaching of patterns using visual approaches was more likely to be applicable and less constraining.

4.6.2 Teachers’ views about the flexibility of the use of visuals across grade levels

4.6.2.1 Involvement of a variety of problems to develop generalisation skills

Both interviewed Grade 7 teachers conceded that they involved a variety of problems to develop generalisation skills. TA7 said the following to this effect: *“The learners could predict terms [and generalise patterns] with ease after interacting intensively with a variety of pattern [problems]. Such were the moments that brought me pleasure and satisfaction especially when learners could do this with minimum help from me.”* TC7 added significance to this indicator when she said: *“My learners had enough knowledge background of arithmetic and a bit of algebra which enabled them to navigate pattern problems that involved pictures.”*

The first signs of the flexibility of using a visualisation approach to teaching patterns can be considered as the teacher’s capability to involve a variety of pattern problems, with the aim of developing generalisation skills. TB9 supported this view in the following words: *“When I presented pattern problems, a number of things changed. First I discovered that visuals assist learners in a great way to understand patterns clearly. Secondly there was widespread interest to learn mathematics that was promoted by the use of visuals. [Thirdly] almost learners from all ability levels benefited from the use of visuals as their participation levels were highly stimulated by the use of visuals. Fourthly learners were free and willing to generalise patterns unlike in situations I did not present problems from a visual point of view in my past teaching experiences. Interest was stimulated and this made my work easy.”*

4.6.2.2 Evidence of exploration with deeper understanding

Exploration with deeper reasoning capacity, learner autonomy and creativity was one of the indicators which depicted flexibility of teaching using a visual approach. TA7 made a contribution to this effect when he said *“There were such moments, really more especially when learners [autonomously] used, knowingly or unknowingly, visual generalisation strategies that were far spaced from the initial terms of the pattern. For instance creatively predicting the twentieth and fiftieth terms of a pictorial term ...*

with very minimal help from me." There were aspects of deeper reasoning on the part of the learners, judging from the teacher's explanation.

The last but not least indicator of flexibility when a visual approach is used to teach patterns was made visible by TC7 in her elaboration about why visuals should be regarded as an effective approach. She had the following to share with me: *"It is more effective because sometimes when you use visuals you stir group and individual interest. A lot is involved not just numbers and pictures which are written in black and white. There are colours as well which make the picture attractive as well as increase visual perception and promote further exploration of patterns."* This aspect is indicative of group exploration and the promotion of progress and visual perception.

When they explore patterns with deeper reasoning, this has a tendency to develop learner autonomy and creativity. When this occurs, teaching is indicated as being flexible. This aspect became visible when teacher TC9 gave the following insight: *"Before the intervention my teaching of patterns was largely teacher centred and could not invoke creativity in my learners. I did not engage my learners enough to take charge of their own learning and construct meaningful rules. After the intervention this changed, my lessons were now learner centred and I could engage my learners enough to construct meaningful knowledge about patterns and their generalisation from a pictorial perspective."* This shows that learner autonomy was boosted by the teacher through the use of visual tools.

4.6.2.3 Learner autonomy and creativity

The teaching of patterns involved a variety of pattern problems to develop intricate generalisation skills in learners. For instance, TA8 supported this aspect when he said: *"I used problems that involved counting, difference rate, and recursive and to some extent whole-object analysis. To my dismay learners were more able to explore pictorial patterns and could readily generalise from a picture context ... I had to guide them through this process."* To a greater extent the teacher's use of a variety of problems helped learners to develop sound generalisation skills. This was quite enabling of a visual approach to teaching patterns.

Pattern exploration was handled by teachers in a way that promoted deeper reasoning and progress in learners, as this inculcated autonomy and creativity in them. This aspect became visible when TC8 made the contribution that *"... learners were reasoning at a deeper level"* in her response in the second interview. Learner autonomy and creativity was further elaborated in the following statement by teacher TA8 when he said *"My learners could [independently] read and understand patterns from pictures, extend them successfully and describe and manipulate them without my help."*

There is no indication of flexibility whatsoever if a visual approach does not take group dynamics to promote learning progress and visual perception into consideration. This aspect was covered in the following statement by TB9. He said *“However the breakthrough moments came in that the majority of the learners, as they work in groups and as individuals, enjoyed using visuals.*

4.6.2.4 Group dynamics that promote visual perception

It was apparent that the group dynamics created by teachers when they used visual approaches to teach patterns and their generalisation promoted learning progress and encouraged the development of visual perception among learners. *“In the past when I taught patterns I did not get the kind of learner engagements I got when I used visual approaches to teach patterns. For instance there was a girl sitting in the middle row, she normally does not participate in class, she is thought of aloof. In this lesson, when I used visuals, surprisingly she showed interest and was actively taking part in lesson proceedings. This shows that visual teaching arouses learning instincts in learners ...”* I understood that one of these learning instincts that were aroused by the use of visuals was visual perception.

“They generalised patterns mostly aided by the context of the pictorial tool. They would say to each other show me in a picture how you arrived at a particular rule. This was quite interesting as these group interactions manifested in improved learner performance in written exercises, better than when I taught without the use of visuals previously.” It can be deduced from the views portrayed in this statement that group dynamics promoted visual perception to a considerable extent. TA9 added to this view when he said *“The learners interacted positively well with me and among themselves. There was freedom and tolerance of each other’s views and learners supported one another in the dynamics of knowledge construction.”*

4.6.3 Teachers’ views about the usability of the use of visuals across grade levels

4.6.3.1 Potentiality to develop cognitive perception

The potentiality to develop cognitive perception is one indicator of the usability of visuals in teaching patterns. This aspect was made visible during the interview process by TA7. The teacher said *“Learners who are taught using visual approaches have a higher knowledge retention capacity [which can be used] for the purposes of constructing meaningful and useful knowledge for use in everyday life.”* There could be no other better potentiality than this to develop cognitive perception in learners.

There was a powerful indication that teaching patterns using visuals has the potential to develop cognitive perception in learners. This was shown in a statement by TA8 when he said *“If learners see pictures they will quickly grasp the concept and internalise it quite effectively. Learners who are taught using visuals develop critical thinking and deeper cognitive understanding.”* This indicated to me that

a visual approach is quite a usable approach to teaching patterns because of its potentiality to develop cognitive perception.

One of the visible indicators of the usability of a visual approach to teaching patterns was its potentiality to develop cognitive perception in learners. *"In more than just one way my visual approaches assisted learners to cultivate cognitive understanding of patterns. This laid conducive grounds for them to see vividly into patterns and be able to deal with generalisation tasks more decisively."* This was the beginning of the injection of ideas that led to the development of vibrant cognitive perception in learners.

4.6.3.2 Encouragement of learner innovation and originality

The encouragement of learner innovation and basic originality is one of the indicators of the usability of a visual approach to teaching patterns. The experience shared by TA7 had far reaching consequences for this indicator. The teacher pointed out that *"Unlike when a teacher uses the traditional algorithmic approach learners tend to learn by rote method and can easily forget what they have learnt. They may use the knowledge for exam purposes after which they forget everything and become blank as if they learnt nothing. The use of visuals is a real game changer. Visuals encourage learners to own knowledge and become innovative with that knowledge."*

It was evident that the encouragement of learner innovation and originality played a crucial role in pattern teaching, as an indicator of the usability of visuals. *"Learners developed skills of organisation through critical analysis of pictorial patterns thus developing critical skills and getting deeper understanding of mathematics concepts."* The meaning I deduced from this statement by TC8 was that visuals promoted learner innovation and originality as learners perfected their organisational skills from pattern learning. The teacher further made the contribution that *"I will encourage [other teachers] to teach patterns using visuals to discover for themselves their usability and effectiveness if handled carefully."*

"Learners' explorative skills showed innovativeness. These skills were sharpened by the use of visuals and I could see learners were clamouring for more challenging tasks involving pictorial problems." I took it from this statement by TA9 that the encouragement of learner creativity and originality was one such factor which indicated that teaching with visuals make visuals usable. This created an enabling environment for teaching patterns and their generalisation.

4.6.3.3 Learner's effective use of algebraic knowledge

TC7 made a claim to the effect that learners' effective use of algebraic knowledge had the potential of advancing symbolic knowledge as she used visuals to teach patterns. This claim of hers was, yet

again, indicative of the usability of a visual approach to teach patterns and their generalisation. In her own words she said *"Everything worked very well. Learners were able to understand and respond from pictorial representations the essence of a pattern [that led to a symbolic representation] and could describe and analyse patterns in an elaborate manner."* This aspect could not have been possible unless learners had sound algebraic knowledge to support them.

Another indicator of the usability of visuals to teach patterns manifested in the idea that learners' effective use of algebraic knowledge to advance symbolic representation is crucial to pattern teaching. TA8 contributed to this aspect in this way: *"To me it was the level of excitement which manifested when learners found a general rule for a given pictorial pattern after being skilfully guided through pattern exploration by me. Learners made connections between what they were taught in elementary algebra and what they experience in everyday life. They constructed knowledge together through sharing of insights and engaging each other in thought provoking discussions about patterns and rule formulation."* This observation by the teacher points to effective use of algebraic knowledge by learners through guided instruction, to advance symbolic representation of pictorial patterns.

When learners effectively make use of previously acquired algebraic knowledge to advance symbolic representations, this becomes an enabling factor of visualisation which is indicative of its usability. This aspect was voiced by TC9 in the following way: *"Visuals helped my learners understand better. As I introduced patterns of geometric figures in my teaching approach and move to numeric representations of the pattern, learners used these figures and the context of the geometric figures to create rules that incorporated symbolic representations. With pictures learners could readily describe patterns explicitly in words, analyse and explore patterns in a variety of ways. This could not have been possible had I used traditional approaches like the numeric approach."* In another instance the teacher lamented that *"... learners took time to come up with symbolic representations of the pictorial patterns."* Had this persisted during the teacher's teaching of patterns, this could have counted as a constraining factor to using visualisation to teach patterns.

4.6.3.4 Time it took to complete tasks

The time it took to complete tasks with adequate knowledge construction is an indicator of the usability of a visual approach. TA7 made a remark to this effect when he said *"The application of visuals created a conducive environment for learners to learn collectively ... as they showcased new gained skills [and knowledge constructions] with regard to the timed tasks they had to do either individually or in groups."* The teacher later confirmed that learners completed the tasks in time in a member checking process I conducted after the interview process.

The time it took to complete tasks with adequate knowledge construction was quite crucial in indicating the usability of visuals to teach patterns and their generalisation. The following statement depicts how TA8 viewed this aspect: *“From what I have experienced after the intervention, teaching with visuals drastically reduces the time it takes to complete teaching patterns. I can say with that the use of visuals to teach patterns fits in well with the curriculum demands and rather allows a teacher to have extra time for revision. Before the intervention I struggled to finish pattern tasks with my learners in time and this would inconvenience me a lot. Teaching with visuals got most of the learners to participate fully in knowledge construction.”*

The time it take to complete tasks with adequate construction of knowledge can be an indicator of the usability of visuals to teach patterns and their generalisation. If it takes longer than necessary to complete a task when visuals are used, this becomes a constraining factor. This aspect became technically visible in the case of TB9. The teacher lamented that *“The challenges I faced were that instead of the visuals helping learners, they caused some form of distraction for my learners. As a result tasks were not completed in time in some cases. The visuals were a source of digression and this paused challenges and counter production in my teaching process.”* On the other hand TA9 expressed a different view to this. He said *“Visual approaches make it possible for the teacher and the learners to complete topic tasks well ahead of the prescribed time and have ample time for revision.”* This statement conclusively ascertained the usability of a visual approach to teach patterns effectively.

4.7 SUMMARY OF THE EMERGENT THEMES FROM THE INTERVIEWS

Analysis of the interview responses revealed that visuals were at the centre of teaching patterns and their generalisation for teachers at all Senior Phase grade levels. Teachers indicated that the use of visual tools provided learners with an opportunity to ‘see from their mind’s eye’. Thus, teaching using visuals was key to the development of visual perception in learners and subsequently the development of critical thinking. It was quite apparent, during the observation process that learners’ cognitive perception developed under the teachers’ instructional guidance and negotiated learning interventions as suggested by Dolittle and Hicks (2003). Teachers echoed this phenomenon in their collective views during the interview process. This evidence is found in the following statement by TB9: *“For some of us the use of visuals changed drastically the way learners perceived and understood patterns and pattern learning.”*

It was noted that teaching using visuals as experienced by participant teachers after the intervention enabled, rather than constrained the teaching of patterns. From the teachers’ views in the interview process, several indicators emerged that supported visualisation as an effective approach to teaching patterns and their generalisation. I gathered enough data from the teachers that favoured the use of

visuals to teach patterns. It was from this source of rich data that I concluded that teachers, across grades, were in agreement that visualisation makes the teaching of patterns applicable, flexible and very much usable for teaching patterns. This phenomenon availed itself as multiple perspectives emerged when learners interacted with a variety of pattern representation content, skills and social contexts (Weinberger, 2011). This aspect resonated conveniently with the fifth theoretical principle of social constructivism as envisaged by Dolittle and Hicks (2003). The principle states that construction of knowledge is integrated more deeply by engaging in multiple perspectives and representations of content, skills, and social realms.

Teachers' perceptions about the use of visuals showed that visualisation is an applicable approach for a number of reasons. Teachers in all three grade levels supported its applicability by making reference to the following: 1) the teachers' constant interactions with a variety of visuals in most cases promoted learner understanding; 2) as teachers' employed a visual approach in teaching patterns, they witnessed learners engaging in problem-solving activities; 3) most of the participants found teaching with visuals quite compatible with the traditional curriculum as their planned activities fitted in well with the prescribed syllabus; and 4) the teachers in several instances proclaimed that their own involvement in the learning process had far-reaching consequences to the general applicability of visualisation approaches they were using to teach patterns and their generalisation. Participant teachers tried their best to minimise their own involvement in the construction of pattern knowledge by learners. This was achieved when teachers acknowledged the fact that learning of pattern concepts occur when one learns from others in a socio-cultural context. Teachers tactfully used culturally relevant manipulates like bottle tops, matchsticks and homemade trinkets to assist learners to readily understand pattern concepts in familiar contexts (Nodding, 2011). The use of a variety of visual tools promoted both visual and cognitive perception in learners (Rahim, 2009).

The views expressed by teachers concerning indications of the flexibility of using visuals to teach patterns included the following insights: 1) as teachers involved a variety of pattern problems they observed that more of their learners developed intricate generalisation skills. 2) Also, as teachers engaged learners in rigorous pattern exploration activities with deeper reasoning, learners developed autonomous tendencies and became explicitly creative. 3) Teachers created an environment where positive group dynamics thrived as they taught patterns using a visual approach. This promoted visible learning progress as learners' visual perception increased for the better. Teachers achieved much by involving a variety of pattern techniques as they engaged learners in pattern exploration. It became manifest in the interview process that an individual's abilities and achievements cannot be understood outside the connection of the social circle and the culture in which are immersed (Adam, 2017).

Teachers explained how they cultivated group dynamics that were supportive of collective knowledge construction.

Lastly, teachers' perceptions in the interview process revealed that several factors indicated that teaching patterns using visuals became visibly usable when the following occurred: 1) the use of visuals exhibited a potential to develop noticeable cognitive perception in learners. Teachers' visual approaches were conducive for learners to see vividly into patterns and be able to perform generalisation tasks more decisively, as one teacher put it. 2) Teachers' use of visuals encouraged learners to take charge of their learning as they became increasingly innovative and were able to express their originality in coming up with new knowledge about the patterns they were learning. 3) In a number of instances, teachers struggled to engage learners to effectively use their algebraic knowledge to advance symbolic representations. This aspect slightly delayed the completion of generalisation tasks, leading to the teachers' over-involvement in the teaching/learning process. This marginally infringed on the constructivist teacher/learner contract as it violated the aspect of collective construction of knowledge. 4) Despite the prevalence of some underlying challenges, most teachers managed the time it took to complete tasks quite well. Tasks were completed in time with adequate knowledge construction.

However, in view of the ideas expressed in the second tenet of social constructivism, an individual's cognition involves collaboration and shared understanding in problem-solving activities (Dolittle & Hicks, 2012). Such was the general sentiment among participant teachers in the focus group interview. The tenet states that cognition organises and makes sense of one's experiences and is not a process to render an accurate representation of external reality. According to the teachers, learners' cognition assisted them to organise pattern concepts as they experienced multiple representations in given pattern problems. Thus, meaning making proceeded from ensuing social processes that included teachers' instructional ingenuity. This involved the application of generalisation strategies and the development of pattern concepts that work for particular situations (Beghetto, 2016).

The teachers' views revealed much about what factors enabled and/or constrained the teaching of patterns and their generalisation when a visualisation approach was used. Mostly, the teachers' perceptions point to a number of enabling factors together with a few constraining factors. In fact, the emergent constraints can be addressed if teachers are adequately trained and conscientised about how best to deal with them, by attending to the enabling factors (Riviera, 2011).

In my observations based on the perceptions of the participant teachers, I noted the following enabling factors of employing a visualisation approach to teach patterns and their generalisation: 1) Teachers' adequate use of a variety of visuals promotes pattern understanding. 2) Teachers' careful

use of visuals encourages learners to employ problem-solving techniques. 3) Teachers' adherence to the syllabus content renders the use of visuals compatible with traditional curriculum. 4) Teachers' minimal involvement in the teaching process (by taking on the role of mentor and guide) develops learner autonomy. 5) Teachers' involvement of a variety of pattern problems develops children's generalisation skills. 6) The teacher's encouragement of learners to explore patterns with deeper reasoning also develops learners' autonomy and creativity. 7) Teachers' creation of a conducive environment leads to the development of positive group dynamics that promotes learning progress with increased visual and cognitive perception. 8) Teachers' methods of using visual strategies encourages learner innovation and originality. 9) Teachers' ingenuity to encourage learners to effectively use their pre-knowledge of algebra advances their ability to use symbolic representations. 10) Teachers' ability to manage time when using visuals results in tasks being completed in time. These, and other factors which were not explicitly stated in the interviews, enabled the teaching of patterns from a visual angle (Rahim, 2009; Whitley, 2004; Gooding, 2009).

Here I present a few of the constraining factors experienced by participant teachers when teaching patterns using visual approaches: 1) Teachers' over involvement in the teaching process hindered autonomous knowledge construction by learners. 2) Learners' poorly developed algebraic knowledge delayed task completion and the advancement to symbolic representations. 3) Teachers' lack of preparation of adequate visual tools limited the learning scope of learners. 4) Teachers' lack of crisis management skills created unnecessary digressions as learners were distracted by inefficiently used visuals and manipulates. It must be noted that when one enabler was either neglected or omitted, the resultant aspect was the prevalence of a constraining factor that hindered the smooth teaching of patterns and their generalisation (Riviera, 2011; Bozkurt, 2017; Kashefi, 2015 and Kalchman & Koedinger, 2001)

4.8 CONCLUSION

In this last, but not least, chapter to the study, I have presented and discussed the data collected from the survey questionnaires, the lesson observations, and interviews. Data from each research instrument has been discussed separately and concluded with a brief summary. The data from the survey questionnaires and interviews was based on perceptions and experiences of the participating teachers. The data from the lesson observations was based on visualisation processes at work as teachers taught patterns and their generalisation. Some of the themes used to categorise and discuss data from the survey questionnaires, the lesson observations and the interviews emerged from the data while others were informed by theory.

Teachers' responses from survey questionnaires revealed that most teachers in the CHW district use visuals to teach patterns but with varying degrees of commitment to attach real value and practicability to the use of visuals. Challenges such as learners' limited knowledge of algebra, the time it takes to prepare visuals and lack of resources were cited as the reasons why some teachers were not effectively using visualisation strategies to extend what they were teaching to build learners' prior knowledge, make connections among concepts in mathematics and represent a concept in multiple perspectives.

Such survey findings necessitated the need for an intervention programme to support teachers in how to use visual tools and strategies in their mathematics classes. The teachers were thus observed after the intervention programme. From the observations, the following aspects of teaching from a visualisation perspective were observed and noted.

Observations of teachers' teaching revealed that they all used pictures and manipulates to create images during their lessons to develop the intended mathematical ideas. All teachers were able to manipulate the images they generated to support their explanations of both far and near generalisation, and constructive and deconstructive generalisation of patterns. They did this with tact and technique as they taught learners how to move from verbally describing a pictorial pattern to rule formulation and final symbolic representation. The visuals were used in line with the teachers' verbal explanations, applied mathematical language and terminology, as teachers assisted learners to make connections between visual and symbolic representations of the mathematics concept they were working on.

I noted that some teachers used visuals to promote mathematics abstraction and generalisation particularly in the second lesson. Teachers displayed their visual tools on the chalk board and on the demonstration table as they jointly examined them with their respective learners. This propelled learners to be actively engaged in the lessons as they developed a deeper understanding of what the teacher was teaching.

Conceptual understanding indicators such as multiple representations, building on learners' prior knowledge and connecting concepts and ideas in mathematics, dominated in the lessons taught by the teachers (Kilpatrick, Swafford & Findell, 2001). Problem-solving that could be extended to real life situations was not entirely visible in the lessons taught by teachers, however some teachers used the classroom floor tile pattern and other techniques to contextualise pattern problems involving square patterning and other natural phenomenon. During the process of analysing data from lesson observations this researcher extensively employed the use of three analytical tools developed for this very purpose.

Interview findings provided me with ample evidence to support data from questionnaires and lesson observations. It was revealed from interview responses that teachers used visuals to help them in demonstrating different pattern ideas that were linked to other mathematics constructs and to real life situations. Teachers made use of visuals to build from learners' background knowledge, so they were able to link concepts like geometry and algebra to patterns and this enhanced their understanding.

Teachers noted that working with visuals prompted learners to be actively involved as learner-to-learner interactions became more visible. They realised that their role shifted from being teacher-centred to that of a facilitator helping learners to construct their own knowledge.

Lastly, it was quite notable from the interview responses that after participating in an intervention programme, teachers were empowered to create and use visuals to teach patterns and their generalisation better than when they relied predominantly on numerical values and pre-determined formulas to teach patterns. From the teachers' collective perceptions, it became clear that visualisation approaches, if used effectively, have the potential to make the teaching of patterns and their generalisation practically applicable. Their collective views were also to the effect that the use of visuals was largely flexible and quite usable in the mathematics classroom. Considering their lesson experiences where they taught patterns using prepared visual tools, teachers recommended that Senior Phase teachers should be equipped with enough well-prepared visual tools to teach patterns and their generalisation. This, teachers suggested, would assist other teachers to overcome underlying challenges if they were encouraged to increase learners' explorative skills using a variety of tools and improve their instructional skills. The findings of this study are clearly presented in the next chapter to paint an overall picture of what this investigation yielded.

CHAPTER FIVE: CONCLUSION AND RECOMMENDATIONS

5.1 INTRODUCTION

The focus of this study was to investigate how widespread was the use of a visual approach to teaching patterns by Senior Phase mathematics teachers in the CHW District, prior to an intervention. It was also to investigate how selected Senior Phase mathematics teachers apply visual approaches to teaching patterns and their generalisation after an intervention programme, and subsequently to investigate the experiences and perceptions of selected teachers in order to establish what factors enable and/or hinder the teaching of patterns before and after the intervention when they used visual approaches. This chapter presents first the summary of findings, then an outline of the contributions of the study, followed by recommendations to scholars of mathematics education and curriculum developers. I also give the limitations of this study and some suggestions for further research. Lastly, I present my own personal reflections emanating from the results of this study.

5.2 SUMMARY OF MY FINDINGS

The summary of the findings of this study are presented per the research question. This is because the research questions were designed to answer crucial aspects of the research study in respect of the teaching of patterns using visuals.

Research Question 1

The intention of this question was to investigate the approaches used by Senior Phase mathematics teachers (before an intervention) in the CHW district to teach patterns and establish whether a visual approach was one of them. Thus, the question was phrased as follows:

How do Senior Phase mathematics teachers in CHW district teach patterns and their generalisation prior to an intervention programme?

Responses from survey questionnaires revealed that teachers in the CHW District acknowledged the importance and value of visualisation in the teaching of patterns and their generalisation. Many of the teachers were either using visuals on a minimal basis or not using them at all. Challenges such as the time it takes to prepare visuals, the difficulty in transferring of visual information to symbolic representations, lack of vital technological resources in schools, time constraints, compatibility issues with syllabus requirements and learners' limited knowledge of algebra, among others, were cited as reasons why teachers were not fully utilising visualisation strategies to extend what they were teaching to real life situations. Despite this, teachers confirmed that visualisation constructs build on

cognitive perception and on learners' prior knowledge and allow them to present mathematical ideas in multiple ways.

Among the teachers who confirmed using visuals, examples of the visual aids they used when teaching patterns and their generalisation were as follows: diagrams, tables, flow charts, pictures and manipulates (such as match sticks, stickers and bottle tops). Common among the teachers' experiences was the use of geometric pictures to represent numerical patterns that depict mathematical ideas. Most of the teachers preferred to use manipulatives instead of other forms of visuals – indicating that these could be used effectively to aid with problem-solving activities, and they do not require much preparation as they are ready made. Teachers who gave reasons for using visuals quite often in teaching patterns, had similar reasons for not using them more often as those given by teachers who preferred not to use visuals. Thus, it turned out that neglecting enabling factors of visualisation becomes a constraining factor.

Research Question 2

This question aimed at finding out how selected participant teachers implemented ideas gained from an intervention process, in order to closely examine how visual strategies were used as the teaching of patterns unfolded. Thus, the question was phrased in the following manner:

How do selected teachers in the CHW District teach patterns and their generalisation after participating in an intervention programme?

The selected teachers whom I observed drew pictures and diagrams as well as arranged objects on the chalkboard and display tables, in order to generate visual images. The images were then used to develop pattern concepts with mathematical ideas. The teachers did not necessarily use strict mathematical rules and definitions or elaborate calculations to define pattern generations. However, they used manipulatives to prompt pattern rule deductions by learners through systematically creating images of pattern ideas. The generated images were then inspected by both teachers and learners to establish emerging regularities. The teachers used the ideas emanating from image generation and image inspection to create platforms for classroom discussions. Teachers used probing and prompting techniques to query and to ascertain learners' prior knowledge and cultivate learners' deeper understanding of patterns and their related concepts. There was a visible connectedness between generated and displayed visual images and the classroom discussions that ensued between both teachers and learners and amongst learners.

Visual information transference to symbolic representations was illustrated through image transformation mostly in the second lesson when generalisation strategies were being examined. The

transformation of images (through construction and deconstruction) gave credence to new pattern ideas teachers developed to promote collective discussions with their respective learners. These transformations arrested learners' interest visually, enhancing their active engagement in the lesson proceedings. Learners were not only excitedly enjoying the lessons but also deepening their cognition of pattern concepts.

Throughout the lesson observations, teachers did not only inculcate visual agility in learners but also unlocked mathematical connectedness between pictorial and symbolic representations in developing pattern concepts. In this way, the teachers assisted learners in making connections between visual and symbolic inscriptions of the mathematical concepts they were learning. The ensuing process helped learners to create mental pictures in their minds, which they used in understanding generalisation techniques. This enhanced their cognitive perception as well as sharpened their critical perspectives.

It was noted that the visuals used were in line with the teachers' verbal instructions and explanations and had a positive bearing on the learners' visual narratives used together with the implied mathematical terminology. For instance, when making emphasis on pattern images teachers used such mathematical terminologies as 'infinite difference', 'constant difference', 'stage number' and 'general rule' among other mathematical language pertaining to patterns and their generalisation.

Furthermore, teachers also used gestures together with the visuals in their lesson presentations. These were prevalent during teachers' interpretation of the role of manipulatives in developing problem-solving skills. Teachers would move their hands and fingers in trying to illustrate how separate elements of a visual figure should be moved to effect a new construction or deconstruction move that lead to a particular generalisation of the original figure. This way, teachers used visualisation that incorporates gestures to promote mathematics abstraction and generalisation.

The teachers' aims were to achieve generalisation that was to be used as a construction agent of summary regularity in a given visual pattern. However, this aspect posed particular problems among teachers and learners since learners' knowledge of algebra was not as sound as what the teachers anticipated.

Lastly, it was observed that teachers attempted to use visuals to connect pattern learning to real life situations with varying degrees of success. Some teachers used real life problems and everyday examples to enhance learner understanding of patterns and their application in the physical world experience. One teacher used an example of the classroom floor tiles forming a checkered and

systematic square pattern. To solve a pattern problem, another teacher used task cards pinned on the notice board. This enabled and prompted learners to understand and solve pattern related problems.

Research Question 3

Research Question 3 aimed at examining the perceptions and experiences of selected teachers both prior to and after the intervention and implementation process. This information ensued from the interview process that occurred during and after selected teachers taught patterns and their generalisation, applying acquired ideas from the intervention programme. As a result, the question was phrased as follows:

What are the selected teachers' experiences and perceptions of teaching patterns and their generalisation prior to and after participating in an intervention programme?

Teachers expressed the general view that teaching using visuals made it possible for learners to be actively involved in lessons. This enhanced teacher to learner and learner to learner interactions. The teachers also voiced the perception that by teaching using visuals their roles shifted from that of being the centre of knowledge and knowledge construction to that of being mediators of learning and assisting learners to collectively construct their own knowledge.

Teachers noted that teaching using visuals enabled them to build on learners' prior knowledge, make sense of the constructed new knowledge and establish the connectedness of certain mathematics concepts. Teachers indicated that visuals, instead of hindering pattern learning, to a greater extent, they enable pattern learning as they have the potential to develop critical thinking and problem-solving tendencies. Teachers also added that visuals open up grounds for learner innovation and learner originality.

Unlike teachers' views prior to the intervention that visuals hamper learner understanding of patterns and are therefore a hindrance to knowledge acquisition, this view shifted dramatically after the intervention and subsequent implementation process. Teachers conceded that visuals have the potential to develop cognitive perception which is necessary in enhancing understanding of mathematics concepts.

Teachers concluded that visuals have the potential to increase learners' knowledge retention capacity which is essential for exam purposes and for use in life-long learning processes in later years. In summary, teachers' experiences and perceptions are indicative of the usefulness and effectiveness of teaching patterns using visual approaches as they support learner understanding of mathematics concepts.

Research Question 4

The main intention of this question was to investigate what selected teachers viewed as visualisation's enablers and/or constrainers on the teaching of patterns and their generalisation. This information was also sieved from the selected teachers' views and interpretations that emanated from the interview process, with particular focus being on the focus group discussions. Following the views of the selected teachers the question was thus phrased as follows:

What are the enabling and/or constraining factors of applying a visualisation approach to teaching patterns and their generalisation?

Teachers considered that the teaching of patterns using visualisation posed some hindrances, but that these are far outweighed by their enablers. First and foremost, it was revealed by the teachers' insights that neglecting any of the enabling factors of visualisation automatically result in a constraint. For instance, if a teacher does not take time to thoroughly prepare his/her visual tools (with his/her objectives in line with his/her target learners) this has the effect of taking unnecessarily longer to deliver in the actual teaching – leading to an inevitable time constraint.

Secondly, if a teacher neglects the all-important process of proper image generation, this has the effect of making the teaching of patterns grossly incompatible with syllabus requirements. Also, if care is not taken during image inspection this might lead to poor knowledge construction which might hinder understanding and the development of cognitive perception among learners.

Teachers who use vague instructions and inappropriate visual pattern terminology and tools risk not being able to develop the anticipated critical thinking abilities and problem-solving skills necessary for pattern learning and the teaching of other related mathematics concepts. This may lead to learners becoming disinterested and resort to using pattern tools (like manipulatives) as sources of learning distraction.

Algebraic knowledge can be developed concurrently with pattern knowledge if the concept of general regularity is taught from a concept connectedness perspective. Thus, learners' knowledge of algebra or lack thereof cannot be used as an excuse for a teaching and learning hindrance when pictorial representations are transformed to symbolic representations through visual teaching of patterns.

Lastly, teaching using visuals can be seen as a flexible and usable approach that can be perfected with practice, perseverance and continual experience – as expressed by the teachers who participated in this research study.

5.3 CONTRIBUTION OF THE STUDY

In context of the South African education curriculum, this research study has the potential to provide mathematics education researchers and curriculum developers with essential recommendations about including visualisation in the design of policy documents and further research efforts. This project did not only improve my knowledge on and practice in visualisation in mathematics education, but it also helped me as a practising mathematics educator and scholar to expand my knowledge on how to use effective instructional practices in teaching patterns and their generalisation and other related mathematics concepts.

It is my lasting hope that the teachers who participated in this study were duly empowered with different approaches that are required to conceptually teach patterns and their generalisation using visualisation. In addition, the results of this study could help institutions of higher learning and training to integrate visual approaches in teacher training initiatives and adequately prepare teacher trainees to embrace the visuals sphere in handling pattern teaching.

Impressions from participants

In my bid to get a comprehensive view of how the research process impacted on my participants I conducted brief personalised discussions with each of the participants. I did this as part of my member checking exercise. Generally, I gathered that the study had a far-reaching positive impact on the teachers' pedagogical practices as it provided a variety of technical strategies for teaching patterns and their generalisation. Some teachers conceded that before the intervention they lacked tactical strategies to make pattern teaching likeable and beneficiary to learners. Furthermore, some participants appreciated the fact that the use of visual approaches such as visual question generation, image use, visual tool inspection and visual tool transformation perfected their teaching techniques to a greater extent. Previously teachers taught patterns and their generalisation without a sound strategic framework to guide them through teaching of patterns. One participant commented that if patterns are effectively taught (as in the way they learnt to teach them in the intervention process) at Senior Phase level, this would impact positively on the teaching of all other related mathematics concepts at higher grades and beyond. Most of the participants concurred that a sound knowledge of algebra (among learners) is a pre-requisite to successful visual pattern teaching. The participants also agreed that teacher control, coupled with meaningful and well-orchestrated learner activities, will avoid unnecessary digressions during teaching time when manipulatives and other visual tools are employed.

Lastly, the findings of this study may help authors of mathematics textbooks to come up with learning materials that are compatible with curriculum demands and to include the use of visualisation strategies in their textbooks. One of the research participants lamented that contemporary textbook

material on patterns was somewhat confusing for both teachers and learners, since they lack the pedagogical ramifications of a visual approach to teaching patterns. If teachers' perceptions and experiences from this research study are to be considered, this may have the impact of improving teaching techniques of patterns, thereby increasing learners' conceptual understanding of patterns and algebra and equipping teachers with ample tools to teach these mathematics concepts and others.

5.4 RECOMMENDATIONS

I am quite aware that although the findings of this study cannot be generally applied to the context of mathematics teaching in the entire South African education system, they can at least contribute to individual teachers who wish to use visualisation as an instructional tool for teaching patterns and their generalisation. Based on the findings of this study, I would like to make the following informed recommendations. After information sharing workshops on the research study findings in the CHW District, mathematics subject advisers should be encouraged to provide workshops on how best to use visualisation strategies (i.e., counting analysis, recursive analysis, whole-object analysis, difference rate analysis and explicit analysis) and visual tools with special emphasis on aspects that promote learner cognitive perception, critical thinking and conceptual understanding of patterns such as:

- Use of multiple visual processes
- Connecting the mathematics of patterns to learners' prior knowledge
- Connecting the mathematics of patterns to real life situations
- Connecting concepts and ideas in mathematics particularly the ones that involve patterns
- Use of multiple representations and information transference.

On the other hand, curriculum developers should consider including and making visualisation strategies, such as the ones examined in this study, compulsory in teaching patterns and their generalisation. Visual teaching approaches that involve visual question generation, image use, visual tool inspection and visual tool transformation need to be developed and included in the curriculum documents. This recommendation is premised on the fact that this study's findings show that the use of such strategies with appropriate visual tools assist learners in understanding and solving pattern problems. Therefore, the use of such visual strategies has to be emphasised in official policy documents.

Furthermore, universities training teachers should ensure teacher trainees are well equipped in generating and utilising visualisation strategies (which includes both far and near, constructive and deconstructive generalisation strategies) to teach patterns and their generalisation. In the course of

teacher training, emphasis should be made on the role that visualisation plays in mathematics education.

In my view and experience, most South African schools have in ample supply such basic resources as manila paper, scissors and markers that teachers can utilise to make a variety of visual tools that they can use to teach patterns and their generalisation, if they get enough professional development training in this respect. To add to that, the DBE has in recent years embarked on equipping most South African Schools and teachers with laptops and every school has received or is in the process of acquiring a projector that can be used in mathematics teaching. At least this can help address the challenge of resources posing constraints for teaching patterns using visual approaches.

All the visual tools that were used in teaching patterns by teachers in this study were hand made by the teachers themselves, with the researcher's collaboration during the intervention programme. Teachers should make and/or be provided with adequate guidance to design visual tools that learners can manipulate in class, instead of having to draw them on the chalkboard during the lesson. This will help to minimise the time it takes to draw them. In using visual tools teachers should afford learners a chance to explore them as they facilitate the learning process. This will enable learners to create pictures in their minds which can reflect on the use of visuals to solve pattern problems. This will ease cognitive pressure on the learners because their processing demands will be split over verbal and visual information processing channels while they explore pattern dynamics in visual figures.

For visual tools to be used optimally, teachers should familiarise themselves them before engaging learners in lessons, so that they are not disappointed when assembling and manipulating them to generate images. Teachers should also consider using visual tools in creating platforms for discussions to ensure that their verbal explanations are in line with the visuals used and the language of mathematics to be developed.

The use of visualisation strategies helps learners in exploring pattern dynamics and representing patterns in their symbolic accuracy. Teachers need to train their learners on how to use these visual strategies and also encourage them to use such strategies when solving pattern problems. This will have the potential of exposing learners to a rich set of mathematical concept connectedness which can be further emphasised for learners to see relationships between the picture and the general rule in symbolic form.

It is of paramount importance for teachers to make their learners aware of the processes to be followed when inspecting and exploring patterns from a visual perspective. This entails understanding the different elements of the pictorial figure and identifying relevant mathematical concepts within

it; formulating the ideas into a verbal explanation that describes the relations involved; deducing the mathematical rule that governs the pattern and reflecting on the solution process; and evaluating and interpreting the mathematical rule in relation to the original figure. These processes need to be explicitly taught to the learners by the teacher.

Common to most mathematics textbooks is the idea that there are geometric patterns that reflect a corresponding numerical pattern. In my view, in order to reflect explicit visualisation, mathematics textbook writers should consider incorporating more expressly visualisation material in their textbooks by creating space for more fascinating visual figures that not only focus on the image but also on the visualisation processes: this will assist teachers and learners in analysing and creating collective and concrete meanings from the visuals.

5.5 LIMITATIONS

With a global pandemic raging at the time of my conducting this research study, I had to deal with multiple challenges that adversely limited my research efforts to conduct a study that involved human subjects. The first hurdle presented itself in that I could not conduct a workshop that I had initially planned to gather baseline data for the study. Covid-19 restrictions included the banning of all human gatherings in both closed and open spaces. I had initially planned to conduct a workshop with a group of forty Senior Phase mathematics teachers from the CHW District. This was aimed to get data about how teachers in the district teach patterns and their generalisation as well as conscientise the teachers on the use of visualisation approaches in teaching patterns. However, this workshop did not take place as planned, and instead I had to conduct the workshop online. It was not easy to get teachers to participate in the online workshop. In the end thirty-seven teachers who had shown interest, agreed to complete a questionnaire to that effect.

I struggled to retrieve the questionnaires. Teachers kept on giving excuses but in the end I managed to retrieve all the questionnaires with the help of the mathematics subject advisor. However, some of the questionnaires were partly completed and had gaps in some of the required areas. I then approached several potential teachers to ask them to participate in an intervention programme and hence in the intended study. Nine teachers were selected and the process of engaging them in an intervention process earnestly began. This was not easy as I had to meet with two teachers at a time (sometimes only one teacher would attend the intervention workshop). It took me more than five weeks to complete the intervention programme.

It was during the lockdown period that I had to conduct classroom visits. It was not easy to be granted permission to enter school premises by school authorities. When I started the visits only Grade 7

Learners were attending school. I managed to visit two Grade 7 teachers twice each during this strict lockdown period. The other Grade 7 teacher did not avail, herself citing work commitments. When lockdown restrictions were finally eased and all grades were attending school I made little progress with my observational visits, because classes were alternating days of attending school and this slowed down my progress quite significantly. Furthermore, the contact time between teachers and learners had been drastically reduced for all subjects including mathematics in all the schools I was carrying out my investigation. A lesson which normally would take fifty minutes had been reduced to thirty minutes, and this impacted negatively on the teaching of patterns using visuals which requires ample time. I had to ask school authorities to grant me ten-minute extensions of the lessons to at least gather credible data from the teaching process. Ideally, more time should have been spent on working with teachers and exposing them to the full potential of the use of visualisation approaches to teach patterns and their generalisation. The results could have been more comprehensive if teachers had used the visual strategies for a longer period of time in their implementation process.

One out of the nine teachers I had selected dropped out of the investigation process, citing difficulties in coping with the demands of the research process. Another teacher allowed himself to be observed twice but did not avail himself for the interview process as he kept on postponing the event. These reduced my collected data quite significantly.

The other challenges I faced occurred during the interview process. The teachers always came up with excuses and this negatively impacted on my interview schedule. However, despite these numerous challenges, useful insights on the use of visualisation approaches in teaching patterns and their generalisation were gathered and analysed.

5.6 SUGGESTIONS FOR FUTURE RESEARCH

Since this study was conducted on a limited scale, I recommend the following areas for future research:

- Conducting research with more widespread research sites and a bigger sample size in order for the findings to be generalisable to a wider context.
- Conducting research on the same topic, but expanded to include teacher training institutions, curriculum developers, textbook writers and learners.
- Conducting research on learners' views and experiences on the use of visualisation processes in learning patterns and their generalisation for critical understanding and problem-solving purposes.

Conducting research on geometry and other areas of mathematics to ascertain if visualisation approaches will also yield positive results.

5.7 PERSONAL REFLECTIONS

My research journey has not been an easy one. It was riddled with drawbacks and unprecedented challenges. There were moments that I strongly felt I could not make it and almost gave up. I was kept going by echoes in my mind of voices such as that of Albert Einstein (n.d) cited in Brainy quote.com (2021) who once said “Life is like riding a bicycle. To keep your balance, you must keep moving.” The other voice that kept me hanging on despite having submitted my draft proposal a total of twenty-one times before I was given the green light to commence my investigation, was that of Tony Robbins (2016) who once said “Stop being afraid of what could go wrong, and start being excited about what could go right.”

This was my first experience in engaging with real and demanding academic research. I spent lots of time sifting through and reading numerous academic articles, journals and textbooks in order to find similarities between past and ongoing research processes and my own study. The readings did not only expand my knowledge and understanding of the topic I was interested in researching, but also took me a step higher in my quest to develop my academic writing skills.

The study gave me a chance to reflect on how I teach patterns and algebra to my learners. It was an experience of a lifetime that cannot be erased from the archives of my living memory. I will thus support and encourage other teachers to embrace the visual world not only in teaching patterns and their generalisation, but mathematics concepts in general.

5.8 CONCLUSION

It is quite evident from this research study that visualisation approaches are essential in teaching patterns and their generalisation. It can be noted from the findings that teaching patterns using visual strategies and visual tools helps learners to create permanent pictures in their minds which they can use to critically reflect on and gain deeper understanding of visual strategies that assist to solve both problems in mathematics learning and in real life scenarios. After presenting a summary of the key findings of the study, discussing the limitations thereon, making recommendations and suggesting possible future research initiatives, I concluded this research study with my own personal reflections. On a concluding note, this researcher wishes to make the following observation: the use of visuals to teach patterns and their generalisation, as reflected in this study, puts the teacher and the learners

on a sound footing that can ease mathematical concept understanding for teaching and learning at higher levels of learning and beyond.

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Eastern Cape Department of Education Curriculum Management.

APPENDICES

Appendix1: Final Ethical Approval letter



Human Ethics sub-committee
Rhodes University Ethical Standards Committee
PO Box 94, Grahamstown, 6140, South Africa
t: +27 (0) 46 603 8055
f: +27 (0) 46 603 8822
e: ethics-committee@ru.ac.za
www.ru.ac.za/research/research/ethics
NHREC Registration no. REC-241114-045

20 March 2020

Simon Chatima

Email: g10C7589@campus.ru.ac.za

Review Reference: 2019-0476-3190

Dear Mr. Chatima

Title: Visualisation in teaching patterns and pattern generalization: Perceptions and experiences from senior phase mathematics teachers.

Principal Investigator: Dr. Clemence Chikiwa

Collaborators: Mr. Simon Chatima,

This letter confirms that the above research proposal has been reviewed and **APPROVED** by the Rhodes University Ethical Standards Committee (RUESC) – Human Ethics (HE) sub-committee.

Approval has been granted for 1 year. An annual progress report will be required in order to renew approval for an additional period. You will receive an email notifying when the annual report is due.

Please ensure that the ethical standards committee is notified should any substantive change(s) be made, for whatever reason, during the research process. This includes changes in investigators. Please also ensure that a brief report is submitted to the ethics committee on the completion of the research. The purpose of this report is to indicate whether the research was conducted successfully, if any aspects could not be completed, or if any problems arose that the ethical standards committee should be aware of. If a thesis or dissertation arising from this research is submitted to the library's electronic theses and dissertations (ETD) repository, please notify the committee of the date of submission and/or any reference or cataloging number allocated.

Sincerely,

Prof Arthur Webb

Chair: Human Ethics Sub-Committee, RUESC- HE

APPENDIX 2: Gatekeeper Approval letter



Province of the
EASTERN CAPE
DEPARTMENT OF EDUCATION

OFFICE OF THE DISTRICT DIRECTOR

Physical Address: The Homestead Building, 02 Limpopo Drive, Laurie Dashwood Park, Queenstown; **Postal Address:** Private Bag X7053, QUEENSTOWN 5320, REPUBLIC OF SOUTH AFRICA, Contact no: 045 808 5712, nrwdebruyne@gmail.com 083 418 8856
e-mail: philisa.mqoboli@gmail.com

CHRIS HANI WEST DISTRICT

TO	:	MR SIMON CHATIMA ABAMBO HIGH SCHOOL
FROM	:	OFFICE OF THE DISTRICT DIRECTOR – CHRIS HANI WEST DISTRICT
SUBJECT	:	YOUR REQUEST TO CONDUCT RESEARCH
DATE	:	20 MARCH 2020

The above mentioned and my conversation with you has reference

You are hereby appointed acknowledge research for your Masters' Degree in schools in the Chris Hani West District.

After some due consideration and consultative processes, I have arrived at the decision to approve your request to conduct research. My approval is subject to the following conditions:

1. That you must not interfere with tuition time
2. No educator or learner name must be mentioned in your final product / research
3. To share the outcome of your findings with the Chris Hani West Education District.

Let me take this opportunity to wish you well in your endeavour.

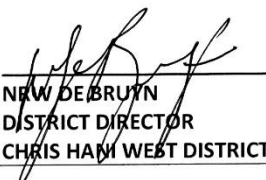
Thank you

Yours in Education

**DEPARTMENT OF EDUCATION
DISTRICT DIRECTOR
CHRIS HANI WEST DISTRICT
KOMANI CMC**

2020 -03- 20

PRIVATE BAG X7053
QUEENSTOWN - 5320


**NEW DE BRUYN
DISTRICT DIRECTOR
CHRIS HANI WEST DISTRICT**

Appendix 3: Principal Approval letter school A



Nkosemntu Motman Senior Secondary School

P.O. Box 456

Queenstown

5320

7 September 2020

Mr Schatima

Abambo High School

Whittlesea

Dear Sir

Subject: LETTER OF APPROVAL

I wish to respond to your request to conduct research at our school.

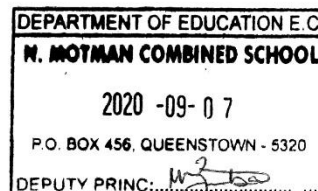
Since what you will be doing is not different from what we are doing, we therefore feel that we must grant you permission to conduct your research.

We promise to give you all the assistance you deserve and we hope you will enjoy your time at our school.

We wish you all the best in your research.

Yours Faithfully

N.V Zimba (Deputy Principal)



Appendix 4: Principal Approval letter school B



MOTMAN PRIMARY SCHOOL

CHRIS HANI WEST DISTRICT

INCIBA CIRCUIT

LETTER OF APPROVAL TO CONDUCT RESEARCH						
TO:	MR SIMON CHATIMA STUDENT AT RHODES UNIVERSITY					
FROM:	THE OFFICE OF THE DEPUTY PRINCIPAL- MOTMAN PRIMARY SCHOOL					
SUBJECT:	YOUR REQUEST TO CONDUCT RESEARCH					
DATE:	04 SEPTEMBER 2020					
The above mentioned and my conversation with you regarding your request.						
You are hereby approved to conduct research for your Masters' Degree at this school.						
After due consideration and consultative processes with the school SGB, we have arrived at the decision to grant you permission to conduct research at our school. Our approval is subject to the following conditions:						
1. That you must not interfere with daily routine of the school. 2. You must adhere to the protocols of ethical research to keep participants anonymous. 3. You must observe strictly all Covid-19 pandemic regulations without failure. 4. You must make a commitment to provide us with feedback at the completion of your research efforts.						
Let me take this opportunity to wish you well in your endeavour.						
Yours in Education	<table border="1"><tr><td>DEPARTMENT OF EDUCATION E.C.</td></tr><tr><td>N MOTMAN PRIMARY SCHOOL</td></tr><tr><td>2020 -09- 26</td></tr><tr><td>P.O. BOX 456 - QUEENSTOWN - 5320</td></tr><tr><td>PRINCIPAL</td></tr></table>	DEPARTMENT OF EDUCATION E.C.	N MOTMAN PRIMARY SCHOOL	2020 -09- 26	P.O. BOX 456 - QUEENSTOWN - 5320	PRINCIPAL
DEPARTMENT OF EDUCATION E.C.						
N MOTMAN PRIMARY SCHOOL						
2020 -09- 26						
P.O. BOX 456 - QUEENSTOWN - 5320						
PRINCIPAL						

Appendix 5: Principal Approval letter school C



Province of the
EASTERN CAPE
EDUCATION

VUSELELA SENIOR SECONDARY SCHOOL

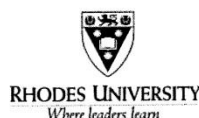
CHRIS HANI WEST DISTRICT

INCIBA CIRCUIT

LETTER OF APPROVAL TO CONDUCT RESEARCH	
TO:	MR SIMON CHATIMA STUDENT AT RHODES UNIVERSITY
FROM:	THE OFFICE OF THE DEPUTY PRINCIPAL- VUSELELA SENIOR SECONDARY SCHOOL
SUBJECT:	YOUR REQUEST TO CONDUCT RESEARCH
DATE:	07 SEPTEMBER 2020
The above mentioned and my conversation with you regarding your request. You are hereby approved to conduct research for your Masters' Degree at this school. After due consideration and consultative processes with the school SGB, we have arrived at the decision to grant you permission to conduct research at our school. Our approval is subject to the following conditions: 1. That you must not interfere with daily routine of the school. 2. You must adhere to the protocols of ethical research to keep participants anonymous. 3. You must observe strictly all Covid-19 pandemic regulations without failure. 4. You must make a commitment to provide us with feedback at the completion of your research efforts. Let me take this opportunity to wish you well in your endeavour. Yours in Education <u>ANDILE P GUANA</u> 	

DEPT OF EDUCATION EMIS NO: 600898 Vuselela Senior Secondary School 07 SEP 2020 P O BOX 1093, QUEENSTOWN 5320 PRINCIPAL: 
--

Appendix 6: Parent/Guardian informed consent



PARENT AND GUARDIAN'S INFORMED CONSENT

INFORMED CONSENT DECLARATION

(Parent or Guardian)

Project Title: Visualisation processes in teaching patterns and their generalisations:
Enabling and constraining factors from mathematics teachers' experiences and perceptions.

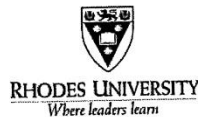
Simon Chatima from the Department of Education, Rhodes University has requested my permission to allow my child to participate in the above-mentioned research project.

The nature and the purpose of the research project, and of this informed consent declaration have been explained to me in a language that I understand.

I am aware that:

1. The purpose of the research project is to investigate how senior phase mathematics teachers teaching patterns and their generalisations using visual approaches. The focus will be directed on the selected teachers' experiences and perceptions about what enables or constrains the teaching of patterns when visual strategies and techniques are used. I understand and consent to the fact that video recordings will be made focusing only on the teachers teaching, and that if by any chance my child gets captured in the video the researcher will make efforts to blur the picture before using it for data analysis purposes.
2. The Rhodes University has given ethical clearance to this research project and I have seen/ may request to see the clearance certificate. [Certificate number]
3. By participating in this research project my child/ward will be contributing, indirectly, towards the necessary professional development for teachers teaching in the district. My child's contribution will also impact directly on the shaping and reshaping of strategies used by teachers to teach patterns and other related mathematical concepts using visual approaches. All this will in some way contribute to improved mathematics results at senior level and beyond.
4. My child/ward will participate in the project by actively taking part as a learner in the lessons that the teacher will prepare to teach for a period of two weeks. My child will not be interviewed or video recorded at any instance during the investigation. My child's role in this study is to be normally involved in lesson proceedings as a learner, since the main focus will be on the teacher and not the learners. If it so happens that my child be visibly captured in the video recordings, the researcher undertakes the responsibility to blur the picture.

Appendix 7: Parent 1 informed consent



12. Any further questions that I might have concerning the research or my participation will be answered by Simon Chatima who will be contacted at mollinchatima@gmail.com or cell number 072 754 3746 (**provide name and contact details**)
13. By signing this informed consent declaration I am not waiving any legal claims, rights or remedies that I or my child/ward may have.
14. A copy of this informed consent declaration will be given to me, and the original will be kept on record.

I, ... have read the above information has been explained to me in a language that I understand and I am aware of this document's contents. I have asked all questions that I wished to ask and these have been answered to my satisfaction. I fully understand what is expected of my child during the research.

I have not been pressurised in any way to let my child take part. By signing below, I voluntarily agree that my child/ward (**insert name of child**), who is years old, may participate in the above-mentioned research project.

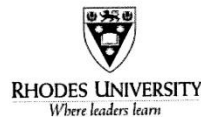
.....
Parent/Guardian's signature

.....
Witness

15/09/2020
Date

Rhodes University, Research Office, Ethics
Ethics Coordinator: ethics-committee@ru.ac.za
t: +27 (0) 46 603 7727 f: +27 (0) 86 616 7707
Room 220, Main Admin Building, Drostdy Road, Grahamstown, 6139

Appendix 8: Parent 2 informed consent



12. Any further questions that I might have concerning the research or my participation will be answered by Simon Chatima who will be contacted at mollinchatima@gmail.com or cell number 072 754 3746 (**provide name and contact details**)
13. By signing this informed consent declaration I am not waiving any legal claims, rights or remedies that I or my child/ward may have.
14. A copy of this informed consent declaration will be given to me, and the original will be kept on record.

I, ... have read the above
inform has been explained to me in a language that
I understand and I am aware of this document's contents. I have asked all questions that I
wished to ask and these have been answered to my satisfaction. I fully understand what is
expected of my child during the research.

I have not been pressurised in any way to let my child take part. By signing below, I voluntarily
agree that my child/ward (**insert name
of child**), who is years old, may participate in the above-mentioned research
project.

.....
Parent/Guardian's signature

.....
Witness

13/09/2020
.....
Date

Rhodes University, Research Office, Ethics
Ethics Coordinator: ethics-committee@ru.ac.za
t: +27 (0) 46 603 7727 f: +27 (0) 86 616 7707
Room 220, Main Admin Building, Drostdy Road, Grahamstown, 6139

Appendix 9: Parent 3 informed consent

APPENDIX J: ISAZISO KUMZALI



RHODES UNIVERSITY
Where leaders learn

Mzaliobekekileyo

MzalindicelaintsebenziswanayakhokunyenomntwanawakhoowenzaiMathematicskulamabanga:
7, 8 okhanye 9 kwiphulolophandoelenziwayokufundisokwezibalo (Mathematics)
kulamabangaengentla.

Oluphandoaluzukunikainkcukachazomntwanawakho
(iziphumozophandazogcinwaziyimfihlo)
ngobalizokubalijolisekutitshalakakhuluobandakanyekayo.

Sovuyiswayintsebenziswanayakhomalunganalomba.

Umsayino *Nm Ngwenyane*..... Umhla
14.09.2020.....

Ozithobileyo

Simon Chatima

Appendix 10: Parent 4 informed consent

APPENDIX J: ISAZISO KUMZALI



RHODES UNIVERSITY
Where leaders learn

Mzaliobekekileyo

MzalindicelaintsebenziswanayakhokunyenomntwanawakhoowenzaiMathematicskulamabanga:
7, 8 okhanye 9 kwiphulolophandoelenziwayokufundisokwezibalo (Mathematics)
kulamabangaengentla.

Oluphandoaluzukunikainkukachazomntwanawakho
(iziphumozophandazogcinwaziyimfihlo)
ngobalizokubalijolisekutitshalakakhuluobandakanyekayo.

Sovuyiswayintsebenziswanayakhomalunganalomba.

Umsayino Nh. Gungu Umhla
15-09-2020

Ozithobileyo

Simon Chatima

Appendix 11: Participant informed consent

TB7



PARTICIPANT INFORMED CONSENT

INFORMED CONSENT DECLARATION

(Participant)

Project Title: Visualisation processes in teaching patterns and their generalisations:
Enabling and constraining factors from senior phase mathematics teachers' experiences and perceptions.

Simon Chatima from the Department of Education, Rhodes University has requested my permission to participate in the above-mentioned research project.

The nature and the purpose of the research project and of this informed consent declaration have been explained to me in a language that I understand.

I am aware that:

1. The purpose of the research project is to investigate how selected senior phase mathematics teachers teach patterns and their generalisations using visual approaches prior to and after an intervention programme. The focus will be on the experiences and perceptions of the teachers about what constrains and what enables the teaching of patterns when a visual approach is used.
2. The Rhodes University has given provisional ethical clearance to this research project and I have seen/ may request to see the clearance certificate.
3. By participating in this research project I will be contributing towards my own professional development and to that of my colleagues through collaborative planning and teaching using visual strategies in our respective classes. The resultant professional development among participants will be shared to teachers in the district and beyond and possibly assist in improving mathematics results at senior level and beyond. My contribution will also go a long way in making the necessary input into mathematics education research by contributing much needed data to the research community.
4. I will participate in the project by actively taking part in an awareness and orientation workshop, participating in an intervention programme that include collaborative planning and teaching in my own classroom. I consent that I will voluntarily fill in a questionnaire and have my discussions with other participants audio recorded. I also consent that the researcher will observe me teaching and video record me in the process. After each stage of the implementation process the researcher will interview me concerning my views and experiences during my teaching, making special

Appendix 12: Teacher 1 informed consent



10. Any further questions that I might have concerning the research or my participation will be answered by Simon Chatima.
11. By signing this informed consent declaration I am not waiving any legal claims, rights or remedies.
12. A copy of this informed consent declaration will be given to me, and the original will be kept on record.
13. Request to take pictures, video and voice recording for this study has been made clear to me and I hereby give my voluntary consent to it.

I, [REDACTED] have read the above information / confirm that the above information has been explained to me in a language that I understand and I am aware of this document's contents. I have asked all questions that I wish to ask and these have been answered to my satisfaction. I fully understand what is expected of me during the research.

I have not been pressurised in any way and I voluntarily agree to participate in the above-mentioned project.


.....
Participant's signature


.....
Witness

06/07/2020
.....
Date

Rhodes University, Research Office, Ethics
Ethics Coordinator: ethics-committee@ru.ac.za
t: +27 (0) 46 603 7727 f: +27 (0) 86 616 7707
Room 220, Main Admin Building, Drostdy Road, Grahamstown, 6139

Appendix 13: Teacher 2 informed consent



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10. Any further questions that I might have concerning the research or my participation will be answered by Simon Chatima.
11. By signing this informed consent declaration I am not waiving any legal claims, rights or remedies.
12. A copy of this informed consent declaration will be given to me, and the original will be kept on record.
13. Request to take pictures, video and voice recording for this study has been made clear to me and I hereby give my voluntary consent to it.

I, [REDACTED] have read the above information / confirm that the above information has been explained to me in a language that I understand and I am aware of this document's contents. I have asked all questions that I wish to ask and these have been answered to my satisfaction. I fully understand what is expected of me during the research.

I have not been pressurised in any way and I voluntarily agree to participate in the above-mentioned project.

Participants signature

Witness

6/7/2020

Date

Rhodes University, Research Office, Ethics
Ethics Coordinator: ethics-committee@ru.ac.za
t: +27 (0) 46 603 7727 f: +27 (0) 86 616 7707
Room 220, Main Admin Building, Drostyd Road, Grahamstown, 6139

Appendix 14: Teacher 3 informed consent



10. Any further questions that I might have concerning the research or my participation will be answered by Simon Chatima.
11. By signing this informed consent declaration I am not waiving any legal claims, rights or remedies.
12. A copy of this informed consent declaration will be given to me, and the original will be kept on record.
13. Request to take pictures, video and voice recording for this study has been made clear to me and I hereby give my voluntary consent to it.

.....have read the above information / confirm that the above information has been explained to me in a language that I understand and I am aware of this document's contents. I have asked all questions that I wish to ask and these have been answered to my satisfaction. I fully understand what is expected of me during the research.

I have not been pressurised in any way and I voluntarily agree to participate in the above-mentioned project.

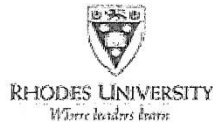
Participant's signature

Witness

09/09/2020
Date

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Ethics Coordinator: ethics-committee@ru.ac.za
t: +27 (0) 46 603 7727 f: +27 (0) 86 616 7707
Room 220, Main Admin Building, Drostdy Road, Grahamstown, 6139

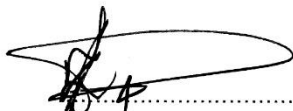
Appendix 15: Teacher 4 informed consent



10. Any further questions that I might have concerning the research or my participation will be answered by Simon Chatima.
11. By signing this informed consent declaration I am not waiving any legal claims, rights or remedies.
12. A copy of this informed consent declaration will be given to me, and the original will be kept on record.
13. Request to take pictures, video and voice recording for this study has been made clear to me and I hereby give my voluntary consent to it.

I, [REDACTED].....have read the above information / confirm that the above information has been explained to me in a language that I understand and I am aware of this document's contents. I have asked all questions that I wish to ask and these have been answered to my satisfaction. I fully understand what is expected of me during the research.

I have not been pressurised in any way and I voluntarily agree to participate in the above-mentioned project.


.....
Participant's signature

Witness


.....

08-09-2020
.....
Date

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Ethics Coordinator: ethics-committee@ru.ac.za
t: +27 (0) 46 603 7727 f: +27 (0) 86 616 7707
Room 220, Main Admin Building, Drostdy Road, Grahamstown, 6139

Appendix 16: Teacher 5 informed consent



10. Any further questions that I might have concerning the research or my participation will be answered by Simon Chatima.
11. By signing this informed consent declaration I am not waiving any legal claims, rights or remedies.
12. A copy of this informed consent declaration will be given to me, and the original will be kept on record.
13. Request to take pictures, video and voice recording for this study has been made clear to me and I hereby give my voluntary consent to it.

I, [REDACTED] have read the above information / confirm that the above information has been explained to me in a language that I understand and I am aware of this document's contents. I have asked all questions that I wish to ask and these have been answered to my satisfaction. I fully understand what is expected of me during the research.

I have not been pressurised in any way and I voluntarily agree to participate in the above-mentioned project.

Participant's signature

Witness

Date

Rhodes University, Research Office, Ethics
Ethics Coordinator: ethics-committee@ru.ac.za
t: +27 (0) 46 603 7727 f: +27 (0) 86 616 7707
Room 220, Main Admin Building, Drostdy Road, Grahamstown, 6139

Appendix 17: Negotiated contract



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I _____ unconditionally agree to participate in this study on voluntary basis, and will undertake to commit myself on purely conscientious terms for the entire duration of the study period. I will not in any way make an intentional attempt to sabotage the researcher's efforts to carry out an investigation that I believe will contribute to existing research efforts in mathematics education by pulling out unceremoniously and without genuine reasons. I therefore append my signature to this document having fully understood and openly negotiated with the researcher without any form of binding compulsion.

Signature of participant

_____ Date _____

Appendix 18: Survey Questionnaire



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1. Tell us, briefly, how you teach patterns and their generalisation in the grade(s) you are assigned to in the Senior Phase?

2. In your teaching of patterns and their generalisation, do you make use of any visual techniques and/or strategies that involve pictures, images, tables, diagrams etcetera? Please explain.

3. Assuming you use visuals in teaching patterns and their generalisation, explain how often and when do you use such.

4. Briefly explain what challenges, if any, do you encounter when teaching patterns and their generalisation using visually inclined approaches?

5. In a like manner as above, tell us what makes the teaching of patterns and their generalisation using visual approaches a likely favourable teaching approach (if it does) based on your individual experience?

Thank you for your time and effort

Appendix19: Interview schedule for study participants

INTERVIEW SCHEDULE A

Tentative questions for one-on-one interviews. The precise nature and detail of the questioning techniques will unfold depending on the content of the video recordings and reflections from the observed lessons.



QUESTIONS DIRECTED TO INDIVIDUAL PARTICIPANTS (For phase A)

1. Can you please shade light on the visual strategies you used to teach patterns and why you preferred such?
2. What would you say worked well with the strategies you used and what could have made it much better?
3. Tell me about the moments that really stood out for you and what that experience did to add real value to your teaching of patterns using visuals.
4. Now tell me about those moments that challenged you the most and express your views about such moments.
5. What is your perception about the perceived moments of breakthrough and the moments of drawback that emerged during the process of teaching patterns using visual approaches?

QUESTIONS DIRECTED TO INDIVIDUAL PARTICIPANTS (For phase B)

1. Would you say with confidence that the generalisation strategies you used worked for the best interest of your planned lessons? Please cite memorable moments in any one of the lessons you taught.
2. What moments, during your teaching of patterns and their generalisation, were posing challenges? Please give examples of any such generalisation technique that posed challenges and refer to such instances in the video recordings.
3. Tell me about what particular generalisation strategies, if any, made it possible for your lessons to be more exciting and smooth flowing. Please refer me to such instances in the video recordings.
4. In your own words tell me about your views concerning the experience you have had teaching patterns and their generalisation using visualisation strategies gained during the intervention process.

INTERVIEW SCHEDULE B



Tentative questions for focus group interview process. The precise nature and detail of the questioning techniques will unfold depending on the content of the video recordings and the reflections from the lesson observations.

QUESTIONS DIRECTED TO THE FOCUS GROUP

1. What common views, if any, do you share about the way you taught patterns and their generalisations before and after the intervention programme?
2. Can you identify the challenges and impediments posed by visualisation in teaching patterns and their generalisations from a common perspective with special reference to individual experiences from the video recordings?
3. In the same light, point out the breakthroughs and necessitating factors that visualisation made possible the teaching of patterns and their generalisation from your collective views based on what transpired during individual teaching sessions. Please make special reference to the video recordings.
4. What then can you say are your collective summary views about the use of visualisation in teaching patterns and their generalisations based on your experiences in this investigation?

APPENDIX 20a: Observation schedule A

OBSERVATION SCHEDULE A: Analytic tool for use in observing the extent to which visualisation is used in teaching patterns and their generalisations after phase A of the intervention process.



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Name of Teacher:		
Topic taught:		Grade:
		Date:
Visual strategy	Description of the strategy	Comments on the use of visual strategies by the teacher
Counting(near generalisation)	Counting with shapes, tables and manipulatives.	
Whole-object analysis (distant generalisation)	Considering a term of the sequence as a unit and using multiples of that unit with adjustments based on the context of the problem.	
Recursive analysis (near generalisation)	Extending the sequence using a common difference using features of the visual figure to build on previous terms.	
Differencing (constructive generalisation)	Using a common difference as a multiplying factor and	

	proceeding to an adjustment of the result.
Explicit analysis (deconstructive generalisation)	Discovering a rule, based on context that allows the calculation of any output value given the corresponding input value.

Visual strategy	Code	Description of the strategy
Counting (near generalisation)	CNG	Counting with shapes, tables and manipulatives.
Whole-object analysis (Distant Generalisation)	WOA	Considering a term of the sequence as a unit and using multiples of that unit with adjustments based on the context of the problem.
Recursive analysis (Near eneralisation)	RAG	Extending the sequence using a common difference using features of the visual figure to build on previous terms.
Differencing (constructive generalisation)	DCG	Using a common difference as a multiplying factor and proceeding to an adjustment of the result.
Explicit analysis (deconstructive generalisation)	EAG	Discovering a rule, based on context that allows the calculation of any output value given the corresponding input value.

Adapted from: Tavares & Cabrita, 2014

APPENDIX 20B: Observation Schedule B.

OBSERVATION SCHEDULE B: Analytic tool for use in observing generalisation techniques used in teaching patterns after phase B of the intervention process

Name of teacher:		
Topic taught:		Grade:
		Date:
Type of generalization technique used	Description of the technique used	Comments on observed use of generalisation techniques by teacher
Whole-object	Carried out with adjustments made on numeric properties and on the context of the problem.	
Recursive	Extension of sequence using common difference was built on numeric relations and on features of the figure.	
Difference rate	Use of common difference as a multiplying factor was done with the teacher proceeding to a final adjustment.	
Explicit	A numerical rule that allows the calculation of any output value given a corresponding input value was done within the context of the problem.	

--	--	--

Type of generalisation technique	Code	Description of technique
Whole-object	WOT	Teacher includes adjustments to the numeric properties and context of the visual figure used.
Recursive	RCT	Teacher extends sequence using a common difference that is built on the numeric relations and on the features of the visual figure.
Difference rate	DRT	Teacher uses a common difference as a multiplying factor and proceeds to an adjustment.
Explicit	EXT	Teacher uses a numerical rule that allows the calculation of any output value when given a corresponding input value within the context of the problem.

Adapted from: Vale & Barbosa, 2015

APPENDIX 20C: Observation Schedule C

OBSERVATION SCHEDULE C: Analytical tool to explore what factors enable or constrain the teaching of patterns when a visual approach is used. (Adapted from Rahim, 2009: Whitley, 2004: Gooding, 2009).



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Name of the teacher:		
Topic taught:		Grade:
		Date:
General category	Description	Comments on observed factors that enable or constrain teaching of patterns
Applicability	-allows constant interaction with a variety of visual tools to promote understanding. -visual figures encourage development of problem solving techniques. - teaching becomes incompatible with the traditional curriculum -teacher's involvement violates the constructivists' teacher/learner contract.	

Flexibility	-visuals can be used flexibly in a variety of pattern problems to develop generalisation skills -visual teaching opens room for learner exploration with deeper reasoning. -classroom culture limits autonomy and creativity for learners. -group dynamics compromise progress as learner may fail to display personal visual perception.	
.Usability	-working with visuals has the potential to develop cognitive perception. -visual teaching allows innovation and encourages originality. -learners' limited knowledge of algebra may hinder the usability of visual tools. -tasks may not be completed within the time limit allocated to do them.	

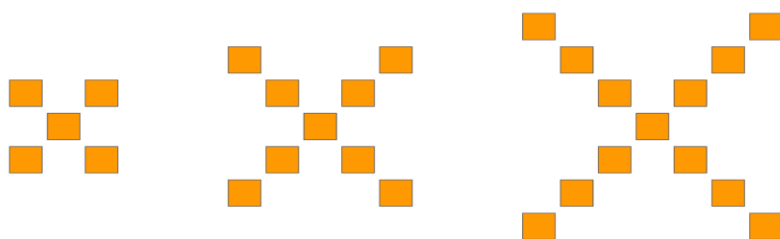
General indicators of whether visualisation enables or constraints the teaching of patterns	Code	Description of indicator
Applicability	APP	Teaching allows constant interaction with a variety of visual tools to promote understanding and encourages problem solving techniques to develop. Teaching is also compatible with traditional curriculum such that the teacher's involvement does not violate teacher/learner contract.
Flexibility	FLX	The visuals used can be utilised in a variety of pattern problems and opens room for learner exploration. Teaching does not limit autonomy, classroom culture or compromise progress in the development of personal visual perception.
Usability	USB	During the teaching process the visuals are accorded the potential to develop cognitive perception that allows innovation and encourages originality. The use of visual tools is not limited by the learners' knowledge of algebra so as to prevent tasks completion within the time allocated to do them.

Appendix 21a: Lesson 1 for Grade 7 teachers

TEACHING PLAN FOR TEACHING PATTERNS USING VISUALISATION

GRADE 7 TEACHING PLAN: LESSON ONE (1)

INTRODUCTION



- Teacher asks learners to count the number of squares at each stage of the pictorial pattern above.
- Teacher prompts learners to first predict then draw the fourth and fifth stages of the pictorial pattern.

LESSON DEVELOPMENT

- Learners write down the numerical values of each stage of the pattern in a sequence.
- Teacher asks learners to explain how each stage of the pictorial pattern is reached.
- Learners discuss the different ways of finding the number of squares at each subsequent stage of the pattern.
- Teacher asks learners to predict the number of small squares at the 10th and 15th stages of the pattern.
- Teacher assist learners to come up with a general rule for the pattern.
- Learners write an exercise from a similar visual tool.

CONCLUSION/REFLECTION

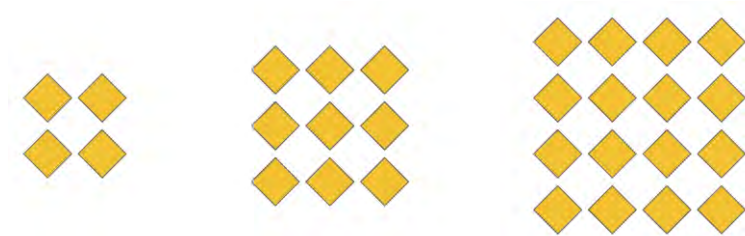
Encouraging learners to come up with similar pictorial patterns, at home, and predicting their fourth and fifth stages. Class discussion on what went right and what went not so right during the lesson and suggest what could have been done to improve the situation.

Appendix 21b: Lesson 2 for Grade 7 teachers

TEACHING PATTERN GENERALISATION: GRADE 7 TEACHING PLAN

LESSON 2

INTRODUCTION



- ❖ Teacher introduces the difference rate visual strategy of generalising a pictorial pattern using the above visual tool.
- ❖ Teacher explains how he/she reached the algebraic rule $4n$

LESSON DEVELOPMENT

- Teacher asks learners to draw the fourth stage of the pictorial pattern.
- Learners use the general rule to calculate the number of small rhombuses in the 10th and 20th stages.
- Teacher introduces two or more similar visual tools and asks learners to derive their respective general rules.
- Learners solve other real life problems involving patterns that need difference rate strategy.
- Learners write a short exercise involving extending, predicting and formulating a general rule.

CONCLUSION/REFLECTION

Discussing simpler ways of formulating algebraic rules for pictorial patterns the context of the given picture.

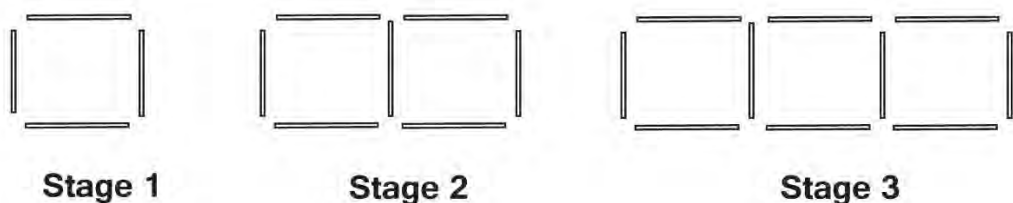
Appendix 22a: Lesson 1 for Grade 8 teachers

TEACHING PLAN FOR TEACHING PATTERN USING PICTURES

Grade 8 LESSON PLAN: LESSON 1

INTRODUCTION

- Teacher asks learners to describe what they see in the pictorial pattern below.



LESSON DEVELOPMENT

- Teacher instructs Learners to extend the pictorial pattern to the fourth and fifth stages.
- Teacher prompts learners to predict the number of sticks that would make the 10th and 20th stages of the pictorial pattern.
- Teacher asks learners to give a narrative description of the rule governing the pictorial pattern
- Teacher helps learners to deduce an algebraic rule for the pictorial pattern by presenting their narratives in symbolic form e.g. $[3n+1]$
- Teacher gives a written exercise which involves two or three visual tools representing pictorial patterns that require counting analysis.
- Teacher initiate group discussions to find out what challenges, if any, learners experienced when patterns were taught from a visual approach.

CONCLUSION/REFLECTIONS

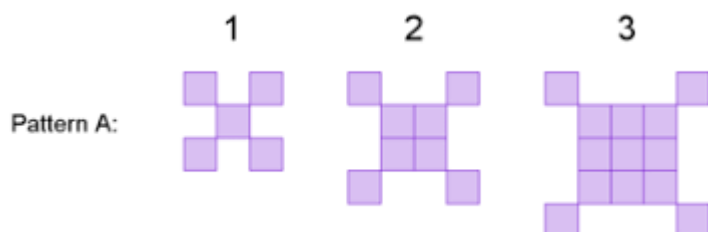
Class discussion on what learners think about using pictures to learn patterns and the formulation of a general rule.

Appendix 22b: Lesson 2 for Grade 8 teachers

PATTERN GENERALISATION: LESSON PLAN FOR GRADE 8

INTRODUCTION

Teacher introduces whole- object analysis for generalising the pictorial pattern below.



- Teacher asks learners to explain verbally what they see happening at each stage of the pictorial pattern.
- Learners break down the elements of the pattern into distinguishable parts.
- Learners count the number of small squares at each stage and make up a numerical sequence out of it.

LESSON DEVELOPMENT

- Teacher prompts learners to discover the trend of the central component of the pictorial pattern.
- Learners deduce the constant value at each stage of the pattern.
- Teacher assist learners to formulate an algebraic expression that describes the pattern at each stage of the sequence $[n^2+4]$.
- Learners use the deduced rule to predict the number of small squares in stages 5 and 8.
- Teacher uses a similar visual tool to create a written activity questions for the learners.

CONCLUSION

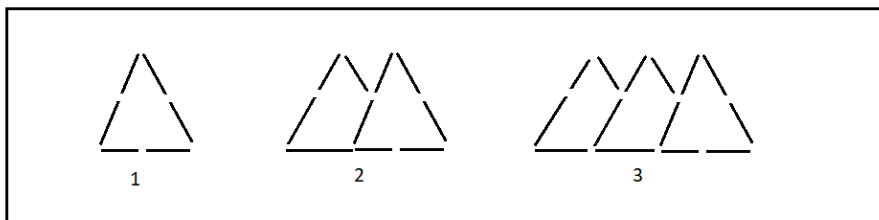
Encouraging learners to create pictorial patterns that require whole-object analysis to generalise as a home work activity

Appendix 23a: Lesson 1 for Grade 9 teachers

TEACHING PROGRAMME FOR TEACHING PICTORIAL PATTERNS

GRADE 9 LESSON PLAN

INTRODUCTION



Teacher asks learners to describe the various pattern formations they can see from the different elements making up the pictorial pattern above. For example the sticks at the base of the triangular pattern starts from 2 and increase by 1 at each stage.

LESSON DEVELOPMENT

- ❖ Teacher asks learners to count the number of sticks at each stage and make a numerical sequence of the count.
- ❖ Teacher prompts learners to predict how many sticks will there be in the 20th and 50th stages of the pictorial pattern.
- ❖ Teacher and learners collectively derive a narrative rule for the pictorial pattern.
- ❖ Teacher demonstrates how to derive an algebraic rule that suits every stage of the pictorial pattern **$[4n+2]$ OR $3n+(n+2)$**
- ❖ Administering a written activity which include at least two visual tools that require counting and/or recursive analysis.

REFLECTION

Whole class discussion on how the different elements of the pictorial pattern contributed to the formulation of the general rule. Teacher initiates discussion on what breakthroughs and what drawbacks were experienced by learners during the lesson.

Appendix 23b: Lesson 2 for Grade 9 teachers

PATTERN GENERALISATION FOR GRADE 9 MATHEMATICS: TEACHING PLAN

INTRODUCTION

Introducing explicit analysis to generalise a pattern from a pictorial format.

Diagram A1

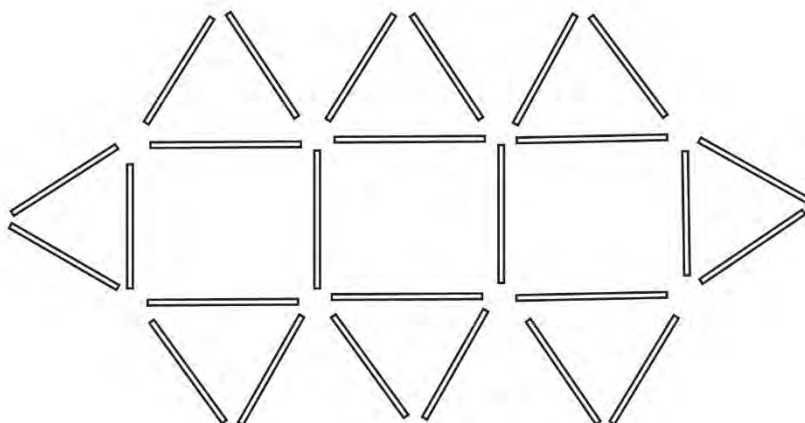
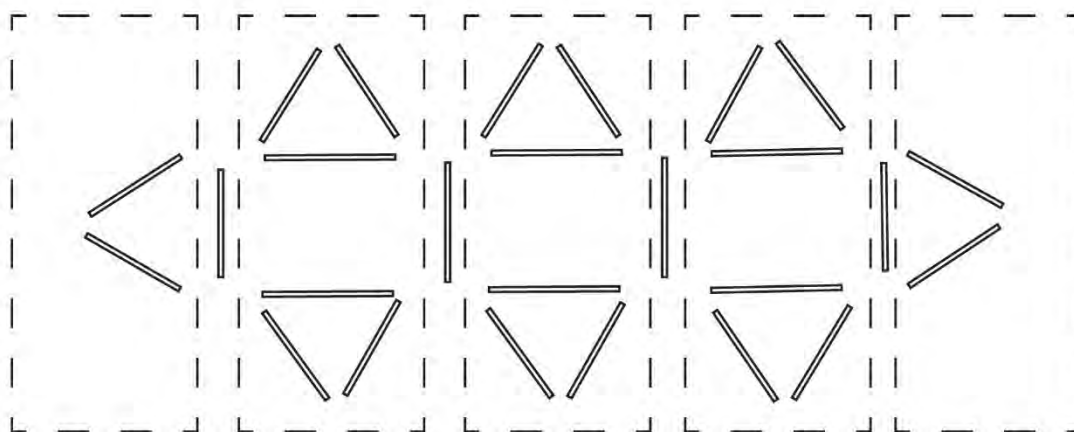


Diagram A2



- Teacher explains how the pictorial pattern in diagram A1 was broken down to its components as shown in diagram A2.
- Teacher explains the derivation of the algebraic rules (i) and (ii) for the pictorial pattern. (i) $6n + (n+1) + 4$ (ii) $[3(n-1) + 4] + [2(2n+2)]$ from the break down process.

LESSON DEVELOPMENT

- Learners describe the different elements making up the pictorial pattern before and after breakdown.
- Teacher describes how each part of the algebraic rule relate to the pictorial pattern.
- Learners draw stages 1 and 2 of the pictorial pattern.
- Learners use either rule (i) or rule (ii) to predict the number of sticks in stages 5 and 6.
- Teacher gives an activity that involves a similar pictorial tool and assist learners to come up with a befitting general rule.

CONCLUSION/REFLECTION

Discussing possible challenges and possible benefits of using explicit analysis to generalise a pictorial pattern. Encouraging learners to work with different pictorial tools at home and proceed to adjust the formulated rule in the context of the given picture.



Certificate of editing

9 July 2021

The thesis "**Visualisation processes in teaching patterns and their generalisation: perceptions and experiences from senior phase mathematics teachers**" by Mr Simon Chatima has been edited by Jean Schäfer.

Editing has consisted of the following:

- Checking house style and consistency with academic specifications
- Proofreading the document
- Checking for typos
- Checking grammar, punctuation and spelling
- Correcting errors accordingly
- Formatting for ease of reading
- Making comments and recommendations for the student to attend to
- Ensuring consistency in the use of phrases, capitalisation and terminology
- Reminding the student about referencing

Jean Schäfer

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Jean Schäfer
Associate Member

Membership number: SCH011
Membership year: March 2021 to February 2022

084 580 7424
jeanschafer@gmail.com
www.jeanschafer-editor.com

www.editor.org.za

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