AN ANALYSIS OF HOW THE USE OF *GEOBOARDS* AS VISUALISATION TOOLS CAN BE UTILISED IN THE TEACHING OF QUADRILATERALS

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ABSTRACT

The relationship between visualisation processes and using manipulatives in the teaching and learning of mathematics is apparent and yet not so vocal in the literature. This could be because of the common mistaken understanding that because manipulatives are visual in nature, then visualisation processes should be obvious. Literature warns that just because something is visual therefore it is transparent, is incorrect. This study argues that the effective use of manipulatives in the teaching of mathematics helps learners to effectively understand mathematical concepts. Research on the teaching and learning of mathematics suggests that physical manipulation experiences, especially of concrete materials concerning shapes, is an important process in learning at all ages. One such teaching tool, the *Geoboard*, a physical manipulative that employs visualisation processes when correctly used, is explored in this study.

The aim of this interpretive case study was to investigate and analyse the use of *Geoboards* as a visualisation tool in the teaching of the properties of quadrilaterals. The study focused on visualisation processes and the use of *Geoboards* through a teaching framework that was informed by the Van Hiele phases of teaching geometry. The study was conducted in the Opuwo circuit of the Kunene region, Namibia, and it involved three selected Grade 7 mathematics teachers, each from a different primary school. It was underpinned by a constructivist theory using the Van Hiele phases of teaching geometry and framed within visualisation processes. The study employed the use of qualitative data collection techniques such as observations and interviews.

The analysis of the findings of this study revealed that *Geoboards* were very useful in demonstrating the visual representations of the properties of quadrilaterals in a cheap and yet novel way in the selected teachers' classes. Moreover, the use of *Geoboards* by the selected teachers effectively fostered visualisation processes such as *concrete pictorial imagery*, *dynamic imagery*, *perceptual apprehension*, *sequential apprehension*, *discursive apprehension* and *operative apprehension*. It was also revealed that *Geoboards* enabled the selected teachers to structure and teach their lessons in a well-planned manner according to the Van Hiele phases, although it was difficult for them to adhere strictly to the hierarchy of the phases.

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"I can do all things through Christ which strengtheneth me" Philippians 4:13

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DEDICATION

This thesis is dedicated to the living memory of my late mom and dad

Manga Esnath Matengu Lweendo

and

Gideon Matengu Daimoni Simonda,

who sadly have not lived to see this successful academic accomplishment of their last-born child.

DECLARATION OF ORIGINALITY

I, **Given Kahale Matengu**, student number **15M8765**, hereby declare that this thesis entitled "An analysis of how the use of *Geoboards* as visualisation tools can be utilised in the teaching of quadrilaterals" is my own work, and a product of my research. It has not been submitted in any form to another institution. Where I have drawn on ideas of people from other publications or other sources, I have fully acknowledged these in accordance with Rhodes University, Education Department reference guide.

Given Matengu 29 November 2018

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LIST OF ACRONYMS

ACE	Advanced Certificate in Education
B Ed	Bachelor of Education
BETD	Basic Education Teachers Diploma
DA	Discursive apprehension
DI	Dynamic imagery
FGI	Focus Group Interviews
JFGI	Jones Focus Group Interview
JSRI	Jones Stimulus Recall Interview
JV	Jones Video (1,2,3)
LCE	Learner Centered Education
MBEC	Ministry of Basic Education and Culture
MEC	Ministry of Education and Culture
MESC	Ministry of Education Sports and Culture
MoE	Ministry of Education
NIED	National Institute for Educational Development
OA	Operative apprehension
PA	Perceptual apprehension
PI	Concrete pictorial imagery
RFGI	Ruth Focus Group Interview
RSRI	Ruth Stimulus Recall Interview
RV	Ruth Video (1,2,3)
SA	Sequential apprehension
SACMEQ	Southern and Eastern Africa Consortium for Monitoring Educational Quality
SATs	Standardised Achievement Tests
SFGI	Smith Focus Group Interview
SRI	Stimulus Recall Interviews
SSRI	Smith Stimulus Recall Interview
SV	Smith Video (1,2,3)

CHAPTER 1

CONTEXT OF THE STUDY

"Tell me and I'll forget;
show me and I may remember;
involve me and I'll understand."

A Chinese proverb

1.1 INTRODUCTION

This is the opening chapter of my research report. In this chapter I present the motivation and explain the background of the problem statement that gave rise to this study. I start by explaining the problem I am interrogating, then review the research goals and questions, outline the research methodology and then point out the significance of the study. Lastly, this chapter provides an overview of the structure of the thesis.

1.2 CONTEXT AND BACKGROUND

Although the Namibian national curriculum for basic education compels mathematics teachers to be "creative and innovative to produce their own teaching and learning materials linked to practice" (Namibia. Ministry of Education, Sports and Culture, [MESC], 2010, p. 6), most mathematics teachers still find it difficult to create and make use of teaching aids and materials. Miranda and Adler (2010) observe that "Namibia is one of the many African countries, in which the use of manipulatives in mathematics classrooms is not a common practice" (p. 17). In my opinion, one possible reason for this phenomenon could be that teachers do not know what types of manipulatives to use and when to use them.

This is the same prevalent situation in the Opuwo circuit of the Kunene region, in the north-west of Namibia. I have observed with great concern that many mathematics classrooms are not activity based. There is limited, or no use made of teaching materials such as manipulatives in most of the mathematics classrooms I have visited. I argue that perhaps this is one of the reasons why learners do not do well in abstract topics such as geometry and algebra. I further argue that one of the mathematics domains that could benefit from using manipulatives is the teaching and learning of basic shapes such as quadrilaterals in geometry.

As a mathematics teacher for the past thirteen years, I constantly experience firsthand that Grade 7 learners lack the conceptual understanding of basic mathematical shapes. Particularly, learners struggle to understand and articulate the properties of quadrilaterals such as those of squares, rectangles, parallelograms, rhombuses, trapeziums and kites. The cause of this problem may well be attributed in part to teaching methods and approaches that do not foreground conceptual understanding and the limited use of concrete manipulatives (such as the *Geoboard*) that can be used to teach geometrical shapes and their properties in a dynamic and exciting way.

It has been shown that the correct and appropriate use of physical manipulatives in mathematics classrooms is a practice that is effective in the teaching and learning of mathematics. Literature shows that teachers who make use of teaching manipulatives, help learners to understand difficult and abstract concepts (Sarama & Clements, 2016). In this study, I have identified the *Geoboard* as a manipulative that can be used by Grade 7 teachers to great effect to teach the properties of quadrilaterals to learners. Kennedy and MacDowell (1998) assert that much of what we want learners to learn about quadrilaterals can be accomplished with geoboard figures.

Geoboards are concrete and physical manipulatives that are designed to teach and learn geometrical shapes. They are made from local materials such as wood and nails, and they allow the construction and formation of basic shapes by placing rubber bands around the nails (Scandrett, 2008). The process of allowing learners to construct and create their own representations of shapes on *Geoboards* is identified as an effective way to help them develop visualisation and spatial reasoning skills (Moyer-Packenham, Salkind, Bolyard & Suh, 2001).

The mathematics education research literature is resounding in its findings that visualisation is key to the development of abstract mathematical concepts (Makina, 2010; Kosslyn, 1994; Presmeg, 1992; Arcavi, 2003; Tall, 2008; Duval 1999). There is a positive connection between the use of physical manipulatives and visualisation processes. Carbonneau, Marley and Selig (2013) found that "the ability for children to capitalise from visual representations should precede symbolic representations" (p. 381). Symbolic representations that are generated from a manipulative tool such as a *Geoboard* can help learners to visualise and develop visual imagery of the shapes and their properties because they can see and relate the concrete and visual representation of shapes to the abstract. According to Zazkis, Dubinsky, and Dautermann (1996), visualisation is an act in which an in individual establishes a strong connection between an internal construct and something to which access is gained through the senses. In this case learners can establish connections between the visual imagery of quadrilaterals and the concrete representations of the same shapes on *Geoboards*.

Moyer-Packenham et al. (2013) add that "however, the way in which manipulatives are used is subject to the interpretations of teachers and students in the environments in which they teach and learn" (p. 19). It would therefore be worthwhile to investigate and analyse a learning environment that can allow teachers to correctly and effectively use *Geoboards* as visualisation tools to teach quadrilaterals to Grade 7 learners. A research study of this kind is needed to help address the critical issue concerning the way teachers can create an environment where learners learn the properties of quadrilaterals in a meaningful way.

Therefore, this study used an intervention programme approach to create an environment that enabled teachers to practice using *Geoboards* in their classes. The intervention programme was aimed at incorporating the use of *Geoboards* with visualisation processes and the Van Hiele model of teaching geometry. The Van Hiele phases were used as a framework to guide the selected teachers in making use of *Geoboards* to employ visualisation processes in their lessons. The intervention programme was designed in such a way that it allowed the creation of a learning environment that was filled with stimuli which encouraged rich construction of quadrilaterals, by learners. The Van Hiele model integrated constructivism and visualisation processes to encourage the involvement of learners in the lessons in order to advance their conceptual understanding of quadrilaterals.

1.3 RESEARCH GOALS AND QUESTIONS

This study was aimed at investigating and analysing the use of *Geoboards* as visualisation tools to teach the properties of quadrilaterals to Grade 7 learners. The goals of the study were threefold, aiming to:

- find out the affordances of the utilisation of *Geoboards* as visualisation tools in developing and enhancing conceptual understanding of the properties of quadrilaterals;
- explore and gain insight into teachers' perceptions and experiences of using *Geoboards* as visualisation tools after the implementation of an intervention programme;
- establish the extent to which the Van Hiele phases of teaching geometry can be applied to teaching quadrilaterals with *Geoboards.*

In pursuing these goals, the following research questions framed this study:

- What are the affordances of the utilisation of *Geoboards* as visualisation tools in the teaching of the properties of quadrilaterals in Grade 7 classes?
- What are selected teachers' experiences of using *Geoboards* as visualisation tools in teaching the properties of quadrilaterals, as a result of participating in an intervention programme?
- How do the participating teachers make use of the Van Hiele phases in their teaching of quadrilaterals using the *Geoboard*?

1.4 RESEARCH METHODOLOGY

This research was undergirded by the interpretive paradigm and employed a qualitative approach to data collection and interpretation. The research used a case study method. The case involved three purposively selected Grade 7 mathematics teachers in the Opuwo circuit of the Kunene region in Namibia. The teachers were identified in this study by their pseudonyms, Mr Jones, Ms Ruth and Ms Smith. These three teachers participated in a three-month intervention program on how to use *Geoboard*s in the teaching of the properties of quadrilaterals. The case study unfolded in five phases which were: securing of research site and selection of participants; designing *Geoboards*, awareness workshops and planning of lessons; piloting of the learning programme and research instruments; implementation and video-recording; and interviews and analysis (videos and interviews). During the intervention programme teachers taught three lessons each and data was collected through lesson observations and interviews. The collected data was qualitatively analysed.

This study is positioned within the constructivist theoretical framework that holds that teaching and learning is a constructive process where learners build an internal representation of knowledge through a personal interpretation of experience (Gupta, 2011). It is a theory of learning that states that individuals construct their own new understandings on the basis of their interactions between what they already know and believe against the ideas and knowledge that they come into contact with (Piaget, 1967). The constructivist approach to teaching with *Geoboards* and the use of visualisation processes in this study were supported by using the Van Hiele phases of teaching geometry. At the core of the study was an intervention programme that enabled the participating teachers to interact and experience the use of *Geoboards*.

1.5 SIGNIFICANCE OF THE STUDY

It is hoped that the findings of this study will inspire Namibian primary mathematics teachers to further strengthen the use of teaching manipulatives such as *Geoboards* to meet the educational needs of primary mathematics. It is also anticipated that educational planners, researchers as well as teachers will find the findings of this study useful and applicable beyond this particular case. I would like to think that this work can contribute towards improving the mathematics curriculum in Namibia, particularly with regard to emphasising the important role of visualisation processes in general and the use of manipulatives in particular.

1.6 OVERVIEW OF THE SUBSEQUENT CHAPTERS

Here is an overview of the chapters that follow in this study:

Chapter 2

This chapter discusses the main concepts used in this study in relation to the research questions. It reviews the relevant and current literature of visualisation processes, manipulatives, *Geoboards*, geometry and quadrilaterals. The chapter also discusses past and current positions of the Namibian curriculum on the use of manipulatives and the teaching of geometry. The constructivist theoretical framework through the Van Hiele phases of teaching geometry is discussed as well.

Chapter 3

This chapter gives a detailed account of the research methodology used in this study. Special reference to the research orientation and research method is made i.e. the interpretive paradigm, case study method, the qualitative approach and how data was collected and generated. The research site, selection of the participants and a short description of the data analysis are presented. The chapter closes by discussing the ethical considerations, validity, limitations and challenges of the study.

Chapter 4

Chapter 4 presents data and discusses the findings of the study. Firstly, the chapter presents data in themes that were generated from the two analytical frameworks found in chapter 3 of the study, as well as themes that emerged out of the teachers' experiences of using *Geoboards*. Secondly, the chapter discusses the findings of the study in line with the reviewed literature on visualisation processes and the Van Hiele phases of teaching geometry.

Chapter 5

This is the concluding chapter of this study. It provides a summary of the main findings of the study, presents the significance of the study and submits several practical recommendations arising from the findings. It also discusses the limitations encountered during the research processes and makes suggestions for further research. The chapter ends the thesis by sharing my personal reflections and experiences as a researcher on this journey of research.

1.7 CONCLUSION

In this chapter I discussed an overview of the study by presenting the problem statement of the research. I also briefly reviewed the research goals and questions, the methodology and the significance of the study. I also outlined an overview of the chapters that follow from this chapter.

The following chapter discusses the review of literature of this study.

CHAPTER 2

LITERATURE REVIEW

2.1 INTRODUCTION

The focus of this literature review is to discuss the use of *Geoboards* as visualisation manipulatives in the teaching of quadrilaterals. While I argue for the promotion of the use of manipulatives in mathematics classrooms, I am equally mindful that there are challenges associated with the use of manipulatives. Hence the reason why, in this chapter, I discuss literature that highlights these challenges as well as how these challenges can be overcome by incorporating visualisation processes and the Van Hiele model of teaching geometry when using manipulatives. In this chapter I present literature for and against the use of manipulatives. My argument for using manipulatives in the teaching of quadrilaterals is supported using visualisation processes and the Van Hiele model of teaching geometry.

To carry out my study on how visualisation processes and the Van Hiele model of learning and teaching geometry can be incorporated on *Geoboards*, I provide an overview of aspects that guide the study, hence this section presents and outlines the following such aspects. Firstly, I discuss the importance and role of teaching with manipulatives, the nature of manipulatives and the use of *Geoboards* as manipulative tools. Secondly, I outline past and current positions of the Namibian curriculum on teaching geometry and the use of manipulatives within the same context. Thirdly, I discuss literature on visualisation in the teaching and learning of mathematics education in general, as well as how visualisation processes can be incorporated on *Geoboards* in particular. Fourthly and lastly, I also discuss the literature on the theoretical framework of my study, which is the constructivist theory of learning. I discuss this learning theory and the use of Van Hiele model of teaching geometrical shapes such as quadrilaterals.

2.2 THE NATURE OF MANIPULATIVES

Manipulatives are handled by learners and teachers to foster conscious and unconscious mathematical thinking. Bartolini and Martignone (2014) describe manipulatives as concrete materials or objects that are used by learners and teachers to explore, acquire, investigate or demonstrate mathematical concepts; or to support the implementation of mathematical procedures in order to perform problem-solving activities drawing on perceptual evidence. Manipulatives have the potential to lead learners to a mathematical awareness and development of concepts. By their very nature they are intended "to represent explicitly and concretely mathematical ideas that are abstract" (Moyer, 2001, p. 176). From my review of the literature on manipulatives, two classifications of manipulatives emerge: (a) physical manipulatives and (b) virtual manipulatives.

(a) Physical manipulatives

Physical manipulatives are concrete objects that can be touched and felt by learners. Their physical and visual aspect allow a more meaningful and an understanding of abstract ideas and symbols (Bartolini & Martignone, 2014). For example, *Geoboards* are physical and thus concrete manipulatives because they can be manipulated by learners through hands-on activities (Moyer, 2001). Thus, the concrete experiences developed with *Geoboards* enable learners to relate abstract ideas to the real world and this is key to early understanding of mathematical concepts.

(b) Virtual manipulatives

On the other hand, virtual manipulatives are not concrete in a physical sense, but they are digital artifacts that can be displayed on computer screens to resemble and simulate physical objects. They are interactive dynamic objects that can be placed on a website (Lee & Chen, 2015). An example of a virtual manipulative is the software such as *GeoGebra*. *GeoGebra* is short for Geometry and Algebra, and according to Mwiikeni (2016) the dynamic nature of *GeoGebra* presents opportunities for learning and teaching mathematics in schools. Virtual manipulatives are very similar to physical manipulatives. They enable as much engagement as physical manipulatives do since they are actual models of physical manipulatives. They can both expand learners' knowledge of key mathematical concepts in similar ways. Each one simply entails a different approach to teaching mathematics; both can convey the same mathematical concepts and are beneficial for diverse learners.

2.3 THE TEACHING OF MATHEMATICS WITH MANIPULATIVES

2.3.1 The benefits of using manipulatives in mathematics classrooms

Literature on the use and importance of manipulatives in mathematics is abundant, and it is mostly positive and supportive. The literature states that learners who use manipulatives in their mathematics classes usually outperform those who do not (Thompson, 2012; Guarino, Dieterle, Bargagliotti, & Mason, 2013). The uses of manipulatives help learners to draw their learning from concrete materials which help their limited mental maturity to grasp abstract mathematical concepts (Piaget, 1952). According to Akkan (2012) the use of manipulatives helps learners to reason mathematically by aiding them to construct meaningful understanding of abstract concepts. Swan and Marshall (2010) also state that manipulatives are visual aids that assist concrete visualisation.

Furthermore, manipulatives can help learners to cope with the stress linked to the learning of mathematics. According to Baloğlu and Koçak (2005) "anxiety has been found to be one of the most prevalent emotional problems associated with mathematics" (p. 1325). It is stated by Hunt, Nipper and Nash (2011) that using manipulatives eases learners' anxiety of learning mathematics. Manipulatives are likely to evoke and heighten learners' interest in the learning of mathematics in a positive manner by creating a learning environment that is fun and enjoyable. In turn, such an environment may help learners to relax and overcome any stress that might have arisen from learning mathematics as a subject. Bujak, Radu, Catrambone, MacIntyre, Zheng, and Golubski (2013) claim that "physical manipulatives are emotionally effective [in overcoming anxiety] because their toy-like forms are familiar and easy to use by children" (p. 542).

At a young age, as they use and interact with physical objectives, learners prepare for later learning at an abstract level. According to Carbonneau et al. (2013), the cognitive benefits of using manipulatives in mathematics are more apparent in younger children who are still developing proficiency with higher representations. Developmental theorists such as Bruner (1964), Montessori (1964) and Piaget (1962) asserted a long time ago that the use of manipulatives to explore mathematical concepts at a young age, are of cognitive benefit to them.

However, I argue that manipulatives should not just be limited to children but should also be used when learners (regardless of age) struggle with abstract reasoning because concepts are difficult to understand. According to D'Angelo and Iliev (2012) the use of manipulatives may help to clarify these difficult concepts to learners. Golafshani (2013) reinforces the idea of using manipulatives to clarify difficult concepts by stating that the uses of manipulatives have some direct effect on struggling learners. This means that if there are concepts learners struggle to understand and comprehend, manipulatives can be effective artifacts to use in scaffolding the learning for learners to understand and comprehend the concepts.

In my experience I have noticed that many Grade 7 learners find it difficult to identify and understand the properties of quadrilaterals. I thus see a need for using concrete manipulatives such as *Geoboards* to help these learners grasp abstract concepts and reinforce them. Therefore, depending on learners' identified learning needs and shortcomings, regardless of their ages, teachers should use appropriate manipulative objects to bring mathematics to life and to make the invisible mathematics concepts visible (Golafshani, 2013). The important part is to make sure that the manipulative object and tasks given to learners are appropriate for the grade level being taught (D'Angelo & Iliev, 2012).

2.3.2 Concerns about using manipulatives

In contrast, there are a few studies that expose some critical but constructive concerns about the use of manipulatives. Laski, Jor'dan, Daoust, and Murray (2015) for example, state that research that examines the advantages of teaching with manipulatives is inconsistent. They claim that some studies found that manipulatives promote learning, whereas others state that they impede learning. Some studies on the learning of geometry reported that learners who used manipulatives did not show improved achievement (Palardy & Rumberger, 2008). The findings suggest that perhaps the teaching did not provide enough connections between learners' representations based on manipulatives and those done by paper and pencil (Sherman & Bisanz, 2009).

Sarama and Clements (2016) caution that although manipulatives can play an important role in the teaching of mathematics, they nevertheless need to be used carefully to create a strong understanding and justification for each step of a procedure. Although they support teaching and can provide mediation of learning, "manipulatives [on their own] do not carry mathematical ideas directly to the learner" (Sarama & Clements, 2016, p. 73). Their nature of being physical does not automatically carry any meaning of the mathematical ideas. Thus, their physicality is not what is important, but what is important is how they can be manipulated by learners and bring about a meaningful understanding to their learning.

It is also important to keep in mind that even though manipulatives are physical in nature, understanding how they represent concepts still requires cognition. MacDonald, Davies, Dockett and Perry (2012) emphasised that the use of manipulatives does not guarantee meaningful learning. Without clear and explicit teaching, learners may not use manipulatives in a way that appropriately represents the mathematical concept being taught to them (Carbonneau et al., 2013). That means even when it comes to *Geoboards,* success of using these manipulatives occurs only when they are used as symbols as opposed to literal representations of what they are (Kosko & Wilkins, 2010). In other words, a manipulative is still just a physical representation of a concept, not the concept itself.

Literature identifies challenges that are associated with the use of manipulatives. A case in point is the study of Golafshani (2013) about teachers' beliefs when using manipulatives in the teaching of mathematics: some of the participating teachers indicated that one of the reasons why they did not use manipulatives was because it takes too much time to learn how to use them and that they take too much time to introduce concepts during the lessons. This suggests that some teachers see manipulatives as time wasting or less important than the "serious" work of learning mathematics. As a result, manipulatives in these situations are not likely used, nor are learners encouraged to make use of them.

Another challenge that is associated with the use of manipulatives has to do with the failure of teachers to match the manipulatives with the right level of instructional guidance as well as the age of learners. According to Carbonneau et al. (2013), at times teachers struggle to choose the right and appropriate manipulatives that correlate with the learners' age or level of understanding. This shortfall negatively impacts on the effectiveness of the manipulatives being used.

Arranging the classroom as a learning environment that allows manipulatives to be effectively used seems to be another concern. Brown, McNeil and Glenberg (2009) attest that when teachers fail to structure the learning environment (classroom) in ways that support learning with concrete materials, learners may equally fail to find the underlying concepts or processes. According to Moyer-Packenham et al. (2013) "the way in which manipulatives are used is subject to the interpretations of teachers and students in the environments in which they teach and learn" (p. 19). What this means is that the classroom set-up should be manipulative-user friendly. Brown et al. (2009) state that the learning environment should be structured in such a way that it grounds the abstract symbol so that learners can use the environment to help guide their thinking.

An important concern of using manipulatives is "failing to connect concrete representations to abstract representations" (Brown et al, 2009). As alluded to earlier in this chapter, the use of manipulatives is meaningless if there is no attempt made to connect the manipulative to the abstract concept. Szendrei (1996) as cited in Miranda and Adler (2010) expressed a similar view when she remarked that teachers should not regard manipulatives as objects that will miraculously "fix all the learning problems that learners experience with mathematics" (p. 17). Instead, teachers should clearly explain and connect a manipulative to the concept learners have a problem with. Unfortunately, many mathematics teachers still fail to master this art of teaching using manipulatives. This in turn confuses and frustrates leaners as they fail to make meaning of why they are using such manipulatives.

2.3.3 Principles of effectively using manipulatives

It is important for teachers to know and understand how to use and administer manipulatives in a meaningful way that benefits learners. To promote the implementation of manipulatives in mathematics classrooms, Laski et al. (2015) suggest four general principles of effectively using manipulatives. The first principle is that manipulatives should be consistently used over a prolonged period. This is to give enough time for learners to link the manipulatives to the abstract concepts they represent. The more they are exposed to the same manipulatives for some time, the more they gain deeper insights into how the manipulatives relate to the concepts. As a result, learners will have a better understanding of the concept.

The second principle is that when using manipulatives, one should start with clear concrete representations and move to more abstract representations over time. The closer in similarity the manipulative is to the concept it represents, the more likely learners understand the relation between the two. Even though many researchers advocate for the use of physical manipulatives in mathematics, it is still important to know that teaching should not only be limited to the use of these concrete materials but should always progress to the use of more abstract representations over time (Laski et al., 2015).

There is a strong parallel that exists in what Laski et al. (2015) advocate on shifting from concrete materials to abstract representation and with what Fyfe, McNeil, Son, and Goldstone (2014) assert. The latter advises that with time, we should move away from manipulatives and instead use drawings and symbols. For example, when using a *Geoboard* as a concrete material to teach the properties of quadrilaterals, the teacher should plan the instruction in such a way that it moves from the concreteness of shapes on the *Geoboard* to the application of abstract thinking on the illustration of the same shapes on dot papers. Dot papers represent a *Geoboard* on paper, where the dots are arranged in a square lattice like the nails on a *Geoboard*. According to Skoumpourdi (2010), concrete objects are not guaranteed to make contributions if not connected with the abstract representations.

The third principle suggested by Laski et al. (2015), is to avoid the use of manipulatives that have irrelevant features to the intended mathematical concept. They also controversially suggest that manipulatives should not resemble everyday objects, particularly when illustrating abstract mathematical ideas, as these may hinder learning. In support of this suggestion, Sarama and Clements (2016) advocate that “when the goal is abstraction, concrete materials may not be particularly helpful” (p. 81). This is because if the manipulative is interesting to play with, learners may spend too much time playing with it rather than finding connections with the concept it intends to represent. As a result, this may elicit ideas that are irrelevant to the content. Therefore, manipulatives should be as basic as possible, without irrelevant perceptual features or references to inappropriate real-life objects, for them to promote effective learning when used in mathematics classrooms.

The fourth principle which is very important is to ensure that there is a clear explanation for the use of the manipulative and the concept related to it. A recent study by Sarama and Clements (2016) reveals that it is possible for learners to hold, move, and arrange manipulatives without thinking about the concept. Thus, learners should model with and use manipulatives to represent mathematical ideas (*ibid.*). The relationship between the manipulative and the concept is very important to underscore because it requires abstract thinking to understand how the manipulative represents concepts.

2.4 GEOBOARDS

2.4.1 Geoboards as teaching manipulatives

A *Geoboard* is a physical manipulative for setting and solving geometry problems. It is made from a flat piece of wood with either small nails or pegs lined up in rows and columns to form an arrangement of repeated squares (Loong, 2014). It was traditionally designed as a pedagogical device to be used for the teaching and learning of geometrical shapes, by placing rubber bands around the nails to form different shapes (Scandrett, 2008). See figure 2.1 below. According to Dzambara (2012), *Geoboards* are simple but effective instruments that can be easily constructed from local materials. They can be used to teach mathematical topics such as plane shapes, translation, rotation, reflection, similarity, co-ordination, counting, right angles, pattern, classification, scaling, position, congruence, area, as well as perimeter (Scandrett, 2008). It is claimed that solving geometrical problems is more successful when learners develop exploratory activities with geoboards (Cotič, Mešinović, Zuljan & Simčič, 2010).

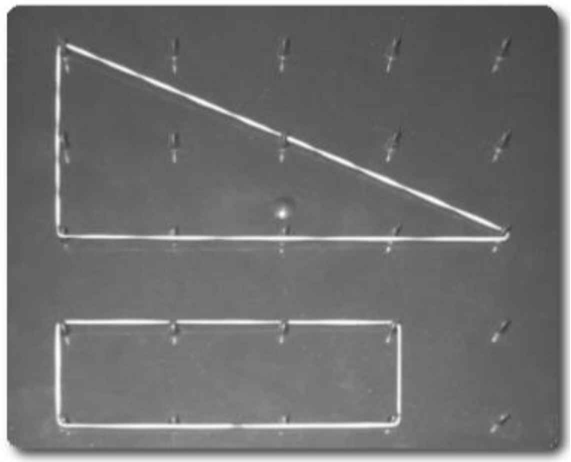

Figure 2.1: Photo of a Geoboard

Interestingly, Furner and Marinas (2011) observed that there has been a decline in the use of *Geoboards*. In my view, this may be due to the widespread impact of technology on nearly every aspect of our lives. Hence, there is a rise and new emphasis on the research on virtual manipulatives rather than on concrete manipulatives. According to Scandrett (2008) “it seems that *Geoboards* have also been forgotten by mathematics educators and there is little reference to them in recent literature” (p. 29). My review of literature seems to suggest that there is a growing research interest and reference to the use of computer-based manipulatives such as *Geogebra* rather than on physical manipulatives such as *Geoboards*.

Earlier research has highlighted the importance of using *Geoboards* as teaching manipulative tools in mathematics. One classical quotation on the use of *Geoboards* in mathematics was asserted by Harkins (1975) who advocated that “there is no manipulative device that surpasses the geoboard as an instrument to cultivate the spirit of exploration in mathematics” (p. 113). Although the use of *Geoboards* is well documented and rather dated, it is still very much effective today. Türegün and Conde (2014, pp. 111, 112) provide a fresh perspective on the use of *Geoboards* in mathematics by asserting that “a concrete Geoboard model provides learners with an increased chance of ownership by allowing them to internalize the model… but it also lays a foundation for a better understanding of the process as we transition subsequently to using technology [such as

Geogebra]". Thus, I see the use of *Geoboards* as fundamental to the teaching of geometry before moving to the use of virtual applications such as *Geogebra*.

My research site is a rural and poorly resourced place where there are schools with no computers for the implementation of virtual manipulatives such as *Geogebra*. I therefore anticipate the use of *Geoboards* to more likely be used in the teaching and learning of quadrilaterals – possibly more so than any other physical manipulative tool, as they can easily be made from local materials.

It is also important to note that the use of *Geoboards* helps learners to be skillful in physical movements (Türegün & Conde, 2014). The device, by its very nature, is a hands-on manipulative that keeps learners busy with both their hands and minds. It has a visual and tactile appeal. It can be manipulated by learners through hands-on activities and experiences (Moyer, 2001). As learners create the shapes with the use of the board and rubber bands they begin to identify concepts that are related to the properties of the geometrical figures. This explains how as learners begin to engage with the *Geoboard* they begin to identify how many sides and corners a square has, for example. Moreover, the concrete experiences developed through *Geoboards* enable learners to relate the abstract to the real world because if the segments of a figure are perpendicular, equal or whatever, they will always look that way on *Geoboards*. Therefore, as visual manipulative tools, *Geoboards* evoke learners' attention and curiosity.

2.4.2 Geoboards and visualisation

Learners may be enabled to see shapes in their minds by the way they are physically constructed on the *Geoboard*. Cotič et al. (2010) assert that *Geoboards* provide learners with visual imagery which in turn helps them to visualise basic mathematical concepts. Early mathematical concepts are strongly connected to preliminary activities involving perception and action within the physical world (Gray, 1999). Thus *Geoboards*, when used as visualisation tools, have the potential to facilitate visual representations to model both fundamental concepts and verbalise abstract ideas (Balka, Hull, & Miles, 2009). For example, it is much easier for learners to explain why the sides of a square are equal because it can be easily confirmed and verified by concretely counting the number of pins on each side of the square.

I therefore argue that the visual nature of *Geoboards* can enable learners to visualise different shapes and figures that are created with them. Hence, visualisation processes are an essential aspect of using *Geoboards* to bring about teaching in the minds of learners through visual imagery and memory (Presmeg, 1986). This is in line with Van Hiele's (1986) model of teaching and learning shapes in mathematics. The model advocates that the initial stage of learning geometry begins with the visual level where learners begin to understand shapes by simply recognising them. The Van Hiele model is discussed in section 2.8 as it forms an integral part of the theoretical framework of my study.

2.5 THE NAMIBIAN CONTEXT

2.5.1 Post-independence education curriculum

This section of the literature review sheds light on the educational system and the development of the school curriculum in Namibia. The Namibian pre-independence education system was characterised by discrimination and inequalities (Namibia. Ministry of Education and Culture [MEC], 1993). The education system was divided along racial and ethnic lines with unequal access to education as well as irrelevance to many. Expressed clearly, Njabili (2004) articulates that "the schools and curriculum were organised and run on a racial basis, the curriculum content and assessment procedures were foreign, the curriculum materials were mainly examination syllabuses prepared in and by the ruling country" (p. 31).

However, after independence in 1990, educational reforms were put in place to address and undo the damages that resulted from the unfair policies and malpractices within the education sector. The MEC (1993) put in place significant reforms that are aimed at providing *access*, *equity*, *quality* and *democracy* to education for all Namibian citizens. The reforms are framed within the country's educational philosophical maxim: "*towards education for all*". Presently, these reforms (access, equity, quality and democracy) serve as the main goals for Namibian education. The goals were set as a response to dissatisfaction expressed towards the lack of relevancy in both the content of the school curriculum and access to education (Njabili, 1995).

In 1993 the Namibian government also introduced the pedagogical idea of Learner-Centered Education [LCE] as a framework for teaching at all levels of primary and secondary schooling. Iipinge and Kasanda (2013) claim that this was an attempt to get rid of the teacher-centered teaching and subject-curriculum. The colonial education system typically viewed learners as empty vessels who came to class with no experience and knowledge of their own. The focus of teaching and learning was solely limited to teachers. At its core, LCE embraces an approach to learning where teachers put the needs of learners at the center of what they do in the classroom. A learner-centered approach presupposes that "learning must begin by using or finding out the learners' existing knowledge, skills and understanding of the topic…Then teachers develop activities that build on and extend the learners' knowledge" (Namibia. Ministry of Basic Education and Culture [MBEC], 1999, p. 2).

Mathematics as a subject also underwent reforms since independence, although with few desired changes (Naukushu, 2016). Since independence the Namibian government has attached great significance to the teaching and learning of mathematics in schools. This is evident in the government's recognition of the role of mathematics in the development of other fields of learning… "Mathematics is indispensable for the development of science, technology and commerce" (MESC, 2010, p.18). The value of mathematics is also seen in one of the goals of vision 2030 of the Namibian government: "…strengthening the teaching of English, Mathematics, Science and Technology at all levels" (MESC, 2010, p. 77). Furthermore, the relevance of mathematics in the school curriculum was demonstrated in the year 2012 by making mathematics as a compulsory subject from Grade 1 to Grade 12 (*ibid.*).

Among the many attempts that were embarked upon to address the weak achievement of learners in mathematics by the government, involved the initiative of joining the Southern and Eastern Africa Consortium for Monitoring Educational Quality (SACMEQ) from 1995. According to Makuwa and Maarse (2013), SACMEQ is a network of ministries of education from 15 different countries that have decided to work together to:

> (a) build the technical capacity of educational planners and researchers in participating Ministries of Education, to monitor and evaluate the conditions of schooling and the quality of their own basic education systems; (b) undertake cooperative large-scale cross-national

> assessments of student learning; and (c) utilize innovative information dissemination approaches and a range of policy-dialogue activities to ensure that SACMEQ research results are widely discussed, debated and understood by all stakeholders and senior decision makers and subsequently used as the basis for policy and practice. (p. 349)

Since its establishment, SACMEQ has conducted three major surveys thus far. The studies were conducted in the periods 1995-1998, 2000-2004 and 2006-2011 respectively. The studies included an extensive assessment of the performance levels of Grade 6 learners and their teachers in the areas of literacy and mathematics to (Makuwa & Maarse, 2013). Namibia participated in all three studies. Based on their performance in international school assessments, Namibia performed lower than its regional neighbors. The SACMEQ study of 2004 revealed that "Namibian learners and their teachers performed poorly in mathematics, as compared to the performance of learners and teachers at the same level in other Southern African countries" (Mateya, Ilukena & Utete, 2016). This shows that mathematics knowledge among both Grade 6 learners and teachers leaves much to be desired. A study by the National Institute for Educational Development (NIED) in 2010 on Grades 5, 6 and 7 learners' performance in mathematics also attributed the deficiency of mathematics knowledge at primary level to the technique's teachers used to teach the subject.

Another notable reform in mathematics after independence was the introduction of Standardised Achievement Tests (SATs) to Grades 5 and 7 in mathematics and English in 1995 (Namibia. Ministry of Education [MoE], (2013). SATs are meant to provide diagnostic information about what learners in Grades 5 and 7 know and can do in key learning competencies in the Namibian curriculum. Unlike other examinations that are administered for promotional purposes, the sole purpose of these tests is for their outcomes to be used for diagnosis rather than for promotion to the next grade. These tests also help to monitor schools' progress from one year to the other in terms of learners' achievement (MoE, 2014).

Thus far, these tests have also exposed the poor performance of Namibian learners in mathematics. According to the MoE (2014), serious concerns over learners' poor performance in Grades 5 and 7 emerged after the publication of the second round of the SATs in 2013. Peters (2016) links the low achievement levels of learners in SATs to teachers. She argues that the learners' poor performance is because of teachers' lack of both confidence and content knowledge to help their learners in their learning of mathematics. Moreover, she reminds us that "in Namibia Lower Primary phase teachers (Junior Primary from 2015 onwards) are not subject experts but 'generalists' that have to teach the foundation of mathematics" (Peters, 2016, p. 68).

2.5.2 The present education curriculum

At present, the Namibian curriculum is experiencing yet another major reform. The government implemented the revised curriculum in 2015. Before the implementation of the new curriculum the ministry conducted a series of training sessions for all primary mathematics school teachers in the country (Tjihenuna, 2014). This was done in line with the recommendations of the ministry that with any curriculum reform, it is important that teachers are helped to take ownership and implement it with commitment (MESC, 2010). With regard to mathematics, the entire curriculum is being changed. The learning content is upgraded to a higher level compared to the old curriculum. For example, most of Grades 8 and 9 content in Mathematics is now moved down to Grade 7. What this means is that Grade 7 learners are now compelled to learn more difficult mathematics content compared to the old curriculum. The implementation of the new curriculum was first implemented for Grade 7 learners in 2016. This year, 2018, the new curriculum is implemented for Grade 9 for the first time and the old curriculum will completely phase out in the year 2020 (*ibid*.).

Table 2. 1 The new curriculum for Geometry for the upper primary phase (Grade 4-7);

	Learning Objectives	Competencies
Grade 4	Understand that 2-D shapes are not affected by their orientation and after simple transformations	- Identify and name the following regular and irregular two-dimensional shapes: triangle, square, rectangle, rhombus, parallelogram, other quadrilaterals, pentagon, hexagon and circles in different orientations. - Identify 2-D shapes found in pictures and diagrams. - Draw 2-D shapes in different orientations.
	Understand line symmetry	- Identify a line of symmetry on symmetrical 2-D shapes. - Complete, draw or construct symmetrical figures, e.g. through paper folding.
Grade 5	Know the basic properties of common regular quadrilaterals	- Identify different kinds of quadrilaterals: square, rectangle, parallelogram and rhombus. - Describe differences and similarities between squares and rectangles; rectangles and parallelograms; squares and rhombuses in terms of - lengths of sides, - angles in shapes (limited to right-angles, angles smaller than or greater than a right angle), - lines of symmetry.
Grade 6	Know the properties of different kinds of quadrilaterals and triangles	- Identify squares, rectangles, rhombuses, parallelograms, trapeziums and kites and describe their properties in terms of their sides, angles and symmetry. - Draw lines of symmetry in quadrilaterals.
Grade 7	Understand the properties of quadrilaterals and triangles	- describe, sort, name and compare different kinds of quadrilaterals (square, rectangle, rhombus, parallelogram, trapezium and kite), in terms of their length of sides, parallel or perpendicular sides, angles (right, acute, obtuse) and symmetry,

When examining the above extract from the syllabus, one notices that in the learning of the properties of geometrical shapes, the competencies cover quadrilaterals such as squares, rectangles, rhombuses, parallelograms, trapeziums and kites. These are the same quadrilaterals whose properties are examined in my study. The quadrilaterals are gradually taught from Grade 4 to Grade 7, for example in Grade 4 and 5 learners are taught squares, rectangles, rhombuses, and parallelograms only. In addition to the first four quadrilaterals learned in Grade 4 and 5, you will notice that in Grade 6 and 7 learners are again taught the properties of two more types of quadrilaterals, namely the trapeziums and kites. Moreover, take note that in Grade 7 the properties of all the six quadrilaterals are taught to learners in detail. One possible reason for this arrangement in the curriculum could be because Grade 7 is an exit grade to the junior secondary phase (MoE, 2015). Thus, learners should be taught and reminded about the properties of all the shapes they have learned from Grade 4 before they move over to the secondary phase.

Mathematics is amongst the five compulsory promotional subjects in Grade 7, including English, Home Language, Natural Science and Social Science. Other optional promotional subjects in Grade 7 are Agriculture, Home Ecology and Design and Technology. Up until now, mathematics has been regarded as one of the important subjects in which learners have to pass to move to the next grade. The latest Grade 4-7 mathematics syllabus explains the reason why mathematics is important in the primary phase by stating that "numeracy is one of the two core features of primary education, the other being literacy" … [Therefore] Mathematics and the languages are thus the most important subjects in the curriculum in this phase" (MoE, 2015).

However, and regrettably, it has happened that since the implementation of the new curriculum from 2015, there are already concerns that the new curriculum is not aligned to learners' level of understanding. One of these concerns emerged from the media newspaper. Several school principals in Windhoek and the northern part of the country shared their grievances that learners are not doing well in the new curriculum because it seems to be too challenging for them (Shapwanale, 2017). Amongst the cited subjects that are said to be difficult for learners are mathematics and English. The school principals also regret the doing away of the F and G symbol and setting the E symbol (40%) as the cut-off point. In my view, it is unfortunate that our new education curriculum is perhaps not well aligned with learners' level of understanding. I therefore see my study as a contribution towards finding a solution to this problem as it attempts to find ways on how the new content, specifically for Grade 7, can be taught in a meaningful way. I argue that it is important that teaching practice is well-aligned with learners' levels of understanding.

Arguably, one possible reason for learners' failure in mathematics in Grade 7 Namibian schools may be attributed to the misalignment of the curriculum and the level of the learners' understanding. If this is so, then this presents a big challenge for teachers. According to Van Hiele (1986), if learners are taught at a higher level than their level of understanding they may not perceive and understand what the teacher sees in a geometric problem or situation. In a situation like this Mason (2009) warns that learners may then simply resort to memorising.

2.5.3 The use of teaching manipulatives in Namibia

Studies that were carried out by the ministry reported that poor performance at the secondary level in mathematics was due to a poor mathematical foundation at the primary level (MESC, 2006). Clegg and Courtney-Clarke (2009) assert that “many of the problems associated with mathematics in Namibia stem from the lower primary phase” (p. 2). Moses (2012) also observed that in Namibia low levels of performance in mathematics learning in the primary phase expand to secondary level. Therefore, to remedy this situation, Peters (2016) suggests that the chief aim of education reform in Namibia should be finding ways “to improve the teaching and learning process in the primary school mathematics classrooms” (p. 4). According to Clegg and Courtney-Clarke (2009), the use of manipulatives as a means of improving teaching and learning in primary mathematics is unfortunately often absent. They found that:

> the use of manipulatives, concrete models and examples and illustrative posters or pictures tended to be limited to lower primary classrooms, … Mathematical concepts develop from the concrete to the semi-concrete to the abstract. The use of manipulative models, pictures, oral language and real-world situations as representations of mathematical concepts are seldom found in Namibian classrooms where the use of written symbols (the most abstract) as representation of concepts is favoured. (p. 23)

The Namibian national curriculum for basic education compels mathematics teachers to be “creative and innovative to produce their own teaching and learning materials linked to practice” (MESC, 2010, p. 6). One such practice is the effective use of “concrete objects as teaching aids to enhance learning in the classroom” (MESC, 2006, p. 6.). One of the primary reasons why mathematics teachers should employ concrete objects is to help learners to relate what they see, touch and experience in the classroom to real life objects and experiences (Brown et al., 2009).

Notwithstanding the call from the government for teachers to make use of teaching and learning aids and materials, many teachers still find it difficult to create and make use of these teaching aids and materials. Miranda and Adler (2010) observed that “Namibia is one of the many African countries, in which the use of manipulatives in mathematics classrooms is not a common practice” (p. 17). I argue that one of the mathematics domains that could benefit from using concrete teaching aids in Namibia is the teaching and learning of geometry, especially when using a visual manipulative tool such as a *Geoboard*.

2.6 THE CASE OF GEOMETRY

2.6.1 Geometry and the school curriculum

Geometry is deemed to be one of the oldest and most established domains of mathematics. Jones (2000) states that “its origins can be traced back through a wide range of cultures and civilizations” (p. 77). The study of geometry is generally concerned with the properties and relations of shapes in space. The current Namibian curriculum views the study of geometry as “the mathematical understanding of space and shapes” (MoE, 2015, p. 1). The importance of geometry has recently brought about a growing call for its rigorous inclusion in the school curriculum. Literature asserts that it should already be included in the early years of schooling (MacDonald et al., 2012; Moss, Hawes, Naqvi & Caswell, 2015). Its teaching only becomes effective and clear to learners if it is intertwined with rigorous spatial reasoning (Clements & Sarama, 2011; Zhang, Ding, Stegall & Mo, 2012). On geometry and spatial thinking, Arcavi (2003) also pointed out that geometric and spatial thinking are not only important in and of themselves, but they also support number and arithmetic concepts and skills. Battista (2007) also asserts that the teaching of geometry provides a major opportunity to develop and increase the deductive reasoning and proving ability of learners.

Furthermore, literature identifies geometry to be an important domain of both mathematics and other fields within the school curriculum. Senechal (2005) as cited in Clements and Sarama (2011) summarised this perspective as follows:

> Geometry should be a focus at every age, in every grade, every year. Mathematics curricula are often criticized for their insularity— ‘what does this have to do with the real world?’ No mathematical subject is more relevant than geometry. It lies at the heart of physics, chemistry, biology, geology and geography, art and architecture. It also lies at the heart of mathematics, though through much of the twentieth century the centrality of geometry was obscured by fashionable abstraction. This is changing now, thanks to computation and computer graphics which make it possible to reclaim this core without loss of rigor. The elementary school curriculum should give the children the tools they will need tomorrow. (p. 134)

Unfortunately, Ginsburg, Cannon, Eisenband and Pappas (2006) lament the fact that geometry and spatial reasoning are the main domains in mathematics where teachers lack preparation, content knowledge and interest. Clements and Sarama (2011) have also concluded that these areas of mathematics are "often ignored or minimised in early education" (p. 133). This has resulted in learners often failing mathematics (Adolphus, 2011). In my view as to why learners do not do well in geometry, is that they find it very difficult to relate their learning of geometry with their day to day living. I accentuate that teachers should help learners relate the conceptual knowledge of the geometry they learn in the classroom to the real world they live in. In this way, I believe, learners will be able to understand mathematics within the context of their environment. Özerem (2012) affirms this thought by stating that effective teaching of geometry "allow[s] students to analyse and interpret the world they live in as well as equip them with tools they can apply in other areas of mathematics" (p. 721).

2.6.2 The teaching of quadrilaterals

Good quality instruction of geometric concepts is of great importance in the teaching of mathematics. Several such concepts are found in quadrilaterals. Hence, the focus of my research project is on quadrilaterals. Quadrilaterals are four sided shapes bounded by straight lines. The topic of quadrilaterals "consists of concepts and definitions of abstract geometry that are required to solve problems related with geometry use in daily life" (Nisiyatussani, Fathurrohman & Anriani, 2018, p. 28). The new Namibian curriculum identifies six different types of quadrilaterals (squares, rectangles, rhombus, parallelograms, trapeziums and kites) whose properties should be taught across Grade 4 to Grade 8 levels (MoE, 2015). One of the objectives of the curriculum (syllabus) on the teaching of quadrilaterals is that learners should be enabled to define and classify, by means of sorting, naming and comparing, the different quadrilaterals in terms of their sides, angles and symmetry.

International literature states that to demonstrate that learners understand the concept of quadrilaterals, they (learners) need to define and classify the quadrilaterals according to their properties (Monaghan, 2000; Fujita & Jones, 2007; Erdogan & Dur, 2014). For example, learners need to define and classify a square in relation to a rectangle. They need to grasp the underlying principle that a square is a special case of a rectangle by comparing their properties. However, Fujita and Jones (2007) state that defining and classifying quadrilaterals "appears to be a difficult topic for many learners" (p. 3). This is also a prevailing challenge even in the primary schools of my research site. To be specific, learners seem to have difficulties with the formal definitions of shapes, let alone the hierarchical classification of these shapes.

To effectively teach quadrilaterals, Akkaş and Türnüklü (2014) recommend three strategies that can be employed by mathematics teachers. The first strategy is to define quadrilaterals. They (*ibid.*) advocate that "defining is an important aspect in learning quadrilaterals" (p. 186). The defining of quadrilaterals may either be done by means of formal definitions or by listing the properties. They also recommended that generally, the definition strategy should be used together with visualisation. The second is the classification strategy. Quadrilaterals can be classified differently in three ways: hierarchal classification, partial classification and transformation classification. The third strategy to teach quadrilaterals effectively is the visual representation of quadrilaterals. This may be done by employing visualisation strategies that include drawing models or the use of concrete models of quadrilaterals. Visualisation strategies can be examined under three sub-categories: "visualisation using examples from daily life, visualisation using materials, and visualisation by drawing" (Akkaş & Türnüklü, 2014, p. 189).

In my study, the participants make use of visualisation strategies in the teaching of quadrilaterals. According to (*ibid.*), the teaching strategy is mainly focused on visualisation by using physical manipulatives. Therefore, the manipulative I advocate and employ in this study is the use of a geometrical board called the *Geoboard*. Other visualisation strategies such as drawing are also used. These strategies complement the defining and classification strategies mentioned above.

2.7 VISUALISATION IN MATHEMATICS

2.7.1 Definition of visualisation

Literature states that visualisation can be mental or physical. In her work, Nardi (2014) expresses this notion by articulating that the physical aspect of visualisation is the idea of inscriptions of various kinds, and this she refers to as external representations, whilst the mental aspect of visualisation refers to visualisation in the mind. What this means is that visualisation is not meant for illustrative purposes only but is also recognised as a key component of reasoning. Makina (2010) describes visualisation as a mental process, visual thinking or reasoning that employs visual imagery in the solution of problems. In their opinion, Kaminski, Sloutsky and Heckle (2008) also recognise visualisation as the ability to form and negotiate a mental image necessary for problem-solving in mathematics. In my reading, most recent researchers seem to concur with Arcavi's (2003) definition of visualisation. He profoundly defines visualisation as:

> the ability, the process and the product of creation, interpretation, use of and reflection upon pictures, images, diagrams, in our minds, on paper, or with technological tools, with the purpose of depicting and communicating information, thinking about and developing previously unknown ideas and advancing understandings. (p. 217)

A paraphrase of this definition, without doing violence to its meaning, would be: visualisation helps us to see the unseen. It helps us to see beyond what meets the eye by not only seeing what is within our sight (physical), but it also enables us to see and interpret what we mentally see in our minds, on paper or technological tools. As a result, we can communicate, think about and expand the understanding of the information and ideas that develop from such a process of visualisation.

Roslan and Ahmads' (2017) view of the definition of visualisation also adds another dimension to the meaning of visualisation. They claim that the term visualisation "means to imagine and describe" (p. 170). They generalise the definition of visualisation as the "ability [of learners] to imagine and design objects that can be altered from multiple views… viewing the object and thinking either in two dimension or in three dimensions" (p. 170).

The application of visualisation to mathematics teaching and learning stimulates learners' minds, helping them to see trends and patterns. According to Rudziewicz, Bossé, Marland and Rhoads (2017), visualisation processes enable learners to "see the unseen", such as concepts, trends and patterns. When learners possess the ability of "seeing the unseen" in mathematical shapes and word problems, they begin to develop a deeper understanding of concepts (Natsheh & Karsenty, 2014). Furthermore, the claim has been advanced by Mudaly (2010, p. 65) that "viewed pictures often create clearer images in our minds because of the symbols attached to what we see, accompanied by other sensory perceptions". A similar view is reinforced by Duval (2014) who reasons that visualisation is a cognitive process through visual reasoning whereby learners think through what they see and hear (including words) in order to coordinate all possible representations of a concept. Therefore, visualisation allows learners to develop deeper and richer concepts than they otherwise might have developed from other types of instructions that do not make use of it. If well administered in the teaching of mathematics, visualisation can "visually evoke real-life experiences" (Figueiras & Arcavi, 2014).

In the context of my study, I embrace Nemirovsky and Nobles' (1997) definition of visualisation who associate it with the process of harmonizing what learners see in their environment to what they think in their minds. I relate this concept of visualisation to what was earlier explained by Cobb, Yackel and Wood (1992). They used the term *dualism* to explain the connection created between mathematics in learners' minds and the mathematics in their environment. Zazkis et al. (1996) also expound on the concept of visualisation as a link between what learners think and what they see by stating that:

> [a]n act of visualization [that is, the process of visualization] may consist of any mental construction of objects or processes that an individual associates with objects or events perceived by her or him as external. Alternatively, an act of visualization may consist of the construction, on some external medium such as paper, chalkboard, [*Geoboard*], or computer screen, of objects or events which the individual identifies with object(s) or process(es) in her or his mind. (p. 441)

Visualisation processes involve the use of visual imagery. Owens (2003) asserts that imagery refers to an abstract formation of concrete or dynamic images that can be taken apart, reconstructed or changed. Kosslyn, Thompson and Ganis (2006) describe working with visual imagery as the ability to form mental representations of the appearances of objects and to manipulate these representations in the mind. The mental images created by learners provide the foundation for insight. When a learner receives first-hand experiences that help him or her to build the necessary mental images, learning becomes deeper and more penetrating (Pellerone, Passanisi, & Bellomo, 2015). Therefore, I gather that the use of *Geoboards* as *visualisation* tools can easily produce visual imagery such as *concrete pictorial* and *dynamic images*. Presmeg (1986) asserts that learners can think in pictures without any linguistic support.

2.7.2 Visualisation processes and the use of Geoboards

The concrete pictorial imagery

Thinking in pictures involves the formation of concrete or pictorial images, and this is crucial for helping learners to progress with learning concepts (Sousa, 2008). Teahen (2015) argues that "the pictorial stage involves providing or creating visual representations that will help students to visualise mathematical operations during problem solving" (p. 28). Learners should be encouraged to use concrete pictorial imagery by forming pictures of the problem or presented situation in their minds. Makina (2010) asserts that this usually encourages learners to solve problems holistically, instead of breaking them into parts. Thus, I see a *Geoboard* as a visualisation tool that can enable learners to think in pictures. The *Geoboard* can facilitate a process of constructing actual shapes in a vivid, concrete and detailed manner so that the visualisation process becomes implicit, and learners are able to work with the shapes physically and in their minds when making sense of their properties, for example.

The dynamic imagery visualisation process

Thomas and Mulligan (1995) contrast dynamic to static imagery by stating that dynamic imagery is a representation that is changing and moving whilst static imagery is a representation that is fixed. Dynamic imagery is important in mathematics learning because it "has the ability to tell an unfolding temporal narrative… it carries us along through time" (Joshi, Mehta, Drucker, Stollnitz, Hoppe, Uyttendaele & Cohen, 2012, p. 251). However, creating dynamic imagery in learners is challenging if it is not aided by professional tools and training (*Ibid*.), such as teaching manipulatives. Again, this brings me to the conclusion that when a *Geoboard* is effectively used as a manipulative tool it can evoke dynamic imagery in learners. The *dynamic* element that is in a *Geoboard* can enable learners to transform and move shapes by placing the elastic bands in different positions. Pinto and Tall (2002) advocate that mental constructions of images involve playing with such manipulatives in various ways.

In this project, I thus argue that the construction and manipulation of shapes on *Geoboards* in different ways may help learners to physically and mentally build up an understanding of the properties of shapes. The *dynamic* aspect of visual imagery can also help learners to 'imagine what it would look like' if they move a shape from one position and another or manipulate shapes to form other different shapes. Mason (1992) profoundly asserted that learners should not just create mental images, but they need to use the images as a mental "screen" in visual thinking in order to comprehend learning in Mathematics.

2.7.3 Cognitive apprehensions and the use of Geoboards

Approaching geometry from a cognitive point of view, Duval (1995) proposes an interesting analytic framework for looking at and analysing the understanding of geometric drawings or images. The framework identifies and distinguishes four types of cognitive apprehensions (understandings) that are at play when interacting with geometric shapes. These are: *perceptual apprehension*, *sequential apprehension*, *discursive apprehension*, *operative apprehension*.

Perceptual apprehension

Perceptual apprehension refers to what is recognised from an image at first glance. It is the very first perception when confronted with a geometric shape or figure, for example. According to Duval (1995), in order "to function as a geometrical figure, a drawing must evoke perceptual apprehension, and at least one of the other three" (p. 143). Perceptual apprehension designates the ability of learners to name figures and the ability to recognise in the perceived figure several sub-figures. It was also asserted by Van Hiele (1986) that perception is central to learners' recognition of basic shapes. Thus, vision plays a major role in helping learners to identify, classify and sort out shapes according to how they are externally seen. Duval (1999) strengthens this point by arguing that "nothing is more convincing than what is seen" (p. 12). Therefore, for learners to understand the properties of any quadrilateral, they need to see how the properties look in order for them to be convinced. For example, learners would know and understand what a square looks like when it is physically presented to them on a *Geoboard*.

According to Mudaly and Naidoo (2015), this first stage requires that learners manipulate concrete objects followed by the pictorial representation. A *Geoboard* as a visual manipulative can be an effective tool that can concretely represent the quadrilaterals. Kalogirou and Gagatsis (2011) state that “the ability to recognise plane shapes by students seems to be influenced by a number of factors, including visual perception” (p. 29). However, it is important to note that learners should move beyond mere perceptual apprehension in order to appropriate the generalisation process. Samson and Schäfer (2011) suggest that one way to move beyond perceptual apprehension is by seeing a diagram or shape in multiple ways. The experiences of using *Geoboards* as manipulative tools have the potential to help learners look at shapes in multiple ways. *Geoboards* allow learners the chance to explore shapes in a variety of ways, and according to Seefeldt and Wasik (2006), this may help them to “think about their world [of shapes] in alternative ways and help them to understand that there are multiple ways to solve problems [involving shapes]” (p. 250).

Sequential apprehension

Sequential apprehension is an understanding that results from constructing a figure or when describing its construction (Duval, 1995). In this case, the figural units depend not only on perception but on mathematical and technical constraints (in the latter case this could be ruler and compasses, or perhaps the primitives in computer software) and/or their properties. In my study the construction of quadrilaterals with elastic bands on *Geoboards* is intended to help leaners to conceptually encounter the properties associated with these shapes. For example, I believe that as learners explore the many ways of constructing a kite on a *Geoboard*, the more they are consequently accorded the opportunity to understand the properties that uniquely makes a kite different from any other quadrilaterals.

Discursive apprehension

According to Duval (1995), in any geometrical representation the perceptual recognition of geometrical properties must remain under the control of statements (such as denomination, definition, primitive commands in a menu). Duval (*ibid.)* furthermore explains that “perceptual recognition depends on discursive statements because mathematical properties represented in a drawing cannot be determined solely through perceptual apprehension; some must first be given through speech and engagement” (p. 31). Thus, in my study, for learners to demonstrate a broad understanding of quadrilaterals, they need to construct the shapes on *Geoboards*, accompanied by descriptions of how the shapes are constructed. For example, learners should show why the angles of a rectangle are 90 degrees each, not only because they look that way but through a deeper and broader understanding of the concept of perpendicular lines. In my opinion, I suggest that this may be best presented by constructing two intersecting vertical and horizontal line segments on the *Geoboard*, followed by discussions of how the two lines are related to the 90-degree angle.

Van Hiele (1986) also stressed that it is important to acquaint learners with the definitions and meanings of the properties of shapes by giving them an opportunity to discuss what they have learned — this should usually happen in the explicitation phase. According to Gagatsis, Monoyiou, Deliyianni, Elia, Michael, Kalogirou, … Philippou (2010) learners should use geometrical definitions to demonstrate an understanding of geometry, and this is possible only if learners infer "geometrical relationships from the construction of the figure" (p. 41). Mudaly (2010) states that "whilst physical and mental images are necessary, words and sentences are essential in the recording of the information acquired through visual means" (p. 66). This statement clearly shows that learners need opportunities to articulate their thinking processes in linguistic form. In other words, verbalisation of concepts may enable learners to understand them more clearly.

Operative apprehension

This involves operating on a figure, either mentally or physically, which can then give insight into the solution of a mathematical problem (Duval, 1995). Operating on a figure depends on the many ways it can be modified, manipulated and transformed. According to Gagatsis et al. (2010) operating on a figure mentally or physically involves three types of modifications and these are the mereologic way, the optic way and the place way. The mereologic way refers to the reconfiguration of the parts of the whole given figure into parts of various shapes and the combination of them into another figure or sub-figures. The optic way is when the operation of a figure results in the figure appearing enlarged or narrowed when it is viewed from a different angle, and the place way refers to the position or orientation variation of the shape.

Deliyianni, Gagatsis, Monoyiou, Kalogirou and Kuzniak (2011) found that "operative apprehension is the one which contributes the most to geometrical figure understanding" (p. 606). This is because operative *apprehension involves* visualisation and according to Duval (1999) "there is no understanding [in geometry] without visualization" (p. 13). Again, I see the use of a *Geoboard* as an effective way of stimulating operative understanding in learners. This is because a *Geoboard* is in a position to provide opportunities to learners for concretely and cognitively modifying, manipulating and transforming shapes with the elastic bands.

The above discussed concepts on visualisation processes (*concrete pictorial imagery*, *dynamic imagery*, *perceptual imagery*, *sequential apprehension*, *discursive apprehension* and *operative apprehension*) are adapted and adopted as observable indicators for the first part of my analytical framework – see Table 3.3 in the methodology chapter.

2.8 THEORETICAL FRAMEWORK

2.8.1 Constructivism

This study is positioned within the constructivist theoretical framework, as the basis of working with *Geoboards* is largely dependent on interactions which assume an activity-based learning approach. The constructivist learning theory holds that "learning is a constructive process in which the learner is building an internal representation of knowledge, a personal interpretation of experience" (Bednar, Cunningham, Duffy & Perry, 1992, pp. 21, 22). It is a theory of learning that derives from a philosophical position that individuals construct their own new understandings based on an interaction between what they already know and believe, against the ideas and knowledge that they meet (Piaget, 1967).

2.8.2 Radical constructivism

Radical constructivism has its roots in Jean Piaget's cognitive constructivism which advocates the personal or individual construction of learning. Ernst von Glasersfeld is the founder and most prominent proponent of radical constructivism (Liu & Chen, 2010). His central argument is that knowledge cannot be passively transferred from the teacher to the learner, but instead a learner must be an active participant in the learning process. Building on this argument, Golafshani (2013) also advocates that "the teacher should not be a transmitter of knowledge but should instead act as a facilitator to the construction of knowledge for all learners" (p. 139). Therefore, in a radical constructivist-based mathematics classroom, Van de Walle (2007) suggests that mathematical ideas should not be 'poured into' a passive learner. Instead, learners must be actively involved in the learning of mathematics by encouraging them to "wrestle with new ideas, to work at fitting them into existing networks, and to challenge their own ideas and those of others" (Van de Walle, 2007, p. 23).

2.8.3 Constructivism and teacher practice

Richardson (2003) acknowledges that "constructivism is a theory of learning and not a theory of teaching" (p. 1629). However, contemplating on Von Glasersfelds' (2001) radical constructivism approach to the pedagogy of teaching, I argue that even though constructivism is not a theory of teaching it is still important that teachers learn and know how to work with it. According to Von Glasersfeld (2001), an approach to 'radical constructivism teaching' should take note of the following points of view:

> (1) teaching involves creating opportunities for students to trigger their own thinking; (2) teachers not only need to be familiar with the curricular content, but they must also have available a repertoire of didactic situations in which such conceptual content can be naturally built up in a way that sparks the students' natural interests; (3) teachers need to realize that students' mistakes are not wrong as such, but are predictable solutions on the way to more adequate conceptualization;(4) teachers need to understand that specialized words in academic disciplines do not have the same meaning for a student as they do for the expert, and teachers must have an idea of the students' present concepts, ideas, and theories; and (5) teachers must realize that the formation of concepts requires reflection, something accomplished by conversations among students and with the teacher. (pp. 10-11)

Teachers are the ones who are in position to help learners construct their mathematical knowledge. It is true that teachers have great control of what they teach, but far less control over what learners learn (Richardson, 2003). Hence, the reason why they should create a learning environment that supports and triggers learners to construct their own learning and meaning making. Joldersma (2011) recommends that for an effective constructivist learning environment to exist, teachers should strive to become constructivists. They should "see both their own and the children's actions from the children's points of view" (Cobb & Steffe, 2011, p. 21). This means that the teachers' actions within a constructivist environment need to be informed by learners' needs.

2.8.4 Constructivism and teaching manipulatives

An important characteristic of a constructivist environment is the common use of teaching and learning manipulatives in the classroom. D'Angelo and Iliev (2012) state that teachers who implement manipulatives in their mathematics lessons create constructivist-based classrooms. Golafshani (2013) also recommends that we need to "recognize that teachers' teaching styles will become more constructivist through instructional practices with manipulatives" (p. 157). The use of manipulatives is also supported by Brooks and Brooks (1999) who suggest that in "constructivist theory curricular activities rely heavily on primary sources of data and manipulative materials… than on textbooks and workbooks" (p. 17). It is further claimed that manipulatives help learners to construct, reinforce and connect several representations of mathematical ideas that are meaningful for learning to occur (Clements, 1999).

2.8.5 The intervention programme and constructivist theory

At the heart of my study is an intervention programme. This programme aims to inspire teachers to incorporate visualisation processes and the Van Hiele phases of teaching geometry with *Geoboards* into their teaching practice to conceptually teach the properties of quadrilaterals. The underpinnings of this programme rest on the constructivist theory because the programme emphasises that learners need to construct their own mathematical meaning of quadrilaterals using *Geoboards.* I find the constructivist theory to be well aligned with my study because the use of *Geoboards* as visualisation tools allows teachers to facilitate activities that actively engage learners in the meaning making of the properties of shapes. Yackel (2001) reinforces the notion that "students construct their own meaning from the words or visual images they see or hear" (p. 41). So, I argue that as learners construct the shapes physically on the *Geoboards*, they concurrently visualise and can construct the same shapes in their minds.

In addition to the above, I also advocate that meaning making is more significant if learners are presented with activities that enable them to construct meaning in an engaging manner. This can be achieved effectively through creating a constructivist learning environment. Thus, I see the decision to use *Geoboards* as manipulatives in this study as critical to creating an environment that stimulates construction of meaning in learners (Moyer-Packenham et al., 2013). A *Geoboard* facilitates activities that help learners to construct new knowledge of the properties of quadrilaterals by building on their current knowledge through interacting with this manipulative. As teachers help learners to interact with *Geoboards*, learners are availed opportunities that allows them to "construct stable generalizable concepts for themselves" (Brown et al., 2009, p. 161).

2.8.6 Constructivist theory and the Van Hiele model of teaching and learning geometry

The use of constructivist theory in this study is supported by the application of Level 1 to two of Van Hiele's (1986) model of geometrical thinking. The model strongly aligns with constructivism, as its foundation is embedded in individual knowledge construction and an activity-based pedagogy. According to the model, learners, assisted by appropriate instructional experiences, move sequentially through five levels in a hierarchal order in their geometry learning (Van Hiele, 1986). The five Van Hiele levels of geometrical reasoning are the Visualisation (Level 1), the Analysis (Level 2), the Informal Deduction (Level 3), the Formal deduction (Level 4) and the Rigour (Level 5), respectively. A description of each level of the model is briefly outlined in Table 2.2 below.

Table 2. 2 Descriptions of the Van Hiele levels of geometrical reasoning

LEVEL	BRIEF DESCRIPTION OF LEVELS
Visualisation	At this level learners recognise geometric shapes by **appearance alone**. They often do this by comparing them to unknown similar prototype. Learners make decisions based on **perception**, not reasoning.
Analysis	At this level learners see shapes as **carriers of properties**. They can recognise and name properties of geometric shapes, but they do not see relationships between these properties. When describing an object, a learner operating at this level might list all the properties the learner knows, but not discern which properties are necessary and which are sufficient to describe the object.
Informal Deduction	At this level learners see **relationships** between properties and between shapes. Learners can create meaningful definitions and give **informal arguments** to justify their reasoning. They can comprehend the relation between shapes and create the relationships. However, the role and significance of formal deduction is not understood.
Formal Deduction	At this level learners can **construct proofs**, understand the role of postulates, theorems and definitions, and know the meaning of necessary and sufficient conditions. Learners should be able to construct proofs such as those typically found in a high school geometry class.
Rigour	At this level learners can be able to make **more abstract deductions**, understand how to work with axiomatic and establish, compare mathematical systems. Learners can understand the use of indirect proof and proof by contrapositive and can understand non-Euclidean systems.

Adapted from Mason (2009, pp. 4-5)

This study is limited to the transition of Level 1 to Level 2; the visual (recognition) and analysis levels of geometrical thinking. My assumption is that the basic competencies of the Grade 7 mathematics curriculum are grounded within these two levels. Most of the competencies require learners to identify and describe quadrilaterals, and moreover learners are supposed to describe the properties of the quadrilaterals. The latter constitutes the transition from Level 1 to Level 2. In the study the visual (recognition) level of Van Hiele's model foregrounds visual perception. Thus, when learners construct shapes on *Geoboards*, their initial appearance becomes the mechanism through which they recognise and operate on shapes before attaching specific properties to them. For example, at Level 1, for a Grade 7 learner the concept of a 'square' is a square because it looks like one. It is through harnessing this level of visualisation and exploring further that the properties of a square become apparent. It is asserted in this study that the use of *Geoboards* can be an appropriate and effective means for these properties to become apparent.

Henceforth, the properties of a square become clear to learners through a successful transition of Level 1 to Level 2. In other words, learners begin with the recognition of shapes as wholes (Level 1), and then they progress to seeing these shapes as carriers of properties (Level 2). Thus, they begin to recognise the shapes (quadrilaterals) by their properties. This transition entails that learners move from observing and identifying shapes to eventually recognising their properties. Mason (2009) suggests that much attention should be given to Level 1 to ensure a successful transition through all the five levels. De Villiers (2010) credits the process of helping learners to transit from Level 1 to Level 2 as a move that enables them to acquire the technical language by which the properties of the concept can be described. He stresses that the process also helps them to recognise "certain new relationships between concepts and the refinement and renewal of existing concepts" (De Villiers, 2010, p. 2).

2.8.7 The Van Hiele phases of the instructional cycle and the use of *Geoboards*

Van Hiele (1986) asserted that the movement of one level to the next is more a result of a learning process rather than a result of age or maturity. Thus, he suggested that to help learners to move from one level to the next level, teachers should make use of a learning process he referred to as the phases of the instructional cycle of teaching geometry. Commenting on the phases, Crowley (1987) also advocated that advancements through each level of teaching geometry is a result of the five phases of teaching. The following is a description of the five phases and how they may play a role in the instruction of the properties of quadrilaterals in this study (Van Hiele, 1999):

Phase 1: *Information*

The first phase is the *Information phase,* which proposes that teachers should first identify what learners already know about the topic at hand, thus helping them to become oriented to the new topic. In the first phase, teaching should begin with finding out what learners already know to lead them to explore and discover certain structures. As learners work through the materials given to them, they become acquainted with what is expected of them. For example, in my study the teacher should construct as many quadrilaterals on the *Geoboard* as possible, differing in size and position, followed by a discussion of how learners visualise these shapes. In each case, learners should be asked to give the name of the shape to determine what they already know about the specific shape. Idris (2009) says that by doing this "students become acquainted with the structure of the material, such as examining examples and non-examples of geometric concepts" (p. 96).

Phase 2: *Guided orientation*

The second phase is the *Guided orientation phase,* which suggests that teachers should allow learners to explore shapes in carefully structured tasks such as folding, measuring, or constructing. In this study learners should explore the properties of quadrilaterals by constructing specific concepts such as parallel, perpendicular, diagonals, equal, adjacent and opposite on *Geoboards* as well as dot papers. For example, the teacher uses a *Geoboard* to demonstrate to learners and for them to observe what happens when an elastic band is put across the opposite vertices (corners) of a square.

Phase 3: *Explicitation*

The third phase is the *Explicitation phase*. In this phase teachers are encouraged to assist and inspire learners to describe and discuss what they have learned about the concepts in their own words. Idris (2009) states that learners "become conscious of the relations, they try to express them as words; they learn the technical language accompanying the subject matter" (p. 97). It is in this phase that learners share ideas about the properties of shapes through group discussions. Van Hiele (1986) strongly considers discussion to be the most important part of the teaching-learning process and without giving learners an opportunity to express what they have learned; the transition is impossible. As discussed earlier in this review of literature, there is a clear parallel between Van Hiele (1986) and Duval's (1995) views on the importance of allowing learners to express their understanding through discussions. They both recognise effective teaching as allowing learners an opportunity of expressing their thoughts and ideas through discussions.

Following Kress' (1994) view that "the child's language may be regarded as a window to the child's conceptual world" (p. 132), I may confidently say that language and mathematics cannot be separated. A clear understanding of mathematical concepts heavily depends on better understanding of the language of instruction being used. So, allowing learners to freely express and discuss mathematical concepts may help them to understand better. The Namibian curriculum also emphasises that to implement learner-centered teaching in any classroom (including mathematics classrooms), teachers need to 'talk less' and allow learners to do 'more talking' through group discussions (MESC, 2010).

Phase 4: *Free orientation*

The fourth phase is referred to as the *Free orientation phase*. In the free orientation phase learners start making their own connections to solve problems and investigate more open-ended tasks. In this study learners are given a variety of tasks on both the *Geoboard* and dot papers on the properties of different quadrilaterals and asked to make connections to other shapes and figures. In so doing, learners learn through general tasks to find their own way in the network of relations of the properties of quadrilaterals. The *free orientation phase* also provides information on how learners are grasping the properties taught to them. The Namibian curriculum advocates that it is important to assess learners' progress to determine where they are doing well and where they need help (MoE, 2015). It is therefore the responsibility of the teacher to plan and facilitate suitable tasks that can be given to learners on both *Geoboards* and dot papers to determine how well they have understood what has been taught to them.

Phase 5: *Integration*

The fifth and last phase is the *Integration* phase. In this phase learners should be able to summarise, reflect on and integrate what they have learned about the properties of squares, rectangles, parallelograms, rhombuses, trapeziums and kites. Idris (2009) states that "in the fifth phase, integration, pupils build an overview of all they have learned of the subject, of the newly formed network of relations now at their disposal" (p. 97). This is the phase that prepares learners for the next level. According to Dongwi (2013), in the *integration phase* "learners summarise the new understanding of the concepts involved and incorporate the appropriate language of the new level"

(p. 25). In the case of my study, it is planned that teachers help learners to make connections between the different properties of the shapes (quadrilaterals) they have learned.

The phases above form the second integral part of my analytical tool to analyse the evidence of how my participating teachers incorporated the *Geoboard* in the observed lessons – see analysis Table 3.4 in the methodology chapter.

2.8.8 The choice of the Van Hiele model and the teaching intervention programme

Van Hiele (1986) asserted that the transition from one level to the next is a learning "process that has to be done by the pupils themselves" (p. 62). The levels are concerned more with learners' understanding, whilst the phases are more about how teachers can help learners to transit from one level to the next higher level. The phases are a cycle that repeats itself through each level until learners "attain the highest possible level of geometric thinking for the content under study" (Dongwi, 2013). As this model is executed within a constructivist learning context in my study, it is expected of the participating teachers that they will guide learners by creating a learning environment that enables them to construct meaning through the medium of the *Geoboard*. This type of environment is only possible if teachers adopt the Van Hiele model of teaching geometry in their teaching practice or programme. Van Hiele (1986) emphasises that the "transition from one level to the following is not a natural process; it takes place under the influence of a teaching-learning program" (p. 50). Therefore, in my study I argue that the choices of activities that the teachers use are critical in ensuring a successful transition from Level 1 to Level 2 through the use of Van Hiele phases. These choices and activities are informed by various roles such as "planning tasks, directing students' attention to geometric qualities of figures, introducing terminology and engaging students in discussions using these terms and encouraging explanations" (Idris, 2009, p. 98).

2.8.9 The Van Hiele phases of the instructional cycle in the Namibian context

Thus far, my review of literature reveals that except for a handful of studies (Muyeghu 2008; Mateya 2008; and Dongwi 2013) there has been an absence of focus on determining the level of geometric conceptualisation at the primary level in Namibia. The concern over the limited focus on the Van Hiele model of teaching and learning geometry is also affirmed by Dongwi (2013) to be a longstanding issue in Namibia. She observes that "the Van Hiele theory is not well researched in Namibia and where research has been done, it has focused on the Van Hiele levels of geometric thinking [and not the phases of instruction]" (p. 5). In my literature review I concur with Dongwi's (2013) observations that there has been little investigation involving the Van Hiele model in Namibia. Moreover, I see that even the three identified researchers who did investigate the use of Van Hiele model in Namibia did not focus their studies at the primary level, but instead their studies were mainly on the Van Hiele model of teaching geometry at the secondary level only (junior and senior phases).

Briefly, Muyeghu (2008) used the Van Hiele model to investigate teaching strategies applied by Grade 10 teachers to teach geometry in Namibia. He found that teachers were able to facilitate teaching and learning that was at Level 1 and 2 of the model. In his study, Mateya (2008) investigated the alignment of the senior secondary phase geometry syllabus to the Van Hiele model of teaching geometry to determine the highest possible level of geometric conceptualisation learners operated on. The study found out that the highest possible Van Hiele level of thinking required by the Grade 12 mathematics curriculum was Level 3, whilst Grade 12 learners' actual level of reasoning in geometry was between Level 1 and Level 2. This finding implies that learners who participated in the study functioned at a level of geometric thinking not corresponding to their mathematics curriculum. A recent study on the Van Hiele model of teaching geometry in Namibia was done by Dongwi (2013) who explored the experiences of senior mathematics teachers (Grade 11) of using the Van Hiele phases of teaching circle geometry. She found that teachers who participated in the study used the Van Hiele phases in their teaching without difficulty.

Although my study does not try to analyse the level of the primary curriculum and geometrical reasoning of learners, I however want to find ways how primary mathematics teachers (especially Grade 7) are able to apply the Van Hiele phases of the instructional cycle of teaching geometry on a *Geoboard* in order to effectively teach the properties of quadrilaterals. De Villiers (2010) claimed that geometry teaching that is not aligned with Van Hiele's model is unlikely to succeed. It is thus important to review the primary school geometry curriculum along Van Hiele lines, and/or implement the Van Hiele model if we desire to see good performance at the primary and secondary levels. It is emphasised by Özerem (2012) that "the Van Hiele model continues to be the best-known theoretical account of students' learning about shapes" (p. 721). Hence, my decision to use this model in my study of how best shapes (quadrilaterals) can be taught by using a manipulative tool such as a *Geoboard*.

On the other hand, De Villiers (2010) is critical of developing a geometric grade-based curriculum using the Van Hiele model because the model is not age-based. It would thus be difficult to infuse that into a typical curriculum structure that is age-based. I however, argue that it is still possible to integrate the model into everyday teaching on a module-by-module, or topic-by-topic basis. The Van Hiele model provides a very practical, rational and logical framework to plan and implement lessons.

2.9 *GEOBOARDS* AND CONCEPTUAL UNDERSTANDING

In this study I share the same sentiments as Kilpatrick et al. (2001), that in any teaching, the extent to which learners have conceptually understood what is taught to them is by means of building meaning and making connections between different concepts and methods in appropriate ways. In the teaching and learning of the properties of quadrilaterals, *Geoboards* can be interesting and effective devices on which to construct shapes and help learners expand their understanding of concepts such as parallel sides, perpendicular sides, diagonals, verticals, horizontals, bisecting lines, opposite sides and adjacent sides. For example, in order for learners to form an image of a concept of parallel lines they need to visualise how its elements look on a *Geoboard*. Therefore, if used appropriately, a *Geoboard* is a device that has the potential to facilitate conceptual understanding of the properties of quadrilaterals in learners.

Cobb et al. (1992) acknowledge that "external representations are typically developed in an attempt to help students construct conceptual meaning" (p. 14). Thus, when the visualisation potential of *Geoboards* is harnessed through visualisation processes and the Van Hiele phases of instructional cycle, learners may conceptually make connections between the different constructed shapes and their properties. This in turn may also encourage critical thinking, meaning making, and forming connections, hence enabling learners to conceptually understand and identify the properties of quadrilaterals.

Teaching for conceptual understanding is directly related to the effective use of concrete objects and the power of visual representations (Eisenhart, Borko, Underhill, Brown, Jones & Agard, 1993). However, the use of any concrete and visual representation is only a tool; the final destination is conceptual understanding. Thus, the embedding of the visualisation processes and the Van Hiele phases of instructional cycle of teaching geometry on *Geoboards* may open opportunities for the learning of new knowledge, and in turn this may develop the conceptual understanding of basic shapes in learners.

2.10 CONCLUSION

In this chapter, I presented an overview of issues that I regard to be key in guiding my study into the analysis of how *Geoboards* can be used as effective manipulative tools in teaching the properties of quadrilaterals. My review of literature centered on aspects such as the use of a *Geoboard* as a physical manipulative, the teaching of geometry, visualisation processes, the constructivist theory, and the Van Hiele model of teaching and learning geometry. After reviewing literature on these aspects, my overall comprehension is that these five aspects need to be jointly used to analyse the effectiveness of their use when teaching the properties of quadrilaterals. I am alerted by Nardis' (2014) warning that:

> the assumption that 'because something is visual, it is [therefore] transparent, and you understand it' is incorrect when there is substantial intermediate work that students need to engage with in order to achieve this understanding. (p. 2017)

In fact, I may confidently advise that as mathematics teachers we should not have an incorrect idea of thinking that just because a *Geoboard* is visual in nature, then it automatically brings about visualisation processes without substantial intermediate work. A *Geoboard* needs to be used properly and with intent for it to be an effective tool to teach and learn.

The next chapter will comprehensively discuss the research methodology used in this study.

CHAPTER 3

METHODOLOGY

3.1 INTRODUCTION

In this chapter I present my research methodology that I adopted for my study. I discuss the appropriateness of the research design and methodology that I have chosen in terms of the main research questions. I start the discussion by explaining the research paradigm that directed and guided my research process. I also broadly discuss the research methods I used to understand the phenomenon under study. Further, I discuss in detail the data collection tools I used for my study as well as how I analysed the data. I end the chapter by discussing and explaining the ethical issues I observed when collecting data.

Before I discuss the research design and methods I used in the research process, I first remind the reader of my research goals and questions:

3.2 RESEARCH GOALS AND QUESTIONS

The purpose of this study is to investigate and analyse the use of *Geoboards* as visualisation tools in the teaching of the properties of quadrilaterals. Specifically, the study intends to:

- find out the affordances of the utilisation of *Geoboards* as visualisation tools in developing and enhancing conceptual understanding of the properties of quadrilaterals;
- explore and gain insight into teachers' perceptions and experiences of using *Geoboards* as visualisation tools after the implementation of an intervention programme;
- establish the extent to which the Van Hiele phases of teaching geometry can be applied to teaching quadrilaterals with *Geoboards.*

This study attempts to answer the following research questions:

- What are the affordances of the utilisation of *Geoboards* as visualisation tools in the teaching of the properties of quadrilaterals in Grade 7 classes?
- What are selected teachers' experiences of using *Geoboards* as visualisation tools in teaching the properties of quadrilaterals, as a result of participating in an intervention programme?
- How do the participating teachers make use of the Van Hiele phases in their teaching of quadrilaterals using the *Geoboard*?

My decision to carry out this investigation was inspired by a suggestion in the literature that as a physical manipulative learning tool, *Geoboards* can provide a means to act upon the world and cognitively scaffold learners to construct knowledge (Salomon & Perkins, 1998) – in this case mathematical knowledge of quadrilaterals through the integration of visualisation processes.

3.3 RESEARCH ORIENTATION

A research study is usually shaped by a paradigm. Patton (2002) described a paradigm as "a world view, a general perspective, and a way of breaking down the complexity of the real world" (p. 69). According to Guba (1990, p. 17), this world view is composed of "a basic set of beliefs that guide action". Therefore, as a researcher the set of beliefs I bring forth in this study are informed by the interpretive paradigm. Literature convinced me that the interpretive paradigm is an appropriate framing for my study because it is guided by the basic philosophical assumption that "meaning which informs human behaviour… and results are created not found, thus the reason why the researcher needs to engage in the situation" (Bertram & Christiansen, 2014, p. 26). Cohen, Manion and Morrison (2007) also advocate that the central purpose of the interpretive paradigm is to "understand the subjective world of human experience" (p. 21).

My study aims to understand human behaviour and action, with reference to the teaching practice of using *Geoboards*. Therefore, I needed to be positioned in a learning environment where teachers actively use *Geoboards* as manipulatives to teach the properties of quadrilaterals. Cohen et al. (2007) assert that interpretive researchers "begin with individuals and set out to understand their interpretations of the world around them" (p. 22). Thus, the interpretive paradigm directly helped me to engage with and interpret data related to the teaching practice that involved the use of *Geoboards*. Moreover, the interpretive paradigm forced me to immerse myself in data that consisted of teachers' experiences of using *Geoboards.* It also underpinned my sense-making and interpretation of the participating teachers' interactions with the *Geoboards* when teaching the properties of quadrilaterals.

3.4 RESEARCH METHODOLOGY

This research adopts a case study method. Stake (1995) defined a case study research as an investigation and analysis of a single or collective case, intended to capture the complexity of the object of study. In the context of this study, the case involved three Grade 7 mathematics teachers in the Opuwo circuit of the Kunene region in Namibia. The case consisted of three teachers' teaching practices of quadrilaterals when using *Geoboards* as visualisation tools. In particular this case study focused on the use of *Geoboards* as physical manipulatives that employed visualisation processes, the Van Hiele model of geometrical thinking in the teaching of quadrilaterals and experiences and perceptions of selected teachers.

This case study methodology paved a way for me to observe the effects of using *Geoboards* in a real context. Cohen et al. (2007) asserted that one of the strengths of case studies is that researchers observe effects in real contexts. Hyett, Kenny and Dickson-Swift (2014) also motivate the use of case studies by reasoning that:

> Case study research has a level of flexibility that is not readily offered by other qualitative approaches such as grounded theory or phenomenology. Case studies are designed to suit the case and research question and published case studies demonstrate wide diversity in study design. (p. 1)

At the core of my study was an intervention programme. The intervention programme was intended to provide the participating teachers with an environment that made it possible for them to experience ‘what it was like’ to teach the properties of quadrilaterals using *Geoboards* as visual manipulatives. This was inspired by Cohen et al. (2000) who posited that “case studies strive to portray ‘what it is like’ to be in a particular situation, to catch the close-up reality and ‘thick description’ of participants’ lived experiences of thoughts about and feelings for, a situation” (p. 182). The unit of analysis for this study was two-fold: firstly, it entailed the teaching practice of the participating teachers, and secondly it entailed the experiences and perceptions of those same teachers. Yin (2009) states that the unit of analysis is the data and the analysis of this data in relation to the research questions.

I decided to use a qualitative research approach because I felt that it aligned well with the interpretive paradigm and suited my research questions most effectively. In a qualitative approach as the researcher I was interested in understanding how participants constructed meaning of a situation under investigation (Merriam, 2002) – in this case the teaching with *Geoboards*. Thus, the use of a qualitative approach helped me to gain an in-depth understanding of how the three selected teachers used *Geoboards* as visualisation tools to teach the properties of quadrilaterals. Kothari (2004) stressed that a qualitative approach to research is “important in behavioural sciences where the aim is to discover the underlying motives of human behaviour” (p. 3), and it is also concerned with “subjective assessment of attitudes, opinions and behaviour” (p. 5).

3.5 RESEARCH DESIGN

This case study unfolded in five phases:

1. Securing of research site and selection of participants,
2. Designing *Geoboards*, awareness workshop and planning of lessons,
3. Piloting of the learning programme and research instruments,
4. Implementing learning programme and video-recording, and
5. Interviewing and analysing.

3.5.1 Phase 1: Securing of research site and selection of participants

In Phase 1 I secured the three research sites (three schools in the Opuwo circuit) by writing consent letters to the selected teachers, the affected learners, the parents, the inspector of education as well as the principals of the three schools where the intervention was planned to take place. The details of how these letters were written and administered are discussed under the ethical considerations at the end of this chapter. The selection criteria for the participants is also discussed under selection of participants in 3.6. The teaching process of the intervention programme was planned to take place at the three respective primary schools of the three participating teachers in the Opuwo circuit.

3.5.2 Phase 2: Designing *Geoboards*, awareness workshop and planning of lessons

Phase 2 involved the implementation of three workshops at my work place. The first workshop was on designing *Geoboards* with the participating teachers. I deemed it necessary to arrange for a local carpenter to help us design and manufacture at least ten ten-by-ten *Geoboards* that were to be used in the intervention programme. I provided all the materials that were needed to make the ten *Geoboards*. I insisted that research participants were involved in the designing and manufacturing of the *Geoboards* in order to capacitate them with the knowledge and skills of how to construct their own *Geoboards* at a later stage. In the second workshop the participants and I discussed possible ways of using *Geoboards* with reference to the visualisation aspects of this manipulative. The purpose of the second workshop was fourfold:

Firstly, to orientate and familiarise teachers on how *Geoboards* can be used to construct and manipulate different types of basic shapes. Secondly, to analyse and discuss literature on visualisation in general and visualisation processes in particular. The presentation focused on visualisation processes that were discussed in the review of literature chapter. Thirdly, the Van Hiele model of geometrical thinking was also discussed. Fourthly, together with the participants we practised the use of *Geoboards* in order to see how visualisation processes and the Van Hiele model of teaching geometry could be related.

In the third workshop, we designed a learning programme that consisted of six lesson plans (two per teacher) on the properties of quadrilaterals. The lessons were allocated as follows amongst the three teachers:

Table 3. 1 Allocation of lessons

Participant	**Lesson No.**	**Type of quadrilateral**
Teacher one	1 2	• Square • Rhombus
Teacher two	3 4	• Parallelogram • Trapezium
Teacher three	5 6	• Rectangle • Kite

The lesson preparations were all planned on the official regional lesson preparation template forms to make teachers feel familiar about the lessons they were planning and about the relevance of this study with respect to the present syllabus (see Appendix F). The only information that was added on the template was the details and the order of the Van Hiele phases of teaching geometry. The use of *Geoboards* as visualisation tools in concurrence with the Van Hiele model were central to the six planned lessons.

All three workshops took place at my own school, which did not coincide with any of the three participating schools.

3.5.3 Phase 3: Piloting of the learning programme and research instruments

The six lesson plans were shared among the three participating teachers. Each teacher piloted one lesson with a class in his or her school. I video-recorded each of the pilot lesson for data collection and testing of the analytical tools. The video-recording process in this phase was also a piloting exercise that allowed me to determine the effectiveness of the video-recording process as well as the learning programme before the actual implementation. For example, I was able to tell if the way I handled the video camera as well as the angle, position and distance from the recorded teacher was not disruptive to his or her actions in the classroom.

The piloting phase also helped me and the participants to reflect on each first lesson for the teacher. Together with all the participants, we reflected on each of the piloted lessons in order to identify and address any shortcomings that might have been experienced before the actual implementation. By so doing, I allowed for reflection of the first lesson of each teacher before proceeding to the next lesson. Luwango and Schäfer (2013) comment that "during reflection, teachers identify their strengths and weaknesses that culminate in better planning as alternative teaching approaches are adopted, eventually meeting the demand of mathematical proficiency" (p. 62). Most importantly from what I have experienced, I was also able to align the analytical tool for the first research question on each pilot lesson to see how effective it would be.

3.5.4 Phase 4: Implementing learning programme and video-recording

In this phase all the six lessons, including the piloted lessons, were implemented. Each teacher taught three lessons: one pilot lesson and the other two actual implementation lessons. This was a total of nine lessons presented for data collection of this study. All the nine lesson presentations were video-recorded for data analysis purposes. The data was used to answer the first research question on the affordances of the utilisation of *Geoboards* as visualisation tools in the teaching of the properties of quadrilaterals. The implementation of the learning programme was scheduled in the afternoons to avoid disturbing the normal teaching time, as was promised in the letters to the Inspector of Education and the three respective school principals, when I requested permission to conduct research.

The schedule of the teaching programme was as follows:

Table 3. 2 Schedule of the lessons

Teacher	Lesson # and Topic	Date
Piloting stage		
Teacher 1	Lesson 1: Properties of a Square	06 March 2018
Teacher 2	Lesson 3: Properties of a Rectangle	07 March 2018
Teacher 3	Lesson 5: Properties of a Trapezium	08 March 2018
Lesson implementation		
Teacher 1	Lesson 1: Properties of a Square	12 March 2018
	Lesson 2: Properties of a Rhombus	14 March 2018
Teacher 2	Lesson 3: Properties of a Rectangle	02 April 2018
	Lesson 4: Properties of a Parallelogram	05 April 2018
Teacher 3	Lesson 5: Properties of a Trapezium	15 March 2018
	Lesson 6: Properties of a Kite	23 April 2018

3.5.5 Phase 5: Interviewing and analysing.

In this phase I carried out two types of interviews: The Stimulus Recall Interviews (SRI) and the Focus Group Interviews (FGI). The interviews generated data to answer the second research question of the study. The details of how each of the interviews was administered are discussed under research techniques. All the recorded videos and interviews were transcribed and qualitatively analysed. Patton (2002) contends that the transcription of data is a transition between fieldwork and full analysis. Thus, as a researcher, when I transitioned from data collection to data analysis I had “a chance to get a feel for the cumulative data as a whole” (p. 441). In other words, the transcriptions of the audio-video-recorded tapes enabled me to immerse myself in the data.

3.6 THE SELECTION OF PARTICIPANTS

This study involved three participants. The selection criteria for choosing these participants were based on the following: firstly, I purposively selected the participating teachers based on their willingness to participate in the research project. Secondly, I selected teachers with at least three years teaching experience. Thirdly, and very importantly, I gave priority to teachers who had attended the new revised curriculum workshop on Grade 7 mathematics conducted by the government. The rationale behind selecting this number of participants was based on the scope of the study and the conviction that they would generate rich data for my study. According to Mertens (2005), researchers typically select samples with a goal of identifying information-rich cases that allow them to study a case in-depth when working within the interpretive paradigm. Therefore, the fact that the participants were Grade 7 teachers from three different schools, convinced me that their participation in my study would provide rich and comprehensive data that would enable me to answer the research questions of the study.

Moreover, each of the participants had a fair amount of teaching experience which was necessary to contribute meaningfully to the study. It was also hoped that their participation in the 2016 regional workshop would ensure that they brought to the intervention some knowledge and sense of the current developments in the curriculum at a national level. The other reason why I chose these participants was for convenience purposes. They all worked in the same cluster system as my school — hence they were located not far from each other.

3.7 DATA COLLECTION TECHNIQUES

Data collection techniques are the means to find answers to the research questions of this study. Thus, to collect in-depth rich data that answered the research questions, I used the following data collection methods:

3.7.1 Observation

Observation was the primary data collection tool of this study. Kumar (1996) explains observation as a "purposeful, systematic and selective way of watching and listening to an interaction or phenomenon as it takes place" (p. 105). Observations provided evidence of how the participants used *Geoboards* in a real time context (Merriam, 2001). All the nine lessons were recorded and analysed for data collection. The lessons ranged from thirty-five to forty-five minutes long. I analysed the recorded videos to find out how the teachers used *Geoboards* in relation to visualisation processes and the Van Hiele model in their teaching of quadrilaterals, and thus answered the first research question.

I subjected the video-recordings to both analytical tools (see Tables 3.3 and 3.4 under data analysis). The analytical tool in Table 3.3 was used to observe how teachers used *Geoboards* as visualisation tools to incorporate visualisation processes such as *concrete pictorial imagery*, *dynamic imagery*, *perceptual apprehension*, *sequential apprehension*, *discursive apprehension* and *operative apprehension*. The analytical tool in Table 3.4 was used to determine the extent to which the Van Hiele phases such as *Information*, G*uided orientation*, *Exiplicitation*, *Free orientation* and *Integration phase* of teaching geometry were applied by the teachers when they taught quadrilaterals with the *Geoboards*.

One possible shortcoming of observations is that "it is practically impossible to observe everything that is happening in any situation, especially one where there are different interactions going on among a number of people (Bertram & Christiansen, 2014, p. 94). This was also experienced in this study, hence the reason why I relied extensively on the video-recordings of the lessons. At times I would look at what learners were doing while focusing on the teacher, and this was sometimes a little distracting and not always satisfactory. Although I might not have captured every move and action of the teacher during the lesson, I still believe that I collected rich enough data to enable me to answer my research questions.

3.7.2 Interviews

Interviews were my other data collection technique. I found interviews to be appropriate for my study as they enabled me to explore the lived experiences of the respondents. Bertram and Christiansen (2014) state that an interview is "a conversation between the researcher and the respondent" (p. 80). Hesse-Biber and Leavy (2011) comment that the conversation between the researcher and the participant is aimed at producing knowledge. Therefore, the use of interviews helped me to collect data and produce knowledge that could not be directly observed in the videos (Patton, 2002). The interviews provided data that answered the second research question which sought to understand the teachers' experiences of the use of *Geoboards* as visualisation tools when teaching quadrilaterals. I used two types of interviews:

Stimulus Recall Interviews

For me to get reliable data, I used stimulus recall interviews (SRI). Lyle (2003) described SRI as "an introspection procedure in which (normally) videotaped passages of behaviour are replayed to individuals to stimulate recall of their concurrent cognitive activity" (p. 861). The use of SRI assisted the participants to recall their actions after viewing the video-recordings of the lessons with me. During the interviews I frequently stopped the video to provide time for the teacher to reflect on his or her lesson. It was in these pauses that I asked a series of structured, but relatively open-ended questions. Most of the questions were based on the analytical framework illustrated in Table 3.3 and Table 3.4. Although I had prepared the questions in advance (see Appendix G), however, during the reflection on the videos, more probing questions followed depending on how the interviewees responded. The precise nature of questions also depended on what transpired in the video-recordings as we observed the particular lesson. The questions revolved around the teachers' experiences and perceptions on the use of *Geoboards* in relation to visualisation processes. All the SRIs were voice recorded and transcribed for data analysis purposes.

Focus Group Interviews

According to Morgan as cited in Cohen et al. (2011), a Focus Group Interview (FGI) is a type of interview that involves an interviewer and a group of interviewees, thus data is collectively collected from more than one person at the same time. By using this type of data collection tool (Appendix H) I sought to validate the participants' points of view on the use of *Geoboards*. The FGIs created a platform where the participants could possibly correct each other and come to a more common agreement should there be a vast gap in opinions as to how they experienced the use of *Geoboards*. I admit that I was aware of the possible shortcomings of the FGIs. Literature informed me that "information collected through a group process may be biased since less assertive participants may be dominated by more outspoken individuals" (Nieuwenhuis, 2007, p. 91). So, during the interviews I made sure that all participants had an equal chance of responding to the question. I directed questions to specific individuals and I also probed everyone on each question asked. Hydén and Bülow (2003) credit the FGI for its ability to yield insight that would not normally be available in straightforward individual interviews. Thus, this form of data collection proved to be very useful in my study.

Bertram and Christiansen (2014) warn that power relations can influence the outcome of the interviews. In my view I mitigated against this in my study by ensuring that the intervention programme was truly collaborative and cooperative. I made sure that I was seen as an equal with my participants. As discussed under the ethics section in the methodology chapter, I strived to build a mutual relationship between us by insisting that we were all working towards a common goal of learning together. Hence, during the interviews it was easy for the teachers to talk freely and openly share their experiences of the use of *Geoboards* with me.

3.8 ANALYTICAL FRAMEWORK

My data analysis was an ongoing process and it started concurrently with data collection as advocated by Coffey and Atkinson as cited in Maxwell (2005), who claimed that "we should never collect data without substantial analysis going on simultaneously" (p. 95). I used two analytical tools for the study. The first tool was adapted from Presmeg (1986) and Duval (1995), as shown in Table 3.3, and it was used to analyse the recorded videos. It consisted of observable indicators on how *Geoboards* were used as visualisation tools. The analytical tool helped me to analyse data in a quest to find answers to the first research question. In particular, the observable indicators sought to find evidence of the affordances of the utilisation of *Geoboards* when teachers administered the six visualisation processes used in this study. The second analytical tool, Table 3.4, was adapted from Van Hiele (1999). The tool was mainly used to analyse the SRIs and FGIs interviews in order to establish the teachers' experiences and perceptions on the use of *Geoboards* in relation to the Van Hiele phases of teaching geometry.

Both the visualisation processes and Van Hiele phases of teaching geometry in the two analytical frameworks were thoroughly discussed in the literature review chapter.

Table 3. 3 Visualisation processes indicators to analyse how Geoboards are used as visualisation tool

Type of visual imagery/ cognitive apprehension	**Visualisation processes indicators**	**Coding**			
		0	1	2	3
Concrete pictorial imagery	There is evidence of the use of *Geoboards* that encourages learners to form mental pictures of the properties of quadrilaterals.				
Dynamic imagery	There is evidence that the teachers encouraged the manipulation of static *Geoboard* figures to dynamic processes by changing the position(s) of rubber bands and number of pegs to transform shapes.				
Perceptual apprehension	There is evidence that the teacher used the *Geoboards* to assist learners to simply recognise basic shapes. These are not necessarily relevant to the constructions of quadrilaterals.				
Sequential apprehension	There is evidence that the teachers facilitated the independent constructing of shapes using the *Geoboards*. Learners are encouraged to construct and describe shapes on their own.				
Discursive apprehension	The teachers encouraged learners to verbally describe the properties of the constructed shapes.				
Operative apprehension.	The teacher sets problems for the learners to solve using the *Geoboard.*				

Adopted from Presmeg (1986) and Duval (1995)

Table 3.3a Coding descriptors for Table 3.3

Coding	**Categories**	**Descriptions** (visualisation process)
0	No evidence	There is no evidence of visualisation processes on the use of *Geoboards*
1	Weak evidence	There is little evidence of visualisation processes on the use of *Geoboards* (1-2 incidences)
2	Medium evidence	There is sufficient evidence of visualisation processes on the use of *Geoboards* (2-3 incidences)
3	Strong evidence	There is abundant evidence of visualisation processes, more than three incidences on the use of *geoboards*

Table 3. 4 Indicators of the evidence of the use of geoboards as visualisation tools to execute the Van Hiele phases of teaching geometry under visual level.

Phase	Description of indicator	Score
Information	*Geoboards* are used as visualisation tools in attempting to find out what learners already know about the properties of shape(s).	
Guided orientation	*Geoboards* are used as visualisation tools to guide learners to form different shapes based on their properties.	
Explication	Teacher(s) give learners an opportunity to discuss the properties of the shapes that are constructed on *geoboards.*	
Free orientation	*Geoboards* provide activities that enable learners to link relevant relationships of the properties of shapes.	
Integration	Properties of geoboard figures are identified and named by learners through the help of the teacher. Thus, to provide evidence that the use of *Geoboards* as a visualisation tool has prepared learners for the next level (descriptive/analysis).	

Adopted from Van Hiele (1999).

Table 3.4a Coding descriptors for Table 3.4

Code/score	Categories	**Descriptions** (Van Hiele phase(s))
0	No evidence	There is no evidence of the phases of Van Hiele used
1	Weak evidence	There is little evidence of the Van Hiele phases on the used (1-2 incidences).
2	Medium evidence	There is some evidence of the phases of Van Hiele used (2-3 incidences).
3	Strong evidence	There is an abundance of evidence of the phases of Van Hiele used.

3.9 RELIABILITY AND TRUSTWORTHINESS

Any research should be reliable and trustworthy in order to be valid. Schwandt (1997) defines validity as how accurately the account represents participants' realities of the social phenomena and is credible to them. Thus, in a qualitative research approach I needed to demonstrate that my study was credible and that I did not manipulate or misinterpret participants' words and actions. Therefore, to interpret the findings and overcome validity threats I used triangulation to blend between my sources of data. Triangulation refers to the collection and analysis of information using a variety of sources and methods (Fielding & Fielding, as cited in Maxwell, 2005). According to Plano-Clark and Creswell (2015), triangulation is done by corroborating evidence of findings from different individuals or data sources. Thus, to validate the data of this study, I used triangulation (observations, SRI and FGI analysis). I used the SRI and FGI to validate the authenticity of the video-recordings. This was done by replaying the videos and asking probing questions to validate the teachers' actions and words in the videos.

I also used member checking validation to determine whether my findings were accurate. Plano-Clark and Creswell (2015) mention that "member checking is a process in which the researcher asks one or more participants to check the accuracy of the findings" (p. 364). I therefore gave all my participants the transcribed SRI and FGI to validate whether what was recorded was indeed what was said. I also requested them to make any changes if they did not agree with what was transcribed. Moreover, I also piloted some of the lessons as well as data collection tools before the actual implementation. Most importantly, the mere fact that data was collected from the lived and perceived experiences of the participants, validation was guaranteed as the participants had first-hand experience of the use of *Geoboards* as visualisation tools.

3.10 ETHICS

Cavan (1977) defined ethics as "a matter of principled sensitivity to the rights of others, and that 'while truth is good, respect for human dignity is better" (p. 810). My study adhered to research ethics by ensuring the following ethical principles:

3.10.1 Respect and dignity

As a researcher I sought permission for access to the three schools where the research was conducted (Cohen et al., 2000). I wrote consent letters to the relevant circuit inspector of education and the principals of the three schools where the research was conducted. After permission was granted from the above-mentioned office bearers, I sought consent from the participating teachers that I recruited for my research project. Before the implementation of the study I emphasised to the participants that their participation was voluntary and that they could withdraw at any time they wished. To protect, guarantee anonymity and safeguard the confidentiality of interviewees in the study I used the pseudonym of Mr Jones, Ms Ruth and Ms Smith, respectively.

Moreover, I informed participants that I was going to video-record their lessons and that the lessons were going to be briefly viewed and discussed with the other participants for reflection purposes. Although I intended to video-record teachers only, I still went ahead and sought permission from parents of learners who belonged to the teachers' mathematics classes. This was done by writing letters, both in English and the local language, Otjiherero (in the event of a parent not being able to understand the English consent letter) - see Appendix E. The letters informed parents about the purpose of the intervention. It was also explained to parents that the focus of my observations was limited to teachers only, thus I was not going to video-record the learners. I assured the parents that their children were not going to feature in the recorded videos. However, some parents did not return the consent letters and I did not include their children in the intervention. I also assured the school principals, the teachers and the parents that the intervention programme was not going to disturb any official school activities as it was planned to be executed in the afternoons.

3.10.2 Transparency and honesty

I demonstrated honesty in this study by clearly explaining the purpose of the research to the participants. The participants were informed about what the study was about. I also informed them about what I was going to do with the findings of the study. Transparency was exercised in that I made sure that I clearly and honestly disseminated all the information related to the study to everyone who was involved; this included parents, teachers and even learners. Before the commencement of each interview I again reminded the participants that they were voluntarily agreeing to the interviews and that they were not compelled to answer questions if they did not feel like it and had a right to stop or withdraw at any time. I ensured total transparency by not giving any false impressions or unrealistic expectations to any of the participants.

3.10.3 Accountability and responsibility

I demonstrated accountability in this research by safe-guarding the collected data during and after the research project. In my interactions with the participants I was conscious of facilitating an atmosphere and ethos that was democratic, non-threatening and trusting. The data of this research will be kept for five years on my computer and at the Education Department of Rhodes University. I will ensure the safekeeping of the data on my computer by protecting it with a case-sensitive password.

3.10.4 Integrity and academic professionalism

Honesty and integrity were my guiding principles in this study. I guarded against plagiarism by adhering to the Rhodes University's Education Department Guidelines for Academic Writing and Referencing of 2016. I presented data the way it was collected without manipulating or influencing it to suit my own assumptions.

3.10.5 Researcher positionality

Although I do not believe that my position as a teacher influenced the participants' actions or responses, I was however conscious of the chances of that happening. So, I made sure that I did not influence nor compromise the participants' conduct and responses by distancing myself from uttering my own thoughts and opinions on what we were doing. Right from the beginning of planning the learning programmes, I made it clear to the teachers that we were colleagues, and that together we were to strive to learn how to use *Geoboards*. Thus, they should not look at me as the 'know it all'. I beseeched them that I was also learning, so we were on the same level. I was explicit that I wished to learn from them, hence the reason why I was studying their practice. I always endeavoured to build a trustworthy relationship with the participants by being democratic and professional. Throughout the research study I upheld high levels of conduct and ensured that I represented the Education Department of Rhodes University to the best of my abilities.

3.11 CONCLUSION

In this chapter I described the methodological approaches I followed to conduct my study. I detailed the research paradigm, the research method, the research design and the data collection techniques I used. I furthermore explained how I analysed the collected data. I also outlined the ethical principles I followed, to adhere to appropriate research ethics principles.

In the following chapter I present, analyse and discuss the findings of the collected data.

CHAPTER 4

DATA ANALYSIS AND PRESENTATION

4.1 INTRODUCTION

In this chapter I report on the findings of the data collected from my investigations. The findings provide the necessary information and argumentation to answer the three main research questions, which are:

- What are the affordances of the utilisation of *Geoboards* as visualisation tools in the teaching of the properties of quadrilaterals in Grade 7 classes?
- What are selected teachers' experiences of using *Geoboards* as visualisation tools in teaching the properties of quadrilaterals, as a result of participating in an intervention programme?
- How do the participating teachers make use of the Van Hiele phases in their teaching of quadrilaterals using the *Geoboard*?

Since my research study sought to investigate the use of *Geoboards* as visual tools, the goals of the research were to attempt to find out: firstly, the affordances of the utilisation of *Geoboards* as visualisation tools in developing and enhancing conceptual understanding of the properties of quadrilaterals; secondly, it also aimed to explore and gain insights into teachers' perceptions and experiences of using *Geoboards* as visualisation tools after the implementation of an intervention programme; and thirdly, to establish the extent to which the Van Hiele phases of teaching geometry can be applied to teaching quadrilaterals with *Geoboards.*

Therefore, I present the findings that emerged out of this study as follows:

I start by giving a short description of the layout of how I analysed the data. Then I give a brief overall description of the analytical tool that was used to analyse the first research question. In presenting data for the first research question, I present a bar chart that depicts the use of visualisation processes. I also briefly summarise how each of the six visualisation process was used with the *Geoboard* during the lesson. Data collected from interviews is also presented to answer the second research question about teachers' experiences on the use of *Geoboards*. Furthermore, I discuss the implementation of the Van Hiele model for all three of the lessons of each teacher, for the provision of data of the third research question. Thereafter I discuss the findings in terms of the similarities and differences in the way the visualisation processes and the Van Hiele phases were used by the teachers.

Since I used the interpretive paradigm to underpin my data analysis I strived to make sense of the huge volume of the data I had collected. I did this by clustering data into themes that were in line with my analytical frameworks, as well as themes that arose with respect to the second research question of the teachers' experiences. I also let the data speak for itself to a large extent in an attempt to portray exactly how the respondents viewed the use of *Geoboards* when teaching the properties of quadrilaterals. Denzin and Lincoln (2011) explain that qualitative analysis transforms data into findings and that the challenge of qualitative analysis lies in making sense of a huge amount of verbal data. Thus, the two analytical processes described above (my interpretation of the data and letting the data speak for itself) happened in parallel.

I profiled and coded my research participants and sources of data. Kothari (2004) describes coding as "the process of assigning numerals or other symbols to answers so that responses can be put into a limited number of categories or classes" (p. 123). Therefore, I deemed it important to present codes and profiles of participants because it helps the reader to understand the data sources based on the data presented. Coding is also important for confidentiality and anonymity of the participants. In the case of my study, I used pseudonyms to protect and hide the identity of my participants.

4.2 DESCRIPTION OF THE CODING OF THE ANALYTICAL FRAMEWORK

Before I discuss the findings of this study, and for ease of reading, it is important for me to explain to the reader the coding that I employed and provide a description of the analytical tools I used. The description presented in this chapter is in a 'vertical' form and it is structured as follows:

1. Profile and coding of the teacher.
2. Brief description of the lesson.
3. Visualisation processes: here I first present a bar chart which shows the extent of evidence to which the visualisation process occurred in the lesson (see analytical tool in chapter 3.3). Thereafter I discuss how each of the six visualisation processes was used by the teacher concerned.
4. Stimulus recall interviews: here I present and discuss the individual stimulus recall interviews with each teacher after the presentation of all the three lessons.
5. The Van Hiele model: here I first present a table to show the extent of evidence to which the Van Hiele phases were used on all the three lessons of each teacher on the *Geoboard*. Then I briefly narrate how each phase was used on the *Geoboard*.
6. Focus group interviews: here I present and discuss the group interview I had with all the three participants after the intervention programme.

Table 4.1 below shows the coding criteria which were used for each of the graphs (numeric data) drawn from the observation instruments:

Table 4. 1 The coding criteria used for the graphs

0	No evidence	There is no evidence of visualisation processes on the use of G*eoboards*
1	Weak evidence	There is little evidence of visualisation processes on the use of G*eoboards* (1-2 incidences).
2	Medium evidence	There is enough evidence of visualisation processes on the use of G*eoboards* (2-3 incidences).
3	Strong evidence	There is abundant evidence of visualisation processes, more than 3 incidences on the use of G*eoboards.*

The visualisation processes are abbreviated as follows: *Pictorial imagery* **(PI),** *Dynamic imagery* **(DI)**, *Perceptual apprehension* **(PA)**, *Sequential apprehension* **(SA)**, *Discursive apprehension* **(DA)** and *Operative apprehension* **(OA)**.

4.3 TEACHER ONE: MR JONES

4.3.1 Profile and Coding

Mr Jones has been teaching mathematics to Grade 7 for about eleven years now. He holds a Basic Education Teacher Diploma (BETD) from Caprivi College of Education, now University of Namibia. He specialised in mathematics education Grades 8–10 and English language education Grades 4–7. Since he started teaching he has been teaching mathematics Grades 4–7. Data for Mr Jones is coded as follows: Video one (**JV1**), video two (**JV2**), Video three (**JV3**), Stimulus Recall Interviews (**JSRI**) and Focus Group Interviews (**JFGI**).

4.3.2 Lesson 1

4.3.2.1 A brief overview of the lesson

Mr Jones taught three lessons. The first lesson was a pilot lesson that aimed at setting the stage for the teacher. Mr Jones taught the properties of a square. The video of the lesson was forty minutes and fourteen seconds long. The objective of the lesson was to teach learners the properties of a square in terms of its sides, angles and symmetry. The teacher used a *Geoboard*. The class was divided into groups of four to five learners each. Each group was given its own *Geoboard* and elastic bands to use.

4.3.2.2 Visualisation processes

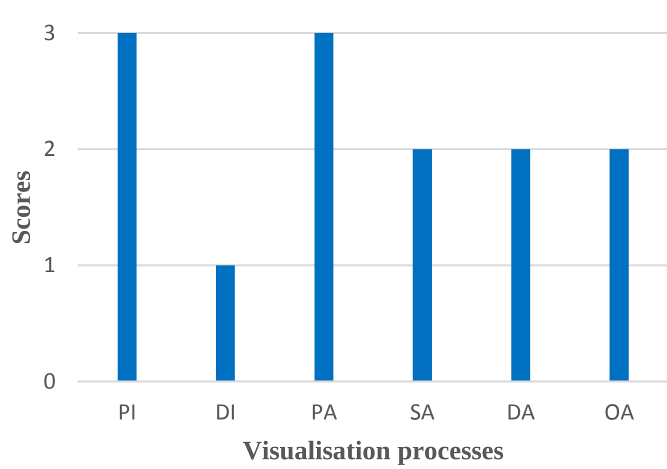

Figure 4. 1: Numerical scores of Mr Jones' use of visualisation processes. Lesson 1.

Figure 4.1 above summarises the numerical scoring for each of the six visualisation processes for lesson one. Each visualisation process is measured by 0–3 points which correspond to the coding criteria in Table 4.1. The analysis of data from Figure 4.1 indicates that there was abundant evidence of incidences where Mr Jones used the *Geoboard* to enable learners to make use of *concrete pictorial imagery* and *perceptual apprehensions*. *Sequential*, *discursive* and *operative apprehensions* were moderately used in the lesson. On the other hand, there was little evidence of the use of *dynamic imagery* in the lesson.

Concrete pictorial imagery: Data (JV1) revealed that Mr Jones used a *Geoboard* to help learners visualise and form the image of a square in their minds. This he did in a step-by-step construction of a square by joining four elastic bands together. The result of this activity helped learners to visualise and see the whole shape of a square even before it was completed. The learners easily completed the whole square after the teacher had first constructed the two perpendicular lines on the *Geoboard*. This suggests that learners could see the complete square in their minds even before it was completely constructed, hence the reason why they managed to complete the shape.

Another incidence happened when one of the learners responded to the teacher's question on the properties of a square, when the teacher was summarising on the chalkboard. This learner responded by using gestures to indicate that a square has four equal sides. Again, this suggests that the learner could visualise how the square was constructed on the *Geoboard* and hence tried to express his thought of how a square looked, through the means of indicating its construction in the air.

Dynamic imagery: Mr Jones rarely made use of a *Geoboard* to help learners employ the *dynamic* type of imagery. The only incidence where it looked like *dynamic imagery* was used, is when he asked learners to transform a square with five pins on each side to a bigger square with seven pins on each side.

Perceptual apprehension: The application of *perpetual apprehension* on a *Geoboard* during the trial lesson of a square was very evident. Data revealed that Mr Jones constructed properties related to a square on a *Geoboard* before engaging learners on the concepts. On several occasions he would refer to the big square drawn on his *Geoboard* for any questions related to a square (see Figure 4.2). He asked learners on how many sides a square has, by pointing to the square that was already constructed on his *Geoboard*, for example. Thus, learners would give their answers based on the square that was constructed on the *Geoboard* of the teacher.

Figure 4. 2: Mr Jones asking questions on squares that were constructed on his Geoboard

Sequential apprehension: On two occasions during the lesson Mr Jones asked learners to construct and complete a square on a step-by-step basis. He constructed two perpendicular lines and asked learners to complete the shape by joining two other elastic bands of different colours in such a way that all the sides were equal. For example, Figure 4.3 below shows a learner who is joining a yellow elastic band to the already constructed blue elastic band. The learner is counting the number of pins, thus making sure that they are equal to the other line segment.

Figure 4. 3: A learner connecting an elastic band to the already constructed one

Discursive apprehension: Mr Jones accorded learners opportunities to discuss the properties of a square such as equal and parallel sides, diagonals and symmetry as they were constructed on his and the learners' *Geoboards*. Learners were asked a series of questions by the teacher. However, data shows that Mr Jones did not create many opportunities for learners to engage with each other and discuss the properties of a square in their respective groups. The only time learners engaged in talking was when the teacher asked a question to the whole class.

Operative apprehension: Mr Jones used the *Geoboard* to help learners who did not understand the concept of diagonal and symmetry. On one occasion he asked learners to construct a diagonal on a square that was on their *Geoboards*. One group that did not comprehend the concept of 'diagonal' constructed a small one-by-one square in the corner of the big five-by-five square (see Figure 4.4). To help this group, Mr Jones explained the meaning of the concept 'diagonal' and later asked learners to show him the opposite-end corners of their square. Then he operated on the figure by removing the wrongly constructed diagonal and used a different rubber band to construct the correct diagonal. Thereafter, he asked learners to place another rubber in the other two opposite corners for them to come up with another diagonal on the same square as see in the figure below.

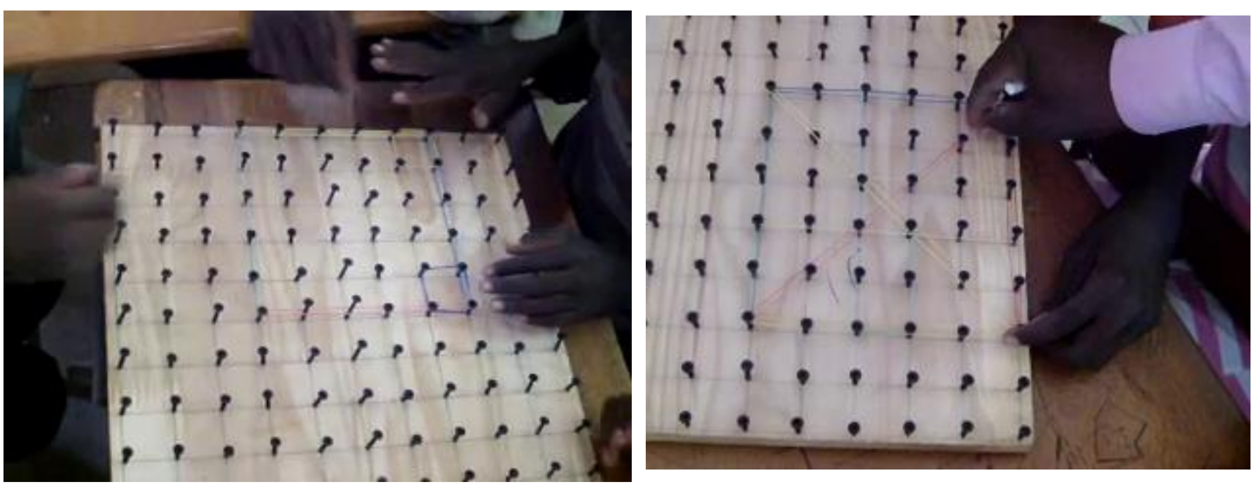

Figure 4. 4: Mr Jones helping learners who wrongly constructed a diagonal line

In another incidence, the same group again had difficulty constructing a line of symmetry because they had used a square that had sides that were composed of even number pins. The square was six-by-six pin array. So, the problem arose from constructing a line that equally divided the square in two congruent parts. This was not possible because the *Geoboard* did not provide them with a set of middle pins for them to place an elastic band across to the other side, but instead there were two pins on the middle. Interestingly, this led them to just hook the rubber band on both the two lines as seen in Figure 4.5. Mr Jones is also seen trying to make the learners see the problem and let them reconstruct a different square with sides of an odd number of pins.

Figure 4. 5: Mr Jones helping learners who failed to construct a line of symmetry

4.3.3 Lesson 2

4.3.3.1 A brief overview of the lesson

After the pilot lesson, we had a reflective discussion with Mr Jones and the other two colleagues. Recommendations for possible changes were made for the improvement of the second lesson. So again, in his second lesson Mr Jones taught the properties of a square. The lesson was forty-five minutes and three seconds long. As indicated earlier, the purpose of the lesson was to teach learners the properties of a square in terms of sides, angles and symmetry. Mr Jones used a *Geoboard*, and the class was divided into groups of four to five learners each. This time around, the learners were reshuffled so that they were grouped differently compared to the grouping of the previous lesson. Each group had its own *Geoboard* and elastic bands to use.

4.3.3.2 Visualisation processes

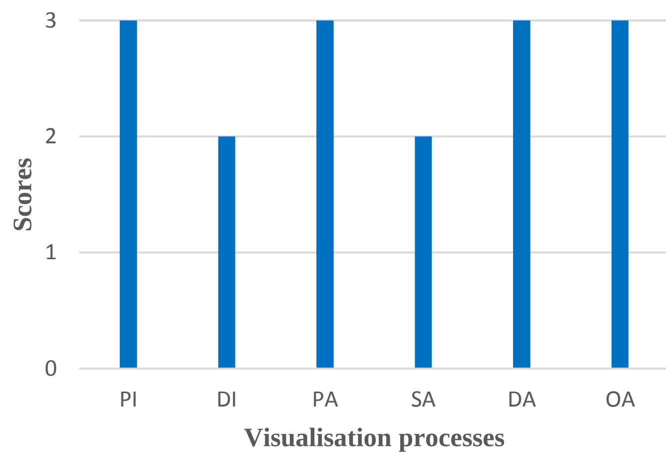

Figure 4. 6: Numerical scores of Mr Jones' use of visualisation processes. Lesson 2.

Figure 4.6 summarises the numerical scoring for each of the six visualisation processes used by Mr Jones in the second lesson. As can be seen from the Figure, there was great improvement in the evidence of incidences where Mr Jones used a *Geoboard* to employ the six visualisation processes. An overview of the bar chart above shows that there was abundant evidence that Mr Jones had used *concrete pictorial imagery*, *perceptual*, *discursive* and *operative apprehensions* in his lesson. The bar chart also shows that Mr Jones made use of the *dynamic imagery* and the *sequential apprehensions* to a slightly lesser extent.

Concrete pictorial imagery: From the data reported in Figure 4.6, *concrete pictorial imagery* is one of the four visualisation processes that was strongly implemented on the *Geoboard* by Mr Jones in his second lesson. This was evident by how he encouraged learners to make use of the *Geoboard* to mentally create images of the properties of a square. On one occasion he challenged learners to form a square from a line segment and line of symmetry that were already constructed on the *Geoboard* (see Figure 4.7). For this, learners managed to complete and construct the square without difficultly. This suggests that they could visualise the image of the whole square from the two given line segments even when it was not physically presented to them.

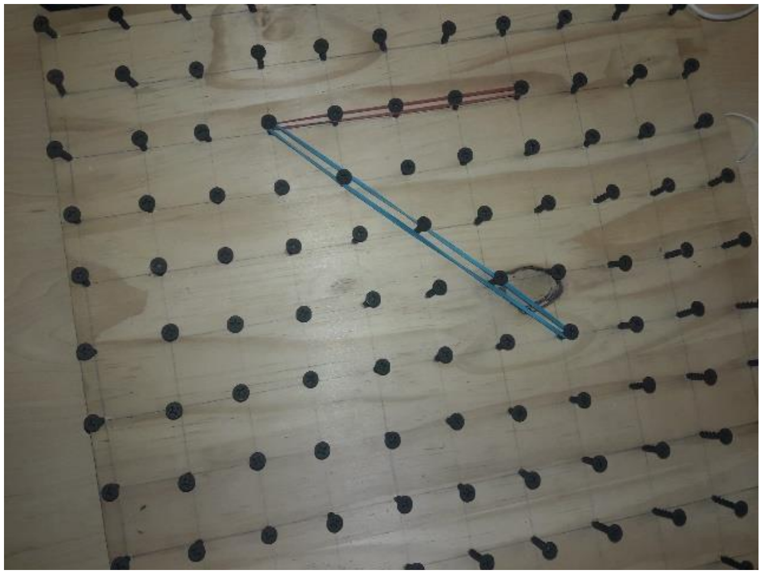

Figure 4. 7: Part of a square with one side of five pins and a line of symmetry

The dynamic imagery: In the second lesson Mr Jones improved on the use of *dynamic imagery* on the *Geoboard*. On two occasions he instructed learners to change the size of a square without dismantling the original shape. In both events, learners pulled up an elastic band from one corner of the shape and stretched it either backwards or forward and placed it on a different pin in such a way that both adjacent sides to the vertex pin had an equal number of pins. However, these are the only incidences where the use of *dynamic imagery* was seen to be evident on the *Geoboard* in this lesson.

Perceptual apprehension: Mr Jones also made use of several constructed line segments to explain the properties of a square on the *Geoboard*. Video data (JV2) shows that he constructed and explained perpendicular, parallel, adjacent, diagonal and symmetry lines on the *Geoboard*. For almost the entire lesson he kept on raising his *Geoboard* to learners for them to see the procedural construction of the square through its properties.

Sequential apprehension: As stated earlier, Mr Jones made use of *sequential apprehension* to help learners visualise shapes on a *Geoboard*. He did this by helping learners to construct a square by means of building it from its respective properties. He also asked learners to copy and reconstruct the same line segments he constructed on his *Geoboard*.

Discursive apprehension: Mr Jones provoked class and group discussions by means of probing questions on the properties of a square. Video data (JV2) shows that most of the time learners were engaged in group discussions while constructing the line segments. What seemed to be interesting is that learners used a *Geoboard* to enhance their discussions by constructing the line segments that were directly related to their discussions (see Figure 4.8).

Figure 4. 8: Learners constructing line segments during the discussions

Operative apprehension: Figure 4.8 above also provides data that shows that Mr Jones gave learners activities that made them operate on squares using *Geoboards*. As clearly seen, learners were physically constructing line segments on a *Geoboard* to establish and understand the properties of a square. Data also revealed that learners were frequently given tasks that required them to either construct lines of symmetry and diagonal lines or find solutions to partly constructed squares by constructing the missing line segments.

4.3.4 Lesson 3

4.3.4.1 A brief overview of the lesson

The third lesson of Mr Jones was on the properties of a rhombus. As planned in the research design, before the implementation of the third lesson we again had a reflective session on the second lesson with all the participants. After reviewing the video of the second lesson, we made some recommendations for the improvement of the third lesson. The lesson was thirty-nine minutes and forty-six seconds long. The lesson aimed at teaching the properties of a rhombus in terms of sides, angles and symmetry.

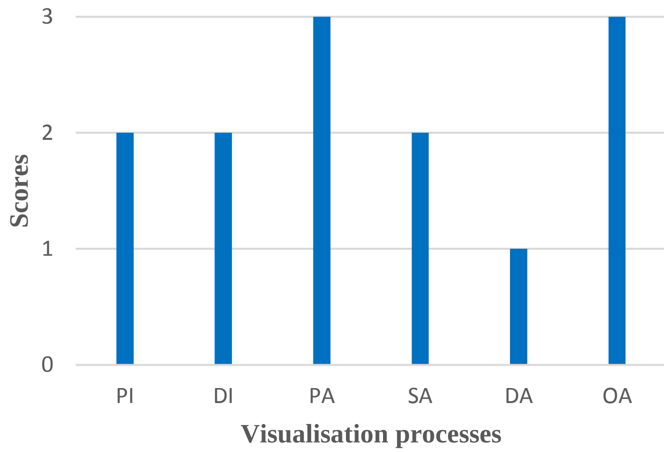

Figure 4. 9: Numerical scores of Mr Jones' use of visualisation processes. Lesson 3.

To sum up the results in Figure 4.9, video data (JV3) shows that there was a decline in the use of visualisation processes in the third lesson of Mr Jones. The *perceptual* and *operative apprehensions* are the visualisation processes that were most used in this lesson. Otherwise *pictorial imagery*, *dynamic imagery* and *sequential apprehension* were used to a lesser extent. *Discursive apprehension* was not used much in this lesson.

Concrete pictorial imagery: Analysing the data in Figure 4.9, we can see that *pictorial imagery* had moderate evidence of being used on the *Geoboard*. In his third lesson, Mr Jones did not encourage learners much to make use of a *Geoboard* for mental image creation. However, one of the few incidences when he seemed to use it, was when he asked learners to identify the shape that was constructed on his *Geoboard*. Data video (JV3) shows that during the introduction to the lesson, Mr Jones constructed a rhombus and asked learners to identify the shape. One of the learners answered that it was a square. Then Mr Jones reminded the learner to remember how a square looked from the previous lessons and then compare it with the shape on the *Geoboard* for her to see if the shapes still looked the same.

Dynamic imagery: Video data (JV3) revealed that the *dynamic* element of the *Geoboard* was explored in this lesson. What came out strongly in this lesson was when Mr Jones asked learners to change a five-by-five side pinned square, to a rhombus. He instructed learners to pull out and stretch one of the two opposite corners. He then demonstrated to learners how to change the square to the rhombus, even though it became a challenge for him to explain how the pins could be counted as the lines were not lying along a straight pattern of pins.

The application of *dynamic imagery* was also seen at one point when Mr Jones turned around the *Geoboard* to make learners understand that a rhombus will remain the same even if the *Geoboard* is turned upside down (as seen in Figure 4.10).

Figure 4. 10: Mr Jones turning the Geoboard upside down

Perceptual apprehension: Mr Jones used geoboard shapes to draw learners' attention to the difference between a square and a rhombus. Learners deduced and compared the similarities and differences between a rhombus and a square by making use of the teacher's *Geoboard*. Moreover, learners identified the properties of a rhombus from what they could see with their own eyes as depicted from the teacher's *Geoboard*. These properties Mr Jones listed on the chalkboard for learners to see.

Sequential apprehension: Mr Jones demonstrated the use of *sequential apprehension* on the *Geoboard* by making learners construct a rhombus in different ways. This was done by instructing learners to construct, in their groups, a rhombus that looked exactly like the one on the teacher's *Geoboard*. Learners were seen constructing and reconstructing the rhombus, by either using one rubber band or by joining four rubber bands of different colours to come up with the shape that looked exactly like the teacher's.

Discursive apprehension: Mr Jones dominated the lesson by doing much of the talking. Data shows that even though he did ask questions to the whole class during the lesson, not all learners responded to these questions. It looked like learners concentrated on constructing shapes on their *Geoboards* instead of discussing the shapes with each other.

Operative apprehension: More time was given to learners to apply the *operative apprehension* visualisation process on the *Geoboard*. Mr Jones let learners explore, both on his *Geoboard* and theirs, the construction of a rhombus in multiple ways. These ways involved tasks such as transforming a square to a rhombus, constructing a rhombus by using only one rubber band and constructing a rhombus by building it from its properties. Figure 4.11 shows the multiple ways learners constructed a rhombus.

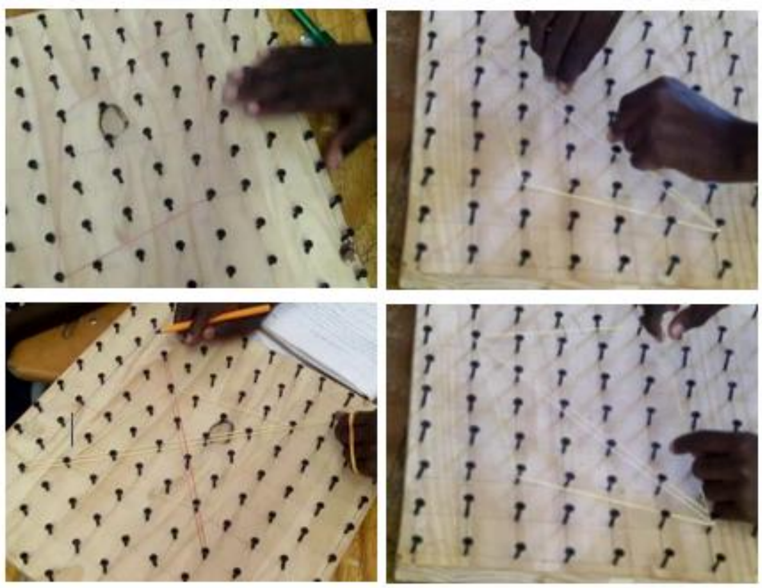

Figure 4. 11: The many ways learners applied the operative apprehension

4.3.5 The teacher's experience of using a *Geoboard*: Stimulus Recall interview

The first three lesson presentations of Mr Jones were followed by a stimulus recall interview that aimed at helping him to reflect on his teaching practice of using a *Geoboard*. The purpose of the interview was to get data on the second research question for the teachers' experiences of the use of the *Geoboard* after the intervention programme. I present data as it emerged, in the following outlined themes:

4.3.5.1 Visual Representation

On the question of the visual aspect of the *Geoboard*, Mr Jones had the following to say:

> *…when you are talking about perpendicular lines they could come and show, prove that this line and this line are perpendicular and where they meet here they form a 90. The opposites and the parallels they could see them on the Geoboard. They could see. They could show even the line of symmetry, but if you are just talking, it is a bit difficult because it is just something that you are talking about but now they could see with their own eyes the way it must be and the way it is.* (JSRI, 219-224).

Mr Jones' response affirms that his experience with the visual aspect of the *Geoboard* was positive. For him, the *Geoboard* helped learners to understand the conceptual definitions of the properties of shapes because they could physically and mentally see these properties through the means of visual representations on the *Geoboard*.

4.3.5.2 Dynamic elements of Geoboards

On the *dynamic* aspect of transforming shapes on the *Geoboard* from one shape to another, Mr Jones said the following:

> *…for example, on the video you could see how we were relating between a square and a rectangle, how to change a square to a rhombus whereby they will just… I mean extend the corners of a square or to put them inside to form a rhombus. So, it is very useful and it's easy to work with* (JSRI, 032-042)

Thus, for Mr Jones, the aspect of changing one shape to another different shape was one of the *dynamic* elements of the *Geoboard*.

4.3.5.3 Geoboards and classroom interaction

With regard to the question of the *Geoboard* and the participation of learners, Mr Jones responded by saying that the learners looked excited during the lessons. He had the following to say:

> "*…everyone was interested, everyone was willing to take part, so it was interesting to them,* [*and*] *they were free to do whatever they had to do there"* (JSRI, 132-133).

Furthermore, Mr Jones preferred the use of a *Geoboard* over a chalkboard by insinuating that a *Geoboard* is more practical than a chalkboard:

> *Ya, you see as I said that this one is more, a Geoboard is more effective compared to the chalkboard because really ne, if the learner talks about equal sides, equal lines, it is easier because they will use the pins to prove that. They can count the pin that; one, two, three, four… one, two, three, four, all the sides have four pins than on the chalkboard where they will just check with their eyes. But here, they will see, and they can prove it physically by counting pins.* (JSRI, 203-208).

In general, Mr Jones seemed to conclude that learners are more active and participative in their interactions when it comes to the use of *Geoboards* compared to the chalkboard. However, he also advised that to ensure proper learning the teacher should always monitor and supervise learners during their interactions with *Geoboards*. His concern was that if not monitored, learners could easily get carried away from the focus of the lesson as they have too much freedom of interaction amongst themselves when playing with rubber bands on *Geoboards*.

4.3.5.4 Challenges with the Geoboard

Not many challenges about the use of *Geoboards* were revealed by Mr Jones in his stimulus recall interview. However, he echoed the same difficulty that I reported on the data of his first lesson on the properties of a square. The problem was more of what might happen when demonstrating lines of symmetry on *Geoboards* with an even number of pins:

> *Ya, there were some hectic somewhere here and there. Especially when it comes the… I mean the lines of symmetry. What I have realised is that if you are making a square using the rubber bands on the geoboard you must be very careful with the pins. Because the pins should equal so that when they use the rubber bands to make the line of symmetry they must put it on the middle ones, on the middle pins. So, if it is not like that, it becomes a problem because it won't give the exact properties* (JSRI, 147-153).

Mr Jones strongly felt that when teaching lines of symmetry on *Geoboards*, it is important to ensure that there are an odd number of pins on each side of the shape. In this way it is much easier to construct lines that are equally going to divide a shape into two congruent shapes.

4.3.6 The Van Hiele phases of teaching geometry

Table 4.2 below shows the numerical scores of the evidence on the use of the Van Hiele phases of learning geometry in all the three lessons of Mr Jones.

Table 4. 2 Numerical scores of the evidence of Van Hiele phases in Mr Jones' lessons

Phase	Description of indicator	Score		
		Lesson 1	**Lesson 2**	**Lesson 3**
Information	*Geoboards* are used as visualisation tools in attempting to find out what learners already know about the properties of shape(s).	3	3	3
Guided orientation	*Geoboards* are used as visualisation tools to guide learners to form different shapes based on their properties.	3	3	3
Explication	Teacher(s) give learners an opportunity to discuss the properties of the shapes that are constructed on *geoboards.*	2	3	1
Free orientation	*Geoboards* provide activities that enable learners to link relevant relationships of the properties of shapes.	2	2	2
Integration	Properties of *Geoboard* figures are identified and named by learners through the help of the teacher. Thus, to provide evidence that the use of *Geoboards* as a visualisation tool has prepared learners for the next level. (descriptive/analysis).	1	2	2

Coding descriptors for Table 4.2

Code/score	Categories	Descriptions (Van Hiele phase(s))
0	No evidence	There is no evidence of the phases of Van Hiele used
1	Weak evidence	There is little evidence of the Van Hiele phases on the used (1–2 incidences).
2	Medium evidence	There is some evidence of the phases of Van Hiele used (2–3 incidences).
3	Strong evidence	There is an abundance of evidence of the phases of Van Hiele used.

Information phase: The table above shows that Mr Jones made use of phase 1 of the Van Hiele model in all his three lessons. Each time he introduced a lesson, he asked learners to identify the shape that was constructed on the *Geoboard*. He told learners the type of shape after they identified it from the *Geoboard*, and only then did he write the topic of the lesson on the chalkboard.

Generally, data from all his video lessons show that he used the *Geoboard* to find out what learners already knew about the shapes before presenting each lesson of the day.

Guided orientation: Video data from all the three lessons shows that Mr Jones made use of the *Geoboard* to guide learners in forming different shapes based on their properties. During the lessons, learners constructed line segments that were related to properties such as opposite, equal, parallel and perpendicular. Moreover, he also was seen walking around the class helping learners to properly construct line segments that were linked to the properties of shapes.

Explicitation phase: The video data of the second lesson (JV2) shows that abundant time was given to learners to discuss the properties of a square. The learners' discussions were in response to the teacher's questions. Learners also focused their discussions on the line segments that were constructed on their *Geoboards*.

Free orientation phase: Mr Jones made moderate use of the *Geoboard* to provide activities that enabled learners to link relevant relationships of the properties of shapes. Data from the second video (JV2), shows Mr Jones giving learners a task of completing the construction of a square when two opposite, and/or adjacent sides are already constructed on the *Geoboard*. Video data in the third lesson (JV3) also shows that Mr Jones gave learners a task of constructing a rhombus from a triangle that was already on the *Geoboard*. In one instance he was seen instructing learners to complete a rhombus from two adjacent sides by using two other rubber bands of different colours.

Integration phase: Data from videos shows that Mr Jones concluded his lessons by asking learners reflective questions. Learners were able to answer the teacher's questions and hence demonstrated that they understood the properties of shapes taught to them. They were able to identify the shapes, and they also demonstrated understanding by listing the properties of the shapes for the teacher to write on the chalkboard.

4.4 TEACHER TWO: MS RUTH

4.4.1 Profile and Coding

The pseudonym of the second participant in my study was Ms Ruth. She has seven years of experience in teaching mathematics to Grade 7 learners. She holds a Bachelor of Education in mathematics and science with the University of Namibia. Ms Ruth is coded as follows: Video one (**RV1**), video two (**RV2**), video three (**RV3**), Stimulus Recall Interviews (**RSRI**) and Focus Group Interviews (**RFGI**).

4.4.2 Lesson 1

4.4.2.1 A brief overview of the lesson

Ms Ruth also had three lessons, where the first lesson was presented during the pilot phase of the intervention programme. The lesson was on the properties of a parallelogram, and it was thirty-eight minutes and four seconds. The lesson objective was to teach the properties of a parallelogram in terms of sides, angles and symmetry. A *Geoboard* was used by both the teacher and learners in this lesson.

4.4.2.2 Visualisation processes

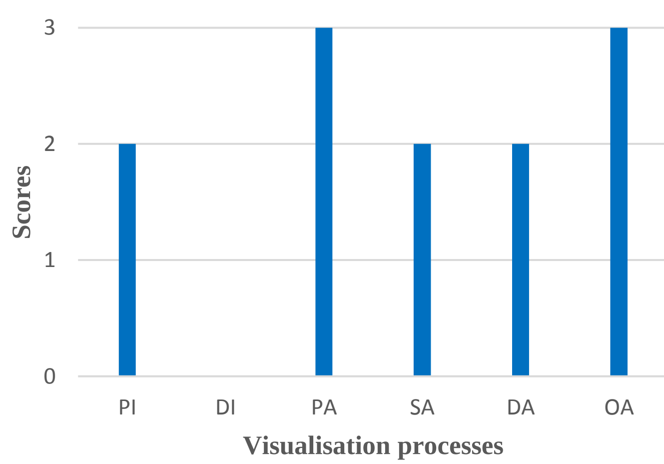

Figure 4. 12: Numerical scores of Ms Ruth's uses of visualisation processes. Lesson 1.

Figure 4.12 above is a summary of the numerical scores for each of the six visualisation processes during the first lesson of Ms Ruth. Each visualisation process is measured by 0–3 points which correspond to the coding criteria in Table 4.1. The graph shows that the use of visualisation processes such as *perceptual* and *operative apprehensions* was more frequent compared to the other three types of visualisation processes (*pictorial imagery, sequential* and *discursive*) which were moderately used during the lesson. *Dynamic imagery* was not used at all in this lesson.

The concrete pictorial imagery: This type of visualisation process was seen to be used by Ms Ruth on the *Geoboard* when she created visual representations of the shapes with different colours to aid learners in identifying a parallelogram (see Figure 4.13). Learners could be seen looking at the teacher's *Geoboard* and raising their hands in an indication that they identified the shape. Indeed, when they answered it was evident that they could identify a parallelogram from the visually represented pictures, by mentioning the colour of the shape.

Figure 4. 13: Visual representation of shapes on a Geoboard

Dynamic imagery: The *dynamic imagery* aspect of a *Geoboard* was not revealed much in this lesson. The only instance when it looked as if it was about to be applied, was when Ms Ruth turned her *Geoboard* that was placed on the chalkboard, upside down. However, she did not say anything about her action to the learners.

Perceptual apprehension: *Perceptual apprehension* was the most-used visualisation process in this lesson. Ms Ruth's *Geoboard* as well as the learners' were vividly decorated with colourful shapes. Learners could identify the properties of a parallelogram by merely looking at the line segments constructed on the *Geoboard* shape. Figure 4.14 shows how the shape appeared.

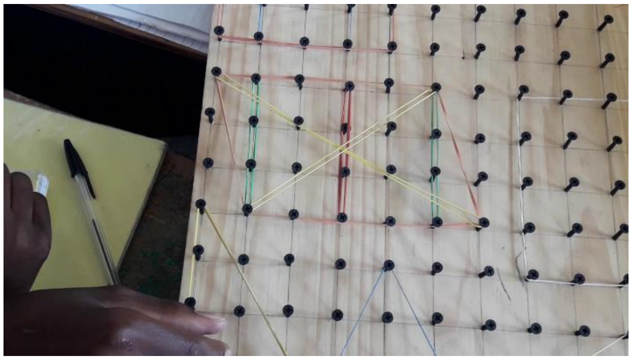

Figure 4. 14: Colourful depictions of shapes on learners' Geoboards

Sequential apprehension: Ms Ruth guided learners to understand the construction of a parallelogram. Video data (RV1) shows that learners were constructing line segments on their respective group *Geoboards*. On one occasion a learner was seen constructing and describing the construction of parallelogram on the teacher's *Geoboard* (see Figure 4.15).

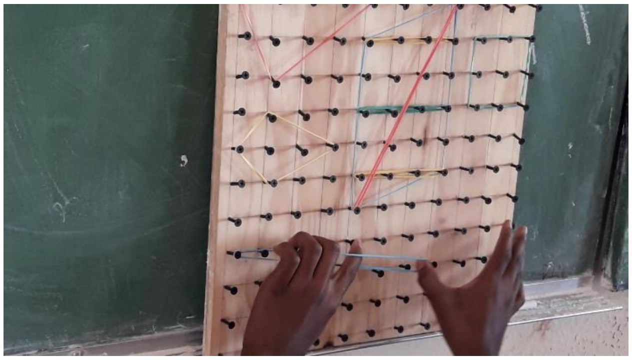

Figure 4. 15: A learner demonstrating the construction of a shape using the Geoboard

Discursive apprehension: The teacher was seen asking learners questions on the different terminologies that are related to the concepts of the properties of a parallelogram. Terminologies such as 'parallel', 'equal', 'opposite' and 'symmetry' were discussed during the lesson.

Operative apprehension: Ms Ruth gave learners a variety of problems that were related to the construction of a parallelogram on the *Geoboard*. She also gave learners activities on dot papers where they answered several questions in relation to the construction of a parallelogram.

4.4.3 Lesson 2

4.4.3.1 Brief description of the lesson

A review of the first lesson was done before the implementation of the second lesson by all the participants. The second lesson was also on the properties of a parallelogram. The lesson was thirty-nine minutes and seventeen seconds. Learners were divided into groups, and the teacher ensured that each group was comprised of different learners from the previous lesson.

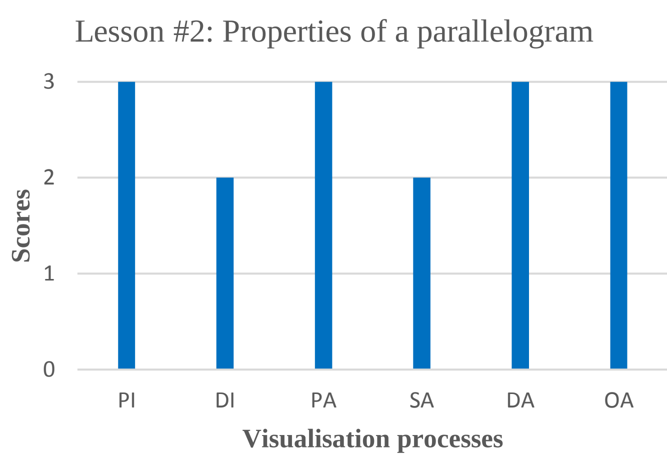

Figure 4. 16: Numerical scores of Ms Ruth's uses of visualisation processes. Lesson 2.

Figure 4.16 above shows that the visualisation processes such as *concrete pictorial imagery*, *perceptual apprehension*, *dynamic imagery* and *operative imagery* were mostly used by Ms Ruth in her second lesson on the *Geoboard*. The graph also shows that *dynamic imagery* and *sequential apprehension* were moderately used.

Concrete pictorial imagery: Video data (RV2) shows that Ms Ruth used visual images on a *Geoboard* to help learners differentiate between a rectangle and a parallelogram. She did this by constructing both a rectangle and a parallelogram on her *Geoboard* and then asked learners to explain the differences between the two shapes as they appeared to them.

Dynamic imagery: The *dynamic* element of the *Geoboard* was seen in this lesson when Ms Ruth requested a learner to change a rectangle to a parallelogram on the teacher's *Geoboard*. On one occasion the video data (RV2) also shows the teacher instructing learners to change the size of a parallelogram to a much bigger one by means of stretching the rubber bands.

Perceptual apprehension: *Perceptual apprehension* was applied by Ms Ruth when she asked learners to identify the shapes that were constructed on her *Geoboard*. Shapes such as a square, a parallelogram and a rectangle were randomly constructed. Learners were able to identify each type of shape without difficulty.

Sequential apprehension: In addition to the application of *perceptual* understanding on the use of *Geoboards*, Ms Ruth also taught and demonstrated the properties of a parallelogram by constructing and explaining 'parallel', 'equal' and 'opposite' lines.

Discursive apprehension: Ms Ruth made learners construct a rectangle and parallelogram on their *Geoboards* and then asked them to discuss the similarities and differences between the two shapes. Learners compared the angle properties of the two shapes.

Operative apprehension: Ms Ruth helped learners to operate on the shapes that were constructed on the *Geoboards*. Learners were seen manipulating rubber bands on their group *Geoboards* to transform and modify parallelograms to rectangles and vice versa.

4.4.4 Lesson 3

4.4.4.1 Brief description of the lesson

This was Ms Ruth's third lesson and it was on the properties of a trapezium. The lesson video was thirty-seven minutes and twenty-six seconds long. A reflective discussion of the previous lesson was done. Learners were again reshuffled to allow the presence of different learners in each group.

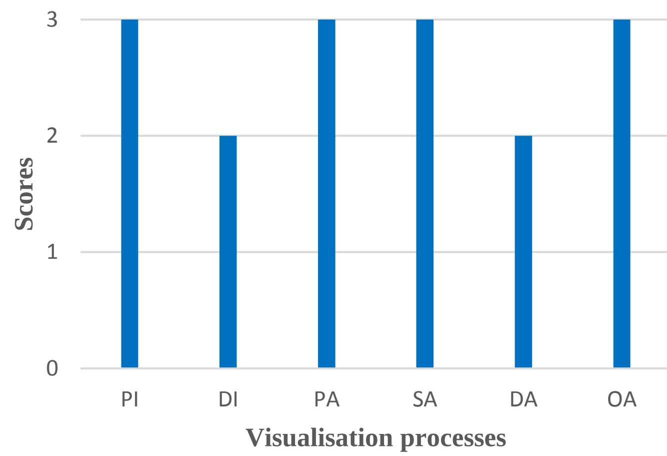

Figure 4. 17: Numerical scores of Ms Ruth's uses of visualisation processes. Lesson 3.

The analysis of Figure 4.17 shows that in her third lesson, Ms Ruth made use of the *Geoboard* on more than three occasions to apply the use of visualisation processes such as *concrete pictorial imagery*, *perceptual*, *sequential* and *operative apprehensions*. The graph also shows that *dynamic imagery* and *discursive apprehensions* were moderately used during the lesson.

Concrete-pictorial imagery: The *concrete imagery* aspect of a *Geoboard* was seen in this lesson when Ms Ruth placed a *Geoboard* trapezium next to the trapezium drawn on the chalkboard, labelled ABCD (see Figure 4.18). Thereafter she asked learners if the two opposite parallel sides were equal (line AB and CD). Learners responded that they were not equal because line AB had four pins, whilst line CD had six pins. Interestingly, learners drew their conclusions from the constructed geoboard figure and not from the chalkboard. In other words, they more easily saw and even confirmed the difference using the *Geoboard* rather than the chalkboard.

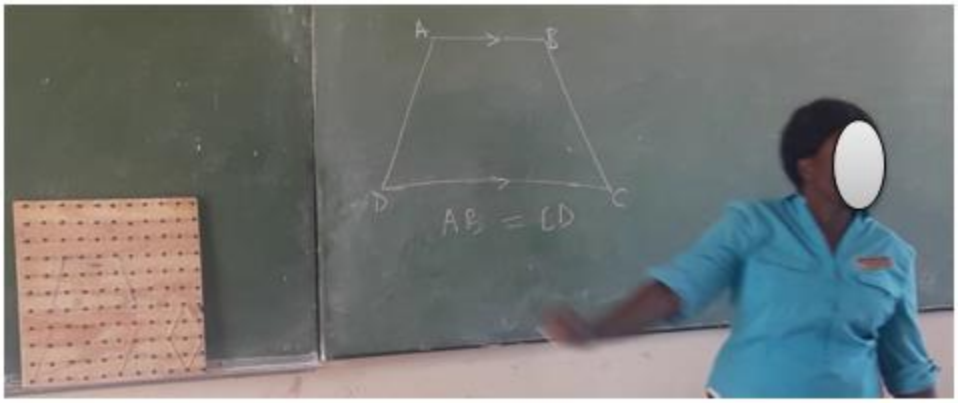

Figure 4. 18: An illustration of a trapezium on both a Geoboard and a chalkboard

Dynamic imagery: The *dynamic* element of the *Geoboard* was seen in this lesson when one learner turned the *Geoboard* upside down. Video data (RV3) shows that on one occasion a learner was challenged by the teacher and other learners to prove to the whole class why she referred to one of the sides as the base of the trapezium. At the end of the discussion, the teacher explained that it mattered not how the *Geoboard* was oriented; a trapezium was still going to be a trapezium.

Perceptual apprehension: Video data shows that Ms Ruth's *Geoboard* figures aroused learners' attention. Both the teacher and learners' *Geoboards* were arrayed with rubber bands of different colours. It was evident that these coloured bands activated learners' sense of sight in identifying the properties of a trapezium.

Sequential apprehension: The teacher guided learners to explore the construction of the properties of a trapezium such as parallel sides, perpendicular height and diagonals. She asked questions to learners that compared a trapezium to a rectangle and a parallelogram.

Discursive apprehension: Within their respective groups, learners discussed the properties of a trapezium. Ms Ruth also asked learners to discuss the properties of a trapezium in comparison to the properties of a rectangle and a rhombus.

Operative apprehension: Video data (RV3) shows that Ms Ruth gave learners an activity on dot papers for them to answer questions that were related to the properties of a trapezium (see Figure 4.19).

Figure 4. 19: Learners demonstrating their understanding of the properties of a trapezium

4.4.5 The teacher's experience of using a *Geoboard*: Stimulus Recall interview

The following data emerged from the stimulus recall interview I had with Ms Ruth:

4.4.5.1 Visual Representation

> *…learners managed to identify the different shapes that they saw* (RSRI, 0218).

To the question of the *visual* element of the *Geoboard*, Ms Ruth stated that learners identified shapes according to how they were visually presented to them on the *Geoboard*. Thus, she testified to the existence of the visual aspect of the *Geoboard*.

4.4.5.2 Dynamic elements of *Geoboards*

Data from SRI parallels with the data from RV3 on the *dynamic* element of the *Geoboard*. Ms Ruth confirmed and recounted the incidence where one learner turned a *Geoboard* upside down, an act that prompted a class discussion on whether a trapezium, turned upside, would still be a trapezium.

> *Although only the position was turned around but still it remained a trapezium, so the leaner looked at it from a different angle* (RSRI, 097-098).

It was turned into a different shape, but, it was still the same shape which is a trapezium. It was only rotated but it was quiet interesting coz the learners looked at it from a different perspective and then she was still answering the questions that were asked from her on the number of pins but then it was still correct (RSRI, 109-113).

4.4.5.3 *Geoboards* and classroom interaction

Ms Ruth said that learners interacted with the *Geoboard* to construct trapeziums during the lesson. Data also shows that *Geoboards* aroused learners' interest and participation in the classroom.

…everyone was interested in doing something, in constructing something, everybody wanted to touch here and there. So, everybody was actively participating (RSRI, 191-194)

…they were all eager to do something. All of them were actively involved to construct something (RSRI, 254-255)

4.4.5.4 Challenges with the *Geoboard*

Ms Ruth experienced a problem related to the size and distribution of pins on the board.

The size of the Geoboards that we had they are a bit small to the extent that the learners who were seated at the back could not see clearly on the structure that the teacher has constructed for them. And, apart from the number, the availability of enough Geoboards there which also played a major role within that lesson, maybe the spaces that we have also the sizes and the spaces and distribution of pins it might have also an effect (RSRI, 272-278).

4.4.6 The Van Hiele phases of teaching geometry

The table below shows the numerical scores of the evidence on the use of the Van Hiele phases of learning geometry in all the three lessons of Ms Ruth.

Table 4. 3 Numerical scores of the evidence of Van Hiele phases in Ms Ruth's lessons

Phase	Description of indicator	Score		
		Lesson 1	**Lesson 2**	**Lesson 3**
Information	*Geoboards* are used as visualisation tools in attempting to find out what learners already know about the properties of shape(s).	3	3	3
Guided orientation	*Geoboards* are used as visualisation tools to guide learners to form different shapes based on their properties.	2	3	2
Explication	Teacher(s) give learners an opportunity to discuss the properties of the shapes that are constructed on *geoboards.*	3	2	1
Free orientation	*Geoboards* provide activities that enable learners to link relevant relationships of the properties of shapes.	2	2	3
Integration	Properties of *Geoboard* figures are identified and named by learners through the help of the teacher, to thus provide evidence that the use of *Geoboards* as a visualisation tool has prepared learners for the next level (descriptive/analysis).	3	1	1

Coding descriptors for Table 4.3

Code/score	Categories	Descriptions (Van Hiele phase(s))
0	No evidence	There is no evidence of the phases of Van Hiele used
1	Weak evidence	There is little evidence of the Van Hiele phases on the used (1-2 incidences).
2	Medium evidence	There is some evidence of the phases of Van Hiele used (2-3 incidences).
3	Strong evidence	There is an abundance of evidence of the phases of Van Hiele used.

Information phase: Data video (RV1, RV2, RV3) shows that Ms Ruth introduced her lessons by first seeking to know what learners already knew about the topics. This was done by constructing shapes on the *Geoboard* and later asked learners to identify the shapes.

Guided orientation: Video data also shows that a *Geoboard* was used to guide learners on how to form shapes through the construction of their properties. At all times Ms Ruth was seen correcting and guiding learners in the correct way of constructing the shapes.

Explicitation phase: In all the lessons the teacher engaged learners in discussions of the properties of shapes. *Geoboards* were used to enhance the discussions as learners would often refer their discussions to the line segments of the geoboard figures.

Free orientation phase: A few activities were administered on *Geoboards* for learners to explore the properties of different shapes. Data from RV3 shows that Ms Ruth gave learners extended activities on dot papers to supplement the use of *Geoboards*.

Integration phase: In one of her lessons Ms Ruth concluded by asking questions to learners. Otherwise data from other two video lessons shows that the integration phase was jumbled between the *Explicitation* and *Free orientation phases*. I also observed that time constraints affected Ms Ruth's conclusions of the other two lessons.

4.5 TEACHER THREE: MS SMITH

4.5.1 Profile and Coding

The third participant was named Ms Smith. She is also a qualified mathematics teacher with seven years of teaching experience. She holds two diplomas in mathematics education. Ms Smith is coded as follows: Video one (**SV1**), video two (**SV2**), Video three (**SV3**), Stimulus Recall Interviews (**SSRI**) and Focus Group Interviews (**SFGI**).

4.5.2 Lesson 1

4.5.2.1 A brief overview of the lesson

The piloting lesson of Ms Smith was on the properties of a rectangle. The duration of the lesson was thirty-six minutes and forty-three seconds. Ms Smith made use of *Geoboards* as teaching manipulatives in this lesson.

4.5.2.2 Visualisation processes

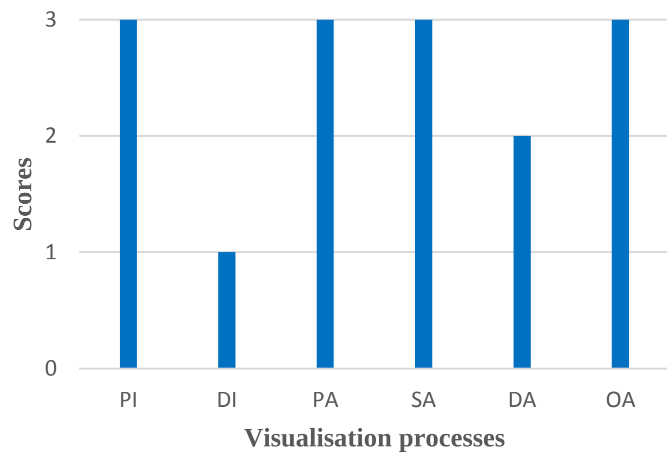

Figure 4. 20: Numerical scores of Ms Smith's uses of visualisation processes. Lesson 1.

The above figure shows how Ms Smith made use of the *Geoboard* to implement visualisation processes in her teaching of the properties of a rectangle. An overview of the graph indicates that there was much use of visualisation processes such as *pictorial imagery*, *perceptual*, *sequential* and *operative apprehensions. Discursive apprehension* was moderately used, but on the other hand not much *dynamic imagery* was used on the *Geoboard*.

The concrete pictorial imagery: Video data (SV1) shows that Ms Smith provided visual representations of rectangle shapes on the *Geoboard*. There is evidence that learners were able to mentally visualise the properties of a rectangle, as they were to complete the constructed shapes on *Geoboards*.

Dynamic imagery: Ms Smith and learners used *Geoboards* to change a square to a rectangle. On one occasion she also directed learners to expand or reduce a six-by-four rectangle.

Perceptual apprehension: The video data (SV1) provides evidence that Ms Smith used a *Geoboard* to help learners recognise basic shapes. This was done during the conclusion of the lesson. She constructed shapes such as a triangle, square, rectangle, kite and parallelogram on her *Geoboard* and later asked learners to identify a rectangle. She also asked learners to construct lines of symmetry on the *Geoboard* (see Figure 4.21).

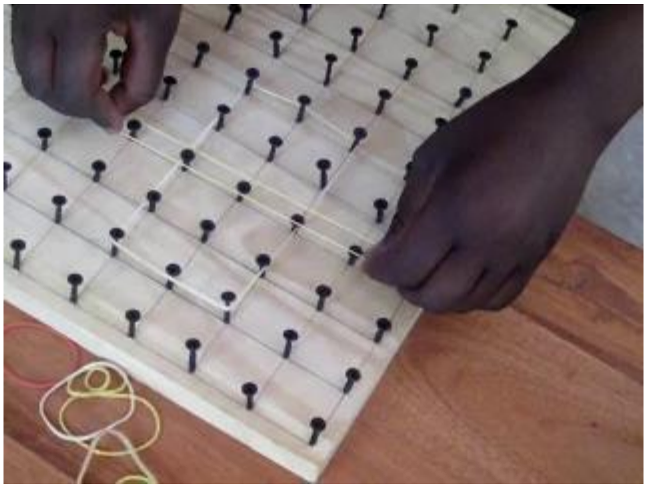

Figure 4. 21: A learner placing a line of symmetry on a rectangle

Sequential apprehension: There is abundant evidence that Ms Smith used a *Geoboard* to explore the properties of a rectangle. Video data (SV1) shows that Ms Smith guided learners to construct properties such as parallel, perpendicular, diagonal and symmetry on their *Geoboards*. Learners were able to explain the construction of a rectangle through the properties.

Discursive apprehension: In their groups, learners discussed the properties of a rectangle through the aid of a *Geoboard*. The teacher asked learners questions on the line segments that were constructed on the geoboard figure.

Operative apprehension: There is evidence in the video data (SV1) that Ms Smith used a *Geoboard* to help learners operate on a rectangle. Learners were given activities such as modifying and transforming a square to a rectangle and changing the size of a rectangle to a bigger or smaller one. Learners also constructed the properties of a rectangle by placing rubber bands, to physically demonstrate how each property appeared.

4.5.3 Lesson 2

4.5.3.1 A brief overview of the lesson

The second lesson of Ms Smith was also on the properties of a rectangle. The total time of the lesson was thirty-five minutes and seventeen seconds. As has been a practice throughout the intervention, a reflection of the previous lesson was done, and suggestions put forth for the improvement of the next lesson.

4.5.3.2 Visualisation processes

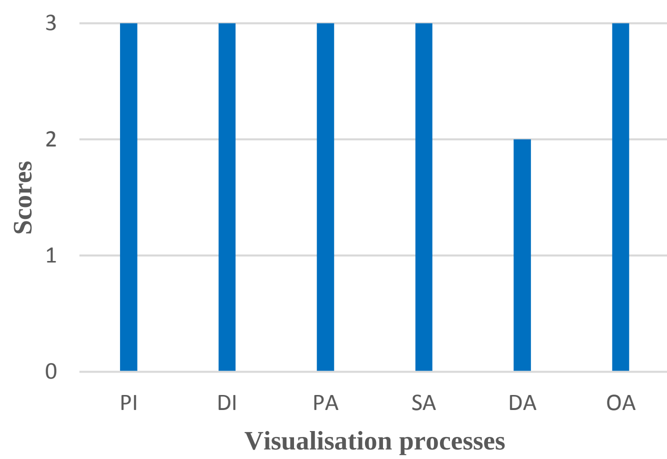

Figure 4. 22: Numerical scores of Ms Smith's uses of visualisation processes. Lesson 2.

The graph above shows that there was enough evidence that the teacher made use of the visualisation processes to teach the properties of a rectangle on the *Geoboard*. Briefly, the graph shows that the *pictorial imagery*, *dynamic imagery*, *perceptual*, *sequential* and *operative apprehensions* were sufficiently used whilst *discursive apprehension* was moderately used.

Concrete pictorial imagery: Ms Smith helped learners to visualise the properties of a rectangle by means of constructing line segments that represented the properties on the *Geoboard*. Evidence from video data (SV2) shows that the *Geoboard* she used was arrayed with rubber bands of different colours depicting each property of a rectangle.

Dynamic imagery: The *Geoboard* was used in the lesson to transform a square to a rectangle, as well as a parallelogram to a rectangle. The video data also shows Ms Smith turning a *Geoboard* in a sideways position to demonstrate that a rectangle remains the same even if the position angle is changed. In this case the rectangle that had a six-by-three pin array, when turned became a three-by-six pin array rectangle.

Perceptual apprehension: Learners were made to understand the properties of a square by means of visual representations on the *Geoboard*. As she explained each property to learners, Ms Smith visually presented the properties on the *Geoboard* for learners to see how the properties looked.

Sequential apprehension: Ms Smith, step by step, demonstrated the construction of a rectangle to learners who followed the connection of separate rubber bands of different colours on her *Geoboard*. Video data (SV2) shows that for each step Ms Smith explained and defined each property that made up the rectangle.

Discursive apprehension: Learners were availed an opportunity to discuss the properties in their respective groups. However, not all learners took part in the discussions.

Operative apprehension: Activities that required learners to construct the properties of a rectangle on *Geoboards* were administered by the teacher, in order to help them understand the properties thereof.

4.5.4 Lesson 3

4.5.4.1 A brief overview of the lesson

The third lesson of Ms Smith in the intervention programme was on the properties of a kite. The lesson had a time duration of thirty-six minutes and five seconds. *Geoboards* were again used in this lesson and learners were reshuffled to allow different learners to form each group.

4.5.4.2 Visualisation processes

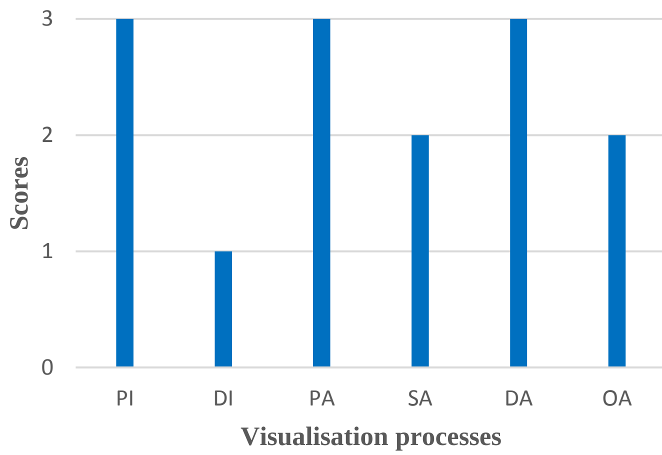

Figure 4. 23: Numerical scores of Ms Smith's uses of visualisation processes. Lesson 3.

Video data (SV3) shows that in the third lesson of Ms Smith there was enough evidence that she made use of visualisation processes such as *pictorial imagery*, *perceptual* and *discursive apprehensions*. On the other hand, the use of *sequential* and *operative apprehensions* was on a moderate level. *Dynamic imagery* was only used on one occasion in this lesson.

Pictorial imagery: There was evidence that the teacher employed the pictorial imagery visualisation process in this lesson by means of using a *Geoboard*. The video data (SV3) shows that Ms Smith constructed four different types of shapes on her *Geoboard* and asked learners to identify a kite from the shapes (see Figure 4.24). Thus, learners were able to identify a kite from the visual shapes that were presented to them on the *Geoboard* by the teacher.

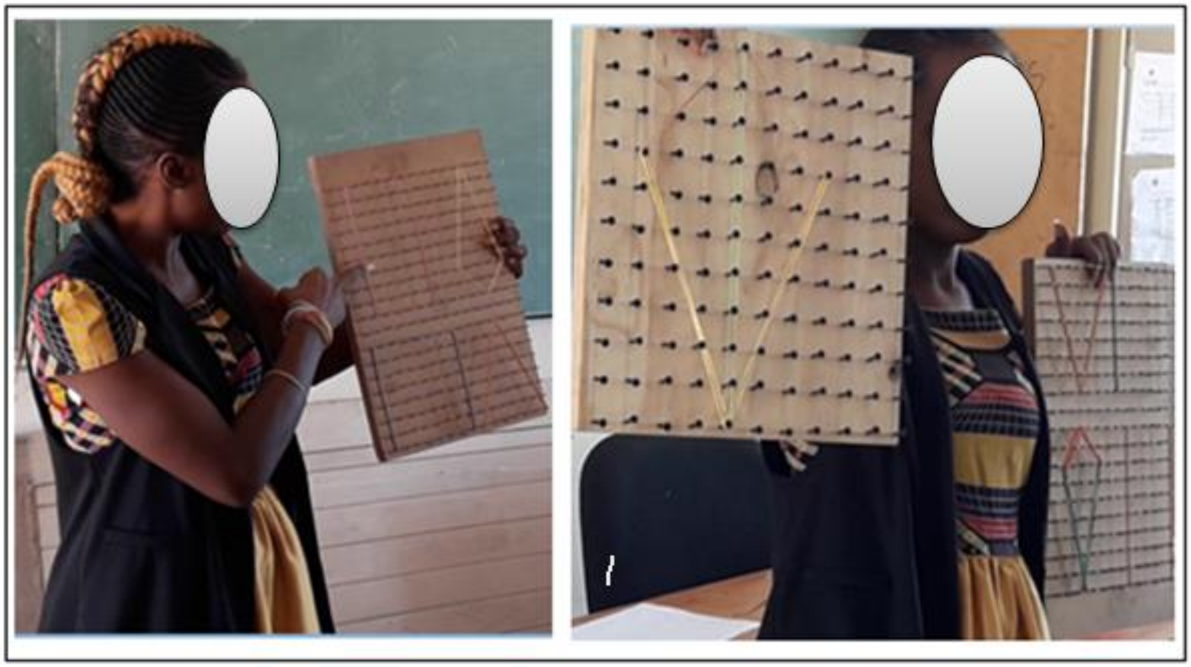

Figure 4. 24: Visually constructed shapes on Ms Smith's Geoboard

Dynamic imagery: Not much evidence was seen in the video recording where the *dynamic* element of a *Geoboard* was explored in the lesson. It was only on one occasion where Ms Smith was seen to instruct learners to change a rhombus to a kite and thus discuss the differences between the two shapes.

Perceptual apprehension: Throughout the lesson, the *perceptual* aspect of the *Geoboard* was seen when Ms Smith kept on pointing her questions at the geoboard figures. She kept on putting up her *Geoboard* during the lesson for learners to see. Both the teacher and learners' *Geoboards* were covered with vividly clear shapes during the lesson (see Figure 4.25).

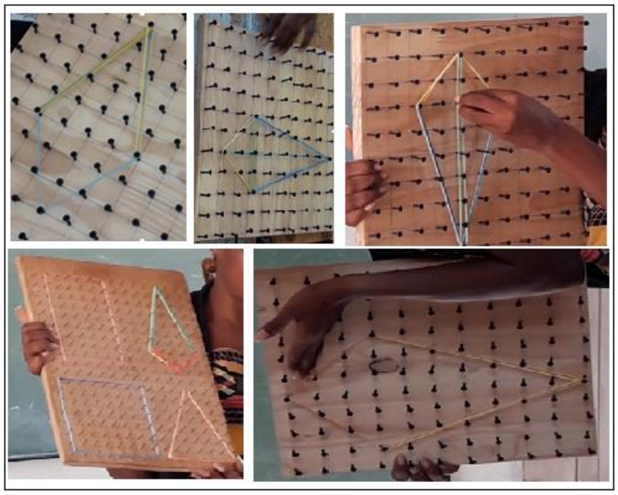

Figure 4. 25: Visually constructed shapes on Ms Smith's Geoboard

Sequential apprehension: Ms Smith constructed and described the construction of a kite by using a *Geoboard*. Step by step, she explained the properties that constituted the construction of a kite on the *Geoboard* by drawing learners' attention to specific properties. She also summarised the properties by listing them on the chalkboard (see Figure 4. 26). In this case learners were telling her what to write.

Figure 4. 26: Ms Smith listing down the properties of a kite on the chalkboard

Discursive apprehension: Ms Smith engaged learners in the discussion of the properties of a kite. She asked questions throughout the lesson, although only a few individual learners were actively responding to her questions. At one point, learners were requested to discuss the diagonals of a kite, even though feedback was not followed up to determine the extent to which learners understood the concepts.

Operative apprehension: Video data (SV3) shows that the *Geoboard* was used to operate on the properties of a kite. Ms Smith gave learners group activities that required them to make use of a *Geoboard*. The activities required learners to construct and explain the diagonals of a kite, complete a half-constructed kite, and determine the number of lines of symmetry as well as the order of rotational symmetry of a kite.

4.5.5 Teacher's experience of using a *Geoboard*: Stimulus Recall interview

After Ms Smith taught her three lessons, I had a stimulus recall interview with her to determine her teaching experience of using a *Geoboard*. The following data emerged out of the interview:

4.5.5.1 Visual Representation

> *They themselves [learners] mentioned that they can see four different shapes and then I asked them to name them and then to identify the one they are mentioning of which respective elastic rubber colour do they have* (SSRI, 027-029).

Data from SSRI agrees with data from the observation video (SV3) on the visual representation aspect of a *Geoboard*. Ms Smith confirmed that learners clearly saw the four different shapes and even explained how each was different from the other.

4.5.5.2 Dynamic element of *Geoboards*

The *dynamic* element of a *Geoboard* was profoundly described by Ms Smith as follows:

> *… with the Geoboard it was very easy to form a kite from other shapes like a rhombus because you just take the rubber bands and you can adjust them even learners could adjust them the way they feel like. I mean to extend one corner of a rhombus or to put it inside to form a kite* (SSRI 121-124).

4.5.5.3 *Geoboards* and classroom interaction

The following expressions from Ms Smith's interview clearly show how she experienced the positive effects of the use of *Geoboards* in her lessons:

> *It is a very useful equipment and learners work out cooperatively without arguments with listening to one another and making possible correction as per what the other learner is saying and in case it does not work out others were coming in with a different input to enable them to make a clear kite because everyone wanted to have a perfect kite. So, it is a very important tool to use as this what enables learners to use their own hands and then it also promotes creativity in them* (SSRI, 100-106).

> ... *counting the pins and whatever so the participation of the learners was quite positive. It allowed learners to work in a group much better than before (SSRI, 145-146).*

According to Ms Smith, the use of *Geoboards* created a learning environment where learners were actively working together in harmony. Furthermore, she also said that learners were able to form the geoboard figures and they became creative in the construction of shapes.

4.5.5.4 Challenges with the *Geoboards*

Ms Smith revealed that the only challenge she experienced with use of *Geoboards* was time constraints.

> *The only problem that I have experienced is that it [use of Geoboard] just takes up time (SSRI, 194-195).*

4.5.6 The Van Hiele phases of teaching geometry

Table 4.4 below shows the numerical scores of the evidence on the use of the Van Hiele phases of learning geometry in all the three lessons of Ms Smith.

Table 4. 4 Numerical scores of the evidence of Van Hiele phases in Ms Smith's lessons

Phase	Description of indicator	Score		
		Lesson 1	**Lesson 2**	**Lesson 3**
Information	*Geoboards* are used as visualisation tools in attempting to find out what learners already know about the properties of shape(s).	3	3	3
Guided orientation	*Geoboards* are used as visualisation tools to guide learners to form different shapes based on their properties.	3	3	2
Explication	Teacher(s) give learners an opportunity to discuss the properties of the shapes that are constructed on *geoboards.*	2	2	2
Free orientation	*Geoboards* provide activities that enable learners to link relevant relationships of the properties of shapes.	1	2	2
Integration	Properties of *Geoboard* figures are identified and named by learners through the help of the teacher. Thus, to provide evidence that the use of *Geoboards* as a visualisation tool has prepared learners for the next level (descriptive/analysis).	2	1	0

Coding descriptors for Table 4.4

Code/score	Categories	Descriptions (Van Hiele phase(s))
0	No evidence	There is no evidence of the phases of Van Hiele used
1	Weak evidence	There is little evidence of the Van Hiele phases on the used (1-2 incidences).
2	Medium evidence	There is some evidence of the phases of Van Hiele used (2-3 incidences).
3	Strong evidence	There is an abundance of evidence of the phases of Van Hiele used.

Information phase: Ms Smith introduced each of her three lessons by constructing shapes on her *Geoboard* and asked learners to say what they knew about the shapes. Thus, she attempted to find out what learners already knew about the shapes before she taught the lessons on quadrilaterals.

Guided orientation: through the process of employing the *sequential apprehension* visualisation process, Ms Smith directed learners step-by-step to understand how to construct a rectangle and a kite.

Explicitation phase: Learners were given an opportunity to discuss the properties of a rectangle and a kite. Video data (SV1 & SV3) shows that Ms Smith attempted to correct learners who misspelled (both in their writing and pronunciation) terms such as ‘trapezium’, ‘parallel’, ‘diagonal’, ‘opposite’ and ‘symmetry’.

Free orientation phase: Learners were given a variety of activities on their *Geoboards* that were meant to provide information to the teacher about how they comprehended the properties of a rectangle and kite that were taught to them.

Integration phase: Video data (SV1) shows that Ms Smith concluded the lesson by asking learners what they had learned, to determine how much they understood what was taught to them. However, video data SV2 and SV3 shows that due to time constraints, Ms Smith could not implement the integration phase in the correct sequence of the phases, thus no questions were asked to learners and no summary was done on each of the two lessons. However, I observed that in the second lesson (SV2), the integration phase was integrated within the *Explicitation* and *Integration phases*.

4.6 FOCUS GROUP INTERVIEW: Teachers’ experiences of using *Geoboards*.

After all the three teachers presented their lessons, and a stimulus recall interview was conducted for each teacher, a Focus Group Interview (FGI) was administered. All the three participants actively participated in the interview and the following data emerged:

4.6.1 Visual representation

To the question on the visual representation of a *Geoboard*, Ms Ruth said that the use of colourful bands was helpful in visually attracting learners’ attention: *it [the Geoboard] became more attractive to the learners simply because we were using colourful rubber bands (RFGI, 198-200).* Mr Jones also added that the visual aspect of the *Geoboard* was evident in that learners easily counted the number of pins as they concretely appeared to them:

…where you could come up with properties like equal. Then you ask them how equal? So, they can see the pins that this side is having 'four' maybe all the sides are 'four' 'four' 'four'. So, will be like when say they are equal they can count those pins (JFGI, 219-222).

Ms Smith also had the following to say on the question of visual representation of a *Geoboard*:

The moment they saw the board that had pins their interest was already aroused (SFGI, 231).

4.6.2 Dynamic element of *Geoboards*

Ms Smith compared a chalkboard to a *Geoboard* by stating that the latter is more *dynamic*:

With the Geoboard you can do a lot of activities, when you compare the Geoboard to the chalkboard whereby on the chalkboard you have to rub and redraw, but on the Geoboard, I will just use those rubbers, adjust them to the shapes of their choices (JFGI, 033-036).

The other two colleagues also attested that the use of elastic bands made it easier to transform shapes on the *Geoboard*.

4.6.3 *Geoboards* and classroom interaction

All teachers stated that the *Geoboard* generated and enabled positive classroom interactions:

So they were more eager to learn since they were constructing their own shapes there. And then at the same time you could see it also create the learning, the good learning environmental for the learners in the classroom between each other because as you can see we have grouped them in the groups and everybody was always eager to do something (RFGI, 021-025).

For me it was useful because you see learners could share ideas and this increases participation of learners, we are talking about learner centered approach so that one really helped the learners to engage in the lesson (JFGI, 088-091).

From my personal perspective it was effective. It was more of putting theory into practice whereby it is more connected to real life situation of stuffs that they created *themselves. It also improved creativity among the learners and then aroused leaners' interest (SFGI, 040-043).*

4.6.4 Challenges with the *Geoboard*

Time constraints emerged from FGI data to be an important challenge in the use of *Geoboards*. Mr Jones indicated that:

One of the challenges was that it was time consuming because it is so attractive each learner wanted to use a Geoboard, to engage with it (JFGI, 259-261).

Even in the FGI data, Ms Smith maintained that the use of *Geoboards* was time wasting.

The negative thing is that when it comes to giving learners time for tasks, supervising them guiding them, apart from it being a good practice, it is just time consuming. It takes up time and then the whole thing geometry is not the only topic. There are a lot of topics that need to be covered and you look at the duration of which you have to finish when using a geometrical board, it is just time wasting. (SFGI, 104-108)

Ms Ruth added that the other problem she thought of was the lack of *Geoboards* to cater for all learners: *the number of Geoboards was limited (RFGI, 256).* She elaborated that even though learners were divided into groups to allow the sufficient use of the few available *Geoboards*, to her the *Geoboards* were still not enough because of the relatively large number of learners. Ms Ruth added that the teacher's *Geoboard* should alternatively be bigger than all other *Geoboards* to give a clear view of shapes to all learners in the class.

4.7 SIMILARITIES AND DIFFERENCES IN THE FINDINGS OF VISUALISATION PROCESSES, VAN HIELE PHASES AND TEACHERS EXPERIENCES ON THE USE OF *GEOBOARDS*.

This section summarises the analysis of the data presented above. The focus is on the similarities and differences that emerged out of the different data sources that were used in the study. I discuss the findings in line with the reviewed literature and I also align the discussion to the three main research questions of this study:

- What are the affordances of the utilisation of *Geoboards* as visualisation tools in the teaching of the properties of quadrilaterals in Grade 7 classes?
- What are selected teachers' experiences of using *Geoboards* as visualisation tools in teaching the properties of quadrilaterals, as a result of participating in an intervention programme?
- How do the participating teachers make use of the Van Hiele phases in their teaching of quadrilaterals using the *Geoboard*?

4.7.1 The use of *Geoboards* and visualisation processes

4.7.1.1 Concrete pictorial imagery

The findings revealed that the *concrete pictorial* visualisation process was effectively used by all three participants on the *Geoboards*. Learners were helped to physically and visually construct shapes with the aid of the *Geoboard* by means of forming shapes with different colours. This finding is in line with Sousa's (2008) point of view, who asserted that the formation of concrete and pictorial images helps learners to understand abstract mathematical concepts. Thus, when teachers constructed geometrical shapes and line segments to represent concepts that explained the properties of different quadrilaterals, according to literature learners can work with these shapes both with their hands and in their minds. As advanced by Duval (2014) in the literature review chapter, it was also clear in the findings that learners developed mental pictures of the properties associated with the quadrilaterals because the quadrilaterals were concretely and visually presented to them on the *Geoboard* by the teachers.

Furthermore, all the three teachers posed questions on the properties of quadrilaterals that induced learners to mentally visualise the shapes. For example, in the data of SV3, Ms Smith asked learners to complete a partly constructed kite with one long diagonal and a shorter adjacent side on her *Geoboard*. The fact that learners were able to complete the formation of the kite strongly suggests that learners had a mental picture of the whole shape even before it was completed, hence the reason why it was easy for them to physically complete it on the *Geoboard*.

4.7.1.2 Dynamic imagery

The *dynamic imagery* visualisation process was also evident in the use of *Geoboards* among teachers. The findings suggest that teachers attempted to make learners understand that shapes can be transformed by means of adjusting rubber bands. Learners managed to change and transform squares into rectangles, rectangles into parallelograms and rhombi and kites. I also observed that as learners were freely playing with the elastic bands on their own, they unconsciously applied *dynamic imagery* without the teacher's instructions. I suppose that the process of externally transforming shapes with rubber bands has likely caused learners to develop the same image shapes in their minds. Nemirovsky and Noble (1997) described this unique situation as the process where learners associate and harmonise the external representation (in this case from the *Geoboard*) to what they think in their minds.

4.7.1.3 Perceptual apprehension

The findings of this study revealed that *perceptual apprehension* was the most dominant visualisation process in all the three teachers' lessons. Perhaps this was because of the nature of the manipulative tool that was used. First and foremost, it was very easy for the *Geoboard* to call to mind the sense of sight in learners. What was verbally said by the teachers was physically represented to learners by means of concrete pictures. Hence learners could see the representation of quadrilaterals for themselves in order to be convinced and understand.

Secondly, the use of colourful elastic bands also added to the *perceptual apprehension* of the *Geoboard*. All the *Geoboards* in all the three teachers' lessons were arrayed with this beautiful sight of colourful elastic bands which I believe aroused learners' curiosity and interest in the lessons. I may emphasise here that the findings of this study and the literature on visualisation processes have convinced me that learners' ability to recognise different quadrilaterals is strongly influenced by visual perception (Kalogirou & Gagatsis, 2011). In this very exceptional circumstance one may agree with the phrase that says, "*seeing is believing"*. There was strong evidence that what was concretely seen and observed by the learners made an impression on them. Indeed, Duval (1999) has rightly stated that "nothing is more convincing than what is seen" (p.

12). The *Geoboard* is thus a powerful instrument to convince learners of the physical appearance and properties of quadrilaterals.

4.7.1.4 Sequential apprehension

Learners were given activities that were aimed at helping them to understand multiple ways in which quadrilaterals could be constructed. This was done by teachers who, step-by-step, constructed the shapes on *Geoboards* for learners to see. At the same time when teachers were constructing the shapes, they were also explaining the construction in words for learners to fully understand. Thus, learners were made to understand the construction as well as describe the construction of quadrilaterals because of teachers showing them how to do it. Once again, the *Geoboard* became the mediating tool that was used to practically show and explain to learners how to construct the quadrilaterals. In the literature review, Duval (1995) emphasised that the understanding of figural units depends not only on perception but on mathematical and technical constraints. Therefore, in this study, to provide significance to the understanding of quadrilaterals, *Geoboards* were used as tools to demonstrate the construction of shapes, as well as provide a rational and linguistic description of the concepts and their properties.

4.7.1.5 Discursive apprehension

In all the lessons, learners were afforded an opportunity to discuss the properties of quadrilaterals. Although most of the class and group discussions were not extensive in engaging learners, it was still clear to me that teachers intended to encourage learners to talk about what they were learning. A *Geoboard* was again used in these discussions as both the teachers and learners were using geoboard figures as a reference in their discussions. The findings suggest that these discussions helped learners to comprehend most of the properties as they could share their thoughts on how the quadrilaterals could be constructed.

Akkaş and Türnüklü (2014) advocate that defining is one of the strategies of teaching quadrilaterals that can be advanced by teachers. Chikiwa and Schäfer (2016) recommend that teachers should create a conducive environment that helps learners to internalise mathematical language. Therefore, according to the findings of this study, I argue that *Geoboards* can create an environment that fosters the use of mathematical language when defining quadrilaterals. When teachers afford learners opportunities to discuss the concepts related to quadrilaterals, defining becomes more meaningful because learners can discuss and define the concepts as they appear on *Geoboards*.

4.7.1.6 Operative apprehension

The many activities that were given to learners were fundamental to the understanding of the properties of quadrilaterals. The findings show that teachers used *Geoboards* to administer activities that allowed learners to operate on quadrilaterals. Duval (1995) stated that operating on a shape depends on the many ways it can be modified, manipulated and transformed. Findings suggest that when learners operated on the quadrilaterals by means of modifying, manipulating and transforming, they gained insight into the properties and were even able to solve problems that were related to the construction and description of the quadrilaterals.

4.7.2 The use of *Geoboards* and the Van Hiele phases of teaching geometry

What was strongly evident in most of the lessons in this study with respect to the Van Hiele model of teaching geometry was that teachers did not sequentially follow the order of the phases as suggested by Van Hiele (1999). The findings of the study show that teachers made use of the first two phases in the order they were intended to be, without difficulty. I observed that the *Information* and *Guided orientation* phases were successfully implemented by all teachers. The findings on the *Information phase* indicates that teachers used *Geoboards* to create shapes that were in turn used to find out what learners already knew about each quadrilateral under discussion. I was particularly impressed by how learners could easily name the shapes that were formed on the *Geoboards*. Much time was also spent in the *Guided orientation phase* where teachers guided and directed learners on how to construct the quadrilaterals. The use of the *Geoboard* once again proved to be a very effective manipulative tool to explicitly demonstrate to learners how each quadrilateral could be formed through the construction of the respective properties.

On the other hand, I also observed that teachers had difficulty in sustaining the sequence of the phases as intended by Van Hiele. It appeared to me that the phases from the *Explicitation phase* down to the *Integrated phase* were jumbled. It was difficult for me to ascertain at which phase the teacher was, in the lesson because it looked like all the three phases (E*xplicitation*, *Free orientatio*n and *Integration phase*) often coincided. For instance, in two of their lessons, Ms Ruth and Ms Smith gave learners activities to work on *Geoboards* in what looked like the *Free orientation phase*. At the same time learners were asked to discuss the shapes in the activities (*Explicitation phase*). Moreover, in the middle of the lessons, learners were made to make connections between the types of quadrilateral they were learning, with other types of quadrilaterals (*integration phase*).

According to Van Hiele (1999), when teaching geometrical shapes, one should always start with the *Information phase,* then the *Guided orientation*, the *Explicitation phase*, *Free orientation phase* and lastly the *Integration phase*. However, from what has emerged from the findings of this study, I argue that at times these phases do not follow a set sequence. Teaching is a complex activity that does not necessarily follow a clean linear progression as Van Hiele suggests. Therefore, I suggest that teachers should not be instructed to follow a set hierarchy of teaching phases – the phases co-emerge and flow into each other, as this study has shown.

The evidence from the discussions of the *Explicitation phase* strongly suggests that learners used terms such as ‘equal’, ‘diagonal’, and ‘vertical’, ‘horizontal’, ‘opposite’ and ‘symmetry’. Though not extensively engaged, the *Explicitation phase* proved to be one of the phases that helped learners to understand the properties of quadrilaterals. It was noteworthy that sometimes learners would use words such as ‘right’ and ‘left’ to describe the shorter rather than the longer side of a shape because they did not have the appropriate vocabulary. Nevertheless, Idris (2009) noted that learners start to be aware of the correct and appropriate technical language to use if they can freely express themselves in the lesson. The teacher should gradually correct learners as they become acquainted with the appropriate language discourse used in the topic. Indeed, language usage is central to the learning of mathematics, and was evident in this study.

4.7.3 Teachers' experiences on the use of *Geoboards*

4.7.3.1 Visual representation

The use of a *Geoboard* was a very useful way to show the visual representation of geometric concepts in a cheap and yet not-so-common way in these classes. All the three teachers affirmed that a *Geoboard* was very useful in visually representing the properties of quadrilaterals. Cotič et al. (2010) asserted that *Geoboards* can be effective tools to teach geometrical concepts because they provide learners with visual imagery to visualise the concepts.

4.7.3.2 Dynamic element of *Geoboards*

Teachers alluded to the dynamism that a *Geoboard* affords to the teaching of mathematics as a visual tool, especially in forming and transforming geometric shapes. Ms Smith identified the flexible nature of a *Geoboard*. An example is the turning and reorientation of the *Geoboard*. Mr Jones also pointed out that the *Geoboard* does not need cleaning, but adjustment of rubber bands if one is correcting a mistake or improving the figure constructed. Both Ms Ruth and Mr Jones noted the *dynamic* element of the *Geoboard* in that quadrilaterals can be transformed and moved by placing the elastic bands in different positions.

All the participating teachers acknowledged that the construction and manipulation of quadrilaterals on *Geoboards* was a lot easier than compared to doing it on the chalkboard. Ms Ruth cited a scenario from her lesson where some learners, after constructing a trapezium, turned the *Geoboard* and an interesting discussion ensued. This is evidence enough to argue that such a *dynamic* element only occurs when movable manipulatives such as *Geoboards* are used. Literature advocates that learners should use images as mental "screens" (Mason, 1992). In other words, this is to mean that learners should mentally see shapes in motion. Therefore, the use of *dynamic* representations of shapes on *Geoboards* by mathematics teachers can help learners to build an awareness that the world is always in motion, including shapes.

4.7.3.3 Geoboards and classroom interactions

In all the teachers' lessons, the *Geoboard* brought about excitement and increased participation during teaching. It also improved learner involvement, livened up the class and everybody was attentive. Moyer (2001) said that a *Geoboard* is a hands-on device that has a tactile appeal. Therefore, I conclude that the findings from the teachers' experiences affirm that the physical and visual nature of the *Geoboard* has greatly influenced learners' participation in the classrooms. The *Geoboard* has aroused learners' attention, interest and curiosity in the learning of quadrilaterals.

Furthermore, when I analyse the teachers' responses on the use of *Geoboards* in line with the classroom interactions, findings suggest that *Geoboards* helped to cater for diverse learners, the shy and not-so-shy ones, the average and the fast learners in the teachers' classes. It can be argued that it was because learners worked in groups. The classroom setup was manipulative user-friendly, hence the reason why it encouraged interaction and participation amongst the learners. Brown et al. (2009) insist that manipulatives are most effective and likely to yield more results in classroom environments that allow learners to think freely through active participation and exploration.

Moreover, I can only but credit the maximised participation of learners in the lessons to the mere fact that they enjoyed a stress-free environment. Stress is often caused by anxiety, and according to literature, a *Geoboard* as a visual manipulative tool has the potential to ease learners' anxiety when learning mathematics (Hunt et al., 2011). Furthermore, the teachers' use of this visual tool brought variety and innovation to learning in these teachers' classrooms.

4.7.3.4 Challenges with the use of *Geoboards*

Teachers in this study experienced some challenges with the use of *Geoboards*. They shared these during the interviews. Ms Ruth had challenges relating to the size of the *Geoboards* in relation to her class size. She suggested that a bigger *Geoboard* would be needed to increase visibility from all corners of the classroom. She also experienced challenges related to the size and distribution of pins on the board.

Some geometric concepts were not so easy and apparent to demonstrate on the *Geoboard*. Mr Jones and Ms Smith gave example of lines of symmetry. Mr Jones suggested that using pins and rubber bands for illustrating the concept of lines of symmetry on the *Geoboard* would work well when an odd number of pins are used. This is because of the existence of a clear set of pins in the middle. He noted that this was not the case when an even number of pins on the sides of a shape were used. I see the scenario of learners failing to locate a line of symmetry on the *Geoboard* to have been caused by teachers. It seems they did not take into consideration the issue of even and odd numbers of pins on the *Geoboard* before giving the activity of symmetry to learners. Sarama and Clements' (2016) advice when it comes to the use of manipulatives is that teachers should make sure that there is a connection between the manipulative and the concept related to its representation. However, it seems this was not heeded to as the anticipation of the problem that could arise regarding the use of an even number of pins was overlooked.

It was also revealed by Mr Jones and Ms Smith that the use of *Geoboards* used much of their teaching time. The review of literature in this study also attested to the concern that the use of manipulatives requires much time during the lesson to introduce concepts (Golafshani, 2013). Nevertheless, all teachers admitted that despite consuming much of their teaching time, the use of *Geoboards* was very effective in the teaching of the properties of quadrilaterals.

4.8 CONCLUSION

In this chapter I presented data and discussed the findings of the study. The presented data and discussed findings were twofold: firstly, I outlined how each lesson was conducted by presenting the data in line with the three overarching research questions; and secondly, I discussed the findings focusing on the visualisation processes, Van Hiele phases of teaching geometry as well as themes that emerged out of the teachers' responses to their perceptions and experiences of using *Geoboards*.

It emerged from the analysis that the use of *Geoboards* as teaching manipulatives is indeed an effective way that can foster visualisation processes such as *concrete pictorial imagery*, *dynamic imagery*, *perceptual apprehension*, *sequential apprehension*, *discursive apprehension* and *operative apprehension*. The findings from the video recordings and interviews showed that teachers were comfortable in using the *Geoboard* because they found it to be convenient and exciting in teaching the properties of quadrilaterals. The *perceptual*, *visual*, *discursive* and *dynamic* aspect of the *Geoboards* greatly contributed to the positive interaction of learners in the classroom.

Although there were challenges in using *Geoboards*, the findings strongly revealed that most of the properties of quadrilaterals that were taught by means of using *Geoboards* were well understood by learners.

Although the participating teachers did not follow the order of the phases exactly as suggested by the Van Hiele theory, it was still evident that each phase found its place in the lessons and was effectively utilised.

In the next chapter I conclude my study by highlighting the main findings and present recommendations for further research in the field.

CHAPTER 5

CONCLUSION AND RECOMMENDATIONS

5.1 INTRODUCTION

This qualitative case study analysed data on the use of *Geoboards* as visual manipulatives in the teaching of quadrilaterals. In particular, the purpose of the study was to investigate and analyse the use of *Geoboards* as visualisation tools when teaching the properties of quadrilaterals, by applying visualisation processes and the Van Hiele phases of teaching geometry.

In this concluding chapter I provide a summary of the main findings of the study, present the significance of the study and submit some recommendations arising from the findings. I also discuss the limitations encountered during the research processes. Since this study has illuminated other researchable areas related to it, I also make suggestions for further research. I end the chapter by sharing my personal reflections on my experience of this study.

5.2 SUMMARY OF THE MAIN FINDINGS

Several themes related to the use of *Geoboards* as visualisation tools arose from the analysis of data from the previous chapter. In this chapter I summarise the findings in order to answer the three research questions. Herewith is the summary of the findings of my study:

Research question 1

What are the affordances of the utilisation of Geoboards as visualisation tools in the teaching of the properties of quadrilaterals in Grade 7 classes?

- The use of *Geoboards* as teaching manipulatives transpired to be an effective way of teaching the properties of quadrilaterals.

- The use of *Geoboards* by the selected teachers effectively fostered visualisation processes such as *concrete pictorial imagery*, *dynamic imagery*, *perceptual apprehension*, *sequential apprehension*, *discursive apprehension* and *operative apprehension*.
- The *Geoboard* proved to be a very appropriate manipulative to teach the properties of quadrilaterals because it is a simple medium to illustrate the properties of quadrilaterals.
- The *Geoboard* enabled the learners to explore and experiment with their constructions using the given rubber bands.
- The construction of quadrilaterals on the *Geoboards* helped learners to develop mental pictures of the properties associated with these quadrilaterals.
- The dynamic element of the *Geoboard* was key to helping learners transform one shape to another by means of adjusting rubber bands.
- The *Geoboard* proved to be a powerful tool that helped learners to discover and 'see' the properties of quadrilaterals.
- The colourful display of rubber bands on *Geoboards* was fundamental to arousing learners' interest and curiosity in the properties of quadrilaterals.
- The *Geoboard* served as a powerful instrument to convince learners of the physical appearance of the properties of quadrilaterals.
- The selected teachers used the *Geoboards* to demonstrate the construction of quadrilaterals, as well as provide a rational and clear description of their properties.
- The *Geoboard* was a suitable device for learners to explore and understand the multiple ways of how quadrilaterals may be constructed.
- The *Geoboard* was used by the teachers as a point of reference for discussion and deliberations about the properties of quadrilaterals.
- Teachers were able to define the properties of quadrilaterals in a meaningful way, as they used the *Geoboards* to translate abstract concepts to concrete and tangible objects.
- The provision of *Geoboards* contributed to creating a conducive learning environment which encouraged exploration and group discussions.

Research question 2

What are selected teachers' experiences of using Geoboards as visualisation tools in teaching the properties of quadrilaterals as a result of participating in an intervention programme?

- All the selected teachers held the common view that *Geoboards* were very useful in demonstrating the visual representation of the properties of quadrilaterals in a cheap and yet novel way in their classes.
- Teachers appreciated the dynamism of *Geoboards* regarding forming and transforming quadrilaterals. They concluded that this was possible because it was easy to adjust and manipulate the rubber bands.
- Teachers made comparisons between the use of a chalkboard and a *Geoboard* in terms of mobility, usage and classroom interactions. The latter was favoured as more effective in the teaching of quadrilaterals because of its unique visual and tactile attributes.
- *Geoboards* were seen by all teachers to be best in increasing learners' participation, curiosity and classroom interactions. It was also reported that *Geoboards* helped to cater for the shy and confident ones, the average and the fast learners in the teachers' classes.
- Some of the teachers did experience challenges with the use of *Geoboards*. Some challenges were related to the size and distribution of the pins on the board. It was difficult, for example, to demonstrate the line of symmetry when an even number of pins defined the length of a shape.
- Teachers also lamented the issue of time constraints when using *Geoboards*. It was commonly viewed that much time is needed to allow for the effective use of *Geoboards*. This is a challenge when it is difficult to complete the Namibian curriculum.
- It was the teachers' opinion that despite the experienced challenges with the use of *Geoboards*, the devices remained the best manipulative tools to use when teaching the properties of quadrilaterals.

Research question 3

How do the participating teachers make use of the Van Hiele phases in their teaching of quadrilaterals using the Geoboard?

- All the teachers attempted to structuring their lessons according to the Van Hiele phases. It emerged however, that the teachers found it difficult to adhere strictly to the hierarchy of the phases. Although the phases enabled the teachers to teach with the *Geoboard* in a well-planned manner, the phases did flow into each other — especially the *Explicitation*, *Free orientation* and *Integration phases*.
- The teachers implemented the *Information phase* to introduce the lessons by means of constructing quadrilaterals on the *Geoboard* and asked questions to solicit learners' prior knowledge on the topic to be taught.
- The *Guided orientation* was employed with the *Geoboard* when teachers, step by step, guided and directed learners to construct quadrilaterals and their respective properties.
- The *Explicitation phase* allowed teachers to work on the vocabulary of learners by having conversations and correcting and explaining the different terminologies related to the conceptual properties of quadrilaterals.
- The *Free orientation* was seen by the teachers as affording the learners opportunities to explore a variety of activities on the *Geoboards*.
- The *Integration phase* was not very evident as time was against the teachers in completing most of the lessons. It was only in one or two lessons where some teachers summarised what was learnt in the lessons.

5.3 SIGNIFICANCE OF THE STUDY

It is hoped that the findings of this study will inspire Namibian primary mathematics teachers to further strengthen the use of teaching manipulatives to meet the educational needs of primary mathematics. My engagement in this study has enlightened me on how a simple visual manipulative such as a *Geoboard* can be used to effectively teach the properties of quadrilaterals. It facilitates a visualisation approach to meaningful teaching, particularly if it is used in conjunction with the Van Hiele phases of teaching geometry. I now see myself as an improved mathematics teacher who is very likely to continue to teach geometrical shapes by using *Geoboards*.

I hold a strong belief that a teacher is central to the teaching of mathematics. Thus, a teaching manipulative tool such as a *Geoboard*, when used correctly, has the potential to evoke a positive learning environment that eases a teacher's approach to teaching, and evokes learners' curiosity and interest in the learning of mathematics in general and the basic geometrical shapes in particular. Although this case study cannot claim generalisability, it is hoped that education planners, educators and researchers as well as teachers will find the findings of this study useful and applicable beyond this particular case. I would like to think that this work can contribute towards improving the mathematics curriculum in Namibia, particularly with regard to emphasising the important role of visualisation processes in general and the use of manipulatives in particular.

5.4 RECOMMENDATIONS

By reflecting on the summary of findings made above, I submit the following recommendations:

- In under-resourced, rural and remote schools, mathematics education officers should advocate and encourage mathematics teachers to make use of visual manipulatives that are easily accessed from local materials. One such visual manipulative is a *Geoboard*.
- Mathematics teachers should endeavor to use visual manipulatives such as *Geoboards* in the teaching of the properties of quadrilaterals to stimulate learners' interest and increase participation.
- This study has provided evidence of the effectiveness of visually oriented lessons using a visual manipulative (*Geoboard*). It is thus recommended that the Namibian mathematics curriculum be visually oriented by emphasising visualisation processes when teaching basic shapes such as quadrilaterals.
- Visualisation processes in mathematics education are an effective yet complex approach to the teaching and learning of mathematics. Therefore, curriculum developers should be

informed and made aware about this approach in order to provide the necessary support to mathematics teachers in Namibia

- Likewise, priority should be directed to the training of mathematics teachers to equip them with effective ways of making use of visualisation processes in their pedagogical practices.
- Mathematics teachers should also be given meaningful and quality support from inside and outside the school boundaries in order to enhance their capacity to improve the use of visual manipulatives beyond the *Geoboards* in Namibia.
- International literature and the findings of this study presented strong evidence on the importance of making use of the Van Hiele model of teaching geometry. The model provides a very practical, rational and logical framework to plan and implement lessons. Therefore, I recommend that the model be anchored in the Namibian curriculum in order to expose teachers to an internationally recognised and effective teaching and learning framework for the teaching and learning process of geometrical shapes.
- Regarding the phases of teaching geometry, Van Hiele (1999) recommends that teachers should follow a clear linear progression of phases starting from *Information phase*, *guided orientation*, *Explicitation phase*, *Free orientation phase* and the *Integration phase*. However, I argue and recommend that teachers need to remain flexible about the sequence of this hierarchy and recognise that these phases overlap and need not be followed in a dogmatic fashion - the phases co-emerge and flow into each other, as this study has shown.

5.5 LIMITATIONS

This study encountered the following limitations and challenges:

Firstly, this study used a small qualitative sample of three teachers only. Therefore, the findings of this study cannot be generalised beyond its case.

Secondly, the use of dot papers was not consistently adhered to in the study. The findings show that only one teacher used dot papers to give extended activities to learners. This limited the provision of more rich data on the use of *Geoboards*.

Thirdly, the data collection of this study was done for a period of two months However, and as emerged out of the findings, this was not sufficient time to make the best use of the *Geoboards*. I have learned that if more time could have been given to the teachers to thoroughly explore the use of *Geoboards*, more rich and detailed data could have been attained.

Fourthly, the study was about the teachers' actions on the use of *Geoboards,* thus only focused on the teachers during the capturing of the videos. However, I learned that a fuller picture of the effectiveness of *Geoboards* when teaching quadrilaterals could have been fully realised if the data also consisted of considerations from the learners of the participating teachers.

5.6 SUGGESTIONS FOR FURTHER RESEARCH

Since this study focused on three primary school teachers only, the recommendations made above need to be seen against this context. Therefore, I suggest that further research is done to include a bigger sample of teachers from a larger variety of schools. It is only then that the results of the recommended studies can be generalised and used against a wider context to improve classroom practice with regard to the use of *Geoboards* and other manipulatives when teaching geometry at the primary phase.

I also recommend the following research areas that could yield promising research questions related to my study:

- The use of *Geoboards* as visualisation tools in the teaching of perimeter and area of geometrical shapes.
- The use of *Geoboards* as visualisation tools beyond the case of geometrical shapes, for example algebra and number sense.
- Aligning visualisation processes to manipulatives other than *Geoboards*.
- Analysing how visualisation manipulatives can aid a successful transition from one Van Hiele level to another in the teaching of geometry.

5.7 PERSONAL REFLECTIONS

This research study was not an easy task for me to complete. I must confess that I initially approached the study with strong personal convictions conditioned by misplaced expectations and assumptions. I initially was under the impression that a research study was just a matter of reading and paraphrasing articles, finding a suitable and convenient research topic and then collect data, write up and submit a thesis. However, this naivety was altered quite rapidly as a result of extensive readings and close guidance from my supervisor on how to carry out and write up a meaningful and worthwhile qualitative research study.

I said to myself "*where there is a will, there is a way*". Despite the many logistical challenges encountered in the study, the successful completion of this thesis was fueled by a great passion and a desire to learn. I personally hold a strong belief that "*knowledge is power*" and that it is only willing and learned humans who can bring about significant and much-needed changes to this world.

For me, this experience was an academic journey worth travelling. First and foremost, it was a great honor to have been granted an opportunity of getting involved in this research project of visualisation in the teaching and learning of mathematics. Mathematics as a subject has been at the heart of my life since childhood. My immersion in the researching of visualisation processes and manipulatives has greatly improved my own knowledge and understanding of the pedagogy of mathematics. I am still young and energetic; hence my motivation and desire to play a role in the development of Namibia's education system, especially in the field of mathematics education. I am committed to positively contributing to the ongoing efforts of improving mathematics education in Namibia and Africa at large.

I see myself as a mathematics researcher and educator. Therefore, all the engagements I underwent in the completion of this thesis has set up an intermediate level of experience in understanding research methodologies. I hope that this experience will form a solid platform to launch my research and writing trajectory. The writing up of this thesis has improved my reading and writing skills enormously. A last thought that thrills, motivates and inspires me is the opportunity to possibly locate my own writing in the context of international literature.

5.8 CONCLUSION

In this chapter, I concluded the whole research processes. I presented a summary of the main findings of the study, discussed the limitations of the study and made recommendations that arose from the findings. I also suggested appropriate researchable areas related to the study and shared my personal reflections on the entire research journey.

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APPENDICES

Appendix A: Ethical clearance

RHODES UNIVERSITY

Grahamstown • 6140 • South Africa

EDUCATION FACULTY • PO Box 94, Grahamstown, 6140

Tel: (046) 603 8383 / (046) 603 8393 • Fax: (046) 622 8028 • e-mail: [illegible]

PROPOSAL AND ETHICAL CLEARANCE APPROVAL

Ethical clearance number 2017.12.08.13

The minute of the EHDC meeting of 05 December 2017 reflect the following:

2017.12.8 CLASS B RESTRICTED MATTERS
MASTER OF EDUCATION RESEARCH PROPOSALS

To consider the following research proposal for the degree of Master of Education in the Faculty of Education:

Mr Given Matengu (15M8765)

Topic: An analysis of how the use of geoboards as visualisation tools can be utilized used in the teaching of quadrilaterals.

Supervisor: Professor M Schäfer
Co-Supervisor: Dr C Chikiwa

Decision: *Approved*

This letter confirms the approval of the above proposal at a meeting of the Faculty of Education Higher Degrees' Committee on the 5 December 2017.

The proposal demonstrates an awareness of ethical responsibilities and a commitment to ethical research processes. The approval of the proposal by the committee thus constitutes ethical clearance.

Sincerely

Ms Zisanda Sanda
Secretariat of the EHDC, Rhodes University
8th December 2017

Appendix B: Approval letter from the Inspector of Education

REPUBLIC OF NAMIBIA
KUNENE REGIONAL COUNCIL
DIRECTORATE: EDUCATION
DIVISION: PROGRAMMES & QUALITY ASSURANCE
SECTION: CIRCUIT OFFICES

Tel: 065 272 950
Fax: 065 273 341
E-mail: uerii.tjiv@gmail.com
Private Bag 3034
Opuwo

Date: 02 January 2018

To: Mr. Given K. Matengu
Student: Rhodes University

SUBJECT: APPLICATION FOR A RESEARCH SITE (CONSENT LETTER)

Your letter on the above subject dated: 30 January 2018 bears reference.

1. First and foremost as one of my teachers in the Circuit I would like to extend my gratitude on the progress you are making towards your study for the Masters' degree in Education.
2. I took note of the schools where you intend to conduct research as well as the envisaged duration.
3. Given the assurance that the study will not cause any disruption to the normal school programme, I hereby unreservedly grant you permission to conduct the research as intended.
4. I sincerely hope that the study will inspire the teachers and have positive impact on the effective use of manipulatives in the mathematics classrooms at the chosen schools.

I wish you all the best.

Faithfully yours,

....
Mr. U. C Tjivikua
IoE: Opuwo Circuit

Appendix C: Approval letters from the school principals

REPUBLIC OF NAMIBIA
KUNENE REGIONAL COUNCIL
MINISTRY OF EDUCATION, ARTS AND CULTURE

[redacted]

TEL: 065-273023
FAX: 065-273153

P O BOX 5
OPUWO

15 February 2018

Mr. Given K. Matengu
Rhodes University
Grahamstown
South Africa

PERMISSION TO CONDUCT RESEARCH

With reference to your letter dated 12 February 2018, I herewith grant you permission to conduct research at [redacted] primary school.

Feel free to contact the school for any further arrangements needed.

Yours in education

[redacted]
PRINCIPAL

KUNENE REGIONAL COUNCIL
Directorate: Education
[redacted]
2018 -02- 15
Tel: 0 [redacted]

KUNENE REGIONAL COUNCIL
DIRECTORATE: EDUCATION,ARTS AND CULTURE
[redacted]

Telefax: 09 264 65 274 804 Private Bag 3004, Opuwo

Enquiries: [redacted]
Mobile: [redacted]
E-mail: [redacted]

School Principal

15 February 2018

Mr Given K. Matengu
Rhodes University
Grahamstown
South Africa

Dear Mr Matengu

SUBJECT: PERMISSION FOR CONDUCTING RESEARCH AT [redacted] PRIMARY SCHOOL

1. Your letter dated 12 February 2018 has reference.
2. Permission is hereby granted for you to conduct research, titled: An analysis of how the Geoboards as visualisation tools can be used in the teaching of quadrilaterals, at [redacted] Primary School.
3. It must be borne in mind that all your activities should not interfere with the normal conduct of classes.
4. It is foreseen that the target group will gain academically from this undertaking.

I wish you the best of success in academic engagements.

Yours faithfully,

[redacted]
Principal

Kunene Regional Council, Education, Arts and Culture
The Principal [redacted]
15 FEB 2018
Private Bag 3004
Opuwo
Tel. 065-274804 Fax. 065-274804

Education is our salvation

TNAP PRIVATE SCHOOL

REPUBLIC OF NAMIBIA
KUNENE REGIONAL COUNCIL
MINISTRY OF EDUCATION, ARTS AND CULTURE

TEL: 065-273023
FAX: 065-273153

P O BOX 5
OPUWO

16 February 2018

To: Mr. Given Kahale Matengu
Rhodes University

Re: Permission letter to conduct research

In response to your request to do research at our school, I am informing you that you have been granted the permission to go ahead and do the research.

I hope and trust that the engagement of our grade 7 mathematics teacher will bring about empowerment in terms of knowledge and teaching methods of Mathematics.

Yours Sincerely

Mr. Simataa R.
PRINCIPAL

MINISTRY OF EDU
2018 -02- 16
TEL: (065) 273009
P.O. BOX 5, OPUWO
REPUBLIC OF NAMIBIA

Appendix D: Participants teachers consent forms

RHODES UNIVERSITY
Where leaders learn

INFORMED CONSENT FORM

Research Project Title:	An analysis of how *Geoboards* as visualisation tools can be used in the teaching of quadrilaterals.
Principal Investigator(s):	GIVEN K. MATENGU

Participation Information

- I understand the purpose of the research study and my involvement in it
- I understand the risks and benefits of participating in this research study
- I understand that I may withdraw from the research study at any stage without any penalty
- I understand that participation in this research study is done on a voluntary basis
- I understand that while information gained during the study may be published, I will remain anonymous and no reference will be made to me by name or student number
- I understand that XXX (other data collection requirements particular to this research, e.g. test results, personal information, video recording) may be used
- I understand and agree that the interviews will be recorded electronically
- I understand that I will be given the opportunity to read and comment on the transcribed interview notes
- I confirm that I am not participating in this study for financial gain

Information Explanation

The above information was explained to me by: MR GIVEN K. MATENGU

The above information was explained to me in English and I am in command of this language: YES

Voluntary Consent

I, [redacted]

her[redacted] consent to participate in the above-mentioned research.

Signature:	Date: 23/02/2018

Investigator Declaration

I, **Given K. Matengu**, declare that I have explained all the participant information to the participant and have truthfully answered all questions asked by the participant.

Signature:	Date: 23/02/2018

RHODES UNIVERSITY
Where leaders learn

INFORMED CONSENT FORM

Research Project Title:	An analysis of how *Geoboards* as visualisation tools can be used in the teaching of quadrilaterals.
Principal Investigator(s):	GIVEN K. MATENGU

Participation Information

- I understand the purpose of the research study and my involvement in it
- I understand the risks and benefits of participating in this research study
- I understand that I may withdraw from the research study at any stage without any penalty
- I understand that participation in this research study is done on a voluntary basis
- I understand that while information gained during the study may be published, I will remain anonymous and no reference will be made to me by name or student number
- I understand that XXX (other data collection requirements particular to this research, e.g. test results, personal information, video recording) may be used
- I understand and agree that the interviews will be recorded electronically
- I understand that I will be given the opportunity to read and comment on the transcribed interview notes
- I confirm that I am not participating in this study for financial gain

Information Explanation

The above information was explained to me by: Given Kahale Matengu

The above information was explained to me in English and I am in command of this language: Yes

Voluntary Consent

I, [redacted]pate in the above-mentioned research.

Signature:	Date: 23/02/2018

Investigator Declaration

I, **Given K. Matengu**, declare that I have explained all the participant information to the participant and have truthfully answered all questions asked by the participant.

Signature:	Date: 23 / 02 / 2018

RHODES UNIVERSITY
Where leaders learn

INFORMED CONSENT FORM

Research Project Title:	An analysis of how the use of *Geoboards* as visualisation tools can be used in the teaching of quadrilaterals.
Principal Investigator(s):	GIVEN K. MATENGU

Participation Information

- I understand the purpose of the research study and my involvement in it
- I understand the risks and benefits of participating in this research study
- I understand that I may withdraw from the research study at any stage without any penalty
- I understand that participation in this research study is done on a voluntary basis
- I understand that while information gained during the study may be published, I will remain anonymous and no reference will be made to me by name or student number
- I understand that XXX (other data collection requirements particular to this research, e.g. test results, personal information, video recording) may be used
- I understand and agree that the interviews will be recorded electronically
- I understand that I will be given the opportunity to read and comment on the transcribed interview notes
- I confirm that I am not participating in this study for financial gain

Information Explanation

The above information was explained to me by: GIVEN K. MATENGU

The above information was explained to me in English and I am in command of this language: YES

Voluntary Consent

I, [redacted]
he[redacted]ed research.

Signature:

Date: 20/02/2018

Investigator Declaration

I, **Given K. Matengu**, declare that I have explained all the participant information to the participant and have truthfully answered all questions asked by the participant.

Signature:

Date: 20/02/2018

Appendix E: Parents' consent forms (English language)

RHODES UNIVERSITY

Where leaders learn

Inquiries: Mr. G.K. Matengu
0812026224

P.O. Box 190, Opuwo.
12 February 2018

Dear Parent

SUBJECT: REQUEST FOR PERMISSION TO CONDUCT RESEARCH ON THE MATHEMATICS TEACHERS OF YOUR CHILD

My name is **GIVEN KAHALE MATENGU**, and I am a Masters (degree) student at Rhodes University (RU) in Grahamstown, South Africa. The research I wish to conduct for my Master's full thesis requires me to observe and video record mathematics teaching at a primary school and interview the teacher of a class. This research will be conducted under the supervision of Prof. Marc Schäfer and Dr. Clemens Chikiwa.

This letter serves to seek formal consent from you as a parent of [redacted] to allow me to enter the classroom of your child in order to video record his/her teacher. The study is on how teachers use a manipulative tool (in this case a geoboard) to teach basic shapes in mathematics. I intend to observe and video record the teacher over a period of three weeks from the 15th February to 16th March 2018. I will not video record any children in the class, but they will be present when the teacher teaches. I further assure you that the intervention will not disturb any official school activities as it is planned to be executed in the afternoons. In the event that you do not wish your child to be part of the class during the time of my research on the teacher, I will not include your child in the intervention. This same assurance was given to the inspector and principal of the school.

This research will be beneficial to the mathematics teacher of your child in that it will enhance the teacher's teaching and in turn benefit your child's learning.

If you require any further queries, please do not hesitate to contact me on the cell number 0812026224 and email address givenmatengu@gmail.com.

Thank you for your time and consideration in this matter.

Yours sincerely

Given K. Matengu (Student: Rhodes University).

I [redacted], parent or legal guardian of [redacted]

hereby give Mr Given K. MATENGU permission to video record my child's teacher for the purpose of his research. It is understood that Mr Given K. MATENGU will not video record my child for any purposes.

Signed: ....

Date: 15/02/2018

Local language: Otjiherero

RHODES UNIVERSITY

Where leaders learn

Inquiries: Mr. G.K. Matengu
0812026224

P.O. Box 190, Opuwo
12 February 2018

Munene Omusuverwa

EPU: OMANINGIRIRO WOUSEMBA OKUTJITA ONGONḒONONENO KOMIṰIRI YOMUATJE WOYE YOVIVARERO.

Owami Given Kahale Matengu, ngu mbi rihonga (omaster degree) Kouniversity yaRhodes mo Grahamstown moSouth Africa. Ongonḓononeno yandje tjinene mai kaṱaṱera kokutara nokukambura onḓengura (oVideo) motjirihongwa ihi ovivarero nawina okukapura omapuriro koMiṱiri yotjirihongwa ho. Ongonḓononeno mai kara hei yomatarero/omatjeverero wozonongo mbari oo Prof. Marc Schäfer na Dr. Clemens Chikiwa.

Orutuu ndwi maru ningire ousemba wokotjiveta kom[redacted] kokutja u ndji yandjere Okuhita metuwo mu mu hita omuatje woye okukayendisa omatarero nomaperendero wandje nga. Omakonḓononeno tjinene maye katara omuano oMiṱiri mbu mai unguriså oviungurisiwa pekepeke (otjitjangero tjovinavikoro) okuhonga ovinavikoro movivarero. Ongonḓononeno ya tyewa okukaehdisiwa koure womueze okuza tji yeri 15 Tengarindi ngaa tji yeri 16 Seninarindi 2018. Hina okukaperenda ngamwa omuatje metuwo ndi posiya ovo mave kara norupa mu ngamwa tji matji ngurwa mo. Me kwizike kutja kapena ovitjitwa vyosikore ma vi kurungisiwa I yomakonḓononeno wandje tjinga owo amaye tjitwa ozoiri zomahatenya.

Omunene ngu hina okuyandjera omuatje we okukara norupa, kena okuningikiziwa kunao kena okurira orupa rwotjitjitwa hi. Omakwizikiro ngayayandjerwa kotjiuru tjosikore nawina omutare wozosikore wosikore ndjo.

Omakonḓononeno nga ouwa owingi koMiṱiri yovivarero yomuatje woye indu oyo tji mai toora onḓjiviro yomuano wokuhonga nu ouwa mbwi harukuru mau ya komuatje woye.

Tji u na ondjivisiro ndji mo vanga okutjiwa, o keyakeya okuhakaena kena ami ponomora yongoze ndji 0812026224 poo keha rorungovi ndwi (givenmatengu@gmail.com)

Me tja okunene okuhepa koruveze rwoye nombango yoye motjito hi.

Owoye

Given K. Matengu (Omuhongwo: poRhodes University).

Ami Mr [redacted] Omunene poo omukuramenepo wa [redacted] mba me yandjere Omutengwa Given K. Matengu omsemba wokutjita ongonḓononeno yomatjira yomuatje wandje. Opakahu kutja Given K. Matengu kena okuḱakambura ovipeperendero (ovideo) komuatje wandje.

Omunwe kembo ....

Omayuva 16-02-2018

Appendix F: Lesson Presentations

Area of a Square

THE LESSON PREPARATION

TEACHER: Mr. Jones	**GRADE:** 7	**DATE:**	**TIME:** 40 min
SUBJECT: Mathematics			
THEME AND TOPIC: Geometry: Quadrilaterals Squares			
TEACHING AIDS AND RESOURCES TO BE USED: *Syllabus, research proposal (M. Ed), Geoboards*			
GENERAL OBJECTIVES (refer to syllabus): Learners will • understand the properties of quadrilaterals and triangles know the terminology for circles	**SPECIFIC COMPETENCIES. Learners should be able to:** • Identify and describe the properties of squares according to the length of sides, parallel or perpendicular sides, angles (right, acute, obtuse) and symmetry.		
• ***If home work was given during the previous lesson, take time to monitor learners´ books before starting with the new lesson, e.g. ask learners to open their books and check randomly $\pm$5 minutes.*** • ***The new lesson should start with a short introduction e.g. probing questions, role play, etc.***			

1. **PRESENTATION OF SUBJECT CONTENT AND LEARNING TASKS:**		
Teachers' activities as aligned to Van Hiele phases of learning Geometry on the *Geoboard*.	**Min**	**Learners' activities as aligned to Van Hiele phases of learning Geometry on the *Geoboard*.**
Information phase The teacher introduces the lesson by identifying what learners already know about squares, thus orientating them to the topic. This will be done by constructing as many shapes as possible on the *Geoboard*, differing in size and shape for learners to identify squares out of them.	5	Learners identify squares from different shapes constructed on the teacher's *Geoboard*. They will furthermore explain to why the said shape is a square.
Guided orientation The teacher allow learners to explore shapes in carefully structured tasks such as folding, measuring, or constructing. In this case the teacher will allow learners to explore the properties of squares by constructing specific concepts such as parallel, equal, perpendicular, diagonals, opposite sides, and line of symmetry on G*eoboards*.	10	Learners make line segments that are perpendicular, parallel, intersecting, and opposite on *Geoboards* by placing rubber bands on the pins. Learners will also be guided on how to work with symmetry (line of symmetry).
Explicitation phase. The teacher assist and inspire learners to describe and discuss what they have learned about the concepts in their own words. The discussion of learners should	5	In groups of five each, learners discuss how they can construct line segments that are perpendicular, parallel, intersecting, and opposite on *Geoboards*. The discussion will be accompanied by a demonstration on the *Geoboard*.

feature on how they have seen the properties on *Geoboards*		
Free orientation phase. The teacher helps learners to make connections between properties of squares by using a *Geoboard* to solve problems and investigate more open-ended tasks. The teacher will give learners a variety of tasks on G*eoboards* on the properties of squares.	15	Learners make connections of the different properties and concepts that are found on a square. For example, such as because all the four sides are equal then the line of symmetry are also four, connections between diagonals and line of symmetry, as well as connections between perpendicular lines and lines of symmetry.
Integration phase. The teacher uses the *Geoboard* to help learners to summarise, reflect on and integrate what they have learned about the properties of squares.	5	Learners summarise and integrate the different concepts/properties that are constructed on *Geoboards*. Learners will be directed to construct squares of different sizes on *Geoboards*.

- ***Incorporate reading and writing activities during lesson delivery***
- ***Accommodate all learners with their different learning abilities & pace of learning, with activities during the lesson or in the afternoon/evening extra classes***

2. INDICATE THE NATURE OF ASSESSMENT TASK(S) GIVEN [after the lesson]: Topic task/practical investigation/project/practical Exercises, etc. e.g. Topic task 1, practical investigation 2, etc.

Will this Assessment Activity form part of CA marks	YES	NO
		X

3. REFLECTION:

NB: The main focus on lesson delivery is <u>''specific competencies (Objectives) turned into class exercises and assessment tasks during and after the lesson''.</u> The teacher should take LESS time on lecturing and MORE time on engaging learners with class exercises during the lesson. This exercise requires that the teacher prepares activities and tasks based on specific competencies (objectives) well IN ADVANCE and ensure that learners are engaged from the beginning to the end of the lesson. Class exercises can be marked through peer marking (by learners), so that quality time is geared towards all specific competencies (objectives). <u>Please attach all assessment activities (questions and answers) given to learners to this lesson plan.</u>

NAME & SIGNATURE **DATE**

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PRINCIPAL/HOD/SEO/SUBJECT HEAD/PHASE HEAD

DATE

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SUBJECT TEACHER

Area of a rhombus

THE LESSON PREPARATION

<table>
<tr><td>TEACHER: Mr. Jones</td><td>GRADE: 7</td><td>DATE:</td><td>TIME: 40min</td></tr>
<tr><td colspan="4">SUBJECT: Mathematics</td></tr>
<tr><td colspan="4">THEME AND TOPIC: Geometry: Quadrilaterals
Rhombus</td></tr>
<tr><td colspan="4">TEACHING AIDS AND RESOURCES TO BE USED: Syllabus, research proposal (M. Ed), Geoboards.</td></tr>
<tr><td colspan="2">GENERAL OBJECTIVES (refer to syllabus): Learners will

• understand the properties of quadrilaterals and triangles know the terminology for circles</td><td colspan="2">SPECIFIC COMPETENCIES. Learners should be able to:

• Identify and describe the properties of a rhombus according to the length of sides, parallel or perpendicular sides, angles (right, acute, obtuse) and symmetry.</td></tr>
<tr><td colspan="4">• If home work was given during the previous lesson, take time to monitor learners´ books before starting with the new lesson, e.g. ask learners to open their books and check randomly ±5 minutes.
• The new lesson should start with a short introduction e.g. probing questions, role play, etc.</td></tr>
</table>

4. **PRESENTATION OF SUBJECT CONTENT AND LEARNING TASKS:**		
Teachers activities as aligned to Van Hiele phases of learning Geometry	**Min**	**CLASS EXERCISES (should be relevant to the specific competencies/objectives)**
Information phase The teacher introduces the lesson by identifying what learners already know about a rhombus, thus orientating them to the new topic. This will be done by constructing as many shapes as possible on the *Geoboard*, differing in size and shape for learners to identify rhombuses out of them.	5	Learners will identify rhombuses out of the many shapes constructed on the *Geoboard* by the teacher. The shapes will also include squares.
Guided orientation The teacher allow learners to explore shapes in carefully structured tasks such as folding, measuring, or constructing. In this case the teacher will allow learners to explore the properties of a rhombus by constructing specific concepts such as parallel, equal, perpendicular, diagonals, and line of symmetry on *Geoboards*. The teacher will show learners on how to change from a square to a rhombus on a *Geoboard*.	10	Learners make line segments that are perpendicular, parallel, intersecting, and adjacent sides on *Geoboards* by placing rubber bands on the pins. Learners will also be guided on how to work with symmetry (line of symmetry) on *Geoboards*.
Explicitation phase. Teachers assist and inspire learners to describe and discuss what they have learned about the concepts in their own words. Furthermore the teacher will construct a square and a rhombus for learners to discuss the similarities and differences that exists between the two.	5	In groups of five each, learners discuss how they can construct line segments that are equal, perpendicular, parallel, intersecting on *Geoboards*. The discussion will be accompanied by a demonstration on the *Geoboard*. Learners will also discuss the differences that exists between a square and a rhombus.

Free orientation phase. The teacher helps learners to start making their own connections on the properties of a rhombus by using a *Geoboard* to solve problems and investigate more open-ended tasks. In this case learners will be given a variety of tasks on G*eoboards* to explore the properties of a rhombus. They will make connections between a rhombus and a square.	15	Learners make connections of the different properties and concepts that are found on a rhombus. For example, connections on why the line segments of a rhombus are all equal on the *Geoboard*. They will also contrast the angles of a rhombus to that one of a square. Learners will use a *Geoboard* to make connections such as the lines of symmetry of a rhombus are in the same position as the diagonals of a rhombus.
Integration phase. In this phase learners should be able to summarise, reflect on and integrate what they have learned about the properties of a rhombus. The teacher will summarise the lesson by asking learners to differentiate between a square and a rhombus.	5	Learners summarise and integrate the different concepts/properties that are constructed on *Geoboards*. Learners will draw rhombuses and explain the properties of a rhombus by using a *Geoboard*.
• ***Incorporate reading and writing activities during lesson delivery*** • ***Accommodate all learners with their different learning abilities & pace of learning, with activities during the lesson or in the afternoon/evening extra classes***		

5. **INDICATE THE NATURE OF ASSESSMENT TASK(S) GIVEN [after the lesson]: Topic task/practical investigation/project/practical Exercises, etc. e.g. Topic task 1, practical investigation 2, etc.**

Will this Assessment Activity form part of CA marks	YES	NO
		X

6. **REFLECTION:**

NB: The main focus on lesson delivery is ''specific competencies (Objectives) turned into class exercises and assessment tasks during and after the lesson''. The teacher should take LESS time on lecturing and MORE time on engaging learners with class exercises during the lesson. This exercise requires that the teacher prepares activities and tasks based on specific competencies (objectives) well IN ADVANCE and ensure that learners are engaged from the beginning to the end of the lesson. Class exercises can be marked through peer marking (by learners), so that quality time is geared towards all specific competencies (objectives). Please attach all assessment activities (questions and answers) given to learners to this lesson plan.

NAME & SIGNATURE **DATE**

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PRINCIPAL/HOD/SEO/SUBJECT HEAD/PHASE HEAD

DATE

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SUBJECT TEACHER

Area of a parallelogram

THE LESSON PREPARATION

TEACHER: Ms. Ruth	**GRADE:** 7	**DATE:**	**TIME:** 40min
SUBJECT: Mathematics			
THEME AND TOPIC: Geometry: Quadrilaterals Parallelogram			
TEACHING AIDS AND RESOURCES TO BE USED: *Syllabus, Research Proposal (M. Ed), Geoboards and dot papers*			
GENERAL OBJECTIVES (refer to syllabus): Learners will • understand the properties of quadrilaterals and triangles know the terminology for circles		**SPECIFIC COMPETENCIES. Learners should be able to:** • Identify and describe the properties of parallelograms according to the length of sides, parallel or perpendicular sides, angles (right, acute, obtuse) and symmetry.	
• ***If home work was given during the previous lesson, take time to monitor learners´ books before starting with the new lesson, e.g. ask learners to open their books and check randomly*** $\pm$***5 minutes.*** • ***The new lesson should start with a short introduction e.g. probing questions, role play, etc.***			

7. PRESENTATION OF SUBJECT CONTENT AND LEARNING TASKS:

Teachers activities as aligned to Van Hiele phases of learning Geometry	**Min**	**CLASS EXERCISES** **(should be relevant to the specific competencies/objectives)**
Information phase The teacher introduces the lesson by identifying what learners already know about parallelograms, thus orientating them to the topic. This will be done by constructing as many shapes as possible on the *Geoboard*, differing in size and shape for learners to identify the parallelograms out of them.	5	Learners identify parallelograms from the constructed shapes on the *Geoboard*. The constructed shapes will include rectangles.
Guided orientation The teacher allows learners to explore shapes in carefully structured tasks such as folding, measuring, or constructing. In this case the teacher will allow learners to explore the properties of parallelograms by constructing specific concepts such as parallel, equal, diagonals, opposite sides, and symmetry on G*eoboards* as well as dot papers. The teacher will change a rectangle to a parallelogram by using a *Geoboard*.	10	Learners make line segments that are equal, parallel, intersecting, and opposite on *Geoboards* by placing rubber bands on the pins. Learners will also be guided on how to work with symmetry (line of symmetry). The angles of a parallelogram will also be explored in comparison to rectangles.
Explicitation phase. Teachers assist and inspire learners to describe and discuss what they have learned about the concepts in their own words. The discussion of learners should feature on how they used *Geoboards* to explore the properties.	5	In groups of 3 to 4 each, learners will discuss why parallelograms does not have 90-degree angles, why it does not have lines of symmetry, as well as why only the opposite angles are equal. The discussion will be accompanied by demonstrations on the *Geoboards*.

Free orientation phase. The teacher helps learners to start making their own connections on the properties of parallelograms by using a *Geoboard* to solve problems and investigate more open-ended tasks. Learners will be given a variety of tasks on both G*eoboards* and dot papers on the properties of parallelograms and they will be asked to make connections to other shapes, for example making connections between a rectangle and a parallelogram.	15	Learners make connections of the different properties and concepts that are found on a parallelogram in comparison to rectangles.
Integration phase. In this phase the teacher helps learners to summarise what they have learned on the properties of a parallelograms.	5	Learners summarise and integrate the different concepts/properties that are constructed on *Geoboards*. Learners will draw parallelograms of different sizes on dot papers.
• ***Incorporate reading and writing activities during lesson delivery*** • ***Accommodate all learners with their different learning abilities & pace of learning, with activities during the lesson or in the afternoon/evening extra classes***		

8. INDICATE THE NATURE OF ASSESSMENT TASK(S) GIVEN [after the lesson]: Topic task/practical investigation/project/practical Exercises, etc. e.g. Topic task 1, practical investigation 2, etc.

Will this Assessment Activity form part of CA marks	**YES**	**NO**
		X

9. REFLECTION:

NB: The main focus on lesson delivery is <u>''specific competencies (Objectives) turned into class exercises and assessment tasks during and after the lesson''.</u> The teacher should take LESS time on lecturing and MORE time on engaging learners with class exercises during the lesson. This exercise requires that the teacher prepares activities and tasks based on specific competencies (objectives) well IN ADVANCE and ensure that learners are engaged from the beginning to the end of the lesson. Class exercises can be marked through peer marking (by learners), so that quality time is geared towards all specific competencies (objectives). <u>Please attach all assessment activities (questions and answers) given to learners to this lesson plan.</u>

NAME & SIGNATURE **DATE**

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PRINCIPAL/HOD/SEO/SUBJECT HEAD/PHASE HEAD

DATE

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SUBJECT TEACHER

Area of a trapezium

THE LESSON PREPARATION

TEACHER: Ms. Ruth	**GRADE:** 7	**DATE:**	**TIME:** 40 min
SUBJECT: Mathematics			
THEME AND TOPIC: Geometry: Quadrilaterals Trapeziums			
TEACHING AIDS AND RESOURCES TO BE USED: *Syllabus, research proposal (M. Ed), Geoboards and dot papers*			
GENERAL OBJECTIVES (refer to syllabus): Learners will • understand the properties of quadrilaterals and triangles know the terminology for circles		**SPECIFIC COMPETENCIES. Learners should be able to:** • Identify and describe the properties of trapeziums according to the length of sides, parallel or perpendicular sides, angles (right, acute, obtuse) and symmetry.	
• ***If home work was given during the previous lesson, take time to monitor learners´ books before starting with the new lesson, e.g. ask learners to open their books and check randomly $\pm$5 minutes.*** • ***The new lesson should start with a short introduction e.g. probing questions, role play, etc.***			

10. PRESENTATION OF SUBJECT CONTENT AND LEARNING TASKS:		
Teachers' activities as aligned to Van Hiele phases of learning Geometry on the *Geoboard*.	**Min**	**Learners' activities as aligned to Van Hiele phases of learning Geometry on the *Geoboard*.**
Information phase The teacher introduces the lesson by identifying what learners already know about trapeziums, thus orientating them to the topic. This will be done by constructing as many shapes as possible on the Geoboard, differing in size and shape for learners to identify squares out of them.	5	Learners identify trapeziums from different shapes constructed on the teacher's *Geoboard*. They will furthermore explain to why the said shape is a square.
Guided orientation The teacher allows learners to explore shapes in carefully structured tasks such as folding, measuring, or constructing. In this case the teacher will allow learners to explore the properties of trapeziums by constructing specific concepts such as parallel, equal, perpendicular, diagonals, opposite sides, and line of symmetry on *Geoboards*. The teacher will also use a *Geoboard* to explore the properties of an Isosceles Trapezium	10	Learners make line segments that are perpendicular, parallel, intersecting, and opposite on *Geoboards* by placing rubber bands on the pins. Learners will also be guided on how to work with symmetry (line of symmetry on an Isosceles Trapezium).
Explicitation phase. The teacher assists and inspire learners to describe and discuss what they have learned about the concepts in their own words. The discussion of learners should feature on how they have seen the properties on *Geoboards*	5	In groups of five each, learners discuss how they can construct line segments that are perpendicular, parallel, intersecting, and opposite on *Geoboards*. The discussion will be accompanied by a demonstration on the *Geoboard*.

Free orientation phase. The teacher helps learners to make connections between properties of trapezium by using a geoboard to solve problems and investigate more open-ended tasks. The teacher will give learners a variety of tasks on *Geoboards* on the properties of trapezium (isosceles trapezium.	15	Learners make connections of the different properties and concepts that are found on a trapezium. For example such as a parallelogram has only one pair of parallel sides, whilst an isosceles trapezium has one pair of parallel sides, and two sides equal thus this makes it to have two base angles equal and one line of symmetry.
Integration phase. The teacher uses the *Geoboard* to help learners to summarise, reflect on and integrate what they have learned about the properties of a trapezium.	5	Learners summarise and integrate the different concepts/properties that are constructed on *Geoboards*. Learners will be directed to construct trapeziums (including isosceles trapeziums) of different sizes on *Geoboards*.
• ***Incorporate reading and writing activities during lesson delivery*** • ***Accommodate all learners with their different learning abilities & pace of learning, with activities during the lesson or in the afternoon/evening extra classes***		

11. INDICATE THE NATURE OF ASSESSMENT TASK(S) GIVEN [after the lesson]: Topic task/practical investigation/project/practical Exercises, etc. e.g. Topic task 1, practical investigation 2, etc.

Will this Assessment Activity form part of CA marks	YES	NO
		X

12. REFLECTION:

NB: The main focus on lesson delivery is <u>''specific competencies (Objectives) turned into class exercises and assessment tasks during and after the lesson''.</u> The teacher should take LESS time on lecturing and MORE time on engaging learners with class exercises during the lesson. This exercise requires that the teacher prepares activities and tasks based on specific competencies (objectives) well IN ADVANCE and ensure that learners are engaged from the beginning to the end of the lesson. Class exercises can be marked through peer marking (by learners), so that quality time is geared towards all specific competencies (objectives). <u>Please attach all assessment activities (questions and answers) given to learners to this lesson plan.</u>

NAME & SIGNATURE **DATE**

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PRINCIPAL/HOD/SEO/SUBJECT HEAD/PHASE HEAD

DATE

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SUBJECT TEACHER

Area of a rectangle

THE LESSON PREPARATION

TEACHER: Ms. Smith	**GRADE:** 7	**DATE:**	**TIME:** 40min
SUBJECT: Mathematics			
THEME AND TOPIC: Geometry: Quadrilaterals Rectangles			
TEACHING AIDS AND RESOURCES TO BE USED: *Syllabus, research proposal (M. Ed), Geoboards and dot papers*			
GENERAL OBJECTIVES (refer to syllabus): Learners will • understand the properties of quadrilaterals and triangles know the terminology for circles		**SPECIFIC COMPETENCIES. Learners should be able to:** • Identify and describe the properties of rectangles according to the length of sides, parallel or perpendicular sides, angles (right, acute, obtuse) and symmetry.	
• ***If home work was given during the previous lesson, take time to monitor learners´ books before starting with the new lesson, e.g. ask learners to open their books and check randomly*** $\pm$***5 minutes.*** • ***The new lesson should start with a short introduction e.g. probing questions, role play, etc.***			

13. PRESENTATION OF SUBJECT CONTENT AND LEARNING TASKS:		
Teachers activities as aligned to Van Hiele phases of learning Geometry	**Min**	**CLASS EXERCISES (should be relevant to the specific competencies/objectives)**
Information phase The teacher introduces the lesson by identifying what learners already know about rectangles, thus orientating them to the topic. This will be done by constructing as many shapes as possible on the *Geoboard*, differing in size and shape for learners to identify rectangles out of them.	5	Learners identify rectangles from the constructed shapes on the *Geoboard*.
Guided orientation The teacher allow learners to explore shapes in carefully structured tasks such as folding, measuring, or constructing. In this case the teacher will allow learners to explore the properties of rectangles by constructing specific concepts such as parallel, equal, perpendicular, diagonals, opposite sides, and symmetry on *Geoboards* as well as dot papers.	10	Learners make line segments that are perpendicular, parallel, intersecting, and opposite on *Geoboards* by placing rubber bands on the pins. Learners will also be guided on how to work with symmetry (line of symmetry). The 90 degrees of a rectangles will also be explored through the concept of perpendicular lines.
Explicitation phase. Teachers assist and inspire learners to describe and discuss what they have learned about the concepts in their own words. The discussion of learners should feature on how they used *Geoboards* to explore the properties.	5	In groups of 3 to 4 each, learners discuss how they can construct line segments that are perpendicular, parallel, intersecting, and opposite on *Geoboards*. The discussion will be accompanied by demonstrations on the *Geoboards*.

Free orientation phase. The teacher helps learners to start making their own connections on the properties of quadrilaterals by using a *Geoboard* to solve problems and investigate more open-ended tasks. Learners will be given a variety of tasks on both G*eoboards* and dot papers on the properties of different quadrilaterals and they will be asked to make connections to other shapes, for example making connections between a square and a rectangle.	15	Learners make connections of the different properties and concepts that are found on a rectangle. For example connections between perpendicular lines and lines of symmetry, connections between a square and a rectangle.
Integration phase. In this phase the teacher help learners to summarise what they have learned on the properties of a rectangle.	5	Learners summarise and integrate the different concepts/properties that are constructed on *Geoboards*. Learners will draw rectangles of different sizes on dot papers.
• ***Incorporate reading and writing activities during lesson delivery*** • ***Accommodate all learners with their different learning abilities & pace of learning, with activities during the lesson or in the afternoon/evening extra classes***		

14. INDICATE THE NATURE OF ASSESSMENT TASK(S) GIVEN [after the lesson]: Topic task/practical investigation/project/practical Exercises, etc. e.g. Topic task 1, practical investigation 2, etc.

Will this Assessment Activity form part of CA marks	**YES**	**NO**
		X

15. REFLECTION:

NB: The main focus on lesson delivery is <u>''specific competencies (Objectives) turned into class exercises and assessment tasks during and after the lesson''.</u> The teacher should take LESS time on lecturing and MORE time on engaging learners with class exercises during the lesson. This exercise requires that the teacher prepares activities and tasks based on specific competencies (objectives) well IN ADVANCE and ensure that learners are engaged from the beginning to the end of the lesson. Class exercises can be marked through peer marking (by learners), so that quality time is geared towards all specific competencies (objectives). <u>Please attach all assessment activities (questions and answers) given to learners to this lesson plan.</u>

NAME & SIGNATURE **DATE**

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PRINCIPAL/HOD/SEO/SUBJECT HEAD/PHASE HEAD

DATE

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SUBJECT TEACHER

Area of a kite

THE LESSON PREPARATION

TEACHER: Ms. Smith	**GRADE:** 7	**DATE:**	**TIME:** 40min
SUBJECT: Mathematics			
THEME AND TOPIC: Geometry: Quadrilaterals Kite			
TEACHING AIDS AND RESOURCES TO BE USED: *Syllabus, research proposal (M. Ed), Geoboards and dot papers*			
GENERAL OBJECTIVES (refer to syllabus): Learners will • understand the properties of quadrilaterals and triangles know the terminology for circles		**SPECIFIC COMPETENCIES. Learners should be able to:** • Identify and describe the properties of a Kite according to the length of sides, parallel or perpendicular sides, angles (right, acute, obtuse) and symmetry.	
• ***If home work was given during the previous lesson, take time to monitor learners´ books before starting with the new lesson, e.g. ask learners to open their books and check randomly ±5 minutes.*** • ***The new lesson should start with a short introduction e.g. probing questions, role play, etc.***			

16. PRESENTATION OF SUBJECT CONTENT AND LEARNING TASKS:

Teachers activities as aligned to Van Hiele phases of learning Geometry	Min	CLASS EXERCISES (should be relevant to the specific competencies/objectives)
Information phase The teacher introduces the lesson by identifying what learners already know about a Kites, thus orientating them to the new topic. This will be done by constructing as many shapes as possible on the *Geoboard*, differing in size and shape for learners to identify Kites out of them.	5	Learners will identify Kites out of the many shapes constructed on the *Geoboard* by the teacher. The shapes will also include Trapeziums.
Guided orientation The teacher allow learners to explore shapes in carefully structured tasks such as folding, measuring, or constructing. In this case the teacher will allow learners to explore the properties of a Kite by constructing specific concepts such as equal, perpendicular, diagonals, adjacent sides, and symmetry on *Geoboards* as well as dot papers.	10	Learners make line segments that are equal, intersecting, and adjacent sides on *Geoboards* by placing rubber bands on the pins. Learners will also be guided on how to work with symmetry (line of symmetry).
Explicitation phase. Teachers assist and inspire learners to describe and discuss what they have learned about the concepts in their own words. Furthermore, the teacher will use a *Geoboard* to construct the lines of symmetry and diagonals of a Kite and let learners discuss the relations.	5	In groups of 3 to 4 each, learners discuss how they can construct line segments that are perpendicular, intersecting, and adjacent on *Geoboards*. The discussion will be accompanied by a demonstration on the *Geoboard*.

Free orientation phase. The teacher helps learners to start making their own connections on the properties of a rhombus by using a *Geoboard* to solve problems and investigate more open-ended tasks. In this case learners will be given a variety of tasks on both *Geoboards* and dot papers on the properties of a Kite and asked to make connections to other shapes and figures. The teacher will furthermore help learners to explore the concept of adjacent sides of a Kite on a *Geoboard*.	15	Learners make connections of the different properties and concepts that are found on a Kite. For example, connections on why the longest diagonal bisects (divides in half) the shortest diagonal by using *Geoboards* or dot papers. They will also look at the relationship between diagonals and lines of symmetry of a kite.
Integration phase. In this phase learners should be able to summarise, reflect on and integrate what they have learned about the properties of a Kite.	5	Learners summarise and integrate the different concepts/properties that are constructed on *Geoboards*. Learners will draw Kites and explain the properties of a Kite by using a *Geoboard*.
• ***Incorporate reading and writing activities during lesson delivery*** • ***Accommodate all learners with their different learning abilities & pace of learning, with activities during the lesson or in the afternoon/evening extra classes***		

17. INDICATE THE NATURE OF ASSESSMENT TASK(S) GIVEN [after the lesson]: Topic task/practical investigation/project/practical Exercises, etc. e.g. Topic task 1, practical investigation 2, etc.

Will this Assessment Activity form part of CA marks	YES	NO
		X

18. REFLECTION:

NB: The main focus on lesson delivery is <u>''specific competencies (Objectives) turned into class exercises and assessment tasks during and after the lesson''</u>. The teacher should take LESS time on lecturing and MORE time on engaging learners with class exercises during the lesson. This exercise requires that the teacher prepares activities and tasks based on specific competencies (objectives) well IN ADVANCE and ensure that learners are engaged from the beginning to the end of the lesson. Class exercises can be marked through peer marking (by learners), so that quality time is geared towards all specific competencies (objectives). <u>Please attach all assessment activities (questions and answers) given to learners to this lesson plan.</u>

NAME & SIGNATURE **DATE**

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PRINCIPAL/HOD/SEO/SUBJECT HEAD/PHASE HEAD

DATE

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SUBJECT TEACHER

Appendix G: Stimulus Recall Interview

Semi-structured questions

1. How did you use the Geoboard to determine the prior knowledge that your learners had about quadrilaterals and their properties? Please provide examples and identify on the video recording where this happened.

2. Did you find the *Geoboards* useful for forming different types of shapes? Please provide examples and identify on the video recording where this happened.

3. How did you provide the learners an opportunity to discuss or report back what they found out about the properties using the *Geoboards*? Please provide examples and identify on the video recording where this happened.

4. How did you use the *Geoboards* to enable learners to make links between the various properties of the quadrilaterals? Please provide examples and identify on the video recording where this happened.

5. Do you think that using the *Geoboards* equipped your learners with the necessary skills to identify and name quadrilaterals according to their properties? Please elaborate and provide examples

6. In your view and experience, did the use of *Geoboards* add to the learners' conceptual understanding of quadrilaterals? Please elaborate and provide examples.

7. In your view, did the use of the *Geoboards* increase learner participation in their learning of quadrilaterals? Please elaborate and provide examples.

8. How do you rate the Geoboard's visual aspect to the visualisation processes we discussed in one of our workshops? Please elaborate.

9. According to your experience what were some of the constraining factors when using the *Geoboards* in your teaching? Please elaborate.

10. Are you going to use the *Geoboard* again in your teaching? Please explain your answer.

11. Please elaborate on how the use of the *Geoboard* enabled you to teach more visually?

12. How did the *Geoboard* assist the learners in visualising quadrilaterals?

Appendix H: Focus Group Interviews
Semi-structured questions

1. Would you please share with me your experiences of using a *Geoboard*?

2. Did you find the *Geoboards* useful for forming different types of shapes? Please explain.

3. How did you use the *Geoboard* to explore the different visualisation processes we have discussed during the workshops?

4. How did you experience the formation and transformation of shapes on the *Geoboards*?

5. How do you relate the use of *Geoboards* to the participation of learners?

6. How did *Geoboards* help you to teach the quadrilaterals in a more visual way?

7. How do you evaluate the class and group discussions during your lessons?

8. What are the challenges that you experienced when you were using *Geoboards* in your teaching? Please mention and elaborate on them?

9. What would you recommend that would enable us to effectively use *Geoboards* in our teaching?