Adaptation of the Mathematics Recovery programme to facilitate progression in the early arithmetic strategies of Grade 2 learners in Zambia

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Declaration of original authorship

The work contained in this thesis has not been previously submitted to meet requirements for an award at this or any other higher education institution. To the best of my knowledge and belief, the thesis contains no material previously published or written by another person except where due reference is made.

………………………… 16th February 2016 …………………………

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Abstract

Research indicates that many children finish primary school in Southern Africa still reliant on inefficient counting strategies. This study extends the research of the South African Numeracy Chair project to early mathematics intervention with Grade 2 learners. It investigated the possible adaptation of the Mathematics Recovery programme to facilitate learner progression in early arithmetic strategies.

This study aimed to investigate the possibility of adapting the Mathematics Recovery programme for use in a whole class setting, and to research the effectiveness of such an adapted programme. This study also aimed to investigate the extent of the phenomenon of unit counting and other early arithmetic strategies used in the early years in Zambia.

This study was conducted from an emergent perspective. A review of the literature indicated that children who become stuck using unit counting face later mathematical difficulties, and that teacher over-emphasis on unit counting in the early years of schooling may be a contributing factor.

This study used a qualitative design research methodology that consisted of a preparation phase, teaching experiment and retrospective analysis. The context of this teaching experiment was a seven week after-school intervention with a class of Grade 2 learners aged seven to eight in a rural Zambian primary school. Data collection and analysis focused on video recordings of a sample of 6 learners. The experimental teaching content focused on the Early Arithmetic Strategies aspect of the Mathematics Recovery programme.

Although limited by time and research focus, this study found that all learners made some progress in early arithmetic strategies, and indicates that the Mathematics Recovery programme has potential for adaptation for early intervention in whole class teaching to address the mathematical education challenges in Zambia and beyond. This study also found that unit counting predominated in the sample learners, but that strategies were not yet entrenched, indicating this was a suitable age for early intervention.

This study makes methodological contributions to a growing body of research into the adaptation of the Mathematics Recovery in Southern African contexts and suggests avenues for possible further research.

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List of Acronyms

BNWS	Backward Number Word Sequence
CMIT	Count Me in Too
DBRC	Design Based Research Council
EAS	Early Arithmetic Strategies
FNWS	Forward Number Word Sequence
GRZ	Government of the Republic of Zambia
IFEN	Instructional Framework for Early Number
LFIN	Learning Framework in Number
MR	Mathematics Recovery
SACMEQ	Southern and Eastern Africa Consortium for Monitoring Educational Quality
SANC	South African Numeracy Chair
TIMSS	Trends in International Mathematics and Science Survey

Chapter 1. Introduction

1.1 Southern African and Zambian mathematics education context

Failing mathematics education is a widespread problem in the Southern African region and much attention has been given to the apparent "educational crisis" in South Africa (Fleisch, 2008; Bloch, 2009). South Africa and Botswana, the two Southern African countries included in the latest Trends in International Mathematics and Science Survey 2011 (TIMSS), both ranked as two of the lowest performing countries (Mullis, Martin, Foy & Arora, 2012). It is possible to see how Zambia, though not a participant in TIMSS, fits into this global picture by cross-referencing with the SACMEQ III mathematics results. Of the 15 countries surveyed[1], Zambia ranked 14th, below South Africa at 10th (Musonda & Kaba, 2011).

International and Southern African research has highlighted the efficacy of targeted early intervention as a means of addressing such failings in mathematics education. Not only does early mathematical performance affect later mathematical performance (Wright, 2003), but the gap between learners' mathematical performance also increases as they progress though school (Wright, Martland & Stafford, 2006; Graven, Stott, Mofu & Ndongeni, 2015). For these reasons, targeted early intervention in numeracy has been shown to be more effective than later intervention, in terms of both learner outcomes and cost (Heckman, 2000; Spaull & Kotze, 2015).

Zambia has made educational investment a development priority over the last decade[2]. This investment has paid off in quantitative terms (IBO, 2008) as Zambia is on track to meet its Millennium Development Goal of universal primary access, with primary school completion rates now at over 90% (IBO, 2008). Despite, or perhaps because of, this expanded access, learner results have remained stagnant at a low level. The National Assessment Survey Report is the main instrument for monitoring progress in basic education within Zambia (IBO, 2008). In 2008, the national mean in mathematics results at the Grade 5 benchmark was 39.4%. Zambia attributes this to the consequent increase in teacher-pupil ratios and class sizes, which has "tended to compromise quality" (GRZ, 2011, p. xix). The government is currently implementing its 2011-2015 3rd Education Sector Plan and the focus is shifting from *quantity* to *quality* of education. Plans highlight improving teaching in mathematics as a "key activity" (GRZ, 2011, p. 43). The government recognises that "teachers hold the key to better

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[1] *The countries represented in the Southern and Eastern Africa Consortium for Monitoring Educational Quality (SACMEQ) III report include Botswana, Kenya, Lesotho, Malawi, Mauritius, Mozambique, Namibia, Seychelles, South Africa, Swaziland, Tanzania, Uganda, Zambia and Zimbabwe*

[2] *Educational expenditure represented almost 20% of the national budget in 2010, an historical high (GRZ, p.xvii)*

quality service provision" (GRZ, 2011, p.xxi), and that educational research is a priority (GRZ, 2011, p. xvi).

1.1.1 The South African Numeracy Chair project

South Africa has also identified research as a key strategy to address the mathematics education crisis (Fleisch, 2008; Bloch, 2009). A government-private partnership has responded by establishing six research and development chairs in mathematics education, co-funded by the FirstRand Foundation (with the Rand Merchant Bank), the Anglo American Chairman's fund, the Department of Science and Technology, and the National Research Foundation. Two of the six chairs focus on early numeracy development in Primary Schools, reflecting increasing awareness of the importance of early numeracy development. Professor Graven holds one such primary chair, the South African Numeracy Chair (SANC) at Rhodes University, where I am a Masters student. Although the work of the SANC project was initially focused on the broader Grahamstown area, it has now extended to other parts of the Eastern Cape region and within South Africa (Stott, 2014). My research will further extend the work of the SANC project to the wider Southern African region. The SANC project aims to: improve learner performance in primary schools as a result of quality teaching and learning, and to research sustainable and practical solutions to the challenges of improving numeracy in schools.

1.2 Rationale

I have been working in mathematics education in primary schools in the Southern Province of Zambia since 2012. During this time, a phenomenon that has struck me time and again is that of learners' reliance on perceptual unit counting, especially the use of tally marks. For example, when faced with an addition problem some learners draw a tally corresponding to each addend and then proceed to count all the marks to derive the sum. The use of this method is not restricted to the lower grades or to simple arithmetic. I observed one child in Grade 5 using this method to solve a multi-digit multiplication sum. Evidence from final primary exam papers in the school also indicated that use of this strategy was not uncommon in Grade 7.

This reliance on perceptual counting strategies is not unique to Zambia. It has been readily observed in mathematics learners in other parts of the world (Gray & Tall, 1994), where it is frequently encountered in learners with 'mathematical difficulties' (Dowker, 2005). However, there is a growing body of evidence (Schollar, 2008; Ensor et al., 2009; Weitz 2012) suggesting that this is in fact a common practice in South Africa, employed by the majority of young learners. Ensor et al. (2009) found that concrete methods of solving problems, including tally counting, are common in the South African schools they investigated. Schollar reported that 79.5% of Grade 5 learners and 60.3% of Grade 7 learners relied on unit counting to solve problems. Of these learners, 38.1% of Grade 5 and

11.5% of Grade 7 learners relied *exclusively* on simple unit counting to solve problems.

Dineen (2014) posits a correlation between a country's ranking in mathematical performance (as measured by the TIMSS[3]) and the emphasis placed on the use of counting strategies in its National Curriculum. No explicit reference is found to the use of concrete, abstract, or mental strategies in the Zambian National Curriculum Documents, 2003. However, the teacher's guide accompanying a common primary textbook used throughout Zambia is more explicit. "Learners must be encouraged to add and count using concrete objects, until such a time that the need for this decreases. This will happen naturally as learners' understanding develops and broadens" (Kachinga & Siatichema, 2004a, p. 29). The above extract from the Grade 1 teacher's guide highlights two key features that will be discussed further below:

1) Teachers are strongly advised to actively encourage the use of concrete objects.

2) Teachers are advised that the transition from concrete to abstract understanding will "happen naturally" without any support from the teacher.

Prior to the commencement of this study, my experience of classroom practice in Zambia was that the use of concrete objects and tallies was encouraged by teachers in the lower grades and actively taught in Grade 1. Figure 1 shows how learners are encouraged to use a tally to solve a double-digit subtraction in the Grade 1 textbook. The teacher's guide further reinforces the modelling of tallies: "while learners are seated at the teaching station, make 20 strokes on the board, strike off 11 and count the remaining ones" (Kachinga & Siatichema, 2004a, p. 43).

Figure 1 Tally use modelled in a Grade 1 Teacher's Guide (source: Kachinga and Siatichema, 2004b, p. 35)

From Grade 2 onwards, a wide variety of other increasingly abstract strategies for addition and subtraction are modelled in the learner books. However, this early exposure to the tally method seems to have long lasting repercussions both for teaching and learning.

There is a broad, international consensus that progression from perceptual to more abstract arithmetical strategies is vital for mathematical success (Carpenter & Moser, 1984; Gray & Tall, 1994, 2007;

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[3] *Dineen analysed the curricula and performance of ten countries: Singapore, Hong Kong, England, United States, The Netherlands, Ireland, Australia, Sweden, New Zealand, Canada.*

Wright, 2003; Schollar, 2008; Ensor et al., 2009). There are serious consequences to failing to make such progression and, once such practices become entrenched, there is "little hope of future development" (Gray & Tall, 1994, p. 18). The body of research into this phenomenon in Southern Africa is currently very limited, but part of a growing research focus. For example, Venkat and Askew (2012) found evidence that primary teachers repeatedly established number bonds through counting-by-ones. This occurred even despite the presence of resources that could be used to model more sophisticated strategies. Given the scope and scale of this problem in Southern Africa it seems possible that it originates in common teacher practices and/or curricular and other guidance, as Spaull and Kotze (2015) suggest. If the problem does have such an origin then the current lack of knowledge and understanding is again an example of the gaps between research and practice in mathematics education, and there is a real need for swift action to rectify the situation. Whatever the cause(s) of these problems, further research is warranted. Although it is beyond the scope of my study to investigate the origins of this problem, it may generate some hypotheses regarding possible causes which could then be investigated further.

1.3 Significance and potential value

This study explores the early arithmetic strategies of Grade 2 learners in a Zambian classroom and investigates the adaptation of the Mathematics Recovery (MR) programme to facilitate learner progress to more sophisticated strategies. The MR programme is used extensively in many countries including Australia, the USA, and the UK, where its results are well proven (Willey, Holliday & Martland 2007; Dowker, 2009). The MR programme has not yet been implemented in Southern Africa on a broad scale and there are several barriers to its use in the original format (Stott, 2014; Wasserman, 2015). In addition to the likely number of learners in any class who would qualify for intervention, another main barrier is the lack of resources available for intensive one-to-one assessments and interventions (Dowker, 2005). Within the SANC project there is a growing body of research on how the MR intervention programme can be adapted to a South African context (Weitz, 2012; Mofu, 2013; Ndongeni, 2013; Stott, 2014; Wasserman, 2015). My study will build upon this research, extending it to Zambia and to a wider Southern African context. It will also add to the growing body of research into early numeracy in Southern Africa. As Dowker (2005) notes, lack of communication between researchers and practitioners is a problem that has "bedevilled the whole area of mathematical development" (p. 241). My study also attempts to bridge this gap by close collaboration with the class teacher during the research.

1.4 Research goals and questions

This research aims to:

1. Investigate the extent of the phenomenon of perceptual counting and other early arithmetic strategies used in Grade 2 in a Zambian school.

2. Investigate the possibility of adapting the MR programme for use in a Grade 2 group/whole class setting to facilitate learners' progression to more facile arithmetic strategies.

3. Research the effectiveness of this adapted programme.

These aims align with those of the SANC project "to improve learner performance in primary schools as a result of quality teaching and learning" and "to research sustainable and practical solutions to the challenges of improving numeracy in schools" (SANC, 2015). The parameters of this latter aim are relevant in the Zambian context, where there are often extreme resource constraints (GRZ, 2011). In line with the above aims three questions will be addressed by this research:

1) What progress, if any, do learners make in early arithmetic strategies using the Mathematics Recovery programme approach?

2) What strategies are used by Grade 2 learners in solving early arithmetic problems?

3) How might the MR programme be adapted to help Grade 2 learners progress in their early arithmetic strategies and what are the advantages and constraints that emerge from the whole class adaptation?

These research questions evolved during the study. I had originally intended to focus my study on learners' use of tally marks and on the possibility of reconceptualising tally marks to facilitate progression as part of early arithmetic strategies in the context of an after-school maths club. However, as I became involved with the class, the opportunity presented itself to study the adaptation of the MR programme for whole class use.

1.5 Thesis outline

This chapter has introduced the context, rational and purpose of my study, and the research questions to be addressed. In Chapter 2 I outline the theoretical framework underlying the study and present a review of the literature. Chapter 3 details the methodological framework and describes the research design of the study. I present the results of my study in Chapter 4. Chapter 5 presents a discussion and analysis of the results. In Chapter Six I present an evaluation of my study, draw conclusions and make recommendations.

Chapter 2. Literature Review and Theoretical Framework

In this chapter I outline the theoretical framework underlying this study, and present a review of the literature.

2.1 Theoretical framework

The theoretical framework underpinning this study is that of an emergent perspective (Cobb & Yackel, 1996). An emergent perspective "coordinates a social perspective on communal activities with a psychological perspective on the reasoning of the participating students" (Cobb, Stephan, McClain & Gravemeijer, 2001, p. 114). It is a pragmatic, modified constructivist view that is distinguished from both strong cognitive and strong social versions of constructivism, drawing on aspects of both. In an emergent perspective, "neither individual students' activities nor classroom mathematical practices can be accounted for adequately except in relation to the other" (Cobb, 2000, p. 310). From an emergent perspective, learning is a constructive process that happens as learners participate in, and contribute to, a learning community (Cobb & Yackel, 1996). This approach has been developed over the last 20 years by mathematics educators and researchers who recognise that constructivist, socio-cultural, and socio-constructivist theories each offer a valuable perspective on learning, but that none of them necessarily gives the full picture (Simon, 2009). The emergent perspective also acknowledges a reflective relationship between theory and practice (Cobb & Yackel, 1996).

Constructivism, based on the work of Piaget, views the learner as constructing their own knowledge though a process of active discovery (Pritchard, 2009). The role of the teacher is that of facilitator or engineer, designing learning environments and guiding the learner in their cognitive knowledge construction. Ernest von Glasersfeld was instrumental in leading the application of constructivist theories to mathematics education in the 1980s (Steffe & Kieren, 1994). The constructivist perspective contrasted strongly with the traditional didactic instruction prevalent throughout much of the twentieth century and before. Traditional instructional approaches can be viewed as broadly behaviourist in nature, with mathematical knowledge transferred from teachers to learners with an emphasis on repetition and reinforcement (Pritchard, 2009). Constructivist theories of mathematics education based on the work of von Glasersfeld influenced the work of many of the mathematics educators and researchers referenced in this study (Steffe & Cobb, 1988; Wright, Martland & Stafford, 2006; Wright, Martland, Stafford & Stanger, 2006; Gravemejier & Van Eerde, 2009).

Socio-cultural theories of learning challenge both these viewpoints. They draw on Vygotsky's idea of learning as a collaborative, social process, in which knowledge is co-constructed (Vygotsky, 1978). In contrast to Piaget's stage theory, Vygotsky views learning as leading development. Askew (2013) applies Vygotsky's theory to mathematical development, suggesting that "children do not develop into a 'stage' whereby understanding the nature of addition becomes possible, it is working 'as though' they understand addition that allows the development to occur" (p. 7). Both constructivist and socio-constructivist theories of learning will be discussed further below in relation to the literature on progression in early arithmetic.

By adopting an emergent perspective in my study, I was able to coordinate features from each of the above theories, using them as a set of tools to give a fuller picture of the learning. Others have paved the way for this approach (see, for example Lerman, 1998; Simon, 2009; Stott, 2014; Dineen, 2014). Lerman (1998) and Simon (2009) both describe such a pragmatic approach using the metaphor of a lens. For Lerman (1998), research is a lens, through which we can focus in and out on the constructivist or socio-cultural features of learning, whilst still being aware of the full picture. For Simon (2009), the theories themselves can be used as different lenses through which to look at the researched learning.

An emergent perspective also enables the synthesis of the theoretical approaches underlying the two projects on which my research is based. The SANC project maths clubs, which inspired the ethos of the research context in my study, are underpinned by a socio-cultural theory of learning (Stott & Graven, 2013). The MR programme has its theoretical origins in the constructivist theory of von Glasersfeld (1978) via Steffe (1992), which in turn draws on Piaget's theory of cognitive development (Wright, 2003).

For my study, the changeable lenses of an emergent perspective proved an especially useful tool when adapting the *individual* intervention activities of the MR programme for use in a social group or whole class setting. A constructivist lens enabled me to focus on individual learners' learning and the possible cognitive process involved, whilst a socio-cultural lens brought out social features of learning that I may otherwise have overlooked.

2.2 Literature review

2.2.1 Theories of early arithmetic strategies and progression

Research undertaken into early mathematical learning in the late 1980s and early 1990s generated several functioning descriptions of the nature of children's early arithmetic strategies (Siegler, 1987; Dowker, 2005; Wright, Martland & Stafford; 2006). Dowker (2005) observes that at any point individual children will use a wide range of strategies.

Working from a constructivist perspective, researchers and educators have found it useful to qualify the 'stages' of early arithmetical strategy progression, and various models have been developed. This was in contrast to the broadly behaviourist traditional view of early arithmetic, which prioritised retrieval of facts learnt through drill and rote learning rather than computation (Cowan et al., 2011). Common to many of these stage models of early arithmetic is the idea that early arithmetic strategies begin with counting-by-ones, which becomes more advanced, before the child progresses to non-counting strategies.

Carpenter and Moser (1984) view children's arithmetical progression as progression from counting concrete objects to doing mental calculation. They identify five counting stages from count-all to count-on-from-largest-addend. Askew and Brown (2003) identify a "well established sequence of development" (p. 6) as children progress through a sequence of counting-all, counting-on from the first number, counting-on from the larger number, using known facts and, finally, deriving number facts. Steffe and Cobb (1988), following von Glasersfeld, identify five learning stages in the development of counting based strategies: 1) the perceptual counting scheme, 2) the figurative counting scheme, 3) the initial number sequence, 4) the tacitly nested number sequence, and 5) the explicitly nested number sequence. Wright and colleagues' model of Early Arithmetic Strategies (EAS) is built on the constructivist work of Steffe, Cobb and von Glasersfeld (Wright, 2003). Following Steffe and Cobb (1988), Wright and colleagues identify six stages of EAS as a part of their Learning Framework in Number (LFIN). EAS describes a "progression of stages in which counting is used in increasingly more sophisticated ways" (Wright, 2013, p. 28). The six broad stages in this domain are: emergent, perceptual, figurative, initial number sequence, intermediate number sequence and facile number sequence.

Steffe and Cobb's model is based upon their seminal longitudinal research study into the early arithmetic progression of six first-grade students in Georgia, USA in the early 1980s. Their aim was to build a "model of cognitive changes in children's initial, informal number sequences" (Steffe & Cobb, 1988, p. vii) and how these sequences progressed over time. The aim was then to create a set of teaching guidance and activities to support development of children's early arithmetic strategies. Their study took the form of a teaching experiment, conducted twice a week for 16 weeks each year, derived from Piaget's clinical interview and aimed at "discovering what might go on in children's heads" (Steffe & Cobb, 1988, p. vii).

In the model that resulted from this research, Steffe and Cobb (1988) use the term 'stage' in a precise sense to encompass four characteristics, following von Glasersfeld and Kelley (1982; cited in Wright, Martland & Stafford, 2006):

1. A characteristic remains constant for a period of time
2. The stages form an invariant sequence
3. Each stage builds on and incorporates the previous stage

4. Each new stage involves a significant conceptual reorganisation (p. 52)

This definition of a stage is adopted by Wright and colleagues in their EAS stage model.

The theories of Steffe, Cobb and Wright and colleagues are strongly constructivist in foundation, following Piaget and von Glasersfeld (Steffe & Cobb, 1988; Wright, Martland & Stafford, 2006; Wright, Martland, Stafford & Stanger, 2006). Within these models, progress between stages is a cognitive process that involves an accommodation or change. They used the term "cognitive reorganisation" to refer to this process, which they define as a significant change in the child's thinking (Steffe & Cobb, 1988, p. 46; cited in Wright, Martland, Stafford & Stanger, 2006, p. 37). Such cognitive reorganisation is often preceded by an extended period of sustained, hard thinking on the part of the child. Given that a stage is a characteristic that remains constant for a long time, following von Glasersfeld's definition, a cognitive reorganisation leading to progress by an EAS stage is regarded as a milestone in child's development (Wright, Martland, Stafford & Stanger, 2006). Wright and colleagues refer to cognitive process using terms from the constructivist work of Steffe and Cobb (1988) and von Glasersfeld (1995). Such terms used include cognitive reorganisation, anticipation, curtailment and re-presentation. As they form part of the teaching guidance for the MR programme, these terms will be used in this study as defined in Wright, Martland, Stafford and Stanger (2006).

In these models of early arithmetic progression, progress initially involves development of counting based strategies, and the development of non-counting strategies is regarded as the final "step". However, as Dineen (2014) notes, researchers have recently been investigating the idea that children's early arithmetic can also be advanced through non-counting or grouping strategies. Askew (2013) describes three approaches used by children in solving early arithmetic problems: counting, decomposition and retrieval. He summarises two broad views from the literature as to how, and in what order, these approaches develop. The first he terms 'the progression view', a broadly Piagetian view of progression though stages of counting, as described above (Askew, 2013). The second he terms 'the number sense view' (after Baroody, 2006) which prioritises the selection of an efficient strategy and which he aligns with a more broadly Vygotskian view of learning. Askew suggests that recent research into children's early arithmetic strategies by Cowan et al. (2011) aligns more with the number sense view suggesting there is "no clear hierarchy of these strategies" (Askew, 2013, p. 4). Despite this, Dineen (2014) found that international curricula prioritise counting based strategies in the first years of schooling. There is also an emphasis on counting based strategies in the early years of schooling in Zambia, as described above.

Within early arithmetic, subtraction is often perceived as more challenging to teach and learn than addition (Baroody, 1984; Fuson, 1984; Haylock & Cockburn, 1989; Kamii, Lewis & Kirkland 2001; Cockburn, 2007). Reasons suggested for this include the increased cognitive demands of subtraction as compared to addition (Baroody, 1984) and that children's strategies for subtraction may be limited to taking away in ones (Haylock & Cockburn, 1989). Although my study into early arithmetic focused primarily on addition some interesting and unanticipated results related to subtraction emerged during the retrospective analysis.

Wright and colleagues' (2006, 2012) EAS model of early arithmetic strategies is the progression model explored in this study as part of the adaption of the MR programme to a whole class context. I used a description of the EAS that combines elements of Wright, Martland and Stafford's (2006) original Stages of Early Arithmetic Learning and the updated Early Arithmetic Strategies from Wright, Stanger, Stafford and Martland (2012). The EAS is the most important aspect of the framework underlying the MR programme (Wright, Martland & Stafford, 2006, p. 414). Although EAS is an example of a progression model, the MR programme's underlying framework also includes other strands which focus on the development of early grouping strategies and known facts (Wright, Martland & Stafford, 2006) connecting with a number sense view. From a Vygotskian socio-constructivist perspective, learning can lead progression in early arithmetic strategies and the available opportunities for learning will, to a certain extent, influence the path of progression. Wright, Martland, Stafford and Stanger (2006) refer to Vygotsky's Zone of Proximal Development when describing learning that is challenging but achievable for the learners with support. In this study, the term "at the cutting edge", as used by Wright, Martland, Stafford and Stanger (2006, p. 59), will be used to indicate such learning.

The interchangeable lenses of the emergent perspective will enable me to reflect on the results of this study from both the progressive and the number sense views of early arithmetic strategy development.

2.2.2 Failure to progress

There is a wide consensus that to be successful in mathematics it is necessary for learners to progress through 'stages' of early arithmetic as described above (Gray & Tall, 1994, 2007; Wright, Martland & Stafford, 2006; Schollar, 2008; Ensor et al., 2009; see also Carpenter, Fennema, Franke, Levi & Empson, 1999). Alternatively, the number sense view (Baroody, 2006; Askew, 2013) conceives of arithmetical progress as the development of efficient strategies. Common to both of these views is the idea that learners' progress in early arithmetic will be characterised by development of efficient and effective strategies for solving problems.

Failure to progress beyond the early stages, or to develop efficient strategies, leads to subsequent overreliance on counting or inefficient strategies. This in turn leads to mathematical difficulties (Siegler, 1988; Geary, Bow-Thomas & Yao, 1992; Gray & Tall, 1994; Ostad, 1997, 1998; Dowker 2005). Such less advanced strategies can become entrenched (Dowker, 2005) and the learners can become "stuck" at an early counting stage. Difficulties then manifest themselves as learners progress through the schooling system and are faced with more challenging mathematics. Gray and Tall (1994) and Baroody, Bajwa and Eiland (2009) suggest that learners who try to employ counting-based strategies to solve higher-level problems (e.g. multi-digit calculations) face an “extremely difficult” task (Grey & Tall, 1994, p. 21) that involves much cognitive effort. They provide evidence to support their claim that those learners with such mathematical difficulties are actually doing a more difficult form of mathematics, which causes the consequent “divergence in performance between success and failure” (Gray & Tall, 1994, p. 1). The growing body of evidence from South Africa suggests that entrenchment of less advanced counting-based strategies is widespread (Ensor et al., 2009). This conception of entrenchment, and the consequences thereof, enabled me to put the results of this study into a wider context.

2.2.3 The role of teaching

There is a recurring suggestion in the literature that teachers’ emphasis on the use of perceptual counting in the early years of school can lead to such strategies becoming entrenched (Gray & Tall, 1994; Dowker, 2005; Wright, Martland & Stafford, 2006; Gray, 2010). Likewise, Schollar (2008), Ensor et al. (2009) and Weitz (2012) suggest that teaching practices may be at the root of the perceptual counting problem in South Africa. In South Africa, Schollar (2008) suggests that this phenomenon is caused by the “application of ineffective learning practices in classrooms” (p. 16). Ensor et al. (2009) are more specific, stating that “learners are restricted from access to more abstract ways of working with number by classroom practices that privilege concrete models of representation” (p. 15). Indeed, this is something that I have noted from my previous work in Zambian classrooms, specifically an emphasis on unit counting (counting-by-ones) and the use of tallies.

Dineen (2014) following Gray (2010) suggests that “students in their third and fourth year of school may be reluctant to use alternative, more mathematically sophisticated approaches … if the sole focus of their schooling prior to this has been on the use of counting strategies” (p. 2). Likewise, Dowker (2005) suggests that strategies can become entrenched, “especially if the child is given too much of the wrong sort of arithmetical practice” (p. 242). Wright, Martland and Stafford (2006) explain that “doing tasks of this kind is likely to encourage or reinforce the use of strategies involving perceptual counting and thus discouraged advancement” (2006, p. 90). Gray and Tall (1994) summarise this in a powerful image, stating that teachers’ persistence in emphasising the use of perceptual counting strategies “leads many children inexorably into a cul-de-sac from which there is little hope of future development” (p. 18). It is likely that in emphasising the use of such perceptual counting strategies, teachers are acting with the best of intentions (Dineen, 2014) and on the recommendation of curricular material, as may be the case in Zambia and South Africa (Kachinga & Siatichema, 2004a, 2004b; Weitz, 2012).

This is not to say that the use of perceptual counting strategies and supporting perceptual items (including fingers) is to be discouraged. Indeed, perceptual counting is a key starting point from which fluent arithmetic strategies can develop. Wright, Martland and Stafford (2006) state that "finger patterns play an important role in early numerical strategies" (p. 26). Moch (2001) suggests that the use of concrete materials is now regarded as international best practice, and their use is recommended by numerous educators (Haseler, 2008; Henderson, Carne & Brough, 2003; Thomas & Allingham, 2008; Williams, 2008, as cited in Dowker, 2009). However, it is not just *that* but *how* these materials are used that makes a difference (Haseler, 2008). Many researchers express the idea that teachers should actively facilitate learner reflection when using such concrete materials (Ball, 1992; Maclellan, 2001; Gravemeijer, 2004).

Collaboration with the class teacher was a key feature of this study. Although investigation of classroom teaching was beyond the scope of this study, the suggestion from the literature of the possible role of teaching in strategy entrenchment provided a focus for discussions between the class teacher in this study and myself, and a focus for analysis of the teaching and learning.

2.2.4 The case for early intervention

International research suggests that learners' early numeracy performance affects, and can even predict, later mathematical achievement (Daraganova & Ainley, 2012; Jordan, Kaplan, Ramineni & Locuniak, 2009, as cited in Wright, 2003). The gap between learners also increases as they progress though school (Wright, Martland & Stafford, 2006). Studies in South Africa have also found this effect. The results of the Annual National Assessments (ANA) show that with each progressive year of schooling more and more learners lag behind meeting the basic numeracy requirements for their grade level (Graven, Stott, Mofu & Ndongeni, 2015). The research suggests that targeted early intervention in numeracy can have a significant impact on learners' performance and confidence (Wright, Martland & Stafford, 2006; Dowker, 2005) and that this is much more effective than later intervention in terms of outcomes and cost (Heckman, 2000; Spaull & Kotze, 2015). Internationally, Williams (2008) put forward a strong recommendation for early numeracy intervention for primary schools in the UK (Dowker, 2005). Within the South African context, Fleisch (2008) stressed that it is imperative to identify and remediate learning gaps early on, before they become insurmountable learning deficits and lead to almost certain failure and drop-out in higher grades. As Weitz concluded in her 2012 Master's thesis, "South Africa has to focus more on the lower grades for progression in the higher grades to be assured" (p. 118). Spaull and Kotze (2015) describe a threefold motivation for early intervention, suggesting it is epistemologically, pedagogically, and economically more effective than later interventions. Wasserman (2015) recommended in the conclusion of her study into the MR programme with Grade 4 learners in South Africa that "ideally the recovery program should be done in Grade 2 to bridge the gaps sooner" (p. 121). Early intervention is one of the key features of the MR programme explored in this study, which will now be described. For these reasons, early intervention was a key factor in the selection of the sample for my study, informing both the selection of Grade 2 learners and the decision to work with the whole class.

2.2.5 The Mathematics Recovery programme

The Mathematics Recovery (MR) programme is an intensive intervention programme in early number learning, developed by Bob Wright and colleagues between 1992 and 1995 at Southern Cross University in New South Wales, Australia. In the original MR programme, individual learners partake daily in 30 minutes of one-to-one teaching over a period of 12 to 15 weeks. It is aimed at low attaining children in the second year of schooling who are perceived as "falling behind … before the gap between their knowledge and that of more able pupils grow[s] too wide and cause[s] them to experience excessive failure" (Dowker, 2009, p. 25). In the Southern African context, it is not one or two learners, but rather the majority of the class who are in danger of falling behind in terms of development of arithmetic strategies (see the South African Annual National Assessment results from the Department of Basic Education, 2013, 2014). Although focused on individual intervention, the programme also makes provision for group and whole class teaching whilst still requiring individual assessments (Wright, Martland, Stafford & Stanger, 2006; Wright, Stanger, Stafford & Martland, 2012).

Both assessments and interventions in the MR programme are underpinned by a constructivist theory of young children's mathematical learning, which leads to a comprehensive and integrated framework for assessment and teaching. As stated above, the MR programme is based on the longitudinal empirical research and resultant theories of Steffe and Cobb (1988). The key features of the MR programme fall under four headings: Early Intervention, Assessment, Teaching and Professional Development (Wright, Martland & Stafford, 2006, p. 2). Thorough diagnostic video-interviews are conducted before the intervention with individual learners. The video-interviews are analysed against Wright and colleagues' Learning Framework in Number (LFIN) to give a detailed picture of the child's current knowledge and understanding (Wright, 2003). The LFIN is a framework of early numeracy, consisting of four parts subdivided into eleven aspects, of which the EAS is the "primary and most significant aspect" (Wright, Martland & Stafford, 2006, p. 21). The six stages of EAS are presented in Figure 2 below.

Stage Number	Stage Descriptor	Characteristics (representing increasing levels of sophistication)
0	Emergent counting	**Cannot count visible items**. The child might not know the number words or might not coordinate the number words with the items
1	Perceptual counting	Can count only **visible** items starting from 1. Including seeing, hearing and feeling
2	Figurative counting	Can count **concealed items** but the learner will 'count all' rather than 'count on'.
3	Initial number sequence	Initial number sequence. The child can **count on** rather than counting from one, to solve + or missing addends. May use the **counting down** to solve removed items. (count-back-from)
4	Intermediate number sequence	**Count-down-to** to solve missing subtrahend (e.g. 17-3 as 16, 15 and 14 as an answer. The child is able to use a more **efficient** way to count down-from and count down-to strategies (count-back-to)
5	Facile number sequence	Uses of range of **non-count-by one strategies**. These strategies such as compensation, using a known result, adding to 10. Commutativity, subtraction as the inverse of addition, awareness of the 10 in a teen.

Figure 2 Overview of the EAS aspect of the LFIN (source: Wright, Martland & Stafford, 2006 p. 21)

In Wright, Martland, Stafford (2006) and Wright, Martland, Stafford and Stanger (2006) this aspect is referred to as Stages of Early Arithmetic Learning (SEAL). This study will use the updated term Early Arithmetic Strategies (EAS) from Wright, et al. (2012). For each EAS stage, typical learner profiles are provided.

The LFIN also contains aspects for Forward Number Word Sequences (FNWS), Backward Number Word Sequences (BNWS), numeral identification, structuring numbers to 20, and early multiplication and division (Wright, Martland & Stafford, 2006). The LFIN is a holistic framework for early numeracy, so each part in turn relates to progression in early arithmetic strategies.

Following individual assessment of learners in the MR programme teachers employ a specially developed instructional approach and distinctive instructional activities. These are primarily applied to individual interventions but can also be used with small groups and whole classes. Teaching consists of a series of carefully designed and highly structured teaching activities underpinned by the Instructional Framework for Early Number (IFEN), as set out in Wright, Martland and Stafford (2006). The holistic MR programme provides an extensive professional development course for teachers. Although analysis of professional development within the MR programme is beyond the scope of my study, consideration of this key feature enabled reflection on my own learning during this study.

2.2.6 Adaptation of the MR programme to a whole class context

Since its development in Australia in the 1990s, the MR programme has been implemented as a one-to-one learner intervention programme in Australia, the USA, New Zealand, Canada, Ireland and the UK. In this time, many of the participating schools have successfully applied elements of MR programme to classroom contexts. As Wright, Martland and Stafford (2006) observed, "in each of the years since its inception, the MR programme has significantly influenced general classroom teaching of mathematics" (p. 7). Research into the MR programme within the SANC project also observed such an effect (Wasserman, 2015). Wright (2003) concluded that MR "accords strongly with current, cutting-edge approaches to mainstream classroom teaching of number" (p. 7).

The MR programme has been systematically adapted to whole class contexts in Australia and New Zealand. The first such adaptation was in 1996 by the New South Wales Department of Education and Training, forming the basis of a systematic initiative in mathematics in the early years of schooling (Wright, Martland & Stafford, 2006). This initiative was called Count Me in Too (CMIT). CMIT focused on the development of teachers' understanding of childrens' strategies in early number, and understanding of how to help children progress to more sophisticated numerical strategies. According to Wright, Martland and Stafford (2006), key aspects of the theory and methods of MR were adapted, including: the guiding framework, the approach to assessment and the assessment tasks, the underlying theory of early numerical learning, the guiding principles for teaching, and approaches to professional development (p. 8).

In 2000 – 2001, the Early Numeracy Project was developed in New Zealand, based upon CMIT. Originally designed for use in primary schools in Years 1 to 3, the project has evolved and now extends to all years of primary schooling, with a growing secondary component (Tozer & Holmes, 2005). Young-Loveridge (2004) identifies three aspects as central to the Early Numeracy Project: the teacher development programme, the framework, and the diagnostic interview. The Early Numeracy Project's number framework has two main sections: knowledge and strategy (Tozer & Holmes, 2005). The strategy section describes the mental processes children use to solve operational problems with numbers. It consists of a sequence of nine global strategy stages, split into counting strategies and part whole strategies (Tozer & Holmes, 2005). The five counting strategy stages are: 0) emergent, 1) one to one counting, 2) counting from one on materials, 3) counting from one by imaging, and 4) advanced counting. The Early Numeracy Project retains features from the original MR programme including the diagnostic interview and ongoing assessment against a framework. It also introduces adaptations, for example, a whole class profile is used to manage assessment information. To enable teachers to respond to the diverse needs and starting points in a class, Tozer and Holmes (2005) explain that "children are grouped according to their strategy stage" (p. 35).

Several features of these systematically adapted programmes informed the adaptation of the MR programme in this study, for example, the grouping of learners by stage and the use of more descriptive stage names.

2.2.7 Research into the MR programme

The MR individual intervention programme has been the focus of several research studies (Wright, 2003; Dowker, 2005; Willey et al., 2007; Williams, 2008; Smith, Cobb, Farran, Cordray & Munter, 2013). These studies have found that the MR programme can lead to considerable improvement in attainment for learners at risk of falling behind (Williams, 2008). Likewise, research into the whole class systematic adaptations CMIT and the Early Numeracy Project suggests that they are highly successful in terms of student learning and achievement and teacher professional development (Christensen, 2003; Thomas, Tagg & Ward, 2003).

There is a growing body of research into how the MR intervention programme can be adapted to a Southern African context (e.g. Weitz, 2012; Mofu, 2013; Ndongeni, 2013; Stott, 2014; Wasserman, 2015) to which my research will add. Wasserman's 2015 study of Grade 4 learners explored the adaptation of MR assessment and teaching to a group context, with a focus on EAS and conceptual place value. Stott's 2014 study investigated Grade 3 learners' progression in all early numeracy aspects in the context of after-school maths clubs. Ndongeni drew on the LFIN to establish Grade 4 learners' levels of conceptual understanding in multiplication. Mofu investigated the effectiveness of the MR programme in developing the multiplicative reasoning of Grade 4 learners in a group context. Weitz's 2012 study investigated the use of the LFIN assessment interview as a tool for assessment of Grade 2 learners. From this it is seen that research into the MR programme can focus on all or selected aspects of the LFIN.

Of the individual MR intervention programmes, Dowker (2009) notes there is a lack of evidence of a correlation between the length of the intervention and the gains made, as interventions are often concluded for the purposes of research (p. 16). Despite this, Dowker (2005) noted of one-to-one interventions "the amount of time given to such individualised work does not need, in many cases, to be very large to be effective" (p. 252). A 2007 study in the UK investigated the impact of one-to-one individual intervention programmes. A total of 20 half-hour sessions were held over a period of five to seven weeks, equivalent to an average total of 35 hours per learner (Willey et al., 2007). The study found that most learners made gains of two SEAL stages. Within the SANC project, Stott's (2014) study worked with 17 Grade 3 learners over the course of a year (an average of 28 hours per learner). Her study revealed that, whilst some learners made progress in EAS, more changes were noticeable in the conceptual place value and early multiplication and division aspects of the LFIN.

2.2.8 SANC project maths clubs

The SANC project has developed a model for after-school mathematics clubs as part of its work to improve mathematics teaching and learning in South Africa. SANC project maths clubs have a well-established, learner-friendly ethos that values active student engagement and promotes reflection on strategy use. They are designed to be 'communities of learning' informed by an underlying socio-cultural learning paradigm (Stott & Graven, 2013). The maths club ethos places value on talk about mathematics and on positive attitudes towards mistakes, with the teacher as facilitator of learning. Table 1 summarises some of the intended features of the SANC project maths club environment as contrasted with regular classroom environments observed from schools participating in research with the SANC project. My study used the ethos of these clubs to inform the socio-cultural norms for the teaching experiment for both learners and teachers and to focus reflection and analysis on social features of learning.

Table 1 Contrasted classroom and club environments (source: Stott & Graven, 2013 p. 2)

Observed mathematics classroom environment	Intended club environment
Compulsory attendance is expected as part of formal schooling (in-school time)	Voluntary participation during out-of-school time
Less learner choice in the activities that they work on and engage with	More learner choice in the activities that they work on and engage with
Curriculum and assessment standards as a prescriptive framework strongly influencing choice of content and activities (i.e. the South African curriculum documents)	Curriculum as contextual guide for what is nationally expected of learners but individual learner numeracy levels guide content and activities
Largely acquisition based and often driven by teaching for/to assessments	Participation based; participants are active and engaged
Teacher led and much whole class teacher–learner interaction	Many interactions are learner led with few whole class–mentor interactions and many one-to-one interactions between mentors and learners
Assessment tends to be summative and results in ranked performance (e.g. South African Annual National Assessments)	Assessment is formative and integrated and used to guide individual learning experiences for participants
Prescriptive, teacher-controlled classroom rules within general school rules	Negotiated socio-mathematical norms which may differ from in-school time rules

2.2.9 Nomenclature

Commentators use a variety of terms to refer to similar aspects of numeracy development. Wright (2013) advocates the development of a common nomenclature to introduce "an element of precision to discussions among professionals in this area" (p. 26). He contrasts the field of numeracy research with that of the more established literacy research, which has an extensive established nomenclature (Wright, 2013). As Wright and colleagues' framework will be used in this research, I use their nomenclature in framing and discussion of the study. In an early arithmetic context, Wright, Martland and Stafford (2006) use the term "perceptual replacements" to define use of fingers to represent and replace the objects (or numbers) to be counted. Tally marks drawn for purposes of unit counting can also be classified as perceptual replacements in this way. Nomenclature considerations informed my discussions with the class teacher and the adaptation of the MR programme.

In this chapter, I have outlined the theoretical framework of this study, and presented a review of the literature. In the next chapter I will outline the methodological approach and describe the research process for my study.

Chapter 3. Methodology

This chapter introduces the methodological framework of the study and describes the research design. The first part of this chapter gives an overview of the design research methodology, and the particular approach used in this study. The second part describes in more detail each of the three phases of design research and presents this study's research plan.

3.1 Overview of methodological orientation and framework

The methodological framework of this study is that of design research, a research method aimed at educational improvement through the exploration of how teaching and learning work. The goals of design research are twofold. Firstly, design research aims to develop an innovative learning intervention. Secondly, it aims to develop empirically grounded theories of how the particular learning intervention works (DBRC, 2003). By exploring the relationship between theory and practice, design research can act as a bridge between them (Dowker, 2005).

In contrast to more traditional research methods, design research provides theories of *how,* not just *whether,* teaching innovations or interventions are effective. Gravemeijer and van Eerde (2009) argue that understanding of how a method works is more useful to teachers, as they may then "take those theories as conjectures, which they can test and modify in their own classrooms" (p. 511). Likewise, Cobb (1996) suggests that more traditional comparative research provides little insight for teachers who wish to adapt interventions for use in their own classrooms. Design research should address complex problems in real contexts, and must lead to shareable theories of learning and teaching (DBRC, 2003). To this end, collaboration with practitioners is key. As such, class teacher collaboration and the classroom context were key features of this study.

In design research, research and development occur through iterative cycles of design, enactment, analysis, and redesign (DBRC, 2003). Design research has three phases: 1) the preparation for the experiment, 2) the teaching experiment, and 3) the retrospective analysis (Gravemeijer & van Eerde, 2009, p. 513). Details of the research design for each of these phases will be presented below.

Design research is underpinned by a broad theoretical base encompassing constructivism, socio-constructivism, and socio-cultural theory (Gravemeijer & van Eerde, 2009). As such, it aligns with the emergent perspective used in this study. This study uses the methodology of a type of design research aimed a developing what Gravemeijer (2004; Gravemeijer & van Eerde, 2009) calls a local instruction theory.

Design research is an evolving paradigm and as such it consists of a series of approaches (Barab &

Squire, 2004). All the approaches share the key principles described above, but may differ in their specific aims and development contexts. According to Gravemeijer and van Eerde (2009), a local instruction theory is "a theory about how students learn a specific topic in mathematics, and how the learning process is supported" (p. 510). As such, this particular design research approach is suited to my research goals of understanding and supporting Grade 2 learners' development of early arithmetic strategies.

Some criticism levied at design research includes suggestions of generally unscientific approaches and questions regarding the generalisability of claims (DBRC, 2003). One of the strengths of the design research paradigm is that it embraces the complexity of learning. Indeed, it poses the challenge that more detached "scientific" methods of research are unable to account for the influence of context, the complex nature of outcomes, and the incompleteness of our knowledge about learning (DBRC, 2003). In defence of small sample research, Adler (as cited in Graven, 2002) suggests that results can be viewed as generative rather than generalisable in that they "generate further research questions and provide explanatory models for a research topic" (Graven, 2002, p. 136).

Shavelson, Phillips, Towne and Feuer (2003) recommend steps to be taken with regards to the scientific rigour of design research including excluding competing conjectures, considering claims with skepticism, and encouraging the exploration of rival hypotheses. Likewise, Cobb, Confrey, diSessa, Lehrer and Schauble (2003) recommend that design researchers work systematically through data and conduct research in a way that promotes transparency and replicability as much as possible. Furthermore, Neuman (2006) recommends the triangulation of data as best practice in any qualitative study. The methods used to support validity of the results in this study are discussed in the data collection section below.

It is imperative that design research be based on prior research and on well-founded design principles (DBRC, 2003). The MR programme was the prior research on which the design research experiment of this study was based. The MR programme is a holistic programme of assessment, teaching intervention, and professional development developed from longitudinal research and refined over years of practice. Design research and the MR programme have much in common, beginning with their constructivist underpinnings, as has been also noted by Dineen (2014). In the sections below I will explain how the MR programme's assessment interviews, LFIN, Instructional Framework in Number (IFEN), and teaching guidance relate to the design research process and to my study.

Within Gravemeijer and van Eerde's (2009) design research method, a set of exemplary instructional activities and materials may also be a product of the research, in addition to a local instructional theory. This feature further reinforced my choice of Gravemeijer and van Eerde's (2009) version of design research for this study where I considered how an already established set of teaching activities from the MR programme might be adapted to a new context.

3.2 Research design

This section describes each of the three phases of my design research study in more detail. For each phase the methodological framework will be presented followed by a description of the research design of this study.

The nature of the data was largely qualitative and interpretive, with some summative data. The qualitative nature of data collected in a design research study is a strength of the design research methodology, as qualitative data enables the complexity of the learning process to be captured. However, it is also a potential source of criticism, as discussed above. As such, care was taken to ensure the validity of the data. I achieved this by thorough documentation of all aspects of the research process through the use of a research journal and thorough triangulation of data. Triangulation involved looking at the data from multiple viewpoints, thereby improving the accuracy of the analysis (Neuman, 2006). Six data collection tools were used at various stages of the research: video recordings, audio recordings, research logs, planning sheets, photographs of learner work, and a research journal. Each data collection tool is described in more detail below, in the section describing the relevant phase of research.

I kept a research journal throughout the three phases of this study. This enabled me to thoroughly document all aspects of the research process as they occurred. I then used my research journal for triangulation of data. In this way, the research journal was a key method of data collection and was used to provide detail and support for the data analysed. For example, it provided supporting data for the case study narratives in Chapter 4. I made both written and audio entries where I recorded my observations, questions, ideas and reflections. Some of the richest entries were the audio recordings I made of my verbal reflections immediately after the teaching experiment sessions. The observations were fresh in my mind for these entries, and the audio recoding format enabled me to freely capture the reflective process.

In design research, data collection and data analysis are closely linked (Gravemeijer & van Eerde, 2009). As such, data analysis occurred in all three phases of this study. Simultaneous analysis of data occurred during the preparation and teaching experiment phases. A comprehensive retrospective analysis of all data was conducted during the final phase of the research. Hatch (2002) supports such recursive data analysis methods in qualitative studies. The MR programme also prescribes simultaneous data collection and analysis of the pre and post assessments interviews, following a process as specified by Wright, Martland and Stafford (2006). The simultaneous and retrospective data analysis methods are described below in the relevant sections.

3.2.1 Phase 1 - preparation for the teaching experiment

I shall adopt the metaphor of a journey in my description of this approach to learning, following Gravemeijer and van Eerde (2009) and Simon (2009). If the learners' current cognitive structures are the starting point of their journey, then the instruction goal is the intended destination. To extend the metaphor, the anticipated local instruction theory becomes the map. Teachers and researchers can then ask "which route will the learners take?". Wright and colleagues refer to "learning pathways", similar to the learning trajectories of the design research paradigm. They suggest that "the LFIN provides a blueprint ... and indicates the likely paths for children's learning" (Wright, Martland & Stafford, 2006, p. 8). During the description that follows, the nomenclature of both design research and the MR programme will be defined and used.

3.2.1.1 Research context

Recall that design research addresses a complex problem in a real context (DBRC, 2003). The complex problem addressed in this study is that of progression in early arithmetic strategies, as introduced in Chapter 1. The context for the research was an out-of-school teaching experiment, the ethos of which was inspired by the SANC project after-school maths clubs. Participants numbered 18 and were the Grade 2 learners from one class of a rural government primary school in the Southern Province of Zambia. The learners were aged seven turning eight, and this was their second year of formal schooling. The study was conducted in the second of three terms, half way through the school year. The teaching experiment was conducted after school once a week, for seven weeks. The sessions were of one-hour duration. To a certain extent, this school was an opportunity sample, as the school is a local one to me, and one that I had had contact with prior to this study as part of my work in primary education in the Southern Province of Zambia.

Collaboration with the class teacher was a key feature of my study, given the aim of investigating the possibility of adapting the MR programme for use in real group/whole class settings in Grade 2 Zambian classrooms. Having identified the school and grade for my study, I invited the Grade 2 class teacher to be involved in the study as she expressed an interest. She was present for the majority of the after-school teaching experiment sessions, apart from when circumstances prevented her from attending. During the sessions we worked together to lead small group teaching, and after each session we discussed and reflected on the learning. Such close collaboration enhanced both my learning and the results of the study. The involvement of the teacher enabled more insight into the learners' early arithmetic strategies and deeper reflection into practical aspects of adaptation of the MR programme to such a classroom context. The after-school teaching experiment was conducted in the learners' regular classroom. In this way, the classroom setting and the involvement of the class teacher enabled a close approximation to "normal" classroom teaching, yet retained the freedom from curricular and other restrains that the after-school format allows. This aligned with DBRC's (2003) criteria that design research should address complex problems in real contexts.

I decided not to focus simply on learners perceived as having 'mathematical difficulties'. At this young age, a broad spectrum of the Grade 2 class may still be at risk developing such difficulties, so it was important that the study investigated methods that could work with a range of learners. It had been my initial plan to work with a smaller group of learners in a group intervention. However, the class teacher and the head teacher encouraged participation of all class members, and the whole class expressed enthusiasm about joining in. This provided me a new and exciting opportunity to research early arithmetic strategy development in a whole class context. As a result, I refocused my aims towards development of early arithmetic strategies in a whole class intervention, which further justified inclusion of the class teacher in the teaching experiment. All 18 Grade 2 learners in the class attended the sessions at some point, although attendance was not consistent for some learners. The gender balance of the class was equal, with nine boys and nine girls. English and ChiTonga were the teaching languages used in the teaching experiment, as they were also the languages used during regular classroom mathematics lessons. The class teacher was fluent in both English and ChiTonga. I am a first language English speaker and do not speak ChiTonga. The class teacher would often translate my English instructions and questions into ChiTonga, or reiterate in English. The learners were encouraged to use any language of their choice during the teaching experiment sessions. The mother-tongue language of the majority of learners was Silozi, although all understood ChiTonga and some English (they were learning English as an additional language).

In their second year of formal schooling, the Grade 2 learners should have had full exposure to the Grade 1 syllabus, including emphasis on perceptual counting strategies (as discussed in Chapter 1). Initially, learners from Grades 1 to 3 had been considered as possible participants for my research. The selection criteria of Grade 2 learners was ultimately chosen as being the one that had the best potential to meet the goals of the research and as being an optimal point for intervention (as discussed in Chapter 2). Now in Grade 2, learners would be expected to make progress towards more sophisticated arithmetic strategies. I anticipated that it might be possible to observe the beginning of the "entrenchment" of perceptual counting at this stage. As such the development of teaching strategies to help learners make this transition would be highly relevant to this age group.

3.2.1.2 Positioning of the Researcher

Prior to the teaching experiment, I had to decide whether to act as a researcher or as a teacher/researcher in the teaching context. I decided to take on the dual roles of teacher and researcher, known as participant/observer in research terms. The participant/observer role is a complex one, being both causal and observational (Barab & Squire, 2004). Cobb (2000) suggests that collaboration with a teacher, rather than acting alone as teacher/researcher, involves a "trade off" (p. 330). In taking on the dual roles of researcher and teacher in collaboration with the class teacher, I hoped to avoid this trade off to a certain extent. Maxwell (2009) cautions the risk of reflexivity, the unavoidable mutual influence of researcher and participants. However, viewed through a Vygotskian lens, the researcher will always have an effect on the research (Smagorinsky, 1995). A key strength of the design research method is that it acknowledges this feature of classroom learning, and turns it into an advantage. As such, reflection on the impact of my role as teacher/researcher is integrated into all phases of this study.

3.2.1.3 Ethical Considerations

Before commencing my research I obtained permission from all relevant authorities (the local District Education Board and the school principal). As this research involved vulnerable participants, special care and responsibility was taken to ensure that the research was conducted in accordance with the principles of respect, dignity, transparency, honesty, accountability, responsibility, integrity, and professionalism. The parents/guardians of all participants were informed of the details of the research in a letter in English and ChiTonga. Permission was obtained from them before the commencement of the study, based on the principle of informed consent. They were informed that participation was voluntary and that their children could withdraw from the study at any time. As the primary data collection method used in this research was video recording, special care was taken to ensure the privacy of all participants. Permission was sought from parents/ guardians for the use of photos and videos, and due steps were taken to obscure the identities of the participants in the write up. My invitation to the class teacher explained the purposes of my research. Anonymity and confidentiality were discussed, as was her right to withdraw at any stage of the research. Rhodes University ethical clearance was furthermore obtained.

3.2.1.4 Establish Conjectured Local Instruction Theory

As stated above, the aim of Gravemeijer and van Eerde's (2009) approach to design research is the development of an improved local instruction theory. In this study, the improved local instruction theory was the adaptation of the EAS and IFEN for use in a Grade 2 Zambian classroom context. In preparation for the teaching experiment, I adopted the EAS aspect of the LFIN, along with the corresponding key topics from the IFEN, as the conjectured local instruction theory. This was the theory of progression in early learning strategies that I would test empirically during the teaching experiment. In the metaphor of a journey, the EAS and the IFEN were the maps that I used to plan the learners' journeys towards more sophisticated early arithmetic strategies.

As noted above, there are many parallels between the MR programme and design research. The first three of the four steps of Wright, Martland Stafford and Stanger's (2006) teaching and learning cycle stated in the form of questions map to the three steps of the preparatory phase of design research (in brackets below):

1. *Where are the students now? (cognitive starting point)*
2. *Where do I want them to be? (instructional goal)*
3. *How will they get there? (hypothetical learning trajectory)*
4. *How will I know when they get there?*

This study adopted the LFIN as a conjectured local instruction theory, investigating the adaptation of the MR programme to a different context. The actual learning trajectory, the learning route or pathway taken by the children in the class may differ from the hypothetical trajectory. In comparing the two, new insights into the children's thinking and learning may emerge, and this may contribute to the development of an improved local instruction theory. Each of these steps will now be described in more detail.

3.2.1.5 Where are the students now?

Having established the underlying research framework, the design researcher must then identify the assumed cognitive starting point of the learners (Gravemeijer & van Eerde, 2009). This corresponds to the first question of the MR programme's teaching and learning cycle, "where are the learners now?" (Wright, Martland, Stafford & Stanger, 2006. p. 52).

Wright, Martland and Stafford (2006) and Gravemeijer and van Eerde (2009) advocate the use of video-recorded individual interviews to establish an assumed starting point for learners. In this study, video-recorded assessment interviews based on the MR programme's assessment interview schedules were conducted individually with six sample learners prior to the commencement of the teaching experiment. The videos of the assessment interviews were then analysed against the EAS in order to establish an initial EAS stage for each learner. See below for details of the interview schedule and analysis process.

Given that I was using video data of assessments and interventions, I decided to select a sample of six learners from the whole class of 18 in order to keep the amount of data manageable. Prior to the commencement of the teaching experiment, I selected six learners that I thought might be representative of the class for the video-recorded pre assessment interviews. Having administered the assessments with these six learners and then reviewed the video to make stage judgements, the class teacher and I selected five of these learners for the study sample. One female learner at Stage 3/4 was deselected (as there were too many representatives of this stage and gender) and an additional male learner from the lower stage was selected and interviewed. This resulted in a sample of six learners whom the class teacher considered to be representative of the class of 18 in terms of gender and mathematical performance. Of the six learners, three were male and three female. There were two learners representative of each of the three mathematical performance levels identified in the class. In order to preserve anonymity, the six sample learners were given a pseudonym: Hendrix, Charles, Memory, Kamwi, Grace, and Mutinta. The sample learners were all aged seven turning eight, apart from Grace, who was eight turning nine. The whole class took part in the after-school teaching sessions, and in my dual role as teacher/researcher I planned for progress of all learners. However, it was the six sample learners who were the focus of data collection and analysis in this study.

The interview schedule was derived from Assessment Interview Schedule 1.1 in Wright, Martland and Stafford (2006). Questions 8 and 9 of Assessment Interview Schedule 1.1 were extracted in their entirety as these were the questions identified by Wright, Martland and Stafford (2006) as being the "significant tasks" (p. 51) for identifying EAS. The assessment interview schedule used in this study thus consisted of a possible total of 27 addition and subtraction problems involving screened and unscreened collections, and is included as Appendix B.

The interviews with the six sample learners were administered and videoed following the recommended method as described in detail in Wright, Martland and Stafford (2006). Limited adaptations were made. Stones were used instead of counters, as these were the available equivalents. As a result, there was no colour coding as advised by Wright and colleagues. The learner and I sat side by side on a bench, in front of the desk, rather than behind it. See Figure 13 in Chapter 4 for a video-still showing the assessment set up. I made this decision because the learners' hands would potentially be hidden from the camera if the learners were seated behind the desk. It was planned that the class teacher should be present for the initial assessment interview, as translator and interpreter where necessary. However, she was unable to do so for personal reasons. As a result, I alone conducted the pre interview in English. This was a limitation in that it may have affected the learners' willingness or ability to describe their strategies. However, it did not seem to influence their understanding of the assessment tasks themselves as English was one of the languages used in classroom mathematics instruction.

Although other researchers in the SANC project (Stott, 2014; Wasserman, 2015) allowed use of pencil and paper during MR programme assessment interviews, I decided to conduct the assessment interviews without pencil and paper. Their distinction from traditional pencil and paper tests is a defining feature of feature of the MR programme's assessment interviews. The original focus of my study had been the use of tally marks as an early arithmetic strategy. Making the decision to exclude pencils from the assessment interviews meant there would be no data on written tally-based strategies from the pre and post assessment interviews. I decided to take this opportunity to explore the strategies used by the six sample learners when pencil and paper weren't available and I reasoned that the teaching experiment sessions themselves would provide opportunities for evidence of written tally use, as pencil and paper would be freely available then.

The nature of the MR programme assessment interview means that not all questions are asked of each learner, as the set of questions asked is dependent on each learner's initial responses. To make the interview easier to administer I created an interview flowchart based on the instructions in Wright, Martland and Stafford (2006).

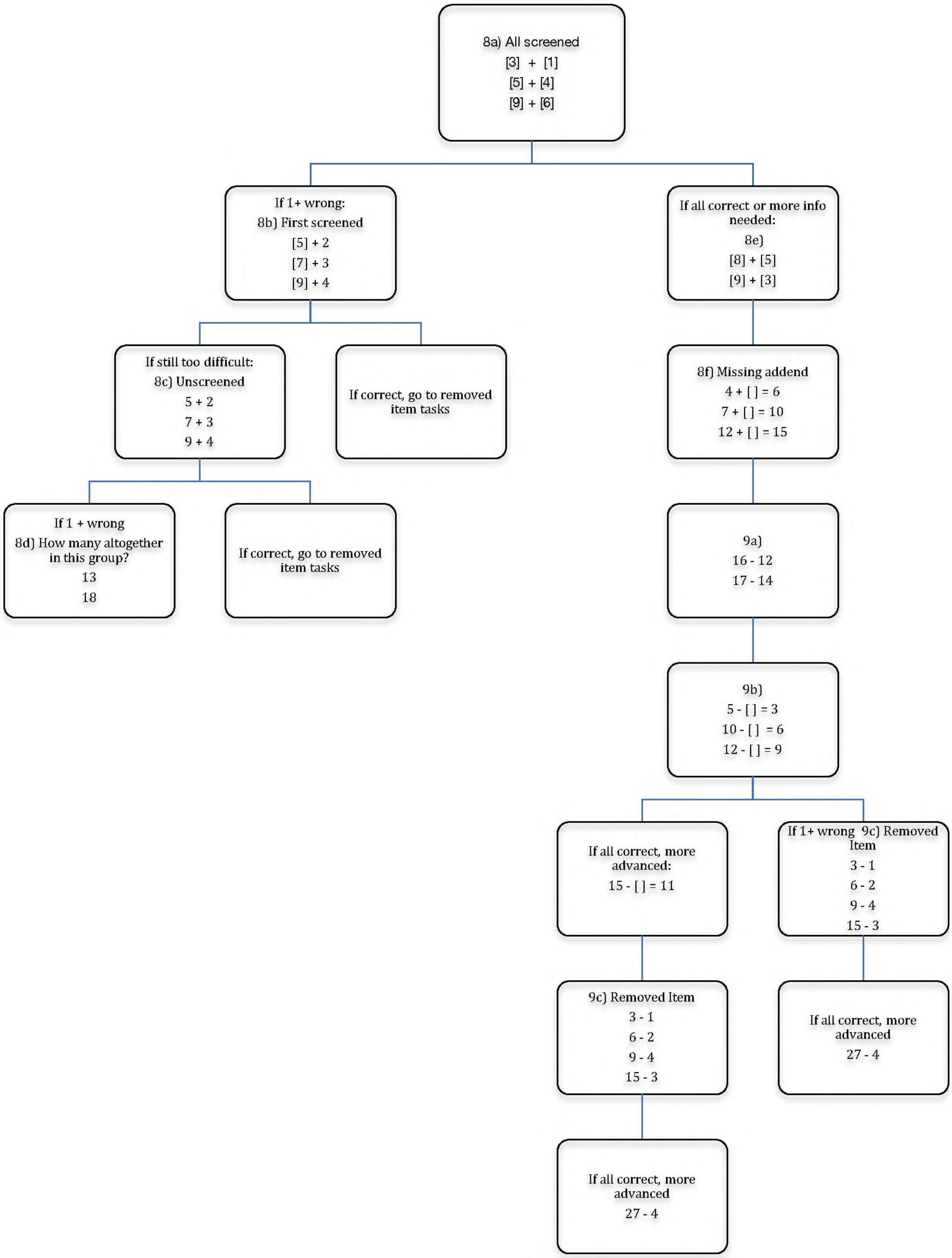


Figure 3 Flowchart of Assessment Interview Schedule 1.1
(adapted from Wright, Martland and Stafford, 2006, p. 159)

The video recordings were analysed against the LFIN framework and an EAS stage judgement was made based on the most advanced strategy used by the learner. This provided the class teacher and me with a starting point for each learner's learning journey. The video of each interview was reviewed and the interview schedule annotated using the MR programme coding schedule provided by Wright, Martland and Stafford (2006). Specific observations of verbal responses and use of fingers were also noted.

Wright, Martland and Stafford note that "analysis of Mathematics Recovery assessments involves a good deal of learning on the part of teachers" (2006, p. 75). As this was my first experience of administering the assessment, I reviewed the videos several times in order to review and secure my judgements. Wright, Martland and Stafford provide extensive guidance about, and examples of, how to make an EAS stage judgement. Nevertheless, I found it quite difficult to make EAS stage judgements for the two children who seemed to be on the borderline between stages. It was difficult to make a judgement as to what strategy a child had used if they made no obvious mouth or finger movements and if, when questioned, they did not explain what they had done.

Consequently, when I systematically reviewed the video recordings of the pre and post assessment interviews during the retrospective analysis phase, I found it helpful to develop a stage judgement checklist, compiling all the instructions from Wright, Martland and Stafford (2006) in one central reference document (see Appendix E). In the retrospective analysis section below I describe in more detail the resultant checklist and how it helped me to make stage judgements.

Five small group focused teaching groups were established for the whole class teaching experiment, centered on the six sample learners. The class teacher used her experience of the learners' mathematical performance to sort the rest of the class into stage groups, based on the judgements of the EAS stages of the six sample learners. In order to create manageable group sizes for teaching, the Stage 1 and Stage 3 /4 learners were spilt into two groups each. This resulted in eight class learners assigned to two Stage 1 groups (three girls and five boys). Three class learners were assigned to a Stage 2 group (one girl and two boys). Seven class learners were assigned to two Stage 3 /4 groups (five girls and two boys). In the MR programme Stage 3 and 4 learners are classified together for teaching (Wright, Martland, Stafford & Stanger, 2006). No learners were judged to be at Stage 0 (emergent counting) so no Stage 0 group was created.

Before teaching started, I took photographs of the sample learners' school mathematics books, looking for evidence of early arithmetic strategies, including use of tallies. I discussed the learners' current strategy use with the class teacher, to provide additional information to support the stage judgements.

3.2.1.6 Where do I want them to be?

Having established the assumed starting points of the learners, the next step of the preparatory phase was to consult the "map" (the EAS and IFEN in this study) in order to set instructional goals for each of the stage groups. The general goal of the MR programme is for all children to develop facile arithmetic strategies, that is, to progress to Stage 5 of EAS. Progression across stages "involves the child using increasingly sophisticated ways to solve number problems" (Wright, Martland & Stafford, 2006, p. 9) on their step-by-step journey up the stages of EAS.

Based on the duration of my study, I decided that the instructional goal of each group would be progression by at least one stage. Such progression could be indicated by an improvement in the EAS stage of the learners.

For those learners assessed at Stage 1, the instructional goal was for them to progress from perceptual counting to counting screened objects from one. For those learners judged to be at Stage 2, the goal was to progress from counting-from-one to counting-on for addition, and counting-back for subtraction. The MR programme suggests that teaching of a learner at Stage 3 should focus on advancing the learner directly to Stage 5, and that it is "not considered crucial to focus teaching on the development of counting-down-to" (p. 67). Therefore the goal for learners at Stage 3 and 4 was to progress from counting-on and back to using a range of facile, non-counting-by-ones strategies.

I decided to focus on progression in strategies for addition, and to set intermediate targets given the limited timescale of the study and the likely MR progression rates (discussed in Chapter 2). These were taken from the "way forward" guidance in Wright, Martland, Stafford and Stanger (2006). The instructional goals and intermediate targets for each group are summarised in Table 2.

Table 2 Instructional goals and intermediate targets

Stage Groups	Starting point	Intermediate targets	Instructional goal for the teaching experiment (as EAS stages)
Stage 1	Perceptual counting (EAS Stage 1)	Consolidate finger patterns up to 5, 10.	Counting-from-one (EAS Stage 2)
Stage 2	Counting-from-one (EAS Stage 2)	Combine and partition numbers in range 1 to 10 in settings involving spatial patterns.	Counting-on and back (EAS Stages 3/4)
Stage 3/4	Counting-on and back (EAS Stages 3/4)	Increment by 10s and 1s on and off the decade. Add one to nine to / from a decade number.	Facile, non-counting-by-ones (EAS Stage 5)

3.2.1.7 How will they get there?

The final step in the preparatory phase of a design research study is to establish the path learners are expected to take as they progressed from their starting points toward the instructional goals. Simon (1995) terms these anticipated cognitive activities the "hypothetical learning trajectory" (p. 135). From a constructivist view of learning, teachers must anticipate the mental activities of learners whilst simultaneously considering the learning goals and how the two are related (Gravemeijer & van Eerde, 2009). Gravemeijer and van Eerde term this initial reflection the "anticipatory thought experiment". The key to the design research process is the researcher's understanding that the learning may not actually follow this hypothetical trajectory, which will be empirically tested in the teaching experiment phase.

The IFEN and the accompanying guidance in Wright, Martland, Stafford and Stanger (2006) fill in the detail of progression from one stage to another. Combined with the EAS, they provided the hypothetical learning trajectory for this study. There is progression within the activities of each key topic in Wright, Martland, Stafford and Stanger (2006). For example, the gradual distancing of the setting as counters are progressively screened from the learner's view in an addition task. Table 3 below indicates the hypothetical learning trajectory for each group, expressed as the associated bank of planned instructional activities.

Table 3 Hypothetical teaching and learning trajectories (expressed as key topics from Wright, Martland, Stafford & Stanger, 2006).

Stage 1 Group	**Stage 2 Group**	**Stage 3 /4 Group**
Key Topic 6.3 Figurative Counting Key Topic 6.4 Spatial Patterns Key Topic 6.5 Finger Pattern	Key Topic 7.3 Counting-on and Counting-Back Key Topic 7.4 Combining and partitioning Key Topic 7.5 Partitioning and Combining Numbers in the range 1 to 10	Key Topic 8.3 Incrementing by Tens and Ones Key Topic 8.4 Adding and Subtracting to and from decade numbers Key Topic 8.5 Addition and Subtraction to 20, using 5 and 10

Having designed and collected data for the preparatory phase, I was now ready for the teaching experiment, which was the second phase of my research.

3.2.2 Phase 2 - teaching experiment

The aim of the second phase of design research is to test and improve the conjectured local instruction theory so that it more accurately reflects the actual learning pathways taken by the learners. In the teaching experiment phase of my study, the EAS and IFEN (as the conjectured local instruction theory) were empirically tested in the context of the after-school teaching experiment. The aim of the teaching experiment was to adapt the EAS as necessary so that it more accurately reflected the actual learning pathways taken by the learners. In design research, the teaching experiment consists of an iterative cycle of design, testing, and revision, which serves the purpose of refining and adjusting the local instruction theory. To continue the journey metaphor, the map was annotated to more accurately reflect the path taken.

The whole class teaching experiment intervention sessions were held once a week for seven weeks, outside of class time after-school. These teaching experiment sessions lasted an hour. There were 18 participants, organised into five small teaching groups of three to five learners, as described above. The six sample learners were distributed amongst these five groups, so that each group contained at least one of the six sample learners.

The model for the teaching experiment session format was that of the SANC project maths clubs. Socio-cultural norms were introduced in Session 1, including listening to one another's ideas and not laughing at mistakes, but rather using mistakes as learning opportunities. In each hour-long session the class learners played pair and small group mathematics games whilst the class teacher and I moved from group to group leading focused teaching activities. All small-group focused teaching was video-recorded with the video focusing on the six sample learners. Figure 4 is a still from the video recording of a small group focused teaching session during Session 3 and illustrates the classroom set up and use of video recording.

Focused teaching lasted an average of seven minutes per group per session, varying between four and twelve minutes (source: teaching experiment video recordings). I led the first small group teaching activities, with the class teacher watching and translating certain instructions into ChiTonga. Once the class teacher felt confident with an activity, she would take over the teaching and I would watch and make observations. We created a folder of activities and a box of resources to use, and during the teaching experiment sessions we would either work separately, leading small group teaching with different groups, or we would work together with the same group. The transcript in Hendrix's case study in Section 4.3.1 contains an example of how we worked together. Working together provided opportunities for sharing of observations, for analysis of learners' strategies, and for reflection on teaching techniques, whilst our working independently enabled each group to experience more focused teaching time. Figure 4 below shows us working independently with different stage groups whilst the remaining groups played mathematics games.

Figure 4 Still from video recording of small group focused teaching with Grace's Stage 3/4 group during teaching experiment Session 3

Table 4 below provides an overview of how the teaching experiment and the pre and post assessment interviews were implemented in terms of timing, participants, researcher role, and data collection.

Table 4 MR programme teaching experiment implementation plan

Date	Event	Participants	Researcher/ teacher involvement	Teaching activities	Research activities	Data obtained	How does data collected inform research?
22/06/2015	Pre assessment interview	Six sample learners	Researcher conducting assessment interviews	No teaching or prompting, hence all strategies observed were spontaneous	Individual assessment interviews video-recorded	1. Transcription of all six individual interviews 2. Annotated Assessment interview Schedule (*see Appendix B*) 3. Each instance of spontaneous strategy use coded and entered into database (*see Figure 8*)	Baseline EAS stages for judgement of EAS progression *(see Section 4.1)* Data on spontaneous strategy use to address Research Question 2 (*see Section 4.2*)
Every Thurs from 25/06/2015 to 06/08/2015	Session 1 and 2	All class learners in mixed groups	Session 1: Researcher led (class teacher absent) Session 2: researcher and class teacher leading small group focused teaching	Session 1: introducing socio cultural norms. Session 2: Small group focused teaching began midway through Session 2. Refer to Table 5 for details of intervention activities	Video recording of all small group focused teaching, with camera centred on sample learners (*see Figure 4*) Audio recording of post session discussion with class teacher Research journal	1. Transcription of video of all instances of spontaneous strategy use before small group focused teaching began (*see Section 3.2.3.2 below*) 2. Transcription of audio recordings of teacher researcher discussions. 3. Refection in research journal 4. Teaching experiment planning sheet	Data on spontaneous strategy use to address Research Question 2 (*see Section 4.2*) Data on teaching and learning to address Research Question 3 - adaptation of MR programme to whole class context *(See Section 4.3)*
	Session 3 to 7	All class learners in EAS stage groups	Researcher and class teacher leading small group focused teaching (Class teacher absent Session 5)	Class learners At least one sample learners in each stage group. Refer to Table 5 for details of intervention activities	Video recording of all small group focused teaching with camera centred on sample learners.	1. Transcription of selected excerpts from videos of small group focused teaching for three case studies (*see Section 3.2.3.2 below*) 2. Transcription of audio recordings of teacher researcher discussions 3. Refection in research journal 4. Teaching experiment planning sheet	Data on teaching and learning to address Research Question 3 - adaptation of MR programme to whole class context *(See Section 4.3.)*
7/08/2015	Post assessment interview	Six sample learners	Researcher conducting assessment interviews	No teaching or prompting, hence all strategies observed were spontaneous	Individual assessment interviews video-recorded	1. Transcription of all six individual interviews. 2. Annotated assessment interview Schedule (*see Appendix B*). 3. Each instance of spontaneous strategy use coded and entered into database (*see Figure 8*).	Comparative EAS stages for judgement of EAS progression *(see Section 4.1)* Data on spontaneous strategy use to address Research Question 2 *(see Section 4.2)*

Gravemeijer and van Eerde (2009) identify 4 steps of an iterative design cycle: *planning, enactment, observation and analysis,* and *evaluation,* shown in Figure 5 below. In this study, the research focus for this iterative cycle was mainly the six sample learners, although in my dual role as teacher/researcher I tried to take into account the learning journeys of all learners in the class. Data collection was simultaneous with analysis during the teaching experiment, given the aim of refining the local instruction theory.

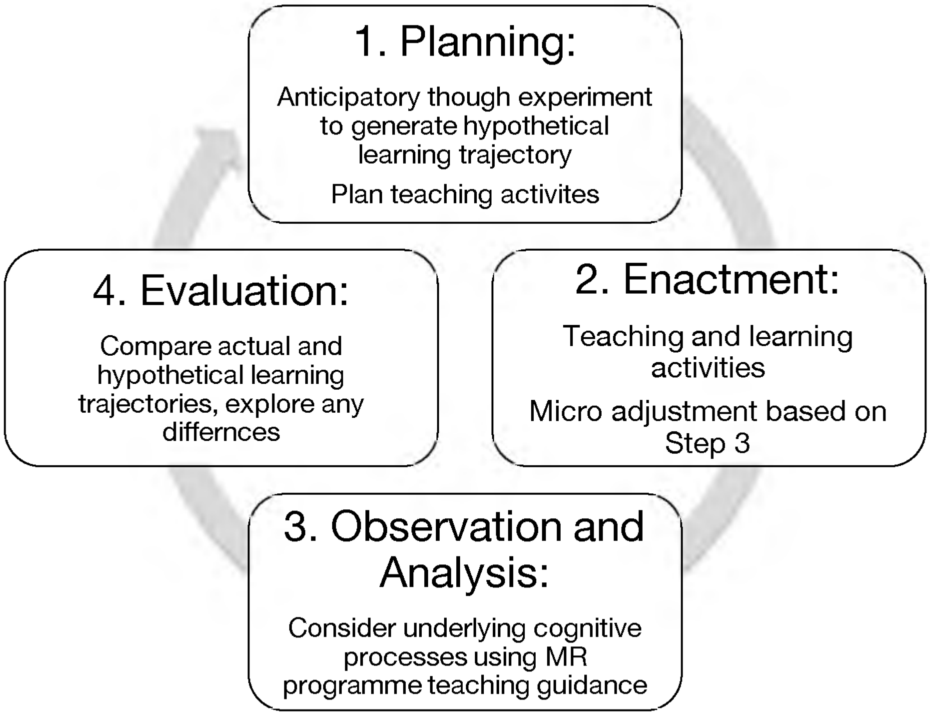


Figure 5 Iterative design cycle used during the teaching experiment phase of study

Gravemeijer and van Eerde (2009) state that it is essential to carefully document what is going on in the classroom and the rationales for making conjectures and revisions (p. 516). To this end, I created a teaching experiment planning sheet with space to record each step of the iterative micro design process. Following each teaching experiment session I completed this document with data and analysis from observation, teacher discussion, and video recordings, as well as my ongoing reflections in my research journal. A worked example from the case study of Hendrix is presented here (see Figure 6) to illustrate how the iterative design cycle worked and as an example of a completed planning sheet. The template is attached as Appendix D.

Session: 2 Group: Green	Child 1 Hendrix
1. Planning: Previous session evaluation: H and C relying on fingers as perceptual replacements for perceptual counting. Dot patterns not yet secure. Planned Activities: 5.4.1, 5.5.3, 6.5.1 **HLT:** Leaners will… *Learners will…* Develop more facile finger and dot recognition patterns, to enable progress to figurative strategy	**2a. Observations:** Dot pattern recognition improving. Had been playing dominoes for the rest of the session. Spatio-motor patterns less secure. Misses dots, counts without pointing, inaccurate patterns. **2b.** Analysis use MR tool) Visualised dot patterns not yet secure.
3. Evaluation: Use flashing of dot pattern cards to develop visualisation. Try using dot cards instead of stones for screened collection tasks?	

Figure 6 Sample of completed teaching experiment planning sheet from Hendrix's group, Session 4

The first step of the iterative design cycle, *planning*, consists of an anticipatory thought experiment. Prior to each teaching experiment session, observation and assessment information from the previous session was reviewed to identify each learner's starting point. As in the preparatory phase, an anticipatory thought experiment was then conducted to generate a hypothetical learning trajectory for the next teaching experiment session, derived from the conjectured local instruction theory (EAS and IFEN). I then selected a set of teaching activities from the bank of activities (see Table 3 above).

For example, following analysis and reflection on Hendrix's strategy use in Session 2, my anticipatory thought experiment led to the hypothetical learning trajectory that developing facile finger patterns might help advance Hendrix's strategies in Sessions 3 and 4. I recorded this in the teaching experiment planning sheet (see example Figure 6 above).

The second step of the iterative cycle is the *enactment* of the planned teaching and learning activities in the learning context. At this stage the researcher must endeavour to forget the hypothetical and react to what is actually going on in the classroom (Steffe & Thompson, 2000). All small-group focused teaching was video-recorded with the video focusing on the six sample learners. Teaching principles from the MR programme guided the teaching. In this phase of design research the teacher is constantly adjusting the instructions and tasks in response to their observations and analysis of the learners' learning. This is similar to what Wright and colleagues refer to as "micro adjusting" (Wright, Martland, Stafford & Stanger, 2006, p. 31).

The third step of the iterative cycle involves the simultaneous *observation* and *analysis* of learners' actions and hypothesising about underlying cognitive process. In this study, observations of learners were analysed against the LFIN and MR teaching guidance. There is extensive guidance for MR teaching in Wright, Martland, Stafford and Stanger (2006) for both individual and whole class teaching, most of which can also be applied to small group teaching. Having analysed the two sets of guidance and identified common themes I created a mind map tool in order to facilitate this planning, observation, and analysis (see Appendix C for the mind map tool). During the small group focused teaching, micro-adjustments, such as screening a collection or decreasing the magnitude of the numbers involved, were made on the basis of this observation and analysis.

For example, during Session 4 I observed that Hendrix's facility at dot pattern recognition had increased, supported by playing dominoes and dice-based bingo when not participating in small group focused teaching. Hendrix was less successful when making spatio-motor patterns to match dot patterns. He often missed dots, counted without pointing, and made inaccurate pattern shapes. Again, I recorded this in the teaching experiment planning sheet (see Figure 6 above).

In the final step of the iterative cycle this analysis of the actual and hypothesised learning trajectories is *evaluated*. If the child did not take the anticipated learning pathway, one asks how did their path vary? In my study the evidence from observation and video recording of actual learner processes and progress was compared to the hypothetical learning trajectory (the EAS and IFEN). The mind map analysis tool developed from the MR programme teaching guidance was used to facilitate such comparisons. Any differences were noted and this information was then fed back to inform the preparation for the next teaching experiment session.

Following each teaching experiment session, the teacher and I shared and analysed our observations and then reflected on the learning in a post session discussion. Audio recordings were made of our reflective discussions. I did not review and transcribe these audio recordings until the retrospective analysis phase. However, the discussions themselves served as a reflective tool for the iterative cycle process. I noted relevant items from these discussions in the teaching experiment planning sheet (see Appendix D). I had full audio recordings from all five teaching experiment sessions at which the teacher was present (apart from Session 3 when there was a software error during the recording). Immediately after this conversation I made an audio research journal recording recalling what was said as best I could. In retrospect, I have learnt that a backup recording would be of benefit if resources allow for this.

Following each teaching experiment session, I noted my observations, and those of the class teacher, into my research journal. I then reviewed the video recordings and made further observations and analysis. The resulting analysis and reflection was entered into the teaching experiment planning sheet in order to facilitate selection of the next session's teaching activities, and so the cycle continued.

For example, following analysis and reflection on my and the teacher's observations of Hendrix, I noted my plans to use flashing of dot patterns to facilitate visualisation, and scaffolding to enable new learning to be extended into the problem-based scenarios (see Figure 6 above). I adapted task 6.3.3 to use the dot cards instead of collections of stones / counters in order to scaffold the development of a visualising based strategies.

3.2.2.1 Actual teaching sequence

Teaching activities for the small group focused teaching were chosen from the bank of suggested activities in the MR programme. A possible bank of activities was identified during the preparation phase, but specific activity choices were made prior to each teaching experiment session, based on evaluation of the previous sessions. Table 5 summarises the actual sequence of small group focused teaching activities for each stage group.

Table 5 Actual sequence of focused small group teaching activities

Session	Stage 1 Groups	Stage 2 Groups	Stage 3 Groups
1	Intro session, no small group teaching, games only		
2	6.3.1[4] Counting items in two collections with first collection screened 6.3.2. Counting items in two collections with second collection screened	7.3.1 Counting items in two screened collections	8.4.1 Adding from a decade
3	5.4.1 Ascribing numerosity to patterns and random arrays 5.5.3 Simultaneous patterns for one to five, fingers seen 6.5.1 Five plus patterns for six to ten.	7.4.1 Combining numbers to five 7.4.2 Partitioning five	8.4.2 Subtracting to a decade
4	5.5.3. Simultaneous patterns for one to five, fingers seen (adapted – flashed dot cards up to six) 5.4.2 Making spatio-motor patterns to match spatial patterns 5.4.3. Making auditory patterns to match spatial patterns 5.5.7 Using fingers to keep track of temporal sequences of sounds	7.4.3 Combining five and a number in the range one to five	8.4.3 Adding to a decade: one to five
5	5.4.1. Ascribing numerosity to patterns and random arrays 6.3.3. Counting items in two screened collections, (adapted to dot cards, spatio-motor emphasis)	7.4.4 Using five to partition numbers in the range six to ten 7.3.1 Counting items in two screened collections adapted to use screened ten and five frames, then combined with objects	8.4.4 Subtracting from a decade: one to five
6	5.4.2. Making spatio-motor patterns to match spatial patterns 5.5.3. Simultaneous patterns for one to five, fingers seen (adapted – flashed dot cards up to six) 6.5.1 5 plus patterns for six to ten. 6.3.3. Counting items in two screened collections, (adapted to dot cards, fingers emphasis) SANC Number Talks Prompts: Dot Cards	7.3.1 Counting items in two screened collections adapted to use screened ten and five frames, then combined with objects	Counting by ten FNWS BNWS 8.4.5 Adding to a Decade: six to nine Flashing and covering from ten - familiarity without counting.
7	6.1.1 Saying short FNWS (not from one) 6.1.2 Saying short BNWS SANC Number Talks Prompts: Dot Cards	7.3.4 Removed Item Tasks 7.4.5 Combining Numbers to ten SANC Number Talks Prompts: Dot Cards	8.5.2 Building numbers to six 8.5.6 Adding by going through ten adapted to ten frames Bridging through ten with empty ten frames and counters/ beans? SANC Number Talks Prompts: Dot Cards
Resources used in the teaching experiment: stones, seeds, bottle tops (as counters), ten frames – printed and hand drawn, five frames- printed and hand drawn, dice – plastic, dominoes – plastic and printed paper.			

A range of dice, domino and paper based games were available for learners to play during each

[4] Numbers refer to activities in Wright, Martland, Stafford and Stanger (2006)

teaching experiment session when they were not involved in small group focused teaching. The games were taken from the group/individual activities in the whole class section of Wright, Martland, Stafford and Stanger (2006) and from the SANC project activities booklet (SANC, 2015). During the first teaching experiment session, I observed that most learners were unfamiliar with the games which included bingo, snap, and dice games, and some were unfamiliar with the dice themselves. As a result, I introduced new games gradually across the seven weeks. Certain aspects of the games were adjusted during the teaching experiment to facilitate progression. For example, I used dice with larger numbers or replaced dot dice with numeral dice.

3.2.2.2 Resources and settings

Given the study's aim of investigating the adaptation of the MR programme to a context where resources may be limited, I tried to use locally available materials as far as possible. At the start of my study the following were available in the classroom: a small collection of large plastic bottle tops, wall displays, and a few interesting devices of the teacher's own construction. I had anticipated that there might have been a large collection of metal bottle tops, as had been my experience in other schools. However the class teacher was using small stones from the gravel path outside the classroom to serve as counters. Following our discussion, the teacher brought a collection of seeds for the next teaching experiment session. The materials were left with the teacher after the sessions when she was present. When I did not leave them after Session 5, she expressed disappointment that they had not been available during the week. During our discussions the class teacher repeatedly mentioned the lack of resources at her school as a barrier to mathematics teaching and learning.

Rows of dots for engendering counting-on were printed but not used. No equivalent was found for the Arithmetic Racks, which are two rows of ten moveable beads (Wright, Martland, Stafford & Stanger, 2006). I thought it might have been possible to improvise them using seeds instead of beads. The class teacher also suggested that two traditional counting games played with stones could potentially be developed into activities, but unfortunately we did not explore that idea further.

Each learner was given an exercise book for the duration of the teaching experiment, which they were free to use as they chose for jotting, workings out, etc. Photographs were taken of these books as well as worksheets used, before they were then returned to the learners to take home.

3.2.2.3 Post Assessment Interview

The teaching experiment culminated in a second assessment of the six sample learners. The post intervention assessment interviews were all conducted on the day following the final teaching experiment session, seven and a half weeks after the pre assessment interviews. The same format and method were used as for the pre assessments, as detailed above in Section 3.2.1. Although the class teacher was available for the post assessment, I conducted the post interviews to ensure consistency across the two. The set of questions asked varied slightly due to the flowchart nature of the assessment interview schedule, as indicated in Figure 3 above. The location of the interview was changed from the library to the empty Grade 2 classroom, which had better light for video recording. The post assessment interview videos were analysed using the same method as detailed above. While I was conducting the post assessment interviews, I did not feel that the learners' performance reflected the progress I had observed during the teaching experiment. However, all data from the pre and post interviews was then systematically analysed during the retrospective analysis phase, and subtle progress was revealed.

3.2.3 Phase 3 - retrospective analysis

The goal of the final phase of a design research experiment varies according to the goal of the particular research study (Gravemeijer & van Eerde, 2009). The main aim of this study was the re-construction of an improved local instruction theory of progression of early arithmetic strategies in a Grade 2 whole class context.

As indicated above, data analysis was simultaneous with data collection during the teaching experiment phase in order to facilitate the revision of the conjectured local instruction theory through iterative micro design cycles. However, the retrospective analysis phase "creates the opportunity for a more thorough and systematic analysis of the same data" (Gravemeijer & van Eerde, 2009, p. 514). Returning to our journey metaphor, it is as if we now redraw the map based on what was observed during the journey. This 'new map' is the improved local instruction theory, an empirically grounded theory of learning and teaching in a specific mathematics topic. This is the primary output of the design research process. In the retrospective analysis phase of this study, each of the three research questions was addressed in turn, within the context of the reconstruction of an improved local instruction theory for development of early arithmetic strategies. In design research, this theory should be shareable (DBRC, 2003) and testable in itself, thus a macro cycle of experimenting emerges (Gravemeijer & van Eerde, 2009). The outcome of the design research may be a new set of hypotheses for further testing, and so the cycle of educational improvement continues. In this study, the resultant theory is a shareable set of suggestions for the adaptation of the MR programme to a whole class context, with recommendations for areas for further investigation. The retrospective analysis process for each question will now be considered in turn.

3.2.3.1 Research Question 1 - What progress (if any) do learners make in early arithmetic strategies using the Mathematics Recovery programme approach?

In addressing Research Question 1, I conducted a retrospective analysis of the pre and post assessment interview data to compare the early arithmetic strategies of the learners before and after the teaching experiment. The following indicators were used to establish this progress: EAS stage, strategy spectra, and frequency of correct responses.

During the retrospective analysis phase, all of the six sample learners' responses to each problem task from the pre and post interviews were coded as per the strategy coding schedule (see Table 8 below) and entered into a database. See Figure 8 and Section 3.2.3.2 below for more detail of this process.

In the preparation and teaching experiment phases, I had analysed the video-recorded pre assessment interviews against the EAS of the LFIN framework following Wright, Martland and Stafford's (2006) extensive guidance. Wright and colleagues specify that EAS stage judgements are made based on the *most advanced strategies* available to the child. The frequency of strategy use should also be taken into consideration, but this varies with task type and the specific stage in question. Only *effective strategies* should be considered, apart from those tasks where a failure to answer a question is indicative of a stage.

As I mentioned earlier, this was my first experience of the MR programme, and as experienced by other students (Weitz, 2012; Wasserman, 2015) I found it quite difficult to make confident judgements using the guidance provided by Wright, Martland and Stafford (2006). To facilitate my judgements I developed a stage judgement checklist of criteria that I then used to secure my EAS stage judgements for the six sample learners in the retrospective phase.

I was then able to apply the stage judgement criteria to the database by applying the following filters: child name, observation period, non-introductory tasks, and correct responses only. Then I made a count of the strategy stage codes (see below Table 8) and applied the following stage judgement criteria:

> There must at least two instances of the most advanced stage strategy for the learner to be judged at that stage, apart from:
>
> i. Missing subtrahend tasks, where at least one correct response meant a Stage 4 judgement, as long as the strategy used was not count-down-from;
> ii. Stage 5 strategies, where three counts were required for a Stage 5 judgement. (The EAS stage judgement checklist is attached as Appendix E).

The aim of the assessment interview was to observe the learners' most *advanced* strategy. For one reason or another, a learner may not have displayed their most advanced strategy in the assessment interview. This was the case with one of the sample learners, Memory. During the retrospective analysis, an additional search was done on the data set, this time to include cases from the pre assessment and spontaneous strategy use from Sessions 1 and 2 (before small group focused teaching began). The same search criteria were applied as above. To be consistent, this was done for all the sample learners. Memory was the only one for whom the pre assessment EAS stage judgement changed as a result.

In order to establish if there had been changes in the type of strategy used between the pre and post assessments, the responses of the sample learners were analysed as EAS stage spectra (see Figure 9 below). A number of researchers in the SANC project have used such spectra to analyse strategy progression (Mofu, 2013; Stott, 2014). For each learner a search was done on the data set for correct responses to the pre and post assessment interviews questions. The set of questions asked in the assessment interview differs slightly between the pre and post interview for each learner due to the flowchart nature of the MR assessment interview schedule as described above. To enable fair comparison between each learner's responses in the pre and post assessment interviews, a subset of "questions common to both assessment interviews" was identified for each learner. This category was added into the database and the responses coded accordingly (see Table 6). Correct responses by stage strategy code (see below) were extracted from both pre and post assessments. These were then displayed as a spectrum so that differences between the pre and post assessment interviews could be analysed. An example is given below in Figure 7. Where no progression was seen in the broad EAS strategy spectrum, a more detailed spectrum was produced based on the specific strategies used by the learners (see below).

Less sophisticated ← **Frequency of strategies by EAS stage used to correctly answer questions in the pre and post assessment interviews** → **More sophisticated**

Learner X (six common questions)	**Stage 1**	**Stage 2**	**Stage 3**	**Stage 4**	**Stage 5**	**Total**
Pre	2	1	1			4
Post		2	3			5

Figure 7 Example of strategy spectrum

For each sample learner, the total frequencies of correct responses for the pre and post assessment interviews were included in the spectrum, as well as the total number of questions asked. This provided data on the accuracy of learners' strategy use in addition to the information on the range of strategies used. For each learner only the subset of questions common to both the pre and post assessment interview was analysed as a spectra so as to enable a fair comparison. For example, if a question was asked on the post assessment interview but not in the pre, then the learner's strategy to solve that question was not included in the spectra analysis. In the example in Figure 7 above, a total of six common questions were asked to learner X in each assessment interview, of which they answered four correctly in the pre assessment interview and five correctly in the post assessment interview. The example spectrum in Figure 7 shows that learner X used more advanced strategies in the post assessment interview than in the pre assessment interview.

3.2.3.2. Research Question 2 - What strategies are used by Grade 2 learners in solving early arithmetic problems?

In addressing Research Question 2, I analysed my observations of the six sample learners against the MR programme's EAS stage framework of early arithmetic strategies. Because of the richness of the data, a wide range of comparisons could be made. I explored various options and decided to keep a strong focus on the research question. As Gravemeijer and van Eerde (2009) suggest is likely to happen, new topics of interest also emerged during the retrospective analysis and are addressed in Chapter 6 as areas for possible future research. The following analyses were selected for inclusion in the results, as they were judged to best address the research question:

All learners:

1. Overall frequency of EAS strategy use for addition and subtraction
2. Frequency of finger use by problem type
3. Tally use

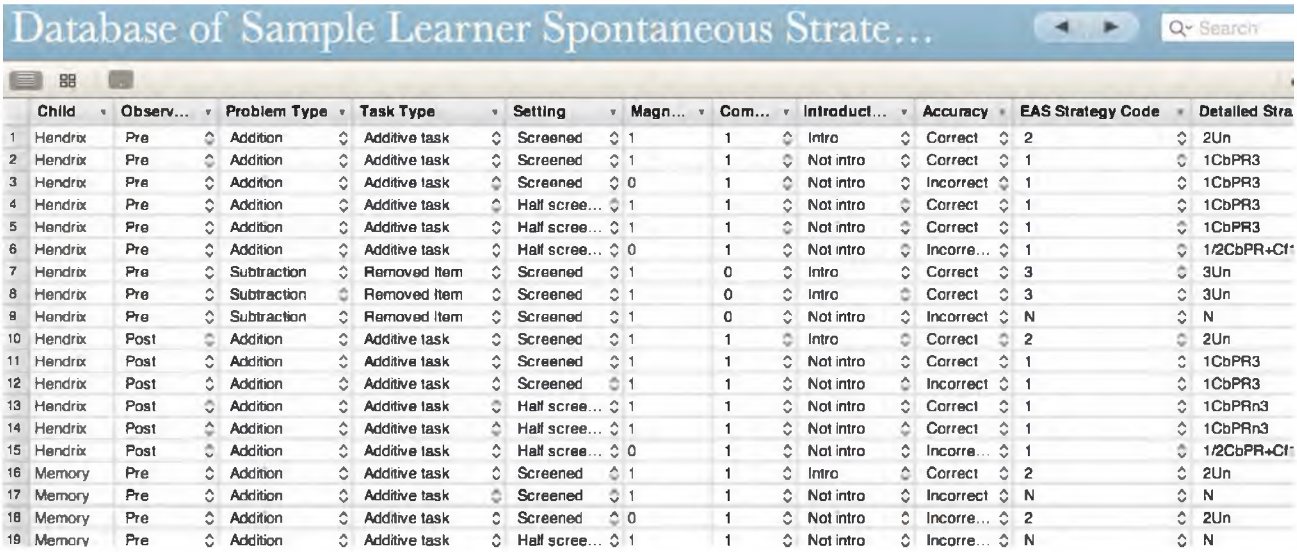
Individual learners:

1. Strategy use by problem type and magnitude of numbers
2. Finger use by problem type, strategy, and observation period

During the retrospective analysis phase, I made a comprehensive and systematic review of all video recordings of the six sample learners from both the pre and post assessment interviews and all teaching experiment sessions. Instances of spontaneous strategy use were noted, by which I mean an occasion on which a learner attempts to solve a problem "unassisted either directly or indirectly by the teacher" (Wright, Martland & Stafford, 2006, p. 9).

I reviewed the video recordings multiple times to produce a written narrative of the observations. I also transcribed sample learner utterances where they occurred. The following verbal and non-verbal indicators were used to inform strategy judgements: mouth movements (vocal and sub vocal), finger, hand movement, eye movements, and response time (Wright, Martland & Stafford, 2006). The aim of analysing these observations was to enable reflection on the underlying mental processes and conceptions of the sample learners (Gravemeijer, 2004). However, it is not possible to state with certainty which cognitive processes were involved. Strategies were more easily identifiable in some sample learners than others. For example, it was easier to identify the strategies of those learners like Mutinta who counted out loud and used their fingers.

A coding schedule evolved simultaneously with this retrospective review of the sample learners' strategy use data, so it was an emergent coding scheme. I created a database (see screen-shot in Figure 8) with categories and codes according to the emergent coding scheme. Across all six sample learners, 174 cases of spontaneous strategy use were identified from the video recordings, of which 71 were from the pre assessment interview, 71 from the post assessment interview, and 32 from teaching experiment Sessions 1 and 2 before the start of small group focused teaching.



Database of Sample Learner Spontaneous Strate…

	Child	Observ…	Problem Type	Task Type	Setting	Magn…	Com…	Introduct…	Accuracy	EAS Strategy Code	Detailed Stra
1	Hendrix	Pre	Addition	Additive task	Screened	1	1	Intro	Correct	2	2Un
2	Hendrix	Pre	Addition	Additive task	Screened	1	1	Not intro	Correct	1	1CbPR3
3	Hendrix	Pre	Addition	Additive task	Screened	0	1	Not intro	Incorrect	1	1CbPR3
4	Hendrix	Pre	Addition	Additive task	Half scree…	1	1	Not intro	Correct	1	1CbPR3
5	Hendrix	Pre	Addition	Additive task	Half scree…	1	1	Not intro	Correct	1	1CbPR3
6	Hendrix	Pre	Addition	Additive task	Half scree…	0	1	Not intro	Incorre…	1	1/2CbPR+Cf
7	Hendrix	Pre	Subtraction	Removed Item	Screened	1	0	Intro	Correct	3	3Un
8	Hendrix	Pre	Subtraction	Removed Item	Screened	1	0	Intro	Correct	3	3Un
9	Hendrix	Pre	Subtraction	Removed Item	Screened	1	0	Not intro	Incorrect	N	N
10	Hendrix	Post	Addition	Additive task	Screened	1	1	Intro	Correct	2	2Un
11	Hendrix	Post	Addition	Additive task	Screened	1	1	Not intro	Correct	1	1CbPR3
12	Hendrix	Post	Addition	Additive task	Screened	1	1	Not intro	Incorrect	1	1CbPR3
13	Hendrix	Post	Addition	Additive task	Half scree…	1	1	Not intro	Correct	1	1CbPRn3
14	Hendrix	Post	Addition	Additive task	Half scree…	1	1	Not intro	Correct	1	1CbPRn3
15	Hendrix	Post	Addition	Additive task	Half scree…	0	1	Not intro	Incorre…	1	1/2CbPR+Cf
16	Memory	Pre	Addition	Additive task	Screened	1	1	Intro	Correct	2	2Un
17	Memory	Pre	Addition	Additive task	Screened	1	1	Not intro	Incorrect	N	N
18	Memory	Pre	Addition	Additive task	Screened	0	1	Not intro	Incorre…	2	2Un
19	Memory	Pre	Addition	Additive task	Half scree…	1	1	Not intro	Incorre…	N	N

Figure 8 Screen-shot of database of sample learner spontaneous strategy use

The coding schedule consisted of five context categories, indicating what varied, and seven learner response categories indicating what was observed. See Tables 6, 7 and 8 for the complete coding schedule.

Table 6 Coding schedule - Context categories

Category	Codes	Comments
Child	1. Hendrix 2. Charles 3. Memory 4. Kamwi 5. Grace 6. Mutinta	For confidentiality purposes, I have given each learner a pseudonym.
Observation Period	1. Pre assessment interview 2. Post assessment interview 3. Teaching experiment Session 1/2	For the purpose of analysis I use the term "observation period" and include the pre and post assessment interviews, as well as spontaneous cases from Sessions 1 and 2 before small group teaching commenced. This is in order to provide a richer picture of the learners' spontaneous strategy use in a range of contexts (Dowker, 2005).
Problem Type	1. Addition 2. Subtraction 3. Perceptual Counting	
Task Type	1. Additive task 2. Missing addend 3. Removed item 4. Missing subtrahend 5. Subtraction sentence 6. Grouped Items	From the task type categories in MR programme assessment interview (Wright, Martland & Stafford, 2006).
Setting	1. Screened collection 2. Semi-screened collection 3. Unscreened collection 4. Number sentence	Terminology from MR programme. Wright, Martland, Stafford and Stanger define settings as "devices or materials which are used in posing tasks to the children" (p. 32). Screened collections are denoted in the text by square brackets e.g. [4] + 5 indicates a semi-screened addition task. Semi screened indicates that first or second collection of objects is screened. Games involving a single die I have counted as semi-screened, as only one addend is visible at any one time.
Magnitude of addend/minuend	1. ≤10 (10 or less) 2. >10	Within or beyond the finger range (Wright, Martland & Stafford, 2006, p. 46).
Introductory questions?	1. Intro 2. Non-intro	Defined in my study as problems within the finger range with addends or minuends of 1 or 2 (following Wright, Martland & Stafford, 2006, p. 34).
Common question to both pre and post assessment interview?	1. Yes 2. No	Due to the flowchart nature of the Assessment Interview Schedule the set of questions varied between each interview, so the subset of common questions was identified to enable fair comparison.

Table 7 Coding schedule - Learner response categories

Category	Codes	Comments
Accuracy	1. Correct 2. Incorrect +/-1 3. Incorrect	The code incorrect +/-1 indicates that the answer given was within one of the correct answer (either one too large or too small).
EAS Stage Strategy Code	See coding schedule in Table 8 below.	
Detailed Strategy Code	See coding schedule in Table 8 below.	
Were fingers used as part of strategy?	1. Yes 2. No	
Finger Use Code	1. PRc - as perceptual replacements, consecutive finger patterns 2. PRsc - as perceptual replacements, consecutive and simultaneous finger patterns 3. PRc - as perceptual replacements, simultaneous finger patterns 4. T – to keep track of counts 5. Subtle 6. None	Finger strategies are derived from aspect C of LFIN (Wright, Martland & Stafford, 2006). They are listed in order of increasing sophistication.
Tally marks	1. Used 2. Not used	

The main function of the coding schedule was to enable the retrospective data analysis. Strategies were coded according to the schedule in Table 8. For each instance of spontaneous strategy use both a broad EAS stage strategy code was given as well as a more detailed strategy code. Strategies are listed in order of increasing sophistication. The source of all strategy terms is Wright, Martland and Stafford (2006), apart from the counting-from-five-or-ten, which is a definition I created following my observation of a particular type of counting-based strategy (discussed in Chapter 5). Strategy codes are my own, following Wright, Martland and Stafford (2006) and Dineen (2014). In my presentation and analysis of results in Chapters 4 and 5, strategies are referred by name and by abbreviations. Note that the strategy code begins with a number. This represents the broad EAS stage with which the strategy aligns. For example, the code 3CO refers to EAS Stage 3 and "counts-on". For the coding of finger patterns I have substituted the term "consecutive" for the original term "sequential" as used by Wright, Martland and Stafford (2006) because of the alliteration with "simultaneous". Such a decision to change vocabulary was not taken lightly, as the need for a common nomenclature in numeracy (Wright, 2013) was a big consideration for me, as I discuss further in Chapter 6. The EAS stage strategy and detailed strategy categories are described in Table 8 below.

Table 8 Coding schedule – EAS strategies

Broad EAS Stage Strategy Code	Strategy Detail Code	Description of response (page numbers are first mention in Wright, Martland and Stafford, 2006)
Stage 0	N	Does not have a strategy to solve the task or seems to be guessing. Does not understand task. Code only applies to incorrectly answered problems.
Stage 1 Perceptually counts available unit items. May or may not count three times	1CP	Counts-by-ones items perceptually available (p. 56) as part of task presentation (e.g. unscreened counters). May or may not count-from-one three times (p. 28).
	1CbPR3	First builds perceptual replacements (p. 74), for example fingers or tally marks, for both collections and then counts the perceptual replacements by-ones-from-one-three-times. Fingers may be raised consecutively (p. 26) or simultaneously (p. 58).
	1CbPRn3	Build perceptual replacements for addends. Does not count-from-one three times. Raises fingers simultaneously to make facile finger patterns (p. 26) as perceptual replacements for the numbers in the problem. As a transition to this strategy, may raise fingers consecutively to build one number from one, whilst building the other simultaneously. This strategy only applies for sums within the finger range (10 or less).
Stage 2 Counts by ones, has to build up to numbers in the problem, from one or another closer point (5/10/20)	2Cf1	Counts-from-one (p. 12) by ones. May use fingers to keep track of counts (p. 19) May make one addend as perceptual replacements using fingers. The equivalent strategy for subtraction involves the learner counting-up-to the minuend, then the subtrahend, and then counting or judging the difference.
	2Cf5/10	A learner counts-from-five-ten-or-twenty to build the minuend, and then counts again to build the subtrahend. May or may not involve perpetual replacements, and counting three times. This is a strategy that I observed in my analysis of the data, and does not relate to any strategy in Wright, Martland and Stafford (2006). The closer the number counted from to the real number, the more sophisticated. This strategy was only observed for subtraction.
Stage 3 Strategies Advanced counting-by-ones	3CO 3CUT 3CDF	Counts-on by-ones (p. 22). Counts-up-to by ones. Counts-down-from by ones.
Stage 4 strategies Solves missing subtrahend tasks	4CDT	Count-down-to (p. 50) in missing subtrahend tasks.
	4CUT	Counting-up-to (p. 48) in missing subtrahend tasks.
Stage 5 Non counting-by-ones strategies	5NC1	Other, using a known fact (p. 72) to derive. Choice of strategy to scenario.
Unclear	1Un 2Un 3Un 4Un	Exact strategy unclear as limited information, but at least stage judgement was made based upon response time, mouth movements, or eye movements.

I made the decision to exclude Wright, Martland and Stafford's (2006) further classification of Stage 2 strategies as either "figural, motor, or verbal" (p. 61). Wright and colleagues categorise figural as low Stage 2, and verbal as high Stage 2. I tried to apply these categories during both the teaching experiment and the retrospective analysis, but found it difficult to decide if a learner's strategy was verbal or figurative when there were few verbal or non-verbal clues. As such, I decided to focus more on recording the specifics of the observed behaviours, for example finger and mouth movements.

Although learner response time is used as an indicator of performance in research into arithmetic strategies (Green, Lemaire & Dufau, 2007) it was beyond the scope of this study to systematically code the response time of each learner. This decision was reinforced by my reflection that any resulting analysis might potentially be circular, as I used response time to inform some of my strategy judgements (Wright, Martland & Stafford, 2006). Where response time was noted in the strategy narrative, the measurement was from the end of questioning to the time the learner answered (although some learners also made use of questioning time to begin solving the problem).

To address the second research question, the full coded data set was then analysed to look for patterns in strategy use. Counts of strategy use are referred to as frequency of strategy use. The data set was analysed both as a whole and by each sample learner. Because of the variability of learners' strategy use (Dowker, 2005) a large, rich data set was needed to make secure judgements and for conclusions drawn to be valid. As I discussed below in Section 4.1, some progress was observed between the two assessment interviews; however, overall the changes were small. Hence the rationale for analysing the combined data from all observation spaces to address the second research question.

3.2.3.3 Research Question 3 - How might the MR programme be adapted to help Grade 2 learners progress in their early arithmetic strategies, and what are the advantages and constraints that emerge from the whole class adaptation?

Recall that the final phase of design research involves the reconstruction of an improved local instruction theory. In my study, this improved local instruction theory takes the form of suggestions for the adaptation of the MR programme to a group/whole class context of the Grade 2 learners. A comprehensive retrospective analysis of all data was conducted to address Research Question 3. The actual learning journeys of the six sample learners were analysed and contrasted with the conjectured local instruction theory, the EAS. The video recordings of interviews and teaching sessions were reviewed and re-analysed using the MR programme tools and the coding schedule (detailed above). Analysis focused in particular on observation of learners' verbal and non-verbal responses, on teacher/researcher processes, and on socio-cultural elements. This data was triangulated with that of the teaching experiment planning sheet, audio recordings of the post session discussions between myself and the class teacher, and my research journal.

Three case study learners were selected from the six sample learners, and the learning journeys of these three case study learners are presented as narrative vignettes in Chapter 4. One case study learner was selected from each stage group as a focal point for the reconstruction of the learning journey of the whole stage group. Selection criteria for the case study learners were the quantity and relevance of the data available. Their learning journeys are analysed and discussed in Chapter 5.

These narratives represent not just the case study learners' learning journeys, but also my own, which is detailed alongside those of the learners in the context in which it occurred. Such retrospective analysis again offered opportunities for my development as researcher/teacher and I identified many areas for improvement in my decision making and selection of teaching activities. In the journey metaphor, this could be related to error or inexperience on the part of the map-reader (i.e. myself), rather than inaccuracies in the map. The resulting suggested improvements are discussed in relation to the adaptation of the MR programme in Chapter 5. I do not discuss the learning of the class teacher in any depth as this was not the focus of my study. However, I do use her responses and input in various transcripts to illustrate the collaborative nature of our work and to highlight interesting areas for future study.

To conclude this chapter, the primary output of the final stage of design research is an empirically grounded theory of learning and teaching in a specific mathematics topic. In my study this improved local instruction theory suggests adaptation of the MR programme to a group/whole class context of Grade 2 learners in Zambia. This theory is shareable and thus fulfils the criteria of the DBRC (2003). This new theory is itself testable and thus a macro cycle of experimenting emerges (Gravemeijer & van Eerde, 2009). In this way, the outcome of the design research may be a new set of hypotheses for further testing and so the cycle of educational improvement continues.

This chapter has described the methodological framework and the research procedure used in this study. The next chapter will present the results of the study.

Chapter 4. Results

In this chapter I present the findings from a detailed microanalysis for each research question. This microanalysis results in a large number of detailed tables for each of the six sample learners. In the discussion in Chapter 5, I will pull all these aspects together and highlight the pertinent aspects from this chapter in narrative form. This chapter is divided into three sections. Section 4.1 compares the pre and post assessment interview performance of the six sample learners, relating to Research Question 1. Section 4.2 presents data on the spontaneous early arithmetic strategies of the sample learners and addresses Research Question 2. Section 4.3 presents the learning journeys of three case study learners, and relates to research Questions 3 and 1.

4.1. Research Question 1 - progress in early arithmetic strategies

Research Question 1 looks at what progress, if any, the learners make in early arithmetic strategies from participating in teaching experiment sessions in which the teacher and myself used the MR programme approach. This section presents the results of the pre and post assessment interviews as they address Research Question 1: *what progress, if any, do learners make in early arithmetic strategies using the Mathematics Recovery programme approach?* Three progress indicators are considered in turn: pre and post assessment EAS stage judgements, strategy spectra, and frequency of correct responses. In order to preserve anonymity, pseudonyms are used to report the results of the six sample leaners.

4.1.1. EAS stage progress

EAS stage judgements for the six sample learners were made before and after the 7-week teaching experiment. These judgements were made of the basis of pre and post assessment interviews. Additional evidence from Sessions 1 and 2 was also taken into account for the pre assessment interview stage judgement (see discussion in Chapter 5 for more details).

Table 9 EAS stages of the six sample learners before and after the teaching experiment

Learner	EAS stage Pre Assessment	EAS stage Post Assessment	EAS stage change
Hendrix	1	1	0
Charles	1	1	0
Memory	2	3	1
Kamwi	2	3	1
Grace	4	4	0
Mutinta	4	4	0

Table 9 shows that, at the start of the teaching experiment, two of the six sample learners were judged to be at EAS Stage 1 (perceptual counting), two were judged to be at Stage 2 (figurative counting), and two at Stage 4 (intermediate number sequence). Following the teaching experiment, two learners, Memory and Kamwi, were judged to have progressed by one stage, whilst the other four learners were judged to have remained at the same stage. The two learners who made EAS stage progress both progressed one stage from Stage 2 (figurative counting) to Stage 3 (initial number sequence).

4.1.2 Strategy spectrum and frequency of correct answers

As the broader EAS stage analysis did not reveal much of the more subtle progression I had seen during the teaching experiment sessions, I needed a more detailed way to show progress. The strategies used by each of the six sample learners in the pre and post assessment are thus presented as spectra in Figure 9 below. These spectra show the range and frequency of strategies used by all learners to *correctly* answer problems during the pre and post assessment interviews and also summarises the frequency of correct responses.

Recall that each strategy code begins with a number designating the broad EAS stage of the strategy, as well as more detailed strategy code. Figure 9 summarises how frequently each learner accurately used a strategy of each stage.

The set of questions answered by each learner in the pre and post assessment interviews varied slightly as a result of the flowchart like nature of the assessment interview schedule (see Figure 3). Therefore, to enable a fair comparison, only data from questions that were common to both interviews for each learner is included. For those learners who do not show much change in broad EAS strategies (i.e. Hendrix and Charles) a more detailed strategy spectrum is presented following description of the broad spectrum.

Less sophisticated	Frequency of strategies by EAS stage used to correctly answer questions in the pre and post assessment interviews					More sophisticated
	Stage 1	**Stage 2**	**Stage 3**	**Stage 4**	**Stage 5**	**Total Correct**
Hendrix (of 6 common questions)						
Pre	3	1				**4**
Post	3	1				**4**
Charles (of 7 common questions)						
Pre	1	1	1			**3**
Post	4					**4**
Memory (of 6 common questions)						
Pre		2				**2**
Post	1	1	3			**5**
Kamwi (of 7 common questions)						
Pre	1	2	2			**5**
Post	1	3	3			**7**
Grace (of 19 common questions)						
Pre		3	11	4	1	**19**
Post	1	5	10	3		**19**
Mutinta (of 17 common questions)						
Pre	2	3	7	2		**14**
Post	3	4	6	1		**14**
Sample Total (of 62 common questions)						
Pre	**7**	**12**	**21**	**6**	**1**	**47**
Post	**13**	**14**	**22**	**4**	**0**	**53**

Figure 9 Broad strategy spectra of six sample learners for correctly answered questions in pre and post assessment interviews (adapted from Mofu, 2013)

Overall, the combined data for all sample learners in Figure 9 shows that the predominant strategies used in both interviews were Stage 3 strategies, with 21 instances in the pre assessment interviews and 22 instances in the post assessment interviews. The frequency of more advanced counting-by-ones strategies (EAS Stage 3 or 4) was higher in the pre assessment interviews than in the post assessment interviews, and the frequency of less advanced initial counting strategies (EAS Stage 1 or 2) was higher in the post assessment interview than the pre assessment interviews. Overall the frequency of correct answers increased for the whole sample from 47 to 53. The frequency of correct responses increased from the pre assessment interview to the post assessment for three of the six learners (Charles, Memory and Kamwi), remaining constant for the other three learners.

Summary spectrum Figure 9 shows that there was no change in Hendrix's EAS stage or in the frequency of his correct responses between the pre and post assessment interviews. Nor was there any change in the broad EAS stage of the strategies Hendrix used to correctly answer 4 out 6 common questions during both assessment interviews. Therefore, a more detailed look at the specific strategies used was necessary.

	Stage 1: Counts perceptual items	Stage 1: Counts perceptual replacements three times	Stage 1: Counts perceptual replacements not three times	Stage 2: Unclear	Stage 2: Counts-from-one
Pre		3		1	
Post		1	2	1	

Figure 10 Strategy spectrum detail for Hendrix - correctly answered questions

The more detailed spectrum in Figure 10 shows a more subtle change for Hendrix in the specific strategies used from the pre to the post assessment interviews. In two instances in the post assessment interview Hendrix counted-perpetual-replacements without counting-three-times, a more advanced strategy that he had not used in the pre assessment interview. In the post assessment interview, he only once counted-perpetual-replacements-three-times, whereas he had used this strategy on three occasions in the pre assessment interview.

Figure 9 shows that Charles used more advanced strategies in the pre assessment interview but answered one more question correctly in the post assessment interview. The frequency of Charles' correct responses increased by one, from three out of seven to four out of seven. Figure 9 shows that Charles used a wider range of strategies in the pre assessment interview, but was more consistent in the post assessment, using only Stage 1 strategies. This necessitates a more detailed look at the exactly strategies he used.

	Stage 1: Counts perceptual items	Stage 1: Counts perceptual replacements three times	Stage 1: Counts perceptual replacements not three times	Stage 2: Unclear	Stage 2: Counts-from-one
Pre	1			1	1
Post	2		2		

Figure 11 Strategy spectrum detail for Charles - correctly answered questions

The detailed strategy spectrum in Figure 11 shows that in the post assessment interview, Charles twice successfully used his fingers to build-perceptual-replacements without counting-three-times. This is the most sophisticated Stage 1 strategy, and one which he had not used in the pre assessment interview. In the post assessment interview he was able to answer the semi-screened collection addition task[5] [7]

[5] *Screened collections are denoted in the text by square brackets. For example [5 - 2] denotes a screened collection removed item task [4] + 5 indicates a semi-screened addition task, 3 + 6 indicates an unscreened addition task.*

+ 3 using this strategy, which he had not done in the pre assessment interview. He attempted to use the same strategy for numbers beyond the finger range, but was unsuccessful.

As Figure 9 shows, Memory used no Stage 3 strategies in the pre assessment interview, whereas these were her predominant strategies in the post assessment interview, used to obtain three of her five correct answers. This evidence supports the result of a change in Memory's EAS stage from the pre assessment interview to the post assessment interview. The frequency of Memory's correct answers also increased by three, from two out of six to five out of six.

To answer the common tasks on the pre assessment interview, Kamwi used Stage 3 strategies twice, but only on the introductory tasks, which means they could not count towards a pre assessment interview Stage 3 judgement. To answer the common questions in the post assessment interview Kamwi used Stage 3 strategies three times, an increase of one, solving a non-introductory screened collection task beyond the finger range ([9 + 6]) that he had not been able to solve before. He also successfully answered one more subtraction question in the post assessment interview than in the pre assessment interview using a Stage 2 strategy. Kamwi's post assessment Stage 3 judgement was confirmed by his use of Stage 3 EAS strategies on further additive and missing addend tasks (these were not common questions to the two interviews, so were not included in the results above). The frequency of Kamwi's correct responses increased from five out of seven to seven out of seven.

Figure 9 shows that Grace used a wider range of strategies in the post assessment interview than in the pre assessment interview, and that overall she used less advanced strategies in the post assessment as judged against the EAS. In the post assessment interview, Grace used one fewer advanced Stage 5, 4 and 3 strategies respectively, and she used Stage 2 strategies on two more occasions. She also used a Stage 1 strategy that she had not used during the pre assessment. Grace answered all 19 common questions correctly in both interviews.

On the whole, Mutinta also used less advanced strategies in the post assessment interview than in the pre assessment interview, as the results in Figure 9 show. She used one fewer Stage 3 and 4 strategies respectively, and one more Stage 1 and Stage 2 strategies respectively. Mutinta answered 14 of 17 common questions correctly in both assessment interviews. In the post assessment interview she answered one missing addend task correctly, which she had failed to understand in the pre assessment interview. In the post assessment interview she answered one missing subtrahend incorrectly, which she had previously answered correctly in the pre assessment interview. In both assessment interviews her answer to the advanced missing subtrahend task [27 – 4] was incorrect by one, and she attempted a counting-from-five strategy on both occasions.

4.2 Research Question 2 - spontaneous early arithmetic strategies of learners

This section presents the results of spontaneous early arithmetic strategy use by the six sample learners related to Research Question 2, which investigates the strategies used by Grade 2 learners in solving early arithmetic problems. Recall that spontaneous strategies are those used by learners "unassisted either directly or indirectly by the teacher" (Wright, Martland & Stafford, 2006, p. 9). To address this question the data set of sample learner spontaneous strategy use was analysed as a whole, looking at instances of spontaneous strategy use from all three observation periods. Strategy use is first analysed by type of problem and magnitude of numbers in problems. Then use of fingers and use of tallies as part of early arithmetic strategies are considered.

4.2.1 Strategy use by problem type (addition and subtraction, and magnitude of numbers)

First, I present a table of the entire data set of the six sample learners over the three observation periods. Then, where there is sufficient data, I present a table of strategy use for addition and for subtraction problems for individual learners. Strategy use is broken down by the magnitude of the numbers in the problem, and by correct and incorrect responses. This provides a detailed picture of spontaneous strategy use for each learner.

Table 10 summarises the strategies used for addition and subtraction by the whole sample of learners, in terms of initial counting-by-ones, advanced counting-by-ones or non-counting-by-ones. Of the 174 cases of spontaneous strategy use identified in this study (see Chapter 3), 162 were cases of spontaneous strategy use for addition and subtraction (the remaining cases were for grouped item problems). Of these 162 cases for addition and subtraction, there were many more addition problems than subtraction problems, 108 compared to 54. As stated in Chapter 3, the number of instances of strategy use varied between sample learners.

Table 10 Summary of frequency of strategies for addition and subtraction for all six learners

Strategy Code (EAS stage)	**Addition**	**Subtraction**	**Total**
Initial counting by ones (Stage 1/2)	60	27	87
Advanced counting by ones (Stage 3/4)	48	26	74
Non counting by ones (Stage 5)		1	1
Total	**108**	**54**	**162**

As Table 10 shows, for all problems, counting-by-ones strategies are the predominant strategies of the six sample learners, with initial counting-by-ones more prominent than advanced counting-by-ones, used for more than half of all problems in 87 out of 162 instances. It is interesting to note that, despite the high proportion of data from Stage 4 learners. Only one instance of Stage 5 non-counting-by-ones strategy use was observed for subtraction, and no instances of Stage 5 strategy use for addition were observed (see Grace's case study below for a description of that strategy).

I now present the strategy use data for addition and for subtraction, by each sample learner. Strategy use is broken down by magnitude of numbers in problem, and by correct and incorrect responses.

Table 11 Summary of Hendrix's addition strategies

		Frequency of strategy use				
		Sum ≤10		**Sum >10**		**All addition problems**
		✔	✘	✔	✘	
1. Builds Perceptual Replacements	Counts three times*	8		2	6	16
	counts all	4				4
	hybrid				3	3
2. Counts from one		2				2
Total		**14**		**11**		**25**

* *counts each addend then counts-all*

Table 11 shows that Hendrix's predominant addition strategies are Stage 1 perceptual counting strategies. He twice answered addition problems correctly without using Stage 1 perceptual counting strategies, but only for sums less than 10 (specifically the [3 + 1] question on the pre and post assessment). Hendrix's main strategy was to build perpetual replacements and then count-from-one three-times (i.e. he counted each addend separately and then counted all), a strategy which he used on 16 occasions. For sums within the finger range this strategy was successful and Hendrix answered all 14 problems correctly.

When extended to sums beyond the finger range, Hendrix's perceptual strategies were mainly unsuccessful. Hendrix's two correct responses to addition problems beyond the finger range involved a strategy of making tally marks in the dust below the desk. This enabled him to extend his strategy of building perceptual replacements beyond his finger range (see Hendrix's case study below). The hybrid strategy that he developed to extend his strategy to numbers greater than 10 was also unsuccessful at this time.

I only have data for three subtraction problems for Hendrix. All three problems were within the finger range. Hendrix answered two introductory removed item tasks correctly without using perceptual replacements (these were [3 – 1] and [5 – 2] and the he failed to answer the non-introductory task [9 – 4], seeming to have no strategy to solve it.

Table 12 Summary of Charles' addition strategies

		Frequency of strategy use				
		Sum ≤10		Sum >10		All addition problems
		✔	✘	✔	✘	
0. No visible strategy			1			1
1.Perceptual Replacements	Counts three times		2			2
	not three times	3	1			4
2. Counts from one		1			1	2
2. Unclear		1			1	2
Total		**9**		**2**		**11**

Table 12 shows that Charles' predominant strategies for addition were Stage 1 strategies that involved building perceptual replacements on his fingers. He counted-perceptual-replacements-three-times on 2 of 11 occasions, and counted-perceptual-replacements without counting-three-times on 4 of 11 occasions. Charles also had access to slightly more advanced strategies that did not involve building perceptual replacements for the addends on his fingers, but these strategies were only effective for sums within the finger range. On the task [5 + 4] in the pre assessment, Charles used a figurative counting strategy where he looked at the screen and seemed to be trying to visualise the screened collection, counting-from-one. However, beyond the finger range, this strategy was not accurate, even when the second collection was unscreened. I have no subtraction data for Charles as the flowchart nature of the assessment interview schedule meant that he was not asked to solve any subtraction problems.

Table 13 Summary of Memory's addition strategies

		Frequency of strategy use				
		Sum ≤10		Sum >10		All addition problems
		✔	✘	✔	✘	
0. No strategy			2		1	3
1. Counts Perceived Items	counts stones				1	1
	PR* three times	1				1
2. Counts from one/five/ten	from one	2		2		4
	unclear	3			2	5
3. Counts on		2		2	1	5
Total		**10**		**9**		**19**

**builds perceptual replacements on fingers.*

Table 13 shows that regardless of the magnitude of addends, Memory used a range of strategies from Stages 1 to 3. She used Stage 2 strategies in 4 and 5 instances, so Stage 2 strategies were predominate for Memory with a total of 9 out of 19 instances. Memory's strategies were more effective for sums within the finger range, of which she answered only 2 of 10 problems incorrectly, compared to answering over half of sums beyond the finger range incorrectly. There is no subtraction data for Memory due to her initial responses during the pre assessment interview and the flowchart nature of the interview schedule.

Table 14 Summary of Kamwi's addition strategies

		Frequency of strategy use				
		Sum ≤10		Sum >10		All addition problems
		✔	✘	✔	✘	
2. Counts from one/five/ten	from one	1				1
	unclear	4		3		7
3. Counts on (unclear*)		2		4		6
Total		**7**		**7**		**14**

** strategy unclear but judged to be at least Stage 3 counting-on*

Table 14 shows that Kamwi's predominant strategies for addition were Stage 2 strategies, which he used on 1 and 7 occasions (a total of 8 out of 14 occasions) followed by Stage 3 counting-on, used on 6 out of 14 occasions. Kamwi's strategies were difficult to judge, as he did not often use his fingers or count out loud. Kamwi was accurate in his addition strategies, both within and beyond the finger range. Table 14 shows that Kamwi used more advanced strategies when solving sums greater than 10, where his predominant strategy was judged to be at least counting-on. In the post assessment, the clearest example of Kamwi counting-on was for [9 + 6], where his mouth moved seven times, consistent with sub vocal counting-on. Kamwi's subtraction strategies are summarised below.

Table 15 Summary of Kamwi's subtraction strategies

	Frequency of strategy use				
	Min ≤10		Minuend >10		All subtraction problems
	✓	✗	✓	✗	
1. Counts PR, not three times	2				2
2. Counts from one (unclear*)			1	1	2
3. Counts on (unclear*)	4				4
Total	**6**		**2**		**8**

* *strategy unclear but judged to be at least named strategy*

Table 15 shows that Kamwi's predominant strategies for subtraction were judged to be at least Stage 3, used on four of eight occasions. However, he also used a Stage 1 strategy, counting-perceptual-replacements without counting-three-times, for subtraction problems within the finger range on two of six occasions. This is a less advanced strategy than he had used for addition problems involving numbers of similar magnitudes. Overall, he also used less advanced strategies for subtraction problems with a minuend greater than 10 than he did for problems within the finger range.

Table 16 Summary of Grace's addition strategies

	Frequency of strategy use				
	Sum ≤10		Sum >10		All addition problems
	✓	✗	✓	✗	
3. Counts on/ up to	9		12		21
Total	**9**		**12**		**21**

Table 16 shows that Grace consistently and effectively used a counting-on strategy to solve all spontaneous addition problems during the study period. This strategy was equally effective for addition problems both within and beyond the finger range. Grace's subtraction strategies are summarised in Table 17 below.

Table 17 Summary of Grace's subtraction strategies

		Frequency of strategy use				
		Min. ≤10		Min. >10		All subtraction problems
		✓	✗	✓	✗	
1. Counts PR	not three times	1				1
2. Counts from one	from one			2		2
	from five/ten			2		2
	unclear	1		3		4
3.Counts down from/on		4		1		5
4. Counts down to/up to		4		3		7
5. Non count by ones				1		1
Total		**10**		**12**		**22**

Tables 16 and 17 show that Grace used a more varied range of strategies to solve subtraction problems than she had for addition problems. Grace used Stage 3 strategies on five occasions and Stage 4 strategies on seven occasions, resulting in a combined total for advanced Stage 3 and 4 counting strategies of 12 out of 22 occasions. However she also used less advanced Stage 1 and 2 strategies on almost half of all occasions, and used only one Stage 5 non-counting-by-ones strategy. Grace's strategy use was more varied for subtraction problems with a minuend beyond the finger range. See Grace's case study below for further description and elaboration of this.

Table 18 Summary of Mutinta's addition strategies

	Frequency of strategy use				
	Sum ≤10		Sum >10		All addition problems
	✓	✗	✓	✗	
1. Counts PR not three times	1				1
2. Counts from one	1				1
3. Counts on/up to	6		11		17
Total	**8**		**11**		**19**

Table 18 shows Mutinta's predominant strategy for addition was counting-on/up-to, which she used in 17 of 19 instances. She used this strategy exclusively for sums greater than 10, in all 11 instances. For sums within the finger range, counting-on was her predominant strategy. She was also effective in her strategy use for addition, answering all questions correctly. Mutinta's subtraction strategies are summarised in Table 19 below.

Table 19 Summary of Mutinta's subtraction strategies

		Frequency of strategy use				
		Min. ≤10		**Minuend >10**		**All subtraction problems**
		✔	✘	✔	✘	
1. Counts perceptual replacements		4				4
2.Counts from one	from one				1	1
	from five/ten			6	1	7
	unclear				1	1
3.Counts down from/on		4				4
4. Counts down to /up to		2		2		4
Total		**10**		**11**		**21**

Tables 18 and 19 show that Mutinta used less advanced strategies for subtraction compared to addition, and that her predominant strategies for subtraction were Stage 1 and 2 strategies. She used the widest range of strategies of all the learners, accurately using strategies from Stage 1 to Stage 4 for sums within the finger range. Mutinta was more consistent but less accurate for minuends greater than 10, where Stage 2 counting-from-one-or-five-or-ten strategies were her predominant strategies, which she used accurately to solve 6 of 11 sums beyond the finger range. She did not count-down-from for any subtraction problem. Mutinta vocally counted up from 11 to 15 to solve a missing subtrahend in the pre assessment interview.

4.2.2 Use of fingers as a part of early arithmetic strategies

Fingers form an important part of learners' early arithmetic's strategies (Wright, Martland & Stafford, 2006). This section presents the results of how the learners used fingers as part of their spontaneous early arithmetic strategies. An overview chart is presented for all learners and then a table is presented with results for each learner. Recall from Chapter 3 that two broad types of finger-based strategies were observed during the retrospective data analysis - fingers used as perceptual replacements, and fingers used to keep track of counting. The second is the more advanced strategy. Figure 12 below summarises use of fingers for all learners, in all three observation periods for both correctly and incorrectly answered questions. Note that the total amount of data varies for each sample learner, due to the flowchart nature of the pre and post assessment interview schedule and variations in the amount of data gathered for each learner in Sessions 1 and 2. This is reflected in Figure 12 in the relative height of each column. The focus rather is on the frequency of different *types* of finger use for each learner.

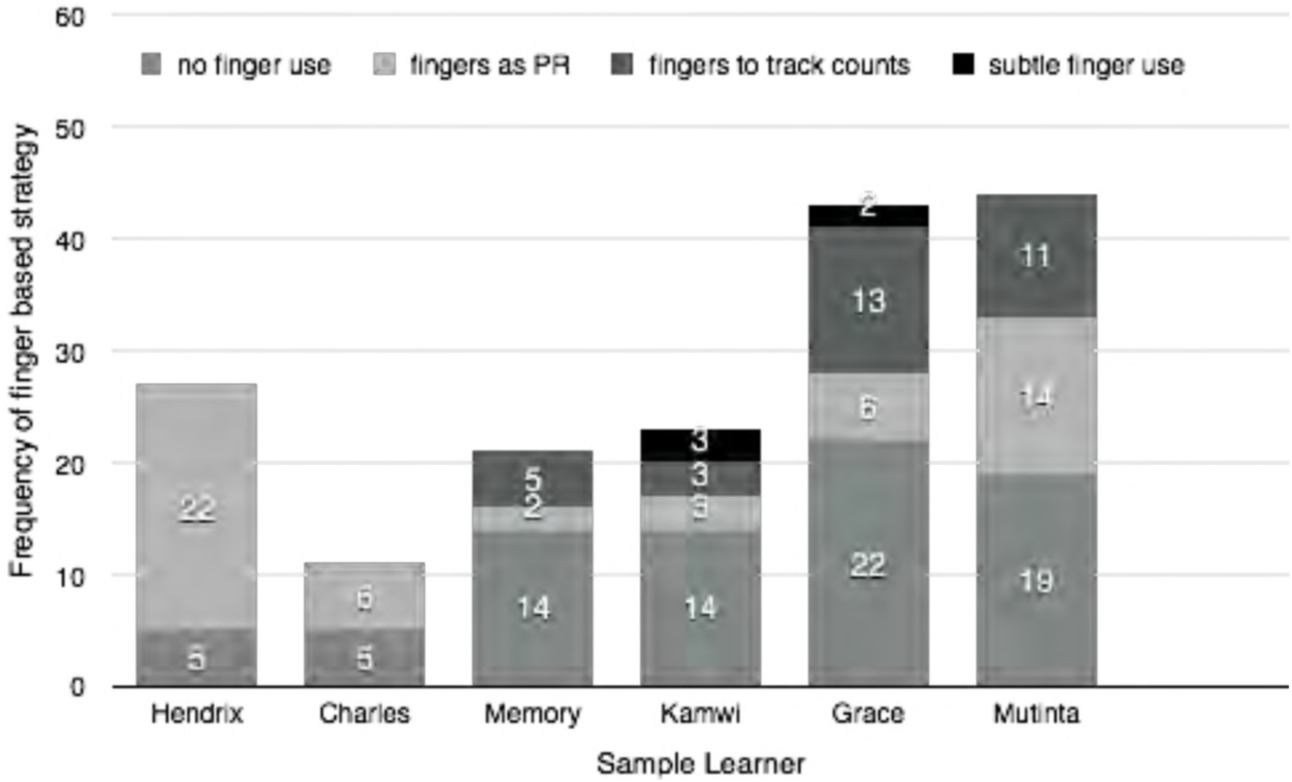


Figure 12 Frequency of finger-based strategies for all learners

Figure 12 shows that there was a variation in the way in which each of the sample learners used their fingers as a part of their early arithmetic strategies. Hendrix (EAS Stage 1) used fingers in the majority of his problem solving attempts, using them exclusively as perceptual replacements and never to keep track of counts. Charles (also judged at EAS Stage 1) used his fingers for roughly half of the problems he attempted. Memory and Kamwi (initially EAS Stage 2) used fingers infrequently, in only 7 of 21 and 9 of 23 instances respectively. Of interest are Grace and Mutinta's (EAS Stage 4) use of fingers as part of early arithmetic strategies. They both used fingers in the more sophisticated way to keep track of counts, but they also used fingers as perceptual replacements on 6 and 14 occasions respectively.

The following Tables 20 to 25 present the frequency with which each individual learner used fingers as part of his or her spontaneous early arithmetic strategies. Finger use is broken down by problem type, by specific finger-based strategy, and by the three observation periods – the pre and post assessment interviews, and instances of spontaneous strategy use in teaching experiment Sessions 1 and 2 before small group focused teaching began.

Table 20 Frequency of Hendrix's finger use for addition, by strategy and observation period

	Frequency of finger use			
	Pre	**1&2***	**Post**	**All**
As PR**, consecutive finger pattern	5	11	3	19
As PR, consecutive & simultaneous finger patterns		1	2	3
No finger use	1		1	2
Total	**6**	**12**	**6**	**24**

** Teaching experiment Sessions 1 and 2; ** Perceptual replacements*

Table 20 shows that Hendrix used his fingers exclusively as perceptual replacements when solving addition problems. Hendrix mainly raised his fingers consecutively, but used simultaneous finger patterns once in teaching experiment Sessions 1 and 2 and twice in the post assessment interview (to represent the addends 2 and 3). The only problem for which he did not use his fingers in either the pre or post assessment interviews was the introductory task [3 + 1], which he answered very quickly. He did not use any fingers at all for the three subtraction problems for which I have data (not included in Table 20). Hendrix's increased use of more sophisticated simultaneous finger patterns in the post assessment interview suggests subtle progress in his early arithmetic strategies.

Table 21 Frequency of Charles' finger use for addition, by strategy and observation period

	Frequency of finger use			
	Pre	**1&2**	**Post**	**All**
As PR, consecutive finger pattern			6	6
No finger use	5			5
Total	**5**		**6**	**11**

Table 21 shows that Charles used his fingers consistently as perceptual replacements on all 6 occasions that he used his fingers. There was a significant difference in Charles' use of fingers by observation period. In the pre assessment interview he used no fingers, and in the post assessment interview he used fingers as part of his strategy to solve all problems. Such a contrast in Charles' use of fingers between observation spaces suggests that socio-cultural factors may have been at play. The pre assessment interview was an unfamiliar context, and Charles may have been unclear as to what was expected of him (discussed further in Section 5.1.3 below). Alternatively, these results could reflect the impact of teaching activities involving finger-based strategies during the teaching experiment.

Table 22 Frequency of Memory's finger use for addition by strategy and observation period

	Frequency of finger use			
	Pre	**1&2**	**Post**	**All**
As perceptual replacements			2	2
To keep track of counts		4	1	5
No finger use	9		5	14
Total	**9**	**4**	**8**	**21**

Table 22 shows that Memory used fingers inconsistently and that there is significant difference in Memory's use of fingers as part of her strategies by observation period. Memory used no fingers in the pre assessment, but in the first teaching experiment sessions she used fingers as part of her strategies to keep track of counts in all four instances. As no small group teaching had yet occurred, this suggests that socio-cultural factors including the unfamiliarity of the assessment interview context may have influenced Memory's varied use of fingers between the assessment interview and teaching experiment contexts, as may also have been the case with Charles. In the post assessment interview Memory used her fingers on a total of 3 out of 8 occasions, twice as perceptual replacements and once to keep track of counts. This suggests that whatever factors had prevented her from using fingers in the pre assessment interview may have been mitigated to a certain extent by the time of the post assessment interview.

Table 23 Frequency of Kamwi's finger use by strategy, problem type and observation period

	Frequency of finger use								
	Addition Problems				**Subtraction Problems**				**All**
	Pre	**1&2**	**Post**	**All**	**Pre**	**1&2**	**Post**	**All**	
As perceptual replacements					1		2	3	3
To keep track of counts	1	1	1	3					3
Subtle finger use			3	3					3
No finger use	4	2	4	10	3		2	5	15
Total	**5**	**3**	**8**	**16**	**4**		**4**	**8**	**24**

Table 23 shows that Kamwi predominately did not use fingers as part of his early arithmetic strategies, using no fingers on a total of 15 of 24 occasions. Kamwi used no fingers for 10 of 16 addition problems and for 5 of 8 subtraction problems. His finger use was more advanced for addition problems than for subtraction problems. He used his fingers to keep track of counts only for addition problems, and used his fingers as perceptual replacements only for subtraction problems. In one instance, Kamwi used a counting-from-one-or-five-strategy for subtraction, using his fingers as perceptual replacements. Such a contrast between the sophistication of Kamwi's use of fingers by problem type suggests that Kamwi's early arithmetic strategies for addition were more advanced than his strategies for subtraction.

Table 24 Frequency of Grace's finger use by strategy, problem type and observation period

	Frequency of finger use								
	Addition Problems				Subtraction Problems				All
	Pre	1&2	Post	All	Pre	1&2	Post	All	
As perceptual replacements							6	6	6
To keep track of counts	3	4	5	12			1	1	13
Subtle finger use					2			2	2
No finger use	5	1	3	9	9		4	13	22
Total	**8**	**5**	**8**	**21**	**11**		**11**	**22**	**43**

Table 24 shows that across all observation periods for addition, Grace's predominant strategy involved the use of fingers to keep track of counts, for 12 of 21 problems. For the remaining 9 of 21 addition problems she did not use fingers as part of her strategy. Grace did not use fingers as perceptual replacements when solving addition problems. For subtraction problems, Grace's use of fingers was more varied. On 13 of 22 occasions her strategies for subtraction involved no use of fingers. When using fingers for subtraction, Grace's mainly used fingers in the less sophisticated way as perceptual replacements (on 6 of 22 occasions). Grace only once used fingers to keep track of counts when solving subtraction problems, whereas she used fingers to solve more than half of the addition problems. Table 24 also shows a difference in Grace's use of fingers by observation period. There is a higher frequency of finger use in the post assessment interview than in the pre assessment interview. In the post assessment interview Grace used her fingers to solve five of eight addition problems, compared to three of eight problems in the pre assessment interview. The same pattern occurs for subtraction problems, for which she used fingers on a total of seven occasions in the post assessment compared to only two occasions in the pre assessment. These results show that Grace's strategies for subtraction were less sophisticated than those for addition, and also suggest that there may have been socio-cultural or contextual factors influencing her use of fingers in the different observation periods, for example the unfamiliarity of the assessment interview context and her perception of the researcher's expectations.

Table 25 Frequency of Mutinta's finger use by strategy, problem type and observation period

	Frequency of finger use								
	Addition Problems				**Subtraction Problems**				**All**
	Pre	**1&2**	**Post**	**All**	**Pre**	**1&2**	**Post**	**All**	
As perceptual replacements			1	1	6		7	13	14
To keep track of counts	2	3	5	10	1			1	11
No finger use	5	5	2	12	4		3	7	19
Total	**7**	**8**	**8**	**23**	**11**		**10**	**21**	**44**

Table 25 shows that Mutinta used fingers as a part of her early arithmetic strategies in the majority of instances, to solve a total of 25 out of 44 problems (she used fingers as perceptual replacements on 14 occasions and to keep track of counts on 11 occasions). Table 25 reveals a clear pattern of finger use by problem type, as Mutinta used fingers in a more advanced way for addition problems than for subtraction problems. To solve addition problems, her predominant use of fingers was to keep track of counts (10 occasions), compared to only one instance where she used fingers as perceptual replacements. In contrast for subtraction problems she mainly used her fingers as perceptual replacements (on 13 occasions), compared to only one occasion where she used her fingers to keep track of counts. That was when she counted up to solve a missing subtrahend task. The results also show that for addition Mutinta used her fingers more frequently in the post assessment interview than the pre assessment interview. There was no significant difference in Mutinta's use of fingers for subtraction between the pre and post assessment interviews. As with Grace and Kamwi, these results indicate that there was a difference in the sophistication of Mutinta's strategy use by problem type, and that her strategies for solving subtraction problems were less sophisticated than her strategies for solving addition problems.

Overall, these results for finger use by the six sample learners provide insight into their early arithmetic strategies and thus help to address Research Question 2. These results show that finger use as a part of early arithmetic strategy varied between learners, but that overall, learners used fingers in less sophisticated ways to solve subtraction problems compared to addition problems. The results also suggest a pattern in use of fingers by observation period, suggesting a possible socio-cultural or contextual influence on learners' use of fingers as a part of early arithmetic strategies. These results, and possible socio-cultural and contextual factors, will be discussed further in Chapter 5.

4.2.3 Tally use as part of early arithmetic strategies

This section presents the evidence for use of tallies as part of the six sample learners' spontaneous early arithmetic strategies. Recall from Chapter 3 that pencil and paper were not available during the pre and post assessment interviews. Therefore, no data on paper and pencil based tally use is available from the pre and post assessment interviews. I anticipated that any tally-based strategies would become clear when pencil and paper were available during the teaching experiment sessions themselves. However, almost no instances of paper based tally mark making were recorded or observed during the teaching experiment sessions. During one small group focused teaching activity, a non-sample learner looked as if he might begin to draw tally, but I redirected him. Limited tally marks were observed on the back and front cover of Mutinta's exercise book when the books were photographed at the end of the experiment, although Mutinta was not videoed using this strategy at all during the sessions. The only video recording of a tally-based strategy is of Hendrix, who is shown extending the perceptual replacements on his fingers by drawing tally marks on the bench in Figure 13. Hendrix's use of tallies is described in more detail in his case study below.

Figure 13 Still from video recording of post assessment interview shows Hendrix making tally marks on the bench during a screened collection task

There is other evidence that tally-based strategies were used by the learners. My sampling of the learners' mathematics books prior to the start of the teaching experiment indicated that tallies were in use by the learners. For example, some evidence of written tally use was found in Grace's classroom mathematics book prior to the commencement of the study. Tally marks were found on the front and back cover, and on certain pages in the book, e.g. tasks involving three digit addition and magic square addition.

Anecdotal evidence from discussions with the class teacher also suggested that use of tallies was encouraged in class. During the first pre assessment interview, the class teacher suggested that if she were to bring a pencil from the classroom, the learner would be able to count by ones to solve a subtraction problem, by making tally marks. During the post Session 2 discussion I asked if there might be any examples of tally counting in the learners' exercise books. The class teacher suggested there might be "because mostly we tell them to count [by ones], it's easy for them to get the answers like that" (source: audio recording of post session discussion, 02/07/2015). This comment suggests more focus on accuracy of answers over developing efficient, increasingly sophisticated methods.

4.3 Research Question 3 - adaptation of the MR programme

This section presents case studies of the learning journeys of three of the sample learners. These case studies show how taking a design research approach to the teaching experiment allowed me to make adaptations to the MR programme that enabled the learners to make progress in their early arithmetic strategies. Of the six sample learners, one case study learner was selected from each stage group, as a focal point for the telling of the learning journey for the whole group. My selection of one sample learner from each stage group was based on the quantity of data available and the relevance of the available data in addressing the research question. The main source of data for the case studies was transcribed video recordings of pre and post assessment interviews and small group teaching sessions. This data was supplemented by and triangulated against data from the teaching experiment planning sheet, my reflections and observations in my research journal and my transcriptions of audio recordings of post teaching experiment session discussions between myself and the class teacher.

For each of the three case study learners, I give an overview of their participation in the pre assessment interview, the teaching experiment, and the post assessment interview. I continue the metaphor of a learning journey for progression in early arithmetic strategies in order to frame the narratives presented here. Recall that in the journey metaphor, the EAS is the map, the pre assessment interview tells us where the learner is starting from, and the instructional goal is the intended destination of each learner. I use terms from the MR programme teaching guidance tool (see Appendix C) to describe each learner's participation and to describe the decision making of the class teacher and myself. These are shown in the text in italics.

4.3.1 Hendrix's Case Study

On the basis of the pre assessment interview Hendrix was judged to be at EAS Stage 1 (perceptual counting). For problems beyond the finger range Hendrix seemed either to have no strategy, or made unsuccessful attempts to use a hybrid strategy that involved building perceptual replacements on his fingers and visualising tally marks on the bench. Hendrix appeared to have difficulty keeping track of the addends whilst counting-from-one three-times. The instructional goal for Hendrix was progression to EAS Stage 2 (figurative counting) and he was placed into a Stage 1 group. My anticipatory thought experiment generated a hypothetical learning trajectory from the EAS and IFEN frameworks. I hypothesised that Hendrix's progress to figurative counting would involve further *cognitive reorganisation*, as he would need to be able to keep track of addends beyond the finger range. By using his fingers to build perceptual replacements, I noted that Hendrix had already taken a small step forward from simply counting perceptual items. I thought that motor unit items (e.g. raising or touching fingers) and visualised items (the tally marks) might be significant on his journey. I reflected that

Hendrix would also have to progress beyond needing to count-three-times, and that secure finger and dot patterns might help with this. I identified relevant key topics from Wright, Martland, Stafford and Stanger (2006) to form a bank of suitable activities (see Table 3 in Chapter 3 above).

My observations of Hendrix from Sessions 1 and 2 reinforced my judgments from the pre assessment interview. Whilst playing a dice game in Session 1, Hendrix seemed unfamiliar with the dice itself, and unclear of how to roll it. A photograph of a 100 square puzzle that Hendrix partially completed in Session 1 shows that his numerals were secure up to, but not beyond, nine (see Figure 14 below).

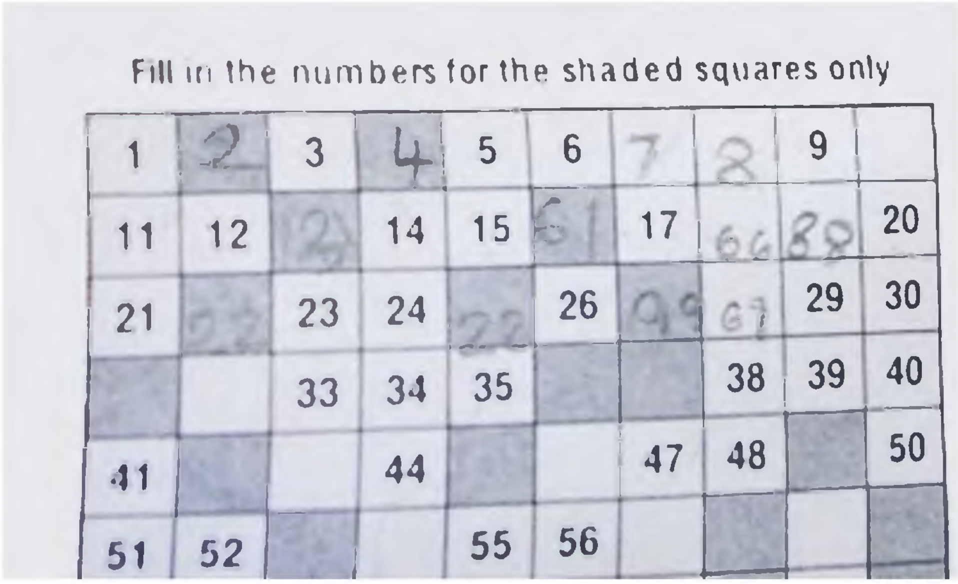


Figure 14 Hendrix's 100 square from Session 1 showing insecure numerals beyond 9

In Session 2 Hendrix was part of a mixed stage group solving semi-screened addition tasks. As in the pre assessment interview, Hendrix's initial strategy was to build perceptual replacements on his fingers. This strategy was successful for sums within but not beyond the finger range. He also displayed a simultaneous finger pattern for two, and solved the introductory task [5] + 2 without counting-from-one three-times. I had hypothesised that Stage 1 learners might make pointing movements, double-count verbally, or visualise the collection, but Hendrix was not observed doing any of these actions.

In the excerpt below, I (indicated as R) had put out and covered eight stones and left three bottle tops uncovered. The class teacher (indicated as CT) *provided sufficient wait time* (40 seconds) for Hendrix to answer, even though the other learners in the mixed group shouted out their answers in the first few seconds.

R: This time, eight stones and three bottle tops.
H: (Counts out eight fingers consecutively). One, two, three, four, five, six, seven, eight.
H: (Pauses, looks up, mouth moves. Looks down at his fingers. Looks under bench and makes marks in the dust on the desk shelf. Looks at the marks and redraws three firm tally marks). One, two, three. (Counts from one again starting on his fingers). One, two, three, four, five, six, seven, eight.
H: (Continues his count on the tally marks drawn in the dust). Nine, ten, eleven.

CT: What is the answer?
H: Eleven (smiles widely).

It seemed that Hendrix then went on to *anticipate* that he could use this strategy to successfully solve the final problem, [10] + 4. Hendrix was *engaged*, and *reflecting on his thinking*. He *developed his own strategy*, which enabled him to solve a range of problems that he had not been able to solve before. In this way, the teaching was *at the cutting edge* for Hendrix, but not for the other group members who could answer quickly and easily. However, the strategy that Hendrix spontaneously developed was not a more advanced strategy, but rather an extension of perceptual counting to higher magnitude sums (as will be discussed in Chapter 5).

In this excerpt from the audio recording of the post session discussion, the class teacher suggested that Hendrix's strategy may have its origins in teacher modelling of tally use and perceptual counting during classroom teaching.

R: So he ran out of fingers, and he used the dust that was there. (They go over to the desk and look). He's wiped it.
CT: He was making lines, because mostly that's what we use, even on the board, for them to get the correct answers. Because of the stage where they are, you find that you make ma-ones, you count 'one, two, three; one, two, three, four' then you count them together 'how many are they?' Then you get the answer.

As part of the iterative cycle of analysis and reflection on Hendrix's strategy use, Sessions 3 and 4 focused on developing facile finger patterns, using finger patterns to keep track of counting, and visual dot patterns. I hypothesised that these would act as stepping-stones to enable the Stage 1 learners to develop figurative counting strategies. Additional activities from Stage 0 (teaching the emergent child) were chosen *from the bank of suitable activities*. This was because my observations indicated a lack of basic finger patters or dot pattern awareness. See Table 5 in Chapter 3 for a summary of the small group focused teaching tasks used in each teaching experiment session.

During Sessions 3 and 4, Hendrix and the other learners' facility at dot pattern recognition increased, supported by playing dominoes and dice-based bingo when not participating in small group focused teaching. Hendrix was less successful when making spatio-motor patterns to match visual dot patterns. He often missed dots, counted without pointing, and made inaccurate patterns. Following analysis and reflection, I planned to use *flashing* of dot patterns to facilitate visualisation, and *micro adjusting* to enable new learning to be extended into more problem-based scenarios.

I returned to the problem solving tasks in Session 5 and 6 to attempt to *engender more sophisticated strategies.* I adapted the counting items in two screened collections task (Task 6.3.3 from Wright, Martland, Stafford and Stanger, 2006) to use dot pattern cards instead of collections of stones or counters. I intended that this would *engender* the development of visualisation based strategies. In the other Stage 1 group, a non-sample learner with more secure spatio-motor patterns had developed a figurative strategy during this adapted activity. This activity was less effective for Hendrix, whose initial strategy was to build perceptual replacements on his fingers, then point vaguely over the screened cards after I had modelled the strategy. It was clear that Hendrix did not have secure and distinct dot patterns, and had difficulty keeping track of the second addend. I reflected on this with the class teacher, and we decided to adapt the activity to build on Hendrix's preference for use of fingers.

In Session 6 the class teacher and I worked together and presented a series of screened additive tasks within the finger range. As before, Hendrix's initial strategy was to build perceptual replacements consecutively on his fingers and then count-from-one three times. During the tasks, the class teacher and I modelled the use of facile finger patterns. This extract from the video transcript, towards the end of the activity shows Hendrix making some progress.

CT: (Places five dot card) How many are these?
H: Five.
CT: (Screens card). Five fingers (models raising five fingers simultaneously).
H: (Raises five fingers simultaneously).
CT: (Places three dot card). How many are these?
H: (Raises three fingers simultaneously on other hand. Then lowers all fingers and raises five then three fingers consecutively, counting). One, two, three, four, five, six, seven, eight.
CT: (Continues to assist other learners and then models checking answer by counting dots).
H: Eight!

A subtle change is noted in the way Hendrix used his fingers in this task. By initially establishing facile finger patterns for 5 and 3, he was able to keep track of the count by raising the recalled finger pattern. This contrasted with his initial strategy where he had to build each addend up from one consecutively on his fingers, and then keep them raised whilst he counted them again. Although not yet a figurative strategy, this was an important step forward for Hendrix toward more sophisticated early arithmetic strategies. Note that although some teacher modelling occurred, Hendrix had *developed his own method.*

Reflections on the observations of this activity led us to continue supporting the development of Hendrix's finger-based strategies, although I did not find much guidance on exactly how to do this at such a level of detail in Wright, Martland, Stafford and Stanger (2006). The class teacher and I planned to continue modelling use of facile finger patterns and consolidating strategy development on sums within the finger range, before extending to larger numbers (*micro-adjusting*). Unfortunately, Hendrix and Charles were both absent for the final, seventh teaching experiment session.

Hendrix was judged to be at EAS Stage 1 (perceptual counting) on the basis of the post assessment interview, and did not achieve the instructional goal of progression to Stage 2 (figurative counting). However, as described above and in Section 4.1, he did make some subtle progress towards figurative counting that was evident in the post assessment. Two interesting examples of Hendrix's strategy use in the post assessment interview are described here for further discussion in Chapter 5.

The first example is Hendrix's strategy for the task [5 + 4]. Hendrix prepared by touching five fingers on his left hand consecutively as I began to introduce the problem. When I then said "four stones here", Hendrix again raised his left hand, opened his five fingers slightly and said "five", as learners using a counting-on strategy to solve a problem with five as the first addend might do. This will be discussed below in Section 5.3.3.

The second example for further discussion is an interesting change in the hybrid strategy that he attempted to use to solve a task beyond the finger range. Task 9 + 4 was posed twice during the post assessment, firstly with both collections of stones screened, then with only the first collection of nine items screened. Hendrix incorrectly answered 14 on both occasions. In the first instance, his strategy involved counting out nine fingers, then drawing four short imagined tally marks on the bench with a finger (see Figure 13) then counting-all-from-one. When counting-all, he miscounted five imagined tally marks instead of four. This extract shows the strategy Hendrix then used when the four stones were unscreened.

R: Nine here, four here. How many altogether?
H: (Counts out fingers consecutively). One, two, three, four, five, six, seven, eight, nine.
(Makes nine tally marks on the bench: six in a row, then runs out of space, and makes three more below. Counts out his nine fingers again).
One, two, three ... nine.
(Touches the bench in a (non-dice) five dot pattern, counting).
Ten, eleven, twelve, thirteen, fourteen!

I hypothesised that Hendrix might have been able to solve this task by using a visualised dot pattern if he had had a secure dot pattern for four, and if he had been able to anticipate that such a strategy would work.

In summary, Hendrix's case study shows how elements of the individual MR programme were adapted for use with a group of learners within a whole class setting, and how this adaptation facilitated subtle learner progression in early arithmetic strategies.

4.3.2 Memory's Case Study

Memory's strategies in the pre assessment interview were unclear, as she made no use of her fingers. Her strategies involved long pauses and looking down but not at the stones. She did not solve the screened collection additive tasks, and answered inconsistently on the semi-screened and unscreened collection tasks. For two additive tasks, her initial answer was a number smaller than the first addend, suggesting she did not understand the task. My analysis of this pre assessment interview put Memory at Stage 1 (perceptual counting). However, during the first teaching experiment session Memory spontaneously and successfully used figurative counting strategies (EAS Stage 2). In the context of a dice game[6] she counted-from-one to solve two addition problems beyond the finger range. Memory used her fingers to keep track of her counts, but did not consistently match the number of counts to the number of finger touches.

On the basis of these observations, I revised my judgement of Memory's baseline EAS stage from Stage 1 to Stage 2. Memory was placed in the Stage 2 group, and the goal for her learning journey was to progress to Stage 3, counting-on for addition. I hypothesised that her learning journey would involve a cognitive reorganisation, as she would no longer needed to start her counting at one when solving early arithmetic problems. On the basis of my observations from the Sessions 1 and 2, I hypothesised that Memory might use her fingers to keep track of counting-on, rather than explicit double-counting or recognising a temporal sequence of counts. Developing accuracy would be key. Wright, Martland, Stafford and Stanger (2006) suggest that partitioning and combining involving five and ten may help Stage 2 learners like Memory to develop number sense and accuracy. Partitioning and combining numbers involving five and ten was therefore established as an intermediate target for Memory. As with Hendrix, I then identified relevant key topics for teaching from Wright, Martland, Stafford and Stanger (2006), detailed in Table 3 in Chapter 3 above.

In Session 2, the class teacher and I co-led small group focused teaching of a screened collection counting activity in which we modelled a counting-on strategy. Memory was one of four girls in the mixed stage group for this session. Analysis of the video of the small group focused teaching showed that while solving this problem the four girls shared their various solutions with each other before raising their hands to answer.

For the first problem of [14 + 3] Memory counted-from-one, keeping track of the counts on her fingers. This extract from the video transcript shows Memory's response to the next problem [23 + 4] following teacher/researcher modelling of counting-on.

> M: *(Pauses for a while, then touches her finger). One.*
> *(Pauses. Restarts her count from twenty three. Touches four of her fingers consecutively whilst looking ahead and counting) twenty three, twenty four, twenty five, twenty six.*
> *(Pauses, touches the middle of her finger again) twenty seven.*
> *(Smiles, covers face with hands).*
> *Twenty eight!*

This extract shows that Memory began to count-from-one, then stopped as she realised she could count-on to solve the problem, a phenomena that Wright, Martland, Stafford and Stanger (2006) call *curtailment*. However, her strategy use was not yet accurate, as she counted-on-by-ones for one number too many. Again, I observed that when keeping track of counts on her fingers, there was

[6] *Bonds to twenty dice game (SANC, 2015).*

inconsistent, inaccurate, one-to-one correspondence between her verbal counts and her finger touches.

Analysis of, and reflection on, these observations suggested that a focus on partitioning activities might help Memory improve her accuracy before more counting-on was encouraged. To this end, small group focused teaching in Sessions 3 and 4 consisted of a range of activities involving five frames, designed to develop pattern recognition and conceptual reasoning (activities 7.4.1, 7.4.2 and 7.4.3). The five frames were unfamiliar to all the class learners, and as a result I increased and then gradually decreased the time interval of the *flashed* cards (following Wright, Martland, Stafford & Stanger, 2006, p. 34). By the end of Session 4 Memory was beginning to be able to ascribe number to the displayed patterns without counting-from-one.

During the small group focused teaching in Sessions 5 and 6 we continued to work with the flashed five frames, and aimed to engender the development of counting-on by *varying the setting* to include the screened five and ten frames and numeral cards. Interestingly, retrospective analysis of the video recording just prior to the start of small group teaching in Session 5 reveals Memory spontaneously and accurately using a count-on strategy when playing a board game. During small group focused teaching in Session 5, Memory was less accurate than the other Stage 2 group learners when counting-on. For example, Kamwi clearly and consistently counted on sub-vocally during Sessions 5 and 6. Memory guessed answers and became distracted by the other learners, as shown in the following extract from the video transcript for the sum [20 + 6].

R: *(Pointing to the whole screen which is covering two ten frames and a six-in-ten frame) Twenty and six, how many?*
M: *(Raises five finger on one hand, counting) twenty, twenty one, twenty two, twenty three, twenty four, twenty five, twenty six.*
(Pauses, is distracted by another child, raises three finger on other hand, counting)
Twenty seven, twenty eight, twenty nine. Twenty nine!

However, during small group focused teaching in Session 6, Memory seemed to be benefit from the involvement of another learner (indicated as L), as the following extract from the video transcript shows.

R: *Twenty and four (screens ten frame cards).*
M: *(Looks at other learner who is counting aloud. Raises two fingers simultaneously then two fingers consecutively and keeps pace counting with other learner)*
Twenty two, twenty three, twenty four!
R: *(Places out and screens three full ten frames and one four-in-ten frame) How many?*
M: *Thirty four.*
R: *(Screens the 34. Places a two-in-ten frame) And how many here?*
L: *Two!*
M: *Thirty six! (Smiles widely, looks at other learner).*

Memory was motivated to keep pace with the other learner and to answer accurately, displaying *engagement* and *enjoyment of the challenge*. Her solution for [34 + 2] also displays features of non-counting-by-ones strategy, as she answered very quickly without obviously counting-on.

As all three learners in the Stage 2 group had displayed counting-on for addition in a range of settings, in Session 7 the class teacher and I decided to focus on subtraction during small group focused teaching. Memory's initial strategy for the removed items tasks was unclear, but seemed to involve initially counting-up-from-one. When we modelled a counting-back-to strategy, Memory and the other Stage 2 learners struggled to keep track of the counts, so we decided to assess their backwards number word sequences (BNWS), which we realised were not secure. Assessment of the rest of the class during Session 7 also revealed a number of learners with insecure BNWS from 10 and 20. The class teacher suggested that she had not been focusing much on counting backwards in class, as it was assumed this had been covered in Grade 1.

Memory was judged to have progressed to Stage 3 in the post assessment interview, confirming the observations from the teaching experiment that she now had access to counting-on strategies for addition. Overall, her strategies in the post assessment interview were more accurate than in the pre assessment interview, and of the two problems she answered incorrectly, her answers were only incorrect by 1. However, her strategy use in the post assessment interview was varied and inconsistent, and she was distracted by other learners outside the classroom. Memory counted on to solve [9 + 3] and [9] + 4, using her fingers to keep track of counts. Memory answered [5] + 2 and [7] + 3 quite quickly and without using her fingers or counting out loud, strategies that I judged to be at least Stage 3 (counting-on). However, as these were sums are based on the numbers 5 and 10, it could also have been that the small group focused teaching activities based on partitioning numbers had helped her to start developing Stage 5 non-counting-by-ones strategies.

In summary, Memory reached her intended learning destination of progression to EAS Stage 3, but she tended to wander off from the path every now and then, and her counting-on strategy use was not yet robust. This case study has shown how elements of the MR programme were adapted to enable such progress towards more sophisticated early arithmetic strategies.

4.3.3 Grace's Case Study

On the basis of the pre assessment interview, it was clear that Grace had access to advanced counting-by-ones strategies and as a result I judged her to be at EAS Stage 4 (intermediate number sequence). Grace was highly accurate, answering all 19 questions of the pre assessment interview correctly. Grace's strategies were difficult to judge as she made few mouth movements and little overt use of her fingers. Judgments were made mainly on the basis of response time and subtle eye, head and finger movements. Grace counted on to solve additive and missing addend tasks. Because of the speed and accuracy of Grace's responses to the first screened collection additive tasks, I initially thought that she might have be using non counting-by-ones strategies. However, careful retrospective review of the video recording revealed that she was quickly counting the stones of the second collection before they were screened. On the missing subtrahend tasks, her specific strategy was unclear and she did not respond when I asked how she had worked it out. I judged that her strategy was at least counting-down-to or up-to. Grace took more than a minute to respond to the first written subtraction sentence task, [16 – 12]. This task was designed to elicit the most advanced strategies. Her strategy was unclear, but she blinked frequently and moved her eyes, finally looking down at her hidden right hand before answering correctly. On the next subtraction sentence, [17 – 4], she answered within two seconds, suggesting that whatever process she had used to answer [16 – 12] had led her to develop a short–cut or non-counting-by-ones strategy with which to solve [17 – 4]. This was the only clear instance of a non counting-by-ones strategy that Grace displayed in the pre assessment interview.

On the basis of the pre assessment interview, Grace was placed into a Stage 3/4 group. The goal of her learning journey was to develop Stage 5 non-counting-by-ones strategies. Intermediate targets on her learning journey were to be able to increment by tens and ones on and off the decade, and add one to nine to and from a decade number without counting by one. I hypothesised that such progress would involve a *cognitive reorganisation*, as she came to no longer rely on counting-by-ones. I anticipated that steps on her journey might include the development of more advanced number knowledge, including number bonds (known facts) and development of the ability to partition numbers and use grouping by five and ten. Relevant key topics involving ten frames were identified as a bank of suitable activities (see Table 3 in Chapter 3 above).

Analysis of Grace's strategies from Session 1 reinforced the stage judgments originally made, as she consistently counted on in the context of dice games (discussed above). Sessions 2 and 3 focused on introducing the ten frames and then using them to support adding to and subtracting from a decade number (activities 8.4.1 and 8.4.2). Grace displayed a relatively secure understanding of tens and units, and was able to solve these problems quickly with relative ease, even when the ten frames were screened. She did not appear to be counting-on or back. My analysis of her responses against Wright, Martland, Stafford and Stanger's (2006) guidance suggested that she had perhaps become aware of the semantic link between, for example, "twenty add four" and "twenty four" (p. 181). In order to ensure the problems were *at the cutting edge* for Grace, I reflected that *distancing the setting* by trying the task with spoken numbers or printed numerals instead of ten frames might provide an appropriate level of challenge.

Following my analysis of the above observations, teaching progressed to more challenging tasks involving adding to and subtracting from decade number in Sessions 4, 5 and 6. For these activities (for example 24 + ? = 30), the semantic link between the addends and the answer no longer applied. Hence, I hypothesised that Grace could be supported in using known facts (number bonds of ten) and reasoning involving the ten frames as visualised patterns. Grace found these activities more challenging, and it became apparent during small group focused teaching in Sessions 4 and 5 that she was still not familiar with the patterns of numbers on the ten frames, especially those from 6 to 9. As a result, I adapted the activities in Session 6 to involve matching pattern cards, to facilitate pattern recognition and hence scaffold reasoning about visualised patterns. In Session 6, Grace displayed increasingly accurate number bonds for 10, and was able to solve addition and subtraction to and from 10 without counting-on or back. Following Wright, Martland, Stafford and Stanger (2006), I hypothesised that she might be able to reason from these known results to solve addition and subtraction problems to and from other decade numbers. The extract from the video transcript below shows Grace's response to the first two problems beyond 10, namely 30 – 2 and 40 – 4 (unscreened).

R: (Places three full ten frames, unscreened). How many here now?
G: Thirty.
R: OK, thirty. Take away two (screens two dots on the third ten frame to make a regular eight dot pattern).
G: (Looks at partially screened card briefly). Thirty eight.
R: Thirty eight?
G: (Looks at cards, smiles, looks at R and CT) Twenty eight.
CT: (Smiles).
R: (Unscreens the cards, places another full ten frame). How many now?
G: Forty.
R: OK, take away four. (Screens four dots on the last card).
G: (Looks at partially screened card, sub vocal) Six. (Out loud) Thirty six!

Grace's initial incorrect response of 38 to the problem 30 - 2 suggests she is reasoning visually or using known facts. When prompted, she reflected on her thinking and was able to self correct to 28, displaying awareness of decade numbers. On the second problem she took a little more time to reflect, and her sub vocalisation shows that she explicitly made use of a known fact (10 – 4 = 6) to solve the problem correctly on the first attempt. In the post Session 6 discussion, the class teacher shared her observation that Grace was now answering the small group focused teaching problems quickly, without counting-on. We decided to try and extend this progress in the next session to sums that involved bridging through the decade.

As a result, during small group focused teaching in Session 7, I adapted activities from Wright, Martland, Stafford and Stanger's (2006) key topic 8.5.6 (addition by going through 10) to involve the ten frames. However, Grace reverted to counting-on by ones to solve these problems. I reflected that more foundational work on partitioning and building numbers was necessary to scaffold her use of non-counting-by-ones strategies before bridging through 10 in this way. In the retrospective analysis, I reflected that an alternative, perhaps more successful, pathway might have been to have first consolidated the non-counting-by-ones strategies on and off the decade by *distancing the setting*. Then we could have moved on to partitioning activities before attempting to bridge through 10. In this respect, I reflected that bridging through 10 was *beyond the cutting edge* for Grace at this point.

At the end of teaching experiment Session 7, Grace joined a mixed stage group taking part in a "number talks"[7] activity, led by the class teacher. In this activity, the class learners were presented with a range of large non-standard dot patterns, and asked to say how many dots there were and how they saw them. The aim was to encourage the class learners to explain and discuss their thinking, with the goal that they would be able to extend these skills to later discussion of their early arithmetic strategies. The class teacher encouraged all learners to share their thinking, and encouraged alternative strategies and positive socio-cultural norms for discussion. Grace observed and listened to the responses of the other learners for the first two tasks, then joined in on the third task to offer her suggestion of how she saw the nine dot pattern.

I judged Grace to be at EAS Stage 4 on the basis of the post assessment interview, therefore no change in Grace's EAS stages occurred between the pre and post assessments interviews. She answered all questions correctly in both assessment interviews, but there was a variation in the range and type of strategies observed. I found that strategy judgments were easier to make in the post assessment interview, as Grace was more overt in her verbal counting and use of fingers than she had been in the pre assessment interview.

Overall, Grace used less advanced strategies in the post assessment interview, as judged against my strategy coding schedule. No instances of non-counting-by-ones strategies were observed in the post assessment interview. As in the pre assessment interview, Grace consistently counted on to solve additive and missing addend tasks, using her fingers to keep track of counts. Her strategies for the subtraction sentence tasks in the post assessment interview were much clearer than in the pre assessment interview, and provided insight into the strategy she may have used in the pre assessment interview. Grace took 30 seconds to solve the subtraction sentence [16 – 12] in the post assessment interview, compared to over a minute to solve the same problem the pre assessment. Grace started by building 16 on her fingers then pointing to her toes inside closed shoes. She then worked backwards from her toes to her fingers, counting-up-to 12, and looking at her four remaining fingers to see what was left. For the next task [17- 4] Grace used a variation on the counting-from-one strategy which I called counting-from-five-or-ten. In this instance, she began her count at 10, building up to 17 on her fingers. She then began again at 10, counting-up-from 11 to 14, and looked at her remaining three fingers before answering "three". On the missing subtrahend and removed item tasks, she used a range of more and less sophisticated strategies. For [15 – 11], she built perceptual replacements on her fingers and toes from 1 up to 15, then from 1 up to 11, then counted the difference. For [12 – ? = 9], she nodded her head 3 times, then paused and raised 3 fingers consecutively without looking at them.

[7] *Number talks are classroom discussions around a mathematics problem focused on sense-making (Stott & Graven, 2015).*

Analysis of this suggest she had either counted-up or down-to by visualising or temporal sequence, then had checked this answer with motor movement of her fingers. However, there is no way to determine if she counted up or down, and she did not respond when questioned.

In summary, Grace did not reach the learning goal of spontaneous non-counting-by-ones within the time frame of the teaching experiment. However, she showed progress towards the intermediate target of adding to and from a decade without counting-from-one, and used non-counting-by-ones strategies during small group focused teaching, although these strategies were not spontaneous or *robust*. This case study showed how the individual MR programme was adapted for group work, and how it could be adapted further to engender more progress in early arithmetic strategies for learners at this 3/4 Stage group.

This chapter has presented the results of the study. The next chapter will discuss the results in light of the three research questions and the literature.

Chapter 5. Discussion of Results

In the previous chapter I presented the results of the study. In this chapter I discuss and analyse the results as they address each of the three research questions.

5.1 Research Question 1 - what progress, if any, do learners make in early arithmetic strategies using the Mathematics Recovery programme approach?

Section 4.1 presented a comparison of the results from the pre and post assessment interviews for the representative sample of six learners from the 18 Grade 2 learners who took part in the teaching experiment. Three indicators of progress in early arithmetic strategies were described: EAS stages, strategy spectra and frequency of correct responses. Section 4.3 presented the learning journeys of the three case study learners. Evidence of progression from the teaching experiment sessions was presented, and the sample learners' responses in the pre and post assessment interviews were described in more detail. In this section, I discuss the progress of the six sample learners from the pre to the post assessment interviews in order to analyse whether or not each learner achieved the instructional goals and targets.

The emergent perspective of this study enables me to use the interchangeable lenses of constructivist, socio-cultural and Vygotskian socio-constructivist theories, which each offer a valuable perspective when analysing learning and progression (Simon, 2009). Likewise, both the progression and the number sense views of arithmetic progression can be applied to help analyse progress in early arithmetic strategies (Askew, 2013). In the preparation phase of this study, instructional goals were set for each of the six sample learners. The instructional goals for the teaching experiment were for each learner to develop more sophisticated early arithmetic strategies, and to progress by one EAS stage. Due to the short duration of the study additional intermediate targets were also set.

In the MR programme, progression in early arithmetic strategies is represented by learners' advancement thorough the six stages of EAS, towards the overall goal of facile non-counting-by-ones strategies. Comparison of the pre and post assessment interview results showed that two of the six sample learners progressed by one EAS stage, while the other four learners remained at the same stage. The two sample learners who progressed by an EAS stage (Kamwi and Memory) were both in the Stage 2 group, progressing from Stage 2 to Stage 3 (that is, from counting-from-one to counting-on). Analysis of the spectra of strategies used by the sample learners in the pre and post assessment interviews provided additional evidence of progress and supported the judgement of stage progression for the Stage 2 sample learners. Analysis of strategy spectra also provided evidence of subtle progression in early arithmetic strategies for one of the Stage 1 sample learners, Hendrix. However, analysis of the strategy spectra of Stage 3/4 sample learners Grace and Mutinta suggested that they both used less sophisticated strategies in the post assessment. The frequency of correct responses showed that accuracy in the pre and post assessments increased for three of the six sample learners, and remained constant for the other three learners. Again, the change in frequency of correct responses was most significant for the Stage 2 learners, Memory and Kamwi, whose accuracy increased by 3/6 and 2/7 points respectively in the post assessment interviews. Of the two Stage 1 learners Charles improved by 1/7 points, whilst Hendrix made no change. At Stage 4, Grace answered all questions correctly in both the pre and post assessment interviews whilst Mutinta showed no change in overall accuracy. Evidence from the teaching experiment supported the evidence from the pre and post assessments, and will be discussed below.

A possible hypothesis for the above is that progression from Stage 1 to 2 and from Stage 3/4 to 5 requires extensive foundational work in other early numeracy aspects beyond EAS, such as structuring numbers and BNWS. It is suggested that this is in contrast to progress from Stage 2 to Stage 3, which can be facilitated by teaching focused on EAS activities alone. Viewed through a socio-cultural lens, it could also be that, in the one-to-one pre and post assessment interviews, learners reverted to their trusted counting-by-ones based strategies, even if they had progressed to more sophisticated strategy use in the teaching context. It can be argued that this variation between strategy use in the assessment and teaching contexts is more likely to be visible in whole class interventions (such as that of this study) as there is a significant socio-cultural difference between the one-to-one assessment interview context and the group teaching context, for example in the presence of other learners and possible difference in the learners' perception of what is expected in the group teaching context as compared to the one-to-one assessment context. In one-to-one interventions the differences between the assessment and teaching contexts is less pronounced, and so may have less of an impact on the results. Another hypothesis for the above is that transitions from Stage 1 to Stage 2 and from Stage 3/4 to Stage 5 require more time and consolidation than transitions from Stage 2 to Stage 3, but further research would be needed to investigate this.

In Wright, Martland and Stafford's (2006) constructivist progression model of early arithmetic each stage is discreet and progress between stages is a cognitive process that involves an accommodation or change. This change they term *cognitive reorganisation* following Steffe and Cobb (1988). However Wright, Martland and Stafford (2006) do discuss progress *within* stages and they suggest that within a stage children may be more or less secure in their use of strategies. This idea is referred to as *robustness* in the MR programme, and will be considered in the following discussion of the results relating to progression of each stage group.

5.1.1 Stage 1 learners - from perceptual to figurative counting

The two sample Stage 1 learners did not achieve the instructional goal of progression from perceptual to figurative counting by the end of the 7 week teaching experiment. At the end of the study, Hendrix and Charles still relied on building perceptual replacements to represent collections of objects in early arithmetic problems. Viewed through Wright and colleagues' constructivist lens, it can be said that insignificant cognitive reorganisation occurred to declare stage progression. However, detailed strategy spectra analysis showed that Hendrix had progressed from predominantly counting-three-times, to not-counting-three-times. He no longer had to build each addend from one and then count-all. Although subtle, this change does represent an important shift in strategy towards the next EAS stage. If this is not a cognitive reorganisation, then it is at least streamlining of the processes involved. Charles' spectra did not show clear progression, but there was a change in his strategy use between the pre and post assessment interviews. In the pre assessment, he had used a variety of strategies from Stages 1 to 3. Although he did not develop more sophisticated strategies, the strategy spectra results suggest he did consolidate his existing strategies and use them more effectively.

The results indicated that the Stage 1 learners developed and consolidated their finger patterns and, to a lesser extent, their visualised dot patterns during the course of the teaching experiment. Both finger patterns (Wright, Martland and Stafford, 2006) and visualisation (Bobis, 1996) have been identified as important aspects for early arithmetic progression. Both Charles and Hendrix made progress towards their intermediate targets, which were the consolidation of finger patterns up to five and ten. Again, the evidence from the teaching experiment sessions and the post assessment interviews confirm this. The finger use results showed that Hendrix used simultaneous finger patterns as part of his early arithmetic strategies on two occasions in the post assessment interviews, compared to no use in the pre assessment interviews. The results showed that Charles' strategy use was much more consistent in the post assessment interview than in the pre assessment interview. He used fingers in his strategies for all problems in the post assessment interview, whereas he had not used his fingers at all in the pre assessment interview. Viewed through a Vygotskian lens, it can be seen that the learning opportunities provided during the intervention may have lead the learning, enabling Hendrix and Charles to make some progress.

Charles' accuracy increased by 1/7, whilst there was no change in Hendrix's accuracy. Neither was able to bridge the finger range barrier, although Hendrix seemed to be developing a hybrid figurative strategy based upon dot patterns that might have worked given more time (see Section 5.3 for further discussion of this).

Considering their starting points, progression to Stage 2 may have been beyond Hendrix and Charles' capabilities within the time frame of the teaching experiment. They first needed to consolidate and develop their figurative understanding of numbers (i.e. visualised patterns, facile finger patterns, counting rhythm). In addition, both learners were absent for the final session, so they only attended 6 of the 7 teaching experiment sessions.

5.1.2 Stage 2 learners - from figurative counting to initial number sequence (counting-on/back)

Both Stage 2 learners achieved their instructional goals and progressed to using counting-on strategies, from counting-from-one. The evidence from the EAS stage judgements, strategy spectra, frequency of correct response, and the teaching experiments all agree with this finding. Through a constructivist lens I can speculate that a cognitive change occurred to bring about this progress. Memory showed the most significant change in strategy use, as Stage 3 strategies became her most frequent strategy, compared to no evidence of use of these before the teaching experiment. Kamwi's spectrum showed similar but less dramatic progression. Of all the sample learners, the change in frequency of correct responses was most significant for Memory and Kamwi. For Memory, this progress represents a clear cognitive reorganisation. The evidence thus suggests she did not have access to the counting-on strategy before the study and that a significant change in her thinking had occurred (Steffe & Cobb, 1988; Wright, Martland and Stafford, 2006).

For Kamwi, the evidence is less clear, as he used his fingers infrequently and did not overtly count out loud. Analysis of Kamwi's strategy spectrum shows that he may have had some access to counting-on before the start of the study, but that the evidence for this was not enough for him to be judged at Stage 3 in the pre assessment. I provide further discussion and evaluation of this in Chapter 6. Clear evidence of Kamwi counting-on sub-vocally in the teaching experiment supports the judgement that he used counting-on in the post assessment interview. His ability to answer missing addend tasks and his increased accuracy in the post assessment suggests that by the end of the teaching experiment, counting-on had become a robust strategy for Kamwi. It is difficult to attempt an analysis of Kamwi's cognitive processes from a constructivist perspective, as his strategies were hard to observe. However, a Vygotskian socio-constructivist lens helps bring his learning into focus. Opportunities that enabled him to work as though he was at the counting-on stage may have facilitated the development observed (Askew, 2013). From a number sense perspective, Kamwi's increased use of counting-on represents increased access to, and selection of, more efficient strategies.

5.1.3 Stage 3/4 learners - from intermediate to facile number sequence

The evidence for progression for the Stage 3/4 learners is the least clear of the six learners. Their instructional goal was to develop non-counting-by-ones strategies. There is little evidence that either Grace or Mutinta developed spontaneous non-counting-by-ones during the study, and neither used any non-counting-by-ones strategies in the post assessment. Neither Grace nor Mutinta changed EAS stage between the pre and post assessments. Video evidence from the teaching experiment sessions clearly showed that, with the aid of ten frames, both Grace and Mutinta were able to solve additive and subtractive tasks without counting-by-ones. However, this strategy did not transfer to spontaneous use in the context of the post assessment interview. Through Wright and colleagues' constructivist lens, the evidence implies that their strategy use was not yet robust. However from a socio-cultural perspective the shift in learning context from the teaching session to the one-to-one post assessment interview could account for the different strategy use in the different contexts. This is discussed further below. Evidence from the teaching experiment shows that both Grace and Mutinta made progress towards their intermediate targets of adding and subtracting to and from a decade number.

In both pre and post assessments, Grace answered all questions accurately and Mutinta showed no net change in accuracy. The strategy spectra results suggest that both Grace and Mutinta used less advanced strategies in the post assessment interview. As Dowker (2005) notes, children will use a range of strategies in different contexts, so EAS stage judgements are made on the basis of the most advanced strategies used (Wright, Martland and Stafford, 2006). Therefore, although Grace and Mutinta used less advanced strategies in the post assessment, this does not mean they had regressed in terms of EAS during the study.

Grace and Mutinta used fingers much more extensively in the post assessment interview than the pre assessment interview, so it may have been that their strategies were more visible in the post assessment interview, and therefore judged as less advanced. It seems likely that Grace was using similar strategies in the pre and the post assessment interviews, but that in the post assessment she used her fingers for convenience, as analysis of her post assessment interviews suggested. As Wright, Martland and Stafford (2006) note, children with more advanced strategies do use less advanced, finger-based strategies for convenience (p. 68). In the pre assessment interview, Grace had limited her finger use, and kept them hidden when she did use them. Viewed through a socio-cultural lens, we can see that the pre assessment interview was an unfamiliar context. The learners may have been unclear as to what was expected of them, and so Grace may have concealed her use of fingers. Analysis of narrative data from Grace's case study suggests that she was quicker when using her fingers in the post assessment. So, although these strategies were *less sophisticated* as judged against my coding schedule, they may have been *more efficient* for Grace because of the speed and fluency with which she was able to use them. A number sense view of arithmetic progression (Baroody, 2006; Askew, 2013), which prioritises the selection of an efficient strategy (rather than identifying a hierarchy of strategies), contextualises these observations.

In summary, the results suggest that all six sample learners did make some progress using the MR programme approach, but that the extent of this progress was limited by the short duration of the intervention, particularly for Stage 1 and Stage 3/4 learners. Progress also varied by stage group. The most significant progress was made in the Stage 2 group, followed by subtle progress in the Stage 1 group and limited progress toward intermediate targets in the Stage 3/4 group.

5.2 Research Question 2 - what strategies are used by Grade 2 learners in solving early arithmetic problems?

This section discusses the results of the six sample learners' spontaneous strategy use in relation to Research Question 2. Unit counting (counting-by-ones) strategies predominated overall. All six sample learners were able to quickly solve some introductory tasks without obviously counting (recall from Chapter 3 that these introductory tasks were defined as problems within the finger range with addends or minuends of 1 or 2). However, beyond the introductory tasks, only one spontaneous instance of non-counting-by-ones was observed (by Grace, in the pre assessment interview).

5.2.1 Less advanced subtraction strategies extended to problems with numbers of greater magnitude

Recall that the study focused mainly on developing strategies for addition rather than subtraction. As a result, there is only sufficient data on spontaneous subtraction strategies for the three more advanced of the six sample learners, Grace, Mutinta and Kamwi.

Strategies for addition problems varied by stage group and by individual learner. All learners used a range of strategies for addition, apart from Grace who consistently counted-on. The results of the Stage 1 learners (Hendrix and Charles) clearly show that they were unable to solve problems beyond the finger range. They relied on building perceptual replacements on their fingers. The only exception were the two occasions when Hendrix drew tally marks in the dust under the desk, thereby extending his strategy of building perceptual replacements to sums beyond the finger range (this is not a more advanced strategy in Wright, Martland and Stafford's (2006) terms, as will be discussed further in Section 5.3 below).

Of the three learners for whom there is subtraction data there was a significant difference between their strategy use for addition and subtraction problems. All three learners used less advanced strategies for subtraction than they used for addition. This phenomenon is clearest in the results of the Stage 3/4 learners Grace and Mutinta, whose strategies for addition were predominantly advanced (Stage 3), but who used less advanced (Stage 1/2) strategies for around half of their subtraction problems. Several researchers have suggested that learners may find subtraction more challenging than addition (Baroody, 1984; Kamii et al., 2001; Cockburn, 2007), and this could explain these results to some extent. An alternative hypothesis is that classroom teaching to date had focused less on the foundational knowledge and skills (including BNWS) necessary for the development of more advanced subtraction strategies (discussed further below).

There is limited discussion in Wright, Martland and Stafford (2006) of less advanced counting strategies that may be used by learners when attempting to solve subtraction problems. They describe one strategy that involves using fingers as perceptual replacements, which they suggest is “viable for subtraction only when the minuend is no greater than 10” (p. 90). Beyond this, Wright, Martland and Stafford suggested that children at less advanced EAS stages tend to guess or use addition in order to solve subtraction problems.

In this study, all three learners for whom there is subtraction data were observed to use a strategy that involved building the minuend on their fingers, then counting-down or up-to the subtrahend or difference, and then counting what was left. The learners seemed to be extending the strategy of building perceptual replacements described by Wright, Martland and Stafford (2006) above to minuends greater than 10. I observed two variations on this strategy. The first variation involved counting-from-one to build the minuend, and the second involved counting from a number nearer the minuend but not the minuend itself (e.g. counting-up from 10 if the minuend was 17).

Wright, Martland and Stafford (2006) note that children judged to be at Stage 3 and above might use fingers as perceptual replacements for subtraction tasks within the finger range. Wright, Martland, Stafford and Stanger (2006) consider that up to six is the range that it is “considered useful for children to become facile at keeping track of counting” (p. 109). As the case study results show, the sample learners in my study extended the use of finger-based strategies to minuends and differences of up to 30.

The use of such counting-by-ones strategies for problems involving numbers of such magnitude requires much cognitive effort (Gray & Tall, 1994; Baroody, Bajwa & Eiland, 2009). The sample learners in this study were keeping track of counts well beyond the range considered useful by Wright, Martland, Stafford and Stanger (2006), and had developed their own relatively cognitively demanding and complex methods to keep track of these counts. This is discussed further in Section 5.2.4.

Wright, Martland and Stafford (2006) suggest that children learn to use counting-down-from for subtraction “at around the same period” (p. 65), as they develop counting-on for addition. Whilst counting-on to solve addition problems was a clearly identifiable strategy used by the sample learners in this study, the results show that no clear observations were made of more advanced counting-down-to strategies to solve removed items tasks. Counting-on strategies were clearly identifiable in all of the four sample learners who used them, judged on the basis of vocal counting and clear finger movements. In cases where an unclear subtraction strategy had been judged as at least Stage 3, there was no way to tell if the learners were counting-down-from or counting-up-to to solve these tasks. My judgements of Stage 3 strategies were made on the basis of the time taken to answer, eye movements and other clues that excluded a counting-from-one strategy judgement as the learners did not respond when I asked "how did you do that?"

The only clear instance of a sample learner using an advanced counting-by-ones strategy to solve a subtraction problem was when Mutinta vocally counted-up-to in order to solve a missing subtrahend task during the pre assessment interview. Use of such counting-up-to strategies for a missing subtrahend task is considered by Wright, Martland and Stafford (2006) as "not a common occurrence" (p. 74). They suggested that in such an instance, children are reconceptualising the subtraction task as an addition task, which in itself is indicative of an advanced strategy, suggesting understanding of the inverse relationship between addition and subtraction. However, Fuson (1984) questioned whether counting-up for subtraction would actually be an easier strategy for children than counting-down, and Baroody (1984) suggested that the difficulties of the simultaneous processes associated with counting-down was a reason why "children tend to supplement counting down with a counting-up procedure" (p. 1).

As mentioned, this study focused on the EAS strategies of the sample learners, and did not formally assess the other aspects of the LFIN which include BNWS. However, the BNWS of the learners were assessed informally in the final teaching experiment session, and the observations indicated that many learners had weak BNWS from 20 and 10. As the class teacher indicated, there had been little focus on counting backwards in class. On the basis of this evidence, I suggest that learners in this study may either have been totally unfamiliar with counting-down strategies, or found them more cognitively demanding, and that as a result they may have infrequently used counting-down-to for subtraction problems.

5.2.2 Extensive perceptual finger use as a part of early arithmetic strategies

Wright, Martland and Stafford (2006) state that "finger patterns play an important role in early numerical strategies" (p. 26). The results of my study show that fingers were used to some extent by all six sample learners. Hendrix used his fingers in the majority of problems. Charles, Grace and Memory used their fingers in around half of their strategies, and Memory and Kamwi used theirs in roughly a third. Two broad types of finger use were observed in this study: use of fingers as perceptual replacements, and use of fingers to keep track of counts. Recall that keeping track of counts is considered the more sophisticated use of fingers. A spectrum of finger patterns for the numbers 1 to 10 was also observed, from consecutively to simultaneously raised finger patterns. As the results show, there was a connection between the more sophisticated finger use and the more sophisticated finger patterns, i.e. learners who were able to keep track of counts also tended to use the more sophisticated simultaneous finger patterns.

There is a pattern in the type of finger-based strategy use by EAS stage of the sample learners. The Stage 1 learners used fingers only in the least sophisticated way, as perceptual replacements. The most extensive use of fingers was also seen in this stage group (by Hendrix). What is interesting is the extent to which Hendrix used his fingers as perceptual replacements. This was clearly a strongly established strategy for him. The Stage 2 learners made the least use of fingers as a part of their early arithmetic strategies. However, analysis of the results suggest that Memory's finger use varied across the observation periods, so there may have been other socio-cultural factors at play (discussed further below). Of all the groups, the Stage 3/4 learners most frequently used fingers in a sophisticated way (to keep track of counts). This corresponded with Wright, Martland and Stafford's (2006) observations that "as children progress across the stages of early arithmetical learning, they typically develop increasingly sophisticated finger strategies" (p. 2). However, what was interesting was the extent to which the Stage 3/4 learners also used their fingers as perceptual replacements (discussed above in the context of subtraction, Section 5.2.1). Use of fingers as perceptual replacements represented approximately a third of Grace's overall finger use and represented the majority of Mutinta's finger use as a part of early arithmetic strategies.

As we have seen from the literature, there is a precedent for more advanced learners using simultaneous finger patterns for convenience to solve subtraction problems within the finger range. However, the extent of such finger use and its extension beyond the finger range is notable in this study. Likewise, so are the limited instances of fingers being used to keep track of counts in subtraction problems. As Wright, Martland and Stafford (2006) note, with increasing progression "one expects that, ultimately, children will no longer rely on using finger patterns" (p. 26). However, this does not seem to be the case in the results observed in the teaching experiment in this study. The implications of these observations are discussed further in Section 5.2.5 below.

The results also reveal a pattern in the use of fingers by observation period. The results show that Memory did not use her fingers at all in the pre assessment interview, but did use them as a part of her spontaneous strategy use in teaching experiment Sessions 1 and 2, and the post assessment interview. This suggests a possible reason as to why the data from the pre assessment interview alone did not accurately reflect her baseline EAS stage. Memory's finger use was inconsistent, yet it seems as if fingers were an important aspect of her early arithmetic strategies. Likewise, the results show that Grace and Mutinta used fingers much more extensively in the post assessment interview. Analysis of Grace's subtle use of fingers in the pre assessment interview, with her hand hidden from view, suggested she had some reason for not showing her finger use where she might have normally have done so. This was discussed in relation to a socio-cultural theory of learning in Section 5.1.3 above. A socio-cultural theory of learning may also provide an explanation for why Memory may not have used her most advanced strategies in the unfamiliar context of the pre assessment interview. This highlights a potential limitation of the pre assessment interview and is discussed further in Section 6.3.2.

5.2.3 Limited use of written tallies

Tally use has been noted as a widespread feature of learners' mathematics strategies in Southern Africa (Schollar, 2008; Ensor et al. 2009). The phenomena is not restricted to early arithmetic, but to learners throughout the primary level. As noted, the use of tally marks is recommended as a teaching method in one of the main primary Grade 1 textbooks in Zambia, and these were the textbooks in use in the Grade 1 at the case study school. The class teacher explained how this method was still modelled to young learners in Grades 2 and above as a way of helping them to solve arithmetic problems. However, as explained in Chapter 3, I made a deliberate decision to avoid pencil and paper use in the assessment interviews, in an attempt to elicit other possible strategies and to move the emphasis away from unit tally counting.

Overall, the results show there was little evidence of tally use in this study. Because of the aforementioned methodological decision, there was no opportunity to collect evidence of written tally use from the pre and post assessment interview observation periods. However, written tally use was not directly observed at all during the teaching experiment sessions where pencils and paper were freely available. Tally marks were found in one leaner's (Mutinta) teaching experiment exercise book at the end of the teaching experiment. Hendrix was the only sample learner observed using tallies in any significant way. Again, these were not written, but drawn in dust or imagined on the table top.

There is a recurring suggestion in the international literature that teachers' emphasis on the use of perceptual counting in the early years of school can lead to such a strategies becoming entrenched (Gray & Tall, 1994; Dowker, 2005; Wright, Martland & Stafford, 2006; Gray, 2010). Within Southern Africa, Ensor et al. (2009) suggest that "learners are restricted from access to more abstract ways of working with number by classroom practices that privilege concrete models of representation" (p. 15) including tally marks.

The results of my study suggest that tally use was yet to have become entrenched in this way with these young Grade 2 learners. At this stage, the learners' methods were much more concrete, including use of fingers and concrete objects, and had not progressed to involve such notation. Neither the class teacher nor myself modelled or encouraged tally use during the teaching experiment. Viewed through a socio-cultural lens, the results suggest that teacher modelling and encouragement may therefore be a factor in learner strategy use and entrenchment in classroom contexts. The class teacher had explained that during regular mathematics lessons she often modelled the use of tallies to learners as a strategy for solving early arithmetic problems, and that this strategy was in use by learners during mathematics lessons and was confirmed by my analysis of learners' mathematics exercise books.

5.2.4 Extension of perceptual counting strategies beyond the finger range

Use of tally marks enabled Hendrix to extend his strategy of creating perceptual replacements to sums beyond the finger range. Wright, Martland and Stafford (2006) discuss the use of toes to extend such a strategy up to 20, but they make no reference to tally marks.

In a similar way, the complex finger-based strategy for subtraction, discussed in Section 5.2.1, enabled the Stage 3/4 sample learners to extend their use of fingers as perceptual replacements to subtraction problems with numbers of larger magnitudes. Rather than using a count-down-to strategy, learners used their fingers as perceptual replacements for larger magnitude numbers beyond the range that is considered useful by Wright, Martland and Stafford (2006).

The six sample Grade 2 learners in this study had developed their own strategies that were enabling them to extend perceptual counting strategies to be able to solve problems of increasing difficulty involving numbers of greater magnitudes. The evidence from other Southern African studies also shows learners' continued use of unit counting strategies in problems with numbers of great magnitude (Schollar, 2008; Askew, 2013). Wasserman (2015) similarly found this:

> It was noted that most learners successfully and unsuccessfully relied on methods of finger counting or tally counting no matter the size of the numbers being added or subtracted. There was little evidence of use of more efficient or abstract methods. (p. 2)

Use of tally marks and such complex invented finger-based strategies may enable children to 'get by' and get the right answer. From a socio-cultural perspective, a classroom atmosphere where the emphasis is primarily on accuracy, not also on strategy, might reinforce this. As the class teacher's comments indicated this may have been the case with the class in this study prior to the commencement of the teaching experiment. Ultimately, entrenchment of such less advanced counting strategies leads to mathematical difficulties in later grades (Siegler, 1988; Geary et al. 1992; Gray & Tall, 1994; Ostad, 1997, 1998; Dowker 2005). The results therefore suggest that the Grade 2 learners in this study were at risk of developing later mathematical difficulties, but that at this young age counting-by-ones strategies such as tally use could not yet be described as entrenched in these learners. Hence, there is the need to help Grade 2 learners redirect their cognitive efforts to enable them make progress to more sophisticated early arithmetic strategies. The next section will discuss how this study adapted the MR programme to a whole class context in order to facilitate such progress.

5.3 Research Question 3 - investigating the advantages and constraints of a whole class adaptation of the Mathematics Recovery programme to facilitate progression in early arithmetic strategies

As stated in Chapter 2, the MR programme is a holistic intervention programme encompassing early intervention, assessment, teaching, and professional development (Wright, Martland & Stafford, 2006). In this discussion I will use three of these headings - early intervention, assessment, and teaching - when considering the possible adaptation of the MR programme to help Grade 2 learners progress to more sophisticated early arithmetic strategies in a whole class context. Unfortunately, discussion of the use of the MR programme for professional development is beyond the scope of this study. The EAS and LFIN will be discussed under the assessment heading, and the IFEN will be considered under the teaching heading. Furthermore, I will discuss the actual adaptations made to the MR programme during the teaching experiment and consider their advantages and constraints. I will also discuss how the results may suggest possible further adaptations of the MR programme for future work with learners at similar stages and in similar teaching contexts.

5.3.1. Early intervention

International and Southern African research suggests that targeted early intervention in numeracy can have a significant impact on learners' performance, and that this is much more effective than later intervention in terms of outcomes and cost (Heckman, 2000; Dowker, 2005; Wright, Martland & Stafford, 2006; Spaull & Kotze, 2015). Early intervention is necessary because the gaps in learners' knowledge and understanding increase as they progress though school (Wright, Martland & Stafford, 2006; Graven, Stott, Mofu & Ndongeni, 2015).

The MR programme intervention is normally targeted at the lowest attainers in the second year of schooling, "before the gap between their knowledge and that of more able pupils grow[s] too wide and cause[s] them to experience excessive failure" (Dowker, 2009, p. 25). In the Southern African context, it is not one or two learners but rather the majority of the class who are in danger of falling behind with regards to more sophisticated arithmetic strategies (see SACMEQ III results in Moloi & Chetty, 2010; Musonda & Kapa, 2011).

The intervention in this study targeted a class of Grade 2 learners in the second year of primary school in rural Zambia (aged seven turning eight). In this context, it was assumed that almost all learners could be at risk of low attainment, so a whole class intervention was planned. Unlike other SANC project research into the MR programme, which explored smaller after-school clubs (Stott, 2014), or out-of class group interventions (Wasserman, 2015), this study adapted the individual teaching activities from Wright, Martland, Stafford and Stanger (2006) for use in small group focused teaching within a whole class context.

Although focused on individual intervention, the MR programme also makes provision for group and whole class teaching (Wright, Martland, Stafford & Stanger, 2006; Wright, Stanger, Stafford & Martland, 2012) and it has been systematically adapted for whole class use in Australia and New Zealand.

One of the main advantages of the adaptation in my study was that we were able to work with the whole class, rather than with only a small group. As the results show, all learners were still reliant on counting-by-ones, and therefore may have been at risk of entrenching such less advanced strategies. In this way the results of the study supported my view that most learners in the class might have been at risk of future low achievement. Such a whole class intervention, if effective, could reduce the risk of later low achievement for all learners, as has been demonstrated in Australia and New Zealand (Christensen, 2003; Thomas, Tagg & Ward, 2003). However, the results seem to point to the need for a longer period of intervention than was investigated in my study.

The timing of this intervention was significantly shorter than recommended for the individual MR intervention programme. Within each of the six teaching experiment sessions (excluding the introductory session), focused teaching with each of the five groups only lasted for an average of seven minutes per group, an average of 42 minutes in total for each group. Working with the whole class in this way did mean that teaching time per child was limited as compared to other individual and small group interventions.

Although all learners made some progress during the study, progress varied by stage group. Over the seven weeks, two of the six sample learners progressed by one stage. Taking duration into consideration, these results are comparable to those of other Southern African MR programme studies (see Chapter 2). Dowker (2005) noted that one-to-one interventions do not need to be very long to be effective. Although the impact of group teaching may have been diluted as compared to a one-to-one intervention, these results suggest that the impact was not proportionally reduced. Being able to use flexible grouping by EAS stage meant that teaching could be more efficient, as all group learners were working from the same stage starting point. New Zealand's Early Numeracy Project uses such grouping to enable targeted individual intervention in classroom contexts where there may be a range of starting points in a class (Tozer & Holmes, 2005).

This was a time-bound research study. In a real classroom context it could be imagined that such an 'intervention' could run over a longer period of time or over the entire academic year. Alternatively, elements of the MR programme could be integrated into day-to-day mathematics teaching, as with the Count Me In Too project in Australia and the Early Numeracy Project in New Zealand. However, the research context of my study was a small class of only 18 learners. Considering that class sizes of 40 or 50 are common in Southern African schools, further research will be needed in order to adapt any such programmes to be effective with large classes.

Another advantage of the use of group rather than individual teaching was the social aspect of learning. In group and classroom learning situations, Gravemeijer and van Eerde (2009) emphasise the importance of considering the actual learning trajectories in light of the wider social context. Thus zooming out from a constructivist view of the child's cognitive processes, we can use a socio-cultural lens to consider how the pathways of the learners interact (Gravemeijer & van Eerde, 2009). The results suggest that learners were co-constructing learning in the small group context: spontaneously sharing answers, self-correcting, and trying to keep pace with one another. Wasserman (2015) similarly found that several learners' progress was enhanced by the motivation resulting from observing efficient strategies of other learners.

Another advantage of such a whole class adaptation was that it enabled co-running of intervention sessions with the class teacher herself. Collaboration with teachers is a key feature of the design research methodology and is a way of bridging the gap between research and practice in mathematics education (Dowker, 2005). During our post session discussions, the class teacher indicated that she had begun to use some of the MR programme elements in her day-to-day teaching. Wasserman also found a similar effect on the class teacher in her 2015 study into the MR programme in South Africa. As Wright, Martland and Stafford (2006) observe, "in each of the years since its inception, the Mathematics Recovery programme has significantly influenced general classroom teaching of mathematics" (p. 7).

5.3.2 Assessment

In the MR programme, assessment takes two forms: an initial video-recorded one-to-one assessment interview followed by continuous assessment during the teaching intervention. Both forms of assessment were used in this study, and the assessment interview was administered again at the end of the teaching experiment in order to assess the effectiveness of the adapted MR programme. Assessment in the MR programme is underpinned by the LFIN, against which observations are analysed and assessments made. This study employed the video-recorded assessment interviews, which are a defining feature of the MR programme. However, limited time, resources and research-scope meant that the assessments were not applied to the whole class, but only to a sample of six learners. For the same reasons, the assessment interviews were not applied in their entirety, but only selected questions that focused on EAS were used from the original MR programme Assessment Interview Schedule 1.1. The class teacher and I then used the EAS stage judgements of the six sample learners to sort the rest of the class into stage groups (as discussed in Chapter 3).

The advantages and constraints of the adaption of the MR programme assessments to a whole class setting will now be discussed, under the following sub headings: video-recorded assessment interviews, the assessment interview schedule, EAS stage judgements for the whole class, and the LFIN.

5.3.2.1 Video-recorded assessment interviews

Both Gravemeijer and Eerde (2009) and Wright, Martland and Stafford (2006) refer to the video recording of teaching sessions to facilitate learner and teacher development. As Wright, Martland and Stafford (2006) suggested, I found there to be several advantages to the video-recorded assessment including the level of detail of the information available, the ability to replay and reassess, and the ability for teachers to review the video together to consolidate judgements. I found the videos to be a useful tool for developing researcher and teacher understanding.

However, one of the major constraints of the individual video interviews were that they were time consuming, even given the small size of the sample in this study. In addition, specialist resources were needed that may not be available in similar school contexts. Other SANC projects have found similar results. Mofu (2013) concluded that the primary disadvantage of the MR programme assessment interview was that it is “labour intensive and time consuming to administer” (p. 67). As a way of addressing this, Wasserman’s (2015) study explored the possibility of adapting the MR programme assessment interview to work in a group context. In this study, the class teacher and I used more informal methods to make strategy judgements for the non-sample class learners, as discussed further in Section 5.3.2.3 below.

5.3.2.2 The assessment interview schedule

Using only the EAS aspect of the LFIN for the pre assessment interviews meant that other key areas of early numeracy progression were not assessed, for example BNWS and structuring of numbers. Had these aspects been assessed, they may have made the subsequent intervention more effective. Use of continuous assessment during the teaching experiment enabled the class teacher and me to fill some of these gaps.

Using the MR assessment interview in its original form also meant there was no data on paper based strategies and notation, the development of which is encouraged in the MR programme. However, this decision did provide an opportunity to focus on developing the learners’ mental strategies, which it seemed were not often practiced.

Furthermore, the sample learners were unfamiliar with both the format of the interview and with me as the interviewer. This seemed to have influenced the learners' responses as discussed above, but there are also inherent disadvantages to using any such isolated assessment. As the results confirm, the learners used additional strategies in the video-recorded small group focused teaching that they had not used in the pre assessment interview, hence the importance of getting a rich picture of their strategy use in a range of contexts (Dowker, 2005).

5.3.2.3 EAS stage judgements for the whole class

The research suggests that early numeracy interventions need to be *targeted* to be most effective (Wright, Martland & Stafford, 2006; Dowker, 2005). From a Vygotskian perspective, targeted learning is learning that is challenging but achievable with support, termed "at the cutting edge" by Wright, Martland, Stafford and Stanger (2006, p. 59). In the MR programme, individual intervention is initially targeted at the cutting edge on the basis of analysis of individual video assessment interviews. As discussed above, there are several disadvantages to the use of such assessments in a Southern African context.

In this study, the class teacher and I used the EAS stage judgements of the six sample learners to sort the rest of the class into stage groups, based on our growing knowledge of the LFIN and the teacher's experience of the class learners' mathematical proficiency and early arithmetic strategies. This enabled us to place all learners into stage groups for targeted teaching without having to conduct lengthy individual video interviews with each learner. In this way, it was intended that all learners would benefit from targeted teaching at the cutting edge, even though we hadn't the time or resources to video assess them all. This adaptation also empowered the teacher to use her existing understanding of the learners' mathematical proficiency, and provided an opportunity for professional development, as we applied our understanding of the LFIN.

An interesting question is the extent to which the class teacher's view of the learners' EAS stages correlated with their researched EAS stage, as assessed against the LFIN. At that point, this was her first introduction to the MR programme, so her grouping was based mainly on ad hoc previous experiences of the learners' mathematical strategies. My informal analysis of video evidence from the teaching experiment sessions showing some non-sample learners suggests that some of the class teacher's judgements were accurate, but that a few learners were operating outside of their assigned stage group (both above and below). Analysis of these results is beyond the scope of this study, but it raises an important aspect when considering adaption of the assessment element of the MR programme in similar contexts, as will be discussed further in the next chapter.

5.3.2.4 The LFIN

The LFIN was used as the basis for initial and continuous assessment in this study. This study focused mainly on the EAS aspect of the LFIN, which is 1 of 11 aspects. The EAS aspect proved to be a powerful model for teacher and researcher understanding and learning. Both the class teacher and I were unfamiliar with the MR programme before the research began, but we were able to pick up the general concepts relatively quickly, even without access to a physical copy of the books (I only had them as eBooks). As well as providing a model of EAS learning, the LFIN provided a framework for rich discussion of teaching strategies, especially those related to perceptual counting.

I found it relatively easy to use the strategy descriptions from the EAS aspect of the LFIN to recognise and classify the individual strategies used by learners to solve particular problems. However, I found it harder to judge the overall EAS stage of a learner, as making EAS stage judgements involved applying specific criteria to observations of a learner's strategy use in a range of problems (see the Results Section 4.2 above).

The class teacher and I found that the EAS Stages 1 and 3 were the easiest to understand and identify. Stage 2 and 4 we found to be the least intuitive. Furthermore, the nomenclature associated with Stage 2 felt quite "jargon heavy". The class teacher and I tended to use strategy names in our discussion (e.g. counts from one) rather than the names of the stages (e.g. figurative counting) as they are found in Wright, Martland and Stafford (2006).

In this study, the sample learners did not always fit well with the "profile of a typical child" (Wright, Martland & Stafford, 2006, p. 9*)*. This was more so at the upper Stages 3 and 4. Recall from discussion of the results above that less advanced strategies for subtraction were observed frequently in both Grace and Mutinta. As discussed in Section 5.2 above, there was limited evidence for counting-down in missing subtrahend tasks, which is the defining feature of Stage 4.

In this regard, I questioned the usefulness of the Stage 3/4 distinction in this context. Of the sample of six learners in my study there were no learners at Stage 3 and two learners at Stage 4. But both of these learners used less advanced strategies for subtraction and neither fitted well with the typical learner profiles as discussed above. Furthermore, the class teacher and I did not make much of the Stage 3/4 distinction in our discussions, because Stage 3 and Stage 4 learners are taught together in the MR programme. Further analysis of the usefulness of this distinction with this age group is beyond the scope of this study, as the other 10 aspects of the LFIN were not formally assessed. Within the SANC project, teachers of older learners have found these distinctions more applicable (Stott, 2014).

This does however raise the question as to whether the LFIN's model of progression is perhaps more influenced by the context of the underlying research study than is acknowledged within the constructivist perspective. The original research on which the LFIN is based is Steffe and Cobb's longitudinal teaching experiment with six first-grade students in Georgia, USA, in the early 1980s.

Steffe and Cobb (1988) were working from a strongly constructivist perspective. However, viewed through a socio-constructivist lens, the teaching may have influenced the progress observed in their study. Cobb and Yackel (1996) acknowledge such a reflexive relationship between theory and practice, and Cobb (2000) suggests that from an emergent perspective, the relationship between an individual learner's action and the wider classroom mathematics context must be taken into account. Further analysis of this issue is beyond the scope of this study, but this observation does suggest possible avenues for future research.

The class teacher's and my understanding of the learners' initial starting points was limited as we only used the EAS element for the pre assessment interview (as the issue of the BNWS from Session 7 showed). This indicates that a more holistic approach involving assessment of all LFIN aspects may have been better but perhaps more time consuming, necessitating a longer duration for this study.

Although the primary focus had been on the EAS aspect, we ended up using the structure of numbers to 20 aspect of the LFIN extensively. The teacher and I found that finger use was a key strategy indicator and a rich source of information. However, there were no levels of finger use described in Wright, Martland and Stafford (2006) so I developed a spectrum (see Table 7, Chapter 3) to clarify this and used this to assess the Stage 1 learners.

5.3.3 Teaching

In this study, the IFEN was used as a framework for planning for learner progression. One-to-one teaching intervention activities from Wright, Martland, Stafford and Stanger (2006) were adapted for use in small group focused teaching within a whole class context. These one-to-one activities were chosen, rather than the whole class teaching activities from Wright, Martland, Stafford and Stanger (2006), as this was an intervention study targeting the whole class who had been judged as being at risk of falling behind. Discussion and notation were two features of the MR programme teaching that this study did not focus on.

As discussed in Section 5.1 above, certain stage groups made more progress than others. Recall that one of the defining features of the design research methodology of this study is that it addresses how an intervention works. In light of this, possible reasons for the difference in progress between stage groups will now be considered.

During the teaching experiment I synthesised the individual and whole class teaching guidance from the MR programme to inform the small group focused teaching. I found this helpful because the individual guidance shed light on cognitive aspects of learning and the whole class guidance shed light on social aspects of learning. Just as the emergent perspective enabled focus on different aspects of learning during this analysis, so my synthesised teaching guidance enabled me to focus on individuals and on the group whilst teaching (as Wright, Martland, Stafford & Stanger, 2006 recommend). However, a constraint of this adaptation was the logistical difficultly of applying some of the one-to-one teaching guidance when working with a group. For example, it was not possible to micro-adjust group activities to meet all individual needs simultaneously.

Small group teaching was focused mainly on the key EAS topics from the IFEN. This meant that the holistic MR programme was not delivered as recommended. As the six learner case studies indicated, this may have impacted on the progress made, especially that of the Stage 3/4 learners. Had the full programme been delivered, the Stage 3/4 learners may have benefited from activities to develop their BNWS for secure counting-down-from for subtraction and from activities to develop partitioning of numbers for the development of non-counting-by-ones strategies.

Another advantage of the MR programme was the flexibility of the teaching activities, in that these can be adapted as necessary. This proved useful with the Stage 1 learners as I found that there was less guidance in Wright, Martland, Stafford and Stanger (2006) as to how to model and scaffold figurative strategies than there was for the development counting-on. As the six case studies show, I adapted certain activities to this end.

A constraint of the IFEN, as applied in this context, was that the "children may" section of the guidance for each activity did not always reflect what was observed. In a similar way, I had found that the actual observations and the typical EAS profile did not always correspond (see above). For example, there is no reference in Wright, Martland and Stafford (2006) to the use of tally marks as a part of early arithmetic strategies, nor is there any mention of the finger-based strategies for subtraction that I observed.

An interesting outcome of the adaptation of the MR programme in this context was the observation that visualisation of dot patterns could offer an alternative pathway to more sophisticated early arithmetic strategies for Stage 1 learners who use tally marks as perceptual replacements. The evidence presented in Hendrix's case study suggested that dot patterns could be visualised as an alternative to tally marks. Looking through a constructivist lens to "discovering what might go on in children's heads" (Steffe & Cobb, p. vii) it seems regular dot patterns may be easier to visualise than tally marks, thereby reducing the cognitive demands of keeping track of counts.

The invariant nature of the EAS sequence was another possible constraint related to the progression of the Stage 1 learners. For Hendrix and Charles, keeping track of two addends, as is required in figurative counting, seemed a demanding task using either fingers or dot patterns. Again, adopting a constructivist lens I hypothesised that rather than aiming for these Stage 1 learners to develop figurative counting strategies, it may have been less cognitively demanding for them to progress directly to counting-on. In this way, the cognitive demands of keeping track of counts would be reduced. As the results in Hendrix's case study show, he did at one point hold the addend 'in his head', rather than build the addend from one. There is a precedent for such a jump in MR programme teaching, as the teaching guidance suggests a jump from Stage 3 to 5, although Steffe and Cobb (1988) stated that the stages form an invariant sequence and that each stage builds on and incorporates the previous stage. Dineen's (2014) study also indicated that the "figurative counting stage is not a necessary prerequisite for counting-on to solve addition tasks" (p. 298). As Askew (2013) noted, current research supports a number sense view of learning, where there is no clear hierarchy of strategies, thus countering Steffe and Cobb's earlier claim about an invariant sequence.

There is also a need to support the development of non-counting-by-ones at all stages, as is provided for within the holistic MR programme. My study's focus on the EAS aspect limited opportunities for such development. Dineen (2014) concludes that early grouping strategies "should constitute an important part of the mathematics curriculum for students in their first year of school" (p. 298). The New Zealand Early Numeracy Project also recognises these two aspects of early arithmetic, as it includes both counting and part-whole strategies in its framework (Tozer & Holmes, 2005). This has implications for potential intervention or systematic programme design in a Southern African context.

According to the class teacher I collaborated with, the lack of resources was one of the biggest barriers to mathematics teaching and learning. Some adaptations were made to the resource materials used in the teaching activities. An advantage of this was that most of the key resources could be improvised using inexpensive, locally available materials, as fits with the SANC project's aims of development of sustainable solutions. However, a constraint was that some of these homemade materials were not very durable.

In this chapter I have discussed the results of the study by each research question in turn. In the next chapter I reflect on and evaluate the study, make recommendations and suggest areas for further research.

Chapter 6. Conclusions, limitations and recommendations

Chapters 4 and 5 presented and discussed the results of the study. In this chapter, I draw conclusions related to each of the three research questions, outline the limitations of the study, and make recommendations as to possible future avenues of research.

The focus of this study was a teaching experiment exploring the adaptation of Wright and colleagues' Mathematics Recovery programme for use with a class of 18 Grade 2 learners in a rural Zambian primary school. The study aimed to answer the following three research questions:

1) *What progress, if any, do learners make in early arithmetic strategies using the Mathematics Recovery programme approach?*
2) *What strategies are used by Grade 2 learners in solving early arithmetic problems?*
3) *How might the MR programme be adapted to help Grade 2 learners progress in their early arithmetic strategies, and what are the advantages and constraints that emerge from the whole class adaptation?*

I now address each research question in turn, drawing conclusions and making recommendations as to avenues for future research and considering implications for future interventions in similar contexts.

6.1 What progress, if any, do learners make in early arithmetic strategies using the Mathematics Recovery programme approach?

The results of this study suggest that all learners made some progress in EAS using the MR programme approach. However, the extent of this progress was limited by the short duration of the intervention, by the teacher/researcher's lack of experience of the MR programme, as well as by design and adaptation factors. Learner progress in early arithmetic strategies varied by stage group. The most significant progress was made in the Stage 2 group, followed by subtle progress in the Stage 1 groups, and limited progress toward intermediate targets in the Stage 3 /4 groups.

The results of the different stage groups provide support to Wright, Martland and Stafford's (2006) conception of progress between EAS stages as discrete rather than continuous, involving periods of consolidation before a "cognitive reorganisation" occurs. However, analysis of learners' strategy use in this study also aligned with a number sense view of early arithmetic progression and questioned the conception of EAS stages as following an invariant sequence. Learners were observed to use strategies that were *efficient* for them but that were not necessarily the most sophisticated strategies that they had access to (as judged against the EAS's hierarchical stage model).

These results suggest areas for further investigation and development when adapting the MR programme to ensure progress for all, as I discuss in Section 6.3.

6.2 What strategies are used by Grade 2 learners in solving early arithmetic problems?

The sample of six learners was considered by the class teacher to be relatively representative of the class of 18 in terms of both gender and mathematical performance. Strategies involving unit counting (counting-by-ones) predominated overall. In this study little spontaneous use was observed of more sophisticated strategies involving non-counting-by-ones. When it was observed, the exact strategies were not clear and were limited to the introductory tasks, with only one exception.

This study showed that the six sample learners were extending less advanced subtraction strategies to problems with numbers of greater magnitude. Perceptual finger-based strategies were used with minuends of large magnitudes, extended beyond the finger range.

A distinctive subtraction strategy was observed and classified as 'counts-from-one-or-five-or-ten'. This strategy involved learners building perceptual replacements for the minuend and the subtrahend on their fingers in order to then find the difference. Counting-down-to for subtraction was not clearly observed. This contrasted with clear evidence of counting-on for addition. I hypothesised that this could be attributable to the learners' weaker knowledge of BNWS.

There was limited observation of spontaneous written tally use by the sample learners in this study, despite tally use being actively encouraged by the class teacher during regular mathematics lessons. I speculated that this might mean that tally use was not yet entrenched in these young learners, and that fingers were still the learners' main perceptual replacements. This observation provided justification for early intervention in this context.

This research adds to the limited but growing body of research on early arithmetic strategies in Southern Africa. Further research into learners' early arithmetic strategy use and into teaching strategies for early arithmetic is justified in a Southern African context, in order to better understand what is going on in early grade classrooms and to inform the design of interventions.

6.3 How might the MR programme be adapted to help Grade 2 learners progress in their early arithmetic strategies, and what are the advantages and constraints that emerge from the whole class adaptation?

Despite this study's limitations, which I address below, the results suggest that the MR programme has the potential to be adapted for use in Grade 2 whole class contexts to facilitate progression in early arithmetic strategies. Its adaptation is especially justified as a practical early intervention facilitating learner and teacher development. By the end of the study, elements of the MR programme were already being used in regular mathematics lessons by the class teacher. Further research is warranted into such an adaptation, as this study focused mainly on the EAS aspect of the holistic MR programme. Based upon this research, I believe that in a Southern African context the MR programme has the potential to be developed into a systematic numeracy programme, as exemplified by the Count Me in Too project in Australia and the Early Numeracy Project in New Zealand.

6.3.1 Early intervention

One of the main advantages of the MR programme adaptation in this study was that the class teacher and I were able to work with the whole class, rather than selected individuals or one group of learners. As the study found, although the six representative sample learners were at different EAS stages, they were all still reliant on unit counting to some extent. As discussed, they were perhaps at risk of entrenching such less advanced strategies, leading to lower achievement in later grades.

The study showed that small group focused teaching within a whole class context (as used by the New Zealand Early Numeracy Project) could be an effective format for adaptation of the MR programme to similar Southern African contexts. Perhaps further research could investigate MR programme whole class plenary teaching, rather than group teaching within a whole class context.

Class teacher involvement was key to this study. My learning was greatly enhanced by the teacher's participation and insights. Likewise, the teacher stated that she learned much from participating in the teaching experiment. Teacher/researcher collaboration is recommended for further such studies given the SANC project's aim of developing sustainable solutions to the crisis in mathematics education in Southern Africa.

6.3.2 Assessment

This study found that the limitations of using video-recorded assessment as an assessment tool for whole class use outweighed the advantages. Limitations of the video-recorded assessments related mainly to the time and specialist resources needed to complete them. Advantages included the level of detail available and the ability to replay and confirm observations. However, the use of video recording for training and professional development of teachers is potentially useful.

Some alternative assessment methods were explored briefly in this study and further research into such assessment methods is justified. For example, research into the use of teacher observation and judgement in conjunction with the LFIN and diagnostic questions could be considered. The continuous assessment element of the MR programme was effective and made it possible to overcome some of the limitations of having formally assessed only the EAS aspect of the LFIN.

The LFIN as a framework for assessment proved to be a powerful model for teacher and researcher understanding and for planning teaching activities. It also framed our discussion of teaching strategies, especially those related to perceptual counting, and provided an opportunity for professional development for the class teacher and myself.

The class teacher and I encountered some difficulties with the terminology of the EAS. For example, we tended to use the predominant strategy name (e.g. counts-on) rather than the EAS stage name (e.g. initial number sequence) when discussing the EAS Stage 3 learners. Furthermore we had difficulty fitting the upper EAS Stages 3 and 4 descriptions to observations of the Grade 2 learners' early arithmetic strategies. For example, the EAS and accompanying literature contains no mention of the less advanced subtraction strategies observed in use by these learners. In this way my study suggests there is an opportunity for further research into the EAS as a "best fit" model in similar contexts.

6.3.3 Teaching

The synthesized individual and whole class teaching guidance in the MR programme was helpful in planning and leading small group teaching. The flexibility of the MR programme teaching activities allowed for adaption as necessary. Although this study focused mainly on the development of counting based early arithmetic strategies, a more holistic approach to teaching as recommended by Wright, Martland, Stafford and Stanger (2006) may have been more effective. This study suggested that dot patterns could be used as an alternative to tally marks, in order to engender more sophisticated strategies.

A key finding was that that the actual observations and the typical EAS profile suggested by Wright, Martland and Stafford (2006) did not always fit. Additionally, my observations did not always reflect the 'children may…' section of the MR programme's teaching guidance for each activity. Findings from this study support Dineen's (2014) findings that certain children might benefit from skipping the figurative stage and moving on to counting-on directly, so as to lower the cognitive demands of keeping track of large numbers of counts. This study recommends that non-counting strategies (e.g. grouping and part-part-whole strategies) be progressed alongside counting based strategies at all stages.

Lack of resources was the biggest constraint as perceived by the class teacher. This study focused mainly on locally available, low cost materials but durability was an issue. Further research could investigate sustainable resources that could be used in similar contexts. It was beyond the scope of this study to address teachers' professional development, which is a key aspect of the MR programme that could be investigated by future research.

6.4 Limitations and validity of the research

In order to ensure the validity of the findings of this study, certain measures were taken. My key data gathering tool, the MR programme assessment interviews, had been assessed for validity across a range of international contexts. It was noted that a common criticism of the design research method is its potential lack of scientific rigour. As such, the steps recommended by Shavelson et al. (2003), and Cobb et al. (2003) were followed with regards to the systematic collection of a wide range of data, which was then triangulated (Neuman, 2006). An explicit chain of reasoning as specified by Gravemeijer and van Eerde (2009) was followed from beginning to end (Middleton, Gorard, Taylor & Bannan-Ritland, 2008) and all steps were carefully documented to promote transparency and replicability (Cobb et al., 2003).

The selection criteria of Grade 2 learners was justified in relation to the goals of the research. Likewise, the goals of the research justified the decision to collaborate closely with the class teacher, which was also ethically driven and based on her willingness to participate. The decision to use and adapt features from the MR programme in a research context inspired by the SANC project maths club model (Stott & Graven, 2013) was justified by the growing body of literature supporting their use in such research (Weitz, 2012; Mofu, 2013; Ndongeni, 2013; Stott, 2014; Dineen, 2014). Despite these methodological considerations, several factors limited the study. I now discuss these factors in turn.

6.4.1 Inexperience of the Mathematics Recovery programme

Neither the class teacher nor myself had used the MR programme before the start of the study, nor had we participated in MR training programmes that are available in other contexts. Recall that professional development is a key feature of the MR programme. I had been learning about the MR programme as part of my master's course at Rhodes University and was supported in my learning by my SANC project supervisors. Unfortunately, formal data collection of the class teacher's professional development was beyond the scope of this study. My learning as teacher/researcher, which is a central aim of design research, was documented alongside that of the sample learners. For example, as indicated in Chapter 3, I felt more confident when analysing the video of the post assessment interview than I had analysing the video of the pre assessment interview during the preparation phase of the study, having spent 7 weeks observing and analysing the videos of small group focused teaching against the LFIN.

Any conclusions related to the progress of learners and effectiveness of the adaptation must take into account this limitation of teacher/researcher and class teacher inexperience. However, the sample learners still made progress in their early arithmetic strategies despite our lack of experience. Given that the class teacher had little training in the MR programme and no access to the MR programme books, these results are encouraging in terms of the sustainability of such a programme in Zambia, where there may be a limit to the teacher training resources available. If I were to repeat this teaching intervention I would also extend the duration of the MR programme intervention and the frequency of teaching sessions.

6.4.2 Duration of study

The duration of the study was limited, to an extent, by my needs as a researcher, as has been the case with other MR programme research (Dowker, 2009). As indicated above, in future research I would extend the period and frequency of interventions for both pedagogical and research reasons whilst maintaining the sample size of six for the research. Working with the whole class together in this 7 week teaching experiment meant each group experienced only a few minutes of small group focused teaching each week. Genuine classroom contexts would enable a more prolonged focus but progress could potentially be affected by the larger class sizes common to Southern African school contexts.

6.4.3 Language

Language was not an issue in the way I had initially anticipated, as the class teacher herself used English as the primary medium of instruction during regular mathematics lessons. I had anticipated a need to translate learner responses from the video recordings, but this was not necessary as the learners mainly spoke in English when in front of the camera. Language barrier issues have also been mentioned in other such SANC project studies (see Wasserman, 2015, for example). Two significant language barriers were encountered in my study.

Learners' understanding of my English language instruction during the assessment interviews could have impacted the results (as in a similar way the unfamiliar context of the assessment interviews may have affected learners' use of fingers). Likewise, there was a lack of data on learners' narratives of their strategies. As Wright, Martland and Stafford (2006) state, the primary aim of the MR assessment interview is to "elicit the most advanced strategy" (p. 8) through careful probing and questioning. The language barrier may have been a limiting factor in this.

The language barrier impacted on the quality of teaching that I provided and on my choice of teaching activities. Discussion is a key feature of group and whole class teaching in Wright, Martland Stafford and Stanger (2006) but this was one of the key features that did not occur so much in my study, as I noted in Chapter 5. The language barrier limited discussion and co-construction of meaning during small group focused teaching. Viewed through the socio-cultural lens, the co-construction of meaning is a key element in learner progress. The class teacher was able to lead such discussions in English and ChiTonga, however I was unable to use ChiTonga during small group focused teaching, and found the children were reluctant or unable to discuss their strategy use in English. The data from the number talks activity led by the class teacher showed the possibility for discussion as a context for learning. Progression results must take such limitations into account. Had language not been a barrier, and if more productive ways were found to draw on learners' home languages in discussion of strategies, more progress may have been seen. This is an issue that researchers and potential educators need to be aware of when working outside their first language context, in order to ensure that this key feature of the MR programme is given sufficient attention.

6.4.4 Notation

The development of learners' notation to support mental strategies was another feature of the MR programme on which this study did not focus. This could have been an interesting area for research. However, the decision was made to exclude pencil and paper from the assessment interviews, which shifted the research focus to the development of mental and practical strategies.

The exclusion of pencil and paper from the assessment interviews generates another research avenue as to whether the availability of pencil and paper during the MR programme assessment interview has an effect on the early arithmetic strategies used by learners. The conclusions of such a study could lead to refinement of the MR programme assessment interview for use in a Southern African context.

A focus on the development of mental and practical strategies during the teaching experiment sidelined opportunities to investigate notation as a tool for facilitating early arithmetic progression. As several research studies in Southern Africa have indicated an emphasis by teachers on the use of tally marks in early arithmetic, future research could explore the reconceptualisation of such written notation in order to facilitate learner progression in early arithmetic strategies.

6.4.5 Research design - data collection and analysis

The strategy coding schedule and checklist for making EAS stage judgements used in this study were of my own design, following Wright, Martland and Stafford (2006). Adaptations to the MR programme coding schedule included, for example, the assignment of stage numbers to the codes and development of new codes.

Some methodological decisions I made impacted on the results. For example, the coding of learners' immediate answers and the classification of [6 - 2] as an introductory question. If these choices had not been made Kamwi, for example, would have just passed as Stage 3 in the pre assessment interview, according to my stage judgement checklist (see Chapter 5 and Appendix E). However, I hope to have included enough detail of the underlying observations and criteria on which such judgement was made to enable other researchers to reconstruct my judgements and to use the tools that I developed through the study.

Were I to use the MR programme assessment interview as an analysis tool again, I would think about keeping the set of questions constant for all learners. Likewise, differences between the sets of questions for each learner in the pre and post assessment interviews (as a result of the flowchart format of the interview) made analysis of the results problematic. This would mean a change to the way the MR programme assessment interview is structured.

There were technical issues associated with data collection. Limited data was captured from teaching experiment Session 1 due to the logistical issue of filming and teaching at the same time. This issue was resolved in subsequent sessions thanks to the loan of a tripod, which freed me to concentrate on teaching. Recording error also meant that certain video and audio recordings were lost, for example the post Session 3 discussion between myself and the class teacher. I therefore recommend that a backup recording would be of benefit for similar studies if resources allow.

6.5 Conclusion

The interchangeable lenses of the emergent perspective from which this study was conducted proved especially useful when I was considering the learning at an individual and whole class level. A constructivist lens enabled me to interpret observations about the sample learners' learning in terms of the potential underlying cognitive processes, for example, cognitive reorganisation. This enabled analysis of, and planning for, individual learning. Adopting the SANC project maths club ethos for this study set up the possibility for such reflection through a socio-cultural lens. Use of a socio-cultural lens enabled me to take account of the interactions within groups of learners and between teachers and learners, which revealed social features of learning that may otherwise have been overlooked if learning had been viewed through only a constructivist lens. Socio-cultural features included peer competition as a motivating factor, and the influence of context on learner strategy use, for example between the assessment interview and the teaching experiment sessions.

This study integrated the design research and MR programme methodologies. I found that the design research paradigm aligned well with the MR programme, because of similarities between the iterative cycle of design research and the MR programme's teaching and learning cycle. The design research process consists of macro cycles, and further hypotheses may be generated for testing as part of the research. In this case, areas of interest and questions for further research were part of the findings of this study.

This study has made methodological contributions to the study of the MR programme as part of the SANC project and the broader Southern African context. Collaboration with the class teacher was a key feature of this research, serving to bridge the gap between research and practice. I found that my dual role of teacher/researcher provided a rich opportunity for learning, and this study documented my own learning about the MR programme and about learners' early arithmetic strategies alongside that of the learners.

This study contributes to the growing body of research into adaptation of the MR programme emerging from the two South African Early Numeracy Chairs' research teams, extending this research to a younger age group and to a broader Southern African context. Although limited by time and research focus, this study found that all sample learners made some progress in early arithmetic strategies as a result of the whole class adaptation of the MR programme intervention. This study also provided a detailed snapshot of the early arithmetical strategies of a sample of Grade 2 learners in a Zambian primary school, adding to the body of research on early arithmetic strategies in Southern Africa. This study found that unit counting predominated in the Grade 2 sample learners, but that strategies were not yet entrenched, indicating this was a suitable point for early intervention. This study outlines areas for further research into child and teacher learning in the context of early arithmetic strategies.

There is a pressing need for early intervention in Southern Africa to address the issue of reliance on perceptual counting, as indicated in a range of studies. This study suggests that the MR programme has potential for adaptation for use in a whole class context to facilitate learners' EAS progression. Elements of the MR programme could be adapted to form part of a whole class intervention programme in Southern Africa, similar to such systematic programmes in Australia and New Zealand.

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Appendix A: Parental Consent Letter

Informed Parental Consent Form for Grade 2 learners participating in After-School Maths Club and research related to these clubs.

Date: 5th June 2015

Dear Grade 2 Parent/ Guardian,

I will be running an after-school maths club at your child's school. The club will run on Thursday afternoons. The club will be a fun place where we will provide ways to support your child's maths learning. We invite them to come and learn. I will be researching how they learn. In the clubs, your child may be interviewed, observed at what they do and may be included in video and voice recordings. I am running the club as part of the research into maths learning at Rhodes University in South Africa, where such clubs are frequently used for such research into maths learning. We are running these clubs because Southern Africa needs confident learners who feel they can do maths so they are able to go on and study mathematics in their further education and, more importantly, who are able to use maths in their everyday lives.

During the maths clubs, your child will be involved with mathematical activities and mathematical work. The clubs will run once a week on Thursdays during the last 8 weeks of Term 2. The club will run for 1 hr from 1330.

All the data recorded is confidential and no one except the club mentors Catherine Young and [class teacher's name] andill have access to the raw data collected during the research. The data will be kept confidential and will be stored in safe place. During and after the study, we may publish some of the research in order that other interested people may learn from it. Any information used about your child in publications will have a number or pseudonym on it instead of his/her name. Your child will not be asked to talk about any personal information about themselves or their home lives.

Neither you nor your child will be provided with any payment to take part in the maths clubs or the research. However, they will be given refreshments during the club session.

You are welcome to come to the first maths club workshop so that you can see how the club works. This will be on the 25th June 2015.

You do not have to agree that your child will join the maths club and take part in research. Before you decide, you can talk to anyone you feel comfortable with or come and talk to me at the introductory session if you prefer, on the 11th June 2015. If you have any questions, please contact Catherine Young: [phone number]

Kind regards

Catherine Young

Masters Student

Studying with the South African Numeracy Chair Project at Rhodes University, South Africa. The work of the South Numeracy Chair at Rhodes is supported by the FirstRand Foundation (with the RMB), Anglo American Chairman's fund, the Department of Science and Technology and the National Research Foundation.

I agree voluntarily for my child to participate in the Maths Club sessions AND in the research study

Name of Child										Print Name of Parent or Guardian	
Date	D	D	/	M	M	/	Y	Y	Y	Y	Signature of Parent or Guardian	

Appendix B: Assessment Interview Schedule

Assessment Interview Schedule

Revised Wright, Martland and Stafford (2006)

Early Arithmetical Strategies

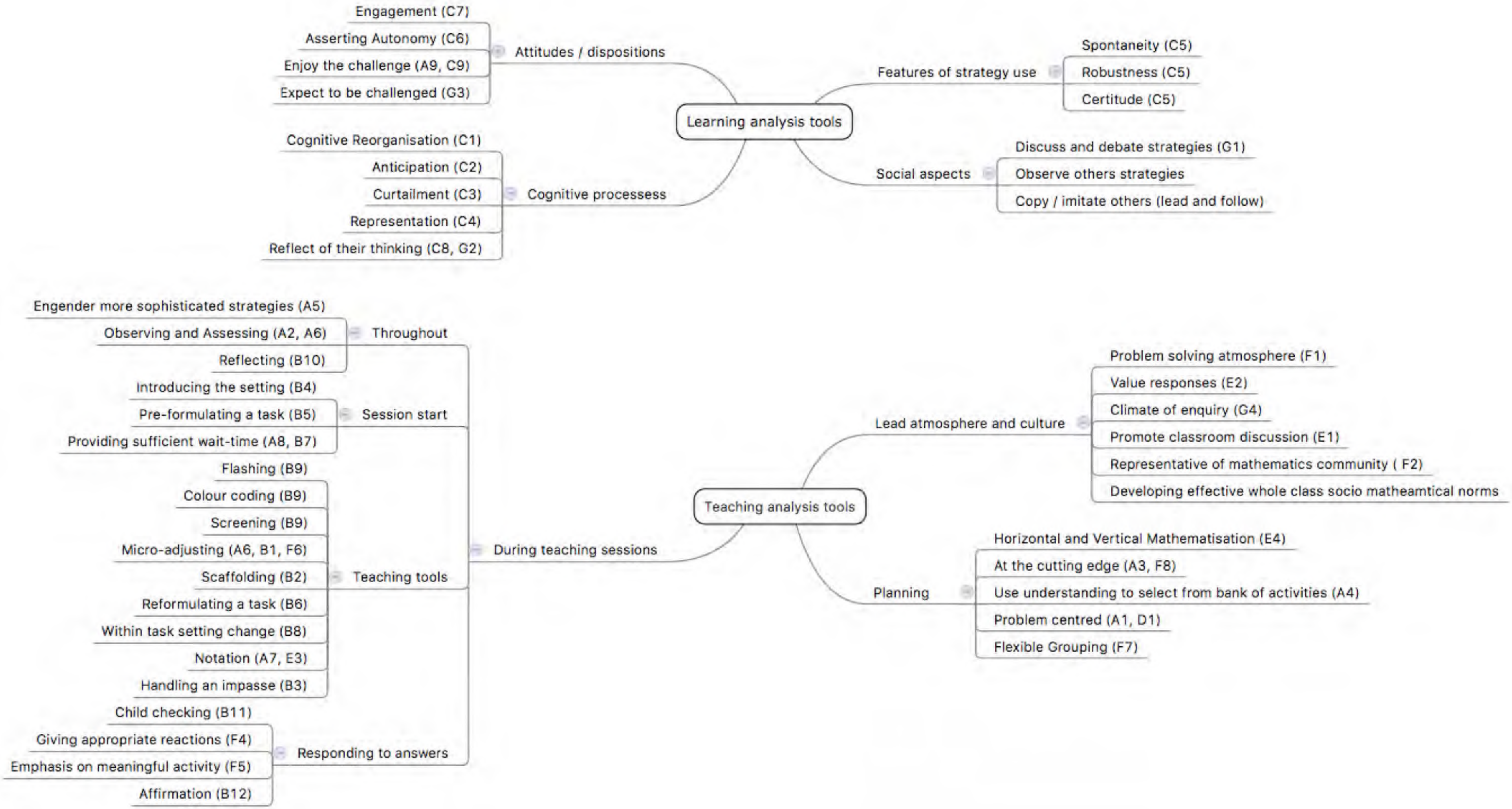
Child's Name DOB Age

Interviewer's name Date of interview Teacher/ Class

Question	Pre Assessment Date:		Post Assessment Date:	
	Narrative	Code	Code	Narrative
1. All screened *[3] + [1]*				
2 [5] + [4]				
3 [9] + [6]				
4. First screened (if 1+ wrong): [5] + 2				
5. [7] + 3				
6. [9] + 4				
7. If still too difficult: unscreened 5 + 2				
8. 7 + 3 •				
9. 9 + 4				
10.Gouped Items •13				
11. •18				
•12. •[8] + [5]				

13. [9] + [3]				
14. Missing addend •*4 + [ ] = 6*	*Counting-up-to/on, or using subtraction?*			
15. •7 + [ ] = 10				
16. •12 + [ ] = 15				
17. Subtraction sentences 16 - 12	*To elicit advanced strategy.*			
18. •17 - 14				
19. Missing subtrahend •*5 - [ ] = 3*	*to elicit 'counting-down-to' (Stage 4+)*			
20. •10 - [ ] = 6				
21. •12 - [ ] = 9				
22. •If all correct, more advanced: 15 - [ ] = 11				
23. Removed items •*3 - 1*	*To elicit counting-down-from (Stage 3+).*			
24. 6 - 2				
25. 9 - 4				
26. 15 - 3				
27. If all correct, more advanced 27 - 4				
EAS Stage	0 1 2 3 4 5			0 1 2 3 4 5

Appendix C: MR Programme Teaching Guidance Tool



Codes in brackets refer to the numbered teaching guidance in Wright, Martland, Stafford and Stanger (2006)

Appendix D: Teaching Experiment Planning Sheet

Design Research Teaching Experiment Planning Sheet Session: ________ Group: ______________

	Child 1	Child 2	Child 3	Child 4	Child 5	Teaching
1. Planning: Previous session evaluation: Planned Activities: **HLT:** Learners will...	**2a. Observations:**					**Observations and Analysis:** (Use MR tool)
Learners will...	**2b.** Analysis *(use MR tool)*					
3. Evaluation:						

Appendix E: EAS Stage Judgement Checklist

1. Review video recording of learner assessment interview.
2. Annotate the assessment interview schedule noting learner's responses using either the Mathematics Recovery coding schedule (Wright, Martland & Stafford, 2006, p. 188) or the coding schedule developed for this study (see Table 8).
3. Exclude incorrect responses.
4. Exclude introductory tasks.
5. Count the remaining strategy codes and the apply the following criteria:
    a. There must at least two instances / occurrences of the most advanced stage strategy for the learner to be judged at that stage, apart from:
        i. Missing subtrahend tasks, where at least one correct response meant a stage four judgement, as long as the strategy used was not count-down-from
        ii. Stage 5 strategies, where three counts were required for a stage 5 judgement.
6. If having following the above steps a learner does not qualify for even Stage 1, then a Stage 0 judgement is made.