INVESTIGATING TEACHERS' EXPERIENCES OF USING MULTIPLE REPRESENTATIONS TO TEACH FRACTIONS FOR CONCEPTUAL UNDERSTANDING IN 'GRADES 5 -7': A NAMIBIAN CASE STUDY

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by

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DECLARATION OF ORIGINALITY

I, **Julia Iiyambo, student no. 11I5240,** declare that this thesis: "***Investigating teachers' experiences of using multiple representations to teach fractions for conceptual understanding in 'grades 5 – 7': A Namibian case study"*** is my own work written in my own words. Where I have drawn on the words or ideas of others, these have been acknowledged using complete references according to Departmental Guidelines.

Julia Iiyambo — December 2014

(Signature) — (Date)

ABSTRACT

This study investigated teachers' experiences of using multiple representations to teach fractions for conceptual understanding in 'grades 5 – 7'. Three teachers were involved in the study and the main goal was to explore how teachers might work with the additional tools of multiple representations to improve the teaching of fractions for conceptual understanding. Different types of multiple representations such as written mathematical symbols, descriptive written words, pictorial representations, manipulations, concrete reality, oral representations, and experience-based representations and videos or power point presentations were examined. This investigation was carried out through a focus group workshop developed by the researcher and though observations of fraction teaching which took place in schools. The research focused on the teachers working to develop their use of multiple representations to teach fractions and how they implemented what they learned in the workshop into their own classroom teaching. This study was conducted using a qualitative case study design and was oriented in the interpretive paradigm.

The three participating teachers in the focus group workshop were observed and interviewed individually. The purpose of observing teaching before the workshop was to observe how the selected teachers taught fractions and in particular to observe which types of representations they used and how these were used to enhance the conceptual understanding of the learners. Post teaching took place after the focus group workshop and the different multiple representations which were used in different lessons were observed and analyzed. The purpose of the interview was to gain an understanding of the teachers' views of using multiple representations when teaching fractions for conceptual understanding.

The data showed that teachers who participated in the focus group workshop worked positively to develop their use of multiple representations to teach fractions. They made sense of fractions and were able to look at representations in different ways to develop the meaning and concepts of fractions. Teachers developed an understanding of working with multiple representations and were able to make connections among concepts and the use of concrete representations. Teachers

also developed their lesson plans effectively to involve a variety of teaching methods and multiple representations, despite the limited time available to them. Moreover, teachers used different modes of representation to improve learner engagement in learning activities. Lastly, teachers used multiple representations to teach in ways that improved the learners' conception of fractions.

On the basis of this research, it can be concluded that in-service workshops for teachers on the integration of multiple representative tools in the teaching of 'Grades 5 – 7' fractions could provide a valuable contribution to further developing the teaching of fractions in schools.

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DEDICATION

This thesis is dedicated to my three daughters (Albertina, Rauna and Junior Julia) and particularly to my husband (Lukas Iiyambo) for the love, care and patience that he has shown throughout the entire period of this research. He inspired me, when I encountered challenges.

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CHAPTER 1

INTRODUCTION OF THE STUDY

1.1 INTRODUCTION

This study sought to explore how teachers working with the additional tools of multiple representations could improve the teaching of fractions for conceptual understanding in grades 5 – 7. This chapter introduces this research by describing the background as well as the context of the study, this is followed by a statement of the goals of the research study. In addition I highlight the significance of the study and close by presenting an overview of the whole thesis.

1.2 BACKGROUND TO AND CONTEXT OF THE STUDY

The way in which learners learn fractions is affected by the way teachers teach fractions. In my experience the teaching of fractions is poor in the upper primary school level (grades 5 – 7). Many teachers only use the prescribed books and follow their procedures exactly, without including any other methods that may make their teaching more effective. Also, many teachers use a single method of teaching fractions and do not consider any other methods that might enhance conceptual understanding.

In my experience the use of multiple representations when teaching fractions enhances conceptual understanding. This research project investigates this possibility by first introducing teachers to ways of using multiple representations when teaching fractions for conceptual understanding and then investigating their experiences as they explore the use of these tools.

According to the Namibian Ministry of Education [MOE] (2010), mathematics is compulsory for all the grades (1 – 12). The Namibian Ministry of Education [MOE] (2010, p. 5) states in the Mathematics Syllabus for grades 5 – 7 that on entry into the Upper Primary Phase in grade 5, all learners are expected to have achieved basic competence in the four operations with whole numbers and fractions. Learners are expected to add, subtract, multiply and divide and also

should be able to compare and order fractions. The syllabus for grades 5 – 7 further requires that all learners should develop mathematical learning with understanding, to build the skills and attitudes that are considered necessary in order to contribute to the development of society.

Yet, learners in upper primary experience problems with a number of basic mathematical competencies, particularly those involving fractions (Namibia. NIED, 2010). The report by NIED in 2010 proposes that for learners to improve their learning of fractions, teachers should adjust their approach to teaching by using a variety of teaching and learning materials.

It was reported by the National Institute for Educational Development (Namibia. NIED, 2010) that Namibian teachers display limited understanding in mathematics teaching. It reports that in-service training institutions are not fully equipping teachers to work with different methods for developing and presenting learning support materials. Amato (2004) has found that teachers' understanding of the concept of fractions as numbers also appears to be limited, possibly due to the learning of these concepts over a short period of time in their initial teacher education (Amato, 2004). This research proposes improving in-service teacher education to improve teachers' conception of rational numbers.

1.3 GOALS OF THE RESEARCH

The goal of this research is to explore how teachers could work with multiple representations, to improve the teaching of fractions for conceptual understanding in grades 5 – 7. To achieve this goal the research will respond to the following two research questions:

1. What are the selected grades 5 – 7 mathematics teachers' experiences in working to develop the use of multiple representations to teach fractions?
2. How do the teachers work to change their practice in order to develop their use of multiple representations to teach fractions for conceptual understanding?

1.4 SIGNIFICANCE OF THE STUDY

The significance of this study is that it will deepen the research into the teaching and learning of fractions in the Namibian context, and into teacher professional development relating to this content domain. In particular, it will investigate the power and effect of using multiple

representations as tools for the teaching of Grades 5 – 7 fractions. It will also generate an awareness of the challenges faced by upper primary mathematics teachers when using these tools in the teaching of Grades 5 – 7 fractions. The study will also develop the proficiency of grades 5 – 7 teachers in solving fractional problems.

1.5 LIMITATIONS OF THE STUDY

This study is a case study and so will have the limitations associated with this method. The results cannot be generalized because of the small number of the participants and the short time period of the project. Due to the time constraint, it was also difficult to investigate a wide variety of multiple representations.

1.6 THE OVERVIEW OF THE THESIS

Chapter 1

Chapter one introduces this research by describing the background as well as the context of the study, this is followed by a statement of the goals of the research study. In addition I highlight the significance of the study, limitations of the study and close by presenting an overview of the whole thesis.

Chapter 2

In Chapter two I highlight the background of upper primary mathematics teaching in Namibia. The chapter then reviews literature relevant to this study and presents a theoretical framework for the research. The chapter is divided into six main sections. The first section focuses on research on mathematics teaching in Namibia and includes three sub-topics, namely; mathematics teaching, curriculum and issues in teaching fractions. The second section focuses on teachers' conceptual understanding of fractions. The third section discusses teachers' mathematical knowledge and content knowledge for teaching which include rule-based teaching and teachers' mathematics experiences. The fourth section discusses fractions. This section is subdivided into five discussion topics: teaching of fractions; what teachers should know and do in the teaching of fractions; teaching of fractions in a constructivist way; learning of fractions and teachers mathematical experiences with the teaching of fractions. The fifth section focuses on ways to improve the engagement in the teaching of mathematics. The last section focuses on multiple

representations and it is further subdivided into four subsections namely: the definition and types of multiple representations; connections through different representations; use of multiple representations to develop and deepen teaching of fractions and effective planning and preparation.

Chapter 3

Chapter three presents the methodology used in this study. It describes the research questions, the research orientation, the selection of the research site and participants, the research design, data collection techniques and instruments used, data analysis procedures, research ethics and validity, and the limitations and challenges encountered.

Chapter 4

This chapter presents the findings and analysis of data obtained in this study. The workshop observation data, the lesson observation data and the individual interview data were analyzed. Many extracts are given to support the presentation. I discuss how the grades 5 – 7 mathematics teachers worked to develop their use of multiple representations to teach fractions for conceptual understanding, and their subjective experience of this work. The similarities and differences of their use of multiple representative tools will be analyzed.

Chapter 5

This chapter presents a brief discussion of the findings which resulted from the data analysis.

A summary of the findings, recommendations, limitations and challenges encountered in the research are discussed in chapter six and suggestions for further research are outlined in this chapter. . The next chapter presents the review of the related literature as a way of informing the research.

CHAPTER 2

LITERATURE REVIEW

2.1 INTRODUCTION

The aim of this study was to investigate the experiences of the teachers when using multiple representations to teach fractions for conceptual understanding when teaching fractions in grades 5 – 7 in a Namibian school.

This chapter reviews literature relevant to this study and presents a theoretical framework for the research. The chapter is divided in to five sections. The first section focuses on research on mathematics teaching in Namibia. The second covers research that focuses on teachers' conceptual understanding of fractions. The third section discusses teachers' mathematical knowledge and content knowledge for teaching which includes rule-based teaching and teachers' experience in teaching mathematics. The fourth section discusses fractions. The last section focuses on multiple representations and the use of multiple representations to develop and deepen the teaching of fractions and effective planning and preparation.

2.2 MATHEMATICS TEACHING IN NAMIBIA

2.2.1 Mathematics teaching

Before independence in Namibia in 1990, mathematics was not regarded as an important subject to teach to black learners, because "the pick and shovel is the natural work of the native", as Mr. Schoolman, a former Minister of Labour in apartheid South Africa, claimed (Zeichner, 1999, p. 10). After independence, the new education system required different approaches and teaching methods. The learner was placed at the center of learning. That is, teaching and learning became learner centered. In the "Pilot Curriculum Guide for Formal Basic Education" (1997, p. 3) it is stated "children learn best when they are actively involved in the learning process, and the teaching methods used should be chosen to encourage the active involvement and participation

of the learners. Teachers should structure lessons appropriately for each task. There should be variation between teacher directed, teacher facilitated and learner directed work, depending on which is the most effective in relation to the learning objectives and content of the lesson".

In addition, the goal of teaching and learning is to enhance understanding, problem-solving and democratic learning (Namibia. MOE, 2010). It is further stated in "Toward Education for All" (1993, p. 60) that the active involvement and participation of the learner is of utmost importance. According to Swartz (1999, p. 252) learner-centered education became a target for critique by many teachers, because of the change to a "new, not so clearly defined" system. This was especially the case for those who had been exposed to a system for many years where everything was dictated from above and "good" results of learners were achieved.

The "Pilot Curriculum Guide for Formal Basic Education" (1997, p. 5) specifies the development of functional numerical and mathematical thinking – changing the syllabus according to the demands of the society. Four of the most important aims included in the syllabus for mathematics for basic education are: Developing positive attitudes toward mathematics, enabling learners to acquire basic number concepts and numerical notations, enabling learners to understand and master basic mathematical concepts and operations and enabling learners to apply mathematics in everyday life. It is also stated that, in order to help learners to make the change from learning in their mother tongue to learning in English, a teacher should use concrete objects, pictures, drawings and gestures to help learners understand (grade four syllabus for mathematics, 1999: 5).The Mathematics and Science Teacher Extension Programme (MASTEP) was introduced in Namibia to meet the needs of mathematics teaching in Namibia, but upper primary education for mathematics was not addressed through this programme.

Peters (2006) stated that the rule based approach is being used by teachers when teaching mathematics. He explained that when teaching mathematics, teachers use rule based structures which are short term and of limited adaptability. Also, because connections are between symbols rather than concepts, learners may apply an appropriate remembered rule to the solution of a problem without knowing why the rule works. Peters gave an example of instrumental teaching,

which “might be a lesson in which the teacher explains the cosine rule and how to apply it. The learner sees no ‘connection’ to real life situations why the rule ‘works’. Such demonstration of using instrumental teaching might be in oral form to the teacher or to other learners, in writing *or* by suitable actions with concrete materials”. Moreover, The Upper Primary Mathematics Facilitators Training Workshop offered by NIED (2006) listed some factors which lead to low performance in mathematics, one of which is the lack of teachers’ subject content knowledge and lack of support teaching materials.

2.2.2 Curriculum

The Namibian education system is organized into four categories namely; lower primary education (grade 1- 4), upper primary education (grade 5 – 7), junior secondary education (grade 8 – 10) and senior secondary education (grades 11 and 12).

According to Peters (2006), mathematics is widely regarded as one of the most important subjects in the Namibian school curriculum. When concern is expressed about the attainment of learners in Namibia, whether legitimate or not, and comparisons are made with learners in other countries, mathematics is singled out as being a particularly worrying problem (Namibia. National Evaluation of the Basic Education Teacher Diploma Subject Areas, 1997: 5).

Curriculum reform was undertaken with the view to developing an educational program relevant to the needs of Namibia. In “Toward Education for All” (1993: 53) it is stated that access, equity, quality and democracy are the overarching goals for education. “The broad goals of basic education are to provide learners with the necessary knowledge, understanding and to be able to develop practical skills which are necessary for a solid foundation for further academic training” (Angula, 1999, p. 21).

2.2.3 Issues in teaching fractions

According to the research done by Nambira, Kapenda, Tjipueja and Sichombe (2009) both teachers and learners perceived ‘common fractions’ as a difficult topic. They stated that treating denominators of fractions as divisors e.g. a/b as a ÷ b is a challenge to learners. So are comparing and ordering mixed numbers and improper fractions in practical situations. Similarly,

recognizing equivalent fractions and calculating fractional parts of quantities is difficult for learners to understand. Though not apparent, comparing and ordering fractions with different denominators including finding a common denominator (e.g. 1/3, ¼ and 1/5) is problematic, in fact, 50% of the teachers indicated that learners struggle with that topic.

In the study done by Vatilifa (2012), not all teachers lacked fraction understanding in their teaching. This was revealed when they managed to correct their learners' misconceptions in different cases, for instance, during the addition and subtraction in a fraction lesson, a student teacher redirected the learners and showed them an appropriate way to complete the task. Moreover she added that some teachers distributed 'teaching aids' to their learners without incorporating the connection of such teaching aids in their teaching. This evidence showed that the student teacher's thinking in this particular case was that by including this 'teaching aid', it would satisfy the criterion of 'using teaching aids 'in the teaching of fractions.

2.3 TEACHERS CONCEPTUAL UNDERSTANDING IN MATHEMATICS

Hiebert and Carpenter (1992) view conceptual understanding in mathematics as "a grasp of the relationships that explain the behavior of the physical world and the relationships between the observable and more abstract or more general scientific concepts". It increases in sophistication as the student progresses through school and develops cognitively, and thus the evidence of understanding will vary across grades. Kilpatrick (2001, p.140) added that "conceptual understanding supports retention, because facts and methods learned with understanding are connected, they are easier to remember and use and they can be reconstructed when forgotten". He states that the need to demonstrate the use of multiple representations of learning content can significantly enhance the conceptual understanding of students.

Grouws (1992, p. 90) defines understanding in terms of the way in which information is represented and structured. "A mathematical idea, or procedure, or fact, is understood if it is part of an interrelated network". Mathematics needs to be represented so that it can be understood by teachers and learners. Grouws (1992) further said that, "many of our learners and teachers today find mathematics challenging and do not understand the concepts. This is because teaching focuses on written mathematical symbols far too early, before learners are ready to understand

and grasp the concepts. The reality is that mathematics is an abstract subject that requires different forms of representations as a bridge between the concrete and the abstract.

2.3.1 Teachers conceptual knowledge

According to Kilpatrick et al. (2001, p. 141), "one of the defining features of conceptual understanding is that knowledge must be connected so that it can be used intelligently. Teachers need to make connections within and among their knowledge of mathematics, students and pedagogy. The kind of knowledge that makes a difference in teaching practice and in students' learning are an elaborated, integrated knowledge of mathematics, a knowledge of how students' mathematical understanding develops and a repertoire of pedagogical practices that take into account the Mathematics being taught and how students learn it". Kilpatrick further explains that for teachers who have already achieved some mathematical proficiency, separate courses or professional development programs that focus exclusively on mathematics, on the psychology of learning, or on methods of teaching, provide limited opportunities to make these connections. This case was revealed in Namibia by Vatilifa (2012, p. 82) who explained that teachers' mathematical content knowledge as well as their pedagogical content knowledge needs to be strengthen, because this affects the ways teachers deliver the fraction lessons to the leaners.

"Teachers need to be able to explain, not only how to solve a problem, but also why the procedure is appropriate and why flawed approaches are inappropriate" (Fazio & Siegler, 2010, p. 21). This type of explanation requires a deep knowledge of fraction computation. Professional development should provide opportunities for teachers to focus on cultivating this deeper level of knowledge. Opportunities should be provided to the teachers to explain why an algorithm works, that allow them to identify concepts they do not yet fully understand. Many teachers do not have an understanding of why procedures are productive. In addition, teachers should know the fraction concepts that are taught at their grade level and should also know the concepts that come before and after. By understanding what students have been taught in earlier grades, teachers can build on what the students already know and better identify the sources of students' misconceptions

Mullis et al. (2003) and Hiebert, and Carpenter (1992) report that students learn more when taught by experienced teachers than they do when taught by teachers with just a few years' experience. However, the relationship between experience and achievement may be affected by many factors. For example, assignment policies within schools may result in the more highly skilled teachers getting specific classes, or older teachers getting higher-tracked classes. The need for long serving teachers to engage in professional development, and the extent to which they do so, can also impact their effectiveness. Kilpatrick (2001) argued that, "Although teachers often look for evidence of conceptual understanding in students' ability to verbalize connections among concepts and representations, their conceptual understanding is not clear and easy to understand. He further added that a significant indicator of conceptual understanding is being able to represent mathematical situations in different ways and knowing how different representations can be useful for different purposes.

Although most of the fractional operations—addition, subtraction, multiplication and division—are covered in upper primary school, they are often revisited in secondary school. Research suggests that students have a procedural knowledge of fractional operations rather than an understanding of underlying concepts (Mack, 1990). For example, to meaningfully add and subtract fractions, Mack (1995) has argued students must interpret fractions as numbers—a value of a quantity—instead of thinking of fractions as two numbers (Kerslake, 1986).

2.3.2 Rule based teaching

An investigative approach to the teaching and learning of mathematics aligns with curriculum reform movements in mathematics education (e.g. National Council of Teachers of Mathematics (NCTM), 2000; Australian Education Council, 1991). A traditional rule-based, skill mastery approach to the teaching of mathematics encompasses formulated goals that includes promoting students' communication skills and problem solving capacities, and enables learners to experience the actual processes through which mathematics develops (Australian Education Council, 1991). The importance of an investigative approach to the teaching of mathematics has been highlighted in recent classroom based research. For example, the TIMSS Video Study (Hollingsworth, Lokan & McCrea, 2003) revealed that in Australian classrooms there was little emphasis on developing deep understanding of mathematical concepts or the connections

between them. Stacey (2003) further commented that this cluster of features constitutes a syndrome of shallow teaching, where learners experience a lot of repetition and problems of low complexity, and opportunities for mathematical reasoning are scarce. Mathematics lessons were often found to offer low levels of intellectual quality and connectedness, suggesting that learners were not given enough time to develop their critical thinking and to deepen their understanding, and to be able to make connections between mathematics and the real world. Goos et al. (2008) also found that teachers often set assessment tasks that were low in intellectual demand and unconnected to the world outside school.

Some researchers suggested that teachers need to give support to the learners so that their engagement in understanding and diagnosing problems are enhanced (NCTM, 2000). The teacher can involve the learners in solving the problem by, for example, demonstrating the procedures of formulating, reformulating or solving the problem, and doing so in such a way that the thinking process is made explicit to the learners; posing questions to the learner to encourage his or her own solutions. McCormick (1997) stresses the importance of not providing learners with solutions, even when learners are not sufficiently skilled or knowledgeable to develop them for themselves. This simply treats procedural knowledge as declarative knowledge and will not help problem solving. The pressures of classroom management of 20 or more learners and the need to ensure that they all make a complete 'working' product often leads to this 'quick fix' approach (McCormick & Davidson, 1995).

2.4 FRACTIONS

2.4.1 Teaching of Fractions

Van de Walle (2007) stresses the different ways in which fractions are viewed and he stated that one way of viewing them is equal-sized portions of a whole or unit. Ma (1999) further added that the topic of fractions is viewed as being difficult to understand, as well as to teach by many teachers. The teaching of fractions is not only seen as difficult, but "in a broader scheme of things, a dismal failure" (Tzur, 1999). Tzur's study suggests that it is important to integrate research on fraction teaching and fraction learning if one is to suggest sensible ways to improve the current state of the teaching and learning of fractions.

Research has found that effective tasks for the introduction of fraction concepts are sharing problems in which there is a remainder which can be divided. These sharing situations elicit the informal knowledge that the children bring to the learning situation and can be used successfully for introducing fractions (Mack, 1990; Empson, 1995; Murray, Olivier & Human, 1996). De Beer and Newstead (1998) state that even grade1 learners have the ability to make sense of such fraction problems, even if their lack of social knowledge prevents them from producing the correct fraction name or symbol.

According to Kamii and Clark (1995), in introducing fractions by sharing problems with remainders, learners are exposed to improper fractions and mixed numbers from the beginning. This is important so that they will think about parts and wholes at the same time. According to the grade 4 and 6 pre–test study which was done by Newstead and Murray (1998), many teachers in the past have introduced learners to common fractions using pictures of repartitioned shapes or actual manipulatives and they pointed out that this approach is dangerous. Not only can it lead to a very limited interpretation of fractions, but the idea of equal portioning is often lost.

Newstead and Murray (1998), in testing grade 4 and 6 learners, found evidence that the traditional teaching of fractions resulted in misconceptions. For example, they found that many learners associated fractions with meaningless 'recipes' (often incorrect), or simply did not have any meaningful association for fractions. The study pointed to the dangers of an approach where teachers introduce learners to common fractions using pictures of pre-partitioned shapes, or actual manipulatives. "Not only can it lead to a very limited interpretation of fractions, but the idea of equal partitioning is often lost" (Newstead & Murray, 1998). Moreover Mcleod & Newmarch (2006) and Kilpatrick et al. (2001) point to the fact that although some misconceptions about fractions may be immediately apparent it is also important to find strategies for uncovering learners thinking processes and seeing errors that might lie below the surface. Sometimes the learners appear to give a correct response, but their reasoning misfires. To see this, it is important to encourage learners to voice their ideas, even if they are based on misconceptions. A specific example of a misconception is where some learners see fractions e.g. $\frac{1}{3}$ and $\frac{1}{4}$ as interchangeable. This may be because the words quarter and third do not suggest the numbers 4 and 3. It is very important to check whether learners have this misunderstanding. Ott,

Snook and Gibson (1991) noted that textbooks and classroom examples further limit the experiences of students and their ability to extend their knowledge of partitive division to division of fractions. Snook and Gibson describes partitive division as the fractional part of a group where the size of a whole group need to be found e.g. Someone have 35 apples to share equally among 7 people. How many apples does each person get?
In contrast to Van Der Walle (2007), Lamon (2001) identifies many possible meanings (interrelated sub-constructs or interpretations) and children need to develop these different meanings (Lamon, 2001). These are identified as quotient (which includes both partitioning and sharing), part-whole, measure, operator and ratio. These are all more complex than counting and Lamon stresses the necessity of using these different kinds of representations to understand fractions.

On the other hand, Van De Walle (1990) claims that pupils need to have a sound knowledge of equivalent fractions before learning the four operations on common fractions especially addition and subtraction. Two fractions are equivalent if they represent the same amount. To acquire the conceptual understanding of equivalent fractions, pupils should use models such as area, length and sets to discover different names for models of fractions. Rectangle slicing can be used to develop an understanding of equivalent fractions. Pupils can be put in groups, given papers, draw several equal squares and then shade the same fraction in different squares using different subdivisions.

In agreement Kamii and Clark (1995) posit that children should be encouraged to generate their own diagrams to represent fractions. These may closely resemble the diagrams that can be found in textbooks, but represent children's own understanding rather than someone else's thinking.

2.4.2 What Teachers Should Know and do in the teaching of Fractions

It is widely accepted that the knowledge teachers hold affects the way they perform all the core tasks of teaching (National Council of Teachers of Mathematics [NCTM], 2000). In particular, a teacher's knowledge of subject matter, student learning, and development and their teaching methods have all been identified as important elements of teacher effectiveness. Focusing on a teacher's knowledge of content, Shulman (1986) defined pedagogical content knowledge (PCK)

as knowledge of a subject "*for teaching*". He differentiated this from pure subject knowledge by describing PCK as, including the best ways to present the subject to learners, the most useful examples to use to illustrate certain points, and an idea of the misconceptions and preconceptions that learners may bring with them to the learning process.

Fractional concepts are important building blocks of the upper primary and secondary school mathematics curriculum. Conceptually based instruction of fractions requires teachers to have a complete understanding of the subject matter. Several researchers (e.g. Ball, 1990; Shulman, 1986;) have proposed theories about teacher knowledge characteristics and structure. In their model, the three knowledge domains central to mathematics teaching are common knowledge of mathematics, specialized knowledge of content, and knowledge of students and their ways of thinking about the content.

Shulman (1986) explains that common knowledge is the knowledge that a mathematically educated adult, not necessarily a teacher, needs to possess to provide correct mathematical solutions. Specialized knowledge of content is the possession of mathematical knowledge and skills such as being able to explain why an algorithm works or being able to provide students with multiple representations addressing diverse learning styles. The upper primary mathematics teachers need both common and specialized knowledge of fractional concepts along with general pedagogical knowledge.

The Namibian research done by Tomas and Ward (2006) showed that a pen-and-paper assessment focusing on teachers' pedagogical content knowledge can be both efficient and effective in differentiating between teachers. In general, teachers scored more highly on questions requiring a response based on their content knowledge than on questions where responses required them to describe the key ideas involved or the actions they would take with a student. Questions that caused teachers difficulty involved the addition of fractions, division with fractions, and proportional reasoning, with approximately one-third of teachers' responses indicating a lack of conceptual understanding in each of these areas.

Since Shulman's definition of PCK, researchers have worked in many subject areas to investigate teacher knowledge and map the precise knowledge a teacher requires to be effective (Hill, Schilling & Ball, 2004). In mathematics, these investigations have included comparing the views of pre-service teachers with those of experienced teachers (Ball, 1990), in-depth interviews with practicing teachers (Harel & Lim, 2004), and comparisons of the differences in teacher knowledge across cultures (Ma, 1999). In focusing on teacher knowledge in mathematics, researchers have made a distinction between teachers that have an algorithmic or rule-based understanding of mathematics and those that have a deep conceptual understanding.
Difficulty with fractions among teachers is well documented in many countries, and many authors consider fractions to be the most difficult area of mathematics covered in primary school (Smith, 2002). Studies into teacher knowledge of fractions have found both procedural and conceptual understandings among teachers, although procedural understandings dominate in this area (Fuller, 1997). Considerable differences have also been found in the explanations teachers provide to students when working with fractions. When looking at the representations that teachers use to present fractions to students, a limited repertoire has been found (Ball, 1990). Circular representations are most commonly used, but these can be problematic because they are unable to illustrate conceptually complex operations with fractions such as division.

The Mathematics Science Research Institute – NSRI (2004) and Kilpatrick et al. (2001) say that in teacher training institutions, teachers cannot learn all they need to know about the mathematics they will teach and how to teach it effectively. Consequently, they recommend that teacher education be seen as a professional continuum, a career-long process. In their initial education teachers are provided with a basis for ongoing learning so that they can adapt to the changes of the curriculum. Teachers can continue to learn by participating in various forms of professional development and mathematicians need to help them to develop an understanding of how to teach mathematics.

2.4.3 Teaching of Fractions in a Constructivist

The role of the constructivist teacher is to guide and support students' invention of viable mathematical ideas rather than transmit "correct" adult ways of doing mathematics. Some see the constructivist approach as inefficient, free-for-all discovery. In fact, even in its least directive

form, the guidance of the teacher is the feature that distinguishes constructivism from unguided discovery. The constructivist teacher, by offering appropriate tasks and opportunities for dialogue, guides the focus of students' attention, thus unobtrusively directing their learning (Bruner, 1986).

Traditional instruction, on the other hand, values only established mathematical techniques and concepts. For example, even though many teachers consistently use concrete materials to introduce ideas, they use them only for an introduction; the goal is to get to the abstract, symbolic, established mathematics. Inadvertently, students' intuitive thinking about what is meaningful to them is devalued. They come to feel that their intuitive ideas and methods are not related to *real* mathematics. In contrast, in constructivist instruction, students are encouraged to use their own methods for solving problems. They are not asked to adopt someone else's thinking but encouraged to refine their own. Although the teacher presents tasks that promote the invention or adoption of more sophisticated techniques, all methods are valued and supported. "Through interaction with mathematical tasks and other students, the student's own intuitive mathematical thinking gradually becomes more abstract and powerful"(Bruner, 1965; Cobb, 1988; Mathematical Sciences Education Board (MSEB), 1989).

2.4.4 Learning of Fractions

Many students experience great difficulty understanding the meaning of fractions (Anthony & Walshaw, 2007; Lamon, 2007; Fazio & Siegler, 2010). For many students who have spent their early mathematics lessons focusing on counting (whole) numbers, recognizing that there are many numbers between those whole numbers called fractional numbers is difficult.

According to Zevenbergen, Dole and Wright (2004), students often comment that they find fractions to be meaningless and confusing. Their learning has frequently been based on rules and procedural computation, while conceptual understanding has often been minimal. Understanding what a fraction means and how to operate with fractions (i.e. addition, subtraction, multiplication and division) is often daunting for many students. Furthermore, Fazio and Siegler (2010) added that added that many students see fractions as a difficult topic, because they lack conceptual

understanding and view the numerator and denominator of a fraction as a separate number instead of one.

Some researchers have shown that concrete materials help students to understand mathematical concepts (Fennel & Rowan, 2001; Zevenbergen et al, 2004). Finding a range of suitable real world models as contexts for teaching mathematical ideas is recognized as part of good teaching. However as Zevenbergen et al. (2004) note, finding accurate real life models that refer to fractions is often difficult.

It is important that learners think about relationships between fractions, rather than just trying to memorize methods for processing them. An introduction to fractions should include a strong emphasis on developing reasoning skills, comparing fractional amounts and exploring equivalence (Fazio & Siegler, 2010). Many learners may feel that listening to the teacher and completing their own worksheets individually is the main way of learning. However, Mcleod and Newmarch (2006) believe that learners learn more if they actually enjoy the activity, have a chance to discuss what they do, explain their work, and reach a shared understanding. "There is now widespread recognition of the value of collaborative work in developing conceptual understanding. Although the activities outlined by Mcleod and Newmarch (2006) can be done individually, most of them will work better as collaborative tasks. This approach may be unfamiliar to many learners, particularly those whose previous mathematics experience was in a more traditional classroom" (Mcleod & Newmarch, 2006, p. 16).

Furthermore, Wu (1999) comments that fractions should be informally introduced as early as the second grade (because, for example, even second graders need to worry about drinking "half a glass" of orange juice!), and there is no harm done in allowing children to get acquainted with fractions in an intuitive manner up to even the fourth grade.

2.4.5 Ways to Improve Engagement in Mathematics

The impact of technology on teaching in many different subjects has been widely researched over the past couple of decades. Looking at general motivation and engagement, Arrowood & Overall (2004) and Chung & Walsh (2006) reported that computers could be useful in promoting engagement and motivation in young children in the teaching of handwriting and literacy. Price

et al. (2003) focused on collaboration and various others have demonstrated that the use of technology in teaching and learning is associated with improved problem solving and language skills (cited in Couse & Chen, 2010). Other examples of the use of technology in teaching that have been widely debated are is the use of 'whiteboards' (Beeland, 2002). Computer desktop virtual reality (Lee, Wong & Fung, 2010) and, with particular reference to this study, work on enthusiasm associated with mathematical achievement (Barkatsas, Kasimatis and Gialamas (2009). However, there is some skepticism concerning the benefits of technology in the classroom, particularly in its current form (Plumm, 2008). Of particular concern is the gap between the school environment, where access to technology is relatively limited, and the child's 'outside' experience where mobile devices are causing a revolution in the way in which information is accessed. In contrast to computer technology, the use of multiple representations may provide a sound link between school and outside experience.

The use of multiple representations and the ability to translate among representational models has been shown to be an important factor in students' abilities to model and understand mathematical constructs (Cifarelli, 1998; Fennell & Rowan, 2001; Goldin & Shteingold, 2001; Kamii, Kirkland & Lewis, 2001; Lamon, 2001; Perry & Atkins, 2002). In a summary of over 100 research studies, Marzano (1998) found that one instructional technique that demonstrated a consistently positive impact on student achievement was the use of graphic/nonlinguistic formats to explore and practice new knowledge. The Lesh Translation Model also highlights the importance of students' abilities to represent mathematical ideas in multiple ways including manipulatives, real life situations, pictures, verbal symbols and written symbols (Lesh, Cramer, Doerr, Post, Zawojewski, 2003). It is very important to combine different types of multiple representations when teaching the learners to increase learners' understanding when teaching fractions. Students have fewer experiences of fractions outside the school and therefore teachers need to provide relevant experiences to enhance learners' understanding in mathematics outside the classroom. Learners should be able to connect procedural knowledge with conceptual understanding (National Research Council, 2001).

2.4.6 Teachers' Mathematical Experiences and Understanding for the Teaching of Fractions

Wu (2010) argues that in the teaching of mathematics, mathematical content dictates pedagogy in a vast majority of cases. This explains the great interest in teachers' content knowledge. There is at present a lack of clarity about what this content knowledge is, and how to enable teachers to acquire this knowledge. Wu explains that, in the 1970s, the educator E.G. Begle made a first attempt to demonstrate a connection between teacher content knowledge and student achievement. To his dismay, he found no statistically significant correlation. Begle measured this knowledge by the number of mathematics courses taken and by teachers' grades (technically: grade point average). Subsequent research also failed to improve on his findings except for teachers in grades 9-12. According to Wu (2010) the research methodology of Begle and his followers is flawed.

Hill, Rowan & Ball (2005) suggest that teachers should have a knowledge and understanding of different fractional concepts in order for them to teach it efficiently to the learners. However, many teachers lack understanding of using visual representations to teach fractional concepts. Teachers must be able to use a number of different representations and be able to represent them in the class when teaching. Teachers must be able to represent different fractional problems using different representations to make learners understand. Teachers must come up with the appropriate representation for each fractional problem. Teachers can be asked to explain why an algorithm works, or they can solve advanced problems that allow them to identify concepts they do not yet fully understand. For example, almost all teachers know that fraction division problems can be solved through the procedure "invert and multiply". However, many teachers lack a deep understanding of why that procedure is effective (Hill, Rowan & Ball, 2005; Ma, 1999; Vamvakoussi &Vosniadou, 2010).

According to Ma (1999), teachers need professional development workshops to share their views and problems on the issues related to fractions and should be able to discuss the appropriate approaches that teachers can use to enhance learners understanding. Teachers should also point out difficulties which learners have when learning fractions Teachers should also discuss the

types of errors that students commonly make and what misconceptions underlie each of the errors.

2.5 MULTIPLE REPRESENTATIONS

2.5.1 Definitions of Multiple Representations

Researchers have understood the need for using multiple representations in learning and teaching. Therefore it is important to look at how researchers describe multiple representations. Edgardo (2001) states that multiple representations include pictures, diagrams, sketches, tables, written symbols, oral representations and the use of manipulatives. A further description by Schultz and Waters (2000) stated that multiple representations include graphs and diagrams, tables and grids, formulas, symbols words, gestures, software code, videos, concrete models, physical and virtual manipulates pictures and sounds. Ainsworth, 1999) confirm that multiple representations in technology include calculators, computers, graphing, games and simulation. Ozgun-Koca (1998) defines multiple representations as "external mathematical embodiments of ideas and concepts to provide the same information in more than one form" (p. 1). He classified representations as external and internal. Each of these researchers provides a considerable number of sub-themes exposed to more and deeper research linked with other fields.

2.5.2 Types of Multiple Representations of Fractions

Some forms of representation—such as diagrams, graphical displays, and symbolic expressions—have long been part of school mathematics. Unfortunately, these representations and others have often been taught and learned as if they were ends in themselves. The National Council of Teachers of Mathematics (2000), stresses that representations should be used as a vital elements in supporting understanding of the learners in terms of mathematical concepts and relationships; in communicating mathematical approaches, arguments, and understandings to one's self and to others; in recognizing connections among related mathematical concepts. New forms of representation associated with electronic technology create a need for even greater instructional attention to representation (National Council of Teachers of Mathematics, 2000).

The Commonwealth of Learning (2001); Lewis, Perry, Friedkin & Baker (2010); Askew (2012); Boaler (2009) and Van de Walle, Karp, all suggest that the use of a variety of representational

models are crucial in the teaching of fractions, not only to introduce concepts but as a means of clarifying ideas that may appear confusing to pupils in symbolic form. It is often valuable to repeat the same activity using different models. Three main types of models can be distinguished: area models, length models and set models. Area models can be used to demonstrate visually that a fraction is part of a whole. Students should be given counter examples to test their conceptual understanding of common fractions. These models represent fractions as part of an area or a region and are useful when exploring fractional parts of 2D shapes. Typical examples of this type of model are: circular "pie" pieces where a full circle represents a whole, rectangular pieces where any piece can represent a whole, shapes/areas created with rubber bands on a geoboard , shapes/areas drawn on dotted paper or grids, pattern blocks – multiple polygons and folded papers.

The Commonwealth of Learning (2001) and Lewis, Perry, Friedkin & Baker (2010) emphasize that length models are similar to area models. The difference is that lengths are compared instead of area. Number lines and physical materials on the basis of length can be compared. Length model are measurement models where lengths are compared and fall into two main categories: 1) *linear*: Lines which are drawn and subdivided (number lines or line segments). The number line is considered to be particularly helpful due to the difference between a) plotting an actual number on a number line and noting its distance from 0 and b) simply comparing one length with another. 2) *2D and 3D:* Concrete materials that can be compared in terms of their length are folded paper strips, ribbon, string (2D) or fraction bars/rods.

Set models also illustrate common fractions as part of a whole. The set of objects make a whole and subsets make up parts of a whole. However the idea of looking at a set of elements as a single entity contributes to making set models difficult for primary pupils. Despite the difficulties faced by pupils teachers cannot do away with the set model interpretation of fractions because it links real life situations to using fractions and ratio concepts. Moreover, set models in this case the whole is considered as a set or a collection of objects (for example, counters) with subsets making up the fractional parts. Typical examples include: partitioned collections of objects and arrays- rows and columns of objects Boaler (2009); Van de Walle et al. (2009).

2.5.3 Connections through Different Representations

According to Businskas (2008), a mathematical connection may be variously referred to as a relationship between mathematical ideas, as a relationship that is constructed by learners and as the process that is part of the activities of doing mathematics. Coxford (1995) clarifies that conceptualized connections are very broad ideas or processes that can be used to link different topics in mathematics. He identified mathematical process like representation, application, and problem solving and reasoning as mathematical connections. For example upper primary learners should develop facilities in moving back and forth among the concrete and pictorial models, the oral name and the symbolic representation of any fraction or decimal (Coxford, 1995, p. 7).

Kilpatrick et al. (2001) commented that the use of concrete materials for teaching mathematics is very crucial, particularly in the elementary grades. Concrete materials should always be seen as a means and not an end in themselves. They require enough time to be used to allow learners to build meaning and make connections. Moreover, Vygotsky (1978) added that, a child can learn successfully when his or her existing knowledge is not too far removed from the knowledge of the community. Vygotsky (1978) postulates a zone of proximal development – an area of cognitive activity where a learner cannot make much progress on his or her own, but can succeed through interactions or collaboration with more capable others. The child then internalizes the strategies at play in the social dimension to become more capable individually.

The CAPS document (Department of Basic Education, 2011a) states that learners should work with relationships and convert flexibly between variables in terms of numerical, graphical, verbal and symbolic representations. They should be able to communicate appropriately by using descriptions in words, graphs, symbols, tables and diagrams. This suggests that learners should be assisted to establish links or connections between and within these multiple representations.

Literature has shown the importance of learners being able to move comfortably *between* and *among* these multiple representations. Weinberg (2001) explained by an example, that learners' conceptual understanding level is directly proportional to the learners' ability to translate and transform the representations of the concept *across* and *within* a wide variety of representational systems.

With specific reference to generalizations in mathematics, literature suggests that all of mathematics is about generalizing patterns (Brown & Coles, 2010). Watson and Mason (2006) for example claimed that generalization was at the heartbeat of Mathematics. If teachers are unaware of its presence, and are not in the habit of getting students to work at expressing their own generalizations, then mathematical thinking will not take place. In the literature, student thinking and ideas are also referred to as the most important item for a teacher to have an awareness of as they look to nurture quality mathematical discourse (Stein, Engle, Smith & Hughes, 2008). Research has shown that the way individuals develop mathematically is via periods of expansion and reconstruction or assimilation and accommodation. These are periods in which the emphasis is on making new connections within existing cognitive structures (Smith, 1984).

Businskas (2008) studied the problem solving activity of a group of pre-service teachers as a process of making connections. He found that the pre-service teachers were engaged in making variety of connections. Pre–service teachers connected ideas using more than one representation.

2.5.4 Use of Multiple Representations to Develop and Deepen Teaching of Fractions

Representations are identified as one part of teachers' pedagogical knowledge. Shulman defined these representations as "including analogies, illustrations, examples, explanations, and demonstrations – in a word, the ways of representing and formulating the subject that make it comprehensible to others" (Shulman, 1986, p. 9). Specifically in mathematics, Ball et al. (2008) also highlighted representations as being part of the 'specialized content knowledge' of mathematics unique to teaching. This specialized knowledge includes selecting representations for particular purposes, recognizing what is involved in using a particular representation and linking representations to underlying ideas and other representations. Teachers need to be able to draw on a variety of representations as there is "no single most powerful form of representation" (Shulman, 1986, p. 9).

According to Rau, Alaven & Rummel (2009), the use of multiple representations can help learners to understand, develop, and communicate different mathematical features of the same object, and make connections between different properties. Multiple representations include graphs and diagrams, tables and grids, formulas, symbols, words, gestures, software code, videos, concrete models, physical and virtual manipulates, pictures, and sounds.

It has been further argued by Ronning (2010) that much research has been done on learning with multiple representations and it is claimed that the use of multiple representations enhances learning. For example, using different examples of representations in fractions, encourages learners to construct a deeper understanding of a given situation.

Research done by Rau et al. (2009) shows that when compared to learning with only a single representation, the use of multiple representations of learning content can significantly enhance student learning in complex domains. However, providing learners with multiple representations does not necessary result in flexible knowledge acquisition. Learners only acquire a deep understanding if they are able to link multiple representations of the same concept and coordinate between them. If the learners fail to coordinate the information in different representations, then their learning will be disrupted. Ainsworth (2006, p. 184) noted that:

> There are number of ways to design multi – representational systems that influence the process and outcomes of learning. An external representation consists of (1) the represented world, (2) the representing world, (3) what aspects of the represented world are being represented, (4) what aspects of the representing world are doing are doing the modeling and (5) the correspondence between the two worlds. So, when considering the effectiveness of a representation both the information provided in the representation and the way it is represented must be considered.

Ainsworth (1999, p. 134) has identified "three main functions of using multiple representations; complementary roles, to constrain interpretations and to construct deeper understanding." He explains that the first function is to use representations containing complementary information. The second function is to use one representation to constrain learners' interpretations of another

representation. The third function has to do with encouraging learners to construct a deeper understanding of a situation.

Rau et al. (2009) carried out an investigation to test whether grade 6 students learn better with multiple graphical representations (MGRs). All the students worked with an Intelligent Tutoring System (ITS) for fractions, designed and created to carry out the research. They either learned with one single graphical representation or with multiple graphical representations. Some students were prompted to explain how the graphical representations corresponded to the symbolic presentation. Students' knowledge of fractions was assessed before and after the experiment. The research points to the fact that students learn better with multiple graphical representations of fractions than they do with a single graphical representation, but only when prompted to explain (Rau, Alaven & Rummel, 2009; Bulgar, 2009). Kamii and Clark (1995) agree that children should be encouraged to generate their own diagrams to represent a fraction.

In another study done by Bulgar (2009), students made use of three methods to solve division problems in mathematics. These methods were: reasoning involving natural numbers, reasoning involving measurement and reasoning involving fraction knowledge. In reasoning involving natural numbers, students changed the division from a fraction problem to a natural number problem. For example, students might convert fractional parts of meters to centimeters. In reasoning involving measurement, students created a measurement tool the size of the divisor and counted how many times they could place the tool along the object that is the dividend. In reasoning involving fraction knowledge, the students made use of their knowledge of the number of unit fractions to solve problems and explain the solutions. Students opted to use different methods when solving problems in the division of fractions. Some students began by using reasoning involving fraction knowledge, but then applied reasoning involving measurement. On the other hand, many students used the method of reasoning involving fractions when solving these problems (Bulgar, 2009).

Some researchers commented that, focusing on the concept of sharing can be helpful when introducing fractions as it meaningfully connects the idea of fair shares with fractional parts. Moreover, it assists the learner in understanding that the larger the denominator the smaller the

fractional part, that is, the more sharers there are, the smaller the portions. It is a good idea to link this work with division. Furthermore, the habitual use of circular items such as pizzas and cake as models to teach fractions has certain shortcomings and a square unit can often be more versatile for teaching fractions (Suggate et al., 2010).

Moreover, the report by Hiebert and Wearne et al. (1991) states that the use of concrete materials enhance learners' understanding of decimal fractions. On the other hand, Omanson and Resnick (1987) reported that the use of base-ten blocks had little effect on upper primary learners understanding. Thompson (1994) indicated that teachers have to be careful when using concrete materials. He said materials may be concrete, but the idea that the teacher intends leaners to see is not in the materials.

Professional Development Service for Teachers (PDST, 2012), suggest that pupils' mathematical ideas can be further challenged by presenting them with wholes that are correctly divided into requested fractional parts, that is, a quarter and those that are not. Pupils should be asked which of these figures are correctly divided into quarters and explain their reasons.

When fractions are compared PDST (2012), Fazio & Siegler (2010) suggest that benchmarks are helpful for pupils to develop their number sense in relation to fractions. Estimation is central to applying benchmarks. Pupils need lots of opportunities to compare fractions to benchmarks. The most important reference points or benchmarks for fractions are 0, $\frac{1}{2}$ and 1. PDST (2012) further suggested that the following activities can be used to develop and consolidate benchmarking.
Furthermore, in order to use the algorithm for adding fractions, common denominators are required. As pupils engage in concrete and pictorial problem-solving activities requiring the addition and subtraction of fractions, they will discover the need for a common denominator. This is due to the fact that the algorithm only works successfully when adding or subtracting fraction parts that are the same size. However, pupils who have strong fraction number sense will often be able to add and subtract fractions with unlike denominators without ever getting a common denominator. The number line is an excellent tool through which fractions can be added or subtracted mentally (PDST, 2012). However, if pupils have developed a deep understanding of the concept of fraction equivalence, then they should more easily see common relationships

between various denominators. Continued reinforcement of the equivalence concept is necessary whilst the pupils are engaged in the addition and subtraction of fractions (Van De Walle et al, 2009; Fazio & Siegler, 2010).

2.5.5 Effective Planning and Preparation

Kilpatrick et al. (2001) stresses that teachers vary in how they manage the time they have, sometimes focusing on one strand of proficiency and ignoring others. According to the research which was done by Kilpatrick between two fourth-grade teachers who used the same mathematics textbook, it was found that they spent their time quite differently. One teacher focused on concepts, and the other emphasized drill and practice of computational skills. Even when the amount of time and the textbook are uniform, therefore, students can encounter different content and have different opportunities to learn it.

Effective teaching preparation and implementation is important to help students understand lessons in the easiest and clearest way possible. With proper preparation and effective classroom management, it is possible to achieve the desired educational outcomes. Teachers have attested that an efficiently organized classroom and an effective approach make it possible for students to have high academic achievements (Kyungsoon, 2010).

Effective implementation is greatly dependent on the kind of lesson preparation made. In teaching, the preparation of materials and instructional tools comes before efficient implementation. The combination of educational materials and supplements with a structured learning format help ensure that lessons are better understood (Kilpatrick et al., 2001).

2.6 CONCLUSION

The literature discussed in this chapter, confirms that the topic of fractions is viewed as being difficult to understand, as well as to teach by many teachers. Research demonstrates that the use of multiple representations of learning content can significantly enhance the conceptual understanding of students. Such representations should be used to support learners' understanding of mathematical concepts and relationships; their communication of mathematical approaches, arguments, and understandings to themselves and to others; and their recognition of

connections among related mathematical concepts. Thus, using different representations of fractions may encourage learners to construct a deeper understanding of this topic. The literature in this chapter supports the use of a variety number of representations in the class when teaching. By representing different fractional problems in different ways, learners will be enabled to develop better understanding of fractions. To do this, effective preparation and implementation of teaching by the teacher, will be important.

CHAPTER 3

METHODOLOGY

3.1 INTRODUCTION

This chapter provides an outline of the research process. More specifically it describes the research orientation, methodology and design, as well as the participants, research site, data collection techniques and analysis tools. The chapter concludes with a brief discussion of the ethical considerations, validity issues and the limitations of the study.

3.2 RESEARCH GOALS

The study was framed by the following two research questions:

- What are the selected grades 5 - 7 mathematics teachers' experiences in working to develop their use of multiple representations to teach fractions?
- How do the teachers work to change their practice to develop their use of multiple representations to teach fractions for conceptual understanding?

3.3 RESEARCH ORIENTATION

This research study seeks insight into mathematics teachers' experiences in developing their practice to use multiple representations when teaching grades 5 – 7 fractions. For this reason the study is located in the interpretivist paradigm (Creswell, 2003). This paradigm is appropriate for this study as it focuses on the subjective experiences and insights of the participating teachers.

3.4 RESEARCH METHODOLOGY

This study took the form of a qualitative case study (Simons, 2009). The case of this study was the engagement of the three teachers in deepening their use of multiple representations of fractions in grades 5 -7. Cohen, Manion and Morrison (2011) describe a case study as a study of a case in a context. The context for this study is that of the practice of mathematics teaching within Namibian education. Teachers' experiences in working to deepen their use of multiple representations when teaching fractions thus constitute the unit of analysis in this study.

This research uses an exploratory approach, because the purpose of the research is to gain insight into teachers' use of these additional tools and into their experience of exploring these tools in a group workshop setting.

This qualitative case study is underpinned by the interpretivist paradigm. This implies that the major focus is on the participating mathematics teachers' subjective experiences of using multiple representations in the teaching of fractions.

3.5 RESEARCH SITE AND PARTICIPANTS

This study was carried out in three upper primary schools which had grades 5 – 7 classes, in a single circuit in Namibia. Three schools were identified which were conveniently situated to my workplace. The participants in this study were three grades 5 – 7 mathematics teachers. The three schools were approached and participants were selected on a voluntary basis from the upper primary teachers who teach mathematics. One teacher volunteered from each school. Two of the participating teachers were female and one was male. All the teachers participating in this study have between six and twelve years of teaching experience.

3.6 RESEARCH DESIGN

This study was divided into three phases:

3.6.1 PHASE 1 - Teaching fractions – pre-workshop observation

The aim of this phase was to observe how the selected teachers taught fractions, particularly with respect to which types of representations they used and how these were used to enhance the conceptual understanding of the learners. The teachers were observed before the focus group workshop in order to gain insight into their practice before the workshop was held. The lesson observations were also used to inform the workshops in Phase 2.

3.6.2 PHASE 2 - Focus group workshop

The purpose of this workshop (see Appendix A) was to explore and develop the notion of teaching with multiple representations with the three participating teachers. The participants were introduced to the approach of teaching fractions using multiple representations. They were presented with additional materials that contained different methods of teaching fractions using

multiple representations, and were asked to explore these as additional methods that might make their teaching of fractions easier and more understandable to their learners. Teachers were asked to interact with one another to solve some of the problems in the workshop handout given (see Appendix D). They then drafted rough plans for two lessons on fractions, incorporating some of the activities and techniques from the material given. The three participating teachers were asked to prepare and present these lessons in their classrooms for observation in phase 3 using multiple representations with their grades 5 – 7 learners.

Some video clips of upper primary mathematics teachers' lessons on using multiple representations in the teaching of upper primary fractions were viewed to facilitate the discussions in the focus group workshop. The focus group interview schedule was used to capture the experiences of the participating upper primary teachers.

3.6.3 PHASE 3 – Teaching fractions – post-workshop observation

After the focus group workshop, the three participating mathematics teachers prepared and presented two lessons of 40 minutes each on fractions to their grades 5 – 7 learners. These lessons were video-recorded.

3.7 DATA COLLECTION TECHNIQUES/TOOLS

For the purpose of triangulation (Cohen et al., 2000), I used multiple methods to collect the data. The data was collected using the following techniques and tools:

1. Observation
2. Interviews
3. Document Analysis

Data collection tools: Observation– Observation data was generated as the participants engaged in teaching. The teaching process and the lesson presentations of the three teachers were video-recorded. During the teaching observations I also made field notes on what was happening in the classroom – an observation schedule was designed for this purpose (See Appendix B). Based on the notes and the video data, a thick description of the observations was generated for each teacher. According to Clarke and Ritchie (2001) a thick description is usually a lengthy

description that captures the sense of actions as they occur. The data collected from the first observation provided me with insight into the teachers' experience and practice.

Data collection tool: Video recording – The participants' interactions with one another in the workshop was video-recorded. The teachers worked in groups and as individuals and both of these interactions were captured on video. These recordings were reviewed after the workshop to offer the teachers the opportunity to observe themselves and identify the additional tools that emerged from the focus group workshop.

Second Classroom observations –Observation data were generated as the participants engaged in teaching. The teaching process and the lesson presentations of the three teachers were video-recorded. During the teaching observations, notes were taken on what was happening, based on the same observation schedule (Appendix B) that was used in the first classroom observation. Based on the notes and the video data, a rich description of the observations was generated. The data collected from the second observation provided me with insight into the teachers' experience and practice after the workshop was conducted.

Semi-structured interviews –In order to gain an understanding of the teachers' understanding of their use of multiple representations when teaching fractions for conceptual understanding, individual interviews were carried out (See Appendix C – Interview transcripts). These provided insight into the first and second research questions. These interviews were carried out after lesson observations were conducted. The interviews focused on the participants' experiences as they explored these tools in the group setting workshop and their experiences in teaching using these tools. The interviews were audio- recorded and transcribed.

Interviews involve the asking of questions that elicit information about attitudes and opinions, perspectives and meanings that help produce rich descriptive data (Trochim, 2000) (See Appendix E – Semi – structured interview questions). Thomas (2011, p. 163) states that semi-structured interviews provide room for further questions for clarity. Van der Mescht (2011) states that semi-structured interviews are very useful in qualitative research because they consist of

structured and unstructured questions. He further states that they are useful because they provide the researcher with the opportunity to probe and search further.

Document analysis – The National Mathematics Syllabus for grade 5- 7 was analyzed and reviewed and I looked specifically at the topic of fractions. I analyzed how the learning objectives are outlined and whether the syllabus clearly states the approaches that teachers should use when solving problems on fractions. Document analysis is unobtrusive, does not interfere with participants, and can be checked for reliability (Ary et al., 2006).

3.8 DATA ANALYSIS

According to Cohen et al. (2007, p. 461), qualitative data analysis is about "making sense of data in terms of the participants' definitions of the situation, noting patterns, themes, categories and regularities". The video-recorded data collected from the classroom observations was transcribed and analyzed to explore the participants' experiences in using multiple representations when teaching fractions. Plowman (2004) states that video recordings make it possible for me to search back and forth throughout the entire recorded corpus for instances of both rare and frequent events and interpretive judgments.

The audio-recordings of the semi-structured interviews were also transcribed and analyzed to explore the participants' experiences in using multiple representations when teaching fractions. Categorizations of themes that emerged from phases one to three (observations, interviews and document analysis) were analyzed, coded and processed. I searched for patterns and connections among the themes within those categories and looked for connections between the various categories. This enabled me to present themes from the interviews and observations in a thematically organized manner.

The three selected teachers were labeled as Teacher 1, 2, and 3. Similar experiences on the use of multiple representations were grouped together. Unusual experiences on the use of multiple representations were also noted and were tabulated with the similar experiences. The transcribed data from individual interviews that reflected similar experiences on each individual representation were colour-coded blue; while unusual experiences were colour-coded pink. The

colour-coded data was read many times and the main ideas from each participating teacher were extracted and tabulated. Similar ideas were easily identified because they appeared repeatedly in the tables In some sections of the analysis, after reading the transcribed data many times, a summary of similar experiences was tabulated (Gay, Mills & Airasian, 2012). This was one way of organizing the experiences of the participating teachers of using multiple representations in the teaching of fractions.

3.9 ETHICS

Before the first observation, on the 28th of February 2014, I visited the three schools. I told them about my research project and asked them whether they would be interested in participating in my study. I also asked permission from the three principals and gave them letters that informed them officially about my research and asked permission to perform the study. I also requested permission from the Ministry of Education and from the circuit inspector to carry out my research and the permission was granted (See Appendix F).

The three teachers were provided with a letter of consent which they completed (See Appendix G). I also informed them of the pre-observation, the focus group workshop date and the entire research programme. I asked permission to carry out this research from the Director of Education, the district inspector, the principals of the three upper primary schools, and the three participating teachers. Letters requesting consent were written and sent to the teachers informing them about the research and also asking for their permission to participate in the research. The names of the participants (three teachers) were not revealed and I labelled them as 1, 2 and 3. The schools also remained anonymous.

The focus group workshop on using multiple representations to enhance conceptual understanding was held in a convenient place that was suitable for all three teachers. The participants were told about the aim of the research, the implications of the research and I ensured that the participants were fully aware of their rights as individuals, that their participation in this study was voluntary and that they would be able to withdraw at any time they wished. Confidentiality of all participants was ensured at all times.

3.10 VALIDITY

Each teacher was given their interview transcript in order to ensure that the transcript represented their genuine experiences of teaching fractions in grades 5 – 7 using multiple representations. I ensured that her experience in using multiple representations did not create a bias in the data collected. I strived to remain honest and transparent throughout the research process by devising and using appropriate instruments, and collecting accurate, representative, relevant and comprehensive data. More than one data collection technique was used to strengthen the validity of the data (Cohen et al., 2011).

3.11 CHALLENGES

I experienced several challenges during the research process. Firstly, it took time for the teachers to volunteer to take part in the study. By the time they volunteered, they were busy preparing for the National English test for all teachers in Namibia. This situation forced me to do the pre-observation a bit later than planned before the focus group workshop. I only managed to observe one lesson before the focus group workshop. Also, because only two lessons were observed after the workshop, teachers did not have the time to make use of all the multiple representations explored in the workshop. I encountered a number of technological challenges with respect to the video-recording process. Technical problems, and the resultant loss of data, resulted in certain observations having to be repeated. On the day of the workshop, one teacher arrived towards the end of the workshop. This resulted in the workshop starting later than planned and thus being shorter than originally intended.

3.12 LIMITATIONS

This study has all the limitations of a case study. The results can therefore not be generalized and can only be used within the context of the selected region in Namibia. The topic of fractions is complex and it is very difficult to make conclusions in a period of two lessons. In hindsight, if I were to do this research again I would do more than one workshop on the use of multiple representations in the teaching of fractions (grades 5- 7). I would also allow the teachers to teach more lessons for observation.

3.13 CONCLUSION

This chapter discussed the methodology. It began with the description of the research goals of the study. This was then followed by a description of aspects of the case study such as the orientation and the particular approach for this case study. The research site and the sample selection in terms of the research participants were described. The tools employed for data collection were identified and discussed. The approach to data analysis was then reviewed (Cohen et al, 2007). The consideration of ethical issues, validity and limitations surrounding this particular study concluded this chapter.

The next chapter presents the data collected and its analysis.

CHAPTER 4

PRESENTATION AND ANALYSIS OF DATA

4.1 INTRODUCTION

This chapter presents the data analysis used to address the two main research questions of the study. This involved investigating the teachers' experiences and the manner in which they worked to change their practice to develop their use of multiple representations to teach fractions for conceptual understanding. The individual interviews, as well as the descriptions of the taught lessons and the workshop were carefully analyzed and categorized in order to identify commonalities among the different modes of data. The process of analyzing and organizing the data involved colour coding of topics, which were then grouped and categorized into themes, which gradually emerged.

The chapter presents the findings from the focus group workshop, the observations of lessons on fractions taught by the three grades 5- 7 mathematics teachers and the individual interviews. A summary of a teaching episode from one lesson of each mathematics teacher is presented. Transcripts of the actual lessons are included in the appendices. Before presenting the analysis of the individual interviews and classroom observations, a short profile of each teacher is provided.

4.2 BRIEF PROFLIES OF THE TEACHERS

4.2.1 Teacher 1

Teacher 1 is female and has been a mathematics teacher for 10 years. She holds a Basic Education Teacher's Diploma (BETD) from the Rundu College of Education and an Advanced Diploma in Education from the North West University. In the BETD programme, Mathematics was one of her major subjects, the other being Natural Science. She currently teaches mathematics in Grades 5, 6 and 7 at school 1. She is the Mathematics subject head for grades 5 – 7 and is currently studying for an Honours degree with the North-West University.

4.2.2 Teacher 2

Teacher 2 is also female and has been a mathematics teacher for 12 years. She holds a Basic Education Teacher's Diploma (BETD) from the Caprivi College of Education and an Advanced Diploma in Education from the North West University. She also completed her Honours with North-West University. In the BETD programme, Mathematics was one of her major subjects, the other being Natural Science. She teaches mathematics in Grades 6 and 7 at school 2. She is currently the Head of Department for Mathematics and Natural Sciences for grades 5 – 10 and is acting principal of the school. It is the opinion of the inspector that she is one of the best teachers in the region and she was appointed by the region to conduct workshops for mathematics from grades 5 – 7.

4.2.3 Teacher 3

Teacher 3 is male and has been a mathematics teacher for 6 years. He holds a Basic Education Teacher's Diploma (BETD) from the Windhoek College of Education. His majors were Mathematics and Natural Sciences (grades 5 – 7). As a teacher of mathematics, Teacher 3 currently teaches the Grade 5 and 6 at school 3.

4.3 PRESENTATION OF THE DATA

4.3.1 Teachers' experiences in teaching fractions using multiple representations

Teachers used different representations in the workshop to represent different fractional ideas. These representations included concrete models and manipulatives like 20 blocks. From the workshop data, it was clear that the teachers became actively involved in the workshop, solving the problems together. The two teachers who were involved in the workshop built fractional relationships within and between the models. They manipulated the blocks into different common fractions like $\frac{1}{2}, \frac{1}{4}, \frac{3}{4}, \frac{1}{3}$ and discussed what they were doing. The following is an extract from their discussion when making halves:

Teacher 1 took two blocks and painted one on top with a red pen to represent a half.

Teacher 1: I have two blocks here, the shaded block represented a half.
Teacher 2: Ooh, yes. If you take four blocks and you paint two blocks with a red pen (busy painting the two blocks with the red pen), it represents the same half. And if you remove one shaded block, you remained with three where one shaded block represented a third ***(Extract 1, Appendix A, P 6)****.*

Teachers demonstrated how to introduce the topic of fractions to the learners using multiple representations. The following extract from the workshop shows how they responded when asked to demonstrate how they could introduce this topic.

Teacher 1: (demonstrating with paper) I use papers, I give them these papers into groups for them to understand easier, or for them to experience it and then they follow my instructions. First of all you show them the full complete paper which shows a whole. So, now to make it a half, because know when you talk of fraction we talk of equal parts, I have to divide this paper into a half, and then open it to show them equal part (two halves), and then I instruct them to fold that paper again into half and open it to see how many parts of a whole are there and so on…. In this way learners should be able to differentiate which one is bigger, which one is small when they are given a questions on comparing of fractions. I can also use an apple as a real object, you cut it into equal parts, one part is a half, and if you cut a half apple further again, you will be having a quarter. When you explain $\frac{1}{2}$*, explain it in different ways for the learners to understand.*

Teacher 2: I use video, where learners can see real objects, like in my case I have a video on fractions, and I usually use it when I introduce that topic to the learners, I show them a video and this video has already different representations. After watching, the video, I either give the learners an exercise to do or I will ask them questions. ***(Extract 2, Appendix A, P 3).***

After the workshop presentation, teachers worked on the worksheets and gave possible representations to those problems. They came up with potential ways of how to present those types of problems to the learners in their classes. Teachers thought of ways that different representations could be used to help students build a conceptual understanding of fractions by linking visual models to numeric representations.

During the interview, teachers were asked to share their feelings about the teaching of fractions. All the three teachers indicated that they felt good when teaching fractions. Teacher 1 responded:

P36.T1: *No, I feel very good; I enjoy it very much, especially by doing it more practical, by using examples papers, flash cards, when you use flash cards learners enjoy so much. Most of the learners are more actively involved. Teaching is boring, it is better if learners are involved* ***(Extract 3 .Appendix B1, P36)****.*

T2 also shared her feelings about the teaching of fractions by saying:

> ***P32.T2:****Oh, I feel good, before we started, then I told the learners that next week we will start with fractions, so learners were so excited, because that is the lesson were they are free to talk, you know to do other things, those are the topics where learners are actively involved. This topic is more practical, that is why the learners like fractions very much* ***(Extract 4.Appendix B2, P32)***.

T3 also expressed his feelings about the teaching of fractions by saying:

> ***P24.T3:*** *As a teacher I always have this excitement in terms of making sure that, I would want to make my learners understand. So I am happy to talk to the learners, I am happy to present this to them, I am also happy to come across some of those challenges that the learners are bringing in the classroom environment, where we are able to say look, if you have this things in particular situation in terms of fractions, how do you solve it? Or how do you approach it? So it makes me feel good. I want to make it more practical, when I am teaching my kids, I don't want to make it an oral issue, and I want to make it a practical issue, where I will be able to give them examples from their houses where they are coming from, the classroom where they are seated. For example you have 20 learners in grade 5A, three of the learners are boys. To present this as a fractions, three is the part of the whole, the whole is 20 learners, so, three over twenty (* $\frac{3}{20}$ *) is the correct expression in terms of fractions. I want to make it more practical, more reality to them, so it makes me feel good when teaching fractions* ***(Extract 5. Appendix B3, P24)***.

All the teachers responded that they felt confident and excited when teaching fractions. Teachers 1 and 3 indicated that they enjoyed the teaching of fractions immensely. Especially when using practical examples such as paper and flash cards. They reported that the learners enjoyed the lesson when they used flash cards. Most of the learners became actively involved in such activities. Teachers indicated that teaching would be boring if learners were not actively involved.

4.3.2 Teaching with multiple representations supports the learning of fractions for conceptual understanding

Each of the three mathematics teachers were observed for a 40 minute double period after the focus group workshop had taken place. Summaries of these lessons are included in the appendices. These observations provided a focused view of the interactions between the teacher and the learners as they used multiple representations in their teaching. The three teachers used different representations to present their lessons to the learners. Brief vignettes from each of these lessons are given below. These show which types of representations were used by each

teacher when presenting fractions. The snapshots also indicate how the teachers engaged with multiple representations to help develop learners' understanding of the concept of fractions.

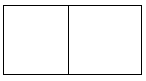
4.3.2.1 VIGNETTE 1: Teacher 1 teaching equivalent fractions (Appendix B1 – Teacher 1: lesson observation)

Teacher 1 used oral representations and pictorial representations when she introduced the topic of fractions to the learners. She said the words aloud (oral representations), like "dividing a square into four equal parts" and drawing the square (pictorial representation) at the same time. The teacher started by asking what a fraction is:

> *What do you understand by the term fractions? What do you do when dealing with fractions? The learners tried to figure out what 'fraction' means. The teacher then asked again try to make the question clearer to the learners. Teacher: What is fraction is my question? Can somebody explain what fraction is all about? Learner 1: Fractions are numbers that are mixed together. Teacher: Repeats what the learner has said, numbers that are mixed together? Hmmmmm, the other people, what do you think? Learner 2: Fractions is when you put a half of a number. Teacher: Very good, another person. Learner 3. A fraction is when you divide a shape. Teacher: A shape, which shape? Learner 3. For example a square, you divide it into four equal parts, then you count how many fractions are there? Teacher: Very good, you divide a square into four equal parts, you see how many parts will come out. The teacher explains while drawing a square on the chalkboard to show what learner 3 has said* ***(Extract 6. Appendix B1, Line2).***

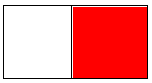
T1 used flash cards showing pictorial representations to present different terms for fractions like a whole, a half, a quarter and an eighth. She said that if you divide a whole into two equal parts, then one part is equal to a half, saying it, writing it, drawing it and showing the flash card to the learners. She used flashcards by saying:

> *Teacher: We are going to look at flash cards today. You were doing this thing in grade 4. She picks up one card and shows it on the chalkboard. Teacher: This is a flash card. This flash card is divided into two equal parts.*



> *Teacher Place the flash cards on the chalkboard. Teacher: How many parts do you see here?*

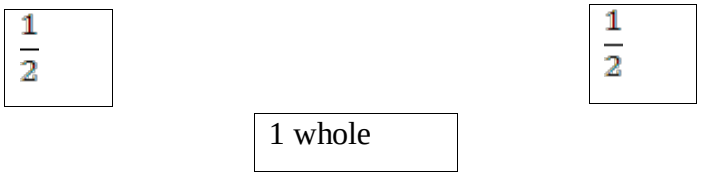
All learners: Two. Then the teacher draws the card (In a form of a rectangular shape) on the chalkboard .Teacher: Out of the two, how many is shaded? Learner: One. Teacher: Yes, then you say it is one.



Teacher: Simply, first of all you find out, how many parts are there in total, then you say is two. Teacher writes on the chalkboard: $\frac{}{2}$*,Teacher: The total is always at the bottom and then you put your fractional line on top of the total. How many parts is shaded? All learners: one. Teacher: You listen very well according to the question. If they ask you the shaded, then you answer the shaded is one, the coloured one (pointing on the chalkboard). What will you say, if they ask you to identify unshaded, then you will give the uncolored one. The teacher showing on the flash card on the chalkboard. Unshaded and shaded show a half, the teacher explains while writing on the chalkboard (**Extract 7. Appendix B1, Line4 - 7).***

She used mathematical symbols when she explained. She would say the word, write it on the chalkboard and draw a diagram or use a flash card to represent the said word or written symbol. Teacher 1 also involved learners in her lesson. She divided them into groups and gave them papers to practice some examples in equivalent fractions. Teacher 1 instructed the learners to cut and fold the paper into two equal parts called halves and then further divide the paper into four equal parts which are called quarters and even further into eight equal parts which are called eighths. Teacher 1 wrote those terms on the chalkboard and did drawings to represent the fraction parts she was describing. In this way she used multiple modes of representation to improve learners understanding.

Teacher says, can we go ahead? Learners: Yes. Teacher explains using a paper written: half, half. Teacher: This is what I am talking about (pointing at the paper). We will come back to the equivalent fractions. If I take two of my half and put on the A4 paper on the chalk board, these two halves will form a whole one.



The teacher divides the learners in pairs and gives them A4 paper. Teacher: I divide this paper into a half (fold it into a half) and I cut it into half. (Takes a paper and cut it into two equal parts) If you put two halves together, they form a whole. Then we go ahead, this paper is a whole, then we divided it into half and now I want to us to divide it again into another half and open it. How many parts can you see here? Learners: 4 parts. Teacher: Let me write it on the chalkboard:

$\frac{1}{4}$	$\frac{1}{4}$
$\frac{1}{4}$	$\frac{1}{4}$

*The teacher shows four parts on the chalkboard. Teacher: $\frac{1}{4}$ is called one quarter. If you take the four quarters and put it on the whole A4 paper on the chalkboard, will they look the same or not? Learners: the same. Teacher: They are the same (**Extract 8. Appendix B1, line 8 – 11)**.*

Learners were also taught how to compare fractions, by comparing different fraction parts constructed from paper. For example, $\frac{1}{2}$ is bigger than $\frac{1}{4}$. Teacher 1 explained clearly using the paper that as the denominator gets bigger the parts on the paper get smaller and as the denominator gets smaller, the parts on the paper get bigger.

This teacher also used the flash cards as a game in the class to match with the fractional poster on the chalkboard. She asked learners from each group to come and display the flash cards that were equivalent to what was on the poster which was displayed on the chalkboard. For example, learners identified cards with symbols **1** or **one** in words for display. Or a card with 'quarter' or 'one-fourth' written on them, or a drawing of a square divided into four equal parts or $\frac{1}{4}$.

Teacher: You are going to play a game (pastes the poster on the chalkboard). We start from the top one. What does the first line represents? Here is the poster: Teacher: Choose one card and stick to the poster. Learner 1: sticking card number 1, which shows a whole number on the first line. Teacher: learner 1 correct? All learners: Yes. Teacher: Good, we go to the next one, yes, one learner? Learner 2 sticks the flash card on the second line and so on.

The picture below shows how the learners in the class pasted the flash cards on the correct places on the poster.

The teacher was not able to finish her lesson in the time for which it was prepared, because she included many tools in her lesson which made her unable to finish on time. The activity that was still to be done, where learners was supposed to be given printable handout to compare fractions, was given as a homework **(Extract 9. Appendix B1, line 19-24).**

4.3.2.2 VIGNETTE 2: Teacher 2 teaching equivalent fractions (Appendix B 2- Teacher 2: Lesson observation)

Teacher 2 in her teaching of fractions used video clips and power point presentations. She used two video clips. The first video was in a form of a song about the teaching of fractions. After the song the teacher used oral representations to ask some questions concerning what was said in the song by saying:

> *Teacher asks the learners, did you see things that you combine together to make a whole? Learners: Yes. Teacher asks the learners how many quarter parts do you need to make a whole ? Learner 1 says three. Teacher repeats what the learner have said, three, three halves you need to make a whole? Other learners: No. Teacher asks again, how many halves? Learner 2. Two. Teacher: Yes, two halves that you need to make a whole* ***(Extract 10. Appendix B2, line 4).***

Although teacher 2 asked learners some questions concerning what was said in the song, she did not use written mathematical symbols to represent her oral information in writing. The second video showed a conversation between a teacher and a learner, explaining what a fraction is. It also gave examples of equivalent fractions. The teacher in the video explained in a very informative way how to divide a whole into pieces, like halves, thirds, quarters and eighths. She also showed different ways to express the same meanings, such as, if you divide a whole into

two equal parts, one part is called a half, or it can be written as one upon two or $\frac{1}{2}$. If you divide a whole into four equal parts one part is called a quarter or one-fourth, or $\frac{1}{4}$, or a drawing of a square which is divided into four equal parts and one part out of the four is shaded. Different representations were used in a video, like half and a quarter in words and $\frac{1}{2}$ and $\frac{1}{4}$ in written mathematical symbols.

After the video, teacher 2 asked some questions, saying:

> *The teacher: before I give an exercise, what did you learn? What did you learn? Who can tell me? The class was quite for some time. Teacher then asked again who can tell me. Learner 1: fractions. Teacher: Fractions, what is a fraction? Learner 2: Fraction is part of the whole. Teacher repeats what the learners said; a fraction is part of the whole. Teacher asks, how many quarters do you need to make a whole? Learner 3: Four quarters. Teacher asks again, how many halves do you need to make a whole? Learner 4: Two halves. Teacher continues with the questions, how many eighths, eighths is what? The teacher continues, eighths, one eighths (saying it orally), how many eighths do you need to make a whole? Learner 5: two. Teacher said eights is one over eight, if I have one over eight, how many parts do I need to make a whole? Learner 6: (not sure) eight parts. The teacher praises the learner, yes, it is eight parts* ***(Extract 11. Appendix B2, line 11).***

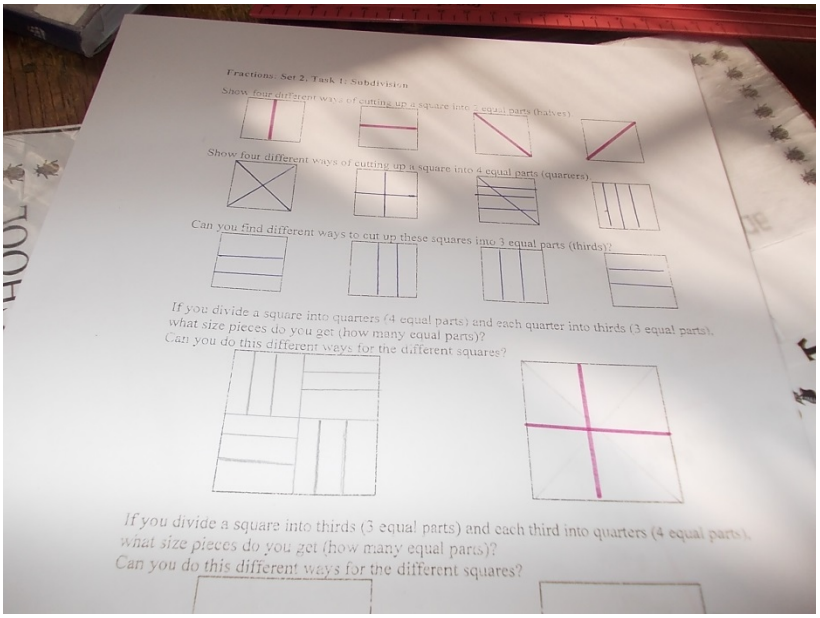
After this, teacher 2 gave the learners an exercise to work through in pairs, using manipulatives such as paper. She requested the learners to fold the papers into two equal parts either vertically or horizontally but not diagonally, since the shape of the paper is in the form of the rectangle, and then into four equal parts.

> *Teacher gives papers to the learners and instructs them to fold them into half into different shapes. Teacher says, if you are seated two, two, then each one must have a different shape, but each shape must be equal to a half. Teacher says, I want you to make a half using this papers, I want to see how you can get a half on a different shape. Teacher said you just make a half. Teacher said right, I did not say make other fractions. Ok, open it; somebody should explain your shape using your paper? One learner: My shape is divided into two equal parts.*
> *Teacher explains, you can fold a paper vertically, horizontally and diagonally, but this paper is not a square, is a rectangle, you cannot fold it diagonally. The teacher showing the learners using her paper, these corners do not touch each other. Teacher instructs the learners to use their paper to make a quarter. And I want to see four parts and I want you to cut one part from that 4 parts: Hold your other three parts and one part, with the part that you have cut. Teacher asks what fraction is it. One Learner: One over four. Teacher asks and the fraction for the uncut? Three over four. Teacher says, I want to see all four parts being cut, how many parts do you have? One learner: four parts. Teacher yes, this four parts came from one whole, is what we say sharing. If they were four learners and you cu that bread into four equal parts, each one is going to get what?*

Learners: one piece. Teacher asks which is what? One learner: one–fourth. Teacher: Yes, one fourth or a quarter. Ok. Teacher says ok now, you take one piece and you fold it three times, don't open it. One part folded three times. Teacher asked the learners to open that one part. Teacher asks the learners, how many parts do you have there?
One learner says six, one says eight parts. Teacher: Ok good, they are eight. Teacher asked the learners to shade three parts, you colour with anything, so long as there is a colour on it ***(Extract 12. Appendix B2, line 13-15).***

Learners were then given worksheets to answer. The first question on the worksheet was to show four different ways of cutting a square into two equal parts (halves).

In this lesson, learners were actively engaged in the lesson because they watched the video on fractions first and after that, the teacher asked them questions (oral representations. She gave the learners papers to work with in pairs to identify different fractional parts like; a half, a quarter and eighths and to cut out some parts on the whole to represent either a third or a quarter and learners were given worksheets to answer. Here are some of the learners, answers when they cut the square into different parts.



4.3.2.3 VIGNETTE 3: Teacher 3 teaching equivalent fractions (Appendix B 3 – Teacher 3: Lesson observation)

Teacher 3 in his teaching of equivalent fractions used power point presentations. His lesson was prepared on slides. He taught the topic of equivalent fractions following the rule based method.

In his presentation, a lot of representations were shown, including written mathematical symbols, pictorial representations, oral representations and concrete or real objects. The teacher said the words and symbols orally when he represented the topic to the learners. Although different representations were shown in the power point representations, he did not write or draw these representations on the chalkboard. Furthermore, teacher 3 did not allow learners to sit in groups so that they can identify different fractional concepts together. Learners were not actively engaged, since they were not given a chance to express their creativity using papers, posters, flash cards and other representations.

4.3.2.4 General discussion on the mathematics teachers teaching fractions using multiple representations

Looking at the three mathematics teachers' classroom vignettes, they provide glimpses of the different types of multiple representations which were used when teaching fractions and the kind of interaction that occurs in these three classrooms. There are more similarities than differences in their lesson presentations. All the teachers used oral representations when they were teaching fractions. Focusing on T1's lesson, she explained the first part of her lesson orally (oral representations), she said the words aloud, while dividing a square into four equal parts and drawing the square at the same time on the chalkboard. T1 used flash cards, a pictorial representation, while explaining different terms for fractions like a whole, a half, a quarter, and an eighth. She said that if you divide a whole into two equal parts, then one part is equal to a half, saying it, writing it drawing it and showing the flash card to the learners.

T2's teaching style is in a way similar to T1's teaching style, where she used oral representations to introduce the topic to the learners. T2 asked questions, like teacher 1. She asked the learners whether they saw the things that were used to combine together to form a whole. T2 asked how many halves and quarters do you need to make a whole. Although T2 asked learners some questions orally concerning what was said in the song which was presented in the first video, she did not use written mathematical symbols to represent her oral information in writing like T1 did, rather she used the video. T1 used mathematical symbols when she was explaining a half, a quarter and an eighth to the learners. T1 would say the word, write it on the chalkboard and draw a diagram or use a flash card to represent the said word or written symbol. For example in her

teaching she used a flash card to represent a whole and a half. T1 and T2 divided the learners in groups and gave them papers to demonstrate different fractional parts like a whole, a half, a quarter, a third and an eighth and they discussed the answers with their learners.

T2 and T3 in their teaching of fractions used video clips and power point presentations respectively. T2 used two video clips. The first video was in the form of a song about the teaching of fractions. After the song the teacher used oral representations to ask some questions concerning what was said in the song. The second video showed a conversation between a teacher and learner. The teacher in the video explained how to divide a whole part into pieces, like into halves, thirds, quarters and eighths. After the video, the teacher asked further questions orally. T2 repeated these questions because she had already asked them already during the first lesson video clip. She did not represent the information in writing but said it orally. Moreover, T2 gave the learners an exercise to work on in pairs, using manipulatives such as paper. T2 gave another exercise where learners were given worksheets to answer. In T2's lesson, learners were actively engaged in the lesson because they firstly watched the video on fractions and after that, the teacher asked them questions (oral representations). Finally, learners were given worksheets to answer.

On the other hand, T3's learners were not actively involved. T3 did not allow learners to sit in groups so that they could identify different fractional concepts together. Learners were not given a chance to express their creativity using papers, posters, flash cards and other representations. Although different representations were shown in the power point presentations, the teacher did not represent them on the chalkboard in terms of writing or drawing or involve the learners to show their creativity.

T1 and T2's learners were actively involved throughout the lesson. Although T1 did not use the video like other two teachers, she prepared her lesson very well. A lot of representations were presented, including oral representations, written math symbols, pictorial representations, and concrete materials like paper. She demonstrated different fractional parts given a whole object like paper. Learners identified different fractional parts given a whole object in groups and through direct observation, I saw how learners enjoyed folding the paper. T3 taught the topic of

equivalent fractions following the rule based method. In his power point presentations, a lot of other different representations were shown, like written mathematical symbols, pictorial representation, oral representations and concrete or real objects.

It is also seen in the vignettes presented earlier that teacher 1 and teacher 2's use of multiple representations when teaching fractions was influenced by the workshop. Despite the challenges, the two teachers who participated in the workshop showed positive effort, working with the different types of multiple representations which were presented in the workshop. They shared their experiences willingly during the focus group workshop. The concrete materials, manipulatives, pictures and posters were provided and teachers were positively engaged with these presentations. Teacher 3 joined the workshop later and it did not appear to influence his way of teaching.

The commonality that was seen through the analysis of data is that both teachers 1 and 2 said that they managed to use multiple representations in their teaching by planning their lessons so that they could use different methods in their teaching effectively. T2 and T3 commented that planning helped them to look for an easier method to make the lesson easier for the learners to grasp and which made their teaching more effective. On the other hand, all three teachers complained that teaching time was insufficient. Because the syllabus contains so much content, it forces teachers to teach theoretically and to focus on one type of representation. Teachers 2 and 3 added that although the time was limited, effective planning makes it possible to cover more methods over a period of time because your aim is to develop the understanding of the learners. Through lesson observation, it was seen that T1 and T2 were not able to finish their lesson on time, but needed to set the remaining work as homework. T3 finished on time because he only used power point presentations. He did not actively involve the learners in his lesson. Although it was a double lesson, in the first, the learners were given only a few questions to answer individually and discuss the results and the second lesson was used for a class activity. On the other hand, T1 and T2 involved their learners throughout the two lessons. Teacher 1 used concrete objects like paper, scissors to cut, posters and flash cards to teach the concepts of fraction and equivalent fractions to her learners. Teacher 2 used video clips to catch the attention of the learners. She also used paper and scissors and her learners were involved in fraction

thinking problems, where they needed to find different ways to divide shapes, and to determine which shapes are divided into fractions.

4.3.3 Interview Analysis

In the interviews, teachers were asked how they managed to use multiple representations in their teaching. Teacher 1 responded as follows to the use of multiple representations:

> ***P20.T1****: By planning my lesson effectively, so that I can use different methods in my teaching. On the other hand, time is not enough and the syllabus contains too much content, it forces teachers to teach theoretically and to focus on one type of representation.* ***P22.T1:*** *Definitely, syllabus requires people to finish earlier during third term, teachers have to rush so that they can finish with syllabus and start with revision* ***(Extract 15. Appendix C1, P20-22).***

Teacher 2 stated that:

> ***P10.T2:*** *Learning plays a very important role, so it gives you more time to plan your lesson, so that it must be effective. On the other hand, time is also limited to use the different methods but if the planning is done, you can be able to cover even less methods, if not on that day, you can still continue the following day to cover all the methods, because what you are looking at is the understanding of the learners. Once the learners catch up then you know it is easier for them to work on it. In addition to this, the syllabus contains too much content which force teachers to teach using one method only, because the time is limited and you are required to finish that syllabus at the end of the year.* ***(Extract 16. Appendix C2, P10-14).***

Teacher 3 in his testimony said that:

> ***P12.T3:*** *I think what is important in that the essence is effective learning, the moment you have to make sure that your planning has been set out into a very strategic way, but yet think now to the learners, it is very good, you won't really say that it is a lot of problems in teaching. When I am presenting a lesson, I have to make sure; I get different materials from various sources. Like from internet for instance you download pictures that has been once used and inducted to make teaching easier, in that way you can be able to manage it.*
>
> ***P14.T3:*** *You do not need to do everything at the same time, you need to focus on the part which you think, and learners will be able to catch up on that particular time. You need to remember as well that, learners have to do activities based on that particular approach. The learners should not only watch the video, but they should also be able to reflect on what they have watched. I think it is also important, if you want to give more work, it is important to organize extra classes, so that you can present more methods.* ***(Extract 17. Appendix C3, P12-14).***

The commonality that was seen through the analysis of data is that both teachers said that they managed to use multiple representations in their teaching by planning their lessons effectively. On the other hand, both teachers complained about the time being insufficient and the amount of content in the syllabus forcing to focus on one type of representation. Furthermore, T3 added that if you want to give more work, it is important to organize extra classes, so that you can present more methods, which would be a good idea because of the limited time that is available.
Furthermore, during the interview, teachers were asked about the types of multiple representations which are mostly used in their classes when teaching mathematics and the ones that are least used. Here are the responses of the teachers:

> ***P30. T1:*** *Mostly, they use descriptive written words and symbols and the one that is mostly neglected a lot is pictures, pictorial.* ***P31.JI:*** *Why do think pictorial is more neglected?* ***P32.T1:*** *Because of maybe teachers do not have time to draw pictures and are ignorant to get information and put them together, I think that is why they prefer always just to write on the board in words or in numbers.* ***P33.JI:*** *You also said earlier that teachers do not have time to draw pictures……****P34.T1:*** *Yes, they do not have time, sometimes for them to go and buy materials is always a problem* ***(Extract 18. Appendix C1, P30 – P34).***

Teacher 2 responded that:

> ***P24.T2:*** *Mostly used are written math symbols, descriptive written words, pictorial presentation, oral presentation and experience- based and a video. E.g.* $\frac{1}{2}$ *in symbols, one out of two (a half) in words and you can relate it in terms of pictures. Among the presentations mentioned, I prefer a video and then a concrete, where learners they use concrete things, because they are using it with their own hands, and so they can see if you talk of the half, you bring something that they see, you tell them to cut, they understand. A video also bring reality in the lesson. What they cannot even find in the class, once you use a video, then they can see what is going on, so it makes it easier than just writing, you draw, this is a half, once you wipe the chalkboard then, they can easily forget what you have said.*
> ***P25.JI:*** *Do you think video is mostly used in schools?*
> ***P26.T2:*** *Not always, because the schools do not have some of the equipment, so videos is only those once are privileged schools, schools neglect the part of having materials, where it is the part that brings reality in the lesson for the learners to understand.* ***(Extract 19. Appendix C2, P24 – P30).***

Teacher 3 also commented on the most common representations which are used in schools when teaching mathematics. He said that:

> ***P18.T3:*** *Looking at the presentation that we have here, I would say, the descriptive written words is used mostly by the teachers and written math symbol, in most cases we also used pictures especially when you are giving an introductory lesson to the learners. Normally, oral, you speak then you use pictures, you connect the oral with the pictures, but I would say looking at the experience that we have in the teaching, we do not use videos and power points. Firstly, this may be that the materials are not readily available in schools; we do not have them at all. Secondly, it is not all learners are exposed to these materials (videos and power point), that could be one of the reason why we do not at all times use videos or power point into our classroom teaching fractions.*
> ***P20.T3:*** *They are very, very excellent to be used in the classroom. That is why I would feel using technology in the classroom like videos and power point presentations is a very crucial method of teaching fractions* ***(Extract 20. Appendix C3, P18 – P20).***

All three teachers indicated that the most used types of representations are: descriptive written words, written mathematical symbols, pictorial presentation, oral presentation and experience-based representations, symbols such as $\frac{1}{2}$ in symbols, phrases such as one out of two (meaning a half) related to pictures. Although teachers 2 and 3 indicated that pictorial representation are mostly used, teacher 1 did not support that, she said pictorial representations are neglected, because sometimes teachers do not have time to draw pictures and do not collect and organize information effectively. She indicated that teachers prefer to write on the board in words or in numbers. Apart from the presentations mentioned, teacher 2 preferred a video and then concrete representations, where learners use concrete objects. Because they are working with their hands, and they can see if you talk about a half, you bring something that they see, you tell them to cut it, they understand. Teacher 2 indicated that a video also brings reality into the lesson and is the best method to use when teaching fractions.

Teacher 3 indicated that in most cases pictures are used, especially with the introductory lessons to the learners. Teacher 3 further indicating that usually you speak (oral representations) then you use pictures, you connect the oral with the pictures. Both teachers indicated that in their experience, teachers do not use videos and power point. Teacher 3 indicated that videos and power point are least used because they may not be readily available in the schools. “That could be one of the reasons why we do not at all times use videos or power point into our classroom teaching fractions”.

All teachers were strongly in favor of the use of videos in the classroom; expressing the wish that they all could have these resources in schools. Teacher 3 added that in his lesson "as you have seen in my lessons that when the learners are able to view what they are learning, it is easy for them to capture, instead of us talking and preaching things to them because they do not have any practical ideas that we are telling them but when they see, it is very easy for them to connect and recall". That is why teacher 3 would feel that using technology in the classroom like videos and power point presentations is a crucial method of teaching fractions.

4.4 EMERGING THEMES

During the process of analyzing and organizing the data, several themes gradually emerged. These themes included:

(a) Introducing and teaching fractions in meaningful ways
(b) Developing understanding of multiple representations
(c) Developing connections
(d) Teachers' planning and preparation
(e) Teachers' use of multiple representations and modes for improved engagement
(f) Teachers use of multiple representations to teach in ways that improved learners' conception of fractions

4.4.1 Introducing and teaching fractions in meaningful ways.

Teachers were able to introduce the topic of fractions and look at representations in useful ways to develop the meaning and concepts of fractions. The multiple representations used during the workshop and teaching practice were: written mathematical symbols, descriptive written words, pictorial representations, manipulatives, concrete reality, oral representations, experience-based representations and videos or power point presentations. In the workshop, teachers engaged positively with concrete materials to make sense of and solve the given fractional problems. The first question explores fractions through sharing concrete materials such as sweets. Teachers used concrete materials (sweets which were provided) to explore the two questions

1. How can 20 sweets be shared among the 5 best learners in your class.
2. Divide a chocolate (with 20 blocks) fairly between a mother and her two daughters (Activity 1. 1. Workshop hand out (Appendix D)).

Teachers worked for some time to respond to the two questions.

> ***Teacher 1:*** *20 sweets and five learners (wrote on the paper), drawing 20 circles on the paper and encircled five small circles.* ***Teacher 2:*** *Yes, you are correct, which means that, each learner will get four sweets, teacher 2 also taking twenty sweets and divided them into five groups to match the number of five best learners in the class.*
> ***Teacher 1:*** *perfect, you see, there are many ways to kill the cat, can you see the two ways that we have used?* ***Teacher 2:*** *yes, yes… we do the same to question 2. For question two we use chocolate or 20 blocks or counters to represent the number of blocks.* ***Teacher 1:*** *Oh, yes, then you divide the bars among the three people, each one will get 6 bars and the remaining 2 bars will be shared equally, by putting them together and divide them into three equal parts (****Extract 21****.* ***Appendix A, P4).***

Teacher 2, responding to the first question**:**

> *I take 20 sweets (teacher taking 20 real sweets which were provided) and divide them equally among the 5 best learners, so each learner will get 4 sweets.*

Furthermore, teacher 1 responded to the second question, where they were asked to divide a chocolate (20 bars) equally that is being shared by a mother and her two daughters.

> **Teacher 1:** *Since, they are three in total and the bars are twenty, firstly each one will get 6 bars and the remaining two bars will be put together and shared equally into three parts.*

Teachers engaged positively in these sharing problems and were able to generate answers. They explored these fractional questions using a variety of concrete materials and they developed a sound understanding of the sharing concept of fractions. Although teachers did come up with possible answers they spent most of their time exploring one question (Appendix A, P5).

In addition, teachers gave the following example of how they could introduce the topic of fractions to the learners using multiple representations. Here is the observation:

> *Teacher 1: (demonstrating with a paper) I use papers, I give them these papers into groups for them to understand easier or for them to experience it and then they follow my instructions. First of all you show them the full complete paper which shows a whole. So, know to make it a half, because know when you talk of fraction we talk of equal parts, I have to divide this paper into a half, and then open it to show them equal part (two halves), and then I instruct them to fold that paper again into half and open it to see how many parts of a whole are there and so on…. Teacher 2: I use video, where learners can see real objects, like in my case I have a video on fractions, I usually use it when I introduce that topic to the learners, I show them a video and this video has already different representations. After watching, the video, I either give the learners an exercise to do or I will ask those questions (****Extract 22. Appendix A, P2).***

The two teachers who participated in the workshop were positively involved in the programme. They shared their experiences willingly during the focus group workshop (Appendix A, P6). The two teachers demonstrated different ways of introducing the topic of fractions to the learners. Teacher 1 presented the situation where learners should be exposed to a wide variety of situations such as folding and cutting of paper. In the activity hand out which was given to the teachers, they shared their experience on how to solve given problems.

Workshop hand out (Appendix D, Activity 1.2.):

1. Hendricks gives a prize to the group in his class that has behaved the best during the week. The prize is the box with 10 chocolate bars.

a) This week Ann's group wins the prize. There are four people in Ann's group. They all want the same amount of chocolate. How much chocolate does each child get?

b) Last week it was John's group that won the prize. There are six people in John's group. How much chocolate does each child get?

Teachers worked for some time to get the correct solutions and how to present this problem to the learners to make it more understandable in the classroom. In some instances teachers tried to get the possible solutions and presented them to each other. These are the teachers' responses when they were asked to answer the two sub-questions of the question above.

> **Teacher 2:** *To answer the first sub-question (a), I use this real chocolate (Teachers were provided with the real chocolates), if there are four learners in a group and each chocolate contains 10 bars, what I do is, each child will get two bars of chocolate.*

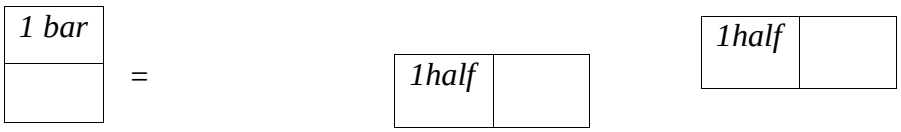
Teacher 2 also drew a rectangular model on the white board to show how the 10 bars of chocolate can be divided equally among the four learners in a group:

1 bar				

10 bar chocolate - shared by four learners
Each learner will get two bars:

1 bar			

The remaining two bars will be shared again equally. Meaning that two learners will share one bar. Each learner will get two full bars and one half bar.



Teacher 1*(explaining the first answer), I will take a whole paper as a chocolate, divide it into ten bars take a scissor and cut out each piece which represent a bar, each learner will get two pieces and the remaining two pieces, I will fold each into two equal parts and cut them out so each learner will get two full pieces and one half piece (****Extract 23. Appendix A, P7 – 8).***

The concrete materials, manipulatives, pictures and posters were provided and teachers were positively engaged with these presentations in the workshop. In the workshop teachers used concrete models and manipulatives like 20 blocks which were manipulated into different fractions like $\frac{1}{2}$, $\frac{1}{4}$, $\frac{3}{4}$, etc. and counters which helped to create meaning. But they found it difficult to build in relationships within and between the models, to represent mathematical ideas. (Appendix A, P3).

In the activity 1.3 – Sharing fractions, teachers were given this problem to solve: Four friends go on a full day outing. At lunch time they are hungry. They have 8 sandwiches to share for lunch. How do these friends share the sandwiches fairly for lunch?

(a) How can you look at this situation to see fractions?

(b) Use different representations to describe the situation above?

Teachers were given a hint to draw pictures and symbols. They used concrete materials such as pens, books, posters and biscuits to represent real sandwiches, papers, scissors, etc. Workshop hand out (Appendix D, Activity 1.3.)

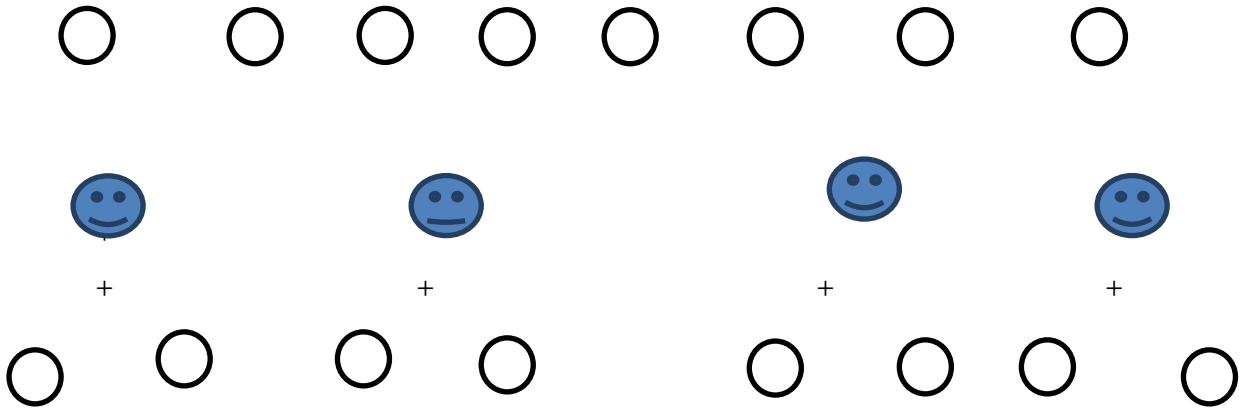
Workshop (P9 – P11) Teachers shared ideas on the activity.

Teacher 2: *where should we see fractions in this questions? We are given four friends and 8 sandwiches and the question is on sharing?* ***Teacher1:*** *Ooh, yes, you know it is four friends sharing 8 sandwiches, it is simple, four friends sharing 8 sandwiches, so they are sharing part of a whole.* ***Teacher 2:*** *Ooh, yaa, you are right, sharing part of a whole is already a fraction.* ***Teacher 2:*** *Ok, let's continue with (b).*

Teacher 2: *We are given a hint to draw pictures, let's take a poster, you know poster is big enough and you can draw clear pictures on it that can be seen, lets draw 8 sandwiches,* ***Teacher1:*** *and then?* ***Teacher 2:*** *and then four pictures of four friends and we draw arrows from the sandwiches to the four friends showing how they are sharing 8 sandwiches.* ***Teacher 1:*** *Yes you are right, since we have even numbers, it is very simple, four friends and eight sandwiches, ooh, each one will get two sandwiches.*

Teacher 2: *Yes, (teacher 2 representing the question in form of drawing, drawing with a pencil).*

Teachers discussed their answers: **(a)** ***Teacher 1*****:** *I can see fractions because, four friends are sharing sandwiches.* ***Teacher1:****Sharing part of a whole answer.* ***Teacher 2****: I take a poster and I draw eight circles which represent sandwiches, I draw four friends. Since they are four friends, each one will get two sandwiches.*



Teacher 2: *So, each learner will get two sandwiches. I will take the poster and paste at the back of the class for the learners not to forget.* ***Teacher1****:I take papers and cut out eight circles to represent sandwiches, I use stones to represent friends, then I show to the learners, that each friend will get two sandwiches, so I put two circles next to the stone* ***(Extract 24. Appendix A, P9 – P11).***

Here teachers used a variety of concrete materials to represent sharing of fractions. Teachers worked for some time to answer question 1.2b. They used different concrete materials to make sense of sharing of fractions. They used a poster to draw pictures and they also used biscuits which were provided to represent sandwiches. Similarly, in the interview teachers were asked to

explain whether they enjoyed teaching fractions and here are their responses. T1 and T 2 testified that they enjoyed teaching fractions when they could use different representations like physical objects (like paper) which were readily available. This can be seen in their testimony.

> *P8.T1: I enjoy teaching of fractions when I use different representation. P9.JI: Hmmm P10.T1: Yes, for example, I can use physical objects like papers which is readily available, I basically like to use papers so that learners can see, because they are also having papers, so, we will do it together with them. P11.JI: How are you doing it? P12.T1: I let them take out one whole paper and then move to a half, meaning that, they divide that paper into a half and then into a quarter and so on. P13.JI: Are you giving them instructions to follow? P14.T1: Yes, I give them instructions* ***(Extract 25. Appendix B1, P8 – P14)****.*

Teacher 2 also commented on the enjoyment of teaching fractions:

> ***P4.T2:*** *I enjoy teaching of fractions, because it gives you a variety of teaching methods, you know, it's too practical, and you use different representations, use physical objects like papers or even anything that they can use, even themselves, you ask them, then they can be able to say……..****P5.JI:*** *Yaaa, because fractions can come anywhere…****P6.T2:*** *Yes, yes that is the good practicality of it that is the good part, learners like working with pictures, you know they like holding, touching so that they should not forget what they have learnt* ***(Extract 26.Appendix B2, P4 – P6****).*

T2 added that the topic of fractions is practical and it is good if you use pictures, because learners like working with pictures. By holding and touching they do not forget what they have learnt. Furthermore, T3 also testified to the enjoyment of teaching fractions, saying:

> ***P6.T3:*** *Ya, …… I enjoy teaching of fractions using flash cards, where the drawings have been given to the learners so that they can see fractions in real life approach. And also it is important when you have a classroom where learners are mostly involved in working with fractions using diagrams of pictures, that has been given to them, in that case, it becomes easier to work on fractions with the learners. ……and also using technology in the classroom. For instance using power point presentation to explain basic fractions and also using videos, for the learners to see how fractions take place in reality. Using video in the classroom could very crucial and very, very critical, because learners are watching how problems are being solved and in this way learners attention will be attracted more and learners will be engaged more on the lesson* ***(Extract 27. Appendix B3, P6 – P8)****.*

Both teachers indicated that they enjoyed teaching fractions when they could use different methods and presentations, to teach fractions in more interesting ways. T1 and T2 indicated that they used physical objects like paper so that learners can see the fractions in reality. They did this

with learners in groups. T1 let them take out one whole paper and divide it into halves and then quarters and so on. T3 enjoyed teaching fractions using flash cards, where the drawings have been given to the learners so that they can see fractions in real life. T3 also mostly enjoyed teaching fractions using technology in the classroom. For instance using power point presentations to explain basic fractions and also using videos, for the learners to see how fractions take place in reality.

During the interview, teachers were asked to give their feelings about the teaching of fractions, and here are their responses:

> ***P36.T1:*** *No, I feel very good; I enjoy it very much, especially by doing it more practical, by using examples papers, flash cards, when you use flash cards learners enjoy so much. Most of the learners are more actively involved. Teaching is boring, it is better if learners are involved* ***Extract 28. Appendix B1, P36)****.*

Teacher 2 responded:

> ***P32.T2:*** *Oh, I feel good, before we started, then I told the learners that next week we will start with fractions, so learners were so excited, because that is the lesson were they are free to talk, you know to do other things, those are the topics where learners are actively involved. This topic is more practical, that is why the learners like fractions very much* ***Extract 29. Appendix B2, P32)****.*

Teacher 3 stated:

> ***P24.T3:*** *As a teacher I always have this excitement in terms of making sure that, I would want to make my learners understand. So I am happy to talk to the learners, I am happy to present this to them, I am also happy to come across some of those challenges that the learners are bringing in the classroom environment, where we are able to say look, if you have this things in particular situation in terms of fractions, how do you solve it? Or how do you approach it? So it makes me feel good. I want to make it more practical, when I am teaching my kids, I don't want to make it an oral issue, I want to make it a practical issue, where I will be able to give them examples from their houses where they are coming from, the classroom where they are sited. For example you have 20 learners in grade 5A, three of the learners are boys. To present this as a fractions, three is the part of the whole, the whole is 20 learners, so, three over twenty (* $\frac{3}{20}$ *) is the correct expression in terms of fractions. I want to make it more practical, more real to them, so it makes me feel good when teaching fractions (****Extract 30. Appendix B3, P24)****.*

All three teachers responded that they felt positive and excited when teaching fractions. Teacher 1 indicated that she enjoyed teaching fractions very much, especially when making use of practical examples such as paper and flash cards. (Appendix B2, p. 36 and Appendix B3, p. 24).

4.4.2 Developing understanding of multiple representations

In the workshop, teachers were asked to define the term 'multiple representations'. The teachers attempted to develop a suitable definition for quite some time. After a while, the researcher gave them a hint. She referred to the example where teacher 2 explained a half in different ways. The responses of the two teachers are given below.

> **Teacher 2:** *Multiple representations is when you use different presentations, we are using multiple representations to make the teaching and learning easier for the learners to understand. If you give one method, then it might be difficult for them to understand. You should have different ways, so that even those who are weak can pick from those methods. They pick which one is easier for them to answer* **Teacher 1:** *Multiple representations are different styles of teaching, which somebody can use to accommodate all the different types of learners that we have in the class.* ***(Extract 31. Appendix A, P1)***.

Again, in the semi-structured interview, the three teachers were asked to explain the term 'multiple representations'. They had been unable to define this term during the workshop, but in the interviews, the teachers drew on the workshop discussion and their teaching and were able to provide a more appropriate definition and give examples of teaching mathematics or fractions using multiple representations. Teacher 3 was not able to provide a comprehensive definition, but he did manage to give examples. The teachers, responses were as follows:

Teacher 1:

> ***P16.T1:*** *The way of using different methods of teaching, it can be any topic. It is used to understand, develop and to communicate different mathematical features of the same object. For example what I was doing in my lesson when I used one out of two and present it in different ways, where I used papers, folding a paper into half and write it on the chalkboard as symbol (*$\frac{1}{2}$*) and in words (**Extract 32. Appendix C1, P16).***

Teacher 2:

> ***P8.T2:*** *Multiple representations are the different methods of teaching. They use to understand, they develop, they create, they communicate, you know… A lot of things, different methods, you can also use technology as part of method of teaching. So, these*

*are words, symbolize and describe a lot of mathematical terms, so it is more of different methods that you are using in your teaching (**Extract 33. Appendix C2, P8).***

Teacher 3:

> ***P10.T3:** It is simple different methods for one topic. You use different approaches in terms of methods in terms of explaining a certain particular lesson. Variety of approaches to represent a single lesson, for example teaching numerators and denominators by flash cards, video or by power point representations. So, when we are using a variety of approaches in terms of presenting that particular lesson, which are multiple representations (**Extract 34. Appendix C3, P10).***

4.4.3 Developing connections.

Teachers were able to make connections among concepts and the use of concrete representations certainly led to deeper understanding. They were able to connect and coordinate mathematical ideas as expressed in language, symbols, and different representations. They worked with a variety of models that yielded multiple representations to describe fractions when dealt with different activities given in the workshop handout (Activity 1 – 4, Appendix D). Teachers were also able to compare fractions (Appendix A, p3 – workshop transcript), and to subdivide fractions (Appendix D- Activity 2- workshop hand out). They also developed the concept of fractions as numbers in a way that highlighted the fraction units and the connections between fractions and whole numbers (Appendix D part – whole diagram Activity 4 - workshop hand out). In the workshop, they were provided with different representations that enabled them to make connections among mathematical ideas. They seemed to remember these ideas and understand the mathematics that they studied during the workshop when they were presenting their lesson.

Data from the semi-structured interview showed the teachers' understanding of how multiple representations could help a child to learn fractions:

> ***P38.T1:** Yes, yes, it opens up their minds. They can be able to represent fractions into different ways into their own way. Learners make more connections, connecting mathematics to real world. When we are talking of real world, a learner might drop out of school and may end up in construction companies where they will use it to measure part of the whole or they can even apply it at home (**Extract 35. Appendix C1, P38).***

Teacher 2 responded:

> ***T2:****It increases the opportunities for learners to develop understanding of fractions, and also it helps them to connect some mathematical ideas using different tools and teaching methods, you know critical thinking is also comes in once they are given something, they have to think for them to come up right, that is the good part. If you give them a practical work, they won't even just do it, they think how should I do it and for me to come right, so at the end what they have done by their hands they will not forget, than what you have written on the chalkboard, so that is the… It increases the higher interaction among learners, you know they will be talking to each other, it is more of practical, yes like what they were doing in the lesson, and you know it is more of group work* ***(Extract 36. Appendix C2, P35-40).***

T3 also responded to the question of whether he thought that multiple representations might help a child to learn fractions:

> ***(P30.)T3:*** *It also helps them to connect mathematical ideas using different tools when we are teaching fractions.****T3:*** *It opens their minds; it does not bind them to one way of using it. It brings them back into reality of doing things like what I said earlier, it tells them that, this is what we are learning in the classroom, that this are not just things that we are talking, but it is something that you are on a day to day approaching or have approached in your real life (****Extract 37. Appendix C3, P30).***

The teachers commented positively on mathematical connections. They stated that when learners represented fractions in different ways and related these to their prior knowledge, they made more connections, connecting mathematics to the real world. Teacher 1 explained the 'real world' as follows: "A learner might drop out of school and might end up in construction companies where they will use it to measure part of the whole. (Appendix C1).

Furthermore, teachers 2 and 3 explained that multiple representations help learners to connect mathematical ideas using different tools and teaching methods. They make connections in such a way that each one in the class can be helped. Learners can be given more examples at home, so that even outside the class fractions are used. This broadens their minds. Moreover, teacher 3 added that you can give examples like: "You have 20 learners in grade 5A; three of the learners are boys. To present this using fractions, three is the part of the whole, the whole is 20 learners, so, three over twenty ($\frac{3}{20}$) is the correct expression in terms of fractions. (Appendix C3). He said that he wanted to make fractions more practical, more real to the learners and so it made him feel good when teaching fractions. Teachers were able to explain mathematical connections but they

were not able to connect mathematical ideas to make the concept of connections clear in their lessons.

In the lesson observations, few mathematical connections were observed, but T1 and T2 did make some mathematical connections in their classes. T1 used a fraction poster to match with the different flash cards which are made up of pictures, numbers and words.

> *"I want you to come and recognize this part, remember we are looking at the equivalent fractions. Fractions which have the same meaning. The teacher put the poster on the chalkboard, with many flash cards on the box. Learners match the card with the fractional poster on the chalkboard. Learners match fraction with the correct shape in this poster and paste (paste activity). Learners recognize or identify the card which is equivalent to what is on the poster" (**Extract 38. Appendix B1, L18).***

4.4.4 Teachers planning and preparation

During the pre-lesson observation, teacher 1 used paper and the chalkboard to introduce her topic of fractions to the grade 5 learners. The lesson for teacher 1 was planned well, although she was not able to finish explaining the concept of fractions to the learners. Teacher 2 used technology; she introduced her topic of fractions using power point presentations. The lesson was prepared well and she was able to finish her lesson on time. Teacher 3 used the chalkboard only when he explained fractions to the learners and the lesson finished on time. It took only 15 minutes to present this lesson. During the second lesson observation (after the workshop), teachers 1 and 2 used a variety of representations to teach equivalent fractions to the learners and they were not able to finish what was prepared within the two lessons. Teacher 3 used technology (power point presentations) to present his lesson and he was able to finish his lesson on time.

Teachers developed their lesson plans effectively to involve a variety of teaching methods and multiple representations. They complained that the time allocated was insufficient to cover all the presentations prepared for that lesson. The data presented below shows how teachers planned their lessons despite the time constraints. They planned their lessons to include more than one representation. During the semi-structured interview, they were asked how they managed to use multiple representations in their teaching. Teacher 1 commented that:

> ***P20.T1:** By planning my lesson effectively, so that I can use different methods in my teaching. On the other hand, time is not enough and the syllabus contains too much*

*content, it forces teachers to teach theoretically and to focus on one type of representation (**Extract 39. Appendix C1, P20).***

Teacher 2 responded:

> ***T2:** Learning plays a very important, so it gives you more time to plan your lesson, so that it must be effective. On the other hand, time is also limited to use the different methods but if the planning is done, you can be able to cover even fewer methods, if not on that day, you can still continue the following day to cover all the methods, because what you are looking at is the understanding of the learners. Once the learners catch up then you know it is easier for them to work on it. In addition to this, the syllabus contains too much content which force teachers to teach using one method only, because the time is limited and you are required to finish that syllabus at the end of the year (**Extract 40. Appendix C2, P10).***

Teacher 2 also responded:

> ***P45.JI:** But, why do you think teachers are not using them? **P46.T2:** Teachers are just lazy to take time to plan; they are only going in the class as long as they taught, so long as the period is over. Teachers are just lazy to sit and plan, because you ask them, they will always say I do not have time but they were supposed to have time to do their planning for the understanding of all the teaching that you have in the class (**Extract 41. Appendix C2, P 46).***

Teacher 3 responded:

> ***P11.T3:** I normally when I am presenting a lesson, I have to make sure; I get different materials from various sources. Like from the internet for instance you download pictures that has been once used and inducted to make teaching easier, in that way you can be able to manage it.**P13.JI:** How do you manage to use a variety of resources if the time is limited to 40 minutes?**P14.T3:** You do not need to do everything at the same time, you need to focus on the part which you think, and learners will be able to catch up on that particular time. You need to remember as well, that learners have to do activities based on that particular approach. If you have presented the two approaches within that particular lesson, I would say with activities, we need to do practical in that approach with the learners. The learners should not only watch the video, but they should also be able to reflect on what they have watched. I think it is also important, if you want to give more work, it is important to organize extra classes, so that you can present more methods that is also good because of the limited time that we have (**Extract 42. Appendix C3, P11-P14).***

In the lessons observed, T1 and T2 did not complete the planned lessons in the time allowed, although, they had a double lesson. They planned their lesson well to include different types of representations, but they were not able to finish. Here is the evidence for teacher 1:

> *Learners continue to do it. Teacher says let us do it fast. Teacher supposed to give another activity, a printable fraction as a handout about equivalent fractions and comparing of fractions, because of time the teacher could not be able to finish the lesson as planned. She gave it as homework (**Extract 43. Appendix B1, L 14).***

Here is the evidence for teacher 2:

> *Teacher: Hurry up, hurry up, just draw a line, a line. Learners were all quiet for some time. After some time teacher asked the learners to explain their answers. Learners explain their answers. One learner: I draw a line horizontally, vertically and diagonally (twice) from right to left and the other one from left to right. Teacher: The first question is done, who can read the second one? One learner: Show four different ways of cutting up a square into four equal parts (quarters). The teacher was busy with this worksheet and she could not finish her lesson (**Extract 44. Appendix B2, L18-L19).***

T3 finished his lesson on time, because he only used one type of representation (power point diagrams) and he did not involve the learners in the same way that T1 and T2 did.

The commonality in the data is that all the teachers said that they managed to use multiple representations in their teaching by planning their lessons effectively. T2 and T3 commented that planning helped them to look for an easier method which learners would not forget and which made their teaching easier however both teachers T1 and T2 complained that the time was too short and the syllabus contained too much content. But T2 and T3 added that although time is limited, planning is the important thing, so that you would be able to cover even more methods. If you did not complete them on one day, you could continue the following day because what you are looking for is the understanding of the learners. In the lesson observation, it was seen that T1 and T2 did not finish their lessons on time and what remained to be done was given as homework. While T3 finished on time, because he only used power point presentations. He did not actively involve the learners in his lesson. On the other hand, T1 and T2 involved their learners throughout the two lessons.

In addition, teacher 2 indicated that planning is very important to make the teaching more practical. Once you make it practical, it opens the minds of the learners. Teacher 3 indicated that teachers are lazy to take time to plan. Teacher 2 further commented that teachers always have an excuse, saying that they do not have time to do their planning. Teacher 3 indicated that proper planning, carried out in a strategic way will lead to effective learning and will reduce problems

in the class. He indicated that he has to make sure that he gets different materials from various sources. “Like from the internet – for instance you can download pictures and you can use them to make teaching easier” (Appendix C3). In addition, teacher 3 indicated that “teachers do not need to do everything at the same time, you need to focus on the part which you think is important, and learners will be able to catch up easily. You need to remember as well that learners have to do activities based on the particular approach chosen” (Appendix C3).

“If you have presented two approaches within that particular lesson, I would say with activities, we need to do practical work in that approach with the learners. The learners should not only watch the video, but they should also be able to reflect on what they have watched” (Appendix C3).Teacher 3 further suggested that because of the limited time, it is important to organize extra classes, so that you can present more methods. All teachers indicated that effective planning should be done, to cover more than one method and to give time to the teacher to come up with possible representations which will be used in the lesson to make it easy for the learners to understand what you are teaching.

During the semi-structured interview, teachers were asked about the important choices that they made when they were planning their lessons. They were also asked if there was something that they would like to change in their teaching practice. Teacher 1 testified that:

> *… I would like to change in the way I plan my lesson and I have to ensure that more than two presentations are used in one lesson* ***(Extract 45. Appendix C1, P45 – P52).***

Similarly, teacher 2 in her testimony stated that:

> ***P55 – P56).P56.T2:*** *The teaching, it is only the planning, I do not know whether we have more sources of teaching we should have, if I have to change it is the way we teachers are planning, for them to have this lessons implemented, we should also make sure that more than two presentations are being used in every, not only in fractions but in every teaching you should have more than two methods to use just to explain and to widen the knowledge of the learners (****Extract 46. AppendixC2, P55 – P56).***

Teacher 3:

> ***P42.T3:*** *I think, the most important thing, like what I have said earlier, I would want to do more planning in terms of the one approach (traditional approach). I would like to do more planning, read more resources, find more ways on how others or even the globe in*

*the whole world have approach the teaching of fractions, that is something I would be willing to gain knowledge on (**Extract 47. Appendix C3, P43).***

All teachers indicated that they wanted to change the way they plan their lessons, to ensure that they included more than two representations in their teaching for the understanding of the learners and to broaden the knowledge of the learners. Teacher 3 added that he wanted to move away from the traditional approach of using one type of tool to using more tools. Teacher 3 made a choice in his planning to include an assessment tool like a worksheet "to make sure that we reflect and would be able to find out to what extent, what impact did my teaching have on the children."

Teachers were also asked how the focus group workshop contributed to their teaching of fractions. Teachers responded that:

P42.T1:** It was good, it opens up my mind in the use of multiple representations, when using different tools when teaching fractions rather than focusing on one. It opens my mind to design my own multiple representations, using physical objects that are affordable (**Extract 48. Appendix C1, P42).

Teacher 2 testified that:

P48.T2:** It was good, because we teachers were more practical, we have learnt a lot of drawings. We have time even to divide things, you know, we were working as if we are learners. It was more interesting, a lot of teaching aids were there meaning, you can have a lot of aids that you can use but at the end of the day you mean one thing. It means that, the more, you use it, the more you understand, so it was really helpful (**Extract 49. Appendix C2, P48).

Teacher 3 also testified to the workshop contribution in his teaching of fraction:

P38.T3:** It was interesting workshop, although I came late but I would to thank you for explaining everything to me. I have learnt different ways in terms of how others are approaching their teaching, it was very crucial. It opens up my minds, so that I can be able to use more other multiple representations into my teaching to make the learning of my children effective, so it was an eye- open up for that matter. It contributed to my development as a teacher in that particular topic of fractions (**Extract 50. Appendix C3, P38).

All teachers indicated the workshop opened up their minds to the use of multiple representations. "It was interesting, a lot of concrete materials were used to mean the same thing, meaning that,

the more you use it, the more you understand" (Appendix C2). It contributed to their understanding of fractions. They stated that more workshops should be given in all the topics of mathematics so that teachers can discuss the possible tools that they can use to make the understanding of mathematics more accessible to the learners.

4.4.5 Teachers used multiple representations and modes for improved engagement

Teachers used different modes of representation to improve learner engagement in the learning activities. Modes included pictorial representations; concrete manipulatives like dice; real objects like papers, flash cards and worksheets; video and power point representations; written mathematical symbols; descriptive written words; oral representations; experience based (real world) problems; and school word problems. During the interview, teachers were asked how they thought multiple representations contributed to the teaching of fractions. Teacher 1 responded:

> ***P24.T1:*** *It helps learners to learn from one another.****P25.JI:*** *You mean when the learners are actively involved, they learn more from one another than from a teacher?****P26.T1****:Yes, because sometimes are afraid to ask the teacher, that is why sometimes, when I am teaching I usually call in one the child to come and solve some problems and explain and you can really see some learners pay more attention when another learner explains* (***Extract 51. Appendix C1, P24-26).***

Teacher 2 stated:

> ***P16.T2:*** *It increases the opportunities for learners to develop understanding of fractions, and also it helps them to connect some mathematical ideas using different tools and teaching methods, you know critical thinking is also comes in once they are given something, they have to think for them to come up right, that is the good part. If you give them a practical work, they won't even just do it, they think how should I do it and for me to come right, so at the end what they have done by their hands they will not forget, than what you have written on the chalkboard, so that is the….. It increases the higher interaction among learners, you know they will be talking to each other, it is more of practical, yes like what they were doing in the lesson, and you know it is more of group work.*
>
> ***P17.JI:*** *Yes, like what they doing in the lesson…* (***Extract 52. Appendix C2, P16-17).***

In the interview, both teachers T1 and T2 indicated that multiple representations increased the interactions among learners and helped learners to learn from one another. Teacher 1 commented that sometimes learners were afraid to ask the teacher questions. That is why when teaching she usually called in one the child to come and solve some problems and explain to the others and

"you can really see some learners pay more attention when another learner explains". Teacher 2 indicated that when learners talk to one another in a group, they are highly engaged in a lesson and they will talk and help each other and they will understand more easily.

Furthermore, all teachers indicated that technology played a role in learners' engagement. In this study, the use of technology in their teaching of fractions appears to have a positive impact on the teaching and learning of fractions for teachers 2 and 3. The videos and power point representations used, allowed the teacher to introduce a wider range of representations that included written math symbols, descriptive written words, pictorial representations, manipulations, real objects, oral representations; experienced based problems and worksheet based lessons. The use of videos and power point presentations appears to have increased learners engagement by providing a resource that promoted interactivity, immediate feedback, challenge and fun. Teacher 1 did not use a video, because the school does not have a machine. Here are the responses of teacher 2 and 3 on the use of technology when teaching fractions.

> ***P42.T2:*** *You know video, kids watch what is going on, and you can even see the attention, learners will listen very carefully because they know afterwards there will be questions, so they enjoy. There was a song in there, they saw how the person was cutting and so on, so they will never forget, once you ask them questions, they will always remember that this question they were doing it when they were cutting and so on, so video is an excellent tool when teaching fractions, because learners are really actively involved (**Extract 53 .Appendix C2, P42).***

Teacher 3 stated:

> *I use technology in the classroom. For instance using power point presentation to explain basic fractions and also using videos, for the learners to see how fractions take place in reality. so it proves that with videos and power point representations, we do really get the impact, and learners are actively engaged in the lesson. Other representations do not give you the real impact that you want. I have decided; let me try video to see because we now normally, learners enjoy watching things in reality, why can't we use them so that we bring the reality to them. That is actually the simple reason why I opted to use videos (**Extract 54. Appendix C3, P8).***

Teacher 1 did not use a video, but in her testimony on the use of technology in the teaching of fractions stated:

> ***P51.JI:*** *Why did you use flashcards and papers but not video?* ***P52.T1:*** *As I said earlier papers and flash cards are readily available but not video, the school does not have*

> *video. I wish the school could have videos so that learners can see how fractional problems are solved in reality and in this way, it encourages the learners to listen attentively and to imitate what was said in the video (**Extract 55. Appendix C1, P51- 52).***

Teacher 2 and 3 indicated that they used videos and power point presentations to present their lessons as it helped learners to engage with the lesson, since they were looking at pictures of real objects. Teacher 2 and 3 indicated that learners see how fractions take place in reality, when using videos and power point presentations. Teacher 2 and 3 believed that learners' engagement with mathematics had improved, with learners displaying increased enthusiasm and higher levels of participation during mathematics lessons. Teacher 2 indicated that learners saw how the person was cutting a whole shape into smaller parts to indicate different fractions that can come out of the whole." In using technology, learners will never forget, once you ask those questions, they will always remember that this question they were doing it when they were cutting and so on". So teacher 2 indicated that video is an excellent tool when teaching fractions, because learners' attention is focused.

Teacher 2 indicated that learners saw in a video how papers and cakes were cut into halves and quarters. And after that she presented the lesson with a second video and the learners listened carefully. After the second video paper was given to the learners to solve some of the problems. They did not have any difficulty explaining them or doing them practically using paper, because they understood the process when they watched the video. The video and power point presentations attracted the interest of the learners: they were quiet and at the end of the lesson, "they were now singing with the people who were singing in the video" (Appendix C2). The purpose of making the presentations was to help learners understand. There was no evidence that learners understood what was taught by the teachers, although they did show an interest in watching the video.

Teacher 3 indicated that in the video which he used there were pictures of flamingos in a certain area, there were five flamingos on one area that was shown in picture and two left the area. The question was how many flamingos are left from a group that was there? So, two went, three were left. In terms of fractions the learners could easily say that is two out of five ($\frac{2}{5}$) left, the area where they were. Using the fractional video in the classroom resulted in learners watching how

problems were solved. In this way the video attracted their attention and learners were engaged in the lesson.

Teacher 1 indicated that although their school does not have video she supported the use of videos in mathematics. It helped learners to see how fractional problems are solved in reality and in this way it encourages the learners to listen attentively and to imitate what was said in the video. The three mathematics teachers encountered the problem of boredom in their mathematics classroom if it was not taught effectively. All three teachers mentioned boredom during the interview at different times when they were answering. Teacher 1 in her response stated:

> *I make it more practical so that most of the learners can be actively involved. Teaching is boring, it is better if learners are involved (**Extract 56 Appendix C1, P35-36).***

Teacher 2 testified on boredom that

> ***P60.JI:** You know, learners are scared sometimes to ask the teacher…..**P61.T2:** Yes, yes, the other thing is that, we are ignoring learner centered, but if we make 90% of the lesson is just learners talking and explaining and then you find that this learners should take control of the lesson and the lesson won't be boring because learners are involved (**Extract 57 Appendix C2, P60-61).***

Similarly, teacher 3 added that:

> *I want to make it more practical, when I am teaching my kids, I don't want to make it an oral issue, and otherwise it will be boring to them. I want to make it a practical issue, where I will be able to give them examples from their houses where they are coming from, the classroom where they are seated (**Extract 58 Appendix C3, P24).***

All three teachers indicated that teacher talk is tedious so learners need to be involved in the lesson. Teacher 2 indicated that learners should be involved throughout the lesson and they must be given a chance to talk and explain to others so the concept of centeredness is not ignored. On the other hand, teacher 3 indicated that fractions should be taught in a practical way, he did not want to make it an oral issue; otherwise it would be boring to the learners.

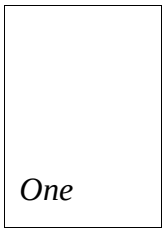
In the lesson observations, it was seen that T1 and T2 used many representations in their lessons to engage learners in their lesson. T1 used papers to engage learners in the lesson. T1 and T2 arranged the learners in groups and in pairs and learners were actively involved. In the lessons of T2 and T3, learners were working independently. Although learners worked independently in the lesson of T3, he did not give the learners an opportunity to discuss their answers, even though they had enough time to do the individual activity.

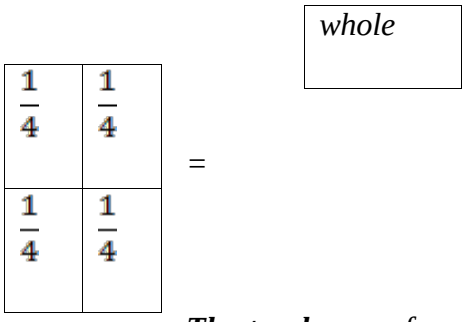
When making sense of ideas, students were given opportunities to work both independently and collaboratively. At times they were given a chance to think and work quietly, away from the demands of the whole class and to be in pairs or small groups so that they could share ideas and learn with and from others. At other times they were given a chance to be active participants in a purposeful, whole-class discussion, where they clarified their understanding and were exposed to broader interpretations of the mathematical ideas that were the focus. The example below shows how teacher 1 used paper to engage the learners in the learning of fractions.

> *The teacher divides the learners in pairs and gives them A4 papers. The teacher explains, I divide this paper into a half (folds it into a half) and I cut it into half. (Teacher takes a paper and cuts it into two equal parts) if you put two halves together, they form a whole. Then we go ahead, this paper is a whole, then we divided it into half and now I want us to divide it again into another half and open it. How many parts can you see here?*
> ***Learners:*** *4 parts.*
> ***Teacher:*** *Let me write it on the chalkboard:*

$\frac{1}{4}$	$\frac{1}{4}$
$\frac{1}{4}$	$\frac{1}{4}$

> ***The teacher*** *shows four parts on the chalkboard,* $\frac{1}{4}$ *is called one quarter. If you take the four quarters and put it on the whole A4 paper on the chalkboard, will they look the same or not? Learners: the same. Teachers repeats, they are the same.*

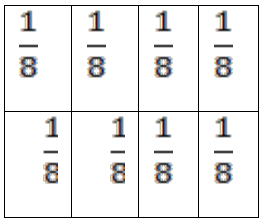




The teacher: *so four quarters will give you the same as a one whole. If you take two quarters and put them together, they will give you a half and it can be expressed as shown on the paper.*

The teacher: *A half is equal to two quarters, they are the same. Teacher: Let's go ahead, still you take one full paper, then you divide it three times and then you open it. How many parts you see here? Learners: They are eight.*

The teacher shows her paper to the learners and counts the eight parts that the paper is divided onto.



Then you say, each part is $\frac{1}{8}$, $\frac{1}{8}$, *... saying together with the learners while pointing on the chalkboard* ***(Extract 59. Appendix B1, L10- L12).***

Learners used paper to demonstrate different fractional problems in groups with the teacher. Similarly, teacher 2 also used paper and video clips to engage her learners in her lesson.

Teacher 2 also used paper like teacher 1 to engage the learners in a lesson after they watched the video.

Teacher: *Now I want you to show some parts of the whole on these papers, you can make a quarter on a square, where else can you make a quarter? Some learners shouting the answer, on a circle, where else …. and a half?* ***Learner 6:*** *on a triangle.* ***Teacher:*** *On a triangle? You can make a half on a triangle? Is it?* ***Learners:*** *yes. Let us use these papers and see. Let see…*

Teacher gives papers to the learners and instructs them to fold them into half into different shapes. ***Teacher:*** *If you are seated two, two, and then each one must have a different shape, but each shape must be equal to a half. Teacher says, I want you to make a half using this papers, I want to see how you can get a half on a different shape. Teacher said you just make a half. Teacher said right, I did not say make other fractions. Ok, open it, somebody should explain your shape using your paper?* ***One learner:*** *My shape is divided into two equal parts.* ***Teacher:*** *You can fold a paper vertically, horizontally and diagonally, but this paper is not a square, is a rectangle, you cannot fold it diagonally. The teacher showing the learners using her paper, these corners do not touch each other. Teacher instructs the learners to use their paper to make a quarter. And I want to see four parts and I want you to cut one part from that 4 parts: Hold your other three parts and one part, with the part that you have cut. Teacher asks what fraction is it?*

One Learner*: One over four. Teacher: And the fraction for the uncut? Three over four. Teacher says, I want to see all four parts being cut, how many parts do you have?*

One learner: four parts. ***Teacher:*** *Yes, this four parts came from one whole, is what we say sharing. If they were four learners and you cu that bread into four equal parts, each one is going to get what?* ***Learners:*** *one piece.* ***Teacher:*** *Which is what?* ***One learner:*** *one – fourth. Teacher: Yes, one fourth or a quarter. Ok. (****Extract 61. Appendix B2, L12-L16).***

4.4.6 Teachers used multiple representations to teach in ways that improved learners' conception of fractions

Teachers used different representations to improve learners' conceptual understanding of fractions. In the interview, teachers were asked why they used a range of examples when they were teaching fractions. The teachers responded like this.

P54.T1: *For them to understand very well, for them to pick up something to make a difference that a half can be represented in words, symbols and pictures and for them to compare e.g.* $\frac{1}{2}$ *and* $\frac{1}{4}$*, theoretically, learners will say* $\frac{1}{4}$ *is bigger than* $\frac{1}{2}$*, because 4 is bigger than 2, but if you show them practically, they would be able to know which one is bigger (****Extract 62. Appendix C1, P54).***

Teacher 2 responded

P64.T2: *Looking at this understanding, you know not everyone who was given those papers got everything correct, you know, other they got it wrong, but you know they learned from their fellow mates, so now, that is why given a lot of examples to do, activities to do and at the end you will find out that all the learners are on the right track. If you give learners questions to try they will get it and some will volunteer to come and*

*explain on the chalkboard, and some will say ooh, this is how he did his answer/ his pattern, next time they should be able to try and get it right (**Extract 63. Appendix C2, P64).***

Teacher 3:

P45. JI:** I observed that when you were teaching fractions, you used power point presentation and you also gave worksheets to the learners to work on, why did you do that? **P46.T3:** It is a culture, it is a norm. It is not good to keep on preaching things and you did not find out, to what extent do they make impact on the learners, that is the reason why I gave them worksheets to check whether the new method that I have decided to include in that particular lesson, the use of power point presentation has worked and to see whether it is helpful for the learners to understand the learning of fractions, that is the reason why I gave them assessment worksheets (**Extract 64. Appendix C3, 45-46).

In the interview, teachers were asked to describe how multiple representations contributed to the learning of fractions. Here are the responses of the teachers:

P24.T1:** It increases opportunities for the learners to develop understanding of fractions. It helps students to connect mathematical ideas using different tools when teaching fractions. It also increases the high interactions among learners. Lastly, it helps learners to learn from one another **(Extract 65. Appendix C1, p. 23-24).

Teacher 2 responded:

P16.T2:** It increases the opportunities for learners to develop understanding of fractions, and also it helps them to connect some mathematical ideas using different tools and teaching methods, you know critical thinking is also comes in once they are given something, they have to think for them to come up right, that is the good part. If you give them a practical work, they won't even just do it, they think how should I do it and for me to come right, so at the end what they have done by their hands they will not forget, than what you have written on the chalkboard, so that is the….. It increases the higher interaction among learners, you know they will be talking to each other, it is more of practical, yes like what they were doing in the lesson, and you know it is more of group work (**Extract 66. Appendix C2, .16).

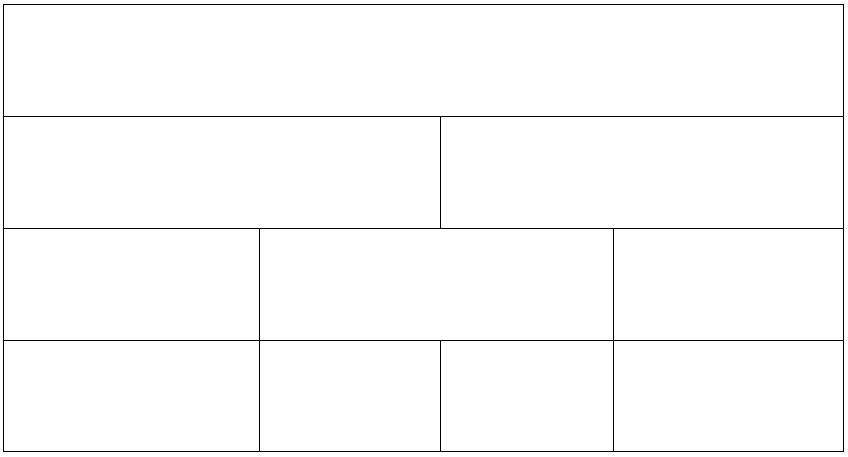
Teacher 3:

***P15.JI:** Okay, How do you think multiple representations contribute to the learning of fractions? Is it contributing? **P16.T3:** Exactly, Firstly, it increases the interactions in the class, learners are more involved, especially when they have watched a video. In normal cases when the learners do not like some of the classes, but if they are watching like my approach of using videos, when they watch, they are more easily controlled and they can be able to interact thereafter when they are done watching the video. It is also help them*

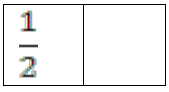
to connect mathematical ideas using different tools when we are teaching fractions ***(Extract 67. Appendix C3, 15-16).***

In the lessons observed, teachers used multiple representations when teaching fractions but it was not proven whether it enhanced the understanding of the learners. Teacher 1 in her teaching used fractional posters and flashcards for the learners to think and match the correct fractional cards to the correct place at fractional poster. T1 also gave a worksheet on fractional thinking problems as homework. Here the learners needed to find different ways to divide shapes and to determine which shapes are divided into fractions. This worksheet was discussed in the teachers' workshop (Appendix A, Activity2). The lesson proceeded as follows:

Teacher: *Paste the poster on the chalkboard. We start form the top one, you see the first line it represent what? Here is the poster:*



Choose one card and stick to the poster. ***Learner 1:*** *Sticking card number 1, which shows a whole number on the first line.* ***Teacher:*** *Is learner 1 correct.* ***All learners:*** *Yes. The teacher: good, we go to the next one, yes, one learner? Learner 2 sticks the flash card on the second line:*



The teacher: *You are correct. Teacher asks any card equivalent to the second line? There are many cards that are equivalent, but they are written differently but mean the same. Learner 3 sticking card which is written one over four on line 2. Teacher says is not correct, you don't put that card there because that line is divided into two parts only which is a half, check nicely. Teacher explains. You count how many parts are there?, one, two and so on. It was first a whole, now the whole is divided into two parts.* ***Teacher:*** *Yes, yes, quiet, quiet, lets continue with another person? This girl is coming.*

The learner search for a correct card. Teacher says count the parts, look at that set and compare it to the one on the chalkboard, remember you consider also if it was written in words, consider the cards in words or in drawing.
Learner placing a card at a correct line Learners clapping hands. ***Teacher:*** *How many parts are there? All learners: three parts.* ***Teacher:*** *Well; let's go ahead; there are still some cards here that belong to this poster or to any line in the poster. Learners coming one by one to stick the cards at the correct places.* ***Teacher:*** *Do not to tell each other's answers (****Extract 68 Appendix B1, L19-L23****).*

Teacher 2 also used the fraction thinking problem which was used by teacher 1 as homework. Teacher 2 gave it as individual work. Learners answered this worksheet individually and the teacher instructed the learners not to copy other learners' work. Here is the observation.

Teacher: *Ok good; let me give you the exercise (worksheets handed out to the learners). Learners do the exercise individually. Student must find different ways to divide shapes, determine which shapes are divided into fractions.* ***Teacher:*** *Let us look at the first question.* ***Teacher reads:*** *Show four different ways of cutting up a square into 2 equal parts (halves). All I need is half; you need four different ways on how to cut a square into two halves. Hurry up, hurry up, and just draw a line, a line. Learners were all quiet for some time. Then the teacher asked the learners to explain their answers.* ***One learner:*** *I draw a line horizontally, vertically and diagonally (twice) from right to left and the other one from left to right.…… (****Extract 69, Appendix B2, L16-17****).*

4.5 CONCLUSION

This chapter presented the findings of the study. Vignettes of the teacher's workshop and the observed lessons were also provided. The data collected through the workshop, observed lessons and the individual interview data was analyzed and presented. I explored teachers work with the additional tools of multiple representations in order to improve the teaching of fractions to grade 5 – 7 learners. Themes merged from the data analysis that provided insight into the mathematics teachers' experiences when working with the additional tool of multiple representations, to make the teaching of fractions easier. I also explored how teachers worked to change their teaching of fractions by using multiple representations for conceptual understanding. The following chapter provides the discussion of the research findings in terms of the themes emerged from the data analysis in relation to the reviewed literature.

CHAPTER 5

DISCUSSION

5.1 INTRODUCTION

This study investigated teachers' experiences of using multiple representations to teach fractions for conceptual understanding in grades 5 – 7. First, it examined their experiences in working to develop the use of multiple representations to teach fractions. Second, it investigated how the teachers worked to change their practice to develop their use of multiple representations to teach fractions for conceptual understanding. The participants were three Grades 5 – 7 mathematics teachers.

5.2 DISCUSSION

This chapter discusses the major findings that address the two sub-questions of this study. It compares these findings with the findings of others who have conducted similar research on this topic. The issues emerging from the data will be discussed further in relation to the literature reviewed in Chapter two.

5.2.1 Introducing and teaching fractions in meaningful ways

Research has found that effective tasks for the introduction of fraction concepts, are sharing problems in which there is a remainder which can be divided. These sharing situations elicit the informal knowledge that the children bring to the learning situation and can be used successfully for introducing fractions (Mack, 1990; Empson, 1995; Murray, Olivier & Human, 1996). De Beer & Newstead (1998) state that even grade 1 learners have the ability to make sense of these fraction problems, even if their lack of symbolic knowledge prevents them from producing the correct fraction name or symbol. The results of this study showed that the two teachers demonstrated different ways of introducing the topic of fractions to the learners. Teachers presented the opportunity where learners should be exposed to a wide variety of situations such as folding and cutting of paper. Teachers solved problems involving sharing situations during the focus group workshop and learners solved similar problems as class activities.

Research by Hiebert and Wearne (1991) concludes that the use of concrete materials enhances learners' understanding of decimal fractions. On the other hand, Thompson (1994) indicated that teachers have to be careful when using concrete materials. He said materials may be concrete, but the idea you intend the learners to see is not merely in the materials. It is very important to help learners understand what they are learning by using those materials. Although the results from the focus group workshop showed that teachers worked with different concrete materials to solve fractional activities, it was not proven that they understood the type of materials that they used and the reasons why they used them. The two teachers who were involved in the workshop used different concrete materials to present their fractional lessons, but it could not be proved that learners understood why they used such types of materials.

The Mathematics Science Research Institute (MSRI, 2004) and Kilpatrick et al. (2001) say that in teacher training institutions, teachers cannot learn all they need to know about the mathematics they will teach and how to teach it effectively. Hence, teachers need a basis for ongoing learning so that they can adapt to the changes of the curriculum. Teachers can continue to learn by participating in various forms of professional development and mathematicians need to help them to develop an understanding of how to teach mathematics using a variety of approaches. In this study evidence shows that by working with the concrete materials that were provided, the teachers appeared to make good sense of the fractional problems given in the workshop. The analysis showed that in the focus group workshop, teachers engaged positively in using the different concrete materials to solve the problems and were be able to generate meaningful answers.

Moreover, the Commonwealth of Learning (2001); Lewis, Perry, Friedkin & Baker (2010); Askew (2012); Boaler (2009) and Van de Walle et al. (2009), suggest that the use of a variety of representational models are crucial in the teaching of fractions, not only to introduce concepts but as a means of clarifying ideas that may appear confusing to pupils in symbolic form. It is often valuable to repeat the same activity using different models.

Teachers added that video can also be used to introduce the topic of fractions to the learners, because learners can see real objects and the video already has different representations. The

research done by Mildenhall, Swan, Northcote & Marshall (2008) shows similar results regarding the use of technology in the classroom. These researchers concluded that technology promotes student interest, more sustained concentration, and more effective learning.

The data analysis in this study showed that fractions may be easier to teach (and teachers may enjoy the teaching of fractions more) when different methods for teaching and different presentations of fractions are used. But this does not imply that fractions are easy to teach. Indeed, there is broad agreement in the literature that the topic of fractions is difficult to understand, as well as to teach (Ma, 1999), and that many learners find this topic difficult to learn. On the other hand, Tzur (1999) in his study suggests that it is important to integrate research on fraction teaching and on fraction learning if one is to suggest sensible ways to improve the teaching and learning of fractions.

5.2.2 Developing understanding of multiple representations

Edgardo (2001) describes multiple representations as including pictures, diagrams, sketches, tables, written symbols, oral representation and the use of manipulatives. Ainsworth, Billy and Wood (2002) confirm that multiple representations in technology include calculators, computers, graphing, games and simulation. Ozgun-Koca (1998) defines multiple representations as "external mathematical embodiments of ideas and concepts to provide the same information in more than one form" (p. 1). In this study teachers were unable to define the term multiple representations when they were asked during the workshop, but during the focus group workshop they were provided with a definition and many examples of multiple representations. Following this the teachers were able to define what a multiple representation is and give examples of teaching mathematics or fractions using multiple representations. T3 was not able to provide a definition. In summary, the workshop appeared to have influenced teachers to define multiple representations during the interviews. They defined multiple representations as different methods of teaching which can be presented in a particular lesson which can be used to understand, develop, create, and communicate different mathematical features of the same objects.

5.2.3 Developing connections

Teachers used concrete materials to solve fraction activities which enabled them to make connections among mathematical ideas to the real world. Teachers worked together to solve some of the mathematics activities which were provided during the workshop. The workshop appeared to positively influence the teachers to use more than one representation when they presented their lessons. This was observed during the lesson. Teachers worked with a variety of models to apply multiple representations to describe fractions (halves, thirds, fourths) in real world situations in their lessons. The literature supports the idea that teachers should emphasize the connection between real-world problems and the fraction notation used to represent the problem, because children are better able to solve fraction arithmetic problems when they are presented in meaningful, real-world contexts (Irwin, 2001; Fazio & Siegler, 2011). Moreover, this idea is also supported by Kilpatrick et al. (2001) who state that the use of concrete materials for teaching mathematics is crucial. Learners require time to build meaning and make connections.

Teachers were also able to compare and subdivide fractions. They developed the concept of fractions as numbers if used in the way that highlights the unit and the connections between fractions and whole numbers (Appendix A). Therefore this study provides observed data that supports the research findings reported by Businskas (2008) that the problem solving activity of a group of pre-service teachers is a process of making connections. He found that pre-service teachers were engaged in making a variety of connections.

All three teachers commented positively on mathematical connections. In the interviews teachers stated that when learners represent fractions in their own way, they make more connections, connecting mathematics to the real world. Teacher 1 explains the real world as; a learner might drop out of school and might end up in construction companies where they will use it to measure part of the whole. Furthermore, teacher 2 and teacher 3 explained that multiple representations help learners to connect some mathematical ideas using different tools and teaching methods. They make connections in such a way that learners relate fractional problems into different representations which mean the same. Learners can be given more examples at home, even outside the class. This evidence is supported by the CAPS document (South Africa. DoBE, 2011)

that learners should work with relationships and convert flexibly between variables in terms of numerical, graphical, verbal and symbolic representations. They should be able to communicate appropriately by using descriptions in words, graphs, symbols, tables and diagrams. This suggests that learners should be assisted in establishing links or connections between and within these multiple representations. The evidence from the data analysis revealed that T1 in her teaching used a fractional poster to match with flash cards which consist of pictures, numbers and words and T2 used videos and learners related what they watched in video to the practical work with paper provided by their teacher.

5.2.4 Teachers planning and preparation

All three teachers indicated that effective lesson planning should be done and should cover more than one representation. This would allow the teacher to come up with possible representations which could be used in the lesson to make it easy for the learners to understand what is being taught. Teachers 1 and 2 indicated that because of the length of the syllabus, teachers are forced to teach theoretically and to finish within a limited period of time.

Both teachers indicated that they wanted to change the way they currently plan their lessons, to ensure that they include more than two representations in their teaching in order to improve the understanding and deepen the knowledge of the learners.

Teacher 3 added that he wanted to move away from the traditional approach of using one type of tool to more tools. Teacher 3 has made a choice in his planning to include an assessment tool like a worksheet to make sure that he can determine what impact his teaching had on the children

Both teachers 1 and 2 indicated that the workshop opened up their minds to the use of multiple representations. It was interesting, a lot of concrete materials were used to mean the same thing, meaning that the more you use different concrete materials, the more you understand. It contributed to their understanding of fractions. They stated that more workshops should be given in all the topics of mathematics so that teachers can discuss the possible tools that they could use to make the understanding of mathematics more accessible to the learners. This evidence from the data analysis is supported by the previous research that professional development

opportunities should be given to the teachers to cultivate and expand their knowledge. Teachers could be asked to explain why an algorithm works, or they could solve advanced problems that allow them to identify concepts they do not yet fully understand. For example, almost all teachers know that fraction division problems can be solved through the procedure "invert and multiply". However, many teachers lack a deep understanding of why that procedure is effective and professional development opportunities could contribute to this development (Hill, Rowan & Ball, 2005; Ma, 1999; Vamvakoussi &Vosniadou, 2010).

5.2.5 Teachers used multiple representations and modes for improved engagement

Teachers used different modes of representation to improve learner engagement in learning activities. Tools like pictorial representations, concrete manipulations like dice, real objects like papers, flash cards, worksheets, video and power point representations, written mathematical symbols, descriptive written words, oral representations, experience based problems and school word problems. This is supported by researchers such as Fennel & Rowan (2001) and Zevenbergen et al. (2004), who have shown that concrete materials help students to understand mathematical concepts. Finding a range of suitable real world models as contexts for teaching mathematical ideas is recognized as part of good teaching. However as Zevenbergen et al (2004) note, finding accurate real life models that refer to fractions is often difficult.

Teacher 1 also used the flash cards as a game in the class to match with the fraction poster on the chalkboard. She asked the learners from each group to come and display flash cards equivalent to what was on the poster which was displayed on the chalkboard. This is in agreement with the previous research that pupils can also play games to identify equivalent fractions. Cards are turned over with the side carrying the name down and the players can identify the set of equivalent fractions. Models can be used to add or subtract common fractions. Group work and teachers directed activities should be used when solving problems to enhance learners' engagement and to further their understanding (Van De Walle et al. 2010). Bruner (1965); and the Mathematical Sciences Education Board (MSEB) (1989) also support the ideas that "through interaction with mathematical tasks and other students, the student's own intuitive mathematical thinking gradually becomes more abstract and powerful".

Learners were involved in fraction activities in the class; they were given paper to fold to represent different fractional entities like a half, a quarter, one-fourth, a whole and others. Learners had a problem with comparing fractions but by using paper, they understood by looking at the two shapes that a half is bigger than a quarter. The teacher also provided the learners with worksheets so that they could show four different ways of cutting a square into two equal parts (halves). The use of paper and worksheets attracted learners' attention. Learners worked in groups and interacted actively with other learners to solve some fractional problems. This idea is in agreement with previous research that maintains that the constructivist teacher, by offering appropriate tasks and opportunities for dialogue, guides the focus of students' attention, thus unobtrusively directing their learning (Bruner 1965).

Furthermore, in the constructivist classroom, students work primarily in groups, and learning and knowledge are interactive and dynamic. There is a great focus and emphasis on social and communication skills, as well as collaboration and exchange of ideas. This is contrary to the traditional classroom in which students work primarily alone, learning is achieved through repetition, and the subjects are strictly adhered to and are guided by a textbook. Class discussion is one of the most important distinctions of constructivist teaching methods which allow students to discuss concepts and ideas in real world context (Jonassen, 1999; Piaget, 1969). This study supports the ideas that multiple representations increase the interactions among learners and help learners to learn from one another. The evidence from the data analysis revealed that sometimes learners are nervous to ask the teacher and that is why it is useful to encourage a child to solve problems and explain their reasoning to others as other learners pay more attention when a peer explains a problem. The evidence from the analysis indicated that when learners talk to one another in a group, they are highly engaged in a lesson and they will talk and help each other and this would aid understanding.

Smart boards in classrooms are the new white board and give teachers a great way to present websites and virtual manipulatives to the whole class, groups, or individuals. Smart boards support interactive learning to students. They promote student interest, more sustained concentration, and more effective learning. Smart boards have the potential to make

manipulatives more accessible to large groups of children, and to use the shared learning experience within the classroom to further enrich students' learning (Mildenhall et al. 2008). Moreover, virtual and concrete manipulatives reinforce mathematical concepts separately but a combination of both is the best way to achieve the best results (Burns & Hamm, 2011). Websites can offer the engagement with lessons and the use of virtual manipulatives through technology creates the opportunity to learners to makes meanings and sees relationships (Moyer et. al., 2002). This is supported by the use of some teachers of both video and concrete materials. Some teachers did not use video, but they combined other manipulatives, like paper, flash cards and posters. Learners interacted with other learners when they were working in groups and some teachers facilitated the process because they moved around the classroom helping learners in groups and discussed their answers together.

Vygotsky (1978) postulates the idea of a zone of proximal development which also supports the idea that in an area of cognitive activity where a learner cannot make much progress on his or her own, but can progress through interactions or collaboration with more capable others. The child then internalizes the strategies at play in the social dimension to become more capable individually.

5.2.6 Teachers used multiple representations to teach in ways that improved learners' conception of fractions

The Lesh Translation Model also highlights the importance of students' abilities to represent mathematical ideas in multiple ways including manipulatives, real life situations, pictures, verbal symbols and written symbols (Lesh, Cramer, Doerr, Post, Zawojewski, 2003). Combining various modes of representation is a feature of the virtual manipulative concept tutorials used during the lessons with these students. The evidence from the data analysis showed that teachers used different representations, but it was not proven that what they used improved learners' conceptual understanding of fractions.

Through interaction with mathematical tasks and other students, the student's own intuitive mathematical thinking gradually becomes more abstract and powerful (Bruner, 1986; Cobb, 1988; Mathematical Sciences Education Board (MSEB), 1989). This is supported by the lesson

observation where teachers used multiple representations to enhance the understanding of the learners. Teacher 1 in her teaching used fraction posters and flashcards for the learners to think about and match the correct fraction cards to the correct place on the fractional poster. T1 and T2 also gave a worksheet as homework on fraction thinking problems, where learners had to find different ways to divide shapes and to determine which shapes are divided into fractions. On the other hand, T3 did not really involve the learners in his lesson; learners did not get a chance to interact with other learners or with the teacher in the classroom. T3 gave a worksheet as homework for learners to shade and identify fractional parts.

Through data analysis teachers felt that learners learn more when they interact with other learners in the classroom. There is now widespread recognition of the value of collaborative work in developing conceptual understanding (Mcleod & Newmarch, 2006).

Van De Walle et al. (1990) claims that pupils require a sound knowledge of equivalent fractions before learning the four operation on common fractions especially addition and subtraction. Two fractions are equivalent if they represent the same amount. To acquire the conceptual understanding of equivalent fractions pupils should use models such as area, length and sets to discover different names for models of fractions. This is supported in the lessons that were observed. This idea is also supported by Cifarelli (1998), Fennell and Rowan (2001), Goldin and Shteingold (2001), Kamii, Kirkland, and Lewis (2001), Lamon (2001) and Perry and Atkins (2002) who all stated that the use of multiple representations and the ability to translate among representational models has been shown to be an important factor in students' abilities to model and understand mathematical constructs.

Moreover Mcleod & Newmarch (2006) and Kilpatrick et al. (2001) point to the fact that, some misconceptions about fractions may be immediately apparent; however, it is also important to find strategies for uncovering learner's thinking processes and seeing errors that might lie below the surface. Sometimes the learners appear to give a correct response, but their reasoning misfires. It is important to encourage learners to voice their ideas, even if these are based on misconceptions, some learners see some fractions e.g. $\frac{1}{3}$ and $\frac{1}{4}$ as interchangeable. This may be because the words "quarter" and third do not suggest the numbers 4 and 3. It is very important to

check whether the learners have this misunderstanding. This I found to be true because in some lessons learners became confused when they were comparing fractions, for example a half and a quarter. Some learners said $\frac{1}{4}$ is bigger than $\frac{1}{2}$ because 4 is bigger than 2. But when the learners folded paper, they realized that $\frac{1}{2}$ is bigger than $\frac{1}{4}$ by looking at the folded parts.

Kilpatrick et al. (2001) claims that, although teachers often look for evidence of conceptual understanding in students' ability to verbalize connections among concepts and representations, conceptual understanding need not to be explicit. This idea fits with the data which was analyzed. Some teachers did not really teach for conceptual understanding, although they thought that learners understood what they had been taught, but the learners were not tested to find out whether they understood or not.

Professional development opportunities should focus on cultivating this deeper level of knowledge in teachers. Teachers can be asked to explain why an algorithm works, or they can solve advanced problems that allow them to identify concepts they do not yet fully understand. However, many teachers lack a deep understanding of why that procedure is effective (Hill, Rowan & Ball, 2005; Ma, 1999; Vamvakoussi & Vosniadou, 2010).

5.3 CONCLUSION

This chapter discussed the data analysis from the focus group workshop, observed lessons and individual interviews related to the literature review from Chapter 2. The discussion of the findings of the study was based on the teachers' experiences of the use of multiple representations to teach fractions for conceptual understanding in grades 5 – 7. It was presented in terms of the emerged themes identified from Chapter 4. The following chapter concludes the research project.

CHAPTER 6

CONCLUSION

6.1 INTRODUCTION

This chapter presents a summary of the research findings that emerged from the data collection process. A brief discussion of the limitations and challenges encountered in this study will also be given. This chapter ends with a discussion of the recommendations and suggestions for future research.

6.2 EXPERIENCES OF THE TEACHERS USING MULTIPLE REPRESENTATIONS

6.2.1 Introducing and teaching fractions in meaningful ways

Teachers demonstrated positively how to introduce the topic of fractions to learners. They held that learners should be exposed to a wide variety of situations such as folding and cutting of paper. Teachers solved problems on sharing situations during the focus group workshop. The three participating teachers saw representations as a useful way to develop the meaning of the concepts of fractions. The multiple representations used during the workshop and teaching practice, such as written mathematical symbols, descriptive written words, pictorial representations, manipulations, concrete reality, oral representations, real- life problems to experience-based problems and videos or power point presentations influenced the teaching and learning positively. Teachers manipulated concrete materials such as posters, pictures, manipulatives, sweets, paper and others to make sense of the fractional problems and to bring reality to the classroom. Both teachers and learners engaged positively in the problem of sharing and were able to come up with answers. Teachers who attended the workshop used what they had learned in a meaningful way with the use of multiple representations when teaching, as compared to the teacher who did not attend. Some of the challenges encountered by the teachers included difficulties of building relationships within and between the models that served to represent mathematical ideas. Working with learners to develop understanding in this way it take time in the lessons observed, teachers struggled to complete planned lessons in the time available.

6.2.2 Developing understanding of multiple representations

It was a challenge for teachers to define the term multiple representations when they were asked to define it during the workshop. The workshop presentations and discussions, as well as the work with specific examples that teachers carried out during the workshop contributed to developing teachers understanding of multiple representations. After the programme, participants were able to formulate a more appropriate definition of multiple representations and give examples on teaching mathematics or fractions using multiple representations.

6.2.3 Developing connections

Teachers were able to make connections among concepts and the use of concrete representations appeared to lead to deeper understanding in the workshop, but in lesson observations, few mathematical connections were observed. Teachers were able to connect and coordinate mathematical ideas as they were expressed in language, symbols, and different representations. Teachers worked with a variety of models to apply multiple representations to describe basic fractions (halves, thirds, fourths). T1, T2 and T3 teachers were not really able to explain mathematical connections and were not able to make the concept of mathematical connections clear to the learners.

6.2.4 Teachers planning and preparation

Teachers developed their lesson plans effectively to involve a variety of teaching methods using multiple representations, despite the time constraints. Time was noted to be one of the challenges encountered by the teachers during lesson planning. They complained that there was insufficient time to cover all the presentations prepared for that lesson. In addition, the syllabus contains too much content, forcing teachers to teach the required content and to focus on one type of representation. Despite this, they found planning to be an instrument which helped them to look for easier methods to increase the learners' understanding and to make their teaching easier. Despite the challenges, both teachers indicated that they want to change the way they plan their lessons, to ensure that they include more than two representations in their teaching to assist the understanding of the learners and to broaden the knowledge of the learners.

6.2.5 Teachers used multiple representations and modes for improved engagement

Teachers used different modes of representation to improve learner engagement in learning activities. Representational tools included pictorial representations, concrete manipulations like dice, real objects like papers, flash cards, worksheets, video and power point representations, written mathematical symbols, descriptive written words, oral representations, experience based problems and school word problems. Both teachers saw multiple representations increase the high level interactions among learners, which helped learners to learn from one another which is supportive of the constructivist ideal. Although teachers indicated that technology is expensive, they saw that the use of videos and power point presentations appears to have increased learners engagement by providing a resource that promoted interactivity, immediate feedback, challenge and fun. It is important to observe that it was not proven that learners understood what was taught by the teachers, although they seem to show an interest in working with representations using ICT.

6.2.6 Teachers used multiple representations to teach in ways that improved learners' conception of fractions

In the lessons, it was observed that teachers used multiple representations when teaching fractions, Kilpatrick et al. (2001) claim that, although teachers often look for evidence of conceptual understanding in students' ability to verbalize connections among concepts and representations, conceptual understanding need not to be explicit. This idea was supported by the data which was analyzed. From their engagement and their verbal responses, it appeared as if learners understood what they were taught, but due to the scope of the research, learners were not tested to provide more formal verification of this observation.

6.3 LIMITATIONS AND CHALLENGES ENCOUNTERED

The first challenge is that, because this was an exploratory case study, the size of the sample of teachers was very small – having only three participants. If the scope of the study was larger, and there was more time I would have encouraged more mathematics teachers to participate in this study. The second challenge I faced was the period of pre–observation, I planned to observe the three participants before the focus group workshop so that I could familiarize myself with the way they teach fractions. But this was not possible in each case, because one teacher was busy

with another topic. The third challenge was the implementation of the focus-group workshop for the programme. This was conducted on one day after school, which was a short time period and as a result, teachers were not able to explore many fraction problems and were not able to represent these problems with a variety of representations. One teacher turned up very late and so missed the first part of the focus-group workshop. The time to collect the data was also very short because each teacher was observed for two periods (40 minutes each).

6.4 SIGNIFICANCE OF THE STUDY

The significance of this study is that it has helped to identify teachers' experiences and the manner in which they changed their practice, as they worked to develop their use of multiple representations to teach fractions for conceptual understanding in upper primary schools in Namibia. It also generated an awareness of the challenges faced by upper primary mathematics teachers when using tools of multiple representations in the teaching of Grades 5 –7 fractions. The rationale of the study was to ask the teachers to explore the possibility of using additional tools when introducing common fractions and decimal fractions and help grades 5 – 7 teachers to develop proficiency in solving fraction problems.

This study is also significant because it revealed that tools of multiple representations can be powerful and effective in the teaching of Grades 5 – 7 fractions. The major purpose for asking teachers to explore the additional tools of multiple representations is to offer mathematics teachers experiences that will allow them to develop strong mental images of fractions that enhance conceptual understanding of fractions, and then to allow them the opportunity to implement use these tools and understanding in their classroom teaching.

6.5 RECOMMENDATIONS AND SUGGESTIONS FOR FURTHER RESEARCH

If I had to do this research over again, there are four major aspects that I would take into consideration. The first one is to increase the sample size of the participants. The second aspect would be the time period of the focus group workshop. Ideally it would have taken place over three to four days instead of one day. This would allow a deeper exploration of multiple representations and of different areas of fractions. The third aspect would be to expand the observation period in schools; so that I could really observe how teachers teach and how they

employ different representations in their teaching and also to see how learners and teachers interact with them. The fourth aspect is that teachers should be given the same topic to cover and the same activity for the learners and learners' activities would be analyzed. The last aspect, in-service workshops for teachers and the integration of multiple representative tools in the teaching of Grade 5 – 7 fractions should be initiated. Therefore, I recommend teachers, educators and policy makers to use multiple representations when teaching or dealing with the topic of fractions.

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APPENDICES

APPENDIX A: WORKSHOP TRANSCRIPTION

P1: Teachers were introduced to the workshop on using multiple representations when teaching fractions to the grade 5 – 7 learners. The workshop was more on discussion. *Teachers were asked to define multiple representations*. Teachers were quiet for some time, trying to figure out the definition of multiple representations. The researcher gave them a hint. She referred to the example which teacher 1 was explaining a half , where she used different ways to explain it. *Here are their responses:*

Teacher 2:*Multiple representations is when you use different presentations, we are using multiple representations to make the teaching and learning easier for the learners to understand. If you give one method, then it might be difficult for them to understand. You should have different ways, so that even those who are weak can pick from those methods. They pick which one is easier for them to answer.*

Teacher 1: *Multiple representations are a different style of teaching, which somebody can use to accommodate all the different types of learners that we have in the class.*

P2: *Furthermore the researcher asked the teachers to give an example on how they can introduce the topic of fractions to the learners using multiple representations. Teachers were quite for one minute and then teacher 1 responded.*

Teacher 1: *(demonstrating with a paper)I use papers, I give them these papers into groups for them to understand easier or for them to experience it and then they follow my instructions. First of all you show them the full complete paper which shows a whole. So, know to make it a half, because know when you talk of fraction we talk of equal parts, I have to divide this paper into a half, and then open it to show them equal part (two halves) , and then I instruct them to fold that paper again into half and open it to see how many parts of a whole are there and so on….In this way learners should be able to differentiate which one is bigger, which one is small when they are given a questions on comparing of fractions. I can also use an apple as a real object, you cut it into equal parts, one part is a half, and if you cut a half apple further again, you will be having a quarter. When you explain* $\frac{1}{2}$*, explain it in different ways for the learners to understand.*

Teacher 2: *I use video, where learners can see real objects, like in my case I have a video on fractions, I usually use it when I introduce that topic to the learners, I show them a video and this video has already different representations. After watching, the video, I either give the learners an exercise to do or I will ask them questions. The researcher was surprised by the teachers' answers.*

P3: *The researcher further gave different types of multiple representations which can be used when teaching fractions or other topics in mathematics. These types are:* written math symbol, descriptive written words, pictorial representations, manipulations, concrete materials, oral representations, experience- based and videos or power point presentations. Before, the workshop handouts, the researcher requested teachers to manipulate the 20 blocks which were provided to teach the concepts of fractions and comparing of fractions such as halves, quarter, three – fourth, etc.

The researcher gave two problems from the first activity of the workshop hand out to the teachers to discuss those questions and give feedback. Here are the two questions:

Workshop handout, Activity 1.1

1. How can 20 sweets be shared among 5 best learners in your class.
2. Divide a chocolate (20 bars) that is being shared by a mother and her two daughters.

P4: Teachers worked sometimes to respond to the two questions.

Teacher 1: *20 sweets and five learners (writing on the paper), drawing 20 circles on the paper and encircle five small circles.*

Teacher 2: *Yes, you are correct, which means that, each learner will get four sweets, teacher 2 also taking twenty sweets and divide them into five groups to match the number of five best learners in the class.*

Teacher 1: *perfect, you see, there are many ways to kill the cat, can you see the two ways that we have used?*

Teacher 2: *yes, yes… we do the same to question 2. For question two we use chocolate or 20 blocks or counters to represent number of blocks.*

Teacher 1: *Oh, yes, then you divide the bars among the three people, each one will get 6 bars and the remaining 2 bars will be shared equally, by putting them together and divide them into three equal parts.*

P5: Teacher2, responding to the first question**:** *I take 20 sweets (teacher taking 20 sweets in reality, because it was provided) and divide them equally among the 5 best learners, so each learner will get 4 sweets.* Furthermore, teacher 1 responded to the second question, where they were asked to divide a chocolate (20 bars) equally that is being shared by a mother and her two daughters.

Teacher1: *Since, they are three in total and the bars are twenty, firstly each one will get 6 bars and the remaining two bars will be put together and shared equally into three parts.* Teachers engaged positively in these problems of sharing and could give the answers.

P6: Furthermore after the workshop presentation teachers were given more worksheets with more problems to work on and to give possible representations to those problems. Teachers were asked to think on how to present those types of problems in their classes, if they are to present them to the leaners. Teachers were given some problems on sharing of fractions. Here is the first problem:

Activity 1.2. Workshop hand out

a) *Hendricks gives a price to the group in his class that has behaved the best during the week. The prize is the box with 10 chocolate bars.*
b) *This week Ann's group wins the prize. There are four people in Ann's group. They all want the same amount of chocolate. How much chocolate does each child get? b) Last week it was John's group that won the prize. There are six people in John's group. How much chocolate does each child get?* **(Appendix D, Activity 1.2.)**

P7: Teachers were working sometimes to get the correct solutions and correct presentations to this problem to make it more understandable and easier to the learners if they are to teach it in the class. Teachers were quite sometimes. These are the teachers' responses when they were asked to answer the question above which has two sub-questions.

Teacher2:*To answer the first sub-question (a), I use this real chocolate (Teachers were provided with the real chocolates), if there are four learners in a group and each chocolate contains 10 bars, what I do is, each child will get two bars of chocolate. Teacher 2 drew also a rectangular model on the white board to show how the 10 bars chocolate can be divided equally among the four learners in a group:*

1
bar

10 bar chocolate - shared by four learners

Each learner will get two bars:

And the remaining two bars will be shared again equally. Meaning that two learners will share one bar. Each learner will get two full bars and one half bars.

$\frac{1}{2}$ $\frac{1}{2}$ $\frac{1}{2}$ $\frac{1}{2}$

P8: Teacher 1 *explains the first answer, I will take a whole paper as a chocolate, divide it into ten bars take a scissor and cut out each piece which represent a bar, each learner will get two pieces and the remaining two pieces, I will fold each into two equal parts and cut them out so each learner will get two full pieces and one half piece.*

P9: In the activity 1. 3 – Sharing fractions, teachers were given this problem to solve: Four friends go on a full day outing. At lunch time they are hungry. They have 8 sandwiches to share for lunch. How do these friends share the sandwiches fairly for lunch?

(a) How can you look at this situation to see fractions?

(b) Use different representations to describe the situation above? Teachers were given a hint to use pictures and symbols (draw), Concrete materials such as pens, books, etc, Posters and Real sandwiches, etc.

P10: Teachers shared ideas on this activity.

Teacher 2 : *where should we see fractions in this questions? We are given four friends and 8 sandwiches and the question is on sharing?*

Teacher1: *Ooh, yes, you know it is four friends sharing 8 sandwiches; it is simple, four friends sharing 8 sandwiches, so they are sharing part of a whole.*

Teacher 2: *Ooh, yaa, you are right, sharing part of a whole is already a fraction.*

Teacher 2: *Ok, let's continue with (b).* Teachers worked sometimes to answer question 1.2b. They used different concrete materials to make sense of fractions in terms of connection. They used a poster to draw pictures papers to cut and they also used biscuits which were provided to represent sandwiches.

P11: Teacher 2: *We are given a hint to draw pictures, lets take a poster, you know poster is big enough and you can draw clear pictures on it that can be seen, lets draw 8 sandwiches, T**eacher1:** and then?*

Teacher 2: *and then four pictures of four friends and we draw arrows from the sandwiches to the four friends showing how they are sharing 8 sandwiches.*

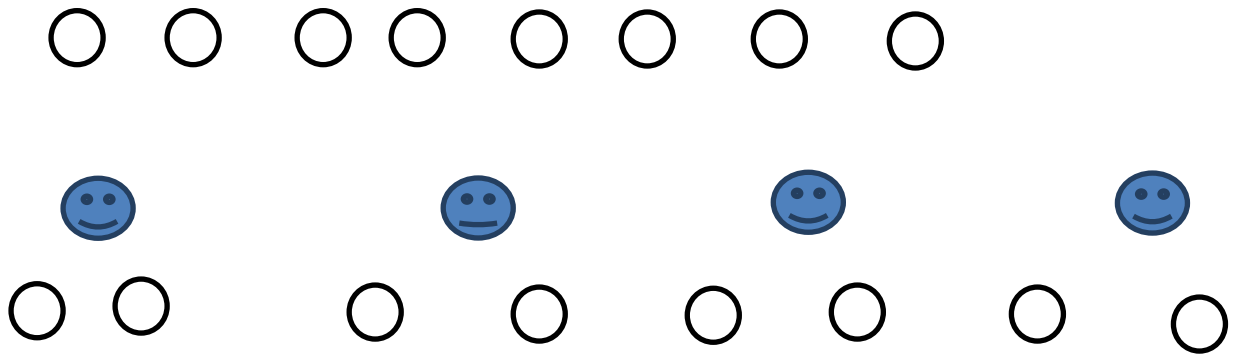
Teacher 1: *Yes you are right, since we have even numbers, it is very simple, four friends and eight sandwiches, ooh, each one will get two sandwiches.*

Teacher 2: *Yes, (teacher 2 representing the question in form of drawing, drawing with a pencil).After that, teacher 1 took 8 biscuits and stones to represent the question as it was presented on the poster.*

P12: Teachers discussed their answers:

Teacher 1. *I can see fractions because, four friends are sharing sandwiches. Sharing part of a whole.*

Teacher2: *I take a poster and I draw eight circles which represent sandwiches, I draw four friends. Since they are four friends, each one will get two sandwiches. Teacher 2.*



So, each learner will get two sandwiches. I will take the poster and paste at the back of the class for the learners not to forget.

Teacher1: *I take papers and cut out eight circles to represent sandwiches, I use stones to represent friends, then I show to the learners, that each friend will get two sandwiches, so I put two circles next to the stone.*

APPENDIX B: DESCRIPTION OF TEACHERS' LESSONS

APPENDIX B 1: Teacher 1

LESSON 2: POST- OBSERVATION TRANSCRIPT

SCHOOL	**: CODE 1**
TEACHER	**: CODE 1**
GRADE	**: 5**
DATE	**: 07 March 2014**
DURATION	**: 1 HOUR AND 20 MINUTES**
TOPIC	**: COMMON FRACTIONS – COMPARING FRACTIONS**

Line 1. *This is the second lesson of teacher 1 after the focus group workshop. The teacher greeted the learners and instructed them to sit down.*

The teacher writes the topic and the date on the chalkboard.

Line 2. *The teacher asks some questions concerning the previous lesson when fractions were introduced to the grade 5 learners. What do you understand by the term fractions? What do you do when dealing with fractions? For quite sometimes, learners tried to figure out what does fraction means. The teacher then asked again try to make the question clearer to the learners.* ***Teacher:*** *What is fraction is my question? Can somebody explain what fraction is all about?* ***Learner 1:*** *Fractions are numbers that are mixed together. Teacher: Repeats what the learner has said, numbers that are mixed together? Hmmmmm, the other people, what do you think?* ***Learner 2:*** *Fractions is when you put a half of a number. Teacher: Very good, another person.* ***Learner 3:*** *A fraction is when you divide a shape. Teacher: A shape, which shape? Learner 3. For example a square, you divide it into four equal parts, and then you count how many fractions* ***are there?***

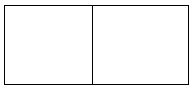
Teacher*: Very good, you divide a square into four equal parts, you see how many parts will come out.*

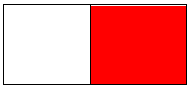
The teacher *explains while drawing a square on the chalkboard to show what learner 3 has said. Learners in the class: Oooooh, responded after the drawing of the teacher on the chalkboard.*

Line 3. The teacher*: This part that you are talking about, how are they going to be, who can tell me? If you divide a square into four equal parts, can you still explain yourself learner 3?*

Learner3: *They are going to be the same.*

Line 4. Teacher: *we are going to look at flash cards today. You were doing this thing in grade 4. The teacher picks up one card to explain to the learners. This is a flash card, showing a card on the chalkboard. Teacher explains, this flash card is divided into two equal parts.*

Line 5. *The teacher*  *placing the flash cards on the chalkboard. The teacher: How many parts do you see here?*

All learners: *Two. Then the teacher*  *draw the card (In a form of a rectangular shape) o the chalkboard*

The teacher: *Out of the two, how many is shaded.*

One learner: *One.*

Teacher: *Yes, then you say it is one. Teacher writes on the chalkboard* $\frac{1}{2}$*, simply, first of all you find out that, how many parts are there in total, then you say is two. The teacher writes it on the chalkboard* $\frac{}{2}$*, the total is always at the bottom and then you put your fractional line on top of the total.*

Line 6. Teacher: *How many partsareshaded.*

All learners: *one.*

Teacher: *You listen very well according to the question. If they asks you the shaded, then you answer the shaded is one, thecolored one (pointing on the chalkboard).*

Line 7. Teacher: *What will you say, if they ask you to identify unshaded, then you will give the uncolored one. The teacher showing on the flash card on the chalkboard. Unshaded and shaded show a half, the teacher explains while writing on the chalkboard.*

unshaded	*Shaded*

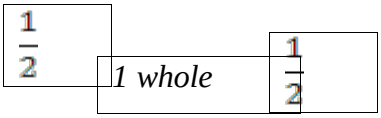
Line 8. Teacher*: Can we go ahead? Learners: Yes. Teacher explains using a paper written, half, half*

$\frac{1}{2}$	$\frac{1}{2}$

This is what I am talking about, pointing at the paper.

Teacher: We will come back to the equivalent fractions.

Line 9. Teacher*: If I take two of my half and put on the A4 paper on the chalk board, this two halves will form a whole one.*



Line 10. *The teacher divides the learners in pairs and gives them A4 papers.*

The teacher: *I divide this paper into a half (fold it into a half) and I cut it into half. Teacher takes a paper and cut it into two equal parts, if you put two halves together, they form a whole.*

Line 11.*Teacher: Then we go ahead, this paper is a whole, then we divided it into half and now I want to us to divide it again into another half and open it.*

Teacher: *How many parts can you see here.*

Learners: *4 parts. Teacher: Let me write it on the chalkboard:*

$\frac{1}{4}$	$\frac{1}{4}$
$\frac{1}{4}$	$\frac{1}{4}$

The teacher shows four parts on the chalkboard, $\frac{1}{4}$ is called one quarter. If you take the four quarters and put it on the whole A4 paper on the chalkboard, will they look the same or not?

Learners: *the same. Teachers repeats, they are the same.*

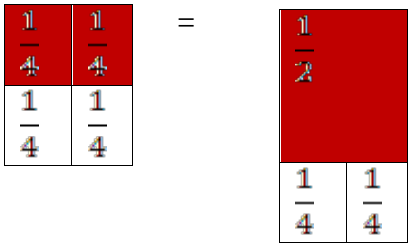
$\frac{1}{4}$	$\frac{1}{4}$
$\frac{1}{4}$	$\frac{1}{4}$

=

One whole

The teacher: *so four quarters will give you the same as a one whole.*

Line 12.Teacher: *If you take two quarters and put them together, they will give you a half and it can be expressed as shown on the paper.*

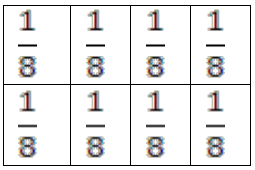


Teacher: Ahalf is equal to two quarters, they are the same.

Line 13. Teacher: *Lets go ahead, still you take one full paper, then you divide it three times and then you open it.*

Teacher:*how many parts you see here. Learners: They are eight.*

The teacher showing her paper to the learners and counting the eight parts that the paper is divided onto.

The you say, each  *part is* $\frac{1}{8}$, $\frac{1}{8}$, ….. *Saying together with the learners while pointing on the chalkboard.*

Line 14. Teacher:*You can see how the parts are getting smaller. Teacher warns the learners that they must pay attention, because when she gives work, she does not want anyone to get zeros.*

Line 15. Teacher:*When I gave you half, you can see that half is bigger than a quarter. If you compare a half with the quarter, a half is bigger than a quarter you can even compare the two using that paper , that a half part is bigger than a quarter.*

Line 16. Teacher: *If you compare a half and a whole number which one is smaller.*

One learner: *The whole is bigger than a half, showing on the two papers. Teacher praises the learners and continues that if you compare a half with a quarter, then do not tell me that a one quarter is bigger than a half, because the denominator is bigger. The more the denominator gets more, the more it is getting smaller. Teacher says you can even see here that* $\frac{1}{4}$ *and* $\frac{1}{8}$*. One over four is bigger than one over eight part.*

Line 17: Teacher: *How many one eighths make up a half.*

One learner: *counting the number of parts using the paper, four eighths.*

The teacher *praises the learner, so four eighths make up a half. In a half we can find how many quarters?*

Learners: *Two.*

Teacher: *Good, they are two, and then they are the same. Ok.*

Line 18. Teacher: *Let's look at this flash cards. This is what you have learnt. I want you to come and give me…… before we continue, I want you to come and recognize this part, remember we are looking at the equivalent fractions. Fractions which have the same meaning. The teacher put the poster on the chalkboard, with many flash cards on the box. Learners match the card with the fractional poster on the chalkboard. Learners match fraction with the correct shape in this poster and paste (paste activity). Learners recognize or identify the card which is equivalent to what is on the poster.*

Line 19.Teacher: *You are going to play a game. Teacher paste the poster on the chalkboard, we start form the top one, you see the first line it represent what? Here is the poster:*

Line 20.Teacher: *Choose one card and stick to the poster.*

$\frac{1}{2}$	

Learner 1: *Sticking card number 1, which shows a whole number on the first line. Teacher: Is learner 1 correct. All learners: Yes. The teacher: good, we go to the next one, yes, and one learner?*

Learner 2 *sticks the flash card on the second line:*

Line 21. Teacher: *you are correct.*

Teacher: *Any card equivalent to the second line? There are many cards that are equivalent, but they are written differently but mean the same. Learner 3 sticking card which is written one over four on line 2. Teacher says is not correct, you don't put that card there because that line is divided into two parts only which is a half, check nicely.*

Teacher:*. You count how many parts are there?, one, two and so on. It was first a whole, now the whole is divided into two parts.*

Line 22. Teacher: *Yes, yes, quiet, quiet, lets continue with another person? This girl is coming. The learner search for a correct card.*

Teacher: *Count the parts, look at that set and compare it to the one on the chalkboard, remember you consider also if it was written in words, consider the cards in words or in drawing. Learner placing a card at a correct line*

Line 23.*Learners clapping hands.*

Teacher: *how many parts are there. All learners: three parts.*

Teacher:*well;let's go ahead, there are still some cards here that belong to this poster or to any line in the poster.* Learners coming one by one to stick the cards at the correct places. Teacher tells them not to tell each other's answers.

Line 24. *Learners continue to do it.*

Teacher : *Let us do it fast.*

Teacher supposed to give another activity, a printable fraction as a handout about equivalent fractions and comparing of fractions, because of limited time the teacher could not be able to finish the lesson as planned. She gave it as homework.

APPENDIX B2: Teacher 2

LESSON 2: POST- OBSERVATION TRANSCRIPT

SCHOOL : CODE 2

TEACHER : CODE 2

GRADE : 6

DATE : 11 March 2014

DURATION : 1 HOUR AND 20 MINUTES

TOPIC : COMMON FRACTIONS

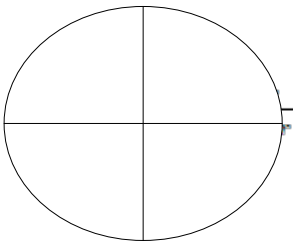
Line 1. Teacher: *Good morning class*

Teacher: *ok, I will give you fewer video clips, there are two video clips that you are going to watch and then after I will ask you questions and give you an activity afterwards, so listen very carefully.*

Line 2. Teacher: *The first video is in a form of a song of fractions, so learners were instructed to listen very carefully. Some words in a song: fractions, fractions the part of the whole. Fractions the word I know. Examples,* $\frac{1}{2}$, $\frac{1}{3}$,……. *Equal parts*

$\frac{1}{4}$	$\frac{1}{4}$
$\frac{1}{4}$	$\frac{1}{4}$

Line 2. Teacher: *Fraction, fraction what you need to know. It takes 3 thirds to make a whole*

Two parts: 

Line 3. *After the song, teacher asks the learners whether they should be able to sing the song.*

Learners: *yes miss.*

Teacher: *Now our lesson has started, I would like you to listen very carefully, that were just an introduction, you should know what a fraction is.*

Line 4. Teacher: *Did you see things that you combine together to make a whole?*

Learners: *Yes. Teacher asks the learners how many quarter parts you need to make a whole.*

Learner 1: *three.*

Teacher repeats what the learner have said, three, three halves you need to make a whole.

Other learners: *No.*

Teacher: How many halves?

Learner 2: *Two.*

Teacher: *Yes, two halves that you need to make a whole.*

Line 5. Teacher: *Ok.*

Teacher: *Now, you are going to listen to the second video, so you listen very carefully, because I am going to give you some work to do. The 2nd video on fractions started. The class was silent.*

Line 6. *The video was about a learner and a teacher on teaching and learning fractions.*

Teacher on a video: *I am going to teach you how to make a mask using paper which looks like the face of the dog. Firstly, you fold the paper into half.*

Learner: *But, this is not a half is looking like a triangle, I am confused.*

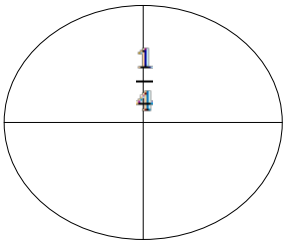
The teacher on a video: *Don't worry; I am here to help you.*

The teacher on a video: *If you fold this paper and the corner touches each other, that means that this paper is divided into equal parts.*

Teacher on a video: *It does not matter whether you fold it horizontally, vertically or diagonally as long as two parts are equal.*

Line 7. *So, remember this if a whole is divided into two equal parts; the two equal pieces of a whole are called halves.*

Learner on a video: *Oh, that is interesting.*

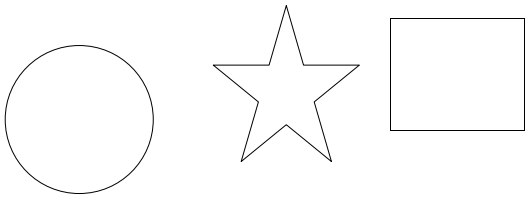


Teacher on a video: *Do you know what a quarter is.*

Learner on a video: *No.*

Teacher on a video: *a quarter is one part of the four equal parts, showing the diagram*

Look at these different shapes:



Line 8. Teacher in a video: *You have a cake, now imagine that you have three pieces out of the four pieces of this cake, what portion of the cake would you have then?*

Learner: *I does not know.*

Teacher: *that is where fractions come in. You just learnt about the fraction* $\frac{1}{2}$ *and* $\frac{1}{4}$ *are some examples of fractions. See a fraction is nothing but part of a whole, you divide a paper into two equal parts or halves or can be shown as* $\frac{1}{2}$ $(one\ upon\ two)$ *or one part out of two equal parts.*

Line 9. Teacher on a video: *When you divided it into quarter, we divided it into four equal parts. That means quarter is shown as one upon four (* $\frac{1}{4}$ *) or part of the four equal parts. If you have a cake and just be divided into four equal pieces and you take three pieces, what fraction of the cake will you have?*

Learner in a video: *I would have three pieces out of four pieces (*$\frac{1}{4}$*) or three – fourth of the cake.*

Teacher in a video : *What fraction of the whole cake is the remaining piece? Learner in a video:* $\frac{1}{4}$ *or one part out of four (one – fourth) or we can say a quarter. A teacher on a video praised the child; I think you have learnt very well.*

Line 10. Summary of the video:

- Fraction is part of the whole.
- If a shape is divided into two equal parts, each part is called a half.
- Half is represented as one upon two ($\frac{1}{2}$)

- If a shape is divided into four equal parts, then each part is called a quarter
- A quarter is also known as one – fourth
- One – fourth is written as one upon four
- Three equal parts of the four total parts is equal to as $\frac{3}{4}$ (3 upon 4)

The video is over.

Line 11. The teacher: *before I give an exercise, what did you learn? What did you learn? Who can tell me? The class was quite for sometimes. Teacher then asked again who can tell me?*

Learner 1: *fractions.*

Teacher: *Fractions, what is a fraction?*

Learner 2: *Fraction is part of the whole. Teacher repeats what the learners have said; a fraction is part of the whole.*

Teacher: How many quarters do you need to make a whole?

Learner 3: *Four quarters.*

Teacher:*how many halves do you need to make a whole?*

Learner 4: *Two halves.*

Teacher:*, how many eighths, eighths is what? The teacher continues, eighths, one eighths (saying it orally), how many eighths do you need to make a whole?*

Learner 5: *two.*

Teacher: *Eights is one over eight, if I have one over eight, how many parts do I need to make a whole?*

Learner 6: *(not sure) eight parts. The teacher praises the learner, yes, it is eight parts.*

Line 12. Teacher: *Now I want you to show some parts of the whole on these papers, you can make a quarter on a square, where else can you make a quarter?*

Some learners: *shouting the answer, on a circle, where else….. And a half?*

Learner 6: *on a triangle.*

Teacher: *on a triangle? You can make a half on a triangle? Is it?*

Learners: *yes. Let us use these papers and see. Let see…..*

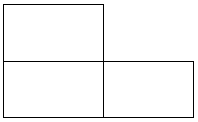
Line 13. Teacher: *gives papers to the learners and instructs them to fold them into half into different shapes.*

Teacher: *if you are seated two, two, then each one must have a different shape, but each shape must be equal to a half.*

Teacher: *I want you to make a half using this papers, I want to see how you can get a half on a different shape. Teacher said you just make a half. Teacher said right, I did not say make other fractions. Ok, open it; somebody should explain your shape using your paper?*

One learner: *My shape is divided into two equal parts*

Line 14. *Teacher explains, you can fold a paper vertically, horizontally and diagonally, but this paper is not a square, is a rectangle, you cannot fold it diagonally. The teacher showing the learners using her paper, these corners do not touch each other. Teacher instructs the learners to use their paper to make a quarter. And I want to see four parts and I want you to cut one part from that 4 parts:*



Hold your other three parts and one part, with the part that you have cut. ***Teacher:*** *what fraction is it. One Learner: One over four. Teacher asks and the fraction for the uncut? Three over four.*

Teacher: *I want to see all four parts being cut, how many parts do you have?*

One learner: *four parts.*

Teacher: *yes, this four parts came from one whole, is what we say sharing. If they were four learners and you cu that bread into four equal parts, each one is going to get what*?

Learners*: one piece.*

Teacher: which is what?

One learner: *one – fourth.*

Teacher: *Yes, one fourth or a quarter. Ok.*

Line 15. Teacher: *ok now, you take one piece and you fold it three times, don't open it. One part folded three times. Teacher asked the learners to open that one part.*

Teacher: *how many parts do you have there?*

One learner: *six, one says eight parts.*

Teacher: *Ok good, they are eight. Teacher asked the learners to shade three parts, you colour with anything, so long as there is a colour on it.*

Line 16. *For quite sometimes learners tried to color the three parts.*

Teacher: *Ok, done, what fraction is it?*

One learner: *three over five.*

Teacher: *is he correct? Some say yes, some say no.*

Teacher: *what fraction is it?*

One learner: *three over eight.*

Teacher: *continues asking questions, why three over eight, can somebody explain?*

One learner: *because the total is eight, you color three parts out of eight parts.*

Line 17. Teacher: *Ok good; let me give you the exercise (worksheets handed out to the learners). Learners do the exercise individually. Student must find different ways to divide shapes, determine which shapes are divided into fractions.*

Teacher: *Let us look at the first question.*

Teacher reads*: show four different ways of cutting up a square into 2 equal parts (halves).*

Teacher explains: *All I need is half; you need four different ways on how to cut a square into two halves.*

Line 18. Teacher: Hurry up, hurry up, just draw a line, a line.

Learners were all quite for some times. After sometimes teacher asked the learners to explain their answers. Learners explain their answers.

One learner: *I draw a line horizontally, vertically and diagonally (twice) from right to left and the other one from left to right.…….*

Line 19. Teacher: *The first question is done, who can read the second one?*

One learner: *Show four different ways of cutting up a square into four equal parts (quarters). The teacher was busy with this worksheet and she could not finish her lesson.*

APPENDIX B3: Teacher 3

LESSON 2: POST- OBSERVATION TRANSCRIPT

SCHOOL : CODE 3

TEACHER : CODE 3

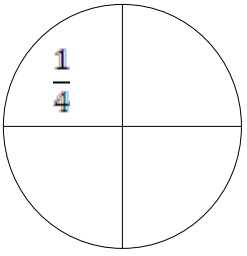
GRADE : 5

DATE : 13 March 2014

DURATION : 1 Hour 45 Minutes

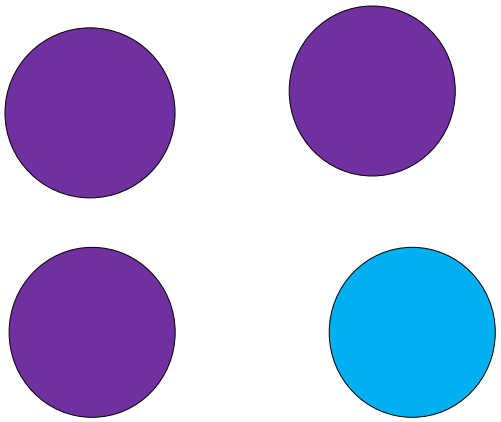
Line 1. The teacher greeted the learners and introduces the topic of fractions. The teacher uses power point presentation. He introduced the topic of fraction by outlining the first two parts of fractions. He gave an example of $\frac{3}{4}$, the teacher explains the parts on top is called numerator which is three , three is just like my nose because it is on top and the part at the bottom which is 4 is the denominator. I use to remember it like this, the numerator is the one on top and the denominator obviously is at the bottom. Three over four can be represented in different ways.

Line3. Teacher: *Three over four looks like this; it can be presented in this form of diagrams (diagrams on power point presentation):*



Teacher: *Three over four can also be represented in a rectangular shape*

Teacher: *Or circles can be used to represent* $\frac{3}{4}$*, where three parts are shaded differently from other one.*



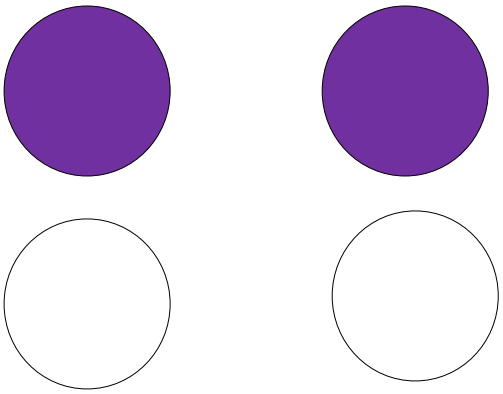
Teacher: This is another example of $\frac{3}{4}$,

Teacher: *Again, three parts are shaded differently but mean the same as the previous example. There are four circles, of which one part represents a quarter. Can you see that children.*

Learners: *yes teacher*

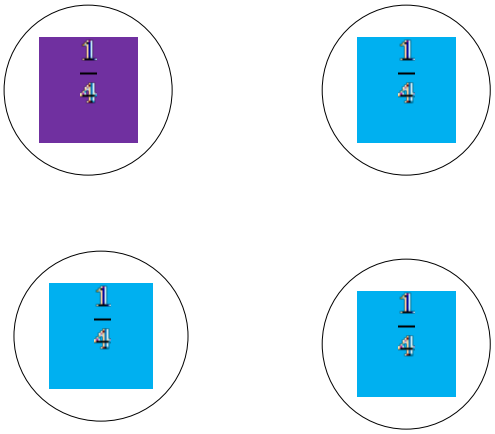
Line4. The teacher discusses the following examples with the learners;

a) What fractions of the balls are purple?



Learner 1 answered it is $\frac{1}{2}$.

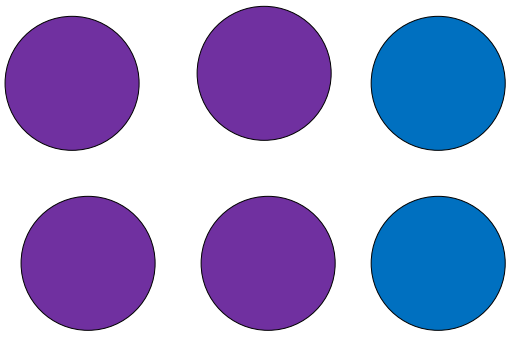
Line 5 b) what fractions of the balls are purple?



Learner 2: *It is* $\frac{3}{4}$.

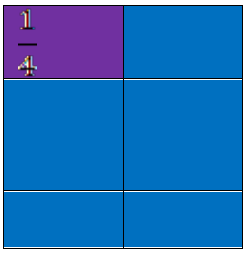
Teacher : *no it is not correct, another person? Learners 3 raised his hands and give the correct answer as* $\frac{1}{4}$. *The teacher continues with the questions.*

Line 6 c) What fractions of the balls are purple?



Learner 4 in the class answered is $\frac{4}{6}$.

Line 7 .d) What fractions of the rectangle is purple



Learner 5 :*it is* $\frac{1}{4}$*. The teacher praises the learners for correct answers.*

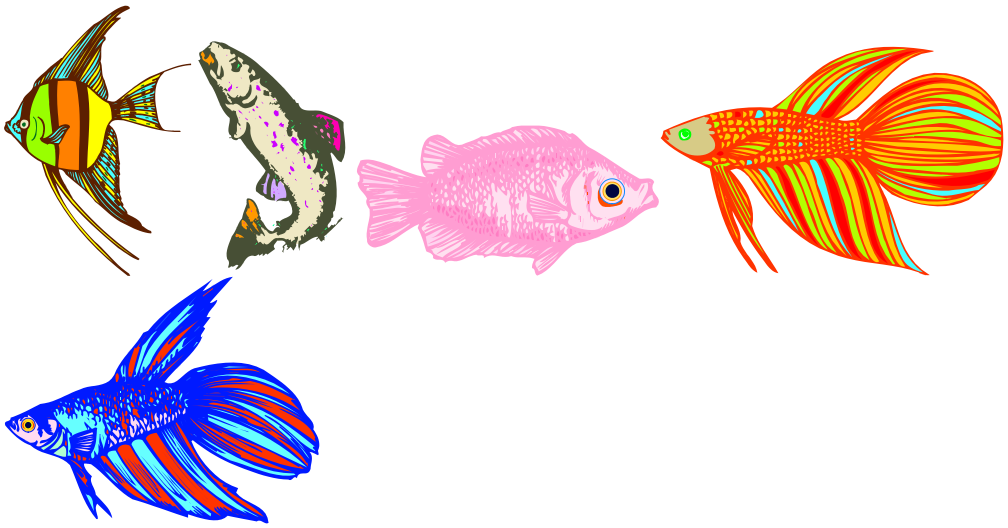
Line 8.Teacher: *What fraction of the musical instruments has strings?*

Learners were quiet for sometimes, looking at which musical instruments have strings.

Learner 6: *It is one over five. Teacher said no, check nicely, is it only one which has strings.*

Learner 7: *they are two out of six. The teacher compliments that learner.*

Line 9. Teacher: *What fraction of the fish has stripes?*



Lines 10*.Learners were a bit confused when they looked at the fish.*

Teacher: *How many fish have stripes?*

Learner 8 unsure: *Two over five.*

Teacher: *Is it two over five? Check nicely children.*

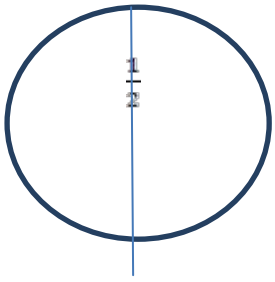
Learner 9: *Three over five.*

The teacher was very happy with the learners' progress but unsure whether the children really understand the content, since he was just asking questions and let the learners answer.

Line 11. Teacher: *I have another slide that we need to discuss before I give the class activity.*

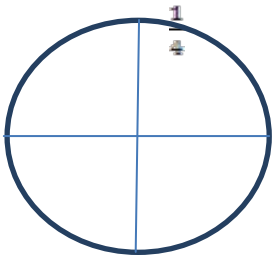
Learners: *yes sir.*

Teacher: *Ok. Let us look at this slide. How many halves are there in whole?*



All learners: *Two halves*

Line 12.Teacher: *Ok, how many quarters are in the whole, when you look at this pie?*



Learners looking at the pie to identify the number of quarters in one whole.

Learner 10: *Four quarters.*

Teacher: Yes it is correct. How many eighths are in a whole?

Learner11: *It is four, learner 12: zero,*

Learner: *13: it is one, learner: it is two.*

The teacher confirmed that the answer is two. The teacher counted the number of parts with learners on the white board.

Line 13. The teacher gives an exercise that learners must fill in the numerators or denominators for each fraction given on the power point presentations and identify the number which is circled. Questions: 1. Fill in the numerators or denominators for each fractions given a) $\frac{}{2}$, b) $\frac{3}{}$, where a) numerator: 1 and b) denominator is 8. 2. Identify the number which is shaded.

a) $\frac{3}{4}$

Line 14. Learners were busy for a while and afterwards the teacher discussed the answers with learners. The hint of the question was given and almost all the learners in the class answered the first questions. Learners could fill in $\frac{1}{2}$, one and $\frac{3}{8}$, eight. For the second questions, teacher gave a chance to one learner to explain the answer to other learners.

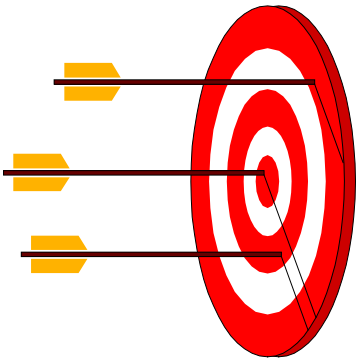
$\frac{3}{4}$. ***Learner 1:*** *the three is a numerator, because it is on top.*

Teacher praises the learner, good and the bottom one?

Learner 2: *four is the denominator.*

Teacher asks learner 2: *why are you saying four is the denominator. Learner 2: because is at the bottom of the line. Teacher praises the learner and continues with the lesson.*

b) What fraction of the arrows hit the bull's eye?



The teacher call in three learners from different groups to come give the answers the last three questions. The bell rang and the lesson came to an end.

APPENDIX C 1: TEACHER 1 (T1) – INTERVIEW TRANSCRIPT

TEACHERS' INDIVIDUAL INTERVIEW TRANSCRIPTION.

P1. JI: Good afternoon mem?

P2. T1: Afternoon.

P3.JI: Welcome to the interview on teaching of fractions using different multiple representations.

P4.T1: Hhhhmm, thank you

P5.J1: I enjoyed your three lessons that I have observed the one before the workshop and the two after the workshop.

P6.T1:` Thank you.

P7. JI: I am going to ask you some questions, so feel free to answer. How do you enjoy teaching of fractions?

P8.T1: I enjoy teaching of fractions when I use different representation.

P9.JI: Hmmm

P10.T1: yes, for example, I can use physical objects like papers, I basically like to use papers so that learners can see, because they are also having papers, so, we will do it together with them.

P11.JI: How are you doing it?

P12.T1: I let them take out one whole paper and then move to a half, meaning that, they divide that paper into a half and then into a quarter and so on.

P13.JI: Are you giving them instructions to follow?

P14.T1: Yes, I give them instructions.

P15. JI: What do you understand by the term multiple representations?

P16.T1: Hmmm, the way of using different methods of teaching, it can be any topic. It is used to understand, develop and to communicate different mathematical features of the same object. For example what I was doing in my lesson when I used one out of two and present it in different ways, where I used papers, folding a paper into half and write it on the chalkboard as symbol ($\frac{1}{2}$) and in words.

P17.JI: Okay

P18.T1: Yes, yes, yes, otherwise they would not know if you do not write it, they will not know what is a half. You have to try all the links for them to understand.

P19. JI: How do you manage to use multiple representations in your teaching?

P20.T1: By planning my lesson effectively, so that I can use different methods in my teaching. On the other hand, time is not enough and the syllabus contains too much content, it forces teachers to teach theoretically and to focus on one type of representation.

P21.JI: Why do you think some teachers are not using multiple representations in their classes? Do you think is because of the syllabus which contains too much content?

P22.T1: Definitely, syllabus requires people to finish earlier during third term, teachers have to rush so that they can finish with syllabus and start with revision.

P23.. JI: How do you think multiple representations contribute to the teaching of fractions?

P24.T1: It increases opportunities for the learners to develop understanding of fractions. It helps students to connect mathematical ideas using different tools when teaching fractions. It also increases the high interactions among learners. Lastly, it helps learners to learn from one another.

P25.JI: You mean when the learners actively involved, they learn more from one another than from a teacher?

P26.T1: Yes, because sometimes are afraid to ask the teacher, that is why sometimes, when I am teaching I usually call in one the child to come and solve some problems and explain and you can really see some learners pay more attention when another learner explains.

P27.Ji: Hmmm. okay

P28.T1: Hmmmm

P29.JI: On school mathematics, which one of the eight representations, the one that you have learn in the workshop are mostly used by the teachers and which ones are more neglected?

P30. T1: Mostly, they use descriptive written words and symbols and the one that is mostly neglected a lot is pictures, pictorial.

P31.JI: Why do think pictorial is more neglected?

P32.T1: Because of may be teachers do not have time to draw pictures and are ignorant to , get information and put them together, I think that is why they prefer always just to write on the board in words or in numbers.

P33.JI: You have also said earlier that teachers do not have time to draw pictures……

P34.T1: Yes, they do not have time, sometimes for them to go and buy materials is always a problem.

P35. JI: How do you feel about teaching of fractions?

P36.T1: No, I feel very good; I enjoy it very much, especially by doing it more practical, by using examples papers, flash cards, when you use flash cards learners enjoy so much. Most of the learners are more actively involved. Teaching is boring, it is better if learners are involved.

P37.. JI: Do you think that multiple representations help the child to learn fractions?

P38.T1: Yes, yes, it opens up their minds. They can be able to represent fractions into different ways into their own way. Learners make more connections, connecting mathematics to real world. When we are talking of real world, a learner might drop out of school and may end up in construction companies where they will use it to measure part of the whole or they can even apply it at home.

P39. JI: Where do you think multiple representations are not helpful?

P40.T1: Nowhere, they are very much helpful. It makes the life easier for the learners to understand fractions. If the learners do not understand this way, then the other way they might understand.

P41.. JI: How did the focus group workshop contributed to your teaching of fractions?

P42.T1: It was good, it opens up my mind in the use of multiple representations, when using different tools when teaching fractions rather than focusing on one. It opens my mind to design my own multiple representations, using physical objects that are affordable.

P43.JI: You mean from now you will start designing your own material?

P44.T1: Yes, exactly

P45.. JI: Is there anything that you want to change or not?

P46.T1: It was really good, I would like to change in the way I plan my lesson and I have to ensure that more than two presentations are used in one lesson

P47.JI: You mean, you would like to include more than two representations when teaching mathematics?

P48.T1: Yes, to make sure that, my learners understand.

P49. JI: In your planning what important choices have you made, when you were planning the lesson of fractions or what tools did you use and why did you use them?

P50.T1: I have used the papers, because they are readily available. I have also used posters and I design my own flash cards.

P53..JI: I have observed that when you were teaching, you used a lot of examples, a lot of representations in one example. Let say in an example of $\frac{1}{2}$and $\frac{1}{4}$, you used flash cards, papers and the chalkboard, why did you use so many examples?

P54.T1: For them to understand very well, for them to pick up something to make a difference that a half can be represented in words, symbols and pictures and for them to compare e.g. $\frac{1}{2}$and $\frac{1}{4}$, theoretically, learners will say $\frac{1}{4}$ is bigger than $\frac{1}{2}$, because 4 is bigger than 2, but if you show them practically, they would be able to know which one is bigger.

P55.JI: Mem, thank you so much for your contribution, our interview has come to an end. Is there anything that you want to add?

P56.T1: I think the university rectors should get involved and encourage the ministry to train more teachers and they must emphasize in the use of technology. It will help a lot because, learners find mathematics very difficult, but if tools are there, then they will not find it difficult.

P57.JI: Thank you mem for your time.

P58.T1: Thank you too

APPENDIX C2: TEACHER 2 (T 2) – INTERVIEW TRANSCRIPT

TEACHERS' INDIVIDUAL INTERVIEW TRANSCRIPTION.

P1.JI: Good afternoon and welcome to the interview using multiple representations when teaching fractions in grade 5 – 7.

P2.T2: Good afternoon and thank you very much.

P3. JI: How do you enjoy teaching of fractions?

P4.T2: I enjoy teaching of fractions, because it gives you a variety of teaching methods, you know, it's too practical, and you use different representations, use physical objects like papers or even anything that they can use, even themselves, you ask them, then they can be able to say……..

P5.JI: Yaaa, because fractions appear anywhere…..

P6.T2: Yes, yes that is the good practicality of it that is the good part, learners like working with pictures, you know they like holding, touching so that they should not forget what they have learnt.

P7.JI: What do you understand by the term multiple representations

P8.T2: Multiple representations are the different methods of teaching. They use to understand, they develop, they create, they communicate, you know……. A lot of things, different methods, you can also use technology as part of method of teaching. So, these are words, symbolize and describe a lot of mathematical terms, so it is more of different methods that you are using in your teaching.

P9.JI: How do you manage to use multiple representations in your teaching?

P10.T2: Learning plays a very important, so it gives you more time to plan your lesson, so that it must be effective. On the other hand, time is also limited to use the different methods but if the planning is done, you can be able to cover even fewer methods, if not on that day, you can still continue the following day to cover all the methods, because what you are looking at is the understanding of the learners. Once the learners catch up then you know it is easier for them to work on it. In addition to this, the syllabus contains too much content which force teachers to teach using one method only, because the time is limited and you are required to finish that syllabus at the end of the year .

P9.JI: Yaa , I know that teachers are rushing to complete the scheme of work but before they rush, they must take it too easy, learners come first, they must make the learners understand the whole content and to do this is only when you use different methods of teaching.

P11.JI: Yes, because for the learners to understand, you need to use different methods, different style for them to understand very well.

P12.T2: Yes, yes, definitely.

P13.JI: Now, like in terms of teaching fractions, how do you manage to use different styles when you are teaching fractions?

P14.T2: As I have said, once I am on the topic of fractions, it gives me time to sit and plan the easier and even look for an easier method which learners cannot forget, even if they are given something to do, so planning is very important on this thing that is why for me to make it more practical, it opens the mind, once you make it practical, it opens the minds of the learners. That's why I have to sit in so that I can plan nicely the methods or the possible ways that I can use to make fractions understandable to the learners.

P15..JI: Okay, How do you think multiple representations contribute to the learning of fractions?

P16.T2: It increases the opportunities for learners to develop understanding of fractions, and also it helps them to connect some mathematical ideas using different tools and teaching methods, you know critical thinking is also comes in once they are given something, they have to think for them to come up right, that is the good part. If you give them a practical work, they won't even just do it, they think how should I do it and for me to come right, so at the end what they have done by their hands they will not forget, than what you have written on the chalkboard, so that is the….. It increases the higher interaction among learners, you know they will be talking to each other, it is more of practical, yes like what they were doing in the lesson, and you know it is more of group work.

P17.JI: Yes, like what they doing in the lesson….

P18.T2: Yes, like what they doing in the lesson, you know it is more of a group work, they will be more talking and helping each other, then at the end they stand up and explain to other fellow learners and they will understand more easily.

P19.JI: Yes, that is good, and the worksheets that you gave, that is saying, cut a square into a half in four different ways and I saw how learners came up with different ideas…….

P20.T2: Yes, yes, they came up with different patterns, how they can cut other patterns……some patterns which I have seen I was not even thinking of, you give them opportunity to try for themselves, then they will come up with something that you not even expecting, so learners are good when they are given the opportunities.**P21.JI:** Yes learners are creative

P22.T2: Yes, definitely, you give of the chances to the learners to show off what they know.

P23. JI: In school mathematics, which of the eight types of representations which you have learnt in the workshop are most often used? Which are neglected? Why?

P24.T2: Mostly used are written math symbols, descriptive written words, pictorial presentation, oral presentation and experience- based and a video. Among the presentations mentioned, I prefer a video and then a concrete, where learners they use concrete things, because they are using it with their own hands, and so they can see if you talk of the half, you bring something that they see, you tell them to cut, they understand. A video also bring reality in the lesson. What they cannot even find in the class, once you use a video, then they can see what is going on, so it makes it easier than just writing, you draw, this is a half, once you wipe the chalkboard then, they can easily forget what you have said

P25.JI: Do you think video is mostly used in schools?

P26.T2: Not always, because the schools do not have some of the equipment, so videos is only those once are privileged schools, schools neglect the part of having materials, where it is the part that brings reality in the lesson for the learners to understand.

P27.JI: Among the eights representations which one do you think are mostly used in schools, not only in your school but other schools also.

P28.T2: Written math symbols, oral representation and descriptive written words also. E.x .$\frac{1}{2}$ in symbols, one out of two (a half) in words and you can relate it in terms of pictures. But videos or power point presentations are the best ways to teach fractions

P29.JI: Really, it is a good way, although many schools does not have videos…….

P30.T2: Yes, but there are many ways to get those things, it is only that schools are neglected some of this things.

P31..JI: How do you feel about teaching of fractions?

P32.T2: Oh, I feel good, before we started, then I told the learners that next week we will start with fractions, so learners were so excited, because that is the lesson were they are free to talk, you know to do other things, those are the topics where learners are actively involved. This topic is more practical, that is why the learners like fractions very much.

P33.JI: You say the learners feel good when they are learning fractions?

P34.T2: Yes, yes because they are more I involved in the lesson of fractions because it is too much practical. If a teacher wants the learners to understand it must be more of practical, it is a nice topic in the book which the learners enjoy.

P35. JI: Do you think that multiple representation help a child to learn fraction? Explain your answer.

P36.T2: Yes it opens up the minds of the learners, they do not focus, you know stereotyped….. it widens their minds. It gives them an opportunity of having a lot of methods that they can pick up and use what they understand. They are also be able to represent fractions in different ways. The learners should also make connections, they make connections in such a way that each one in the class can be helped, so it also develop the deep understanding of the learners in that different ways of teaching.

P37.JI: They can also use it at home……..

P38.T2: Yes, yes you give them more examples, then they say oh, even at home, even outside the class fraction is also being used, so it widens their minds.

P39.JI: Can you give other examples outside the classroom?**P40.**

P40.T2: A half of water in a cup, dividing a birthday cake, chocolate…..

P41.JI: Why did you use video clips in your teaching fractions? Do you think learners will learn more from the video clips?

P42.T2: You know video, kids watch what is going on, and you can even see the attention, learners will listen very carefully because they know afterwards there will be questions, so they enjoy. There was a song in there, they saw how the person was cutting and so on, so they will never forget, once you ask them questions, they will always remember that this question they were doing it when they were cutting and so on, so video is an excellent tool when teaching fractions, because learners are really actively involved.

P43. JI: Where do you think multiple representations are not helpful?

P44.T2: Nowhere, they are very much helpful; there is no where I can say multiple representations are not helpful, because that is the best way of teaching fractions, they are helpful once you try them, you will be able to make everything easier for you, as far as your lesson is concerned, and it makes it easier also for the learners to understand

P45.JI: But, why do you think teachers are not using them?

P46.T2: Teachers are just lazy to take time to plan; they are only going in the class as long as they taught, so long as the period is over. Teachers are just lazy to sit and plan, because you ask them, they will always say I do not have time of which they were supposed to have time to do their planning for the understanding of every teaching that you have in the class.

P47. JI: How is the focus group workshop contributed to your teaching practice?

P48.T2: It was good, because we teachers were more practical, we have learnt a lot of drawings. We have time even to divide things, you know, we were working as if we are learners. It was more interesting, a lot of teaching aids were there meaning, you can have a lot of aids that you can use but at the end of the day you mean one thing. It means that, the more, you use it, the more you understand, so it was really helpful.

P49.JI: Meaning that the materials that were used in the focus group workshop contributed a lot to your teaching practice?

P50.T2: Definitely, definitely.

P51.JI: Ooh, and you are planning even to go use them in your teaching……

P52.T2: Yes, yes, I have started using them, I gave them some activities, so they are really good.

P53.JI: So, you mean that your practice has been developed after the workshop?

P54.T2: Yes, yes because it means that even if you do not have a video in the class, also those practical that we did there, you can apply them in your class, learners can still be good in that topic.

P55. JI: Is there something that you want to change or not in your teaching practice?

P56.T2: Hmmmmm, the teaching, it is only the planning, I do not know whether we have more sources of teaching we should have, if I have to change it is the way we teachers are planning, for them to have this lessons implemented, we should also make sure that more than two presentations are being used in every, not only in fractions but in every teaching you should have more than two methods to use just to explain and to widen the knowledge of the learners.

P57. JI: Ooh, that is good, In your planning, what important choice did you make and why?/ What tools did you use and why did you use them?

P58.T2: Actually, I used a video, I started with the video, where there was a song and in the song, a song of fractions, learners enjoyed well, listening to this and see, how things are being cut into half into quarter while there is a song in it. And then after that I came up with the real lesson, also with the video where they were listening, that is why after the lesson when I gave them some handouts to do, they did not have any problem to explain or to put it on paper, because they have understood. Learners have seen what was happening on the video that was the main purpose to attract the interest of the learners. Video and power point presentations attracts the interest of the learners, they were so quiet and at the end of the lesson, especially the song, they were now singing with the people who were singing in the video. So the purpose of making a lot of presentations is to make learners understand.

P59.JI: You have also used worksheets/ handouts and they were also busy doing it and you were asking some of them to present the answers…….

P60.T2: Yes, yes sometimes if you ask the learners to stand up and present it motivates the learners, they say; ooh even me I can stand in front of others and explain and even

those ones who are weak can say, even my classmate can do that, why not me?, it motivates them. You find that the next question, everyone will raise their hands to be able to present or explain. They understand much better when they explain to one another than the teacher.

P61.JI: You know, learners are scared sometimes to ask the teacher…..

P62.T2: Yes, yes, the other thing is that, we are ignoring learner centered, but if we make 90% of the lesson is just learners talking and explaining and then you find that this learners should take control of the lesson and the lesson won't be boring because learners are involved.

P63. JI: I observed that when you taught fractions you used a lot of examples (used a lot of presentations to one example), like in the video you have also used a lot of examples, you used papers, you asked learners to stand up and say out any example of fractions that they have in their pencil cases or wherever in the class ,where learners did a lot of drill practice of different tools to a given example, both individually and group work, why did you do that, what encouraged you to do this?

P64.T2: Looking at this understanding, you know not everyone who was given those papers got everything correct, you know, other they got it wrong, but you know they learned from their fellow mates, so now, that is why given a lot of examples to do, activities to do and at the end you will find out that all the learners are on the right tract. If you give learners questions to try they will get it and some will volunteer to come and explain on the chalkboard, and some will say ooh, this is how he did his answer/ his pattern, next time they should be able to try and get it right.

P65.JI: Why did you do it that way? After one question, you finish and then you discuss it?

P66.T2: So that next time, they should not repeat the mistakes which they did in the first. So we discussed first questions and then in discussion learners are correcting their mistakes which they did, then the next example, then they will not do the mistakes again which they did on the first one. So it helps them to correct and to see where

they are wrong to correct immediately, at the end of the lesson all of them not even 100% but at least 80% to 90% understand what was going on, that was the purpose to let everyone to contribute in the lesson.

P67. JI: I noticed that in your teaching of fractions you emphasized on learner centered, learners were actively involved the whole period. What motivated you to do this?

P68.T2: That is the only way to let the learners understand, especially if it is you who is talking, the learners will get bored and they will not even understand well because they are not involved, so that is why I make by all means that every lesson a learner at least 80% of my lesson/ teaching, learners are involved in it. They will be able to understand more than me standing there just explaining till the end of the period , it is focusing more on understanding.

P69.JI: Thank you mem, our interview has come to an end.

P70.T2: Thank you very much also.

APPENDIX C3: TEACHER 3 (T 3) – INTERVIEW TRANSCRIPT

TEACHERS' INDIVIDUAL INTERVIEW TRANSCRIPTION.

P1. JI: Good afternoon sir

P2.T3: Good Afternoon

P3.JI: Welcome to the interview on the teaching of fractions using multiple representations to the grade 5 learners. Feel free to answer the questions which I am going to ask you. You are more than welcome to add or to ask me, if there is anything that is not clear.

P4.T3: Thank you.

P5. JI: How do you enjoy teaching of fractions?

P6.T3: Yaa, actually is a quiet interesting topic , you know, when you as teacher tend to use valid methods for valid teaching ideas of presentations to teaching fractions to make it more interesting. And of course, I enjoy teaching of fractions using flash cards, where the drawings have been given to the learners so that they can see fractions in real life approach. And also it is important when you have a classroom where learners are mostly involved in working with fractions using diagrams of pictures, that has been given to them, in that case, it becomes easier to work on fractions with the learners.

P7.JI: Which method do you mostly use to enjoy teaching of fractions; I know there are different methods, which is mostly appropriate to you, which makes you to enjoy teaching of fractions?

P8.T3: Like what I have said earlier, it is very good to teach fractions using flash cards and also using technology in the classroom. For instance using power point presentation to explain basic fractions and also using videos, for the learners to see how fractions take place in reality. For instance, in the video which I have used there were pictures of flamingos in a certain sea area, there were five flamingos on one area that was shown in picture and two left. The question was how many flamingos are left from a group that was there? So, two went, three were left. In terms of fractions the learners could easily present to say that is two out of five ($\frac{2}{5}$) left, the area where they were. Using video in the classroom could very crucial and very, very critical, because learners are watching how problems are being solved and in this way learners attention will be attracted more and learners will be engaged more on the lesson.

P9.JI: What do you understand by the term multiple representations?

P10.T3: It is simple different methods one topic. You use different approaches in terms of methods in terms of explaining a certain particular lesson. Variety of approaches to represent a single lesson, for example teaching numerators and denominators by flash cards, video or by power point representations. So, when we are using a variety of approaches in terms of presenting that particular lesson, which is a multiple representations.

P11.JI: How do you manage to use multiple representations in your teaching focusing on mathematics and then on fraction itself?

P12.T3: I think what is important in that essence is effective learning, the moment you have make sure that your planning has been set out into a very strategic way, but yet think now to the learners, it is very good, you won't really say that it is a lot of problems in teaching. Meaning applying multiple representation and also you should be able or I normally when I am presenting a lesson, I have to make sure; I get different materials from various sources. Like from internet for instance you download pictures that has been once used and inducted to make teaching easier, in that way you can be able to manage it.

P13.JI: How do you manage to use variety of resources if the time is limited to 40 minutes?

P14.T3: You do not need to do everything at the same time, you need to focus on the part which you think, and learners will be able to catch up on that particular time. You need to remember as well that, learners have to do activities based on that particular approach. If you have presented the two approaches within that particular lesson, I would say with activities, we need to do practical in that approach with the learners. The learners should not only watch the video, but they should also be able to reflect on what they have watched. I think it is also important, if you want to give more work, it is important to organize extra classes, so that you can present more methods, that is also good because of the limited time that we have.

P15.JI: Okay, How do you think multiple representations contribute to the learning of fractions? Is it contributing?

P16.T3: Exactly, Firstly, it increases the interactions in the class, learners are more involved, especially when they have watched a video. In normal cases when the learners do not like some of the classes, but if they are watching like my approach of using videos, when they watch, they are more easily controlled and they can be able to interact thereafter when they are done watching the video. It is also help them to connect mathematical ideas using different tools when we are teaching fractions.

P17. JI: In school mathematics, which of the eight types of representations which you have learnt in the workshop are most often used? Which are neglected? Why?

P18.T3: perhaps I should start we the most used. Looking at the presentation that we have here, I would say, the descriptive written words is used mostly often by the teachers and written math symbol, in most cases we also used pictures especially when you are giving introductory lesson to the learners. Normally, oral, you speak then you use pictures, you connect the oral with the pictures, but I would say looking at the experience that we have in the teaching, we do not use videos and power points. Firstly, this could be may be the materials are not readily available in schools; we do not have them at. Secondly, it is not all learners are exposed to these materials (videos and power point), that could be one of the reason why we do not at all times use videos or power point into our classroom teaching fractions.

P19.JI: And do not you think that power points and videos are good to be used in classes, although we do not have them in our schools?

P20.T3: They are very, very excellent to be used into the classroom; sometimes I wish we all have these resources in our schools. Actually, I have loaded and as you have seen in my lessons that when the learners are able to view what their learning, it is easy for them to capture, instead of us talking and preaching things to them because they do not have any practical ideas that we are telling them but when they see and it is very easy for them to connect and recall. That is why I would feel using technology in the classroom like videos and power point presentations is very crucial method of teaching fractions.

P23.JI: How do you feel about teaching of fractions?

P24.T3: As a teacher I always have this excitement in terms of making sure that, I would want to make my learners understand. So I am happy to talk to the learners, I am happy to present this to them, I am also happy to come across some of those challenges that the learners are bringing in the classroom environment, where we are able to say look, if you have this things in particular situation in terms of fractions, how do you solve it? Or how do you approach it? So it makes me feel good. I want to make it more practical, when I am teaching my kids, I don't want to make it an oral issue, I want to make it a practical issue, where I will be able to give them examples from their houses where they are coming from, the classroom where they are sited. For example you have 20 learners in grade 5A, three of the learners are boys. To present this as a fractions, three is the part of the whole, the whole is 20 learners, so, three over twenty ($\frac{3}{20}$) is the correct expression in terms of fractions. I want to make it more practical, more reality to them, so it makes me feel good when teaching fractions.

P25.JI: Good, so, what do you think are the feelings of the learners? Do they feel good? Do they like the topic itself? Is it difficult or easy to them? Are the learners actively involved when you teach them?

P26.T3: Like I said earlier, when we use pictures like in the video, for instance and power point, you know that animation is normally enjoyable to the learners. The test the I have tested in the class is that, when I use pictures in whatever video in the classroom kids tend to enjoy more than when we just talking. In other words the kids are fine learning fractions using different methods. So, it is just upon us the teachers to make sure you put in too much effort in terms of making sure that the kids do not become bored when you are teaching them fractions.

P27.JI: But, some teachers are saying fraction is very difficult to teach, what is your view on this?

P28.T3: No, no, that is a perception that can be overwhelmed in terms of our approach.

P29.. JI: Do you think that multiple representations help a child to learn fraction? Explain your answer.

P30.T3: Yes, it opens their minds; it does not bind them to one way of using it. It brings them back into reality of doing things like what I said earlier, it tells them that,

this is what we are learning in the classroom, that this are not just things that we are talking, but it is something that you are on a day to day approaching or have approached in your real life. So, in that way it opens up their mind in terms of how their view the learning of fraction itself.

P31.JI: In the lesson that I have observed, you used videos / power point presentations only. Why did you use that one, why not other multiple representations?

P32.T3: We have been teaching with other approaches like oral presentation, so it proves that with videos and power point representations, we do really get the impact, and learners are actively engaged in the lesson . Other representations do not give you the real impact that you want. I have decided, let me try video to see because we now normally, learners enjoy watching things in reality, why cannot we use them so that we bring the reality to them. That is actually the simple reason why I opted to use videos.

P33. JI: Where do you think multiple representations are not helpful?

P34.T3: If not used effectively, they might not be helpful. I can say 90% of the multiple representations can be helpful, if we use them effectively or if we do our planning effectively. So, I would say, it is very much useful in many cases, if we present them in an effective way.

P35.JI: So, you are only saying they are not helpful if they are not used properly?

P36.T3: Exactly

P37. JI: How did is the focus group workshop contributed to your teaching practice? How did your teaching developed after the workshop?

P38.T3: It was interesting workshop, we have to learn different ways in terms of how others are approaching their teaching, it was very crucial. It opens up my minds, so that I can be able to use more other multiple representations into my teaching to make the learning of my children effective , so it was an eye- open up for that matter. It contributed to my development as a teacher in that particular topic of fractions.

P39.JI: You are saying the multiple representations that you have learnt in the workshop; you will use them in your teaching?

P40.T3: Exactly, I will be able to use them in my class

P41. JI: Is there something that you want to change or not in your teaching practice?

P42.T3: I think, the most important thing, like what I have said earlier, I would want to do more planning in terms of doing so reluctant on one way of approach (traditional approach). I would like to do more planning, read more resources, find more ways on how others or even the globe in the whole world have approach the teaching of fractions, that is something I would be willing to gain knowledge on.

P43. JI: Ooh, that is good, In your planning, what important choice did you make and why?/ What tools did you use and why did you use them?

P44.T3: The choice that I have made in my planning is on assessment tool like worksheet to make sure that we reflect and would be able to find out to what extent, what impact my lesson which I have with my learners had impact on what we have been doing. So, assessment as the tool that I have used, the reason is very simple, to make sure that I find out to what extent did my teaching have an impact on the children.

P45. JI: I observed that when you were teaching fractions, you used power point presentation and you also gave worksheets to the learners to work on, why did you do that?

P46.T3: It is a culture, it is a norm. It is not good to keep on preaching things and you did not find out, to what extent do they make impact on the learners, that is the reason why I gave them worksheets to check whether the new method that I have decided to include in that particular lesson, the use of power point presentation has worked and to see whether it is helpful for the learners to understand the learning of fractions, that is the reason why I gave them assessment worksheets.

P47.JI: Thank you so much sir, I hope you enjoyed the questions that I have asked you.

P48.T3: Thank you very much too.

APPENDIX D: WORKSHOP HANDOUT

WORKSHOP HANDOUT

ACTIVITY 1: SHARING OF FRACTIONS

- Leaning equivalent fractions

  Teachers can use this interactive tool to help students build a conceptual understanding of fractions by linking visual models to numeric representations. Students learn about equal parts of a whole (denominators) and shaded parts (numerators). This page includes a video demonstration of the tool and sample lessons from the Conceptual curriculum. Free registration is required to use the tool. A paid subscription is necessary to access full curriculum and allow full student use.

1. a) How can 20 sweets be shared among 5 best learners in your class.
   b) Divide a chocolate (20 bars) that is being shared by a mother and her two daughters.

2. Mr Hendircks gives a price to the group in his class that has behaved the best during the week. The prize is the box with 10 chocolate bars.

a) This week Ann's group wins the prize. There are four people in Ann's group. They all want the same amount of chocolate. How much chocolate does each child get?

b) Last week it was John's group that won the prize. There are six people in John's group. How much chocolate does each child get?

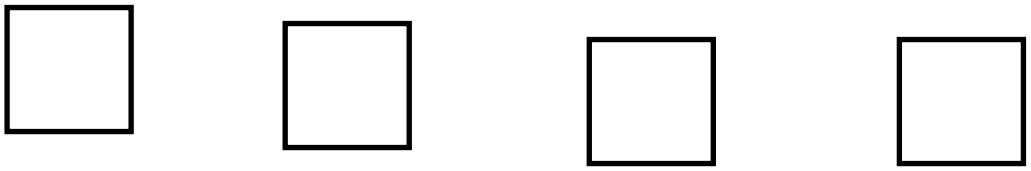
3. Four friends go on a full day outing. At lunch time they are hungry. They have 8 sandwiches to share for lunch. How do these friends share the sandwiches fairly for lunch?
   a) How can you look at this situation to see fractions?
   b) Use different representations to describe the situation above?
      - Hint use pictures and symbols (draw)
      -Concrete materials such as pens, books, etc
      - Posters

- Real sandwiches, etc

4. Selma has three friends. Her mother bought a cake for her birthday. She shares the cake fairly with her three friends and the mother. Describe this situation into different representations to see fractions?

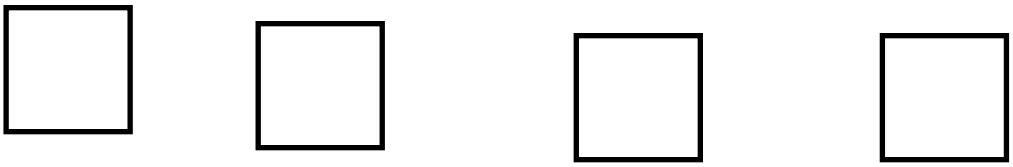
ACTIVITY 2: FRACTIONS – SUBDIVISION

1. Show four different ways of cutting up a square into 2 equal parts (halves - $\frac{1}{2}$).



2. Show four different ways of cutting up a square into four equal parts (quarter - $\frac{1}{4}$).

3. Can you find different ways to cut up these squares into three equal parts (thirds - $\frac{1}{3}$).



ACTIVITY 3

Task Rotation- Math/ Fractions

Core Content: Apply multiple representations to describe fractions (halves, thirds, fourths)

HOOK- Think about our study of fractions and how you have learned to use and identify them in real life situations.

Mastery Task	Interpersonal Task
Thinking of fractions strips, draw ½, $^1/_3$, and ¼. Then put them in order of least to greatest	Write a letter to your parents telling them why you would rather have ¾ of your favorite cookie rather than ¼.
Understanding	**Self- Expressive**
Compare how 3/6 is the same as ½. Draw pictures to explain.	Design and draw a pizza divided into fractions. All parts must be the same and must equal 1.

ACTIVITY 4 – PART – WHOLE DIAGRAMS

Develop the concept of fractions as numbers if used in the way that highlights the unit and the connections between fractions and whole numbers.

4.1. Part – whole diagrams for fractions less than one unit.

4.2. Diagrams for fractions equal to one unit

4.3. Mixed numbers e.g. 2 units and $\frac{3}{4}$

4.4. Addition of proper fractions

a) Represent $\frac{3}{4} + \frac{2}{3}$ in the form of diagrams.

b) Represent $\frac{5}{8} + \frac{7}{8}$ in the form of the diagram.

c) Represent $\frac{7}{10}$ in the form of the diagram.

APPENDIX E: INTERVIEW QUESTIONS

A) Introductory Questions

1. How do you enjoy teaching of fractions?
2. What do you understand by the term multiple representations?
3. How do you manage to use multiple representations in your teaching (Focus on maths and then fractions)?
4. How do you think multiple representations contribute to the learning of fractions?
5. In school mathematics, which of the eight types of representations which you have learnt in the workshop are most often used? Which are neglected? Why?
6. How do you feel about teaching of fractions?
7. Do you think that multiple representations help a child to learn fractions? Explain your answer.
8. Where do you think multiple representations are not helpful?

B Questions arised from the focus group workshop

9. How was the focus group workshop contributed to your teaching practice/ how is your practice developed after the workshop?

10. Is there something that you want to change or not?

11. In your planning, what important choice did you make and why?/ What tools did you use and why did you use them?

12. I observed that when you taught fractions you used a lot of examples (used a lot of presentations to one example), where learners did a lot of drill practice of different tools to a given example, both individually and group work, why did you do that, what encouraged you to do this?

13. I noticed that in your teaching of fractions you emphasized on _____ why and what motivated you to do this

APPENDIX F: REQUEST TO CARRY OUT RESEARCH

Enquires: Julia Iiyambo 08 January 2014

To: The Director, Inspector, Principals and Teachers

Dear …….

RE: Request for permission to conduct research

Iam Julia Iiyambo, a head of department (HOD) for Mathematics and Sciences at J. G. Van Der Wath Secondary School. I am currently a part time student with Rhodes University, Grahamstown, in South Africa, (Student no. 11I5240). I have been studying for the Master's Degree in Mathematics Education since January 2013, For this degree, I am required to undertake a research project in my field of study, Mathematics Education.

The goal of my research is to explore how teachers may work with multiple representations to teach fractions for conceptual understanding in grade 5 – 7.The research will utilize narrative methodologies including interviews of teachers, observation of teachers, video-recordings and audio recordings. The focused teachers will be provided with participants consent forms in advance before carrying out the data collection process. Participants' names will not be used in this research nor will any activities pose any risk of physical, mental or social harm to either participants or the school itself.

The purpose of this letter is to request your permission to carry out this research in your school. The research will involve a focus group interview and a number of individual interviews with each teacher, as well as a number of videotaped classroom observations of lessons by the teacher. Videos will only be viewed by me as a researcher and supervisor.

I will thank you for your support in advance and would be most grateful if you grant me permission.

Yours Sincerely

……

APPENDIX G: TEACHERS' CONSENT FORM

Student informed consent form

I _____ agree voluntarily to partake in Mrs Julia Iiyambo's research project. I am aware that the information to be collected will be reflected in her report, but I am being assured that confidentiality and anonymity will be strictly considered throughout the handling of such information. I am aware that I can withdraw my participation any time during the research process.

…….. ……

Signature of participant Date