AN INVESTIGATION OF TEACHERS' MATHEMATICAL TASK SELECTION IN THE ZAMBIAN CONTEXT

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Evaristo Kangwa

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ABSTRACT

This research sought to investigate the sources and type of tasks used in the teaching of trigonometry in Zambia's secondary schools, and to investigate the criteria used and decisions made by teachers in their selection and implementation of tasks. The study was conducted in three different school types located in high cost, medium cost and low cost respectively. One participant was chosen from each of the different categories of schools.

The research was located within an interpretive paradigm. Data were collected through semi-structured interviews, lesson observations and document analysis which include: lesson plans for five consecutive days, pupils' activity books and three textbooks predominantly used by the teachers. Document analysis was informed by the task analysis guide and essential themes which were used to tease out teachers' task practice with regard to criteria used and decisions made in the selection and implementation of tasks. Essential themes that were qualitatively established were validated and explicated by the qualitative analysis.

The findings of the study indicate that teachers picked tasks from prescribed textbooks. The study further suggests that teachers selected a mix of low and high level tasks, procedures without connections and procedures with connections tasks to be specific. There were no memorisations and doing mathematics tasks. Their choice of tasks was based on the purpose for which the task was intended. Some tasks were selected for the purpose of practicing the procedures and skills, other tasks for the promotion conceptual development. Most of high level tasks decline to low level tasks during implementation. The findings also indicate that teachers selected and implemented a variety of tasks and concepts. Furthermore, teachers presented tasks in various forms of representations and in a variety of ways.

However, the results of this study could not be generalized because of the small sample involved. The results presented reflect the views and task practices of the target group. A possibility for future study would be to consider a large population, drawn across the country.

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Finally, special thanks go to my wife Ireen for her unwavering support and encouragement throughout the period of my study. I also thank my children for the support and patience, especially the times I was not there for them whenever they needed paternal guidance.

DEDICATION

I dedicate this work to mum and dad for raising me up into a responsible parent.

DECLARATION OF ORIGINALITY

I EVARISTO KANGWA declare that this project is my own work written in my own words. Where I have drawn on the words or ideas of others, these have been acknowledged using complete references according to the departmental guidelines

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ACRONYMS

AIEMS	Action for Improving of English, Mathematics and Sciences
JICA	Japanese International Co-operation Agency
MOE	Ministry of Education
SMASTE	Strengthening Mathematics, Sciences and Technology Education
VVOB	Flemish Office for International Co-operation and Technical Assistance
ZSSSM	Zambia Secondary School Syllabus Mathematics

CHAPTER ONE: INTRODUCTION

1.1 Introduction

The teaching of mathematics is characterised by the use of manipulatives, calculations, activities, an assortment of tasks and solving of problems (Doyle, 1983; Shimizu, Kaur, Huang & Clarke, 2010). For this reason, classroom instruction is largely and generally organised and orchestrated around mathematical instructional tasks. This means that students in mathematics classrooms spend much of their day-to-day time working on tasks, activities or problems. Task selection becomes very important in this respect, because the tasks with which students are engaged determine what they learn about mathematics and how they learn it (Stein, Remillard, & Smith, 2007). This is underscored by Kilpatrick, Swafford and Findell"s (2001) statement that "what is learned depends on what is taught" (p. 333). Teachers affect students" learning by choosing the content, deciding how to present it, and determining how much time to allocate to it.

This chapter describes the context of this study, and attempts to give its significance in relation to task practice. The goals and objectives are also discussed here. In the first section the background to the problem to be investigated, is given, as it is perceived in relation to the teaching and learning of trigonometry in the Zambian context.

According to Kilpatrick et al. (2001), teachers make choices and decisions in their day-to-day classroom practice. The decisions they make are based on a number of factors, which include the content, the students and the ways in which students learn mathematics. For instance, teachers make instructional decisions about what to teach, how to represent it, and how to deal with problems of student misunderstandings. Teachers also make decisions about what kind of tasks are central to students" learning; tasks that shape not only their opportunity to learn but also their view of the subject matter. Each of these decisions requires careful consideration of the mathematics at stake and the instructional options and purposes at play. It is therefore reasonable to argue that improvements in task selection and implementation will lead to improvements in students" learning.

From the above argument, it is apparent that good selection of tasks by teachers, is important for promoting learning in mathematics. The types of tasks the teachers select have a strong influence on the kind of classroom environment and the opportunity to learn that is created. Kilpatrick, et al. (2001) state that students learn best when they are presented with challenging work that focuses on sense making and problem solving, as well as skill building. It has been acknowledged widely in literature that effective instruction and eventual student achievement is dependent upon the type of tasks student are engaged with (Kilpatrick, et al., 2001; Stein et al., 1997; Doyle & Carter, 1984)

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1.2 Research context

The majority of mathematics teachers in the world over rely on textbooks and curriculum materials as the main tool for teaching (Stein, et al. 2007; Valverde, Bianchi, Wolfe, Schmidt & Houng, 2002; Pepin & Haggarty, 2001). According to Pepin (2008), "teachers often rely heavily on textbooks in their day-to-day teaching, and they decide what to teach, how to teach it, and the kinds of tasks and exercises to give to their students" (p. 1). Textbooks are a key part of the curriculum materials used for guiding students‟ acquisition of culturally valued concepts, procedures, intellectual dispositions, and ways of reasoning (Battista & Clements, 2000). Textbooks define and represent the subject for many students, and they influence how those students experience mathematics. Pepin (2009) reports that "the use of curricular materials such as textbooks, together with the selection of mathematical tasks, impacts to a large extent on the mathematical diet offered to students" (p. 2507). Indeed textbooks are the primary resources for both students and teachers in the classroom. The teachers in Zambian schools are no exception in this regard.

1.2.1 Education in Zambia

The Education system in Zambia, before 1964 and for two decades after independence was predominantly western in orientation. Secondary schools then followed the Cambridge syllabus. This meant that textbooks and other curricular materials which were used in most public and privately owned schools had to come from abroad. The Zambian government, realising that its students were being deprived of the opportunities to learn due to textbooks and materials whose contexts were foreign to the learners, decided to introduce a Zambian secondary school

mathematics syllabus. This syllabus was developed; primarily to "address the factors that inhibit children‟s comprehension and understanding of the mathematics they are trying to learn, by suggesting methods and strategies to suit the different situations and environments in which mathematics is taught" (Mukuyamba, Shamapango, Sichilima, Mwanakatwe, Banda, Nkalamo & Mumbula, 1995, p. iii).

The introduction of the Zambian secondary school syllabus mathematics was hoped to change the way mathematics was being taught and learned in schools. It was envisaged that with that change, students‟ performance in mathematics and the learning of mathematics in general would improve. Nevertheless, students‟ performance in mathematics kept on declining despite the introduction of this syllabus. The Ministry of Education then undertook a number of initiatives in order to improve student performance in mathematics and sciences. For instance in 1997, the Ministry of Education established the Action for Improvement of English, Mathematics and Sciences Programmes (AIEMS) as an initiative to improve the teaching of English, Mathematics and Sciences at all levels (MOE, 1996, P.173). The Ministry of Education has since been working in areas of mathematics and sciences along with partners such as the Flemish Office for International Co-operation and Technical Assistance (VVOB) of Belgium and Japanese International Co-operation Agency for Strengthening Mathematics, Sciences and Technology Education (JICA SMASTE). However, Zambian students‟ performance in mathematics for more than a decade now has not been remarkable. In fact, education Minister, Geoffrey Lungwangwa is quoted as saying "the overall performance of pupils in mathematics, science and technology has been very poor" ("Mathematics is difficult-Education Minister", 2009). Inspite of a wide range of resources to which both the teachers and students are exposed, there has been little or no significant improvement recorded. Poor task selection may be one of the factors contributing to these poor results. This study aims to investigate the source and type of tasks teachers select for use in their classrooms, the criteria used in selecting tasks and the decisions teachers make with regard to task selection and implementation.

1.2.2 Task selection in Zambian classrooms

Good selection of tasks by teachers, to promote learning in mathematics, is critical. But, in my personal experience as a teacher of mathematics in a secondary school, I must admit that there were times when I conveniently picked tasks from textbooks without scrutinising their suitability and presented them as such in class. I often did this when I was not comfortable with and did not prepare adequately in a particular topic. I suspect that many teachers in Zambia have at one time or the other fallen into a similar trap.

The experience I have gained so far as a teacher trainer, with student teachers on their teaching practice is in agreement with my presumption about teachers‟ understanding of tasks and task selection. Each time I have gone out monitoring my students, I have seen the same pattern of mathematical tasks, in terms of their levels of cognitive demand. My experience of this stereotypical nature of tasks selected by student teachers for use in their classrooms, led me to believe that they may be drawing these tasks without careful selection, from particular resources. Additionally, my interaction with a few experienced teachers led me to believe that this may be the trend not only for trainee teachers, but also for the experienced ones. Many of these teachers appear to pick tasks from textbooks without paying attention to the nature and the purpose for which the task should be selected. It is therefore prudent to argue that poor task selection and their eventual implementation in class could be contributors to Zambian students‟ poor achievement in mathematics.

However, this does not necessarily mean that all mathematics teachers in Zambia are poor at task selection. There are, of course, some teachers whose performance is very good and have always produced good results with the pupils they have handled. Questions such as „what is the contribution of task selection to the performance in these teacher‟s practice? or what does good practice of task selection and implementation entail?‟ motivated this study.

1.2.3 Task selection in trigonometry

In order to thoroughly investigate good task practice in promoting student learning through task selection and implementation, I decided to narrow my study to the specific topic of trigonometry rather than the entire subject of mathematics. I have chosen this topic because it is one of the

topics that pose difficulties for students; it rests on other topics like geometry, functions and algebra, and it is a foundation for other branches of mathematics such as calculus. According to Weber (2005), "trigonometry is one of the earliest mathematics topics that links algebra, geometry and graphical reasoning, it can serve as an important precursor towards understanding of pre-calculus and calculus" (p. 91)

The difficulties that students have with trigonometry are often associated with difficulties in learning mathematics in general. But what makes this topic even more difficult for students to comprehend is the fact that terminologies and functions such as the sine, cosine and tangent are new to them. According to Maharaj (2008), student "difficulties relate to ... conversions of algebraic syntax, and the gap between arithmetic, algebra and geometry" (p. 403). For this reason, the teacher needs to carefully select appropriate trigonometry tasks that will develop student"s understanding of the topic.

Little research has been conducted on the teaching of trigonometry in Zambia. This is not only the case for Zambia, but perhaps even beyond. Weber (2005) reports that, "despite the documented difficulties with learning trigonometric functions the education research literature in this area is sparse" (p. 91). This study is therefore significant in that it will tease out good practices in task selection choices that may promote effective teaching. Additionally, this study will shed light on some of the critical decisions impacting learning that teachers make during classroom instruction. The landscape for this study cuts across the kind of resources and type of tasks teachers select for their students. More importantly, it includes the criteria used and decisions made in the selection and implementation of tasks that impact student learning.

1.3 Research Goals

The goals of this study are to investigate the source and type of tasks used, the process of mathematical task selection and the reasons for these choices, in the teaching of trigonometry by Grade 11 teachers who show consistently good results.

To achieve these objectives I intend to answer the following questions:

1. From what resources do these teachers draw tasks?

2. What type of tasks do these teachers select for use in teaching trigonometry?
3. What criteria do the teachers use to select their tasks?
4. What other decisions do these teachers make in their selection of tasks?

1.4 Thesis Overview

This section provides a summary of the thesis. The thesis consists of six chapters. Chapter one introduces the study, starting from the description of the context of the study and the need to carry out an investigation and the goals.

A review of related literature about the problem of task selection and implementation is set out in chapter two. This chapter also reviews literature which is not directly connected but may provide insight to the problem that is being investigated. For instance, literature on teachers‟ mathematical knowledge for teaching and the teaching of trigonometry in general is reviewed. The frameworks for analysing data are discussed in this chapter as well. Three frameworks which are useful to analyse data are considered for this study. These are: instructional task categories (Kaur, 2010); Stein et al.‟s (1996) framework for task analysis and Kilpatrick et al. (2001) framework for mathematical proficiency.

In chapter three, the research methodology is described in detail; explicating the goals for the study and procedures and techniques involved thereof. The research sites and participants are described in this chapter as well. Chapters four and five provide an analysis of the research findings. The chapters also discuss the findings in the context of the reviewed literature. Chapter six presents the discussion and conclusion of what emerged from the findings. Recommendations for further investigation are also provided in this chapter.

CHAPTER TWO: LITERATURE REVIEW AND CONCEPTUL FRAMEWORK

2.1 Introduction

Studies on mathematical tasks have increasingly received a lot of attention from educators and curriculum designers. A number of researchers of mathematics education for the past one and a half decades have been attracted to various investigations in this area. Investigations ranging from academic tasks to specific classroom tasks have been their focus. This focus on the type of mathematical tasks in textbooks and curriculum materials has arisen, partly because of the evidence suggesting that the type of tasks students engage with set the parameters for student opportunities to doing mathematics and understanding the subject matter (Doyle, 1983; Hiebert & Wearne, 1987), and because of the evidence from analysis of nature of mathematical tasks literature suggesting that mathematical tasks with high-level of cognitive demand are more likely to induce high level thinking in the students which in turn enhances student understanding.

Despite this widespread interests and concern, there still remains a lot to better understand the nature of tasks that evoke student thinking and reasoning and their impact on student achievement. For instance, Stein, Remillard, and Smith (2007) looked at how curriculum influences student learning. They report that "the introduction of human interaction with curriculum materials brings variation to the information and styles of learning to which students will be exposed" (p.352). In a study of the relationship between mathematics instruction and students" thinking, Stein, Grover and Henningsen (1996) observed distinctions between tasks teachers planned and their implementations of them in classrooms. These researchers found that teachers adjusted particular features and cognitive demands of reform-oriented tasks while students worked on them, illustrating the interactive and emergent nature of the written curriculum. In another study, finding suggests that teachers were selecting and setting up the kind of tasks that reformers argue should lead to students" capacities. During implementation, the task features remained consistent with how they were set up, but the cognitive demands of high-level tasks had a tendency to decline. However, these studies were not specific on how cognitive demand of high-level tasks relate to student achievement.

It is evident from the above that student‟s opportunities to learn are provided partly through the tasks they (learners) are exposed to and the decisions teachers make before and during classroom instruction. The decisions teachers make contribute greatly to the nature of classroom environment that is created thereof. To this end, it suffices to state that student learning is impacted by task choices and effective implementation of these tasks.

2.2 Tasks that promote student learning of mathematics

Despite the different terms used for mathematical tasks, such as "academic tasks" (Doyle, 1983) and "instructional tasks" (Hiebert & Wearne, 1993), they generally refer to questions, constructions, applications, and exercises in which students engage. Mathematical tasks are cardinal to students‟ learning because "tasks convey messages about what mathematics is and doing mathematics entails" (NCTM, p. 24).

The tasks in which students engage provide the contexts in which they learn to think about the subject matter, and different tasks may place different cognitive demands on students (Doyle, 1983; Hiebert & Wearne, 1993). Thus, "the nature of tasks can potentially influence and structure the way students think and can serve to limit or broaden their views of the subject matter with which they are engaged" (Henningsen & Stein, 1997). Hiebert, Carpenter, Fennema, Fuson, Wearne, Murray, Oliver, and Human (1997) argue that students build understanding by reflecting and communicating and tasks should allow and encourage these processes. Similarly, if students spend their time reflecting on why things work the way they do, how ideas are connected to their prior knowledge, or how ideas and procedures compare and contrast, then they are likely to be constructing new relationships and new understanding of mathematics (Hiebert at el., 1997). They use the term "appropriate" to describe the nature of mathematical tasks that build understanding.

It has been widely acknowledged by researchers that tasks are the main part of mathematics activities that comprise the opportunities student are offered during and outside the classroom. Also apparent in the literatures is the notion that critical to learning and understanding of mathematics is the quality of learning experiences the learners are rendered with (Doyle, 1983;

Hiebert & Wearne, 1987). To a large extent, tasks in a mathematics classroom determine the amount of learning and learning experiences the students go through.

Doyle (1983) posits that "tasks regulate the selection of information and the choices of strategies for processing that information" and that "students will learn what a task leads them to do" (p. 162). In this sense, tasks can enhance or impede students" chances of experiencing the doing of mathematics. The role of mathematical tasks to engage students in thinking and reasoning about important mathematical ideas has been explored by many researchers (Doyle, 1988; Stein et al., 2000).

According to Hiebert et al. (1997) such tasks should have the following features:

> First, the task must allow the student to treat the situation as problematic, as something they need to think about rather than as a prescription they need to follow. Second, what is problematic about the task should be the mathematics rather than other aspects of the situation. Finally, in order for students to work seriously on the task, it must offer students the chance to use skills and knowledge they already possess Tasks that fit these criteria are tasks that can leave behind something of mathematical value (p. 18).

The Professional Standards for Teaching Mathematics (NCTM, 1991) argues that students" opportunities to learn are afforded by engaging them in mathematical activities and experiences that are worthwhile (p.1). In order to improve the teaching and learning of mathematics, NCTM (1991; 2007) encouraged the use of worthwhile mathematical tasks (NCTM, 2007, p. 32). Not all tasks are created equal; some tasks engage students in mathematics more than others. Therefore, in order to promote student learning, a teacher must implement worthwhile mathematical tasks. NCTM (2007) goes further to provide a list of characteristics of such tasks and succinctly state that:

> The teacher of mathematics should design learning experiences and pose tasks based on sound and meaningful mathematics that...
>
> - Engage students" intellect;
> - Develop mathematical understanding and skills;
> - Stimulate students to make connections and develop a coherent framework for mathematical ideas;
> - Call for problem formulation, problem solving, and mathematical reasoning;
> - Promote communication about mathematics;

- Represent mathematics as an ongoing human activity; and display sensitivity to, and draw on students‟ diverse background experiences and disposition. (p. 32-33).

Tasks of this nature sufficiently challenge students to develop important concepts through communication and problem solving thereby promoting desired student mathematical learning.

High level mathematical tasks provide critical opportunities for students to develop mathematical understanding as well as learn about the nature of mathematics and how one engages in it. Such tasks however, are difficult for teachers to implement effectively. High level mathematical task is one that requires students to put forth some cognitive effort as they work to understand, make connections to, and build upon, mathematical concepts and ideas. In that sense, students are required to represent the mathematical ideas in multiple ways, and make connections between representations of underlying mathematical ideas. In short, high level tasks, promote thinking, reasoning and mathematical sense making (Stein, Smith, Henningsen, & Silver, 2000).
Stein, Grover, and Henningsen (1996) argue that "authentic opportunities for students are created when teachers use tasks that are problematic, that have multiple solution strategies, that demand explanation and justification, and that can be represented in various ways" (p. 457).

2.3 Task selection for mathematical teaching and learning

Learning mathematics is more meaningful in a classroom when the learners are provided with the necessary and appropriate learning opportunities. Critical to learning and understanding of mathematics is the quality of learning experiences the learners are provided with. To a large extent, tasks in a mathematics classroom determine the amount of learning and learning experiences the students go through. In other words, tasks serve as a context for students‟ thinking, during and after instruction. Doyle (1983) argues that "tasks influence learners by directing their attention to particular aspects of content and by specifying ways of processing information" (p. 161).

The tasks with which students become engaged in the classroom form the basis of their opportunities to learn what mathematics is and how one does it (Doyle, 1983, p. 162). Lappan and Briars (1995) contend that " there is no decision that teachers make that has greater impact

on students‟ opportunities to learn and on their perceptions about what mathematics is than the selection or creation of the tasks with which the teacher engages students in studying mathematics" (p. 138).

The teachers need to select tasks that afford students with an opportunity to learn by engaging them in mathematical activities and experiences that are worthwhile. Worthwhile mathematical tasks are described by NCTM as ones that do not separate mathematical thinking from mathematical concepts and skills that capture students‟ curiosity and that invite them to speculate and pursue their hunches (Stein, Remillard, & Smith, 2007, p. 348). Stein, Grover, and Henningsen (1996) argue that "authentic opportunities for students are created when teachers use tasks that are problematic, that have multiple solution strategies, that demand explanation and justification, and that can be represented in various ways" (p. 457). Therefore, tasks that present and/or engender features described above are likely to promote student learning.

2.4 Different categories and characteristics of tasks

Tasks set parameters for what is to be learned and for how the learner is to engage with that content. According to Doyle (1988), a task refers to the academic work students do in the classroom and consists of four components:

> (a) a goal state or end product to be achieved; (b) a problem space or set of conditions and resources available to accomplish the task; (c) the operations involved in assembling and using resources to reach the goal state or generate the product, and (d) the importance of the task in the overall work system of the class (Doyle, 1988, p. 169).

Doyle et al. (1988) also describe two different categories of tasks: novel and familiar. The classification of tasks as novel or familiar depended on the students‟ previous experiences. A novel task could become familiar if the teacher made the task routine in some way. Extensive research on mathematical tasks has been conducted by Stein and her colleagues. Stein et al (2000) defined a task as "a segment of classroom activity devoted to the development of a mathematical idea" (p.7). Stein et al. (2007) explain that "tasks include expectations regarding what students are expected to produce, how they are expected to produce it, and the resources available for so doing" (p. 346).

Stein et al. (2000) classify mathematical tasks according to task features and level of cognitive demand. Task features "refer to the aspects of tasks that mathematics educators have identified as important considerations for the development of mathematical understanding, reasoning and sense making" (Stein & Henningsen, 1997, p. 529). For instance, can a task be solved in a variety of ways, through the use of multiple representations, or does it provide opportunities for mathematical communication, explanations and justification. The level of cognitive demand refers to the type of thinking employed when working on a task.

Stein, et al. (2000) also used four hierarchical categories of cognitive demands to assess mathematical tasks. They posit that tasks are appropriate if they meet the levels of cognitive demand that helps in building students‟ mathematical understanding. Tasks are appropriate for students‟ level of ability if they relate closely to current knowledge and skills to be understood but be different enough to extend students‟ thinking. If tasks are too easy or too hard they are not motivating and are unlikely to engage students in deep thinking. It is reasonable to argue that tasks that are too easy or too hard have limited cognitive value. In other words, a set of tasks needs to cater for all students. Therefore, the appropriate levels of cognitive demand that induce learning are necessary.

Different mathematical tasks place different demands on students‟ thinking. A number of frameworks used today for analysing the levels of cognitive demands of the tasks in textbooks, or as set up by the teacher and as enacted in classroom draw from Doyle‟s (1988; 1983) for describing and analysing students‟ academic work. Doyle (1988) defines academic work as "the cognitive processes students are required to use in accomplishing the task" (p. 170). Doyle (1983) classified instructional tasks into four categories (memory tasks, procedural or routine tasks, comprehension/understanding tasks, and opinion tasks), which he reduced to two cognitive levels of academic work. The lower level of academic work include memory tasks and procedural or routine tasks. These tasks often involve the memorisation, reproducing of facts and concepts or application of formulas or algorithms (Doyle, 1988). Higher-level cognitive academic work engage students with cognitive processes such has comprehension, interpretation, flexible application of knowledge and skills, choosing strategies to solve problems, making

connections between several sources of information to accomplish the task, drawing inferences, and formulating and testing conjectures (Doyle, 1988).

2.5 Teaching of trigonometry

Studies on trigonometry in mathematics education research for the past one and half decades have concentrated on ways of how the topic can effectively be taught in class in order to promote student understanding; student mistakes and misconceptions in the learning of trigonometry; and more recently, the incorporation of technology in the teaching of the topic. For example, Kendal and Stacey (1996) compared the two methods of teaching basic trigonometry to find out which promotes better understanding of the underlying concepts and mastery of skills. Their conclusion suggest that "the ratio method of teaching introductory trigonometry is a better choice for schools than the unit circle method" (Kendal & Stacey, 1996, p.327), as exemplified by better performance recorded by students of all ability levels with the ratio method in their study. In a study to investigate the students" understanding of the relationship between real numbers and trigonometric concepts, Orhun (2010) used radian concept and trigonometric function concept to measure student knowledge. He reported that existence of the gap between the real numbers and trigonometric relations was characterised by students" lack of knowledge of an earlier introduction of the topic.

Weber (2005) explored students" understanding of trigonometric functions involving two groups. One group was taught using traditional methods (lecture based), the other group was taught using experimental instruction. Weber"s (2005) study reveals that "students who received standard instruction did not develop a strong understanding of trigonometric functions" (p. 103). These students were generally unable to justify why trigonometric functions had the properties that they did and they were unable to form reasonable estimates for what the output of trigonometric function should be (Weber, 2005, 103).

In other study, Kissane and Kemp (2009) explored the ways in which the teaching of trigonometry is affected by technologies for both students and teachers. The focus of this study was „opportunities for learning and teaching" that were not readily available before the

availability of technology. Kissane and Kemp (2009) argue that "prior to the availability of technology, graphing trigonometric functions was a tedious undertaking for students, with little practical purposes" (n. p). Moore (2009) devised an instructional sequence to promote students‟ foundational understandings of and connections between right angle and unit circle trigonometry. He reports on both the design and implementation of a lesson intended to develop student conceptions of angle measure such that these conceptions enabled coherence between right triangle and unit circle trigonometry (Moore, 2009, p. 1480). Central to his lesson design and implementation was the use of technology to help in the development of student understandings. Kissane and Kemp (2009) claim that the availability of technologies that learners or teachers may have access to change the teaching and learning of trigonometry in many ways, such as, (1) it may change the opportunity provided by technology to help students engage with trigonometric concept; (2) students can interact directly with trigonometric ideas in a more active way than with paper and pencil alone. None of these studies has dealt with task selection in trigonometry.

There is little or no research conducted on task selection in trigonometry. Studies on task selection with regard to specific topics are scanty. For instance, Saani‟s (2009) study on teacher‟s task practice in relation to teachers‟ knowledge, investigated tasks and task selection in geometry. Modau (2007) investigated teachers‟ selection and use of tasks in the old and new curriculum. This study was not specific on one topic, but compared topics in the old and new curriculum which also includes trigonometry. Therefore, this study is inevitable in that it will shed some light on the selection of tasks in trigonometry that can enhance students‟ understanding of the topic.

2.6 Conceptual frameworks

2.6.1 Instructional tasks

Drawing partly from Kaur‟s (2010) classification of instructional tasks and Saani‟s (2009) analysis of use of tasks in classroom, this study classifies trigonometry tasks selected and implemented by teachers as examples, class exercises and homework. In his investigation of mathematical tasks, Kaur (2010) classified mathematical tasks as "instructional tasks," if the mathematical tasks was used to promote the students‟ learning of mathematics, and "assessment tasks," if the mathematical tasks was intended to generate information about the student or about effectiveness of instruction (p. 10).

He further classified tasks into four categories: learning, review, practice and assessment tasks. *Learning* tasks refers to tasks teacher uses to teach student new concepts, definitions and terminologies; *review* tasks are tasks used to review previously learned concepts or skills to facilitate learning of new concepts and skills. Using familiar tasks or students‟ prior knowledge; *practice* tasks refer to tasks used during the lesson to either illuminate concept or demonstrate skill further and the teacher asks students to work through tasks during the lesson either in groups or individually or during out of class time. Routine or non-routine, based on what has been learned; and *assessmen*t tasks refers to tasks used to assess students‟ performance or effectiveness of the lesson, which can be formative or summative (Kaur, 2010, p. 10).

Similarly, Saani (2009) posits that "tasks were used in at least one of the three different ways during the lesson: as examples, class-work, and homework" (p. 145). He defined examples as tasks that the teacher set up in class to illustrate procedures or explain concepts to students, usually solved by the teacher; class-work referred to tasks that were set up for students to do during the lesson and homework meant tasks that were given to students to do at home (Saani, 2009, p. 145).

From these classifications, it is reasonable to argue that there are basically two purposes that each mathematical task serves; instructional or assessment. It is difficult therefore to delineate these categories, for they can serve more than one purpose at the same time. Discussion of the

categories of instructional tasks is beyond the scope of this study. In this study, I propose to adapt Saani‟s (2009) categories of classroom tasks as follows: *examples* will refer to learning and review tasks; the tasks the teacher gives in class either to be used to teach new concepts, definitions and terminologies or tasks teacher uses to review previously learned concepts and skills to facilitate learning of new concepts and skills; *class exercises* refer to practice tasks which are the tasks that the teacher gives during the lesson and students work on them either individually or in groups. What the teacher does is to monitor the students work on the tasks and see if they are making any progress. In my framework *homework* refers to the extra work, other than an exercise, given to students to be completed outside a mathematics lesson, either at school or at home. The categories proposed were so prominent in the teachers‟ lesson plans as well as students‟ exercise books.

In this study therefore, trigonometry tasks were set up for one of the following purposes: examples which the teacher used to teach trigonometric concepts, algorithms, procedures and skills; classroom exercises, these were set up for pupils to practice the procedure and skills further and mastery of concepts; and homework which served the same purpose as class exercises, but students completed these tasks in more flexible conditions and environment.

2.6.2 Mathematical task framework

Drawing partly from Henningsen and Stein‟s (1997) Mathematical Task Framework and Stein et al.‟s (2000) Task Analysis Guide, the study utilised the mathematical task framework which helped to identify three phases during which teachers‟ decisions and actions seem to affect the cognitive level at which the content is experienced in mathematics classrooms. The framework suggests that teachers can influence student learning by the tasks they select, by the way in which they present these tasks, and by the manner in which they work on these tasks with their students (Henningsen & Stein, 1997, p. 528). The study utilised the notion of mathematical task framework to ascertain the type of tasks selected or modified for use in classroom. The task analysis guide was employed to track down the tasks that teachers set up and how they unfolded as they were implemented in class. Task Analysis Guide classifies tasks into four categories to capture the cognitive level of the tasks. These are:
Lower level tasks which include:

(i) memorisation – tasks that may involve recalling, stating and naming, and
(ii) procedures without connections to understanding, meaning or concepts.

Higher level tasks are classified as:

(i) procedures with connections to understanding, meaning and concepts, and
(ii) doing mathematics – that is, solving non routine problems, conjecturing, generalising and proving (Modau & Brodie, 2008; Smith & Stein, 1998).

The levels of cognitive demand are described in the following paragraph.

Stein et al. (2000) developed a framework for analysing cognitive demands of tasks. They elaborate the levels of cognitive demand of the mathematical tasks as memorisation, if tasks involve reproducing previously learned facts, rules, formulae, or definitions. Memorisation tasks do not require any explanation from learners; they are straight forward and learners use well known facts to solve them. Procedures without connections to meanings require reproduction of a procedure but without connections to underlying concepts or meaning (Stein et al., 2000). Such tasks are directed towards producing correct answers rather than building mathematical understanding. Procedures with connection to understanding meaning focus learners‟ attention on the use of procedures for the purpose of developing deeper understanding of mathematical concepts and ideas. Such tasks develop in learners the flexibility and fluency of using procedures in solving mathematics problems in a meaningful way. Doing mathematics, also known as problem solving, does not require any procedure to be followed. There exists no predictable way of solving the problems. Such tasks have more than one way of solving the problem. Students working with such tasks call for deep thinking, they have to curiously and critically analyse the situation, examine the task constraints that may limit possible strategies and solutions. The description of the four levels of cognitive demand of mathematical tasks are summarised in table 2.1 below.

Table 2.1: The levels of cognitive demands

Lower-Level Demands	Higher-Level Demands
Memorization Tasks • Involves reproducing previously learned facts, rules, formulae, or definitions OR committing facts, rules, formulae, or definitions to memory. • Cannot be solved using procedures because a procedure does not exist or because the time frame in which the task is being completed is too short to use a procedure. • Are not ambiguous – such tasks involve exact reproduction of previously seen material and what is to be reproduced is clearly and directly stated. • have no connection to concepts or meaning that underlie the facts, rules, formulae, or definitions being learned or reproduced.	**Procedures With Connections Tasks** • Focus students‟ attention on the use of procedures for the purpose of developing deeper levels of understanding of mathematical concepts and ideas. • Suggest pathways to follow (explicitly or implicitly) that are broad, general procedures that have close connections to underlying conceptual ideas as opposed to narrow algorithms that are opaque with respect to underlying concepts. • Usually are represented in multiple ways (e.g., visual diagrams, manipulatives, symbols, problem situations). Making connections among multiple representations helps to develop meaning. • Require some degree of cognitive effort. Although general procedures may be followed, they cannot be followed mindlessly. Students need to engage with the conceptual ideas that underlie the procedures in order to successfully complete the task and develop understanding.
Procedures Without Connections Tasks • Are algorithmic. Use of the procedure is either specifically called for or its use is evident based on prior instruction, experience, or placement of the task. • Require limited cognitive demand for successful completion. There is little ambiguity about what needs to be done and how to do it. • Have no connection to the concepts or meaning that underlie the procedure being used. • Are focused on producing correct answers rather than developing mathematical understanding. • Require no explanations, or explanations that focus solely on describing the procedure that was used.	**Doing Mathematics Tasks** • Requires complex and non-algorithmic thinking (i.e., there is not a predictable, well-rehearsed approach or pathway explicitly suggested by the task, task instructions, or a worked-out example). • Requires students to explore and to understand the nature of mathematical concepts, processes, or relationships. • Demands self-monitoring or self-regulation of one‟s own cognitive processes. • Requires students to access relevant knowledge and experiences and make appropriate use of them in working through the task. • Requires students to analyze the task and actively examine task constraints that may limit possible solution strategies and solutions. • Requires considerable cognitive effort and may involve some level of anxiety for the student due to the unpredictable nature of the solution process required.

Source: Stein and Smith (2000) the four levels of cognitive demand

Doyle (1983) argues that what students learn is largely defined by the tasks they are given. Tasks differ in the demand they make on comprehension, strategy development, procedural skills, and so on. Doyle (1988) also argues that "tasks with different cognitive demands are likely to induce different kinds of learning" (p. 395). The cognitive demands of mathematical tasks can change as tasks are introduced to students and/or as tasks are enacted during instruction (Stein, Grover, & Henningsen, 1996).

Stein, et al., (2007) however, noted that all tasks do not provide the same opportunities for students‟ thinking and learning. In their study of the relationship between tasks set up and task implementation, Stein, et al., (1996) sought to "examine mathematical tasks in terms of features and cognitive demands" (p. 461). They referred task features to "the existence of multiple solution strategies, the extent to which the task lends itself to multiple representations, and the extent to which task demands explanations and/or justifications from students," and cognitive demand to "the kind of thinking processes entailed in solving the task as announced by the teacher" (Stein, et al., 1996, p. 461).

2.6.3 Teachers' knowledge

It is widely accepted that teachers of mathematics need deep understanding of mathematics (Ball, 1993; Grossman, Wilson, & Shulman, 1989; Ma, 1999). A teacher‟s mathematical knowledge should consist of deep understanding of mathematical principles that underlie mathematical procedures and of the webs of ideas connected to particular mathematical topics (Ball, Lubenski, & Mewborn, 2001). Ma (1999) refers to this „profound understanding of mathematics‟ as a connected, structured and coherent knowledge core of mathematical concepts, in order to understand different representations of a mathematical concept, different solution strategies for solving a problem, and students‟ thinking or misconceptions about mathematical concepts and procedures.

Teachers play a pivotal role in ensuring that students are afforded opportunities to learn. By selecting, adapting or generating mathematical tasks, the teachers make decisions based on a numbers of factors, which include the content, the students and the ways in which students learn

mathematics. The decisions teachers make about what tasks are appropriate to help students‟ understanding of mathematical concepts are also critical to the kind of classroom environment that a teacher wants to create. Selecting appropriate tasks requires a teacher knowing the topics well and how they relate to each other from prior grade to succeeding grades.

Literature on teachers‟ knowledge identifies and distinguishes different domains of teacher knowledge and posits such domain as important requisites for effective teaching (Ball, thames, & Phelps, 2009; Ball, Lubienski, & Mewborn, 2001). This resonates with aspects of teachers‟ mathematical knowledge referred by Ball and her colleague as "specialised knowledge of content, unique to individuals engaged in teaching," and different from "common knowledge of content" that would be utilised by mathematicians or other adults in general (Ball & Hill, 2004, pp 332-333). Sanni (2009) argues that teachers‟ knowledge is an important aspect of successful teaching and learning as well as improved learner achievement (p. 42). Hill, Rowan, and Ball (2005) found that teachers‟ mathematical knowledge for teaching positively predicted student gains in mathematics achievement during the first and third grades (p. 399).

While it has been widely acknowledged in literature that the quality of mathematics teaching depends on teachers‟ knowledge of the subject, Shulman (1986) argues that "mere content knowledge is likely to be as useless pedagogically as content-free skill" (p. 8). He further argues that "knowing a subject for teaching requires more than knowing its facts and concepts" (Shulman, 1986, p.8). The same argument is held by Ball, Thames, and Phelps (2009) that knowing mathematics for teaching requires knowing in detail the topics and ideas that are fundamental to the school curriculum, and beyond. Shulman (1986) uses the term pedagogical content knowledge to refer to

> the most useful forms of representation of those ideas, the most powerful analogies, illustrations, examples, explanations, and demonstrations. In other words, the most useful ways of representing and formulating the subject that makes it comprehensible to others. . . . Pedagogical content knowledge also includes an understanding of what makes the learning of specific topics easy or difficult: the conceptions and preconceptions that students of different ages and backgrounds bring with them to the learning of those most frequently taught topics and lessons. (p. 9).

Ball et al. (2007 build on Shulman‟s notions of pedagogical content knowledge further by including knowledge of student and knowledge of curriculum which Shulman considered as separate categories. In their quest to develop a theory which is practice-based, for what it entails to have broader mathematical knowledge for teaching, Ball and her colleagues identify the mathematical knowledge required and the skills involved in teaching. They elaborate on Shulman‟s pedagogical content knowledge which they divided into two sub-domains, namely, "knowledge of content and student, and knowledge of content and teaching" (Ball, Thames, & Phelps, 2007, p. 2). Knowledge of content and student is the domain of knowledge which is the amalgam of knowing students and knowing mathematics, while knowledge of content and teaching combines knowing about teaching and knowing about mathematics. Knowledge of content and student is about knowing students and being able to anticipate what might be their difficulties and challenges and what could be their conceptions and misconceptions to them. Therefore, a deeply organised and connected knowledge of mathematics enable the teacher to consider effective methods and representations for engaging students with mathematical ideas, to recognise the mathematics in students‟ alternative strategies or ways of thinking, and take advantage of opportunities for students to make meaningful mathematical connections (Stein, Baxter, & Leinhardt, 1990). Additionally, knowledge of content and teaching relates to designing of instruction, from planning and sequencing activities to choice of tasks and decisions on what to do with the tasks. Stein et al. (2007) theorise that teacher knowledge, among other factors, contribute to the transformation of tasks between phases. They point out that teacher knowledge likely impacts how tasks are used in instruction.

Knowledge of student is equally important because classroom instruction depends of "the interaction among the teachers, students and content in an instructional triangle" (Kilpatrick, Swafford & Findell, 2001, p.313). When the teacher has the knowledge of the student, he or she will be able to select tasks that are appropriate for his or her students. He/she will also be able to anticipate and prepare in advance, how to manage learning difficulties that might arise during the course of students‟ engagement with the chosen tasks. As Henningsen and Stein (1997) noted that

> Teachers must know their students well in order to make intelligent choices regarding the motivational appeal, difficulty level, and degree of task explicitness

> needed to move students into the right cognitive and affective space so that high-level thinking can occur and progress can be made on the task (p. 537).

Teaching requires knowledge beyond that being taught to students. This statement illuminates the importance of knowledge of content. But knowing more than what is being taught is not sufficient. Ball et al. (2009) argue that teaching involves breaking down of compressed mathematical knowledge so that it can be taught directly to students as they develop understanding. Nevertheless, the biggest challenge that teachers face is that students must develop fluency with compressed mathematical knowledge so that eventually they should be able to use sophisticated mathematical ideas and procedures. They further argue that teachers need, to hold unpacked mathematical knowledge because teaching involves making features of particular content visible to and learnable by students (Ball et at., 2009). The idea that students must develop fluency with compressed mathematical knowledge which the teacher must unpack to enable certain features be visible to the student can be likened to what Kilpatrick, Swafford & Findell (2001) described as mathematical knowledge necessary for effective teaching practice and coined this as "mathematical teaching proficiency" (p.380). They came up with five strands that are interdependent and intertwined to describe fully mathematical proficiency. The five strands are under the following labels: conceptual understanding, procedural fluency, strategic competence, adaptive reasoning, and productive disposition. They feel that teachers‟ pedagogical understanding represents their mathematical conceptions. Teachers‟ mathematical conceptions influence the decisions they make about what mathematics to teach, how to represent it and how to handle students‟ (mis)understanding of problems.

In the following section I discuss the five strands of mathematical proficiency which provide a lens for analysing the impact of implemented task on student learning (effectiveness of task implementation).

2.6.4 The five strands of mathematical proficiency

Kilpatrick et at. (2001) describe mathematical proficiency in terms of the kinds of cognitive changes that should be developed in children so that they can be successful in learning mathematics. Mathematical proficiency represents five inter-dependent strands, which are intertwined. The teaching and learning of mathematics is viewed as a product of interaction

between the teacher, students, and mathematics in an appropriate context (Kilpatrick et al., 2001, p. 371). In order to teach effectively, a teacher ought to have sufficient pedagogical content knowledge that helps students to develop proficiency. For example, "choosing content, deciding how to present it and determining how much time to allocate to it are ways in which learning is affected by how the teacher interacts with content." (Kilpatrick et al., 2001, p. 333).

Furthermore, teachers need to select tasks that keep students engaged in deep thinking and motivated throughout when working on tasks. Engaging students in cognitively demanding tasks is critical to the quality of student learning (Hiebert & Wearne, 1993; Stein & Lane, 1996). However, cognitively demanding tasks are not self-enacting; what determines student learning is rather how these tasks are set up and worked on during instruction. Teachers‟ role remains critical in introducing and enacting these tasks. NCTM (2000) suggests that

> Worthwhile tasks alone are not sufficient for effective teaching. Teachers must decide what aspects of a task to stress, how to organise and orchestrate the work of the students, what questions to ask to challenge those with varied levels of expertise, and how to support students without taking over the process of thinking for them (p. 19).

The fact that teachers make instructional decisions about what to teach, how to represent it and how to deal with problems of student misunderstandings, entails that their decisions requires coordination between the mathematics at stake and the instructional options and purposes at play. Shulman (1986) claimed that teachers needed to experience mathematics in ways that allowed for rich understandings of conceptual underpinnings of the content in order for them to clearly and effectively explain to others. The kind of knowledge described here is similar to what Kilpatrick et al. (2001) describe as "teaching for mathematical proficiency" (p. 313). I now turn to the five strands of mathematical proficiency which are interwoven and inter-dependent, these are: conceptual understanding, procedural fluency, strategic competence, adaptive reasoning and productive disposition. Impact of implemented tasks on student learning is closely associated with extent to which tasks or class activities promote mathematical proficiency in students.

Kilpatrick et al. (2001) define "conceptual understanding refers to an integrated and functional grasp of mathematical ideas" (p. 118). Conceptual understanding involves making connections, relations and sense making of a mathematical concept. For a teacher, conceptual understanding

involves making connections between mathematical subject knowledge and using it intelligently with an understanding of pedagogical and student knowledge and context (Kilpatrick et al., 2001).

Procedural fluency refers to a set of skills, which are used in carrying out procedures flexibly, accurately, efficiently and appropriately (Kilpatrick et al., 2001). Procedural knowledge is about the knowledge of procedures, when and how to use them properly. Kilpatrick et al. (2001) do not consider procedure fluency for a teacher as the knowledge that involves fluency in performing procedures but also the fluency in performing instructional practiced. Teachers with procedural fluency are able to readily draw upon their knowledge when interacting with mathematics content, students and the teaching context in an instructional triangle. Fluency in performing classroom instructional tasks helps teachers to immediately detect and react to situations arising in their practice more accurately, efficiently and appropriately (Carpenter, 1988). A teacher with procedural fluency should be able to flexibly and accurately decide whether or not a student‟s response to specific question is fully or partly correct.

Strategic competence, according to Kilpatrick at el (2001) refers to the ability to formulate, represent and solve. To formulate mathematical problems and represent them accurately, teachers must understand the purpose and underlying features of that problem. Teachers encounter a lot of situations in classroom environments, they experience different problems and many times they have to first discover what the problem is and then find ways to solve the problem. Some of the problems are not ready made but they arise from the interactions among the teacher, student and content in the classroom environment. Carpenter (1998) refers teaching to a problem solving business. Kilpatrick, Swafford & Findell (2001) propose that for teachers to be able to give appropriate solutions to students, questions they need to figure out what they students know. Teachers will be able to represent the problem or solution to the students clearly and accurately if their conceptual understanding is rich. Teachers need to understand the situation first and be able to see relationships amongst concepts, whether implicit or explicit.

Kilpatrick, Swafford & Findell (2001) define adaptive reasoning as the “capacity to think logically about the relationships among concepts and situation” (p. 129). They further describe

adaptive reasoning as “the glue that holds everything together” (p. 129). If the students are to be able to understand the concepts and algorithms of mathematics they need to be able to explain and justify them as well as use them in different problem situations. The teachers‟ role is to pose questions that will prompt students to able to explain and make justifications. A teacher needs to allow students to challenge each other, and as students struggle to make justifications learning is taking place. Teachers depend on their understanding to enhance their ability to analyse and think through students‟ explanations and justifications. This combined with the procedural fluency and strategic competence enables the teacher to see problem strategies in different ways. They are quick to tell whether a given answer is correct or wrong. It is obvious that one cannot analyse a concept and explain it clearly if one does not understand. Therefore, if the teacher cannot logically, flexibly, and accurately move from one representation to another, he/she will not be able to make good choices of the tasks that help to develop student understanding of mathematics.

Productive disposition refers to the inclination toward seeing sense in mathematics, to perceive it as both useful and worthwhile, and to believe that steady effort in learning mathematics is rewarding (Kilpatrick, et al., 2001, p. 131). Productive disposition is manifested when a student begins to see oneself as an effective learner and doer of mathematics. Kilpatrick, Swafford & Findell (2001) state that “frequent opportunities to make sense of mathematics, to recognise the benefits of perseverance, and to experience the rewards of sense making in mathematics is required in developing a productive disposition” (p. 131). In order to develop the abilities of teachers and students in the four strands of mathematical proficiency discussed above, teachers and students first and foremost should see the value in mathematics, that it is worth spending time on and that steady effort on mathematics is rewarding.

The five strands are relevant to my study because they form a conceptual frame for understanding the development of mathematical reasoning and provide a lens through which task implementation and its impact on learning can be viewed. The five strands complemented the cognitive levels of demand to help me ascertain the extent to which selected and implemented tasks promote mathematical reasoning in classrooms in order to promote proficiency. Saani (2009) argues that “teachers‟ emphasis on different strands in mathematics classroom is

discernable from the type of tasks selected and how these are implemented" (p. 40). Therefore, these strands of proficiency are important as an analytical frame in that they will help me to understand the extent to which the nature of tasks used in classrooms and the decisions teachers make during classroom instruction enhance the development of student learning.

2.7 Conclusion

In this chapter I have presented a review of literature on tasks. I have dealt with literature which relates directly or indirectly to task selection and implementation. Directly implicated with task selection and implementation is teacher knowledge and mathematical teaching proficiency discussed in the chapter. I have also discussed the analytical frameworks that I have worked with in this study.

The next chapter presents and discusses the design of this research. The focus of this chapter is a detailed description of methodology adopted, participants, and research cites, instruments for data collection, and data analysis.

CHAPTER THREE: METHODOLOGY

3.1 Introduction

This chapter discusses the research approach. Described in this chapter are the research strategies; including the methods used for the study in order to improve the reliability of the research findings. Teachers of mathematics for grade 11 participated in this study. The selection of participants and relevant information about them is discussed in detail. The data sources included interviews, teachers‟ lesson plans, pupils‟ activity books, textbooks and lesson observations. These instruments are presented in detail in this chapter as well. A discussion of how data were analysed concludes the chapter.

3.2 Research approach

In this study, an interpretive research paradigm was adopted because the study was aimed to understand teachers‟ task selection and implementation processes, important decisions they make before, during and after classroom instruction and criteria guiding task selection choices that impact student learning. According to Cohen, Manion and Morrison (2000), “an interpretive paradigm gives the researcher an opportunity to understand and interpret the world in terms of its actors (p.180).” This approach was appropriate for my study because my research questions were located within this paradigm. My study sought to understand the source and type of tasks teachers selected for use in classroom, important decisions they made and criteria used concerning task selection choices.

A case study methodology was adopted for this research. According to Kombo and Tromp (2006), “A case study seeks to describe a unit in detail, in context and holistically” (p. 72). I employed the case study design, because I sought to have in-depth understanding the processes of task selection of a small number of teachers “in detail, in context and holistically” (Kombo and Tromp, 2006, p. 72). Several possible designs exist for conducting qualitative research and each of them has its advantages and disadvantages. Case study enables a researcher to have in-

depth understanding of the subjects as well as the topic under investigation. It also permits the use of different data collection methods (triangulation) which improved the reliability of data that was collected and analysed. As Collins and Hussey (2003) explicate that "the methods used to collect data in a case study include documentary analysis, interviews and observation" (p. 69).

3.3 Research sites and participants

The study involved three teachers of mathematics, selected purposively from public high schools within Mufulira district in Zambia. Kombo and Tromp (2006) define purposive sampling as "a sample method, where the researcher purposely targets a group believed to be reliable for the study" (p. 82). This sample was selected from a group of teachers that had consistently produced good results at grade 12 for a period of five years or more. In addition to producing good results, these teachers held a first degree. Head Teachers and Heads of Department were used to help identify these teachers. One participant was selected from each of the different categories of schools: high cost, medium cost and low cost. I chose different school types in order to gain more insight and provide a broader coverage of school types. All the schools in the sample were easily accessible from my work place and that significantly cut down travelling costs.

3.4 Research Instruments and techniques

Data were collected using different instruments and techniques. By allowing the researcher to examine cases from several points of view (triangulation), multiple data sources provided information in the context, whereby providing rich data for analysis. In view of these provisions and the research questions that I intend to answer in this study, the following instruments and techniques were used for data collection: semi-structured interviews, lesson observations, document analysis (textbooks, teachers‟ lesson plans, and pupils‟ activity books). The choice of various techniques is advantageous in that the limitation and inadequacy of any one of the tools is complemented by the other, thereby eliciting rich and comprehensive data (De Vos, Strydom, Fouche,` & Delport, 2005, p. 315).

The research was conducted in four stages: phase 1: pre-lesson interview; phase 2: document analysis; phase 3: perusal of textbooks and phase 4, post-lesson interview

3.4.1 Interviews

The first interview focused on the identification of the resources that the teachers used for their teaching of trigonometry, how they used them (resources), and the type of tasks they selected. The interview revealed that teachers used the Zambia Secondary School Syllabus Mathematics 11 and 12 (ZSSSM 11 and 12) which are prescribed textbooks, and that New General Mathematics supplemented the two prescribed textbooks. The second interview was a follow up to the first interview and focused mainly on the implementation of the tasks. The second interview came shortly after lesson observation. This was aimed at eliciting the major decisions teachers make when choosing tasks and criteria used in their task choices. Both interviews were tape recorded and transcribed. The information gathered contributed greatly in answering the research questions raised in this study. Although some of the information did not directly contribute to answering the questions, it provided insight into the problem.

The data obtained from interviews were analyzed by identifying themes. The themes were subjected to further analysis by relating and inter-relating them. Six major themes were used as a lens through which teacher‟s task practice was discerned. Further data was generated by analyzing lesson plans, pupils‟ activities books and lesson observations theme by theme for each teacher. Important and interesting issues arising from this analysis were cross checked with available literature regarding teacher‟s task practice.

3.4.2 Document analysis

3.4.2.1 Lesson plans and pupils activity books

The document study, comprised lesson plans, pupils‟ activity books and textbooks. The focus in this phase was on type of trigonometry tasks both in the teachers‟ lesson plans and the pupils‟ activity books. The data collected from teachers‟ lesson plans were the tasks set up by the teachers referred to in this study as selected tasks and data collected from pupils activity books were the tasks enacted by pupils referred to in this study as implemented tasks. Tasks were delineated into categories according to the purposes for which they were intended. They

included: examples, class exercises and homework. From the documents, two sets of tasks were identified. These are the selected tasks and tasks not selected. The first set consisted of all tasks selected by the teacher, whether the task was used in classroom or not. This includes all tasks as they appear in the teachers‟ lesson plans. The second set of tasks consists of tasks that were selected but not used in classroom or tasks which were not selected at all, but had the same features and were at the same level of cognitive demand as the selected and implemented tasks. The tasks were then classified according to level of cognitive demands. This data enabled me to ascertain the nature and type of trigonometry tasks teachers selected and implemented in their classrooms. The tasks were then subjected to Stein, et al.‟s (1996) framework for analysing mathematical tasks. It was observed that tasks set up at low level remained at that level even during implementation. However some of the tasks that were set up at high level of cognitive demand (mainly procedures with connections) denigrated to low level demand.

3.4.2.2 Textbooks

Three textbooks and/or instruction materials that were prominently used by teachers for their trigonometry tasks were considered. Tasks selected from these textbooks were recorded and cross checked with those which appeared in the teachers‟ lesson plans. It is worth noting that besides tasks selected by the teachers, tasks with similar features and at the same level of cognitive demands as the tasks not selected and implemented were also considered. Textbook perusal was useful in the sense that they enabled me to ascertain the origin and the nature of the tasks presented in the teachers lesson plans..

3.4.3 Lesson observation

At least one lesson observation was conducted for each of the three teachers. The purpose of the lesson observation was to provide a basis from which classroom practices of individual teachers could be discerned with respect to selection and implementation of tasks. The observations lasted between 40 and 80 minutes and were video recorded, so that I could go back to them during data analysis. During this time I also jotted down interesting points that arose as the lesson progressed, specifically attending to the instructional tasks, how the instructional task was presented to students, the interactions that occurred as students worked on the instructional tasks, and the exchanges that occurred during any whole group discussion.

3.5 Data analysis

In this study, data collected from interviews were coded into themes and grounded into categories. The categories were aligned to the research questions. The themes were coded under the following headings: resources from which tasks are drawn, type of tasks used in classrooms, criteria used in the selection of tasks and teachers‟ decisions before and during classroom instructional practice. The data were then displayed according to these themes using descriptive statistics and narratives.

Three frameworks informed my data analysis. To answer the first and second research questions I used the categories of instructional tasks drawing from Kaur‟s (2005) four categories of instructional tasks earlier discussed in chapter two and Saani‟s (2009) categories of classroom activities. Tasks were put in the following categories: examples, class exercises and homework. Distributions of tasks in each of the three categories were analyzed for each teacher. To explore what type trigonometry tasks teachers used in their classrooms, the tasks were further subjected to an analysis using stein et al.‟s (2000) framework by considering their features and the levels of cognitive demand in them. The tasks were analyzed separately; chosen tasks and tasks not chosen. Chosen tasks are the tasks which were selected and implemented in class and tasks not chosen are the tasks which were selected but not implemented or task not chosen at all from the textbooks but were similar in features and cognitive demands to the chosen tasks. In my analysis I matched the chosen and unchosen tasks that were similar, and considered only the selected tasks which were similar to tasks not chosen.

The third and four research questions were addressed by critically examining teachers‟ lesson plans, pupils‟ activity books and lesson observation using essential themes as a lens through which good task practice can be discerned. Essential themes were crafted from major themes that emerged from teacher interviews. Analysis of lesson plans, pupils‟ activity books and lesson observations was orchestrated theme by theme. Important issues and patterns in teaching practice patterning to what it entails for good tasks practice were noted and these were exemplified from the literature. The two frameworks, “task analysis guide” (Stein et al., 2000) and “mathematical

proficiency" (Kilpatrick et al., 2001) proposed in chapter two provided the basis on which task practice was discernible.

3.6 Limitations

Despite their advantages, case studies have limitations. As stated earlier, a case study is specific to a study involving a small sample, therefore its results cannot be generalised.

3.7 Validity and reliability

Qualitative research is more complex than quantitative research when it comes to maintaining rigour. Rigour includes the notions of reliability and validity. Guba and Lincoln (1985) use four criteria that can be useful to ensure rigour in research: credibility, transferability, dependability, and conformability. I employed rigour to the research by using a variety of data sources and involving a number of analytical tools. Rigour in this study was also enhanced by employing different frameworks so that the weakness of one framework may be compensated by the strength of another.

3.8 Ethical issues

This study is not about making value judgment. Therefore, whatever findings I come up with shall be made known to the participants. Participation in this study was by voluntary agreement and participants were given the right to withdraw from the study at any time. All information obtained from the participants was treated confidentially. Participant anonymity was also guaranteed throughout the stages of this research.

3.9 Conclusion

In this chapter I have discussed the methodology I used to collect data. I have also explained how and why I arrived at the sampling method (purposive sampling). The sites where the study took place and participants that took part are described in this chapter. The study investigated

teachers‟ source of tasks; the type of task selected and implemented in classroom; the criteria used and decisions they made before and during classroom instruction. The findings of the study could not be generalized to all teachers because of the methodology which was used. Nonetheless, it might be beneficial to teachers and school managers in these schools to ensure good task practice that promotes and maintains student motivation, thereby enhancing student learning of mathematics.

CHAPTER FOUR: ANALYSIS OF MATHEMATICS TASKS

4.1 Introduction

The data analysis is reported in this and the following chapter, organized in four sections that correspond to the four research questions presented in chapter 1. This chapter consists of two sections. The first section describes the sources of tasks selected by teachers. The second section presents the analysis of tasks teachers chose for use in classroom. The types of tasks were analyzed using Stein et al.‟s (1996) task features and the levels of cognitive demand.

4.2 Sources of tasks selected by teachers

This section identifies the most prominent resources from which teachers draw trigonometry tasks and the type of tasks they select for use in classrooms. This information was gathered from the interviews I had with the teachers. When asked about the most prominent textbooks used in their teaching of trigonometry, the teachers indicated that they frequently used the Zambia Secondary School Syllabus Mathematics (ZSSSM) for grades 11 and 12 because these are prescribed textbooks for the syllabus. The ZSSSM is a series of prescribed textbooks for Zambia mathematics syllabus, for secondary school from grades 8 to 12. In trigonometry, the ZSSSM 11 deals with angle measures, trigonometric ratios and applications of trigonometric ratios, in two and three dimensions. The ZSSSM 12 is a continuation from book 11, this book covers the cosine and sine rules with their applications. The teachers also indicated that besides the prescribed textbooks, New General Mathematics (NGM) was another frequently used textbook.

All three teachers revealed that they picked trigonometry tasks from ZSSSM 11 and 12. Two teachers used New General Mathematics 3 to supplement the prescribed textbook. Teacher 1 also supplemented the prescribed textbook with „Thinking Process‟ the book which contains Cambridge past examination questions and answers. In addition, the sources of the tasks selected for use in classroom, teachers‟ lesson plans for five consecutive days were collected and the tasks in them were noted. The pupils‟ activity books were also collected and tasks that were implemented in class were scrutinised and recorded. These tasks were further scrutinised in order to ascertain their sources. The source of each task from the lesson plans was noted and these were compiled in table 4.1 below.

Table 4.1: The sources of tasks

SOURCE OF TASK	Teacher 1	Teacher 2	Teacher 3
Zambia Secondary School Syllabus Mathematics 11 (ZSSSM 11)	17	13	12
Zambia Secondary School Syllabus Mathematics 12 (ZSSSM 12)	19	22	21
New General Mathematics Book 3	-	2	7
Thinking Process: Cambridge past Exam questions	5	-	-
TOTAL	**41**	**37**	**40**

The interviews with teachers revealed that all three teachers used ZSSSM 11 and 12 because the language used in these textbooks was simple for the students. They all attested to the fact that the ZSSZM textbooks also acted as a guide to the breadth and depth of content to be covered, because these were written in line with the syllabus. Besides textbooks being written in line with the syllabus and acting as a guide, Teacher 2 and Teacher 3 strongly felt that the coverage of content in these textbooks was also adequate. Additionally, Teacher 3 alluded to the fact that the examples given in these textbooks were related to the local environment and as such they were easy to follow.

The tasks presented in table 4.1 were further delineated according to the purpose for which each task was used. To do this both teachers‟ lesson plans and the pupils‟ activity books were used to discern the actual intention of each task. Three categories of tasks were identified in this research. These are: examples, class exercises and homework:

- Examples which the teacher used to teach trigonometric concepts, algorithms, procedures and skills;
- Class exercises, these were set up for pupils to practice the procedure and skills further and master the concepts; and

- Homework which served the same purpose as class exercises, but students completed these tasks in more flexible conditions and environment.

The pupils‟ activity books were used to locate the tasks in each of the mentioned categories.

Table 4.2 shows the distribution of trigonometry tasks selected by the teachers and the purpose for which the task was intended.

Table 4.2: Categories of selected trigonometry tasks

Teacher	NUMBER OF TASKS			
	Examples	Class exercises	Homework	Total
Teacher 1	9 (22%)	19 (46%)	13 (32%)	41
Teacher 2	14 (39%)	11 (30%)	12 (31%)	37
Teacher 3	9 (23%)	17 (43%)	14 ((34%)	40

It is worth mentioning that the picture of selected tasks shown in table 4.2 was different from the picture at implementation. When pupils‟ books were scrutinized and tasks in them were compared with tasks set up by teachers, a disparity emerged between the selected tasks and the implemented tasks. Some selected tasks in the lesson plans were not enacted in classroom. Another notable shift involves the task categories, for instance, some tasks set up as classroom exercises ended up being executed as examples. The causes of shifts of tasks from one category to another and omission or inclusion of certain tasks will be discussed later. The following section focuses on the type of tasks selected and implemented by the teachers. To understand the type of tasks teachers selected and used in their classrooms, analysis of tasks using task features and levels of cognitive demands in them was conducted. I now turn to the analysis of the tasks.

4.3 Task analysis

In this section, tasks were analyzed using the task analysis of Stein et al. (2000). Two sets of tasks were identified; the „chosen" and „not chosen" tasks. Chosen tasks, refer to all tasks selected by the teacher as they appear in the lesson plan, and tasks „not chosen", comprise all tasks selected but not implemented or tasks not selected at all. The set of all tasks selected but not implemented in classroom forms the first layer of tasks not chosen. The second layer of tasks „not chosen" refers to all tasks in textbooks which are similar to the implemented tasks, but were not picked at all. The two sets of tasks were arrived at by matching similar chosen tasks with tasks not chosen in textbooks and similar pairs were chosen for analysis. In this study, the tasks that were analyzed comprised only those tasks implemented in classroom which were similar in many ways with tasks which were not chosen in terms of task features and levels of cognitive demands. Later in this section, I discuss how some tasks set up at high-level demands declined to low level demands and how other tasks remained unchanged during implementation.

The tasks were analysed using Stein et al"s (2000) task analysis guide. The analysis is organized according to task features and levels of cognitive demand.

4.3.1 Procedures without connections tasks

Procedures without connections tasks are tasks which are algorithmic; require limited cognitive effort for successful completion and they do not necessarily demand making connections to the meaning that underlies the procedure being used or to the mathematical concepts in focus. Instead they are focused on producing correct answers by memorizing procedures rather than developing mathematical understanding.

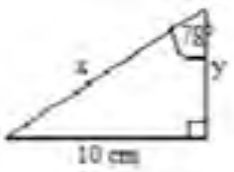
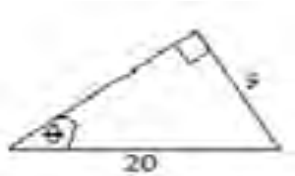
Tasks coded as procedures without connections were put in three groups according to „task type". Task type refers to the level of difficulty (complexity) that the task poses, coupled with the amount of time required to complete the task. The three task types identified were: Type one - those tasks which require very little time in order to be completed, and involve a single step procedure to solve them. Besides students do not need to make connections to the meaning that underlies the procedure or concepts being used, this type of task could easily be completed by calling upon single step procedures and algorithms. Type two tasks require a bit more time than

type one and are a bit more challenging than type one tasks. These tasks may involve two or more steps or procedures in order to solve them. Although they do not require students to make connections to meaning that underlies the procedures or concepts, they may require more time in order to be worked out. Students may be required to perform repeated operations in order to arrive at the correct answer. Type three tasks require more time and demand some cognitive effort. There is some ambiguity in this type of task. The tasks may require that students relate to previously learnt concepts or procedures.

A total of nine procedures without connections tasks which were selected by all the teachers and implemented in three different grade 11 lessons are considered in this section. All the tasks were procedural and algorithmic in nature despite varying times and cognitive efforts required to complete them. Some tasks evoke limited cognitive effort to be completed. They do not necessarily demand making connections to the meaning underlying the procedures being used or the mathematical concepts being reviewed. The fact that such tasks are procedural and algorithmic, they demand production of a correct answer through memorized formulae and procedures. Therefore, engaging with each task requires well rehearsed procedures and recitation of an appropriate trigonometric ratio or rule in order to arrive at the correct answer. In the following section I analyze and discuss the three task types.

In this section the three groups of tasks according to task type are analyzed and discussed.

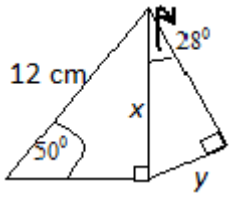
Figure 4.1: type one task

1.1 Calculate the value of x and y correct to 2 decimal places	**1.2 Calculate to the nearest $0.1°$ the size of θ**
	

In order to find x and y in task 1.1 and θ task 1.2, the student needs to recall the appropriate trigonometric ratio. For example, in task 1.1 the student is expected to use either the sine ratio in order to find x or the tangent ratio to find y. Once the student has found x or y, he/she can then use Pythagoras theorem to find y or x respectively. But use of Pythagoras theorem in this case may not appeal to the student, in which case they may use the other trigonometric ratio. In the second task (task 1.2), to find θ requires that the student uses the sine ratio. Both tasks are procedural and algorithmic, solved by the application of fixed rules or procedures in order to arrive at the answer. In both cases, a single step procedure is involved to find the answer. They require very little cognitive effort, since only straightforward application of a single concept is needed to solve them. They are both focused on producing correct answers and no explanations or justifications are required. Therefore, the two tasks are Type 1 procedures without connections tasks.

Figure 4.2: Type 1 multi-step task

1.1 Calculate the value of x and y



Task 1.3 is more involving compared to tasks 1.1 and 1.2. Although this task appears to be more involving than the first two tasks, it demands almost the same level of cognitive engagement as the others. The only difference here is perhaps the time required to complete this task. This task will require a little more time and some cognitive effort than the two tasks discussed in the previous paragraph, in the sense that this task poses little ambiguity about what needs to be done first before proceeding to the next stage. The learner is required to first find x that is when s/he can find y. To find x the learner needs to use the sine ratio. Once the learner has calculated the value of x, he/she can use the appropriate ratio to find y. From this discussion, it is very clear that

Task 1.3 is focused on routine procedures to produce the correct answers. Therefore, Task 1.3 may well be classified as Type one procedures without connections task.

Figure 4.3: Type two tasks

2.1 A town Y is 200 km from town X in a direction 040^{o}. How far is Y east of X?

2.2 A wall is h metres high. A ladder, leaning from the top of the wall to the ground is 15 metres long. The ladder makes an angle of elevation of 54^{o} with the horizontal ground. Find:

a) The distance of the foot to the bottom of the wall.
b) The value of h.

The two tasks presented above require limited cognitive effort for successful completion. Engaging with each of the tasks requires the use of some well established and possibly memorized trigonometric ratios for finding the unknown side or angle of the right angled triangle. There is also little ambiguity in them. The task requires that the student first transforms the information into a sketch drawing. The learner needs to employ concepts learned previously to be able to accomplish drawing of the diagram correctly. The student needs to make a sketch drawing as a means for deciding what trigonometric ratio and/or procedure to use. For example in task 2.1, to find how far town Y is east of town X requires the learner to use previously learned concepts or ideas about bearings. By making a sketch drawing, of course bearing in mind that a right triangle is formed, and by using the appropriate ratio (sine ratio) the learner is able to find the required distance. In Task 2.2, a sketch drawing is used in order for the learner to see the relationships between the vertical wall and horizontal ground. Once the learner has correctly interpreted the information into a diagram, it becomes easy to choose the right procedure for finding the height of the wall. Even though the two tasks involve interpretation of text information by transforming text into a sketch drawing and applying appropriate procedures and algorithms, both tasks do not require any explanations that focus on describing the procedure that was used. Therefore, both tasks could not be considered as a high level task, instead they fit the description of procedures without connections tasks in Stein et al‟s (2000) classifications.

The pair of tasks presented below exemplifies type three tasks. Each of the tasks has two questions, as shown in the figure below. I now analyze and discuss the two tasks

Figure 4.4: Type three tasks

3. In the triangle ABC, calculate the sides and angles in each of the following:

3.1 c = 9.4 cm, $\angle$ A = 48^o, $\angle$ C = 56^o. Find a.

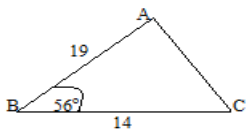
3.2 a = 14.3 cm, b = 12.6 cm and $\angle$ B = 41^o. Find $\angle$ A

4. In the triangle ABC, calculate the side and/or angles of the following:

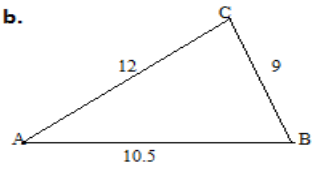
4.1 a = 14 cm, c = 19 cm and $\angle$ ABC = 56^o

4.2 a = 9 cm, b = 12 cm and c = 10.5 cm

a.



b.



Task 3 has two subtasks, which include Tasks 3.1 and 3.2. Both tasks require that students use the sine rule to find the indicated side and angle respectively. To find the side „a" the students needs to use the ratio involving side „a" to sine of the angle A and side „c" to sine of an angle C. This can be presented symbolically as $\frac{a}{sinA} = \frac{c}{sinC}$ or the equivalence, where a and c are sides of the triangle and A and C are angles opposite the two sides respectively. In task 3.2 the student is required to find the angle A. This task is slightly different from task 3.1 in that student employs the sine rule in order to find the angle. To find the angle A, the student employs the ratio $\frac{sinA}{a} = \frac{sinB}{b}$ or the equivalence. The two subtasks involve procedures and algorithms. To complete these questions, the student needs to memorise the sine rule and substitute in the values.

Like the previous task, task 4, has two subtasks namely, tasks 4.1 and 4.2 respectively. Both tasks are procedures without connections task.

Task 4 differs with the previous task in the sense that the cosine rule is used to find the unknown side or angle. For instance, in order to find side b in task 4.1 the learner is expected to use the formula $b^2 = a^2 + c^2 - 2acCosB$, where a and c are the known sides and b is the unknown side and B is the included angle.

In both tasks 3 and 4 use of procedures is called for. All tasks require limited cognitive effort for their successful completion. The tasks are more cognitively involving than type one and type two tasks because they involve more complex relationships. Type three relates a pair of ratios, involving four component quantities, whereas tasks one and two involve only two component quantities. Therefore, type three tasks are more cognitively involving than the other two types because of the complexity of relationships involved. That differentiates this type of tasks from type one and type two tasks. Even if the tasks involve more complex relationships, they are procedures without connections tasks, because the tasks have fixed or determined rules to follow in order to arrive at the correct answer. They do not require making connections to the meaning that underlies the procedures being used or the mathematical concepts in focus. In that sense, only well established cosine and sine rule suffices for the successful completion of the tasks.

The nine tasks presented here are procedures without connections tasks. The tasks were extracted from the pupils‟ books. All the nine tasks do not require students to make connections; neither can they be solved using several solution methods they all depend on correct procedures and algorithms in order to get the correct answers.

4.3.2 Procedures with connections tasks

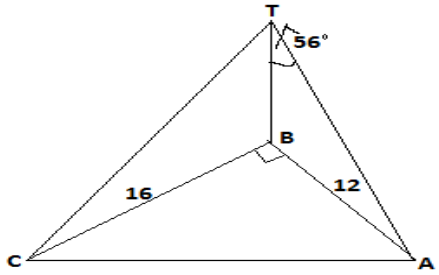
In procedures with connections tasks, learners‟ attention is focused on the use of procedures or algorithms for the purpose of developing deeper levels of understanding of the mathematical concepts and ideas in the task. Even though procedures might have been previously learnt, they cannot be followed mindlessly. Students need to engage with the conceptual ideas that underlie the procedures in order to successfully complete the task and consequently develop understanding. These tasks require some degree of cognitive efforts.

This paragraph discusses procedures with connections trigonometry tasks chosen by the teachers and enacted by the pupils in different grade 11 classrooms.

Figure 4.5: Procedures with connections tasks

5.1 Three points A, B and C are on a horizontal ground as shown in the diagram below. T is vertically above B. If $\angle$ ATB = 56^o, BC = 16 m and AB = 12 m, calculate:

a) BT
b) TC
c) $\angle$ BTC
d) Calculate the shortest distance from B to AC

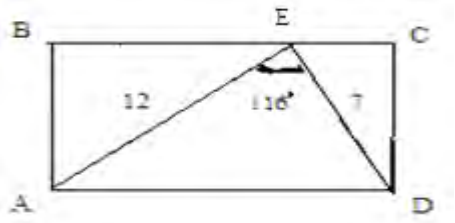


Task 5.1 involves application of trigonometric ratios. To engage with this task requires that the student makes connections between concepts or procedures. For instance, in order to solve this problem, the student needs to use trigonometric ratio in order to find BT. The student will be required to connect BT (the vertical height) to triangle BTC which forms a right angled triangle and then use Pythagoras theorem in order to find TC. To find the shortest distance, the student needs to understand that the perpendicular distance is the shortest distance. The student can choose from a host of methods by connecting procedure to find the shortest distance. Although the task may have suggested pathways, connections are needed between different concepts and procedures, such as trigonometric ratios, Pythagoras theorem and area of triangle. The task also requires that student utilises the relationships among the multiple procedures and concepts. That is the relationship between the shortest distance and the height in the area of a triangle. The student needs to know why the procedure is being used and not to use it mindlessly. This task fits

the description of procedures with connections task. Therefore, it is a procedure with connections task.

Figure 4.6: Procedures with connections task

5.2 In the diagram, ABCD is a rectangle, AE = 12 cm, BE = 7 cm and ∠ AEB = 116°. Find the length and breadth of the rectangle.



The above task involves application of both cosine rule and trigonometric ratios. To find the sides requires that the learner unpacks the rectangle into triangles. By considering triangle AED, the cosine rule can be used to find AD which is the length of the rectangle. Breadth of the rectangle which happens to be the height of triangle AED, can be found by first finding the area of the triangle using the formula, $A = \frac{1}{2}absin\theta$. Then connect this area to $A = \frac{1}{2}bh$, in which the base is AD and solve for the height h. The task requires some degree of cognitive effort to be solved. This task also focuses students on developing deeper levels of understanding of mathematical concepts and ideas. That is students need to understand why and how different ways of finding area of triangle can be utilised to find the height (breadth of rectangle) by relating different concepts and procedures. The task involves making connections between multiple relationships to underlying meaning of the concepts or procedures being used. For example, by using cosine rule to find the length and relating different formulae for finding area of the triangle to find the breadth of the rectangle. Students with rich conceptual understanding of trigonometric concepts would be able to complete the task successfully. This task fits well into procedures with connections task.

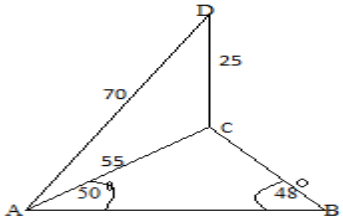
The task presented in the figure below is more complex than the two procedures with connections tasks described in the previous paragraph. Task 5.3 has several questions which are closely linked. To engage with this task requires that student make connections with the underlying meaning of the concept or procedure being used. For instance, in order to find (i) and (ii), the student should understand what procedures are appropriate. That is the sine and cosine rules should be used for (i) and (ii) respectively. To engage with (iii) the student needs to understand the concept of bearings and connect this to the acute angle found in (ii). The tasks can be solved using a range of techniques. For instance, the student will be required to use cosine and sine rules, and the concept of complementary angles to find the bearing. The trigonometric ratio can then be used to locate the position of a point equidistant from A and C. The task is also open to many interpretations. To successfully complete this task requires good conceptual understanding of the underlying meanings to the procedures, and flexibility in applying these procedures. The student cannot mindlessly use the procedures without understanding why it is being used to successfully complete this task.

Figure 4.7: Procedures with connections task

5.3 A, B, C and D are four points on level ground with B due east of A. It is given that AC = 55 m, CD = 25 m, AD = 7 m, $\angle CAB = 50^o$ and $\angle ABC = 48^o$

a) Calculate:
   (i) AB
   (ii) $\angle CAD$
   (iii) The bearing of D from A

b) A man walks due east from A until he reaches a point P which is equidistant from A and C. Calculate the distance AP



All three tasks presented above are procedures with connections tasks. They all require some degree of cognitive effort to be completed because the tasks are open to many solution routes.

General procedures may be followed for component parts of each solution, but there is no procedure for the complete solution. All the tasks require that student make connections among different relationships, and can be presented in several ways. They also suggest path ways to be followed either explicitly or implicitly, which are broad general procedures that have close connections to underlying conceptual ideas as opposed to narrow algorithms that are obscure with respect to underlying concepts.

4.3.3 Tasks implemented in classrooms

Having analyzed different sets of tasks, I now turn to tasks selected and implemented in the three classrooms. Tasks implemented in class refer to all tasks given as examples, class exercises and homework. The analysis of both pupils‟ work-books and the lesson observations revealed that there were no memorization and doing mathematics tasks. Teacher 1 implemented a total of 37 out of 41 selected tasks of which 2 tasks were of type one, 7 tasks were type two, 5 tasks were type three, and the remaining 23 tasks were procedures with connections tasks. Teacher 2 implemented 35 out of 37 selected tasks of which 13 were procedures without connections tasks consisting of 3 type one tasks, 5 of each of type two and type three respectively and 22 were procedures with connections tasks. Teacher 3 implemented 35 out of 40 selected tasks of which 14 were procedures without connections tasks constituting 2 type one tasks, 6 type two tasks, 6 type three tasks and 21 procedures with connections tasks. Actual implementation of tasks has been dealt with in chapter 5.

The summary of levels of cognitive demands of the selected and implemented tasks is presented in the tables below.

Table 4.3: Cognitive demands of tasks as set up by the teacher

Cognitive Level / Teacher	Memorization	Procedures Without connections				Procedures with connections	Doing mathematics
		Type 1	Type 2	Type 3	total		
TEACHER 1	0	2	7	5	14	23	0
TEACHER 2	0	3	5	5	13	22	0
TEACHER 3	0	2	6	6	14	21	0

Table 4.3 shows that Teacher 1 implemented 14 procedures without connections tasks representing 38% and 23 procedures with connections tasks representing 62% of tasks implemented in class. Teacher 2 implemented 35 tasks, constituting 22 procedures with connections and 13 procedures without connections tasks representing 63% and 37% respectively of the implemented tasks. Teacher 3 also implemented a total of 35 tasks with 21 procedures with connections and 14 procedures without connections tasks, representing 60% and 40% respectively.

Any discussion of task selection and implementation is not separable from discussion of the three phases through which tasks pass. Stein, Smith, Henningsen and Silver (2000) propose three phases, as they appear in textbooks, as they are set up or interpreted and planned by the teacher and as they are enacted by both the teacher and pupils in classroom. For a better understanding of teachers‟ task practice, there is a need to track down the cognitive levels of demands of the tasks. I argue that teachers‟ task practice is closely tied to the manner in which they carry out their classroom instruction. In the following section attention is drawn to how tasks set up at high level declined or remained at high level and where they declined, to the reasons for their decline.

4.3.4 Decline of task

The tasks were scrutinized in the pupils‟ activity books. Analysis of both pupils‟ activity books and actual teaching revealed some shifts in the levels of cognitive demands from high level task to low level tasks. Firstly, disparities between tasks in lesson plans and pupils‟ books indicate that there was an increase in the number of tasks given as examples in class. For example, Teacher 1 planned for 9 tasks as examples, but during implementation gave 16 examples. Teacher 2 initially prepared 14 tasks for examples; during implementation he used 19 tasks. Similarly, Teacher 3 had prepared 9 tasks to be used as examples, but at implementation he used 15. This change in the use of the task does to some extent remove the challenging aspects of the task because it is highly likely that the teacher can step by step guide the students to the solution.

Secondly, the teachers removed challenging aspects of the tasks by not giving students‟ time to think about and struggle with the problems. For instance, Teacher 2 dominated much of the lesson as exemplified from his lesson vignette. The teacher often took large proportions of class time assisting students to do the tasks meant for them. The teacher literally solved the problems for the pupils whenever students hesitated in responding to his questions. He did not press for explanations from the students. Thirdly, the tasks in the pupils‟ work books for Teacher 2‟s class, unlike other classes, showed a lot of similarities in the solution methods used. That is an indication that students were not challenged to experience mathematics on their own either in or outside the classroom due to the sameness of logical thought presented in their books. That may be a possible cause of the decline of the levels of cognitive demands of trigonometry tasks. The shift in tasks types was also noted in Teacher 2‟s lessons. Two type 2 tasks declined to type 1 tasks by way of making procedures to be used very explicit by the teacher. It is worthy noting that all procedures without connections tasks remained at the same level during implementation. The levels of cognitive demands of the tasks at implementation are shown in the table below.

Table 4.4: Levels of cognitive demands at implementation

	MEMORISATION	PROCEDURES WITHOUT CONNECTIONS	PROCEDURES WITH CONNECTION	DOING MATHEMATICS
TEACHER 1	0	26	11	0
TEACHER 2	0	27	8	0
TEACHER 3	0	25	9	0

Table 4.4 shows the decline of procedures with connections tasks to procedures without connections tasks. At task set up, Teacher 1 presented 23 procedures with connections tasks, at implementation 12 tasks were reduced to low level tasks. Teacher 2 presented 22 high level tasks, during implementation, only 8 tasks remained unchanged, while 14 tasks were reduced to low level tasks. Teacher 3 presented 21 high level tasks, but when it came to implementation, 9 tasks declined to low level tasks with 9 tasks remaining unchanged. While some tasks set up at high level demands declined to low level demands, all tasks which were set up at low level remained unchanged during implementation. But the change in the type was evident in Teacher 2‟s lesson where two type 2 tasks were transformed to type 1 task by removing the slight ambiguity in them.

A range of factors is noted in literature that influences the decline of students‟ thinking processes into the use of procedures without connection to meaning. These include the removal of challenging aspects of the tasks, shifts in focus from understanding to the correctness or completeness of the answer, and inappropriate amounts of time allotted to the task (Henningsen & Stein, 1997, p. 535), as well as tasks that are too difficult.

To track the decline of levels of cognitive demands of tasks, Stein et al.‟s (2000) task analysis guide was used. A remarkable decline in levels of cognitive demands of tasks was noted during the lesson observations. During task implementation about 60% of the tasks which were set up at high level declined to low level demands. And only 40% of the tasks were maintained at high level demand. In one of the lessons, it was observed that the teacher reduced the demanding aspects of the task by guiding the pupils, showing them all necessary techniques and by carrying

out step by step procedures thereof. Teacher 2, for instance, spent 25 minutes on giving examples of which one task was meant for class exercise and 10 minutes was left for students to work on class exercises individually. There was not much interaction amongst students. In this lesson pupils were not afforded an opportunity to think and reason about trigonometry concepts being developed and procedures being used. In addition to this lesson pupils were not given enough time to grapple with the tasks and to challenge one another. Teacher 2 dominated much of discussions during his lesson. He asked questions regularly but did not wait to get different views from students. The teacher permitted few responses from the students.

Teacher 1 was patient and encouraged his pupils to contribute freely. He acknowledged both correct and wrong answers and used wrong answers as learning points. The teacher used 20 minutes on three examples and then in groups of four students spent eight minutes to work on a task. After discussion of the problem which lasted 22 minutes, the teacher gave three tasks for students to work individually for about 30 minutes. The teacher during the last 10 minutes together with his pupils worked through the tasks.

Teacher 3 encouraged his pupils to contribute freely as well. He allowed his pupils to work collaboratively in small groups and as whole class. The teacher spent 12 minutes on giving examples. After two examples the teacher gave two tasks for students to work in small groups for 10 minutes. Thereafter, the groups presented their solutions showing their solution methods clearly. The teacher pressed his students to compare the different strategies and to make justifications for use of a strategy. He made sure that the solutions presented to the class were clear. He spent 25 minutes on class discussions. The last 15 minutes students worked on the problems individually.

From this analysis, I noted that Teacher 2 over simplified tasks for his students by giving a lot of examples of the same kind and with little variation. As Stein, Grover and Henningsen (1996) state, “teachers can wittingly or unwittingly change the nature of tasks by stressing less or more challenging aspects of the task or by altering the resources available to students” (p.460). Thus, denying their students opportunities to learn mathematics.

All three teachers indicated that they stuck to teaching what was prescribed in the syllabus because of fear of failure of their students. Teachers feared that their students would not do well in their examinations if they did not cover the syllabus. One teacher pointed out that "the time was limited, to go through all that in order to cater for all students. Because if you do not finish the syllabus, when the examination comes you disadvantage your students" (Teacher 2: first interview). The sentiments given by the teachers may be a contributing factor for the decline of some of the cognitively challenging tasks to low level tasks, because teachers stressed less or more on challenging aspects of the tasks thereby altering the mathematical challenge for students as independent thinkers.

The following section addresses the third and forth questions. What criteria do teachers use and what decisions do teachers make in their selection and implementation of tasks?

4.4 Conclusion

The chapter has laid out the results of the analysis of tasks and teachers‟ task practice in regard to the criteria used and decisions made. The analysis of tasks revealed that teachers predominantly selected tasks from the prescribed books. Very few tasks were modified or adapted from these textbooks. Results of the analysis of the tasks selected and implemented by the teachers in classrooms indicate that teachers selected more procedures with connections tasks than procedures without connections tasks and there was no memorization or doing mathematics task. Furthermore, all procedures without connections tasks remained unchanged at implementation while about 60 percent of procedures with connections tasks declined to procedures without connections.

The decline of high level tasks can be attributed to fear of failure of the students. Teachers feared that if they did not finish the syllabus their students would fail the examination. Teachers also tended to oversimplify tasks for the students by giving a lot of examples and inadvertently taking over students‟ responsibility of solving mathematical problems. The teachers, especially Teacher 2 did not allow students enough time to struggle with trigonometric tasks.

CHAPTER FIVE: ANALYSIS OF TASK PRACTICE

5.1 Introduction

In this chapter the analysis of teacher task practice is presented. The chapter is divided into two sections that attempt to address the third and fourth research questions. The focus of this chapter is good task practice as a consequence of the criteria and decisions made by teachers before, during and after classroom instruction.

5.2 Criteria guiding teachers' tasks choices

In order to develop a clear understanding of teachers‟ task practice and to explore factors influencing teachers‟ task choices, I interviewed the three teachers who have consistently produced good results with the classes they have handled. All teachers have been teaching mathematics for more than five years and they hold first degrees. Interviews were held with each participant at two different occasions. The pre-lesson interview focused on general information about teachers‟ task selection choices and their sources. Post-lesson interviews focused on the implementation of tasks. The post-lesson interview was conducted shortly after the lesson observation. Both interviews sought to obtain insight into each teacher‟s task practice by considering criteria used and decisions made by the teacher during the whole process of task selection and implementation. A number of reasons and factors were raised by participants as having influenced their selection and implementation of trigonometry tasks.

This section gives the profiles of the three teachers. The profiles include a brief discussion of the criteria used and decisions made by these teachers in their selection and implementation of trigonometric tasks.

5.2.1 Teacher 1

Teacher 1 has been teaching for sixteen (16) years. He started with a secondary teacher‟s diploma qualification before he obtained a Bachelor‟s degree in mathematics. The teacher has been teaching at this school since his initial training. Therefore, the teacher is well qualified and experienced. His inclusion in my study was due to the recommendations I received from his supervisors, the Head of Department and the Head Teacher.

The interview with the participant in regard to the criteria used and decisions made in his choice of trigonometry tasks is summarised under the following themes:

Time available: The teacher reported that the duration of mathematics periods was a determining factor in his choice of tasks. Depending on how much time was available to him, he could choose either to do or not to do that task. He said "I might choose not to do a certain task because of the limited time." (Interview 2, Res 9)

Availability of materials: Other than availability of time, the teacher mentioned that materials guided his choice of tasks. He said "when planning I consider the availability of materials to be used in class" (Interview 1, Res 7).

Student ability: The teacher explained that he considered the abilities of pupils in class when choosing tasks. He went on to say that certain type of tasks can only be used in certain classes. According to teacher 1, "I might choose for the first class which is the pure class, I might choose to prove the formula, because I know they will grasp the concept. But when I am teaching the last class, I might skip and might just give the formula. Because I know even if we go through the formula, it will just threaten them more." (Interview 1, Res 6).

Teaching aims: The teacher informed me that the tasks that he had picked had the aims for which they were selected. The teacher said when he picks a task he should analyse it first. He said "and the aim of giving a task is either to enhance understanding or supplement explanation." (Interview 1, Res 10)

Bring out student attainment: Teacher said lesson objectives guided the selection of tasks and that objectives were set to guide the teacher on how much and how far one should go. When asked about how the task contributions influence his task choices from textbooks he had this to say "I write the lesson objective, even the tasks I will choose them in line with the lesson objectives. So that when they are done successfully, I will say the objective of the lesson has been achieved."(Interview 1, Res 11).

Variety: The teacher said he selected tasks that provided different ways of working with the concepts to be taught, in that way pupils benefited a lot from the tasks.

Simple and challenging tasks: The teacher mentioned that the tasks that he selected were at the level of the students. When the teacher was asked to comment on the tasks that he had selected he had this to say "I chose my tasks based on the previous work. I started with simpler tasks so that all students could participate. When I introduced the difficult ones, you saw how student participation reduced drastically. I think my choice is based on what students already know and I prepare tasks in such a way that we move from easy to difficult tasks. On another occasion the teacher said "those that are challenging (referring to challenging tasks), normally I use assignments." (Interview 2, Res)

Building blocks for challenging task: On the issue of tasks as building blocks for challenging tasks, the teacher said he prepared his task in such a way that he started with what students had an idea about, in order to link to new situations. He said "the aim of the task is either to enhance understanding or supplement explanation." He went on to say "even if I give them information different from the way the example was given, they are able to apply the concepts" (Interview 1, Res 8). Related to this theme is another incident where the teacher mentioned that the objective of his selected tasks was to see to it that students were able to use the cosine rule in unfamiliar situations. He talked about a situation where pupils were required to use the cosine rule in order to find the length and breadth of a rectangle from an inscribed triangle. The task described here is task 5.2 which has been analysed in chapter 4.

Teacher's knowledge of the pupil: The teacher said knowing his pupils helped in his teaching, because by knowing pupils it was easy for him to tell if there was a problem. He did not have to wait until the problem was big. He said "as a teacher you need to know your pupils very well, you will know when they are ready for this and that task." He gave an example, "If you are teaching, suddenly student develop cold feet, you will quickly change your approach if you really know your pupils."

Sequencing of tasks: The teacher said the type of questions or tasks one asks and how they are ordered matters in achieving one‟s set goals. He said tasks should be sequenced in such a way that simple concepts are taught first and then complex or abstract concepts.

5.2.2 Teacher 2

Teacher 2 has been teaching mathematics at his school for six (6) years. He holds a first degree. The teacher has not been teaching for too long, unlike the other two participants, however, he is experienced enough to handle all grades in secondary school. The interview with the participant on what criteria guided his choice of tasks and some critical decisions that he made during the process of task selection and implementation revealed the following issues:

Availability of time: The teacher was sceptical about availability of time to cover all important activities in mathematics. He mentioned that tasks which take a lot of time to be completed were usually avoided and so his pupils could not be exposed to a variety of tasks and concepts. He chose to do this because he wanted to cover the syllabus. He said "the syllabus does not provide for all such. Also the time is limited to start going through all this (referring to different types of tasks), you will find that maybe you do not finish the syllabus, and when the exam comes you disadvantage your students." (Interview 1; Res 5)

Tasks that provoke deep thinking: The teacher speaking about the tasks in the textbooks from which he selects tasks said "the people who wrote those books knew what they were doing, because the exercises in them (books) really provoke one into deep thinking." He further went on to say that "the examples and explanations that have been given in those books seem not to provoke somebody to simply apply what they have learned or have been taught, they provoke somebody into deep thinking" (Interview 1, Res 11).

Related/ connected concepts –relevance: The teacher explained how he chooses the tasks whose concepts are connected. He said "concepts in the tasks to be taught in a lesson need to be closely related." The teacher emphasised that his choice of tasks is based on how closely related the concepts are. He explained that "I will pick the exercise that is so related to what I was talking about in the lesson."

Content/topic coverage: The teacher selected tasks that could be easily dealt with in class or during the period allocated for doing mathematics. Teacher explained how he went about choosing the tasks that he used in class against background of limited time. He said "you know our time table is kind of rigid and you will be required to finish the syllabus at the end of the day." He said "I normally choose tasks that do not consume a lot of time, so that I can cover as much work as possible" (interview 2, Res 10).

Multiple representations and modelling: The teacher described the type of tasks that he likes using. He said "I like using the ones where pupils are just given a statement of a question and then the pupil has to come up with a mathematical model like a diagram" (Interview 1, Res) He cited an example for this: A bird is on top of a flag pole and it is looking down at the boy at an angle of depression of 55^0. A boy is 5 metres from the flag pole. Calculate the height of the pole. So they have to come up with the diagram or a model.

Level of difficulty of concepts: The teacher said he took into consideration level of difficulty of the concepts to be taught when selecting the tasks. He said that, he always started with simple tasks and gradually moved to challenging tasks.

Familiar concepts against unfamiliar concepts: The teacher said that in choosing the tasks to be taught he looked at how familiar the students were with that topic. So when choosing the tasks, he chose them in such a way that he begun with what was familiar and then moved to unfamiliar situations. He is quoted saying, in interview 2, Res 10, "like example that I gave, I started with what the pupils already know, I started from the known to the unknown. Pupils tend to understand when you begin with simpler examples and then go to challenging examples."

Student ability: The teacher chose tasks that catered for the ability of most of pupils. Teacher said that "tasks need to be chosen in such a way that they are manageable by most of the pupils." (Interview 2, Res10)

Achievement of objective: The teacher chose tasks based on the objectives of his lessons. According to the Teacher "examples and exercises should be prepared in line with the objectives of that lesson." He further explained that the activities and examples he uses always come from the same textbooks. The only difference was in the explanations used and the objectives that are set.

5.2.3 Teacher 3

Teacher 3 has been teaching for over twenty (20) years. He taught in primary school before he upgraded his qualification to a degree. He has taught secondary school mathematics in three different schools for over 14 years, of which 8 years has been spent in his current school.
The interview with the teacher as regards criteria used and decisions made in his selection and implementation of tasks revealed the following issues as having influenced his selection choices:

Build from what students already know: The teacher stated that some tasks are primarily chosen to connect new knowledge to the existing knowledge of the pupil. He said "when preparing a lesson I look at what my pupils are able to do and then I select the examples and exercises from that angle, in short I start with simple and the move to challenging activities" (Interview 2, Res 9).

Vary level of difficulty of task (simple vs difficult task): When teacher 3 is planning he bases his selection of tasks on the level of difficulty. When asked how he chooses his tasks when planning, the teacher had this to say "I look at the level of difficult of the questions. Then I will mix the simple and challenging questions" (Interview 1, Res 5).

Attainment of objectives: The teacher reported that the tasks contribute to the success of the lesson if the tasks require that children are involved. If I select a task it is not for me to do, but requires that children attempt it. He said "when (children) they are involved, and they are given question, they will definitely answer them. If they answer them successfully, then I will know that I have achieved my objectives." He went on to say that "tasks are aimed at achieving the objectives that are set when I plan the lesson. In short, tasks must be relevant in order to help me to achieve objectives that I have set." (Interview 1, Res 10)

Real life and modelling: The teacher described as best, those tasks which require application of trigonometry. He explained that "especially tasks which deal with real life situation." He gives an example that "you ask children to say a ladder is leaning on the wall. The man working on electricity is on top of the ladder. Find the distance from the top of the ladder where the man is to the foot of the wall, if the foot of the ladder makes an angle of 48^0 with the horizontal ground and is 2 metres from the foot of the wall." He said the children will need to sketch and the transform the entire real thing into a triangle which is a right angled triangle.

Relevance and connection between different tasks: The teacher selected tasks that were related in one way or another. In the second interview Teacher 3 said "I expect the children to be able to apply trigonometric ratios in situation like that of a triangle, so that when I look at applications, they will be able to use the same knowledge to other mathematical situations."

Multiple representations: The teacher described how the children present word problem into different forms of representations. He said "the children will need draw the diagram and later transform into symbols. For instance they will sketch the wall as vertical and ladder slanting. Then they will transform the entire thing into a triangle. He concluded by saying children will picture the real thing and then bring it to the classroom, for them to find the solution or answer to the problem.

Construct own tasks: The teacher in talking about how he modified some of the tasks from the textbook, also alluded to the fact that some tasks he just constructed them. He said and I quote: But sometimes I just make questions from my head. Meaning he constructed some tasks. From the task analysis, two tasks were found to have been adapted by the teacher from the textbook.

Student ability: The teacher's statement "I will mix the questions; the simple and the challenging to see if those slow learners can attempt the challenging ones" may imply taking into account student abilities. In another instance the teacher mentioned that he deliberately chooses tasks to cater for all ability levels.

Enhance learning: The teacher said a task is suitable if it enhances learning in pupils. He said "I use challenging tasks when I want my students to learn something, because when students are allowed to discover the method to the answers on their own, they internalise what they have discovered rather than when they are provided with the formula" (Interview 2, Res 10)

Stimulate thinking: The teacher pointed out that it is not each and every task that is suitable; some tasks are not good in the sense that they do not evoke thinking in pupils.

Motivation: The teacher mentioned that he deliberately chooses tasks to cater for all ability levels. According to the teacher, he selects questions that are difficult and questions that are a bit simple so that pupils can experience success. That way they get motivated. He said, when slow learners experience some success they get stimulated to do even more. So I mix questions based on student strength. The simple ones which I am sure everyone will get right and then mix with those which are difficult (Interview 2, Res 5)

5.3 Themes for the criteria used in the selection of tasks

By carefully considering themes identified from the participants‟ interviews and by relating these themes within and across the participants, a new pattern of themes begun to emerge. This process was repeated more than six times to ensure that general themes were clearly identified and classified. Themes which seemed to have similar characteristics were collapsed into one theme. Consequently, all themes that emerged from the three interviews were collapsed into 12 major themes, which fall under the following headings in no particular order: time availability; student ability; multiple representations; provoke deep thinking; related to real life and modelling; motivation; building blocks for challenging tasks; variety; different levels of difficult; connectedness of concepts and ideas; attainment of objectives and familiar and unfamiliar tasks. The themes and sample comments extracted from the teacher‟s stories are presented in the table below.

Table 5.1: Categories and sample comments from the participants

Theme	Sample of teacher's comment
Time availability	• I might choose not to do certain tasks because of time (Teacher 1, first interview) • Time is limited to start going through all this (Referring to all the tasks selected), (Teacher 2, first interview)
Attainment of objectives	• Tasks selected are aimed at achieving set objectives (Teacher 3, first interview) • When I give an exercise, I will choose tasks in such a way that objectives come out (Teacher 1, first interview).
Variety	• I select tasks that that provide different ways of working with the concepts taught (Teacher 1, first interview). • I would give different but closely related tasks (Teacher 2, first interview)
Connections with other concepts and ideas	• Choice of tasks is based on how closely related the concepts are (Teacher 2, first interview). • I select task that are related in one way or another (Teacher 3, first interview)
Promote deep thinking	• The exercises in them (books) really provoke one into deep thinking (Teacher 2, first interview). • Task that enhances learning (Teacher 1, first interview; Teacher 3, second interview)
Multiple representation	• The ones (tasks) where pupils are just given a statement of a question and then the pupil has to come up with a mathematical model like a diagram or formula (Teacher 2, first interview)
Real life and modelling	• Children will picture the real thing and then bring it to the classroom, for them to find the solution or answer to the problem (Teacher 3, first interview)
Familiar and unfamiliar tasks	• I started with what students already know, to unknown (Teacher 2, interview 2). • The example that I gave I started with what the pupils already know (Teacher 3, second interview).
Different levels of difficulty	• I prepare tasks in such a way that we move from easy to difficult tasks (Teacher 1, second interview) • I will mix them (tasks) simple and challenging questions (Teacher 3, first interview)
Building block for challenging tasks	• The aim of the task is to either enhance understanding or supplement explanation (Teacher 1, first interview) This quote is interesting as related to teaching for understanding, but I am not convinced it relates to

	building blocks for challenging tasks.. You will need more here to justify this.
Motivation	• I start with simpler tasks so that all students can participate (Teacher 1, first interview) • Select questions that are simple and a bit difficult so that some pupils can experience success (teacher 3, interview 1).
Student ability	• I choose tasks to cater for all ability levels (Teacher 3, second interview). • I might choose to prove the formula, because I know they will grasp the concept. But when I am teaching the last class, I might skip and might just give the formula (Teacher 1, first interview)

5.4 Further analysis

In this section I classify themes into two distinct classes: *general* and *essential* themes. First, general themes include: attainability of objectives, availability of time and student ability. Time and lesson objectives are called general themes because they are clearly seen and implied at all stages in teachers‟ instructional design and implementation. Student ability is coded general theme because of the relative nature of this theme. To establish clearly the ability of a student requires some empirical evidence, which may be achieved through a series of assessments in different areas and over a period of time before a claim can be made. Besides, different student abilities in classroom is the reason why instruction in designed in a particular way. For this reason, student ability is not an essential tool in my analysis, although this theme was frequently mentioned by the teachers. .

Second, essential themes which include: variety, promote deep thinking, connectedness of concepts and other mathematical ideas, real life and modeling, building blocks for challenging tasks, multiple representations, motivation, familiar and unfamiliar concepts or tasks, and different levels of difficult. These themes are essential in the sense that they constitute key elements necessary for any mathematics instruction which is aimed at promoting student understanding. Therefore, essential themes are used in this section as a lens through which good task practice can be discerned.

Teachers‟ lesson plans and pupils‟ activity books for a period of five days and one lesson observation for each teacher were analyzed using essential themes. This analysis of lesson plans, activity books and lessons was orchestrated theme by theme. The summary of the findings is presented below.

5.4.1 Analysis of lesson plans

5.4.1.1 Variety

An examination of the teachers‟ lesson plans revealed that all three teachers used a variety of tasks. Variety refers to an assortment of tasks such as similarities and differences between the complexity of tasks; whether or not context of a task is familiar to the students‟ environment; use of concepts or skills, rules and procedures to problems of the same sort or different kind; tasks involving a range of techniques and/or procedures; tasks eliciting a single or multiple solutions methods.

In the lesson plans presented by Teacher 1, a mix of procedural and conceptual tasks was noted. Procedural tasks involve the use of memorized sequence of procedures or algorithms. Students often solve such kind of tasks by using appropriate remembered rules (algorithms) without knowing why they work. Whereas in conceptual tasks often procedures to be used may not be explicit. Besides, procedures are not used blindly without understanding why they are being used. These tasks require that student relate different concepts and rules in order to solve them. Sixteen tasks which rely heavily on computations that do not necessarily require understanding of the underlying domain were identified in the teacher‟s lesson plans. Although the tasks were procedural, the amount of time and effort required to solve them was different. Some tasks required one or two step procedure(s), while other tasks required multiple step procedures to solve them. All tasks presented on day 1 involved single step procedure. The tasks involved resolving of right angled triangles by the use of trigonometric ratios. On day 2, the tasks presented involved the applications of trigonometric ratios. This set of tasks required making some connections by linking new concepts to previously learned concepts and skills. More than one step procedure was required to solve such tasks. Seventeen tasks coded conceptual tasks were also noted in the teacher‟s lesson plans as well. These are tasks which involved computations that required students to know why they were using such rules and procedures.

A mix of routine and non routine tasks was also evident in the teacher‟s lesson plans. Routine tasks normally stress the use of a set of known or prescribed procedures (algorithms) to solve problems. Non routine tasks involve a search for a strategy that seeks to find the solution. From the analysis fifteen tasks were coded routine because the emphasis was on practicing the procedures and skills. Non routine or challenging tasks were also found in teacher‟s lesson plans. This emphasizes use of heuristics which do not guarantee a solution to a problem and often requires little use of algorithms. It should be noted from the onset that in this report a thin line has been drawn between procedural tasks and routines tasks, and between conceptual and non routine tasks. This is due to the fact that routine problems are procedural in nature and non routine tasks rely on conceptual understanding.

The analysis further revealed that some tasks selected by Teacher 1 involved use of several procedures and techniques in order to solve them. For instance, the problem involving the calculations of perpendicular height, the angle between an edge and the base, and the angle between a sloping face and the base, when given a pyramid with a square base (Task 3.1, Day 3). This task involved the use of several procedures that relate from one procedure to another and required students to know how and when to use the procedure. Therefore such a task evokes procedural and conceptual skills in the learner. I found 20 tasks that made use of several techniques to successfully complete them. Other than tasks that required a number of techniques to be performed in order to solve them were those tasks that needed to be transformed in many ways (multiple representations). 10 tasks were subject to multiple representations. These tasks are also open to several interpretations and therefore, present a number of solution pathways. Tasks open to a number of interpretations and whose solution paths are not easily accessible fit the description of tasks relating to real life. All the eleven tasks found in the teacher‟s lesson plans were context embedded.

The tasks selected by Teacher 2 as presented in the lesson plans were a mix of procedural and conceptual tasks. A thorough analysis of tasks in the lesson plans reveals that there were more tasks that required use of procedures than those that involved conceptual knowledge. There were more tasks that involved straight forward (multiple) procedures than those that required making connections between concepts and procedures. I found that nineteen tasks were routine and

involved the use of procedures ranging from single to multi-step procedures. 9 tasks were coded non routine. It is interesting to note that Teacher 2 presented similar tasks in terms of levels of cognitive engagement required to do them and concepts and skills being learnt were the same within the lesson unit. But complexity (in terms of cognitive effort and length) of tasks across the lesson plans was slightly different. That is the complexity of tasks increased from one lesson to the other. However, that was not the case for Teachers 1 and 3. The tasks within and across lesson units for both teachers were varied in terms of complexity.

Unlike Teacher 1, the teacher included very few tasks with multiple representations. The teacher utilized more routine tasks, where he gave a number of examples and then gave students similar tasks to work on. From the five lessons Teacher 2 had planned, only four tasks were related to real life. The teacher presented very few challenging tasks compared to the two teachers.

Teacher 3 like the other two teachers presented an assortment of tasks in his lesson plans. That is tasks ranging from routine to non routine, and tasks that have multiple representations. From the analysis, I found that eighteen tasks were routine and involved the use of procedures and 14 tasks were non routine. Unlike lesson plans for Teacher 2, the teacher‟s lesson plans were composed of simple and challenging tasks within and across the lesson plans. The tasks selected by the teacher were balanced to cater for different abilities in class, which was similar to the planning of Teacher 1. In Teacher 3‟s lesson I found nineteen tasks that required use of a range of techniques in order to solve them. I further found that the teacher utilized nine tasks that were related to real life.

5.4.1.2 Connectedness of concepts and tasks

Connectedness of concepts and procedures within and across tasks were observed in the teachers‟ lesson plans. Connections will refer to making connections to what students already know; making connections with underlying concepts and relations being learnt; making connections in terms of linking and carrying out procedures accurately and appropriately; making connections within the topic and across topics and subjects; making connections with real life and connecting different representations.

Investigation into connections and connectedness of concepts and tasks, revealed that tasks in the lesson plans for Teacher 1 were carefully sequenced in such a way that they were building on the current knowledge of the students. Tasks presented were such that they started with simple concepts and gradually moved to abstract concepts. The teacher‟s choice of tasks involving multiple steps showed that the teacher picked tasks which required relating different procedures by linking one procedure to another in order to solve the problem. The teacher gave examples connected to the students‟ prior knowledge and instead of giving similar tasks as class exercises; he used tasks which required students to apply learned concepts.

For the purpose of helping students develop conceptual understanding, the teacher employed task contexts which were familiar to the students‟ environment and the mathematics being learnt. Teacher 1 included context embedded tasks in his lesson plans. This helped students to learn important mathematics in context that are closely connected to real life and those that are purely mathematical.

The lesson plans for Teacher 2 reveal carefully sequenced tasks starting with simple and gradually move to difficult tasks. Like Teacher 1 the examples given by the teacher were connected to the students‟ current knowledge. The teacher did not utilize context embedded tasks like Teachers 1 and 3. The tasks were textbook mathematics tasks. Unlike Teachers 1 and 3 who presented a wide range of tasks which provided students with variety of learning experiences within and across the lessons, Teacher 2 presented similar tasks within but differed across the lessons. Sixteen tasks were noted to be connecting to students‟ prior knowledge.

The tasks in the lesson plans that Teacher 3 presented had connections within and across the lesson. Sequencing of the tasks was similar to the other teachers. The teacher chose tasks that connected the concepts and procedures in multiple ways to other mathematical ideas. Two tasks which had connections to other subject areas were found in the lesson plans. That is the major difference with the planning of other teachers. Like the other teachers, Teacher 3 utilized tasks that required relating different procedures and/or concepts in order to solve them.

All three teachers presented their tasks in such a way that they started from the known and progressively moved to unknown. The tasks chosen by the teachers connected students‟ current knowledge to their prior knowledge. The tasks presented by Teacher 2 did not evoke strong connections between tasks within the lesson unit compared to the tasks presented by the other teachers, because of similarity of tasks within the lesson units. Teacher 1 utilized more context embedded tasks than the other teachers.

5.4.1.3 Multiple representations

Multiple representations are ways to symbolize, to describe and refer to the same mathematical entity. Multiple representations here refer to tasks used to understand, to develop and to communicate different mathematical features of the same object or operation, as well as connection between different properties for example: graphs or diagrams, tables, formulae, symbols and words.

The findings of the analysis of tasks in the lesson plans with regard to different representations in them reveal that all tasks relating to real life had multiple representations. Other tasks involving multiple representations include: the tasks that require use of a range of techniques to solve them; the tasks which were open to many interpretations; tasks that involve constructing formula or modeling.

Analysis reveals that Teacher 1 utilized tasks that required the use of a range of strategies to solve them. For instance, drawing and calculating. Twenty tasks were noted as involving a range of techniques and strategies. Furthermore, two tasks involving mathematical modeling to represent, describe and provide solution to the problem were found and a total of eighteen tasks were coded multiple representational.

Teacher 2 did not select as many tasks with multiple representations as the other teachers. In addition to the four tasks which are context embedded, six tasks that involved use of a variety of strategies to solve them were found. Ten tasks in the five lesson plans were coded multiple representational. This is much lower than eighteen selected by Teacher 1 and thirteen found in the lesson plans for Teacher 3.

Like Teachers 1 and 2, all tasks in Teacher 3‟s lesson plans which relate to real life had multiple representations. Other than tasks subject to many forms of representations, Teacher 3 used four tasks that needed several strategies of presenting the solutions. For example, use a combination of trigonometry ratios, sine rule, cosine rule and construction to resolve the problem. Thirteen tasks were coded multiple representational.

5.4.1.4 Promoting deep thinking

Challenging tasks are reported in literature to promoting higher level thinking and reasoning in students as opposed to routine or procedural tasks. Tasks that are likely to encourage higher thinking and reasoning in students include: tasks whose solution methods are not readily accessible; tasks which can be solved using many different ways; tasks which can be presented in multiple ways; tasks which have multiple representations, tasks related to real life and tasks which require explanation and justification

Findings in regard to tasks promoting deep thinking indicate that Teacher 1 presented tasks which elicited a range of solution methods. For example, tasks relating to real life required use of different forms of representations in order to solve. These tasks are also open to many interpretations. Because these tasks are open to many interpretations, they can be solved using several solution methods. The findings of the analysis revealed that 23 tasks were high level tasks likely to promote deep thinking.

Although Teacher 2 presented 22 procedures with connections tasks in his lesson plans, the tasks were purely textbook problems. There were very few tasks for which the solution methods were not readily accessible and few tasks could be solved using a range of techniques. There was no task that required students to make explanations or justifications.

On the contrary, Teacher 3 had included tasks that required students to justify the use of a particular method or formula. Two tasks that pressed for justification from the student for using a procedure or formula were found. Also apparent in the teachers‟ lesson plans were tasks with

several solution methods. Tasks for which students did not have easy access to solution methods were suitable for promoting deep thinking.

5.4.1.5 Building blocks for Challenging tasks

Under the theme „building blocks for challenging tasks", examples are used for the purpose of either linking concepts or procedures to students" prior knowledge or current knowledge and use then to relate to novel situations.

Teacher 1 started with familiar tasks in his lesson plans and then gradually moved onto challenging tasks. The examples presented were connected to class exercises. Tasks across lesson units were presented in such a way that concepts or procedure learned in the preceding lesson were pre-requisites to concepts or procedures to be learned in the succeeding lesson. From the analysis, tasks within and across lessons can be described as hierarchical in that they were carefully sequenced so that tasks were building on students" current knowledge. The teacher used class exercises to consolidate the skills and concepts learned from the exampled by extending this to tasks of the same or different kind.

Teacher 2 prepared tasks in such a way that they built on students" prior knowledge. For instance, the examples in his lesson plans were always connected to previously learned concepts and procedures. The lessons started with the previous work, with further examples provided building on the current knowledge. Like Teacher 1, the sequence of tasks was carefully done, but complexity of the tasks did not vary much.

Like the other two teachers, Teacher 3 carefully sequenced the tasks. He started with familiar tasks and gradually moved onto challenging tasks. Familiar tasks created a context to prepare for challenging tasks. The teacher included a lot of problems on applications of both trigonometric ratios and sine and sine and cosine rules.

5.4.1.6 Relate to real life

Tasks relating to real life refer to the context of the task which may be the real world or imagined situation in which a mathematical task is embedded. All teachers utilized some tasks which were

related to real life. However, the number of context embedded tasks used by teachers varied from teacher to teacher. There were 11 tasks which were found in lesson plans for Teachers 1. Teacher 3 presented 9 tasks and Teacher 2 had the lowest with only 4 tasks.

5.4.2 Analysis of pupils' activity books

5.4.2.1 Variety

Findings from the analysis of pupils‟ books indicate that pupils‟ books as expected contained a variety of tasks just like in the lesson plans. I found both procedural and conceptual tasks in the pupils‟ books in all the three classes. Furthermore, tasks of different levels of difficulty were found in the books.

Students‟ work in Teacher 1‟s class indicates that students employed similar strategies on most of the procedural tasks found in their books. The sameness of strategies used among the students was clearly seen in tasks presented as examples. Although the strategies employed by students looked similar, subtle differences in solutions and solution methods between students was evident, especially in the exercises and homework. Variation in the way solutions strategies were orchestrated by students was greatest among challenging tasks. I also found that students in this class utilized different techniques and strategies in tasks of the same sort and tasks of different kind. Whereas most students often made use of diagrams to interpret tasks in order to solve them, few students utilized the formulae by drawing on their own strategies.

Interestingly, in Teacher 2‟s class there seemed to be little difference in the way students‟ work was presented. The sameness of solution methods among the students may be attributed to considerable amount of time spent on whole class discussion. Consequently, students ended up doing more examples than class exercises. It is possible that the students in this class were not given a lot of freedom to practice procedures and skills learned. This is because students‟ work in their activity books did not present a range of techniques. I observed that students work in this class relied on formulae to solve the problems. That is an indication that students may be depending on memorized procedures and formulae.

There was an assortment of tasks in students‟ books in Teacher 3‟s class. Like in the teacher‟s lesson plan, students‟ exercise books contained both procedural and conceptual tasks. There were tasks that required a range of techniques to solve them and those which needed to be represented different forms of representation. In my analysis, I found that students employed different solution strategies on tasks of the same kind. I also found that students gave a range of answers to some tasks. For instance, some students used diagrams, while others used algebraic formula to resolve a trigonometric task that was given as an exercise. The fact that student employed different solution methods also gave rise to a range of solutions that was seen in the pupils books.

5.4.2.2 Connectedness of concepts and tasks

Findings of the analysis of the pupils‟ books as regards the connections and connectedness of concepts, procedures and tasks in pupils‟ books indicate that the classes for Teacher 1 and 3 had tasks whose concepts were connected with other concepts. The tasks were carefully sequenced so that concepts were connected to the current knowledge of the student. Unlike Teacher 2 who presented a set of similar tasks within the lesson, the other teachers utilized tasks of same sort or different kind within the same lesson in order to elicit a range of techniques.

As I have pointed out earlier, tasks within the lesson in the pupils‟ books for Teacher 2 were similar and thus did not elicit a lot of concepts or strategies. The connections between concepts and connectedness of tasks in classes for Teachers 1 and 3 was high compared to Teacher 2 for the same reasons given in the teachers‟ plans.

5.4.2.3 Multiple representations

The findings of analysis of pupils‟ books in terms of multiple representations tasks are similar to the results from the analysis of lesson plans. Like in teachers‟ lesson plans, all tasks relating to real life in the pupils‟ books were multi representational. Additionally, tasks that involved use of a range of techniques to solve them were also open to different forms of representations.

In the pupils‟ books in Teacher 1‟s class, I found that in addition to tasks relating to real life tasks that involved a range of techniques to solve were open to a number of representations.

Students in this class employed different strategies to solve the problems they were given in class. I noticed that students employed different representations in the solution of problems, such as drawings and symbols.

I have already reported that Teacher 2 did not select as many tasks with multiple representations. Therefore, there were correspondingly few tasks in the pupils‟ books that could be presented in different forms. Pupils‟ work was characterized with similar strategies employed by students.

The same pattern of task as in Teacher 3‟s lesson plans was noted in pupils‟ books. Like in the teacher‟s lesson plan all tasks relating to real life involved multiple representations. The students in this class presented their solutions using many forms of representations such as scale drawing, using algebraic symbols as well as sketching of diagrams as aids.

5.4.2.4 Promoting deep thinking

The findings of the analysis of pupils‟ books with regard to tasks promoting deep thinking show that in class for Teacher 1 students presented tasks which prompted use of a range of solution methods. I found that several solution strategies were presented by students in tasks relating to real life. There were also challenging tasks where students used different methods because they procedures to be used were not explicitly stated. The tasks provoked deep thinking because there was some ambiguity in them.

The picture of tasks in pupils‟ books in Teacher 2‟s class is similar to that of lesson plans. Like the lesson plans, very few tasks were recorded to have been promoting deep thinking. For instance, there were very few tasks for which the solution methods were not readily accessible.

In the students‟ books in Teacher 3‟s class there were some tasks that required students to explain and justify the use of procedures. Students‟ responses on such tasks show that about sixty percent of the class exhibited high level thinking and reasoning in their justifications of solution methods. Also found in the pupils‟ books were tasks that could be solved using a range of solution strategies. It is evident from the tasks described here that Teacher 3 selected tasks that provoked student thinking and reasoning

5.4.2.5 Building blocks for Challenging tasks

Findings of the analysis of pupils‟ books in terms of tasks serving as building blocks for challenging tasks, I found that there was not much disparity between tasks presented in the lesson plans and those in the pupils‟ books. The tasks in the pupils‟ books were carefully sequenced such that simple concepts or procedures came first before the complex (abstract) concepts or procedures.

All three teachers planned their work in such a way that tasks were connected to prior knowledge of the student. The lessons started with simple tasks and progressed to challenging tasks.

5.4.2.6 Relate to real life

All tasks which were related to real life were enacted in the three classrooms. Teacher 1 presented all 11 tasks which were related to real life. Teacher 2 had his four tasks implemented in class. Teacher 3 implemented the 9 tasks which were related to real life as well. The tasks in the pupils‟ books did not differ from tasks as they appear in the lesson plans.

5.4.3 Analysis of lesson observations

5.4.3.1 Teacher 1

The lesson which focused on the applications of the sine and cosine rules lasted for 80 minutes. The teacher gave four examples before giving the class exercise. The first example was a revision task. The task involved solving a non right triangle, using the cosine rule. The second example was similar to the first example, but this task required the use of both cosine and sine rules in order to solve the triangle completely. The third example was a word problem which needed to be transformed into diagram and then come up with an appropriate formula to solve the problem. The fourth example involved finding the area of a rectangle in which two sides and an included angle of an inscribed triangle were given.

The teacher drew a rectangle on the board (see Figure 4.6). He asked his pupils to be in groups of four. Ten groups were formed. The teacher asked students in groups to find the area of the rectangle. After 8 minutes of group work, the teacher asked group by group to give their answer.

Group 1 started: Through its secretary it gave the answer, then group 2, then group 3 until the last group. Four different answers emerged from the ten groups. The teacher then asked the groups to defend their answers.

Teacher 1: Can we have the secretary for Groups 1 to come and show as their working on the board." (Day 5, lesson 2)

Group 1: The answer was 75.5 cm^2. The group used the cosine rule to find the length of unknown side of the inscribed triangle which is also the length of the rectangle. That is the formula $c^2 = a^2 + b^2 - 2abcosC \implies c^2 = 266.66 \therefore c = 16.33\,cm.$ Then relating the formulae for finding the area of a triangle ($A = \frac{1}{2} base \times height$) and $\left(A = \frac{1}{2} absin\theta\right) = \frac{1}{2} \times 12 \times 7sin116°$, and equating $\frac{1}{2}(16.33) \times height = \frac{1}{2} \times 12 \times 7sin116°$, he calculated the height (4.62 cm) which is equal to the breadth of the rectangle.

Each group was then given a chance to explain its solution method in defense of the solution provided.

The same working for Group 1 was repeated by Groups 2, 5, 8 and 10.

Group 3: Used the cosine rule correctly and found the length but messed up in relating the different formulae for area of the triangle.

Group 4: Managed to find the length of the rectangle, but failed to proceed from there.

Group 6: Found the answer correctly. The group used the cosine rule to find the length of the rectangle. Then they used the cosine rule to find an acute angle of the triangle in which all sides are known. That is $\cos C = \frac{a^2+b^2-c^2}{2ab}$ and then used trigonometric ratio to find the opposite side ($height = 12SinC$) which is the breadth or the rectangle.

Group 7: Used cosine rule to find the length. Like group 3, they messed up on the area of the triangle to find the height.

This exercise lasted for 22 minutes. The teacher then asked the students to choose most reasonable result for the area of the rectangle. Students responded that 75.5 cm^2 was an answer.

From the lesson observation, it was evident that the teacher used tasks of different cognitive engagement. The tasks given as examples were carefully sequenced, starting with a routine task and then gradually moved to a complex task.

The teacher also used a variety of teaching methods by employing various combinations of teacher centered and student centered approaches. Teacher centered included worked examples, explanations, demonstration and structured questioning. Student centered strategies included collaborative group work, practical tasks, problem solving, investigation by comparing, and student presentation

A range of assessments was also used by the teacher, which include verbal explanations (question and answer), written (paper and pencil) and self assessment. The students also applied various methods to solve the tasks that they were given.

In order to successfully complete the task the students were required to make connections between concepts learnt previously within and across the topic. For instance, in the above vignette the problem demanded that students relate cosine rule and trigonometric ratios and making connections between different ways of finding the area of the triangle in order to solve the problem. Students solved the problem by linking procedures and concepts to other procedures and concepts. The teacher assisted the students to make connections by asking them to compare their solution methods with others.

Students used a range of techniques to solve the problem. The different groups approached the solution of the problem differently. It is interesting to report that several solution methods came up from the group presentations. For example, Group 1 compared the two formulae for finding area of triangle in order to find the height. Whereas Group 6 employed cosine rule to find and acute angle which it then used to find the height of the triangle. However, all the Groups used cosine rule to find the length of the rectangle. This is an indication that the task was open to many interpretations.

To engage with the task, students needed to know how to find the area of a triangle using different representations on one hand, and knowledge of relationship between trigonometric ratios and cosine or sine rules on the other hand. Knowledge of area of triangle using different

mathematical representations is a necessary building block to solving this task. The students used a combination of strategies and techniques which they had acquired previously.

The task challenged the students to think and reason about the mathematical ideas embedded in the problem. The problem did provoke deep thinking in students in that there was some ambiguity in it. This means that the solution method was not readily available to the students. In addition, students needed to have good conceptual understanding to be able to see the relationships among different concepts and procedures. This is exemplified from the different ways students approached the solution and the relationships student had to make to navigate through the solution method.

The tasks students engaged with during the lesson were set in mathematics context. In particular example number four employed aspects from other topics. Although the task did not relate to real life, the context in which the task was embedded was familiar to the students. The task required that students draw concepts from different topics, such as mensuration and trigonometry.

5.4.3.2 Teacher 2

The lesson observation was conducted on the fourth day. The lesson lasted 40 minutes. The lesson for the day was focused on the sine rule. This was the first lesson under this subtopic. The teacher drew a non right triangle on the board and labeled it ABC. The sides corresponding to the angles A, B and C were labeled a, b and c respectively.

Having drawn the triangle the teacher went on to state the sine rule as "the ratio of sine angle A to the side a is equal to the ratio of sine of angle B to side b and it is also equal to the ratio of sine of angle C to side c." Stated algebraically the sine rule is $\frac{sinA}{a} = \frac{\sin B}{b} = \frac{sinC}{c}$. He writes it on the board. The teacher stated the formula without explaining where it was coming from. He then gave two conditions which ought to be satisfied in order to use the sine rule.

Teacher 2: The sine rule is used when (i) two sides and an angle opposite the any of the two sides are known. This is a condition referred to as angle, side and side (ASS),
(ii) two angles and a side opposite any of the angles are known. The condition referred to as angle, angle and a side (AAS).

The teacher gave the first example. The example involved finding unknown side given two sides and an angle opposite a known side. The teacher worked through the problem showing all the steps involved in finding the angle. He then went on to find the remaining angle and a side. The teacher showed every step that was involved very clearly. He then asked his pupils to copy the example in their exercise books.

Example number two satisfied the second condition in which two angles and an opposite side were given. The teacher asked his pupils to identify the conditions satisfied with the information provided on the triangle. One pupil said the second condition had been met because two angles and a side opposite the angle (AAS) were given. The teacher stated the formula and then asked the pupils for values to substitute in. He then simplified and solved for the unknown side.

In the third example, the teacher wrote the question on the board. The tasks involved solving the triangle completely. The teacher invited one of the students to go and work out the problem on the board. The student hesitated for a while, but with encouragement from the teacher he made his way to the chalk board. The student was assisted by his colleagues through all stages of calculation.

Example number four, was a whole discussion. The teacher wrote the question on the board. The question in presented below

Example 4: A man starts off from town A to town B 15 km apart and then connects to town C which is 20 km from B. Given that C is on the bearing of 060^0 from A.

By drawing a sketch, calculate

(i) The distance from A to C

(ii) The bearing of C from B

Teacher read out the question and posed a question.

> Teacher 2: Can we have someone to come and present this information in form of a diagram? (Day 4, lesson 1)
>
> The class remained quiet for some time, and then suddenly a hand went up.

Teacher 2: Yes Tina (Not a true name) (Day 4, lesson 1)

Tina: Moved to the board and sketched a triangle

Teacher 2: is that right? (Day 4, lesson1)

Chorus answer: No

Teacher 2: What is wrong with Tina‟s diagram? (Day 4, lesson 1)

Peter: The angle, I mean the bearing.

Teacher 2: Okay, Peter come on and show Tina where she has gone wrong. (Day 4, lesson 1)

Peter: Drew the diagram correctly

Teacher 2: Which rule are we going to use? Is it cosine or sine rule?

Sam: I think it is sine rule

Teacher 2: Fine, we are going to use sine rule. Can someone state the formula for sine rule? (Day 4, lesson 1)

Jean: Sine rule is equal to $\frac{a}{SinA} = \frac{b}{sinB}$

Teacher 2: Took over and lead the students into the solution of part 1 of the problem.

Teacher 2: Who can do the second part? (Day 4, lesson 1)

Peter: went to the board and worked out the bearing.

Teacher 1: Good (Day 4, lesson 1)

The teacher instructed the students to copy the worked example. Since there was not so much time left before the end of the period the teacher gave the class two questions to do in class and three questions as homework.

From the foregoing classroom discussion, it is evident that the teacher did not give his students enough time to think and reason about the problem. The questions utilised by the teacher were more or less directing student thinking in a particular way and not thought provoking. For example, questions such as “which rule are we going to use?”

There was not much variety of tasks used in the classroom. As can be seen from the lesson observation, the teacher gave a set of similar examples, with only a difference in complexity which gradually increased from first example to the fourth example. The tasks were carefully

sequenced in such a way that the first two examples involved single procedure while the other two examples involved several procedures and techniques. Although there was some variety in teaching methods, that is teacher centred and student centred methods, the teacher seemed to have dominated the class discussion.

The tasks had connections with other tasks within the lesson. This is evident from the way tasks were presented. The teacher started with the simple tasks which involved a single procedure and gradually moved to tasks that involved several techniques to solve. For the tasks which had multiple questions the concepts or procedures were linked to one another. The fourth example had some connections to other topics. For instance, part two of the example required finding the bearing of town C to town B, which falls under a different topic.

The teacher did not exploit the power of multiple representations to enhance understanding. Apart from inviting students to sketch the triangles, there was no moment during the lesson that indicated that students were making their own interpretations. The lesson had a lot of areas where multiple representations could have played a vital role in exploring alternative methods. However, the teacher directed learning by imposing his strategies and techniques on the students.

As I have already pointed out the tasks were carefully sequenced starting with tasks which acted as building blocks to other tasks. The first two examples were the building blocks for the fourth example, in the sense that the student needed to understand sine rule before they can engage in deciding the appropriate formula to use in a new situation.

There was some evidence of task relating to real life. In example four, the task was context embedded, and required students to come up with the locations of the three towns. However, during the lesson presentation, the emphasis to make the situation appear real was eluded. The task was reduced to a routine task with emphasis placed on correct answers by carrying out procedures and algorithms accurately and appropriately.

5.4.3.3 Teacher 3

The lesson was 80 minutes long; it was the third lesson in the series. The lesson was on the applications of trigonometric ratio. The teacher gave three examples, of which the first example involved finding the height of the building, given the angle of elevation and the distance the man in car from the building. This task was similar to the task that was given as part of homework in the previous lesson. The teacher invited his students to attempt the question on the board.

Teacher 3: Shall we have someone to come and solve this question?

The class was quiet for a few minutes, then one student raised up his hand.

Teacher 3: Bwalya come forward. (Day 2, lesson 3)

Bwalya: Makes his way to the chalkboard. He goes through the question and the sketches the building, the car at a distance from the building and joins the top of the building and the car with a straight line.

Teacher 3: Asks the class. Is Bwalya"s drawing correct? (Day 2, lesson 3)

Student: Chorus answer. Yes

Teacher 3: Yes Bwalya, go ahead. (Day 2, lesson 3)

Bwalya: Now we draw a right angle triangle, so that we can find the height. The reference angle is 60^{o} (angle of elevation), the opposite is the height of the building and the adjacent is 20 metres. He states the tangent ratio and substitutes the values to find the height.

Teacher 3: Is that right class?

Students: Yes

The second example involved finding the angle ACB; the bearing of town A from town C and the bearing of A from B. Given that A, B and C are three towns such that B is due east of A, C is 50 km from A on a bearing of 135^{o} and that C is equidistant from A and B.

The class was asked to work in pairs on their desks. A few minutes later, the teacher asked for volunteers to go and work out the problem for the class on the board.

The first volunteer presented her solution. She made a sketch drawing of the locations of the towns and showed that the triangle ABC was right angled in which angles ABC = BAC = 45^o. She then found the bearings. The second and the third volunteers did pretty the same thing.

The teacher asked whether there was anyone with a different answer. The class was quiet.
Teacher 3: Fine there seems to be no problem, and went on to the third example.

The third example involved finding the height of the building. The teacher wrote the question on the board. "A and B are two points on the horizontal ground 80 m apart, the angles of depression from A and B of the top of the building are 12^o and 20^o respectively. Find the height of the building."
The teacher asked the class to be in groups of five. He asked each group to choose a secretary to record the group discussions. The teacher gave the class 10 minutes to discuss the problem in groups. After 10 minutes, each group was asked to present its solution on the board.

Group 3: The secretary for the third group started, he asked for a rule and protractor. He drew a horizontal line and labeled one end A. He measured 4 cm (to represent 80 m) from A and marked it B. Using a protractor he measured angles 12^0 and 20^0 at A and B respectively. He then drew straight lines one from A through 12^o mark another from B through 20^o mark. The intersection of the two lines was the top of the building. He measured the length of the slant height and used his scale to find the actual length which happened to be 196 m. To find the height of the building he used the sine ratio. That is $h = 196sin12° = 40.75\ m.$

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Group 6: The secretary sketched the triangle and labels it ACD, with B lying on AC. She labeled the distance AB = 80 m and BC = x m. She used algebra to relate the height. That is $h = (80 + x)tan12°$ and $h = xtan20°$. She equated the two expressions and solved for $x \approx 112m.$ Finally, she used the value of x to find the height of the building
$h = 112tan20° = 40.76m.$

.

Group 8: approached the solution in the same way as group six. The difference was that instead of using $h = xtan20°$ to find the final answer, they used $h = (80 + 112)tan12° = 40.76m$. (Day 2, lesson 3)

The other groups did not present because the teacher just sampled three groups. After going through the three solutions together with the students, the teacher gave an exercise. Three tasks formed the class exercise that students were asked to do in class and two other tasks were given toward the end of the lesson to be done by students at home. The teacher during this time was going round the class correcting and helping those who had problems understanding the concepts.

From the classroom vignette it is evident that the teacher used a mix of procedural and conceptual tasks. The tasks also varied in the level of complexity. The three tasks presented by the teacher as examples required different concepts and skills in order to complete them.
The teacher employed a variety of methods, ranging from teacher demonstrating to pupils through student working individually or collaborating in pairs to working in small groups. Different solution methods were also noted during whole class discussion.

Although the teacher used a whole range of different tasks as examples the concepts and skills being developed and practiced were the similar. In short, there were connections between concepts and procedures being learned. The concepts in the third example had connections with other concepts within and across the topic. The students needed to use either the algebraic relationship or construction method in order to find the height of the building.
In regard with multiple representations, all three examples including tasks given as class exercises and homework were open to many representations. The tasks could be solved using a range of techniques and methods. The students also presented their solution strategies in many different ways.
From the varieties of strategies used to solve these tasks, it could be argued that the tasks were open to many interpretations. Tasks which have some ambiguity in them are open to many interpretations and this kind of task promotes deep thinking. Example 3, for instance, required students to realize that in order to find the height of the building, the relationship between the

two sides are associated to the height. The task was cognitively demanding on the part of the students.

As can be observed from the lesson, the teacher orchestrated his lesson in such a way that tasks were arranged according to the levels of difficulty. The concepts in these tasks were building on students‟ current knowledge. All eight problems implemented on this day were tasks relating to real life. The contexts of the tasks were also familiar to the class.

The summary description of the tasks in terms of criteria used and sample tasks are presented in the table below.

Table 5.2 Description of tasks in terms of the criteria used and sample of selected tasks

Criteria used	Description of task	Sample tasks
1. Variety of tasks and concepts	• Mix of task: - Procedural tasks and conceptual tasks - Routine and non routine • Relate to real life or context embedded • Sequencing of tasks, progress from known to unknown • Multiple representations • Hierarchical tasks: from simple to complex tasks	T-1.2; 1.3; ….,1.6: Resolving right angled triangle given reference angle and a side or when given two sides only. (Teacher 2, Day 1) T- 2.3: A man is standing 70 m from the foot of the trees. He spots a bird on top of the tree at the angle of elevation of 55^o. Calculate the height of the tree. (Teacher 1, Day 2) T-5.4: Find the the shortest distance given three points A, B and C where AB = 12 cm, AC = 32 cm and an included angle of 28^o (Teacher 3, Day 5)
2. Connectedness of concepts or tasks	• Linking existing knowledge to new knowledge • Applying learnt concepts to other situations • Connect task of one unit to the other • relating concepts between different tasks • relating to real life	T- 2.7: The angles of depression of two spots on the ground from a helicopter are 20^o and 44^o respectively. Given that the vertical distance of the helicopter above the ground is 3 km, calculate the distance between two spots. (Teacher 1, Day 2) T- 3.6: A square pyramid has a square base PQRS of 8 cm. The diagonals intersect at N, the vertex T is vertically above N and its length is 10 cm. L is the mid-point of PQ. Calculate a) length TL b) $\angle QTN$ (Teacher 2, Day 3).
3. Tasks with multiple representations	• Task which lend to many forms of representation • Elicit several solution pathways • A range of techniques required in	T-5.5: Calculate the length and breadth of a rectangle ABCD where, AE = 12 cm, BE = 7 cm and $\angle AEB = 116°$. (Teacher 1, Day 5)

	solving the task • Require justifications for use of procedure or formula	T-5.5: P, Q and R are fishing camps along the banks of the lake Kariba joined by straight paths PQ, QR and RP. P is 7.6 km from Q and Q is 13.2 km from R and $\angle PQR = 120°$. Calculate (i) the distance PR (ii) the area of triangle PQR (iii) the shortest distance from Q to PR (Teacher 3, Day 5)
4. Relate to real life and modeling	• Context embedded tasks • Usually have no prescribed solution routed • They are open to many forms of representation and interpretations • Require transformation into diagrams, symbols and modeling	T-3.5: Farai walks 5 km from A to B on a bearing of 035°. She then walks 6 km from B to C on bearing of 125°. Calculate the a) the distance and b) the bearing of C from An (Teacher 2, Day 3) T-5.6: An aeroplane leaves an airport and flies due north for $1\frac{1}{2}$ hours a 500 km/h. It then flies 400 km on a bearing of 053°. Calculate the distance and bearing of the plane from the airport.
5. Promote deep thinking and reasoning	• Challenging; not explicitly stated with some ambiguity in them. • Elicit a range of techniques • Tasks open to multiple representation • Tasks that relate to real-life	T-3.5: A and B are two points on the horizontal ground 80 m apart. The angles of depression of A and B from top of the building are 12° and 20° respectively. Find the height of the building. (Teacher 3, Day 3) T-3.3: A rectangular box 7 cm long, 6 cm wide and 6 cm high. A rod 12 cm long rests with its lower end in one bottom corner and is supported by the opposite top corner. Calculate (i) The inclination of the rod to the horizontal (ii) The length of the rod from top corner beyond the box.
6. Building blocks for other tasks in future	• Application of learned concepts and skills to new situations • Tasks that building on students prior knowledge • Tasks of appropriate level of difficult, to encourage pupils to work toward the challenge	T-5.2: solve the non right triangle, given the two sides and the angle opposite a side. (Teacher 2, Day 4) T-4.1 Finding a side of the triangle given a side and two angles in a non right triangle. T-1.1; 1.2; ….; 1.8: Resolving right angled triangles. (Teacher 2 Day 1)

5.5 Comparing teaching practice

Analysis of classroom practice of the teachers in the previous section indicates that all three teachers implemented a variety of tasks. The tasks were carefully sequenced. All the teachers started from the known and gradually moved to the unknown. Teacher 1 gave four tasks as examples in class which were of varying complexity. Teacher 3 gave three examples, in which the tasks presented were of different kind, but the concepts and skills developed were closely connected. Teacher 2 also gave four examples just like Teacher 1. Although Teacher 2 gave the same number of examples as Teacher 1, the duration of his mathematics lesson was 40 minutes which was far much shorter than that of Teacher 1 who had 80 minutes. Teacher 2 therefore compressed a lot of work in the short space of time.

The time spent on actual teaching varied from class to class. Whereas Teacher 2 spent a lot of his time (25 minutes) demonstrating to the students on the board, the other teachers divided time in such a way that few minutes were spent on teacher demonstrations and the remaining time on group work and class discussion. For instance, Teacher 1 spent 8 minutes on demonstration, while Teacher 3 spent 10 minutes. Teachers 1 and 3 gave their students more time, 25 and 30 minutes respectively, to practice the skills and the concepts learned in class.

All three teachers employed different teaching strategies, ranging from teacher centered, which included teacher demonstration, explanations and structured question to student centered strategies which included group work, practical tasks, investigations and student presentations. Although student centered strategy was noticed in all three classes, the level of student interactions varied from class to class. Teacher 3‟s class recorded the highest level of student interaction followed by Teacher 1. In Teacher 3‟s class, the students were given freedom to interact amongst themselves and to share their experiences and the teacher facilitated this interaction by engaging students in discussions. The atmosphere in Teacher 1‟s class as regards student-teacher and student-student interaction was similar to that of Teacher 1. On the contrary, Teacher 2 did not give his students enough time to interact with one another as can be observed in his lesson vignette.

Both Teacher 1 and Teacher 3 implemented challenging tasks in comparison with the tasks implemented by Teacher 2. Both teachers employed at least two tasks that were open to many interpretations. The teachers also encouraged their students to present their own solution methods. In Teacher 1 and Teacher 3‟s lessons, a variety of solution methods was observed in both students‟ work and teachers‟ presentations.

The tasks utilized by all three teachers were familiar to the contexts of the students. The development of mathematical ideas in all the three lessons progressively moved from simple to more difficult ones. Although tasks were presented from simple to difficult ones in all cases, each teacher emphasized more on what they felt was important to their students. For example, Teacher 1 stressed more on developing conceptual ideas as can be observed in example 4. Teacher 2‟s emphasis was on student acquisition of procedural fluency, by emphasizing getting right answers. Teacher 3 tried to strike a balance between student understanding of concepts and the use of procedures accurately and appropriately.

5.6 Conclusion

Results of further analysis indicate that all three teachers selected and implemented procedural and conceptual tasks. The tasks were selected for the purposes of practicing routines algorithms or practicing newly taught skills or procedures and promoting thinking and reasoning. Although all teachers selected and implemented procedural and conceptual tasks, differences in cognitive demands and conceptual knowledge required working on the tasks were clearly seem from one teacher to the other. Results show that Teacher 1 selected and implemented challenging tasks, while Teacher 2 predominantly implemented procedural tasks. Teachers 1 and 3 made use of tasks that were context embedded. For instance, one quarter of the tasks selected by Teacher 1 were contextually embedded.

The results further indicate that Teachers 1 and 3 engaged their students more in solving problems that promoted both procedural and conceptual understanding than did Teacher 2. Classroom vignettes also illuminate how the three teachers utilized their time and the varieties of tasks and strategies they employed. Teacher 1 used much of his time discussing with his

students, while Teacher 2 dominated class discussion. Teacher 3 divided his time equally between demonstration and group work.

In the following chapter a discussion of how the findings inform mathematics teachers‟ tasks practice based on the criteria used and decisions made in selection and implementation of tasks is presented. The discussion draws from literatures on good tasks practice to justify the findings.

CHAPTER SIX: DISCUSSION AND CONCLUSION

6.1 Introduction

The study sought to investigate good task practice through selection and implementation of tasks. This included exploring the sources and type of tasks selected by teachers, the criteria used and decisions made by teachers before, during and after classroom instruction. The research provided sufficient data to show teachers‟ classroom instructional practice and related the findings of the analysis to relevant literature on good tasks practice.

6.2 Type of tasks and their sources

The study shows that teachers 1 and 3 selected trigonometric tasks from the prescribed textbooks (ZSSSM 11 and 12) and New General Mathematics Book. Teacher 1, however, supplemented the two textbooks with „Thinking process‟ the textbook containing Cambridge past examination questions and solutions. Careful scrutiny of the tasks revealed that teachers selected most if not all the tasks from the textbooks. There was little evidence suggesting that teachers adapted or modified tasks chosen from these books. Two tasks were found though to have been adapted from the ZSSSM 11 by Teacher 1. It is evident from the above findings that teachers relied heavily on textbooks for selection of tasks. This is in line with other research findings that teachers use textbooks heavily for their selection of tasks (Freeman & Porter, 1989; Pepin & Haggarty, 2001) and that “most teachers decide what to teach, how to teach it, and what sorts of exercises to assign to their students largely on the basis of what is contained in the textbooks authorized for their course (Robitaille & Travers, 1992, p. 706).

The analyses of tasks show that teachers selected slightly more procedures with connections tasks than procedures without connections. While Teacher 1 selected 61% procedures with connections tasks, Teachers 2 and 3 selected 59% and 58% procedures with connections tasks respectively. I have already pointed out that not all tasks selected were implemented in class. One teacher felt that time was not adequate to go through all the tasks he had planned for. He added that if he was to adhere to the planned tasks in all his lessons he would not cover many topics and that would affect students‟ performance in examinations.

From the analysis it is evident that teachers selected cognitively demanding tasks, but when it came to implementation the tasks declined to lower levels of cognitive demand. The factors that inhibited the maintenance of high level tasks included: teachers over simplified the tasks for students by giving too many examples, teachers were impatient to hold back whenever students pressed for clarification In many cases teacher took over the thinking and reasoning meant for students. For instance, Teacher 2 did not give his students enough time to struggle with mathematical tasks. The teacher did not press for explanations and justifications from the students. This coincides with some of the factors that Stein, Grover, and Henningsen (1996) associated with the decline of high-level cognitive demands.

6.3 Task practice

The study indicates that all three teachers used a variety of tasks in terms of carrying out procedures and concepts embedded in them. Although all teachers used a variety of tasks, the levels of engagement required to do the tasks varied from teacher to teacher. I have shown in my analysis that Teacher 2 presented similar tasks within the lesson by giving related examples followed by an exercise to practice the procedures and skills taught. While the other teachers presented tasks of different kind within and across the lessons, some of which elicited different concepts and procedures. This suggests that teachers 1 and 3 provided students with the opportunity to engage with different tasks and a range of concepts and procedures.

The analysis of trigonometry tasks indicates that teachers selected and used tasks for different purposes. Some tasks were chosen for the purpose of practicing procedures and skills. Other tasks were selected and used to develop higher thinking and reasoning and to promote conceptual understanding. The tasks that provoke students to make connections between concepts and procedures; required identifying the characteristics of concepts, recognizing the similarities and differences among concepts according to these characteristics, and constructing relations among them promote conceptual understanding. The question is what should be the right proportion of tasks that will promote the development of procedural fluency and conceptual development? While there might not be a specific proportion that is appropriate and while it might be

contextual, “a classroom where all the tasks that the learners do are low level tasks is not likely to support both conceptual and procedural learning” (Saani, 2009, p.260).

This idea is supported by the statement that “the degree of students‟ conceptual understanding is related to the richness and extent of the connections they make” (Hiebert & Carpenter, 1992, p.69).

The study showed that teachers carefully sequenced tasks in such a way that concepts and mathematical ideas were connected to students‟ prior knowledge. The study also indicates that teachers prepared their tasks in such a way that they started with what students had an idea about in order to link to new situation. Furthermore, the teachers chose tasks which were building blocks for challenging tasks. This is supported by research findings suggesting that Students learn new mathematical concepts and procedures by building on what they already know (Kilpatrick, et al., 2001; NCTM, 1991; Hiebert & Carpenter, 1992). Another study indicated that the degree of students‟ conceptual understanding is related to the richness and extent of the connections they make (Hiebert & Carpenter, 1992). I argue that conceptual knowledge is effectively developed in a learner when past experience is relevant to new situations, and when tasks provide an appropriate challenge.

In line with task features and levels of cognitive demands of the tasks described by Stein and her colleagues, as the kind of thinking that involved as the tasks passes through the three phases of instruction (Stein et al., 1996). Task features refer to aspects of whether the task can be solved through a variety of strategies, through the use of multiple representations and whether the task provides opportunities for mathematical communication, explanations, and justifications. The levels of cognitive demand refer to the types of thinking involved in solving the task. The study has shown that teachers selected and implemented more procedures with connections tasks than low level tasks. It is also evident from the lesson observations that some tasks enacted in class had features described above. However, none of the teachers except Teacher 3 had utilized tasks that pressed for explanations and justifications, and he used only two. It is also interesting to note that at implementation some high level tasks declined to procedures without connections. Although that was the case, there were immense opportunities that teachers could have exploited

to maintain tasks at high level cognitive demand, such as pressing students for explanations and justifications, allowing them enough time to think and reason about the tasks, and scaffolding instead of demonstrating and telling (Kilpatrick et al., 2001). Furthermore, the analysis of documents and lesson observations have shown that teacher selected tasks with features such as having multiple representations, and that can be solved using a variety of techniques.

Kilpatrick at el. (2001) write that "a significant indicator of conceptual understanding is being able to represent mathematical problems in different ways and knowing how different representations can be useful for different purpose" (p. 123). Research also suggests that using multiple representations in both teaching and learning support the development of mathematical understanding. There is evidence from the study that teachers presented tasks that could be presented in many different ways. For instance, example 3 presented by Teacher 3 during lesson observation, involved a range of techniques and was capable of being represented in different forms. Both Teacher 1 and Teacher 3 utilized tasks involving several techniques on the same or different task. The lesson observation also revealed that Teacher 1 allowed the students to explore different ways of solving a problem, by connecting different concepts and mathematical ideas.

6.4 Process of mathematical task selection

Having discussed teachers‟ task practice, I now turn to the process through which task choices were made. I have earlier indicated that not all tasks selected by the teachers were implemented in class and that ninety percent of the tasks came from the prescribed textbooks. Interviews with teachers and document analysis indicate that teachers‟ selection process was largely inspired by objectives from the textbooks on one hand and their mathematical knowledge on the other hand. All three teachers followed the syllabus content in the prescribed textbooks and made little or no modifications to the tasks they selected.

Teachers‟ tended to select tasks that they were more comfortable with. This claim is exemplified by the large number of procedural tasks. There was no doing mathematics tasks and yet there were tasks involving problem solving in those textbooks. There was also little evidence

concerning the recasting of tasks in situations where students did not do well. The selection process was linear and hierarchical in that teachers strictly followed the syllabus.

6.5 Teachers' practice

Analysis of both teachers‟ task practice and classroom practice indicate that the tasks that the teacher presented and eventually enacted in their classrooms were closely connected to their lesson objectives. The teachers utilized the tasks differently. From the three lesson vignettes, it is reasonable to argue that Teacher 1 and Teacher 3 were trying to develop conceptual understanding in their students by engaging them in solving tasks that involved the use of several strategies by drawing on and relating concepts and procedures from different topics.

From the analysis of lesson observation, it is evident that the time each teacher spent on enactment of tasks considerably differed from one teacher to the other. Teacher 2 spent 25 minutes giving examples. There is no incident where students were allowed to discuss and challenge one another. The other teachers spent few minutes, between 8 and 15 minutes, of their lesson times on demonstrating to their students. Both Teacher 1 and 3 allowed their students to communicate their solution methods and to challenge each others strategies. Furthermore, Teacher 3 guided his students by probing their thinking and reasoning.

The analysis also indicates that teachers employed different teaching strategies, ranging from teacher centered to student centered strategies. However, the degree of student centeredness of these lessons strategy varied from class to class. The analysis indicated that Teacher 3‟s class had the highest level of student interactions. The students in this class shared their mathematical experiences freely. Similarly, student-teacher and student-student interaction in Teacher 1‟s class was warm and cordial.

Analysis of both teaching practice and tasks practice suggest that Teachers 1 and 3 implemented challenging tasks. Besides presenting challenging tasks, the teachers encouraged their students to present their own solution methods. Additionally, analysis indicates that there was variety of both solution methods and presentations of solutions.

The analysis further suggests that the tasks employed by the teachers were familiar to students, in the sense that the language and level of difficulty of tasks were appropriate. Teachers placed emphasis on what they perceived as important aspects of mathematics. Teacher 1 placed more emphasis on developing conceptual understanding. While Teacher 2 emphasized more on algorithm and the process of getting correct answers, Teacher 3 orchestrated tasks that promoted both procedural fluency and conceptual understanding.

6.6 Conclusion

This study reported here indicates that teachers selected a variety of tasks with different levels of cognitive demands. Teachers also implemented the tasks using a range of techniques and in a variety of ways. I argue that these varieties of tasks in terms of task features, levels cognitive demands, and the different ways of representing them help in developing student‟s mathematical proficiency. This is supported by Stein et al (1996 that the types of mathematical tasks to which students are exposed influence the kinds of thinking processes in which they engage, their level of engagement, and therefore the learning outcome achieved. They emphasize that tasks that are more likely to promote deep thinking and reasoning are meaningful and worthwhile tasks. They further contend that such tasks are characterized by features such as having more than one solution strategy; being capable of being represented in multiple ways; demanding that students communicate and justify their processes and understandings in written and/or oral form.

I have shown in chapter four that different tasks are used for different purposes and that tasks offer different learning opportunities to different students. Therefore, a task that might be high-level for one group might be routine for another group and impossible yet to another group. Therefore, by providing a mix of tasks, for instance, procedural and non procedural tasks at appropriate levels of challenge, diversity is catered for. In other words mathematical tasks should cater for a range of students in terms of previous mathematical achievement and interest, and different ways of thinking and, learning and working mathematically (Freedman, Delp, and Crawford, 2005). This means that tasks must have multiple entry points and multiple solution pathways. In addition, although low-level tasks many provide an opportunity for students to develop procedural fluency, this is only one strand of mathematical proficiency that students

must develop, the other strands are conceptual understanding, strategic competence, adaptive reasoning and productive disposition (Kilpatrick, et al., 2001).

The study also showed that teachers selected high level tasks but when it came to implementation 60% of procedures with connections tasks declined to procedures without connections. The teachers did much of the work for the students, they did not press students to critically think about trigonometric tasks, by not asking them to explain and justify their solution methods. Furthermore, teachers did not give students time to challenge one another‟s thinking.

It is evident from the study that teachers relied on the prescribed textbooks for their selection of tasks. While the role of the teacher is to select, revise and develop tasks that are likely to foster the development of understanding and mastery of procedures that also promote the development of proficiency in mathematics, the teachers hardly modified the tasks from the textbooks and did not go outside of the prescribed books. The challenge that the teachers seem to have is to select high level tasks such as doing mathematics, and to maintain high level cognitive demands.

The results of this study cannot be generalized because of the small sample involved. Therefore, the results reflect the views and task practices of target group. A possibility for future study would be to consider a large population, drawn across the nation. The study should be conducted over a reasonable period of time, say three years, so that teachers‟ task practice can be generalized.

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APPENDIX A

INTERVIEW QUESTIONS: PHASE 1

1. For how long have you been teaching trigonometry?
2. Mention some of the challenges you have had teaching this topic.
3. Could you comment on the challenges that teachers (in general) face teaching trigonometry?
4. What are the sources of these challenges?
5. How do you use textbooks in your teaching?
6. Do you have specific books/materials that adequately cover the topic of trigonometry?
7. Mention three textbooks that you have predominantly used for teaching trigonometry
8. What reasons can you give for choosing these books
9. What type of activities/tasks do you find interesting in these books?
10. When planning your lessons, how do you choose activities/tasks to be included in the lesson?
11. What significant contributions do tasks have on the success of your lessons?

    Eg.a) What type of trigonometry tasks do you think work best?

    b) What trigonometry tasks do you think will enhance student learning?

    c) How do these contributions influence your choice of tasks from the textbooks?

APPENDIX B

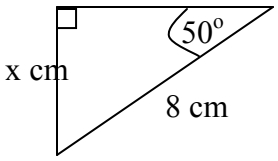
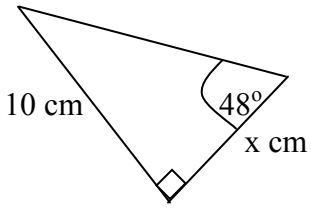
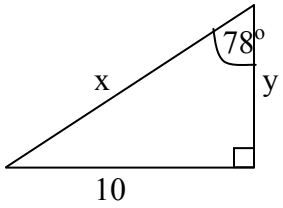
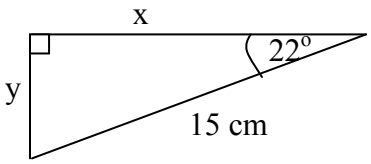
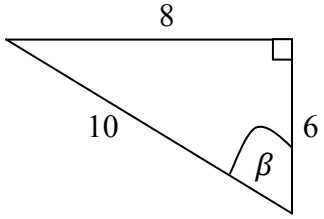
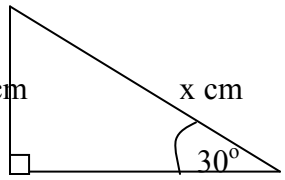
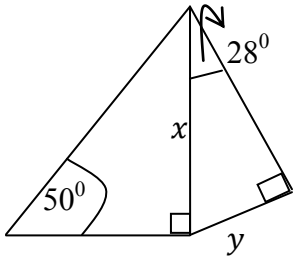
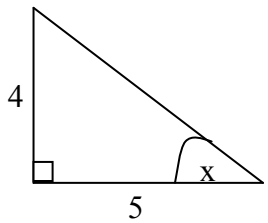
INTERVIEW QUESTIONS: PHASE 2

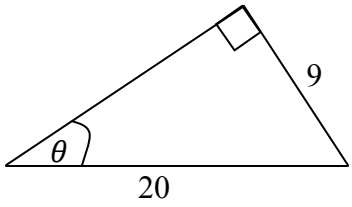
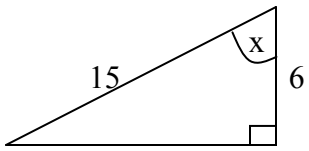
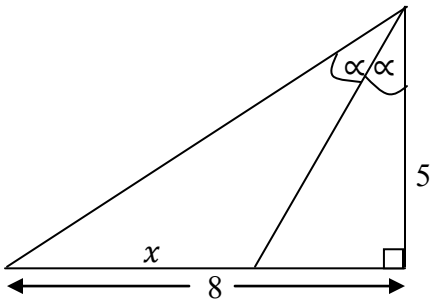
1. In a nutshell, could you talk about the tasks that you selected and implemented in the classroom?
2. What criteria guided the selection of the tasks you used in class?
3. Which tasks worked well for the purpose they were indented?
4. Did you have a good task for instance that did not work well as expected?
5. What learning did you expect to arise from this task? How? (Probe understanding)
6. What learning actually arose from the task? How? (Probe process and understanding)
   Here are the Frameworks
7. Do you think they relate to your task selection choices?
8. Could you please comment on the frameworks?
9. Are there other places you could have used them?

APPENDIX C

Teachers use tasks for three or more purposes in class.
Type 1: tasks teacher uses to teach student new concepts, definitions, & terminologies or review previously learned concepts and skills to facilitate learning of new concepts and skills referred to as **examples**
Type 2: tasks that the teacher gives to students to be completed during the lesson and students work on them either independently or in groups and the teacher is able to monitor them, called **class exercises**
Type 3: extra work, other than an exercise, given to students to be completed outside the lesson time, either at school or at home, known as **homework**.
Some tasks have features of more than one type.

Study the table very carefully and answer the questions that follow.

A. Tasks selected and implemented in class	**B. Tasks selected but not implemented in class**	**C. Tasks not selected**
1. calculate the value of x a)  b)  2. Find x and y  	1. calculate the value of x and y  2. calculate to the nearest 0.1^0, the size of the labelled angle the fig. below:  3. A helicopter flies vertically above a canoe on Lake Kariba. The canoe is 1 km from the shore. The angle of elevation of the helicopter from the shore is 36^o. Calculate the vertical distance from the canoe	1. calculate the value of x and y a)  b)  2. Calculate the size of the angle labelled x a) 

3. Calculate to the nearest 0.1^o, the size of θ  4. A wall is h metres high. A ladder, leaning from the top of the wall to the ground is 15 metres long. The ladder makes an angle of elevation of 54^o with the horizontal ground. Find: c) The distance of the foot of the bottom of the wall. d) The value of h.		b) 
5. A town Y is 200 km from town X in a direction 040^o. How far is Y from X? 6. In the fig. below a) Write down the value of tan2∝. Hence calculate b) ∝ c) x  		3. A ship sails 16 km due West and then 26 km due North. Find is present bearing from its starting point 4. The angle of depression of two spots on the level ground from a helicopter above an above are 42^o and 28^o respectively. Calculate the distance between the two spots if the helicopter is 2 km above the ground.

1. Indicate in the table below the tasks that best describe the way you would choose the tasks when teaching mathematics and why? For example: column B task 1 can be indicated as B1, meaning Type B task 1, then you give the reason why you think the task is suitable say suitable as an example, exercise etc. State at least two reasons for each choice

Type 1 tasks	**Reason (s)**
e.g B1 and C1 etc.	- as an example, this tasks have basic ideas about trigonometric ratios - they are ease for students to ..... - the tasks are .....
Type 2 tasks	**Reasons**
Type 3 tasks	**Reasons**

2. What are the major considerations for your choice? Give at least three reasons
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3. What criteria guide the selection of tasks for use in class? Give at least three reasons
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4. There are times when you do not implement some of the tasks in class that you had chosen for one or the other. Why and how do you arrive at such decisions? Explain in detail the reasons for your actions.
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5. What other decisions guide the selection and sequencing of task the way you do?
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6. What advice would you give to student teachers with regard to choosing and sequencing of classroom tasks?
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