

THE USE OF SMARTPHONES AND VISUALISATION PROCESSES
FOR CONCEPTUAL UNDERSTANDING OF MENSURATION: A CASE
STUDY OF THE MATHCITYMAP PROJECT IN NAMIBIA

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ABSTRACT

The aim of this study was to investigate and analyse the potential use of smartphones as visualisation tools by learners to enhance conceptual understanding through mathematics trails developed using the MathCityMap (MCM) project. This research study is part of the VIPROmaths project which seeks to research the effective use of visualisation processes in mathematics classrooms in South Africa, Namibia, Zambia, Switzerland and Germany.

This study adopted a case of twelve purposively selected Grade 9 learners from a school in the Eheke circuit of the Oshana region, Namibia. This study was framed within a social constructivist perspective and sought to investigate visualisation processes as well as conceptual understanding of learners as they conceptualised the MCM tasks in new, outdoor and collaborative learning situations. The MCM app was installed on selected learners' smartphones to access and to walk the MCM trails located in various places in the schoolyard. Three MCM trails based on three key themes of mensuration (perimeter, area and volume) were created. Each consisted of four tasks that were sourced and developed in line with the Grade 9 Namibian mathematics syllabus. This study is oriented in an interpretive paradigm and employed video-recorded observations and focus group interviews as qualitative data collection methods.

Data collected were analysed first using the themes developed from Ho's (2010) work on visualisation processes and Kilpatrick, Swafford, and Findell's (2001) conceptual understanding. During analysis, some themes emerged from the data and were considered.

Findings from this study revealed that smartphones afforded learners ample opportunities to enhance the visualisation processes that they went through as they learned the concepts of measurement. In addition to this, some learners were initially pessimistic regarding the use of smartphones for learning purposes. This study recommends that resources such as MCM be effectively be used in formal school settings. The learning of measurement can be advanced in outdoor settings where learners have physical and spatial access to the learning content. Smartphone technology can be used as an additional tool to integrate Information and Communication Technology (ICT) in learning mathematics within the Namibian context.

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DECLARATION OF ORIGINALITY

I, Liina Shimakeleni, Student number g17S8213, declare that this thesis entitled: *The use of smartphones and visualisation processes for conceptual understanding of mensuration: A case study of the MathCityMap project in Namibia*, is my own work, written in my own words. Where I have drawn on words or ideas of others, these have been acknowledged according to Rhodes University Education Department referencing guidelines.



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31/01/2022

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LIST OF ABBREVIATIONS AND ACRONYMS

2-D	Two Dimensional
3-D	Three dimensional
app	application program
CCP	Connecting to a previously solved problem
CMP	Connecting Mathematics to Pre-knowledge
CRW	Connecting ideas and concepts in mathematics to the Real World
CVR	Constructing a visual representation
EAP	Encoding the Answer to the Problem
FGI	Focus Group Interview
GPS	Global Positioning System
ICT	Information and Communication Technology
JES	Justifying and Explaining mathematical ideas and Solutions
MCM	MathCityMap
MOEAC	Ministry of Education Arts and Culture
NelNet-eLC	Namibia Open Learning Network trust e-Learning standard Committee
RMC	Representing Mathematical Concepts in different ways
SADC	Southern African Development Community
SMS	Short Message Service
UNICEF	United Nations International Children's Emergency Fund
USR	Understanding Spatial Relations of the element
UVP	Using Visualisation to solve the Problem
VITALmaths	Visual Technology for the Autonomous Learning of Mathematics

CHAPTER 1 INTRODUCTION

1.1 INTRODUCTION

This chapter introduces the study focusing on how smartphones can be used as visualisation tools to develop conceptual understanding of mathematics in Grade 9 learners. The chapter begins with the research background and context of the study, in which I discuss how my study used the MathCityMap (MCM) application and related it to the learning of measurement in the Namibian context. The chapter further outlines the research goals and questions that frame this study. The chapter closes with an overview of the structure of the thesis.

1.2 BACKGROUND AND CONTEXT OF THE STUDY

In recent years, there has been a growing research interest in the use of smartphones for learning mathematics. Fabian, Topping, and Barron (2018) for example, investigated the effects of using smartphone technologies on learners' attitudes and achievements in mathematics. Positive results were noted from this research as participants' attitudes and performance in mathematics were recorded to have improved. In another study conducted in South Africa by Ngesi, Landa, Madikiza, Cekiso, Tshotsho, and Walters, (2018) that involved the distribution of smartphones to 44 selected Grade 9 learners, results showed that smartphones were effective tools for supporting off-campus and after school hours' teaching and learning.

In South Africa, Ndafenongo (2011) researched how VITALmaths video clips on smartphones could be used in teaching and learning mathematics. Results showed that the video clips on the smartphones helped learners develop conceptual understanding of the Pythagorean theory, the clips were visually appealing and they enhanced learner autonomy. However, this study did not observe the learners' actions and interactions with smartphones to show how conceptual understanding was enhanced.

In Namibia, the policy guidelines for education prohibits learners from bringing tools such as cell phones and tablets into classrooms (Namibia Press Agency, 2007). Learners' use of these gadgets on school premises is restricted rather than regulated. Research conducted by the United Nations International Children's Emergency Fund (UNICEF, 2016) however, identified the mobile phone as the most accessible ICT tool in Namibia, especially for the young generation. In his speech, the then Namibian Minister of ICT reported that Namibia was amongst the few African countries that

has more mobile phone subscriptions than its total population (Semali & Asino, 2014). An exploratory research study conducted in five regions in Namibia during 2016 by UNICEF reported an increased use of smartphones and the internet by children under the age of sixteen. The rapid growth of access to and use of mobile phones around the world, Africa and in these five regions in particular, have the potential of improving learning (Semali & Asino, 2014). While the study acknowledges that the use of smartphones was practiced in some educational regions of Namibia, it reiterates the need to help young people use these gadgets responsibly and for their mental growth (UNICEF, 2016).

This study intended to contribute to the narrative to assist learners to use smartphones responsibly for their learning of mathematics. The learning of mensuration at my school relies heavily on traditional, abstract approaches where learners are expected to complete lists of exercises in the textbooks and have them checked for the correct or wrong answer by the teachers. I argue that exciting opportunities for learning mathematics can be created in collaboration with this MCM project. This is a mobile math application that allows for mathematics problems to be solved in real-life places that can be located with a Global Positioning System (GPS) using smartphones (Ludwig & Jesberg, 2015). The MCM app is a freely available tool that can be harnessed to promote learning of mathematical concepts such as mensuration. I intend to use the MCM app as a visual learning tool in a school that is located in the rural area of Namibia. In particular, I wish to investigate how smartphones used by twelve Grade 9 learners can be utilised as a visualisation tool for conceptual learning of mensuration.

In this study, learners were required to interact with physical artefacts in their environment (i.e. on their trail) as they learn measurement. Gutiérrez (1996) stated that mental images are created from sensory cognition of concrete ¹materials that learners engage with when using their sense organs. These images may then be expressed or communicated in the form of diagrams, pictures, drawings,

¹ In this thesis, the word has ‘concrete’ has two meanings, depending on the context in which it is used: 1)the physical form of a material; and 2) building material made from a mixture of gravel, sand, cement and water.

gestures and discussions. Thus, taking a visual approach to learning mathematics, especially in basic and key concepts like measurement, can provide effective features of how and why mathematics works (Söbbeke, 2005; Steinbring, 2006). According to Makina (2010), visualisation incorporates those mental processes rooted in concrete experiences that make use of, or are characterised by, visual imagery, visual memory, visual processing, visual relationships, visual attention and visual imagination. Arcavi (2003) further said that “the visual display of information enables us to 'see' the story, to envision some cause-effect relationships, and possibly to remember it vividly” (p. 218). This study made use of smartphone technologies (Mitchell, Cistic & Maxl, 2007) as visual tools, while observing these learners’ behaviour (Khen, 1999) during their learning of perimeter, area and volume concepts.

1.3 PURPOSE OF THE STUDY

The learning of mensuration at my school has had its fair share of difficulties in relying heavily on traditional, abstract methods. With these approaches, learners normally complete lists of exercises in textbooks and have them checked for the correct or wrong answers by their teachers. Wijers, Jonker and Drijvers (2010) suggest that this fault may be augmented through the use of real, locally found objects. Ramaley and Zia (2005) emphasise the importance of learning environments which are active, social, and learner-centered.

The goal of this study was twofold: firstly, the study discovered the potential use of smartphones for supporting conceptual learning of mensuration of real-life tasks developed using the MCM project. Secondly, this study explored the selected learners' experiences and views regarding the use of smartphones as visualisation tools in the learning of mensuration of shapes outside their classrooms.

1.4 RESEARCH QUESTIONS

The study is guided by the following research questions.

1. How can smartphones be used as visualisation tools in developing conceptual understanding in the learning of mensuration using the MCM app in grade 9?
2. What are the selected learners' experiences of learning mensuration of shapes using smartphones as visualisation tools outside the classroom?

1.5 SIGNIFICANCE OF THE STUDY

This study intends to promote the learning of mathematics outdoors using locally available visual and tangible artefacts within a given environment through the MCM application that will be installed on smartphones. It is therefore particularly significant in the Namibian context as it aligns well with integrating learning with ICT, an approach advocated by the Namibian Curriculum.

It is hoped that through this research, learners will get inspiration that, in addition to socialising and leisure, smartphones can also be used for learning purposes. It is also hoped that teachers and subject advisors who read this study will acknowledge the use of smartphones as well as the inclusion of physical activities in the mathematics learning process. Zender, Ludwig & Gurjanow, (2017) stated that embodied mathematics helps learners develop sensory thoughts. This kind of mediation does not only extend learning outside the classroom but it also provides users with the opportunity to personally control their learning as well as to creatively own their learning processes and easily communicate with their peers (Laurillard, 2009).

1.6 THEORETICAL FRAMEWORK

This study is positioned within the social constructivist framework, as it regards learning as a situated social endeavor facilitated by conversations between people and mediated through tool use.

It is argued that being able to visualise mathematical ideas is an inherent component in discovering ideas and concepts. Kim (2001) further alluded to the fact that social constructivism views learning as a social process; it does not take place only within an individual, nor as a passive development of behaviours that are shared by external force. The social constructivist theory recognises the inherent interaction of individuals with societies, by which they fashion instruments and direct their energy on infinite circumstances of time and space (Vygotsky, 1978).

The social constructivist perception of learning that “knowledge and understanding can be developed whether or not the teacher is present” (Simon, 1995, p.116). Meaningful learning occurs when individuals are engaged in social activities that are happening in the community that they may find themselves. Gupta (2011) adds that social constructivism provides tools and tasks with

which learners infer ideas, formulate solutions and test their ideas in a collaborative learning environment.

Ramaley and Zia (2005) maintain that the newer forms of technologies such as smartphones “enrich traditional forms of learning and serve as links between active and passive learning, individual and group learning, and the transmission and generation of knowledge” (p. 10) Other proponents of social constructivism recognize that the learning environment and the MCM application promote the social constructivist orientation which advocates that learning is not necessarily restricted to an individual learner making decisions in isolation.

1.6 RESEARCH METHODOLOGY

This study takes the form of a case study. Cohen, Manion and Morrison (2011, p. 253) define a case study as an “exploration or in-depth analysis of a specific instance that is designed to involve multiple sources of information that are rich in content”. In this study, the case is a set of twelve purposefully selected Grade 9 learners. The unit of analysis associated with this case is the learners' engagement with the MCM application on their smartphones in their school environment and peers, with reference to the visualisation processes employed when solving the given MCM tasks.

The participants for this study were twelve of my Grade 9 learners who I purposively selected and introduced to the MCM application. These learners ran the MCM trails, which contained mathematics tasks on the measurement topics for perimeter, area and volume. Data presented and analysed in this chapter was drawn from results generated by two research techniques, namely observation and focus group interviews (FGI).

Firstly, I observed learners as they ran the MCM tasks. I then analysed the group tasks in order to foreground the manifested visualisation processes through an analytical framework developed from Ho's (2010) work. I also analysed the data for conceptual understanding using a framework adapted from Kilpatrick et al. (2001), that became evident (or not) when each group visited the site for each task. I then conducted the FGI in order to help learners reflect and discuss their experience and views that learners had during the running of the tasks.

1.7 STRUCTURE OF THE STUDY

Chapter 1: Introduction

This chapter introduces the context and background, followed by the goals and questions that guided this study. This chapter also provides a brief discussion for the methodology, design and theory that underpin this study. It then concludes with a general outline for the entire study.

Chapter 2: Literature Review

This chapter provides an elaborate discussion of concepts that were used in relation to the research questions. The opening concepts are measurement and visualisation, which lead to the discussion on the MCM project, which advocates the use of smartphones in the outdoor learning environment. I then provide a synopsis on how these concepts lead to conceptual understanding. The chapter then concludes with a discussion about the theory of social constructivism.

Chapter 3: Methodology

This chapter discusses the research methodology that I used in this study. Special reference was made to the case study method under the interpretive paradigm that were used in this research. Thereafter, a detailed account of how the two instruments, namely observations and focus group interviews, were used to collect data. I further explain how the MCM task review process helped me to publish valid and reliable tasks. The chapter concludes by considering ethical issues that pertain to this research.

Chapter 4: Data presentation, analysis and discussion

This chapter presents and analyses the information that was obtained from data. The data obtained from the observation discussed the visualisation processes that learners went through as they learned the measurement concepts that are incorporated in the MCM project. The chapter also considers the learners' views regarding their use of smartphones to learn mathematics in an outdoor environment.

Chapter 5: Conclusions and Recommendations

This chapter summarises the key findings sourced from the analysed data. The chapter also considers the significance of the study, followed by an examination of the limitations that this study endured. The chapter then ends by proposing recommendations and directions for further research.

1.8 CONCLUSION

This introductory chapter, provides the background and context in which the study was carried out. The purpose and research questions that provided a guide to this study are given. Thereafter, the significance for the promotion of learning measurements facilitated by smartphone use outside the classroom is provided. The theoretical framework was later followed by the methodology applied in this study. Lastly the outline of each chapter of the is provided.

CHAPTER 2

LITERATURE REVIEW

2.1 INTRODUCTION

The purpose of this chapter is to review literature that relates to and that informs this study. It begins with a discussion of the definitions of measurement, a topic in the Grade 9 Namibian mathematics syllabus (Namibia. Ministry of Education, Arts and Culture [MEAC], 2015). The chapter further defines visualisation and the roles that it plays when using a smartphone to learn mensuration. It then discusses how mathematics learned in the classroom is linked to real life through outdoor activities, as advocated by the MCM project. Conceptual understanding is another key concept reviewed in this literature. The chapter defines conceptual understanding and how smartphones can be used as visual tools to enhance the conceptual understanding of measurement. The chapter concludes with a discussion of social constructivist theory and how it underpins this study.

2.2 MENSURATION/ MEASUREMENT

Measurement is a component of mathematics that has a long history and important place in the subject (Smith, Van Den Heuvel-Panhuizen & Teppo, 2011). History has it that measurement originated from Egypt. Ancient Egyptians depended on a natural method to measure dimensions such as the arm, which was used as a measure of length (Waziry, 2020). In addition, ancient Egyptians were well-informed in many sciences such as geometry, surveying, astronomy and mathematics. Today, measurement plays a significant role in describing and understanding the properties of shapes (Thompson & Preston, 2004). Also, in the context of education, measurement concepts and skills are directly applicable to the world in which learners live (Ontario Ministry of Education, 2005).

Van De Walle, Karp and Bay-Williams (2014) define measurement as “a number that indicates a comparison between the attributes of an object” (p. 398). Previously, Van de Walle (2004) gave a more elaborate understanding of measurement as a mathematics concept that “... involves a comparison of an attribute of an item or situation with a unit that has the same attribute” (p. 316). According to Ontario Ministry of Education (2005), measurement is an element of mathematics that involves identifying an attribute to be measured such as length, mass, area and then using

definable, consistent units to find the ‘how muchness’ of the attribute. Such a process of identifying and measuring an attribute can be done procedurally without learners developing a conceptual knowledge of measuring and measurement.

Measurement quantifies phenomena like length, mass, weight, temperature, pressure, speed and brightness, among other words that describe the world around us. Thus, Clements and Battista (1986) define measurement as the process of assigning a number to the physical property of an object to compare and assign a geometric quantity of length, area, and/or volume to the given object. Key measurement activities include, most fundamentally, the act of measuring, as well as conversion and computation (Preston & Thompson, 2004). In some school curricula, the treatments of area, perimeter and volume/capacity are called ‘measurement’ topics and ‘geometry’ topics because they are essentially measurements of geometric figures. Mensuration is a topic in mathematics which thus deals with measurement. In this study, the terms mensuration and measurement will be used interchangeably.

The learning of perimeter, area, and volume have often focused exclusively on formulas to be memorised and applied in routine problems, with far less attention to the development of meaning for the formulas and the associated concepts (Bright & Hoeffner, 1993). As a result, the conceptual understanding of these topics from the learners’ point of view will remain a challenge that needs to be addressed. I shall now define the terms **perimeter**, **area** and **volume** as key topics in the theme of mensuration. In each discussion I will conclude with common misconceptions that are provided by literature.

2.2.1 Perimeter

The term ‘perimeter’ is defined as “the distance around the region” (Dickson, 1989, p. 79). Tall (1991) clarified that the act of measuring perimeter involves finding the length of the outer boundary of a 2-D shape. For example, the perimeter of a circle is referred to as a circumference. According to Sarama and Clements (2009) the perimeter of shapes may be calculated in two different ways: the first one being the arithmetic method which is obtained by adding up all the dimensions around the shape – this method is effective when measuring either regular or irregular shapes; the second method is to multiply the length of a dimension with the number of sides that a shape has, and this method is only applicable to calculating the perimeter of regular (equal sided)

shapes. For example, to find the perimeter of a pentagon that measures five cm on each side, one can simply multiply five cm with the five sides.

A study conducted by Tan-Sisman and Aksu (2016) found that most of the participants' conceptual understanding of measurement was marginally developed. This means that, when carrying out perimeter tasks, learners measured perimeter by counting the squares around the outside of a shape drawn onto a grid instead of the linear units (Tan-Sisman & Aksu, 2016). Consequently, this often brings confusion to learners who think that the more the squares a shape has, the larger its perimeter.

As learners ascertain concepts about perimeter, they are also expected to discover that perimeter is not only found on drawn shapes but also on real life objects, including solid objects (Helen & Monicca, 2005). This means that conceptual understanding of perimeter does not only involve learning of shape outlines but is also about finding and analysing geometric properties and features of shapes that give perimeter for both drawn and real-life objects (Helen & Monicca, 2005; Van de Walle et al., 2014). According to Sarama and Clements (2009), the discovery of shape according to its outer boundary facilitates the discovery of the inside region of a shape. This is an important stepping stone to learning area and to growing understanding from the perimeter to the area of a shape.

2.2.2 Area

Cavanagh (2008) defines area as an iteration of a unit until a flat surface is completely covered, with no gaps or overlaps. Thus, the area of a given region is found on a ground within an enclosure of common two-dimension shapes such as triangles, quadrilaterals and polygons (Huang & Witz, 2011).

Baturo and Nason (1996) explain that generalisation of concepts occurs when learners rely on the basic formula with which they are most familiar. For example, learners would identify any two measures of the length on a given shape and use them in a formula that will give a result in area unit. On the other hand, Cavanagh (2008) cautioned that learners who can use a formula to calculate area do not necessarily have the conceptual understanding for these measurements. It is possible that learners may solely rely on procedures without any conceptual understanding on how and why they work (Cavanagh, 2008). Outhred and Mitchelmore (2000), agree, saying that poor

performance in area measurement is often attributed to the “tendency to learn the formula by rote” (p. 145).

Tan-Sisman and Aksu’s (2016) study on learners’ misconceptions and errors in conceptually carrying out measurement tasks revealed that learners could not recognise how length units produced area units. Learners seemed to confuse lengths of sides with the area of a two-dimensional shape. For example, learners were able to recite the area formula of a rectangle and a triangle but could, however, not explain how the area of a right-angled triangle is derived from half of the area of its rectangle when the same triangles are doubled.



Figure 2.1: The area for a right-angled triangle is half the area of its rectangle

Another example is when learners can calculate the area of a circle when provided with the radius but fail to calculate the area of a circle when only provided with the circumference. This possibly can be attributed to not knowing that the value of r can be deduced and obtained from the circumference formula.

The second misconception, according to Tan-Sisman and Aksu (2016), is that learners simply generalise and thus find it hard to identify the two dimensions that can be used to find the formula of area. These misconceptions show that learners lack a conceptual understanding of measurement. To learn area measurement, learners need to develop a notion of what area is, as well as a more formal understanding of the geometric properties of the shapes that define the dimensions involving calculations of volume (Baturu & Nason, 1996), before laying the foundation that enables them to use the formula (Cavanagh, 2008; Sarama & Clements, 2009).

2.2.3 Volume

According to Cross, Woods and Schweingruber (2009), volume is the amount of three-dimensional space occupied by an object. An exemplary definition is that “the volume of a box is the number

of those cubes (all identical) it takes to fill the box without any gaps” (Cross et al., 2009, p. 36). An example of common geometric 3-D shapes are cubes, prisms, cylinders, pyramids, cones and spheres (Van de Walle et al., 2014). Many common objects in real life are approximate versions of these ideal, theoretical shapes (Sarama & Clements, 2009). For example, a pole takes the shape of a cylinder while a concrete slab resembles a cuboid.

The inner surface of a 3-D shape is usually invisible, unless one cuts the shape open, or the shape is made of clear plastic, or the shape is hollow and a face can be removed to look inside. Sarama and Clements (2009) were quick to advise that in case one of these options is beyond reach, one must usually imagine and visualise the inside. One exception is rooms, which are often in the shape of a rectangular prism, and which one may experience from the inside. The ability to mentally transform (Battista, 2004) and distinguish (Barrett, Clements & Sarama, 2017) the inside of a 3-D shape from its outer surface is an especially important foundation for understanding the underlying concepts of measuring the volume of shapes (Battista, 2004; Machaba, 2016).

Kohar, Fachruddin & Widadah (2021) say that an effective way of teaching volume is making use of learner’s experience of comparing objects: to compare two or more objects by asking which one is greater. Questions such as “which one requires more water to its fullest, a bath tub or a toilet tube can be explored” (Kohar et al., 2021, p. 30). These types of questions can trigger the learners’ thinking in terms of what object holds more water compared to another, for example. Study findings (e.g., Van den Heuvel-Panhuizen & Buys, 2008), found this type of approach to be useful in achieving the conceptual understanding of volume. After building up a basic understanding of volume through the comparison of objects, Kohar et al. (2021) say that learning can then continue by investigating how to find the volume of a solid.

Volume Misconceptions

Van de Walle et al. (2014) assert that the successful learning of measurement is not well achieved when learners concurrently learn the formulas of perimeter, area and volume. The simultaneous learning of these concepts can cause confusion that may result in learners developing misconceptions.

The Namibian Grade 9 Revised Curriculum requires learners to determine the perimeter and area of regular and irregular plane figures as well as to find the volume of solid shapes in theoretical situations and applications in everyday life (MEAC, 2016). Both the act of measurement (finding the actual quantifiable aspects of objects using a variety of instruments and processes to accurately determine measurements) and the use of actual physical units of measurements are an integral part of the learning of this topic. According to Burns (2006), “children are exposed to visual thinking through words, numbers, pictures, images, patterns, signs and symbols from a very early age” (p.16). With regard to measurement, learners bring to school a wealth of experiences gained from informal settings such as the family, the community, and through interaction with the environment. As learning progresses, learning becomes more formal and learners are expected to do more complex procedures and develop and use formulas for determining the measures of attributes like area and volume that are not easily measured directly. This enables learners to develop indirect measurement techniques. For example, the use of similar triangles to determine the height of a tree or a building or find the radius of a circle by measuring and using the circumference of a tree or cylinder.

The Namibian school curriculum promotes the learners’ understanding of measurable attributes that include and expand measures of a whole variety of physical phenomena (sound, light, pressure) as well as for a consideration of rates as measures (pulse, speed, radioactivity) [MEAC, 2016]. All these concepts of attributes are better understood if the aspect of spatial thinking, understanding of space and its relationship to actual objects are well developed using various techniques from an early stage of learning (Kilpatrick et al, 2001; Presmeg & Barrett, 2003). According to Smith, Wiser, Anderson and Krajcik (2006), “given the centrality of measurement in science [and mathematics] and the ways measurement can contribute to, it is important to start early in developing a rich understanding of the measurement of important physical quantities” (p. 33).

Measurement of tangible and directly experienced quantities such as length, mass/weight and capacity/volume lead quickly to the measurement and study of composite quantities such as speed as a rate and science aspects such as density and force (Smith et al., 2011). Measures also stretch to various topics of measurement, including geometry that deals with studies of geometric figures, shapes and forms as elements. Also, topics of proportions, differences, angle positions and

transformation can involve measurements (Choudhary, Dogne & Maheshwari, 2016). All the attributes of measurements in mathematics follow the International System (IS) of units. Hence, the learning of measurement is strictly based on IS units such as metre, metre square and metre cube for length (m), area(m^2) and volume(m^3) respectively. However, studies within the Namibian context (Nambira, Kapenda, Tjipueja, & Sichombe 2009) show that the majority of learners' struggle with IS symbols, particularly with the main attributes of length, mass/weight and capacity/volume. This situation could be attributed to several challenges faced by learners in learning measurements.

The teaching of mathematics has been a challenge in the Namibian context since independence in 1990, whereby learners have consistently displayed unimpressive performances in mathematics (Mateya, Utete, & Ilukena, 2016). This poor understanding, coupled with a negative attitude towards the subject, could lead to a poor performance in the understanding of measurement as an independent topic within the content of mathematics (Barrett, Clements & Sarama, 2017).. According to Panthi and Belbase (2017), challenges of learning mathematics range from social to political and technological. Panthi and Belbase (2017) further outlined that the diversity of language as well as the use of technology and affordability may influence the way learners perceive concepts in measurement. Despite a wide range of studies about challenges of teaching mathematics (e.g., Panthi & Belbase, 2017; Mateya et al., 2016; Suurtamm, 2014), only some of the available literature has addressed the aspect of challenges faced by teachers and learners in teaching and learning measurements in mathematics.

When appropriately taught, Barrett, Clements, and Sarama (2017) argue that measurement should provide conceptual images of quantities that are necessary for learners to engage with other mathematic topics such as algebra and geometry. Thus, combining measurement and the use of what has been measured supports a conceptual approach to the teaching and learning of the topic of measurement that goes beyond the ability to simply memorise applications and formulae (Smith, Silver & Stein, 2005). In my research study, learners are provided with an opportunity to use smartphones as visualisation tools, to access and process physically collected measurements from authentic physical objects and use such measurements to do related mathematical computations involving formulae and algorithms.

2.3 VISUALISATION

2.3.1 Definition of visualisation

The term visualisation is defined in various ways. According to Van de Walle et al. (2014), “visualisation is the recognition of shapes in the environment, developing relationships between two- and three-dimensional objects” (p. 427). They (ibid.) further state that visualisation is the ability to create images of shapes and turn them around into different viewpoints, using the mind’s eye. On the other hand, Lohse, Biolsi, Walker and Rueler (1994) define visualisation as “the study of mechanisms in computers and in humans, which allow them in concert to perceive, use, and communicate visual information” (p. 37).

Related to the two definitions provided above is Arcavi’s (2003) definition of visualisation, which states that it is:

the ability, the process and the product of creation, interpretation, use of and reflection upon pictures, images, diagrams, in our minds, on paper or with technological tools, to depict and communicate information, thinking about and developing previously unknown ideas and advancing understanding. (p. 217)

Arcavi (2003) identifies the process of creation, the use of and reflection upon, pictures, images and diagrams as visualisation. This indicates that visualisation happens when learners draw diagrams on paper during the solving of tasks, as well as when using technological tools at their disposal during this same process.

The above-provided definitions suggest that technological tools such as smartphones may be important visual tools at various levels and stages in the process of learning mathematics. This study adopts the definition of Arcavi (2003), as it strongly resonates with both physical and technological access as two key visualisation tools in advancing conceptual understanding of measurement in mathematics.

2.3.2 The role and purpose of visualisation

Visualisation plays an important role as a powerful tool for learning mathematics and can be helpful when solving mathematical problems (Rösken & Rolka, 2006). According to Makina (2010), visualisation incorporates those mental processes rooted in concrete experiences, that make use of, or are characterised by, *visual imagery*, *visual memory*, *visual processing*, *visual*

relationships, visual attention and visual imagination. Arcavi (2003) further states that “the visual display of information enables us to 'see' the story, to envision some cause-effect relationships, and possibly to remember it vividly” (p. 218). Visualisation tools may assist in enhancing communication (Arcavi, 2003), increasing levels of engagement in learning activities (Ndafenongo, 2011), giving hints when solving tasks (Ludwig, Jesberg & Weiß, 2013) and enhancing learners’ interaction with physical environments (Zender, et al., 2017).

One of the key roles of visualisation is to help learners construct mental representations that precisely reflect mathematical ideas with the presented ideas located outside the mind using technological tools (Radford, Demers, Guzman & Cerulli, 2003). Visualisation then becomes an important aspect of mathematical understanding, insight and reasoning, which in turn enhances learners’ critical thinking (Gal & Linchevski, 2010). In this study, learners will be required to interact with physical artefacts in their environment (that is, on their trails) as they learn measurement. Gutiérrez (1996) stated that some mental images are created from sensory cognition of concrete materials that learners engage with when using their sense organs. These images may then be expressed or communicated in the form of diagrams, pictures, drawings, gestures and discussions. Thus, taking a visual approach to learning mathematics, especially in basic and key concepts like measurement, can provide effective evidence of how and why mathematics works (Söbbeke, 2005; Steinbring, 2006).

Piggott and Woodham (2009) identify three purposes of visualisation: namely, to step into a problem, to model the problem and to plan ahead. Stepping into a problem uses visualisations to help learners understand what the problem is about. This helps learners to create ‘deep’ details about the task before any generalisations can be made. For example, to clarify the objectives and know what tools may be required to support their understanding.

The second purpose of visualisation is to model a situation. According to Piggott and Woodham (2009), this is particularly useful when the situation is physically unattainable, and in this case, learners use visualisation to see the unseen-able. Figure 2.1 below presents an example where learners have to use visualisation to see the unseen. Here learners can only see the part of the tyre that is physically appearing before their eyes. It is through the harnessing of visualisation that learners can be able to use their ‘minds’ eye’ to see the underground part of the tire. This in turn,

helps them to model the entire tyre and be able to calculate its geometric properties such as radius and circumference.



Figure 2.1 A car tyre represents a circle

Piggott and Woodham (2009) present the third purpose of the use of visualisation, which is to plan and think ahead when solving problems considering options, benefits and consequences that may be faced. This purpose is a strong resource for alternative thinking and makes learners take into consideration measures that may bring various consequences to their actions.

Piggott and Woodham (2009) further gave a clear description that visualisation is not only about pictures and diagrams but also a dispensable tool to initiate the development of ideas, facilitate communication of results and support understanding. I thus argue that when learners use actual objects or images on paper or technological devices, their conceptual knowledge is retained for a longer period than when only verbal or written words are used during learning (Makina, 2010; Zimmermann & Cunningham, 1991).

2.3.3 Visualisation processes

Ho (2010) asserts that when presented with mathematical tasks², learners go through the following five processes when solving the tasks: (i) understanding the spatial relations of the elements in the problem; (ii) connecting to a previously solved problem; (iii) constructing a visual representation; (iv) using visualisation to solve the problem; and (v) encoding the answer to the problem. What follows next is a discussion of each of these visualisation process.

2.3.3.1. Understanding spatial relations in the problem

This process demonstrates how learners can correctly perceive the given task visually. According to Khan and Khan (2011), good perception of spatial outlines and properties that a task entails, creates a link between physical and geometric properties of that task. Learners who use their senses to perceive a spatial outline of the task are in a better position to produce effective insights into the task, thereby engaging with real objects. Engagement with real objects aids learners to develop sensory thoughts as they discover the spatial entity for the task. According to Moyer-Packenham, Salkind, Bolyard and Suh (2013), learners use spatial representations to facilitate comparison and pattern recognition, detect change, and other cognitive processes when they manipulate the object tasks.

2.3.3.2 Connecting to a previously solved problem

During this process, learners are exposed to more than one method for a solution as they identify a simpler version of the task, thereby identifying methods, tools and techniques that would work best for the given problem. At some point, learners may even do a trial-and-error method to find a solution to the task. An example is the MCM app that “sends hints on demand” (Ludwig & Jesberg, 2015, p. 6) when learners are simplifying and working on MCM project tasks and problems. As learners make use of the stepped-in hints, they are expected to reflect on previous mistakes or derive simpler methods from a complex method to solve the tasks (Ludwig & Jesberg, 2015). For

² A ‘task’ is both a piece of work and the material that learners measure in order to find the solution

example, calculating the volume of a solid cylinder when the radius is not given is a complex task which requires learners to simplify the task to ways and methods that can help solve the problem. In this case, one way of simplifying the task is to first measure the circumference of the cylinder circle, and this will eventually help them find the radius from the equation $C=2\pi r$, by making the subject of the formula.

2.3.3.3 Constructing a visual representation

The process of constructing a visual representation is also known as transforming the task into mathematical form. During this process, learners generate diagrams on paper, tablets, smartphones, mentally and/or through gestures and other forms. According to Eisenberg and Dreyfus (1991), it takes cognition to be able to read diagrams or perceive cues from the provided visual representation to make a conceptual understanding of the given task. For example, given the task to calculate a volume of a solid, learners need to recognise the need to draw the given shape and label its three dimensions. This process will allow learners with better representation skills to help and demonstrate their skills to others. The process of constructing a visual representation of task can also involve the use and proper understanding of the communicating language.

2.3.3.4 Using visualisation to solve the problem.

The process of using visualisation focuses on how learners use the various tools to translate the visualisation that they have generated, into a solution. According to Friedrich and Mandl (1992), learners use visualisation as a scaffolding mechanism to focus, paraphrase, elaborate and activate the need to solve a given task. For Cheng (2013), using visualisation tools increases the relevance of the tasks, to using mathematics in everyday situations as they imprint reality in learner's minds. Mobile applications on smartphones such as a calculator, Google search engine and dictionaries are but a few tools that can be utilised by learners to make sense of, or see clarity within the given task. Learners can also use the tools to develop the visualisation of what intrigues them, to solve a given task.

2.3.3.5 Encoding the answer to the problem

The process of encoding a solution to a problem focuses on the link created between the visual representation and the solution to the task. When learners have access to a solution to a given task,

they have a better chance of gauging how reasonable their answer is. According to Dongwi (2018), converting a mathematical problem into an encoded form causes learners to have thorough knowledge of the problem as it involves interpreting the knowledge perceived and encoding it into visual forms. This means that learners are able to code when they can compare their suggested solution with the visual representation that they have generated while solving the problem. Learners reread the question and revisit the visual representation that they have just crafted. The encoding process allows learners to convert the sensory thoughts into virtual thoughts and vice versa.

This study uses Ho's (2010) five ideas to identify and classify visualisation processes evident in the learners' interaction with the learning environment, as they use the smartphones to learn about measurement of 2-D and 3-D shapes at Grade 9 level, using the MCM application.

2.3.4 Visualisation and code switching.

Another important aspect of this study is the use of language and visualisation. Literature sources (Vygotsky 1978; Dowling, 2002) show that language as a mediating tool of teaching and learning, plays an important role in the learning of mathematics. Considering that English as a medium of instruction is a foreign language to the participating learners of this study, it follows that during the solving of the tasks, learners could switch languages through code switching to help them solve the given tasks, as well as effectively understand the concepts embedded within the tasks.

Code switching refers to the alternate use of two or more languages in one conversation, where the participant in the conversation speaks more than one language (Adler, 2010; Mudaly & Naidoo, 2015; Maluleke, 2019). Neville-Barton and Barton (2005) posit that code switching is a common practice between learners with the same mother tongue, and that in most cases learners use code switching in their personal and mathematical conversations. According to Shilamba (2012), Namibia is one of the countries where learners and teachers often share a common mother tongue, and hence it is likely that communication occurs in both English and the mother tongue.

In his study, Jegede (2011) found that learners prefer being taught mathematics in their mother tongue rather than in English (their second language) for clearer understanding. This shows that learners who are taught in a language other than their mother tongue will always opt to code switch to their mother tongue for the effective understanding of difficult concepts, and this may be the

case in this study. When learners code switch to their mother tongue in mathematics, according to Straehler-Pohl and Gellert (2013), words, can become contextualised visualisations. Chikiwa and Schäfer (2019) add to this, saying that the use of the mother tongue in the teaching of mathematics helps learners to visualise mathematical ideas. Thus, in the context of this study, the co-emergence of language and visualisation through code-switching (Chikiwa & Schäfer, 2019) can help learners to conceptually understand difficult concepts of area, volume and measurement.

2.3.5 Challenges for visualisation

Despite positive results noted in literature, several challenges concerning the use of visualisation are articulated. When solving tasks, learners do not always use visualisation, although this does not however imply that learners are reluctant to visualise in mathematics. Firstly, Khan and Khan (2011) found that some learners may find it hard to understand the perceptual cognitive details of the given task. A major cause of this is when learners do not correctly interpret the visual presented to them. Presmeg (1986) describes such a phenomenon as the inability to “concretise the referent” (p. 54). This means that a challenge occurs while embodying an abstract idea in a concrete image. For example, a learner who cannot concretise the referent would not even be able to assign dimensions of a solid figure in order to calculate its volume.

The second challenge pertains to the inability to pair visualisation with analytic thought (Khan, & Khan, 2011). This means that learners are not able to see, or even if they see, they may not voluntarily use visualisation when solving the task (Presmeg, 1986). According to Phillips, Norris and Macnab (2010), learners with this defect are unable to transfer the task into their imaginations. Learners then become prone to bringing to their thoughts irrelevant details concerning the task. An example to this challenge is given by Fernández, De Bock, Verschaffel and Van Dooren (2014) in which the authors found that some learners tend to use all four dimensions of a rectangle to calculate area when only two adjacent sides are required to carry out this computation.

The third challenge is when learners lack prior knowledge of conceptual traits that the task entails, even when provided with scaffolding visualisation. According to Khan and Khan (2011), learners who lack prior knowledge do not effectively understand the information that they are looking for and are unable to manipulate visualisation effectively and effortlessly. Ginsburg (1977) claims evidence which is later supported by Zeromska (2010), that learners with marginalised prior

conceptual knowledge demonstrate rote memorisation, often at the expense of conceptual understanding. Additionally, Kordaki and Potari (2002) say that a pre-occupation with formulae denies learners the opportunity to access other tools to represent measurement concepts. For example, if a learner cannot recall the formula for calculating the circumference of a circle, that learner may not achieve the objective of finding a radius when only measurements for circumference are available.

All three of the challenges discussed above are related to what Fischbein (1987) referred to as an intervening conceptual structure. This means that when learners are given a visual, it is not necessarily guaranteed that they will use it to solve the mathematics problem. In most cases, it is not that the learners are reluctant to visualize, but that they need to have a robust collection of the knowledge that enables them to exploit the considerable potential for visualisation, and to use the same visualisation to support their conceptual understanding (Khan & Khan, 2011).

Mathematical visualisation is known to arise when working with objects that learners manipulate (Spencer, 2008). This means that the more opportunities that are provided to learners to concretely go through mathematical tasks such as measurement, the more they may be exposed to possibilities of constructing mental representations that precisely reflect mathematical relationships located outside their minds (Ho, 2010; Presmeg, 2006).

2.3.6 Visualisation in the outdoor environment

Most mobile technology devices and applications for mathematics learning have only been employed in regular indoor classroom teaching settings (Wijers, Jonker & Drijvers, 2010). Examples of these are VITALmaths (Ndafenongo, 2011) and (Kadry & Ghazal, 2019). It is thus necessary to explore the potential use of mobile technology for learning mathematics concepts in outdoor settings. Mobile phones in this study are to be utilised as visual devices to engage learners in outdoor mathematical activities that have the potential to promote conceptual understanding. The use of a combination of locally found objects and virtual reality is expected to contribute to learner engagement, interaction and increased motivation (Wijers et al., 2010). Zender et al. (2017) state that embodied mathematics helps learners develop sensory thoughts.

It is hoped that through this research, learners will learn that, in addition to socialising and leisure, smartphones can also be used for learning purposes. It is also hoped that teachers and subject

advisors who will read this study will acknowledge the use of smartphones as well as the inclusion of physical activities in the mathematics learning process. This kind of mediation not only extends learning to outside the classroom, but it also provides users with the opportunity to personalise and control their learning, creatively owning their learning processes and easily communicating with their peers (Laurillard, 2009).

2.4 SMARTPHONES AS MOBILE TECHNOLOGY IN MATHEMATICS EDUCATION

A smartphone has become a multipurpose ICT mobile device in many societies, including those in the developing world. According to Hilbert and Aravindakshan (2018), a smartphone can be regarded as a handheld computer with an intrinsic connection to mobile networks. Sharples, Taylor, & Vavoula, (2010) explain that a smartphone's ability to access mobile networks gives the phone advantages as a visual tool that can be used to learn mathematics in and outside the classroom. Thus, smartphones can provide suitable learning platforms as they have a lot of applications that learners may use in environments other than the classroom, for their academic activities (Fabian et al., 2018). Herrington, Mantei, Herrington, Olney, and Ferry (2014) assert that smartphone devices can be used to create polished and worthwhile products such as learning tasks that can be shared, solved in groups, published and used to generate discussions.

2.4.1 The use of smartphone technology and visualisation to learn mathematics.

This research recognises learners' use of smartphones in many instances of their everyday lives and their potential use for learning measurement concepts in mathematics. Recently the mathematics education research community has started to focus on the smartphone as a tool to access learning (Hilbert & Aravindakshan, 2018; Machmud, 2018; Drijvers, 2019). Smartphones allow mobility during learning (Zender et al., 2017) and this, in turn, may promote visualisations rooted in evoked images of real and everyday objects learners have interacted with before. Learners thus may be prompted to think about their own experiences and share with their peers during solving problems during the trails. As noted by Cahyono (2018), when learners use their smartphones, they "... have the opportunity to construct their own mathematical knowledge by solving the prepared tasks on the math trail supported by their interaction with the environment, including the digital environment" (p. 43). The smartphone's ability to encourage active learner interaction and integration with the environment, allowing learners to leave the boundaries of the classroom and engage with hands-on mathematics activities is central in this study which

researches measurement concepts. The interaction with smartphones in a mathematics learning environment is made possible by mobile applications such as the MCM.

Ndafenongo (2011) says that with visualisation, “the overall goal of instruction is to help learners construct mental representations that correctly or accurately mirror mathematical relationships in instructional representation located outside the mind” (p. 29). Visualisation then becomes an important aspect of mathematical understanding, insight and reasoning, which in turn enhances the students’ critical thinking.

The smartphone today has found pervasive and inconspicuous ways to suggest itself into the mathematics classroom (Garrison & Anderson, 2003; Prensky, 2005; Kolb, 2008). For Selanikio (2008) smartphones are versatile and have the most access to the internet compared with other technological devices such as laptops and desktop computers. For example, smartphones in the classroom can be used to conveniently find a formula for a certain computation, perform a calculation or search for the meaning of certain concepts on the internet.

On the other hand, In Namibia, the policy guidelines for Namibian education prohibits learners from bringing technological tools such as cell phones and tablets into classrooms (Namibia Press Agency, 2007; MOEC, 2018). This is because the use of smartphones in the classroom is currently considered a disruptive practice as learners could be tempted to open unauthorised applications on the smartphone and stop following the teaching in class. To ensure effective use of the smartphones and minimise disruption, it is advised that teachers set acceptable rules with the learners (Pulliam, 2017; Uğur & Koç, 2015). For example, an agreement between the teacher and learners can be reached to prevent distractions such as those from ringtones or the transfer of messages during the lesson. In addition, teachers should employ methods that assure learners to know that they are under supervision, even when they are allowed to use cell phones on the school premises.

These technologies influence the teaching and learning process in mathematics and research has been conducted to unearth the vast development of smartphone use. Kalloo and Mohan (2012) designed the MobileMath application in search of possibilities and contributions of mobile phones to assist learners in the Caribbean to learn mathematics through mobile learning. The MobileMath application presents lessons, tutorials, quizzes and examples to solve problems under the topic of algebra. Results from research reveals that learners were able to improve their performance in

algebra. Learners were also motivated and adapted to learning anywhere and anytime using mobile phones.

Batista and Barcelos (2014) investigated the use of mobile phones in the context of Brazilian schools. The views collected from mathematics students and teachers were used as data to justify the two-sided practicality of using smartphones as learning tools. Firstly, the smartphones were used to make the teaching and learning of mathematics more alluring. Secondly, the advantage of smartphones possessing a great portability could be used to promote learning both inside and outside the physical space of educational settings. The research then concluded that the use of mobile phones provides opportunities for ubiquitous access to learning (Batista & Barcelos, 2014).

Ndafenongo's (2011) study investigated how mobile phones can be used in the teaching of mathematics in South Africa. In his studies, he used five VITALmaths video clips on Pythagoras' Theorem. The videos were sent to learners' phones and were used in class to support understanding of the subject. Class participation was improved, content was developed, and peer interaction enhanced the much-anticipated learner autonomy. The investigation showed that mobile phones can be a useful resource to support teaching and learning in the classroom.

Within Namibia, a small number of projects have reported success in attempts to use cell phone technology to support the learning of mathematics. Kachepa and Jere (2014) explored to find out if mobile users learn any mathematics when playing games. This study came into being when the researchers discovered that the young generation spend most of their time playing games. The researchers wanted to learn about the relationship between the mobile games and mathematics learning. This research was conducted in two different contexts: the first one was conducted at three Namibian high schools in the rural areas. The other study was conducted at Polytechnic of Namibia (now Namibia University of Science & Technology, NUST) and it involved first year students at the institution. Based on their findings, the researchers recommend that the key stakeholders to learning, namely parents, teachers, learners, mobile application developers and the government of Namibia, should consider using existing mobile applications to enhance mathematics learning.

A mobile electronic learning pilot project was conducted in Usakos, a remote town in Namibia. Twenty out-of-school youth were introduced to a three-day workshop, which was aimed at finding

ways in which mobile phone technology can facilitate ICT skills development for participating youth in the remote town of Namibia. The project was organised by Maurice Nkusi, the chairperson for Namibia Open Learning Network trust e-learning standard committee (NelNet-eLC) in 2015. Activities in the project ranged from accessing content and resources that are compatible with mobile phones, collaboration among students (the youth) and tutors, and searching of information through the Internet. All these activities were collectively given a technical name: Tele-teaching. Participants were equipped with skills that enabled them to apply learning using mobile phones. It was envisaged that after the pilot project, the system would be extended to other youth in the rest of Namibia.

2.4.2 The MathCityMap project

The MCM project is a protocol-based application centered on the principles of mathematics trails whereby users follow a set mathematics trail with the aid of a smartphone, stopping at strategically given waypoints to solve a given mathematical task (Cahyono, 2018). The MCM project is supported by the use of a GPS and is based in Germany at the Goethe University (Ludwig, Jesberg & Weiß, 2013). The MCM app can be accessed through the MCM web portal (www.mathcitymap.eu), which is downloaded and installed in smartphones. The project allows for the integration of mathematics and its learning beyond the boundaries of a classroom, using smartphones (Cahyono & Ludwig, 2019). The mathematics that is learned from the classroom (textbooks and chalkboard) is linked to reality through outdoor activities in the outdoor environments, through math trails. A math trail is defined as “a walk to discover mathematics” (Shoaf, Pollak, & Schneider, 2004, p. 10). The walk forms a route, which leads to special locations where interesting mathematical problems, called MCM tasks (for example, see Appendix A), can be found and solved (Ludwig, Jesberg & Weiß, 2013).

The idea of math trails originated some decades ago. In the 1980s, the first documented math trails were created in Melbourne by the Australian mathematics educators Blane and Clarke who prepared thought-provoking mathematics tasks that could be solved by families while on holiday (Blane & Clarke, 1984). Thereafter, mathematics educators took advantage of the existence of these trails by including them in their instructional programmes. The MCM project made use of the already existing idea of mathematics trails in the educational context with the help of new

technologies (Ludwig et al., 2013). This project is worldwide and freely accessible, provided there is internet access at the preferred task location.

The project has four broad components that include outdoor tasks, mathematical city or outdoor trips, the map-based app and the MCM community. As explained by Ludwig and Jesberg (2015) the physical math trail tasks created by the task authors can be accessed via a mobile app bridged through a web portal. Summarised by Cahyono (2018), this portal is useful for four reasons: first is receiving mathematical tasks; second is for storing created tasks so that they can be reused whenever need arises; the third reason is to create and store the routes that have locations with tasks; and lastly it is to provide, at most, three stepped hints for each task that authors have uploaded.

These tasks are accessed through the GPS coordinates on a math trail map installed on a smartphone. Trail data and task spots are available on the MCM app map. Moreover, the app gives direct feedback on entered solutions (Zender et al., 2017), and offers opportunities for the user to ask for stepped hints (Ludwig et al., 2013). These stepped hints are important for helping learners once they fail at their first attempt to find the solution. The app provides learners with up to three opportunities to get a hint. The concept of stepped hints is an especially important component of the MCM project. According to Friedrich and Mandl (1992), one of the rewarding functions that stepped hints have on learning is motivation and enhancing visualisation. This means that when learners get more chances of attempting to solve a task, they can employ other methods, such as sketching, probing questions and engaging in physical contact with the task material so that they seek more ways of representing the task problem, before going to other tasks in the trail (Zender et al., 2017).

My study views the MCM project as providing a vast potential to use smartphones for mathematical purposes in Namibia and possibly later in other parts of the Southern African Development Community (SADC). It is core to my study to provide environments where learners can experience, discover and learn mathematical concepts of measurement outdoors for conceptual understanding, using the latest technology.

2.5 MATHEMATICAL PROFICIENCY

In today's educational settings, there is need to identify and adopt a comprehensive view of learning mathematics proficiently and successfully. An analysis of how mathematics is learned was made, resulting in the development of a framework by Kilpatrick et al. (2001) and is called mathematical proficiency. This framework consists of goals which, according to the authors, have acceptable indicators towards learning mathematics. Kilpatrick et al. (2001) mention that "recognising that no term captures completely all aspects of expertise, competence, knowledge, and facility in mathematics, we have chosen mathematical proficiency to capture what we believe is necessary for anyone to learn mathematics successfully" (p.116). Below, Table 2.1 shows a brief description of the five strands of mathematical proficiency from Kilpatrick et al.'s (2001) point of view.

Table 2.1 The five strands of mathematics proficiency

Strand	Description
<i>Conceptual understanding</i>	comprehension of mathematical concepts, operations, and relations
<i>Procedural fluency</i>	skill in carrying out procedures flexibly, accurately, efficiently, and appropriately
<i>Strategic competence</i>	ability to formulate, represent, and solve mathematical problems
<i>Adaptive reasoning</i>	capacity for logical thought, reflection, explanation, and justification
<i>Productive disposition</i>	habitual inclination to see mathematics as sensible, useful, and worthwhile, coupled with a belief in diligence and one's own efficacy.

Table 2.1 shows the framework for the five strands of mathematical proficiency. The framework consists of five strands which provide a discussion/guide for knowledge, skills, abilities and beliefs that constitute mathematical proficiency. Although Kilpatrick et al. (2001) identified five strands of mathematics proficiency, my study will only focus on conceptual understanding, as this is central to my study. This study specifically looked at how the use of smartphones aided conceptual understanding of mensuration concepts in Grade 9.

2.5.1 Conceptual understanding

A concept is a general idea, understanding or thought embodying a set of things that have one or more properties in common (Barr, Graham, Hunter, Keown, & McGee, 1997). Therefore, conceptual understanding can also be called conceptual knowledge, which Hiebert and Lefevre (1986) refer to as a set of connected webs of knowledge and relationships. Thus, when reviewing literature for this construct, I use the terms conceptual understanding and/or conceptual knowledge interchangeably.

Rittle-Johnson and Schneider (2015) define conceptual understanding as acquiring an abstract or generic idea generalised from particular instances. Balka, Hull and Harbin (2012), share the same sentiments and describe conceptual understanding as a reflection of learners' abilities to reason in settings that demand the careful application of concept definitions, relations, or representations. For Gabriel, Coche, Szucs, Carette, Rey, and Content (2012), conceptual knowledge refers to "the explicit or implicit understanding of the principles ruling a domain and the interrelations between the different parts of knowledge in a domain" (p. 137). The above given working definitions lead to observable traits/indicators that can demonstrate learners' conceptual understanding of what has been taught to them.

Balka et al. (2012) further suggest that learners demonstrate conceptual understanding in mathematics in this way:

They provide evidence that they can recognise, label, and generate examples of concepts; Use and interrelate models, diagrams, manipulatives, and varied representations of concepts; Identify and apply principles; Know and apply facts and definitions; compare, contrast, and integrate related concepts and principles; Recognise, interpret, and apply the signs, symbols, and terms used to represent concepts. (p. 2)

Berger (2006) argues that conceptual understanding of mathematics does not only cover what is known but also what can be known, thereby creating a link between the pre-knowledge and new understanding that can be obtained. Kilpatrick et al. (2001) believe that "when learners have acquired conceptual understanding in an area of mathematics, they see the connections between concepts and procedures and can give arguments to explain why some facts are consequences of others" (p.119).

The following are key conceptual indicators adapted from Kilpatrick et al. (2001):

2.5.1.1 Connecting mathematics to prior knowledge

This indicator involves adapting acquired knowledge to new situations and using it to move toward more efficient approaches when solving mathematics tasks. Prior knowledge includes what learners learnt in previous lessons, as well as what they can visually recall and use from their home environment (Chikiwa & Schäfer, 2019). Learners who arrive at the avenue of learning with visualisation demonstrate the ability to see connections between the mathematics that they are learning and what they already know (Kilpatrick et al., 2001). This implies that learners should learn mathematics with understanding, actively building new knowledge from past experiences.

2.5.1.2 Justifying and explaining mathematical ideas and solutions

This indicator refers to the ability to provide evidence for clearly explaining and articulating mathematical concepts and ideas. Learners can manipulate representations, compare concepts, and apply facts and definitions to justify solutions to mathematical tasks. Brodie (2010) explains that learners who discuss their finding with others afford themselves ample opportunities which enable them to justify their solutions. When learners justify and explain their work, they develop a positive self-concept as well as confidence in their learning of mathematical concepts. Justifying and explaining one's ideas outsmarts rote learning. Moreover, when learners are required to explain why they have chosen a certain method for solving a task rather than just jotting down an answer, they gain crucial reasoning skills.

2.5.1.3 Representing mathematical concepts in different ways

This indicator refers to learners' abilities to show different representations of the same mathematical concepts. In this study, different representations include different diagrammatic and visualised forms of mathematical concepts of measurement. No single representation can depict all aspects of a mathematical concept. For instance, texts are better suited for abstract contexts or for asserting assumptions, whereas pictures are better suited to represent spatial relationships (Clements, 1999).

Learners who get more opportunities to represent concepts in different ways can “correctly form and integrate rich web of concepts and therefore develop a relational understanding” (Kilpatrick et al., 2001, p. 24). Additionally, Lesh, Cramer, Doerr, Post and Zawojewski (2003) listed five

different representations of mathematical ideas, which are pictures, written symbols, oral language, real world situations and manipulative models.

2.5.1.4 Connecting ideas and concepts in mathematics to the real world

This indicator is about the ability of learners to connect and link mathematical knowledge to the outside world and see the practical relevance of the knowledge thereof. Technology plays a linking role in integrating virtual concepts with real life objects. In this research, smartphones enabled learners to actively integrate with the real world by leading learners to task locations around the school ground (Cahyono, 2018). Integrating real life objects is identified by Lesh et. al. (2003) as one of the different ways of promoting relational understanding. Real life objects allow learners to think about and test emerging ideas in ways that are familiar to them.

The conceptual understanding indicators by Kilpatrick et al. (2001) are used in this study to explore how smartphones were used as visual tools to enhance conceptual understanding. Thus, the indicators form an integral part of my analytical framework – see Appendix I.

2.6 THEORETICAL FRAMEWORK

2.6.1. Social Constructivism

This study is informed by Vygotsky's (1978) social constructivism theory. The social constructivism theory states that learning is an active meaning-seeking process by the learner (Vygotsky, 1978) rather than a passive transfer of learning content from teacher to learner (Wenger, 1998). The theory views mathematics learning as a participation in a community where learners are taught to conceptualise situations, communicate about these situations and to use available resources for learning and communicating (Li & Lam, 2013). Gupta (2011) added that social constructivism provides tools and tasks with which learners infer ideas as well as formulate solutions and test their ideas in a collaborative learning environment.

The theoretical views of social constructivism suggest that it is an epistemological theory that asserts that knowledge is first formed between people before it becomes internalised within learners (Daniels & Walker, 2001). Amineh and Asl (2015) affirmed that the constructivist learning orientation seeks to understand how learners create their own knowledge structures and what these structures mean for understanding their influence on thought processes. According to Woolfolk, Davis and Anderman (2013), meaning making is a creation of knowledge that cannot

be separated from the social environment in which it is formed through communication. The way learners experience learning is largely influenced by interaction with the learning environment (Cobb, Yackel & Wood, 1992; Ling & Donner, 2009), thereby promoting interactive learning and facilitating the control of the learning process and its relationship with the real world (Hayes & Scassellati, 2013).

A key element in this research study pertains to enhancing learners' ability to use smartphones and visualisation processes as an inherent component in the learning of measurement concepts. The blend/conflux of smartphone technology, the working in groups and having physical access to learning tasks sits neatly within the basis for social constructivism. This is because in this study, successful solutions to the tasks will be attained when participants interact with peer learners while on a math trail, with the learning environment as well as with the MCM app using smartphones. The physical materials and/or diagrams that learners encounter play an embodiment role that enables learners to grasp measurement tasks meaningfully (Clements & Sarama, 2014). All these aspects promote the social constructivist orientation which advocates for learning as socially situated, and knowledge being created through interactions with others (McKinley, 2015). In other words, learning is not necessarily restricted to an individual learner making decisions in isolation, nor as an empty vessel, but is an interaction between learners and teachers through active and meaningful communication (Li & Lam, 2013).

Proponents of social constructivism (e.g., Ramaley & Zia 2005; Ramalho, Adams, Huggard & Hoare, 2015; Dagar & Yadav, 2016) emphasise the importance of learning environments which are active, social, and learner centered. Ramalho, Adams, Huggard and Hoare (2015) suggested that “learners should be engaged in tasks, seen as ends in themselves and consequently having implicit worth” (p. 251). Kim (2001) stated that social constructivists view of learning as a social process; it does not take place only within an individual, nor as a passive development of behaviour shared by external force. Kim (2001) pointed out that social constructivism is based on specific assumptions, which are *reality*, *knowledge*, and *learning*. I now discuss each of these assumptions below.

The first assumption of social constructivism is *reality*, which states that reality does not exist in advance; instead, it is constructed through human activity. Noddings (2018) asserts that learning

that is embedded in reality seeks “to understand the effects of physical and cultural environments on people” (p. 15), thereby focusing on the learner’s purposive interaction with those environments.

The second assumption is *knowledge*, which places the social and cultural constructs at the centre of learning. This means that when learners are given a task they are expected to create meaning when they interact with others as well as with their learning environment (Ernest, 1999; Gredler, 2012).

Lastly, the *learning* assumption for constructivism stresses that learning is a social process. Learning does not take place only within an individual, nor is it passively developed by external forces (McMahon, 1997). This means that interaction and collaboration is required before learners can internalise the knowledge that they construct.

Chikiwa and Schäfer (2019) argue that “any lack of connection-making between school mathematics and the day-to-day lives of its recipients detaches mathematics from reality, thus making it an abstract and detached subject” (p.126). Meaningful learning occurs when individuals are engaged in social activities that are happening not only in the classroom but also at home or in the community, fashioned by instrument use and directing their energy in infinite circumstances of time and space (Vygotsky, 1978). Gupta (2011) adds that social constructivism provides tools and tasks with which learners infer ideas as well as formulate solutions and test their ideas in a collaborative learning environment.

Ramaley and Zia (2005) maintain that the newer forms of technologies such as smartphones “enrich traditional forms of learning and serve as links between active and passive learning, individual and group learning, and the transmission and generation of knowledge” (p. 10). All these aspects promote the social constructivist orientation which advocates that learning is not necessarily restricted to an individual learner making decisions in isolation, nor considered as an empty vessel.

I see measurement as one of the topics that involves engagement and interactions from the learners. On the other hand, this study requires learners to work in groups of three and this means that for them to gather correct and accurate measurements of the dimensions of objects, there is a need for

them to collaboratively work together. Collaboration is one of the attributes of the social constructivist theory, where knowledge is first constructed in real social contexts before it becomes internalised and personalised by individual learners (Jaramillo, 1996).

Further, the learner's experiences of using smartphones in their everyday lives is a reality that suits this study within the social constructivism theory. Although using smartphones for learning purposes may be something new to many learners, smartphone technology and its implications on their everyday life is real. Furthermore, tasks created within the MCM framework links mathematics with reality through outdoor activities with smartphones (Cahyono & Ludwig, 2019). Thus, the potential effect of such a learning approach is to create a learner-centered environment where learners actively engage and collaborate with each other and/or the teacher as well as the materials (e.g. smartphones).

Proponents of mobile learning advocate that the activities used in mobile learning are social constructivists, since the learners work in collaborative groups (Isenberg, Elmqvist, Scholtz, Cernea & Ma, 2011), thereby promoting interactive learning, and facilitating the control of the learning process and its relationship with the real world (Hayes & Scassellati, 2013).

2.7 CONCLUSION

In this chapter I presented literature by defining mensuration, firstly as a discipline in mathematics and secondly, in the context of the Namibian broad curriculum and Grade 9 mathematics syllabus. Also, as part of this literature I discussed the role of visualisation in learning measurement as well as the processes that learners go through when solving mathematics tasks involving measurement. This was followed by the discussion of smartphones as a tool to learn mathematics at mobile places other than the classroom. The discussion engaged literature that pertains smartphone use and visualisation.

My engagement with literature enabled me to introduce the MCM app in the quest to help learners to conceptually understand the topic of mensuration as a central element of proficiency in Grade 9. Further, with the review of this literature, I was able to adopt observable indicators for the analysis of this study. Lastly, this chapter emphasises the relationship that exists between conceptual understanding, social constructivism and how they inform the learning of measurement in Grade 9.

CHAPTER 3

METHODOLOGY

3.1 INTRODUCTION

The main goal of this chapter is to describe the methodology that I used to conduct this research. The chapter begins by introducing the research goals and questions that frame the research. This is followed by the design and the tools that were used to collect data.

I also provide an outline for the analysis that I conducted in order to seek evidence of how smartphones were used for visualisation processes, leading to the development of conceptual understanding. The chapter then ends with descriptions on how I addressed the ethical issues. This involves a thorough description on how the external review process was conducted in order to harvest valid and worthwhile MCM tasks.

3.2 RESEARCH GOALS AND QUESTIONS

The goal of this was twofold. The first goal was to investigate the role that smartphones play as visualisation tools in developing conceptual understanding in the learning of mensuration using the MCM app in Grade 9. The second goal was to explore the selected learners' experiences in using smartphones as visualisation tools in the learning of mensuration of shapes outside their classrooms. To achieve these goals, the following two research questions were developed:

1. How can smartphones be used as visualisation tools in developing conceptual understanding in the learning of mensuration using the MCM app in Grade 9?
2. What are the selected learners' experiences of learning mensuration of shapes using smartphones as visualisation tools outside the classroom?

3.3 RESEARCH ORIENTATION

3.3.1 Interpretive paradigm

This research project was oriented in the interpretive paradigm. Creswell and Creswell (2017) define an interpretive paradigm as an orientation that is focused on the participants' constructed human interaction and experiences. Bertram and Christiansen (2015) explain that interpretive research "describes and understands how people make sense of their world and how they make

meaning of particular actions” (p. 26). Additionally, Creswell and Creswell (2017) said that researchers in this paradigm tend to rely upon the participants' views of the situation being studied in order to maintain the focus on the phenomenon under investigation. In the context of this study, my aim was to understand how participants shared their learning experiences with their peers as they solved the MCM tasks.

Following an interpretive approach to this study was important because it allowed me to understand the role that smartphones played as visualisation tools from the participants' (learners') perspectives. In order to perceive learners' understandings, I had to enter the worlds of selected learners and observe them solving the problems, as well as probe them about their experiences and reflections regarding their use of smartphones to learn measurement in mathematics.

3.3.2 Qualitative research

This study employs a qualitative approach. According to Ross and Onwuegbuzie (2012), a qualitative approach to research is defined as an approach that aims to understand the meaning of phenomena under study as well as the relationships among naturally occurring variables. Moreover, Merriam (2001) suggests that in qualitative research, the emphasis is on “understanding how people interpret their experiences, how they construct their worlds, and what meaning they attribute to their experiences” (p. 14). This research focused on observing how participants interacted with the smartphone in order to arrive at the concrete measurement learning tasks in their natural environment. Another supporting fact is that I interacted with the research participants through interviews in order to find out about and interpret their experience regarding the use of smartphones at home and for the purpose of learning. The qualitative approach also gave me the opportunity to observe learning as a social practice and how the participants explain their own worlds (Jackson, 1995).

3.4 METHODOLOGY: A CASE STUDY

Creswell (2002, p. 4) defines a research methodology as the underpinning philosophical framework and fundamental assumptions that frame any research studies. This study took the form of a case study. According to Zainal, "a case study is an empirical inquiry that investigates a contemporary phenomenon within its real-life context; when boundaries between phenomenon and context are not clearly evident, and in which multiple sources of evidence are used" (2007, p. 2).

This research is a case study because I undertook an in-depth analysis of the learners' use of smartphones to learn mathematics. Another definition that literature provided is that a case study is an "exploration or in-depth analysis of a specific instance that is designed to involve multiple sources of information that are rich in content" (Cohen et al., 2011, p. 253). Yin (2011) adds that the objective of case study research is to explain, explore and describe in detail what happened in the study programme and why. The case study method enabled me to study the use of smartphones beyond quantifying the data and to deeply understand the learners' perspectives. In this study, the case was a set of twelve purposefully selected Grade 9 learners.

This study conforms with 'descriptive' and 'exploratory' as two of the types of case studies postulated by Zainal (2007). I now explain how the two resonate with my studies. Firstly, the descriptive case study describes the natural phenomena that are articulated at the outset of learning. For example, in this research, I observed the kinds of prior knowledge that learners brought to new, outdoor situations in which they found themselves as they solved the MCM tasks. The case of interest for me was to study and understand how learners followed MCM trails beyond the classroom boundaries, in order to discover the reasons for the way they did as they engaged with measurement of shapes that were naturally found on the school grounds.

The second type that this study takes is the exploratory case study, which aims to explore any event in the data which serves as a point of interest to the researcher (Zainal, 2007). For example, in this research, I asked learners to express their feelings and concerns regarding the use of the MCM application on the smartphones to carry out the measurement tasks.

The unit of analysis associated with this case was the learners' engagement with the MCM app on their smartphones in their school environment and peers, with reference to the visualisation processes employed when solving the given MCM tasks.

3.5 SAMPLE OF THE STUDY

3.5.1 Purposive Sampling

In this section, I discuss purposive sampling as the technique that was used to select the sample for this study. Rule and John (2011) define purposeful or purposive sampling as a method in terms of which the people selected as research participants are deliberately chosen because of their suitability in advancing the purposes of the research (p. 64). The first reason for using purposive sampling was to allow me to have ample time to run the tasks with my learners who I knew well and who would be willing to give me the required time, instead of working with learners from a different school who could perhaps not have trusted me.

The second reason for using purposive sampling is that it helped me to select learners that met the specific criteria. The selection criteria for the participants for this was based on learners (i) who were in Grade 9; (ii) have expressed interest in volunteering to take part in a smartphone-run project. My intention was also to work with participants that (iii) own a smartphone or have access to one at home. This is because I wanted learners to share their views regarding their personal experiences of smartphone use in settings other than those of learning.

The sample

In this study, twelve learners were selected using the criteria given above. The twelve participants were allocated to four groups of three learners each. This was to allow easy distribution of roles played by group members. In each group, one learner was (i) holding the smartphone to navigate the trail; another was (ii) measuring the shapes; and the third member (iii) writing down the measurements of shapes and scribbling sketches or calculations on paper when it was necessary. Learners changed roles as they moved from one task to the next.

3.6 RESEARCH DESIGN

This research was conducted in four phases: 1. Selection of research site and participants; 2. Introduction and orientation to the MCM app; 3. Pilot Trail; and 4. Implementation of the MCM trails.

3.6.1 Phase 1: Selection of research site and participants

This case study took place at a school located in the Oshana region in the northern part of Namibia. The place is conveniently selected because I teach there. The selected participants were twelve

Grade 9 learners. The participants were purposively selected on the basis that they were in Grade 9, had expressed interest in this study, volunteered to take part in the project and they own or have access to a smartphone at home.

3.6.2 Phase 2: Introduction and orientation to the MCM app

I introduced the twelve selected learners to the MCM app. The introduction and orientation session took place on three afternoons for two hours each afternoon. The purpose of the introduction and orientation to the MCM project was to familiarize learners with:

- (i) the experience of using smartphones as an instructional device; and
- (ii) to familiarize themselves with the Math City Map app.

The use of cell phones on the school premises is restricted rather than regulated. This means that instead of being guided on application that learners may use, the possession and use of smartphone is strictly prohibited. As a result, this was likely to be the first time that learners were using the device freely. It was also important that the learners became familiar with the other additional applications—that were going to be used in the study. At this point, I made sure that that any technical problems were dealt with. During this orientation phase, learners were freely allowed to operate the smartphones on their own and ask if there was anything that was not clear to them before the commencement of the running of the MCM trails.

The discussion of questions and suggestions, and agreement to the solution was done collectively by all group members. Each group was expected to use only one smartphone, provide their preferred group name, install the MCM app and download the school trail. Only one group at a time walked a trail. This was to allow full coverage of video recording while learners interacted amongst themselves and with the smartphones.

3.6.3 Phase 3: Piloting the trails

Three learners who were not part of the twelve already chosen, were requested to participate in the pilot running of all the tasks that I published. Piloting helped me check the clarity of the set tasks, solutions and hints on the trail. The pilot running of the trails also helped me to pick up errors that neither I, nor they, were able to pick up. For example, I took the wrong measurements when I designed the tasks for *The tap block*. The pilot runners were constantly getting wrong answers

despite exhausting the hints and solutions. I rectified this error by taking correct measurements and re-uploading the same task. Another example was the was *The old sewage dam*, where a picture could not be used to identify the exact dam. This was because there were similar dams at the site. In order to correct this mistake, a different dam with a unique shape was published.

In both of the two given examples, the reviewers could not pick up these errors because they were based in Europe and were not physically present at the site for the tasks.

3.6.4 Phase 4: Implementation for the MCM trails

This phase took place in three cycles: Cycle 1 focused on perimeter, Cycle 2 on area and Cycle 3 on volume, as illustrated in Figure 3.1 below:

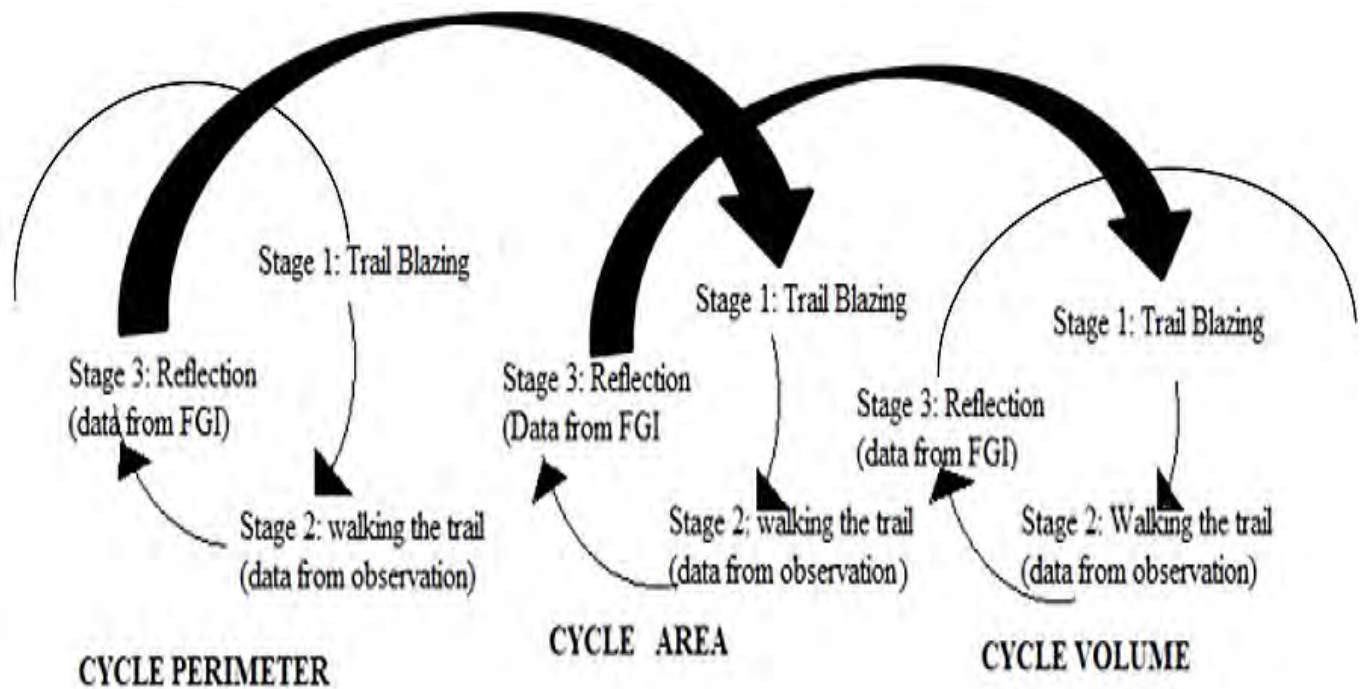


Figure 3.1: Phase 3 cycles.

Figure 3.1 above shows an overview of how the three trails for this project were established. This phase for implementing the MCM trail consisted of three cycles, namely the perimeter, area and volume trail cycles. Each of these cycles consists of three stages. I shall now explain each of the stages.

Stage 1

In the first stage, I designed the perimeter, area and volume trails, one at a time, which consisted of four destination points (waypoints) each, with a task with respective hints and solutions, and uploaded these tasks on the MCM app. I had the trails published on the app so that the tasks in these trails were “visible to all registered MCM users” (Jablonski, Ludwig & Zender, 2018, p. 117). This was to curb any unforeseen technical problems before the project began.

Stage 2

In the second stage, the four groups used the GPS on their smartphones to locate the tasks around the school grounds. I used an observation schedule (see Appendix H) to jot down how the participants made use of visualisation processes as proposed by Ho (2010), as they solved each task. I also used an observation schedule (see Appendix I) to identify Kilpatrick et al. (2001)’s conceptual understanding for learners of each group as they solved the MCM tasks. I observed how they shared responsibilities among group members.

I used the aforementioned indicators to observe how learners used the smartphones to follow the trails on the school grounds to locate the MCM tasks. I also observed whether they used the smartphone collectively or not, and how they shared ideas to access the app, tasks, hints and other useful apps such as calculator and search engines like Google. I also checked if they reread the questions, applied other ways of finding solutions, if they compared their answers with those provided in the sample solution, and how they reacted when they got correct or incorrect answers. My interest was also in how and when the learners used the hints provided on the app.

I observed the learners as they interacted with the task, moving from waypoint to waypoint to measure the dimensions given in the task. Each group was provided with a measuring tape and a folding ruler. I checked how the participants measured the relevant objects identified in the given task. Each group was provided with a notepad on which they did all their workings, sketches and scribbles during their walk of the trail. The notepads were submitted to me after walking the trail. I asked my former learner who runs his own photography, to help me with video recording. This former learner is well familiar with the school environment and thus it was beneficial provide a clear and professional footage of the videos. To allow for full coverage of the learners’ activities, only one group walked the trail at a time. The purpose of the video was to provide first-hand information that would later be used for reflection.

Stage 3

The third stage of a cycle involved reflections that would be done in the form of focus group interviews. I met all the participants and asked them questions regarding their views on the use of smartphones for learning. Reflections were done weekly, soon after learners completed a trail, so that I could get their responses while the events were still fresh in their memories.

During the interviews, I found out from the learners what they enjoyed most, what challenges they faced when solving tasks and how they navigated these challenges. I also asked participants how they shared roles when solving the MCM tasks. The interviews were conducted in an open-ended format, which allowed me to have follow-up questions regarding the hints they used; their reactions to the unforeseen situations they faced during their trails; how the hints helped them to arrive at correct solutions for the tasks; and any other aspects that emerged during their walking through the trails. The participants' activities did not disturb teaching time as observations were scheduled for the afternoons.

3.7 RESEARCH INSTRUMENTS

Two research instruments were used in this study: observations and a focus group interview.

3.7.1 Observations

Observation is defined as “the systematic process of recording the behavioural patterns of participants, objects and occurrences without necessarily questioning or communicating with them” (Maree, 2015, pp. 83-84). The distinctive feature of observation in a research process allows the researcher to gather first-hand data because the researcher goes to the site of the study (Bertram & Christiansen, 2015). In the case of this study, the site is the school grounds, where the MCM trail with its tasks are. Observation was suitable for collecting data in this study because I wanted to focus on the participants' discussions, how they used the smartphones to locate the tasks and discover which of the participants suggested ideas. I used probing questions in the process, regarding their engagement with the actual object, the diagrams/sketches they produced and how they used the smartphone for hints and verifying their solutions. A camera was used to record the learners as they solved each task. Since the participants are minors, their faces were blurred in the video. The video recording took place over a three to four-week period. Table 3.1 below shows the schedule for the running of the trails for each group.

Table 3.1: The schedule for the running of trails for each group.

	Week 1		Week 2		Week 3	
	Observation: Perimeter Trail	Reflection: FGI	Observation Area Trail	Reflection FGI	Observation Volume trail	Reflection FGI
Pilot	8/06/2020	08/06/2020	22/06/2020	22/06/2020	29/06/2020	29/06/2020
Group 1	9/06/2020	9/06/2020	23/06/2020	23/06/2020	30/06/2020	30/06/2020
Group 2	10/06/2020	10/06/2020	24/06/2020	24/06/2020	01/07/2020	01/07/2020
Group 3	9/06/2020	9/06/2020	25/06/2020	25/06/2020	02/07/2020	02/07/2020
Group 4	11/06/2020	11/06/2020	26/06/2020	26/06/2020	03/07/2020	03/07/2020

As shown in Table 3.1, all four of the MCM groups got a chance to complete a trail each week. The running of each attribute i.e. perimeter, area and volume were completed over one week. Once the cycle for perimeter was completed, the two following weeks were allocated to area and volume trails, respectively.

3.7.2 Focus group interview

I conducted semi-structured focus group interviews in this study (see Appendix J). According to Cohen et al. (2007), a semi-structured interview is “where a schedule is prepared that is sufficiently open ended to enable the content to be recorded, digressions and expansions made, new avenues to be included and further probing to be undertaken” (p. 187). According to Henning, van Rensburg and Smit (2011), a focus group is a group of individuals brought together for an open-ended discussion about a certain issue, in order to obtain understanding of the larger community’s attitudes. Semi structured focus group interviews are helpful in “activating forgotten details of experience and releasing inhibitions that may otherwise discourage participants from disclosing information” (Creswell et al., 2012. p. 90). In the case of this study, the focus was directed to all of the twelve of the participants, of which the four groups of three learners each were interviewed at a time.

After each group's completion of the trail, all of the twelve learners got a chance to attend to the group's reflection (see Table 3.1) in the form of semi structured interviews in their respective focus groups. I asked the learners about their experiences, feelings, interests and concerns regarding the use of the MCM app on the smartphones, to carry out the mensuration tasks. The discussion during this interview was audio recorded and transcribed.

3.8 DATA ANALYSIS

Qualitative analysis was used on this research. Cohen et al. (2011) define qualitative data analysis as a process of organizing, accounting for and explaining data in terms of the situation, taking notes of patterns, themes and categories. The analysis of data in this study was multileveled: this would allow me to substantiate data from one unit of analysis that was not provided by another level. The first level was analysing data from observations and the second level was analysing data from the focus group interviews. I now explain each level of analysis.

At the first level, I used inductive analysis in which I identified the visualisation processes through an analytical framework developed from Ho (2010)'s work (see Appendix H). The key focus of this analysis was to find evidence of the visualisation processes that the learners used when they solved MCM tasks. I then coded the themes that I got from the observed data. Cohen et al. (2011) define coding as the ascription to a piece of data of a category or label that is either decided on in advance, or responds to the nature of the data itself (p. 559). It was important that I classified the findings and coded the data so that I could make sense from the data generated.

Below, Table 3.2 shows the analytical template adapted from Ho (2010), used to observe visualisation processes.

Table 3.2: Analytical template to observe visualisation processes (adapted from Ho, 2010).

Visualisation process and code	Observable indicators
Understanding spatial relations of the element (USR)	USR1: learners make use of physical features of the task USR2: learners explain their ideas to reason solutions.
Connecting to a previously solved problem (CPP)	CPP1: learners use the hint on the app CPP2: learners reflect on their mistakes to make new decisions. CPP2: learners work on their own before they request for a hint.
Constructing a visual representation (CVR)	CVR1: learners construct visual images CVR2: learners observe and comment on the visual images CVR3: learners explain the images to themselves or others.
Using visualisation to solve the problem (UVP)	UVP1: learners use calculators on smartphones. UVP2: learners use search engines such as Google. UVP3: learners use a dictionary to make meaning of new words
Encoding the answer to the problem (EAP)	EAP1: learners form and make use of cues to link to the solution. EAP2: learners reread the question just before submitting a solution to the task.

Table 3.2 shows an analytical template that I used to observe visualisation processes. For example, the code USR is used to identify understanding for spatial relations. This indicator seeks evidence of how learners interacted with the learning material and the kinds of ideas and reasoning they generated when they came into contact with the real-life learning situation.

On the same level, I analysed results that emerged from the observation for conceptual understanding using a framework adapted from Kilpatrick et al. (2001). I used the analytical framework in Appendix H to observe the actions that learners exhibited, as guided by the four

conceptual understanding indicators suggested by Kilpatrick et al. (2001). Table 3.3 below shows an analytic template that I used to observe for conceptual understanding, adapted from Kilpatrick et al. (2001).

Table 3.3: Analytical template for conceptual understanding (adapted from Kilpatrick et al., 2001).

Conceptual understanding and code	Observable indicators
Connecting mathematics to pre-knowledge (CMP)	<p>CMP1: learners make use of what they already know.</p> <p>CMP2: learners use an analogue for a representation.</p> <p>CMP3: learners use hints to adapt a concept that they have suggested and agreed upon.</p>
Justifying and explaining mathematical ideas and solutions (JES)	<p>JES1: learners extend the question into a suggestion.</p> <p>JES2: learners use various ways to find a solution to the task.</p> <p>JES3: learners ask questions or respond to others</p>
Representing mathematical concepts in different ways (RMC)	<p>Learners use sketches to:</p> <p>RMC1: to elaborate on questions in the task.</p> <p>RMC2: to suggest a solution to the task.</p> <p>RMC3: to represent concepts.</p>
Connecting ideas and concepts in mathematics to the real world (CRW)	<p>CRW1: learners link abstract mathematical assumptions</p> <p>CRW2: learners use hints to revisit the task model</p> <p>CRW3: learners sense the attribute being measured.</p>

As shown in Table 3.3, I used codes to make sense from the data, for example, the indicator for connecting mathematics to prior knowledge is coded as CMP and this indicator looks at how learners used the knowledge that they had to solve new tasks in new situations. These indicators were coded. According to Saldaña (2015), the provisional codes utilize the preset codes derived from literature and are subject to be modified or revised during the data analysis.

During the analysis for the observed data, I looked for evidence of how smartphones were used for visualisation processes leading to the development of conceptual understanding. Learners' discussions, together with their drawings, sketches, gestures and calculations on notepads were analysed together with the video recordings. Data from this level was used to answer the first research question.

The second level of analysis was on the views of participants. The FGI and their transcripts were qualitatively analysed using thematic analysis, which assisted me in categorizing the participants' responses. The themes were firstly adapted from Ho's (2010) work and thereafter, from Kilpatrick et al.'s (2001) conceptual understanding indicators.

I looked for similarities and categorized the data with similar information; for example, I asked learners from the different groups about the problems that they encountered and classified these responses into themes, such as problem related to problem-solving skills, problems related to the novel use of smartphones and technical problems that they encountered.

I also respected other themes that emerged from the data, regarding the participants' views on the use of smartphones and visualisation processes for the learning of mensuration. I then made sense of how these categories influenced (or not) the learners' understanding of the conceptual learning of perimeter, area and volume.

3.9 VALIDITY AND RELIABILITY

In this study, I used triangulation (Gay, Mills & Airasian, 2011) to validate the data collected. Data triangulation was achieved through observing learners' written work (scribbles, calculations and sketches made when learners solved the MCM tasks), video recordings during the running of the tasks and audio recordings of FGIs during the reflection sessions.

Another technique that preserved the validity of this project was the review process that was done before the tasks could be made available on the MCM portal. All of the tasks were scrutinized by the skilled and experienced personnel managing this app.

I now summarize the criteria that each task needed to meet in order for it to warrant its publication. It is required that a picture of the material is used to identify the object and that the provided picture may not contain enough clues (Ludwig et al., 2013). This means that finding a solution for the

task should require participants to physically attend to the material and perform the measurement on it. Moreover, the task should define clear objectives, calling for relevance and interest in learning. The task is then uploaded and is accompanied by a request for publication. The process of reviewing tasks for the purpose of publication is outlined in Figure 3.2 below:

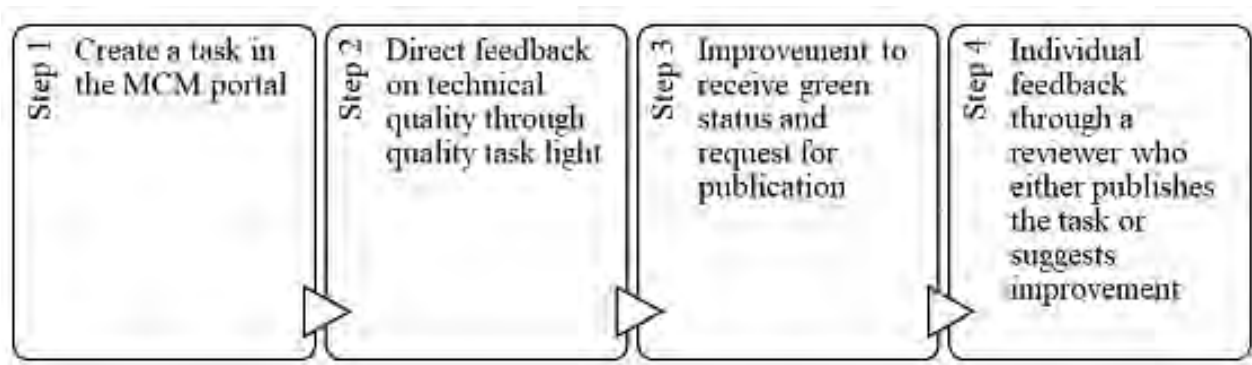


Figure 3.2: The MCM task review process (Jablonski, Ludwig & Zender, 2018, p. 117).

As seen in Figure 3.2, the task author creates and uploads the task, and the reviewers validate and check the task against any errors. The errors may be technical, for example, an incorrect calculation or an inappropriate picture. An invalid task may also result from didactical errors such as unclear objectives or the task may contain information that is irrelevant for learning. A substandard task may result in a decline for publication. An example of how one of my tasks could not reach validation is outlined below in Table 3.4:

Table 3.4: Task publication declined until criteria is met

Task name	Details of the review	Task publication status
The gas cylinder storage	<p>First review date : 2020-01-09</p> <p>Reviewer comment : Decline publication</p> <p>One word is missing from the definition of the task, I assume it is a word "part", one bracket is missing, too. Check the sample solution, please. It is not clear which number is size and which one is a perimeter.</p> <p>The drawing of the object with bolded highlights around the doors would be helpful for learners to know they are working with perimeter of the doors, too</p>	Task was not published
	<p>Second REVIEW DATE : 2020-01-11</p> <p>Reviewer comment : Decline publication</p> <p>Again: Nice task but the initial photo must be replaced with the photo covering only part of the task.</p>	Task was not published
	<p>Third REVIEW date : 2020-01-22</p> <p>Reviewer comment : Accept publication</p> <p>Thank you, everything is now fine.</p>	Tasks published after correction was made

From Table 3.4 above, it is shown that the task had many errors. The reviewers commented on the points that needed to be rectified. Only after the task met the standard could it be published. The

outline of the task publication on first attempt or after the first correction was done is provided in Table 3.5 below:

Table 3.5: The comments as suggested by the system reviewers

Task name	Details of the review	Task publication status
Age of a mopane tree	<p>First review date : 2020-01-09</p> <p>Reviewer comment : Decline publication</p> <p>Good exercise. The solution should be reviewed, as it skips some steps. Somehow it should be indicated that the diameter is going to be divided between the growth of the tree, because now it puts an equality that is not true: $d = c / \pi = 175 / \pi = 29.3$ years (the last step is not true)</p> <p>Second review date : 2020-01-12 Reviewer : Simone Jablonski Reviewer comment : Accept publication</p> <p>Thank you</p>	Task was published after the correction was made.
The school garden gate	<p>First review date : 2020-07-01</p> <p>Reviewer comment : Accept publication</p>	Task was published on first attempt

As seen in Table 3.5, the task had far fewer errors that needed to be rectified. After I and the reviewer reached our contentment with the task, it was then published and visible to all users.

The third technique that I used to ensure valid results for this project was piloting. I piloted the trails with three learners who would not be among the twelve participants. Focus group interview

schedules and observation schedules were also piloted. The pilot was also used to test video and audio recording instruments in order to ensure effective and productive recording.

3.10 ETHICAL CONSIDERATIONS

This section is a summary of the ethical issues that I considered in my research study. According to Cohen, et al. (2011), conducting educational research involves the researcher to address issues of confidentiality, anonymity, informed consent and gaining access to and acceptance within the research setting. In this section, I present a summary of ethical issues that I took into consideration, mainly to safeguard my research participants' identities and rights and the responses they made to this study.

3.10.1 Respect and Dignity

To earn respect from my participants, I established a positive rapport between myself and them. I also obtained the Ethical Clearance certificate from Rhodes University, the gatekeepers' written consent from the Regional Director of Education as well as from the school principal (see Appendices B, C and D respectively) before conducting this research. I was aware that my participants were minors and thus I obtained written assent from the research participants as well as from guardians of the participants. In order to ascertain confidentiality, learners' names were assumed anonymous in the final report. Learners were informed that their participation was voluntary and that they were allowed to withdraw from the study at any time without negative consequences.

3.10.2 Transparency and academic professionalism

To ensure that activities in my research entailed transparent work, I explained the objectives of the study to the learners as well as to the principal and the Regional Director of Education. I also informed learners of the possible contribution they were making regarding the use of smartphones for learning.

In order to conform to the standards of academic professionalism, I used two data collection techniques. The first was that I observed learners as they ran the tasks. The second data collection technique was that I conducted interviews after each group completed running the MCM trails.

3.10.3 Researcher positionality, and confidentiality

I acknowledged the fact that the participants in this research were my learners and that this fact might affect the learners' behaviour and responses during the research. I therefore encouraged learners to share their outmost sentiments and honest feelings. During their participation in this research, learners were allowed to respond at their own pace and in the language of their choice. Cohen et al. (2011) warn that researchers using interviews have to be careful with the informants' views before any interpretations and analysis of data is conducted. I adopted a strategy of member checking (Merriam, 2001), that involves meeting with the participants in order to verify the content of the interview transcripts. This verification exercise provided space for participants to correct data which they felt was inaccurate (Rule & John, 2011, p. 108).

All the data collected will remain confidential between me and my supervisors and will be kept by the supervisor for five years. The findings of this study were documented despite whether they were in conflict with my initial expectations and assumptions or not. Furthermore, the final product of this project shall be made available to the public (teachers, curriculum designers and policy implementers) to serve its intended purpose of sharing the findings on the significance of using smartphones in mathematical learning. I acknowledge that the thesis is my own work and that I adhered to the protocols of the Rhodes University Referencing Guide to reference the work of others appropriately.

3.11 CONCLUSION

In this chapter, I first provided the research goals and questions for this case study. I further described the methodology used in this study. I then discussed the key issues for the research design, including the following: selection of the research site and participants as well as the three cycles that I used to pilot and implement the MCM trails. This chapter concluded with a summary of research instruments for collecting data, as well as the ethical considerations that I employed in this research.

CHAPTER 4

DATA PRESENTATION, ANALYSIS AND DISCUSSION

4.1 INTRODUCTION

The focus of this study was to explore and describe how smartphones were used for visualisation processes and for enhancing conceptual understanding when Grade 9 learners solved measurement related tasks incorporated within the MCM project. This chapter presents and discusses the findings of the study.

Data presented and analysed in this chapter was drawn from results generated by two research instruments, namely observation and focus group interviews. I engaged a two-leveled analysis because I wanted to interrogate data from observation and from FGIs. On the first level, I conducted an inductive analysis by separately analysing group tasks in order to foreground the manifested visualisation processes and conceptual understanding that became evident (or not) when each group visited the site for each task. On this same level of analysis, the transcripts from observations and as well as the video records were analysed to check the promotion of conceptual understanding. I was also interested in the visualisation process exhibited (or not) when learners walked and engaged in the solving of trails of perimeter, area and volume.

As explained in Chapter 3, the twelve participants were coded according to the learner number and the instrument from which data was sourced. For example, L1O represents information provided by Learner 1 from the Observation whereas L7FGI denotes information from Learner 7 from the Focus Group Interview.

Findings from the observation were meant to shed light on how learners solved the MCM trail tasks. Data from observations specifically provided answers to the first research question which is: *How can smartphones be used as visualisation tools in developing conceptual understanding in the learning of mensuration using the MCM app in grade 9?* To answer this research question, data was analysed using Analytical tools 1 and 2 (see Appendices H & I).

The analytical tool developed from Ho's (2010) work (see Appendix H), was used to analyse how participants made use of visualisation processes. In addition, Kilpatrick, et al.'s (2001) indicators

for conceptual understanding (see Appendix I) were used to analyse the observations made when learners were solving each of the MCM tasks on the perimeter, area and volume trails.

The second level of analysis entailed using thematic analysis which assisted me to categorise the participants' responses from the various FGI sessions. The findings of the FGI were used to answer the second research question of this study, which is: *What are the selected learners' experiences of learning mensuration of shapes using smartphones as visualisation tools outside the classroom?* I then made sense on how these categories influenced (or not) the visualisation processes as they learned the concept of measurement. Data is presented and analysed first following the themes that were developed from literature and later those that emerged during analysis.

4.2 VISUALISATION PROCESSES IN THE LEARNING OF PERIMETER CONCEPTS

This section presents the analysis of the observed visualisation processes that learners went through as they ran the perimeter trail. The visualisation processes in the learning of perimeter were categorised as follows: Understanding spatial relations of the element; Connecting to a previously solved problem; Constructing a visual representation; Using visualisation to solve the problem and Encoding the answer to the problem.

4.2.1 Understanding spatial relations of the element

In this study learners were introduced to the spatial outline and properties of objects in their environment during the perimeter trail. For example, Figure 4.1a and b below show snapshots from two different video recordings in which learners interacted with the physical artefacts of the task that they solved. In Figures 4.1a and b, learners are measuring the perimeter of given objects.



Figure 4.1(a): Age of the mopane task



Figure 4.1(b): The hollow concrete task

Learners moved around the tree in order to measure the circumference of the trunk as seen in Figure 4.1 (a); as the tree was a much bigger structure. On the other hand, in Figure 4.1 (b) shows that the learners did not need to move around as they did with the tree, because the size of the concrete block was smaller. The hints were provided by the MCM application to advise and instruct the learners on how to interact with the spatial relations of the learning material at hand.

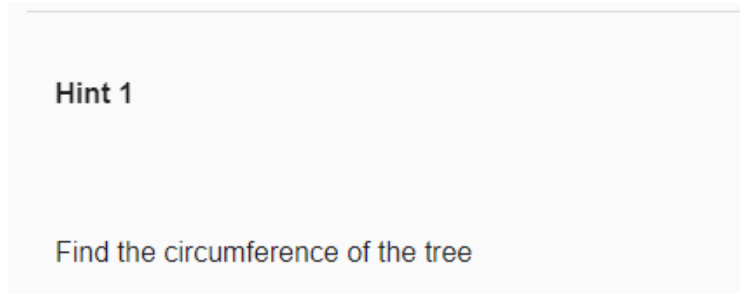


Figure 4.2: The hints provided by the app shows the outlines of the task objects

The role that smartphones played in building a spatial relationship is that the hints on the app sent learners off to physically engage with the tasks. In this case, the hints played the central role to instruct learners the meaning of perimeter as a measure around the given object. Hence, the learners enacted with the physical outlines of the objects in order to measure perimeter for the tasks that they solved.

Learners were connected to the physical features of the tasks. For them to carry out measurements for the calculations of perimeter of the object tasks, I observed that the learners had to move around

the object that was larger (as they embarked on the age of mopane tree task), and/or move their fingers around the smaller object (the hollow concrete cuboid task) in order to discover the outlines as a perimeter around the object. The use of actual objects in this case reiterated what Khan et al. (2015) said: “a renewed study of visualisation in mathematics education must attend to the significant role of the body in space with other bodies” (p. 278). The hints that learners used during the solving of the tasks helped them to develop sensory thoughts as they discovered the spatial relations through touching. Learners used the spatial representations to recognise the shapes of the task that they worked with.

4.2.2 Connecting to a previously solved problem

In this study, the MCM app encourages the use of hints. These hints were mainly used by learners to help them remember previously learnt concepts and ideas that are then useful to solve the task at hand. Learners from Group 2 provided evidence of connecting ideas to previously solved problems when they made use of stepped hints from the MCM app through their smartphone. Prior to this, learners discussed the hints from smartphones on how they would interact with the materials (see Table 4.1) to obtain the required measurements for the solving of the given tasks. I now present a snapshot that shows how L4O from Group 2 made use of the stepped hints to assist them to use different methods for solving the second task of the perimeter trail. After getting an incorrect solution for calculating the cost of the bar for the gas cylinder storage, the conversation in Table 4.1 ensued:

Table 4.1: Learners use stepped hints to improve their method

Learner	What was said by learners	Researcher’s observations
L4O	Hint number one says: First, you need to calculate the total cost of the metal... now how can you use the metal length (to calculate the cost)?	L4O use the smartphone to read the first hint of the task
L5O	Ooo, so now you have to subtract ...6.12 minus 4.12?	The learner is inspired by the hint to say this statement.
L6O	<i>To subtract peni ano? Eenima atudhi addinga ... eyamukulo ndiyaka lyo 4.12 otatu li plus no 6.12 and then atukeli multiply nawa no 12.99</i> [What is there to subtract? We are adding the things (lengths) together ...on that length of 4.12 we add 6.12 then the answer	Out of excitement L5O quickly grabbed the scientific calculator from L6O in order to perform the alternative method for correcting the mistake that they have made while doing their first attempt for solving this task.

we get, we multiply it with the 12.99 (which is the cost of the bar per metre)].

Having received the task's hint, it somehow improved the learners' confidence and it enlightened them to further explore better ways to find the total length of the bar. The stepped hints helped learners strengthen the way in which they processed their visualisations as observed during their discussion as seen in Table 4.1. The reading of the hint prevented the group from subtracting the lengths of the bars. In the same vein, the contents of the hints reminded learners of relevant prior knowledge – which is that adding up all the lengths represented the perimeter for the bar. The recently activated prior knowledge was necessary for the successful solving of the given task. In this task, learners used their past experiences for applying at least methods for calculating the cost of the metal as provided by the hints for the the task.

On the other hand, learners in Group 3 used hints to connect their learning to a previously solved problem as a visualisation process when they solved task 4 of the perimeter trail. Table 4.2. Below provides another excerpt that depicts how the learners of Group 3 used the hints to reflect on a previously learned mathematical idea.

Table 4.2: Excerpt of learners using prior knowledge to confirm with the hints

Learner	What was said by learners	Researcher's observations
L80	<i>O perimeter oyo rectangle?</i> [Is this the perimeter for rectangle?]	Although the shape of the dam clearly looked like a rectangle, L80 still showed the need to confirm the shape of the task.
L70	I think so, because hint three says the outer perimeter and inner perimeter of the rectangle.	L70 started to contribute towards finding the solution to the task soon after she used the smartphone to confirm with the third stepped hint.

L70 used his prior knowledge to support the assumption made by L80 that the shape that they were dealing with in the task was a rectangle. L70 also used the hint to confirm the conventional knowledge that the shape that they were dealing with was a rectangle. The learners then used this

information to confirm the geometrical shape that they were dealing with in the task, which was a rectangle. Once they knew the geometrical shape, they were able to meet up with the task objective, which was to find the perimeter of the rectangular shaped dam.

The reading of the hint was necessary for learners from Groups 2 and 3 in two ways: for Group 2, the hint prevented them from making a mistake of subtracting when calculating the perimeter, while in Group 3 learners used the hint to confirm and thus promote the method that they suggested among themselves. All in all, the hint played the role as a knowledge activator for the learners as they observed their already existing knowledge for the formula for calculating the perimeter of a rectangle in a rather new, outdoor situation.

4.2.3 Constructing a visual representation.

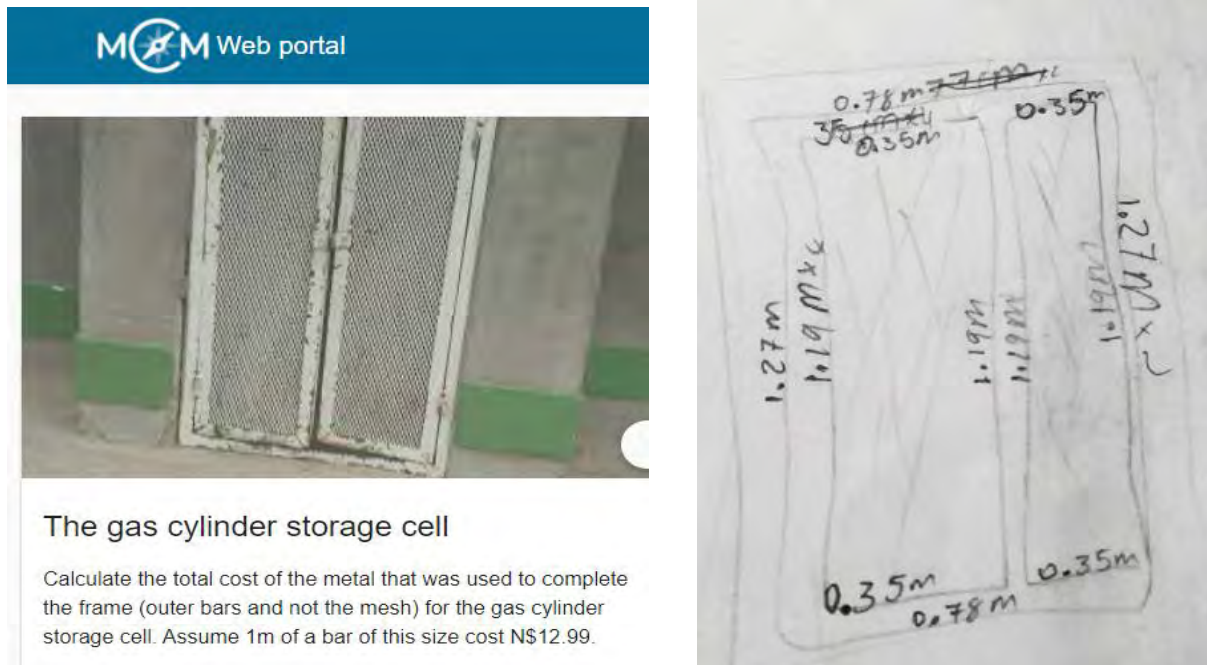
The need for construction of a visual representation emerged when learners from Group 1 were solving the task for the gas cylinder storage container. On first attempt, learners took measurements and used these measures in their calculation. There came a time when they needed to grasp a record of these measurements. The excerpt in Table 4.3 began with a discussion on how L12O realised that they did not record the measurement that they took of the doors of the gas cylinder storage container. He advised a sketch as a way forward in solving this task.

Table 4.3: L12O suggests a sketch and remeasurement of the object

Learner	What was said by learners	Researcher's observations
L12O	<i>Aaye, gents... shila natu drawingeni okaima hono mani</i> [No, fellows... unless we draw this thing mani].	L12O insisted that a sketch be drawn. The word <i>mani</i> is said when someone insist on something, especially to his or her peers.
L11O	<i>Kandiwete twa recordingeni eemeasurement dhetu nawa kaa.</i> [I'm not sure if we recorded our measurements correctly].	L10O took over the paper from L11O and started to sketch the outline of the frames that made up the gas cylinder storage. L10O then copied the measurements and wrote them next to the dimensions of the drawn sketch (see Figure 4.3)
L12O	<i>Paije natu metulululeni aike iinima aishe</i> [now let us start over with the whole measuring operation]	Suggest that they repeat the measurement activity.

The transcripts from Table 4.3 show an excerpt of learners discussing the sketching of the problem task and how to label the necessary information on the sketch. One learner (L12O) insisted that a

sketch was important and that the sketch would be used in solving the task. Learners agreed to start over with the task by repeating the same dimensions to measure as they did on their first attempt but this time, writing down each measurement that they took from the door. The sketch that learners made in the second attempt is displayed below in Figure 4.3.



The gas cylinder storage cell

Calculate the total cost of the metal that was used to complete the frame (outer bars and not the mesh) for the gas cylinder storage cell. Assume 1m of a bar of this size cost N\$12.99.

Figure 4.3: A sketch to represent the door frame of the gas cylinder storage container

L100 used the image on the smartphone to perceive a mental image that was later reproduced into the visual image seen in Figure 4.3. The fact that the learners were at the site of the actual object in the task, made it easy for them to produce a sketch of the object that was in front of them. They could produce the sketch, record measurements of it and improve the sketch as they saw fit. This helped to make learning real and authentic to these learners.

There are qualities related to constructing visual representations that this outdoor learning task presented to the learners. One is that the labelling of the readings adjacent to the measurements made, seems to have assisted the learners with the remaining calculations on this task. Second is that the use of the drawn sketch further opened up opportunities for learners to discuss how the task could be solved, while they were still physically present at the concrete cuboid. The drawing

of the image helped learners to activate their process of thinking about methods of how to find a solution to the task. While on site, the learners checked, re-examined their sketches and used the drawn diagram as a visualisation tool to solve the task. Outdoor tasks in this trail thus provided learners with opportunities to practice elementary mathematical modelling.

4.2.4 Using the smartphone as a visualisation to solve the problem

The use of the smartphone as a visualisation tool to solve the problem was evident when L50 first read the task question from the smartphone, and second, used the same smartphone to search for meaning of words such as ‘mesh’ and ‘bar’ from Google search engine as reported in Table 4.4 below. Thirdly, as mentioned earlier, learners searched for hints from the app as well.

Table 4.4: Excerpt of Group 2 learners using the smartphone to learn new words from internet

Learner	What was said by learners	Researcher’s observations
L40	First you (we) need to calculate the total cost of the metal... now how can you (we) use the metal length in order to find the total cost of the bar?	Suggest a method that they can use to find the total length. L40’s suggestion ends by reciting the int of the task from the smartphone.
L50	Multiplied by four? ... One, two three, four. Ok.	Suggests the four horizontal, inner and outer parts that made up the top and bottom frames of the storage door.

The excerpt in Table 4.4. shows data that pertains to the solving of the MCM perimeter tasks using discussion to solve the problem as a visualisation strategy. Learners were using the visual hint from their smartphone to link it with the actual object which was the door frame. This finding shows that the discussion among learners was enriched by the use of the smartphone during problem solving. In the middle of their discussion, learners needed to refer to the dimensions for the door. It was at this point that realised the task instructions had words that were rather unfamiliar to them. For example, learners were discussing and helping each other identify unfamiliar parts on the door frame such as ‘mesh wire’ and ‘metal bar’ (see Table 4.5) below.

Table 4.5: Group 3 learners used hints to learn unfamiliar words in Task 3

Learner	What was said by the learners	Researcher’s observations
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L80	<i>O bar oyo nee ndjino? Lesha utale ohint number two. [Could this be the bar? Kindly read hint number two.]</i>	Confirms the newly learned vocabulary.
L70	Hint number two shows these yellow lines around the door.	Opens the second hint from the smartphone, this time the hint was a diagram as shown in Figure 4. 4 below



Figure 4.4: A screenshot from the smartphone to show the hint provided by the MCM app

Data provided in Table 4.5 above shows that the main objective in learners' discussion was to verify and connect the assumptions that they had made, with the new information provided by the hint through the use of smartphone.

In this task, the smartphone played a role as a visualisation tool on three fronts to solve the problem. First by providing the question for the task, then by translating unfamiliar words into meaningful definitions and lastly through the provision of hints. The use of a smartphone as a visual tool thus can be considered here as having promoted the solving of the task and hence by inference, providing access to perimeter concepts.

4.2.5 Encoding the answer to the problem

In solving the fourth task of the perimeter trail, learners encoded the solution to the problem to ensure that their responses towards the task were leading to reasonable solutions. For example, L60 wanted to confirm whether the units used in the task were millimetres or centimetres. Before submitting the solution to the task, a discussion of another problem about encoding came up (see Table 4.6) below.

Table 4.6: Group 2 members use repeated confirmations to solve the task

Learner	What was said by the learners	Researcher's observations
L40	First, we need to find this perimeter	L40 suggest to others that they first work out the outer perimeter.
L60	In centimetres?	L60 confirms the units used in the task.
L40	Hmmm.	Learner checks on the smartphone and affirms that the units used are centimetres.
L50	You said (we are measuring) in centimetres ndishi?	Again, L50 confirm with the units while measurement is still going on.
L40	Yes. Remember to write the following measurements in simplified form: outer perimeter to inner perimeter	L40 reads the second hint.
L50	The ratio of the outer perimeter to inner perimeter... we need to simplify...	Echoes part of the hint that was read by L40.
L60	Wait, don't put (enter) the answer yet, we need to simplify our work	Stops L40 from entering a solution. This gave a chance to share a more reasonable idea
L50	If we have 74;166...then 74 goes into itself once and in 166 is two point...	Continues to share his concern with solution with a decimal number.
L40	Then we just round it, I think. I don't think we need a decimal answer kaa... the answers her ABCD none of them have a decimal answer	Gets back on track with others and help contribute towards a reasonable solution
L40	Remember to write the following measurements in simplified form; outer perimeter to inner perimeter.	Continues to read the third hint

The discussion in Table 4.6 took place among learners in order to solve the problem. The discussion was used as a starting point to discover the type of approach that they needed to use in solving the problem tasks. There came a moment where learners ended up with the solution 77:166. Simplifying this ratio resulted in a solution with decimal answers, and it was hard for them to turn a set of two solutions into a ratio format. This led the learners to make use of repeated confirmations of the units of measurement just to check whether they had taken the correct measurements. Moreover, learners exhausted all three of the hints that were provided in this task. The discussion among the learners provided evidence that they were using the smartphone as an essential learning tool in order to solve the problem.

Because learners were on site and they thus had the actual object of discussion in front of them, they could tally their solutions to the question. This, to some extent, invoked in their minds some elements of what Kilpatrick et al. (2001) refer to as adaptive reasoning. Learners were thus made to think logically about the relationships between the solutions they got and the actual object, and were guided into justifying and proving on their own, the correctness of the mathematical procedures or assertions that they had taken.

4.2.6 Conclusion to the visualisation processes during the perimeter trail

During the running of the perimeter trail, learners used their senses to perceive the outlines for the objects that they measured. The smartphones played a visual role, in the sense that opportunities were opened for learners to engage themselves in deliberations and discussions to solve the tasks. Learners used their mental and physical perceptions from the task objects to in order to explore more properties – this was a typical way in which learners came up with suggestions for solving the tasks. Another role played by the smartphones was that to simplify the learning content that learners encountered, smartphones provided learners with concepts that they already knew but needed to be translated into knowledge structures that learners were already familiar with.

4.3 LEARNERS' CONCEPTUAL UNDERSTANDING DURING PERIMETER LEARNING

This section contains a discussion of the learners' understanding of measurement concepts as they solved the perimeter-based tasks. Usiskin (2012) explained that the first step in learning the measuring concept for perimeter is identifying the shape and its properties. The running of the perimeter trail in this project aimed at providing learners with an extraordinary opportunity to understand the concepts contained in the 2-dimensional properties of shape as they learned to identify shapes and properties observed from objects around the school grounds.

4.3.1 Connecting mathematics to prior knowledge

In this section I present and analyse data that relates to how learners connected mathematics concepts that they were learning to prior knowledge. I will draw on observations made as learners were working on the *school garden gate* task. This task required learners to use their prior knowledge in order to recognise the two shapes seen on this garden gate, namely the square and

the rhombus (see Figure 4.5 below). Evidence for this is provided by the discussion between L1O and L2O in Table 4.7 below.

Table 4.7: Group 1 learners formulate an algorithm for a task

Learner	What was said by the learners	Researcher's observations
L2O	<i>One point seven six.... naapa naapa...one point seven six.</i> [One point seven six, also here too, and also here...one point seven six.]	Points to the smartphone and move her fingers so as to zoom in the picture for the task in order to show all the sides of the rhombus are made up of four equal parts
L1O	<i>Shino? Tala L2O, shino oshili opposite naashino, hasho?</i> [And this? Look L2O, this one (side) is opposite with this, right?]	Not fully convinced but ready to listen and learn from others.
L2O	<i>Naapo. Apeshe aike opo, eeside adishe.</i> [There too, the lengths are just the same]	Continues to discuss and start to draw a sketch. (See Figure 4.5 below).
L1O	<i>Tashiti uunima auhe outhike owala pamwe ... ok. Paife atu ningi nee one point seven six m... multiplied by four.</i> [Meaning all these lengths are all equal ... ok. Now we take one point seven six... multiplied by four].	Finally agrees to the concept after measuring and getting the same length.

As shown in the discussion in Table 4.7 above, learners used the prior knowledge in which they recognised the rhombus and square shapes that were used to frame the school garden gate. This prior knowledge was used to discover the geometrical properties of the rhombus and square shapes. L2O was the first to disclose to others that a rhombus has four equal sides. L2O further explained her prior knowledge about the length for the rhombus by suggesting that the single length that was measured. represented the lengths for all other sides and could simply be multiplied by the four sides from which this rhombus is made up.

The pre-knowledge that a rhombus has four equal sides became apparent when L1O reversed her intention to continue taking measurements on the other three sides but was still not fully convinced about the shape having sides of equal length. This discussion activated L2O's construction of knowledge. This was evident when L2O made a sketch on the paper that justified her argument that a rhombus has equal sides and there was only the need to measure one side.

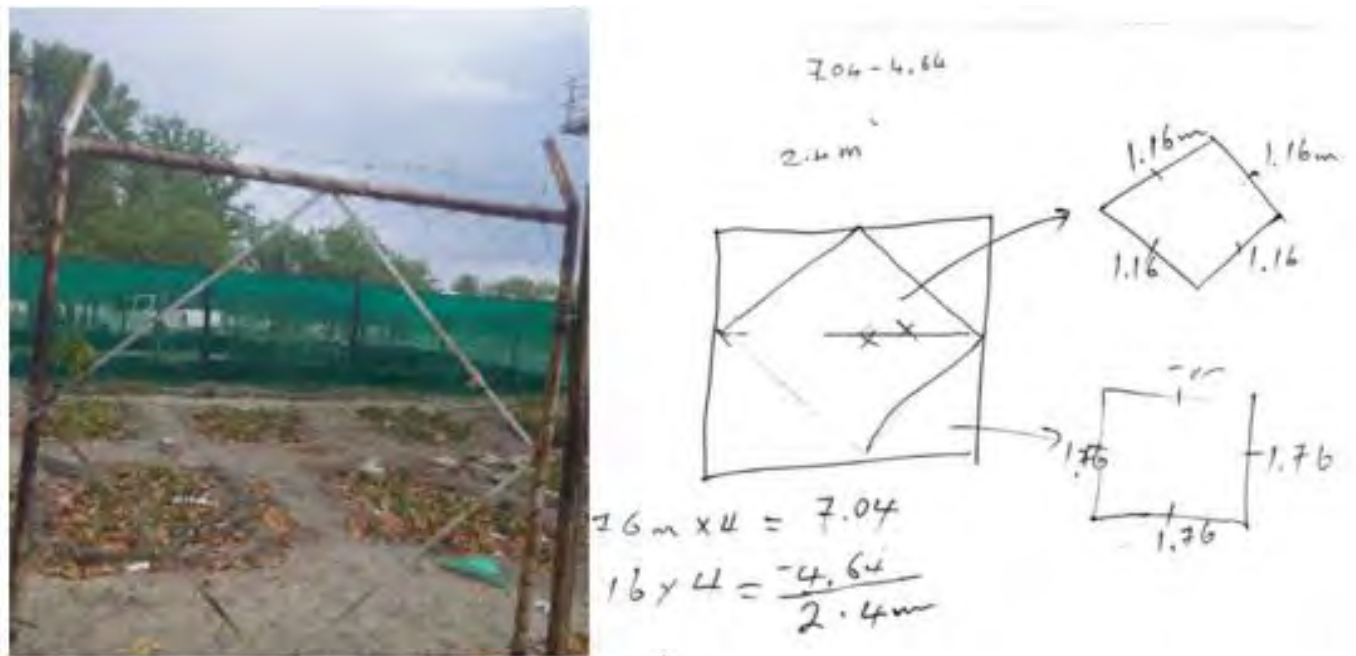


Figure 4.5 School garden gate and its sketch.

Figure 4.5 shows a real-life picture of the school garden gate as well as the sketch drawn by the learner to indicate that she already knew that a rhombus had four equal sides. The set of knowledge, together with the hint provided, was also used to deconstruct the complex shape for the door into a simple square and a rhombus. This made it easy for learners to obtain separate measures in which they subtracted the perimeter for the inner rhombus from the perimeter for the outer square.

The learners' inherent use for measurement units was another interesting observation made regarding the use of prior knowledge in a given situation. When one takes a closer look at the discussion in Table 4.7, none of the learners mentioned any unit of the measures that they took. The task required that the solution be provided in metres as the unit of length. Surprisingly, the units of measurement appear next to some of the dimensions in the sketch as observed in Figure 4.5.

The fact that learners used the ruler and that they devised a measurement strategy, helped them finish off the task with a meaningful learning experience to describe perimeter. This was evident when learners linked the property of a rhombus when describing how to measure its perimeter.

4.3.2 Justifying and explaining mathematical ideas and solutions

A remarkable observation that I made was when learners used a smartphone to help them gain conceptual understanding of the keywords. Here, learners were confronted with a task that encouraged them to explain rational ideas to themselves and to others in order to make their reasoning clear. This task began with L1O reading the question, and the rest of the group members did not readily start discussing, nor act towards solving the task. After a short while looking on each other's' faces (doing nothing), L1O suggested that L2O should open the smartphone dictionary in order to find out definitions of the words they could not understand.

Table 4.8: Learners accessed the smartphone dictionary to learn a new word

Learner	What was said by the learners	Researcher's observations
L2O	Mesh: a material made of a network of wire or thread.	Read the definition of the term 'mesh' from the dictionary
L1O	Between this one and this one Then this is a mesh...because the network is here	Explain on the structure and determines which is the mesh, using her hand to wave crosses to indicate a network.
L2O	Yes, and this must be the bars	Infers on which part of the door could be regarded as the bars

Data provided in Table 4.8 above shows the definitions that were read from the dictionary to find meanings for the keywords in the task. The keywords in the question were 'bar' and 'mesh'. It seemed that reading aloud the definition for the word 'mesh' alone was sufficient for learners to get going.

Learners took advantage of their physical presence at the site of the task to translate the definition from the smartphone into the parts of the door frame in real life as shown in Figure 4. 6 below.



Figure 4. 6: L1O waves at the mesh of the gas cylinder storage

As seen in Figure 4.6, the smartphone was helpful for L1O to access the dictionary and find the definition for mesh. L1O used her hands to wave it into 'cross' formed gestures. This was her way of giving reasons why that part of the door should be referred to as mesh. Learners from Group 1 were therefore afforded an opportunity to actively link these concepts into a meaning that made sense to them. This was evident when L1O used this explanation to her group members to support her own assumption, in order to confirm the meaning of the key word for the task. This assumption was used by the learners to determine the side that learners would take measurements of, in order to find the total length that was used to make the storage cell or container.

4.3.3 Representing mathematical concepts in different ways

Multiple representation was evident when learners from two different groups were able to use more than one method to find the perimeter of the same task. Clements (1999) explains that concrete learning material helps learners to construct several representations of mathematical ideas that are meaningful to their learning. Multiple ways of representation were evident in the two scenes from this study. Firstly, Group 2 learners measured the outer edges of each of the sides of the concrete hollow cuboid.



Figure 4.7: Group 2 learners measure outer perimeter

Figure 4.7 shows learners taking measurements of one of the outer edges of the concrete cuboid. They added all the measurements to get the outer perimeter. The learners were later observed taking measurements from the inner dimensions of the concrete hollow cuboid as seen in Figure 4.8 below.



Figure 4.8 Shows Group 2 learners measure inner perimeter

Figure 4.8 shows the same Group 2 learners measure one of the lengths of the inner sides of the hollow concrete cuboid separately and then add all the measurements to find the inner perimeter of the concrete cuboid.

Figure 4.7 and Figure 4.8 above give evidence for the first scenario where learners took separate measurements for each side of the concrete cuboid. They recorded the corresponding side measurements on a piece of paper. Thereafter, the learners added up all four measurements to find the total length that made up the outer perimeter and paired it against the total length of the inner perimeter of the cuboid. Evidence for the Group 2 learners' work is provided further on as I compare the work with the Group 3 members, outlined below.

The Group 3 learners approached the same task with a different representation, hence they used a different method to solve the task. The Group 3 members simply wound the measuring tape around the concrete block in order to get the outer perimeter at once, as shown in Figure 4. 9 below:



Figure 4.9 Learners from Group 3 measure the outer perimeter

Figure 4.9 above shows how Group 3 members used a method different from Group 2 that I previously described. The Group 3 learners determined the perimeter of the block by putting the measuring tape around the entire object, then they applied the same method to take a single measure for the inner perimeter of the hollow cuboid. Figure 4. 10 below illustrates how the two methods that were used varied and how they all arrived at almost the same solutions.

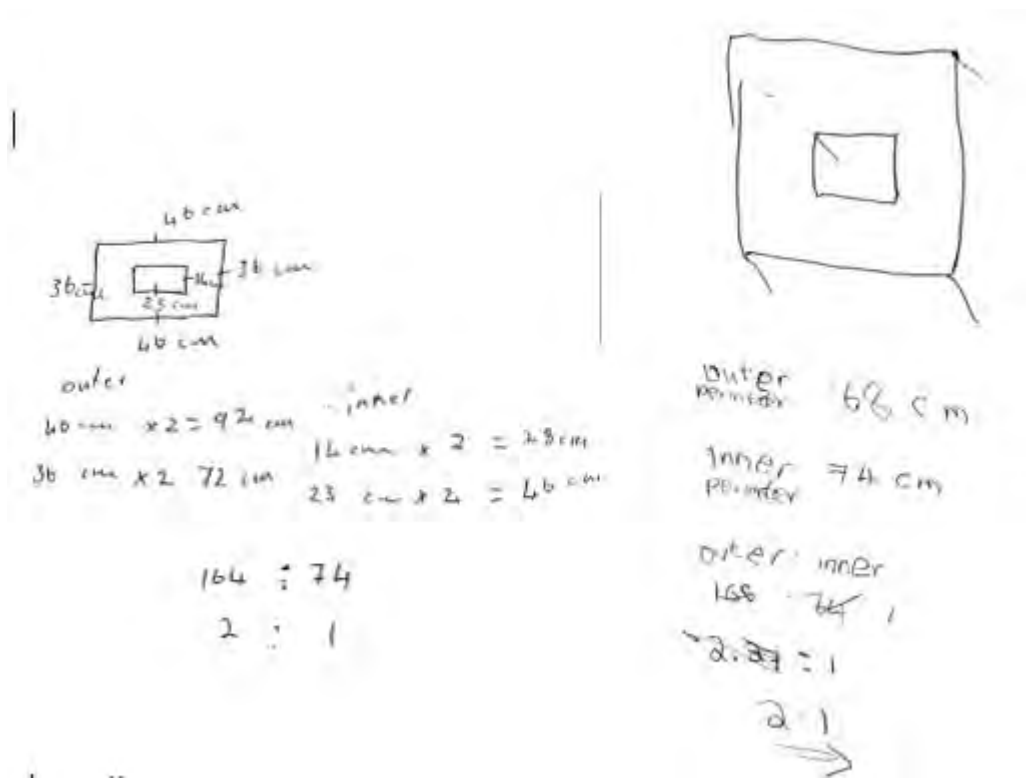


Figure 4.10 (a): Calculations of learners from Group 2

Figure 4.10 (b): Learners from Group 3 measured the perimeter by winding the tape around the whole shape

Figure 4.10 shows approaches used by learners from Group 2 in (a) and from Group 3 in (b) to find the perimeter of the concrete cuboid task.

In Figure 4.10, I present some evidence of learners' work done by the two groups to show how they demonstrated the concept of perimeter in two different ways. According to Sarama and Clements (2009), perimeter of a square may be calculated in two different ways: the first one is by the arithmetic method which is obtained by adding up all of the four dimensions; and the second method is to multiply the length of a dimension with the number of sides that a shape has. In this study, learners from Group 2 in Figure 4.10 (a) found the perimeter of the concrete cuboid task by taking separate measures of the sides of the rectangular shape, which they then added together to

form the inner and outer perimeters respectively. The alternative method is evidenced in the work by the Group 3 as illustrated in Figure 4.10 (b). It shows that learners measured the whole of the outer perimeter by placing the measuring tape around the entire object at once in order to measure the outer perimeter. The learners then repeated this method to measure the inner perimeter of the block.

As learners manipulated the learning material, in this case the hollow cuboid, they perceived facts that were most appealing them by moving around the edges of the cuboid in order to obtain the various measurements. This manipulation imprinted mental images on the learners' minds, which could then be seen through their actions while solving the task. The existence of the varying methods for measuring perimeter around the concrete block espouses the two approaches into one definition of perimeter, as the distance around a given shape.

4.3.4 Connecting ideas and concepts in mathematics to the real world

The connecting of imaginary concepts into real world concepts was evident when learners linked abstract mathematical assumptions (in this case, age), to models they could see and touch (in this case, the circumference of the tree). For example, in the first task of the perimeter trail, after getting several wrong solutions in a row, learners engaged themselves in a discussion to check the reasonableness of their answers regarding the age of a tree. The following table is an extract of how I questioned the reasonableness of the age that the learners got and how it makes sense to them.

Table 4. 9. Discussion between participants and researcher to justify real world situation

Participant	What was said by the learners	Researcher's observations
Researcher	What answers are you getting?	I prompted learners to say the answer they had.
L10	We got four years	Not yet realising what it meant for quite a big tree to be just four years old.
Researcher	So, you're in Grade nine, if this mopane tree is four years that means you didn't find it at this school, only when you were around fifth grade then that's when it started growing?	I wanted learners to make sense of the answer that they ended up having

L2O	Noooh!, that's a big [an old] tree, Miss, it [the tree] can't be four years.	L2O finally picked up where this mistake was when transformed the solution into a real-life context
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Although the task for calculating the age of a tree seemed easy to learners, age in itself is an intangible measure and therefore the solution needed them to conceptually understand the question and apply tactical skills. Learners were expected to link abstract mathematical assumptions, such as age, to things that they see or touch, such as the diameter of the given tree. Kilpatrick et al. (2001) mention that justification is an effective way to make their reasoning clear as when learners hone their reasoning, they improve their conceptual understanding. This means that learners develop appropriate reasoning skills, as they continue to make sense from the results of their actions, and in this way, they improve their learning.

4.3.5 Conclusion of learners' conceptual understanding during perimeter learning

During the running of the perimeter trail, I observed learners using reasoning to help themselves and others discover geometrical properties for the physical shapes that they were measuring. According to Brodie (2010), learners who discuss their findings with others afford themselves ample opportunities to justify their solutions. In this study, learners justified their answers by engaging in negotiations that reflected their knowledge structures that were simpler and more meaningful to them. Learners engaged themselves in discussions that promoted the use of more than one method to find a solution to one task.

My learners used the smartphone to access applications such as the dictionary, in order to confirm the assumptions that they had on concepts that were not known to them. Moreover, they were observed drawing sketches on paper. The discussions, together with the sketches that learners made, manifested their mental perceptions regarding the physical materials that they engaged with as they sought the solutions to the task. These mental perceptions were brought into reality through the drawings and the discussions that learners involved themselves in.

4.4 VISUALISATION PROCESSES IN THE LEARNING OF AREA CONCEPTS

This section provides analysis for the visualisations made by learners as the MCM app introduced the concept of area. According to Khan et al. (2015), the continued engagement of the learners with the environment in which they live influences their learning. In this trail, learners were observed solving tasks related to the concept of area in measurement. A trail comprising tasks designed within and using objects in the school grounds were used.

4.4.1 Understanding spatial relations of the element

The understanding of spatial relations became evident when learners from Group 4 arrived at the poles arranged in a triangle poles that form the fence post of the school garden. Discussions about the dimensions began as soon as the task question was read from the smartphone. L100 made a sketch on paper and then initiated the measurements for the dimensions of the triangle shape. Table 4.10 below shows the transcript of the learners' discussions.

Table 4.10: Excerpt of learners' discussions as they discussed the dimensions of a triangle in real life.

Learner	What was said by the learners	Researcher's observations
L100	<i>Tala, ndjino o base, moes? Ndjino o length... nenge oyo ndjino, walye. [look, this is the base, right? This is the length ... or this one, whatever].</i>	L100 describe the base and length (height) as the essential dimensions to calculate area of a triangle, but unsure of which dimension she could assign as the length.
L120	The base is this one, base have to be straight.	L120 when learner stated that base is <i>straight</i> , he intends to explain the actual property for a right-angled triangle that base should be <i>perpendicular</i> with height and not the slanted pole.
L10	This side is the one which is not straight	Points at the hypotenuse of the triangle
L100	Ok. So, this is the side we don't use in the calculation. Yah. Let's measure then.	L100 pointing at the hypotenuse of the triangle. L110 then took a measuring tape and started to measure

For as long as L100 was allowed by the group members to proceed with measuring, the assumption was confirmed that the height of a triangle is always perpendicular to its base. Learners were quick to exclude measuring the slanting side of the triangle as seen in Figure 4.11 below:



Figure 4.11: Learners 'holding onto' the fence pole of the school garden

Figure 4.11 shows L11O holding the hypotenuse part of the triangle. As observed in their conversation in Table 4.10, L11O used the term 'not straight' to incorrectly assume that if a line is neither perpendicular nor vertical, then it is referred to as 'not straight'. This caused a confusion regarding which parameters could be included when calculating the area of a triangle. This was evident when a state of confusion aroused among the group member who then that the hypotenuse length is not included in calculating the area of a triangle.

Learners identified the base, height and hypotenuse of the triangle while physically standing in front of the shape. The smartphone offered a linking role: as learners looked at the shape displayed on the smartphone they also matched the dimensions that they saw on the phone screen with real life. This identification linked the ideas between the thinking that was going on in their minds, with the physical features necessary to calculate the area of a triangle. Nascent awareness of the concept of area was also evident through the learners' explicit use of a formula to calculate the area. By doing this, they linked the mental construction with spatial constructs as they performed measurement on the physical objects that they measured.

4.4.2 Connecting to a previously solved problem

During the solving of the task on the area trail, learners got an incorrect solution on the first attempt of the task of calculating the area of a trapezium. They then resorted to reading the hints provided by the smartphone in order to find a working approach to the solution of the problem. Below is a discussion on how learners used the stepped hints as an aid to rectify their mistakes for the tasks that they got wrong.

Table 4.11: Group 2 learners use hints to reflect on their mistake

Learner	What learner said	Researcher's Observations
L50	<i>Guys, eyamukulo ola puka</i> [Fellows, the answer is wrong.]	Worried that they got a wrong solution for the task.
L50	Hint number two says the formula for trapezium is A is equal to half multiplied by h multiplied by a plus b , where h is the perpendicular height between the parallel lines a and b .	L50 reads the second hint using the smartphone.
L60	<i>Hasho nga twa ninga?</i> [isn't that what we did?]. Oooo, the problem that we have is the brackets.	Identify where the mistake that they have been doing, which is that they were not using the brackets in their calculation.
L40	Half... multiplied ... by height... multiplied by a plus b.	Echoes on what L50 said as the formula was read, also trying to emphasise to L60 that the formula they used didn't include the measurement for height.
L50	Please calculate the straight lines first, ... make twenty plus nine centimeter	L50 uses the phrase <i>straight lines</i> to refer to the opposite, perpendicular lines of the trapezium.

L60 paved the way in identifying the error in their first calculation, of omitting the step of first adding up the parallel pair of lengths on the trapezium. This is shown in Figure 4.12 below.

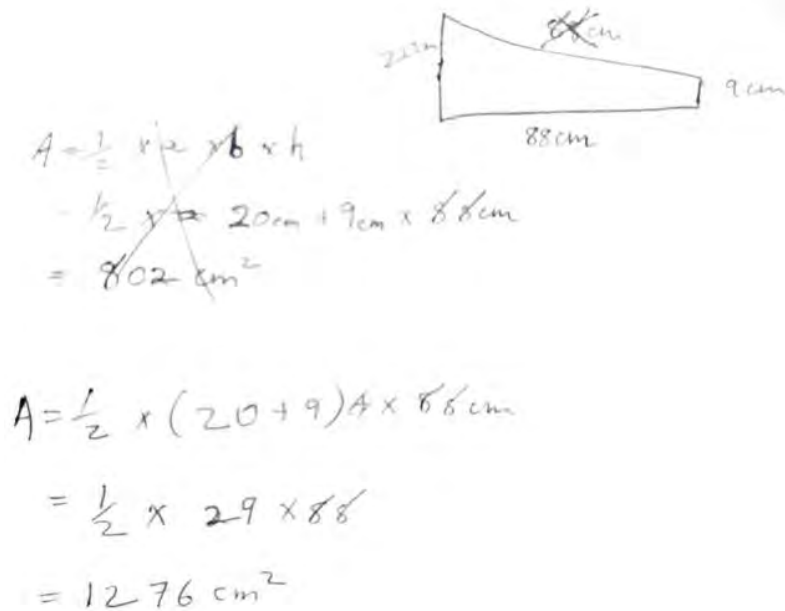


Figure 4.12: Learners redo the task

When looking at the first attempt for the calculation that was done by learners in Figure 4.12, there is evidence that they recalled the formula used to calculate the area of a trapezium. The learners found out that the solution was not correct. They then agreed to use a different strategy to correct their mistake. It was in the second attempt that the learners became careful enough to work out the sides of a and b first before they computed the rest of the attributes in the formula. This improved strategy led learners to a correct solution. In this situation, the prior knowledge was the formula for the trapezium that learners had previously learnt.

The second hint for this task provided by the smartphone reminded the learners of the formula, which they used to confirm their uncertainty; because they did not know whether they had used the correct formula. The provision of hints in this case provided learners with a chance to relook at their calculations and reflect on how to turn a calculation that was previously at fault into a correct one. The sketch that was drawn in this task played a role as a visual representation that entered the learners' process of thinking through the steps, which helped them to assign the dimensions of a trapezium. The drawing of the sketch that learners produced helped them to actively spot the mistakes that they made in prior calculations, and were thus able to re-enter their thinking process in order to correct the mistake.

4.4.3 Constructing a visual representation.

The construction of a visual representation was evident in this study when learners from Group 1 used the smartphone to look for the task location as there were many old sewage dams within the same area. Learners had to use the image on the app to locate the specific old sewage dam that was in the task. When they arrived at the task location, a discussion on how learners were going to solve the task was initiated as provided in Table 4.12 below.

Table 4.12: Excerpt of learners' discussion to solve the task for the concrete dam.

Learner	What learner said	Researcher's observations
L2O	The question said we should calculate the area covered with what?	L2O ask for clarity on the objective of the task.
L3O	Calculate the area covered with concrete and not by metal	Re-reads the question on the smartphone
L1O	So, this means...	Trying to make sense out of the re-read objective of the task
L3O	First, we measure the area of the whole concrete (concrete and metal) then we calculate area of metal (only).	L3O suggest his ideas to other group members, building on the sketch that L1O has drawn.

During their discussion, as seen in Table 4.12, L1O did not complete his sentence but rather started to draw the prominent dimensions of the dam i.e. the inner and outer outlines of the dam. Later on, the labelling for some of the dimensions were observed as seen in Figure 4.13 below:



Figure 4.13: A sketch of the concrete dam

The sketch in Figure 4.13 that L10 produced was copied from the smartphone, which she later assimilated to the dam in real life. This assimilation was evident when I observed the learner turn the phone around so that the dimensions of the dam displayed on the smartphone faced in the same direction as the dam in real life. Again, L10 looked at the dam displayed on the smartphone and then at the dam sketched from real life.

The action for comparing between the two images provided convincing evidence that the picture on the smartphone was being visually represented by the diagram on paper. I observed that having produced the visual representation in the form of a sketch made it easy for learners to record the lengths for the outer and inner dimensions of the dam.

4.4.4 Using the smartphone as a visualisation to solve the problem

The smartphone includes a scientific calculator which is an essential tool used as they solved the tasks. The use of a calculator was a useful tool among the Group 3 learners to share new skills on how multiplying a number by its half is linked to dividing that same number by two. The learners' discussions give evidence of this new learning opportunity as provided in Table 4.13 below:

Table 4.13: Excerpt of how Group 3 members learned new skills.

Learner	What learner said	Researcher's observations
L7O	Ninga... divide [press the divide sign.]	L7O then oriented L9O how to use the calculator to enter the key [$\frac{1}{2}$] that is multiplied by base and height in order to calculate area of triangle.
L8O	Divide?	Not sure why division was used when the formula is based on multiplying half by base and height.
L7O	<i>Ee, mpeya pena one out of two owafa owala to ti one divided by two.</i> [Yes, that one out of two is like you are calculating one divided by two.]	L7O explaining to L8O the relationship between the $\frac{1}{2}$ sign and half
L9O	Oo, then multiplied by base, which is 32cm and then by height which is 129cm. I'm getting 40635cm.	L9O was not satisfied with the answer he got
L7O	Mmhm	L7O was again in disagreement.

Table 4. 13 above shows how learners shared invaluable mathematical knowledge that to half a number means to multiply with the half sign ($\frac{1}{2}$) that is available on the smartphone calculator. Learners were able to use this knowledge and put into practice the multiple ways of representing half as either a fraction ($\frac{1}{2}$), or as a decimal (0.5) on a scientific calculator.

The inherent integration of the smartphone in this task was evident when Group 3 learners used a smartphone calculator to help solve the problem. In this regard, the smartphone is considered as a visualisation tool because it is used as a scaffolding to link the visual images of what learners see, with the numerals in the form of the measurements that they have taken.

4.4.5 Encoding the answer to the problem

When encoding the answer to a problem, learners need a thorough experience of that problem because encoding involves perceiving information and decoding it into visual information (Dongwi, 2018). In this study, encoding the answer to the problem was evident when Group 4 learners arrived at the task for the classroom door. L12O read the task question and was later observed reading the first hint while L11O drew a rectangle to represent the classroom door. L12O held the tape measure at the top corner of the door while L11O bent to reach the bottom corner, and in this way, the two learners measured the height of the door. L12O entered the measurement

on the scientific calculator. He then multiplied the height and base of the door. L12O entered the solution on the app. A wrong solution was obtained on first attempt. L10O requested L12O to reread the question and what she discovered and shared with others is shown in Table 4.14 below.

Table 4.14: Group 4 learners review their methods to solve the task.

Learner	What learner said	Researcher's observations
L10O	<i>Gents, otwa pulwa tuyalule mee metre, tse otwa yalula centimeter</i> [Fellows, we are asked to work in metres and we worked in centimeters].	Suggesting what is needed in the task, after re-reading the question on the smartphone.
L12O	<i>Oooh, now atu metuula ile atu convert mee metre?</i> [Oh, now do we re-measure or convert to meters?]	Suggest that they repeat the measurement procedure in order to provide the solution in the desired.
L11O	<i>Aaye, shapo atu metulula</i> [No, unless we remeasure].	L10O and L11O started re-measuring the length and height of the door. Later on, L12O took a paper and converted the answer they got first from centimeters to meters

Data in Table 4.14 shows that learners collectively decided to correct their mistake in order to get the correct solution to the task, and this was done by relooking at how they had earlier solved the task. Further, I also observed that repeating the measurement as well as converting the final answer into the required units, became a double check of their previous solution in order to find out where they made a mistake.

4.4.6 Conclusion on visualisation processes in the learning of area concepts

During the running of the trail, learners were observed using various visualisation processes as they solved the area tasks. This trail consisted of pictures and diagrams as hints. Learners used these picture-oriented hints to help them reconstruct the diagrams that they had previously made in their minds.

The approach of illustrating the answer using visual representation was evident. The most common illustration done by learners were sketches drawn on paper. The smartphones helped learners to demonstrate their reasoning when given a geometry task, through the use of diagrams or mental thoughts that they processed in their discussions during the running of the area trail. These mental

thoughts were encoded and communicated using words that were easy for them to use. According to Battista (2007) learners who identify and describe the properties of shapes based on their appearance indicate a nascent level of geometric thinking. This was evident in the case of this trail, where learners referred to words like ‘straight’ for perpendicular and ‘not straight’ to indicate the hypotenuse of a right-angled triangle. I assume that this is how the various shapes looked in their mind and thus, this is how they communicated to their fellow group members.

4.5 LEARNERS’ CONCEPTUAL UNDERSTANDING DURING AREA LEARNING

The understanding of area requires the learner to recognise that it is about understanding the space covered within certain boundaries of a flat surface without any overlaps. While the use of formulae is important, Cavanagh (2008) cautioned that learners who are able to use a formula to calculate area do not necessarily have the conceptual understanding of these measurements. In this section, I analyse the learners’ conceptual understanding for area by making sense of how their solving of area tasks was influenced by the physical presence of the objects and the learners’ interaction with these tasks. It was also in my interest to see how they made use of the applications and hints provided to them through the smartphones that they used while running the trail.

4.5.1 Connecting mathematics to prior knowledge

Learners attempted Task 3 for the area trail and the question for the task was read by L4O. An interesting observation that I made here was when the procedure for solving the task came to halt because learners missed the unit as it was being read to them. The excerpt in Table 1.15 shows L5O insisting that the unit be confirmed in order to proceed with the measurement.

Table 4.15: An excerpt of how L5O demanded to know the unit of measurement

Learner	What learner said	Researcher’s observations
L5O	In ...?	Asks the unit in which the measurement was made.
L4O	What?	Pardons for she does not know what was being asked.
L5O	<i>O unitaaa!</i> [the unit]	Reiterates that question is about the unit in which measurement is needed
L6O	(facing to L5O) Lesha vati o unit [Apparently give the unit]	Explains once more, L5O seemed not to know what was being asked.

L5O	<p>Ooo meemeter kaumeee.. then it is one point four meters.</p> <p><i>[Ooh... it is meters, dear, then it is one point four meters].</i></p>	<p>Finally gets the point in question. Explain to others that the units being used are meters and re-read the measure in the desired unit.</p>
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Table 4.15 shows that L5O, who was at the reading end of the measuring tape, wanted to know the unit of measurement that the task required them to work in. I observed the constant probing for the unit to be used in the task and I assumed this was the prior knowledge that L5O and L6O had in mind: when taking measurements to solve a task, it is important to know the unit in which the measure is required. In this case, learners wanted to know whether the measurements needed to be taken in centimetres or metres. L4O did not have this prior knowledge and if she did, she would have understood the need to determine the unit in a given measurement.

This prior knowledge needed to be activated in L4O's mind when the other two members kept on insisting on the measurement unit for the task. For L4O, his prior knowledge was used to influence his activation of new knowledge. As for L5O and L6O, prior knowledge was applied to solve the problem entailed in the task. In this task, L4O's prior knowledge was activated by the constant probing of the unit that was needed in the task. The unit for length to be transferred into the unit area was necessary because L5O wanted to provide the solution with a reasonable unit of area.

4.5.2 Justifying and explaining mathematical ideas and solutions

In the context of this study, justification was observable when learners ran the task to find the area of a classroom door. It is significant to note that no initial discussion took place among these group members. Soon after the correction was done, measurements were underway. I wanted all things to flow naturally so I did not intervene. I was glad to observe that L9O from the group was a little behind in understanding the concepts behind this task. The queries that seek justification are outlined in Table 4.16 below:

Table 4.16: L8O justified an alternative approach to measure the base of the door

Learner	What learner said	Researcher's observations
L9O	<i>Tetiii... okwa tiwa o area of the door... ne omwa mete owala kekuma kakushi komweelo kwokwene?</i> [It was said 'area of the door' ... I see you measure on the wall, is it not on the door?]	Concerned about the place where the measure tape was place on the wall and not on the door itself. The word 'Tetiii' is used to seek for attention from her group members.
L7O	<i>Same iike, moes nande otwa meta mhano is just the same length.</i> [It is just the same. Even if we measure here, the measures are just the same]	Explain that as long as the measure is next to the door the it is just fine.
L9O	<i>Tu mete ee side adishe vakwe</i>	Insist that all of the four sides are measured and not only two sides (a length and a breadth).
L8O	<i>O area oya pumbwa owala ee side mbali...mahno naamhano</i>	Explains that the attribute area only accounts for two adjacent sides
L9O	Ok	Finally agrees to the two explanation provide to L8O
L8O	<i>Kuwete.....the app says its correct</i>	Use the app feedback to confirm that the method used was correct, because it led to a correct answer.

Scrutinising this excerpt allowed me to identify the two events of justification. The first justification is provided by L7O, with the purpose of explaining that measurement that was taken next to the object would still give the correct answer, provided that the ends of the tape were next to that of the door. He tried to instill the property for congruency of shapes in L9O's mind. This justification is provided by Figure 4.14 below:



Figure 4.14: Two dimensions required to measure the area of the classroom door

The second event was when L8O responded to the query again, made by L9O. L8O reassured him that the attribute for area only accounts for two adjacent dimensions, for example the breadth and the height of the door. I have inserted black arrows in the two diagrams to help point out the two justifications: firstly, a measure taken next to the object was congruent and secondly only two sides are needed to calculate area of a rectangle. The feedback from the MCM portal confirmed the task solution as correct.

Explaining shared among learners demonstrates the ability to form and maintain images with certain attributes for conceptual representation. The learners' physical presence to the object and task facilitated the formation and maintenance of the concepts of the shape, which learners constantly referred to when they explained themselves to their group members. Another conclusion that I would like to present is that the feedback that learners received from the app was used to ascertain the methods learners could use to solve the task and the explanations about the tasks were given at the time this task was solved.

4.5.3 Connecting ideas and concepts of mathematics into the real world

During the solving of the trail tasks, it emerged that some learners came into physical contact with the learning material specified for the task.

According to my observations, the outdoor nature of the tasks presented learners with opportunities to interact with the dam that learners were supposed to conduct the measurements on, mainly by touching it, so as to strengthen the importance of the dimensions that they needed to work with. Learners relied on the smartphone to verify the link between the dimensions of the shapes and the geometrical properties of the shape of the task. Table 4.17 below gives evidence of this.

Table 4.17: An excerpt of learners translating the task on the smartphone into real life

Learner	What learner said	Researcher's observations
L5O	First, we have to find the area of the whole thing (concrete dam and metal), then we find the area of the metal...then we subtract the area of the metal from the area of the whole thing (concrete dam and metal).	L5O suggest the approach to solve the task. L5O highlights the part needed for measurement as he points at the central part of the dam, which is made up of metal only.
L6O	Subtract the area of the metal top from that of the concrete top. ... <i>ee, ouli mondjila</i>	Reads out the hint and finally confirms to what L5O suggested.

L40	<i>Ee area adishe atudi mete mee centimeter?</i> [Are we supposed to measure both are in centimeters?]	L40, who was at the reading end of the measuring tape wanted to know the unit of measurement task.
L50	<i>Ee, omee centimeter. Kwata hoka tu mete orea yosamende manga</i> [In centimetres, yes. Hold the measure tape so that we can measure the concrete area].	Learner uses the hint on the smartphone to narrate that they need to make sense of the measurements before they perform any calculations.

Table 4.17 shows how L50 used a smartphone to discover the connection between the dimensions of the task on the smartphone. Learners were able to visualise imagery of the shapes and their properties because they could see and relate the concrete and visual representation of shapes to the abstract. L50 achieved this connection by first crafting the relation between the measurements that were taken on the real object, in this case, a dam, and the calculation methods that were applied in order to solve the task.

Data in Table 4.17 shows that L50 discloses the procedure for calculating the area of the concrete part of the dam. L50 unpacked the set of actions that they needed to take, in order to get the solution for the task. The data from Figure 4.14 also shows that I observed L60 reading aloud the hint from the smartphone to confirm with what L50 had said. The reading of the hint in the form of a confirmation was necessary to consolidate the method of calculation that they were going to carry out.

After this discussion I then observed learners translate their words into actions: they began by identifying the metal and concrete as the key parts of the objectives for this task. the identification of the concrete is displayed in Figure 4.15 below.



Figure 4.15: L5O spreads her hands to point out the concrete top of the sewage dam

As observed in Figure 4.15, L5O spread her hands on the concrete section of the dam. This was a way to distinguish the metal from the concrete part of the dam. Another significant observation from Figure 4.15 was the rubbing of the hands in a circular movement (as in rubbing the top surface), gave me a hint to conclude that the learners conceptually knew that they were working with the area attribute for measurement.

The discussion between L5O and L4O is evidence that L5O0 connected the mathematical knowledge of addition/subtraction of areas of shapes to find the required or asked area in real life. The spreading of the hand was regarded as a form of manipulating the physical learning material that supported the development of strategies for conceptual understanding of what was required of the task. The presence of the smartphone made L5O's explanation worthwhile, because the image was previously seen on the smartphone screen, so it was a simple matter for L5O to reproduce this explanation of the objective in real life.

4.5.4 Conclusion of conceptual understanding in the learning of area

It was remarkable to note how learners translated their problem tasks into sketches and diagrams. In agreement, Hearst (2016) emphasises that visualisation translates abstract information into the visual form, which provides new insight about that information. The use of smartphones was essential in providing visual images that learners could relate to real-life materials that they used

to solve the problems. This is a handy means for transforming tasks from complex into simpler forms as learners solved the MCM tasks.

4.6 VISUALISATION PROCESSES IN THE LEARNING OF VOLUME

This section presents the findings for the visualisation processes evident in the learners' interactions with the learning environment and smartphones when running the volume trail. The tasks in this trail entailed the solving of substances and materials in their 3-dimensional shapes. Analysis is provided regarding how the physical materials incorporated in the task helped learners to understand the links between the geometric features of the shape. Tan-Sisman and Aksu (2016) advise that to understand how geometric shapes are related to the volume they contain, learners need to go through the visualisation processes associated with learning area concepts. In this trail, the task required learners to develop from learning 2-D to solid 3-D shapes, through the use of smartphones to access the MCM app.

4.6.1 Understanding spatial relations of the element

A very rich spatial perception is observed when learners came into physical contact with material involved in the task. The sensory perception that learners obtained from the hint read from the smartphone, imprinted images on L50's mind. This finding was revealed when the learner was tempted to use his hand to fold around the pole, which he perceived as a cylinder. Table 4.18 demonstrates this incident.

Table 4.18: An excerpt showing how learners perceive a pole as a cylinder

Learner	What learner said	Researcher's observation
L40	The pole is a cylinder	L40 read the first hint of the task.
L50	Pole...	Repeating part of the first hint for the task that was read by L40 tempted L50 to touch the pole in a way of folding his hand around the pole as seen in Figure 4.16. The rest of the group members concentration their focus on the pole which L50 was trying to acquaint himself with.

Data provided in Table 4.18 shows that the reading aloud of the first hint intuitively coerced L50 to use his sense of touch to perceive the pole as a physical artefact. The evidence that this learner's

mental cognition was developing, was that as soon as the hint of the task was read, the following was observed as shown in Figure 4.16 below:



Figure 4.16: L50 folds his fingers around the pole

The observed snapshot of L50's hand, as seen in Figure 4.16, shows his hand wrapped around the pole. This way for holding the pole helped other learners to get an idea of what was required concerning the task of the pole. For example, the way L50 closed his fingers around the pole and mentioned the word 'cylinder' could have suggested to other learners that the pole was in the shape of a cylinder and that they needed to identify circumference as the first dimension that they needed to measure. Figure 4.17 below shows the group then measuring the circumference of the pole.



Figure 4.17: L40 joins L50 in measuring the cylinder of a pole

Figure 4.17 shows learners wrap the tape measure around the pole in order to obtain the circumference of the object. The visual-spatial experience of the pole contributed to L5O's understanding of the pole as a cylinder, as provided by the hint that accompanied this task. It became apparent to the Group 2 members to recall that the circumference was needed to be measured in order to find the radius, which they were not able to measure directly.

Touching the pole imprinted this image in the learners' mind. This physical perception allowed learners to make sense of the objective of the task. This sense-making then led to the link between the geometric shape of the task and the onset of the measurement activity that learners employed to solve the task.

4.6.2 Connecting to a previously solved problem

Learners demonstrated adoption of past experiences into a new situation in which they found themselves. During the running of this task I observed learners use more than one formula to solve a single task. Previously, learners would solve a task about finding the volume of a cylinder and then volume of a cuboid in separate occasions. In this task the learners were confronted with a task that required them to solve the volume of the cylinder and that of a cuboid concurrently. In Table 4.19 below, I present evidence on how learners discussed finding a way of transforming such a complex task into a much simpler one.

Table 4.19: An excerpt of how learners used prior knowledge to discover a new formula

Learner	What learners said	Researcher's observations
L4O	First, we need to find the volume of concrete...	Suggest the first step to be taken for the task.
L5O	...then, we find the volume of the cylinder, then we subtract the volume of the cylinder from the volume of concrete block.	Without aid from any hint learners discussed the method that they may follow in order to solve the task.
L6O	Hint number one: The volume of cuboid is equal to length times breadth times height and volume of a cylinder equals to pie radius square times height.	L6O read the first hint which provided a clue on the formulae for cuboid as well as cylinder, but the group members were less concentrated and began wondering around the concrete block with the measuring tape.

L5O and L4O did not pay much attention when the hint was being read by L6O. Instead, they were more concerned with how they were going to measure the dimensions for the two shapes into which the concrete cuboid with a hollow cylinder is made. This was evident when learners started with the measurement activity while L6O was busy contemplating the hints that the smartphone provided. This means that learners were looking forward to gaining some knowledge from the trial-and error approach so that they could make sense of the task.

Making connections to a previously solved problem came into play when learners used the hints provided by the smartphone in order to confirm the assumptions that they had about the formula they were going to use. The formula suggested by the hint was already in the learners' existing body of knowledge. What was left was for the learners to link these formulae to a new, real-life situation in which they found themselves.

4.6.3 Constructing a visual representation

Learners from Group 1 constructed the formula for calculating the volume of a cylinder. It was clearly observed how L2O wrote the formula for calculating the volume a cylinder on the ground. I noticed how this captivated the attention of others as they seemed to welcome the idea of writing on the ground. Consequently, I observed that this encouraged the other group members to actively contribute to the completion of a correct formula by loudly telling L2O what to write next on his ground sketch. Table 4.20 below describes this discussion.

Table 4.20: Learners from Group 1 draw the formula of a cylinder on the ground.

Learner	What learners said	Researcher's observation
L3O	Volume equals to pie times r squared times	L3O began to read aloud the incomplete formula as L2O wrote it down.
L2O and L1O height	In unison, L2O and L1O added the symbol to the formula that was suggested by L3O.

Within a short time, a correct formula for volume of a cylinder was generated. In an attempt to note what they discussed, L2O started to write the formula on the ground as seen in Figure 4.18 below:

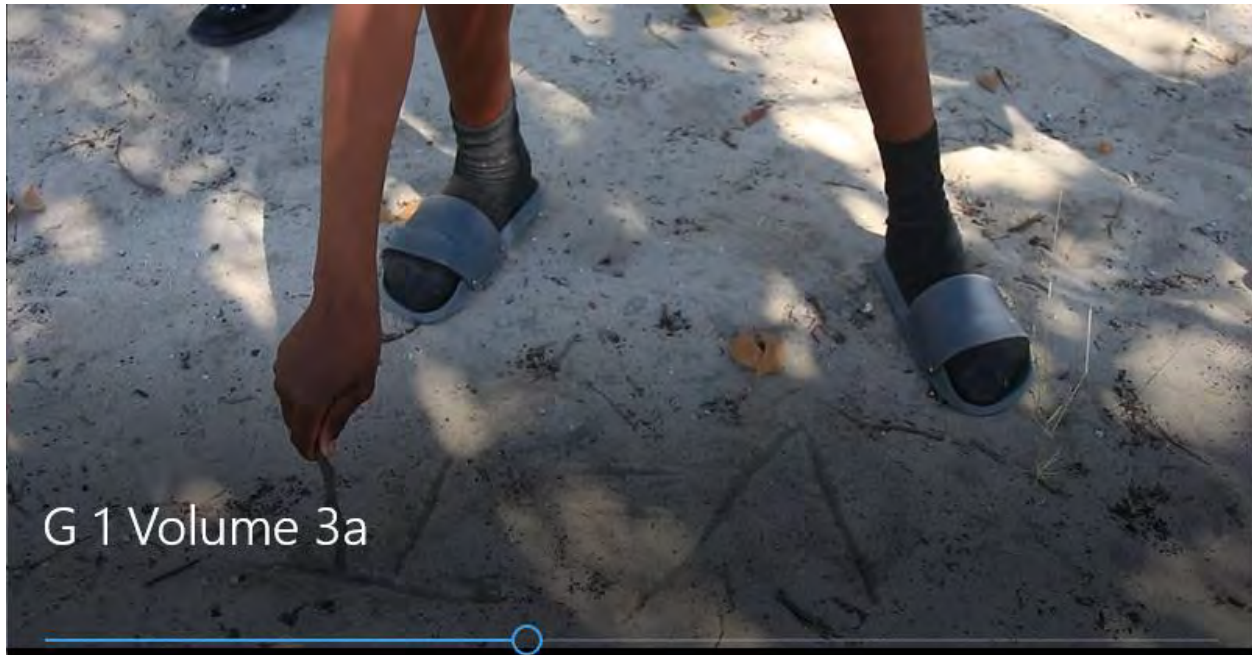


Figure 4.18: Learner writes the formula on the ground

Learners used a reasonably distributed effort to come up with a formula that they later used to calculate the volume of a cylinder. The formula represented the mode of operation that learners were going to use. The learners from Group 1 then substituted the symbols in the question with the measurements they took.

Writing the formula on the ground was a powerful visual representation that Group 1 members generated as they kept on revisiting the formula for cylinders that they wrote on the ground. This formula transformed the task into a mathematical form that activated the calculation process, as they started to substitute the symbolic encryptions into numbers that they computed in order to solve the task.

4.6.4 Using the smartphone as a visualisation to solve the problem

Data findings show how Group 4 members discussed a method to calculate the volume of the pole. After both the question and hints were read, it was particularly interesting to see how learners decided to use both the calculator on the smartphone as well as the scientific calculator to perform the calculations of volume. During the solving of the problem task, L100 realised that there was a mistake, and Table 4.21 shows the excerpt of how Group 4 members arrived at a correct solution after the first failed attempt.

Table 4.21: An excerpt of learners from Group 4 telling others what they have failed to do.

Learner	What learners said	Researcher's observation
L10O	<i>Gents, so far, otwa valuleni ashike o radius inatu yeni vali no volume.</i> [Fellows, so far, we have only calculated the radius and we're supposed to proceed with volume (of the pole)]	Identifies a shortcoming that group members have made.
L12O	<i>Neh?</i> [Is it?]	L12O surrenders the smartphone on which he was calculating by tucking it under the scientific calculator which he continued to calculate with.
L3O	<i>Lesha o formula yo volume... length times breadth times.</i> [read the formula for volume]	Insists that the formula for volume is read, starts with it but still seeks for a completed formula and gets the smartphone calculator and prepares for calculation.
L2O	Volume of a cylinder equals to pi times radius square, time height	Reads the hint as requested.

The action of constant shifting between using the calculator on the smartphone to that of a scientific calculator helped the learners to iron out some uncertainties they had, regarding the smartphone calculator. An example of these uncertainties is that the smartphone calculator may not have as many entries as those on the scientific calculator, which is a more familiar tool used in class. The smartphone played a twofold role during the solving of this task: firstly, it was used as a calculator – a useful tool to find solutions to this activity; and secondly, a tool to augment the formula for the volume of a cuboid that he transferred into the appropriate formula for a cylinder.

4.6.5 Encoding the answer to the problem

When Group 4 members arrived at Task 4 of the volume trail, they had no idea of how to solve the task. This was evident when I observed them take random measurements around the dam. A point in came when L12O requested that L10O reread the formula given in the hints. Evidence that learners were becoming attuned to the objective of the task became clear when they used the smartphone to modify the formula that was provided to them in the hints.

Table 4.22: An excerpt of how Group 4 learners modified the formula in order to solve the task.

Learner	What was said by the learners	Researcher's observation
L12O	Look, if we have pie (π), height we can measure	Trying to suggest the method for generating the formula hearing the hint read from the smartphone.
L11O	Therefore, we need to find r and square it.	Having a formula at hand, L11O attempts on modifying this formula, making radius that subject of the formula
L12O	r, we get it from formula given un the first hint.....	Continues to modify the formula in such a way that it suits the solving of the task.
L11O	Kindly re-read the first hint. [<i>lesha natango o hint yo first</i>]	

In their discussion above, learners first generated their own formula and thereafter used the given formula provided in the hints to produce a suitable solution to the task. The method of manipulating the formula is real evidence that the smartphone provided the Group 4 learners an opportunity to encode the task from its physical attributes that they saw on the smartphone screen, into visual attributes that they wrote on paper as they solved the task.

4.6.6 Conclusion on visualisation processes in the learning of volume

It was well noted how learners used the smartphone to enable them run the volume trail: the smartphones offered the learners timely feedback that allowed them to act accordingly. Learners were observed engaging with the physical dimensions of the object which, as suggested by Tan-Sisman and Aksu (2016), helped them understand the link between the geometric features of the shape and its formula. The learners also used the geometrical and spatial appearance of the object to confirm their assumptions and previous knowledge that they had, as well as to reflect on the task once they ended up with a wrong solution. Learners also adapted previous experiences to new situations when they came across rather unfamiliar situations in the MCM volume tasks.

4.7 LEARNERS' CONCEPTUAL UNDERSTANDING DURING VOLUME LEARNING

The analysis in this section focuses on the conceptual learning perceived by learners from running the volume trail. Berger (2006) argues that conceptual understanding of mathematics does not only include what one in learning, but also entails using what is already known in order to advance to

new forms of understanding. My focus was to analyse how learners used the smartphone to access learning that transforms (Battista, 2004) and distinguishes (Barrett, Clements & Sarama, 2017) the inside of a 3-D shape from its outer surface.

4.7.1 Connecting mathematics to prior knowledge

The use of prior knowledge to adopt new learning was evident when learners solved Task 3 of the volume trail. During the solving of this task, learners generated a formula for calculating the formula for a cylinder.

The tasks began with L2O reading the task question. L2O then started the discussion regarding the method that could be used and helped suggest the formula that they could use to find solution to the task. Table 4.23 shows how the learners collectively generated a method to calculate the volume of a cylinder.

Table 4.23: Learners collectively generated the method to calculate volume of a cylinder.

Learner	What was said by the learners	Researcher's observation
L3O	Only circumference can be measured. radius cannot be measured only be calculated	Read the first hint from the smartphone.
L2O	Circumference equal to radius multiplied by 2π .	L2O initiates the formula by himself, without copying from anywhere.
L3O	Radius is equal to circumference divided by 2π .	L3O simplifies further on how to find the value for radius.
L1O	(circumference is equal to) twenty divided by.....	L1O re-states the formula by substituting variables with the measurements obtained.
L3O	2π	L3O completes the formula for finding the value of radius given the measure for circumference.
L1O	Six point.....six point two eight three.	L1O who had the role for doing calculation read the solution from the smartphone calculator display.
L3O	do we now multiply it by ...? We <i>moes</i> got the radius.	L2O trying to suggest the way forward. The word <i>moes</i> is used as a suggesting word.
L2O	hmmmm (in consensus)we didn't record the height! Where is our height?	L2O opened the first hint noticed there was a value for height that need to be integrated in the formula. This brought so much surprise to all of the group members

because these learners anticipated to substitute values but the measure for the height was not taken.

Data in Table 4.23 shows how my learners used their prior knowledge of manipulating a formula in order to find the unknown measurement of the task that they were dealing with. Learners wound the tape measure around the pole in order to measure its circumference. The collective discussion was made by all three participants. The fact that they used their prior knowledge was evident because they did not copy this formula from anywhere. The learners also had previous knowledge on how to make the radius the subject of the formula for calculating the circumference of a circle, as shown in Figure 4.19 below:

The figure shows a hand-drawn diagram of a cylinder with a height of 200 cm. To the right of the diagram, the following handwritten work is visible:

$$C = 2 \times \pi \times r$$
$$\frac{C}{2\pi} = \frac{2\pi \times r}{2\pi}$$
$$\frac{C}{2\pi} = r$$
$$\therefore r = \frac{C}{2\pi}$$
$$\frac{200}{2 \times 3.14} = 31.8 \text{ cm}$$
$$V = \pi (r^2) \times h$$
$$= \pi (31.8)^2 \times 200$$
$$= 7878.72 \text{ cm}^3$$

Figure 4.19: Learners make the radius the subject of the formula

As observed from Figure 4.19 above, learners used the formula for calculating the circumference and manipulated it in order to find the value for the radius. This was made possible by making the radius the subject of the formula. It was still in their proximal circle of knowledge that the value of r is needed in the calculation for the volume of a cylinder. This was evident when Group 1 immediately started to collectively generate the formula for calculating the volume of the cylinder.

They used this hint so that they could be sure that they had all the measurements from the pole that could help them to come up with a complete solution.

They then encountered a mishap, where they failed to measure the height of the pole. Under ordinary situations, the height would have been provided along with the question. It was in this new, outdoor situation that the measure for height was needed to calculate the volume of the cylinder. Learners then stretched out the tape measure to find the height of the pole – a measurement that was necessary to include in their calculation.

In this task, the hint played a vital purpose in the sense that it helped the learners to recall the importance of the dimensions of the cylindrical pole that they were dealing with, in the task. The constant revisiting of the sketch provided more opportunities to think about the diagram and the corresponding measurements of the dimensions that helped learners to find a solution to the task.

4.7.2 Representing mathematical concepts in different ways

Learners from Group 1 encountered multiple methods for solving the task from the volume trail. After taking the measurements, the learners added the lengths together and L2O asked for the first hint. While L3O was reading the hint, others paid very little attention to listening to the hint that they had requested earlier, and rather concentrated on the calculations that they had done.

Most of the group members measured the ratio of the outer perimeter: inner perimeter as 168:74 simplifying this ratio, they ended up with decimal numbers. L3O then opened the hint on the smartphone to compare it to the different but close solutions that were a result of the different methods used by the learners. Below, Table 4.24 gives a description of the multiple representations the learners used.

Table 4.24: Multiple representations used by the learners

Learner	What was said by the learners	Researcher's observations
L1O	Equals to two point two ... to one...	L1O uses a calculator to simplify the ratio of 168: 74.
L2O	Seventy-four (74) goes into itself once and in one sixty-eight (168) two point two seven (2.27) ...times	Complements the attempt on simplifying the 168:74 ratio.
L3O	<i>Ndjoka kaishi o whole number.</i> [That is not a whole number.]	To L3O, the answer obtained by others was worrisome because none of the multiple-choice answers had a possible solution of a decimal number.
L1O	So, we need to <i>convert</i> to whole numbers	By the word <i>convert</i> , L1O intended to say they <i>round off</i> their answer to a whole number.
L2O	...then is just two	As he rounds 2.27 off to a whole number.

For learners to get the correct solution to this task, they had to use multiple representations of numbers – either a decimal or a whole number. Table 4.24 shows the discussion that took place among Group 1 members as they crafted a different way of representing these numbers.

The process of simplifying a ratio when a number is a decimal, was hard for my learners. It was even more challenging that the solution was multiple choice where they were limited to choosing from only four options, of which none of these solutions was a decimal number. To L3O, the hint was used to confirm whether they were on the right track. It took a while before L1O suggested that they resort to writing the number in a different format, which effectively entailed rounding it off to the nearest whole number.

According to Zender, Ludwig and Gurjanow (2017) the physical engagement with the task allowed learners to regard the learning material from different angles and this allowed them to seek a multitude of ways to represent tasks. This gives learners at more opportunities to learn, as they keep trying different methods to find the most effective one.

4.7.3 Connecting ideas and concepts in mathematics to the real world

When L9O read the questions, the group members looked unsure about what to do. The other two learners (L7O and L8O) signalled the need for a hint. L9O seemed to understand what the others wanted, and she promptly read the first hint to them. Next, the group members took some measurements. In order to determine the radius of the cylinder, the learners needed to determine the centre of the circle, which was hollow. L9O used her pen to virtually determine the center of the circular shaped as shown in Figure 4.20.



Figure 4.20: L9O used a pen to determine the center of the hollow cylinder

The learners then used this mechanism of finding the radius in order to calculate the radius of the circle as shown in the Figure 4.21 below:

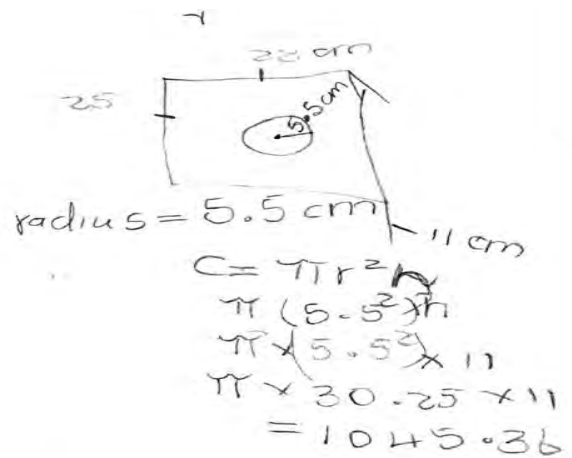


Figure 4.21: L90's calculation derived from determining a measurement virtually

The spatial connection between the task object (in this case, the hollow cylinder) and the learners, showed what was going on in L90's mind as he determined this rather virtual centre of the circle by physically engaging with the task object.

Soon after the learners calculated the radius of the circle, they were again required to calculate the height of the hollow cylinder which was inside a concrete, rectangular block. This height was another essential dimension necessary for calculating the volume of the cylinder. It was interesting



was to observe L90 use her body to visualise the height of the concrete block (Figure 4.22) by using her feet to step along the height of the cuboid.

Figure 4.22: L90 uses her feet to point to the height of the concrete cuboid

The learner's physical contact with concrete block as a learning material enhanced her thoughts about where to take the measurement of the height of the block. The dimensions for the concrete block that were seen on the smartphone were brought into reality when learners started to physically measure a material that they had first encountered on smartphones and eventually physically encountered.

4.7.4 Conclusion on learners' conceptual understanding during volume learning

In summary, data from observations show how the learners' constant use of smartphone provided an integral tool for solving tasks on the volume trail. The smartphone use was handy in the running of the volume trail in the way that learners used it to support meaningful learning, for example, the hint to remind them about their forgotten prior knowledge of a diagram to help them perceive visual artefacts, helped them build onto the solid materials that they encountered.

In the next section I discuss code switching as a theme that gradually emerged during the analysis of the video recorded sessions when learners solved the measurement problems at the various MCM task sites.

4.8 EMERGING THEME FROM OBSERVATIONS

4.8.1. Code switching

A noticeable series of events that occurred during my observations of the running of the trails was that learners constantly switched between English and the vernacular language, which is Oshiwambo. In this study, I observed learners code switch in the following two circumstances:

The first circumstance was lack of fluency in either of the languages that learners used: low proficiency in English and/or a limited ability to use a complete sentence using only words/vocabulary in their own mother tongue. For example, when L50 confirmed that she was finally getting a clue about what the task was on, she said the following to the others:

Mother tongue: *Oooh,tashiti otwa pumbwa oku findinga o radiusaa?*

Translation: *Oooh, so this means we were supposed to find the radius?*

The word *findinga*, which means *to find*, was directly be translated into the Oshiwambo language. I assume the learner found it more efficient to borrow the word from English. Concurrently, the use of the word *radiusaa*, which is a direct translation for radius, was also incorporated into her statement. This suggests that the mathematical word for radius is rather hard to find in the vernacular language of Oshiwambo.

The second circumstance when learners found themselves code switching was when they wanted to illustrate salient information and instruction. Studies (Probyn, 2015; Barwell, 2018), found that learners code switch so that they can create meaning, foster conceptual understanding, as clarify visualisations that they have internally made, to others. According to my observation, when learners verbally interacted with each other, their code switching was supported by visualisations and interpretations, which in turn helped them to clarify concepts between themselves and others:

Mother tongue: *O circumference oyo ndjika.*

Translation: *Here is the circumference.*

As this learner illustrated to others that the shape that they were dealing with in the task was a circumference, he made use of the physical features of the pole to link his ideas to introduce what

was required of the task. This was evident when he bent his fingers around the pole, thereby demonstrating a conceptual understanding for what circumference is to himself and to the other group members.

4.8.1.1 Code switching to enhance visualisation

Finding from the observations revealed how learners from various groups inherently used code switching to enhance the visualisation processes that they were going through. Below, Table 4.25 gives some excerpts of learners' code switching which enhanced their visualisation.

Table 4.25: Excerpts of learners' code switching to enhance visualisation

Learner	Learners' language	Translation	Visualisation process
L10	<i>Gents, otwa pulwa tuyalule mee metre, tse otwa yalula centimeter.</i>	Fellows, we are asked to work in meters and we worked in centimeters.	Learner used a smartphone to read a hint in order to alert other group members that the unit into which the solution would be produced. Instead of reading the hint as is, L10 happened to convey this message in Oshiwambo. Communicating in mother tongue made it easier for generating reasonable and acceptable ideas
L40	<i>Paije nyoluhuliike mha (pointing on paper). Ndjino o cylinder, ndjino o radius.</i>	Now just rewrite here (pointing on paper). This is the (measurement for) cylinder and this one here is the radius.	L40 advised that they re-write the method that they would use to solve the question. The hint from the smartphone was used to not only reflect on the misconception but also pave way on how the task would be solved.
L90	<i>Otatu kongo o radius ... oya pumbwa owala oku za po centre yokangonga</i>	We are looking for the radius ... we just measure starting from the centre (of the hollow cylinder)	L90 mentioned the terms radius and the center as prominent aspects for measuring the diameter of a circle. Emphasis was made when he used these words in English
L100	<i>tala, ndjino o base, moes? Ndjino o length... nenge oyo ndjino, walye.</i>	look, this is the base, right? This is the length ... or this one, whatever.	L100 pointed the position of the of the base and height. This was a way for demonstrating the visual understanding before he would share this knowledge with her group members

Table 4.25 above shows the learners' code switching on various topics, during their conversations as they solved the MCM tasks. The observations made from code switching primarily focussed on identifying the visualisation processes that learners went through as they solved the various MCM

tasks. Code switching was used by my learners to make corrections from the identified miscalculations and misinterpretations of the task questions. Furthermore, the hints sourced from the smartphones were used as a starting point to identify the parts that needed to be measured. Through this, the smartphone played a scaffolding role for learners to understand the measurement of physical attributes in the school grounds.

4.8.1.2 Code switching to enhance conceptual understanding

In this subsection, I describe how code switching helped learners to advance the conceptual understanding obtained as they solved the MCM tasks. Sadoski and Paivio (2013) agree that when ideas are verbally shared, they encompass concepts that help bring clarity to participants in the given verbal discussion. I shall now describe how learners code switched to enhance their conceptual understanding of measurement of the perimeter, area and volume tasks. Table 4.26 below shows how code switching enhanced the learners' conceptual understanding.

Table 4.26: Excerpts of learners' code switching to enhance conceptual understanding

Learner	Learners' language	Translation	Conceptual understanding
L70	<i>Ninga owala π, multiplied by...ninga! (looks at her friend if she is following). Ndjino oyili nale nawa, ninga owala π multiplied by 3.18 square. Yah, paife atu findinga o height.</i>	You take π , multiply it with... please do! (looked at her friend if she is following). This is already fine, just take π and multiply it with 3.18 square. Yah, now we find the height.	This learner demonstrated to other learners on the formula to be used and procedure to be followed. The calculator was used as a visualisation tool to learner's prior knowledge and use it in new situation for calculating volume of a cylinder.
L100	<i>Ndjino o lengthaaaa? Maara o length oyo nгаа ndjika Tu mete oku, okuli straight nawa oku.</i>	So, is this the length? I assume this is the length here. Let's measure here, it is more even here.	Learner wanted to confirm from others; something she assumes she knows but still want to ensure she is on the right track. Constant re-looking on the smartphone helped this learner to suggest to others the most similar edge onto which measurement needed to be taken from.
L10	<i>Meta ko side kwii...</i>	Measure that side	L10 instructs another learner to measure the other side of the tree. This measurement was needed to calculate the diameter of a tree which was rather intangible. It was therefore necessary to link abstract mathematical assumptions things (models) they can see or touch.

L50	<i>O side ndjino oweyi multiply nga lwaali? Because they (the sides) are two.</i>	Did you multiply the length of this side by two? Because the sides are two.	Learners extend the question into a suggestion so that he can confirm his prior knowledge with others.
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The excerpt in Table 4.26 show how learners on various occasions used words from the vernacular and as well as from English in order to comprehend their conversations. During their verbal communication, I observed a sharing mathematical concepts and ideas through the concurrent use of words from Oshiwambo and English

It was also observed that learners swapped the two languages in order to clearly understand each question and what was supposed to be done, in order to find the correct solution to the problem task, and learners used their mother tongue to help them conceptually comprehend the question(s). For example, when L100 said the following in Oshiwambo: “*So, is this the length? I assume this is the length here. Let’s measure here, it is more even here*”, I observed that the learner visualised the length of the object by mentally urging his colleagues to affirm to what he was thinking. Further, L100 used hand gestures to demonstrate where the height was, and to me, this related to spatial relations of shapes.

The combination of the use of both the vernacular language and the movement of hands to show the height was instrumental in helping the learners to understand what the task needed them to do. As a result, they were able to find the right solution to the task because of how it was visualised and comprehended in the language they could understand. Data from observations shows that visualisation, code switching, and conceptual understanding are closely related to each other. To conclude this discourse, it can confidently be understood that the verbal language was used as a tool used by learners to visually and conceptually learn the three measurement topics.

4.9 SUMMARY OF DATA FROM OBSERVATIONS

During the running of all of the trails, I observed learners use the smartphone to read, explain and receive suggestions on how they could go about solving the tasks. The smartphone was the central tool used to facilitate the visualisation processes as learners explored their understanding of the concepts that were learned as they carried out the measurement tasks.

I also observed that learners did not arrive at the tasks as empty vessels. They employed the knowledge that they already had and used this knowledge to solve the tasks that were brought to their attention in the new, outdoor situation. In this project, learners acquainted themselves with the objectives that the task entailed and carried out the tasks by measuring, labelling and calculating to find solutions to the tasks. This shows a transfer of conceptual knowledge from virtual imagination to sensory forms of thought. Learners used the smartphone to access pictures and hints, which they perceived as visual information. The perception was demonstrated when they represented the real-life objects with sketches to match the spatial features of the concrete material. These visuals facilitated the foundation of knowledge structures that learners made as they solved the MCM tasks.

I also took note of how learners inherently switched their use of language between English and Oshiwambo. On one occasion, I observed that learners borrowed words from either of the two languages so that they polish their proficiency and make their verbal communication worthwhile. Another reason behind the learners' code switching was that they wanted to clarify their findings regarding the visual perception that they obtained from the task objects. In their verbal discussions, the learners used words that were accompanied by visualisations and interpretations, which in turn helped them to clarify concepts between themselves and others.

The smartphones presented pictures of tasks to learners, which prompted them to create what Sadoski and Paivio (2013) calls a symbiosis of words and images, as they use words that they find easy to use as they explain, interpret and clarify the concepts.

4.10 THE LEARNERS' EXPERIENCES

In this section I present the FGI in which I asked the participants about their views regarding the use of smartphones to learn mathematics. The FGI followed an open-ended approach which allowed me to have follow-up questions regarding their personal experiences, such as difficulties that the learners encountered or how the smartphones influenced their learning during their walking of the trails.

All of the recorded interviews were transcribed prior to analysing them into themes. According to Kawulich (2017), it is important that the researcher transcribes responses from interviews in order to ensure an appropriate and quality analysis of data. I then analysed data from these transcriptions

by classifying them into themes, such as the problems related to problem-solving skills; issues related to the novelty use of smartphones; and technical problems that they had.

4.10.1 Learners' perceptions concerning the use of smartphones for learning.

Nine of the twelve participants were in support that smartphones could be used for learning purposes. To support their argument, the learners insisted that, with reasonable internet coverage, learning can become easy when using smartphones. They said that smartphones were user-friendly and not just a portable, but a fast tool to use for learning almost any subject. Another supporting point for the use of smartphones in learning contexts is that the device is an all-in-one gadget. The following was revealed during the FGI with Group 2 learners:

For example: if you have a book for math in your bag, it's just a math book that you have in your bag. If you have a dictionary, you just have that dictionary...if you have a watch on your body then that's what you have. But all these things you can have them all if you have a smartphone. Now if you can receive questions and you can answer using phones and then if you have a question you can just put in (enter) then you'll have your answer...everything you can just do it when you have a smartphone? [L6FGI, line 54]

This participant thought/believed that a smartphone can be useful for various purposes, given the many applications that it encompasses. On the other hand, three participants were pessimistic about using smartphones to learn mathematics measurement. One of them raised her concern that smartphones might not be a useful tool for learning:

Smartphones are a good thing to have, but they have some disadvantages the katoshe phones are nice because the battery doesn't finish up quickly. Smartphones they can be used for these nice projects but the problem is the battery that can finish up very quickly. If you give a learner a smartphone they will first play music, go to Facebook. I don't think learners will find enough time to fit in their learning. [L10FGI, 128]

A 'Katoshe phone' is the vintage form of cellphone. The term 'katoshe phone' is derived from a torch-phone and is named after the torch light as the most useful application that this type of phone has. The torch-phone can also be used for the many other basic applications such as calling, sending messages and as a radio, but does not have an internet connection. This participant alluded to the phone battery getting flat easily, and one might find it hard to charge the phone, especially if one lives in villages where there is no electricity. Another participant brought forth his view, ruling out the issue of the battery getting flat, and pointing out that there are actually many places where smartphones can be charged, by saying the following;

It is good to use in common because electricity is coming in our community, examples schools, cuca shops, clinics, even most of my neighbours have electricity in this way smartphone can become a very useful tool. [L5/FGI, line 48]

The views from learners regarding the use of smartphones were both positive and negative. These lived experiences are worth considering because using data can be used to craft better opportunities for integrating the effective use of smartphones, and in order to leverage the potential for smartphones to enhance visual and conceptual learning of measurement in outdoor environments.

4.10.2 Ownership and frequency of smartphone use by learners at home.

Only five of the twelve participants admitted to having their own smartphones. The other six participants indicated that they did not have smartphones of their own, but rather had access to smartphone services through their relatives at home. Dejectedly, one of the participants indicated that she did not own a smartphone nor have access to any sort of smartphone use from her relatives.

In summary, this study found that there were sixteen different smartphone activities or applications that learners used when at home. These applications ranged from social entertainment to educational media applications.

I wanted to find out which applications that learners use on their smartphones; and whether there were any education/learning related programs on the applications that learners used on the smartphones at home. With general use of smartphones by learners at home, learners identified a variety of activities for which they used the smartphone. I classified these sixteen uses into five main categories, according to the similarities between these applications. This classification is

presented in the pie chart in Figure 4.23 below, where I have represented the above information.

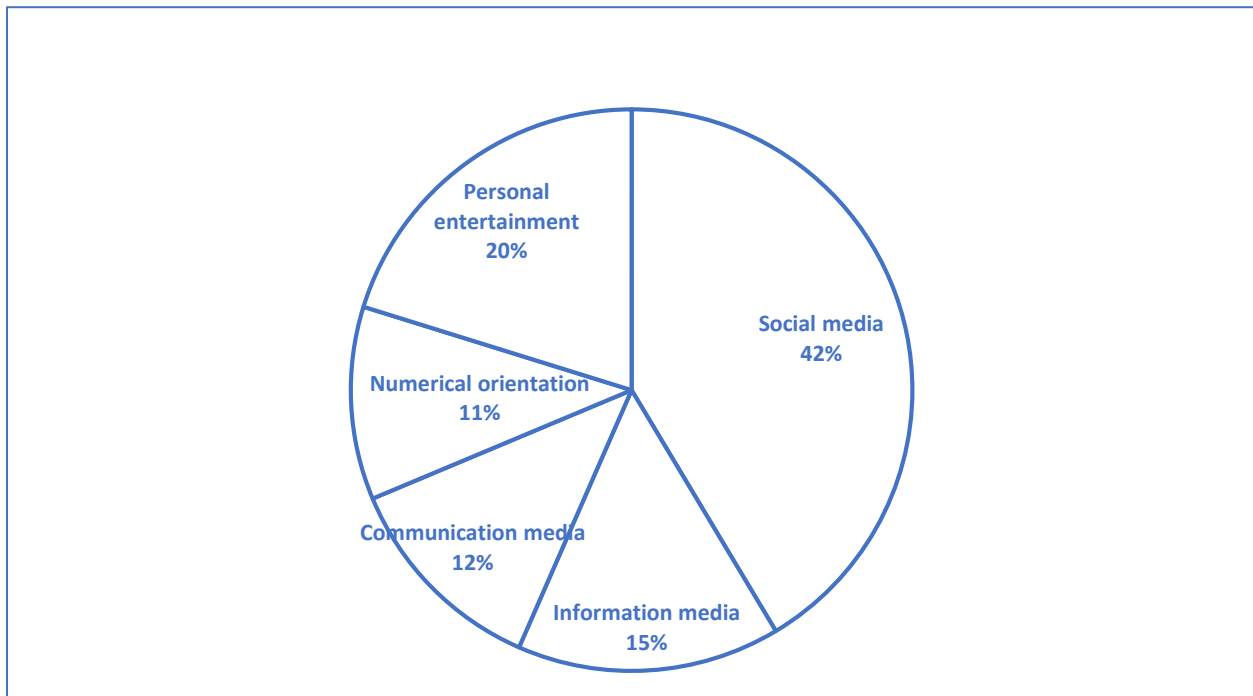


Figure 4.23: Pie chart showing percentages for the five categories of smartphone use

From the sixteen mentioned smartphone apps, the activities ranged from socialising to academic activities, and I have only selected the academically related activities mentioned by learners, which are Google, YouTube, short messaging service (SMS) and offline as well as online dictionaries. I now report on each category.

4.10.2.1 Social Media

The social media category for smartphone use includes the interactive use of sharing mostly entertainment, ideas and interests. The applications under this category are: WhatsApp, YouTube, Facebook, Instagram and Ticktock. This was the most selected category of use and it means that most of the learners use the smartphone at home to access their preferred social platforms. The extract from the interview shows how L2 explained what she uses YouTube for:

On that YouTube I also ask questions and the videos are there with answers. [L12FGI, line 131]

Another learner, L8 mentioned what she uses the smartphone for, this is what she had to say:

My friend showed me how to create and we're going to create my account today after school. I also use it for WhatsApp and Facebook. [L8FGI, Line 97]

The responses from two of the learners indicate that learners indeed make use of the smartphone at home and that they mostly use it for browsing on the internet.

4.10.2.2 Personal entertainment

Personal entertainment consists of applications that allow for one to use the smartphone for entertainment. This class of activities ranged from playing both online and offline music and games, to using the camera to take pictures

My uncle bought us a cell phone and he said we are not allowed to play games. But we use to play at night when alone in our room. [L8FGI, Line132]

The following are names of games that the learner mentioned he uses the smartphone for playing:

Candy crush, Bubble Shooter, Morgz, Solitaire, uhmm... Quest Temple and so many more. [L8FGI, Line134]

I asked L8 if there were any more activities, apart from games, that she and her brother do on the smartphone. This was her response:

We go on WhatsApp, send messages or our father can call us on the phone. We also watch a series like Troll Face Quest. [L8FGI, Line136]

I found out that the series that L8O watches is also game based; an indication that she invests most of the time either playing or watching games. My purpose for asking about the types of games was to find out whether I could find any games related to learning or whether they were exclusively for entertainment purposes. The Solitaire game mentioned by L8 entailed playing cards, which could afford her an opportunity to attune her skills to solving problems in statistics and probability. I could not however establish if there were any games related to measurement in mathematics.

4.10.2.3 Seeking information

The information-seeking category entails the use of smartphones to receive information or clarity. Under the information category, Google search was found to be the most-used app on smartphones. Learners mentioned that they use smartphones at home to access the Google search engine to find out multidisciplinary information concerning various school subjects. The following are extracts from the FGI sessions:

My sister has a cell phone and I use it to Google my information. [L4FGI, Line 64].

I asked about the kind of information and this is what learner had to say in response:

We can Google search for information that one might want to use...you carry it, is like you have a whole bunch of things with you. Anything you want you just go to Google, it's there on the phone. [L6FGI, Line 52]

This excerpt provides data that shows that learners use Google not only to essentially search for information but also for convenience's sake.

Another vital use for smartphones in the information seeking category is the use of the dictionary. The extract below shows how a learner uses the dictionary:

Today we wanted to use a dictionary. Just imagine if there was no cell phone then we had to run to the class where there is a dictionary to search for the word. [L7FGI, Line 98]

I also asked about the word that they looked up in the dictionary:

Mesh. [All, Line 100]

We were struggling with the meaning of the word mesh and we found the meaning from the smartphone. [L7FGI, Line 101]

I then asked whether the dictionary was offline or online. This is what L8O had to say:

The dictionary is already within the phone. [L8FGI, line 103]

From the discussion with the learners, I found that learners make use of smartphones to access applications that extend their learning. To synthesise this response, it is evident that learners make use of the smartphone that may facilitate independent learning. They may access these applications by browsing on the internet or make use of the offline applications.

4.10.2.4 Smartphones for communication

The communication media involves using a smartphone to send and receive SMS's.

I and my brother have a smartphone and we use it to SMS our cousin if I have a question in my homework. [L6FGI, line 104]

When I asked L6 about the content of the SMS's, here is what s/she had to say:

Sometimes to say hi to them or to ask them about a homework. Like when we have a homework on subjects like history and Basic information science and the smartphone help us to find information that is up-to-date. [L6FGI, line 106]

When learners confirmed that they use smartphones at home it is likely that they knew more than what they learned from the introductory workshop they received prior to the running of MCM trails. This means that learners already knew how to work with smartphones, and this enabled them easily solve the MCM tasks in their groups.

4.10.2.5 Numerical orientation

This category is about using a smartphone for getting numerical information or orientation, such as watching the time, setting an alarm and using the built-in calculator. This was the category least mentioned by learners, despite the fact that almost all content in this category are commonly used.

According to the learners' responses, WhatsApp, Facebook and YouTube were the most preferred apps used by learners on smartphones. Interestingly, applications such as the calculator, offline and online dictionaries were found to be less used. This reveals that learners lacked a culture of using smartphones for educational purpose at home, and as a result this affected the way they related to the MCM app in this study.

The activities mentioned by learners were only limited to the home context because cellphones are prohibited at school. However, according to Wijers et al. (2020), effective learning happens when learners create links between what is previously learned with what they can visually recall from their home learning environments. This means that successful learning is most likely to happen if there is a link between the learning gained from outside the classroom setting with classroom learning. Thus, integrating learning with smartphone use, whether at home or at school, can enhance learning in immeasurable ways.

4.10.3 The link between the mathematics learned in the classroom and that was learned in MCM tasks.

This section required learners to describe whether there was a link (or not) between the mathematics they learned when running the trails and the classroom mathematics. In their responses, learners found some similarities between when they solved the MCM tasks outside the classroom, and when in inside the classroom. Similarities articulated here are shapes, methods of

calculations, formulae and units, as well as their conversions. The following are responses from two learners to resonate with this statement:

The shapes are of the objects are same. In this project, we were dealing with shapes like rectangle, trapezium and triangle ... just the same as the ones in the class. [L2FGI, Line 148].

Another thing, the units are just the same and they are converted in the same way. Especially the units we were working with are square metres and square centimetre.....plus also the rulers we used. For example, rulers and measuring tape. Therefore, I think there is a link. [L1FGI, Line 149].

On the other hand, the findings reveal that there are things that were done outside the classroom that differed from what learners usually do when learning inside the classroom. The difference that when learning outside is that in a mathematics classroom, learners would be given tasks where a diagram of shapes is already labelled with measurements. Contrary to the already labelled shapes, the running the MCM trails helped learners to use tools (i.e. measuring tapes) to find the measurements for the solving of the tasks. This helped to evoke learners' visualisation abilities for better conceptual understanding of abstract concepts. Also, measuring and own labelling of shapes has been found to have the advantage of consciously avoiding any possible human error, compared to using the already labelled shapes, even if they are wrong.

4.10.4 Problems encountered when learners were running the MCM trails.

This question required learners to state any experienced problems or challenges when participating in the MCM project. Where possible, the learners were also required to show how they overcame or mitigated against the challenges that they experienced. The data pertaining to the challenges was categorised as follows: challenges related to problem-solving; and challenges associated with the use of smartphones; and technical challenges.

4.10.4.1 Challenges related to problem-solving.

When learners ran the perimeter trail, the majority of them could not solve the age of the tree task. Out of four groups, only one group got the correct solution to the age of the tree task. This problem did not entirely lie in the fact that they were getting it wrong but rather that learners were finding it hard to derive the value of radius of a circle from the circumference that they measured. For example, participants in Group 1 found it hard to work out the age of the tree. This was evident when participants from Group 1 had a little disagreement on where to put the measuring tape

around the tree trunk. Although the group members could not talk about it, I observed it in their actions: L3 was pulling the measuring tape below while L1 was pulling it a bit high.

The two arrows in Figure below illustrate this *disagreement*.



Figure 4.24: Learners who had different views on how to measure the circumference of the tree

The learners had to meet halfway in order to finally come up with a single measure of the circumference of the tree. The fact that the learners were not able to directly measure the diameter brought them a lot of discomfort. As a result, did not find the right method of calculating the diameter after they acquired the measurements for the circumference.

*We didn't know where to put **a** and **b** and we got help from the hints. [L10FGI, line 232]*

The excerpt shows another challenge related to problem-solving that was identified by Group 4: when solving the area of a trapezium, the learners had difficulty in identifying the two perpendicular lines, despite the question stating that the shape was considered to be in a standing position.

4.10.4.2 Challenges associated with the use of smartphones.

One of the participants indicated that she had a problem operating the smartphone and appreciated the assistance she got from one of her group members:

I was struggling with the phone and my friend (L1) helped me out. [L3FGI, Line 38].

Another participant who had a problem of entering the answer on the smartphone said the following:

I was struggling where to put (enter) the answer, when I pressed in the screen then L5 [L3] helped me and showed me where I can press to put (enter) my answers. [L6FGI, line 75].

Most of the problems in these cases was due to the novelty of using a smartphone for learning, and under permitted circumstances. However, as learners progressed from the perimeter to the volume trail, they gradually began to show improvement in how they used smartphones to solve the tasks. In most cases learners shared ideas and skills within their respective groups, and the more they spent time together working on the trails, the more comfortable they became with the use of smartphones. The following is a response from one of the participants at the time of reflecting after running the volume trail:

Even though we got a training, I was not comfortable with the app at the beginning. I was putting myself on the side of the measurement and calculation. But when my chance for being with the phone came I was a bit scared to use it. But now I'm a professor! Hahah. [L2FGI, Line 235]

4.10.4.3 Technical challenges

This aspect required learners to describe the technical challenges related to the measurement of objects within the perimeter, area and volume trails. The participants commented that the two main problems faced were related to taking wrong measurements and calculating incorrectly.

My problem was that we could not measure the parts of the shapes directly. For example, with the cylinders We need to get radius from the circumference. Opo nee [only then] we use that answer to calculate the volume. [FGIL9, line 283].

The challenge provided by this learner emerged from the fact that they could not directly obtain the measurement from the pole. This compelled learners to use a formula in a novel manner. Cavanagh (2008) cautioned that learners who are able to use a formula, do not necessarily have the conceptual understanding for these measurements. To overcome this challenge, learners needed to firstly understand the objectives of the task.

4.10.4.4 Learners' general comments

The participants of Group 1 were happy with the tangible, tactile quality brought to them by the MCM project. This is what one of the members had to say:

It is good to be provided with things that we can touch and not just see them on paper. With this project, we saw the real things we touched the real things. [L3FGI, Line 254].

Further, Group 2 learners appreciated the nature of the MCM app in that it allowed them to provide on-the-spot responses and it also gave them the opportunity to retry the task of the concrete hollow cuboid. Also, as with Group 1, Group 2 echoed the same sentiment in saying that that it was not easy for them to enter answers onto the smartphone. Surprisingly, one member from Group 3 was sceptical of smartphone use among youth, by saying that he thought smartphones are not quintessential for learning:

I'm not sure...because I don't think a person can have a smartphone and just all they do is learn. I think they will just be playing games. [L4FGI, Line 272].

The learners' views discussed above are likely to influence the way in which smartphones might affect their learning. It is not that the learners are reluctant to use smartphones but that learners need to have robust knowledge and confidence in using the smartphone for learning purposes.

4.11 CONCLUSION

In this chapter, I presented data and analysed the findings collected from observations and focus group interviews. Information from each of the two data collection methods is analysed separately. The observation data stemming from the trails is discussed, based on visualisation processes as postulated by Ho (2010). Subsequently, Kilpatrick et al.'s (2001) conceptual understanding that was learned as learners ran each of the trails, was also analysed. Data was presented and analysed, first following, the themes that were developed from literature. Thereafter, the theme that emerged during the running of the tasks was analysed.

Firstly, the tasks in this MCM project were introduced to, and run by learners to help them learn a conceptual understanding of measurement. Visualisation was also part of the analysis where the role played by smartphones to learn measurement and the processes that learners go through when solving mathematics tasks, was investigated. The smartphone was explored as a tool learn mathematics around objects found in the school yard.

Secondly, I discussed and analysed the findings from the FGIs. Analysis of this data was based on the learners' views regarding the use of smartphones to solve the measurement tasks encompassed by the MCM project. Two major findings were identified and analysed. One was the view in support of the use of smartphones for the purpose of learning. The learners who were in support stated that the smartphone had built-in applications such as a calculator, a dictionary and a search

engine which helped them to consolidate the concepts that they constructed. Contrary to this view were learners with pessimistic views regarding smartphone use. These learners stated that the smartphone was not a good tool for learning because its use was accompanied by limitations such as battery life and various technical challenges that learners faced due to the novel use of the smartphone.

CHAPTER 5

CONCLUSIONS AND RECOMMENDATIONS

5.1 INTRODUCTION

The purpose of this study was to investigate how selected Grade 9 learners use smartphones through the MCM application to develop conceptual understanding of measurement. In this chapter, I present the summary of key findings, according to the two instruments which were used to collect the findings: namely observation and focus group interviews, as discussed in Chapter 3. Results from the observed data were meant to discover how learners used smartphones and visualisation processes to solve the MCM tasks. Thereafter, the findings from the focus group interviews were analysed to provide answers to the second research question, which discussed learners' views and experiences regarding the use of smartphones for learning.

This chapter also presents the limitations that were encountered when I conducted this study. I thereafter provide recommendations and personal reflections. I conclude the chapter by presenting some possible avenues for future research prompted by this study and that may be pursued at a later stage.

5.2 SUMMARY OF THE MAIN FINDINGS

The summary of findings to answer the two research questions are presented below, each question in turn.

Research Question 1

How can smartphones be used as visualisation tools in developing conceptual understanding in the learning of mensuration using the MCM application in Grade 9?

I now present the key findings that I observed among learners as they solved the perimeter, area and volume trails.

The learners used hints to connect to connect to previously solved problems in the following situations: firstly, they used hints to confirm the geometrical shapes, which they used to meet the

task objective – which was to find the perimeter around the given object; secondly, learners used hints in the perimeter trail to prevent them from calculating incorrectly. Overall, the learners' use of hints helped to activate their previous knowledge for calculating the perimeter of shapes in new, outdoor situations in which these objects were found.

Understanding of the spatial elements was one of the visualisation processes that learners went through as they solved the tasks in the perimeter trail. As learners engaged with physical outlines of the materials that they measured in the school grounds, they moved around the objects that were large. Alternatively, learners ran their fingers around the smaller objects, in order to discover the outline as a perimeter around the object. When learners came into physical contact with the objects they measured for the tasks, their sensory thoughts were stimulated as they discovered the ultimate definition of perimeter as the distance around a given shape.

The conceptual understanding of measurement for perimeter was observed when learners drew several sketches representing the outlines of the shapes and where they recorded the measurements they had taken, in order to calculate the perimeter of the shapes. The learners were physically present at the site of the task and this enabled them to use more than one method to calculate the perimeter in the same task. The two arithmetic methods that were proposed by Sarama and Clements (2009) were adding the lengths together and that of multiplying lengths with equal sides. When learners represented measurement concepts in multiple ways, it showed how they were able to reflect on the mental mathematical ideas that they had built, with the assistance of the outdoor tasks.

My learners were also observed simplifying the problem at hand, for example, by deconstructing a complex shape into two, simpler and more manageable shapes, that they used in order to solve the task. I argue that this was possible because these learners were exposed to actual physical objects that they could dissect and examine while on site. In an attempt to simplify their work, learners were also observed designing several sketches and diagrams in order to create simplified versions of the task and identifying a method that would work best for the problem. This process of breaking down and simplifying the task was done together with discussions while on site. The discussions that learners took part in were crucial to transform tasks into more understandable

forms. This was evidenced by the diagrams that they generated on paper or on the ground, as well as the formulae that they scribbled in their work notes.

During the running of the area trail, learners used their own words in order to make sense of what they were learning. The outdoor learning situations helped learners use the physical situations that afforded them opportunities to build their own learning context from the interactions they had with the tasks. They repeated this technique with both the tasks that they were solving, creating their own learning contexts, specific to their group. For example, they used words to describe what they saw such as '*not -straight*' to describe the hypotenuse for calculating the area of a right -angled triangle.

When running the volume trail learners were noted showing better and more refined understanding of the concepts that were related to the 3-D properties of solid objects forming part of the trail. Their prior knowledge was activated as they made connections between what they already know with the newly learned mathematics concepts. For example, some of the cylindrical objects had hollow insides while others were solid. Despite this, learners had to activate their prior knowledge in order to find the methods that they could use to measure or calculate the much-needed value for the radius to calculate the volume of the given cylinder.

I now present the common traits that I found among the learners. It is worth to point out that these findings overlap and were common among the majority of the tasks that they solved on the perimeter, area and volume trails.

Firstly, learners went through the processes of encoding visual and physical information that they identified from the tasks. For example, they engaged with the material in order to understand the links between the spatial geometric properties and the formulae used to calculate the volume of the given task. Moreover, learners used the geometrical and spatial properties of objects to confirm the assumptions that they had, regarding their physical appearance. They adapted some of their past experiences into the new situations in which they found themselves. The learners' discovery of the outdoor situations helped them to initiate discussions and draw sketches that during the problem-solving processes.

The learners were able to reflect on the tasks that they had solved previously. There were times when they ended up with incorrect solutions. In such cases, the learners had opportunities to re-examine their calculations in order to identify their mistakes, while still on site. The use of the MCM application on their smartphones aided this process as it allowed them to check for hints and cross check their solutions with the one on the application while on site. I argue that in this study, such opportunities affordances were only possible because of the benefits to learners of the well-structured outdoor tasks that had physical objects directly associated with these tasks. Learners were also observed re-structuring the sketches that they made so that they could assign the dimensions that they measured, to simplify their calculation methods. These newly-formed sketches helped learners to apply polished methods for solving the tasks.

The key conceptual understanding that was learned during the MCM tasks was when learners translated the task problems into their own methods and visual representations that helped them to discover the solutions to the tasks. These new discoveries closed the gap between physical and abstract information, which they translated into visual forms providing new, clear information that they could related to in the authentic objects where the tasks were located.

Another common finding was that learners were able to learn and understand unfamiliar terminology that they encountered within their groups. This was evidenced when they accessed various platforms such as dictionaries on internet sites to find definitions to key words that were unfamiliar to them. The reading of definitions helped the learners to gain rational ideas of the concepts at hand. This placed the learners in a better position for explaining methods and justifying the solutions that they had found. Learners used their explanations in order to provide proofs to make arguments and comparisons, and also to identify the geometric properties of objects – which they then used to craft solutions.

This sub-section is dedicated to the central role played by smartphones in the running of all the trails in this study. Firstly, smartphones were used to access the pictures of the tasks that learners could view during the solving of the task. While solving the tasks, the learners also had an opportunity to request a stepped hint. In most cases of this study, the learners were observed requesting a hint after at least trying the task on their own. These hints played a role in enhancing visualisation by activating the concepts that were previously learned as well as what Ludwig et al.

(2003) refer to as independent learning. This means that learners were able to distinguish what they knew, from what they did not know. The use of hints encompassed by the MCM portal was significantly employed by learners as scaffolding mechanisms to generate reasonable methods and to solve the tasks. The use of hints in all of the trails prompted learners to make connections with prior learning, thereby promoting conceptual understanding through collecting and correcting the mistakes that they made when solving the tasks.

Lastly, the MCM tasks in this project were accessed and solutions were sent through the GPS coordinates, on a map, installed on a smartphone. All of these features intrigued the learners' motivation, as stated by Zender and Ludwig (2016). During the running of the tasks in this project, learners were the main custodians of their own learning. When they got an incorrect solution on their first attempt, learners were more curious to try more methods aligned with the stepped hints that were provided by the MCM.

Research Question 2

What are the selected learners' experiences of learning mensuration of shapes using smartphones as visualisation tools outside the classroom?

First, I had to establish learners' ownership and use of smartphones at home so that I could get an insight regarding the experience that learners had with smartphones. In summary, this study found that learners used smartphones outside school for personal entertainment, numerical information and orientation, social media, seeking information and communication.

The learners' experiences are first summarised here relating to observed learners' successes and advantages in using the smartphone; and second, according to their responses to challenges and/or struggles that relate to their use of smartphones to do perimeter, area and volume tasks. Learners had a common view that using smartphones while in groups made it easy for them to share skills and methods that they used in solving measurement tasks. Also, the learners confirmed that their sharing of skills among the group members improved their ability to operate the smartphones and eliminate the novel use of the smartphones.

The majority of the learners expressed positive experiences regarding the use of smartphones for learning purposes. Most of the learners highlighted that smartphones were portable and that they

could suit learning in almost any location. They also stated that the MCM had applications such as hints, a dictionary, search engines and a scientific calculator that made the processes of solving measurement tasks easier. With this, learners indicated that they were able to find a link between the mathematics that they learned when they solved the MCM tasks outside the classroom with the shapes, calculation methods, formulae and unit conversion, that they did in the classroom. They also enjoyed receiving on-the-spot responses. This advantage allowed learners to do instant reflections and learn from their mistakes.

On the other hand, some learners had their reservations regarding the use of smartphones for learning purposes. Firstly, they indicated that they were not fully convinced that smartphone use was worthwhile for learning. While they thought that smartphones can be used for entertainment, it was encouraging to see that learners acknowledged that they now understood how the smartphone could be used for mathematics learning purposes.

Another challenge that learners mentioned was that of problem solving. In groups, they to solved tasks in different ways and found it hard to enter into negotiations. Some felt that their ideas were undermined and this had a negative impact by playing ‘blame games’ when they could not achieve a correct solution. Another challenge experienced by learners were the two main constraints that they had with regard to their use of smartphones for solving the MCM tasks. Firstly, learners mentioned that the novelty of the smartphone was a major challenge, and they faced problems with their input keys on the smartphone. Some keys were rather unfamiliar, according to the learners, and these technical challenges resulted in incorrect solutions that they were not comfortable with.

Successful learning is most likely to happen if the link between the learning gained from outside the classroom setting is linked with that of classroom learning. Thus, integrating learning with smartphone use, whether at home or at school, can enhance learning in immeasurable ways. It was clear from the patterns of learners’ responses on the MCM tasks that the five visualisation processes cannot exist independently from one another. They are interconnected, and as Ho (2010, p. 3) reiterates, “visualisation plays different functions or roles as students use it to solve problems”. The learners’ responses on the MCM tasks also revealed that conceptual understanding can be achieved when learners are given smartphones to solve tasks in locations where they can physically interact with physical artefacts of the learning material.

5.3 RECOMMENDATIONS

Learners should be introduced to authentic objects within their reach that are of varying shapes and sizes. This could help them to apply various methods so that they are able to discover spatial properties through touching the dimensions of the objects that they are measuring. The use of real objects in this case reiterates what Khan et al. (2015) said: “a renewed study of visualisation in mathematics education must attend to the significant role of the body in space with other bodies” (p. 278).

I now outline three general recommendations regarding this study. Firstly, the running of the trails has presented potential benefits of conceptually learning the topics of perimeter, area and volume. Although this study was conducted in one rural school in the Oshana region, it could be publicised in a broader education society and increase the production of measurement tasks using the MCM project on various school properties. Regarding the language, I suggest that the tasks be published using Oshiwambo. The MCM application already caters for other home languages, so Oshiwambo could be one of these languages.

Secondly, I recommend that instead of only running the math trails after school (as noted under limitations), the MCM trails could be run during the learning of aforementioned themes for measurement topics. The portability and connectivity of smartphones has been found to take learning beyond the classroom boundaries (Sawaya & Putnam, 2015). The integration of MCM projects into the mathematics curriculum, will allow for an embodied approach to learning mathematics.

Lastly, I recommend that the education fraternity embrace the use of smartphones in formal learning contexts, by first allowing schools to relax the rule that prohibits the use of smartphones on school grounds and secondly, by educating learners (and teachers) about the efficient use of smartphones in order to circumvent cyberbullying, prolonged use of smartphones, crime, overuse of internet sites and social media applications. The use of smartphones could then be considered as an alternative tool to integrate ICT into learning mathematics, especially within the Namibian context – as it is still not a common practice in many schools.

The presented recommendations, if well accepted and implemented could help to enjoy the potential benefits of using smartphones in educational contexts.

5.4 THE SIGNIFICANCE OF THE STUDY.

The findings of this research have the potential to inform and encourage active learner interaction with learning content that is set in an outdoor environment. During this project, learners crossed the boundaries of the classroom to learn formal mathematics. This study helped me as a mathematics teacher and researcher to expand knowledge about measurement. This study also has the potential to allow for investigations of how mathematics learned in the classroom can link to real life through outdoor tasks. as advocated by the MCM project.

It is hoped that the implementation of this project evokes learners' awareness and interest in the learning of mathematics, particularly in the use of smartphones to learn mathematics concepts. Although this case study cannot claim generalisability, it is hoped that the Ministry of Education Arts and Culture incorporate visualisation initiatives and programmes that involve the integration of technology such as the MCM project.

5.5 LIMITATIONS OF THE STUDY

This study used a single case study under a very small spectrum; a very small sample size of twelve learners from a single rural school. Results could be more generalisable if there could be more findings from different schools in Namibia. Factors such as connectivity and economic background that influences learners' ownership and familiarity with smartphones may differ, due to a different geographical location for other schools in Namibia. Data was collected at different times and days, after school hours. There is a high possibility that learners could have been exhausted and therefore did not offer consistent participant input. Another factor is that there was a difference in power between me and my learners who happened to be my participants. This rapport might have influenced the learners' responses and shaped my results in a sense that learners might not have shared their outmost sentiments and honest feelings.

The use of smartphones at the school where this study was conducted, is strictly prohibited under normal teaching and learning circumstances. So, it is possible that even after learners were assured that they were permitted to freely use smartphones during the running of the MCM project, learners might still have been limited in fully using the smartphones because of the school rules prohibiting them.

The Covid -19 pandemic was the predominant limit for me to complete this study on time. Schools were closed due to the national lockdown. I was not able to enter the school premises and thus could not collect the data in time.

5.6 PERSONAL REFLECTIONS

This study has helped me to understand that conducting research effectively requires patience, focus and diligence. I also learned that literature engagement is of an essence in order for one to academically grow in the given research field.

I therefore look forward to engaging myself in the integration of smartphone use within the field of mathematics education by sharing the skills and expertise I have gained from this research journey with other teachers during mathematics fairs and conferences.

5.7 AVENUES FOR FURTHER RESEARCH

The results of this study show that smartphones have the potential to be used effectively as visualisation tools for the conceptual learning of measurement. So, in the future I would like to conduct research on how teachers visualise when setting mathematics tasks for learners using smartphones. This has led to a realisation that research on smartphone use in outdoor activities needs to be explored and developed further.

5.8 CONCLUSION

In this chapter, I presented the summary of findings, the significance of the study, limitations of the study and recommendations of avenues for further research, as well as personal reflections. The project has also advocated the suitability and usefulness of this project in various Namibian school environments and situations. This research revealed that learners' use of smartphones in taking part in the MCM smartphone project leveraged the potential for conceptual learning of measurement concepts in mathematics.

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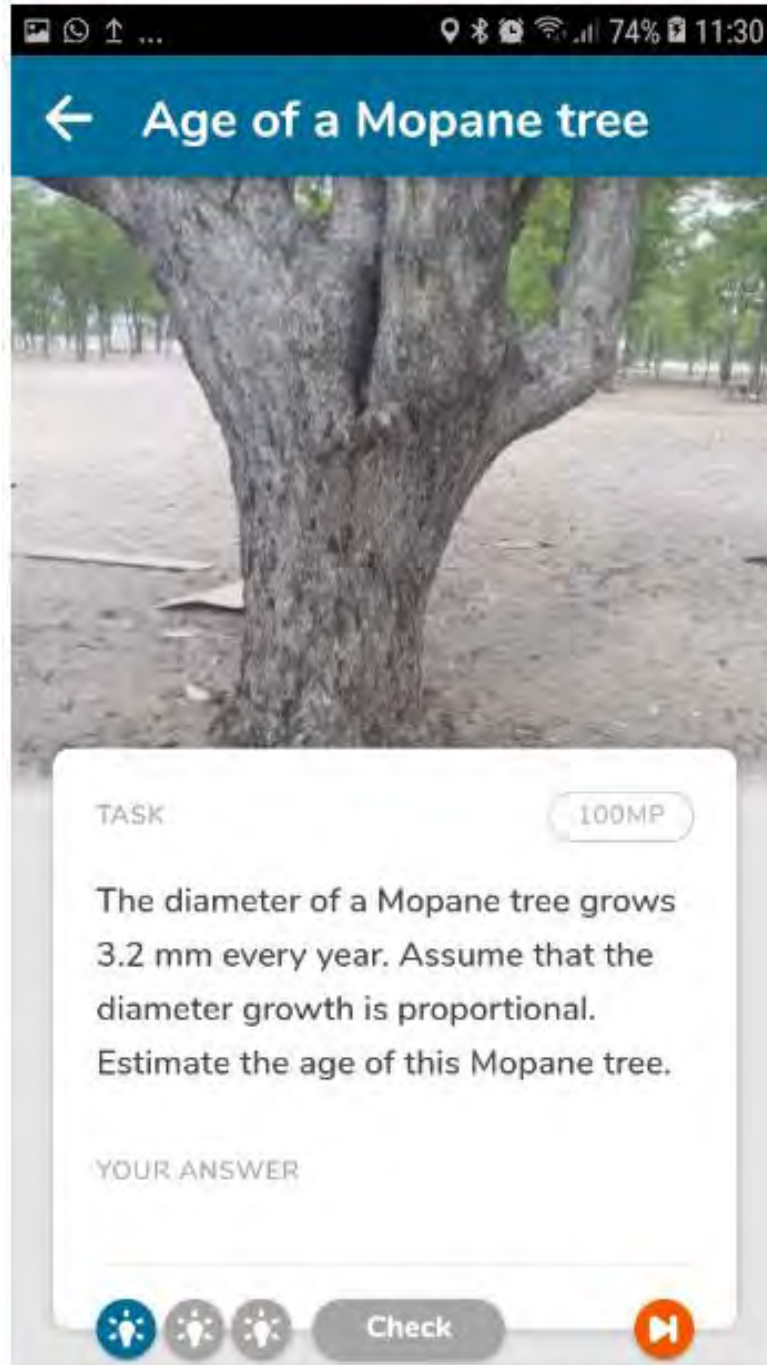
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APPENDICES

APPENDIX A: A SCREENSHOT OF AN MCM TASK



The screenshot shows a mobile application interface. At the top, there is a status bar with icons for camera, clock, location, and battery (74%), and the time 11:30. Below the status bar is a blue header with a white back arrow and the text "Age of a Mopane tree". The main content area features a photograph of a large, thick tree trunk in a field. Below the photo is a white task card with the following text: "TASK" (with a "100MP" badge), "The diameter of a Mopane tree grows 3.2 mm every year. Assume that the diameter growth is proportional. Estimate the age of this Mopane tree.", and "YOUR ANSWER" with a text input field. At the bottom of the task card are three lightbulb icons, a "Check" button, and a red play button icon.

APPENDIX B: ETHICAL CLEARANCE



Human Ethics subcommittee
Rhodes University Ethical Standards Committee
PO Box 74, Grahamstown, 6140, South Africa
t: +27 (0) 46 603 8055
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e: ethics-committee@ru.ac.za

www.ru.ac.za/research/research/ethics
NHREC Registration no. REC-241114-045

4 May 2020

Lina Shimakeleni

Email: g17s8213@campus.ru.ac.za

Review Reference: 2020-0927-3323

Dear Dr. Chikiwa

Title: The use of smartphones and visualization processes for conceptual understanding of mensuration: A case study of the MathCityMap project in Namibia.

Principal Investigator: Dr. Clemence Chikiwa

Collaborators: Mrs. Lina Shimakeleni, Professor Mare Schafer

This letter confirms that the above research proposal has been reviewed and **APPROVED** by the Rhodes University Ethical Standards Committee (RUESC) – Human Ethics (HE) sub-committee.

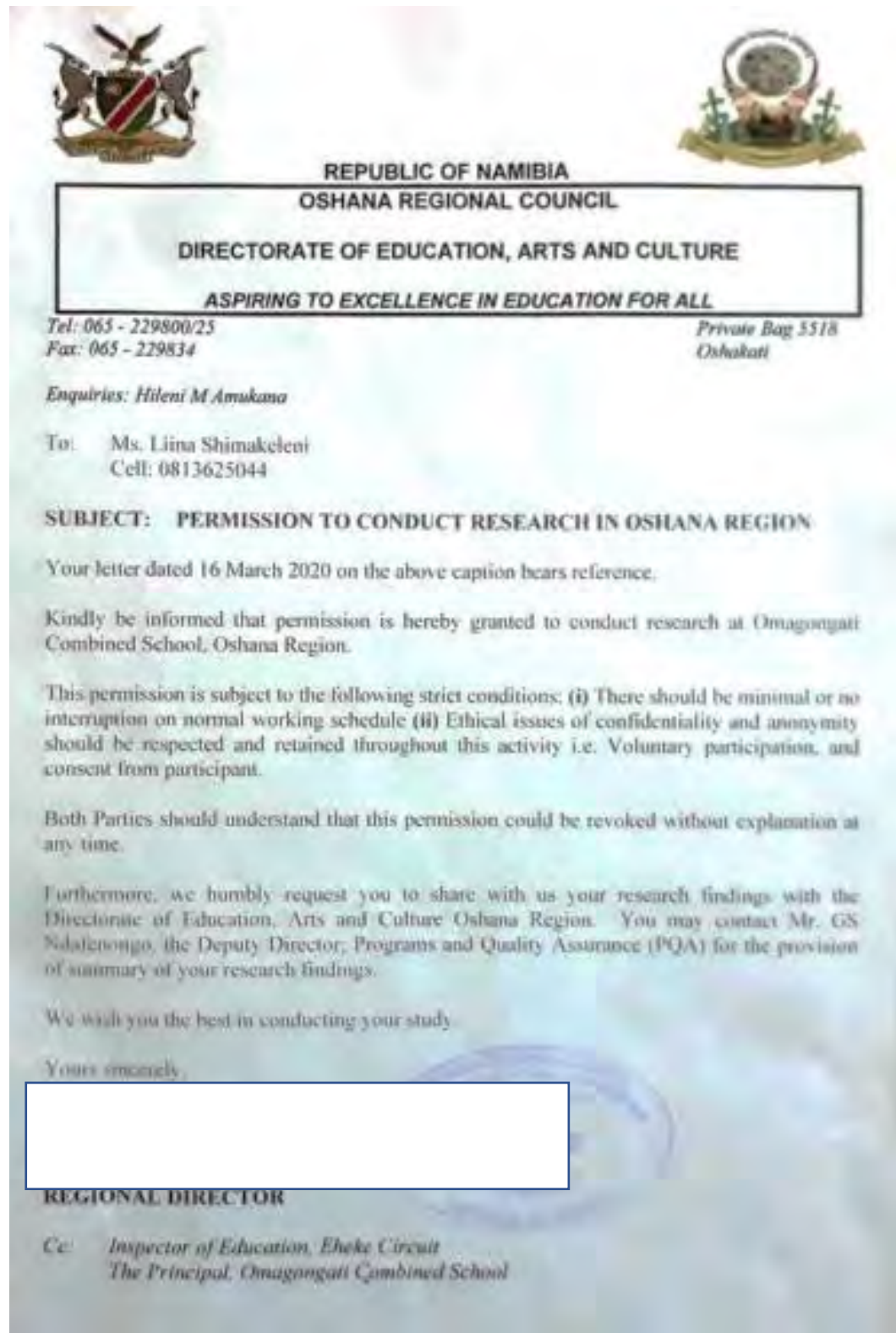
Approval has been granted for 1 year. An annual progress report will be required in order to renew approval for an additional period. You will receive an email notifying when the annual report is due.

Please ensure that the ethical standards committee is notified should any substantive change(s) be made, for whatever reason, during the research process. This includes changes in investigators. Please also ensure that a brief report is submitted to the ethics committee on the completion of the research. The purpose of this report is to indicate whether the research was conducted successfully, if any aspects could not be completed, or if any problems arose that the ethical standards committee should be aware of. If a thesis or dissertation arising from this research is submitted to the library's electronic theses and dissertations (ETD) repository, please notify the committee of the date of submission and/or any reference or cataloging number allocated.


Sincerely,



APPENDIX C: APPROVAL LETTER FROM THE DIRECTOR OF EDUCATION



APPENDIX D: APPROVAL LETTER FROM THE SCHOOL PRINCIPAL


Republic of Namibia

MINISTRY OF EDUCATION
OSHANANA REGION
EENKUI CIRCUIT
OMAGONGATI COMBINED SCHOOL

P O BOX 3253
ONDANGWA

TEL/FAX 065245516

Inq: Mr Haimbala J H 10 JUNE 2020

0813935403

Mrs Liina Shimakaleni
Rhodes University
Student

Dear Mrs Shimakaleni

RE : Permission to conduct research at Omagongati combined school

This letter serves to acknowledge receipt of your letter dated 16 March 2020 on the above caption.

Kindly be informed that permission is hereby granted for your research to be carried out at our school under the theme : **The use of Smartphone's and visualization processes for conceptual understanding of mensuration : A case study of the mathcityMap project in Namibia.**


This permission is subject to a following strict conditions :

1. There should be minimal or no interruption on normal school activities i.e. teaching and learning.
2. Both parties should understand that this permission could be revoked without explanation at any time.
3. Adhere to covid 19 protocols.

Thank you in advance for your understanding.

Your Sincerely

School Principal



APPENDIX E: PARENT CONSENT LETTER (ENGLISH VERSION)



Appendix F

PARENT AND GUARDIAN'S INFORMED CONSENT

INFORMED CONSENT DECLARATION

(Parent or Guardian)

Project Title: *The use of smartphones and visualization processes for conceptual understanding of mensuration: A case study of the MathCityMap project in Namibia.*

Llina Shimakeleni, a master of education student from the Department of education, Rhodes University has requested my permission to allow my child to participate in the above-mentioned research project.

The nature and the purpose of the research project and this informed consent declaration have been explained to me in a language that I understand.

I am aware that:

1. The purpose of the research project is to explore the potential use of smartphones for supporting conceptual learning of mensuration through the mathematics trails developed using the Math City Map (MCM) project.
2. The Rhodes University has given ethical clearance to this research project and I have seen/ may request to see the clearance certificate. [Certificate number: 2020-0727-3323]
3. By participating in this research project, my child will be contributing to the use of smartphones as a visualization tool for supporting conceptual understanding using the MCM app. As a result of this project, learners will also learn better ways of interacting with their peers as they solve the MCM tasks.
4. My child will participate in the project by using the smartphone to access the MCM app and walk four trails consisting of mathematical tasks on the topic of mensuration around the schoolyard.
5. My child's participation is entirely voluntary and if my child is older than seven (7) years, s/he must also agree to participate.
6. Should I or my child at any stage wish to withdraw my child from participating further, we may do so without any negative consequences.
7. My child may be asked to withdraw from the research before it has finished if the researcher or any other appropriate person feels it is in my child's best interests, or if my child does not follow instructions.
8. Neither my child nor I will be compensated for participating in the research.

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9. There may be risks associated with my child's participation in the project. I am aware that:
 - a) the following risks are associated with participation:
 - i) my child's face may appear in the videos
 - ii) my child may be interviewed in opening other applications on the smartphone
 - iii) the following steps have been taken to prevent the risks:
 - i) the video camera will be placed from behind to avoid capturing my child's face.
 - ii) All other applications in the smartphone will temporarily be locked during the time that my child is solving the tasks.
 - iii) there is a 35% chance of the risk materializing.
 10. The researcher intends publishing the research results in the form of a thesis. However, confidentiality and anonymity of records will be maintained and that my or my child's name and identity will not be revealed to anyone who has not been involved in the conduct of the research.
 11. I will receive feedback in the form of a thesis regarding the results obtained during the study.
 12. Any further questions that I might have concerning the research or my participation will be answered by Llina Shimakeleni @llinashimako@ru.ac.za. Cell: 081 3625044. My supervisor's contact details: C.Chibanda@ru.ac.za. Cell: +27 78 9477 970.
 13. By signing this informed consent declaration I am not waiving my legal claims, rights or remedies that I or my child may have.
 14. A copy of this informed consent declaration will be given to me, and the original will be kept on record.


I, _____ have read the above information / confirm that the above information has been explained to me in a language that I understand and I am aware of this document's contents. I have asked all the questions that I wished to ask and these have been answered to my satisfaction. I fully understand what is expected of my child during the research.


I have not been pressurized in any way to let my child participate. By signing below, I voluntarily agree that my child _____ (name of child), who is 15 years old, may participate in the above-mentioned research project.

Shimakeleni Llina Shimakeleni 04/05/2020
 Parent/Guardian's Signature Witness Date

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APPENDIX G: LEARNER ASSENT FORM


CHILD PARTICIPANT'S ASSENT FORM
INFORMED CONSENT DECLARATION


 (Child participant)

Project Title: The use of smartphones and visualization processes for combinatorial understanding of measurement. A case study of the MathCityMap project in Namibia.

Researcher's name: Lara Stronkeleni

Name of participant: [Redacted]

1. Has the researcher explained what she will be doing and wants you to do?
 YES NO


2. Has the researcher explained why she wants you to take part?
 YES NO

3. Do you understand what the research wants to do?
 YES NO

4. Do you know if anything good or bad can happen to you during the research?
 YES NO

5. Do you know that your name and what you say will be kept a secret from other people?
 YES NO

6. Did you ask the researcher any questions about the research?
 YES NO



7. Has the researcher answered all your questions?
 YES NO

8. Do you understand that you can refuse to take part if you do not want to take part and that nothing will happen to you if you refuse?
 YES NO


9. Do you understand that you may pull out of the study at any time if you no longer want to continue?
 YES NO

10. Do you know who to talk to if you are worried or have any other questions to ask?
 YES NO

11. Has anyone forced or put pressure on you to take part in this research?
 YES NO

12. Are you willing to take part in the research?
 YES NO

[Redacted] 04/06/2020
 Signature of Child Date


 Rhodes University, Research Office, Ethics
 Ethics Coordinator: Sinyinda Manguiso
 ethicscommittee@ru.ac.za +27 (0) 48 903 7727 f: +27 (0) 86 616 7707
 Room 220, Main Admin Building, Doreddy Road, Grahamstown, 6109

APPENDIX H: ANALYTICAL TOOL 1 FOR VISUALISATION PROCESSES ADAPTED FROM HO (2010).

Visualisation process	Code	Definition	Observable indicators
Understanding spatial relations of the element	USR	This is concerned with the link between the task and the visualisation crafted by the learners. I will look at how the ideas generated lead to reasonable answers.	USR1: learners make use of the physical features of the task link their ideas with the task definition. USR2: learners explain their ideas to reason solutions.
Connecting to a previously solved problem	CPP	This process is concerned with how learners reflect and build on previous mistakes and misconceptions in an attempt to find solutions to the tasks. Also, how the learners will use the hints on the app to propose other suggestions in an attempt to find a solution to the task.	CPP1: learners make use of the hint on the app CPP2: learners reflect on previous mistakes to make new decisions. CPP2: learners attempt to finding solutions on their own just before they request for another hint.
Constructing a visual representation	CVR	This process looks at the visuals that learners construct when solving the MCM tasks. I will also look at the images that learners explain to others as they construct diagrams on the ground, thoughts made in their minds as well as sketches they scribble on paper.	CVR1: learners construct visuals images (sketches, gestures, verbal descriptions) CVR2: learners observe and comment on the visual images (sketches, gestures, verbal descriptions) made by others. CVR3: learners explain the images to themselves or others.
Using visualisation to solve the problem	UVP	This process is concerned with how learners use the applications on smartphones such as calculators and search engines to help them solve the tasks. I will also look at how learners use visual representations to scaffold mechanism used to solve the tasks.	UVP1: learners use calculators on smartphones. UVP2: learners use search engines such as Google. UVP3: learners make use of a dictionary to make meaning of unfamiliar words.
Encoding the answer to the problem	EAP	Concerned with the link between the solution and the visualisation crafted by the learners. I will look at how the ideas generated lead to reasonable solutions to the problem.	EAP1: learners form and make use of cues to link to the solution. EAP2: learners re-read the question just before submitting a solution to the task.

APPENDIX I: ANALYTICAL TOOL 2 FOR CONCEPTUAL UNDERSTANDING ADAPTED FROM KILPATRICK (2001).

Conceptual understanding	Code	Definition	Observable indicators
Connecting mathematics to pre-knowledge	CMP	This is concerned with the link between the solution and the visualisation crafted by the learners. Learners I will look at how the ideas generated lead to reasonable answers.	<p>CMP1: learners make use of what they already know.</p> <p>CMP2: learners identify and explain an analogue that suits the representation.</p> <p>CMP3: learners use hints to adapt a concept that they have suggested and agreed upon.</p>
Justifying and explaining mathematical ideas and solutions	JES	This process is concerned with how learners reflect and build on previous mistakes and misconceptions in an attempt to find solutions to the tasks. The process also looks at how the learners will use the hints on the app to propose other suggestions in an attempt to find a solution to the task.	<p>JES1: learners extend the question into a suggestion.</p> <p>JES2: learners use various ways to find solution to the task. E.g. formula for calculating Circumference of a circle $C = 2\pi r$ and $C = \pi d$.</p> <p>JES3: learners ask questions. Others respond to support understanding of what is being asked in the task.</p>
Representing mathematical concepts in different ways	RMC	This process looks at the visuals that learners construct when solving the MCM tasks. I will also look at the images that learners explain to others as they construct diagrams on the ground, thoughts made in their minds as well as sketches they scribble on paper.	<p>RMC1: Learners use pictures sketches and models simulations to elaborate questions in the task.</p> <p>RMC2: Learners use pictures sketches and models simulations to suggest a solution to the task.</p> <p>RMC3: learners make use of signs and symbols and terms to represent concepts.</p>

<p>Connecting ideas and concepts in mathematics to the real world</p>	<p>CRW</p>	<p>This process is concerned with how learners use the applications on smartphones such as calculators and search engines to help them solve the tasks. I will also look at how learners use visual representations to scaffold mechanism used to solve the tasks.</p>	<p>CRW1: Learners link abstract mathematical assumptions things (models) they can see or touch.</p> <p>CRW2: Learners use hints provided by the app to revisit the physical model of the task.</p> <p>CRW3: Learner moves around, on or through an object to emphasize the attribute under measure.</p>
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APPENDIX J: SEMI- STRUCTURED QUESTIONS FOR FOCUS GROUP INTERVIEW

1. What are your views regarding learning mathematics using smart phones?

2. How often do you use smart phones to learn? What apps do you go to?

3. Explain, if you saw any, the link between the mathematics you learned from in the classroom and that you did in your MCM trails/ tasks.

4. What challenges did you experience when carrying out the MCM project?

5. What support did you give and receive at the time of carrying out the MCM tasks?

6. Do you have any comment about using smart phones for learning?
