

AN INVESTIGATION OF A MATHEMATICS
RECOVERY PROGRAMME FOR
MULTIPLICATIVE REASONING TO A
GROUP OF LEARNERS IN THE SOUTH
AFRICAN CONTEXT: A CASE STUDY
APPROACH

By

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DECLARATION

I declare that this Research Project represents my original work. It is being submitted for the degree of Master of Mathematics Education at Rhodes University, Grahamstown. It has not been submitted for any degree or examination at any university.

(Signature of candidate)

_____ Day of _____ 20_____ in _____

DEDICATION AND THANKS

I dedicate this thesis to my family, my mother, Maggie Kalipile, my son Axolile Mofu, and two daughters, Anelisa Mofu, and Cwenga Mofu who supported me, giving me the courage and strength to complete this journey.

Whilst undertaking this research study I had an opportunity to present aspects of my work to two regional conferences; the Eastern Cape SAARMSTE conference and the Eastern Cape AMESA conference. I was also able to run workshops with teachers and district officials in the Eastern Cape Department of Education on Multiplication and Division. I produced a handbook called “Developing Fluency in Multiplication and Division” for these workshops.

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ABSTRACT

This thesis describes an intervention using the Mathematics Recovery programme in a South African context with a small sample of Grade 4 learners. The study uses a qualitative case study approach. The data collection included video recorded one-to-one oral interviews with the learners. I used the Learning Framework in Number (LFIN) developed by Wright, Martland, Stafford and Stanger (2006) to profile the learners using pre and post intervention interview data and to determine their levels of multiplicative reasoning. The analysis showed the positive impact of the Mathematics Recovery programme on the improvement of multiplicative reasoning. The study contributes to the use of Mathematics Recovery programmes in South Africa from both a teacher and teacher educator perspective.

ACRONYMS USED IN THE STUDY

Acronym	Meaning
ANA (ANAs)	Annual National Assessment (s)
BWNS	Backward Word Number Sequences
CAPS	Curriculum and Assessment Statement
FNWS	Forward Number Word Sequences
LFIN	Learning Framework in Number
MR	Mathematics Recovery
SMT	School Management Team

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CHAPTER ONE: INTRODUCTION, PURPOSE & RATIONALE

INTRODUCTION

In this opening chapter I provide an outline of the research study. I begin with the rationale for the study and the research problem I addressed. The significance and context of the research are also described. The research aims, research questions and an overview of the research methodology are sketched out. Finally the structure of this research thesis is presented.

RATIONALE OF THE STUDY

South African education is faced with a huge challenge in mathematics as most learners struggle with the basic concepts of numeracy (DOE, 2011). One of the main areas of concern is multiplication. The South African Annual National Assessment (ANA) 2011 report and 2012 Eastern Cape Province grade 3 question by question analysis report indicated that multiplication is one of the specific areas in mathematics where South African learners performed poorly (DOE, 2011, DOE, 2012). This fact, plus my own experience as a mathematics teacher motivated me to examine the need for appropriate interventions in the teaching and learning of multiplication as interventions can be effective in reducing disparities in mathematics achievement (Bobis, Clarke, Clarke, Thomas, Wright, Young-Loveridge & Gould, 2005).

The early childhood years are crucial in forming the basics of mathematics. According to Wright, Martland, Stafford and Stanger (2006), learners who are low-attaining in the early years tend to remain so throughout their schooling and the knowledge gap between low attaining learners tends to increase over the course of time. Having looked at a Mathematics Recovery Programme (Wright et al., 2006), where one of its components is a mathematical intervention on multiplicative reasoning I saw that such a programme had the potential to assist in going some way towards addressing the problem.

At a recent Early Childhood Development conference in Grahamstown (September 2012) which I attended, there was a discussion with Bob Wright regarding what it would mean to use the Mathematics Recovery (MR) Programme in a group context rather than on a one-to-one basis. It was mentioned that in the South African context, due to lack of resources, it would not be feasible to use the programme as an individually focussed intervention the way it has been used successfully in other countries. Bob Wright responded that research looking at using it in a group setting would be valuable.

It is thus my aim to explore the use of the Wright et. al. (2006) MR programme with a group of learners; with the hope that this study could point to the possibility of using the MR programme in whole class situations and open up further avenues for research.

The MR programme has been used and tested in Australia, New Zealand, the UK and the United States (Wright, 2003). To date, it has not been widely researched in the South African context, although this is beginning to change. See for example Graven and Stott, (2012a); Stott and Graven (2013a); Wietz (2012). This research aims to contribute towards filling this gap in the MR programme literature.

In addition, my study aims to contribute to the body of research in primary numeracy education since this is an under-researched in South Africa (Venkat & Graven, 2013). I am a part time Masters student in Rhodes University within the South African Numeracy Chair project which is supported by the First Rand Foundation, Anglo American Chairman's fund, the Department of Science and Technology and the National Research Foundation. The South African Numeracy Chair Project focuses on research and development aimed at improving the quality of numeracy teaching at primary level and improving learner performance in primary schools.

PROBLEM DESCRIPTION

My experience in the classroom confirms that learners experience difficulties with multiplication. I have observed that when working with multiplication, my Grade 5 learners are still counting visible objects in ones. Some learners, when performing multiplication

tasks draw circles or small lines for counting and some just add the numbers. This attests to the fact that their multiplicative reasoning is not fully developed.

CONTEXT OF THE STUDY

South Africa faces many educational challenges; it has been reported as underperforming when compared to other countries (Howie, 2004; Maree, Aldous, Hattingh, Swanepoel & Linde, 2006). According to the International Association for Evaluation of Education Achievement (IEA), South Africa was the lowest performing amongst the developing countries (DOE, 2011 p. 34).

Working with the information from the recent report on the analysis of the 2012 Annual National Assessments (ANAs), (DOE, 2012) the results clearly show that multiplication is one of the specific areas in mathematics where South African learners perform poorly. A question-by-question analysis of the 2012 Eastern Cape Provincial performance reveals that only 26% of learners got the multiplication problems correct (DOE, 2012).

In the 2010 State of the National Address, the South African President stated that the education system plays a vital role in improving productivity and mentioned that children and youth need to be better prepared by their schools to read, write, think critically and solve numerical problems. His speech was based on the foundations laid down in the curriculum where it is clearly stated that the education system must enable all learners to achieve to their maximum ability and the learners should be able to reflect and explore a variety of strategies to learn more effectively (DOE, 2002).

Kilpatrick, Swafford and Findell (2001), in their model of mathematics learning argue that mathematics education should focus on the ways in which learners represent and connect mathematical knowledge, the ways in which they understand mathematical ideas and use them in solving problems. This focus is necessary because learning with understanding is argued to be more powerful than simply memorizing.

My concern as a Grade 5 teacher focuses particularly on the poor multiplicative understanding of learners coming from Grade 4. Teaching multiplicative reasoning begins in Grade 1 as indicated in the National Curriculum Statement (DOE, 2002) and progresses

across the grades. This corresponds with Wright, Martland and Stafford (2006) Mathematics Recovery Programme where learners improve their multiplicative reasoning across all levels.

Graven (2011) argues that after school smaller group sessions with learners provide rich opportunities for the development of more participatory identities. She argues that these are potentially powerful spaces for remediation work especially with learners from disadvantaged backgrounds. It is partly for these reasons that I chose to research learners multiplicative thinking and progress within an after school environment. Stott & Graven (2013c) drawing on their earlier Graven & Stott (2012b) work provide the following summary table of contrasting opportunities for working with learners in a classroom versus an after school environment such as a club. Figure 1.1 below illustrates this contrast.

Observed mathematics classroom environment†	Intended club environment
Compulsory attendance is expected as part of formal schooling (in-school time)	Voluntary participation during out-of-school time
Less learner choice in the activities that they work on and engage with	More learner choice in the activities that they work on and engage with
Curriculum and assessment standards as a prescriptive framework strongly influencing choice of content and activities (i.e. the South African curriculum documents)	Curriculum as contextual guide for what is nationally expected of learners but individual learner numeracy levels guide content and activities
Largely acquisition based and often driven by teaching for/to assessments	Participation based; participants are active and engaged
Teacher led and much whole class teacher–learner interaction	Many interactions are learner led with few whole class–mentor interactions and many one-to-one interactions between mentors and learners
Assessment tends to be summative and results in ranked performance (e.g. South African Annual National Assessments)	Assessment is formative and integrated and used to guide individual learning experiences for participants
Prescriptive, teacher-controlled classroom rules within general school rules	Negotiated socio-mathematical norms‡ which may differ from in-school time rules

Source: Adapted from Graven, M., & Stott, D. (2012). Design issues for mathematics clubs for early grade learners. In D. Nampota, & M. Kazima (Eds.), *Proceedings of the 20th Annual Meeting of the Southern African Association for Research in Mathematics, Science and Technology Education* (pp. 94–105). Lilongwe: SAARMSTE

†, It is important to note that items on the left-hand side are not considered negative but rather in line with school norms whilst much more freedom is available in the clubs.

‡, Cobb and Yackel (1996); Hunter (2008).

Figure 1.1: Contrasted classroom and club environments (Stott & Graven, 2013c, p. 2)

PURPOSE OF THE STUDY

The purpose of the study is to inform teaching in my school and to find ways to support primary school teachers at large in developing the strategies to teach and remediate multiplication reasoning. The study tests the possible effectiveness of the use of the multiplicative aspect of the MR programme in a South African context. In addressing the problem the following research questions will be examined.

RESEARCH QUESTIONS

1. What level of multiplicative reasoning is displayed by the learners?
2. How effective will the use of the Mathematics Recovery programme be in a South African context and implemented to a group of learners?

OVERVIEW OF THE RESEARCH METHODOLOGY

An interpretive research paradigm was used to investigate multiplicative thinking and ways to support and to remediate the learning of multiplication.

I used a qualitative case study approach on five Grade 4 learners in my school. Lietz, Langer and Furman (2006) explain that a qualitative method focuses on the co-construction of meaning between the researcher and the participants. It represents the meaning of its participants and acknowledges the role of social construction in establishing meaning. These learners were invited to participate in an after-school intervention aimed at supporting and remediating multiplicative reasoning.

For data collection I conducted Wright et. al.'s (2006) individual orally administered interviews to assess the learners' level of multiplication reasoning. I later analysed the learners' strategies when they responded to the pre and post interview assessment as well as the interview assessments. Wright et al.'s (2006) interviews have been tested for validity and reliability but my analysis was cross checked by my supervisor and co-supervisor in order to support inter-rater reliability.

In order to comply with the ethical requirements of any research project, permission was requested and granted from the principal, teachers and parents of the learners in the Grade 4 class. Consent forms were signed allowing for learner participation, and I undertook to use pseudonyms in the research to protect the confidentiality of the information collected. I explained to the learners that they could withdraw from the intervention and / or research at any point.

In terms of giving something back in lieu of this opportunity to conduct research, I undertook to share my research findings with other teachers at my school and following my research I have offered to provide a series of voluntary after school sessions focused on supporting the development of multiplicative reasoning for all Grade 4 learners.

OUTLINE OF THIS THESIS

This study consists of five chapters. In this section I have outlined the structure of the study as well as presenting the purpose and rationale of the study.

Chapter Two reviews firstly the historical and contextual framework and secondly, provides a perspective on learning using constructivism as the paradigm underlying this intervention. Thirdly, literature relevant to this study on mathematical proficiency is described as a way to think about mathematics learning in that it encompasses the key ways of knowing and doing mathematics. Lastly the literature relevant to the context of this research on supporting multiplicative reasoning and the Mathematics Recovery Programme is reviewed.

Chapter Three describes the research paradigm, research design and research methods applied in this study. In this chapter I also provide an overview of the intervention programme and my methods of analysis. Ethical and validity issues are also interrogated here.

Chapter Four reports on the analysis and findings of the data obtained in this study and also presents the major research findings as they relate to the research questions. This research, being qualitative in nature, includes a quantifiable aspect in the analysis of the learners' levels for pre assessment and post assessment and learners methods for pre and post assessment.

Chapter Five concludes the study by summarising the discussion and focusing on the key contributions of the study. Additionally I discuss the implications of the study and engage with the limitations and opportunities for further research.

CHAPTER TWO: LITERATURE REVIEW

INTRODUCTION

My research focuses on the problem of numeracy in South Africa, and looks at ways to improve the learner performance in number development particularly in multiplication. My study will attempt to find ways to contribute to improving the poor learner performance in numeracy in my district, province and perhaps in the country as a whole. In this chapter, I will look at the history of the South African curriculum and its current state as it has been informed by the Annual National Assessment, which is a measure of the progress of the learners from grades one to six and nine. I will look at multiplicative reasoning and the literature pertaining to it and at the strategies that the learners in my study may use in their multiplicative reasoning. Finally, I discuss the Mathematics Recovery (MR) Programme, how it has been successfully implemented in other countries and its structure.

HISTORICAL AND CONTEXTUAL FRAMEWORK

SOUTH AFRICAN CURRICULUM

South African education has been hindered by the apartheid era which brought with it prolonged segregation by race, and also by language. Its legacy of division is still strong and is often reinforced by economic inequalities. The South African schooling system has been making a conscious effort to heal the division by providing opportunities that will break down the deep inequalities that still prevail in the society. The South African curriculum aims to ensure that children acquire and apply knowledge and skills in ways that are meaningful to their own lives, creating learners who are able to identify and solve problems and make decisions using critical and creative thinking. Since 1994 South Africa has tried to improve the implementation of the curriculum by repackaging the curriculum in various ways in order to improve the quality of teaching and learning. The current curriculum which is the Curriculum and Assessment Policy Statement (CAPS) is regarded as having

simplified and made clear content coverage (Graven, Venkat, Westaway & Tshesane, 2013).

The South African curriculum aims at ensuring that children acquire and apply knowledge, skills and values necessary for self-fulfilment and participation in society. For people to participate effectively in society they must know basic mathematics. Mathematics is therefore regarded as a very powerful “gate keeper”. It is critical for children to develop strong number sense, to be able to perform basic operations, to know the basic number facts and to perform mental arithmetic with confidence (DOE, 2002). For children to use and apply the mathematics they learn at school they need to experience mathematics as a meaningful, interesting and worthwhile activity. By improving performance in mathematics, the learners will benefit from a higher quality of education and the nation as a whole will benefit (DOE, 2002).

SOUTH AFRICAN LEARNER PERFORMANCE ON MATHEMATICS

South Africa participated in the Third International Mathematics and Science Study (TIMSS) in 2003 and the Southern and Eastern Africa Consortium for Monitoring Educational Quality (SACMEQ) in 2002 and 2007. The results of both national and international studies show that South African did not achieve the required international average scores and was placed at the bottom of the lists. The results of the assessment revealed that more emphasis is placed on children being able to think mathematically than on children being able to calculate (Howie, 2004; Maree, Aldous, Hattingh, Swanepoel & Linde, 2006).

The top priority for the South African government on which the Department of Basic Education (DBE) has to deliver is to improve the quality of basic education. The Annual National Assessment (ANA) is a critical measure to assess the progress in learner achievement. Poor performance of learners in numeracy has been an on-going problem in South Africa. South Africa’s Systematic Evaluation programme which tested Grade 3 learners using a standardised test in 2007 and found that the performance of learners in mathematics was at 43 percent (DOE, 2012). The key problems that were identified appeared to be in the classroom, the incapacity of teachers to identify and apply appropriate

teaching methods, teachers with insufficient training and learners who were given too few opportunities to solve problems on their own. Lack of solid numeracy skills will hinder learner's effective learning in other fields of knowledge (DOE, 2011). Other factors that contribute to poor performance are socio-economic factors, demographics and the historical realities of South Africa. Learners in schools with high poverty levels perform poorly in mathematics (DOE, 2012) and consistently achieve much lower learning outcomes than their counter parts in the urban areas. Also in the predominantly rural and historically disadvantaged provinces there is evidence of poor learner achievement. The school where I teach has been affected by a shortage of classrooms and since it started in 2005 until 2011, it has been operating on double shifts with teaching of Foundation Phase in the morning and Intermediate and Senior Phases in the afternoon. Notional teaching time was thus a problem and the lack of infrastructure contributes to the poor learner performance.

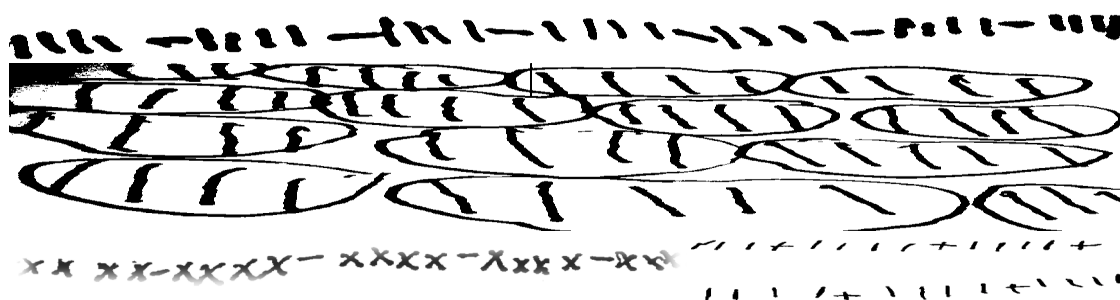
The South African government has introduced many intervention strategies with regard to poor mathematics performance (DOE, 2012). The introduction of Foundations for Learning (FFL) in 2008 to improve mathematics represented a major shift towards providing better methodology and guidance to teachers and ensuring the learners have the materials they need. The National Department of Basic Education (DBE) workbooks were introduced to support the quality of teaching and learning. In 2008 and 2009 trials runs of the Annual National Assessment (ANA) were conducted with a special focus on exposing teachers to better assessment practices and to monitor the extent to which the outcomes of improving the quality and the levels of education are achieved.

The 2011 ANA provided the first national baseline to benchmark annual targets and achievement of 60 percent learner attainment by 2014. It produced sufficiently standardized data in order to allow for the analysis that aimed to enable provinces and districts to support the schools at Foundation Phase.

The key findings were that learners displayed poor computational skills in solving problems involving multiplication of two-digit by one-digit numbers, with only 35 percent of learners in Grade 3 and 40 percent in Grade 4 in the Eastern Cape showing competency (DOE, 2012). Supporting this, Kilpatrick et al. (2001) suggest that mathematics education should focus on the ways in which learners represent and connect mathematical knowledge, the

ways in which they understand mathematical ideas and use them in solving problems. In the ANA's learners revealed an inability to translate problems that are posed in words and to write the problems out in various ways to enable them to solve the problem using mathematical techniques. In a question-by-question analysis of the 2012 Eastern Cape Provincial performance reveals that a mere 26% of learners got the multiplication problems correct (DOE, 2012). Realising that this problem was systemic from the Foundation Phase onwards, the Basic Education Department has focused on improving the schooling in the Foundation Phase.

As a Grade 5 teacher in my school I found that many learners are particularly poor in numeracy especially in multiplication, which indicates perhaps that the learners lack the required foundations in mathematics. I found that my learners struggle with multiplication. Multiplicative teaching is introduced in the early grades at the beginning of Grade 1 as repeated addition as indicated in the National Curriculum Statement (DOE, 2002) and continues up to Grade 3. At that stage learners should have developed more efficient strategies for calculations, which would indicate that they are progressing across the grades. The multiplication tables are stressed in Grade 3. In Clark and Kamii's (1996) study it was noted that multiplicative thinking appears in Grade 2 and multiplication tables are stressed in Grade 3. They also pointed out that learners should not be given multiplication that they cannot do, but should be allowed to solve the problem on their own. In their view they highlighted the point that if a teacher is to teach multiplication they must first understand the nature of the multiplicative thinking of the learners. It is evident that my learners in my class lack the ability to make the shift from concrete counting based strategies to more abstract strategies, i.e. learners cannot find the answer to a multiplication problem without using concrete objects (either counting with fingers or using tallies or small circles).



Learners seem to lack multiplicative reasoning strategies. As a lead teacher working closely with the district I noticed that the problem does not only apply to my school it seems the district is experiencing the same problem when it comes to multiplication. For this reason, my research is therefore grounded on an eagerness to find strategies that could help to improve the multiplicative reasoning of the learners.

Having looked at the Mathematics Recovery (MR) Programme designed by Wright et al. (2006) it seemed to have the potential to help to address the problem. It is thus my aim to explore the use of the MR programme with a group of learners; with the intention that this study could also point to the possibility of using the MR programme in whole class situations, to demonstrate that MR as a programme helps in number early learning and to open up further avenues for research in both South Africa and internationally.

MATHEMATICAL PROFICIENCY

As discussed earlier, South African education needs to improve mathematics education especially in the early years of schooling to enable learners to think critically and solve numerical problems. Kilpatrick et al. (2001) used the term *mathematical proficiency* to capture what it means to learn mathematics successfully. Mathematical proficiency provides a way to think about mathematics learning in that it encompasses the keys of knowing and doing mathematics. For them mathematical proficiency indicates learners who understand basic concepts, are fluent in performing basic operations, exercise a selection of strategic knowledge, reason clearly and flexibly, and maintain a positive outlook toward mathematics. The five interwoven strands of mathematical proficiency are thus:

- *Conceptual understanding:*
an integrated and functional grasp of mathematical ideas;
- *Procedural fluency:*
a knowledge of procedures, when to use them, skills and flexibility in using them accurately and efficiently;
- *Strategic competence:*
the ability to formulate mathematical problems, represent them and solve them;

- *Adaptive reasoning:*
the ability to think logically about the relationships amongst concepts and situations and to justify and prove the correctness of mathematical procedures;
- *Productive disposition:*
the ability to see sense in mathematics, seeing it as useful, worthwhile and encourage learners to believe that they are capable of learning it.

Kilpatrick et al. (2001) use these strands of mathematical proficiency in an integrated manner, so that each reinforces the others. Askew (2013) reinforces the integration of the strands by indicating that fluency in calculation and reasoning about the number system are mutually entwined. If all the strands are involved in teaching and learning it will assist in developing multiplicative reasoning in learners. Learners will acquire higher levels of mathematical proficiency when they have the opportunity to use mathematics to solve multiplication problems fluently. This perspective on proficiency resonates strongly with my aim to explore how learners develop their fluency in strategies when responding to interview questions based on multiplicative reasoning.

PERSPECTIVE ON LEARNING

My research will be framed by a constructivist view of learning. Constructivism draws on the developmental work of Piaget (1977) who asserts that learning occurs by an active construction of meaning. Constructivism embodies the metaphor of learning as construction of knowledge which results from the process of making sense of experience. Piaget (1985), concerned with the way children construct knowledge, recognized the importance of the self-regulation process in individual learning. Anghileri (1989) explains that during the construction of knowledge the learner's skills are integrated simultaneously in working memory.

In constructivism, learning is a cognitive re-organization of thinking which involves a structural shift in cognition. The learner has to organize their thinking, make sense of the new knowledge and accommodate it in order to achieve a higher level of thinking. Constructivism's central idea is that human learning is constructed, that learners build new knowledge upon the foundation of previous learning. Fosnot (2003) explained that in the

constructivist theory of learning, learners do not take in and absorb information, rather they interpret, organize, and infer about it with the cognitive structures they have previously constructed. Learning occurs when learners are actively involved in a process of meaning making by cognitive re-ordering of new concepts with prior concepts by reflecting on their actions therefore knowledge is constructed rather than passively received.

Von Glasersfeld (1989) emphasises the point that learners should construct their own understanding and that they do not simply mirror and reflect what they read. In constructivism, knowledge is constructed from experience, that learning is a personal interpretation of the world and an active on-going process. Piaget (1985) believed that the cognitive development in children depended on the experience with their physical and social environment. Conceptual growth comes from the negotiation of meaning, the sharing of multiple perspectives and the changing of our internal representations through collaborative learning. Anghileri (1989) refers to the process of learning mathematics as active meaning making. In the constructivist theory of learning learners search for the pattern, raise questions; construct their own models, and the big ideas and strategies that the learners demonstrate are discussed.

Fosnot (2003) refers to a 'landscape of learning' which is the journey of cognitive development comprised of big ideas, strategies and models. Learners need to be encouraged to share their big ideas and strategies and during this process their schema are developed, modified or reinforced through accommodation and assimilation. Sharing their strategies promotes confidence and a positive disposition is developed (Kilpatrick et al., 2001). Fosnot (2003) suggested that mathematics classrooms should be like a workshop that encompasses the model of constructivism based reflection and inquiry, generalization and problem discourse where learners are actively involved in their process constructing knowledge. Wright (2003) also supports that classroom practices based on constructivism demands that the teacher assists the learners in their understanding of the content to be learnt and helps the learner to make their own generalization as they explore problems in the classroom. A teacher is referred to as a facilitator who provides guidance and creates the environment necessary for the learners to question and reach their own conclusions, not to lead children

to solutions, but to monitor and ask the questions. A facilitator must provide opportunities to investigate the adequacy of learner's present understandings.

Constructivists believe that learners need to be challenged with tasks that lie just beyond their level of mastery. This increases motivation and builds on their previous successes. Constructivism says that in terms of the sequencing of subject matter, the basic ideas of a topic should first be introduced before being built on further (Sullivan, 2011). Fosnot (2003) explains that the big ideas and strategies serve as important landmarks for the teachers to use as they plan the journey of development with the learners. As they move on the same path, their development depends on individual differences, hence they have their own trajectories and the teacher needs to acknowledge the difference in learner's thinking (Kühne, Lombart, & Moodley, 2013). When children discover patterns and relationships themselves, they are more likely to understand and remember the concept being developed.

The Mathematics Recovery Programme has been designed from a constructivism perspective of learning and it allows the learners to explore in order to mathematise. Mathematisation is a step in the journey to construct the mental maps that will eventually become tools for thinking. According to Fosnot (2003) mathematics should be thought of as a human activity of mathematizing, not as a discipline of structures to be transmitted, discovered but schematizing, structuring and modelling the world mathematically. Wright (2003) and Wright et al. (2006) have based their framework for assessing multiplicative reasoning (and number knowledge) and their MR programme on the constructivist view of learning.

MULTIPLICATIVE REASONING

Importance and issues

In the last decade or so a number of researchers have written about multiplicative reasoning. See for example (Jacob & Willis, 2003; Mulligan, 2002; Fosnot & Dolk, 2009). Multiplicative reasoning is characterised by a capacity to work flexibly and efficiently with an extended range of numbers for example, larger whole numbers. It requires the ability to recognise and use strategies to solve a range of problems involving multiplication or

division (Mulligan & Mitchelmore, 1997). The learner must have the means to communicate multiplication effectively in a variety of ways for example, words, diagrams and symbolic expressions.

Mulligan (2002) highlighted the importance of multiplicative reasoning in that it is essential in the development of concepts and processes such as ratio and proportion, it is therefore imperative to develop multiplicative structures in the early years so as not to impede the general mathematical development of learners in secondary school. Vergnaud (1983 cited in Clark and Kamii, 1996) indicated that learners who have difficulty with computing often have a problem with multiplication. Teachers need to have mental image of a developmental trajectory along which they could expect children to develop and to understand the nature of multiplicative thinking in order to support children along the path of gradual sophisticated multiplicative reasoning (Wright et al., 2006).

What is multiplication?

When honing in on multiplication reasoning researchers have contrasting ideas for explaining multiplication; some researchers consider multiplication as a faster way of doing repeated addition while others say that repeated addition is an implicit, unconscious and primitive intuitive model for multiplication (Clark & Kamii, 1996). (Anghileri, 1989) indicated that addition forms the basis of multiplication, addition theory processes support the learner to transfer from counting meaning to the cardinal meaning. When referring to multiplication, Clark and Kamii (1996) used Piaget's 1987 work as a point of reference which shows that multiplication is not just a faster way of doing addition but is an operation that requires higher-order multiplicative thinking that children construct out of their ability to think additively. Piaget differentiated addition from multiplication in that addition is the construction of number which is accomplished by the repeated addition of ones, whereas multiplication is a more complex operation that is constructed out of addition at a higher level of abstraction based on a conceptual pattern in the mind which is a schema.

The level of abstraction shows the developmental trajectory of learners being able to solve problems from a concrete level of using manipulative to an entirely abstract level where the learner uses verbal arithmetic. Anghileri (1989) added that for multiplication a learner must

possess some schema for keeping track of the numerosity of the group to be repeated. Similarly, Clark and Kamii (1996) state that multiplication requires the construction of new elements through reflexive abstraction, in abstraction the parts together becomes the new whole and multiplicative structures are seen as a conceptual field that involves many interconnected concepts. Multiplication uses a dimensional model based on the notion of ratio and that multiplication structures rely on addition structures, but they have their own intrinsic organization which is not reducible to additive structures.

Anghileri uses the term of learners being “meaning makers” in the process of learning mathematics where construction of abstract composite units takes place (Anghileri, 1989, p.367). Meaning is understood to be the result of humans setting up relationship, reflecting on their actions, and modelling and constructing meaning. During meaning making some learners showed the recognition of the composite nature of a number, while others used one finger to tally each group that they counted, whereas some showed unitary counting which develops to rhythmic counting in groups and later number pattern. When doing rhythmic counting some interim numbers are progressively internalized which can be detected by whispering, a silent mouthed acknowledgement, which indicates that counting has happened. In the number pattern stage learners relate cardinality of the union of sets. It needs less mental processing and the number procedure involves only two simultaneous counts.

To understand multiplication the following pieces of information need to be coordinated: the number of elements in each group, how many groups and the process for executing the product. The initial idea that needs to be developed for multiplicative reasoning is making and naming equal groups. There are a number of ways that this can be done but two of the most useful ways appear to be counting large collections efficiently for example, by twos, fives or tens and organizing the count and systematically sharing collections (Mulligan & Mitchelmore, 1997). The issue of the language and recording associated with this idea is also important. Talking about “groups of” or “lots of” can get in the way of understanding what is going on, which is actually a count of a count. This explains to some extent why this idea can be so difficult for some children who are expected to move from one-to-one counts like one, two, three ...to counting in a one-to-many count like 1 three, 2 threes, 3 threes

(Mulligan, 2002). Recent research by Anghileri (1989) reported the results of her observations of the behaviours and successful solution strategies of learners as they carried out multiplication tasks is that learners must have mathematical understanding prior to the formal instruction which is grounded by everyday situations to which the children have been exposed and she refers to this as a “framework of knowledge” (Anghileri, 1989, p.367).

Multiplicative reasoning

Many learners have an informal understanding of multiplication as adding the same things together. Anghileri (1989) indicated that learner’s early multiplicative reasoning results from cognitive re-organization of learner’s counting to increasingly sophisticated groups to abstract composite units. Learners need to develop strategies that lead to more efficient mental strategies that build on from the known, e.g. doubling and addition strategies. For developing multiplicative reasoning, Mulligan (2002) has identified counting, subitising, grouping, partitioning and sharing as essential elements of multiplicative structure. Fosnot and Dolk (2009) and Fosnot (2003) present some strategies which they term “Big Ideas”. These are characteristic of a shift in perspective, in logic and paradigmatic shifts in reasoning strategies which include skip counting, using a doubling strategy, using and understanding the distributive and commutative properties. Anghileri (1989) described how learners should develop multiplicative reasoning in that they progress in stages from unitary counting or counting by ones, skip counting and repeated addition by understanding multiplication facts and their application. Mulligan (2002) highlighted that learners move from one-to-one counting, additive composition, many-to-one counting, multiplicative relations, and operating on the operators. Learner’s should establish the value of equal groups by exploring more efficient strategies for counting large collections using composite units and sharing collections equally items in groups (1 three, 2 threes, 3 threes, 4 threes ...), they focus on the number in the group (3 ones, 3 twos, 3 threes, 3 fours), on the number of groups (three groups of fives) and should be able to rename the number of groups e.g. 3 fours can be the same as 4 threes which is twelve (Mulligan & Mitchelmore, 1997).

Mulligan (2002) showed multiplicative reasoning as a mathematical structure which is described as spatial organization of objects such as arrays and squares, and that these are ways of promoting multiplicative reasoning where the whole and equal groups are reinforced by visual images.

My understanding of multiplicative reasoning is guided by the five levels detailed by Wright et al. (2006) which are related to equal grouping, counting strategies include rhythmic, double, skip counting where a learner is simultaneously aware of both the composite and unitary aspects. These levels are discussed in detail in the following sections.

EARLY NUMERACY INTERVENTION

Learners who have difficulty with number sense will have a problem in mathematics progression (Askew, 2013) and therefore need early intervention to alleviate the problem. In support of this, Wright et al. (2006) described a need for early numeracy intervention of learners at an early age because difficulty in numeracy can affect performance even in other aspect of the curriculum. Intervention in the early childhood years can be effective in reducing disparities in mathematics achievement (Bobis et al., 2005).

The early years of schooling are a crucial period to foster the basic skills and love for numeracy (DOE, 2011). For learners to be successful in later mathematics activities and to use mathematics effectively in life, they must have a sound understanding of elementary mathematics concepts, and to develop a positive attitude towards the learning of mathematics, and the belief that an understanding of mathematics is attainable (DOE, 2011). Early intervention requires a teacher to play a vital role in the learner's development, and it needs to be carefully planned and to cater for learners from different backgrounds.

Learners with a sound foundation in mathematics develop cognitive abilities such as patterns, reasoning, processing speed and working memory for undertaking mathematics (Anghileri, 1989). Early intervention can prevent the development of negative attitudes and mathematical anxiety in learners and promotes a productive or positive mathematical disposition.

One such programme of intervention is that designed by Wright et al. (2006). I describe this below.

MATHEMATICS RECOVERY PROGRAMME

The notion of early intervention in numeracy can be problematic for educators if they are unable to diagnose what the source of the learner's difficulties is. Teachers need a diagnostic tool to identify the specific problems that they are experiencing with learners and one which can be used to profile learner strengths and weaknesses, a tool that will target particular learner's misconceptions and less sophisticated strategies. Wright et al. (2006) have developed the Mathematics Recovery (MR) programme framework as a learning pathway in an effort to increase learner achievement in number concepts and to provide tools such as these. Specifically, the MR programme includes assessment interviews (including tasks and schedules), a teaching framework and teaching resources and a learning progression model for early number learning.

The MR programme has been applied to a multitude of situations and contexts (Wright et al., 2006). Although Wright et al.'s (2006) MR approach was originally created for intervention in number learning and focused mainly on students in the second year of school (six and seven year olds), the programme has since been extended to include both early and advanced multiplication and division. I will however, only focus on the multiplication aspect as it is my main concern and the focus of this study. The implementation of the multiplication aspect of the MR programme will assist in addressing my second research question: How effective will the use of the Mathematics Recovery Programme be in the South African context? The main focus in my research is on assessment and teaching in the form of an intervention in multiplication.

Countries like Australia, the United Kingdom, Ireland and the United States have used the Mathematics Recovery Programme and have found that it has provided opportunities for developing confident and capable mathematics in learners in the early years of schooling (Bobis et al., 2005). Wright (2003) explains that the MR Programme as a form intervention accords strongly in terms of both theory and practice with the current cutting-edge approaches in classroom teaching. The underlying theory and approaches of the programme

are equally as applicable to average and able learners as to those who are experiencing learning difficulties. The MR Programme is grounded by constructivist teaching experiments where learners construct their own knowledge. The methodology used in the Mathematics Recovery Programme was designed by Steffe (1992, cited in Wright, 2003).

The MR programme involves intensive, individualized teaching and is an approach to early number learning that integrates interview-based assessment and a model of early number learning. The assessment allows the teacher to document a learner's current knowledge and plan subsequent instruction (Wright, 2003).

THE FRAMEWORK FOR ASSESSMENT AND LEARNING

As indicated above, the MR Programme includes assessment interviews (including tasks and schedules), a teaching framework and teaching resources and a learning progression model for early number learning. I will discuss each of these elements below.

MR Programme: Learning Framework in Number (LFIN)

The learning framework (called the Learning Framework in Number or LFIN) provides essential guidance for assessment and teaching in early number. The use of a framework is to enable profiling of student's current knowledge and levels that indicate numeracy development of learner's knowledge (Wright et al., 2006).

According to Wright et al (2006), for a learner to be able to develop multiplicative reasoning he must go through different stages of cognitive development. Wright et al.'s (2006) levels of assessing multiplicative reasoning correspond with Piaget's development stages which proposes four key stages: sensory-motor (concrete objects), pre-operational (mastery of symbols), concrete (how to reason) and formal operation (formal logic) where learners in each stage think and reason differently.

The following five levels in Early Multiplication in the LFIN will assist me in assessing the multiplicative thinking levels of my sample of learners:

- *Level 1: Initial Group*

a learner uses perceptual thinking to establish the numerosity of collections of equal

groups when items are visible and counts by ones not in multiples, i.e. the child uses perceptual counting to make groups of specified size from a collection of items, the learner does not count in multiples.

- *Level 2: Perceptual Counting in Multiples*
a learner uses multiplicative counting strategies to count visible items in equal groups that involve counting in multiples. Counting strategies include rhythmic, double, skip counting; the child relies on visible items.
- *Level 3: Figurative Composite Grouping*
a learner uses multiplicative counting strategies to count items in equal groups in cases where the individual items are not visible.
- *Level 4: Repeated Abstract Composite Grouping*
the learner counts composite units in repeated addition or subtraction, that is, uses the composite units a specified number of times. The learner is simultaneously aware of both the composite and unitary aspects.
- *Level 5: Multiplication as Operations*
a learner can regard both the number in each group as a composite unit, and can immediately recall many basic facts for multiplication and division. A learner is able to use a known fact to work out an unknown fact, the learner use $3 \times 6 = 18$ to work out $18 \div 3$.

Assigning learners to these levels in pre and post assessment interviews will help me to answer research question one in determining what level of multiplicative reasoning is displayed by the learners.

In my research I used these five levels to assess the multiplicative reasoning of the learners I have sampled and will use the recording schedule proposed by Wright et al. (2006) which I discuss in Chapter Three. Having understood the level of understanding of the learners the teacher needs to intervene by developing teaching sequences that will link to learner's current understanding of mathematical reasoning. The learning framework indicates where to take the learner in terms of remediating multiplicative reasoning.

MR Programme: Assessment Interviews

Interviews that follow a schedule of set procedures are used in assessing the learner. These interviews have a social aspect where the teacher provides a supporting and encouraging environment. According to Wright, the interview-based assessment has two purposes:

“Firstly, it should provide a rich, detailed description of the student's current knowledge of early number. This rich picture is necessarily in terms of the aspects of early number knowledge that are reflected on the schedule of assessment tasks. Secondly, the assessment should lead to the determination of level of a learner by looking on the relevant levels in the framework of assessment and learning”
(Wright, 2003, p.8).

Wright et al. (2006) describe their assessment “as a diagnostic assessment that aims to provide extensive and detailed information about the child’s numerical knowledge” (p. 30). Diagnostic assessment involves a teacher or a mediator assessing a child’s understanding of a concept, by looking at the strategies the child uses to find the answer. The teacher is not only interested in the answer of the child, but also in the methods that the child uses to get the answer. The assessment aims to provide more formative ways of addressing problems in early numeracy. The MR assessment takes the form of an orally conducted interview. The instrument is administered to each individual learner in a one-to-one interview lasting between 45 to 60 minutes (Wright et al., 2006).

The interview results do not result in scoring but focus rather on understanding the strategies used by learners when solving number problems. Further, they aim to promote mental computational strategies. Some of the items on the assessment are structured in such a way that if a learner answers the question correctly, the assessment leads on to a more advanced question. The aim of the assessment is to profile the learner and to find out what level a learner operates at based on the strategies the learner uses to find the answer. Through the use of profiling, a model of student development can be constructed which will reveal what form of intervention is necessary and which follow up instructional activities can be applied. The assessment interviews contain a description of assessment tasks that are

closely linked to the levels on each model as indicated above. For my research I focused on Early Multiplication.

Assigning learners to these levels in pre and post assessment interviews will help me to answer research question one in terms of what level of multiplicative reasoning is displayed by the learners?

MR Programme: Teaching Framework

Another key aspect of the MR programme is thus a comprehensive teaching framework that is used for planning interventions that aim to remediate multiplicative reasoning.

The teaching framework takes account of the learner's current knowledge in terms of the LFIN and draws from a bank of instructional settings (resources and manipulatives) and activities. The LFIN determines where the learner is developmentally and the teaching framework indicates where to take the learner. MR teaching is informed by an initial comprehensive assessment and on-going assessment through teaching where the teacher is informed about the learner's current knowledge and problem solving strategies.

MR teaching has the following guiding principles: the teaching approach is inquiry based where a learner is presented with tasks which are problematic for them, so they are engaged with problems when trying to solve them. Over an extended period the learner tries hard to think about the solving strategies for a problem a process of cognitive re-organization and anticipation occurs. On-going assessment plays a critical role through teaching which keeps the teacher informed of the learner's progress and supports the learner. Teaching is focused on individual learner for extending learner's current knowledge (Wright, 2003). The teacher supports and builds on the strategies that the child demonstrates. Sufficient time is provided to the learner to solve the problem where they will be engaged in thinking and encouraged to reflect on the result of their thinking. The teacher continually assesses the learner's progress during the teaching session through careful observation and review of the teaching sessions. I took these principles into account when planning interventions and during interventions.

Within this framework, the key elements of teaching as stated by Wright (2003) are the processes of micro-adjusting and scaffolding. He refers to micro adjusting as the on-going process of presenting each task, which relates to the previous task. Scaffolding refers to the provision of support in the form of access to materials or teacher modelling, which is gradually withdrawn according to the student's responses. I took these processes into account when planning my interventions.

CONCLUDING REMARKS

In this literature review chapter, I situated my study within the broader South African context and reviewed a range of literature relating to multiplication. I elaborated on the framework of mathematical proficiency, constructivism as a perspective of learning, the MR programme including MR framework for assessment and learning.

In the next chapter I outline the research design and the methodology used in this research which allowed the research questions to be addressed.

CHAPTER THREE: RESEARCH DESIGN & METHODOLOGY

INTRODUCTION

The purpose of the study is to inform mathematics teaching in my school and find ways to support primary school teachers at large in developing the strategies to teach and remediate multiplication reasoning. In addressing the problem the following research questions will be examined.

1. What level of multiplicative reasoning is displayed by the learners?
2. How effective will the use of the Mathematics Recovery programme in the South African context be when implemented to a group of learners?

This chapter describes the research design and methodology employed in this research study and includes a discussion of the research sample, research methods, and sources of data, data collection, data analysis, quality criteria (validity and reliability), ethical considerations and the limitations of the study. Figure 3.1 outlines the presentation of the chapter.

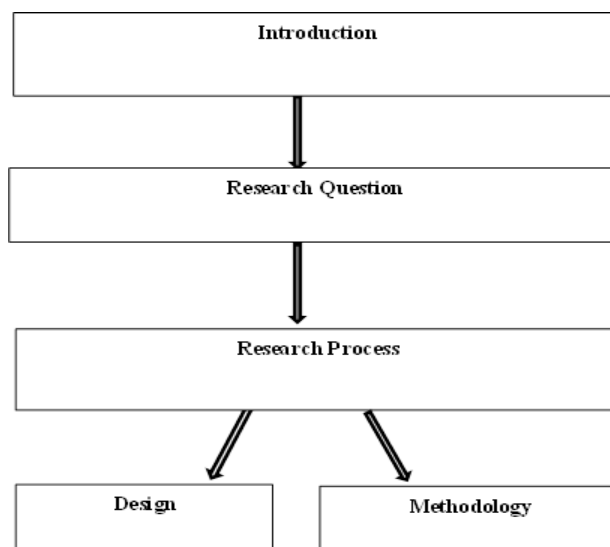


Figure 3.1: Outline of the Research Design and Methodology

RESEARCH QUESTIONS

This study is guided by these research questions:

1. What level of multiplicative reasoning is displayed by the learners?
2. How effective will the use of the Mathematics Recovery programme be in the South African context when implemented to a group of learners?

Table 3.2 below presents a summary of the research design and describes how the research design links to the research questions.

Table 3.2: Research Design links Research Questions

Key Research Questions		<ul style="list-style-type: none"> • What level of multiplicative reasoning is displayed by the learners? • How effective will the use of the Mathematics Recovery programme be in the South African context when implemented to a group of learners? 	
Research Design of Study		Case study (interpretive)	
Nature of data collected		Qualitative with some quantifiable aspects	
ACTION PLAN	Data Collection Instruments	Individual learner interviews	Participant-observer
	Data Source	Videos of interviews and intervention sessions Interview scripts Assessment schedules	Researcher journal
	Data Analysis	Time series analysis	
Ethical Considerations		Confidentiality and anonymity, informed consent	
Strengths of Research		To inform teaching in my school and find ways to support primary school teachers at large in developing the strategies to teach and remediate multiplication reasoning and using the MR programme of intervention.	

SAMPLING

In this study I used a purposively selected sample of six Grade 4 learners. To select these learners I administered a basic written assessment instrument to a class of Grade 4's which specifically looked at assessing their knowledge and understanding of multiplication. This class was a convenience sample as I used one of the Grade 4 classes in the school where I was teaching. I used the scored results from the test to select six learners as my sample: two top scoring learners, two middle scoring learners and the two bottom scoring learners. These learners were invited to participate in an after school intervention programme aimed at supporting and remediating multiplicative reasoning. One of the sampled learners was not available for the interviews and for that reason; I had a final sample of five learners.

RESEARCHER

At the time of conducting this study I was employed at a school in the Eastern Cape as a Grade 5 to 7 mathematics teacher. Therefore I brought to this research study an understanding of the problems of mathematics in the primary school.

RESEARCH METHODOLOGY

RESEARCH PARADIGM

A qualitative, interpretive research paradigm was used to investigate multiplicative thinking and ways to support and to remediate this.

It has been our view for some time that the processes of education, teaching and learning are so complex and multifaceted that to focus only on cause and effect, products and outcomes or correlations in research on schools is of limited value. The complexity of education demands the use of many different research techniques and models. The most productive approach we believe is a qualitative one (Hitchcock & Hughes, 1995, p. 25).

Since learners are unique they interpret things differently so I observed the meaning that the learners constructed as they interacted with me in the application of the various recovery strategies.

RESEARCH DESIGN: A CASE STUDY

I used a qualitative case study approach of five Grade 4 learners in my school. Lietz, Langer and Furman (2006) explain that the qualitative method focuses on the co-construction of meaning between the researcher and the participants. It represents the meaning of its participants and acknowledges the role of social construction in establishing meaning. A case study uses a small group in order to learn more about social realities in a particular context. It allows the researcher to probe with the necessary depth and recognition of the context and hopes to find out knowledge that will be applied to address the social problem (Janse van Rensburg, 2001). Case study research enables one to arrive at an understanding of a complex situation and it can add value to what is already known through previous research. This is substantiated by Merriam (1998), who posits that “investigators use a case study design in order to gain an in-depth understanding of the situation and its meaning for those involved” (p.xii). Case study research is ideal for understanding and interpreting observations of educational phenomena (Merriam, 1998). As is the case in this research study, case study research generally answers questions of a “How?” nature.

RESEARCH METHOD

I use mostly qualitative methods with some quantifiable aspects to produce a rich data set. Qualitative data was used in the pre-assessment and post assessment in relation to allocating levels. The results of the pre-assessment were used to determine whether a need for such interventions was necessary. They were also used to determine learner’s mathematical needs with regard to multiplication. Those needs were then addressed by an intervention designed on the MR programme principles. The results of the post assessment were compared with the results of the pre-assessment, to determine whether the learners had progressed from one level to another. Quantifiable and visual data in the form of graphs and matrices allowed me to track changes in multiplicative proficiency over time, and to gain insight into whether

individual learners had progressed or not in the time between administrations of the assessment instrument. Quantifiable data was recorded on an Excel spread sheet, which was also used to create the graphs and matrices.

RESEARCH TIMESCALE

The timeframe for the study was from March 2013 to April 2013 and focused on the use of the MR programme starting with pre assessment, teaching using the intervention strategies for multiplication and the post assessment. I carried out four intervention sessions over a period of four weeks, with one session each week of approximately one hour in duration.

DATA COLLECTION METHODS

One key aspect of the MR programme is the videotaping of assessment and teaching sessions. Thus (Wright, 2003) argues:

The process of videotaping serves several fundamental and important purposes: first, it is the basis of the distinctive approach to assessment (as described earlier) and it provides permanent records of the assessment process. Second, teacher's viewing their own and colleagues' videotapes is a key component of teacher professional development. Finally, videotaping in the way it is used in MR, is critical for teachers' individual and collective professional learning (Wright, 2003, p. 11).

Videos taken by myself, focused on strategies used by the learners when engaged in multiplication activities and the way learners responded to the oral tasks during the interviews. The learners seemed comfortable with the use of this technology and the use of video recording allowed myself as the researcher to focus on facilitating the interviews whilst issues like gestures, body language and so on were recorded for later viewing.

DATA COLLECTION INSTRUMENTS

I used three instruments to gather my data:

1. A pre and post assessment instrument based on Wright et al.'s (2006) individual orally administered assessments in the form of interviews to assess the learners' level of multiplication reasoning.
2. A research journal
3. Reflection on videos

INTERVIEWS

The research interview can be understood as a two-person conversation initiated by the interviewer for the purpose of gathering research-related information (Cohen & Manion 1980). While this definition of a research interview makes sense, it limits the possibility that a research interview may have a dual purpose. "Rich data" is referred to as long-term involvement and intensive interviews that will enable thick descriptions of what is going on (Maxwell, 2004). This rich data in my study (Maxwell, 2004) was provided by video-recorded interviews which gave me an in depth understanding of the issues learners in Grade 4 have with multiplicative reasoning. The interviews lasted approximately 60 to 75 minutes for each learner.

The interviews were interesting conversations stimulated by a set of items and probes in order to find the strategies used by the learners. Before the interviews I did a trial run with my supervisor and co-supervisor on how to conduct the interviews and video record at the same time.

All interviews were conducted individually in the classroom. If a learner gave an answer instantly a probing question was asked. Learner responses were noted on an assessment schedule based on Wright (2003) (see Appendix B) and were video recorded for later analysis. The video-recording and noting of learner's responses on the assessment schedule would allow me to profile the learner in terms of the LFIN levels after the interview had taken place. The assessment interviews focused on understanding the strategies and methods used by learners when solving number problems during the interviews. Items in the

interview are structured in such a way that if a learner answers the item correctly, the interview leads on to a more advanced item. Each set of interview items informs an LFIN level at which the learners are operating. The LFIN assessment interviews helped me to address both of my research questions in finding out what level a learner operated at based on the strategies demonstrated by the learner to get the answer and to see the impact of the MR on the learners.

Both interviews took the form of a structured, question-response interview. The LFIN items are a fixed set of items as indicated in table 3.3 in a fixed order, and according to Breakwell (1995), this constitutes a structured interview. She also states that research interviews require a particularly systematic approach to data collection in order to maintain validity and reliability (Breakwell, 1995). Wright et al. (2006) advised that the interviewer does not change the items or the order of the items and I ensured that I followed this suggestion when administering the interviews. As such the interview items were the same for both interviews. I was not only interested in the answer provided, but also in the method that the learner used to arrive at the answer. Both assessment interviews aimed at providing more formative ways of addressing problems in multiplication.

The interview for multiplication consists of five major tasks. Table 3.3 indicates which items fall under each task group and the focus of the task is explained.

Table 3.3: Interview tasks indicating intended learner strategies

INTERVIEW ITEMS	ITEM NUMBERS	FOCUS OF THE TASKS / INTENDED STRATEGIES
Forming equal groups	1a, b	Does the child make the groups by moving one counter at a time or by moving multiples of counters? Able to know the total in each group without counting. The learner starts from one when checking the number in a group.
Tasks involving FNWS of multiples	2a, b, c, d	Checking whether the child has the FNWS in multiples and where the sequence of skip counting begins to break down the number in a group

Tasks involving visible items in arrays	3 a, b, c	Uses visible items arranged in arrays. Allows making a slight movement of the counters constant with touching counters in one to one correspondence activities. The task items are linked with conservation and commutative principle in that reversing the order of two factors does not change the product
Task involving equal groups of visible items	4a, b, c, d (i), (ii)	The task emphasizes counting in multiples but in a different context is checking the mode of counting. The child has to first meet partition division and quotient division
Tasks involving screened items	5a, b, c, d, e	Assessment task where items are screened and will see if the child uses multiplicative counting strategies when they cannot see the objects

The advanced tasks are shown overleaf.

Table 3.3 continued: Interview tasks indicating intended learner strategies

ADVANCED INTERVIEW ITEMS			
Tasks presented verbally without visible or screened items		1a, b, c, d	Checking if the child can count in multiples by using knowledge of abstract composite units. The learner is able to recall or quickly derive many of the basic facts and be aware of the communicative principle of multiplication
Task of model for multiplication and division	commutative principle of multiplication, inverse relationship between multiplication and division	2a, b, c, d, e	Tasks to exhibit composite units, understanding commutative principle of multiplication, inverse relationship between multiplication and division and multiplication facts to derive division facts
	Area Multiplication	3	Check whether the learner reason in terms of a unit of area

RESEARCH JOURNAL

In my role as researcher, I observed learners whilst facilitating intervention sessions. Cohen et al. (2000, p. 305) point out that in observations, researcher roles can lie on a continuum. At one end lies the complete participant, moving to the participant-as-observer, thence to the observer-as-participant, and finally to the complete observer. In order to carry out these observations, metaphorically I move from ‘complete participant’ to ‘participant-as-observer’ and back again. As a participant-observer, a research journal served to consolidate and record research ideas that emerged out of reflections from my research presentations and intervention sessions as a teacher. I also recorded findings and observations after each assessment interview I observed and when I had discussions with my supervisor and co-supervisor.

REFLECTION ON VIDEOS

Reflecting on the videos helped me to analyse both the pre and post assessment interviews using Wright et al.’s (2006) framework. I began by noting overall performance on the two assessments using quantifiable data based on levels and stages for the LFIN. Then I compared the results of the two assessments and looked for differences and similarities between the two.

Learners were allocated to levels and stages of the LFIN based on this analysis and using the assessment schedule and video data. I used a descriptive narrative to describe each learners pre and post assessment in detail.

In the next section, I explain how I planned the intervention programme.

OVERVIEW OF INTERVENTION PROGRAMME

I used the teaching programme described in Chapter Two and was guided by the principles of teaching using the MR programme and the key elements (page 32). The intervention was planned as a teaching approach where learners were engaged in thinking deeply to solve the problems that challenged them. The teaching was informed by results of the pre assessment interview and on-going assessment in each session.

For the intervention, I selected from the bank of teaching procedures and resources suggested by Wright et al (2006) in their book. These varied depending on-going observations of the learners' development. The intervention was done as a whole group for one hour once a week over the period of four weeks. Some examples of Wright et al.'s (2006) task are shown below whilst the full range is available in Appendix D.

EQUAL GROUPS

1.1 Making equal groups

Learners are given 12 counters and are asked to make three groups with four in each group.

How many groups? / How many in each group?

1.2 Describing equal groups

Counters/ unifix cubes/ containers

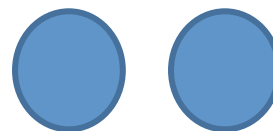
Individuals

Each learner is given 10 counters and two containers, ask them to share it equally into the containers, how many counters in each containers?

Use the same activity but increase the number range

1.3 Combining and counting equal groups

Place out ten 2-dot cards, put each 2-dot card and let the learner count putting each card after the other.



Similar with three dot, four dot and five dot cards

1.4 Determining the number in an equal share

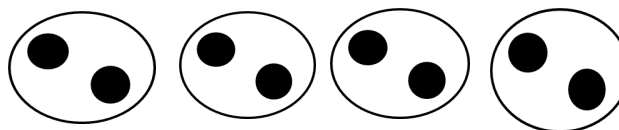
Resources: dots cards/ unifix cubes

Learners are given six counters. Share them amongst three people.
How many will each get?

Use 10 and 2, 12 and 6, 18 and 3

1.5 Determining the number of equal groups

Resources: dots cards/ unifix cubes



Place out four 2-counter cards:

How many counters are there on each card?

How many cards are there?

How many counters are there altogether?

APPROACH TO ANALYSIS

In this section I outline how I approached the analysis of data that I collected. I looked at:

- **Preparing the data for analysis** - Coding data by assigning numeric values
- **Exploring the data** - Visually inspecting data

PREPARING THE DATA FOR ANALYSIS

PROFILING OF LEARNERS

Analysis of the MR assessment interview involved reviewing the videotaped interviews in conjunction with the written interview schedule for each learner. An explicit feature of the MR assessment is that they are not only concerned with whether the answer is correct or not, but also with how the learner arrives at the answer. Each interview was transcribed using a coding system and a written summary of the assessment interview. For coding the assessment I used Wright et al.'s (2006) coding system which is used to derive the maximum information from the child's performance in the assessment interview. The codes indicated how the child responded and the way they gave answers and are shown in Table 3.4 below.

Table 3.4: Assessment schedule codes (Wright et al. (2006))

✓✓	correct with certitude	??	needs time to think
✓	correct	x✓	initially incorrect and correct
? ✓	needs time to think and correct	Rev	assessor revisit an item
Λ	omission of a number in FNWS	IDK	I don't know
Red	teacher prompt		

This coding also allowed me to allocate each child to a level or stage of the LFIN framework for multiplication (the focus of this study) as indicated in table 4.1 (in Chapter Four) and to address question one of my research. I analysed the videos of the interviews of the five learners and coded each of them (see table 3.5 for an example of one learner). This

profiling provided me with a model of development of in multiplication for each learner in my study and through which the teaching programme for this study (in the form of an intervention) was designed. Table 3.5 shows an example of coding for Learner A. The coding for the other learners is available in Appendix B.

Table 3.5: Example for Learner A: Assessment schedule

STRATEGIES	Perceptual counting by ones		Visible and count in multiples														Multiplicative strategies where items are screened					Abstract composite and unitary aspect					Coordinate two composite units				
	1a	1b	2a	2b	2c	2d	3a	3b	3c	3d	4a	4b	4c	4d i	4d ii	5a	5b	5c	5d	5e	ADVANCED MULTIPLICATION										
TASK QUESTIONS	1a	1b	2a	2b	2c	2d	3a	3b	3c	3d	4a	4b	4c	4d i	4d ii	5a	5b	5c	5d	5e	1a	1b	1c	1d	2a	2b	2c	2d	3		
PRE-ASSESSMENT RESPONSES	✓	✓	∧	✓✓	✓✓	✓✓	✓	✓	✓	✓	✓✓	x✓	??	Rev✓	?✓	✓	??	Rev??	?✓	??	IDK	IDK	IDK	IDK	IDK	IDK	IDK	IDK	IDK		
POST ASSESSMENT RESPONSES	✓	✓	✓	✓	✓	✓✓			✓	✓✓	✓✓	✓✓	✓✓	✓	✓	✓✓	✓✓	✓✓	✓✓	✓✓	IDK	IDK	IDK	IDK	IDK	IDK	IDK	IDK	IDK		

NOTE: questions 1a to 2c are level-one questions with regards to the LFIN. In other words, they are structured in such a way that the learner is expected to answer at that level. Questions 2d to 4d are level-two questions. Here the learner can answer using a lower or a higher-level strategy. Questions 5a to 5e and the tasks marked ‘advanced multiplication’ are where learners are expected to answer using higher order level strategies.

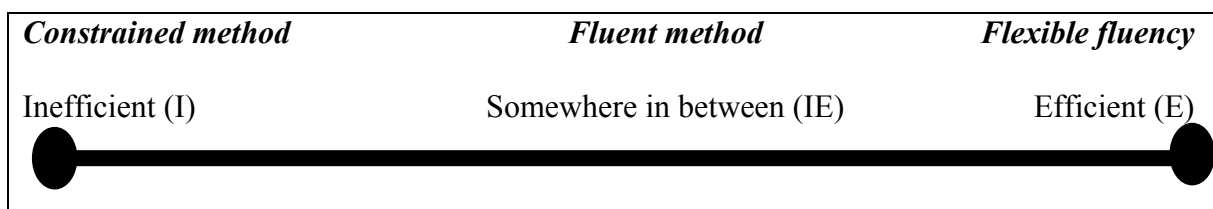
LEARNERS MULTIPLICATIVE PROFICIENCY

For analysis of multiplicative proficiency I quantified the qualitative data. Although the interviews are not meant to result in scores (Wright, 2003) in South Africa researchers are beginning to work with scores as a way of showing a picture of learner progress when working with the MR programme. See for example Graven and Stott (2012a) and Stott and Graven (2013b).

When using the same Wright et al interview tasks, Graven & Stott (2013b) explained that although the oral interview instruments and scripts show the methods the learners used to solve a task, coding responses simply as correct or incorrect, fails to reveal whether the learner has used a more flexible method or is more fluent in answering a question. Thus they developed an efficiency spectrum for procedural fluency that ranged from restricted / constrained procedural fluency towards elaborated and fully flexible fluency.

The strategies that my learners used confirmed the notions of efficiency and fluency I have coded and analysed in the oral interview and showed an overlap of learner strategies/responses. My data displayed a range of responses from restricted / constrained procedural fluency towards elaborated and fully flexible fluency. This resonated with my own sense that learner’s multiplicative proficiency or fluency needed to be captured in its own right. Thus I adapted Graven & Stott’s (2013b) spectrum for procedural fluency into multiplicative spectrums for *each task* in the interview to help me understand learner progress. This progress would be evident when learners moved to the middle or upper end of the spectrum. Figure 3.6 below shows my adapted spectrum of multiplicative proficiency.

Figure 3.6: Spectrum of multiplicative proficiency for Constrained, Fluent and Flexible fluency



(Adapted from: Graven & Stott, 2013b)

Firstly, for each task, I used the LFIN levels to determine where the strategy would fall on the spectrum. For each learner, I mapped the methods they used to answer questions onto the task spectrum. (The full set of matrices is available in Appendix C.) I counted the number of different methods they used. I then worked out their predominant method by comparing the values in the matrix.

EXPLORING THE DATA

LFIN Assessment

In the LFIN assessment, the strategy the child uses to answer a question demonstrates the level of development of his or her conceptual multiplicative understanding. The questions in the LFIN assessment are structured to cover a wide range of problem types and number ranges in order to ascertain the extent to which learners can apply more or less abstract strategies (see table appendix A). As I mentioned previously, profiling learner knowledge in this way formed a basis for the intervention used in this study, targeted to teach at each learner's current level of knowledge and strategies. LFIN levels 1 to 5 were allocated based on the strategy used by the learners to solve tasks in the pre and post assessments.

In this I was guided by Wright et al. (2006) as follows: For the learner to be in *level 1* she or he must establish the numerosity of a collection of equal groups when items are visible and counts by ones when doing so. To be in *level 2*, a learner is able to count in multiples when items are visible. For *level 3*, a learner is able to count in multiples when items are invisible. *Level 4* can demonstrate the repeated abstract composite grouping, by counting composite units in repeated addition or subtraction using the composite a specified number of times. For the learner to be at *level 5* they must understand the communicative principle of multiplication and the inverse relationship between multiplication and division.

VALIDITY AND RELIABILITY

“It [validity] refers to the truthfulness of findings and conclusions [...] and the degree to which explanations are accurate...” (McMillan & Schumacher, 2010, p. 104).

In qualitative research the issue of quality can be addressed by dealing with validity and reliability and might be addressed through “the honesty, depth, richness and scope of the data achieved” (Cohen, Manion & Morrison, 2000, p. 105) and should indicate “the intensive personal involvement and in-depth responses of individuals” (Cohen, Manion & Morrison, 2000, p. 110). Validity and reliability in terms of the MR programme instruments have been established through extensive testing in Northern New South Wales and has been adopted on a relatively significant scale in school systems in several other countries (Wright, 2003). Maxwell (2004) argues that within qualitative research one needs to think about specific validity threats and to find strategies to deal with them. He also suggests respondent validation, although my learners were too young for respondent validation. Breakwell (1995) states that research interviews require a particularly systematic approach to data collection in order to maintain validity and reliability. Wright et al. (2006) advise that the interviewer does not change the items or the order of the items. I complied with this suggestion in administering the interview. I have also drawn on my supervisor and co-supervisor to interrogate the extent to which my interpretation of the data and learner levels is recognizable to support inter-rater reliability. This is also consistent with what Wright suggested we do when he visited South Africa in September 2012.

ETHICS

“Because most educational research deals with human beings, it is necessary to understand the ethical and legal responsibilities of conducting research. [...] Ethics generally are concerned with beliefs about what is right and wrong from a moral perspective. [...] Researchers should generally be open and honest with participants about all aspects of the study. This usually involves a full disclosure

of the purpose of the research...”McMillan & Schumacher., (McMillan & Schumacher, 2010, p. 117).

I requested a meeting with the SMT and a Grade 4 teacher and explained the purpose of the study. I further asked permission from the School Governing Body. Once this permission was granted, I obtained informed consent from principals, teachers, learners and parents through sending letters which explained the focus of my study. I talked to the learners of Grade 4 so that they all understood what I was doing before and after administering the first assessment. I explained that participation was voluntary and that they could withdraw at any time without prejudice. I obtained consent forms signed by parents or guardians before I started to collect the data. I ensured that arrangements were made that were not disruptive to the learners or their families and worked to find suitable times for conducting my research after school. I used pseudonyms in the research, and assured everyone of the confidentiality of the information collected. I committed to ensure their anonymity during the research process. I also explained that only I and my supervisors would have access to the videos.

CHAPTER FOUR: DATA ANALYSIS AND FINDINGS

INTRODUCTION

This chapter reports on the analysis of the data and findings obtained in this study. The analysis involved a comparison of the performance of the learners over time between the pre and post assessments. The LFIN framework enabled me to profile the learners and provided a way of seeing whether there was progression from one level to another. I used the spectrum of mathematical proficiency discussed in Chapter Three. Translating qualitative data into spectrums and visual summaries enabled me to track the notion of multiplicative proficiency progress as discussed in Chapter Three. Analysing the data enabled me to answer my research questions:

- What level of multiplicative reasoning is displayed by the learners?
- How effective will the use of the Mathematics Recovery programme be in the South African context when implemented to a group of learners?

When analysing the data I looked at the following:

- Learner progress in LFIN levels over time from pre and post assessment.
- Learner multiplicative proficiency using the types of strategies employed by the learners for all the tasks.

LEARNER PROGRESS IN LFIN LEVELS

Learner progress in LFIN levels data was analysed qualitatively to get a broader picture of how the learners had progressed across the five levels on the basis of strategies used to solve multiplication and division tasks. Guidelines for profiling of learners onto LFIN levels were provided by Wright (2000). As detailed in Chapter Three, for the learner to be classified as *level 1* the learner cannot produce a number word sequence of multiples of threes and when counting visible collection of equal groups counts by one rather than by multiples. *Level 2* is characterised by the use of multiplication strategies, counting in multiples where the items

are visible. *Level 3* learners use skip counting and repeated addition to establish the numerosity of screened arrays and a partially screened array and by doing so counts equal groups by multiples. For a learner to be profiled to *level 4* and *5* which is advanced multiplication, the learner must have developed an understanding of an abstract composite unit and no equipment is used, the tasks are presented verbally and the learner understands the concept of community and inverse relationship and area multiplication.

To address question one of my study: What level of multiplicative reasoning is displayed by the learners, I look firstly at learners overall progress over time and then drill down to look at the detailed responses and methods used by each learner.

Table 4.1 below gives an overall picture of how the learners progressed in terms of the LFIN levels from the pre-assessment to the post assessment.

Table 4.1: Learners overall progress in LFIN levels over time from pre-assessment to post-assessment

	Learner A		Learner B		Learner C		Learner D		Learner E	
	PRE	POST	PRE	POST	PRE	POST	PRE	POST	PRE	POST
LEVELS	2	3	2	3	1	3	3	4	4	5

Looking at table 4.1 Learner A and B were at level 2 in the pre-assessment and both progressed to level 3 in the post assessment. In the pre-assessment, learner C was at level 1, as she was only learner who relied on counting visible items in ones. Learner D was able to count screened items and showed competency at level 3. Learner E was at level 4. Following the intervention, the data shows that all learners progressed at least one LFIN level in multiplication. Learner C progressed by two levels from level 1 to level 3 showing good progress. Three learners (A, B and C) achieved level 3 as their final level. Level 3 indicates that the learner can use multiplicative strategies where items are screened, can count in multiples or use addition and subtraction to be able to solve the problem. Learner D achieved a final level of 4 indicating that this learner was able to solve problems such as 5

x 3 and was also aware of the 3 as an abstract composite and unitary aspect. The learner coordinated two composite units in the text of multiplication. Learner E was the only learner to achieve the highest LFIN level of 5, which revealed that the learner could recall or quickly derive many of the basic facts and was aware of the communicative principle of multiplication, inverse relationship and area of multiplication the multiplicative reasoning is well developed.

INDIVIDUAL LEARNER ANALYSIS

LEARNER A

Pre-assessment

According to data analysis Learner A could form equal groups taking the counters one by one and when asked how many counters in each group, he started counting from one not in groups. Learner A could produce a number sequence of multiples of 2's, 10's, 5's and 3's fluently. It was difficult for the learner to answer the question on arrays as he lacked an understanding of the row and columns. When given 2×10 , 3×5 and 4×5 arrays he counted the dots in ones and they were turned 90° he started counting from 1. The learner was not confident when giving the answer and seemed uncomfortable in handling the manipulative. He used mostly fingers to count and sometimes was observed counting in ones silently. The learner could not estimate and for partition division he struggled to give the answer. He used his fingers when sharing $24 \div 3$ and got the answer correct. For sharing 15 counters equally among three children he could do that instantly without using counters and could explain that he counted by 3's. For quotient division the learner could count in groups of 4's and 5's. When he was sharing the objects to the containers he did that in ones not in groups. For the screened arrays the learner was able to give the answer, only question 5c stumped him where he was given 30 and seven containers but expected to use only 20 and five containers. The learner could not respond to the questions for advanced multiplication and division. The learner seemed to be operating in level 2 according to LFIN as indicated in table 4.1.

Post assessment

In the post assessment learner A showed evidence of progressing from counting in ones when making equal groups. He made groups of 4's immediately and confirmed the number in each group by counting in multiples of 4's. For FNWS multiples he was able to count in 2's, 3's, 5's from any number e.g. when asked to count from 30. When counting in 10's he could count correctly from 10 to 100 and from 100 he counted 101, 102, and later made a correction and counted correctly. When counting the counters arranged in 2 x 10 arrays at first when he counted 2 x 10 he counted in 2's and when asked how he counted he counted in 10's. For the 3 x 5 array he counted in 5's and 3's. For 4 x 5 arrays he counted in 5's and 4's. In partition division the learner showed an improvement by not taking the counters one by one. Rather he took them in one group of three and one of two to make five in each group. The learner could make groups of 4's and 2's when sharing 24 counters amongst 4 children. His understanding of equal groups was evident when given seven containers and asked to use five and 20 counters he could put the two counters he was not going to use upside down. When counting the screened arrays he could confidently give the correct answers. It was evident that he progressed to level 3 because he could use multiplicative counting strategies where items were screened, he counted in 3's, 4's and 5's as there was no counting in ones but when counting in multiples with the visible objects abstract thinking was not yet developed but progressed when it came to counting in ones and fingers. In both the pre and post assessment this learner could not do the advanced tasks (see table 4.1).

LEARNER B

Pre-assessment

In the pre-assessment learner B moved the counters one by one when forming the groups and after forming the groups she was able to count in 4's to confirm the groups: 4, 8, 12 and able to explain that three counters were left. She could produce counting in multiples of 2's, 5's, and 10 but when counting in 3's learner B omitted the numbers and could not keep the number sequence, she counted 3, 5, 9, 12, 16, 19, 22, 24. When counting the 3 x 5 array she could count silently in 1's and when it was turned 90⁰ she could count in 5's to tell how many dots there were altogether, for 10 x 2 she counted $10 + 10 = 20$. For the multiplication

tasks the learner at first counted in 1's silently and later count in 3's after being probed. When asked to estimate, the learner used her fingers while counting in ones. The learner could share the counters in the containers but could not recall how many counters she put in each container. She wanted to recount the counters in the container. She used the tactic of counting in ones more and counted visible objects. The learner was operating on level 2 and needed to develop confidence in what she was doing as showed in table 4.1.

Post assessment

The learner moved the counters in groups of 4's, and confidently explained how she formed the groups and counted from 4 to 12 fluently. She counted fluently in multiples of 2's, 5's and 10's but with 3's she paused after each count. She instantly gave the answer in the 2 x 10 arrays and even explained that she counted in 10's to get 20 dots. In the 4 x 5 array she gave the answer instantly as 20 and when asked how she got the answer she could explain her reasons fluently when counting in 5's but counting in 4's she used rhythmic counting. She could give the answer to 12 counters in four groups of 3's. When executing the task on partition division and quotient division with equal groups she could not do it without the counters but she could share them by taking the counters in groups. When putting the counters in a group she put them in groups not in ones. For the screened array the learners could give the answer correctly with confidence. After the intervention teaching the learner showed progression to level 3 but the learner could not do the advanced multiplication and division.

LEARNER C

Pre-assessment

Learner C moved the counters one by one when forming the groups and after forming the groups she continued to count in 1's when confirming the groups. The learner was able to count fluently in 2's, 5's, and 10 but when counting in 3's she counted from 3 to 12 and could not keep track of her counting in 3's. For the arrays the learners had no understanding of rows and columns and it was left up to me to remind her. When counting the 3 x 5 array she counted internally in 1's, even when the array was turned 90° . For 10 x 2 she counted

in 1's but gave the answer of 19 instead of 20, and did the same with 4×5 where she answered 19. For multiplication the learner could count in multiples of 3's when probed but could not go further than 12. The learner took one counter at a time and counted them in 1's to find the number in each group and counted the counters three times to give the answer. The learner could share the counters in the containers equally but could not recall how many counters she put in each container. The learner demonstrated level 1 strategy, as she depended on concrete counting when solving the tasks.

Post assessment

After the intervention teaching the learner could move the counters in groups of 4's, and confidently explain how she formed the groups and counted from 4 to 12 fluently. She counted fluently in multiples of 5's. Her Forward Number Word Sequence (FNWS) had improved and she no longer omitted numbers. She instantly gave the answer in the 2×10 arrays and even explained that she counted in 10's to get 20 dots. In the 3×5 and 4×5 arrays she instantly gave the answer and could explain that she counted in 5's. She could answer the 12 dots in four groups of 3 counters in each group. When executing the task on partition division and quotient division with equal groups she could not do it without the counters but she could share them by taking the counters in groups of 4's at the same time. For $24 \div 3$ she first took the group of 4's and added the second group of 4's to make eight in a group, here she displayed an understanding of commutative property. When sharing the counters into the containers the learner could not do this without the counters, and when putting the counters in a group she put them in groups not in ones. When asked how many counters were in a group the learner gave the wrong answer without reasoning about it, I repeated the question and she then gave the correct answer. The learner demonstrated her competence to be at level 3, the learner was challenged by item 5(c) where she needed to recall how many counters she put in each container. She could answer all the questions on screened arrays and enjoyed the arrays. Even though the learner could not do advanced multiplication and division, she had progressed from level 1 to level 3 according to LFIN.

LEARNER D

Pre-assessment

During the pre-assessment interview Learner D moved the counters one by one when forming the groups and after forming the groups she continued counting in 1's to confirm the groups. The learner was able to count fluently in 2's, 5's, and 10 but when counting in 3's, she paused after each multiple of 3. Anghlieri (1989) points out that when time lapses between counts, that indicates that counting in 1's is taking place. When counting the 3 x 5 array she lifted each finger as each finger represented one dot. She counted internally in 1's and when the array was turned 90° she re-counted in 1's. For the 10 x 2 array she silently counted in 1's. When the learner was given a pile of counters to share equally among three learners, she did not touch the counters she looked at them and lifted three fingers and counted until she got to 15. When given a pile of 24 counters to share equally among three children she pointed to the counters one by one without touching them and could share them correctly. She quickly answered $24 \div 4 = 6$. The learner could share the counters in the containers and could give the total number in each container. With the visible array of 3 x 5 she gave the answer fluently. The learner could respond to the advanced tasks but before she could give the answer she needed more time to think and then gave the correct answer. The learner was operating at level 3.

Post assessment

She moved the counters in groups of 4's, and confidently explained how she formed the groups and counted from 4 to 12 fluently. She counted fluently in multiples of 2's, 5's and 10's. When counting in 3's from any number she lost track and still omitted the numbers, 18, 22, and 24 but later corrected herself. She instantly gave the answer for the 2 x 10 arrays and explained that she counted in 10's to get 20 dots. In the 3 x 5 arrays she instantly gave the answer of 15. In the 4 x 5 arrays she gave the answer instantly as 20 and when asked how she got the answer she could explain that she counting in 5's. She could answer the 12 dots in four groups of 3 counters in each group.

When executing the tasks on partition division and quotient division with equal groups she looked at the counters and shared correctly without touching the pile of counters. For $24 \div 3$

she explained that she would have taken seven counters and added one to make a group. The learner could correctly estimate the number of counters that would be in each group and when putting the counters in containers she put them in groups. She responded fluently about the screened array. Learner D could answer the advanced question in task group 1 items from 1 to 4, correctly. The learner had improved also in pausing before giving the answer and she just gave the answer immediately and with confidence. After the teaching intervention the learner was able to progress to level 4.

LEARNER E

Pre-assessment

The learner moved the counters one by one when forming the groups and after forming the groups she continued counting in 1's to confirm the groups. The learner was able to count fluently in 2's, 5's, 10's and 3's. When counting in 5's she lifted five fingers. She also she did that when counting in 3's and tapped her fingers when counting. She could immediately give the answer and when probed as to how she got the answer she indicated the knowledge of counting in multiples of 10's, 4's and 5's and could apply that knowledge to answer other questions confidently. She did not do concrete forming of equal groups. She just looked at the counters and gave the correct answer. This suggested that her multiplicative reasoning had been developed. For task 5(c) she did not put the counters in the container; she looked at them and gave the correct answer. She had an understanding of division of equal groups.

She was one learner who was given the advanced tasks in the pre-assessment. When she was asked to share the 14 cookies amongst 3 children her answer was "each will get $3\frac{1}{2}$ cookies" which demonstrated her ability to do fractions. She mostly gave the answers confidently and she had the knowledge of multiples and their application. She could not proceed to tasks that used composite units, understanding commutative principle of multiplication, inverse relationship between multiplication and division and multiplication facts to derive division facts. In all the tasks that the learner answered she was on the average level or above average level. The learner was operating in level 4, her solutions indicated an understanding of abstract composite units, and she could keep track of the number of times she made counts.

Post assessment

She moved the counters in groups of 4's, and confidently explained how she formed the groups and counted from 4 to 12 fluently. She counted fluently in multiples of 2's, 5's, 10's and 3's. She instantly gave the answer in 2×10 arrays and even explained that she counted in 10's to get 20 dots. In the 3×5 arrays she instantly gave the answer of 15. In the 4×5 arrays she gave the answer instantly as 20 and when asked as to how she got the answer she explained that she counting in 5's and in 4's. She could answer the 12 dots in four groups of

3 counters in each group. When executing the tasks on partition division and quotient division with equal groups she listened to the instruction and gave the answer immediately without looking at and touching the pile of counters. For $24 \div 3$ she explained the strategy of using the known facts that if 3 children get five each that makes 15 and subtracted 15 from 24, and got the answer nine which she further divided it into 3 and added the 3 to 5 to make 8 counters for each person.

For $24 \div 4$ she already knew that $20 \div 4 = 5$ and minus the 20 from 24 and left with 4, which she later divided amongst four and added the one to the five to make six for each person. The learner could correctly estimate the number of counters that would be in each group when putting the counters in containers she put them in groups. When given 30 counters and seven containers and was asked to put 20 counters in five containers she put aside the TWO containers that she was not going to use and counted 10 counters from 30 counters and put them aside and put the 20 counters in five containers. She counted the counters in the container herself without being asked to do so. She responded fluently about the screened array.

For the advanced tasks the learner was confident in giving the correct answer and when probed she could explain that $5 + 5 + 5 + 5 + 5 + 5 + 5 = 30$, $5 \times 6 = 30$ and $2 + 2 + 2 + 2 + 2 + 2 = 12$, $12 \div 2 = 6$. It was easy for her to give the answers with the remainder. When given the task of 9×7 she could explain that was seven groups of 9's. In 8×4 and $32 \div 8$ she was able to explain that 32 was the answer to 8×4 and 8 is the answer to $32 \div 4$. She solved the problem of $56 \div 8 = 7$ correctly and explained that seven groups of 8 make 56. She could instantly give the number of squares, which was 21, and her justification for the answer indicated an understanding of rows and columns. She showed evidence of progression in the way she gave answers about the strategies that she used. The learner progressed to level 5; she used automatised multiplication facts to solve different multiplicative tasks.

LEARNER'S MULTIPLICATIVE PROFICIENCY

In order to analyse multiplicative proficiency I quantified the qualitative data to track possible progress using the spectrum discussed in Chapter Three. The progress of the learner was evident when they moved to the middle or upper end of the spectrum which indicates increased fluency, flexibility and efficiency.

Table 4.2 below indicates the positions of each learner according to the methods each used on the spectrum for the pre and post assessments starting with constrained (I-Inefficient) on the left, fluent (IE) methods in the middle and flexible fluency (E-Efficient) on the right. The values are the number of tasks where the learners showed the usage of different methods.

Table 4.2: Summary of spectrum methods for all learners across seven tasks

	I Constrained	IE Fluent	E Flexible fluency
LEARNER A			
PRE	5	1	1
POST	2	0	5
LEARNER B			
PRE	4	3	0
POST	2	0	5
LEARNER C			
PRE	6	1	0
POST	2	2	3
LEARNER D			
PRE	3	1	3
POST	0	1	6
LEARNER E			
PRE	1	2	4
POST	0	0	7

In conjunction with the graphs shown in figures 4.1 and 4.2, a visual picture of the data for learner progress in multiplicative proficiency emerges. Shifts in learner responses over time are evident. In the pre-assessment Learners B, C was split between using constrained methods and flexible methods, with Learner C using mostly constrained method. Learner A was split in all the methods but with more of constrained method. Learner D was split using all the methods but least of fluency. Learner E was split between using constrained and flexible methods but more with flexible fluency. Most learners seemed to rely most on using constrained methods as compared to fluently and flexible fluency. Learners B and C did not use any flexible fluency methods at all.

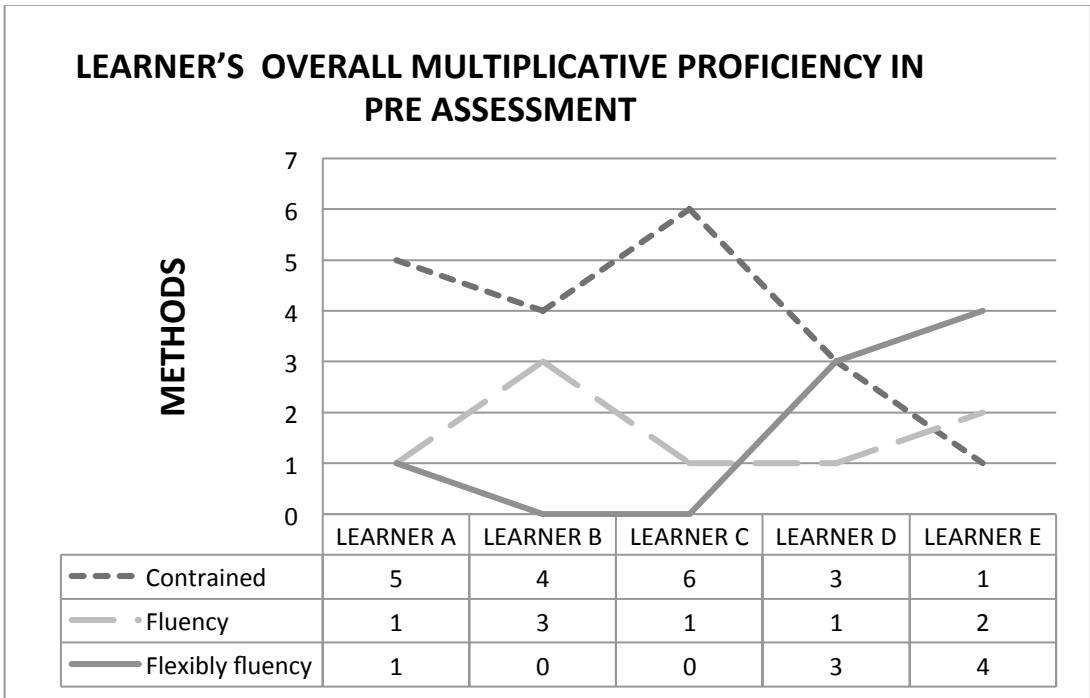


Figure 4.1: Learners' overall multiplicative proficiency in pre assessment

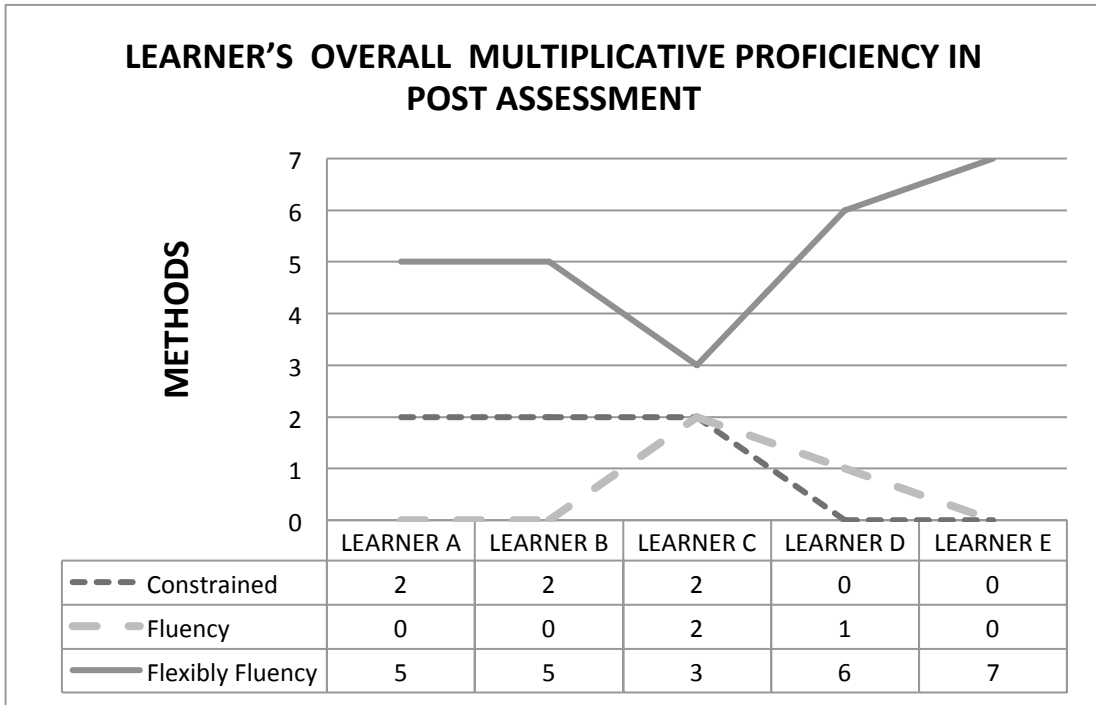


Figure 4.2: Learners' overall multiplicative proficiency in post assessment

The post assessment indicated an overall improvement in multiplicative proficiency for all the learners. There was decreased use of constrained methods and an increase in fluent and flexibly fluency. Although Learners A and B are still using some constrained methods in the post assessment, their biggest shift is to using more flexible methods.

Comparing the graphs shown overleaf (for the pre-assessment and post assessments), it is clear that for all learners, flexibility and fluency in methods used increased. In the next section I focus on the findings for the second research question.

ADDRESSING THE SECOND RESEARCH QUESTION

When comparing the LFIN levels of the learners for pre and post assessment I noticed a positive shift from one level to the next. All learners progressed at least one level within four weeks and one learner progressed two levels (Learner C). Since the MR programme was implemented over only four weeks, it is reasonable to assume that the MR programme rather than the learners continued school learning has a positive on shifting these learners up the levels.

The visual, quantifiable data in the form of spectrums and graphs combined with qualitative data assisted in presenting a richer, deeper analysis of this data and showed that they complement each other. A summary of learners' proficiency levels over time contributed to the potential value of the MR programme. From the levels data and methods data we note that all learners progressed in both aspects indicating the closeness of the relationship between these aspects. For example, Learner C (as in the levels), made good progress in methods; i.e. from a ratio of incidences of constrained to incidences of flexible fluency of 6 : 0 in the pre-assessment to 2 : 3 in the post assessment. I believe that there is evidence that the MR programme can be effectively implemented in the South African context to a small group of learners, which addresses the second of the research questions. Whether the MR programme could be successfully implemented with a larger group of learners remained to be researched further.

The intervention was limited to four weeks due to the scope and time frame restrictions of the research. Further research and intervention over a longer period of time could investigate the possible movement of learners through two or three levels. The fact that the learners in this study showed good progress highlighted the need for continued intervention.

CHAPTER FIVE: CONCLUSION

For learners to develop multiplicative reasoning they need to learn to identify the number in each group, the number of groups and the total in a range of multiplicative situations and come to know that it is the unknown quantity that makes the situation a multiplication and/or a division problem. This enables them to use the inverse relationship and move flexibly between multiplication and division. The use of the Maths Recovery (MR) Programme which included profiling of learners to LFIN levels, teaching in the form of a short intervention and conducting pre and post interview assessments made it possible for the learners in my sample to progress in terms of multiplicative reasoning. The MR programme made it possible to understand that when teaching the learners, one should understand the levels that the learners are operating at so as to assist them in their learning trajectory. During the intervention, the researcher gave the learners guided support, helping learners to think about multiplication and division, encouraging them to use their strategies and make mistakes. Learners were encouraged to enter into discussion in keeping with the constructivist notion that learners are not consumers of content but rather they are constructors of content. Keeping learner-centeredness, by engaging the learners in activities that involved active learning, problem solving and critical thinking were considered in the teaching strategies in the MR programme.

Given the relatively short intervention in this study, four sessions over five weeks, progress made by learners from level one to another level was one of the most important results for both myself as researcher (and teacher) and the learners themselves. The data showed that in the pre-assessment, learners were counting in ones (positioning them at level 1) and relying on using constrained methods. After the intervention, the post assessment shows that this decreased to 0 and learners were able to count in equal groups and use more efficient and fluent methods to solve the tasks. As Table 4.1 in the previous chapter showed, the rate of progression in my study was far greater than I expected; all learners progressed at least one level. Learner C progressed from level 1 to level 3 in the short time, which represents a significant shift in her multiplicative reasoning.

An overview of comparison of both assessments for each level showed significant improvement on learner performance and progress from one level to another. However, not all learners were able to progress to level 5, except for Learner E. One out of five of the learners from the sample was in level 1 and 2 out of five learners were in level 2, One learner was in level 3 and one learner was in level 4. The learners could progress from the level where they were to the next level as indicated in table 4. 1

This study also showed that learners were motivated to do mathematics and this increased their self-confidence and hence, their belief in their ability to do mathematics. This was identified when the learners were better at explaining their reasoning and strategies. As a result, they enjoyed the mathematics sessions, looked forward to mathematics time and were eager to be tested on what they learnt previously. Learners were motivated when they experienced level of success. Von Glasersfeld (1989) believes that a learner's motivation to learn is strongly dependent on the learner's confidence in their potential for learning. Learners gained confidence and motivation to embark on more difficult challenges by virtue of their successful completion of prior challenging tasks.

IMPLICATIONS FOR MY WORK

A key aim of this research was for me as a teacher (and more recently as a teacher educator) to explore the extent to which the MR programme could be used to support learners in developing multiplicative reasoning and proficiency. This is a key outcome of this research. As a teacher I have learnt the importance of providing learning tasks that allow collaboration with peers, having access to concrete materials like arrays for multiplication and division. As a teacher I have seen that the MR programme offers rich learning activities for teachers to use in interventions. I have also seen the usefulness of learning as an educator from the interview and see it as a useful developmental tool. Wright (2013) himself urges teachers and teacher educators to find ways to “incrementally trial and implement” (p.38) MR programme approaches. He believes that this professional learning is a “pathway to profoundly strengthening children's learning of basic arithmetic” (p.38) and that this can lead to young children achieving at significantly higher levels.

Research-learner interaction was crucial in creating an effective learning culture. The suggestion of the use of after school small group learning opportunities for supporting and enabling both student and teacher learning (Graven , 2011; Graven & Stott, 2012b; Stott & Graven, 2013c) helped me to ensure a non-threatening learning environment, which established open dialogue where questions and discussions were encouraged in a non-judgmental way. My enthusiasm and passion for mathematics also likely inspired the learners. The social interaction of the learners served to support, challenge and encourage one another.

Half way through my Masters research in March 2013, I was promoted to be Deputy Chief Education Specialist as a Curriculum Planner for Foundation Phase Mathematics in the Eastern Cape Department of Education. This gave me an opportunity to share my study with 230 lead teachers and 23 subject advisors who were astonished by the teaching of multiplication and division using arrays and later indicated that learners enjoyed them. They also later indicated anecdotally that learners had improved in multiplication as a result of their use of these conceptual resources. In future working with the teachers in the districts I plan to use the Mathematics Recovery Programme as one way of improving learner performance and also improving teacher understanding of learners multiplicative reasoning.

LIMITATIONS OF THE STUDY

Although progression for the learners was evident and with all its advantages, the key disadvantage of the MR Programme is that it is labour intensive and time consuming to administer for more than a few learners. The assessment interviews took approximately one and a half hours for each learner. Additional time was spent coding and allocating learners to LFIN levels. Thus while I would recommend that teachers conduct the interviews with a range of their learners in order to gain in-depth insight into learner levels and difficulties in multiplicative reasoning, it is not feasible to assess all learners in this way. However the implementation of the multiplication part of the MR programme to a group of learners holds potential for work in class and I have conducted a fruitful workshop in this regard with the Foundation Phase teachers in the Eastern Cape.

RECOMMENDATIONS FOR FURTHER RESEARCH

The findings of the efficacy of this study point to an Extended MR programme in a classroom situation but conducted over a longer period of time, perhaps the three years as Wright (2003) has suggested, for all six aspects of the framework.

More research needs to be done on intervention strategies to improve the state of mathematics in South African schools.

Further useful research would be a comparison of the findings of this study with similar research to investigate the resonance of these findings across other parts of South Africa.

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APPENDICES

APPENDIX A: INTERVIEW SCRIPT

R: I have a pile of 15 counters; make 3 groups with four in each group

L: (Takes each counter slowly one by one making 3 rows with four counters)

R: How many did you use?

L (Pointing in each counter silently counting in 1's) 12

R: Count by 2's I will tell you when to stop.

L: (Both hands on the desk, extend one finger from the left hand whilst counting 2, 4, fold the same hand and make a fist and continue counting 6, 8, 10 , 12, 14, 16, 18 , 20, 22

R: Stop and count by 10's I will tell you when to stop.

L: (while she had her fist) 10, 20, 30, 40, 50, 60, 70, 80, 90, 100, 110, 120.

R: Stop, count by 5's I will tell you when to stop.

L: (Both hand are put freely on the table whilst counting) 5, 10, 15, 20, 25, 30, 35, 40, 45, 50, 55, 60, 65, 70, 75, 80

R: Count in 3's I will tell you when to stop.

L: 3, 6, 9, (Start lifting the third figure), 12, (pause) 15, 18, 24

R: Stop, (Display a 10 x 2 array of dots and shows the rows and column), how many dots are there altogether?

L: (Looks in the dots while counting silently in 1's), 20.

R: Okay, Display a 5 x 3 array, there are three columns and five rows how many are there altogether?

L: (Look at the array while counting in 1's silently), 15.

R: Turn the array through 90°, how many dots altogether now?

L: (Look at the array while counting in 1's silently), 15.

R: Okay, Place four plates and put three counters on each plate, while the learners is looking on the other side, How many plates

L: 4

R: There are three counters under each plate, how many counters are there altogether?

L: *Silently nod the head indicating that there was counting going on, 12*

R: Place a pile of 15 counters, I want to share them equally amongst 3 children, and tell me how many will each get.

L: *(Sit up straight and look at the counters without touching them, pause for ten second), 5*

R: Here are the 12 counters; I want you to share amongst children so that each child must get 4, how many children will get counters?

L: *(Looked at the counters for ten second) 3.*

R: Place 24 counters, I want you to share amongst 3 children, how many will each child get?

L: *Looked at the counters, how many counters mam?*

R: I said 24 counters, how many will each get?

L: *(Use the fingers to point at the counters one by one), 5*

R: Its 24 counters I want you to share amongst 4 children how many will each get?

L: *(Looked at the counters without touching it), 6*

R: Place four plates with three counters under each plate, how many counters are there altogether?

L: *(Instantly give the answer) 12.*

R: Put 12 counters and 3 containers, put the counters equally into these containers and tell me how many counters you put into the containers. (You are not allowed to count the counters after you have put them in)

L: *(Take the counters one by one into each container), 4*

R: Okay, Place a pile of 30 counters and seven containers, use 20 counters and five containers, and tell me how many counters in each container.

L: Was confused just put the counters in all the containers

R: Use 3 x 5 array, cover the two upper rows for a second and uncover the lower three rows for another second, how many dots are there altogether?

L: Pause, show three fingers, 15

R: Turn the array 90⁰, how many dots?

L: Pause, silently, 15

R: Place a 6 x 2 array, cover five rows and leave the first row uncovered. How many dots in a row

L: Looked at the array, 2.

R: There are 12 dots how many rows are there altogether?

L: Pause for ten seconds, 6

R: Six children have five marbles, how many marbles are there altogether?,(Repeated the question three time)

L: the learner could not give the answer

END OF THE INTERVIEW

APPENDIX B: CODING OF LEARNERS

LEARNER A

STRATEGIES	Perceptual counting by ones		Visible and count in multiples												Multiplicative strategies where items are screened					Abstract composite and unitary aspect					Coordinate two composite units									
	TASK QUESTIONS	PRE-ASSESSMENT RESPONSES	POST ASSESSMENT RESPONSES	1a	1b	2a	2b	2c	2d	3a	3b	3c	3d	4a	4b	4c	4d i	4d ii	5a	5b	5c	5d	5e	ADVANCED MULTIPLICATION										
																									1a	1b	1c	1d	2a	2b	2c	2d	3	
	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	Rev✓	✓	✓	✓	✓	✓	✓	✓	IDK	IDK	IDK	IDK	IDK	IDK	IDK	IDK	IDK	IDK

LEARNER B

POST ASSESSMENT	PRE-ASSESSMENT RESPONSES	TASK QUESTIONS	STRATEGIES																														
			Perceptual counting by ones	visible and count in multiples																													
			multiplicative strategies where items are screened	abstract composite and unitary aspect																													
					ADVANCED MULTIPLICATION																												
					1a	1b	2a	2b	2c	2d	3a	3b	3c	3d	4a	4b	4c	4d i	4d ii	5a	5b	5c	5d	5e	1a	1b	1c	1d	2a	2b	2c	2d	3
✓ ✓	✓	1a	Perceptual counting by ones	visible and count in multiples	multiplicative strategies where items are screened	abstract composite and unitary aspect	coordinate two composite units	✓																	IDK	IDK	IDK	IDK	IDK	IDK	IDK	IDK	IDK
✓	✓	1b						✓																	IDK	IDK	IDK	IDK	IDK	IDK	IDK	IDK	IDK
✓	✓	2a						✓																	IDK	IDK	IDK	IDK	IDK	IDK	IDK	IDK	IDK
✓	✓	2b						✓																	IDK	IDK	IDK	IDK	IDK	IDK	IDK	IDK	IDK
✓	∧	2c						∧																	IDK	IDK	IDK	IDK	IDK	IDK	IDK	IDK	IDK
∧	✓	2d						∧																	IDK	IDK	IDK	IDK	IDK	IDK	IDK	IDK	IDK
✓	✓	3a						✓																	IDK	IDK	IDK	IDK	IDK	IDK	IDK	IDK	IDK
✓	✓	3b						✓																	IDK	IDK	IDK	IDK	IDK	IDK	IDK	IDK	IDK
✓	x✓	3c						x✓																	IDK	IDK	IDK	IDK	IDK	IDK	IDK	IDK	IDK
✓	x✓	3d						x✓																	IDK	IDK	IDK	IDK	IDK	IDK	IDK	IDK	IDK
✓	✓	4a						✓																	IDK	IDK	IDK	IDK	IDK	IDK	IDK	IDK	IDK
✓	x✓	4b						x✓																	IDK	IDK	IDK	IDK	IDK	IDK	IDK	IDK	IDK
x✓	??	4c						??																	IDK	IDK	IDK	IDK	IDK	IDK	IDK	IDK	IDK
x✓	??	4d i						??																	IDK	IDK	IDK	IDK	IDK	IDK	IDK	IDK	IDK
✓	x✓	4d ii						x✓																	IDK	IDK	IDK	IDK	IDK	IDK	IDK	IDK	IDK
✓	✓	5a						✓																	IDK	IDK	IDK	IDK	IDK	IDK	IDK	IDK	IDK
✓	✓	5b						✓																	IDK	IDK	IDK	IDK	IDK	IDK	IDK	IDK	IDK
✓	✓	5c						✓																	IDK	IDK	IDK	IDK	IDK	IDK	IDK	IDK	IDK
✓	✓	5d						✓																	IDK	IDK	IDK	IDK	IDK	IDK	IDK	IDK	IDK
x✓	✓	5e						x✓																	IDK	IDK	IDK	IDK	IDK	IDK	IDK	IDK	IDK

LEARNER C

POST ASSESSMENT	PRE-ASSESSMENT RESPONSES	TASK QUESTIONS	STRATEGIES																														
			Perceptual counting by ones	Visible and count in multiples																													
			Multiplicative strategies where items are screened	Abstract composite and unitary aspect																													
					ADVANCED MULTIPLICATION																												
					1a	1b	2a	2b	2c	2d	3a	3b	3c	3d	4a	4b	4c	4d i	4d ii	5a	5b	5c	5d	5e	1a	1b	1c	1d	2a	2b	2c	2d	3
✓	??	1a	Perceptual counting by ones	Visible and count in multiples	Multiplicative strategies where items are screened	Abstract composite and unitary aspect	Coordinate two composite units	??																	IDK	IDK	IDK	IDK	IDK	IDK	IDK	IDK	IDK
✓	✓	1b						✓																	IDK	IDK	IDK	IDK	IDK	IDK	IDK	IDK	IDK
✓	✓	2a						✓																	IDK	IDK	IDK	IDK	IDK	IDK	IDK	IDK	IDK
✓	✓	2b						✓																	IDK	IDK	IDK	IDK	IDK	IDK	IDK	IDK	IDK
✓	∧	2c						∧																	IDK	IDK	IDK	IDK	IDK	IDK	IDK	IDK	IDK
∧	∧	2d						∧																	IDK	IDK	IDK	IDK	IDK	IDK	IDK	IDK	IDK
✓	x✓	3a						x✓																	IDK	IDK	IDK	IDK	IDK	IDK	IDK	IDK	IDK
✓	??	3b						??																	IDK	IDK	IDK	IDK	IDK	IDK	IDK	IDK	IDK
✓	??	3c						??																	IDK	IDK	IDK	IDK	IDK	IDK	IDK	IDK	IDK
✓	??	3d						??																	IDK	IDK	IDK	IDK	IDK	IDK	IDK	IDK	IDK
✓	??	4a						??																	IDK	IDK	IDK	IDK	IDK	IDK	IDK	IDK	IDK
✓	✓	4b						✓																	IDK	IDK	IDK	IDK	IDK	IDK	IDK	IDK	IDK
✓	✓	4c						✓																	IDK	IDK	IDK	IDK	IDK	IDK	IDK	IDK	IDK
✓	? ?	4d i						? ?																	IDK	IDK	IDK	IDK	IDK	IDK	IDK	IDK	IDK
✓	??	4d ii						??																	IDK	IDK	IDK	IDK	IDK	IDK	IDK	IDK	IDK
✓	✓	5a						✓																	IDK	IDK	IDK	IDK	IDK	IDK	IDK	IDK	IDK
✓	??	5b						??																	IDK	IDK	IDK	IDK	IDK	IDK	IDK	IDK	IDK
✓	✓	5c						✓																	IDK	IDK	IDK	IDK	IDK	IDK	IDK	IDK	IDK
✓	✓	5d						✓																	IDK	IDK	IDK	IDK	IDK	IDK	IDK	IDK	IDK
x✓	IDK	5e						IDK																	IDK	IDK	IDK	IDK	IDK	IDK	IDK	IDK	IDK

APPENDIX C: SPECTRUM FOR MULTIPLICATIVE PROFICIENCY

For the learners in pre and post assessment for all the tasks

FORMING EQUAL GROUPS						
	Counting by ones and confirm the number in groups by counting in ones (I)		Counting in ones and confirm and confirm counting in groups (IE)		Counting in groups and confirms in group counting (E)	
	PRE ASSESSMENT	POST ASSESSMENT	PRE ASSESSMENT	POST ASSESSMENT	PRE ASSESSMENT	POST ASSESSMENT
LEARNER A	I					E
LEARNER B	I					E
LEARNER C	I					E
LEARNER D	I					E
LEARNER E	I					E

TASK INVOLVING FNWS OF MULTIPLES						
	Count in 2's, 5's. 10' and 3's omitting some numbers in all the multiples of 10's and 3's (I)		Count in 2's, 5's. 10' and 3's but only omitting multiples of 3 (IE)		Count in 2's, 5's. 10' and 3's Fluently (E)	
	PRE ASSESSMENT	POST ASSESSMENT	PRE ASSESSMENT	POST ASSESSMENT	PRE ASSESSMENT	POST ASSESSMENT
LEARNER A			IF			E
LEARNER B			IF			E
LEARNER C	I			IF		
LEARNER D					E	E
LEARNER E					E	E

TASK INVOLVING VISIBLE ITEMS ARRANGED IN ARRAYS						
	Solve the task count one by one (I)		Solve the task counting by one and some counting by multiples (IE)		Solve the task by counting using multiples of 3's, 4's and 5's (E)	
	PRE ASSESSMENT	POST ASSESSMENT	PRE ASSESSMENT	POST ASSESSMENT	PRE ASSESSMENT	POST ASSESSMENT
LEARNER A	I					E
LEARNER B	I					E
LEARNER C	I					E
LEARNER D			IE			E
LEARNER E					E	E

TASK INVOLVING EQUAL GROUPS OF VISIBLE ITEMS						
	Solve the task count one by one (I)		Solve the task counting by one and some counting by multiples (IE)		Solve the task by counting using multiples of 3's, 4's and 5's (E)	
	PRE ASSESSMENT	POST ASSESSMENT	PRE ASSESSMENT	POST ASSESSMENT	PRE ASSESSMENT	POST ASSESSMENT
LEARNER A			IE			E
LEARNER B			IE			E
LEARNER C					E	E
LEARNER D					E	E
LEARNER E						E

TASK INVOLVING SCREENED ITEMS						
	Counting using fingers to keep track of groups and count (I)		The learner is able to count the counters after having shared (IE)		Solve the task by counting using multiples or using addition or subtraction for quotient division with an array (E)	
	PRE ASSESSMENT	POST ASSESSMENT	PRE ASSESSMENT	POST ASSESSMENT	PRE ASSESSMENT	POST ASSESSMENT
LEARNER A	I					E
LEARNER B			IE			E
LEARNER C	I			IE		
LEARNER D					E	E

LEARNER E					E	E
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ADVANCED MULTIPLICATION AND DIVISION TASK PRESENTED VERBALLY WIHOUT VISIBLE OR SCREENED ITEMS						
	Exhibit knowledge of composite units		Exhibit knowledge of communicative principle of multiplication		Exhibit knowledge of inverse relationship between multiplication and division and multiplication facts to derive division facts	
	(I)		(IE)		(E)	
	PRE ASSESSMENT	POST ASSESSMENT	PRE ASSESSMENT	POST ASSESSMENT	PRE ASSESSME NT	POST ASSESSMENT
LEARNER A	I	I				
LEARNER B	I	I				
LEARNER C	I	I				
LEARNER D	I					E
LEARNER E			IE			E

**ADVANCED MULTIPLICATION AND DIVISION
TASK ON COMMUNITATIVITY AND INVERSE RELATIONSHIP AND AREA
MULTIPLICATION**

	Count in multiples by using knowledge of abstract composite units (I)		Count in multiples by using knowledge of abstract composite units, aware of both the composite and unitary aspect (IE)		Keep track of the counts and the total number of counts (E)	
	PRE ASSESSMENT	POST ASSESSMENT	PRE ASSESSMENT	POST ASSESSMENT	PRE ASSESSMENT	POST ASSESSMENT
LEARNER A	I	I				
LEARNER B	I	I				
LEARNER C	I	I				
LEARNER D	I			IE		
LEARNER E			IE			E

APPENDIX D: SAMPLE OF INTERVENTION TASKS

Sample of intervention tasks

EQUAL GROUPS

1.1 Making equal groups

Learners are given 12 counters and are asked to make three groups with four in each group.

How many groups? / How many in each group?

1.2 Describing equal groups

Counters/ unifix cubes/ containers

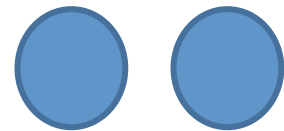
Individuals

Each learner is given 10 counters and two containers, ask them to share it equally into the containers, how many counters in each containers?

Use the same activity but increase the number range

1.3 Combining and counting equal groups

Place out ten 2 – dot cards, put each 2- dot card and let the learner count putting each card after the other.



Similar with 3 dots, four dots, five dots

1.4 Determining the number in an equal share

Resources: dots cards/ unifix cubes

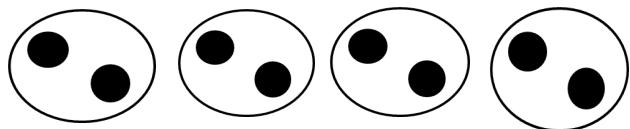
Learners are given six counters. Share them amongst three people.
How many will each get?

Use 10 and 2, 12 and 6, 18 and 3

1.5 Determining the number of equal groups

Resources: dots cards/ unifix cubes

Place out four 2-counter cards:



How many counters are there on each

card?

How many cards are there?

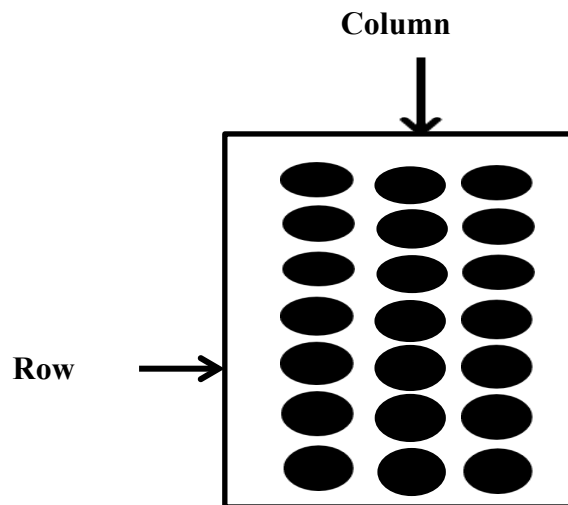
How many counters are there altogether?

1.6 Describing visible arrays

Resources: Arrays

Arrays

The arrays are explained to the learners - that it has rows and column. The learners were showed the rows and columns.



How many rows?

How many dots are altogether?

1.7 Developing counting in groups of 3's/ 4's/ 5's using screen items

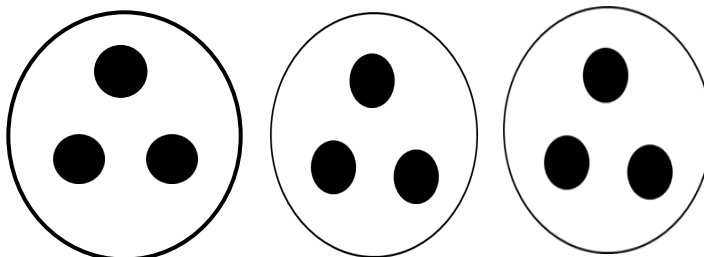
Resources: two dots / 4 dots/ 5 dots cards

Place out a plate each containing three dots in it. Tell the learners that one plate has three dots, place another plate then and ask the learners how many dots are there altogether in two plates. Put more plates under a screen and ask how many dots are altogether

1.8 USE DOT UNDER THE CARD A SCREEN.

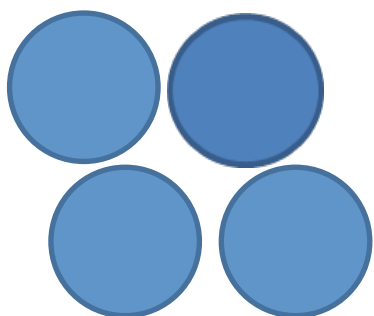
Resources: dot cards

There are three dot cards, under each card there are three dots.

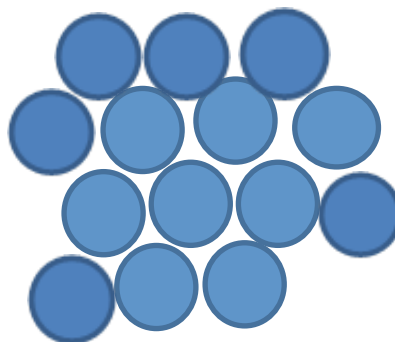
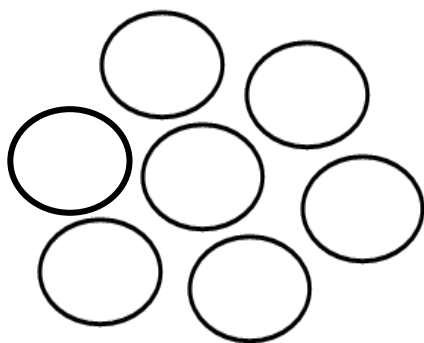


How many dots are there altogether?

The same activity was used but increases the number of dots under each card.



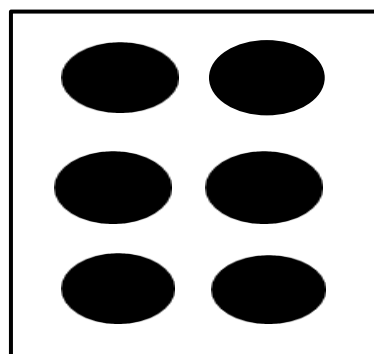
The teacher tells the learner that there are seven cards with 14 dots altogether. Ask how many dots in each card? Learners will not be allowed to touch the dots.



The same activity was used but increases the number range on the dot card

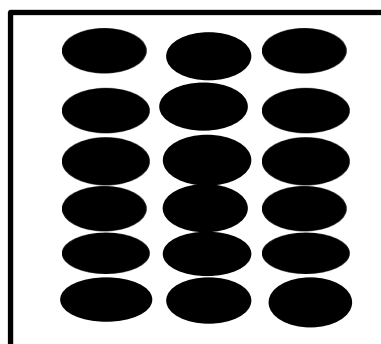
1.9 Resources: Arrays

Display the first row for a second while other rows are covered. Let the learners look at the first row and then show the others for another second or two, ask learners as to how many dots are altogether.



The same activity was used with different arrays

- The teacher unscreens the first row and screens the rest and tells the learners that there are six rows altogether, how many dots are there altogether.

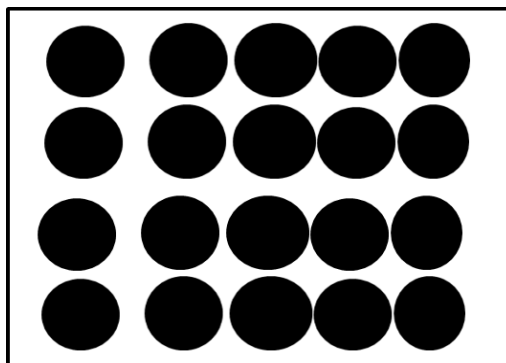


The same activity was used with different arrays. Place a 4 x 5 array and covers one row,

- How many dots are altogether?
- How many rows are there?
- How many columns?

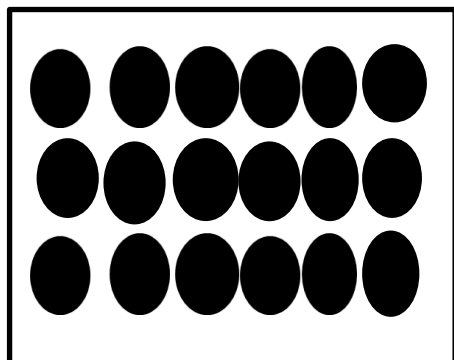
The teacher turns the array at 90°

- How many dots are altogether?
- How many rows are there?
- How many columns?



The same activity was used with different arrays

There are 18 dots altogether and there are 3 rows, how many dots in a row.

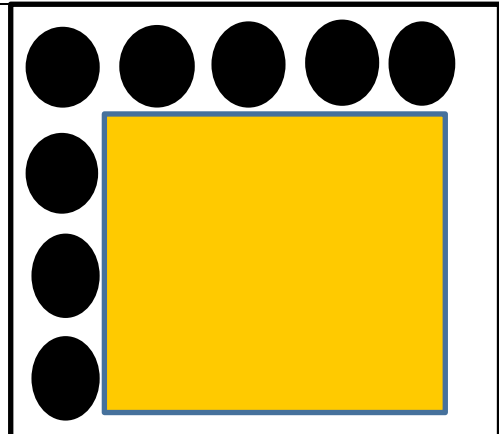


The same activity was used but increases the number

range on the dot cards.

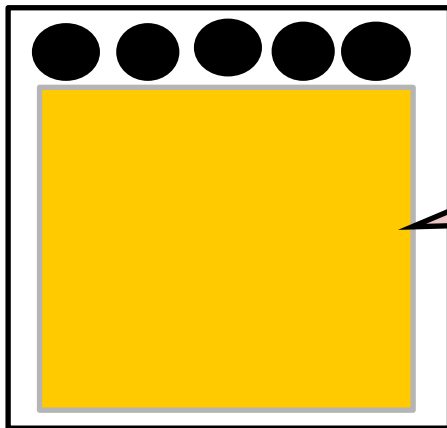
1.10 USE OF A COVERED ARRAY

- How many dots are there altogether?
- Explain how did you get the answer?

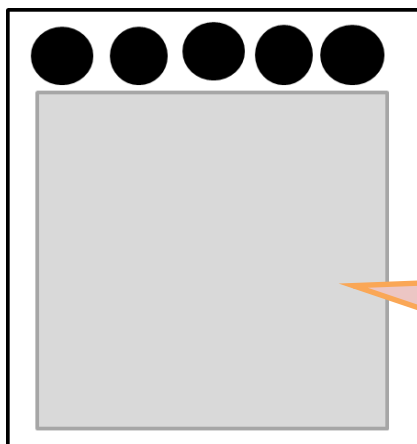


the same activity was used but increase the number range.

Use the following array to answer the following question



If there are six rows with five dots in each, how many dots are altogether?



There are 15 dots altogether. Each row has five dots. How many rows are there?

Word problems: Multiplication and division

- There are four children and they each have three books. How many books are there altogether

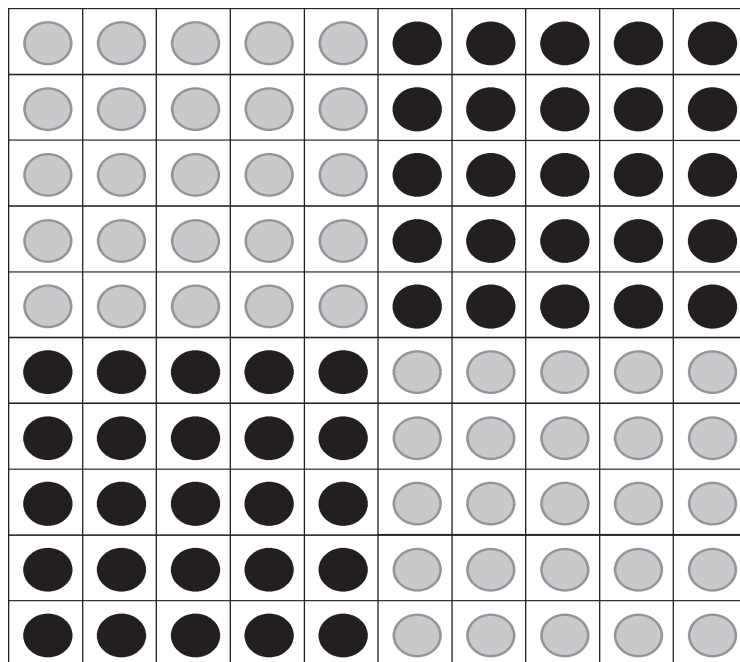
learner must keep the track of the number of ones in each three, the number of threes and the total number of counts

- Fourteen pens are put into groups of two's, how many groups of two are there?

learner count in multiples and keep track of the number of multiples

Multiplication Facts

Use an array in assisting the learning of multiplication facts 10 x 10 dot array



Learners are asked to come to show what 5 x 8 look like and are showed 5 x 8 array about turns it to demonstrate 8 x 5

***** END *****