AN ECONOMETRICAL ESTIMATION OF THE DEMAND FOR MONEY IN SOUTH AFRICA

(The Long-run Function During the Period 1918 - 60)

by

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Thesis presented to the Faculty of Arts of
Rhodes University in fulfilment of the requirements
for the Degree of Doctor of Philosophy.

Grahamstown

May, 1969

INTRODUCTION

"Men are largely motivated by three basic desires - food, love and money - and of these, the least understood is money." [1]

In recent years there has been a marked upsurge in the output of literature dealing with the demand for money, but with the exception of the North American Continent, empirical research has lagged distressingly far behind the voluminous output of theory. This dearth of empirical results has had a restrictive influence, since many of the controversial points which are being disputed by theoreticians can only be resolved by recourse to empirical methods. The restriction of empirical research to the North American Continent has further meant that the various points under dispute have had only a limited qualification and consequently monetary theorists have had no indication as to the universalisability of their conclusions. There is thus a great need for empirical studies in other countries so that the validity of the rival theories can be tested under different conditions.

It was with these thoughts in mind that the present study was undertaken. Its objectives are strictly national and no pretence of strict international comparability is made. Further, great care has been taken to avoid the pitfall so beloved of econometricians, the fallacy of reduction wherein strictly limited results are uncritically universalised. Thus no attempt has been made to draw conclusions which will have universal validity.

The theoretically vital points which are going to be examined in the light of South African experience are:

1. The feasibility of distinguishing idle from active balances, and if this proves possible, the determination of the wealth and interest elasticities of these balances.

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[1] Time, (Jan. 15, 1968), p. 30.

2. Dropping the explicit distinction between idle and active balances to

(a) determine the role of interest rates,

(b) determine the appropriate constraint on the demand function,

(c) determine what effect different definitions of money have on (a) and (b).

3. To examine the stability of the demand function over time.

Truth is, of course, many-sided and any uniform presentation can only aspire to present a one-sided picture, just like a photograph cannot hope to do justice to the full grandeur of nature, merely presenting a one-dimensional representation of a many dimensioned object. In spite of this restriction which is inherent in all econometrical studies, this one-sided picture seems to be justified in view of the lack of any unified and coherent treatment of the demand for money in South Africa.

Chapter Headings.

CHAPTER I

A. The Historical Development of the Concept of the Demand for Money.

1. The Concept Prior to Keynes.

The upsurge of interest in the demand for money, both as a theoretical concept in its own right, and as an important determinant of the effectiveness of monetary policy, is a comparatively recent development. In fact one might say, without too great a fear of committing a travesty of historical justice, that it received its baptism in its present volitional form in Keynes' magnum opus, The General Theory.[1] For prior to Keynes' work, the demand for money was accorded a rather mechanistic treatment in a small subsection of classical economic theory which was grafted on, almost as an afterthought, to the main body of theory concerning the workings of the real sector in order to explain the annoying fact that although the real supply and demand equations determined relative prices, they were completely unable to determine the general price level. In its role as the determinant of the general price level, monetary equations appeared in two guises.

(i) The Fisher Equation of Exchange. This was developed at the turn of the century by Irving Fisher[2] and took the form

$$MV + M'V' = PT$$

where M is the quantity of money (notes and coins), V is its velocity of circulation, M' is the quantity of chequable deposits, V' its velocity of circulation. P is the general price level of all transactions and T is the physical volume of transactions. The equation is envisaged as holding during an arbitrarily defined time period, and must necessarily hold true as an accounting identity on account of its tautological nature. Fisher did not, however, envisage it in this light, and accorded it the

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[1] J.M. Keynes, The General Theory of Employment, Interest and Money (London, 1936).

[2] I. Fisher, The Purchasing Power of Money (New York, 1911).

status of a behaviouristic equation linking the six, supposed independent, variables. V and V', which are the respective transactions velocities, were treated as institutionally determined constants and hence the demand for money relative to the volume of transactions was treated as an entity entirely independent of any volitional factors.

(ii) The Cambridge Cash Balances Equation. On account of the fact that the approaches to the quantity theory of Marshall, Pigou and the early Keynes were quite similar, and since they were all at Cambridge, this approach has come to be known as the Cambridge Theory. As Marshall[3] has shown, this theory has a long history, going back to Petty. Various algebraic representations of differing complexity have been presented, culminating in that of Pigou's[4] viz:

$$P = \frac{kR}{M}\left[c + h(1-c)\right] \quad \text{or} \quad M = \frac{kR}{P}\left[c + h(1-c)\right]$$

However, all the representations had as their basis the concept that an individual desires to keep a certain proportion (k) of his resources in the form of money (M). Instead of concentrating on the volume of transactions, they envisaged real national income (Y) as representing the country's resources, so the elements of the Cambridge theory can be represented by the equation M = kY. None of the Cambridge school made any detailed analysis of the factors influencing k, although it was supposed to depend in some unspecified manner on the wealth of the community, its habits and the institutional framework. Thus it can be seen that the pre-Keynesian treatment of the demand for money was essentially mechanistic, paying only lip-service to volitional elements.

2. The Keynesian Treatment.

Keynes' major contribution to the theory was his explicit consideration of the role of money as a store of value in an economy

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3 A. Marshall, Money, Credit, and Commerce (London, 1923), p. 47.

4 A.C. Pigou, "The Value of Money", Quarterly Journal of Economics, Vol. 32, (Nov., 1917), pp. 38 - 65.

characterised by uncertainty. Previous economists, although in some cases having explicitly recognised the effects of uncertainty on the demand for money, had not placed great emphasis on it and consequently had not paid much attention to money's role as a store of value. On account of the fact that these observations were not assimilated into the main body of theory, further discussion would only detract from the main line of thought and so the matter has been relegated to appendix A. Keynes' approach paid attention to the determinants of expenditure but concentrated on the special characteristics of money as a form of holding wealth. He classified the motives for holding money into three groups <u>viz.</u> the transactions demand; the precautionary demand which, as its name suggests, is the desire to hold an asset whose value is fixed in terms of money in order to meet unforeseen future expenses; and the speculative demand which is the desire to hold money in order to speculate on the future capital value of bonds. He postulated that the amount of money demanded (M_1) to meet the transactions and precautionary motives was mainly a function of the level of real income, and that the amount of money demanded for speculative purposes (M_2) "mainly depends on the relation between the current rate of interest and the state of expectation";[5] and then proceeded to demonstrate that it would bear an inverse relationship to the rate of interest. Keynes' demand function may thus be written:

$$M = f(r,Y) \text{ where } \frac{\partial M}{\partial r} < 0, \frac{\partial M}{\partial Y} > 0 \text{ and } M = M_1 + M_2.$$

In the process of obtaining a more explicit version of the function, he considered M_1 and M_2 separately, asserting that it was "a safe first approximation to regard the amounts of these two sets of cash-holdings as being largely independent of one another."[6] Under this assumption, the demand function becomes:

$$M = M_1 + M_2 = L_1(Y) + L_2(r)$$

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[5] J.M. Keynes, op.cit., p. 199.

[6] ibid., p. 199.

where L_1 is the liquidity function corresponding to M_1 and L_2 the liquidity function corresponding to M_2.

Out of fairness to Keynes, especially in the light of the further developments of the theory and criticisms of his work, it must be emphasised that he viewed the above function merely as a workable first approximation, for he did not believe that in practice money held for the three purposes could actually be segregated into water-tight compartments since "the same sum can be held primarily for one purpose and secondarily for another."[7] Further he explicitly mentioned the possibility of transactions and precautionary balances being interest elastic for they "will partly depend on the cheapness and the reliability of methods of obtaining cash, when it is required, by some form of temporary borrowing."[8]

Empirical attempts of a verification of the Keynesian hypothesis have followed two general lines of approach. The first of these is typified by the work of Latané.[9] He started from the equation $M = f(r,Y)$, made the simplifying assumption that the demand function was homogeneous of degree one in income, and hence obtained the result:

$$M = f(r)Y \quad \text{i.e.,} \quad M/Y = f(r).$$

The term on the left-hand side of the second equation can be interpreted as a macro-economic measure of the reciprocal of income velocity, and hence his empirical investigations consisted in estimating the relationship between income velocity and the rate of interest. He plotted M/Y against $1/r$ and obtained a hyperbolic relationship which he regarded as verifying the Keynesian hypothesis.

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7 ibid., p. 195.

8 ibid., p. 196.

9 H.A. Latané, "Cash Balances and the Interest Rate - A Pragmatic Approach", Review of Economics and Statistics, Vol. 36, (Nov., 1954), pp. 456 - 60 and H.A. Latané, "Income Velocity and Interest Rates : A Pragmatic Approach," Review of Economics and Statistics, Vol. 42, (Nov., 1960), pp. 445 -49.

A far more sophisticated approach is that exemplified by Tobin,[10] who attempted an estimation of the liquidity preference schedule proper. He estimated idle balances by subtracting from total money balances an estimate of active balances derived from the maximum recorded velocity of circulation. The results showed an approximately hyperbolic relationship between idle balances and the rate of interest which he interpreted as representing Keynes' speculative demand schedule.

The central theme of these investigations was the influence of interest rates on the public's demand behaviour. The evidence so obtained was consistent with the role attributed to interest rates by Keynes. However, it should be noted that none of these investigations went beyond an inference drawn from a given sample to evaluate a Keynesian hypothesis against a null hypothesis. Thus the evidence is not the result of a critical examination of rival systematic hypotheses. The problems involved in attempting empirical verification of a Keynesian hypothesis and a survey of the pertinent literature are fully discussed in Chapter IV, so no further mention will be made here. Instead we shall divert our attention to some of the important developments which have their origins in Keynes' work.

3. Developments Subsequent to Keynes.

One of the first developments was a movement towards the dropping of the distinction between idle balances and active balances and its implicit assumption of a topological dichotomy in the effects of the variables. For Keynes' theory of the demand for money is essentially a rather ungainly hybrid of two theoretically inconsistent approaches, with the transactions demand being regarded as technologically determined and the asset demand being treated as a matter of economic choice. Out of fairness to Keynes, we must once again note that in fact in his text he anticipated some of the developments whilst regarding the disaggregated function essentially as a first approximation.

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[10] J.R. Tobin, "Liquidity Preference and Monetary Policy", Review of Economics and Statistics, Vol. 29, (May, 1947), pp. 124-31.

Subsequent thought has tended to follow two lines of development. The first has been to aggregate the demand function on the grounds that the public does not hold two stocks of money, but instead holds just one stock of money, though it holds this stock for various motives which include the transaction and asset demands. The second has been to develop a capital theoretic approach to the transactions demand, the idea being that money is a kind of working capital and money holdings form an inventory serving that function in the process of production and consumption. Just as there is maximising behaviour with respect to inventories of goods, so there is maximising behaviour with respect to the inventory of money. Baumol[11] and Tobin[12] formulated the idea that transactions balances were determined by the volume of transactions within an environment of pre-determined payment schedules. Both derived a function relating transactions balances to the volume of transactions, interest rates and some cost elements associated with the conversion of financial assets. The payment schedules are considered as entities independent of volitional factors, simply appearing in the guise of imposed constraints within which the economic agent must operate. The only choice available is in the time pattern of conversions between money and other financial assets, since the time pattern of payments between money and non-financial commodities is in the nature of an external constraint. The relevance of the above analysis cannot be assessed without some auxiliary specifications, on account of the absence of a semantic rule characterising transactions balances in terms of observable entities and the absence of an explicit merger of a sub-hypothesis bearing on transactions balances with a sub-hypothesis covering asset balances.

The next development was the rehabilitation of the precautionary motive, albeit in a different guise, which in Keynes' treatment paled into insignificance compared to the speculative motive, and

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11 W.J. Baumol, "The Transactions Demand for Cash : An Inventory Theoretic Approach", Quarterly Journal of Economics, Vol. 66, (Nov., 1952), pp. 545 - 56.

12 J.R. Tobin, "The Interest-Elasticity of Transactions Demand for Cash", Review of Economics and Statistics, Vol. 38, (Aug., 1956), pp. 241 -47.

was relegated to a minor position along with transactions balances. On account of the speculative motive's strong dependence on inelasticity of expectations, it cannot be more than a short-run phenomenon and a highly psychological one at that. It is the dependence on this latter factor that led Hicks to dismiss it as a theory which is left "hanging by its own bootstraps".[13] Because of these objections, subsequent Keynesian writings have stressed the role of risk-avoidance in the face of uncertainty as to the future value of the rate of interest. This approach has been elegantly formulated by Tobin,[14] Matthews[15] and others using the concepts of portfolio analysis; the former showing that risk avoiding behaviour implies the existence of an inverse relationship between asset holdings of money and the rate of interest and the latter showing that even if people regarded bonds as being less risky than money on account of them yielding a certain nominal stream of income, the above conclusion still holds.

A further attack on Keynes arose from his restrictive assumption that bonds were the only financial assets other than money. The presence of short term interest bearing assets of certain capital value has made the existence of speculative holdings of money tenuous in the extreme, since the rational investor who has sold bonds in anticipation of a rise in the rate of interest is no longer likely to hold a barren asset such as money when interest bearing assets of equivalent certainty are available. Thus the choice facing the speculator is not so much whether to hold bonds or money, but if he has decided not to hold bonds, in what form of liquid assets he should then hold his wealth. Little has been published by economists subscribing to this school of thought, the best known expository articles being those

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[13] J.R. Hicks, Value and Capital (Oxford, 1939), p. 164.

[14] J.R. Tobin, "Liquidity Preference as Behaviour Towards Risk", Review of Economic Studies, Vol. 25, (Feb., 1958), pp. 65 - 86.

[15] R.C.O. Matthews, "Expenditure Plans and the Uncertainty Motive for Holding Money", Journal of Political Economy, Vol. 71, (June, 1963), pp. 201 - 18.

by Hicks,[16] Duesenberry[17] and Tobin.[18] An attempt has been made by Hicks[19] to synthesize the theory of risk avoidance and the existence of short term liquid assets, but it is too early to comment on the ramifications of this synthesis.

The major advance has been the introduction of wealth, which itself depends on the rate of interest, as an explicit determinant of the demand for money. This inter-connectedness of wealth and interest has raised many conceptual and econometrical problems via the introduction of multicollinearity between the variables, but in spite of this, it represents a major advance because it brings money into the fold of capital theory. Keynes made allowances for wealth effects in his treatment of the consumption function, but he did not consider it in his demand for money. Subsequent developments have followed one of two paths, depending on the particular definition of wealth adopted.

The majority of authors adopted the conventional definition of wealth, restricting it to non-human assets. Metzler[20] showed that if the wealth effect was significant, the liquidity preference curve would be different for a change in the quantity of money brought about by fiscal policy than for a change brought about by open-market operations, on account of the wealth effect of the latter. In addition, the point arises that the speculative demand curve traced out by open-market operations will differ depending on the size of the units used. The wealth effects of discontinuities in open-market operations are

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16 J.R. Hicks, "A Suggestion for Simplifying the Theory of Money", Economica (N.S.), Vol. 2, (Feb., 1935), pp. 1 - 19.

17 J.S. Duesenberry, "The Portfolio Approach to the Demand for Money and Other Assets", Review of Economics and Statistics, Vol. 45 (Supplement), (Feb., 1963), pp. 9 - 24.

18 J.R. Tobin, "Money, Capital and Other Stores of Value", American Economic Review (Proceedings), Vol. 51, (May, 1961), pp. 26 - 37.

19 J.R. Hicks, Critical Essays in Monetary Theory (Oxford, 1967), Chap. 1 - 3.

20 L.A. Metzler, "Wealth, Saving, and the Rate of Interest", Journal of Political Economy, Vol. 59 , (April, 1951), pp. 93 - 116.

exploited by Weintraub[21] and Davis[22] in their contention that the speculative demand curve is irreversible.

A radical departure from the orthodox approach adopted above is provided by Friedman's[23] restatement of the quantity theory and his Permanent Income hypothesis. The main tenets of this theory are firstly that the quantity theory is a theory of the demand for money, not of output, income or prices; and secondly that money is a capital good, so that the demand for it is a problem in a capital theory. The theory takes as its starting point a broad concept of wealth which includes human beings, and relates the demand for money to total wealth and the expected future streams of money income obtainable by holding wealth in alternative forms. Then by a process of mathematical simplifications, economic simplifications and re-arrangements of variables, he arrives at a demand function which depends on the price level, bond and equity yields, the rate of change of the price level, the ratio of human to non-human wealth, income and a taste variable. However, in his empirical work, he postulates a simplified function of the form: $M = kYp^{\delta}$ where $\delta > 0$ and Yp is Permanent Income. Further, he emphatically states "that the demand for money is highly stable - more stable than functions such as the consumption function."[24]

However, Friedman's work still leaves a lot to be desired for not only does he fail to deal satisfactorily with the conceptual problem of the relationship between the rate of interest and wealth but also fails to deal adequately with the econometrical problems raised by his concept of Permanent Income. Permanent Income is evisaged as being a weighted average of past and future money incomes, and is defined as

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21 S. Weintraub, An Approach to the Theory of Income Distribution (Philadelphia, 1958), pp. 156 - 62.

22 R.M. Davis, "A Re-examination of the Speculative Demand for Money", Quarterly Journal of Economics, Vol. 73, (May, 1959), pp. 326 - 32.

23 M. Friedman, "The Quantity Theory of Money - A Restatement", in M. Friedman (ed.), Studies in the Quantity Theory of Money (Chicago, 1956).

24 ibid., p. 16.

the flow of income arising from a stock of wealth, this wealth being defined so as to include human wealth. The empirical measure of Permanent Income which he uses is called expected income and consists of a weighted average of past incomes. However, by bringing lagged variables into the function several econometrical problems are encountered, a fact which Friedman apparently fails to take cognisance of.

The most important implication of Friedman's analysis, however, does not concern the formulation of monetary theory *per se*, but the nature of the concept of income relevant to monetary analysis, which should correspond to the notion of the expected yield on wealth, rather than the conventions of national income accounting. By explicitly considering the human element as part of total wealth, he has raised many important questions as to the role played by human wealth in economic theory. However, as will be shown in later chapters, the concept of human wealth is extremely hard to quantify and consequently most of the questions posed are as yet unsolvable.

B. A Theoretical Analysis of the Motives for Holding Money.

4. An Analytical Distinction Between Money and Other Liquid Assets.

A theoretical analysis of the demand for money does depend on the distinction between individuals choosing rationally to hold money balances and not merely possessing what money balances they happen to have, which may be entirely dependent on the institutional framework. Before getting any deeper embroiled in the theory of the demand for money, a decision must be taken as to the proper definition of money. Definitions *per se*, are not "right" or "wrong"; they are merely adequate or inadequate for the purpose in mind. In economic literature dealing with the theory of money, three main methods of defining money have emerged.

i) A strict accounting or legalistic definition. Although this is the definition to which we must ultimately turn in order to obtain an empirical measure of the quantity of money, as it stands it is unsatisfactory because it takes no account of the motives for using money nor of any underlying economic theory.

(ii) A statistical definition. One strand of this approach seeks clues to substitutabilities among assets by studying how sensitively holdings of currency, demand deposits and other liquid assets depend on such variables as income, wealth and interest rates. The other strand seeks that quantity which correlates most closely with variables such as income, wealth and interest rates in demand equations statistically fitted to historical data. Two overriding methodological criticisms of this approach spring immediately to mind. The first is that it is not obvious why the quantity which correlates most closely with other quantities deserves overriding attention. For example, the correlation between consumption and national income might be closer than that between money and national income, yet it is the latter that is most interesting for monetary theorists. The other criticism is of a subtler nature and has been overlooked by several economists, especially those subscribing to the modern version of the quantity theory, but is none the less of sufficient import to make the use of this definition inappropriate. Implicit in this definition is the assumption that the demand for money is stable over time. This in fact is one of the major tenets of the modern quantity theorists. Thus the definition of money is critically dependent on the assumed stability of the function and as will be shown later this is by no means always the case. Even if the demand function was stable, no indication is given as to which of the many alternatives should enter the function as arguments and so it is easily seen that this method lays itself open to a charge of circularity of logic. A further practical point arises when we consider whether the statistically fitted function is sensitive enough to determine the margin between money and near-money.

Both the above definitions suffer from excess rigidity in that they take no account of actual economic processes, instead forcing an exogeneously determined rigid behaviour pattern onto the living inhabitants of a changing economy. Furthermore, by ignoring economic motivation, the above definitions become spatially and temporally restricted. It should by now be obvious that what is required is a definition that is centred around the economic functions of money.

(iii) A functional definition of money. From the time of Aristotle[25] money has been defined as an asset which has the properties of acting as a unit of account (or a measure of value), a medium of exchange, and a store of value. If one were to select at random a modern introductory textbook on the theory of money and to turn to the section where the author defines money, the reader will find the above properties mentioned, perhaps followed by a brief discussion of these properties often not taking up more than a paragraph, then the author hurries on to less mundane subjects such as what assets have actually served as money. It might be surmised from this brevity that the functions are obvious and ipso facto not warranting further discussion. However, after a little reflection it is not immediately obvious exactly what these functions are and to what extent they are mutually dependent on each other.

The first function, viz. that of acting as a unit of account or a measure of value, presents no great difficulty. It is merely acting as a yardstick whereby relative values in exchange can be calculated. This is a passive function of money, and need not necessarily occur in conjunction with the other two. An example of this occurs in the United Kingdom where some prices are expressed in guineas whereas there is no actively circulating unit of currency corresponding to it, the unit of currency being the pound Sterling.

The second two functions, those of a medium of exchange and a store of value are far more difficult conceptually. The medium of exchange function arose from the urge to avoid the inconvenience of the double coincidence of wants inherent in direct barter. Thus money came to be used as the third or intermediate good in the process of barter, and in so doing enabled the hitherto instantaneous acts of buying and selling to be separated in time. Money is thus capable of acting as a means of storing value over time, and it is this property that is referred to as the store of value function.

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[25] Aristotle, Nichomachean Ethics Bk. 5, Chap. 5, para 10.

Any analytic discussion of money's role as a medium of exchange is usually cast in the framework of static general equilibrium theory, comparing two equilibrium positions after the elapse of an arbitrarily specified period of time, e.g. "Hicksian week". Money which is used to transfer purchasing power from one such time period to succeeding time periods is then regarded as acting as a store of value. Thus any definition of the temporal aspect of money acting as a store of value depends on the duration of the time period specified. As this is abstract and of no fixed duration in terms of physical time, the distinction becomes extremely blurred at the margin. If money which acts as a temporary abode of purchasing power over a period in excess of the period taken for exchange to occur, which itself is an unspecified period, is regarded as a store of value, the definition can be seen to rest on extremely tenuous roots. The solution to this dilemma must lie in the fact that the functions of money as a medium of exchange and a store of value are analytically inseparable. An extreme case can be considered to illustrate this point. Money must act as a store of value between exchanges considered as taking place within the chosen time period, for if it did not it would not be used as a medium of exchange; and at the other extreme, if a store of value such as diamonds, which satisfy the requirements for an asset to be a store of value, was not used as a medium of exchange it could not be regarded as money. Thus these two properties of money are functionally interrelated and although an individual might give prominence to one over the other in certain situations, one function cannot be considered in isolation from the other. A striking analogy can be drawn between the above and the dual nature of light as conceived by modern theoretical physics. Physicists have come to regard the physical phenomenon of light as either having an atomic nature, i.e. consisting of a stream of discrete corpuscles called photons, or as a form of wave motion. Here are two apparently different properties attributed to the same physical entity, yet neither of them considered in isolation from the other is capable of a satisfactory explanation. This interdependence of these two functions of money will be further discussed when we attempt to join together the motives for holding money with the functions of money.

In recent years, as a consequence of the prominence given to the asset function of money by Keynes in his writings, a controversy has arisen over the problem of distinguishing between money and near-money. This problem was first officially aired in the Radcliffe Report[26] which gave prominence to the amount of liquidity present in the economy rather than the conventional concept of the supply of money. No universally accepted definition of near-money exists, though a typical one is that given by Gerhard de Kock; "assets held by the private sector which can be monetised en masse, conveniently, within a reasonably short period and without significant loss".[27] If we look at the status of assets as a store of value from the point of view of the owner, the monetary quality consists in the ability to exercise the purchasing power which these assets represent, by right and without significant penalty or delay. Thus financial assets constitute a class within which the individual components are differentiated only by the point in time at which they possess the characteristic of entitling the owner to means of payment. Eminently satisfactory as this may be for the purposes of monetary control where the time factor is often of prime importance, it does not provide a very satisfactory basis for pure monetary analysis.

The above discussion is couched in terms of money's function as a store of value to the almost complete eclipse of its property of acting as a medium of exchange. As was shown above, exclusive consideration of one property with the consequent neglect of the other cannot give a balanced picture since the two functions are inextricably inter-woven.

It is thus the medium of exchange property to which we must turn in order to remove ourselves from the horns of this dilemma. The

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26 Committee on the Working of the Monetary System (Her Majesty's Stationary Office, Command 827).

27 G. de Kock, "Money, Near-Money and the Monetary Banking Sector", South African Reserve Bank Quarterly Bulletin. No. 79, (March, 1966), p. 12.

analysis so far has been couched in micro-economic terminology since we have considered the functions of money from the point of view of the individual. The time has now come to consider money from a different point of view, namely the macro-economic viewpoint. It is by no means obvious that micro-relationships will carry over unchanged on aggregation to a macro-form on account of the well-known fallacy of composition. It is indeed this, considered in conjunction with the medium of exchange property that will enable us to distinguish functionally between money and near-money.

On aggregation, the problem arises as to whether certain assets which are undoubtedly a form of wealth for their owners, are still wealth for the economy when aggregated. The controversy as to what assets are not wealth on aggregation and the distinction between inside and outside money, interesting and important as they may be in their own right, has no direct bearing on the problem under consideration and thus will receive no further mention. We shall adopt the concept of neutrality as developed by Newlyn[28] to enable us to distinguish functionally between money and near-money. Newlyn develops his neutrality distinction between money and near-money by enquiring whether a payment financed from a holding of an asset does or does not tend to change either its total quantity or its price in the appropriate markets. He classifies an asset as money if the effect of disposing of some of it to make a payment is neutral in the sense of neither changing the total amount of that sort of asset in existence nor disturbing directly its market. A non-neutral effect occurs when the person making the payment either sells some asset or draws upon his claim on a financial intermediary causing a corresponding reduction in the total of such claims and a consequential sale of an asset by the intermediary. Newlyn shows that the two most important assets that pass his neutrality criterion and consequently count as money are currency, used to denote notes and coins, and bank deposits. This

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28 W.T. Newlyn, "The Supply of Money and its Control", Economic Journal, Vol. 74, (June, 1964), pp. 327 - 46.

approach has earned the approval of Yeager[29] who further traces out the differing macro-economic consequences of an excess demand for money and for near-money in the absence of any specific price or market for money. The adoption of this criterion enables us to distinguish satisfactorily for the requirements of this study between money and near-money. Unless otherwise stated, money will now be considered to be any financial asset which satisfies the above-mentioned three functions and further satisfies Newlyn's criterion for neutrality.

5. The Existence of Time as a Reason for Holding Money.

The next step in the analysis is to attempt to discover some sort of relationship between the functions of money and the motives for holding money. This is inherently a micro-economic problem and it will be dealt with as such apart from an occasional foray into the field of macro-economics. Hicks has attempted to establish such a relationship in his lectures on "The Two Triads",[30] but as will be shown below his treatment of the transactions demand is lacking in completeness. Consequently we shall adopt an alternative approach. We shall show that there are two fundamental reasons why an individual should rationally choose to hold money balances; the existence of time and the presence of uncertainty. The conceptual problems involved in linking up the functions of money with the reasons for holding money are extremely complex, and so we shall adopt a procedure of hierarchal partial equilibrium model building with each succeeding model more complex than its predecessor. The end result is not intended to be an all-encompassing representation, but merely one of a number of such alternative representations. We shall attempt to link up the two active functions of money with the three reasons given by Keynes for holding money.

Let us start by considering a world where there is no uncertainty and consequently people do not use money as a store of value

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29 L.B. Yeager, "Essential Properties of the Medium of Exchange", Kyklos, Vol. 21, (Fasc. 1, 1968), pp. 45 - 69.

30 J.R. Hicks, op.cit.

to counter this uncertainty. It is generally accepted that money as a medium of exchange plays no part in the final position of the general equilibrium models of such economists as Pareto and Walras. Admittedly Walras does make use of one good as a numéraire to determine relative prices but the choice of this good is arbitrary and it does not have the required properties of money. Furthermore, the prices that are determined are not intrinsic to the system being merely a set of accounting prices impressed upon the system by the observing economist in order to explain its workings. As Hicks[31] has shown it is possible to have a form of fiat money circulating to avoid the inconvenience of barter but as this does not enter directly into the utility functions of the individuals, no rational individual will choose to hold money in the final equilibrium state. Thus if the market is considered in isolation, Hicks is quite correct in his assertion that money holdings are a dis-equilibrium phenomenon. A long and highly mathematical controversy has been sparked off by Patinkin[32] as to the consistency of classical general equilibrium theory and money, and its offspring the classical dichotomy in the process of which more has been learned about the mathematics of general equilibrium and of the difficulty in obtaining economic conclusions from mathematical equations than about the economics of the problem itself. The outcome is that although it is possible to construct a consistent general equilibrium system with money balances entering explicitly into the utility functions of the individuals, this is a phenomenon restricted to the short run since in long run equilibrium when more than one payments period is considered, these money balances will disappear. Interesting and informative as it may be in its own right, this controversy does not have any immediate bearing on the present line of argument, and consequently no further mention will be made of it.

The stationary nature of the classical general equilibrium theory makes it patently unsuitable as a model for the description of

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[31] ibid.

[32] D. Patinkin, "Relative Prices, Say's Law, and the Demand for Money", Econometrica, Vol. 16, (Apr., 1948), pp. 135 - 54.

the dynamic world, so the next step is to allow for the presence of time in our model. It is not sufficient merely to superimpose a progression of classical stationary states on top of each other and to use this as a model with a temporal dimension as Hicks has done, for in doing so we are ignoring the crucial fact that there will now be imperfect synchronisation between payments and receipts. In this case money acts as a store of value between receipts and payments, but if it is the only liquid store of value available then imperfect synchronisation between receipts and payments is not in itself sufficient to provide a volitional motive for holding money. This point is clearly made by Schumpeter who wrote that "If people get their 'incomes' each Saturday and spend them on consumer's goods each succeeding Monday - transactions between firms being excluded - then the money will lie about in the vaults of the firms from Monday to Saturday, not because there is any demand for cash holdings, but because the institutional arrangement so wills it."[33] The assumption that money is the only liquid store of value is extremely restrictive and so must be dispensed with as we proceed up the hierarchy of models. Once we allow for other liquid stores of value we must be certain not to abstract from consideration of the conversion costs between money and other liquid assets. Of these costs the most important are (i) the costs of investing and disinvesting a small sum for a short period which can easily exceed the interest earned on it; and (ii) the leisure lost by having to think and give instructions about investing small sums which may have a higher utility than the interest which can be earned on them. Both these costs have their origins in the fact that human action takes time. A.W. Marget was one of the first to recognize this fact when he wrote: "Even in a world in which everything were perfectly foreseen, a lack of synchronisation between the receipt of income and its outlay would give rise to a need for cash-balances so long as there are not perfect facilities for the borrowing of money in anticipation of receipts and the investment of money during the period elapsing

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[33] J.A. Schumpeter, Business Cycles (New York, 1939), Vol. 2, p. 547.

between receipt and outlay."[34]

An analytic treatment of the transactions demand showing the volitional elements, cast in the framework of imperfect synchronisation of payments and receipts with imperfect credit facilities has been developed by Baumol,[35] Johnson[36] and Tobin.[37] In view of the fact that their work consists of an analytic presentation of the ideas developed above and does not add any new dimensions to the discussion, a detailed presentation would only detract attention from the main aurgument. A simplified analytical synthesis of the above ideas is presented in section 1 of the mathematical appendix to this chapter.

6. The Existence of Uncertainty as a Reason for Holding Money.

The next step up the hierarchy consists in allowing for the presence of uncertainty. By uncertainty we mean uncertainty as to the future capital value of marketable interest bearing assets, commonly called by the generic title of bonds. In allowing for uncertainty we have opened up the field for an explicit consideration of money's function as a store of value. Keynes postulated two motives for holding money as a store of value, the precautionary motive and the speculative motive.

The precautionary motive is the desire to have available for future requirements and unforeseen contingencies a certain proportion of total resources in the form of money. Keynes thus considered it alongside with the transactions motive, postulating that the amount of money held would be largely insensitive to changes in the interest rate and would be dependent on the volume of transactions, and consequently that the precautionary motive could not be a source of interest elasticity in money holdings. However, as will be shown later, it is just this

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34 A.W. Marget, "The Monetary Aspects of the Walrasian System", Journal of Political Economy, Vol. 43, (April, 1935), pp. 160 - 61.

35 W.J. Baumol, op.cit.

36 H.G. Johnson, Essays in Monetary Economics (London, 1967), Chap. 5.

37 J.R. Tobin, "The Interest - Elasticity of Transactions Demand for Cash", op.cit.

motive for security which is the most plausible source of interest-elastic money holdings.

Keynes was never clear in his writings as to the exact mechanism which lay behind the aggregative liquidity schedule, but the following explanation seems to be the most plausible. The object of speculation is to secure profit from knowing better than the market what the future will bring forth. As Kaldor[38] has shown, the market for bonds and securities is an ideal market for speculation. Each individual has a subjective impression as to what the future capital value of bonds and hence the rate of interest is going to be and he holds this impression with complete certainty. It is this certainty, which has been termed inelasticity of expectations, on which the whole theory stands. Thus anyone whose opinion "differs from the predominant opinion as expressed in market quotations may have a good reason for keeping liquid resources in order to profit if he is right."[39] The individual investor is thus envisaged as either holding all his surplus wealth in the form of money, or as investing it in bonds depending on the relation between his expectations and the market quotations. This does not imply that there exists a smooth relationship between the individual's speculative money holdings and the rate of interest; rather it implies a discontinuous relationship between money holdings and the rate of interest on account of the nature of the choice facing the individual. However, Keynes was quite correct in assuming that because each individual had a different subjective appraisal of the future, aggregation of the differing discontinuous schedules would yield a continuous relation between the rate of interest and speculative money holdings. That this is essentially Keynes' idea can be seen from the following quotation: "It is interesting that the stability of the system and its sensitiveness to changes in the quantity of money should be so dependent on the existence of a <u>variety</u> of opinion about what is

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38 N. Kaldor, "Speculation and Economic Stability", Review of Economic Studies, Vol. 7, (Oct., 1939), pp. 1 - 27.

39 J.M. Keynes, op.cit., p. 169.

uncertain."[40] However, Keynes and most subsequent authors have erred in regarding the liquidity preference schedule as having a roughly hyperbolic shape, for on aggregating what is essentially a unimodal distribution, a S-shaped curve must result. Either Keynes and his followers were unaware of this fact or else they chose to ignore it. The former is perhaps the most likely alternative in view of the lack of rigour in Keynes' treatment of the speculative motive.

Two very important criticisms of the above theory come immediately to mind: (i) it cannot explain diversification, i.e. why an individual holds both money and bonds simultaneously without having recourse to either the transactions or precautionary motives, and (ii) it relies critically on the assumption of inelasticity of expectations. It was the failure to meet these two objections that led to renewed interest in the precautionary motive and its eventual re-emergence as the principal reason underlying the observed inverse relationship between holdings of money for purposes other than that of transactions and the rate of interest. The concept was originally formulated analytically by Tobin[41] in a path-breaking article. The argument in this article is mathematical, but the essential ideas can be presented quite simply.

Suppose that individual wealth-holders have no reason to assume that the rate of interest is any more likely to change in one direction than the other, but that they recognise that it will probably change. If they hold money, they can neither gain nor lose from changes in the interest rate. If, on the other hand, they hold bonds they are taking a risk in that they stand either to lose or gain. However, holding bonds not only involves risk but also a gain. The higher this gain, the more individuals there will be for whom the prospect of gain makes them willing to assume risk; or for any individual, the larger the percentage of his wealth that he is willing to commit in

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40 ibid., p. 172.

41 J.R. Tobin, "Liquidity Preference as Behaviour Towards Risk", op.cit.

the risky form and the smaller the percentage he will prefer to hold in cash. Since the gain obtained from holding bonds depends on the rate of interest, the demand for cash as an asset will bear an inverse relationship to the rate of interest. This explanation has the advantage of not depending on inelasticity of expectations and of explaining diversification. It also has, however, the additional property of being far more stable than Keynes' speculative demand schedule on account of the fact that changes in expectations have no effect. An analytic presentation of the ideas developed above is given in section 2 of the mathematical appendix to this chapter.

7. Summary.

It has been demonstrated that a volitional demand for money exists as a consequence of two factors; the fact that human action takes time, and the presence of uncertainty as to the future capital value of bonds. Lack of synchronisation between payments and receipts in the presence of perfect conversion facilities between money and near-money is a necessary but not sufficient condition for a volitional transactions demand, since account must be taken of the costs of conversion between money and near-money, which arise as a consequence of the fact that human action is not instantaneous. Money's certainty as to its future nominal value in the face of uncertainty about future bond prices gives rise to its position in the portfolio of a risk avoiding investor who is adjusting the composition of his portfolio so as to balance risk against expected gain.

APPENDIX A. Some Un-orthodoxies in the Classical Treatment of the Demand for Money.

Orthodox Classical economists paid little attention to money's role as a store of value in the face of uncertainty and although certain economists anticipated future developments, their conclusions did not become part of the generally accepted theory of the Classical school.

a. Henry Thornton, writing in 1802 on paper credit, realised that the subjective valuation of money would depend to some extent on the state of confidence concomitent with expectations of the future. Writing on the incentive to hoard and the consequential variations in the velocity of circulation of bank notes, he observed that "a high state of confidence contributes to make men provide less amply against contingencies. At such a time, they trust, that if the demand upon them for a payment, which is now doubtful and contingent, should actually be made, they shall be able to provide for it at the moment; and they are loth to be at the expence (sic) of selling an article, or of getting a bill discounted, in order to make the provision much before the period at which it shall be wanted. When, on the contrary, a season of distrust arises, prudence suggests, that the loss of interest arising from a detention of notes for a few additional days should not be regarded." [42]

b. In an essay written in 1829/30, John Stuart Mill discussed the neutrality of money as well as the influence of the precautionary motive on money holdings. Regarding the differences between a monetary and a barter economy, he argued that the use of money yielded utility on account of the fact "that it enables this one act of interchange to be divided into two separate acts or operations; one of which may be performed now, and the other a year hence, or whenever it

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[42] Henry Thornton, An Enquiry into the Nature and Effects of the Paper Credit of Great Britain (1802) edited by F.A. v Hayek, (London, 1939), pp. 96 -97.

shall be most convenient." [43] Thus underlining the fact that the presence of money breaks the inescapable link between the buyer and seller in a barter economy by allowing these operations to take place at different points in time. He did not think that depression was caused by an increased demand for money, but rather that if a period of excessive speculation had forced prices down, it was this increased demand for money that transferred the transitory drop in prices into prolonged deflation. The process of deflation was envisaged as that state of affairs where "persons in general, at that particular time, from a general expectation of being called upon to meet sudden demands, liked better to possess money than any other commodity. Money, consequently, was in request, and other commodities were in comparative disrepute." [44]

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[43] John Stuart Mill, "Of the Influence of Consumption upon Production", in Essays on Some Unsettled Questions of Political Economy, (London, 1844), p. 70.

[44] ibid., p. 72.

APPENDIX B. A Mathematical Treatment of the Transaction and Asset Demands.

No claim to originality is made for the theory presented below, that of section 1 being a synthesis of work done by Baumol,[45] Johnson[46] and Tobin,[47] whilst that of section 2 being a synthesis of the theories developed by Hicks[48] and Tobin.[49]

1. The Transactions Demand.

The analysis will be framed in a time period of unit duration. Suppose an individual receives an amount T in money at the beginning of the period and spends it in a steady stream until it is exhausted at the end of the period. Suppose also that there exist short term interest bearing assets which we shall distinguish by the generic title of bills which bear interest at the rate of i% per period, no matter how long invested. These bills are characterised by absolute security and do not serve as a medium of exchange. Conversion from money to bills and vice-versa is not costless, these costs being comprised of a fixed charge and a variable charge depending on the size of the transfer. More explicitly, let Y be the sum being transferred, then the cost of the transfer

i) from money to bills is $B + KY$

and ii) from bills to money is $b + kY$.

The rational transactor will obviously not convert all his income into bills immediately on receipt because he has to start financing expenditure the instant the period starts, so he would then at once have to convert back into money in order to meet his immediate transactions needs and in so doing he would incur a double penalty.

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45 W.J. Baumol, op. cit.

46 H.G. Johnson, op.cit.

47 J.R. Tobin, "The Interest-Elasticity of Transactions Demand for Cash", op. cit.

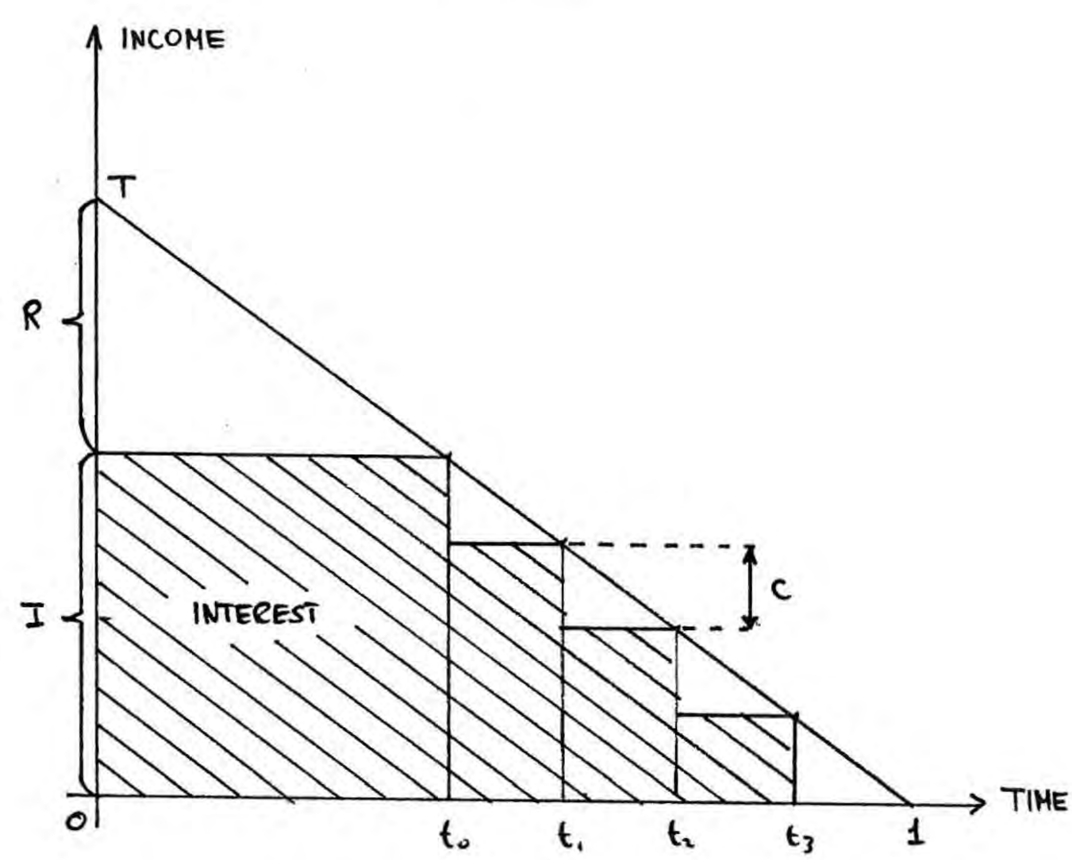
48 J.R. Hicks, op.cit., chap. 6.

49 J.R. Tobin, "Liquidity Preference as Behaviour Towards Risk", op.cit.

He will thus retain an amount R of his income T in the form of money so as to meet his immediate needs and convert the rest I = T-R into bills at once. It should be immediately obvious that (i) he will do so immediately on receipt of his income, for to delay the conversion would entail an interest loss, and (ii) a conversion from bills into money should not occur until the money balance is zero, for to make the transaction before it is necessary only loses interest that would be earned by holding bills for a longer time.

It should also be intuitively obvious that once R has been exhausted, the most efficient conversion policy is to draw out cash in evenly spaced equal amounts. [A rigorous proof of this is given by Tobin op.cit.] A graphical representation of the proposed scheme is shown in diagram 1, where once his initial holdings of money have been depleted, he finances his expenditure by converting his initial holding of bills I into money by a series of four equally spaced withdrawals of an amount C.

DIAGRAM 1.



For convenience in exposition, we shall split up the payments period into two sections, from $t = o$ to $t = t_o$ where expenditure is being financed by R; and from $t = t_o$ to $t = 1$ where expenditure is being financed by drawings on I.

A. Since the payments are continuous, R will serve for a period $\frac{R}{T} = \frac{T-I}{T}$. The average cash holding for this period is $\overline{R} = R/_2 = \frac{T-I}{2}$, therefore the cost of holding R, as measured by the interest lost is $\frac{T-I}{2}\frac{T-I}{T}i$. The cost of investing an amount I is B + KI, and hence the total cost for the first period is:

$$\frac{T-I}{2}\,\frac{T-I}{T}i \;+\; B \;+\; KI \tag{1}$$

B. He now has to start converting I into money to meet his payments. Suppose he makes n withdrawals of an amount C i.e. $n = I/C$. His average cash-balance over the period is $\overline{C} = C/2$ and the effective rate of interest is $\frac{I}{T}i$. Hence the cost of holding this amount of money, as measured by the interest lost is $\frac{C}{2}\,\frac{I}{T}i$. He makes n withdrawals at the cost of $b + kC$ per withdrawal and the total cost of the conversion is:

$$nb + knC \;=\; b\,I/C + kI \;=\; (b + kC)\,\frac{I}{C}\,.$$

Thus the total cost for the second period is:

$$\frac{C}{2}i\,\frac{I}{T} \;+\; (b + kC)\,\frac{I}{C}\,. \tag{2}$$

The total cost (X) over the whole transactions period of obtaining cash is the sum of (1) and (2) viz:

$$X = \frac{T-I}{2}\,\frac{T-I}{T}i + B + KI + \frac{C}{2}i\,\frac{I}{T} + (b + kC)\,\frac{I}{C}$$

The problem can once again be split into two parts, corresponding to the previous division.

1. To determine the optimum size C of the cash withdrawals. This can be obtained by partially differentiating X with respect to C and setting the result equal to zero.

$$\frac{\partial X}{\partial C} \;=\; \frac{iI}{2T} \;-\; \frac{bI}{C^2} \;=\; 0$$

$$\text{i.e. } C \;=\; \sqrt{\frac{2bT}{i}}\,. \tag{3}$$

2. To determine the optimum size of R, the amount of income initially retained in the form of money, differentiate X partially with respect to I and set the result equal to zero.

$$\frac{\partial X}{\partial I} = -\frac{T-I}{T}i + K + \frac{Ci}{2T} + \frac{b}{C} + k = 0$$

$$R = T-I = \frac{C}{2} + \frac{bT}{Ci} + T\frac{K+k}{i}$$

On substituting from (3) we obtain:

$$R = C + T\frac{K+k}{i} \qquad (4)$$

Both the average cash holdings in the first period $\bar{R} = R/2$ and the average cash holding in the second period $\bar{C} = C/2$, exhibit an inverse relationship with the rate of interest i, _ceteris paribus._ It should be intuitively obvious that the average cash-holding, considering the period as a whole, should also exhibit this type of relationship. However, it is an easy matter to give an explicit verification of this.

The average cash balance held over the whole period is simply the weighted average of the average cash-holdings of the two periods. It thus consists of an amount R/2 held for $\frac{T-I}{T}$ and C/2 held for I/T. Making all the necessary substitutions we obtain:

$$\frac{C}{2}\frac{I}{T} + \frac{R}{2}\frac{T-I}{T} = \frac{C}{2}\frac{I}{T} + \left[\frac{C}{2} + \frac{T}{2}\frac{K+k}{i}\right]\frac{T-I}{T}$$

$$= \frac{C}{2} + \frac{T-I}{2}\frac{K+k}{i}$$

$$= \frac{C}{2} + \frac{1}{2}\cdot\left[\frac{C}{2} + \frac{bT}{Ci} + T\left(\frac{K+k}{i}\right)\right]\frac{K+k}{i}$$

$$= \frac{C}{2} + \frac{1}{2}\left[C + T\left(\frac{K+k}{i}\right)\right]\frac{K+k}{i}$$

$$= \frac{C}{2}\left[1 + T\left(\frac{K+k}{i}\right)\right] + \frac{T}{2}\left(\frac{K+k}{i}\right)^2$$

$$= \sqrt{\frac{bT}{2i}}\left[1 + T\left(\frac{K+k}{i}\right)\right] + \frac{T}{2}\left(\frac{K+k}{i}\right)^2$$

which clearly shows the inverse relationship with the rate of interest.

2. The Asset Demand.

Suppose that the investor has an amount K to invest in n alternative financial securities for a specified period and that at the end of this period he realises all his investments. Furthermore, assume that an amount x_j; $j=1 \ldots n$, is the amount invested in the jth security. There are m mutually exclusive outcomes of the various investments, one of which must occur. Let a_{ij} be the outcome of one currency unit placed in the jth security if the ith eventuality materialises. Which of these eventualities will actually occur is not known, but the coefficients a_{ij} which form the matrix (a_{ij}) are supposed known. Further, suppose that the investor is operating in a perfect market characterised by the property that the expected outcomes are unaffected by his actions, i.e. the matrix (a_{ij}) is independent of the amount invested in the jth security, x_j. We must also assume that the investor is capable of assigning subjective probabilities, p_i; $i=1 \ldots m$, to the m possible outcomes.

i) let $v_i = \sum_{j=1}^{n} a_{ij}x_j$ be the outcome for the whole portfolio if the ith eventuality materialises,

ii) let $e_j = \sum_{i=1}^{m} p_i a_{ij}$ be the mean value or expectation of the outcome of one unit placed in the jth security,

iii) let $E = \sum_{i=1}^{m} \sum_{j=1}^{n} p_i a_{ij} x_j = \sum_{i=1}^{m} p_i v_i = \sum_{j=1}^{n} e_j x_j$ be the expectation of the outcome of the whole portfolio.

The problem now is to find an index, which must refer to the total prospect, which the investor can realistically be represented as maximising. Three alternatives present themselves.

a) The simplest alternative is the maximisation of E. In spite of its appealing simplicity, this method suffers from a failure to allow risk-spreading and thus is unnecessarily restrictive. For since $E = \sum_{j=1}^{n} e_j x_j$, and under our assumption the e_j are given to the investor, maximisation of E subject to $\sum_{j=1}^{n} x_j = K$ is the maximisation of a linear form against a linear restraint; so that E will be maximised if the whole capital is put into that investment where e_j is the highest.

b) The Bernoullian principle takes as its maximand the expected utility. The investor is supposed to maximse $U = \sum_{i=1}^{M} p_i u(v_i)$ where $u(v)$ is a function with a positive first derivative and a negative second derivative. This method is extremely difficult to use apart from in the simplest cases and further suffers from the disadvantage that it is rather mechanistic, so yielding results of dubious economic significance.

c) The third alternative is to visualise the investor as choosing between alternative probability distributions of his entire portfolio. A probability distribution more often than not requires more than two moments to specify it. For the sake of representational simplicity we shall restrict the analysis to the first two moments of the distribution, ignoring moments of the third degree or higher. Since the third moment is a measure of the kurtosis or skewness of the distribution, this restriction means that we are ignoring any possible effects of kurtosis. This in reality is a significant assumption, for in the case of a symmetric distribution the investor is prepared to face an equal loss or gain with complete equanimity. However, in real life this is not always the case as the disasterous effects of a loss will often outweigh the advantages of an equal gain. Thus this restriction means that our investor is indifferent between losses and gains. This might be considered as an unduly severe assumption, but the magnitude of the task involved in working with three parameters far outweighs the disadvantage of the restriction; hence for reasons of survival we shall let it stand.

The first two parameters of a distribution are the expected value and the variance. Since the units in which the variance is expressed are the square of those of the mean and in view of the difficulties inherent in assigning any economic significance to the square of a sum of money, we shall work instead with the square root of the second moment, i.e. the standard deviation. (If we were considering the third moment as a parameter we would accordingly have to take its cube root for it to have any economic significance). A choice which has been reduced to between two parameters <u>viz.</u> E the expected value and S the standard deviation can be represented on a two dimensional diagram, but further analysis is needed before such a representation is profitable.

The investor is assumed to be maximising a utility function of the form,

$$U = U(E,S) \quad \text{where} \quad \frac{\partial U}{\partial E} > 0 \quad \text{and} \quad \frac{\partial U}{\partial S} < 0$$

Consider the case where the investor is assumed to have put some of his capital into each of the n possible securities. For the investor to be in an equilibrium position, the principle of equimarginal returns must apply which in turn implies that:

$$\frac{\partial U}{\partial x_j} = \text{constant}; \quad j = 1 \ldots n.$$

$$\frac{\partial U}{\partial x_j} = \frac{\partial U}{\partial E}\frac{\partial E}{\partial x_j} + \frac{\partial U}{\partial S}\frac{\partial S}{\partial x_j}$$

but $E = \sum_{j=1}^{n} e_j x_j$ and so $\frac{\partial E}{\partial x_j} = e_j.$

$S^2 = \sum_{j=1}^{n} \sum_{k=1}^{n} r_{jk} \; s_j s_k x_j x_k$ where s_j is the standard deviation of one unit placed in the jth security i.e. the jth column of (a_{ij}) and r_{jk} is the correlation coefficient between the outcomes of the jth and kth investments, i.e. between the jth and kth columns. Thus since the matrix (a_{ij}) is known, the s_j's and r_{jk}'s are also known.

We shall now make a further simplifying assumption viz. that the prospects of the various investments are uncorrelated i.e. that

$$r_{jk} = 0, \quad j \neq k$$
$$= 1, \quad j = k.$$

Once again this might appear to be an unnecessarily restrictive assumption to make, but it does simplify the mathematics enormously and further does not effect in any essential manner this approach to portfolio selection any more than the presence of complements and substitutes invalidates the marginal theory of consumer behaviour. This point is made very clearly in the graphical analysis where the only two assets are money which is assumed to be risk free and bonds which are considered the risky investment, in which case there can be no correlation between the outcomes of the investment.

Under these assumptions $S^2 = \sum_{j=1}^{n} s_j^2 x_j^2$ (1)

and hence $S \frac{\partial S}{\partial x_j} = s_j^2 x_j$ i.e. $\frac{\partial S}{\partial x_j} = \frac{s_j^2 x_j}{S}$

Thus we obtain $$\frac{\partial u}{\partial x_j} = \frac{\partial u}{\partial E} e_j + \frac{\partial u}{\partial S} \frac{s_j^2 x_j}{S}$$

These must be equal to the same constant for all j. Put $W = -\frac{\partial u}{\partial S} \Big/ \frac{\partial u}{\partial E}$, the marginal rate of substitution between E and S. Then:

$$e_j - W \frac{s_j^2 x_j}{S} = M \qquad j = 1 \ldots n. \qquad (2)$$

Two cases now arise:

1. There is no investment, the outcome of which is certain. Under this assumption all the s_j are positive.

On solving (2) for x_j we obtain:

On substitution into (1) we get: $x_j = \left(\frac{e_j - M}{s_j^2}\right)\left(\frac{S}{W}\right)$; $j = 1 \ldots n$.

$$S^2 = \sum_{j=1}^{n} s_j^2 x_j^2 = \sum_{j=1}^{n} \left[\frac{s_j^2 (e_j - M)^2}{s_j^4} \frac{S^2}{W^2} \right] = \sum_{j=1}^{n} \frac{(e_j - M)^2}{s_j^2} \frac{S^2}{W^2}$$

which gives $W^2 = \sum_{j=1}^{n} \frac{(e_j - M)^2}{s_j^2}$ (3)

Substituting for x_j in the equation $E = \sum_{j=1}^{n} e_j x_j$ yields

$$WE = \sum_{j=1}^{n} \frac{e_j (e_j - M) S}{s_j^2} \qquad (4)$$

Substituting for x_j in the equation $K = \sum_{j=1}^{n} x_j$ gives

$$WK = \sum_{j=1}^{n} \left(\frac{e_j - M}{s_j^2}\right) S \qquad (5)$$

It follows from (3) that

$$W^2 S = \sum_{j=1}^{n} \left(\frac{e_j - M}{s_j}\right)^2 S = WE - WKM$$

$$\text{i.e. } WS = E - KM \qquad (6)$$

In this case neither W nor M are known and therefore cannot be assumed constant. They can, however, be readily eliminated from equations (4), (5) and (6) to yield a quadratic relation between E and S. This general case has no immediate applicability to the problem under consideration, namely the demonstration of the existence of an inverse relationship between money holdings and the rate of interest. In order to remedy this, we shall have to impose one further condition, namely that there is one asset, the outcome of which is certain.

2. In this case there is one investment namely money, the outcome of which is certain. Since the investment period under consideration is longer than the exchange period, the asset function will take precedence over the medium of exchange function and consequently Newlyn's criterion will prove too restrictive. Money in this context is visualised as any liquid asset the outcome of which at the end of the period is certain, so warranting the inclusion of near-money in the definition of money. We will take this to be the nth investment so that $s_n = o$ and $e_n = 1$. Under these conditions M is determined and equal to unity. By analogy with equation (3) we see that:

$$W^2 = \sum_{j=1}^{n-1} \left(\frac{e_j - 1}{s_j}\right)^2$$

and since this expression depends only on the e_j's and s_j's, i.e. the prospects of the individual securities which are assumed known, W becomes a known quantity. Then equation (6) viz. WS = E-KM becomes a linear relation between E and S. Thus by applying marginal advantage theory to an investor's choice we have obtained his opportunity locus, which is analogous to the budget restraint in the theory of demand. The next step is to bring together his opportunity locus which represents all possible combinations of securities which yield maximum advantage or utility, and some representation of his tastes in order to ascertain which of the infinity of optima he will actually adopt.

While it is possible, with the use of much ingenuity to formulate a graphical analysis for the above problem in the case of multiple alternative assets, nothing is lost in the essentials and a tremendous amount gained in simplicity if presentation is restricted to one asset other than money, which for convenience we shall assume to be a bond. This brings the analysis into line with Keynes' treatment of the speculative motive and has the advantage of making comparisons easier.

Suppose that the investor has one unit to invest and that asset number 1 is money, which as pointed out above is characterised by absolute certainty, and asset number 2 is bonds. Then $s_1 = o$ and as a consequence $e_1 = 1$. Thus e_1 is held with absolute certainty. Then $S^2 = s_2^2 x_2^2$

$$\text{i.e.} \quad S = s_2 x_2 \tag{7}$$

where s_2 is the standard deviation of bonds.

The equation $E = WS + KM$ then reduces to $E = (e_2-1)x_2 + 1$ where e_2 is the expected outcome of placing one unit in bonds.

An investor facing this choice at the beginning of the period will earn interest at the going rate of r% on his investment in bonds. However, if the price of bonds changes before the end of the period he will experience a capital gain or loss of amount g on redemption. We shall assume that the investor has no certain idea as to the outcome at the end of the period, (this is merely a re-statement of the initial assumptions and shows quite clearly that his expectations are not inelastic) and in the absence of any other information we shall suppose that he regards a loss or gain as equally likely. (This would imply assuming a symmetrical distribution which is perfectly compatible with the discussion on kurtosis above).

Thus $e_2 = E\left[r+g\right] = r + E[g] = r$

The equation $E = (e_2-1)x_2 + 1$ can be rewritten as $(E-1) = (e_2-1)x_2$.

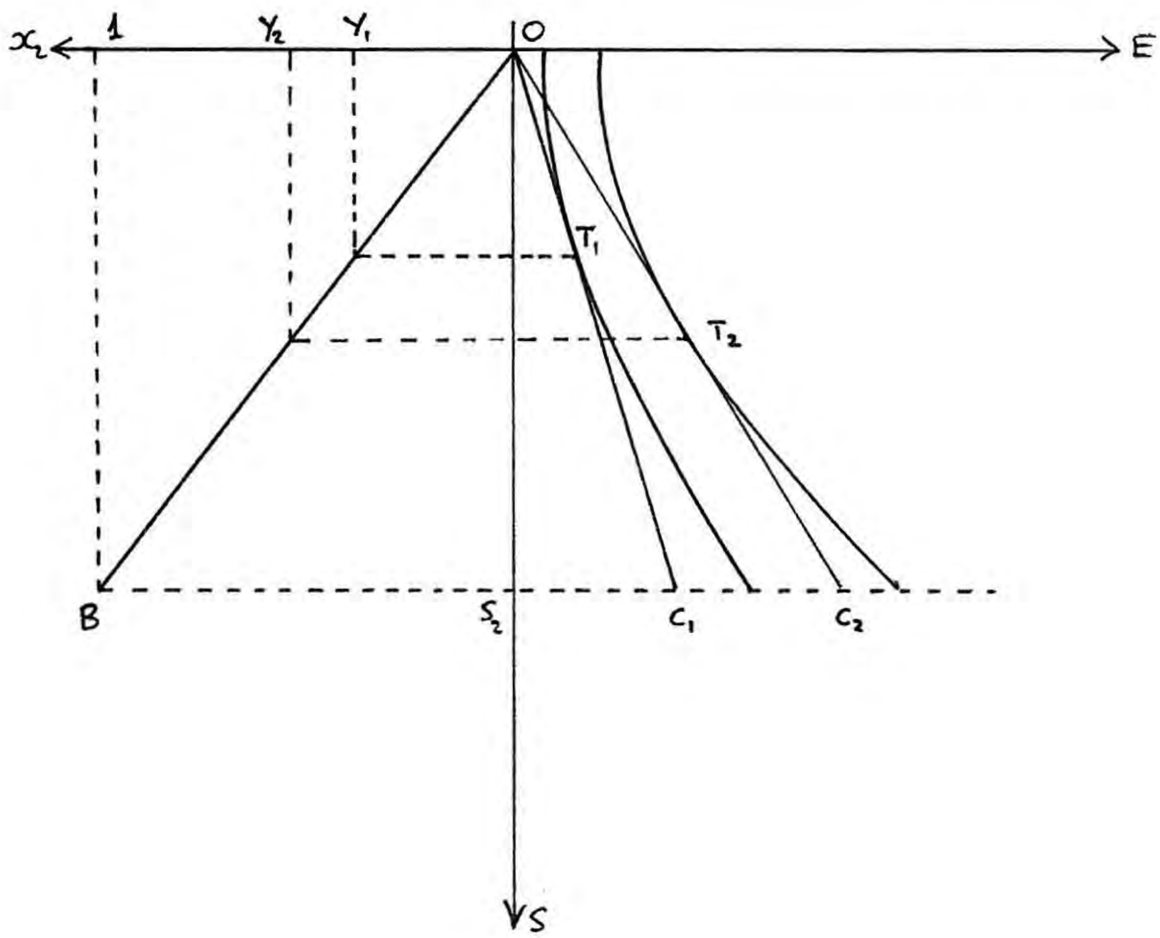
It will help the graphical presentation immensely if we write $E = E-1$ and $e_2 = e_2-1$ which merely represents a shift of the origin. In this case, making the appropriate substitutions we get:

$$E = e_2 x_2 = \frac{e_2}{s_2} s_2 x_2$$

$$\text{i.e.} \quad E = \frac{r}{s_2} S. \tag{8}$$

With the help of equations (7) and (8) we can now present a graphical analysis. We shall measure E along the right hand horizontal axis and x_2 $(x_2 \leqslant 1)$ the amount held in bonds along the left hand horizontal axis. Since S represents a form of disutility we shall measure it along the negative vertical axis.

DIAGRAM 2.



The opportunity locus equation (8) is depicted as a straight line OC_1, drawn for an interest rate r_1. The relationship between total risk and bond holding equation (7) is depicted by the straight line OB. It can be shown that the concept of an indifference curve between E and S has significance and further that for risk-averters they are necessarily negatively sloped. This is a consequence of the fact that a risk-averter will not accept more risk without the prospects of more gain. A negatively sloped curve may be either

i) convex in a south-easterly direction,

or ii) convex in a north-westerly direction.

The latter case which can easily be shown to result in the person holding either all cash or all bonds is thus too restrictive for our purpose.

The point of tangency of the indifference curve with the opportunity locus drawn for the interest rate r_1 is T_1. This represents his actual portfolio composition. At this point he holds an amount Y_1 of bonds and hence a corresponding amount $(1-Y_1)$ of money. A natural consequence of the choice of the probability distribution is that there are two parameters that can vary, (i) the rate of interest r and (ii) his subjective interpretation of the riskiness of bonds s_2. The former is of relevance in this analysis. An increase in the rate of interest from r_1 to r_2 is represented by a movement of the opportunity locus from OC_1 to OC_2. The point of equilibrium has moved to T_2 and the corresponding bond holding has increased to OY_2.

It would thus appear that we have demonstrated the existence of an inverse relationship between the rate of interest and money holdings. However, it is not completely cut and dried on account of the conflict between the income and substitution effects. An increase in the rate of interest is an incentive to take more risk and the substitution effect then implies a shift from security to yield. But an increase in the rate of interest also has an income effect in that it gives the opportunity to enjoy security along with more yield. If the income effect outweighs the substitution effect, money holdings will bear a positive relationship to the rate of interest.

Normally in demand theory very little can be said about this case apart from commenting on the rarity of its occurrence in practice,[50] but in this case since the two commodities have such well defined properties, an additional point can be made. Money holders will experience no capital loss and the rise in r will make them better off only insofar as they switch out of cash into bonds, so the income effect

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[50] G.J. Stigler, "Notes on the History of the Giffen Paradox," Journal of Political Economy, Vol. 55, (Apr., 1947), pp. 152 -56.

cannot actually make them hold more cash. It is thus very unlikely that the income effect will outweigh the substitution effect except under abnormal conditions. A more detailed analysis extending over several time periods confirming this conclusion has been presented by Matthews.[51]

[51] R.C.O. Matthews, op. cit.

CHAPTER II

A. Theoretical Formulation of the Demand of Function

1. Introduction.

Having shown in the previous chapter that a positive demand for money exists at the micro- level, i.e. that a certain proportion of the consumer's wealth will be volitionally allocated to money holdings, general demand theory can best be used to guide our further considerations. However, before analytically formulating such a function, a brief glance at the evolution of demand theory for both monetary and non-monetary goods will not be amiss.

i) Non-monetary goods.[1] The marginalist school, as exemplified by Marshall, considered demand as being influenced by relative prices i.e. market conditions and being unaffected by the individual's initial endowment. Thus in the theory as so formulated the consumer's "demand for any commodity is independent of his income."[2] The modern school, as exemplified by Hicks, allowed for income effects and so explicitly considered the consumer's initial endowment as well as market conditions in the demand equation.

ii) Money.[3] Pre-Keynesian economists considered the initial endowment to the complete exclusion of market conditions as the factor determining the demand for money. This is clearly demonstrated by the form of the Cambridge cash-balances equation viz. $M = kY$. Keynes considered the demand for money to depend not only on income but also on the rate of interest which is a market variable.

We can thus see that although having the same final form at present, the two theories have evolved from different origins. In the

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[1] For a general survey of demand theory, the reader is referred to : G.L.S. Shackle, The Years of High Theory (Cambridge, 1967), Chap.8; J.R. Hicks, Value and Capital (Oxford, 1939), Part I, and J.R. Hicks, A Revision of Demand Theory (Oxford, 1956), Part I.

[2] J.R. Hicks, Value and Capital op.cit., p. 26.

[3] For a discussion of the relevant theories at Cambridge, the reader is referred to E. Eshag, From Marshall to Keynes (Oxford, 1963).

case of non-monetary goods it was the refinement in analytical techniques occasioned by the abrogation of marginal utility in favour of indifference analysis that was responsible for the evolution, whereas in the case of money it was the growing awareness of its role as a store of value that was responsible. As a consequence of this confluence of the two streams, the monetary demand equations as formulated by theorists, whether avowedly Keynesian or capital, are operationally identical.

Thus general demand theory contends that it is the transformation conditions offered on the market and the range of opportunities open to the individual transactor that determines the amount of any particular asset desired. The transformation conditions and the opportunity range must first be specified. The latter may be inferred from a description of the individual's asset portfolio, since at any moment of time he inherits from the past an asset position and is confronted with evolving market conditions indicating the relative prices of the various assets. The individual is then visualised as adjusting his asset position to the changing market conditions so as to maximise the utility obtained from money holdings, such adjustment taking place within the constraints imposed by his initial asset position.

2. Formulation of a General Demand Function.

For convenience in grouping the variables that enter into the demand function, we shall consider separately the transactions demand based primarily on money's function as a medium of exchange and the asset demand based primarily on its function as a store of value. In view of the discussion in chapter I on the non-separability of the primary functions of money, it must be emphasised that this distinction has been made purely for convenience and it does not imply that we can split money into water-tight compartments, or that the effect of any one variable is necessarily confined to one particular function of money.

(i) The Transactions Demand. The most important single variable affecting the transactions demand is obviously income and the functional form of this relationship will depend on the nature of the assumed payments schedule, although we may consider it safe to assume that it is an increasing function of income. On general grounds it has long been argued that an increase in the rate of interest has a twofold effect. Firstly it increases the cost of existing money balances, and secondly it raises the costs of borrowing. The former will induce the transactor to hold a smaller stock of money whereas the latter will induce him to keep a larger stock of money so as to avoid the higher cost of borrowing. These two effects are assumed to cancel each other out and hence the transactions demand for money is supposed interest inelastic. However, as was shown in the mathematical appendix to chapter I, by applying an inventory theoretic approach to the transactions demand, transaction balances can be shown to be interest elastic. The essential element in the above analysis is that there should be no chance of fluctuations in the capital value of the assets alternative to money, so indicating that if transactions balances were interest elastic it would be the short term rate of interest that is appropriate.

(ii) The Asset Demand. This can conveniently be sub-divided into two groups.

(a) The Precautionary motive. An analysis of the precautionary demand has been developed in the mathematical appendix to chapter I in which it was demonstrated that the most plausible motive underlying the precautionary demand is the desire for security in the face of uncertainty. In the analysis proffered the demand for money was influenced by two factors. Firstly, the rate of interest. The analysis was framed in terms of bonds as the alternative asset which would imply that the long term rate was the appropriate rate. However, in the general analysis bonds were only one of a number of alternative assets. In fact the term of the appropriate rate will depend on the duration of the investment period and since this is conceived as being of a longer duration than the exchange period, this would suggest that the long term rate is the appropriate rate. Secondly, the investor's subjective estimation of the riskiness of bonds. Although it was not demonstrated explicitly in the analysis, it is a simple matter to show that money holdings will vary directly with the investor's estimate of riskiness,[4]

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[4] J.R. Tobin, "Liquidity Preference as Behaviour Towards Risk", Review of Economic Studies, Vol. 25, (Feb., 1958), p. 80 ff.

which in turn depends on the degree of uncertainty. Keynes viewed the precautionary motive as income constrained, but it is clear from the above analysis that wealth is the appropriate constraint.

(b) The Speculative motive. As originally formulated by Keynes, the individual is envisaged as holding money balances in order to gain from an anticipated change in the capital value of bonds. Hence speculative holdings will be dependent on the relation between the current rate of interest and the expected rate. The individual may also speculate on the future value of claims to real wealth, and hence the present and expected rates of return on equities will influence speculative balances. In this case there is little doubt that wealth is the appropriate constraint.

Since the value of money and hence its efficiency as a store of value, is inversely proportional to the aggregate price-level, we would expect that the asset demand for money would depend on the expected rate of change of the general price-level. Money holdings will also be affected by the tastes and habits of the individuals as well as by the state of evolution of the financial sector.

In the light of the above, we can now set out algebraically the demand function for an individual transactor. We postulate a function of the form:

$$m = f\left[r_s, r_b, r_b^e, r_e, r_e^e, (dp/p)^e, Y, W_T, T, I, U\right] \qquad (1)$$

Where:

m	=	real stock of money held by the individual,
r_s	=	short term rate of interest,
r_b	=	long term rate of interest (i.e. bond rate),
r_b^e	=	expected long term rate of interest,
r_e	=	rate of return on equities,
r_e^e	=	expected rate of return on equities,
$(dp/p)^e$	=	expected rate of exchange of general price-level,
Y	=	real income,
W_T	=	total wealth,

T = tastes and habits,

I = institutional limitations, and

U = uncertainty.

3. Some Simplifications.

Starting from this demand equation after making some assumptions of a general nature, we shall show that by making certain additional assumptions, mostly concerning the relationship between Y and W_T, we can obtain the various specialised demand functions postulated by different authors. The first point to notice about (1) is that it is formulated in terms of real quantities and not in terms of nominal values. By this we explicitly rule out the possibility of any money illusion, which is justifiable since we are expressly considering a long run demand function. A second problem is the representation of the variables I and T. Since we can have no empirical measure of them we are forced to assume that either they remain constant or else that they have a negligible effect over the period under consideration. The validity of these assumptions will be investigated when the function is examined for stability in subsequent chapters.

An issue of crucial importance is the representation of the expected values.[5] The obvious candidates in order of increasing complexity are:

(a) extrapolation of the present value,

(b) extrapolation of the trend of immediate past values,

(c) some version of a rational expectation hypothesis, and

(d) some probability distribution of expectations based on the first three.

Studies of the role of price changes in the demand for money seem to indicate that it is negligible except in the case of hyper-inflations.[6] Thus we can feel justified in ignoring the term $(dp/p)^e$.

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[5] For a discussion of this problem, the reader is referred to: S.A. Oźga, Expectations in Economic Theory (London, 1965).

[6] Two examples of such studies are: A Hynes, "The Demand for Money and Monetary Adjustments in Chile", Review of Economic Studies, Vol.34, (July, 1967), pp. 285-93, and P. Cagan, "The Monetary Dynamics of Hyperinflation" in M. Friedman (ed.), Studies in the Quantity Theory of Money (Chicago, 1956).

However, we still have to consider the terms r_b^e and r_e^e, the effects of which can hardly be supposed minimal. The objection to using the first assumption is obvious, since if $r_b^e = r_b$ and $r_e^e = r_e$ there would be no speculative demand for money. For simplicity, in fact almost out of necessity due to the impossibility of constructing a satisfactory empirical measure, we shall accept the first alternative. In spite of the above objections it will still prove useful as an explanatory device for examining how much of actual behaviour can be explained by a hypothesis of this kind. A problem which is very closely linked with the above is that of representing the degree of uncertainty facing the transactor. For similar reasons we are likewise forced to assume that the degree of uncertainty remains unaltered or else undergoes no significant change during the period under consideration.

The next simplifying assumption is that both the long run and the short run rates of interest as well as the rate of return on equities show sufficiently high covariance to be subsumed into one common rate of interest denoted by r_f, the financial rate of interest. This rate of interest would be a weighted average of its three components, and which of the available measures of the rate of interest serves best as an indicator of this rate is essentially an empirical decision. A further point to be borne in mind is that in the absence of any measure of the rate of return on equities, the final result must be a financial rate of interest. Under the above assumptions the demand equation now becomes:

$$m = f\ (r_f, W_T, Y). \qquad (2)$$

4. An Analytic Distinction Between Human and Non-Human Wealth.

In the above form the function is not satisfactory on account of the fact that it contains two constraints, wealth and income, which are not conceptually independent. In fact wealth is defined as the capitalised value of income where income is the flow of goods and services that give rise to satisfaction, regardless of whether such goods and services are capable of being brought under the measuring rod of money. Thus total wealth W_T is defined as $W_T = Y/r$ where

Y is income and r the rate of capitalisation. It follows that this definition of wealth includes human wealth.

Economists have long realised the existence of human wealth and the theoretical need for its inclusion in total wealth, as exemplified by Marshall's assertion that "the most valuable of all capital is that invested in human beings."[7] However, the question being discussed here is not whether human wealth should be included in the definition of wealth or not, but whether total wealth should be included in the demand equation as one homogeneous unit or whether there are any grounds for a division of wealth into different categories.

We shall assume that the behaviour of a consumer is affected by the degree of liquidity (here used in the sense of exchangeability) exhibited by his assets since it will influence their effectiveness as a store of value. Thus the individual is envisaged as being capable of ordering his assets in order of increasing or decreasing liquidity and naturally this ordering will depend on the institutional arrangements. Within this ordering there is one primary division, namely between those assets which are exchangeable and those which are not. Thus for a given value of total wealth the individual's behaviour will be influenced by the ratio of exchangeable to non-exchangeable wealth in his portfolio. As yet this is not sufficient to provide an economic reason for categorising wealth since the margin between exchangeable and non-exchangeable assets is institutionally determined.

In order to determine such a division, we shall have to consider the sources of the income streams that are capitalised to form wealth. The fundamental division here is between human and non-human sources.[8] In terms of the above, the wealth equation becomes,

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[7] A. Marshall, Principles of Economics (8th edn., London, 1920), p.564. Note that here Marshall uses the word capital in the financial sense. This in no way invalidates the point made in the text.

[8] If this division is not intuitively obvious, it would be possible to categorise the sources by means of the "gap method" as described in G. Ryle, "Categories" in A.G.N. Flew (ed.), Logic and Language (2nd Series) (Oxford, 1953).

$$W_T = \frac{Y_h}{r_h} + \frac{Y_n}{r_n}$$

where the symbols have the obvious meaning.

To show that this division will influence consumer behaviour in the manner indicated above, we shall first have to examine the process of exchange in some detail. It is vital in the argument that follows to distinguish clearly between stocks and flows as well as between legal and physical possession of the stocks and flows. In demand theory, flows enter the utility functions of the individuals and hence the motive for demand has its origin in flows. The actual process of exchange, however, takes place at an instant of time and ipso facto involves stocks. Let us now examine human and non-human wealth in the light of the exchange process postulated above.

(i) Non-human wealth. In the case of all exchanges involving non-human wealth the seller yields possession, both legal and physical, of the object to the buyer. In other words the seller no longer has any use of the object being exchanged whereas the buyer has.

(ii) Human wealth. Marshall provided a clue to this distinction when he asserted that "the worker sells his work, but he himself remains his own property."[9] The distinguishing feature of transactions involving human wealth is the fact that it is impossible to separate the stock of wealth from its owner since it is an integral part of him and separation would thus cause it to lose its capacity to generate income. However, it is possible in the case of a slave society for physical and legal ownership of the stock of wealth to become divorced. Friedman[10] has considered the case of a slave society and came to the conclusion that the phenomenon of non-exchangeability of human wealth falls away in the case of the slaves but not for the slave owners. However, by failing to distinguish between physical and legal possession of the stock of wealth, his analysis of the problem is at best incomplete.

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[9] A. Marshall, op.cit., p.560.

[10] M. Friedman, Price Theory (London, 1962), p. 201-2.

It is worthwhile to examine the exchange process in a slave society in more detail and to contrast it with the labour market in a non-slave society. In the latter case the purchaser has legal and physical possession of the flow of services but does not have physical or legal possession of the stock of human wealth. In purchasing a slave, the owner has similarly obtained legal and physical possession of the flow of services emanating from the slave as well as legal possession of the stock of wealth. However, on account of the fact that the source of human income, whether it be muscle power or mental energy, cannot be physically separated from its owner (in this case the slave) without causing the flow to cease and hence the value of the stock to disappear, the owner of the slave does not have physical possession of the stock of human wealth.

This distinction is brought out very clearly when we consider the phenomenon of death. On account of the physical non-separability of human wealth from its owner, the stock of human wealth goes out of existence the moment its owner dies, whereas non-human wealth still retains its existence after the death of its owner.[11]

In the treatment of non-human wealth, it is generally accepted[12] that in an economy without uncertainty equilibrium will result in the equalisation of the rate of return on all types of non-human capital. This conclusion has often been extended uncritically to include the case of human capital, but as shown above there is sufficient reason to cast a shadow of doubt over this conclusion.

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[11] The problem now arises of the case of a copyright or patent which can still yield income after the death of its conceiver. The problem is very similar to that for the slaves in that although the conceiver still has physical possession of the knowledge, he no longer has any legal claim to it. A further point to be borne in mind is that a copyright or patent is a form of claim to a future monetary income which is a different method of classification of wealth from the one above and there is no *a priori* reason why the two methods of classification should coincide.

[12] For example see F.H. Knight, *On the History and Method of Economics* (Chicago, 1956), p. 193.

In lieu of the physical and in most cases legal non-separability of human wealth from its owner, let us attempt to discover what additional requirements are necessary to ensure equality of the two rates of capitalisation. The non-separability of human wealth becomes a problem only for those owners of human capital who desire to divest themselves of part of their human capital before it has depreciated. Thus for any owner of human capital who is perfectly certain that he will not want to consume his capital prior to his death the non-separability of human wealth is irrelevant. The non-separability of human wealth becomes a problem only for those owners of human capital who desire to divest themselves of part of it before it has depreciated completely, i.e. on the death of its owner. Hence for any owner of human capital who is perfectly certain that he will not want to consume it prior to his death, the non-separability poses no problem. Thus perfect knowledge of future consumption patterns is a necessary condition for equality of the rates of return. However, this is not a sufficient condition for consider the case of an individual who desires with perfect certainty an income stream for a period of n years. Assume initially that the production functions for either type of capital are identical and further that they yield capital that will produce the desired income stream at the lowest cost only if this capital has a life of (n + t) years. In this case we can see that even with perfect knowledge, non-human capital will still be preferable to human capital since at the end of the nth year the unused portion of the former can be sold, whereas the unused portion of the latter cannot. Hence under conditions of perfect certainty, if the two types of capital are to be equivalent, the production functions must be such that the cost of producing capital of any durability is always equal to the discounted stream of future income yielded by this capital. The invalidity of the above requirement is obvious once we consider the disparity in the amount of energy and time required to obtain a skill (for example riding a bicycle) for one afternoon compared to that for the whole of one's lifetime.

We may say in the light of the above that of two incomes of equal size, one having its origin in human sources and the other in non-human sources, the former must be inferior to the latter and hence the rate of capitalisation applicable to human income must be higher than that for non-human income. There is a further empirical reason why the measured rates of capitalisation should differ. All the empirical measures of non-human wealth express only the net income yielded by such sources whereas economists have as yet been unable to distinguish satisfactorily between gross income and net income yielded by human sources on account of the difficulty of obtaining or even defining any measure of human operating costs. Thus, even if we were able to overcome the problem of non-separability, we would still find that the measured rate of capitalisation of human income would be higher than that for non-human income on account of the fact that the former is measured gross of depreciation and the latter net of depreciation.[13]

5. The Reduced Forms of the Demand Equation.

We see now that it is not only the stock of the individual's total wealth that influences his behaviour, but the ratio of human to non-human wealth as well. Since the individual inherits his asset position from the past, at any one instant in time there exists some division between human and non-human wealth in his portfolio. Let $W_h/W_n = k$. He may be able to change k over time, but it is given at any instant of time. In view of the utter dearth of empirical evidence concerning this division we can be justified in making the simplest possible assumption, viz. that k remains constant over time. This is not so unrealistic as it may seem at first in view of the difficulties inherent in making the switch compared to the time period under consideration.

Since $Y = rW_T$, if we assume a sufficiently high correlation between r and r_f we can eliminate either Y or W_T from the demand equation. If we eliminate W_T we shall obtain an income constrained function of

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[13] For empirical attempts to measure the rate of return on human capital see G.S. Becker, Human Capital (N.Y., 1964); M. Blaug, Economics of Education Vol. 1 (Penguin Books Ltd., 1968); and the articles in Journal of Political Economy, Vol. 70 (Supplement), (Oct., 1962), pp. 1-153.

the form

$$m = f(r_f, Y). \qquad (3)$$

In the above, the empirical measure of income has purposely been undefined. It could either be some measure of current income or some form of "permanent income" as proposed by Friedman.[14] In a later paper,[15] he applied the permanent income hypothesis to the problem of the demand for money. On the basis of empirical evidence he postulated a function of the form $M = f(Y_p)$. His exposition of the function is not completely compatible with the approach adopted so far in that it is conceived of as a macro-function. It is thus necessary to reformulate it on a micro-level in such a way as to preserve the essentials.[16] The function now takes the form

$$m = k(Y_p)^{\delta} \qquad (4)$$

where: m = real stock of money,

k = a constant,

Y_p = permanent real income,

δ = a positive constant.

It should be noted that the rate of interest does not appear in the above formulation as Friedman found that its effect was "too small to be statistically significant."[17] This does not mean that he considers the demand for money to be interest inelastic for he emphatically states in a later article that he knows of "no empirical student of the demand for money who denies that interest rates affect the real quantity of money demanded - though others have misinterpreted me as so asserting."[18]

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[14] M. Friedman, A Theory of the Consumption Function (Princeton, 1957).

[15] M. Friedman, "The Demand for Money, Some Theoretical and Empirical Results", Journal of Political Economy, Vol. 67, (Aug., 1959), pp. 327-51.

[16] It is only possible to do this on account of his explicit assumption of a homogeneous population and a function that is homogeneous of degree one in this population. (ibid., p. 335).

[17] ibid., p. 329.

[18] M. Friedman, "Interest Rates and the Demand for Money", Journal of Law and Economics, Vol. 9, (Oct., 1966), p. 72.

However, if we substitute Friedman's definition of permanent income, (viz. "Income is generally defined as the amount a consumer unit would consume (or believes that it could) while maintaining its wealth intact"[19]) into the above equation we obtain an anomalous result. Since $Y_p = rW_T$, we can write $m = k(rW_T)^{\delta}$

i.e. $\ln.m = \ln.k + \delta \ln.r + \delta \ln.W_T.$

This implies that the wealth and interest elasticities have the same sign which is contrary to accepted economic theory. If we were to accept Friedman's findings that the effect of the rate of interest was too small to be statistically significant it would imply that δ was of the order of zero which is in sharp contrast to his empirical findings that $\delta = 1.8$. It would thus appear that in neglecting the effect of interest rates, Friedman has embodied a contradiction in his function.

If we were to eliminate income from appearing explicitly in the equation we would obtain a wealth constrained function of the form

$$m = f(r_f, W_T).$$

However, when we come to test this equation empirically, we immediately encounter the difficulty of obtaining a measure of W_T since this includes W_h, a quantity which is almost impossible to measure. Hence for practical reasons it would be desirable to obtain a wealth constrained function which does not contain human wealth as an explicit argument. We have earlier made the assumption that the ratio of human to non-human wealth is constant. Hence we can write,

$$Y = r_h kW_n + r_e W_n = (kr_h + r_e) W_n$$

since r_n and r_e are essentially identical. Making the usual assumptions about r_e and also assuming that either r_h and r_f are sufficiently correlated or else that variations in r_h are negligible, we can write $Y = g(r_f)W_n$ and hence obtain a wealth constrained equation in which W_n appears as the explicit argument, viz.

$$m = f(r_f, W_n) \qquad (5)$$

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19 M. Friedman, A Theory of the Consumption Function op.cit., p. 10.

This equation is identical to the one postulated by Meltzer.[20] However, he derived his equation by starting from the hypothesis that r_h is constant. He maintains that $r_h = Y_h/W_h$ and that if we write Y_h^* as the expected income from a stock of human wealth we can write r_h as $r_h = \frac{Y_h}{Y_h^*} \cdot \frac{Y_h^*}{W_h}$. The first term measures the short run deviations of the actual from expected income, i.e. it is an index of the transitory component of human income. He assumes that even though in the short run wealth holders may adjust the composition of their portfolios in response to changes in this index, in the long run it will equal unity. The second term is the ratio of expected human income to its capitalised value which has been assumed constant. It should be noted that since both hypotheses lead to the same equation their validity cannot be determined statistically and in view of the dearth of any empirical measure of the rate of return on human wealth we shall have to regard either hypothesis as being equally plausible.

6. Summary.

It has been shown that all the equations which are going to be empirically tested in later chapters are in principle derivable from a generalised demand function. By the use of some general simplifying assumptions the complexity of the function was greatly reduced, but reduction to the final form depends on a vital distinction between human and non-human wealth. Once the economic consequences of this distinction were explored, it proved possible to reduce the equations to simple wealth or income constrained models.

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20 A.H. Meltzer, "The Demand for Money : The Evidence from the Time Series", Journal of Political Economy, Vol.71, (June, 1963), pp. 219-46.

CHAPTER III

The problem of Aggregation and the Specification of the Variables

A. The Aggregation Problem.

1. Contradictions Between Micro- and Macro-Relations.

The form of the demand function for money considered in chapter II has been that for an individual transactor, i.e. the function has been expressed in the form of a micro-relation. However, the concept most amenable to empirical analysis is the macro-formulation which represents the aggregate or sum of the individual demands. The problems encountered when aggregating micro-demand functions to form macro-demand functions have been dealt with in detail by several authors, notably Allen[1] and Thiel,[2] so it will not be of particular relevance for the purposes of this study to make a detailed analysis of this problem. On the other hand, it will prove informative to make some comments of a general as well as of a specific nature concerning aggregation of monetary demand functions.

The obvious first approximation is to assume that the aggregate demand function for money is of the same form as the micro-function. This is the procedure adopted by Friedman[3] which follows as a consequence of his assumption of a function which is homogeneous with respect to population. For transactions balances, this is not an unreasonable assumption. For speculative balances, the effects of this type of aggregation are much more serious on account of the diversity of expectations. It is clear from the discussion in Keynes' General Theory that it is the difference in views about expected future interest rates which accounts for the existing rate being what it is. Since a liquidity function is defined for a given set of expectations, aggregate demand must then clearly be a function of the distribution of expectations. Quite apart from the problem of measurement, there seems to be no satisfactory method of representing

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1 R.G.D. Allen, Mathematical Economics (London, 1956).

2 H. Thiel, Linear Aggregation of Economic Relations (Amsterdam, 1954).

3 M. Friedman, "The Demand for Money : Some Theoretical and Empirical Results", Journal of Political Economy, Vol. 67, (Aug., 1959), pp. 327-51.

such a distribution as an aggregate. However, by virtue of our simplifying assumptions this problem falls away and so we do not have to consider it when aggregating the reduced form of the micro-equation.

On aggregating micro-relationships of the form $m_i = f(r,y_i)$ where m_i is the amount of money the i^{th} individual wishes to hold, r the rate of interest (assumed the same for all individuals), and y_i the income of the i^{th} individual, one meets the problem of identifying which is (are) the exogeneous variable(s). Clearly in the above micro-formulation there is no doubt. The individual takes as given the level of his income and the rate of interest on the market and then adjusts his holdings of money so that the level is consistent with the independent variables income and the interest rate. But the aggregate variable $M = \sum m_i$ is clearly not the endogeneous variable. The total amount of money is determined by the government, either directly or indirectly by the government creating and regulating reserves and the various financial institutions determining the quantity of money. On the other hand, the variables Y and r are now candidates to be considered as exogeneous. Thus we see that the chain of causation has been altered. It is now primarily from money stock to incomes and interest rates.

(a) If we were to consider Y as the endogeneous variable we would obtain a simple version of the money multiplier

$$Y = g(r,M).$$

This formulation suffers from the fact that the rate of interest is still endogeneous to the system and yet must occur amongst the independent or exogeneous variables.

(b) If we consider r as the endogeneous variable we may write

$$r = h(Y,M).$$

which suffers from an analagous difficulty with Y. This problem has been dealt with by several authors notably A.A. Walters[4].

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[4] A.A. Walters, "Money Multipliers in the United Kingdom, 1880-1962", Oxford Economic Papers, Vol. 18, (Nov., 1966), pp. 270-83, and A.A. Walters, "The Demand for Money - The Dynamic Properties of the Multiplier", Journal of Political Economy, Vol. 75, (June, 1967), pp. 293-98.

There are, however, a number of methods of avoiding this impasse.

i) We can simply ignore it by assuming that money supplied is equal to money demanded, which is possible in the long run. This is the approach that has been adopted implicitly by the vast majority of authors. If, however, the observed points are not equilibrium points, the identification problem makes it very unlikely that the estimated function will represent a pure demand function. Rather it will be a hybrid, composed partly of a demand function and partly of a supply function.

ii) We can assume that the demand function is exclusively a real relationship and the money supply to be nominal. This has been explicitly assumed by Friedman.[5] He points out that it is essential to note the difference between the determinants of the nominal stock of money on the one hand and the real stock of money on the other. The nominal stock of money is determined in the first instance by the monetary authorities and cannot be altered by the non-bank holders of money. The real stock of money is determined in the first instance by the holders of money since the price level which determines the value of money is considered as being determined by the spending decisions of the money holders themselves.

iii) We can adopt a pragmatic approach by acknowledging the existence of the multiplier and attempting to distinguish between the demand equation and the multiplier by the purely statistical measure of goodness of fit. This approach is exemplified by Latané.[6] One obvious criticism of this approach is that it attempts to distinguish between the two concepts by the purely statistical measure of goodness of fit which is not a powerful test on account of the fact that the goodness of fit may be significantly affected by the definition of the variables.

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[5] M. Friedman, op.cit.

[6] H.A. Latané, "Cash Balances and the Interest Rate - A Pragmatic Approach", Review of Economics and Statistics, Vol. 36, (Nov., 1954), pp. 456 - 60.

Furthermore, to obtain a reasonably sophisticated formulation of the multiplier we would have to incorporate some dynamic hypothesis about the money supply which lies outside the scope of the present study.

iv) The fourth approach and perhaps the most fruitful is to deal with the interaction of demand and supply of money by means of a simultaneous equation approach. This approach is exemplified by Teigen[7] and Brunner and Meltzer.[8] Basically the approach consists of formulating simultaneous demand and supply hypotheses and estimating them statistically by means of simultaneous equations. The major advantage of this method lies in the fact that it attacks the identification problem at its very roots. On account of the fact that it entails a supply hypothesis it falls outside the scope of this study.[9]

In spite of these objections we shall assume that the macro-function is of the same form as the micro-function and further that it still represents a demand function.

2. Differences Between Groups of Consumers.

A further problem which arises out of aggregation is that of different groups of consumers. Cross-sectional studies[10] indicate that the motives and the relevant elasticities differ between groups of consumers such as business firms and households. As a consequence of this, one should really consider separate demand functions for the various groups. In South Africa we have the additional problem of

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[7] R.L. Teigen, "Demand and Supply Functions for Money in the United States : Some Structural Estimates", Econometrica, Vol. 32, (Oct., 1964), pp. 476-509.

[8] K. Brunner and A.H. Meltzer, "Some Further Investigations of Demand and Supply Functions for Money", Journal of Finance, Vol. 19, (May, 1964), pp. 240-83.

[9] In a forthcoming paper I shall attempt a comparison of the demand functions obtained by ordinary least squares and by a simultaneous equation approach.

[10] J.J. McCall, "Differences Between the Personal Demand for Money and the Business Demand for Money", Journal of Political Economy, Vol. 68, (Aug., 1960), pp. 358-68.

having a dual society, where a subsistence economy exists alongside an industrialised capitalist sector. Quite apart from differences between business and household holdings, the motives for holding money will differ between the developed and undeveloped sectors. One could reasonably expect that the motives for holding money for transactions purposes would not differ radically between the two sectors since money is used as a medium of exchange for basically the same purposes. The differences most probably will arise out of money's function as a store of value and a measure of value.[11] The rural Bantu would not have an access to nor a knowledge of the various financial institutions of the developed sector and thus one would expect that if they desired to hold wealth in liquid form, as opposed to livestock and other real assets, they would either hoard the money (especially notes and coins) or possibly deposit it in a Post Office or a Building Society savings account. As a consequence of the motives behind such hoarding, one would not expect such balances to be interest elastic and they definitely should not be regarded as speculative balances. On the other hand, the inhabitants of the industrialised sector which would have the most significant effect on the demand for money (Whites, Coloureds and Urban Bantu) have access to and a knowledge of the financial institutions and consequently we would not expect that money in a narrow sense would be regarded as such a good store of value as in the former case.

As a consequence of the above, serious doubts can be cast as to the significance of the aggregate demand function. However, as Grunfeld and Griliches[12] have pointed out, there may in some cases be an "aggregation gain" measured in terms of goodness of fit arising

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11 The problem arises here of determining whether it is a qualitative distinction _per se_ or whether the quantitative difference in the use of money between the two sectors is so great that it produces an apparent qualitative distinction. This could only be decided by an appeal to empirical evidence which is lacking in the case of South Africa, although surveys have been done on the savings habits of rural Bantu, for example, D. Hobart Houghton and E.M. Walton, _Keiskammahoek Rural Survey_ Vol. 2, (Pietermaritzburg, 1952), chap. IV.

12 Y. Grunfeld and Z. Griliches, "Is Aggregation Necessarily Bad?", _Review of Economics and Statistics_, Vol. 42, (Feb., 1960), pp. 1-13.

out of the incorrect specification of the micro-function. Due to the simplifying assumptions made in obtaining the reduced form of the micro-equation this effect may be of consequence. However, for lack of relevant evidence we shall assume as an approximation sufficiently accurate for the purpose of this study, that the macro-function has the same form as the micro-function and furthermore that it remains a demand function.

B. The Specification of the Variables.

The variables which enter the various demand functions being considered can conveniently be divided into three groups.

(i) The dependent variable: in this case money.

(ii) The constraints: in this case, current income, Permanent Income and wealth.

(iii) The "market" variables: in this case the various interest rates.

Each of these three groups will now be discussed in some detail. It should be noted at the outset, that unless otherwise specified, figures are quoted for the end of the calendar year and real figures were obtained from nominal figures by using a G.D.P. deflator.[13]

3. Money.[14]

In keeping with the earlier theoretical discussion of money we shall adopt two empirical definitions of money, a "narrow" concept denoted by M_1 and a "broad" concept denoted by M_2.

A. M_1 is defined to consist of:

i) Notes and coins outside the commercial banks and the Reserve Bank.

ii) Demand deposits at commercial banks.

The problem now arises as to how to deal with deposits at the Reserve Bank, which in some cases count as money. B. van Staden[15] has given

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[13] Provided by officials of the South African Reserve Bank, (1958 = 100).

[14] All monetary data was obtained from the appropriate issues of the South African Reserve Bank Quarterly Bulletin.

[15] B. van Staden, "A Monetary Analysis for South Africa", South African Reserve Bank Quarterly Bulletin, No. 67, (March, 1963), pp. xiv - xxviii.

some convincing reasons why government deposits at the Reserve Bank should not be counted as part of the money supply. It was decided to obviate the problem as to which deposits at the Reserve Bank should be counted as money by excluding all such deposits from the definition of money.

Unfortunately the geographical coverage of the monetary statistics presented by the Reserve Bank changed at the end of 1958. However, by means of the procedure outlined below a fairly comparable series can be obtained.

For the period 1918 - 1958, the data is available directly. For the period January 1959 - December 1960 the following procedure was adopted.

a) Coin holdings were estimated by subtracting holdings of coin by commercial banks (in South Africa and South West Africa) and the Reserve Bank from the total amount of coin issued. This latter figure includes coin in circulation in South West Africa and the High Commission Territories and thus it is not strictly comparable with earlier figures.

b) Notes in circulation were estimated by subtracting holdings of notes by commercial banks (in South Africa and South West Africa) from notes issued by the Reserve Bank, including notes of other banks for which the Reserve Bank has assumed liability. This series is not strictly comparable with the previous one on account of the volume of notes in circulation in South West Africa.

c) Figures for demand deposits for commercial banks in South Africa alone are available, and consequently this series presents no problem.

As can be seen from the above the difference in the series arises from the fact that notes and coins in South West Africa as well as coins in the High Commission Territories have been included in the latter period whereas they do not appear in the former. This difference was assumed to be negligible.

B. M_2 is defined as consisting of the sum of M_1 and the following components.

i) Savings deposits at commercial banks.

ii) Savings deposits at Permanent Building Societies.

iii) Savings deposits at the Post Office.

In this case, the problems that arise are not geographical but temporal.

a) Figures for savings deposits at commercial banks are available as of 31st December for the period under consideration.

b) Up to and including 1944, figures for Post Office savings were only available as of 31st March. From 1945 onwards these are available for the 31st of December.

c) Up to and including 1944, figures of building society savings were only available for the end of the financial year and there was no common financial year. From 1945 onwards these figures are available for 31st December. For convenience in calculation, the end of the financial year was taken to be the 31st of March, although a note in the South African Reserve Bank Quarterly Bulletin of Statistics indicated that the end of the financial year may be taken to be 1st May. This was done in order to place these statistics on an equal footing with Post Office savings. The error so involved is assumed negligible.

As a consequence of this difference in temporal definition, the figures up to 1944 were adjusted from 31st March to 31st December of the previous year by means of a linear approximation which took the form,

$$S_t' = S_{t+1} - \frac{1}{4}\left[S_{t+1} - S_t\right]$$

where S_t' is the adjusted value of savings deposits as of 31st December, and S_{t+1} and S_t are the corresponding values as of 31st March.

4. **Constraints.**[16]

The next group of variables that has to be considered is the constraint. This takes the form of either a stock as in the case of

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[16] All income and wealth data were obtained directly from the South African Reserve Bank.

wealth, or a flow as in the case of current and Permanent Income. In either case the problem of the existence of the government sector arises. We have seen that if we chose to ignore government deposits in the definition of money, the corresponding problem arises as to how to treat the government in estimating the constraint. There are essentially four ways of dealing with the presence of the government: (i) include it both in the definition of money and in the constraint, (ii) exclude it from the definition of money but include it in the constraint, (iii) exclude it from the definition of money and the constraint, and (iv) include it in the definition of money but exclude it from the constraint. In the light of our discussion of the role of the government in the definition of money we see that our choice is restricted to alternatives (ii) and (iii). Selden[17] has maintained that the prime requirement must be that of consistency while in addition Gordon[18] has presented reasons for excluding the government sector from the income constraint. Thus it would appear that the appropriate choice would be (iii). However, in practice the choice is dictated by the availability of statistics and we are thus forced to adopt concept (ii). It should be noted in passing that economists are by no means unanimously in favour of any particular one of the concepts.

We shall now consider each of the three constraints in turn.

A. Current Income. The macro-accounting concepts available are:

i) Disposable Income.

ii) Net Domestic Product.

iii) Net National Product.

iv) Gross Domestic Product.

v) Gross National Product.

All the above have been used by various authors and there is no unanimity of opinion as to which would prove to be the correct

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17 R.T. Selden, "Monetary Velocity in the United States" in M. Friedman (ed.) Studies in the Quantity Theory of Money (Chicago, 1956).

18 R.A. Gordon, "The Treatment of Government Spending in Income Velocity Estimates", American Economic Review Vol. 40, (March, 1950), pp. 152-59.

choice. Further, no author has attempted a theoretical justification of the concept he has chosen. In view of the above, the choice remains a fairly open one and thus will have to be made in the light of available data. We shall adopt Gross Domestic Product measured at market prices to be the macro-measure of current income.

B. Permanent Income. It was felt that, in view of the fact that no Permanent Income figures have been constructed for South Africa, it would prove informative to spend some time discussing these figures. In his book on the consumption function[19] Friedman develops the idea that consumption in a particular year t depends not on income received in that year (i.e. current income) but on some sort of Permanent Income, a measure of wealth, which may be regarded broadly speaking as an average of the annual income which the individual expects to receive over his life-time. It is obvious that Friedman has applied this reasoning to money balances in his treatment of the demand for money.[20] By analogy we see that not all of an individual's money holdings on a particular day are based on receipts for that day. Similarly, even a year may be too short a period for the individual to consider in planning his money holdings. Hence in planning for year t, the individual takes into account the whole of his expected earnings and adjusts his money holdings in such a way as to maximise his utility, not only in a particular year but also over a longer time-span. Money holdings in any given year are therefore a function of the individual's life-time resources and not of his current income and the latter influences money holdings only in so far as it affects the individual's expectations.

It is obvious that Permanent Income as envisaged by Friedman is an ex-ante concept whereas any empirical measure must of necessity be ex-post. As a consequence of this we shall follow Friedman's lead in developing an ex-post concept called Expected Income to distinguish

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19 M. Friedman, A Theory of the Consumption Function (Princeton, 1957).

20 M. Friedman, "The Demand for Money : Some Theoretical and Empirical Results", op.cit.

it from the ex-ante concept of Permanent Income[21]. Expected Income is considered to be a weighted average of past incomes with the weights declining through time. At once two problems arise. (i) What is the appropriate measure of income? and (ii) what is the appropriate weighting pattern?

Friedman[22] constructed his Permanent Income figures from a series of Net National Product figures at constant prices. To keep in line with our earlier discussion we shall use a series of Gross Domestic Product at constant prices to calculated Expected Income.

The choice of the weighting pattern, however, raises some far more serious problems. Expected Income is defined as:

$$Y_E(T) = \int_{-\infty}^{T} W(\theta - T) y(\theta)\, d\theta$$

where $y(\theta)$ is current income and the weights are so chosen that

$$\int_{-\infty}^{T} W(\theta - T) d\theta = 1.$$

The above weighting pattern as it stands is unacceptable for two reasons.

i) Income figures are only available annually, and hence the weights must be expressed in the form of a discrete sum rather than in the form of an integral.

ii) On account of the finite life-time of an individual, unless we assume as Friedman does that all individuals are homogeneous, and on account of the limitations of available data, the time span of the weighting pattern must be finite and preferably of a short duration.

Tong Hun Lee[23] has assumed a weighting pattern of the form

$$Y_E(T) = \alpha Y_{(T)} + \alpha\lambda Y_{(T-1)} + \alpha\lambda^2 Y_{(T-2)} + \dots$$

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21 However, in order to keep in line with the convention adopted by the majority of economists we shall drop this distinction in later chapters, it being assumed that the distinction between the concepts can be inferred from the context.

22 M. Friedman, ibid., p.339, footnote 13.

23 Tong Hun Lee, "Income, Wealth and the Demand for Money : Some Evidence from the Cross-section Data", Journal of the American Statistical Association, Vol. 59, (Sept., 1964), pp. 746-61.

where α and λ are unknown parameters such that $\alpha > 0$ and $0 < \lambda < 1$. If we were to take $\lambda = e^{-\alpha}$ we would obtain

$$Y_E(T) = \alpha Y(T) + \alpha e^{-\alpha} Y(T-1) + \alpha e^{-2\alpha} Y(T-2) + \cdots$$

If this was transformed back to integral form we would obtain

$$W(\theta - T) = \beta e^{\beta(\theta - T)}$$

where to keep in line with subsequent notation, α has been replaced by β. It is easily seen that

$$\beta \int_{-\infty}^{T} e^{\beta(\theta - T)} d\theta = 1.$$

and that the larger the value of β, the shorter the effective weighting period becomes. One method of measuring the effective time span is to compute the weighted average time span between the observations that are weighted and the present.

$$T - \bar{t} = \beta \int_{-\infty}^{T} e^{\beta(\theta - T)} (T - \theta) d\theta = 1/\beta$$

This is the average time lag between the estimated permanent income and the observations from which it is estimated. Twice this time lag may be called the "effective weighting period".

One obvious defect of this approach is that it makes no allowance for secular growth. Being an average of earlier observations, the estimated Y_P is necessarily between the lowest and the highest of these observations, so that this method of estimation applied to a series with secular growth will yield estimated values with a downward bias. To allow for this, we can suppose Y_E to be estimated in two parts.

i) A trend value which is taken to grow at a constant rate viz. that of the growth rate of the G.D.P. figures, namely α %.

ii) A weighted average of adjusted deviations of past values from the trend, the adjustment being made to allow for the trend itself and thus to put all deviations on the same level.

With the above modification, the definition of expected income becomes

$$Y_E(T) = \beta \int_{-\infty}^{T} e^{(\beta - \alpha)(\theta - T)} y(\theta)\, d\theta .$$

The next problem that arises is how to break down the above integral into a finite sum that is suitable for empirical calculation. Data for current income is only available in discrete annual form. On account of this we shall divide up the time period $-\infty$ to T into intervals of one year and make the assumption that for $t-1 \leq \theta \leq t$, $y(\theta)$ can be approximated to by y_t.

When we come to the weighting pattern we find that there are two possible avenues to explore.

i) We can make an assumption about the weights similar to that for income and assume that for $t-1 \leq \theta \leq t$, $W(\theta-T)$ can be approximated to by $W(t-T)$. Under this assumption we obtain:

$$Y_E(T) = \beta \sum_{t=-\infty}^{T} e^{(\beta-\alpha)(t-T)} y_t$$

This is the simplest assumption we can make.

ii) We can follow the method suggested by Cagan.[24] Split the time $-\infty$ to T up into yearly periods and consider the integral as being composed of an infinite sum of integrals over a period of one year.

$$\text{i.e. } \beta \int_{-\infty}^{T} e^{(\beta-\alpha)(\theta-T)} y(\theta)\, d\theta = \beta \sum \int_{t-1}^{t} e^{(\beta-\alpha)(\theta-T)} y(\theta)\, d\theta$$

$$\text{now } \int_{t-1}^{t} e^{(\beta-\alpha)(\theta-T)} y(\theta)\, d\theta = y_t \int_{t-1}^{t} e^{(\beta-\alpha)(\theta-T)}\, d\theta$$

$$= y_t \frac{e^{(\beta-\alpha)(t-T)}}{\beta-\alpha} \left[1 - e^{-(\beta-\alpha)}\right]$$

If we now sum the above over the entire period we shall obtain:

$$Y_E(T) = \beta \sum_{t=-\infty}^{T} y_t \frac{e^{(\beta-\alpha)(t-T)}}{\beta-\alpha} \left[1 - e^{-(\beta-\alpha)}\right]$$

$$\text{i.e. } Y_E(T) = \frac{\beta}{\beta-\alpha} \left[1 - e^{-(\beta-\alpha)}\right] \sum_{t=-\infty}^{T} y_t \; e^{(\beta-\alpha)(t-T)}$$

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24 P. Cagan, "The Monetary Dynamics of Hyperinflation", in M. Friedman (ed.) Studies in the Quantity Theory of Money, op.cit.

It will prove informative to compare these last two estimates of expected income. They are equal if and only if

$$\frac{\beta}{\beta-\alpha}\left[1 - e^{-(\beta-\alpha)}\right] = \beta$$

$$\frac{1}{\beta-\alpha}\left[1 - e^{-(\beta-\alpha)}\right] = 1$$

$$e^{-(\beta-\alpha)} = 1-(\beta-\alpha)$$

On comparison with the Maclaurin expansion of e^x viz.

$$e^x = 1 + x + \frac{x^2}{2!} + \cdots$$

we see that the difference between them amounts to neglecting second and higher order powers in the expansion. Thus the difference between them will be negligible if $\beta-\alpha$ is small. However, with a short weighting period β will not be small and thus the two estimates will diverge.

The final decision which has to be made concerns the length of the weighting period. It seems reasonable to assume that events of more than, say, four years ago will not have any significant effect on present money holdings. This assumption may not be valid if the monetary system has undergone a violent upheaval during the life of an individual since there is some evidence that German memories of the great inflation of the early 1920's and American memories of the price collapse of 1929-33 still exert an influence on patterns of monetary behaviour. However, Friedman has estimated that "the horizon is on the average something like three years."[25] For this reason, as well as for reasons of availability of data, a time horizon of four years was chosen in the construction of Expected Income figures.

It will prove most informative to make a visual inspection of the three alternative weighting patterns over a four year weighting period.

[25] M. Friedman, "Windfalls, the 'Horizon', and Related Concepts in the Permanent-Income Hypothesis" in C.F. Christ et al, Measurement in Economics (Stanford, 1963), p.7.

The three patterns are:

i) $\beta \sum_{i=-3}^{0} e^{\beta i}$ (β = 0.423 ; t' = 4.73)

ii) $\beta \sum_{i=-3}^{0} e^{(\beta-\alpha) i}$ (β = 0.390 ; t' = 5.13)

iii) $\frac{\beta}{\beta-\alpha}\left[1 - e^{-(\beta-\alpha)}\right] \sum_{i=-3}^{0} e^{(\beta-\alpha) i}$ (β = 0.754 ; t' = 2.65)

In the above α = 0.044[26] and β was found by trial and error so that the sum of the weights was unity. The value of β in each case is given in parenthesis along with the effective weighting period t'.

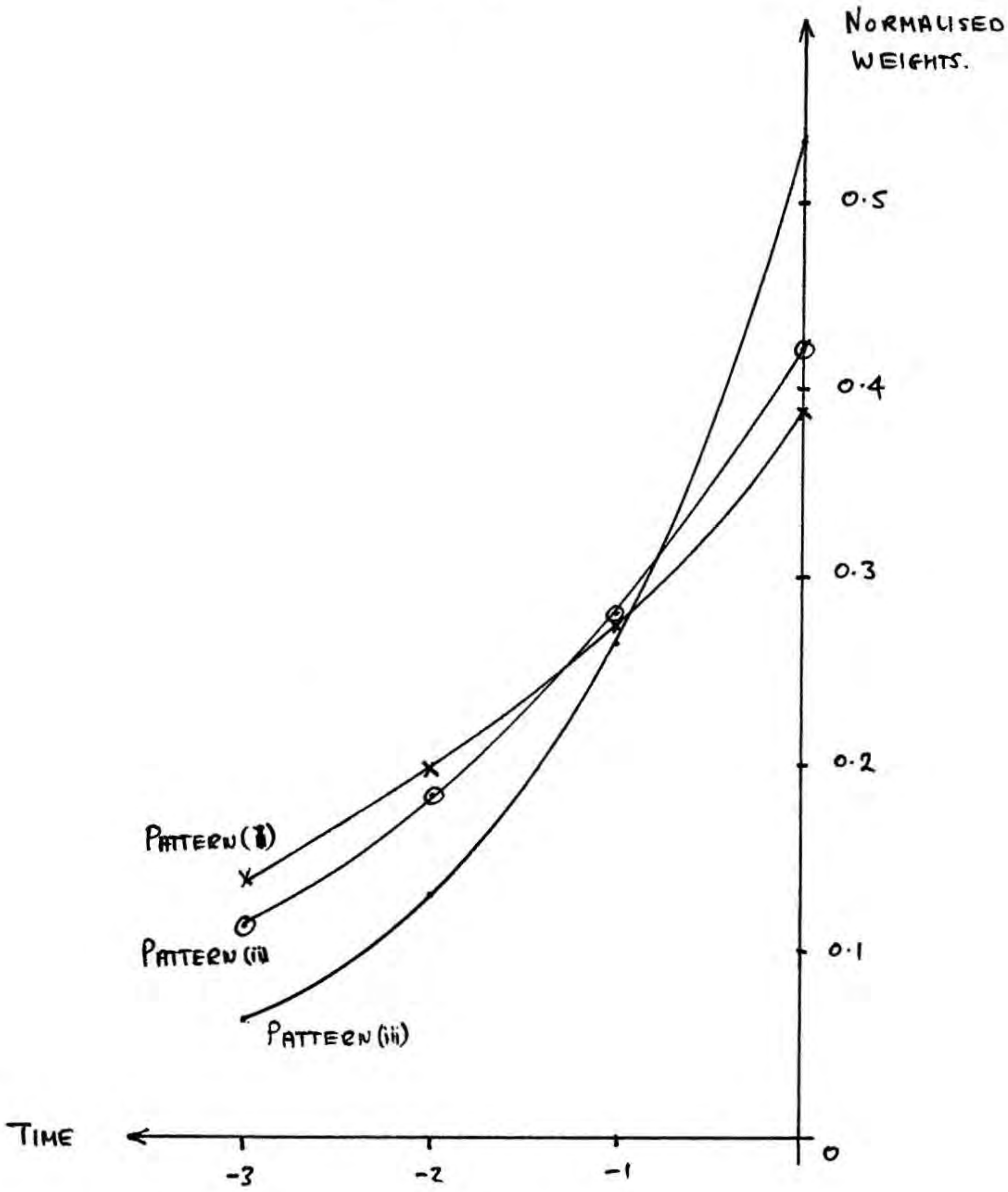
A graphical depiction of the actual weights is given in diagram 1. We here see visually the different characteristics of the weighting patterns as hinted to by their theoretical effective weighting periods. Pattern (ii) has the longest weighting period and as would be expected, it gives a relatively low emphasis to the early years and consequently it has a rather abrupt cut off at the end of the four year period. Pattern (i) gives more importance to earlier years and has a less abrupt cut off than the previous pattern. However, it is unsuitable in that it ignores secular growth. Pattern (iii) with the shortest weighting period has the most acceptable characteristics, giving the highest importance to early years and having the least abrupt cut off. For these reasons pattern (iii) was adopted when calculating expected income figures. The actual formula used to calculate $Y_E(t)$ is:

$$Y_E(t) = 0.539860\, Y_t + 0.265419\, Y_{t-1} + 0.130492\, Y_{t-2} + 0.064155\, Y_{t-3}$$

The sum of the weights differed from unity by 7/1000 of 1% which was considered negligible.

26 α was estimated by a log-linear regression on real G.D.P. figures.

Diagram 1



C. Wealth. In chapter II, we regarded wealth as the capitalised value of an income stream and arrived at the following result:

$$W_T = \frac{Y_h}{r_h} + \frac{Y_n}{r_n}.$$

This definition, useful as it has been for theoretical analysis, is not very satisfactory for empirical purposes on account of the difficulties encountered in obtaining a measure of r_h and r_n and hence will have to be replaced by a more pragmatic concept. Perhaps the most useful and generally accepted classification is that used by Goldsmith[27] in his various studies. As an immediate consequence of

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27 R.W. Goldsmith, A Study of Saving in the United States Vol. 3 (Princeton, 1956) esp. tables W1 and W9 - W16; and R.W. Goldsmith and C. Saunders (ed.) The Measurement of National Wealth; Income and Wealth Series Vol. 8 (London, 1959).

the above definition, three important questions arise.

i) Should human wealth be treated along with non-human wealth as a constraint in the determination of asset equilibrium?

ii) Should non-human wealth be consolidated or combined?

iii) How do we treat the assets and liabilities of the government?

Each of the above will now be subjected to a brief analysis, but it must be noted that the questions are essentially of academic interest as ultimately the choice will be dictated by the more mundane requirement of the availability of data.

The first point to be remembered when drawing up an empirical measure of wealth is that the relevant concept of wealth is the financial one and not the concept of productive capital. Thus financial wealth comprises the value of all assets belonging to an individual. These assets are valuable because they can be exchanged for money, and hence the concept depends to a large extent on the nature of the institutions governing the transfer process. In drawing up our empirical measure of wealth we shall start from the individual and determine the value of his assets that act as a constraint bearing in mind the fact that these assets must be marketable and have value in exchange.

An answer to the first question can now be obtained in the light of the above discussion. On account of the non-separability of human capital from its owner we see that human capital does not play an important role in an individual's portfolio of financial wealth. A pragmatic point against the inclusion of human wealth is the difficulty experienced in obtaining a suitable measure of human wealth. It is for these reasons as well as those expressed in chapter II that we do not include human wealth in the accounting concept of wealth.

The answer to the second question is inextricably linked with the aggregation problem. On the micro-level an individual can hold non-human wealth either in the form of tangible assets or intangible assets. Of these, intangible assets are the most liquid. However, on aggregation, we see that for the nation as a whole, a significant proportion of domestic intangible assets are balanced by corresponding liabilities. A strong case can be put forward for combining the wealth statement on aggregation instead of consolidating it. If this was the case, the appropriate wealth concept would entail a substantial amount of double counting upon aggregation to form the combined statement.

In the case where we are considering a combined wealth statement, the problem arises of how to deal with the assets and liabilities of the government. The liabilities of the government to the public are considered by the public as a means of holding wealth and hence they should be considered in the definition of wealth. On the other hand, it is extremely unlikely that the public's cash holdings will be affected to any significant degree by the value of government assets. However, the distinction between government and private ownership is by no means clear as is exemplified by the existence of public utility companies like Escom. The problem of the influence of government wealth is to a large degree unexplored both theoretically and empirically, although most economists[28] feel that government assets should be excluded and government liabilities included in the combined wealth statement.

In practice, of course, the final choice must be dictated by the availability of data. In South Africa, consolidated wealth figures comprise the value of private and governmental holdings of:

i) Buildings and Constructions

ii) Machinery, Plant and Equipment

iii) Inventories.

This figure may be taken to represent the value of reproducible tangible assets in a consolidated wealth statement.

The shortcomings of the above measure can best be pinpointed by considering the relationship between the accounting concepts of

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28 For an aphoristic discussion of the problem see A.H. Meltzer, "The Demand for Money : The Evidence from the Time Series", Journal of Political Economy, Vol. 71, (June, 1963), pp. 219 - 46.

income and wealth in the light of the theoretical relation $Y = rW_T$. No important differences arise in the relationship between the theoretical concept of income and the accounting concept. However, this is by no means the case for wealth. To show this we need to disaggregate total wealth into human wealth W_h and non-human wealth, and the latter into intangible assets W_i, non-reproducible tangible assets W_{t_n}, and reproducible tangible assets W_{t_r}. Thus we have:

$$Y = r_h W_h + r_i W_i + r_{t_n} W_{t_n} + r_{t_r} W_{t_r}$$

where the various rates of return have the obvious meanings. As was shown in chapter II, the various rates of return on wealth are not necessarily the same. When we substitute our empirical measures of wealth and income into the above, we see that due to lack of data on wealth, the equation becomes:

$$G.D.P. = \ldots + \ldots + \ldots + r_{t_r} W_{t_r}.$$

This obviously does not hold and thus we cannot use wealth and income interchangeably in our empirical demand functions. As a consequence of this we would expect to obtain different results from wealth constrained and from income constrained functions.

A point of further interest would be to calculate the ratio of G.D.P. to measured wealth and to observe the behaviour of this ratio. This has been done in diagram (2). The behaviour of this ratio shows no significant relationship to that of any of the interest rates in diagrams (3) and (4) as would be expected from theoretical considerations. This divergence is most probably due to the inadequacy of the empirical wealth measure used.

5. Interest Rates.[29]

These can conveniently be divided into short term rates and long term rates.

A. Short term rates. We shall use three alternative measures of the short term rate of interest.

[29] Unless otherwise stated, all data was obtained from the South African Reserve Bank Quarterly Bulletin.

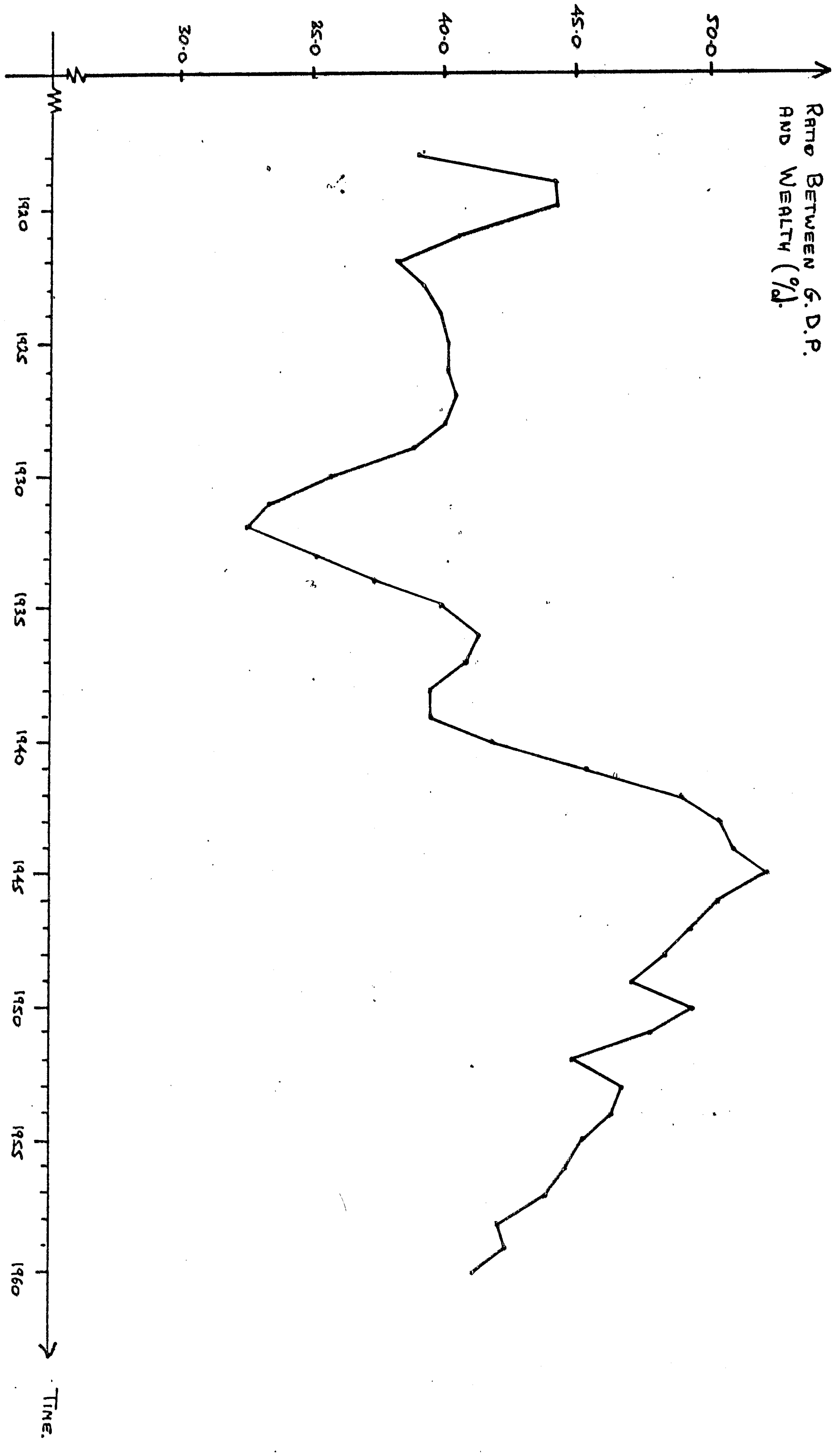

Ratio Between G.D.P. and Wealth (%).
50·0
45·0
40·0
35·0
30·0
1920
1925
1930
1935
1940
1945
1950
1955
1960
Time.

Diagram 2.

DIAGRAM 3

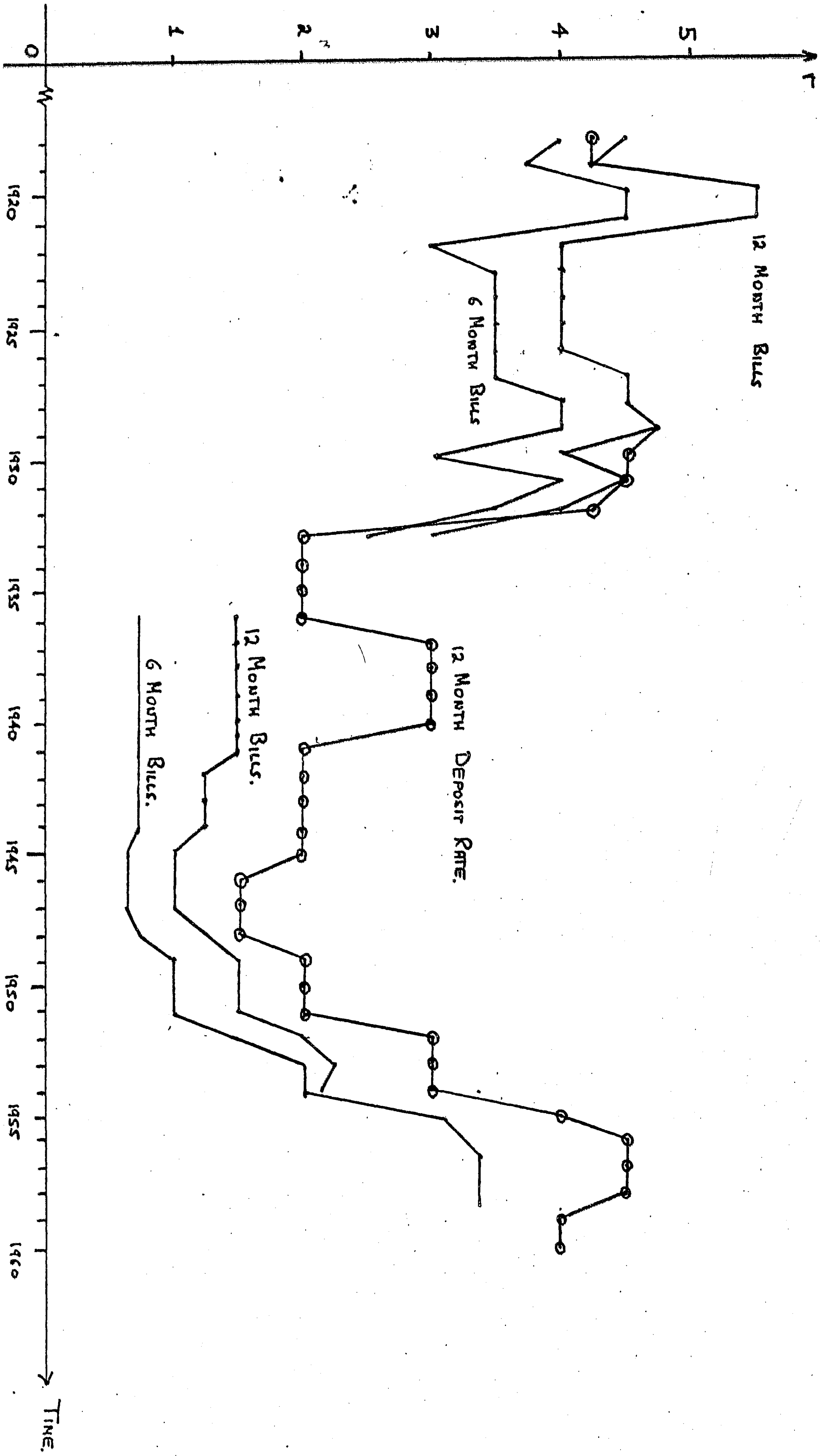
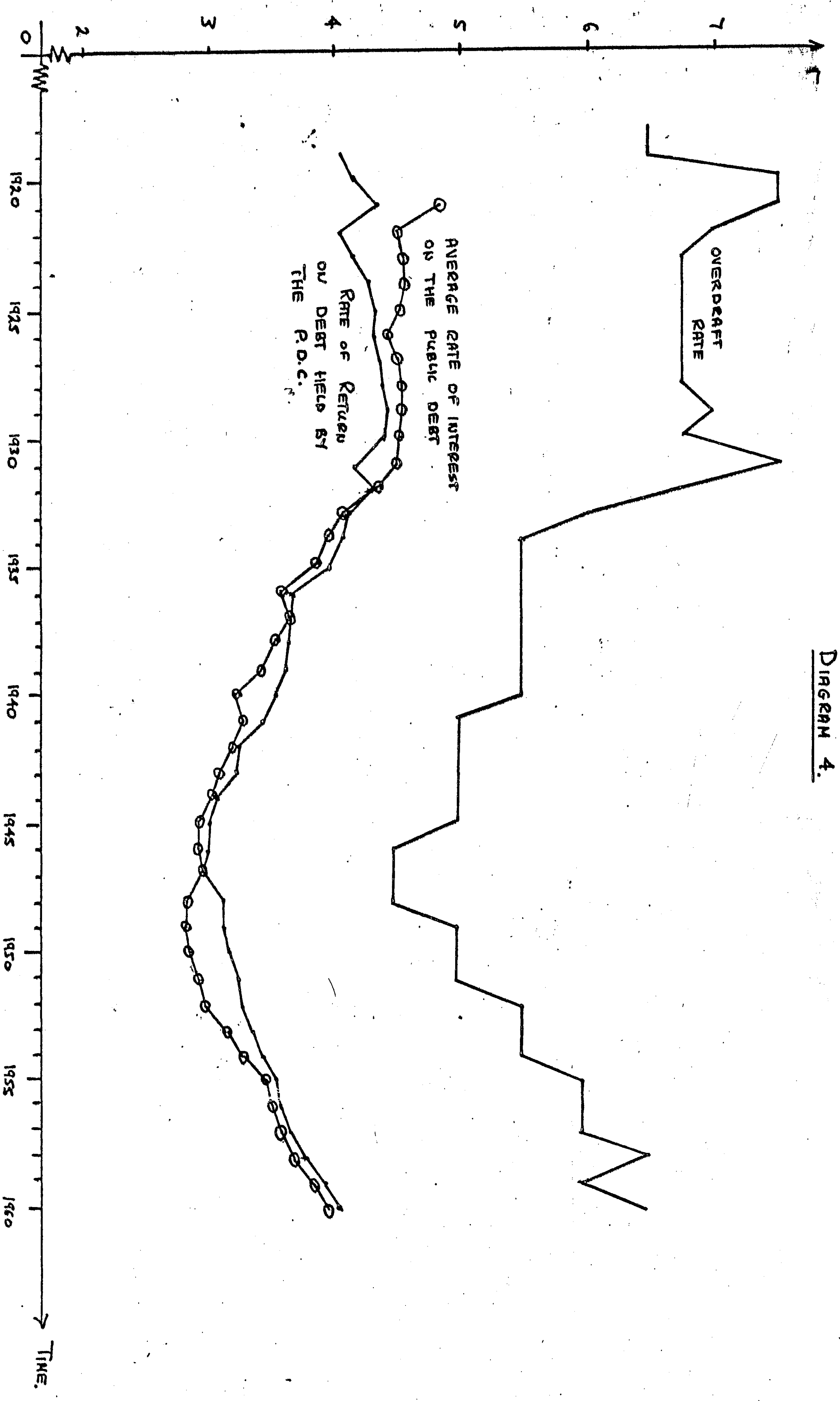

r
0
1
2
3
4
5
1920
1925
1930
1935
1940
1945
1950
1955
1960
TIME.
12 MONTH BILLS
6 MONTH BILLS
12 MONTH DEPOSIT RATE.
12 MONTH BILLS.
6 MONTH BILLS.

DIAGRAM 4.


OVERDRAFT RATE
AVERAGE RATE OF INTEREST ON THE PUBLIC DEBT
RATE OF RETURN ON DEBT HELD BY THE P.D.C.
0
2
3
4
5
6
7
1920
1925
1930
1935
1940
1945
1950
1955
1960
TIME.

i) The 6 month Treasury Bill rate. Figures are available for this up to December 1958 when they were discontinued, with the following exception. South Africa went off the gold standard on 28th December 1932 and this caused a sudden increase in liquidity. As a consequence of this excess of liquidity, the Treasury found that it had no need to borrow money and consequently stopped issuing both 6 month and 12 months bills on 13th February 1933. Both types of bills were first re-issued on 2nd June 1936 except that now the rate quoted was the discount rate and not the rate of interest as previously.[30]

ii) The 12 month Treasury Bill rate. Figures are available up to the end of 1954 when they were discontinued. The same caveat applies for the period 1933-36.

As a consequence of this excess of liquidity in the thirties it was felt that, at least for a few years after the re-issue of Treasury Bills, the rate quoted would not accurately reflect the true cost of short term borrowing. To counter this objection a third measure of the short term rate of interest will be used.

iii) The rate of interest on 12 month deposits at commercial banks. These figures are available in the form of a consistent series for the whole period under consideration. The behaviour of these various short term interest rates over time is shown in diagram (3).

B. Long term rates.

We shall consider 4 alternative measures of the long term market rate of interest, none of which is entirely satisfactory.

i) The Government Bond Yield. This would prove an eminently satisfactory measure of the long term rate of interest if it were not for the fact that no figures prior to December 1937 are available.

ii) The minimum overdraft rate at commercial banks. Figures for this are obtainable in a consistent form for the period under consideration. However, there are two objections to using this as an index

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[30] G. de Kock, A History of the South African Reserve Bank (1920 - 52) (Pretoria, 1954).

of the long term market rate of interest. Firstly, the duration of the overdraft is not specified and secondly it would be rather sluggish in changing in response to market forces.

iii) The rate of return on the debt held by the Public Debt Commissioners.[31] This measure has three major failings. Firstly, part of the assets held by the commissioners do not earn interest and part are held in the form of short term paper. Thus the figure quoted will have some elements of a short term rate embodied in it. However, the ratio of the short term debt held by the commissioners is usually in the region of about 6% of the total debt. Secondly, for policy reasons, the commissioners will tend to hold the short end of long term government stock, and thus the figure would represent a medium rather than a long term rate of interest. Finally, the rate quoted will not represent the rate of interest reigning in that particular year, but rather a weighted average of past rates. This arises on account of the fact that the current market rate will only affect the quoted rate to the extent that funds are available to purchase existing securities at the market rate quoted that year. A further problem that arises is that the figures are quoted for the end of the financial year, viz. 31st March and not for the end of the calendar year. However, it was felt that the error involved in taking this figure to correspond to 31st December of the previous year was small enough to be ignored.

iv) The average rate of return on the Public Debt.[32] This figure suffers from some of the objections to the previous figure in that it contains significant amounts of non-interest bearing and short term debt. The major objections to this figure arises from the method of calculation adopted in compiling it. To derive it, the face value of the total debt has been divided into the value of the fixed annual

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[31] Figures for this were obtained from the appropriate Report of the Public Debt Commissioners as published by the government printer.

[32] Figures for this were obtained from the appropriate Report of the Controller and Auditor-General as published by the government printer.

contractual interest payments, and thus the figure obtained is to a large extent independent of the market rate of interest. It will be affected by it to the extent that the nominal rate of interest on new issues is influenced by it. In addition there are two other definitional problems. The first of these, the fact that the figures are quoted for the end of the financial year, was dealt with in the same way as previously. The second is that up to and including 1950 a figure was quoted for the average rate of return on the debt held internally, whereas from 1951 onwards the figure was quoted for all the debt. As of 31st March 1951, 95.66% of the total debt was held internally and so the error involved is likely to be small. The behaviour of these long term interest rates over time is shown in diagram (4).

6. A Methodological Problem.

One last point that must be raised before we proceed to test empirically the various functions developed in chapter II is a methodological one.

The method adopted in the theoretical formulation is that of partial equilibrium static analysis. In this method we start from a set of basic assumptions, not descriptions, and then proceed by the laws of Aristotlean logic to a conclusion which is then empirically tested. All other factors that could conceivably affect the final form of the function have been defined away by means of a ceteris paribus assumption. Thus the demand schedule, by making no allowance for the sequence of events, requires that the alternative points that comprise the schedule be in existence all at the same time. This requirement contradicts one of the laws of Aristotlean logic, viz. the law that a thing cannot both be A and not-A at the same time. It would now seem that on account of the above contradiction we could never obtain an empirical representation of a theoretical demand schedule.

As a consequence of this, we shall be forced to adopt a more pragmatic approach than that proposed in chapter II. The data from which the functions are estimated are in the form of a time series and the points so obtained are assumed to be the equilibrium points of supply and demand. The various demand functions will be fitted to the time-series data by the method of least squares and that function which yields the best fit as measured by the correlation co-efficient, qualified by the Durbin-Watson statistic being non-critical, will be taken as representing the demand function.

7. Summary.

The problems inherent in aggregating from the micro-level to the macro-level were first discussed along with the assumptions necessary to obviate them. The the empirical measures of the variables entering the demand function were discussed and the differences between the empirical measures and the theoretical concepts were explored. Finally a methodological problem arising from the attempt to estimate a demand function from time-series data was noted. In view of the above difficulties which are inherent in any econometrical study of this nature, it was decided to adopt what is essentially a pragmatic approach in the empirical estimation of the various demand functions.

CHAPTER IV

Keynesian Liquidity Function

1. The Relation Between Income Velocity and the Rate of Interest.

We shall use as the starting point of our empirical study of Keynesian liquidity functions, the crude version of the Cambridge Quantity Theory, viz:

$$M = kY. \tag{1}$$

This theory assumes that cash balances are a fixed proportion of income and that this proportion is independent of the interest rate. If we take M to be M_1 as defined and Y to be G.D.P. at market prices we can then test the above equation by calculating the ratio Y/M which is the reciprocal of the Cambridge k and observing its constancy. This has been done in diagram 1. A cursory inspection of this diagram reveals the non-constancy of 1/k, and thus we shall not give this equation any further consideration. It should be noted that 1/k is equal to V_y, the income velocity of money.

Before extending the analysis, there are two definitional problems which have to be faced. The first of these is the appropriate measure of velocity, whether it is transactions velocity or income velocity that is the relevant concept, and the second is how to take account of the government sector.

The velocity concept which is going to be considered is that of income velocity V_y, and not that of transactions velocity V_t. There are two reasons for this.

i) There is a lack of data on the volume of transactions for the economy as a whole.

ii) Even if this data was available, it may still be desirable to concentrate on V_y in a study directed towards the income implications of liquidity preference because V_y explicitly relates the quantity of money to the level of income.

Selden points out that in answering the question whether the demand for money should pertain to desired money relative to all payments or only to income payments, we must answer the following question:

"Suppose that the cost of holding money is constant but changes in other variables cause V_t and V_y to diverge. Will this divergence of velocities take the form of (1) a decline in V_y, with no change in V_t; (2) a rise in V_t, with no change in V_y; or (3) both a decline in V_y and a rise in V_t?".[1] If (1) is correct, then the V_t approach is correct. But if (2) is correct then the V_y approach is preferable.

There are four methods of treating the government in V_y estimates:

i) include the government in both terms of the V_y ratio;

ii) include the government in the numerator only;

iii) include the government in the denominator only; and

iv) exclude the government from both terms in the ratio.

Selden,[2] as well as Bronfenbrenner and Mayer[3] have used method (ii). Even though Gordon[4] gives some excellent reasons for using only private spending instead of G.N.P., we shall adopt method (ii). This, it should be noted, is consistent with the discussion in the previous chapter.

The next step in the analysis is to try to explain the reasons behind the non-constancy of V_y. The most common variable that is used to explain variations in V_y is the interest rate. If the money supply is divided into active balances M_t, having a constant transactions velocity V_t, and idle balances M_a, having a velocity of zero, then if the liquidity preference theory is empirically sound, the velocity at which the money supply circulates will vary directly with the interest rate.

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[1] R.T. Selden, "Monetary Velocity in the United States", in M. Friedman (ed.), <u>Studies in the Quantity Theory of Money</u> (Chicago, 1956), pp. 196-97.

[2] R.T. Selden, op.cit.

[3] M. Bronfenbrenner and T. Mayer, "Liquidity Functions in the American Economy", <u>Econometrica</u>, Vol.28, (Oct., 1960), pp. 810-34.

[4] R.A. Gordon, "The Treatment of Government Spending in Income-Velocity Estimates", <u>American Economic Review</u>, Vol. 40, (March, 1950), pp. 152-59.

$$V_y = V_t \left(\frac{M_t}{M_t + M_a} \right)$$

As M_a increases, the velocity V_y will decrease. If, contemporaneously with a decline in V_y interest rates consistently fall, then the liquidity preference theory is, at least to some extent, substantiated. For convenience, instead of plotting V_y against the rate of interest, we shall plot the reciprocal of V_y against the interest rate, and then if the theory was empirically sound we would expect to obtain a scatter diagram which approximated to a hyperbola. Thus in effect we are going to test the equation

$$1/V_y = f(r). \tag{2}$$

It should be noted that, as shown in chapter II, this equation corresponds to a Keynesian type demand function which is homogeneous of degree 1 in income.

Since in the above treatment, income velocity is supposed to depend on the magnitude of idle balances, it will prove fruitful to examine the factors influencing idle balances. A more comprehensive treatment will be given later, and it will suffice for the present to observe that idle balances are a means of storing wealth and as such are likely to be influenced by the wealth of the holders.

Stedry[5] recognises this and postulates that the long-run asset demand for money should be expressed as:

$$M_a = \mu(r) . W = \mu(r) . \frac{W}{T}.T$$

Where W is the aggregate wealth of the community, μ the proportion of aggregate wealth held in the form of cash to satisfy the liquidity or asset motive, and T the volume of transactions. He assumes that the ratio $W/T = \beta$ is constant in the long-run. At the same time, he feels that the transactions demand for money should be written as a function of the rate of interest, viz.

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[5] A.C. Stedry, "A Note on Interest Rates and the Demand for Money", Review of Economics and Statistics, Vol. 41, (Aug., 1959), pp. 303-7.

$$M_t = k(r) . T$$

$$M = M_t + M_a = \left[k(r) + \mu(r) . \beta \right] T$$

$$\text{i.e. } M/T = \frac{1}{V_t} = g(r)$$

It must be noted that the velocity concept used here is that of transactions velocity, but the argument could equally well have been framed in terms of income-velocity. The above equation is operationally identical to equation (2), and hence this method of dealing with the influence of wealth on the demand for idle balances is not empirically significant.

It would appear that the best method of taking into account the influence of wealth would be to bring it explicitly into the velocity equation. This would result in an equation of the form:

$$1/V_y = f(r,W) \qquad (3)$$

M. Friedman[6] has further extended the concept of income velocity in his study of the demand for money in the United States. He set out to explain the observed discrepancy in the secular relation between income velocity and real income per capita and the cyclical relation between them. He observed that income velocity declines secularly as real income rises, whereas during the cycle income velocity rises as real income rises and falls as real income falls. He postulated the concept of "Permanent Velocity" which was a function of Permanent Income alone to explain the above paradox. His original formulation of the velocity equation was homogeneous of degree 1 in population, but so as to keep in line with the approach adopted in this study we shall reformulate it as an aggregative relation. It should be noted that none of the essential ideas are altered by this reformulation. Friedman's velocity equation will then take the form:

$$V_p = f(Y_p), \qquad (4)$$

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[6] M. Friedman, "The Demand for Money : Some Theoretical and Empirical Results", Journal of Political Economy, Vol. 67, (Aug., 1959), pp. 327-51.

where V_p is permanent velocity and Y_p is permanent aggregate income in real terms.

A further extension to the concept of Permanent Velocity has been presented by Meltzer.[7] He uses a simplified form of a wealth constrained demand function viz:

$$M = k r^{\alpha} A^{\beta}$$

where : r is the rate of interest

A is some measure of wealth

$\alpha < 0$ and $\beta > 0$

Permanent Income is defined as rA. Hence dividing both sides of the above by Permanent Income we obtain

$$1/V_p = k r^{\alpha - 1} A^{\beta - 1} \qquad (5)$$

He then assumes that $A^{\beta - 1}$ can be neglected because $\beta - 1 \approx 0$ and hence obtains a relationship between permanent velocity and the rate of interest.

The last three equations considered, viz. (3), (4) and (5), whilst representing modern thinking on the determinants of velocity, have admittedly transgressed the original subject material of this chapter. It is for this reason, as well as for the reason that the same concepts can better be handled as variations in the functional form of the demand for money that we shall not consider them any further in this chapter, leaving their empirical testing to subsequent chapters. This also means that we shall ignore all considerations of stability for the present with the exception of the following case.

In a paper by Minsky,[8] the hypothesis that the velocity-interest rate schedule was not stable was first proposed. He started from a Kalecki-type[9] velocity-interest schedule which is essentially a ceteris

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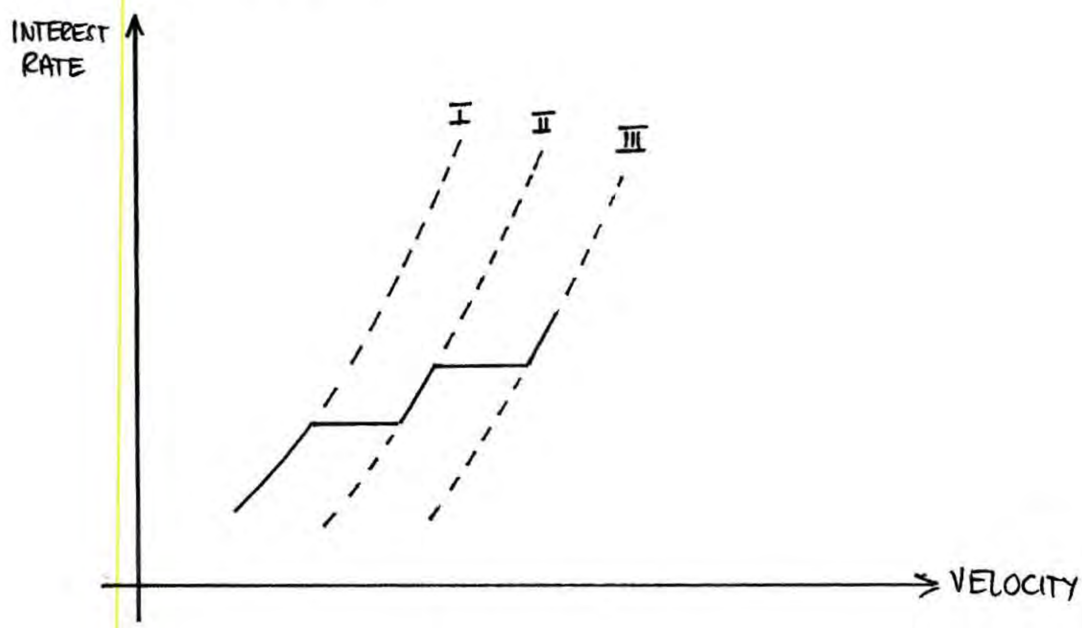
7 A.H. Meltzer, "Yet Another Look at the Low-Level Liquidity Trap", Econometrica, Vol. 31, (July, 1963), pp. 545-49.

8 H.P. Minsky, "Central Banking and Money Market Changes", Quarterly Journal of Economics, Vol. 71, (May, 1957), pp. 171-87.

9 M. Kalecki, Theory of Economic Dynamics (London, 1954).

paribus relation. An important source of disturbance of the ceteris paribus assumptions is the relative availability of money substitutes. What Minsky has suggested is that the provision of money substitutes is systematically connected with movements along the velocity-interest schedule, since high interest regimes give the incentive to innovate in constructing new financial institutions and in creating new financial arrangements. Thus the shift of the curve is not a once-for-all occurrence, but a consequence of the fact that high interest rates themselves destroy the ceteris paribus assumptions. Minsky argues that the resultant velocity-interest rate relation is a step function of the form shown below.



If this hypothesis is true, the observed schedule will be much flatter than the individual curves. It is not immediately clear from Minsky's article whether the relevant velocity concept was income velocity or transactions velocity. Rousseas[10] and Bernstein,[11] using quarterly data during the period 1951-9, claim to have shown that in America

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10 S.W. Rousseas, "Velocity Changes and the Effectiveness of Monetary Policy 1951-7", Review of Economics and Statistics, Vol. 42, (Feb., 1960), pp. 27-36, and "Velocity to Interest Rate Changes : A Rejoinder", ibid., (Nov., 1960), pp. 455-57.

11 P.L. Bernstein, "The Response of Income Velocity to Interest Rate Changes : A Comment", Review of Economics and Statistics, Vol. 42, (Nov., 1960), pp. 453-55.

higher income velocities have come to be associated with given interest rate levels, so lending some empirical substance to Minsky's a priori reasoning.

A shift in the velocity-interest rate schedule could be caused by three factors. The income velocity-interest schedule is of the form

$$V_y = V_t \left(\frac{M_t}{M_t + M_a} \right)$$

where M_t and V_t are assumed to be independent of the rate of interest, whereas M_a and V_y are assumed to be functions of the rate of interest. Thus a shift in the velocity-interest rate schedule could be caused by:

i) A shift in the asset demand schedule, ceteris paribus. The determinants of the asset demand for money will be dealt with subsequently in this chapter, but it would not be too presumptious to assume an asset demand function of the form:

$$M_a = f(r, W) \quad \text{where} \quad \frac{\partial M_a}{\partial W} > 0.$$

However, under these assumptions, we would expect an inwards shift of the velocity-interest schedule. An increase in wealth would cause an outwards shift of the entire asset demand schedule unless money was regarded as an inferior asset. Thus for any given interest rate M_a would be larger than previously and consequently the value of V_y would decrease. Hence a ceteris paribus increase in wealth would cause an inward shift of the velocity-interest schedule.

ii) A change in V_t, ceteris paribus. V_t is determined to some extent by the financial institutional framework in the economy and one would suspect that Minsky's argument was based on changes in V_t.

iii) Some combination of (i) and (ii).

As there is not as yet any adequate statistical technique to differentiate between the three possible alternatives, the cause of any observed shift in the income velocity-interest rate schedule must remain a matter of conjecture.

2. Empirical Results.

The results of plotting the reciprocal of income velocity, defined as the ratio between M_1 and G.D.P. are depicted in diagrams 2 - 7.

A. Short term rates.

i) The reciprocal of income velocity is plotted against the 6 month Treasury Bill rate in diagram 2. A cursory inspection of the graph reveals a roughly hyperbolic shape which is in accordance with Keynesian theory. It would appear that the year 1919 as well as 1936 -9 differed from the general pattern displayed by other observations. 1919 was a year of post World War I inflation, whereas the years 1936-9 were the first four after the re-issue of Treasury Bills, and as a consequence of the excess of liquidity in South Africa during that period they very probably would not indicate the true cost of borrowing.

ii) The reciprocal of income velocity is plotted against the 12 month Treasury Bill rate in diagram 3. The same comments made above apply without exception to this graph.

iii) The reciprocal of income velocity is plotted against the 12 month deposit rate at commercial banks in diagram 4. Once again the graph shows hyperbolic tendencies but it is more disjoint than the previous graphs on account of the discontinuous nature of the changes in the interest rate. It is also not as obvious as in the previous two graphs as to which are the delinquent years.

By virtue of the hyperbolic shape exhibited by all the above graphs we can say that Keynesian liquidity preference theory has been to some extent justified. The absence of the delinquent years 1936-9 in diagram 4 supports the argument about the Treasury Bill rate not being a good indicator of the true cost of borrowing during that period.

B. Long term rates.

i) The reciprocal of income velocity is plotted against the minimum overdraft rate for commercial banks in diagram 5. This graph approximates to a disjoint hyperbola on account of the discrete nature of the changes in the interest rate. It is also characterised by the absence of any immediately obvious delinquent years with the possible exception of 1919.

ii) The reciprocal of income velocity is plotted against the rate of return on the debt held by the P.D.C. in diagram 6. The graph shows only faint hyperbolic tendencies but that of itself does not invalidate Keynesian liquidity preference theory because high income velocities are still associated with high interest rates and low income velocities with low interest rates. The years 1919 and 1933-5 appear as delinquent years though not as strongly as in the case of short-run interest rates. A further interesting feature is that the graph for the post-war period exhibits a higher degree of curvature than the graph for the whole period.

iii) The reciprocal of income velocity is plotted against the average rate of return on internal public debt in diagram 7. This graph shows a more hyperbolic tendency than the previous one and once again the years 1933-5 could be regarded as delinquent years. An interesting feature of the graph is that the relationship over the period 1939-1945 appears to differ significantly from that during 1946-1960, and once again the graph for the post-war period exhibits a higher degree of curvature than the graph for the whole period.

While all the graphs have varying shapes depending on the interest rate used they all support the Keynesian liquidity hypothesis. The presence of the delinquent years is not as noticeable as in the case of short-term interest rates and the absence of the delinquent years 1933-5 in diagram 5 supports the excess liquidity hypothesis advanced in the previous section.

<u>In Summa.</u>

i) All the graphs have a negative slope and thus the data fails to refute the Keynesian liquidity hypothesis.

ii) The degree of curvature of the graphs varies with the duration of the interest rate, and short term rates being associated with higher degrees of curvature than the long term rates which implies that the interest elasticity is greater for long term rates than for short term rates.

iii) The presence of the delinquent years 1936-9 in diagrams 2 and 3 and 1933-5 in diagrams 6 and 7 seem to indicate that the greatest shock on the financial system during the period 1918-60 was caused by the collapse of the gold standard.

iv) There has been no noticeable tendency for given income velocities to be associated with increased interest rates over time. Thus the data provide no evidence of Minsky-type shifts having occurred.

3. The Relation Between Idle Balances and the Rate of Interest.

The concept of income velocity has been strongly attacked by such diverse critics as A.W. Marget and J.M. Keynes. Marget considers that "it is possible to raise the question whether the concept of 'income velocity', as it has often been used, does not in reality pretend to provide a solution for a problem which, after the concept is adopted, is still left as much unsolved as it was before." [12] Keynes made a distinction between the velocity of income deposits which is "the relationship between the total annual receipts of income-receivers and the average stock of money held by them"[13] and the velocity of cash deposits which is "the relationship between the total flow of transactions for all purposes and the average stock of money held for all purposes."[14] However, he stated that "the relationship between the total annual receipts of income-receivers and the average stock of money held for all purposes is a hybrid conception having no particular significance",[15] yet it is this very concept that is normally used as a macro-measure of income velocity. Further he has stated that the use of this latter concept "has led to nothing but confusion."[16] A

[12] A.W. Marget, The Theory of Prices Vol. 1 (New York, 1938), p. 365.

[13] J.M. Keynes, A Treatise on Money Vol. 2 (London, 1930), p. 24.

[14] ibid.

[15] ibid.

[16] J.M. Keynes, The General Theory of Employment, Interest and Money (London, 1936), p. 299.

further criticism of the velocity concept, albeit transactions velocity, has come from the Radcliffe Report where they say that there is no reason for supposing "that there is any limit to the velocity of circulation; it is a statistical concept that tells us nothing directly of the motivation that influences the level of total demand."[17]

A second method of studying liquidity preference in which V_y is relegated to a secondary role, thus obviating the above objections was first mooted by A.J. Brown.[18] He worked with transactions velocity but his method is equally well applicable to V_y. The method consists of calculating income velocity in a period in which it is assumed that idle balances do not exist: this would be a period of maximum velocity. This velocity is then assumed to be constant for the entire period under consideration and idle balances are then computed as the difference between total balances and the ratio of income to the maximum velocity, viz:

$$L_t = M_t - \text{Min}\left[M/Y\right] \cdot Y_t \qquad (6)$$

where L_t = idle balances at time t.

Y_t = income at time t.

M_t = money supply at time t.

Properly stated, the liquidity preference theory explains the relationship between the rate of interest and the demand for idle balances in real terms, hence all the quantities in the above equation are real. This approach is the most commonly used one in the estimation of idle balances and it has been used in various guises by such authors as J. Tobin[19]; L.S. Ritter[20]; A.C. Stedry[21] and M. Bronfenbrenner and T. Mayer,[22] all of whom obtained good results in their attempts to

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17 Committee on the Working of the Monetary System (Her Majesty's Stationery Office, Command 827), para. 391.

18 A.J. Brown, "Interest, Prices, and the Demand Schedule for Idle Money", Oxford Economic Papers, Vol. 2, (May, 1939), pp. 46-69.

19 J.R. Tobin, "Liquidity Preference and Monetary Policy", Review of Economics and Statistics, Vol. 29, (May, 1947), pp. 124-31.

20 L.S. Ritter, "Income Velocity and Anti-Inflationary Monetary Policy", American Economic Review, Vol. 49, (March, 1959), pp. 120-29.

21 A.C. Stedry, op.cit.

22 M. Bronfenbrenner and T. Mayer, op.cit.

discover close relationships between the interest rate and the demand for idle balances.

Up to this point we have related idle balances to an interest rate without specifying the term of the rate, and have thus implicitly assumed that the interest structure has remained unchanged; i.e. that other interest rates, both short term and long term, rise and fall proportionately with the above rate. But we know that although interest rates often, though not always, rise and fall together they do not necessarily move in any fixed proportion to each other.[23]

Before showing the impact of various interest rates on the asset demand, a few prior thoughts about the appropriate rate of interest are in order. Kalecki feels that the velocity of circulation, and hence liquidity preference, depends on the short term rate. "Indeed the higher the short-term rate the greater is the inducement to invest money for short periods rather than to keep it as a cash reserve. ... The higher the short-term rate of interest the more expensive is this convenience as compared with the alternative of investing in short-term assets. ...

The reason for the singling out of the short-term rate in this context is as follows. The short-term rate of interest is the remuneration for foregoing the convenience of holding cash in its pure form. When holding money is compared with holding short bills, the only difference is that the bill is not directly usable for settling transactions and that it yields interest."[24]

But Kalecki does not want to over-emphasise the importance of the short term rate in this regard, and adds that he does not mean that all additions to cash at the disposal of the individual will tend to be invested entirely in bills. Changes in bond prices have also to

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23 This can be seen by a study of diagrams 3 and 4 in chapter III; it has also been confirmed by direct regressions between the various rates.

24 M. Kalecki, op.cit., pp. 73-74.

be considered. Tobin,[25] Behrman[26] and Bronfenbrenner and Mayer[27] prefer the use of short term rates whereas Selden,[28] Warburton[29] and Latané[30] prefer long term rates for this purpose. All of these writers conclude that, while they prefer one interest rate, they realise that a study of this kind cannot disregard other rates. Some kind of over-all index of interest rates, which takes into account the structure as well as the level of the rates, might be the best way of measuring the effect of interest rates on the demand for idle balances. However, lacking this, we shall have to be satisfied in plotting the relationship between idle balances and several interest rates.

The results of plotting idle balances as defined by equation (6) against the various rates of interest are shown in diagrams 8 - 13.

A. Short term rates.

i) The relation between idle balances and the 6 month bill rate is shown in diagram 8. A cursory inspection of the graph shows that large holdings of idle balances were associated with low interest rates and small holdings of idle balances were associated with high interest rates. A more detailed inspection of the graph, however, reveals that in the post-war period, the schedule has undergone a significant shift in that larger balances are associated with the same value of

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25 J.R. Tobin, "A Rejoinder", Review of Economics and Statistics, Vol. 30, (Nov., 1948), pp. 314-16.

26 J.N. Behrman, "The Short-Term Interest Rate and the Velocity of Circulation", Econometrica Vol. 16, (Apr., 1948), pp.185-90, and "The Short-Term Interest Rate and the Velocity of Circulation; Addendum", ibid., (Oct., 1948), p. 370.

27 M. Bronfenbrenner and T. Mayer, op.cit.

28 R.T. Selden, "Monetary Velocity in the United States", op.cit.

29 C. Warburton, "Monetary Velocity and the Rate of Interest", Review of Economics and Statistics, Vol. 32, (Aug., 1950), pp.256-57.

30 H.A. Latané, "Cash Balances and the Interest Rate - A Pragmatic Approach", Review of Economics and Statistics, Vol.36, (Nov., 1954), pp. 456-60, and "Income Velocity and Interest Rates : A Pragmatic Approach", Review of Economics and Statistics, Vol. 42, (Nov., 1960), pp. 445-49.

the interest rate than in the pre-war period. It should be noted that the dates associated with the shift of the schedule are by no means exact. The Second World War was taken as a convenient dividing line which corresponds sufficiently well to the observed shift. These comments should be taken as applying without exception to any subsequent diagrams purporting to show shifts in the schedule.

ii) The relation between idle balances and the 12 month bill rate is shown in diagram 9. The same sort of relationship as above is obtained, though due to the lack of data the shift between the pre- and post war periods is not as noticeable as in the previous diagram.

iii) The relation between idle balances and the 12 month deposit rate at commercial banks is shown in diagram 10. A similar relationship is again obtained, with the shift in the schedule being clearly discernible.

B. Long term rates.

i) The relation between idle balances and the minimum overdraft rate at commercial banks is shown in diagram 11. A result similar to those of section A is obtained in which the shift is clearly discernible.

ii) The relation between idle balances and the return on debt held by the P.D.C. is shown in diagram 12. A result similar to that above is obtained. Further inspection fails to reveal any immediately discernible shift of the schedule, thus apparently indicating a relationship which is stable over time. It should be noted at this point that the apparent stability of the schedule may be merely illusory and caused by the presence of statistical complications as discussed in chapter V.

iii) The relation between idle balances and the average rate of return on internally held public debt is shown in diagram 13. A result similar to that of diagram 12 is obtained except that a movement of the schedule is now apparent.

In view of the comments made above on the relationship between idle balances and wealth, it would be interesting to discover whether there is any relation between them. The relationship between idle balances as defined in equation (6) and national wealth has been plotted in diagram 14 and a cursory inspection of this diagram fails to reveal any stable relationship. Further examination reveals the presence of three separate relationships corresponding to the pre-war, war-time and post-war periods, analogous to the shifts in the schedules observed above.

<u>In Summa</u>.

i) In all graphs, large holdings of idle balances are associated with low interest rates and small holdings of idle balances are associated with high interest rates, which is in accordance with Keynesian theory.

ii) The relationship does not appear to be stable over time, differing between pre- and post-war periods. The degree of stability depends on the term of the interest rate, showing the greatest instability with the short term rates and in particular with the 6 month bill rate, perhaps the most volatile of all the rates, and the least instability with the long term rates, although as previously mentioned this stability may only be illusory due to the possibility of the presence of significant statistical complications.

iii) The instability of the relations between idle balances and interest rates and idle balances and national wealth indicates that neither of these two variables is on its own sufficient to explain the behaviour of idle balances.

R. Eisner[31] has suggested a modification to the above technique to take account of the fact that there may be idle balances in the year of maximum velocity. He has suggested that idle balances be written as:

$$L_t' = M_t - \text{Min}\,[M/Y]\, Y_t + L_o$$

Where L_o are the minimum idle balances that exist even in years of maximum velocity. He did not, however, specify any method of

[31] R. Eisner, "Another Look at Liquidity Preference", <u>Econometrica</u>, Vol. 31, (July, 1963), pp. 531-38.

estimating L_o. This modification could be combined with the method used by Klein viz. "Cash and demand deposits were regarded as active balances, related primarily to income, while time deposits were regarded as idle balances related primarily to the interest rate."[32] Combining the two together we would obtain the following formulation for idle balances,

$$L_t^* = M_t - \text{Min}\ [M/Y]\ Y_t + S_t \qquad (7)$$

where S_t are savings deposits.

The results of plotting idle balances as defined by equation (7) against the various rates of interest are shown in diagrams 15 - 20. The order of presentation is the same as previously viz. 6 month bills, 12 month deposit rate at commercial banks, minimum overdraft rate at commercial banks, rate of return on the debt held by the P.D.C. and the average rate of return on internally held public debt. Unless explicitly mentioned, this order will be adhered to in the presentation of all other results.

The results on the whole are very similar to those obtained previously, confirming in a general way the Keynesian hypothesis. The instability of the relation is more marked than before though the relation between the degree of instability and the term of the interest rate is preserved. The instability of the relationship between idle balances and national wealth (diagram 21) once again indicates that neither the interest rate nor national wealth considered alone, is capable of explaining the behaviour of idle balances.

4. Adjustment of Idle Balances for Wealth.

The liquidity preference function is often regarded as a function of other variables in addition to the rate of interest, usually either income or wealth. As Lydall[33] points out, Keynes was

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32 L.R. Klein, "Empirical Foundations of Keynesian Economics", in K. Kurihara (ed.), Post-Keynesian Economics (London, 1955), p. 303.

33 H.F. Lydall, "Income, Assets and the Demand for Money", Review of Economics and Statistics, Vol. 40, (Feb., 1958), p. 1.

not clear on the question of whether the asset demand for money was affected by income and/or wealth. This is brought out by two quotations from The General Theory dealing with the precautionary motive: "the desire for security as to the future cash equivalent of a certain proportion of total resources"[34] which suggests a relationship to the stock of wealth, and also that "in normal circumstances the amount of money required to satisfy the transactions-motive and the precautionary-motive is mainly a result of the general activity of the economic system and the level of money-income."[35]

But if we regard idle balances as one of several alternative means of wealth holding, we must be led to the conclusion that idle balances are a function of wealth rather than income. Keynes himself provided support for this view when he stated that "it is in respect of his stock of accumulated savings, rather than of his income, that the individual can exercise his choice between liquidity and illiquidity."[36] Further support for this view comes from Brechling who states that "the amount of money which people desire to hold as a store of wealth depends not only on the rate of interest but also the total amount of wealth available."[37] Statistical support for outwards movements of the asset-demand schedule in response to increases in the stock of wealth has been provided by Lydall.[38] In a cross-sectional study of 2,461 income units, Lydall found no significant correlation between gross income and liquid assets held by individuals, but he did find a significant positive correlation between total wealth and holdings of liquid assets.

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[34] J.M. Keynes, The General Theory of Employment, Interest and Money op.cit., p. 170.

[35] ibid., p. 196.

[36] ibid., p. 194.

[37] F.P.R. Brechling, "A Note on Bond-Holding and the Liquidity Preference Theory of Interest", Review of Economic Studies, Vol. 24, (June, 1957), p. 197.

[38] H.F. Lydall, "Income, Assets and the Demand for Money", op.cit.

Now that we have surmised that the asset-demand schedule shifts in response to changes in wealth, if we want to retain a two-dimensional representation we must overcome the problem of how to adjust the schedule in order to compensate for wealth changes. Horwich has suggested that: "An independent (wealth) movement ... is allocated between the asset-demand schedules as follows: each schedule receives a relative share of the total movement equal to the ratio of the existing quantity of the particular asset to total wealth."[39] This suggestion can easily be shown to amount to an assumption as to the form of the demand function for idle balances, namely that it is homogeneous of degree one in wealth.

Let L_t be idle balances as calculated and L_{tw} be their value after being corrected for wealth changes. Then by the above reasoning

$$L_{tw} = \frac{L_t}{W} = f(r)$$

i.e. $$L_t = f(r).W$$

which is equivalent to assuming that $f(r,W)$ is homogeneous of degree one in wealth. This assumption will be tested in the subsequent chapter when the asset-demand function is empirically analysed. An assumption which is essentially of the same nature but which has the advantage of not altering the numerical value of the idle balances as drastically, is to assume that the demand function is homogeneous of degree one in the ratio of wealth in any year to wealth in a particular reference year. For convenience we shall take this reference year to be the year in which income velocity was a maximum. Suppose the value of national wealth in that year was W_o. Then the demand function for idle balances would take the form

$$L_t = f(r, W/W_o)$$

$$= f(r).\ W/W_o$$

$$\frac{W_o}{W}.L_t = f(r)$$

$$\text{Let } L_{tw} = \frac{W_o}{W}.\ L_t$$

39 G. Horwich, "Money, Prices and the Theory of Interest Determination", Economic Journal, Vol. 67, (Dec., 1957), pp. 629-30.

Then we shall obtain $L_{tw} = f(r)$.
This latter procedure is the one which we shall adopt in adjusting idle balances.

The results of plotting wealth corrected idle balances as defined by equation (6) against the various interest rates are shown in diagrams 22 - 27. The results are of the same type as obtained previously with the exception that the shift has become less discernible and in some cases negligible, so lending credence to the hypothesis that idle balances are wealth dependent.

The results of plotting wealth corrected idle balances as defined by equation (7) against the various interest rates are shown in diagrams 28 - 33. Similar results are obtained with the exception that the shift in the function although lessened by the wealth correction process is still clearly discernible. This factor would suggest that the assumption of the asset demand function being homogeneous of degree 1 in wealth is not as good a first approximation as for idle balances alone or else that holdings of savings deposits are motivated by factors other than the rate of interest or wealth.

A point which has been mentioned in the previous chapters but has been made explicitly by Brown is well worth restating. This is that: "liquidity-preference schedules are, ideally, static in the sense in which a Marshallian-demand curve is static. That is to say, they characterise the system of 'tastes' existing at a single moment....

The statistical data from which schedules must be constructed, however, are time-series. If equilibrium is assumed to exist at every moment for which we can measure a set of coordinates, then each such set represents a point which is common to the schedule of the investor under consideration and the corresponding supply-schedule of the assets in the market in which he deals."[40] This above comment is particularly relevant in the light of recent work by Weintraub[41] and

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40 A.J. Brown, "The Liquidity - Preference Schedules of the London Clearing Banks", Oxford Economic Papers, Vol. 1, (Oct., 1938), pp. 52-53.

41 S. Weintraub, An Approach to the Theory of Income Distribution (Philadelphia, 1958), pp. 156-60.

Davis[42] on the irreversibility of the speculative demand schedule for money.

5. **An Alternative Method of Calculating Idle Balances.**

It has been assumed in determining idle balances by the above method that transactions velocity has remained constant at its maximum value <u>viz.</u> 7.13 in 1930. The point now to be determined is how crucial an assumption this is: i.e., if we allow V_t to vary secularly, will idle balances still exhibit an inverse relationship to the rate of interest of the type described by the liquidity preference theory? Basically there are three possible alternatives.

i) Constant V_t. This is the case that has already been discussed.

ii) Secularly rising V_t. For simplicity, assume that V_t has a secular growth rate of 4% per annum which is the growth rate of real G.D.P. In order to facilitate comparisons with the other graphs, we shall assume that $V_t = 7.13$ in 1930. In this case, idle balances would be computed by means of the formula

$$L_t = M_t - \left(\frac{1}{V_{1930} + 0.04t}\right) Y_t$$

where t = o in 1930.

iii) Secularly declining V_t. We shall assume that V_t has been declining at a rate of 4% per annum from a value of 7.13 in 1930. Under these assumptions, idle balances would be computed by means of the formula

$$L_t = M_t - \left(\frac{1}{V_{1930} - 0.04t}\right) Y_t$$

where t = o in 1930.

There is not very much <u>a priori</u> reasoning dealing with the secular trend of V_t. Fisher[43] lists several factors which affect V,

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42 R.M. Davis, "A Re-Examination of the Speculative Demand for Money", <u>Quarterly Journal of Economics</u>, Vol. 73, (May, 1959), pp.326-32.

43 I. Fisher, <u>The Purchasing Power of Money</u> (New York, 1911), p.79 ff.

most of them causing an upward influence on V. On the other hand, Garvy[44] concludes that there have been structural and institutional aspects of the economy tending to prevent both income and transactions velocity from rising to levels established in the late twenties and perhaps even causing velocity to decline slightly since that period. In view of this paucity of a priori reasoning we shall have to choose between the three alternatives on purely empirical grounds.

The relationships between the various definitions of idle balances computed under assumptions of secularly rising or falling V_t and the various rates of interest have been calculated, and on account of the fact that they did not yield significantly different results, it was decided not to incorporate them into the text. These graphs exhibited the same general pattern as before in that large holdings of idle balances are associated with low interest rates and small holdings with high rates. Further examination revealed that the shift is more pronounced for idle balances computed under the assumption of a secularly rising V_t than for a constant V_t and the shift is less pronounced, in fact sometimes negligible, for idle balances computed under the assumption of a secularly falling V_t.

6. Summary of Results.

A. Velocity Diagrams.

i) Every diagram exhibited an inverse relationship between income velocity and the rate of interest, so tending to confirm the Keynesian hypothesis. With the exception of a few delinquent years, the diagrams exhibited a reasonable degree of stability which indicates that the assumption of a demand function which is homogeneous of degree one in income is an adequate first approximation.

B. Idle Balance Diagrams.

i) Every diagram depicts the type of relationship where large volumes of idle balances are associated with low interest rates

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44 G. Garvy, "Structural Aspects of Monetary Velocity", Quarterly Journal of Economics, Vol. 73, (Aug., 1959), pp. 429-47.

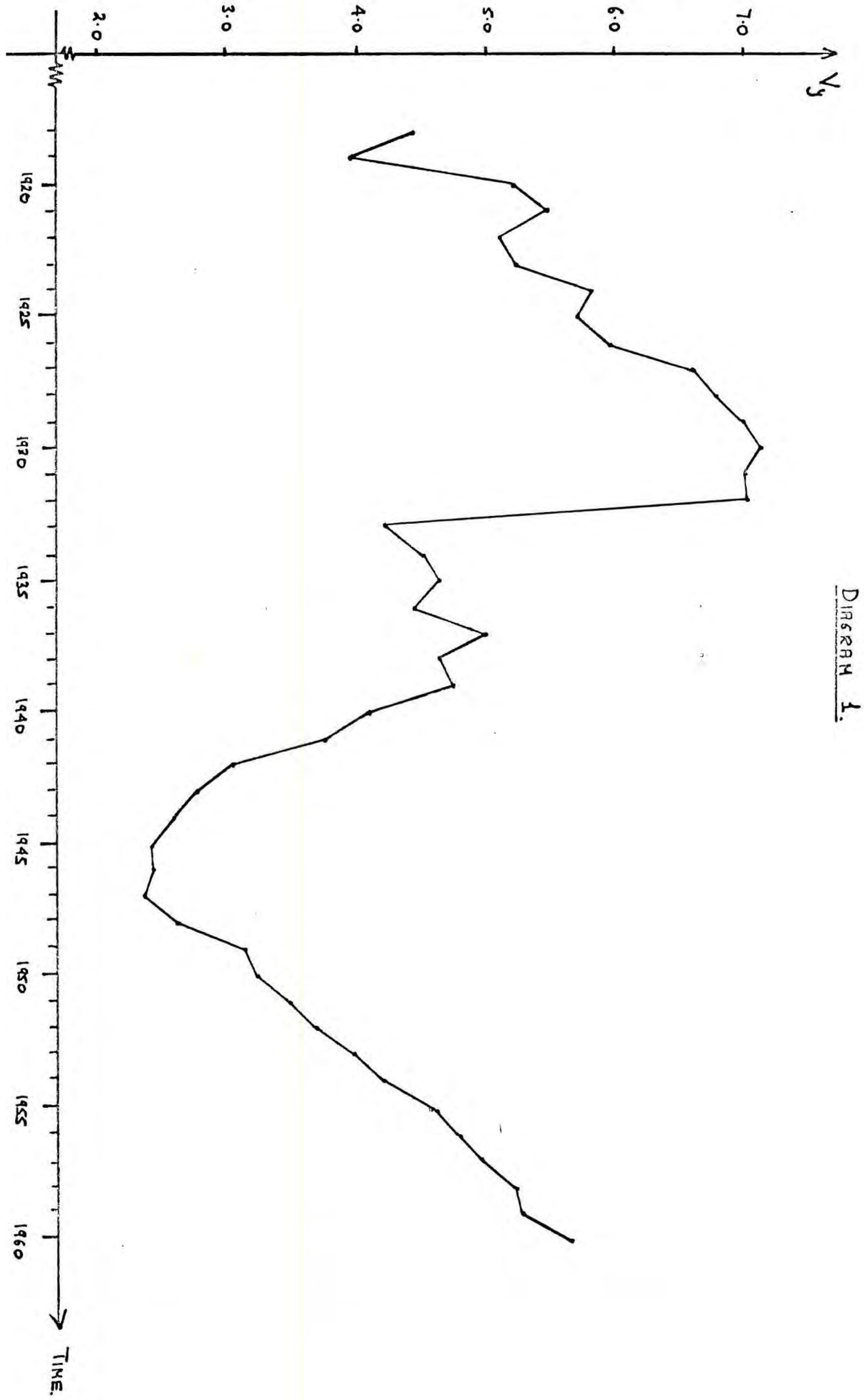
and small volumes of idle balances are associated with high interest rates. This type of behaviour is consistent with a Keynesian hypothesis and so none of the results refutes this hypothesis. This would also imply that the particular definition of idle balances adopted was not a critical factor in the empirical investigation of the Keynesian hypothesis.

ii) The stability of the functional relationship is naturally dependent on the particular definition used, and two major behaviour patterns have manifested themselves.

a) For any interest rate, the assumption of a secularly declining V_t gave the relationship which exhibited the greatest degree of stability over time, followed by a constant V_t, and then a secularly increasing V_t.

b) For any given method of calculating idle balances, the degree of stability of the functional relationship increases with the lengthening of the term of the interest rate, although as pointed out earlier, this stability may be merely illusory.

iii) Adjustment of the idle balances for the effect of wealth, assuming an asset demand function that is homogeneous of degree 1 in wealth, results in most cases in an improvement in the stability of the function, the degree of improvement in stability following the same pattern as in ii(a). The improvement in stability of the sum of idle balances and savings deposits is not as great as in the case of idle balances alone. This would seem to suggest that either the assumption of an asset demand function homogeneous of degree 1 in wealth is not as good a first approximation as in the previous case, or else that savings deposits are not determined by wealth and the interest rate alone but perhaps by certain institutional factors as well.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1955
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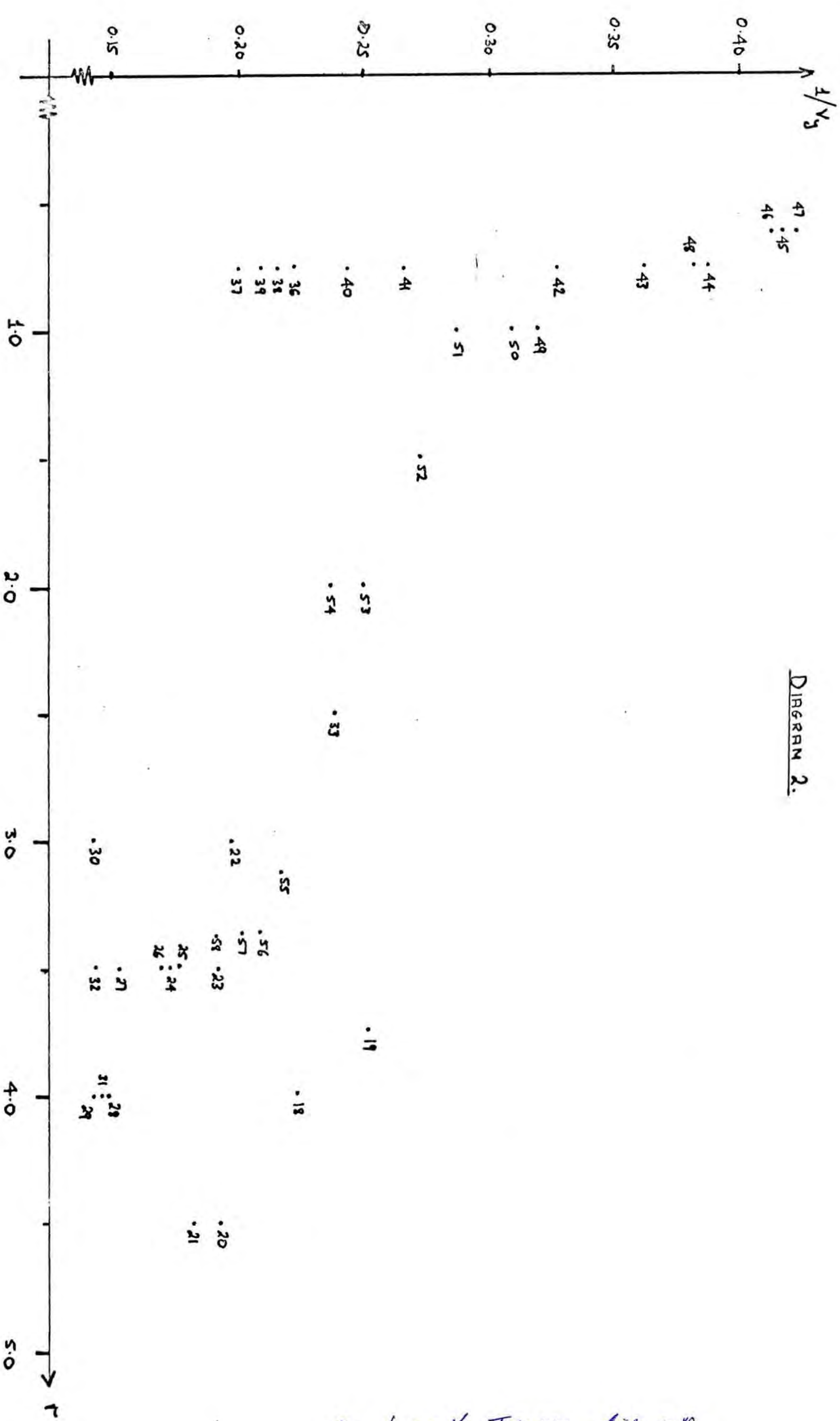
TIME.

DIAGRAM 1.



r = 6 month Treasury Bill rate.

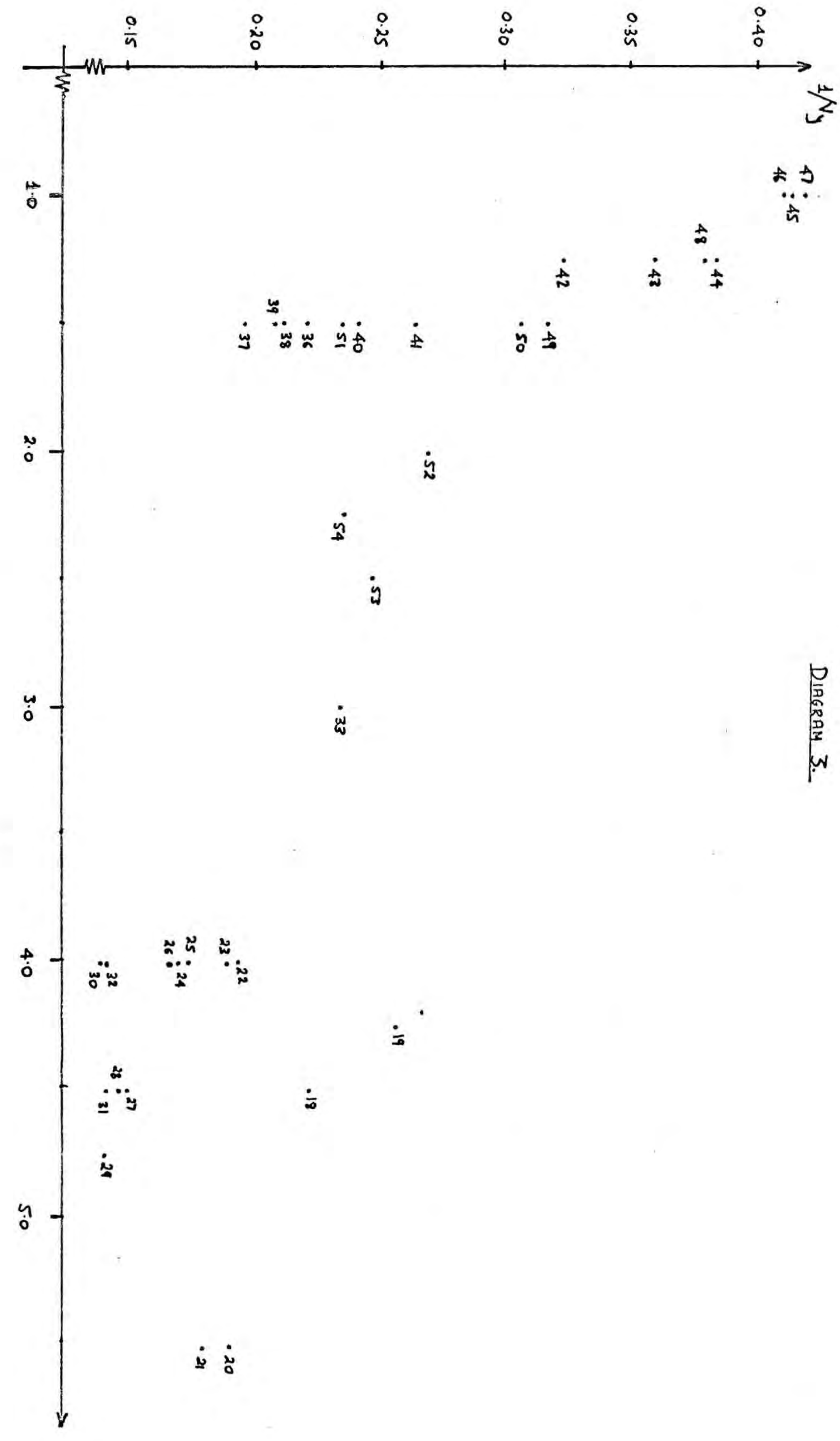


DIAGRAM 3.

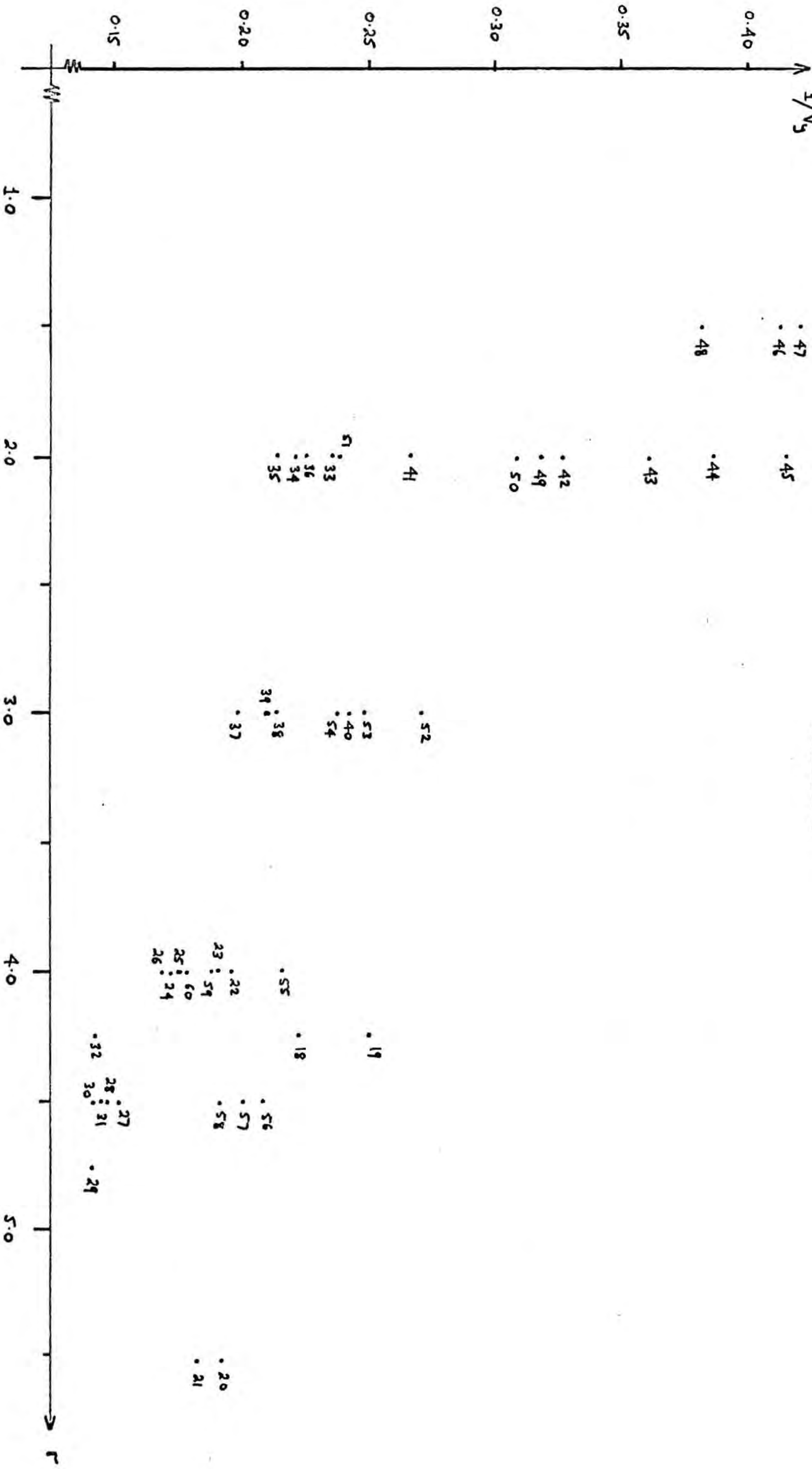


DIAGRAM 4.

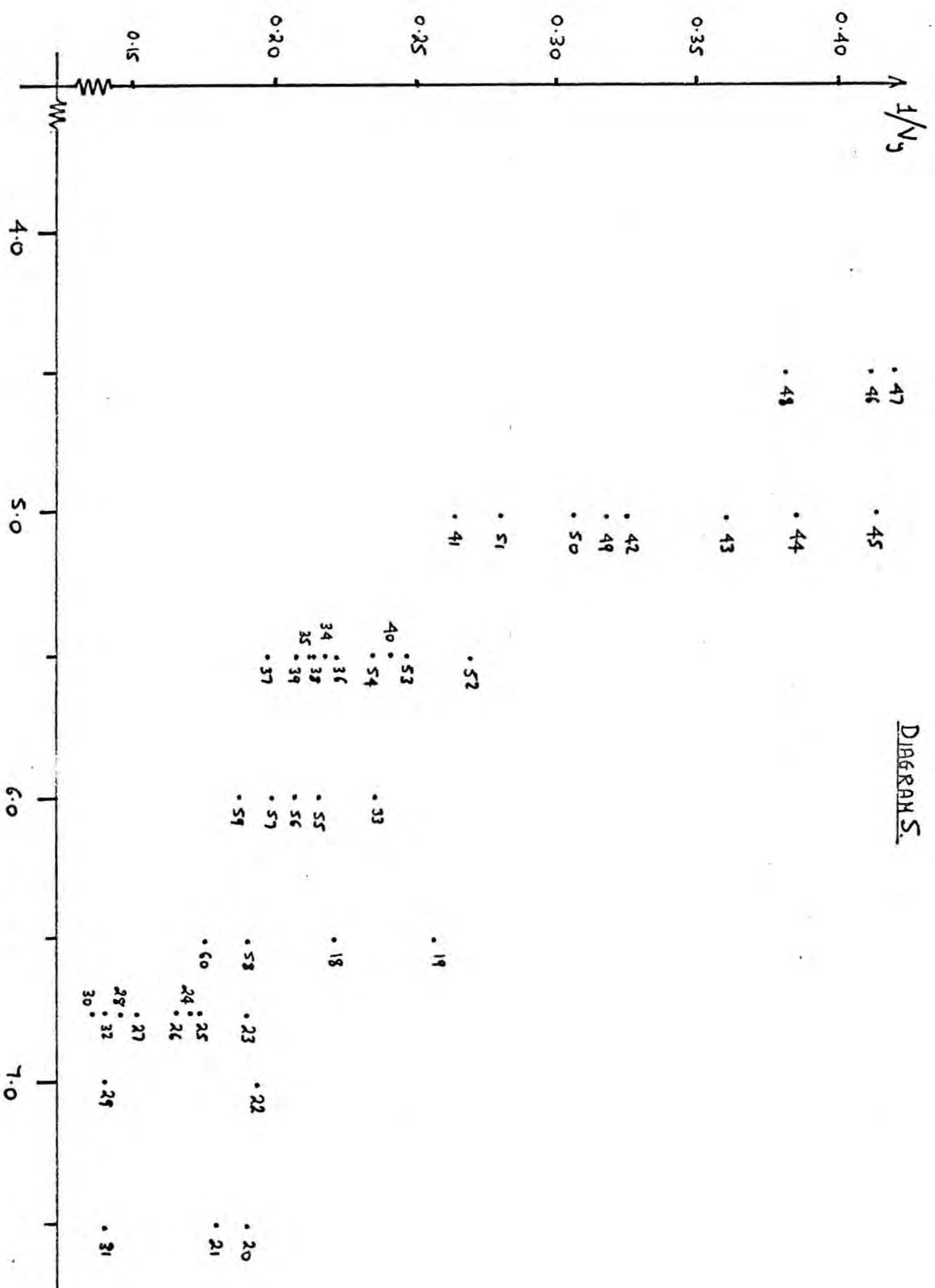
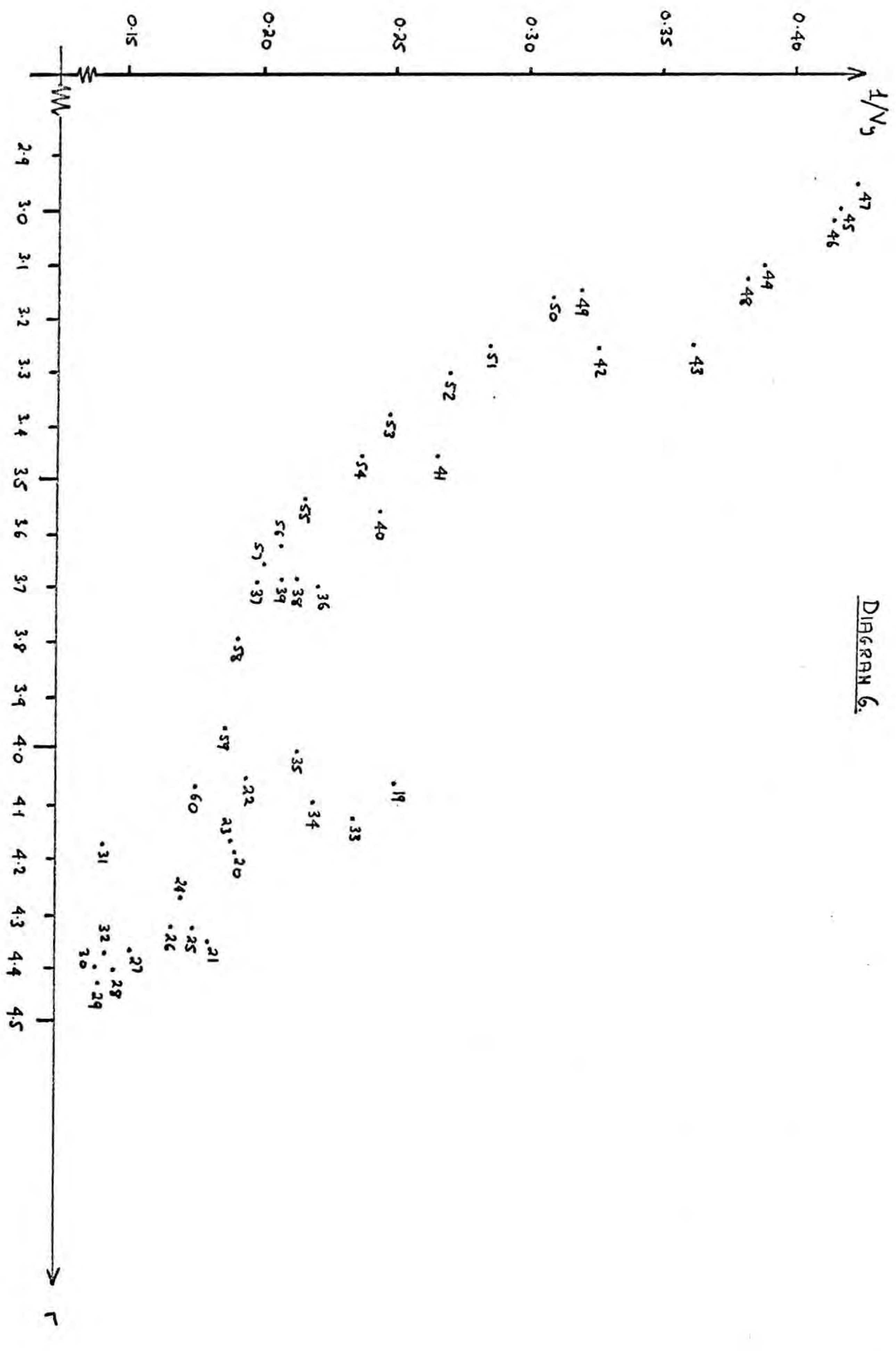


DIAGRAM 5.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r

DIAGRAM 6.

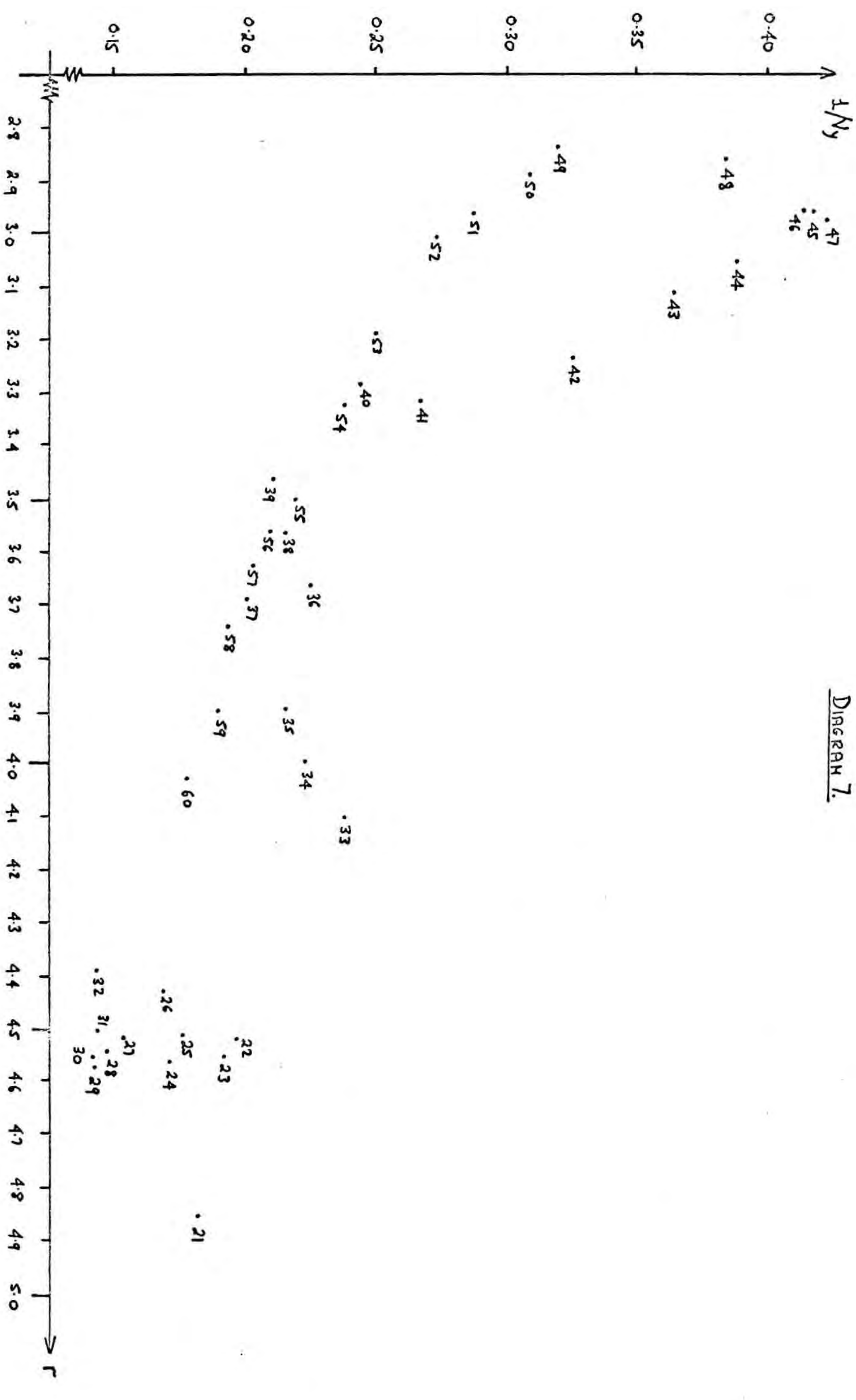
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DIAGRAM 7.

DIAGRAM 8.

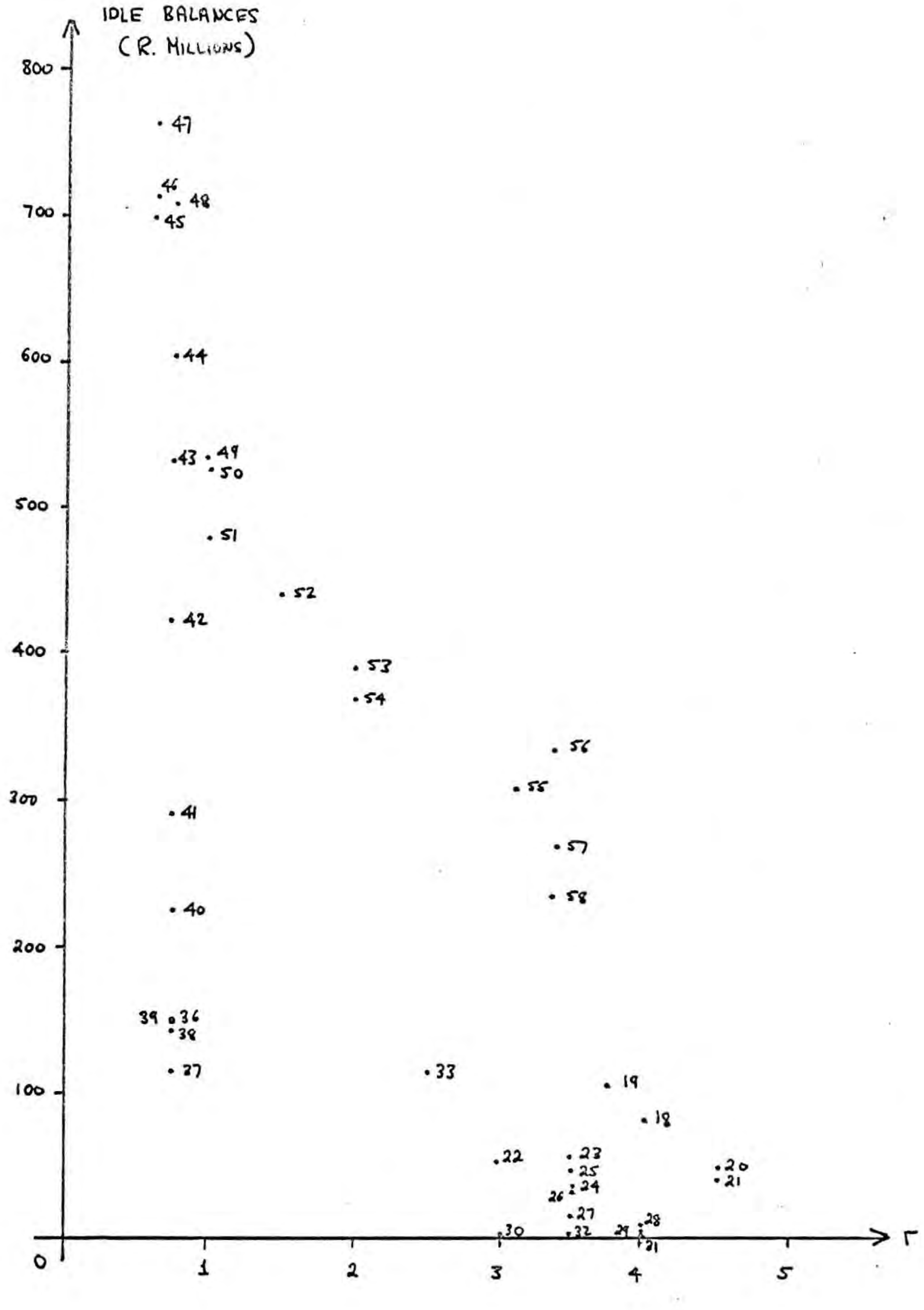
IDLE BALANCES
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DIAGRAM 9.

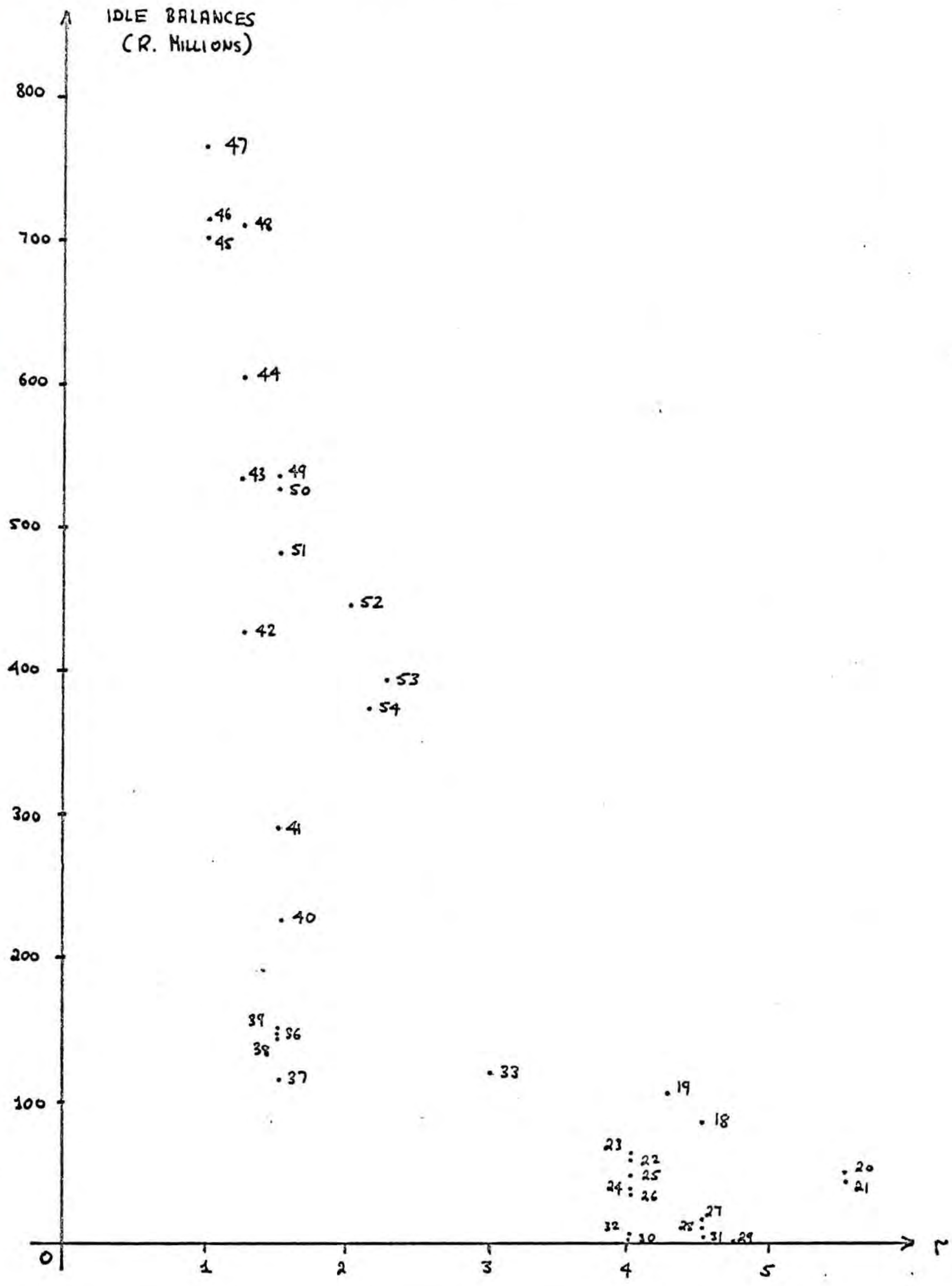
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DIAGRAM 10.

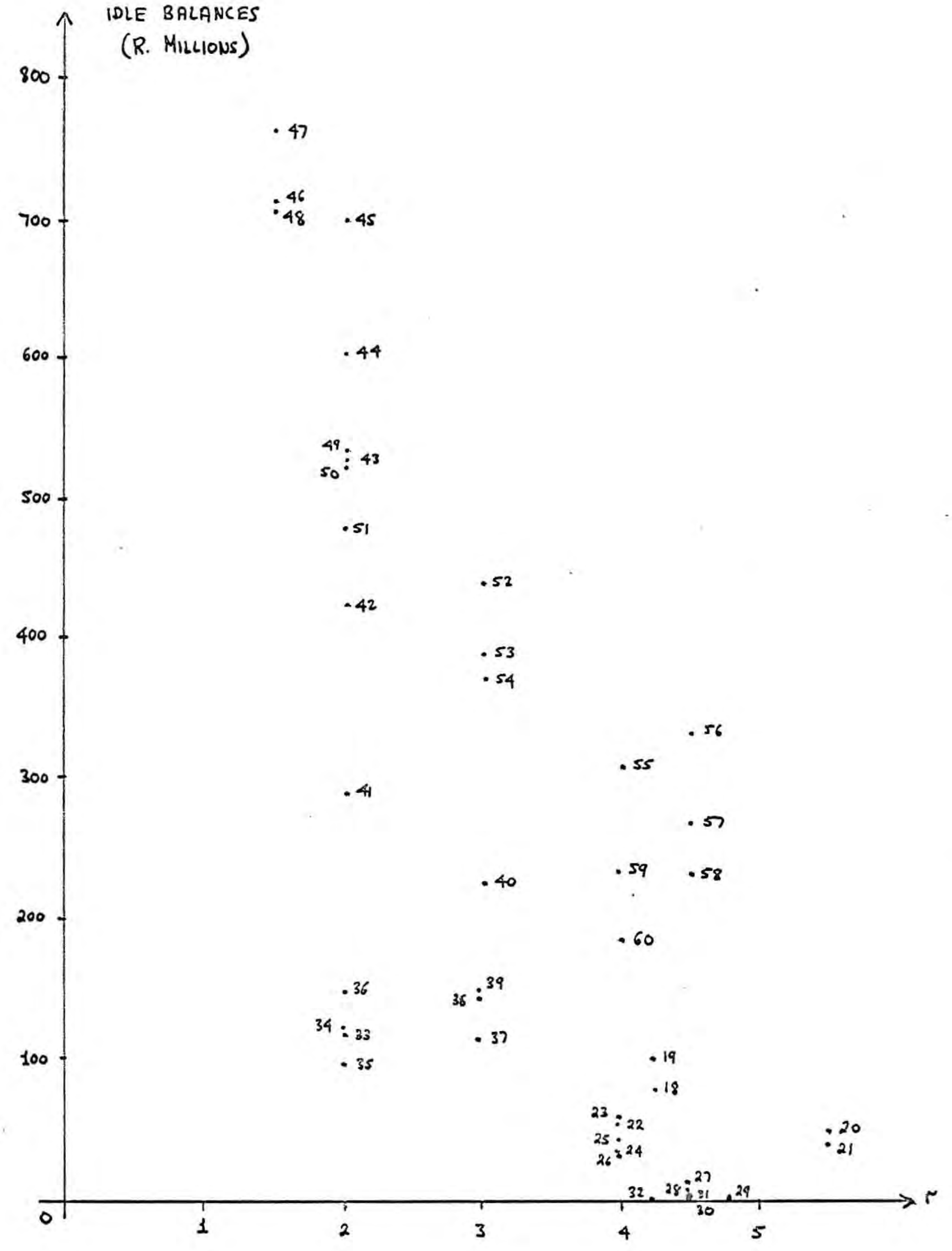
IDLE BALANCES
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DIAGRAM 11.

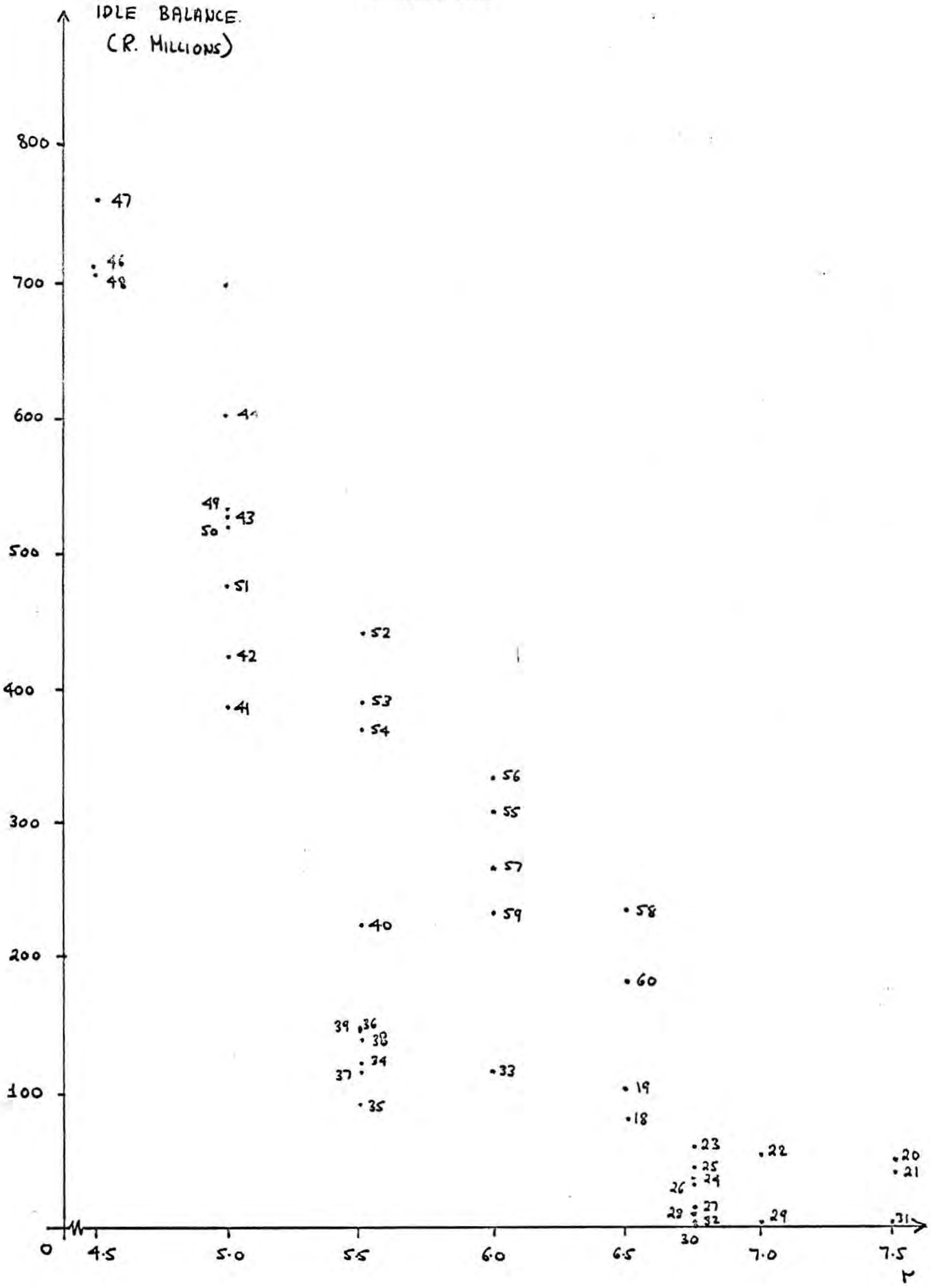
IDLE BALANCE
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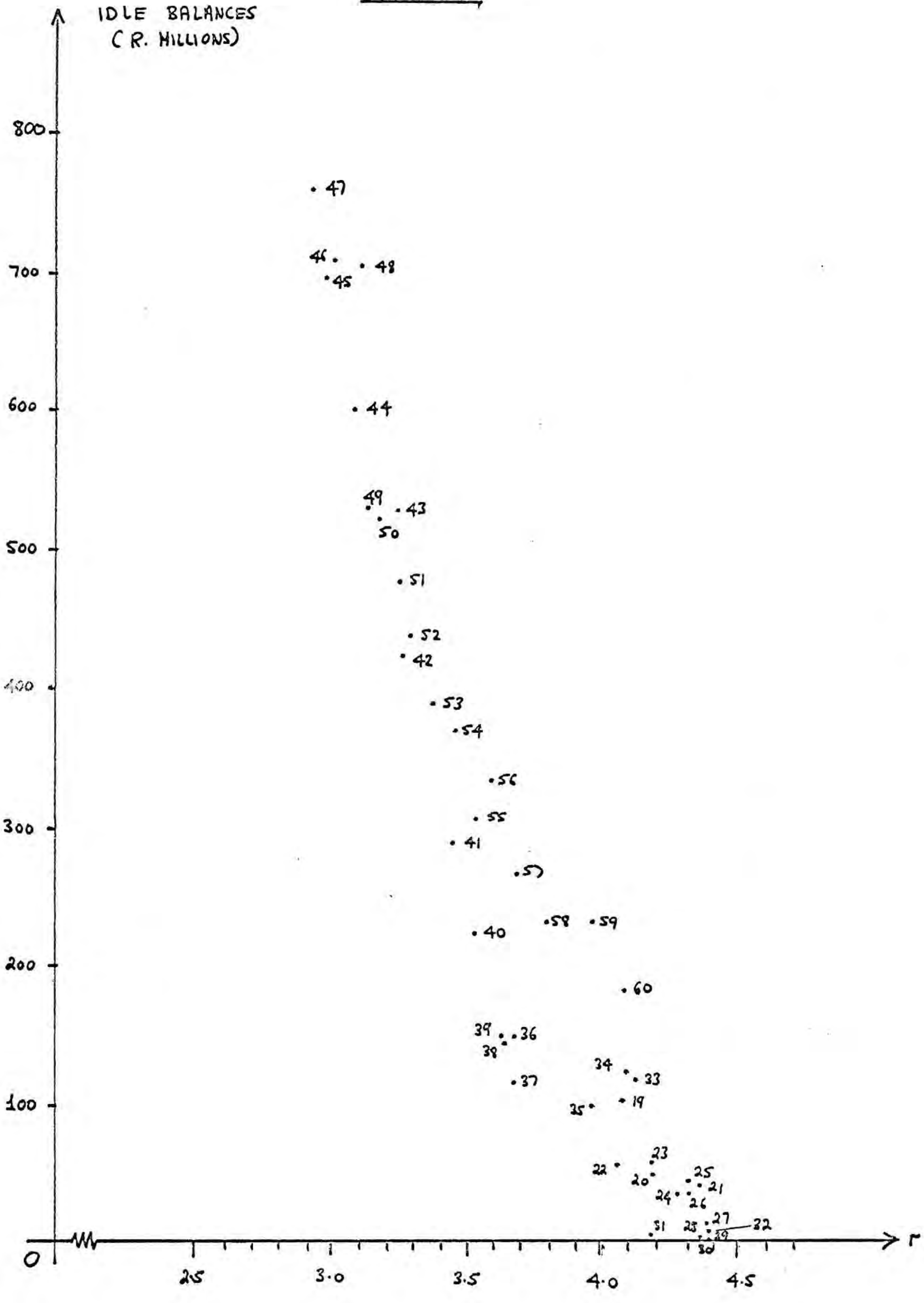
DIAGRAM 12.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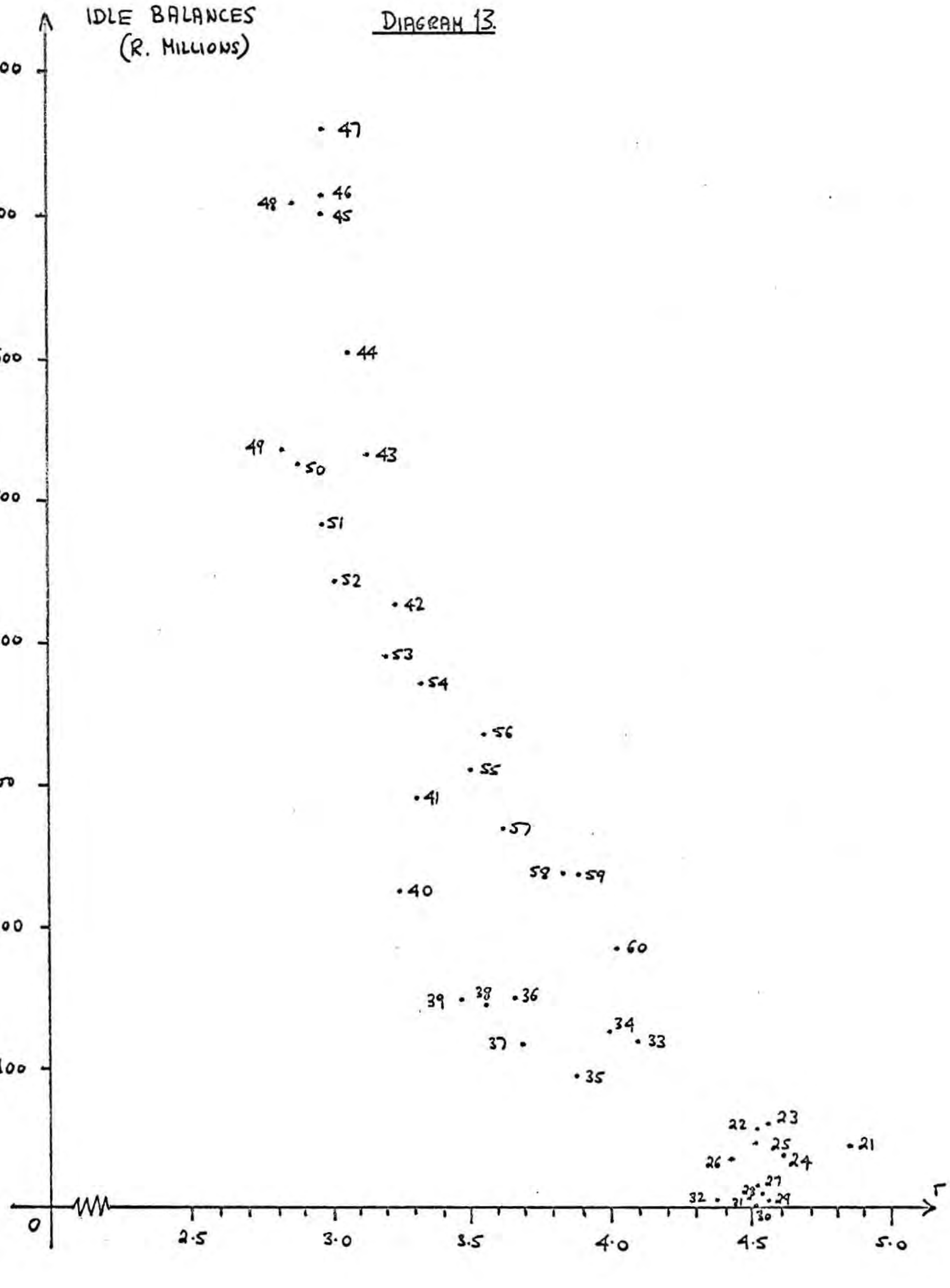
IDLE BALANCES
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DIAGRAM 13.
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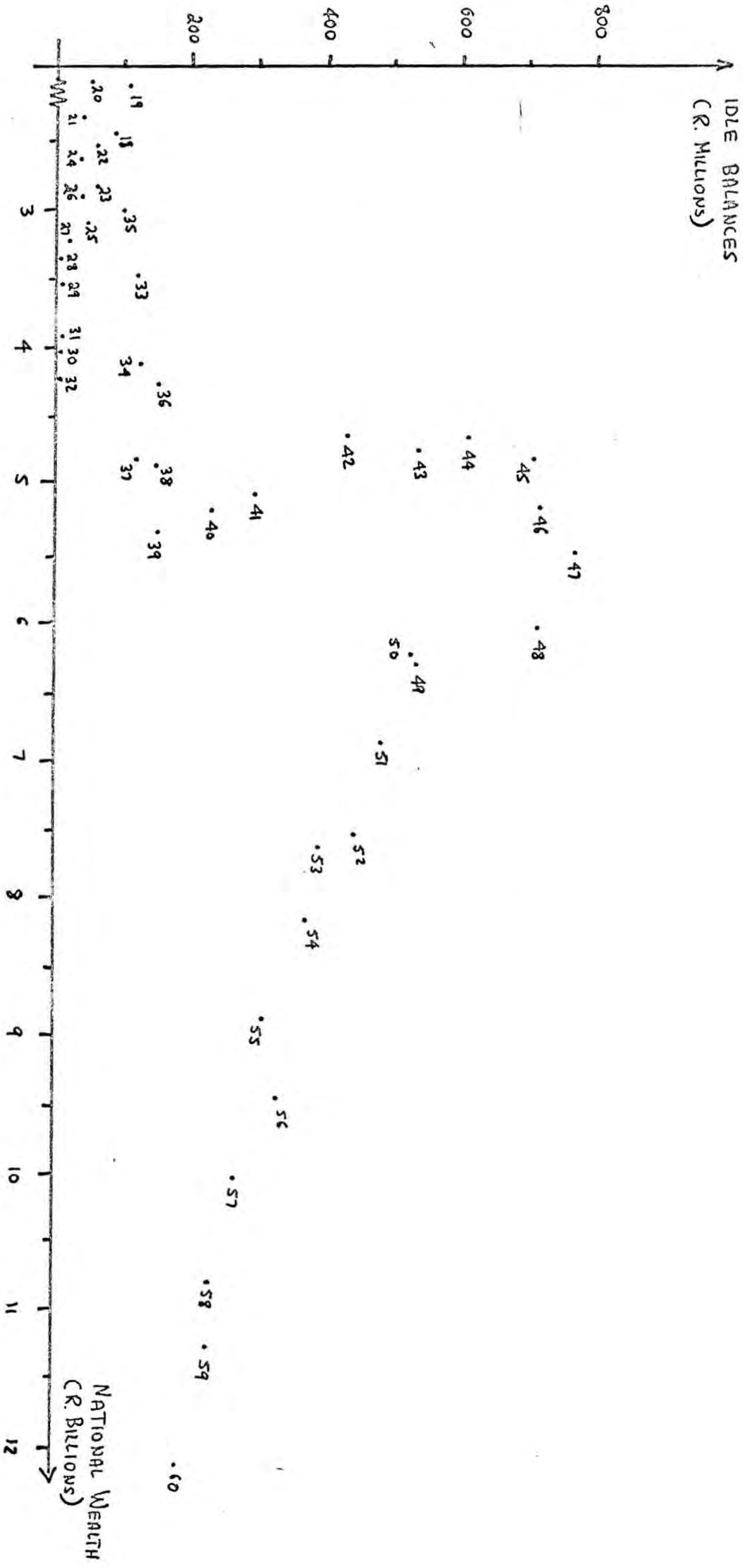
DIAGRAM 14.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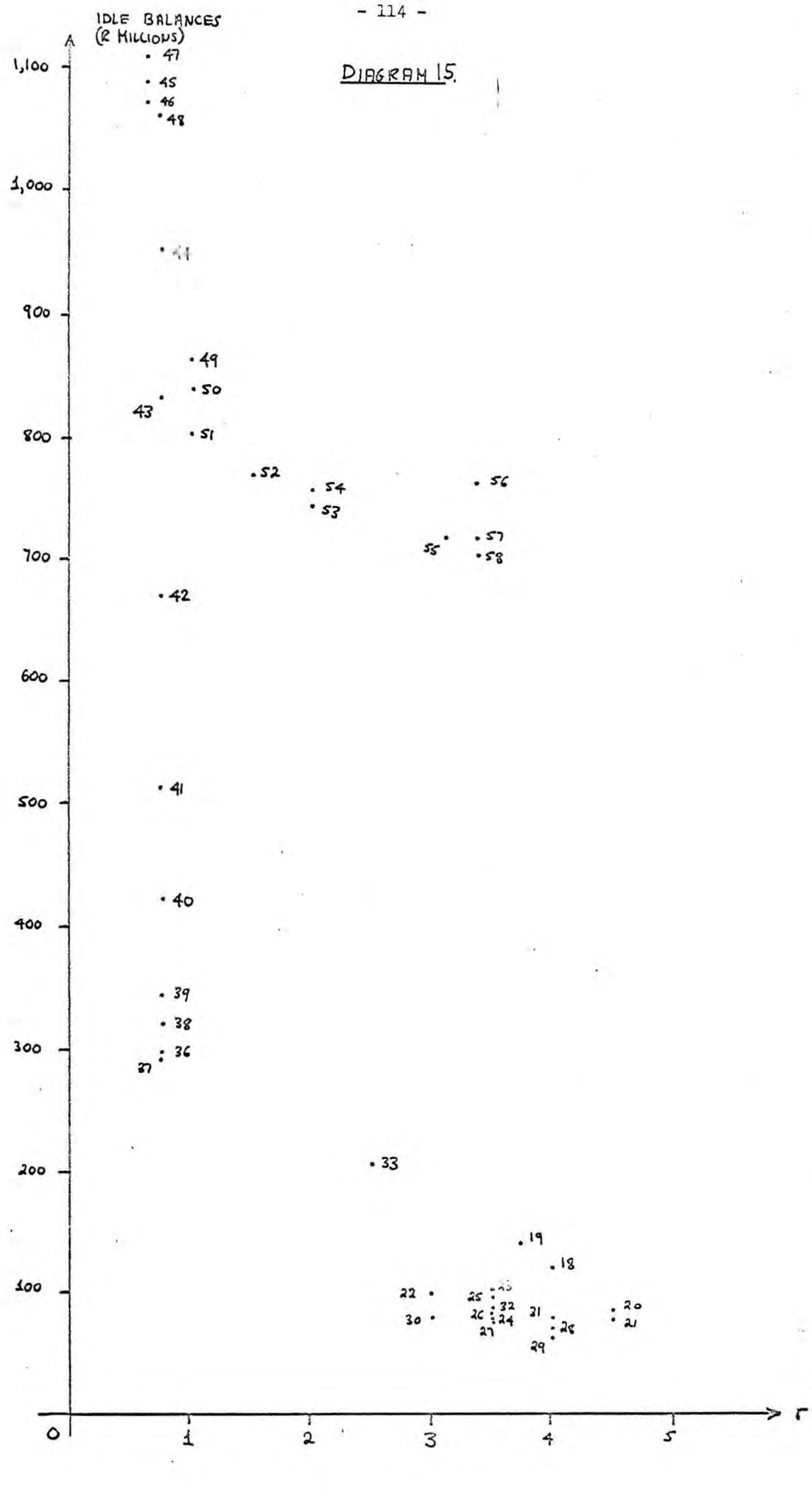
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DIAGRAM 15.

DIAGRAM 16.

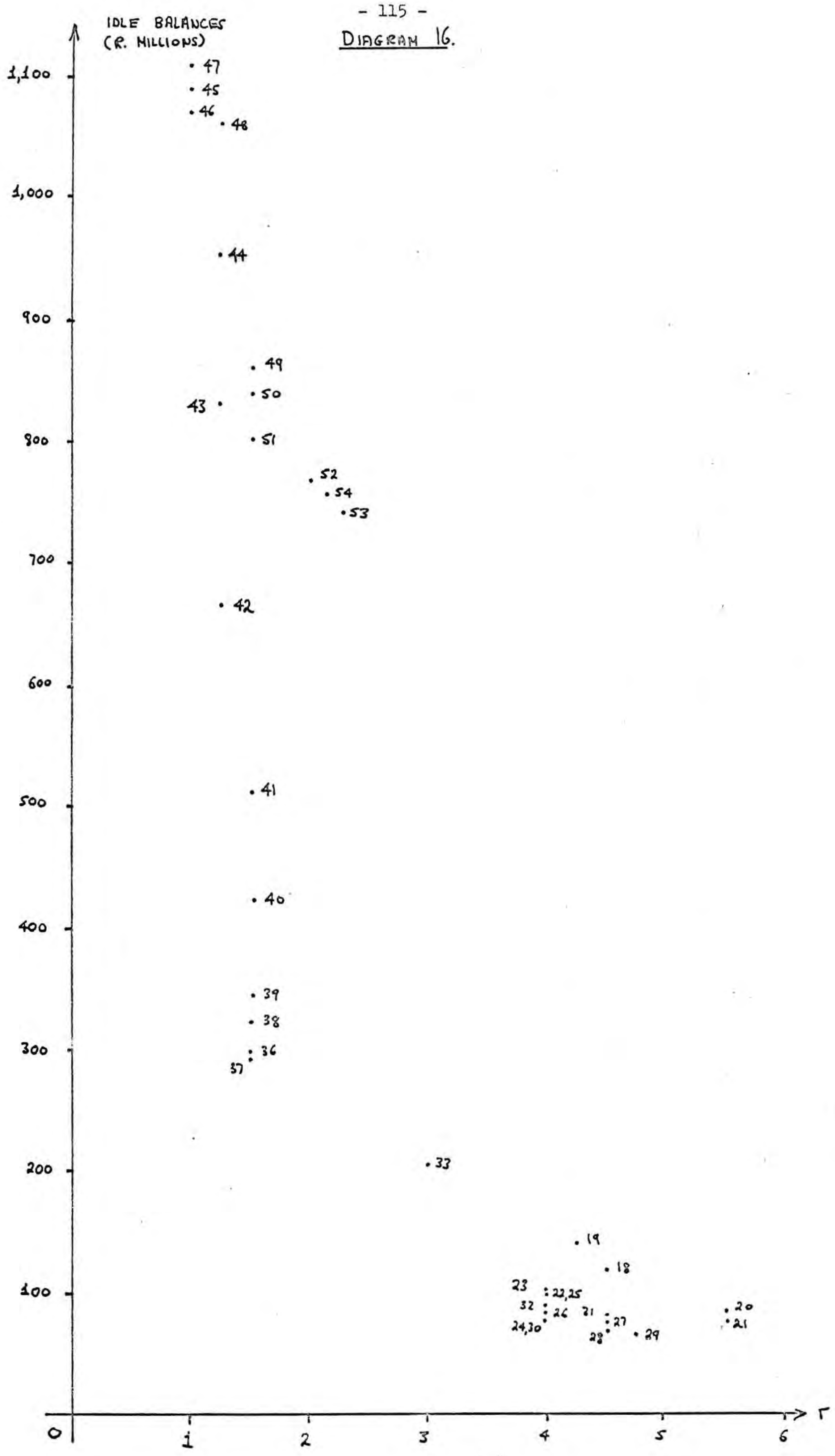
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DIAGRAM 17.

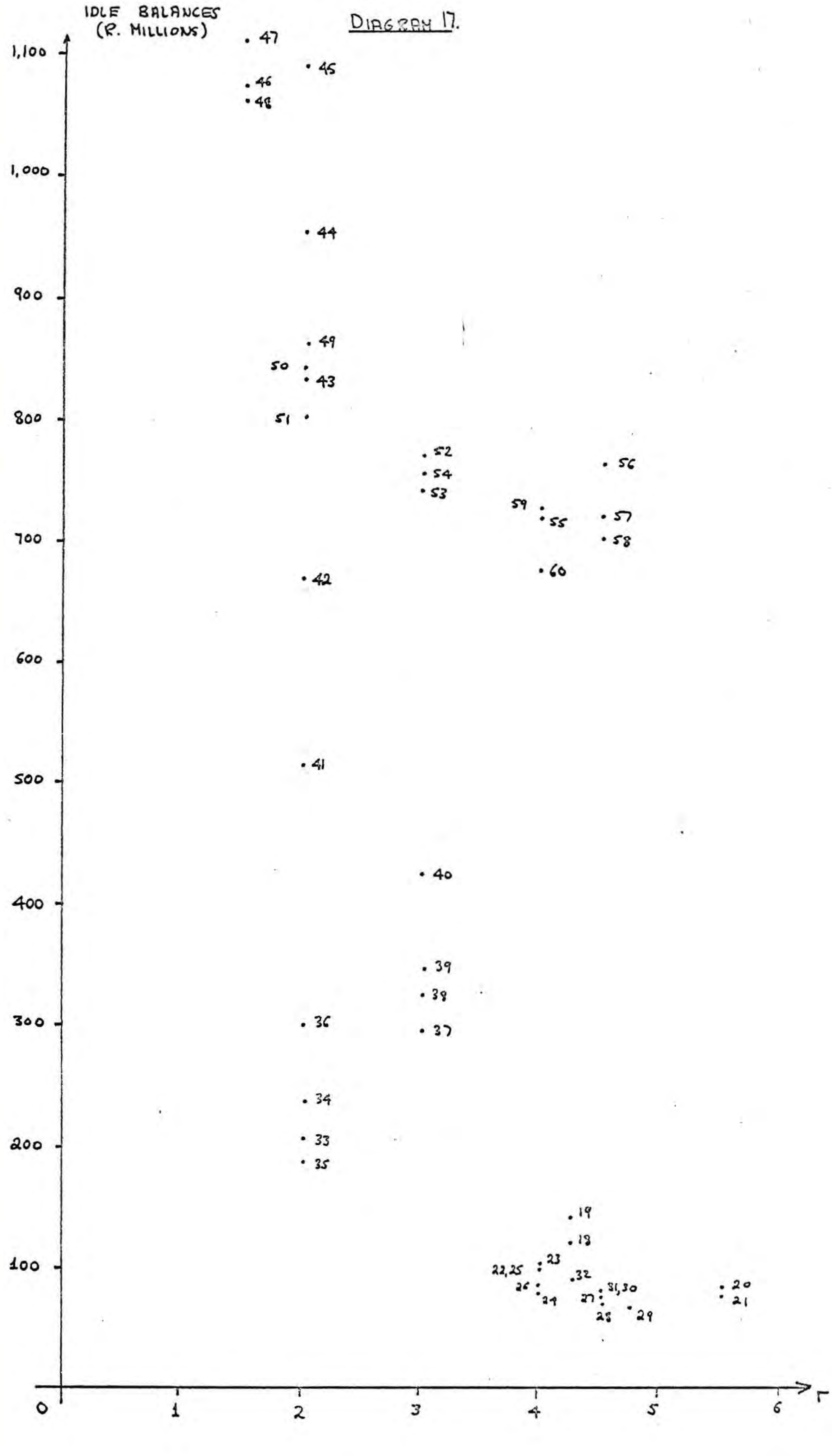
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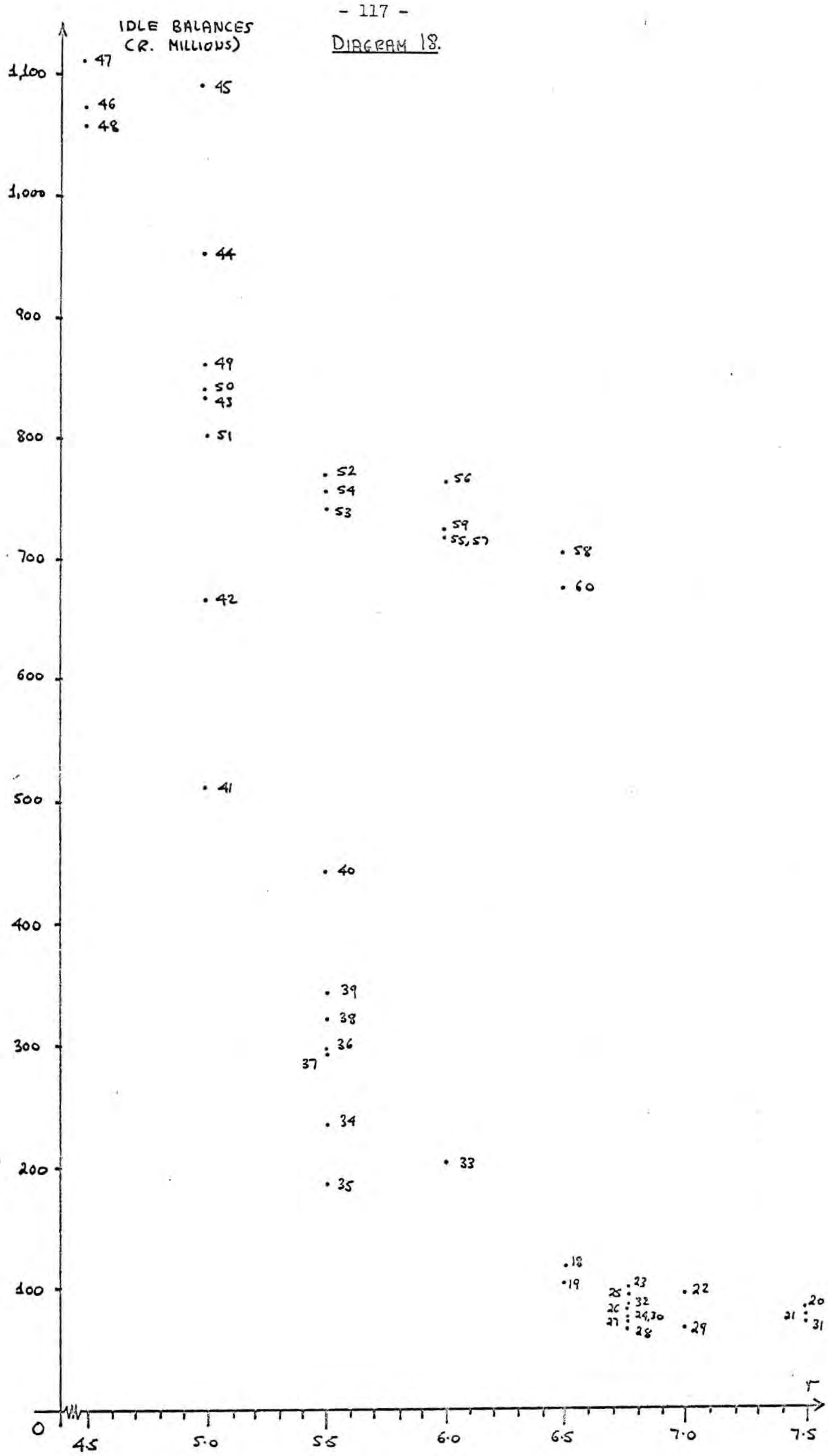
DIAGRAM 18.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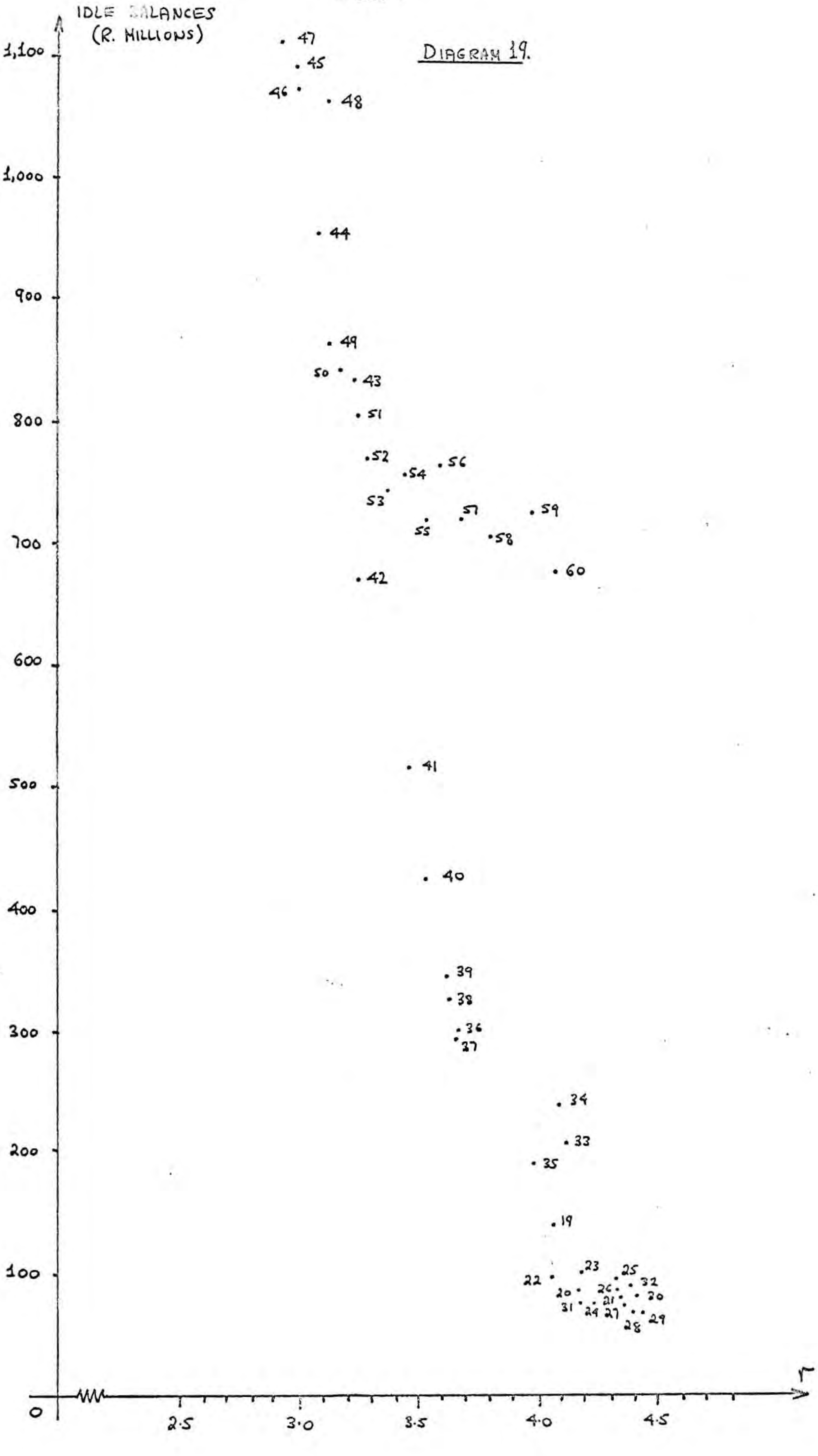
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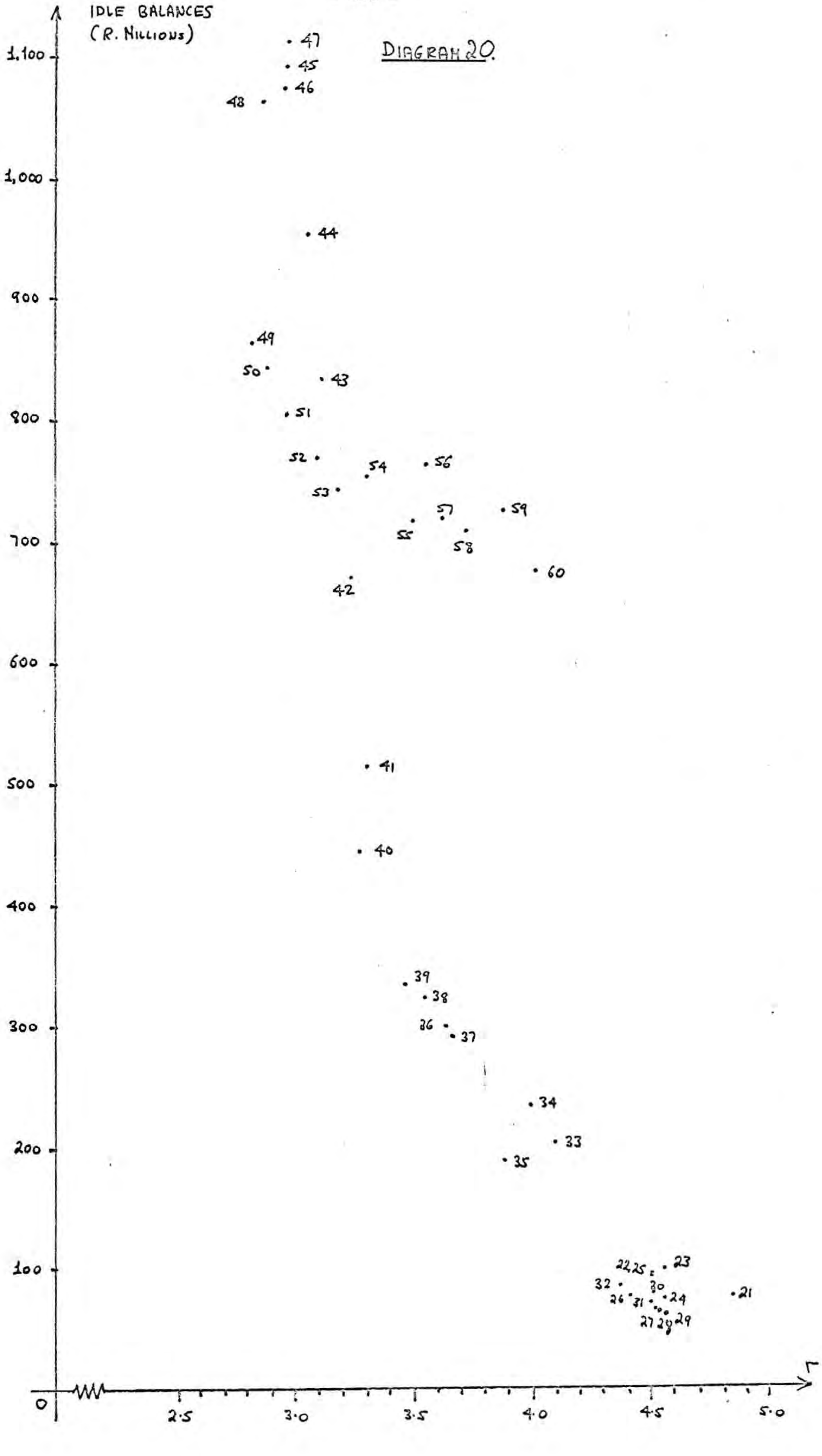
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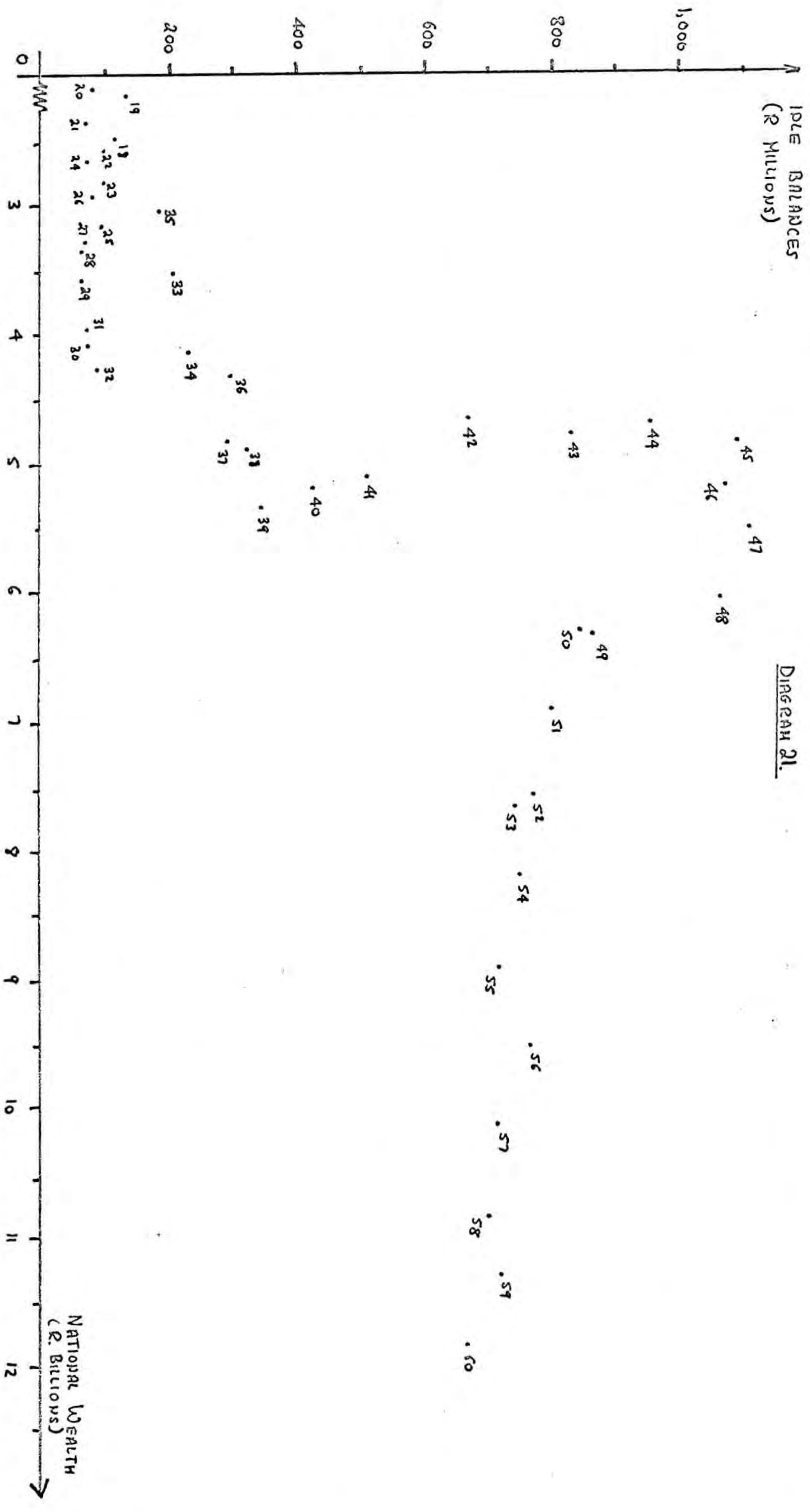

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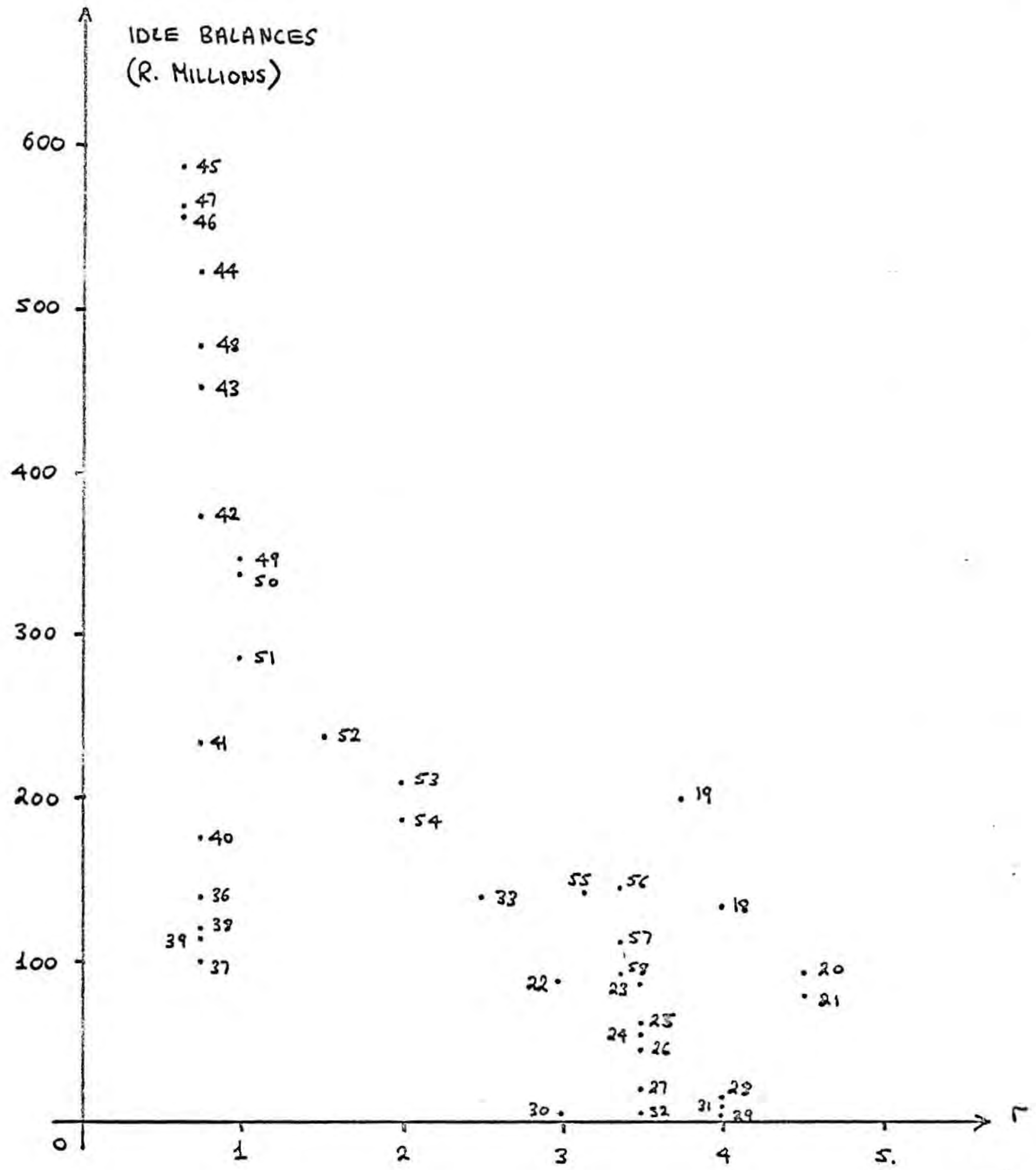
DIAGRAM 22.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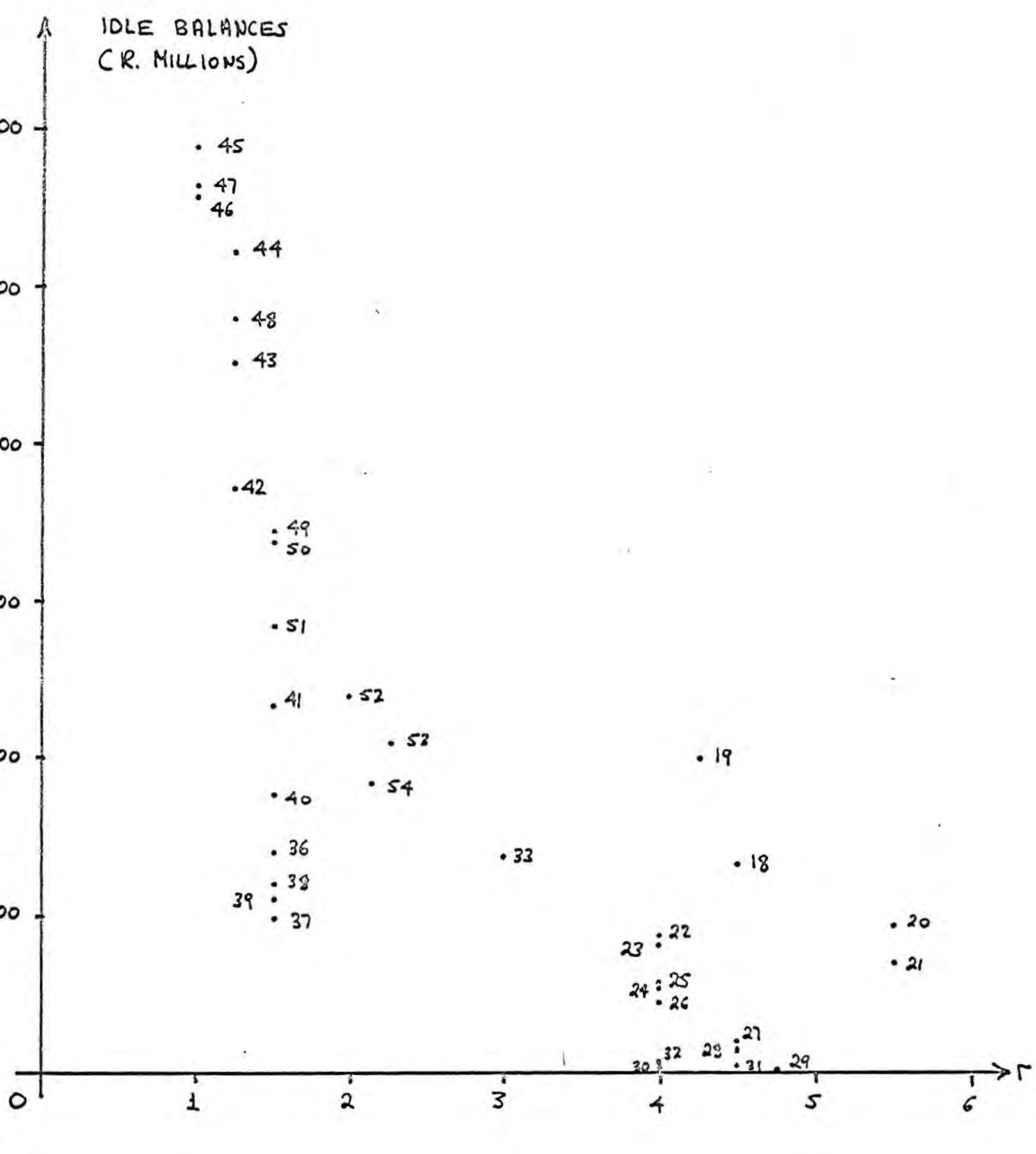
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DIAGRAM 24.

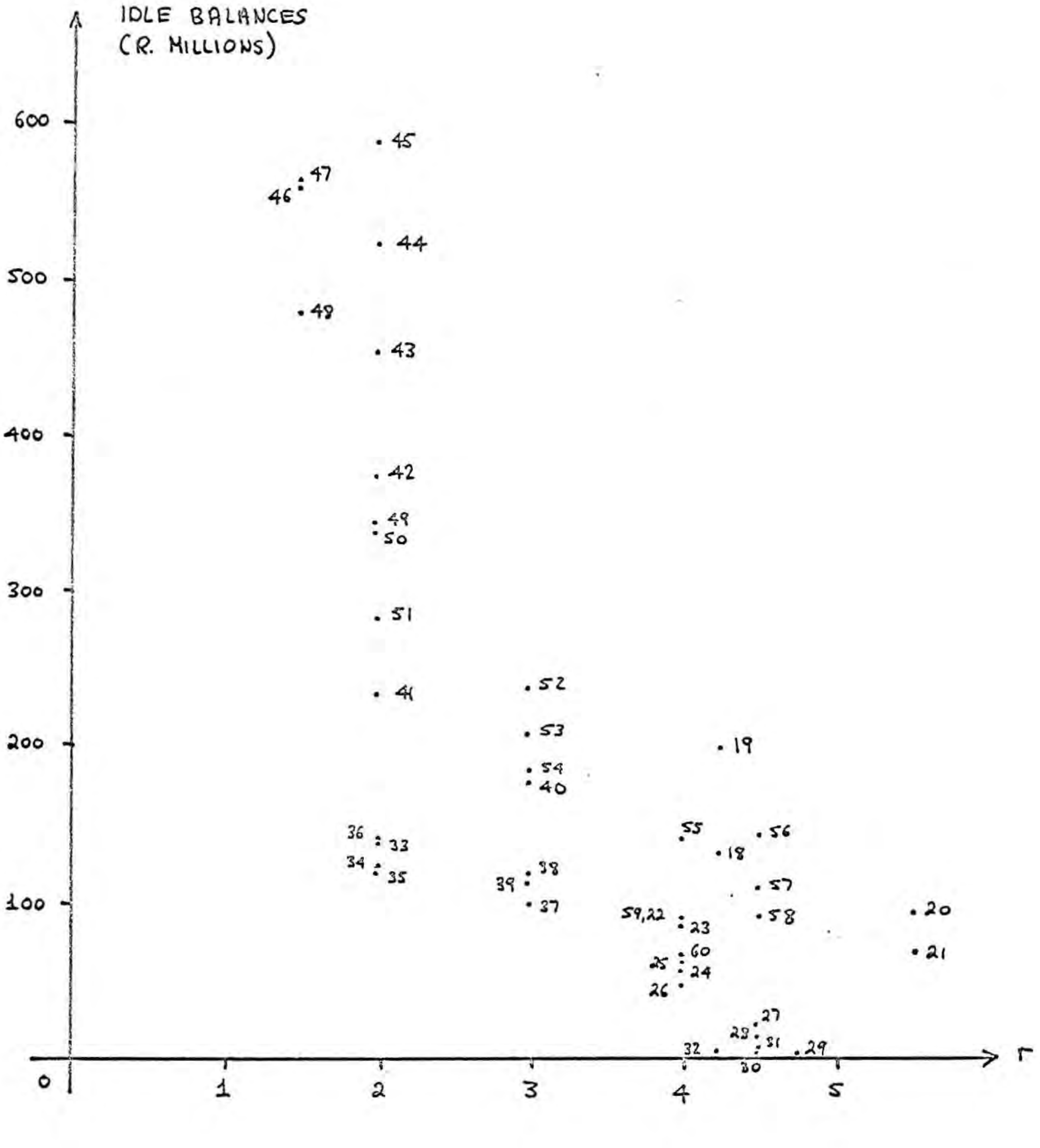
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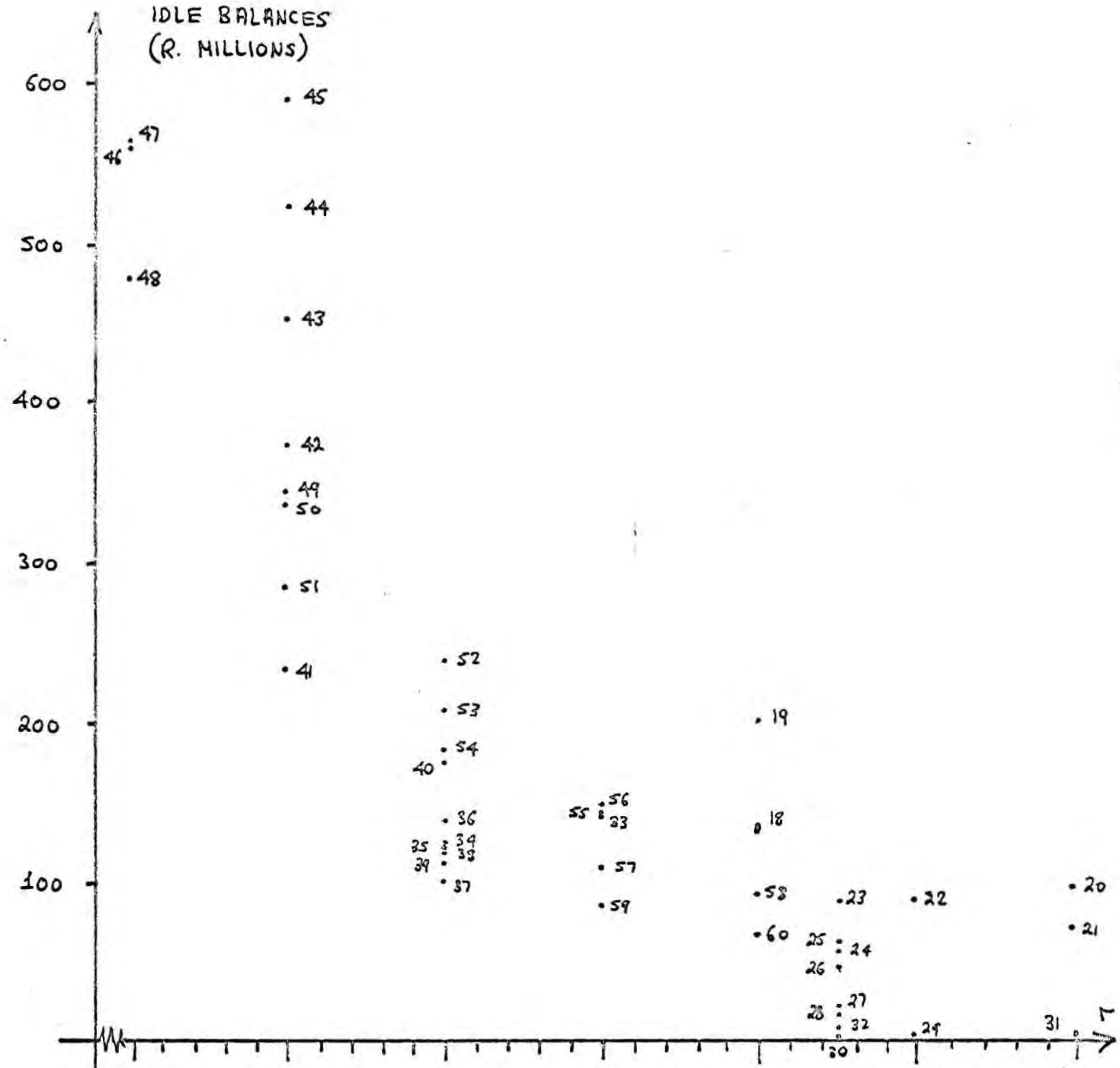
DIAGRAM 25.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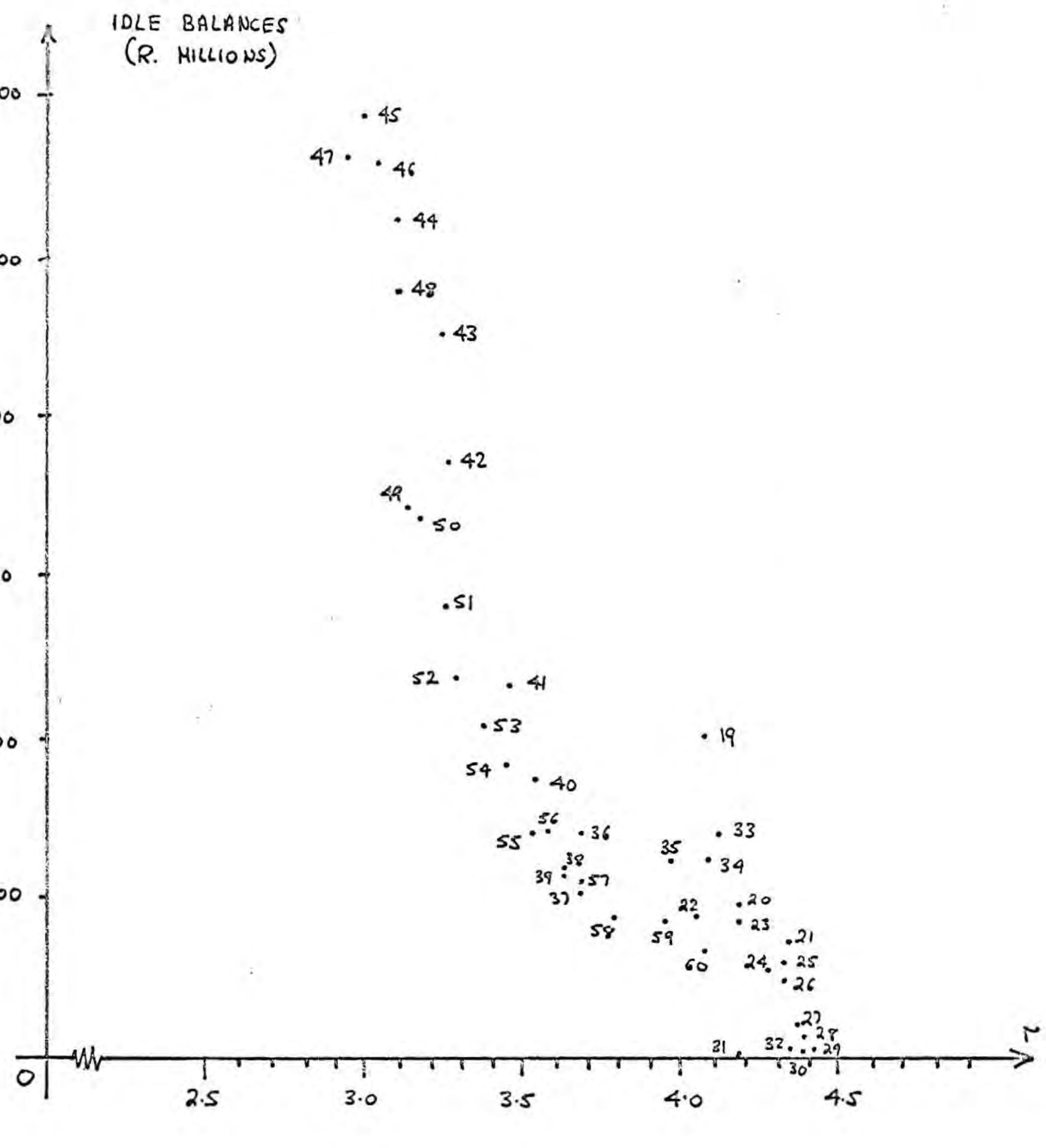
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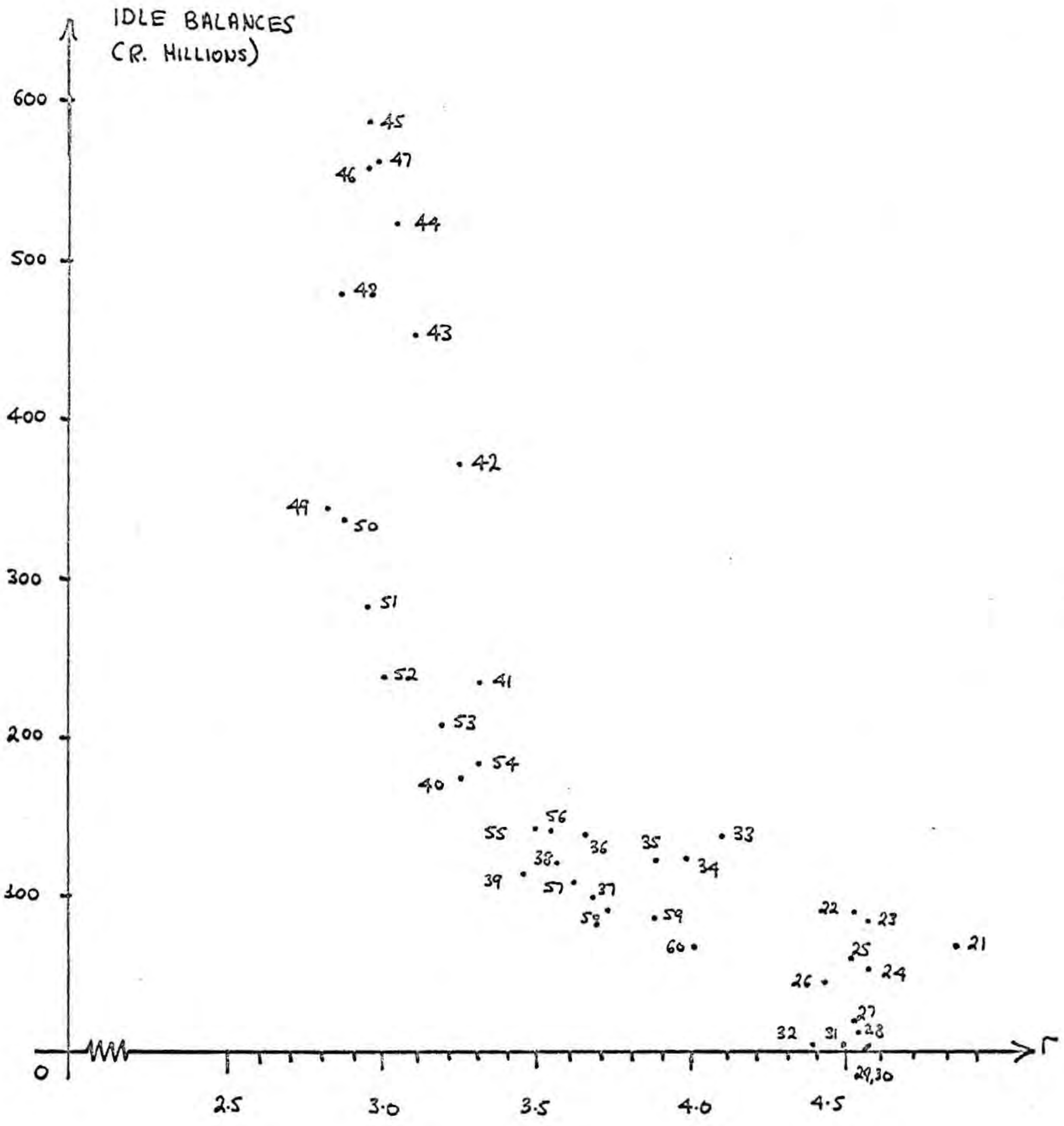
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IAGRAM 28.

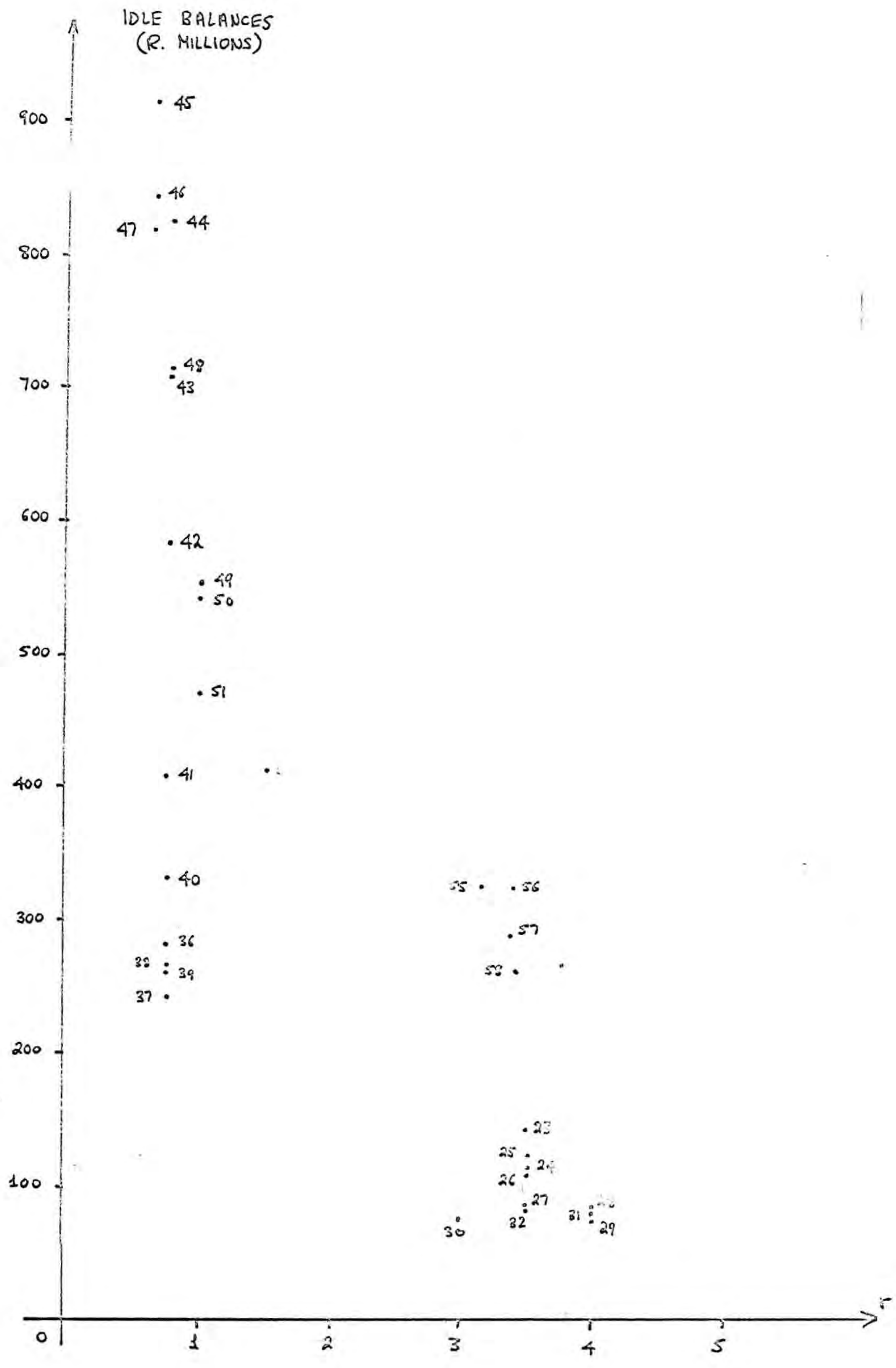
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DIAGRAM 29.

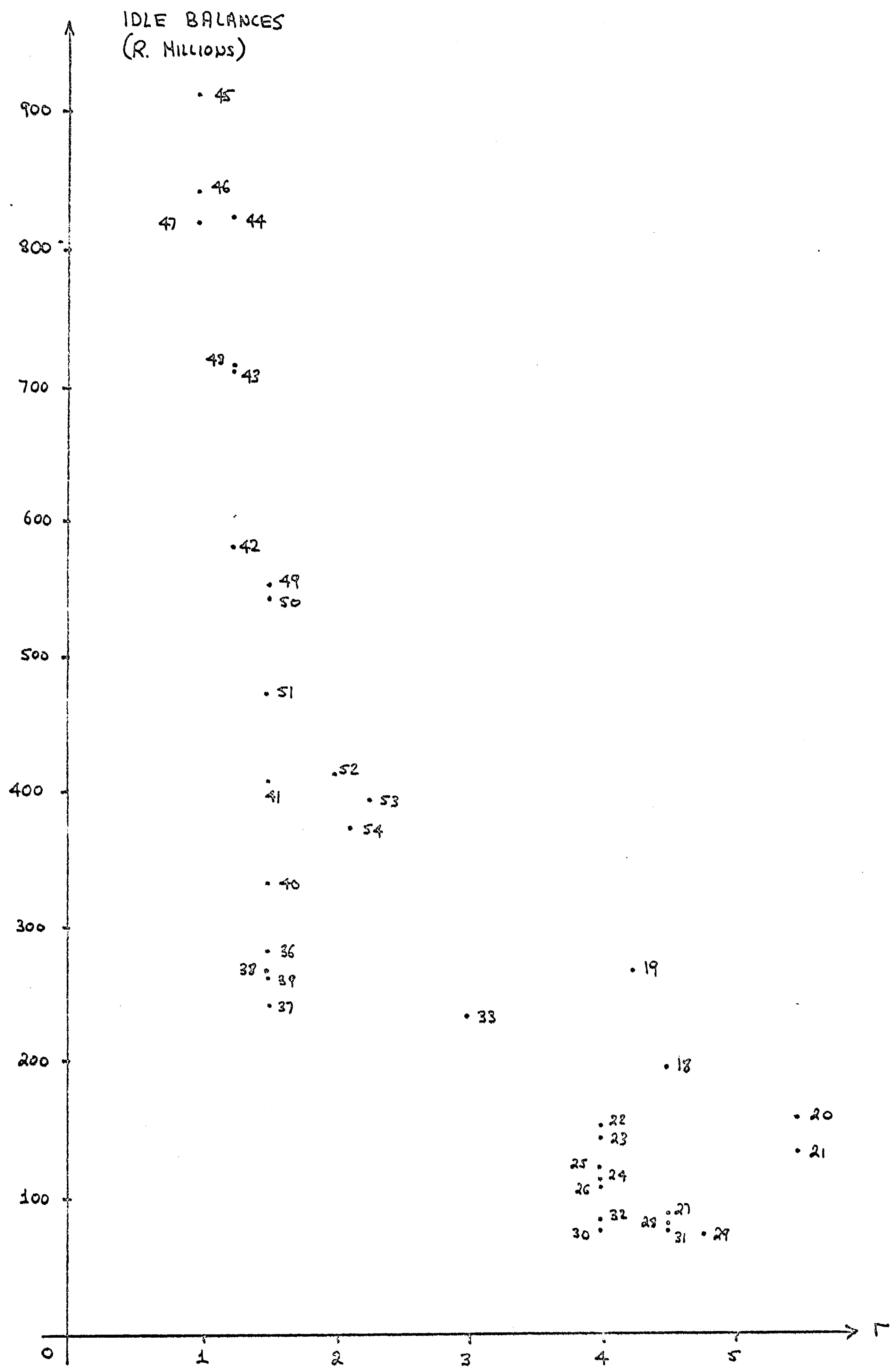
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DIAGRAM 30

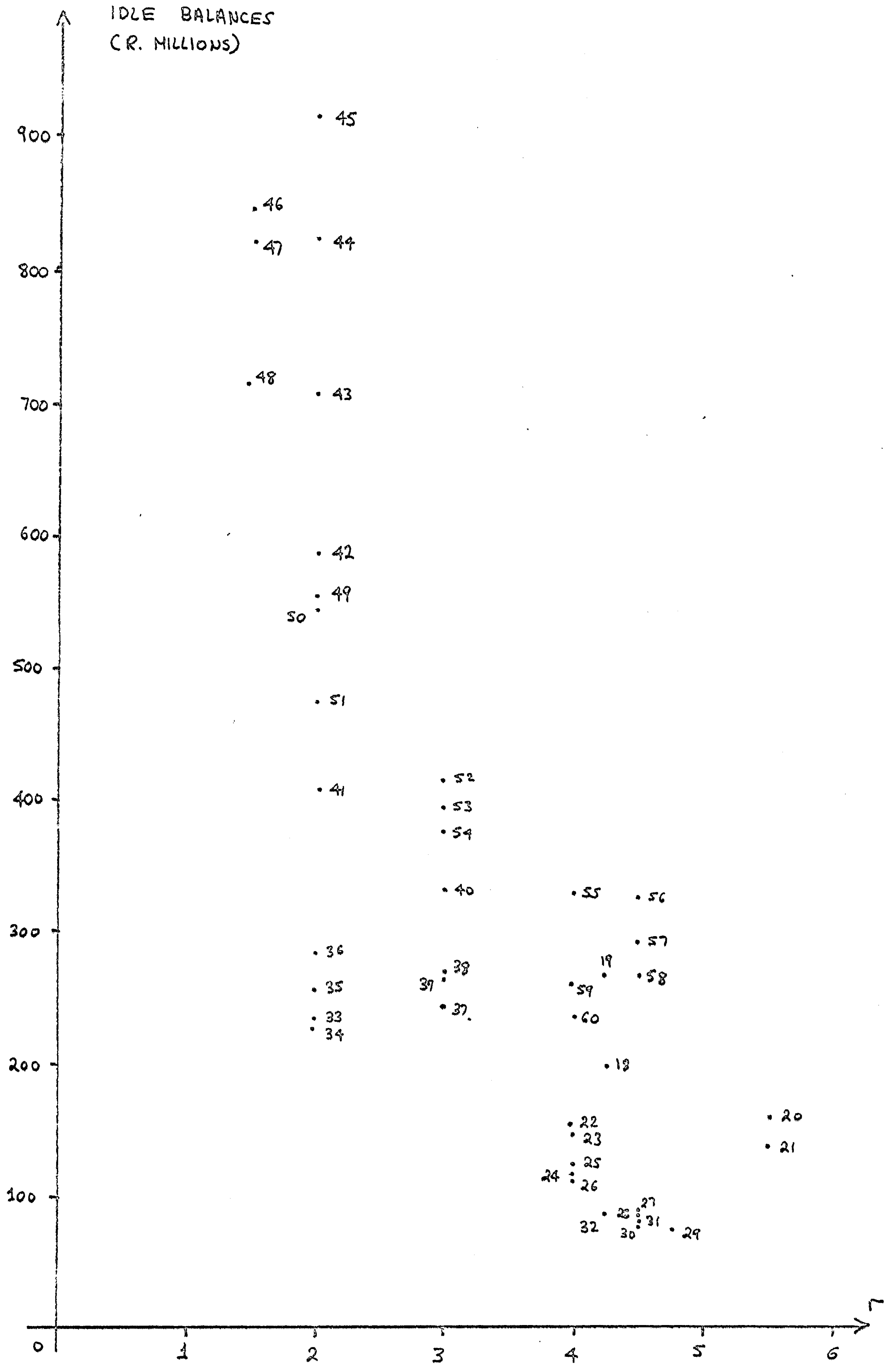
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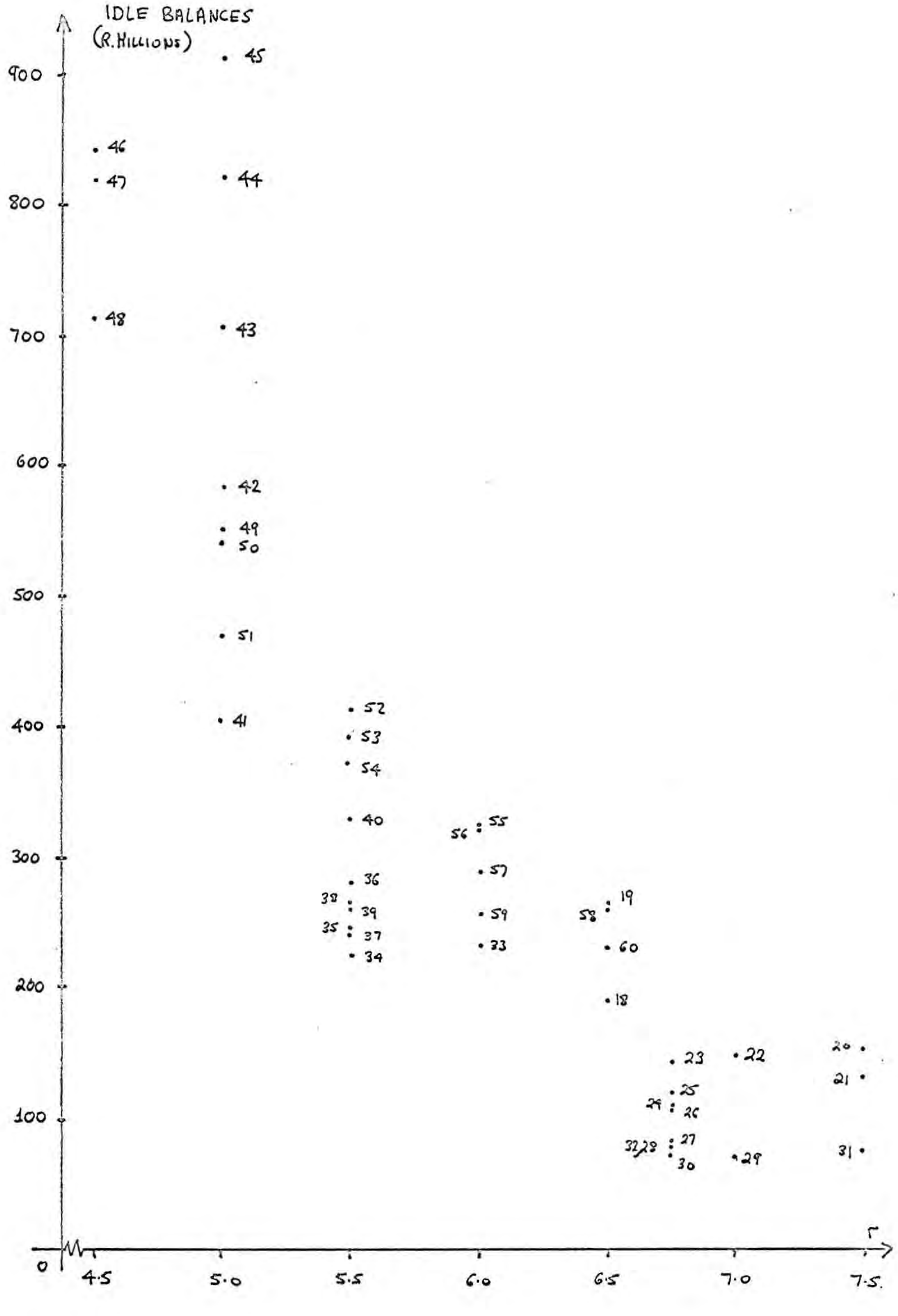
DIAGRAM 31.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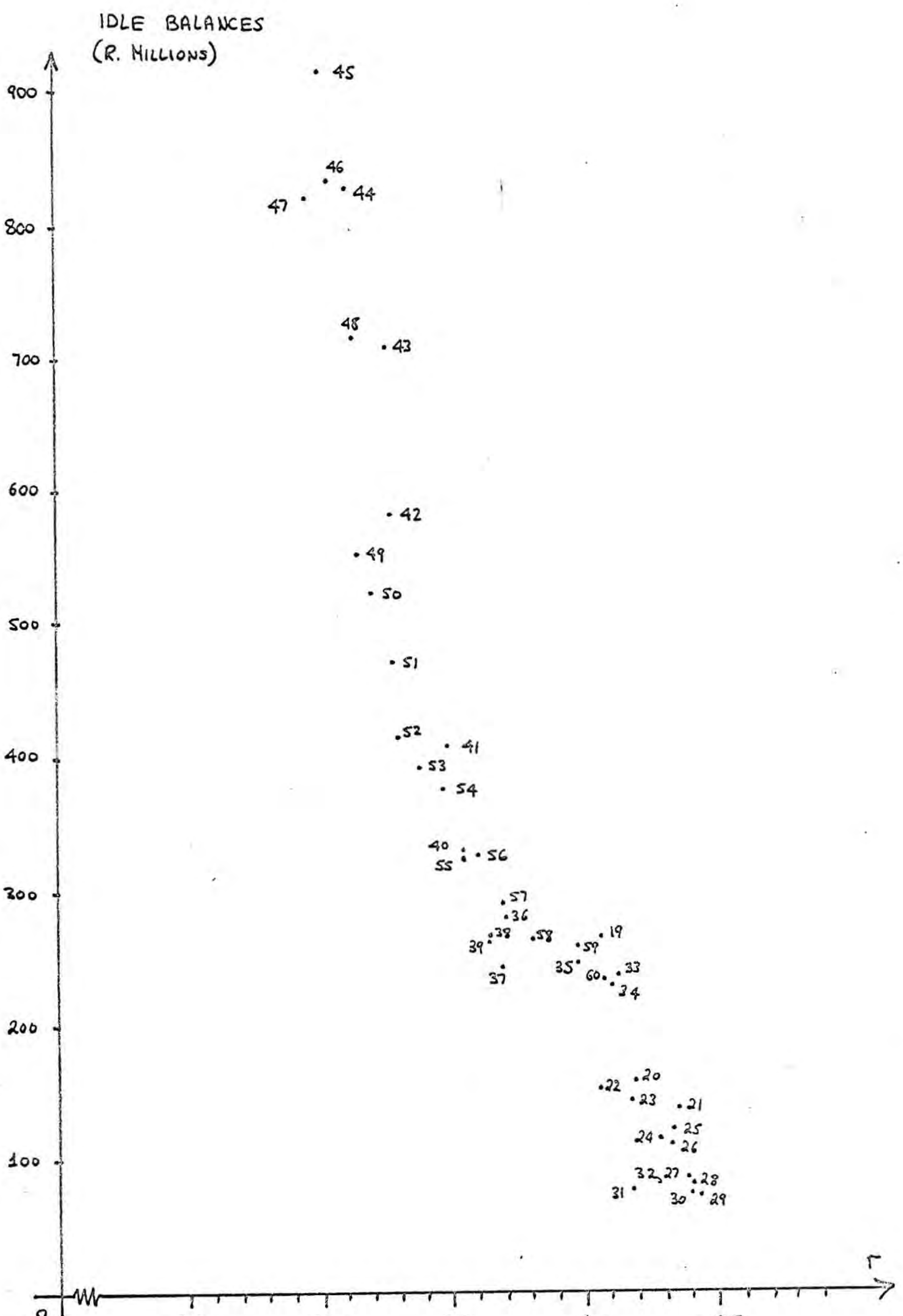
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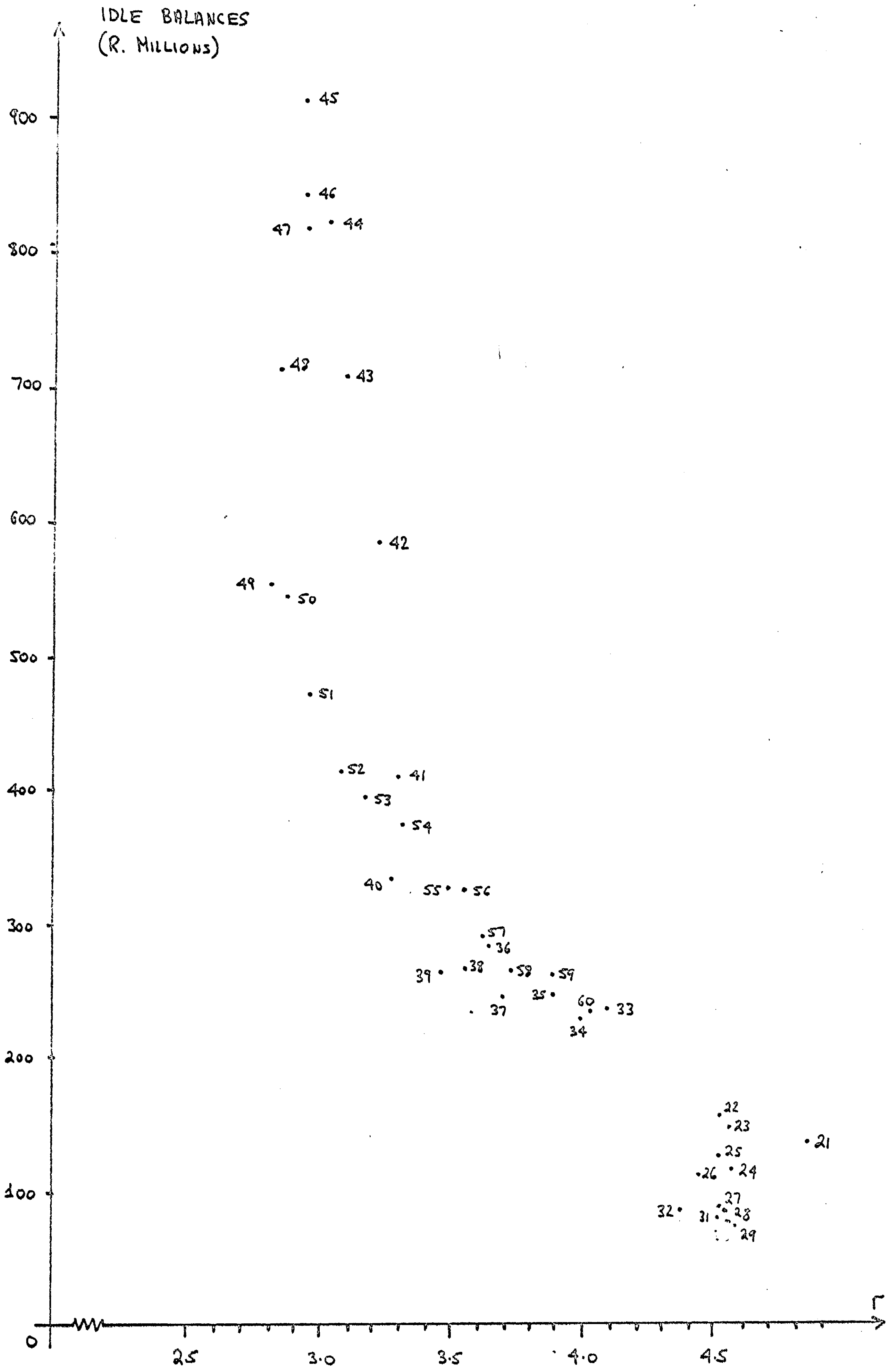
DIAGRAM 33.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CHAPTER V

Statistical Estimation of the Demand Function

1. Introduction.

For the reader who is not conversant with the esoteric doctrines of econometrics a simplified account of the methods used and the problems encountered is now presented. A logarithmic function was used, the reason underlying this choice being the conventional one that the coefficients are the respective elasticities. The function takes the form:

$$\ln M = \beta_1 + \beta_2 \ln r + \beta_3 \ln A$$

or arithmetically: $$M = \beta_1' \, r^{\beta_2} A^{\beta_3} \qquad (1)$$

where: M ~ the monetary variable

r ~ the interest rate

A ~ the appropriate constraint

β_1, β_1' ~ constants

β_2 ~ the interest elasticity

β_3 ~ the constraint elasticity.

The data was fitted by means of an ordinary least squares regression and two-tailed significance tests were performed on the appropriate coefficients. These were $H_0 : \hat{\beta}_2 = 0$ and $H_0 : \hat{\beta}_3 = 0$ which test whether the respective variables play a significant role in the regression and $H_0 : \hat{\beta}_3 = \pm 1$, the positive sign being chosen if $\hat{\beta}_3$ was positive and the negative one if $\hat{\beta}_3$ was negative. This tests whether the fitted function is homogeneous of degree one in the constraint. The correlation coefficient was calculated and was regarded as an adequate indication of the goodness of fit.

2. The Problem of Autocorrelation.

One further statistic was computed, viz. the von Neumann ratio or as it is sometimes called the Durbin-Watson statistic, which is the sum of squares of the first differences of the least squares estimated disturbances divided by the sum of squares of the estimated disturbances themselves. This statistic is commonly used to provide an indication of the degree of autocorrelation present in the residuals.

Durbin and Watson[1] give a table of critical values for the von Neumann ratio, but this table suffers from a zone of ignorance within which the tables are unable to discriminate between the two alternative cases. If autocorrelation is found to be present, a number of alternative procedures suggest themselves.

1) Ignore it. This is the simplest and least satisfactory of all the alternatives. The reason for dissatisfaction in this approach lies in the fact that autocorrelation tends to understate the sampling variances of the coefficients, and consequently to affect the value of the correlation coefficient. As will be seen below when the empirical results are presented, these effects can be extremely significant making any conclusions drawn from uncorrected data untrustworthy in the extreme.

2) The ideal and usually unattainable method entails exact knowledge of the variance - covariance matrix of the disturbance term. In the absence of such information we shall have to make a simplifying assumption common to all the methods of correction employed in practice that the disturbances u_t follow a first order autoregressive scheme of the form:

$$u_t = \rho u_{t-1} + \epsilon_t \qquad (2)$$

where $|\rho| < 1$ and ϵ_t satisfies the assumptions

$$\left.\begin{aligned} E[\epsilon_t] &= 0 \\ E[\epsilon_t, \epsilon_{t+s}] &= \sigma_\epsilon^2 \quad s = 0 \\ &= 0 \quad s \neq 0 \end{aligned}\right\} \text{ for all } t.$$

Once these assumptions have been made, the following practical methods are available.

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[1] J. Durbin and G.S. Watson, "Testing for Serial Correlation in Least Squares Regression", Part 1, Biometrica, Vol. 37, (Dec., 1950), pp. 409-28, and Part 2, Biometrica, Vol. 38, (June, 1951), pp. 159-78.

3) A rather complicated two-stage estimation procedure which yields values which are asymtotically correct has been developed by Durbin.[2] In view of the asymtotical nature of the solution, the validity of its application to small samples remains, however, at best uncertain.

4) A direct estimation of the autocorrelation coefficient in equation (2) by a regression on the least squares estimated residuals. This estimate $\hat{\rho}$ is then used to compute the transformed variables $(Y_t - \hat{\rho} Y_{t-1})$, $(X_{2,t} - \hat{\rho} X_{2,t-1})$ $(X_{3,t} - \hat{\rho} X_{3,t-1})$ ..., and simple least squares again applied to estimate the relation between these transformed variables. This method, simple as it may seem, suffers from the disadvantage that if the residuals from the transformed variables are still significantly autocorrelated, the estimation procedure must be repeated until a random set of residuals results. Each time this procedure is repeated, one observation is lost and consequently in the absence of any a priori information as to the number of repetitions required, this is not the most desirable procedure to adopt in the case of small samples.

5) The most practical method and the one which has been adopted is that suggested by Thiel and Nagar.[3] This method consists of a direct calculation of the autocorrelation coefficient of equation (2), the transformation of the variables by this coefficient as above, and then a regression on the transformed variables. This method has the advantage of being one stage, thus minimising the information loss. The authors have further computed a table for the von Neumann ratio based on a transformation of the beta distribution for testing the presence of autocorrelation. This table is superior to that presented by Durbin and Watson in that it has no zone of ignorance and consequently

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[2] J. Durbin, "Estimation of Parameters in Time-Series Regression Models", Journal of the Royal Statistical Society (Ser. B), Vol. 22, (No. 1, 1960), pp. 139 - 53.

[3] H. Thiel and A.L. Nagar, "Testing the Independence of Regression Disturbances", Journal of the American Statistical Association, Vol. 56, (Dec., 1961), pp. 793 - 806.

significance tests take the form of a one tailed test of the null hypothesis that there exists no autocorrelation against the alternative hypothesis that there is significant autocorrelation. Whenever this table indicated the presence of significant autocorrelation, the above method was then used to correct for it and the regression repeated on the corrected data to obtain unbiased results.

3. Tests for Stability.

The regression was computed for the entire period and was then repeated with the war years (1940-44) omitted to see whether the abnormal conditions of war time had any effect on the function. One of the important tenets of students of the demand for money is the hypothesis that the demand function has remained stable over the years, for if this was not so the concept of a long run demand function would have extremely limited significance, and hence any predictions would be meaningless. The principal test for instability centres around the pre-war (1918-39) and post-war (1945-60) periods. In order to have some definite indication as to whether a shift in the function has occurred between the periods under consideration, a test devised by Chow[4] was adopted. This test, henceforth called the Chow test, consists of computing the sum of squares of the deviations for all observations and then repeating the calculation for each group separately. From these sums of squares a F-ratio can be computed and if it exceeds a chosen critical value, the null hypothesis that all the observations came from the same model is rejected. If the residuals have a random distribution the test has immediate applicability. However, if significant autocorrelation is present, serious difficulties arise which are further exacerbated by the absence of any literature dealing with the influence of autocorrelation on the Chow test.

If autocorrelation is found to be significant, three alternatives present themselves.

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[4] G.C. Chow, "Tests of Equality Between Sets of Coefficients in Two Linear Regressions", Econometrica, Vol. 28, (July, 1960), pp. 591-605.

1) Do nothing about it. In view of the serious consequences of autocorrelation, this would be an extremely unwise procedure to adopt, especially since autocorrelation will in addition affect the number of degrees of freedom of the sums of squares.

2) To correct all the data for autocorrelation and then to repeat the test on the corrected data. This procedure, while undoubtedly superior to the former, is open to some serious reservations.

i) If there was a discontinuity in the function, there is no a priori reason to assume that the autocorrelation coefficient has remained unaltered as is implicitly assumed. On the contrary it would be far more realistic to assume that the coefficient changes and hence has a different value for each of the groups.

ii) A consequence of the method adopted to correct for autocorrelation is that the distinction between the endpoint of the first group and the initial point of the second group becomes rather blurred, a factor which might be of importance in view of the absence of any theory dealing with the sensitivity of the Chow test to changes in end points.

3) To correct all the data for autocorrelation, to correct each group separately and only then to apply the test to the corrected data. This method, whilst countering the latter objection to the method above can still be criticised on several points.

i) The constancy of the autocorrelation coefficient is still assumed when both groups of data are considered together, but dropped when they are considered separately.

ii) Because each group of data has been corrected separately, a total of two observations has been absorbed. Quite apart from the information loss a problem arises in the calculation of the number of degrees of freedom of the F-ratio. If there are three coefficients (including the constant term) in the regression equation, m observations in the first group and n in the second, the F-ratio has 3 and (m+n-6) degrees of freedom respectively. Once all the observations have been corrected, the denominator of the F-ratio becomes (m+n-5), whereas if each group had been corrected separately and then summed, the denominator would have become (m+n-4). Since the test is centred around the two groups of observations separately, it was decided to adopt the latter conversion, though bearing in mind the assumptions made whenever a borderline case is encountered.

Neither of the latter two alternatives is completely satisfactory though the last mentioned seems to be superior and was thus adopted. An interesting point arises in connection with the assumed constancy of the autocorrelation coefficient. As was pointed out above, it is a priori extremely difficult to justify its constancy if the function is discontinuous. However, the test is logically based on a null hypothesis of constancy and if this is not rejected the assumption is acceptable. In view of this and of the absence of any satisfactory alternative, the assumption of a constant autocorrelation coefficient when converting all the data grouped together seems unavoidable. As a consequence of the aura of uncertainty surrounding the test, all borderline cases will be treated with due caution. If the F-ratio indicates a discontinuity in the function, separate regressions will be performed on the two groups of data.

4. Method of Presentation.

In order to spare the reader the chore of struggling through a seemingly unconnected and endless maze of results, a system of classification, similar to that of chapter IV has been adopted. Each function being tested has been assigned a three digit number, e.g. 1.2.3. The first digit indicates the monetary variable used: $1 \sim M_1$; $2 \sim M_2$; $3 \sim L_t$ and $4 \sim L_t^*$. The second digit indicates the interest rate used: $1 \sim 6$ month treasury bill rate; $2 \sim 12$ month treasury bill rate; $3 \sim 12$ month deposit rate; $4 \sim$ minimum overdraft rate; $5 \sim$ rate of return on the debt held by the P.D.C. and $6 \sim$ average rate of return on the public debt. The third digit indicates the constraint used: $1 \sim$ wealth; $2 \sim$ current income and $3 \sim$ Permanent Income. Thus for example, the equation numbered

1.2.3. would represent the regression of M_1 against 12 month treasury bills and Permanent Income. The results are presented grouped first by the independent variable i.e. the various definitions of money, then for each particular constraint, the interest rate is allowed to change through all six possible rates, ranging from short term to long term.

With the object of simplicity of presentation in mind, the following layout of the calculations performed on each particular function has been adopted. Five such calculations were performed for each equation <u>viz</u>:

i) a regression over all observations,

ii) the above repeated with the war years (1940-4) omitted,

iii) a Chow test on the pre-war and post-war data,

iv) a regression over the pre-war years,

and v) a regression over the post-war years.

The results presented below follow this ordering. The letter 'a' attached to any of the above calculations indicates that they have been performed on the raw uncorrected data whereas the letter 'b' indicates that the data has been corrected for autocorrelation before the calculation was performed.

The layout of results (i), (ii), (iv) and (v) is identical and so the description below can be taken as pertaining equally to all four. The results for the raw data are presented first. The estimated equation is written down in full, followed by the square of the correlation coefficient; the number in parenthesis on the right hand side is the number of observations. The Durbin-Watson statistic is given on the next line followed by the results of the hypothesis test. If they indicate that the null hypothesis of independence should be rejected, no further calculations are performed; instead the data is then corrected for autocorrelation and the calculation repeated. If initially, or after correction, the Durbin-Watson statistic indicates that the estimated deviations are independent, or in the fortunately rare case of the correction procedure not being sufficiently powerful to remove all the autocorrelation, the t-values along with the conclusions from the hypothesis tests about the coefficients are presented. The first represents $H_0 : \hat{\beta}_2 = 0$, the second $H_0 : \hat{\beta}_3 = 0$ and the third $H_0 : \hat{\beta}_3 = \pm 1$. If the test indicates that $\hat{\beta}_3$ is not significant, no further tests on $\hat{\beta}_3$ are undertaken.

The only other remaining calculation to be explained is number (iii), the result of the Chow test. As above, the result labelled 'a' gives the F-ratio and the results of the significance test performed on the uncorrected data and the result labelled 'b' gives it for the data corrected in the manner described above.

A similar method of presentation has been adopted for the presentation of the empirical tests of the Friedman-type function. The only difference being that since $\beta_2 = 0$, the t-values are given for hypothesis tests on $\hat{\beta}_3$ which corresponds to δ in Friedman's formulation.

5. Some Other Econometrical Problems.

Two other vitally important points of overwhelming relevance to this study about which unfortunately very little definite can be said due to the relative newness of econometrics <u>per se</u> as a field of study, are the effects of multicollinearity and of lagged variables.

Multicollinearity is the name given to the problem which arises when some or all of the explanatory variables in a relation are so highly correlated one with another that it becomes impossible to disentangle their separate influences and obtain a reasonably precise estimate of their relative effects. This will be the case for wealth and interest on account of their interconnectedness. In view of the problems encountered in the specification of empirical measures of income and wealth, the actual data employed is unlikely to obey the exact relationship predicted in theory and under these circumstances the analysis undertaken by Frisch[5] is of relevance. However, even with the help of modern electronic computers his suggested procedure would entail far more effort than could reasonably

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[5] R. Frisch, <u>Statistical Confluence Analysis by Means of Complete Regression Systems</u> (Oslo, 1934).

be justified and in addition because of the still dubious applicability of his method to econometrics, this phenomenon will not receive empirical investigation in the results presented below.

Lagged variables, as in the case of permanent income and of the two interest rates involving public debt, on their own present a serious difficulty since they introduce a bias into the least squares estimators which in the case of small samples may be extremely serious. If lagged variables and autocorrelation appear simultaneously the position becomes well nigh hopeless and very little reliance can be placed on any results since even after correction for autocorrelation the bias will still remain and there does not as yet exist a satisfactory procedure for estimating this bias.[6]

With these caveats in mind, the empirical results of fitting a logarithmic function to the data are presented below in the order described above.

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[6] For an example of the type of problem encountered see L.R. Klein, "The Estimation of Distributed Lags", Econometrica, Vol. 25, (Oct., 1958), pp. 553-65, and E. Malinvaud, "The Estimation of Distributed Lags: A Comment", Econometrica, Vol. 29, (July, 1961), pp. 430-33.

Eqn 1.1.1.

i) (a) $\ln M_1 = -4.405 - 0.404 \ln r + 1.104 \ln W$ $R^2 = 0.850$ (39)

d = 0.275 reject at 1% and 5%.

(b) $\ln M_1 = 1.461 - 0.284 \ln r + 0.394 \ln W$ $R^2 = 0.344$

d = 1.357 reject at 1% and 5%.

(-3.698) (1.893)

reject at 1% accept at 1% and 5%

---

ii) (a) $\ln M_1 = 4.796 - 0.387 \ln r + 1.121 \ln W$ $R^2 = 0.857$ (34)

d = 0.556 reject at 1% and 5%.

(b) $\ln M_1 = 0.577 - 0.368 \ln r + 0.807 \ln W$ $R^2 = 0.463$

d = 1.822 accept at 1% and 5%.

(-3.235) (3.469) (-0.827)

reject at 1% reject at 1% accept at 1% and 5%

---

iii) (a) F = 148.105 (b) F = 74.372

reject at 1% reject at 1%

---

iv) (a) $\ln M_1 = 20.772 - 0.468 \ln r - 0.048 \ln W$ $R^2 = 0.928$ (20)

d = 2.105 accept at 1% and 5%

(-10.208) (-0.443)

reject at 1% accept at 1% and 5%

---

v) (a) $\ln M_1 = 18.873 - 0.157 \ln r + 0.082 \ln W$ $R^2 = 0.796$ (14)

d = 0.934 reject at 1% and 5%

(b) $\ln M_1 = 6.821 - 0.179 \ln r + 0.156 \ln W$ $R^2 = 0.532$

d = 1.836 accept at 1% and 5%

(-2.247) (0.645)

accept at 1% accept at 1% and 5%

reject at 5%

Eqn 1.2.1.

i) (a) $\ln M_1 = 4.658 - 0.779 \ln r + 0.716 \ln W$ $R^2 = 0.887$ (35)

$d = 0.261$ reject at 1% and 5%

(b) $\ln M_1 = 1.756 - 0.572 \ln r + 0.252 \ln W$ $R^2 = 0.505$

$d = 1.636$ accept at 1% and 5%

(-5.263) (1.185)

reject at 1% accept at 1% and 5%

---

ii) (a) $\ln M_1 = 3.724 - 0.747 \ln r + 0.756 \ln W$ $R^2 = 0.883$ (30)

$d = 0.444$ reject at 1% and 5%

(b) $\ln M_1 = 2.660 - 0.882 \ln r + 0.346 \ln W$ $R^2 = 0.600$

$d = 1.873$ accept at 1% and 5%

(-5.541) (1.296)

reject at 1% accept at 1% and 5%

---

iii) (a) $F = 130.876$ (b) $F = 49.136$

reject at 1% reject at 1%

---

iv) (a) $\ln M_1 = 22.157 - 0.715 \ln r - 0.091 \ln W$ $R^2 = 0.948$ (20)

$d = 2.044$ accept at 1% and 5%

(-12.297) (0.979)

reject at 1% accept at 1% and 5%

---

v) (a) $\ln M_1 = 13.802 - 0.457 \ln r + 0.313 \ln W$ $R^2 = 0.842$ (10)

$d = 1.908$ accept at 1% and 5%

(-2.633) (1.054)

accept at 1% accept at 1% and 5%

reject at 5%

Eqn 1.3.1.

i) (a) $\ln M_1 = 4.234 - 0.794 \ln r + 1.125 \ln W$ $R^2 = 0.875$ (43)

$d = 0.229$ reject at 1% and 5%

(b) $\ln M_1 = 0.390 - 0.486 \ln r + 0.756 \ln W$ $R^2 = 0.538$

$d = 1.483$ reject at 1%, accept at 5%

(-5.523) (5.112) (-1.646)

reject at 1% reject at 1% accept at 1% and 5%

---

ii) (a) $\ln M_1 = 4.578 - 0.729 \ln r + 1.113 \ln W$ $R^2 = 0.883$ (38)

$d = 0.371$ reject at 1% and 5%

(b) $\ln M_1 = 0.115 - 0.713 \ln r + 0.900 \ln W$ $R^2 = 0.577$

$d = 2.033$ accept at 1% and 5%

(-5.678) (4.893) (-0.544)

reject at 1% reject at 1% accept at 1% and 5%

---

iii) (a) $F = 34.186$ (b) $F = 38.632$

reject at 1% reject at 1%

---

iv) (a) $\ln \bar{M}_1 = 9.376 - 0.525 \ln r + 0.483 \ln W$ $R^2 = 0.733$ (22)

$d = 0.393$ reject at 1% and 5%

(b) $\ln M_1 = 0.262 - 0.586 \ln r + 0.840 \ln W$ $R^2 = 0.751$

$d = 1.186$ reject at 1% and 5%

(-5.783) (5.508) (-1.051)

reject at 1% reject at 1% accept at 1% and 5%

---

v) (a) $\ln M_1 = 21.117 - 0.188 \ln r - 0.011 \ln W$ $R^2 = 0.795$ (16)

$d = 0.936$ reject at 1% and 5%

(b) $\ln M_1 = 7.780 - 0.215 \ln r + 0.094 \ln W$ $R^2 = 0.567$

$d = 2.241$ accept at 1% and 5%

(-2.870) (0.627)

reject at 5% accept at 1% and 5%

accept at 1%

<u>Eqn 1.4.1.</u>

i) (a) $\ln M_1 = 4.417 - 2.576 \ln r + 0.901 \ln W$ $R^2 = 0.917$ (43)

$d = 0.451$ reject at 1% and 5%

(b) $\ln M_1 = 1.196 - 1.657 \ln r + 0.781 \ln W$ $R^2 = 0.635$

$d = 1.801$ accept at 1% and 5%

(-5.376)	(5.864)	(-1.646)
reject at 1%	reject at 1%	accept at 1% and 5%

---

ii) (a) $\ln M_1 = 3.741 - 2.449 \ln r + 0.921 \ln W$ $R^2 = 0.922$ (38)

$d = 0.735$ reject at 1% and 5%

(b) $\ln M_1 = 1.628 - 2.192 \ln r + 0.864 \ln W$ $R^2 = 0.731$

$d = 2.231$ accept at 1% and 5%

(-5.716)	(6.325)	(-0.992)
reject at 1%	reject at 1%	accept at 1% and 5%

---

iii) (a) $F = 56.224$ (b) $F = 34.851$

reject at 1% reject at 1%

---

iv) (a) $\ln M_1 = 18.319 - 2.217 \ln r + 0.233 \ln W$ $R^2 = 0.888$ (22)

$d = 1.526$ reject at 1%, accept at 5%

(b) $\ln M_1 = 12.122 - 2.073 \ln r + 0.311 \ln W$ $R^2 = 0.840$

$d = 1.921$ accept at 1% and 5%

(-6.339)	(2.250)	(-4.991)
reject at 1%	accept at 1% reject at 5%	reject at 1%

---

v) (a) $\ln M_1 = 21.603 - 0.672 \ln r + 0.009 \ln W$ $R^2 = 0.792$ (16)

$d = 0.821$ reject at 1% and 5%

(b) $\ln M_1 = 6.514 - 0.700 \ln r + 0.154 \ln W$ $R^2 = 0.602$

$d = 2.514$ accept at 1% and 5%

(-3.481)	(1.037)
reject at 1%	accept at 1% and 5%

Eqn 1.5.1.

i) (a) $\ln M_1 = 6.906 - 3.411 \ln r + 0.787 \ln W$ $R^2 = 0.969$ (42)

$d = 0.682$ reject at 1% and 5%

(b) $\ln M_1 = 1.433 - 3.135 \ln r + 0.887 \ln W$ $R^2 = 0.874$

$d = 1.575$ reject at 1%, accept at 5%

(-10.380) (9.922) (-1.270)

reject at 1% reject at 1% accept at 1% and 5%

---

ii) (a) $\ln M_1 = 6.403 - 3.306 \ln r + 0.803 \ln W$ $R^2 = 0.969$ (37)

$d = 0.813$ reject at 1% and 5%

(b) $\ln M_1 = 2.052 - 3.378 \ln r + 0.862 \ln W$ $R^2 = 0.906$

$d = 1.744$ accept at 1% and 5%

(-11.457) (9.775) (-1.570)

reject at 1% reject at 1% accept at 1% and 5%

---

iii) (a) $F = 12.867$ (b) 7.209

reject at 1% reject at 1%

---

iv) (a) $\ln M_1 = 15.331 - 3.416 \ln r + 0.401 \ln W$ $R^2 = 0.884$

$d = 1.523$ reject at 1%, accept at 5%

(b) $\ln M_1 = 8.316 - 2.978 \ln r + 0.558 \ln W$ $R^2 = 0.853$

$d = 1.566$ accept at 1% and 5%

(-5.607) (3.847) (-3.051)

reject at 1% reject at 1% reject at 1%

---

v) (a) $\ln M_1 = 29.531 + 0.417 \ln r - 0.412 \ln W$ $R^2 = 0.687$ (16)

$d = 0.993$ reject at 1% and 5%

(b) $\ln M_1 = 11.551 - 0.054 \ln r - 0.230 \ln W$ $R^2 = 0.304$

$d = 1.669$ accept at 1% and 5%

(-0.059) (-0.614)

accept at 1% and 5% accept at 1% and 5%

Eqn 1.6.1.

i) (a) $\ln M_1 = 7.013 - 2.644 \ln r + 0.735 \ln W$ $R^2 = 0.947$ (40)

$d = 0.401$ reject at 1% and 5%

(b) $\ln M_1 = 0.967 - 2.461 \ln r + 0.676 \ln W$ $R^2 = 0.512$

$d = 1.606$ accept at 1% and 5%

(-4.941)	(4.382)	(-2.100)
reject at 1%	reject at 1%	accept at 1% reject at 5%

---

ii) (a) $\ln M_1 = 6.148 - 2.529 \ln r + 0.767 \ln W$ $R^2 = 0.954$ (35)

$d = 0.759$ reject at 1% and 5%

(b) $\ln M_1 = 2.560 - 2.973 \ln r + 0.757 \ln W$ $R^2 = 0.845$

$d = 1.789$ accept at 1% and 5%

(-9.014)	(6.457)	(-2.076)
reject at 1%	reject at 1%	accept at 1% reject at 5%

---

iii) (a) $F = 43.215$ (b) $F = 27.605$

reject at 1% reject at 1%

---

iv) (a) $\ln M_1 = 22.207 - 2.939 \ln r + 0.061 \ln W$ $R^2 = 0.936$ (19)

$d = 2.506$ accept at 1% and 5%

(-9.581)	(0.455)
reject at 1%	accept at 1% and 5%

---

v) (a) $\ln M_1 = 30.344 + 0.475 \ln r - 0.450 \ln W$ $R^2 = 0.734$ (16)

$d = 1.136$ reject at 1% and 5%

(b) $\ln M_1 = 17.026 + 0.657 \ln r - 0.567 \ln W$ $R^2 = 0.426$

$d = 1.907$ accept at 1% and 5%

(1.338)	(-2.293)	(1.751)
accept at 1% and 5%	accept at 1% reject at 5%	accept at 1% and 5%

Eqn 1.1.2.

i) (a) $\ln M_1 = 3.599 - 0.302 \ln r + 1.107 \ln Y$ $R^2 = 0.924$ (39)

d = 0.421 reject at 1% and 5%

(b) $\ln M_1 = 0.306 - 0.231 \ln r + 0.866 \ln Y$ $R^2 = 0.598$

d = 1.811 accept at 1% and 5%

(-3.179) (5.348) (-0.827)

reject at 1% reject at 1% accept at 1% and 5%

---

ii) (a) $\ln M_1 = 3.582 - 0.294 \ln r + 1.106 \ln Y$ $R^2 = 0.926$ (34)

d = 0.677 reject at 1% and 5%

(b) $\ln M_1 = 1.079 - 0.260 \ln r + 1.091 \ln Y$ $R^2 = 0.734$

d = 2.110 accept at 1% and 5%

(-2.954) (7.225) (0.601)

reject at 1% reject at 1% accept at 1% and 5%

---

iii) (a) F = 72.409 (b) F = 43.910

reject at 1% reject at 1%

---

iv) (a) $\ln M_1 = 21.077 - 0.474 \ln r + 0.064 \ln Y$ $R^2 = 0.928$ (20)

d = 2.105 accept at 1% and 5%

(-8.353) (-0.422)

reject at 1% accept at 1% and 5%

---

v) (a) $\ln M_1 = 17.526 - 0.169 \ln r + 0.146 \ln Y$ $R^2 = 0.797$ (14)

d = 0.895 reject at 1% and 5%

(b) $\ln M_1 = 5.311 - 0.200 \ln r + 0.302 \ln Y$ $R^2 = 0.543$

d = 1.704 accept at 1% and 5%

(-2.563) (1.021)

accept at 1% accept at 1% and 5%

reject at 5%

Eqn 1.2.2.

i) (a) $\ln M_1 = 0.259 - 0.528 \ln r + 0.940 \ln Y$ $R^2 = 0.935$ (35)

d = 0.396 reject at 1% and 5%

(b) $\ln M_1 = 1.137 - 0.506 \ln r + 0.665 \ln Y$ $R^2 = 0.666$

d = 1.945 accept at 1% and 5%

(-4.482) (3.554) (-1.788)

reject at 1% reject at 1% accept at 1% and 5%

---

ii) (a) $\ln M_1 = 0.030 - 0.510 \ln r + 0.950 \ln Y$ $R^2 = 0.932$ (30)

d = 0.595 reject at 1% and 5%

(b) $\ln M_1 = 0.799 - 0.601 \ln r + 0.821 \ln Y$ $R^2 = 0.766$

d = 2.198 accept at 1% and 5%

(-4.175) (3.937) (-0.856)

reject at 1% reject at 1% accept at 1% and 5%

---

iii) (a) F = 69.724 (b) F = 34.838

reject at 1% reject at 1%

---

iv) (a) $\ln M_1 = 22.036 - 0.715 \ln r - 0.089 \ln Y$ $R^2 = 0.947$ (20)

d = 1.986 accept at 1% and 5%

(-10.025) (-0.720)

reject at 1% accept at 1% and 5%

---

v) (a) $\ln M_1 = 14.850 - 0.383 \ln r + 0.275 \ln Y$ $R^2 = 0.829$ (10)

d = 1.664 accept at 1% and 5%

(-2.355) (0.733)

accept at 1% accept at 1% and 5%

reject at 5%

Eqn 1.3.2.

i) (a) $\ln M_1 = -2.868 - 0.625 \ln r + 1.097 \ln Y$ $R^2 = 0.944$ (43)

$d = 0.287$ reject at 1% and 5%

(b) $\ln M_1 = -0.159 - 0.474 \ln r + 1.008 \ln Y$ $R^2 = 0.731$

$d = 1.972$ accept at 1% and 5%

(-6.760) (8.622) (0.068)

reject at 1% reject at 1% accept at 1% and 5%

---

ii) (a) $\ln M_1 = -2.899 - 0.590 \ln r + 1.096 \ln Y$ $R^2 = 0.947$ (38)

$d = 0.381$ reject at 1% and 5%

(b) $\ln M_1 = 0.783 - 0.626 \ln r + 1.162 \ln Y$ $R^2 = 0.806$

$d = 2.349$ accept at 1% and 5%

(-7.468) (9.619) (1.341)

reject at 1% reject at 1% accept at 1% and 5%

---

iii) (a) $F = 17.626$ (b) $F = 29.720$

reject at 1% reject at 1%

---

iv) (a) $\ln M_1 = 5.847 - 0.464 \ln r + 0.669 \ln Y$ $R^2 = 0.813$ (22)

$d = 0.364$ reject at 1% and 5%

(b) $\ln M_1 = 0.225 - 0.595 \ln r + 1.024 \ln Y$ $R^2 = 0.870$

$d = 1.604$ accept at 1% and 5%

(-8.179) (8.665) (0.202)

reject at 1% reject at 1% accept at 1% and 5%

---

v) (a) $\ln M_1 = 21.137 - 0.189 \ln r - 0.013 \ln Y$ $R^2 = 0.795$ (16)

$d = 0.938$ reject at 1% and 5%

(b) $\ln M_1 = 7.698 - 0.210 \ln r + 0.107 \ln Y$ $R^2 = 0.568$

$d = 2.178$ accept at 1% and 5%

(-3.056) (0.628)

reject at 1% accept at 1% and 5%

Eqn 1.4.2.

i) (a) $\ln M_1 = 3.710 - 2.023 \ln r + 0.925 \ln Y$ $R^2 = 0.956$ (43)

$d = 0.602$ reject at 1% and 5%

(b) $\ln M_1 = 0.681 - 1.482 \ln r + 0.944 \ln Y$ $R^2 = 0.816$

$d = 2.212$ accept at 1% and 5%

(-5.772) (9.762) (-0.578)

reject at 1% reject at 1% accept at 1% and 5%

---

ii) (a) $\ln M_1 = 3.459 - 1.951 \ln r + 0.930 \ln Y$ $R^2 = 0.959$ (38)

$d = 0.868$ reject at 1% and 5%

(b) $\ln M_1 = 0.922 - 1.742 \ln r + 0.972 \ln Y$ $R^2 = 0.875$

$d = 2.378$ accept at 1% and 5%

(-6.005) (10.722) (-0.307)

reject at 1% reject at 1% accept at 1% and 5%

---

iii) (a) $F = 28.564$ (b) $F = 19.516$

reject at 1% reject at 1%

---

iv) (a) $\ln M_1 = 15.881 - 2.029 \ln r + 0.343 \ln Y$ $R^2 = 0.901$ (22)

$d = 1.529$ reject at 1%, accept at 5%

(b) $\ln M_1 = 9.910 - 1.848 \ln r + 0.448 \ln Y$ $R^2 = 0.860$

$d = 1.977$ accept at 1% and 5%

(-5.492) (2.881) (-3.554)

reject at 1% reject 1% reject at 1%

---

v) (a) $\ln M_1 = 21.585 - 0.670 \ln r + 0.010 \ln Y$ $R^2 = 0.792$ (16)

$d = 0.819$ reject at 1% and 5%

(b) $\ln M_1 = 6.481 - 0.660 \ln r + 0.159 \ln Y$ $R^2 = 0.596$

$d = 2.374$ accept at 1% and 5%

(-3.620) (0.944)

reject at 1% accept at 1% and 5%

Eqn 1.5.2.

i) (a) $\ln M_1 = 6.144 - 2.751 \ln r + 0.813 \ln Y$ $R^2 = 0.981$ (42)

d = 0.843 reject at 1% and 5%

(b) $\ln M_1 = 1.769 - 2.573 \ln r + 0.889 \ln Y$ $R^2 = 0.938$

d = 1.592 reject at 1%, accept at 5%

(-10.785) (14.091) (-1.767)

reject at 1% reject at 1% accept at 1% and 5%

---

ii) (a) $\ln M_1 = 5.881 - 2.682 \ln r + 0.821 \ln Y$ $R^2 = 0.981$ (37)

d = 0.928 reject at 1% and 5%

(b) $\ln M_1 = 2.106 - 2.664 \ln r + 0.877 \ln Y$ $R^2 = 0.948$

d = 1.628 accept at 1% and 5%

(-10.812) (13.659) (-1.910)

reject at 1% reject at 1% accept at 1% and 5%

---

iii) (a) F = 5.698 (b) F = 2.736

reject at 1% accept at 1% and 5%

---

iv) (a) $\ln M_1 = 12.654 - 3.039 \ln r + 0.521 \ln Y$ $R^2 = 0.898$ (21)

d = 1.443 reject at 1%, accept at 5%

(b) $\ln M_1 = 5.945 - 2.577 \ln r + 0.686 \ln Y$ $R^2 = 0.864$

d = 1.510 reject at 1%, accept at 5%

(-4.747) (4.427) (-2.026)

reject at 1% reject at 1% accept at 1% and 5%

---

v) (a) $\ln M_1 = 33.983 + 0.674 \ln r - 0.643 \ln Y$ $R^2 = 0.700$ (16)

d = 1.209 reject at 1% and 5%

(b) $\ln M_1 = 14.637 - 0.142 \ln r - 0.248 \ln Y$ $R^2 = 0.395$

d = 1.644 accept at 1% and 5%

(-0.135) (-0.472)

accept at 1% and 5% accept at 1% and 5%

Eqn 1.6.2.

i) (a) $\ln M_1 = 6.069 - 2.135 \ln r + 0.778 \ln Y$ $R^2 = 0.967$ (40)

d = 0.488 reject at 1% and 5%

(b) $\ln M_1 = 1.154 - 2.214 \ln r + 0.835 \ln Y$ $R^2 = 0.808$

d = 1.721 accept at 1% and 5%

(-6.932)   (8.301)   (-1.639)

reject at 1%   reject at 1%   accept at 1% and 5%

---

ii) (a) $\ln M_1 = 5.688 - 2.059 \ln r + 0.790 \ln Y$ $R^2 = 0.970$ (35)

d = 0.799 reject at 1% and 5%

(b) $\ln M_1 = 2.087 - 2.333 \ln r + 0.821 \ln Y$ $R^2 = 0.902$

d = 1.818 accept at 1% and 5%

(-8.152)   (8.998)   (-1.957)

reject at 1%   reject at 1%   accept at 1% and 5%

---

iii) (a) F = 26.048   (b) F = 15.225

reject at 1%   reject at 1%

---

iv) (a) $\ln M_1 = 20.560 - 2.797 \ln r + 0.133 \ln Y$ $R^2 = 0.938$ (19)

d = 2.353 accept at 1% and 5%

(-7.894)   (0.839)

reject at 1%   accept at 1% and 5%

---

v) (a) $\ln M_1 = 34.043 + 0.598 \ln r - 0.640 \ln Y$ $R^2 = 0.760$ (16)

d = 1.406 reject at 1%, accept at 5%

(b) $\ln M_1 = 24.507 + 0.811 \ln r - 0.793 \ln Y$ $R^2 = 0590$

d = 2.272 accept at 1% and 5%

(1.737)   (-2.822)   (0.737)

accept at 1% and 5%   reject at 1%   accept at 1% and 5%

Eqn 1.1.3.

i) (a) $\ln M_1 = -5.006 - 0.324 \ln r + 1.174 \ln Y_p$ $R^2 = 0.943$ (37)

d = 0.398 reject at 1% and 5%

(b) $\ln M_1 = -0.385 - 0.259 \ln r + 1.034 \ln Y_p$ $R^2 = 0.620$

d = 1.742 accept at 1% and 5%

(-4.077)   (5.780)   (0.188)

reject at 1%   reject at 1%   accept at 1% and 5%

---

ii) (a) $\ln M_1 = -4.930 - 0.321 \ln r + 1.171 \ln Y_p$ $R^2 = 0.946$ (32)

d = 0.724 reject at 1% and 5%

(b) $\ln M_1 = -2.149 - 0.304 \ln r + 1.226 \ln Y_p$ $R^2 = 0.799$

d = 1.991 accept at 1% and 5%

(-4.021)   (8.812)   (1.624)

reject at 1%   reject at 1%   accept at 1% and 5%

---

iii) (a) F = 65.014   (b) F = 44.909

reject at 1%   reject at 1%

---

iv) (a) $\ln M_1 = 16.775 - 0.429 \ln r + 0.138 \ln Y_p$ $R^2 = 0.952$ (18)

d = 2.663 accept at 1% and 5%

(-9.247)   (1.005)

reject at 1%   accept at 1% and 5%

---

v) (a) $\ln M_1 = 19.907 - 0.137 \ln r + 0.038 \ln Y_p$ $R^2 = 0.794$ (14)

d = 0.944 reject at 1% and 5%

(b) $\ln M_1 = 6.327 - 0.186 \ln r + 0.228 \ln Y_p$ $R^2 = 0.538$

d = 1.688 accept at 1% and 5%

(-2.199)   (0.691)

accept at 1% and 5%   accept at 1% and 5%

Eqn 1.2.3.

i) (a) $\ln M_1 = -1.839 - 0.521 \ln r + 1.039 \ln Y_p$ $R^2 = 0.952$ (33)

$d = 0.407$ reject at 1% and 5%

(b) $\ln M_1 = -0.104 - 0.485 \ln r + 0.976 \ln Y_p$ $R^2 = 0.698$

$d = 2.047$ accept at 1% and 5%

(-4.743) (4.729) (-0.118)

reject at 1% reject at 1% accept at 1% and 5%

---

ii) (a) $\ln M_1 = -1.956 - 0.508 \ln r + 1.044 \ln Y_p$ $R^2 = 0.952$ (28)

$d = 0.657$ reject at 1% and 5%

(b) $\ln M_1 = -0.872 - 0.576 \ln r + 1.085 \ln Y_p$ $R^2 = 0.831$

$d = 2.229$ accept at 1% and 5%

(-4.639) (5.656) (0.441)

reject at 1% reject at 1% accept at 1% and 5%

---

iii) (a) $F = 57.735$ (b) $F = 35.151$

reject at 1% reject at 1%

---

iv) (a) $\ln M_1 = 18.418 - 0.655 \ln r + 0.079 \ln Y_p$ $R^2 = 0.962$ (18)

$d = 2.563$ accept at 1% and 5%

(-10.564) (0.628)

reject at 1% accept at 1% and 5%

---

v) (a) $\ln M_1 = 15.748 - 0.366 \ln r + 0.234 \ln Y_p$ $R^2 = 0.823$ (10)

$d = 1.623$ accept at 1% and 5%

(-1.882) (0.516)

accept at 1% and 5% accept at 1% and 5%

Eqn 1.3.3.

i) (a) $\ln M_1 = -4.098 - 0.647 \ln r + 1.156 \ln Y_p$ $R^2 = 0.954$ (41)

$d = 0.394$ reject at 1% and 5%

(b) $\ln M_1 = -1.399 - 0.515 \ln r + 1.301 \ln Y_p$ $R^2 = 0.758$

$d = 2.056$ accept at 1% and 5%

(-6.836) (9.622) (2.228)

reject at 1% reject at 1% accept at 1%
reject at 5%

---

ii) (a) $\ln M_1 = -4.115 - 0.614 \ln r + 1.155 \ln Y_p$ $R^2 = 0.958$ (36)

$d = 0.500$ reject at 1% and 5%

(b) $\ln M_1 = -2.114 - 0.697 \ln r + 1.373 \ln Y_p$ $R^2 = 0.846$

$d = 2.346$ accept at 1% and 5%

(-7.814) (11.484) (3.121)

reject at 1% reject at 1% reject at 1%

---

iii) (a) $F = 14.035$ (b) $F = 21.648$

reject at 1% reject at 1%

---

iv) (a) $\ln M_1 = 2.150 - 0.453 \ln r + 0.845 \ln Y_p$ $R^2 = 0.843$ (20)

$d = 0.616$ reject at 1% and 5%

(b) $\ln M_1 = -3.755 - 0.609 \ln r + 1.612 \ln Y_p$ $R^2 = 0.854$

$d = 1.991$ accept at 1% and 5%

(-7.160) (7.831) (2.973)

reject at 1% reject at 1% reject at 1%

---

v) (a) $\ln M_1 = 21.306 - 0.185 \ln r - 0.020 \ln Y_p$ $R^2 = 0.795$ (16)

$d = 0.938$ reject at 1% and 5%

(b) $\ln M_1 = 7.564 - 0.217 \ln r + 0.123 \ln Y_p$ $R^2 = 0.569$

$d = 2.159$ accept at 1% and 5%

(-2.862) (0.647)

accept at 1% accept at 1% and 5%
reject at 5%

Eqn 1.4.3.

i) (a) $\ln M_1 = 2.808 - 2.031 \ln r + 0.968 \ln Y_p$ $R^2 = 0.962$ (41)

$d = 0.683$ reject at 1% and 5%

(b) $\ln M_1 = -0.016 - 1.672 \ln r + 1.070 \ln Y_p$ $R^2 = 0.844$

$d = 2.419$ accept at 1% and 5%

(-6.436)	(11.055)	(0.727)
reject at 1%	reject at 1%	accept at 1% and 5%

---

ii) (a) $\ln M_1 = 2.564 - 1.960 \ln r + 0.973 \ln Y_p$ $R^2 = 0.964$ (36)

$d = 0.992$ reject at 1% and 5%

(b) $\ln M_1 = 0.436 - 1.918 \ln r + 1.046 \ln Y_p$ $R^2 = 0.907$

$d = 2.565$ accept at 1% and 5%

(-7.104)	(12.703)	(0.564)
reject at 1%	reject at 1%	accept at 1% and 5%

---

iii) (a) $F = 19.402$ (b) $F = 14.344$

reject at 1% reject at 1%

---

iv) (a) $\ln M_1 = 14.635 - 1.999 \ln r + 0.400 \ln Y_p$ $R^2 = 0.897$ (20)

$d = 1.663$ accept at 1% and 5%

(-6.118)	(2.304)	(-3.462)
reject at 1%	accept at 1% reject at 5%	reject at 1%

---

v) (a) $\ln M_1 = 21.615 - 0.667 \ln r + 0.008 \ln Y_p$ $R^2 = 0.792$ (16)

$d = 0.818$ reject at 1% and 5%

(b) $\ln M_1 = 6.296 - 0.689 \ln r + 0.183 \ln Y_p$ $R^2 = 0.598$

$d = 2.387$ accept at 1% and 5%

(-3.446)	(0.982)
reject at 1%	accept at 1% and 5%

Eqn 1.5.3.

i) (a) $\ln M_1 = 5.539 - 2.732 \ln r + 0.841 \ln Y_p$ $R^2 = 0.982$ (41)

$d = 1.103$ reject at 1% and 5%

(b) $\ln M_1 = 2.482 - 2.592 \ln r + 0.878 \ln Y_p$ $R^2 = 0.951$

$d = 1.808$ accept at 1% and 5%

(-11.528)	(14.388)	(-2.003)
reject at 1%	reject at 1%	accept at 1% and 5%

---

ii) (a) $\ln M_1 = 5.280 - 2.665 \ln r + 0.848 \ln Y_p$ $R^2 = 0.982$ (36)

$d = 1.182$ reject at 1% and 5%

(b) $\ln M_1 = 2.745 - 2.624 \ln r + 0.872 \ln Y_p$ $R^2 = 0.957$

$d = 1.810$ accept at 1% and 5%

(-10.956)	(13.965)	(-2.046)
reject at 1%	reject at 1%	accept at 1% reject at 5%

---

iii) (a) $F = 3.760$ $F = 2.664$

accept at 1% reject at 5%	accept at 1% and 5%

---

iv) (a) $\ln M_1 = 11.002 - 2.978 \ln r + 0.596 \ln Y_p$ $R^2 = 0.892$ (20)

$d = 1.622$ accept at 1% and 5%

(-5.937)	(3.861)	(-2.613)
reject at 1%	reject at 1%	accept at 1% reject at 5%

---

v) (a) $\ln M_1 = 36.313 + 0.920 \ln r - 0.763 \ln Y_p$ $R^2 = 0.722$ (16)

$d = 1.241$ reject at 1% and 5%

(b) $\ln M_1 = 17.871 + 0.333 \ln r - 0.497 \ln Y_p$ $R^2 = 0.447$

$d = 1.671$ accept at 1% and 5%

(0.349)	(-1.040)
accept at 1% and 5%	accept at 1% and 5%

Eqn 1.6.3.

i) (a) $\ln M_1 = 5.946 - 2.136 \ln r + 0.784 \ln Y_p$ $R^2 = 0.966$ (40)

$d = 0.608$ reject at 1% and 5%

(b) $\ln M_1 = 1.133 - 2.161 \ln r + 0.883 \ln Y_p$ $R^2 = 0.831$

$d = 1.901$ accept at 1% and 5%

(-6.922) (8.062) (-1.066)

reject at 1% reject at 1% accept at 1% and 5%

---

ii) (a) $\ln M_1 = 5.623 - 2.068 \ln r + 0.795 \ln Y_p$ $R^2 = 0.969$ (35)

$d = 0.901$ reject at 1% and 5%

(b) $\ln M_1 = 2.130 - 2.247 \ln r + 0.842 \ln Y_p$ $R^2 = 0.906$

$d = 1.925$ accept at 1% and 5%

(-7.771) (8.769) (-1.647)

reject at 1% reject at 1% accept at 1% and 5%

---

iii) (a) $F = 26.899$ (b) $F = 20.987$

reject at 1% reject at 1%

---

iv) (a) $\ln M_1 = 24.431 - 3.099 \ln r - 0.031 \ln Y_p$ $R^2 = 0.936$ (19)

$d = 2.718$ accept at 1% and 5%

(-8.315) (-0.170)

reject at 1% accept at 1% and 5%

---

v) (a) $\ln M_1 = 34.340 + 0.610 \ln r - 0.655 \ln Y_p$ $R^2 = 0.776$ (16)

$d = 1.342$ reject at 1%, accept at 5%

(b) $\ln M_1 = 23.636 + 0.836 \ln r - 0.823 \ln Y_p$ $R^2 = 0.605$

$d = 2.267$ accept at 1% and 5%

(1.901) (-3.051) (0.656)

accept at 1% and 5% reject at 1% accept at 1%

Eqn 2.1.1.

i) (a) $\ln M_2 = -7.283 - 0.407 \ln r + 1.247 \ln W$ $R^2 = 0.890$ (39)

d = 0.285 reject at 1% and 5%

(b) $\ln M_2 = 0.943 - 0.270 \ln r + 0.602 \ln W$ $R^2 = 0.440$

d = 1.144 reject at 1% and 5%

(-3.887) (3.227) (-2.137)

reject at 1% reject at 1% accept at 1%

reject at 5%

---

ii) (a) $\ln M_2 = -7.702 - 0.384 \ln r + 1.264 \ln W$ $R^2 = 0.899$ (34)

d = 0.613 reject at 1% and 5%

(b) $\ln M_2 = -0.699 - 0.358 \ln r + 1.024 \ln W$ $R^2 = 0.592$

d = 1.821 accept at 1% and 5%

(-3.466) (5.034) (0.119)

reject at 1% reject at 1% accept at 1% and 5%

---

iii) (a) F = 115.384 (b) F = 70.648

reject at 1% reject at 1%

---

iv) (a) $\ln M_2 = 14.772 - 0.456 \ln r + 0.237 \ln W$ $R^2 = 0.952$ (20)

d = 1.796 accept at 1% and 5%

(-10.472) (2.312) (-7.439)

reject at 1% accept at 1% reject at 1%

reject at 5%

---

v) (a) $\ln M_2 = 20.119 - 0.067 \ln r + 0.040 \ln W$ $R^2 = 0.297$ (14)

d = 0.534 reject at 1% and 5%

(b) $\ln M_2 = 2.204 - 0.125 \ln r + 0.404 \ln W$ $R^2 = 0.267$

d = 2.016 accept at 1% and 5%

(-1.807) (1.550)

accept at 1% and 5% accept at 1% and 5%

Eqn 2.2.1.

i) (a) $\ln M_2 = 1.882 - 0.778 \ln r + 0.853 \ln W$ $R^2 = 0.920$ (35)

$d = 0.282$ reject at 1% and 5%

(b) $\ln M_2 = 1.302 - 0.548 \ln r + 0.478 \ln W$ $R^2 = 0.588$

$d = 1.388$ reject at 1%, accept at 5%

(-5.638)	(2.541)	(-2.779)
reject at 1%	accept at 1%	reject at 1%
	reject at 5%	

---

ii) (a) $\ln M_2 = 0.842 - 0.740 \ln r + 0.899 \ln W$ $R^2 = 0.920$ (30)

$d = 0.503$ reject at 1% and 5%

(b) $\ln M_2 = 1.838 - 0.789 \ln r + 0.587 \ln W$ $R^2 = 0.680$

$d = 1.854$ accept at 1% and 5%

(-5.827)	(2.497)	(-1.754)
reject at 1%	accept at 1%	accept at 1% and 5%
	reject at 5%	

---

iii) (a) $F = 97.802$ (b) $F = 42.919$

reject at 1%   reject at 1%

---

iv) (a) $\ln M_2 = 16.011 - 0.692 \ln r + 0.200 \ln W$ $R^2 = 0.962$ (20)

$d = 1.543$ accept at 1% and 5%

(-12.028)	(2.164)	(-8.673)
reject at 1%	accept at 1%	reject at 1%
	reject at 5%	

---

v) (a) $\ln M_2 = 16.815 - 0.314 \ln r + 0.191 \ln W$ $R^2 = 0.759$ (10)

$d = 1.342$ reject at 1% and 5%

(b) $\ln M_2 = 8.445 - 0.300 \ln r + 0.253 \ln W$ $R^2 = 0.512$

$d = 1.990$ accept at 1% and 5%

(-1.804)	(0.706)
accept at 1% and 5%	accept at 1% and 5%

Eqn 2.3.1.

i) (a) $\ln M_2 = -7.183 - 0.776 \ln r + 1.270 \ln W$ $R^2 = 0.905$ (43)

$d = 0.283$ reject at 1% and 5%

(b) $\ln M_2 = 0.161 - 0.415 \ln r + 0.877 \ln W$ $R^2 = 0.583$

$d = 1.125$ reject at 1% and 5%

(-4.903) (6.488) (-0.910)

reject at 1% reject at 1% accept at 1% and 5%

---

ii) (a) $\ln M_2 = -7.544 - 0.707 \ln r + 1.281 \ln W$ $R^2 = 0.915$ (38)

$d = 0.447$ reject at 1% and 5%

(b) $\ln M_2 = -0.510 - 0.638 \ln r + 1.047 \ln W$ $R^2 = 0.630$

$d = 1.949$ accept at 1% and 5%

(-5.244) (6.347) (0.282)

reject at 1% reject at 1% accept at 1% and 5%

---

iii) (a) $F = 26.786$ (b) $F = 43.846$

reject at 1% reject at 1%

---

iv) (a) $\ln M_2 = 3.958 - 0.519 \ln r + 0.742 \ln W$ $R^2 = 0.811$ (22)

$d = 0.296$ reject at 1% and 5%

(b) $\ln M_2 = 0.079 - 0.471 \ln r + 0.889 \ln W$ $R^2 = 0.740$

$d = 1.186$ reject at 1% and 5%

(-4.947) (6.091) (0.760)

reject at 1% reject at 1% accept at 1% and 5%

---

v) (a) $\ln M_2 = 19.021 - 0.117 \ln r + 0.093 \ln W$ $R^2 = 0.190$ (16)

$d = 0.451$ reject at 1% and 5%

(b) $\ln M_2 = 1.755 - 0.152 \ln r + 0.451 \ln W$ $R^2 = 0.379$

$d = 2.437$ accept at 1% and 5%

(-2.510) (2.108)

accept at 1% accept at 1% and 5%

reject at 5%

Eqn 2.4.1.

i) (a) $\ln M_2 = 1.243 - 2.511 \ln r + 1.052 \ln W$ $R^2 = 0.939$ (43)

d = 0.514 reject at 1% and 5%

(b) $\ln M_2 = 0.527 - 1.603 \ln r + 0.942 \ln W$ $R^2 = 0.729$

d = 1.501 reject at 1%, accept at 5%

(-5.613)	(7.969)	(-0.495)
reject at 1%	reject at 1%	accept at 1% and 5%

---

ii) (a) $\ln M_2 = 0.484 - 2.366 \ln r + 1.074 \ln W$ $R^2 = 0.946$ (38)

d = 0.867 reject at 1% and 5%

(b) $\ln M_2 = 0.406 - 2.126 \ln r + 1.034 \ln W$ $R^2 = 0.826$

d = 2.111 accept at 1% and 5%

(-6.238)	(8.971)	(0.291)
reject at 1%	reject at 1%	accept at 1% and 5%

---

iii) (a) F = 42.341 (b) F = 42.562

reject at 1% reject at 1%

---

iv) (a) $\ln M_2 = 12.856 - 2.200 \ln r + 0.492 \ln W$ $R^2 = 0.925$ (22)

d = 1.170 reject at 1% and 5%

(b) $\ln M_2 = 5.553 - 1.901 \ln r + 0.601 \ln W$ $R^2 = 0.863$

d = 1.805 accept at 1% and 5%

(-5.982)	(4.447)	(-2.948)
reject at 1%	reject at 1%	reject at 1%

---

v) (a) $\ln M_2 = 19.718 - 0.365 \ln r + 0.084 \ln W$ $R^2 = 0.166$ (16)

d = 0.400 reject at 1% and 5%

(b) $\ln M_2 = 1.233 - 0.484 \ln r + 0.555 \ln W$ $R^2 = 0.478$

d = 2.695 accept at 1% and 5%

(-3.113)	(2.492)	(-1.996)
reject at 1%	accept at 1% reject at 5%	reject at 1%

Eqn 2.5.1.

i) (a) $\ln M_2 = 3.555 - 3.284 \ln r + 0.943 \ln W$ $R^2 = 0.977$ (42)

d = 0.698 reject at 1% and 5%

(b) $\ln M_2 = 0.505 - 2.964 \ln r + 1.017 \ln W$ $R^2 = 0.901$

d = 1.566 reject at 1%, accept at 5%

(-10.674) (12.393) (0.203)

reject at 1% reject at 1% accept at 1% and 5%

---

ii) (a) $\ln M_2 = 2.898 - 3.144 \ln r + 0.964 \ln W$ $R^2 = 0.978$ (37)

d = 0.896 reject at 1% and 5%

(b) $\ln M_2 = 0.894 - 3.171 \ln r + 1.003 \ln W$ $R^2 = 0.935$

d = 1.818 accept at 1% and 5%

(-12.112) (13.093) (0.044)

reject at 1% reject at 1% accept at 1% and 5%

---

iii) (a) F = 10.023 (b) F = 8.714

reject at 1% reject at 1%

---

iv) (a) $\ln M_2 = 10.100 - 3.419 \ln r + 0.652 \ln W$ $R^2 = 0.925$ (21)

d = 1.364 reject at 1%, accept at 5%

(b) $\ln M_2 = 3.799 - 2.839 \ln r + 0.811 \ln W$ $R^2 = 0.890$

d = 1.517 reject at 1%, accept at 5%

(-5.617) (5.816) (-1.359)

reject at 1% reject at 1% accept at 1% and 5%

---

v) (a) $\ln M_2 = 30.574 + 1.202 \ln r - 0.485 \ln W$ $R^2 = 0.218$ (16)

d = 0.748 reject at 1% and 5%

(b) $\ln M_2 = 5.941 - 0.094 \ln r + 0.105 \ln W$ $R^2 = 0.219$

d = 1.797 accept at 1% and 5%

(-0.127) (0.314)

accept at 1% and 5% accept at 1% and 5%

Eqn 2.6.1.

i) (a) $\ln M_2 = 3.794 - 2.551 \ln r + 0.888 \ln W$ $R^2 = 0.959$ (40)

$d = 0.385$ reject at 1% and 5%

(b) $\ln M_2 = 0.827 - 2.337 \ln r + 0.843 \ln W$ $R^2 = 0.693$

$d = 1.285$ reject at 1% and 5%

(-6.091) (6.872) (-1.276)

reject at 1% reject at 1% accept at 1% and 5%

---

ii) (a) $\ln M_2 = 2.774 - 2.413 \ln r + 0.925 \ln W$ $R^2 = 0.966$ (35)

$d = 0.802$ reject at 1% and 5%

(b) $\ln M_2 = 1.458 - 2.775 \ln r + 0.902 \ln W$ $R^2 = 0.882$

$d = 1.751$ accept at 1% and 5%

(-9.365) (8.554) (-0.924)

reject at 1% reject at 1% accept at 1% and 5%

---

iii) (a) $F = 57.907$ (b) $F = 46.156$

reject at 1% reject at 1%

---

iv) (a) $\ln M_2 = 16.845 - 2.956 \ln r + 0.319 \ln W$ $R^2 = 0.970$ (19)

$d = 2.363$ accept at 1% and 5%

(-12.307) (3.021) (-6.455)

reject at 1% reject at 1% reject at 1%

---

v) (a) $\ln M_2 = 29.579 + 0.932 \ln r - 0.424 \ln W$ $R^2 = 0.555$ (16)

$d = 1.088$ reject at 1% and 5%

(b) $\ln M_2 = 13.916 + 0.764 \ln r - 0.354 \ln W$ $R^2 = 0.203$

$d = 1.758$ accept at 1% and 5%

(1.747) (-1.588)

accept at 1% and 5% accept at 1% and 5%

Eqn 2.1.2.

i) (a) $\ln M_2 = -5.835 - 0.296 \ln r + 1.225 \ln Y$ $R^2 = 0.953$ (39)

d = 0.466 reject at 1% and 5%

(b) $\ln M_2 = -0.527 - 0.211 \ln r + 1.060 \ln Y$ $R^2 = 0.742$

d = 1.710 accept at 1% and 5%

(-3.428) (8.095) (0.461)

reject at 1% reject at 1% accept at 1% and 5%

---

ii) (a) $\ln M_2 = 5.827 - 0.288 \ln r + 1.224 \ln Y$ $R^2 = 0.956$ (34)

d = 0.774 reject at 1% and 5%

(b) $\ln M_2 = 2.245 - 0.247 \ln r + 1.229 \ln Y$ $R^2 = 0.846$

d = 2.086 accept at 1% and 5%

(-3.451) (10.549) (1.967)

reject at 1% reject at 1% accept at 1% and 5%

---

iii) (a) F = 43.510 (b) F = 33.696

reject at 1% reject at 1%

---

iv) (a) $\ln M_2 = 13.652 - 0.432 \ln r + 0.300 \ln Y$ $R^2 = 0.950$ (20)

d = 1.740 accept at 1% and 5%

(-7.878) (2.136) (-4.985)

reject at 1% accept at 1% reject at 1%

reject at 5%

---

v) (a) $\ln M_2 = 18.296 - 0.088 \ln r + 0.125 \ln Y$ $R^2 = 0.302$ (14)

d = 0.493 reject at 1% and 5%

(b) $\ln M_2 = 0.609 - 0.139 \ln r + 0.786 \ln Y$ $R^2 = 0.452$

d = 1.516 reject at 1%, accept at 5%

(-2.394) (2.540)

accept at 1% accept at 1%

reject at 5% reject at 5%

Eqn 2.2.2.

i) (a) $\ln M_2 = -1.463 - 0.533 \ln r + 1.034 \ln Y$ $R^2 = 0.961$ (35)

d = 0.448 reject at 1% and 5%

(b) $\ln M_2 = 0.374 - 0.460 \ln r + 0.880 \ln Y$ $R^2 = 0.788$

d = 1.850 accept at 1% and 5%

(-4.847) (5.804) (-0.790)

reject at 1% reject at 1% accept at 1% and 5%

---

ii) (a) $\ln M_2 = -1.695 - 0.515 \ln r + 1.044 \ln Y$ $R^2 = 0.961$ (30)

d = 0.690 reject at 1% and 5%

(b) $\ln M_2 = -0.202 - 0.541 \ln r + 0.994 \ln Y$ $R^2 = 0.861$

d = 2.194 accept at 1% and 5%

(-4.579) (6.051) (-0.038)

reject at 1% reject at 1% accept at 1% and 5%

---

iii) (a) F = 43.726 (b) F = 25.886

reject at 1% reject at 1%

---

iv) (a) $\ln M_2 = 14.602 - 0.654 \ln r + 0.274 \ln Y$ $R^2 = 0.963$ (20)

d = 1.534 accept at 1% and 5%

(-9.485) (2.276) (-6.045)

reject at 1% accept at 1% reject at 1%

reject at 5%

---

v) (a) $\ln M_2 = 15.406 - 0.320 \ln r + 0.262 \ln Y$ $R^2 = 0.763$ (10)

d = 1.301 reject at 1%, accept at 5%

(b) $\ln M_2 = 7.455 - 0.296 \ln r + 0.318 \ln Y$ $R^2 = 0.516$

d = 1.820 accept at 1% and 5%

(-2.115) (0.878)

accept at 1% and 5% accept at 1% and 5%

Eqn 2.3.2.

i) (a) $\ln M_2 = -5.275 - 0.591 \ln r + 1.221 \ln Y$ $R^2 = 0.965$ (43)

$d = 0.375$ reject at 1% and 5%

(b) $\ln M_2 = -0.618 - 0.413 \ln r + 1.125 \ln Y$ $R^2 = 0.813$

$d = 1.650$ accept at 1% and 5%

(-6.337)	(11.961)	(1.331)
reject at 1%	reject at 1%	accept at 1% and 5%

---

ii) (a) $\ln M_2 = -5.303 - 0.557 \ln r + 1.220 \ln Y$ $R^2 = 0.969$ (38)

$d = 0.473$ reject at 1% and 5%

(b) $\ln M_2 = -1.295 - 0.529 \ln r + 1.236 \ln Y$ $R^2 = 0.859$

$d = 2.288$ accept at 1% and 5%

(-6.988)	(12.702)	(2.428)
reject at 1%	reject at 1%	accept at 1%
		reject at 5%

---

iii) (a) $F = 11.785$ (b) $F = 31.651$

reject at 1% reject at 1%

---

iv) (a) $\ln M_2 = 0.389 - 0.459 \ln r + 0.941 \ln Y$ $R^2 = 0.890$ (22)

$d = 0.241$ reject at 1% and 5%

(b) $\ln M_2 = -0.179 - 0.477 \ln r + 1.058 \ln Y$ $R^2 = 0.872$

$d = 1.639$ accept at 1% and 5%

(-7.271)	(9.711)	(0.534)
reject at 1%	reject at 1%	accept at 1% and 5%

---

v) (a) $\ln M_2 = 18.146 - 0.125 \ln r + 0.136 \ln Y$ $R^2 = 0.197$ (16)

$d = 0.439$ reject at 1% and 5%

(b) $\ln M_2 = 1.095 - 0.134 \ln r + 0.637 \ln Y$ $R^2 = 0.467$

$d = 2.034$ accept at 1% and 5%

(-2.580)	(2.668)
accept at 1%	accept at 1%
reject at 5%	reject at 5%

Eqn 2.4.2.

i) (a) $\ln M_2 = 0.896 - 1.904 \ln r + 1.060 \ln Y \qquad R^2 = 0.937$ (43)

$d = 0.721$ reject at 1% and 5%

(b) $\ln M_2 = -0.167 - 1.415 \ln r + 1.083 \ln Y \qquad R^2 = 0.897$

$d = 1.986$ accept at 1% and 5%

(-6.461)	(14.149)	(1.084)
reject at 1%	reject at 1%	accept at 1% and 5%

---

ii) (a) $\ln M_2 = 0.620 - 1.825 \ln r + 1.066 \ln Y \qquad R^2 = 0.976$ (38)

$d = 1.053$ reject at 1% and 5%

(b) $\ln M_2 = -0.183 - 1.642 \ln r + 1.096 \ln Y \qquad R^2 = 0.937$

$d = 2.244$ accept at 1% and 5%

(-7.089)	(16.083)	(1.404)
reject at 1%	reject at 1%	accept at 1% and 5%

---

iii) (a) $F = 17.035$ (b) $F = 22.056$

reject at 1% reject at 1%

---

iv) (a) $\ln M_2 = 9.726 - 1.936 \ln r + 0.640 \ln Y \qquad R^2 = 0.940$ (22)

$d = 1.173$ reject at 1% and 5%

(b) $\ln M_2 = 3.722 - 1.635 \ln r + 0.761 \ln Y \qquad R^2 = 0.889$

$d = 2.060$ accept at 1% and 5%

(-5.318)	(5.354)	(-1.682)
reject at 1%	reject at 1%	accept at 1% and 5%

---

v) (a) $\ln M_2 = 18.977 - 0.391 \ln r + 0.123 \ln Y \qquad R^2 = 0.171$

$d = 0.391$ reject at 1% and 5%

(b) $\ln M_2 = 0.587 - 0.411 \ln r + 0.775 \ln Y \qquad R^2 = 0.571$

$d = 2.290$ accept at 1% and 5%

(-3.189)	(3.180)	(-0.924)
reject at 1%	reject at 1%	accept at 1% and 5%

Eqn 2.5.2.

i) (a) $\ln M_2 = 2.934 - 2.524 \ln r + 0.962 \ln Y$ $R^2 = 0.986$ (42)

d = 0.737 reject at 1% and 5%

(b) $\ln M_2 = 0.470 - 2.298 \ln r + 1.024 \ln Y$ $R^2 = 0.943$

d = 1.559 reject at 1%, accept at 5%

(-10.128)	(16.822)	(0.399)
reject at 1%	reject at 1%	accept at 1% and 5%

---

ii) (a) $\ln M_2 = 2.577 - 2.430 \ln r + 0.937 \ln Y$ $R^2 = 0.986$ (37)

d = 0.832 reject at 1% and 5%

(b) $\ln M_2 = 0.643 - 2.396 \ln r + 1.017 \ln Y$ $R^2 = 0.955$

d = 1.651 accept at 1% and 5%

(-10.502)	(16.651)	(0.279)
reject at 1%	reject at 1%	accept at 1% and 5%

---

iii) (a) F = 3.899 (b) F = 3.137

accept at 1% and 5% accept at 1% and 5%

---

iv) (a) $\ln M_2 = 6.633 - 2.888 \ln r + 0.811 \ln Y$ $R^2 = 0.934$ (21)

d = 1.141 reject at 1% and 5%

(b) $\ln M_2 = 1.685 - 2.338 \ln r + 0.941 \ln Y$ $R^2 = 0.886$

d = 1.498 reject at 1%, accept at 5%

(-1.685)	(-2.338)	(0.941)
accept at 1% and 5%	accept at 1%	accept at 1%
	reject at 5%	accept at 5%

---

v) (a) $\ln M_2 = 34.638 + 1.377 \ln r - 0.696 \ln Y$ $R^2 = 0.266$ (16)

d = 0.873 reject at 1% and 5%

(b) $\ln M_2 = 5.041 - 0.615 \ln r + 0.389 \ln Y$ $R^2 = 0.060$

d = 1.467 reject at 1%, accept at 5%

(-0.688)	(0.813)
accept at 1% and 5%	accept at 1% and 5%

Eqn 2.6.2.

i) (a) $\ln M_2 = 3.112 - 1.975 \ln r + 0.920 \ln Y$ $R^2 = 0.976$ (40)

$d = 0.421$ reject at 1% and 5%

(b) $\ln M_2 = 0.507 - 1.917 \ln r + 0.943 \ln Y$ $R^2 = 0.825$

$d = 1.491$ reject at 1%, accept at 5%

(-6.481)	(10.165)	(-0.609)
reject at 1%	reject at 1%	accept at 1% and 5%

---

ii) (a) $\ln M_2 = 2.654 - 1.885 \ln r + 0.935 \ln Y$ $R^2 = 0.979$ (35)

$d = 0.723$ reject at 1% and 5%

(b) $\ln M_2 = 0.872 - 2.132 \ln r + 0.958 \ln Y$ $R^2 = 0.919$

$d = 1.755$ accept at 1% and 5%

(-8.143)	(11.432)	(-0.506)
reject at 1%	reject at 1%	accept at 1% and 5%

---

iii) (a) $F = 34.812$ (b) $F = 20.699$

reject at 1% reject at 1%

---

iv) (a) $\ln M_2 = 15.308 - 2.774 \ln r + 0.394 \ln Y$ $R^2 = 0.971$ (19)

$d = 2.083$ accept at 1% and 5%

(-10.062)	(3.201)	(-4.928)
reject at 1%	reject at 1%	reject at 1%

---

v) (a) $\ln M_2 = 33.075 + 1.049 \ln r - 0.604 \ln Y$ $R^2 = 0.604$ (16)

$d = 1.365$ reject at 1%, accept at 5%

(b) $\ln M_2 = 21.009 + 1.010 \ln r - 0.595 \ln Y$ $R^2 = 0.324$

$d = 2.047$ accept at 1% and 5%

(2.391)	(-2.328)	(1.585)
accept at 1%	accept at 1%	accept at 1% and 5%
reject at 5%	reject at 5%	

Eqn 2.1.3.

i) (a) $\ln M_2 = -7.040 - 0.318 \ln r + 1.283 \ln Y_p$ $R^2 = 0.966$ (37)

d = 0.425 reject at 1% and 5%

(b) $\ln M_2 = -1.079 - 0.248 \ln r + 1.202 \ln Y_p$ $R^2 = 0.759$

d = 1.577 reject at 1%, accept at 5%

(-4.771) (8.591) (1.442)

reject at 1% reject at 1% accept at 1% and 5%

---

ii) (a) $\ln M_2 = -6.970 - 0.315 \ln r + 1.280 \ln Y_p$ $R^2 = 0.970$ (32)

d = 0.841 reject at 1% and 5%

(b) $\ln M_2 = -3.230 - 0.295 \ln r + 1.329 \ln Y_p$ $R^2 = 0.892$

d = 1.961 accept at 1% and 5%

(-4.978) (12.979) (3.212)

reject at 1% reject at 1% reject at 1%

---

iii) (a) F = 37.894 (b) F = 32.920

reject at 1% reject at 1%

---

iv) (a) $\ln M_2 = 9.330 - 0.399 \ln r + 0.504 \ln Y_p$ $R^2 = 0.971$ (18)

d = 2.326 accept at 1% and 5%

(-9.517) (4.061) (-3.992)

reject at 1% reject at 1% reject at 1%

---

v) (a) $\ln M_2 = 20.938 - 0.054 \ln r + 0.004 \ln Y_p$ $R^2 = 0.296$ (14)

d = 0.551 reject at 1% and 5%

(b) $\ln M_2 = 1.415 - 0.142 \ln r + 0.628 \ln Y_p$ $R^2 = 0.305$

d = 1.584 accept at 1% and 5%

(-2.009) (1.765)

accept at 1% and 5% accept at 1% and 5%

Eqn 2.2.3.

i) (a) $\ln M_2 = -3.120 - 0.532 \ln r + 1.113 \ln Y_p$ $R^2 = 0.973$ (33)

$d = 0.459$ reject at 1% and 5%

(b) $\ln M_2 = -0.835 - 0.459 \ln r + 1.144 \ln Y_p$ $R^2 = 0.826$

$d = 1.933$ accept at 1% and 5%

(-5.621)	(7.435)	(0.934)
reject at 1%	reject at 1%	accept at 1% and 5%

---

ii) (a) $\ln M_2 = -3.225 - 0.520 \ln r + 1.117 \ln Y_p$ $R^2 = 0.975$ (28)

$d = 0.782$ reject at 1% and 5%

(b) $\ln M_2 = -1.692 - 0.539 \ln r + 1.177 \ln Y_p$ $R^2 = 0.914$

$d = 2.218$ accept at 1% and 5%

(-5.652)	(8.533)	(1.286)
reject at 1%	reject at 1%	accept at 1% and 5%

---

iii) (a) $F = 36.254$ (b) $F = 23.085$

reject at 1% reject at 1%

---

iv) (a) $\ln M_2 = 10.818 - 0.608 \ln r + 0.451 \ln Y_p$ $R^2 = 0.977$ (18)

$d = 2.176$ accept at 1% and 5%

(-10.783)	(3.949)	(-4.805)
reject at 1%	reject at 1%	reject at 1%

---

v) (a) $\ln M_2 = 17.134 - 0.287 \ln r + 0.183 \ln Y_p$ $R^2 = 0.751$ (10)

$d = 1.280$ reject at 1% and 5%

(b) $\ln M_2 = 7.714 - 0.286 \ln r + 0.280 \ln Y_p$ $R^2 = 0.477$

$d = 1.825$ accept at 1% and 5%

(-1.683)	(0.613)
accept at 1% and 5%	accept at 1% and 5%

Eqn 2.3.3.

i) (a) $\ln M_2 = -6.349 - 0.609 \ln r + 1.273 \ln Y_p$ $R^2 = 0.970$ (41)

$d = 0.496$ reject at 1% and 5%

(b) $\ln M_2 = -2.024 - 0.444 \ln r + 1.361 \ln Y_p$ $R^2 = 0.844$

$d = 1.749$ accept at 1% and 5%

(-6.530)	(13.426)	(3.561)
reject at 1%	reject at 1%	reject at 1%

---

ii) (a) $\ln M_2 = -6.365 - 0.577 \ln r + 1.272 \ln Y_p$ $R^2 = 0.974$ (36)

$d = 0.610$ reject at 1% and 5%

(b) $\ln M_2 = -2.721 - 0.550 \ln r + 1.403 \ln Y_p$ $R^2 = 0.895$

$d = 2.266$ accept at 1% and 5%

(-7.395)	(15.222)	(4.369)
reject at 1%	reject at 1%	reject at 1%

---

iii) (a) $F = 9.002$ (b) $F = 33.701$

reject at 1% reject at 1%

---

iv) (a) $\ln M_2 = -3.984 - 0.439 \ln r + 1.149 \ln Y_p$ $R^2 = 0.905$ (20)

$d = 0.515$ reject at 1% and 5%

(b) $\ln M_2 = -4.463 - 0.537 \ln r + 1.928 \ln Y_p$ $R^2 = 0.900$

$d = 2.080$ accept at 1% and 5%

(-7.984)	(10.939)	(5.266)
reject at 1%	reject at 1%	reject at 1%

---

v) (a) $\ln M_2 = 18.293 - 0.122 \ln r + 0.130 \ln Y_p$ $R^2 = 0.192$ (16)

$d = 0.443$ reject at 1% and 5%

(b) $\ln M_2 = 0.930 - 0.169 \ln r + 0.691 \ln Y_p$ $R^2 = 0.441$

$d = 1.918$ accept at 1% and 5%

(-2.843)	(2.496)	(-1.115)
accept at 1%	accept at 1%	accept at 1% and 5%
reject at 5%	reject at 5%	

Eqn 2.4.3.

i) (a) $\ln M_2 = 0.180 - 1.918 \ln r + 1.095 \ln Y_p$ $R^2 = 0.976$ (41)

$d = 0.793$ reject at 1% and 5%

(b) $\ln M_2 = -0.889 - 1.583 \ln r + 1.182 \ln Y_p$ $R^2 = 0.918$

$d = 2.181$ accept at 1% and 5%

(-7.350)	(15.781)	(2.430)
reject at 1%	reject at 1%	accept at 1%
		reject at 5%

---

ii) (a) $\ln M_2 = 0.077 - 1.844 \ln r + 1.101 \ln Y_p$ $R^2 = 0.979$ (36)

$d = 1.177$ reject at 1% and 5%

(b) $\ln M_2 = -0.732 - 1.794 \ln r + 1.151 \ln Y_p$ $R^2 = 0.954$

$d = 2.439$ accept at 1% and 5%

(-8.607)	(18.949)	(2.491)
reject at 1%	reject at 1%	accept at 1%
		reject at 5%

---

iii) (a) $F = 10.370$ (b) $F = 13.889$

reject at 1%   reject at 1%

---

iv) (a) $\ln M_2 = 7.636 - 1.889 \ln r + 0.736 \ln Y_p$ $R^2 = 0.935$ (20)

$d = 1.385$ reject at 1%, accept at 5%

(b) $\ln M_2 = 2.073 - 1.722 \ln r + 0.936 \ln Y_p$ $R^2 = 0.898$

$d = 2.388$ accept at 1% and 5%

(-4.979)	(4.919)	(-0.337)
reject at 1%	reject at 1%	accept at 1% and 5%

---

v) (a) $\ln M_2 = 19.063 - 0.383 \ln r + 0.118 \ln Y_p$ $R^2 = 0.166$ (16)

$d = 0.396$ reject at 1% and 5%

(b) $\ln M_2 = 0.467 - 0.512 \ln r + 0.828 \ln Y_p$ $R^2 = 0.535$

$d = 2.296$ accept at 1% and 5%

(-3.451)	(2.901)	(-0.602)
reject at 1%	accept at 1%	accept at 1% and 5%
	reject at 5%	

Eqn 2.5.3.

i) (a) $\ln M_2 = 2.419 - 2.510 \ln r + 0.987 \ln Y_p$ $R^2 = 0.986$ (41)

d = 0.986 reject at 1% and 5%

(b) $\ln M_2 = 0.599 - 2.305 \ln r + 1.030 \ln Y_p$ $R^2 = 0.955$

d = 1.875 accept at 1% and 5%

(-10.565) (17.161) (0.493)

reject at 1% reject at 1% accept at 1% and 5%

---

ii) (a) $\ln M_2 = 2.077 - 2.422 \ln r + 0.997 \ln Y_p$ $R^2 = 0.986$ (36)

d = 1.061 reject at 1% and 5%

(b) $\ln M_2 = 0.716 - 2.337 \ln r + 1.025 \ln Y_p$ $R^2 = 0.962$

d = 1.907 accept at 1% and 5%

(-10.251) (16.846) (0.406)

reject at 1% reject at 1% accept at 1% and 5%

---

iii) (a) F = 3.478 (b) F = 2.644

accept at 1% and 5% accept at 1% and 5%

---

iv) (a) $\ln M_2 = 4.237 - 2.821 \ln r + 0.921 \ln Y_p$ $R^2 = 0.933$ (20)

d = 1.503 reject at 1%, accept at 5%

(b) $\ln M_2 = 2.220 - 2.577 \ln r + 0.961 \ln Y_p$ $R^2 = 0.890$

d = 1.871 accept at 1% and 5%

(-4.543) (4.882) (-0.200)

reject at 1% reject at 1% accept at 1% and 5%

---

v) (a) $\ln M_2 = 37.179 + 1.645 \ln r - 0.827 \ln Y_p$ $R^2 = 0.285$ (16)

d = 0.931 reject at 1% and 5%

(b) $\ln M_2 = 8.785 + 0.100 \ln r - 0.019 \ln Y_p$ $R^2 = 0.060$

d = 1.509 reject at 1%, accept at 5%

(0.120) (-0.043)

accept at 1% and 5% accept at 1% and 5%

Eqn 2.6.3.

i) (a) $\ln M_2 = 2.948 - 1.975 \ln r + 0.929 \ln Y_p$ $R^2 = 0.975$ (40)

$d = 0.562$ reject at 1% and 5%

(b) $\ln M_2 = 0.107 - 1.894 \ln r + 1.042 \ln Y_p$ $R^2 = 0.857$

$d = 1.798$ accept at 1% and 5%

(-6.603) (10.225) (0.412)

reject at 1% reject at 1% accept at 1% and 5%

---

ii) (a) $\ln M_2 = 2.551 - 1.894 \ln r + 0.942 \ln Y_p$ $R^2 = 0.978$ (35)

$d = 0.840$ reject at 1% and 5%

(b) $\ln M_2 = 0.609 - 2.018 \ln r + 0.998 \ln Y_p$ $R^2 = 0.924$

$d = 1.906$ accept at 1% and 5%

(-7.617) (11.268) (-0.017)

reject at 1% reject at 1% accept at 1% and 5%

---

iii) (a) $F = 29.460$ (b) $F = 23.051$

reject at 1% reject at 1%

---

iv) (a) $\ln M_2 = 16.922 - 2.938 \ln r + 0.329 \ln Y_p$ $R^2 = 0.963$ (19)

$d = 2.610$ accept at 1% and 5%

(-9.125) (2.103) (-4.294)

reject at 1% accept at 1% and 5% reject at 1%

---

v) (a) $\ln M_2 = 33.330 + 1.058 \ln r - 0.617 \ln Y_p$ $R^2 = 0.633$ (16)

$d = 1.305$ reject at 1%, accept at 5%

(b) $\ln M_2 = 20.271 + 1.031 \ln r - 0.619 \ln Y_p$ $R^2 = 0.362$

$d = 2.070$ accept at 1% and 5%

(2.599) (-2.531) (1.558)

accept at 1% accept at 1% accept at 1% and 5%

reject at 5% reject at 5%

Eqn 3.1.1.

i) (a) $\ln L_t = -10.702 - 1.183 \ln r + 1.348 \ln W$ $R^2 = 0.569$ (38)

d = 0.525 reject at 1% and 5%

(b) $\ln L_t = 2.039 - 1.092 \ln r + 0.501 \ln W$ $R^2 = 0.173$

d = 2.164 accept at 1% and 5%

(-2.549) (0.547)

accept at 1% accept at 1% and 5%

reject at 5%

---

ii) (a) $\ln L_t = -11.273 - 1.157 \ln r + 1.373 \ln W$ $R^2 = 0.540$ (33)

d = 0.573 reject at 1% and 5%

(b) $\ln L_t = 1.756 - 1.157 \ln r + 0.576$ $R^2 = 0.180$

d = 2.181 accept at 1% and 5%

(-2.374) (0.593)

accept at 1% accept at 1% and 5%

reject at 5%

---

iii) (a) F = 39.899 F = 18.532

reject at 1% reject at 1%

---

iv) (a) $\ln L_t = 141.895 - 2.970 \ln r - 5.557 \ln W$ $R^2 = 0.793$ (19)

d = 2.469 accept at 1% and 5%

(-7.815) (-6.115) (-5.015)

reject at 1% reject at 1% reject at 1%

---

v) (a) $\ln L_t = 38.095 - 0.267 \ln r - 0.796 \ln W$ $R^2 = 0.957$ (14)

d = 1.518 reject at 1%, accept at 5%

(b) $\ln L_t = 31.644 - 0.194 \ln r - 1.099 \ln W$ $R^2 = 0.928$

d = 2.121 accept at 1% and 5%

(-1.147) (-2.285) (-0.206)

accept at 1% and 5% accept at 1% accept at 1% and 5%

reject at 5%

Eqn 3.2.1.

i) (a) $\ln L_t = 29.666 - 2.604 \ln r - 0.404 \ln W$ $R^2 = 0.667$ (34)

d = 0.547 reject at 1% and 5%

(b) $\ln L_t = 9.220 - 2.467 \ln r - 0.664 \ln W$ $R^2 = 0.335$

d = 2.326 accept at 1% and 5%

(-3.781) (-0.602)

reject at 1% accept at 1% and 5%

---

ii) (a) $\ln L_t = 30.236 - 2.646 \ln r - 0.428 \ln W$ $R^2 = 0.631$ (29)

d = 0.542 reject at 1% and 5%

(b) $\ln L_t = 10.156 - 2.660 \ln r - 0.867 \ln W$ $R^2 = 0.364$

d = 2.362 accept at 1% and 5%

(-3.734) (-0.721)

reject at 1% accept at 1% and 5%

---

iii) (a) F = 36.051 (b) F = 20.122

reject at 1% reject at 1%

---

iv) (a) $\ln L_t = 155.667 - 4.671 \ln r - 6.053 \ln W$ $R^2 = 0.849$ (19)

d = 2.730 accept at 1% and 5%

(-9.492) (-7.572) (-6.321)

reject at 1% reject at 1% reject at 1%

---

v) (a) $\ln L_t = 22.659 - 0.769 \ln r - 0.100 \ln W$ $R^2 = 0.928$ (10)

d = 1.737 accept at 1% and 5%

(-2.383) (-0.173)

accept at 1% accept at 1% and 5%

reject at 5%

Eqn 3.3.1.

i) (a) $\ln L_t = -10.853 - 2.265 \ln r + 1.436 \ln W$ $R^2 = 0.594$ (42)

d = 0.342 reject at 1% and 5%

(b) $\ln L_t = 0.440 - 2.582 \ln r + 0.839 \ln W$ $R^2 = 0.426$

d = 2.021 accept at 1% and 5%

(-5.309) (1.138)

reject at 1% accept at 1% and 5%

---

ii) (a) $\ln L_t = -11.380 - 2.150 \ln r + 1.452 \ln W$ $R^2 = 0.571$ (37)

d = 0.351 reject at 1% and 5%

(b) $\ln L_t = 0.082 - 2.914 \ln r + 0.952 \ln W$ $R^2 = 0.497$

d = 1.972 accept at 1% and 5%

(-5.711) (1.237)

reject at 1% accept at 1% and 5%

---

iii) (a) F = 8.434 (b) F = 5.509

reject at 1% reject at 1%

---

iv) (a) $\ln L_t = 67.572 - 3.350 \ln r - 2.093 \ln W$ $R^2 = 0.447$ (21)

d = 0.480 reject at 1% and 5%

(b) $\ln L_t = 2.880 - 3.771 \ln r + 0.361 \ln W$ $R^2 = 0.602$

d = 1.776 accept at 1% and 5%

(-5.062) (0.326)

reject at 1% accept at 1% and 5%

---

v) (a) $\ln L_t = 49.907 - 0.172 \ln r - 1.314 \ln W$ $R^2 = 0.962$ (16)

d = 1.567 accept at 1% and 5%

(-1.160) (-6.008) (-1.436)

accept at 1% and 5% reject at 1% accept at 1% and 5%

<u>Eqn 3.4.1.</u>

i) (a) $\ln L_t = 14.418 - 7.478 \ln r + 0.782 \ln W$ $R^2 = 0.675$ (42)

d = 0.609 reject at 1% and 5%

(b) $\ln L_t = 4.680 - 6.442 \ln r + 0.638 \ln W$ $R^2 = 0.320$

d = 2.345 accept at 1% and 5%

(-3.953) (1.016)

reject at 1% accept at 1% and 5%

---

ii) (a) $\ln L_t = 13.722 - 7.347 \ln r + 0.802 \ln W$ $R^2 = 0.652$ (37)

d = 0.622 reject at 1% and 5%

(b) $\ln L_t = 5.220 - 6.914 \ln r + 0.603 \ln W$ $R^2 = 0.335$

d = 2.415 accept at 1% and 5%

(-3.826) (0.895)

reject at 1% accept at 1% and 5%

---

iii) (a) F = 16.376 (b) F = 4.354

reject at 1% accept at 1%, reject at 5%

---

iv) (a) $\ln L_t = 121.643 - 13.654 \ln r - 3.597 \ln W$ $R^2 = 0.685$ (21)

d = 1.601 accept at 1% and 5%

(-6.250) (-3.996) (-2.885)

reject at 1% reject at 1% reject at 1%

---

v) (a) $\ln L_t = 44.694 - 1.352 \ln r - 0.993 \ln W$ $R^2 = 0.976$ (16)

d = 1.811 accept at 1% and 5%

(-3.227) (-5.373) (0.379)

reject at 1% reject at 1% accept at 1% and 5%

Eqn 3.5.1.

i) (a) $\ln L_t = 19.001 - 9.579 \ln r + 0.547 \ln W$ $R^2 = 0.729$ (41)

d = 0.636 reject at 1% and 5%

(b) $\ln L_t = 5.763 - 10.256 \ln r - 0.602 \ln W$ $R^2 = 0.426$

d = 1.652 accept at 1% and 5%

(-4.859) (0.967)

reject at 1% accept at 1% and 5%

---

ii) (a) $\ln L_t = 18.926 - 9.553 \ln r + 0.549 \ln W$ $R^2 = 0.707$ (36)

d = 0.633 reject at 1% and 5%

(b) $\ln L_t = 5.330 - 9.906 \ln r + 0.634 \ln W$ $R^2 = 0.439$

d = 1.666 accept at 1% and 5%

(-4.677) (0.939)

reject at 1% accept at 1% and 5%

---

iii) (a) F = 9.857 (b) F = 3.134

reject at 1% accept at 1%, reject at 5%

---

iv) (a) $\ln L_t = 106.268 - 21.384 \ln r - 2.675 \ln W$ $R^2 = 0.650$ (20)

d = 1.144 reject at 1% and 5%

(b) $\ln L_t = 49.358 - 19.179 \ln r - 2.128 \ln W$ $R^2 = 0.481$

d = 1.579 accept at 1% and 5%

(-3.801) (-1.520)

reject at 1% accept at 1% and 5%

---

v) (a) $\ln L_t = 33.543 - 3.221 \ln r - 0.430 \ln W$ $R^2 = 0.974$ (16)

d = 2.336 accept at 1% and 5%

(-2.913) (-1.103)

accept at 1% accept at 1% and 5%

reject at 5%

Eqn 3.6.1.

i) (a) $\ln L_t = 17.592 - 7.480 \ln r + 0.482 \ln W$ $R^2 = 0.722$ (39)

$d = 0.645$ reject at 1% and 5%

(b) $\ln L_t = 6.690 - 9.236 \ln r + 0.412 \ln W$ $R^2 = 0.407$

$d = 1.978$ accept at 1% and 5%

(-4.665)	(0.609)
reject at 1%	accept at 1% and 5%

---

ii) (a) $\ln L_t = 16.207 - 7.289 \ln r + 0.532 \ln W$ $R^2 = 0.703$ (34)

$d = 0.657$ reject at 1% and 5%

(b) $\ln L_t = 6.840 - 9.457 \ln r + 0.413 \ln W$ $R^2 = 0.433$

$d = 1.976$ accept at 1% and 5%

(-4.555)	(0.568)
reject at 1%	accept at 1% and 5%

---

iii) (a) $F = 17.748$ (b) $F = 5.719$

reject at 1% reject at 1%

---

iv) (a) $\ln L_t = 188.112 - 20.925 \ln r - 6.402 \ln W$ $R^2 = 0.752$ (18)

$d = 2.250$ accept at 1% and 5%

(-6.641)	(-4.634)	(-3.909)
reject at 1%	reject at 1%	reject at 1%

---

v) (a) $\ln L_t = 49.474 - 0.734 \ln r - 1.265 \ln W$ $R^2 = 0.963$ (16)

$d = 1.677$ accept at 1% and 5%

(-1.367)	(-5.666)	(-1.187)
accept at 1% and 5%	reject at 1%	accept at 1% and 5%

Eqn 3.1.2.

i) (a) $\ln L_t = -13.569 - 1.004 \ln r + 1.530 \ln Y$ $R^2 = 0.628$ (38)

d = 0.611 reject at 1% and 5%

(b) $\ln L_t = -0.975 - 1.019 \ln r + 1.047 \ln Y$ $R^2 = 0.235$

d = 2.255 accept at 1% and 5%

(-2.451) (1.337)

accept at 1% accept at 1% and 5%

reject at 5%

---

ii) (a) $\ln L_t = -13.556 - 0.988 \ln r + 1.529 \ln Y$ $R^2 = 0.600$ (33)

d = 0.641 reject at 1% and 5%

(b) $\ln L_t = -2.159 - 1.023 \ln r + 1.219 \ln Y$ $R^2 = 0.245$

d = 2.297 accept at 1% and 5%

(-2.143) (1.464)

accept at 1% accept at 1% and 5%

reject at 5%

---

iii) (a) F = 33.719 (b) F = 14.594

reject at 1% reject at 1%

---

iv) (a) $\ln L_t = 175.412 - 3.636 \ln r - 7.372 \ln Y$ $R^2 = 0.791$ (19)

d = 2.155 accept at 1% and 5%

(-7.717) (-6.079) (-5.253)

reject at 1% reject at 1% reject at 1%

---

v) (a) $\ln L_t = 45.438 - 0.226 \ln r - 1.159 \ln Y$ $R^2 = 0.958$ (15)

d = 1.629 accept at 1% and 5%

(-1.293) (-1.941)

accept at 1% and 5% accept at 1% and 5%

Eqn. 3.2.2.

i) (a) $\ln L_t = 12.099 - 2.157 \ln r + 0.386 \ln Y$ $R^2 = 0.666$ (34)

$d = 0.573$ reject at 1% and 5%

(b) $\ln L_t = 7.572 - 2.461 \ln r - 0.331 \ln Y$ $R^2 = 0.341$

$d = 2.291$ accept at 1% and 5%

(-3.420) (-0.310)

reject at 1% accept at 1% and 5%

---

ii) (a) $\ln L_t = 12.156 - 2.156 \ln r + 0.384 \ln Y$ $R^2 = 0.631$ (29)

$d = 0.575$ reject at 1% and 5%

(b) $\ln L_t = 8.200 - 2.677 \ln r - 0.448 \ln Y$ $R^2 = 0.366$

$d = 2.326$ accept at 1% and 5%

(-3.273) (-0.376)

reject at 1% accept at 1% and 5%

---

iii) (a) $F = 27.289$ (b) $F = 9.539$

reject at 1% reject at 1%

---

iv) (a) $\ln L_t = 183.091 - 5.474 \ln r - 7.583 \ln Y$ $R^2 = 0.810$ (19)

$d = 1.912$ accept at 1% and 5%

(-8.200) (-6.497) (-5.827)

reject at 1% reject at 1% reject at 1%

---

v) (a) $\ln L_t = 32.153 - 0.599 \ln r - 0.541 \ln Y$ $R^2 = 0.933$ (10)

$d = 1.677$ accept at 1% and 5%

(-2.048) (-0.801)

accept at 1% and 5% accept at 1% and 5%

Eqn 3.3.2.

i) (a) $\ln L_t = -11.729 - 2.013 \ln r + 1.521 \ln Y$ $R^2 = 0.652$ (42)

d = 0.404 reject at 1% and 5%

(b) $\ln L_t = 0.124 - 2.546 \ln r + 0.952 \ln Y$ $R^2 = 0.428$

d = 2.174 accept at 1% and 5%

(-5.321)   (1.193)

reject at 1%   accept at 1% and 5%

---

ii) (a) $\ln L_t = -11.771 - 1.938 \ln r + 1.517 \ln Y$ $R^2 = 0.630$ (37)

d = 0.402 reject at 1% and 5%

(b) $\ln L_t = -1.222 - 2.810 \ln r + 1.304 \ln Y$ $R^2 = 0.521$

d = 2.072 accept at 1% and 5%

(-5.781)   (1.889)

reject at 1%   accept at 1% and 5%

---

iii) (a) F = 3.858   (b) F = 5.355

accept at 1%   reject at 1%

reject at 5%

---

iv) (a) $\ln L_t = 48.887 - 2.978 \ln r - 1.517 \ln Y$ $R^2 = 0.368$ (21)

d = 0.389 reject at 1% and 5%

(b) $\ln L_t = 0.656 - 3.873 \ln r + 0.849 \ln Y$ $R^2 = 0.620$

d = 1.959 accept at 1% and 5%

(-5.257)   (0.704)

reject at 1%   accept at 1% and 5%

---

v) (a) $\ln L_t = 58.408 - 0.135 \ln r - 1.748 \ln Y$ $R^2 = 0.964$ (16)

d = 1.746 accept at 1% and 5%

(-0.925)   (-6.319)   (-2.704)

accept at 1% and 5%   reject at 1%   accept at 1% and 5%

Eqn 3.4.2.

i) (a) $\ln L_t = 10.732 - 6.767 \ln r + 0.926 \ln Y$ $R^2 = 0.698$ (42)

d = 0.652 reject at 1% and 5%

(b) $\ln L_t = 3.621 - 6.128 \ln r + 0.841 \ln Y$ $R^2 = 0.355$

d = 2.380 accept at 1% and 5%

(-3.757)   (1.432)

reject at 1%   accept at 1% and 5%

---

ii) (a) $\ln L_t = 10.386 - 6.665 \ln r + 0.933 \ln Y$ $R^2 = 0.676$ (37)

d = 0.658 reject at 1% and 5%

(b) $\ln L_t = 3.523 - 6.445 \ln r + 0.881 \ln Y$ $R^2 = 0.369$

d = 2.443 accept at 1% and 5%

(-3.542)   (1.399)

reject at 1%   accept at 1% and 5%

---

iii) (a) F = 10.558   (b) F = 3.378

reject at 1%   accept at 1%

reject at 5%

---

iv) (a) $\ln L_t = 123.167 - 14.126 \ln r - 3.787 \ln Y$ $R^2 = 0.624$ (21)

d = 1.254 reject at 1% and 5%

(b) $\ln L_t = 54.935 - 11.744 \ln r - 2.489 \ln Y$ $R^2 = 0.458$

d = 2.062 accept at 1% and 5%

(-3.704)   (-1.718)

reject at 1%   accept at 1% and 5%

---

v) (a) $\ln L_t = 50.866 - 1.274 \ln r - 1.314 \ln Y$ $R^2 = 0.978$ (16)

d = 1.753 accept at 1% and 5%

(-3.102)   (-5.670)   (-1.355)

reject at 1%   reject at 1%   accept at 1% and 5%

Eqn 3.5.2.

i) (a) $\ln L_t = 16.484 - 8.920 \ln r + 0.646 \ln Y$ $R^2 = 0.737$ (41)

$d = 0.656$ reject at 1% and 5%

(b) $\ln L_t = 5.461 - 9.783 \ln r + 0.668 \ln Y$ $R^2 = 0.440$

$d = 1.684$ accept at 1% and 5%

(-4.526) (1.137)

reject at 1% accept at 1% and 5%

---

ii) (a) $\ln L_t = 16.392 - 8.891 \ln r + 0.648 \ln Y$ $R^2 = 0.715$ (36)

$d = 0.656$ reject at 1% and 5%

(b) $\ln L_t = 5.297 - 9.388 \ln r + 0.661 \ln Y$ $R^2 = 0.450$

$d = 1.697$ accept at 1% and 5%

(-4.224) (1.033)

reject at 1% accept at 1% and 5%

---

iii) (a) $F = 7.603$ (b) $F = 2.361$

reject at 1% accept at 1% and 5%

---

iv) (a) $\ln L_t = 109.804 - 22.448 \ln r - 2.891 \ln Y$ $R^2 = 0.616$ (20)

$d = 1.122$ reject at 1% and 5%

(b) $\ln L_t = 41.745 - 18.393 \ln r - 1.688 \ln Y$ $R^2 = 0.447$

$d = 1.452$ reject at 1%, accept at 5%

(-3.459) (-1.121)

reject at 1% accept at 1% and 5%

---

v) (a) $\ln L_t = 37.308 - 3.053 \ln r - 0.626 \ln Y$ $R^2 = 0.947$ (16)

$d = 2.476$ accept at 1% and 5%

(-2.527) (-1.147)

accept at 1% accept at 1% and 5%

reject at 5%

Eqn 3.6.2.

i) (a) $\ln L_t = 14.249 - 6.923 \ln r + 0.623 \ln Y \quad R^2 = 0.730 \quad (39)$

$d = 0.670$ reject at 1% and 5%

(b) $\ln L_t = 6.355 - 8.836 \ln r + 0.490 \ln Y \quad R^2 = 0.422$

$d = 1.995$ accept at 1% and 5%

(-4.377) (0.765)

reject at 1% accept at 1% and 5%

---

ii) (a) $\ln L_t = 13.268 - 6.735 \ln r + 0.656 \ln Y \quad R^2 = 0.711 \quad (34)$

$d = 0.680$ reject at 1% and 5%

(b) $\ln L_t = 6.571 - 9.008 \ln r + 0.476 \ln Y \quad R^2 = 0.444$

$d = 1.990$ accept at 1% and 5%

(-4.136) (0.684)

reject at 1% accept at 1% and 5%

---

iii) (a) $F = 10.016$ (b) $F = 2.811$

reject at 1% accept at 1% and 5%

---

iv) (a) $\ln L_t = 183.881 - 21.683 \ln r - 6.437 \ln Y \quad R^2 = 0.662 \quad (18)$

$d = 1.631$ accept at 1% and 5%

(-5.092) (-3.428) (-2.890)

reject at 1% reject at 1% reject at 1%

---

v) (a) $\ln L_t = 58.544 - 0.505 \ln r - 1.734 \ln Y \quad R^2 = 0.964 \quad (16)$

$d = 1.928$ accept at 1% and 5%

(-0.901) (-5.811) (-2.460)

accept at 1% and 5% reject at 1% accept at 1% and 5%

Eqn 3.1.3.

i) (a) $\ln L_t = -19.421 - 1.085 \ln r + 1.803 \ln Y_p \quad R^2 = 0.692$ (36)

d = 0.694 reject at 1% and 5%

(b) $\ln L_t = -6.634 - 1.073 \ln r + 1.811 \ln Y_p \quad R^2 = 0.323$

d = 2.272 accept at 1% and 5%

(-2.752)	(2.330)	(1.043)
reject at 1%	accept at 1%	accept at 1% and 5%
	reject at 5%	

---

ii) (a) $\ln L_t = -19.322 - 1.074 \ln r + 1.798 \ln Y_p \quad R^2 = 0.669$ (31)

d = 0.724 reject at 1% and 5%

(b) $\ln L_t = -7.768 - 1.079 \ln r + 1.931 \ln Y_p \quad R^2 = 0.336$

d = 2.292 accept at 1% and 5%

(-2.405)	(2.352)	(1.134)
accept at 1%	accept at 1%	accept at 1% and 5%
reject at 5%	reject at 5%	

---

iii) (a) F = 22.628 (b) F = 11.148

reject at 1% reject at 1%

---

iv) (a) $\ln L_t = 162.126 - 3.232 \ln r - 6.767 \ln Y_p \quad R^2 = 0.768$ (17)

d = 2.396 accept at 1% and 5%

(-6.652)	(-4.686)	(-3.994)
reject at 1%	reject at 1%	reject at 1%

---

v) (a) $\ln L_t = 50.192 - 0.169 \ln r - 1.378 \ln Y_p \quad R^2 = 0.963$ (14)

d = 1.795 accept at 1% and 5%

(-1.018)	(-2.398)	(-0.658)
accept at 1%	accept at 1%	accept at 1% and 5%
	reject at 5%	

Eqn 3.2.3.

i) (a) $\ln L_t = 3.372 - 2.105 \ln r + 0.790 \ln Y_p$ $R^2 = 0.717$ (32)

$d = 0.675$ reject at 1% and 5%

(b) $\ln L_t = 0.756 - 2.281 \ln r + 0.838 \ln Y_p$ $R^2 = 0.400$

$d = 2.415$ accept at 1% and 5%

(-3.233) (0.764)

reject at 1% accept at 1% and 5%

---

ii) (a) $\ln L_t = 3.292 - 2.091 \ln r + 0.793 \ln Y_p$ $R^2 = 0.687$ (27)

$d = 0.676$ reject at 1% and 3%

(b) $\ln L_t = 1.162 - 2.422 \ln r + 0.782 \ln Y_p$ $R^2 = 0.414$

$d = 2.460$ accept at 1% and 5%

(-2.990) (0.638)

reject at 1% accept at 1% and 5%

---

iii) (a) $F = 20.013$ (b) $F = 8.981$

reject at 1% reject at 1%

---

iv) (a) $\ln L_t = 178.830 - 5.035 \ln r - 7.415 \ln Y_p$ $R^2 = 0.799$ (17)

$d = 2.074$ accept at 1% and 5%

(-7.303) (-5.304) (-4.599)

reject at 1% reject at 1% reject at 1%

---

v) (a) $\ln L_t = 36.061 - 0.523 \ln r - 0.722 \ln Y_p$ $R^2 = 0.935$ (10)

$d = 1.703$ accept at 1% and 5%

(-1.542) (-0.913)

accept at 1% and 5% accept at 1% and 5%

Eqn 3.3.3.

i) (a) $\ln L_t = -16.514 - 2.128 \ln r + 1.748 \ln Y_p$ $R^2 = 0.702$ (40)

$d = 0.441$ reject at 1% and 5%

(b) $\ln L_t = -5.666 - 2.773 \ln r + 2.244 \ln Y_p$ $R^2 = 0.500$

$d = 2.196$ accept at 1% and 5%

(-5.775)	(2.812)	(1.559)
reject at 1%	reject at 1%	accept at 1% and 5%

---

ii) (a) $\ln L_t = -16.559 - 2.055 \ln r + 1.745 \ln Y_p$ $R^2 = 0.685$ (35)

$d = 0.436$ reject at 1% and 5%

(b) $\ln L_t = -5.952 - 2.998 \ln r + 2.348 \ln Y_p$ $R^2 = 0.579$

$d = 2.101$ accept at 1% and 5%

(-6.118)	(2.931)	(1.683)
reject at 1%	reject at 1%	accept at 1% and 5%

---

iii) (a) $F = 3.219$ (b) $F = 7.618$

accept at 1%	reject at 1%
reject at 5%	

---

iv) (a) $\ln L_t = 45.888 - 3.077 \ln r - 1.175 \ln Y_p$ $R^2 = 0.421$ (19)

$d = 0.411$ reject at 1% and 5%

(b) $\ln L_t = -11.833 - 4.669 \ln r + 4.521 \ln Y_p$ $R^2 = 0.708$

$d = 2.419$ accept at 1% and 5%

(-6.016)	(1.999)
reject at 1%	accept at 1% and 5%

---

v) (a) $\ln L_t = 60.702 - 0.088 \ln r - 1.857 \ln Y_p$ $R^2 = 0.969$ (16)

$d = 1.903$ accept at 1% and 5%

(-0.621)	(-6.881)	(-3.175)
accept at 1% and 5%	reject at 1%	reject at 1%

Eqn 3.4.3.

i) (a) $\ln L_t = 6.702 - 6.762 \ln r + 1.112 \ln Y_p$ $R^2 = 0.726$ (40)

$d = 0.698$ reject at 1% and 5%

(b) $\ln L_t = 0.437 - 6.817 \ln r + 1.363 \ln Y_p$ $R^2 = 0.413$

$d = 2.529$ accept at 1% and 5%

(-4.033)	(2.216)	(0.590)
reject at 1%	accept at 1% reject at 5%	accept at 1% and 5%

ii) (a) $\ln L_t = 6.193 - 6.620 \ln r + 1.124 \ln Y_p$ $R^2 = 0.706$ (35)

$d = 0.704$ reject at 1% and 5%

(b) $\ln L_t = 0.324 - 7.146 \ln r + 1.404 \ln Y_p$ $R^2 = 0.432$

$d = 2.587$ accept at 1% and 5%

(-3.795)	(2.146)	(0.618)
reject at 1%	accept at 1% reject at 5%	accept at 1% and 5%

iii) (a) $F = 10.701$ reject at 1% (b) $F = 4.550$ reject at 1%

iv) (a) $\ln L_t = 147.620 - 15.163 \ln r - 4.865 \ln Y_p$ $R^2 = 0.659$ (19)

$d = 1.226$ reject at 1%, accept at 5%

(b) $\ln L_t = 65.526 - 13.967 \ln r - 3.141 \ln Y_p$ $R^2 = 0.543$

$d = 2.512$ accept at 1% and 5%

(-4.064)	(-1.681)
reject at 1%	accept at 1% and 5%

v) (a) $\ln L_t = 52.668 - 1.127 \ln r - 1.409 \ln Y_p$ $R^2 = 0.979$ (16)

$d = 1.941$ accept at 1% and 5%

(-2.682)	(-5.880)	(1.707)
accept at 1% reject at 5%	reject at 1%	accept at 1% and 5%

Eqn 3.5.3.

i) (a) $\ln L_t = 14.432 - 8.879 \ln r + 0.738 \ln Y_p$ $R^2 = 0.748$ (40)

d = 0.674 reject at 1% and 5%

(b) $\ln L_t = 4.820 - 9.572 \ln r + 0.768 \ln Y_p$ $R^2 = 0.435$

d = 1.633 accept at 1% and 5%

(-4.357) (1.180)

reject at 1% accept at 1% and 5%

---

ii) (a) $\ln L_t = 14.182 - 8.815 \ln r + 0.746 \ln Y_p$ $R^2 = 0.728$ (35)

d = 0.674 reject at 1% and 5%

(b) $\ln L_t = 4.679 - 9.187 \ln r + 0.759 \ln Y_p$ $R^2 = 0.446$

d = 1.647 accept at 1% and 5%

(-4.041) (1.066)

reject at 1% accept at 1% and 5%

---

iii) (a) F = 8.001 (b) F = 3.053

reject at 1% accept at 1%

reject at 5%

---

iv) (a) $\ln L_t = 126.230 - 23.443 \ln r - 3.610 \ln Y_p$ $R^2 = 0.636$ (19)

d = 1.120 reject at 1% and 5%

(b) $\ln L_t = 58.714 - 20.460 \ln r - 3.115 \ln Y_p$ $R^2 = 0.478$

d = 1.483 reject at 1%, accept at 5%

(-3.581) (-1.479)

reject at 1% accept at 1% and 5%

---

v) (a) $\ln L_t = 41.691 - 2.578 \ln r - 0.852 \ln Y_p$ $R^2 = 0.976$ (16)

d = 2.481 accept at 1% and 5%

(-2.195) (-1.591)

accept at 1% accept at 1% and 5%

reject at 5%

Eqn 3.6.3.

i) (a) $\ln L_t = 14.900 - 6.986 \ln r + 0.597 \ln Y_p$ $R^2 = 0.728$ (39)

$d = 0.658$ reject at 1% and 5%

(b) $\ln L_t = 5.350 - 8.750 \ln r + 0.614 \ln Y_p$ $R^2 = 0.419$

$d = 1.986$ accept at 1% and 5%

(-4.250)   (0.864)

reject at 1%   accept at 1%

---

ii) (a) $\ln L_t = 14.032 - 6.815 \ln r + 0.627 \ln Y_p$ $R^2 = 0.709$ (34)

$d = 0.666$ reject at 1% and 5%

(b) $\ln L_t = 5.591 - 8.917 \ln r + 0.592 \ln Y_p$ $R^2 = 0.441$

$d = 1.983$ accept at 1% and 5%

(-4.000)   (-0.768)

reject at 1%   accept at 1% and 5%

---

iii) (a) $F = 23.938$ (b) $F = 9.965$

reject at 1%   reject at 1%

---

iv) (a) $\ln L_t = 243.608 - 25.510 \ln r - 9.027 \ln Y_p$ $R^2 = 0.802$ (18)

$d = 1.978$ accept at 1% and 5%

(-7.532)   (-5.544)   (-4.931)

reject at 1%   reject at 1%   reject at 1%

---

v) (a) $\ln L_t = 59.478 - 0.463 \ln r - 1.781 \ln Y_p$ $R^2 = 0.970$ (16)

$d = 2.018$ accept at 1% and 5%

(-0.909)   (-6.500)

accept at 1% and 5%   accept at 1% and 5%

Eqn 4.1.1.

i) (a) $\ln L_t^* = -13.198 - 0.698 \ln r + 1.488 \ln W$ $R^2 = 0.841$ (39)

d = 0.354 reject at 1% and 5%

(b) $\ln L_t^* = 0.824 - 0.477 \ln r + 0.672 \ln W$ $R^2 = 0.333$

d = 1.530 reject at 1%, accept at 5%

(-3.518) (1.979)

reject at 1% accept at 1% and 5%

---

ii) (a) $\ln L_t^* = -13.804 - 0.665 \ln r + 1.514 \ln W$ $R^2 = 0.843$ (34)

d = 0.558 reject at 1% and 5%

(b) $\ln L_t^* = -1.121 - 0.587 \ln r + 1.080 \ln W$ $R^2 = 0.440$

d = 1.902 accept at 1% and 5%

(-3.366) (3.028) (0.223)

reject at 1% reject at 1% accept at 1% and 5%

---

iii) (a) F = 58.323 (b) F = 27.730

reject at 1% reject at 1%

---

iv) (a) $\ln L_t^* = 32.254 - 0.992 \ln W - 0.581 \ln W$ $R^2 = 0.876$ (20)

d = 1.617 accept at 1% and 5%

(-8.800) (-2.189) (1.580)

reject at 1% accept at 1% accept at 1% and 5%

reject at 5%

---

v) (a) $\ln L_t^* = 27.680 - 0.118 \ln r - 0.312 \ln W$ $R^2 = 0.826$ (14)

d = 0.641 reject at 1% and 5%

(b) $\ln L_t^* = 4.341 - 0.196 \ln r + 0.119 \ln W$ $R^2 = 0.356$

d = 1.756 accept at 1% and 5%

(-1.915) (0.338)

accept at 1% and 5% accept at 1% and 5%

Eqn 4.2.1.

i) (a) $\ln L_t^* = 3.998 - 1.363 \ln r + 0.748 \ln W \quad R^2 = 0.893$ (35)

$d = 0.365$ reject at 1% and 5%

(b) $\ln L_t^* = 2.046 - 1.027 \ln r + 0.363 \ln W \quad R^2 = 0.529$

$d = 1.819$ accept at 1% and 5%

(-5.420) (1.034)

reject at 1% accept at 1% and 5%

---

ii) (a) $\ln L_t^* = 2.825 - 1.322 \ln r + 0.799 \ln W \quad R^2 = 0.885$ (30)

$d = 0.455$ reject at 1% and 5%

(b) $\ln L_t^* = 2.705 - 1.296 \ln r + 0.349 \ln W \quad R^2 = 0.610$

$d = 2.040$ accept at 1% and 5%

(-5.817) (0.877)

reject at 1% accept at 1% and 5%

---

iii) (a) $F = 47.231$ (b) $F = 17.571$

reject at 1% reject at 1%

---

iv) (a) $\ln L_t^* = 35.572 - 1.529 \ln r - 0.690 \ln W \quad R^2 = 0.912$ (20)

$d = 1.513$ reject at 1%, accept at 5%

(b) $\ln L_t^* = 23.044 - 1.441 \ln r - 0.513 \ln W \quad R^2 = 0.882$

$d = 2.321$ accept at 1% and 5%

(-8.748) (-1.853)

reject at 1% accept at 1% and 5%

---

v) (a) $\ln L_t^* = 21.858 - 0.486 \ln r - 0.047 \ln W \quad R^2 = 0.908$ (10)

$d = 1.460$ reject at 1%, accept at 5%

(b) $\ln L_t^* = 14.093 - 0.428 \ln r - 0.098 \ln W \quad R^2 = 0.768$

$d = 1.843$ accept at 1% and 5%

(-1.795) (-0.195)

accept at 1% and 5% accept at 1% and 5%

Eqn 4.3.1.

i) (a) $\ln L_t^* = -12.870 - 1.338 \ln r + 1.521 \ln W$ $R^2 = 0.863$ (43)

$d = 0.272$ reject at 1% and 5%

(b) $\ln L_t^* = 0.012 - 0.852 \ln r + 0.917 \ln W$ $R^2 = 0.510$

$d = 1.474$ reject at 1%, accept at 5%

(-5.821) (3.885) (-0.351)

reject at 1% reject at 1% accept at 1% and 5%

---

ii) (a) $\ln L_t^* = -13.364 - 1.239 \ln r + 1.537 \ln W$ $R^2 = 0.868$ (38)

$d = 0.338$ reject at 1% and 5%

(b) $\ln L_t^* = -0.459 - 1.133 \ln r + 1.061 \ln W$ $R^2 = 0.569$

$d = 1.974$ accept at 1% and 5%

(-6.225) (3.844) (0.221)

reject at 1% reject at 1% accept at 1% and 5%

---

iii) (a) $F = 14.531$ (b) $F = 23.785$

reject at 1% reject at 1%

---

iv) (a) $\ln L_t^* = 10.016 - 1.239 \ln r + 0.465 \ln W$ $R^2 = 0.682$ (22)

$d = 0.255$ reject at 1% and 5%

(b) $\ln L_t^* = -0.034 - 1.118 \ln r + 0.930 \ln W$ $R^2 = 0.711$

$d = 1.322$ reject at 1%, accept at 5%

(-6.334) (3.411) (-0.257)

reject at 1% reject at 1% accept at 1% and 5%

---

v) (a) $\ln L_t^* = 28.541 - 0.152 \ln r - 0.345 \ln W$ $R^2 = 0.835$ (16)

$d = 0.652$ reject at 1% and 5%

(b) $\ln L_t^* = 5.803 - 0.205 \ln r - 0.045 \ln W$ $R^2 = 0.397$

$d = 2.087$ accept at 1% and 5%

(-2.025) (-0.176)

accept at 1% and 5% accept at 1% and 5%

<u>Eqn 4.4.1.</u>

i) (a) $\ln L_t^* = 1.438 - 4.285 \ln r + 1.152 \ln W$ $R^2 = 0.910$ (43)

d = 0.565 reject at 1% and 5%

(b) $\ln L_t^* = 0.502 - 3.010 \ln r + 1.034 \ln W$ $R^2 = 0.641$

d = 1.880 accept at 1% and 5%

(-5.918) (5.071) (0.166)

reject at 1% reject at 1% accept at 1%

---

ii) (a) $\ln L_t^* = 0.404 - 4.085 \ln r + 1.182 \ln W$ $R^2 = 0.912$ (38)

d = 0.736 reject at 1% and 5%

(b) $\ln L_t^* = 0.457 - 3.548 \ln r + 1.100 \ln W$ $R^2 = 0.707$

d = 2.263 accept at 1% and 5%

(-6.064) (5.275) (0.478)

reject at 1% reject at 1% accept at 1% and 5%

---

iii) (a) F = 22.930 (b) F = 17.932

reject at 1% reject at 1%

---

iv) (a) $\ln L_t^* = 28.677 - 4.901 \ln r - 0.043 \ln W$ $R^2 = 0.846$ (22)

d = 1.063 reject at 1% and 5%

(b) $\ln L_t^* = 9.766 - 4.005 \ln r + 0.307 \ln W$ $R^2 = 0.724$

d = 1.922 accept at 1% and 5%

(-5.480) (0.985)

reject at 1% accept at 1% and 5%

---

v) (a) $\ln L_t^* = 28.347 - 0.618 \ln r - 0.298 \ln W$ $R^2 = 0.840$ (16)

d = 0.565 reject at 1% and 5%

(b) $\ln L_t^* = 4.040 - 0.775 \ln r + 0.156 \ln W$ $R^2 = 0.521$

d = 2.609 accept at 1% and 5%

(-3.197) (0.628)

reject at 1% accept at 1% and 5%

Eqn 4.5.1.

i) (a) $\ln L_t^* = 4.603 - 5.486 \ln r + 0.994 \ln W \quad R^2 = 0.954$ (42)

d = 0.644 reject at 1% and 5%

(b) $\ln L_t^* = 0.409 - 5.080 \ln r + 1.117 \ln W \quad R^2 = 0.814$

d = 1.605 reject at 1%, accept at 5%

(-9.325) (6.902) (0.721)

reject at 1% reject at 1% accept at 1% and 5%

---

ii) (a) $\ln L_t^* = 3.753 - 5.300 \ln r + 1.020 \ln W \quad R^2 = 0.952$ (37)

d = 0.664 reject at 1% and 5%

(b) $\ln L_t^* = 0.568 - 5.271 \ln r + 1.104 \ln W \quad R^2 = 0.831$

d = 1.706 accept at 1% and 5%

(-9.695) (6.393) (0.601)

reject at 1% reject at 1% accept at 1% and 5%

---

iii) (a) F = 8.398 (b) F = 2.823

reject at 1% accept at 1% and 5%

---

iv) (a) $\ln L_t^* = 22.202 - 7.642 \ln r + 0.330 \ln W \quad R^2 = 0.859$ (21)

d = 1.240 reject at 1% and 5%

(b) $\ln L_t^* = 7.589 - 6.277 \ln r + 0.669 \ln W \quad R^2 = 0.775$

d = 1.485 reject at 1%, accept at 5%

(-5.335) (2.046)

reject at 1% accept at 1% and 5%

---

v) (a) $\ln L_t^* = 38.472 + 0.808 \ln r - 0.832 \ln W \quad R^2 = 0.821$ (16)

d = 0.819 reject at 1% and 5%

(b) $\ln L_t^* = 8.856 - 0.693 \ln r - 0.160 \ln W \quad R^2 = 0.369$

d = 1.904 accept at 1% and 5%

(-0.630) (-0.333)

accept at 1% and 5% accept at 1% and 5%

Eqn 4.6.1.

i) (a) $\ln L_t^* = 4.892 - 4.297 \ln r + 0.909 \ln W$ $R^2 = 0.938$ (40)

$d = 0.433$ reject at 1% and 5%

(b) $\ln L_t^* = 1.362 - 4.354 \ln r + 0.836 \ln W$ $R^2 = 0.630$

$d = 1.552$ accept at 1% and 5%

(-6.703) (3.962) (-0.776)

reject at 1% reject at 1% accept at 1% and 5%

---

ii) (a) $\ln L_t^* = 3.386 - 4.090 \ln r + 0.963 \ln W$ $R^2 = 0.942$ (35)

$d = 0.613$ reject at 1% and 5%

(b) $\ln L_t^* = 1.799 - 4.898 \ln r + 0.888 \ln W$ $R^2 = 0.781$

$d = 1.732$ accept at 1% and 5%

(-8.647) (4.440) (-0.558)

reject at 1% reject at 1% accept at 1% and 5%

---

iii) (a) $F = 29.193$ (b) $F = 13.576$

reject at 1% reject at 1%

---

iv) (a) $\ln L_t^* = 39.047 - 6.662 \ln r - 0.492 \ln W$ $R^2 = 0.927$ (19)

$d = 1.713$ accept at 1% and 5%

(-10.509) (-1.767)

reject at 1% accept at 1% and 5%

---

v) (a) $\ln L_t^* = 40.452 + 0.973 \ln r - 0.926 \ln W$ $R^2 = 0.877$ (16)

$d = 1.214$ reject at 1% and 5%

(b) $\ln L_t = 22.220 + 0.825 \ln r - 0.886 \ln W$ $R^2 = 0.653$

$d = 1.815$ accept at 1% and 5%

(1.290) (-2.795) (3.596)

accept at 1% and 5% accept at 1% reject at 1%

reject at 5%

<u>Eqn 4.1.2.</u>

i) (a) $\ln L_t^* = -12.181 - 0.556 \ln r + 1.496 \ln Y$ $R^2 = 0.904$ (39)

$d = 0.514$ reject at 1% and 5%

(b) $\ln L_t^* = -1.693 - 0.427 \ln r + 1.241 \ln Y$ $R^2 = 0.573$

$d = 1.932$ accept at 1% and 5%

(-3.359)	(4.781)	(0.929)
reject at 1%	reject at 1%	accept at 1% and 5%

---

ii) (a) $\ln L_t^* = -12.205 - 0.539 \ln r + 1.496 \ln Y$ $R^2 = 0.902$ (34)

$d = 0.664$ reject at 1% and 5%

(b) $\ln L_t^* = -3.701 - 0.452 \ln r + 1.460 \ln Y$ $R^2 = 0.665$

$d = 2.169$ accept at 1% and 5%

(-3.081)	(5.750)	(1.813)
reject at 1%	reject at 1%	accept at 1% and 5%

---

iii) (a) $F = 38.866$ (b) $F = 20.528$

reject at 1%	reject at 1%

---

iv) (a) $\ln L_t^* = 39.379 - 1.118 \ln r - 0.941 \ln Y$ $R^2 = 0.892$ (20)

$d = 1.828$ accept at 1% and 5%

(-8.572)	(-2.819)	(1.746)
reject at 1%	accept at 1% reject at 5%	accept at 1% and 5%

---

v) (a) $\ln L_t^* = 29.204 - 0.120 \ln r - 0.392 \ln Y$ $R^2 = 0.823$ (14)

$d = 0.666$ reject at 1% and 5%

(b) $\ln L_t^* = 3.809 - 0.215 \ln r + 0.260 \ln Y$ $R^2 = 0.388$

$d = 1.637$ accept at 1% and 5%

(-2.176)	(0.596)
accept at 1% and 5%	accept at 1% and 5%

Eqn 4.2.2.

i) (a) $\ln L_t^* = -1.884 - 1.064 \ln r + 1.041 \ln Y \quad R^2 = 0.923$ (35)

d = 0.487 reject at 1% and 5%

(b) $\ln L_t^* = 0.700 - 0.957 \ln r + 0.806 \ln Y \quad R^2 = 0.658$

d = 2.077 accept at 1% and 5%

(-4.797)	(2.588)	(-0.621)
reject at 1%	accept at 1% reject at 5%	accept at 1% and 5%

---

ii) (a) $\ln L_t^* = -2.229 - 1.036 \ln r + 1.056 \ln Y \quad R^2 = 0.916$ (30)

d = 0.569 reject at 1% and 5%

(b) $\ln L_t^* = 0.348 - 1.075 \ln r + 0.891 \ln Y \quad R^2 = 0.716$

d = 2.292 accept at 1% and 5%

(-4.572)	(2.580)	(-0.317)
reject at 1%	accept at 1% reject at 5%	accept at 1% and 5%

---

iii) (a) F = 37.794 (b) F = 16.486

reject at 1% reject at 1%

---

iv) (a) $\ln L_t^* = 41.725 - 1.689 \ln r - 1.004 \ln Y \quad R^2 = 0.923$ (20)

d = 1.675 accept at 1% and 5%

(-10.506)	(-3.586)	(-0.014)
reject at 1%	reject at 1%	accept at 1% and 5%

---

v) (a) $\ln L_t^* = 23.496 - 0.460 \ln r - 0.124 \ln Y \quad R^2 = 0.909$ (10)

d = 1.452 reject at 1%, accept at 5%

(b) $\ln L_t^* = 14.174 - 0.430 \ln r - 0.115 \ln Y \quad R^2 = 0.766$

d = 1.887 accept at 1% and 5%

(-2.109)	(-0.224)
accept at 1% and 5%	accept at 1% and 5%

Eqn 4.3.2.

i) (a) $\ln L_t^* = -11.037 - 1.109 \ln r + 1.484 \ln Y$ $R^2 = 0.922$ (43)

$d = 0.340$ reject at 1% and 5%

(b) $\ln L_t^* = -1.034 - 0.848 \ln r + 1.251 \ln Y$ $R^2 = 0.651$

$d = 1.850$ accept at 1% and 5%

(-6.530)	(6.220)	(1.249)
reject at 1%	reject at 1%	accept at 1% and 5%

---

ii) (a) $\ln L_t^* = -11.079 - 1.052 \ln r + 1.482 \ln Y$ $R^2 = 0.924$ (38)

$d = 0.344$ reject at 1% and 5%

(b) $\ln L_t^* = -1.662 - 1.036 \ln r + 1.428 \ln Y$ $R^2 = 0.737$

$d = 2.239$ accept at 1% and 5%

(-7.399)	(6.805)	(2.039)
reject at 1%	reject at 1%	accept at 1% reject at 5%

---

iii) (a) $F = 5.995$ reject at 1%   (b) $F = 15.460$ reject at 1%

---

iv) (a) $\ln L_t^* = 4.360 - 1.140 \ln r + 0.749 \ln Y$ $R^2 = 0.723$ (22)

$d = 0.218$ reject at 1% and 5%

(b) $\ln L_t^* = -0.278 - 1.127 \ln r + 1.128 \ln Y$ $R^2 = 0.766$

$d = 1.533$ reject at 1%, accept at 5%

(-7.159)	(4.296)	(0.486)
reject at 1%	reject at 1%	accept at 1% and 5%

---

v) (a) $\ln L_t^* = 30.443 - 0.150 \ln r - 0.444 \ln Y$ $R^2 = 0.833$ (16)

$d = 0.668$ reject at 1% and 5%

(b) $\ln L_t^* = 6.092 - 0.208 \ln r - 0.065 \ln Y$ $R^2 = 0.409$

$d = 2.095$ accept at 1% and 5%

(-2.232)	(-0.223)
accept at 1% reject at 5%	accept at 1% and 5%

Eqn 4.4.2.

i) (a) $\ln L_t^* = 0.600 - 3.584 \ln r + 1.179 \ln Y$ $R^2 = 0.939$ (43)

$d = 0.678$ reject at 1% and 5%

(b) $\ln L_t^* = -0.476 - 2.729 \ln r + 1.203 \ln Y$ $R^2 = 0.769$

$d = 2.149$ accept at 1% and 5%

(-5.982) (7.362) (1.241)

reject at 1% reject at 1% accept at 1% and 5%

---

ii) (a) $\ln L_t^* = 0.145 - 3.445 \ln r + 1.189 \ln Y$ $R^2 = 0.939$ (38)

$d = 0.783$ reject at 1% and 5%

(b) $\ln L_t^* = -0.747 - 2.952 \ln r + 1.244 \ln Y$ $R^2 = 0.806$

$d = 2.396$ accept at 1% and 5%

(-5.816) (7.543) (1.480)

reject at 1% reject at 1% accept at 1% and 5%

---

iii) (a) $F = 12.548$ (b) $F = 10.216$

reject at 1% reject at 1%

---

iv) (a) $\ln L_t^* = 27.920 - 4.856 \ln r - 0.013 \ln Y$ $R^2 = 0.846$ (22)

$d = 1.043$ reject at 1% and 5%

(b) $\ln L_t^* = 8.391 - 3.820 \ln r + 0.418 \ln Y$ $R^2 = 0.725$

$d = 1.968$ accept at 1% and 5%

(-4.935) (1.167)

reject at 1% accept at 1% and 5%

---

v) (a) $\ln L_t^* = 29.917 - 0.619 \ln r - 0.379 \ln Y$ $R^2 = 0.839$ (16)

$d = 0.566$ reject at 1% and 5%

(b) $\ln L_t^* = 4.193 - 0.732 \ln r + 0.130 \ln Y$ $R^2 = 0.514$

$d = 2.528$ accept at 1% and 5%

(-3.308) (0.460)

reject at 1% accept at 1% and 5%

Eqn 4.5.2.

i) (a) $\ln L_t^* = 3.942 - 4.683 \ln r + 1.015 \ln Y$ $R^2 = 0.959$ (42)

$d = 0.642$ reject at 1% and 5%

(b) $\ln L_t^* = 0.357 - 4.357 \ln r + 1.124 \ln Y$ $R^2 = 0.934$

$d = 1.602$ reject at 1%, accept at 5%.

(-8.067) (7.609) (0.841)

reject at 1% reject at 1% accept at 1% and 5%

---

ii) (a) $\ln L_t^* = 3.413 - 4.543 \ln r + 1.030 \ln Y$ $R^2 = 0.957$ (37)

$d = 0.641$ reject at 1% and 5%

(b) $\ln L_t^* = 0.364 - 4.388 \ln r + 1.124 \ln Y$ $R^2 = 0.850$

$d = 1.618$ accept at 1% and 5%

(-8.041) (7.105) (0.783)

reject at 1% reject at 1% accept at 1% and 5%

---

iii) (a) $F = 6.400$ (b) $F = 14.333$

reject at 1% reject at 1%

---

iv) (a) $\ln L_t^* = 21.058 - 7.432 \ln r + 0.385 \ln Y$ $R^2 = 0.858$ (21)

$d = 1.176$ reject at 1% and 5%

(b) $\ln L_t^* = 6.198 - 5.918 \ln r + 0.757 \ln Y$ $R^2 = 0.765$

$d = 1.443$ reject at 1%, accept at 5%

(-4.780) (2.138) (-0.685)

reject at 1% accept at 1% accept at 1% and 5%
reject at 5%

---

v) (a) $\ln L_t^* = 45.593 + 1.114 \ln r - 1.201 \ln Y$ $R^2 = 0.825$ (16)

$d = 0.966$ reject at 1% and 5%

(b) $\ln L_t^* = 10.029 - 1.000 \ln r - 0.071 \ln Y$ $R^2 = 0.465$

$d = 1.809$ accept at 1% and 5%

(-0.741) (-0.102)

accept at 1% and 5% accept at 1% and 5%

Eqn 4.6.2.

i) (a) $\ln L_t^* = 4.047 - 3.695 \ln r + 0.948 \ln Y \quad R^2 = 0.949 \quad (40)$

d = 0.460 reject at 1% and 5%

(b) $\ln L_t^* = 1.002 - 3.887 \ln r + 0.935 \ln Y \quad R^2 = 0.698$

d = 1.669 accept at 1% and 5%

(-6.500) (4.975) (-0.343)

reject at 1% reject at 1% accept at 1% and 5%

---

ii) (a) $\ln L_t^* = 3.233 - 3.538 \ln r + 0.975 \ln Y \quad R^2 = 0.950 \quad (35)$

d = 0.564 reject at 1% and 5%

(b) $\ln L_t^* = 1.134 - 4.220 \ln r + 0.971 \ln Y \quad R^2 = 0.799$

d = 1.763 accept at 1% and 5%

(-7.336) (5.192) (-0.156)

reject at 1% reject at 1% accept at 1% and 5%

---

iii) (a) F = 27.155 (b) F = 10.868

reject at 1% reject at 1%

---

iv) (a) $\ln L_t^* = 43.343 - 7.095 \ln r - 0.689 \ln Y \quad R^2 = 0.933 \quad (19)$

d = 2.134 accept at 1% and 5%

(-9.919) (-2.158) (0.974)

reject at 1% accept at 1% reject at 1%

reject at 5%

---

v) (a) $\ln L_t^* = 47.653 + 1.190 \ln r - 1.297 \ln Y \quad R^2 = 0.897 \quad (16)$

d = 1.574 accept at 1% and 5%

(3.230) (-6.617) (-1.515)

reject at 1% reject at 1% accept at 1% and 5%

Eqn 4.1.3.

i) (a) $\ln L_t^* = -14.889 - 0.594 \ln r + 1.623 \ln Y_p$ $R^2 = 0.935$ (37)

$d = 0.562$ reject at 1% and 5%

(b) $\ln L_t^* = -3.691 - 0.473 \ln r + 1.560 \ln Y_p$ $R^2 = 0.676$

$d = 1.831$ accept at 1% and 5%

(-4.386)	(6.340)	(2.308)
reject at 1%	reject at 1%	accept at 1%
		reject at 5%

---

ii) (a) $\ln L_t^* = -14.828 - 0.583 \ln r + 1.620 \ln Y_p$ $R^2 = 0.936$ (32)

$d = 0.771$ reject at 1% and 5%

(b) $\ln L_t^* = -6.231 - 0.519 \ln r + 1.704 \ln Y_p$ $R^2 = 0.781$

$d = 2.042$ accept at 1% and 5%

(-4.319)	(7.942)	(3.282)
reject at 1%	reject at 1%	reject at 1%

---

iii) (a) $F = 25.503$ (b) $F = 13.678$

reject at 1% reject at 1%

---

iv) (a) $\ln L_t^* = 27.607 - 0.961 \ln r - 0.390 \ln Y_p$ $R^2 = 0.912$ (18)

$d = 2.077$ accept at 1% and 5%

(-8.126)	(-1.112)
reject at 1%	accept at 1% and 5%

---

v) (a) $\ln L_t^* = 33.205 - 0.069 \ln r - 0.576 \ln Y_p$ $R^2 = 0.832$ (14)

$d = 0.711$ reject at 1% and 5%

(b) $\ln L_t^* = 5.124 - 0.196 \ln r + 0.103 \ln Y_p$ $R^2 = 0.400$

$d = 1.654$ accept at 1% and 5%

(-1.782)	(0.216)
accept at 1% and 5%	accept at 1% and 5%

Eqn 4.2.3.

i) (a) $\ln L_t^* = -6.076 - 1.035 \ln r + 1.237 \ln Y_p$ $R^2 = 0.949$ (33)

$d = 0.587$ reject at 1% and 5%

(b) $\ln L_t^* = -2.366 - 0.904 \ln r + 1.340 \ln Y_p$ $R^2 = 0.762$

$d = 2.145$ accept at 1% and 5%

(-5.196) (4.630) (1.174)

reject at 1% reject at 1% accept at 1% and 5%

---

ii) (a) $\ln L_t^* = -6.335 - 1.010 \ln r + 1.247 \ln Y_p$ $R^2 = 0.946$ (28)

$d = 0.699$ reject at 1% and 5%

(b) $\ln L_t^* = -3.036 - 0.966 \ln r + 1.376 \ln Y_p$ $R^2 = 0.818$

$d = 2.350$ accept at 1% and 5%

(-5.028) (4.610) (1.259)

reject at 1% reject at 1% accept at 1% and 5%

---

iii) (a) $F = 22.228$ (b) $F = 10.963$

reject at 1% reject at 1%

---

iv) (a) $\ln L_t^* = 31.674 - 1.478 \ln r - 0.540 \ln Y_p$ $R^2 = 0.932$ (18)

$d = 1.845$ accept at 1% and 5%

(-9.467) (-1.707)

reject at 1% accept at 1% and 5%

---

v) (a) $\ln L_t^* = 27.382 - 0.385 \ln r - 0.304 \ln Y_p$ $R^2 = 0.912$ (10)

$d = 1.444$ reject at 1%, accept at 5%

(b) $\ln L_t^* = 16.042 - 0.376 \ln r - 0.265 \ln Y_p$ $R^2 = 0.769$

$d = 1.915$ accept at 1% and 5%

(-1.572) (-0.429)

accept at 1% and 5% accept at 1% and 5%

Eqn 4.3.3.

i) (a) $\ln L_t^* = -13.291 - 1.150 \ln r + 1.592 \ln Y_p$ $R^2 = 0.942$ (41)

$d = 0.393$ reject at 1% and 5%

(b) $\ln L_t^* = -3.063 - 0.883 \ln r + 1.714 \ln Y_p$ $R^2 = 0.709$

$d = 1.703$ accept at 1% and 5%

(-7.136) (7.696) (3.207)

reject at 1% reject at 1% reject at 1%

---

ii) (a) $\ln L_t^* = -13.322 - 1.095 \ln r + 1.589 \ln Y_p$ $R^2 = 0.945$ (36)

$d = 0.386$ reject at 1% and 5%

(b) $\ln L_t^* = -3.834 - 1.056 \ln r + 1.942 \ln Y_p$ $R^2 = 0.801$

$d = 2.027$ accept at 1% and 5%

(-8.448) (8.765) (4.251)

reject at 1% reject at 1% reject at 1%

---

iii) (a) $F = 4.463$ (b) $F = 23.446$

reject at 1% reject at 1%

accept at 5%

---

iv) (a) $\ln L_t^* = -3.552 - 1.107 \ln r + 1.123 \ln Y_p$ $R^2 = 0.793$ (20)

$d = 0.253$ reject at 1% and 5%

(b) $\ln L_t^* = -3.412 - 1.291 \ln r + 2.833 \ln Y_p$ $R^2 = 0.862$

$d = 1.527$ reject at 1%, accept at 5%

(-9.733) (6.652) (4.304)

reject at 1% reject at 1% reject at 1%

---

v) (a) $\ln L_t^* = 31.288 - 0.132 \ln r - 0.484 \ln Y_p$ $R^2 = 0.837$ (16)

$d = 0.666$ reject at 1% and 5%

(b) $\ln L_t^* = 6.238 - 0.200 \ln r - 0.092 \ln Y_p$ $R^2 = 0.410$

$d = 2.114$ accept at 1% and 5%

(-1.932) (-0.282)

accept at 1% and 5% accept at 1% and 5%

Eqn 4.4.3.

i) (a) $\ln L_t^* = -1.171 - 3.580 \ln r + 1.262 \ln Y_p$ $R^2 = 0.949$ (41)

$d = 0.727$ reject at 1% and 5%

(b) $\ln L_t^* = -2.004 - 2.915 \ln r + 1.414 \ln Y_p$ $R^2 = 0.809$

$d = 2.239$ accept at 1% and 5%

(-6.626)	(8.890)	(2.602)
reject at 1%	reject at 1%	accept at 1%
		reject at 5%

---

ii) (a) $\ln L_t^* = -1.656 - 3.445 \ln r + 1.273 \ln Y_p$ $R^2 = 0.951$ (36)

$d = 0.854$ reject at 1% and 5%

(b) $\ln L_t^* = -2.242 - 3.161 \ln r + 1.421 \ln Y_p$ $R^2 = 0.853$

$d = 2.531$ accept at 1% and 5%

(-6.685)	(9.329)	(2.763)
reject at 1%	reject at 1%	reject at 1%

---

iii) (a) $F = 7.747$ reject at 1% (b) $F = 7.943$ reject at 1%

---

iv) (a) $\ln L_t^* = 24.845 - 4.677 \ln r + 0.118 \ln Y_p$ $R^2 = 0.851$ (20)

$d = 0.989$ reject at 1% and 5%

(b) $\ln L_t^* = 3.704 - 3.830 \ln r + 0.842 \ln Y_p$ $R^2 = 0.723$

$d = 2.469$ accept at 1% and 5%

(-4.397)	(1.676)
reject at 1%	accept at 1% and 5%

---

v) (a) $\ln L_t^* = 30.697 - 0.554 \ln r - 0.420 \ln Y_p$ $R^2 = 0.842$ (16)

$d = 0.566$ reject at 1% and 5%

(b) $\ln L_t^* = 4.122 - 0.751 \ln r + 0.146 \ln Y_p$ $R^2 = 0.514$

$d = 2.516$ accept at 1% and 5%

(-3.075)	(0.460)
reject at 1%	accept at 1% and 5%

Eqn 4.5.3.

i) (a) $\ln L_t^* = 2.799 - 4.641 \ln r + 1.066 \ln Y_p$ $R^2 = 0.963$ (41)

$d = 0.651$ reject at 1% and 5%

(b) $\ln L_t^* = 0.026 - 4.139 \ln r + 1.163 \ln Y_p$ $R^2 = 0.823$

$d = 1.604$ reject at 1%, accept at 5%

(-7.657) (7.165) (1.002)

reject at 1% reject at 1% accept at 1% and 5%

---

ii) (a) $\ln L_t^* = 2.229 - 4.495 \ln r + 1.083 \ln Y_p$ $R^2 = 0.961$ (36)

$d = 0.645$ reject at 1% and 5%

(b) $\ln L_t^* = 0.024 - 4.188 \ln r + 1.165 \ln Y_p$ $R^2 = 0.842$

$d = 1.616$ accept at 1% and 5%

(-7.630) (6.653) (0.942)

reject at 1% reject at 1% accept at 1% and 5%

---

iii) (a) $F = 5.129$ (b) $F = 0.873$

reject at 1% accept at 1% and 5%

---

iv) (a) $\ln L_t^* = 17.433 - 7.162 \ln r + 0.539 \ln Y_p$ $R^2 = 0.863$ (20)

$d = 1.141$ reject at 1% and 5%

(b) $\ln L_t^* = 6.016 - 5.880 \ln r + 0.752 \ln Y_p$ $R^2 = 0.736$

$d = 1.548$ accept at 1% and 5%

(-4.315) (1.486)

reject at 1% accept at 1% and 5%

---

v) (a) $\ln L_t^* = 49.368 + 1.509 \ln r - 1.397 \ln Y_p$ $R^2 = 0.843$ (16)

$d = 1.016$ reject at 1% and 5%

(b) $\ln L_t^* = 14.718 - 0.192 \ln r - 0.523 \ln Y_p$ $R^2 = 0.522$

$d = 1.752$ accept at 1% and 5%

(-0.159) (-0.834)

accept at 1% and 5% accept at 1% and 5%

Eqn 4.6.3.

i) (a) $\ln L_t^* = 3.873 - 3.694 \ln r + 0.957 \ln Y_p$ $R^2 = 0.949$ (40)

$d = 0.469$ reject at 1% and 5%

(b) $\ln L_t^* = 0.227 - 3.820 \ln r + 1.096 \ln Y_p$ $R^2 = 0.703$

$d = 1.675$ accept at 1% and 5%

(-6.387)	(4.976)	(0.436)
reject at 1%	reject at 1%	accept at 1% and 5%

---

ii) (a) $\ln L_t^* = 3.134 - 3.548 \ln r + 0.982 \ln Y_p$ $R^2 = 0.949$ (35)

$d = 0.560$ reject at 1% and 5%

(b) $\ln L_t^* = 0.359 - 4.102 \ln r + 1.097 \ln Y_p$ $R^2 = 0.797$

$d = 1.744$ accept at 1% and 5%

(-6.991)	(5.157)	(0.457)
reject at 1%	reject at 1%	accept at 1% and 5%

---

iii) (a) $F = 35.000$ (b) $F = 14.800$

reject at 1%   reject at 1%

---

iv) (a) $\ln L_t^* = 49.798 - 7.494 \ln r - 0.970 \ln Y_p$ $R^2 = 0.944$ (19)

$d = 2.072$ accept at 1% and 5%

(-11.149)	(-2.971)	(0.919)
reject at 1%	accept at 1% reject at 5%	accept at 1% and 5%

---

v) (a) $\ln L_t^* = 48.076 + 1.198 \ln r - 1.318 \ln Y_p$ $R^2 = 0.909$ (16)

$d = 1.553$ accept at 1% and 5%

(3.522)	(-7.206)	(-1.739)
reject at 1%	reject at 1%	accept at 1% and 5%

Eqn 1.0.3.

i) (a) $\ln M_1 = -7.564 + 1.285 \ln Y_p$ $R^2 = 0.832$ (41)

$d = 0.172$ reject at 1% and 5%

(b) $\ln M_2 = -0.008 + 0.937 \ln Y_p$ $R^2 = 0.184$

$d = 1.788$ accept at 1% and 5%

(2.923) (-0.196)

reject at 1% accept at 1% and 5%

---

ii) (a) $\ln M_1 = -7.153 + 1.264 \ln Y_p$ $R^2 = 0.853$ (36)

$d = 0.319$ reject at 1% and 5%

(b) $\ln M_1 = 1.451 + 1.364 \ln Y_p$ $R^2 = 0.430$

$d = 1.779$ accept at 1% and 5%

(4.987) (1.331)

reject at 1% accept at 1% and 5%

---

iii) (a) $F = 30.577$ (b) $F = 13.527$

reject at 1% reject at 1%

---

iv) (a) $\ln M_1 = -5.852 + 1.198 \ln Y_p$ $R^2 = 0.669$ (20)

$d = 0.624$ reject at 1% and 5%

(b) $\ln M_1 = -2.418 + 1.307 \ln Y_p$ $R^2 = 0.406$

$d = 1.967$ accept at 1% and 5%

(3.411) (0.801)

reject at 1% accept at 1% and 5%

---

v) (a) $\ln M_1 = 28.341 - 0.349 \ln Y_p$ $R^2 = 0.696$ (16)

$d = 0.962$ reject at 1% and 5%

(b) $\ln M_1 = 12.746 - 0.321 \ln Y_p$ $R^2 = 0.325$

$d = 1.793$ accept at 1% and 5%

(-2.500) (-5.288)

accept at 1% reject at 1%

reject at 5%

Eqn 2.0.3.

i) (a) $\ln M_2 = -9.617 + 1.394 \ln Y_p$ $R^2 = 0.873$ (41)

d = 0.145 reject at 1% and 5%

(b) $\ln M_2 = -0.174 + 1.067 \ln Y_p$ $R^2 = 0.258$

d = 1.694 accept at 1% and 5%

(3.638) (0.228)

reject at 1% accept at 1% and 5%

---

ii) (a) $\ln M_2 = -9.220 + 1.374 \ln Y_p$ $R^2 = 0.892$ (36)

d = 0.281 reject at 1% and 5%

(b) $\ln M_2 = -1.598 + 1.490 \ln Y_p$ $R^2 = 0.507$

d = 1.741 accept at 1% and 5%

(5.823) (1.914)

reject at 1% accept at 1% and 5%

---

iii) (a) F = 27.404 (b) F = 14.445

reject at 1% reject at 1%

---

iv) (a) $\ln M_2 = -11.726 + 1.491 \ln Y_p$ $R^2 = 0.782$ (20)

d = 0.486 reject at 1% and 5%

(b) $\ln M_2 = -2.798 + 1.520 \ln Y_p$ $R^2 = 0.511$

d = 1.866 accept at 1% and 5%

(4.215) (1.441)

reject at 1% accept at 1% and 5%

---

v) (a) $\ln M_2 = 22.921 - 0.087 \ln Y_p$ $R^2 = 0.096$ (16)

d = 0.518 reject at 1% and 5%

(b) $\ln M_2 = 4.117 + 0.153 \ln Y_p$ $R^2 = 0.040$

d = 1.859 accept at 1% and 5%

(0.733)

accept at 1% and 5%

Eqn 3.0.3.

i) (a) $\ln L_t = -27.344 + 2.145 \ln Y_p$ $R^2 = 0.447$ (40)

$d = 0.411$ reject at 1% and 5%

(b) $\ln L_t = -1.431 + 1.198 \ln Y_p$ $R^2 = 0.030$

$d = 1.845$ accept at 1% and 5%

(1.078)

accept at 1% and 5%

---

ii) (a) $\ln L_t = -26.238 + 2.087 \ln Y_p$ $R^2 = 0.456$ (35)

$d = 0.471$ reject at 1% and 5%

(b) $\ln L_t = -4.261 + 1.719 \ln Y_p$ $R^2 = 0.076$

$d = 1.869$ accept at 1% and 5%

(1.623)

accept at 1% and 5%

---

iii) (a) $F = 3.766$ (b) $F = 1.572$

accept at 1% and 5% accept at 1% and 5%

---

iv) (a) $\ln L_t = -10.257 + 1.316 \ln Y_p$ $R^2 = 0.040$ (19)

$d = 0.588$ reject at 1% and 5%

(b) $\ln L_t = 1.844 + 0.513 \ln Y_p$ $R^2 = 0.002$

$d = 1.856$ accept at 1% and 5%

(0.168)

accept at 1% and 5%

---

v) (a) $\ln L_t = 64.033 - 2.013 \ln Y_p$ $R^2 = 0.968$ (16)

$d = 1.962$ accept at 1% and 5%

(-20.503) (-10.318)

reject at 1% reject at 1%

Eqn 4.0.3.

i) (a) $\ln L_t^* = -19.457 + 1.820 \ln Y_p$ $R^2 = 0.764$ (41)

d = 0.162 reject at 1% and 5%

(b) $\ln L_t^* = 0.373 + 0.701 \ln Y_p$ $R^2 = 0.043$

d = 1.628 accept at 1% and 5%

(1.300)

accept at 1% and 5%

---

ii) (a) $\ln L_t^* = -18.739 + 1.783 \ln Y_p$ $R^2 = 0.790$ (36)

d = 0.254 reject at 1% and 5%

(b) $\ln L_t^* = -2.020 + 1.677 \ln Y_p$ $R^2 = 0.254$

d = 1.719 accept at 1% and 5%

(3.353) (1.354)

reject at 1% accept at 1% and 5%

---

iii) (a) F = 13.859 (b) F = 7.318

reject at 1% reject at 1%

---

iv) (a) $\ln L_t^* = -23.098 + 1.987 \ln Y_p$ $R^2 = 0.507$ (20)

d = 0.388 reject at 1% and 5%

(b) $\ln L_t^* = -1.722 + 1.354 \ln Y_p$ $R^2 = 0.127$

d = 1.692 accept at 1% and 5%

(1.571)

accept at 1% and 5%

---

v) (a) $\ln L_t^* = 36.287 - 0.717 \ln Y_p$ $R^2 = 0.823$ (16)

d = 0.720 reject at 1% and 5%

(b) $\ln L_t^* = 11.078 - 0.565 \ln Y_p$ $R^2 = 0.340$

d = 1.997 accept at 1% and 5%

(-2.588) (1.993)

accept at 1% accept at 1% and 5%

reject at 5%

CHAPTER VI

Conclusions from Results

1. Introduction.

The two most striking features of the results presented in the previous chapter are firstly that without exception all the regressions over the whole period, as well as those excluding the war years, show the presence of significant and in some fortunately rare cases extremely tenacious autocorrelation, and secondly that in all cases with two possible exceptions, the Chow test indicates that the demand function for the pre-war years differs from that for the post-war years. Further in every case, correction of the data for autocorrelation changes the estimates of the coefficients significantly so indicating the presence of a residual bias, in some cases upwards and in other cases downwards, in the estimators themselves. If the autocorrelation follows a first-order Markov autoregressive scheme as is assumed in the correction procedure, then the well known result that the estimators are unbiased holds. However, if the autoregressive structure does not follow a first-order Markov scheme or if other effects such as lagged variables are present, the estimators will be biased and correction for autocorrelation will then merely alter the magnitude and possibly the sign of the bias without removing it completely. Further, there is no known method of obtaining an estimate of this residual bias and consequently there is no method of discovering whether the corrected estimators still contain an element of bias or not. Bearing in mind the inherent intractability of economic data it would be more realistic to assume that the corrected estimators are in fact biased. A further consequence of these complications is that the correlation coefficient and the t-value for the hypothesis tests are almost certain to be affected and so must be regarded with due caution.

An important fact emanating from the above remarks is that it would not be econometrically advisable to place reliance on the magnitude, or in extreme cases even on the sign, of any of the estimated coefficients of a single equation considered in isolation.

In view of this unfortunate fact, the most satisfactory alternative would be to examine the results for any pattern or trends that are observable, since it is extremely unlikely that the residual bias will be of the same magnitude and sign for every equation. Any single result presented in the previous chapter is characterised by four independent parameters, viz. the time period considered, the definition of the monetary variable, the interest rate used, and the constraint adopted. We may visualise each of these factors as changing independently or in conjunction with one or more of the others so generating possible trends in the results. If we had no indication a priori as to which of the myriad of possible combinations and permutations to inspect, we would be faced with a thankless task which would also involve the added hazard of the possibility of obtaining spurious results. However, for economic reasons the following four possibilities are the most interesting.

i) Keep the definition of money fixed whilst allowing the term of the interest rate to change for each of the possible constraints. This will show the effects of changes in the term of the interest rate on the function.

ii) Keep the definition of money and the interest rate fixed and allow the constraint to vary and then to repeat for the different definitions of money and interest rates. This will show the effects that the various constraints have on the function.

iii) Keep the constraint and the interest rate fixed whilst allowing the definition of money to change and to repeat for all the different constraints and interest rates. This will determine the effect of different definitions of money on the function.

iv) To repeat each of the above for the four different time periods considered.

These in fact are the possibilities that are going to be examined. The order of presentation will be the same as above with the exception that the fourth possibility will be considered concurrently with each of the first three.

2. Analysis of Results.

For expositional reasons we shall consider the two definitions of money separately from the two definitions of idle balances.

A) M_1 and M_2.

i) Regressions over all years. As the term of the interest rate lengthens there is a strong tendency for the interest elasticity to increase and for the fit to improve. Apart from a few exceptions the function is homogeneous of degree one in the constraint.

ii) Regressions excluding the war years. The same general pattern as above is exhibited with the exception that in all cases the interest elasticity is higher and the fit better than the corresponding regression over all the years. There is also a weak but noticeable tendency for the constraint elasticity to follow the same pattern, but the function is still homogeneous of degree one in the constraint.

iii) Regressions over the pre-war years. The fits are in all cases better than the corresponding fits in either of the above two cases. The interest elasticity once again tends to increase with a lengthening of the term of the interest rate but the goodness of fit bears no such relation.

iv) Regressions over the post-war years. The fits are in all cases worse than the corresponding fits for the regressions over the pre-war years. Compared to the equivalent values for the pre-war regressions, the interest elasticities are smaller although still significant and the constraint elasticity is smaller and often not significant, though in some cases it is significantly negative. There is no observable relation between the size of the elasticities, the goodness of fit and the term of the interest rate.

B) L_t and L_t^*.

i) Regressions over all the years. The interest elasticity increases with the term of the interest rate and whenever the constraint is significant, the function is nearly always homogeneous of degree one.

ii) Regressions excluding the war years. The same pattern as above is exhibited. Generally there is an increase in the interest elasticity and goodness of fit compared to the equivalent values in the corresponding regressions over all the years.

iii) Regressions over the pre-war years. Compared to the corresponding values for the above two regressions, the interest elasticity and the goodness of fit has increased. However, the constraint is often not significant and whenever it is significant it is negative.

iv) Regressions over the post-war years. For L_t the results are better than for the pre-war regressions whereas for L_t^* the converse holds true. The constraint is often not significant and in the cases where it is significant it has a negative sign.

Since there is no economic hypothesis concerning the effects on the functions of the different constraints, no pattern should be expected <u>a priori</u>. The only pattern which did emerge was that the Permanent Income constrained models had better fits and higher constraint elasticities than the income constrained models which in turn bore the same relationship to the wealth constrained models. This trend was observed for the regressions over all the years as well as for those excluding the war years. The pre-war and post-war regressions did not exhibit any immediately discernible relationship with the exception of a slight tendency for the pre-war data to exhibit the above pattern.

We shall now let the definition of money change from a broad concept to a narrow one, <u>viz</u>. from M_2 to M_1 to L_t to L_t^* . For each equation there is a tendency for the interest elasticity and its significance to increase and for the constraint elasticity and its significance to decrease as the monetary variable is changed as above. This pattern is strongly exhibited for the regressions over all the years and excluding the war years. The pre-war and post-war regressions fail to show any such pattern, although there is a weak tendency for it to appear in the pre-war results.

The only other group of equations that remain to be examined are those representing Friedman's empirical demand function. The results are so bad that they do not warrant further analysis, more often than not indicating that there is only a spurious relation between money and permanent income.

3. Conclusions from the Above Analysis.

The most salient features of the demand for money in South Africa have now become apparent.

i) All the estimated coefficients with the possible exception of the idle-balance equations are of a magnitude that would be expected in the light of experience and of other studies. This suggests that the residual bias is not strong enough to completely invalidate the results.

ii) Comparison of the non-interest elastic functions with interest elastic functions shows conclusively that the demand for money is interest elastic. Hence the monetary policy measures suggested by the Chicago school would be invalid in a South African setting.

iii) The best fits over all the years were given by Permanent Income constrained models and then by Keynesian-type functions. The majority of these functions are homogeneous of degree one in income and so the velocity diagrams of chapter IV would provide an adequate visual representation of the long run demand for money. Since the empirical measure of Permanent Income bears a closer relation to current income than to the theoretical concept of a flow arising from a stock of wealth, one might be tempted to suggest that the capital theoretic approach to the demand for money is inferior to the income-expenditure approach. However, this deduction cannot be conclusively justified by the results on account of the discrepancy between the empirical measures and the theoretical concepts.

iv) The omission of the war years from the regressions caused an increase in the interest elasticity and the goodness of fit, which in turn indicates that the abnormal conditions of the war years had an effect on the demand for money. This is only to be expected on account of the forced savings induced by the wartime controls on expenditure.

v) Examination of the function for stability revealed that in every case there had been a distinct movement of the function. The evidence overwhelmingly refutes the suggestion that the long run demand function is stable.[1]

vi) Both the constraint elasticities and the goodness of fit were higher for the pre-war regressions than for the post-war regressions which would suggest that there has been a change in money holding habits. Since the constraint elasticities and the fit are lower in the post-war years than in the pre-war years, it would appear that the money holding habits of the post-war years are more volatile than those of the pre-war years and hence more susceptible to exogenous influences. The shift in the function is undoubtedly caused by the forced saving induced by the war years but if the ceteris paribus assumptions had not been violated the function should have returned to its initial form after the passage of a few years. The fact that it did not do so shows that there have been permanent changes in money holding habits. These have most probably been caused by the rapid post-war growth of financial intermediaries and their offspring, money substitutes.[2]

vii) The relation between the term of the interest rate and the interest elasticity shows a surprising result, namely that higher interest elasticities are associated with longer term interest rates. Further the theoretically expected relation between the interest

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1 The limitations inherent in the method adopted must be remembered in any appraisal of this result. A rigorous justification of the stability could only be obtained from a simultaneous equation estimation, though in view of the volume of evidence amassed in favour of instability, it seems extremely unlikely that a simultaneous equation approach would yield significantly different results.

2 For an excellent survey of the South African financial sector, the reader is referred to: G.F.D. Palmer, "The Development of a South African Money Market", South African Journal of Economics, Vol. 26, (Dec., 1958), pp. 239 - 52; A.B. Dickman and G.F.D. Palmer, "The South African Money Market - Some Further Developments", South African Journal of Economics, Vol. 28, (Dec., 1960), pp. 354 - 69; D.W. Goedhuys, "Money on the Wing, or the Rise of Non-Bank Lenders", South African Journal of Economics, Vol. 32, (Sept., 1964), pp. 211 - 18, and A.B. Dickman, "The South African Money Market - Progress and Problems since 1960", South African Journal of Economics, Vol. 33, (Sept., 1965), pp. 213 - 36.

elasticity and the various definitions of money has been observed since M_2 is the least interest elastic and L_t^* the most interest elastic.

viii) The idle balance equations exhibit the expected relationships which suggest that the empirical method of calculation adopted is acceptable. Considering the pre-war and post-war periods separately, idle balances whilst still being very interest elastic do not tend to be responsive to changes in the constraint, often being completely inelastic and sometimes having a negative elasticity, so indicating that they are regarded as an inferior asset. Whether this phenomenon is merely a by-product of the method of calculation in which idle balances appear as a residual, or whether it is a genuine economic relationship is extremely hard to decide. However, in view of the rapid growth of alternative liquid stores of value the above result is not suprising, for money being used as a temporary store of value will very likely be regarded as inferior compared to an interest bearing asset of equivalent certainty. The satisfactory accordance between these results and the conclusions drawn in chapter IV indicates that the diagrams there provide an adequate visual representation of the behaviour of idle balances. However, in spite of this accordance with theory the nagging possibility still remains that they are in actuality merely a mechanistically determined residual having no fixed relation to idle balances and bearing only a spurious relation to the rate of interest.

4. Implications for Monetary Policy.

Since the action of monetary policy is essentially a short run phenomenon, the appropriate function to consider would be the short run function. However, since the short run and long run functions are not conceptually independent, it is possible to draw some conclusions concerning the impact of monetary policy in the light of the above characteristics of the long run function.

Due to the paucity of literature dealing with the modern quantity theorists' views on the action of monetary policy, one is forced to read between the lines in order to obtain their conception of the interaction between the monetary and real sectors. Since they envisage the demand function as being stable, any discrepancy between actual and desired holdings of money must be accommodated by a change in one or more of the arguments of the function. If, as Friedman asserts, the function is not only stable but also interest inelastic, then the price level and the volume of output are critically dependent on the stock of money.[3] However, not all quantity theorists regard the demand function as being interest inelastic, and in this latter case the stock of money and the rate of interest are the operative policy variables.

If, however, the demand function is not stable, the above view of the action of monetary policy is invalid, since any discrepancy between desired and actual holdings need not necessarily cause a change in the arguments of the function - the function itself may shift. Hence once we allow for the possibility of shifts in the function itself, the quantity theorists' views of the action of monetary policy become invalid and the impact of policy measures will not remain immutable over time. Since, given a constant state of expectations, the most likely single factor causing a parametric shift of the demand function is the state of evolution of the financial sector, monetary policy must be envisaged as acting within an institutional framework capable of evolution. Hence there is no reason to suppose that the impact of policy measures will remain the same regardless of the circumstances under which they are applied.

5. Comparisons with Other Studies.

All that finally remains is to make a brief comparison of the above results with those obtained by other authors. The only published study on the long run demand for money in South Africa is that

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[3] For a statement of this view see M. Friedman, "The Supply of Money and Changes in Prices and Output", in U.S. Congress Joint Economic Committee, The Relationship of Prices to Economic Stability and Growth : Compendium of Papers Submitted by Panelists (Washington, D.C., 1958), pp. 241 - 56.

by Heller.[4] An exact comparison between the above results and Heller's results is not possible on account of i) the different definitions used, ii) the different time periods under consideration, iii) the different statistical techniques adopted and iv) his failure to take account of the presence of autocorrelation. As a consequence of this latter fact his results have limited credibility and will have to be compared with the uncorrected results of chapter V. However, in spite of this several striking similarities are apparent. First he finds that the appropriate constraint is current income and not wealth, and secondly that the functions when considered over sub-periods are different from those considered over the entire period; both of which are in agreement with the above results. Any more detailed comparisons such as to the magnitude or sign of the co-efficients would be extremely dangerous on account of the differences between the two studies. Any comparisons with results obtained in similar studies in other countries are so fraught with hazards as to make any conclusions untrustworthy in the extreme, and so regrettably the fascinating problem of international comparisons of demand functions must be left in abeyance until suitable econometric techniques have been evolved.

6. Conclusion.

The moral of overriding importance which emerges from this study is that in the case of South Africa, it is incorrect to regard the demand for money as an entity independent of its environment and hence stable, but that it must be considered in a framework of changing financial institutions and a consequently growing spectrum of liquid assets.

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[4] H.R. Heller, "The Demand for Money in South Africa", South African Journal of Economics, Vol. 34, (Dec., 1966), pp. 335 - 40.

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