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MAKING PUPILS THINK -
THE DEVELOPMENT OF A MICROCOMPUTER-INSPIRED ADAPTATION OF THE STANDARD 7 MATHEMATICS CURRICULUM

HALF-THESIS

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ABSTRACT

This half-thesis gives an overview of the influence of the microcomputer on the way in which mathematics is done, taught and learnt. The nature of mathematics and the nature of the tutor, tool and tutee modes of microcomputer usage are discussed as background. A case is made for the use of action research methods and a classroom-based curriculum development model to facilitate innovation and the integration of the microcomputer into the mathematics classroom. A curriculum development cycle of situation analysis, planning, trial and evaluation is advocated.

This approach is used to develop a microcomputer-based course aimed at enhancing the reasoning skills of standard 7 pupils. Pupils, working in groups of three, interact with the PROLOG system to build up databases of facts and rules. The microcomputer is used in tutee mode. In "teaching" this tutee, pupils discover the need for formal language and logical reasoning. Active learning is promoted by pupils' interaction with the PROLOG system and by discussions within groups. In this environment the teacher becomes a consultant and constructive critic rather than a lecturer.

Findings suggest that the microcomputer plays an important role in terms of pupil motivation and that the microcomputer-based course enables pupils to experience formal language usage and logical reasoning as relevant activities. Pupil databases provide evidence of the pupils' ability to make appropriate use of rules and to distinguish between *and*-conditions and *or*-conditions. The objective of making pupils think was largely achieved.

It is recommended that the course be incorporated in the standard 7 or standard 8 curriculum to complement or replace parts of the Euclidean geometry sections as a vehicle for developing logical reasoning skills. Suggestions for the further use of the microcomputer as an investigative tool in mathematics classes and for further microcomputer-inspired courses are also made. The provision of appropriate training to enable teachers to make effective and innovative use of the microcomputer in mathematics lessons is advocated.

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CHAPTER ONE - MATHEMATICS AND MICROCOMPUTERS

1.1 INTRODUCTION

The growing use of computers in schools and in society raises the issue of how the mathematics curriculum might need to change. Romberg (1990) sees the shift from an industrial to an information society as being a fundamental reason for a need for change in school mathematics. Higgo (1984), likewise, claims that "in the long run computers may even affect mathematics as a discipline" and that computers will influence what parts of mathematics are needed by society and hence what should be taught in schools. Gardiner (1988) provides an opposing view and concludes: "...at an introductory level (age 0 - 16), every mathematical objective I have ever understood can be achieved more simply without hardware than with it". Crosswhite (1990), however, argues that the influence of the "new technology" is so profound that even mathematical objectives need to change and that this technology "has changed the very nature of the problems important to mathematics and the methods mathematicians use to investigate such problems".

To chart a course between these sometimes conflicting views it is necessary to explore the nature of mathematics and clarify the objectives in teaching the subject. It is also necessary to distinguish between different possible roles for the computer in its interaction with mathematics education. This will be done in parts 2, 3 and 4 of this chapter. In parts 5 and 6 the need for adaptations to the mathematics curriculum will be justified by referring to relevant literature. Constraints to implementing such adaptations will be discussed in part 7.

In Chapters 2 and 3 a case is made for the use of a classroom-based curriculum development model and action research techniques to develop a course module which aims at exposing standard 7 pupils to microcomputer usage and facilitating the acquisition of logical reasoning skills. The motivation for using the PROLOG system in the above course and investigating pupils' interactions with the system and with one another, is provided in Chapter 4. The design and implementation of the course and action research project are described in Chapter 5, while the research findings are analysed and discussed in Chapter 6. Recommendations regarding the use and improvement of the course module developed as part of this research project and as regards further possible microcomputer-inspired adaptations to the secondary school mathematics curriculum are made in Chapter 7. Concluding remarks about the way forward are made in Chapter 8.

1.2 THE NATURE OF MATHEMATICS

Atiyah (1986) describes mathematics as an "art" and more specifically the "art of avoiding brute-force calculation by developing concepts and techniques which enable one to travel more lightly". He sees the purpose of mathematical and scientific activity as being the "understanding" rather than just the description of the physical world and feels that a brute-force computer solution to a problem (eg. the famous four colour mapping theorem) does not reach the "understanding" dimension. To some extent this view supports that of Gardiner (1988) who distinguishes between an "engineering approach" to solving a problem ("get the answer, I don't care how") and a truly mathematical approach which emphasizes "elegance and clarity". He sees methods using algorithms and the computer as falling into the first category and is prepared to accept this "engineering" approach only as an intermediate stage. It is perhaps unfortunate that Gardiner appears to make value judgements as regards the "engineering" and what might be termed "elegant" approaches as both are vital components of mathematics. Atiyah's view is broad enough to acknowledge this by indicating how the computer can be used as a tool in the exploratory stage of mathematical research. Polya (1973) also touches on this issue by pointing out that mathematics "presented in the Euclidean way appears as a systematic, deductive science; but mathematics in the making appears as an experimental, inductive science."

The "untidy" aspect of mathematics is also highlighted by Steen (1986) who describes mathematics as "a living, evolving component of human culture" and sees this as being exemplified by the fact that computing forces researchers to develop new mathematics. Complexity Theory which looks at the complexity of algorithms is an example. During the developmental phase, mathematics is "untidy" and, as indicated above, the computer is a useful tool in stimulating and testing conjectures. The phase of elegant proof which follows, however, also has a place and provides rigour and a means of generalizing solutions. It is significant that the abstract parts of mathematics can be used in establishing the theoretical basis of other disciplines. Atiyah points out that mathematical logic filled this role for computing. Consider, for example, the role of the Turing machine and of Boolean algebra.

Mathematics may also be viewed as a language. Just as in a language, poetry is studied for its aesthetic value, so in mathematics there needs to be room to appreciate the beauty of an ingenious proof - as Kline (1953) put it : "an elegantly executed proof is a poem in all but the form in which it is written". On the other hand, a language also has to meet the needs of its users in a variety of situations and spheres of use. Similarly, mathematics as the "language" of science and now computers (cf. Papert's (1981) assertion that computers are "mathematics-speaking beings") also has a utilitarian function and needs to meet the varied needs of its users. Van Rooy (1988) specifies a number of areas in which mathematics has this role. These range from everyday usage for mensuration and interpreting numerical data to its use in specific careers and in other disciplines.

In planning the mathematics curriculum for schools there is a need to take a broad enough view of the subject to encompass all aspects mentioned above. Only if they are exposed to all aspects of the subject does it seem possible for pupils to do what Papert (1980) terms "creative mathematics". Pupils need to experience not only the beauty of the rigorous aspects of the subject, but also the "untidiness" of exploratory stages and the ability of mathematics to serve their needs as its users. To do less, would be to emaciate the subject.

1.3 THE NATURE OF THE MICROCOMPUTER

Adams (1988) describes the use of the computer in education as a "coat of many colours" and uses Taylor's (1980) model of tutor, tool and tutee as a framework for discussing the role of computers in education. When used in tutor mode, the control of the learning process rests largely with the program author. The program presents material to which the pupil responds. The effectiveness of using the computer in this mode is therefore strongly influenced by the quality of the software. In tool mode, control rests with the user who utilises the computer for routine tasks, so enhancing his ability to manipulate, analyse, synthesise and represent data. The use of generic software such as spreadsheets, word processors and data bases play a big role in this mode. In tutee mode the pupil teaches the computer. Taylor (1980) sees this mode as having the greatest general educational benefit by shifting "the focus of education in the classroom from end product to process". This aspect will be discussed more fully in Chapter 4.

Marsh (1989) distinguishes five categories of computer application in discussing the use of the computer as a tool in mathematics education. These categories are: games and simulations, tutorial programs, problem solving using supplied programs, problem solving in a general purpose computational environment and problem solving by programming. Given the variety of possible roles for the computer it appears appropriate also to distinguish between different computer roles for the investigation of the computer's influence on the mathematics curriculum.

The following division is likely to be useful:

1.3.1 the computer as a teaching aid in modelling situations or individualising instruction;

1.3.2 the computer as an investigative tool for use in generating and solving mathematical problems and testing solutions;

1.3.3 the computer as a "replacement mathematician" and producer of answers to mathematical questions;

1.3.4 the computer as a user of mathematics.

The above categories are likely to be relevant in considering changes in teaching methods and will also be used in considering how curriculum content is influenced.

1.4 AIMS IN TEACHING MATHEMATICS

The aims of mathematics teaching are an important determinant of both the content and methods of mathematics courses. A philosophical view as regards the nature of mathematics is also likely to be reflected in the aims. The aims therefore provide a frame of reference when considering computer-inspired adaptations to course content.

It is significant that the broad view of mathematics suggested in part 2 is compatible with the aims of the current (1984) Cape Education Department mathematics syllabus. These aims are:

> To develop a love for, an interest in and a positive attitude towards Mathematics by presenting the subject meaningfully (author's underlining)
>
> To enable pupils to gain mathematical knowledge and proficiency
>
> To develop clarity of thought and the ability to make logical decisions
>
> To develop accuracy and mathematical insight
>
> To instill in pupils the habit of estimating answers where applicable and, where possible, of verifying answers
>
> To develop the ability of pupils to use mathematical knowledge and methods in other subjects and in their daily life (author's underlining)

To provide basic training for future study and careers.
(author's underlining)

The computer can play a significant role in achieving the above objectives by promoting the development of logical thought and promoting the "meaningful" presentation of the subject. Including new topics or adapting the methods used to study existing topics in order to promote pupils' computer competence also appears entirely compatible with the underlined parts of the aims - particularly in view of the "information society" within which pupils will have to pursue their future careers and studies. The present aims of the local mathematics syllabus can therefore be viewed as broad enough to encompass adaptations in content and method in keeping with the so-called information age. The new junior secondary syllabus of the Cape Education Department (1991) and draft senior secondary syllabus of the Department of Education and Culture (1991) also advocate aims and approaches which appear to allow for such adaptations. These are discussed later in this section.

Wright (1987) indicates how the microcomputer can assist in achieving the aims set out in the British DES report "Mathematics from 5 to 16". He follows Papert in seeing computers as being able to provide a bridge between mathematics and the real world and so foster the "communication" aim. The computer in the role of investigative tool is also seen as a means of promoting the achievement of the aims of enabling pupils to notice "overall relationships", of "working independently", adopting a "systematic approach" and doing "in-depth study". The "utilitarian" aim is seen as needing to include the use of application software packages as they are likely to be most used in pupils' future workplaces. Other aims such as promoting "fascination", "initiative" and "confidence" are also seen as being able to benefit from the computer's ability to individualize pupils' mathematical experiences.

The microcomputer also plays a fundamental role in the *Curriculum and Evaluation Standards for School Mathematics* of the National Council of Teachers of Mathematics (NCTM) in the USA. Crosswhite (1990) points out that consideration of what it means to be mathematically literate in an information society forms a central basis for the *Standards* and that amongst the basic beliefs underlying the *Standards* are the views that every student "should have access to a computer for individual and group work" and that they should "learn to use the computer as a tool for processing information and performing calculations to investigate and solve problems". The microcomputer is seen as playing a role in the implementation of each of the first four curriculum standards of the NCTM. These are: *Mathematics as Problem Solving; Mathematics as Communication; Mathematics as Reasoning* and *Mathematical Connections* (which refers to recognising the relationships between different topics in mathematics and between mathematics and other areas of the curriculum). The course module described in later chapters will aim at fostering skills encompassed by the second and third of these standards by facilitating pupils' use of language, symbols and deductive reasoning whilst they interact with the microcomputer.

The new syllabus for standards 5 -7 of the Cape Education Department (1991) includes approaches and aims similar to those discussed in the above paragraphs. The new syllabus document is explicit about its view of mathematics in relation to the overall school curriculum. It sees mathematics as "an essential element of communication", as having an "extensive utilitarian value", as providing a "powerful tool for solving problems", as having "universal value in that it provides a broader insight into the patterns and relationships in the natural and man-made environment", as contributing to "the development of logical thinking" and as having an "inherent appeal which can provide pleasure and satisfaction". These statements reflect a view of mathematics which is in line with the broad view advocated in part 1.2 and international trends mentioned above. Two of the new aims are also significant in this regard. They are:

> To enable pupils (individually or in groups) to solve routine and non-routine problems
>
> To develop the ability to understand, interpret, read, speak and write mathematical language.

The implementation of the aims and beliefs of the 1991 syllabus is likely to require "new" teaching strategies and approaches. Wright's (1987) comments about computers being able to promote the "communication" aim also apply here, while Marsh's (1989) discussion of computer applications in problem solving provides clear evidence of the role the computer can play as a tool in promoting the problem solving aim. The syllabus itself for the first time gives specific recognition to the microcomputer by recommending the use of spreadsheets in the statistics section.

The Draft Core Syllabus of the Department of Education and Culture (1991) for standards 8 -10 goes further by specifically including amongst its aims, "...to develop in pupils...the ability to use a range of mathematical methods and technological aids sensibly and appropriately". It is, however, still necessary to consider specific changes to the content and methods of curricula to provide for this "sensible" interaction with the microcomputer.

1.5 CHANGING APPROACHES TO MATHEMATICS

Romberg (1990) argues that although "most developments at the forefront of a discipline cannot generally be expected to have a major effect on the early years of education, the changes in mathematics brought about by computers and calculators are so profound as to require readjustment in the balance and approach to virtually every topic in school mathematics". In support of this argument, he mentions Rheinboldt's assertion that computers and calculators have affected not only "what mathematics is important", but also more fundamentally, "how mathematics is done". He concludes that the most important change needed in school mathematics is a change in teachers' and pupils' views of what "the mathematical enterprise is all about". It is perhaps in these methodological aspects that the microcomputer can facilitate the greatest reform. A survey of recommendations for change contained in the relevant literature can provide some perspective when evaluating Romberg's views and when considering possible adaptations to the local mathematics curriculum.

Murakami and Hata (1986) see computers as in the first instance affecting the methodology of mathematics education with the teacher, for example, being able to harness the computer as an aid in providing drill and practice or in developing certain concepts. Benn (1990) points out that there is a significant shortage of affordable, high quality educational software. To support this contention he quotes Komoski's finding that only 5% of educational software could be considered to be of good quality. Smith and Keep (1988) also allude to good software being "acknowledged to be in the minority". This problem constitutes a major constraint on the effective use of the computer in tutor mode in particular. Fong (1989), although reporting on a vast number of studies which found computer use effective for drill and practice activities, nevertheless also regards the varying quality of software as being a possible contributing factor to the "mixed" results from studies of the effectiveness of the use of computer tutorial programs for teaching concepts.

Murakami and Hata (1986) see pupils as being able to use the computer as a tool in verifying solutions, comparing alternative strategies for solving a problem and developing proficiency in checking the validity of computer produced results. Marsh (1989) highlights the use of the computer as a tool in mathematics education, for use both in teaching and investigative learning (cf. the categories mentioned in part 3 above), while Fong (1989) gives examples of how the computer can be used as an aid to enable teachers to better perform their teaching tasks. The position paper of the National Council of Supervisors of Mathematics (1989) in the USA also pleads for computer use to move "toward meaningful involvement by students in problem solving and concept development".

Higgo (1984) refers to the computer's role as an investigative tool by suggesting that computer assistance will make the investigation of real problems possible. In the same vein, the authors of the "Will Mathematics Count?" report (1986) suggest that the use of microcomputers in teaching is likely to encourage "flexible problem-solving skills" and promote investigative learning. Lynch, Fischer and Green (1989) in their report on teaching in a "computer-intensive algebra curriculum" also conclude that although more difficult to teach in the short term, student-directed computer methods proved worthwhile because of the improvement in pupils' problem solving skills and broadened concept of mathematics.

There appears to be a strong consensus that the computer can be a powerful aid to mathematics education when used in the role of investigative tool and that it can assist in developing pupils proficiency at solving real problems - so promoting a number of the objectives mentioned in part 1.4. Some caution is, however, necessary and one should be careful not to regard the computer as a simple panacea to all the methodological problems of mathematics education. Gardiner's (1988) warning in criticizing the "Will Mathematics Count?" report's views on investigative learning is relevant here. He points out that the "responsibility to think in order to pose an appropriate problem is not somehow magically removed by the decision to use a machine". The ability to make what Crosswhite (1990) terms "informed choices" between the use of mental computation, conventional paper-and-pencil methods, calculators or microcomputers when calculations are needed to solve problems, is a further skill that pupils will have to acquire. This skill is non-trivial and is likely to be new to many teachers as well.

Many teachers are also likely to require training in the use of the computer as a teaching aid or investigative tool. Furthermore the production or selection of software packages to facilitate the use of the computer in these roles also requires the acquisition of new skills by teachers. In this regard, Smith and Keep (1988) point out teachers' difficulties in having to "exercise professional judgements for which their training and experience have left them unprepared". A related constraint, mentioned by Fong (1989), is the need for teachers to be able to judge whether use of the microcomputer to achieve certain objectives is cost-effective.

The potential advantages of using the computer in the roles mentioned above appear so great that ways need to be found to overcome the problems, particularly if via the computer it is made easier for pupils to encounter the "suitable collection of examples" which psychologists such as Skemp (1971) see as crucial in the formation of mathematical concepts. The constructivist approach to learning which is advocated by the NCTM *Standards* and by implication in the new Cape Education Department *Junior Secondary Course Syllabus* (1991) when it calls for "activity based learning" will be discussed in Chapter 4. Within the constructive learning environment, Papert (1980) points out that the computer can be used to "open up a natural path along which the student can drive himself or herself". In Chapter 4 the appropriateness of the tutee mode of computer use to achieve this goal will also be discussed. The creation of situations suitable for learning in this way is again dependent on teachers adopting new approaches and acquiring the expertise to facilitate the constructive learning process.

1.6 CHANGES TO THE CONTENT OF THE MATHEMATICS CURRICULUM

The changes in the mathematics curriculum advocated thusfar fall largely in the realm of the methodology of teaching, learning and doing mathematics. As regards the actual content of the mathematics curriculum, the recommendations for change contained in the literature surveyed can be divided into two major types. These are:

- the omission of certain topics in view of the computer being able to play the role of substitute mathematician; and
- the inclusion of "new" topics which will still enable existing mathematical objectives to be achieved, while also promoting computer communication.

Higgo (1984) raises as an issue the notion that "some routine manipulation need no longer be taught". The suggestion is that calculators made the teaching of topics such as long division and logarithms superfluous and that the computer could likewise make the learning of algebraic manipulation and calculus techniques unnecessary. The "Will Mathematics Count?" report goes further and suggests that computer graphics packages will remove the need for pupils to be able to draw their own graphs.

The above assertions are presented without much by way of supporting evidence and appear to beg the question of whether "understanding" of a topic is desirable and if so whether it can be achieved by merely letting the computer act as a substitute mathematician. Murakami and Hata's (1986) answer to this question appears sensible. They conclude that it is not possible to achieve the objective of a pupil understanding a topic "without first making efforts to solve exercises by hand" and see a need to differentiate within a topic between "essential material" and "inessential material" where the computer can be used as a tool. This approach is compatible with Crosswhite's (1990) comments about the need for pupils to be able to make "informed choices" regarding methods. The situation as regards the teaching of logarithms as part of the Cape Education Department senior certificate syllabus can serve as an example. The calculator has removed the need for teaching tedious and inessential logarithmic table calculations, but the need for knowing about logarithms in order, for example, to solve equations of the form $a^x = b$ for x (x not an integer) which arise when considering situations of exponential growth has been recognized and is still taught. It could also be argued that long division needs to be taught in order for pupils to understand the division algorithm and later the remainder theorem.

There would therefore appear to be a need to exercise some caution when using the computer role of substitute mathematician as a reason for omitting topics from the syllabus. Perhaps a better way to proceed would be to determine what aspects of a topic are crucial to achieving the objectives of teaching it and how the computer used as a tool can promote the achievement of these objectives. This could include removing the need to study inessential, tedious aspects of the topic. Symbolic-manipulation programs can then as Heid (1989) suggests be used as tools to aid mathematics teachers in achieving their goals. Also as regards making room for programming courses the warning of the Mathematics Association of America, as quoted by Laridon (1990), that "although computer programming is an important tool for all college-bound students, courses in computer programming should not be regarded as substitutes for college preparatory mathematics", is significant.

The "Will Mathematics Count?" report suggests a range of topics such as aspects of discrete mathematics, iterative numerical techniques, number theory and matrices as candidates for inclusion in the curriculum and points out how learning to use spreadsheet and database software can promote the integration of mathematics with other subjects. Laridon (1990) mentions the inclusion of topics such as "statistics, probability, linear programming and numerical methods" as resulting from the "technological/information explosion". Probability and statistics also feature in Romberg's (1990) list of topics which are "considered increasingly important" and are included in the new Cape Education Department syllabus (1991) for standard 7. Topics such as : linear programming, numerical methods and again statistics and probability are included as optional modules in the standard 10 part of the draft syllabus for mathematics of the Department of Education and Culture (1991). The Department of Education and Culture syllabus writers therefore appear to be taking some account of the above trends.

The "new" topics most frequently recommended in the literature surveyed are algorithms and programming. Higgo (1984) suggests that including programming in a mathematics course would promote the learning of the concept of a variable and also of processes such as iteration and generalising. He also points out that research by each of Hart (1982) and Tall (1983) suggests that basic algebra can be learnt via programming and quotes Michie's view that "schoolchildren could learn algebra by teaching it to their computers". The computer in this case fills the role of a patient tutee. The Pendley Manor report (1983) suggests that "mathematical programming should be a staple part of mathematics courses in the future." The editors of "Will Mathematics Count?" also encourage the study of algorithms, while locally Laridon (1986) recommends algorithms in BASIC as "appropriate exposure to computing within mathematics".

Some assessment does have to be made as to whether the study of programming and algorithms is likely to promote the achievement of the objectives of mathematics curricula. Gardiner (1988) adopts a negative view, seeing programming and algorithms merely as a "kind of 'pre-mathematics', which it is the duty of mathematics teachers to transcend". Jones and Lamb (1985), in contrast to this, see programming as an "essential component in the development of reasoning skills". They make a plausible case for the study of programming being able to "incorporate traditional mathematics concepts in a logical structure demanding both deductive and inductive reasoning skills" and compare the set of rules which pupils will need to internalize as being "closely akin to geometric theorems".

Gardiner's view of algorithms appears to be too narrow and out of line with the approach to mathematics recommended in part 1.2. Indeed, even the assessment of a course on programming and algorithms reported by Johnson and Anderson (1985) indicates that there is more to such a course than mere "pre-mathematics". They report for example that areas which need special attention in the course are step-by-step processing in reading algorithms, checking by means of test data and modifying algorithms -

techniques which are necessary in mathematics. It is also significant that, as reported by Graf (1985), the informatics courses introduced into the German lower secondary mathematics curriculum relied heavily on problem solving via algorithms and programming. This is also an aspect highlighted locally by Marsh (1989) who suggests that programming should be seen as "a method for problem solving within the existing curriculum".

A broad consensus that the mathematics curriculum needs to be adapted to meet the demands of the information age is evident from the literature surveyed. Strong arguments are made for the use of the computer as tool in problem solving and for the inclusion specifically of courses in algorithms and programming in the mathematics curriculum. These courses should be able to meet the current Cape syllabus objectives of developing "clarity of thought" and "logical decisions" while also promoting pupils' ability to communicate with the computer and so meet the parts of the aims underlined in part 1.4. The use of the microcomputer as a tool in problem solving also seems to conform with the emphasis of the new draft syllabuses on a problem solving methodology and on activity-based learning.

1.7 CONSTRAINTS AND DEVELOPMENT

Two major constraints need to be explored when considering adaptations to the mathematics curriculum to take account of the advent of the microcomputer. These are the accessibility of microcomputers to pupils and the top-down nature of syllabus development in South Africa.

Most of the literature surveyed in previous sections provides examples from developed countries where access to microcomputers is likely to be easier than in South Africa with its limited financial resources. If a future syllabus were to prescribe microcomputer usage this could have implications such as funding the provision of microcomputers in schools. Given the backlogs in areas such as teacher training, textbook provision and mere classroom provision in the country as a whole, it seems unlikely that such a financial outlay by the state will be feasible.

Laridon (1990) describes syllabus development in South Africa as top-down because it is "decreed by committees at the top of the educational hierarchy". He sees inputs to these committees as coming "largely from academics or researchers who are out of contact with what is happening in the classroom or out of contact with mathematics". This approach can have the effect of stifling classroom based attempts at curriculum innovation as teachers and pupils try to cope with what they perceive as an overloaded prescribed syllabus. Norton (1991) in the summary report of an investigation into pupils' mathematics results in the Cape Senior certificate, for example, records a finding that 40% of teachers were unable to finish the Higher Grade syllabus within school hours. He also reports that a significant proportion of teachers attributed pupil failure on Higher Grade to "the fullness of the syllabus". These findings become more striking when read in conjunction with the fact that more than 90% of the teachers surveyed were "suitably qualified" (two years of university mathematics) and that their curricular efficiency was highly rated in terms of official departmental criteria. Having to cope with the pressures of completing such an official syllabus, can result in teachers becoming alienated by the ever increasing volume and simply omitting sections of the syllabus which are not directly relevant to an eventual external exam. In this way the aims of the syllabus writers can be lost. In making proposals for syllabus adaptations it is necessary to bear this pitfall in mind. Even the fostering of a problem solving approach to the learning of mathematics would appear to be at risk if an overfull syllabus is prescribed. The reason being that as implied by Polya's second rule of discovery - "sit tight until you get a bright idea" (quoted by Schmalz, 1989) - problem solving is time consuming.

A possible manner in which to overcome some of the problems mentioned above is to opt for a classroom-based model of curriculum development. Such an approach would be in line with the "bottom-up" strategy for curriculum development which Laridon (1990) sees as essential for the country. The methods employed in such an approach should also meet the fundamental principles of curriculum development which Laridon (1990) highlights, namely "situation analysis as a starting point" and "classroom research and evaluation as a basis from which to continue to develop". This form of curriculum development will be more fully discussed in Chapter 2 and will form the basis of the research project described in subsequent chapters.

Encouraging classroom-based curricular research can overcome the access problem in that such experimentation is likely only where relevant to the needs of a particular school environment. Schools and pupils with access to microcomputers are likely to perceive adaptations in the curriculum which involve the use of these as relevant. Such research can build up a body of knowledge which is also likely to be of use to schools who obtain access to microcomputers at a later stage. Schools without access to microcomputers should in turn be free to experiment with adaptations which meet the needs of their particular environment at a particular time.

Curriculum development at school level might in itself appear to be a daunting task for teachers, particularly if it involves the use of new or unfamiliar technology. Rudduck (1984), however, points out that a view of the curriculum as problematic is "the key to the improvement of the art of teaching through research". By doing this research, however daunting, the teacher also acquires new skills and knowledge which enable him to develop strategies to cope with changing demands.

CHAPTER TWO - CURRICULUM DEVELOPMENT

2.1 BACKGROUND

In Chapter 1 a classroom-based approach to curriculum development is suggested as a means of adapting the mathematics curriculum to take cognisance of the advent of microcomputers and overcome associated problems. Although the content of the curriculum in South Africa is largely prescribed in the manner indicated by Laridon (1990) and mentioned in 1.7, the situation within the Cape Education Department (CED) is less gloomy than the picture painted by Laridon. The CED does allow a certain amount of latitude and schools are encouraged to experiment. The CED's publication, Guidelines for Curriculum Development (1987), sees curriculum development at school level inter alia as dealing with

- "adapting the provincial syllabus to the needs of the school"
- "experimental testing and revision of learning content, techniques and teaching media"
- "action research"
- "the generation of suggestions for the improvement of the various subject curricula and the submission of these suggestions to the department"

These departmental goals for curriculum development on the "micro level" suggest that mechanisms do exist for the "bottom-up" development advocated by Laridon. Two provisos being that the CED reacts in a meaningful way to curricular "suggestions" originating from such school or classroom-based activity and that teachers have the capacity and are able to find time to undertake the necessary research projects.

Teachers are likely to find such experimentation more difficult in senior standards where the stricture, mentioned in 1.7, of covering a large volume of compulsory content for an eventual external exam still exists. Even in these standards, however, methodological aspects can be investigated and recommendations made to the department for possible inclusion in future notes accompanying the syllabus. In primary standards the departmental policy is that the syllabus may be amended "according to special needs and circumstances" of the school. Similar latitude has been allowed in the junior secondary phase.

The Cape Education Department's efforts at encouraging school based innovation have been channeled through their so-called "laboratory school" system in terms of which designated "laboratory" schools have not only been allowed to, but in fact have been expected to experiment with ways of making the curriculum more meaningful and to report findings to the Department. Working at such a school, the researcher has over a number of years had no difficulty in obtaining permission from the relevant authorities to adapt syllabus contents or apply curriculum compacting in order to create time for experimenting with new approaches. The curriculum development project described in later chapters exemplifies this approach.

The use of classroom-based curriculum development for integrating computer use into the curriculum is advocated by a variety of authors. Fong's (1989) model for "integrating computers to teach mathematics in the classroom" contains steps which match the planning, implementation and evaluation stages of the curriculum development model chosen in section 2.2. Woodhouse and Jones also argue that computer considerations should find a place at each stage of the curriculum development cycle so that the whole curriculum does not become computer education, but that computers are used in an appropriate manner to facilitate the achievement of the aims of a particular development project. In this way classroom-based curriculum development can assist in integrating the computer as a normal and natural part of pupils' learning environment.

2.2 METHODS

Eggleston (1980) describes school-based curriculum development as

> "a process in which the detailed strategies for a curriculum appropriate to the needs of the individual children in the specific school, or even in the specific unit of a school, are developed by co-operative discussion, planning, trial and evaluation."

He also points out that this process "reflects the assets and liabilities both of the school and the community it serves". Given this contention, it seems possible to overcome the constraint of access to microcomputers, mentioned in 1.7, by basing the research in schools and communities which include microcomputers amongst their "assets".

The methodological aspects outlined in Eggleston's description appear appropriate. They can be compared to Polya's (1957) steps in problem solving. The "co-operative discussion" can lead to understanding the problem. Devising a plan, carrying it out and evaluating the results are also the next steps in Polya's approach. Romiszowski (1981) goes further and in describing his systems approach to instructional design, specifically relates it to Polya's problem solving steps. The Cape Education Department guidelines (1987) for curriculum development at school level also proceed from "situation analysis" via "planning" and "teaching" to "evaluation".

The curriculum development method described above also appears to fit in well with the action research techniques which will be advocated in Chapter 3. The following curriculum development steps will therefore be used in the study described in subsequent chapters:

- a situation analysis, including pupil needs, school assets, logistical considerations, syllabus aims and relevant learning theories
- planning, including the specification of course objectives, the selection of teaching approaches, content and software
- trial (the implementation of the course)
- evaluation.

It should be noted that evaluation does not constitute a final step in curriculum development, but merely the end of a particular cycle of the process and that it is therefore also the starting point for the next cycle.

Woodhouse and Jones (1988) point out a further implication of such classroom-based curriculum development. They argue that any "increase in school-based planning entails an increase in information dissemination" and report on the attempts to set up mechanisms to accomplish this in Australia. Structures for accomplishing this locally within the Cape Education Department would seem to be the Department's curriculum and educational guidance services as well as Teachers' Centre study groups in various areas of the province. In this way "fruitful cross-fertilisation" (cf. Woodhouse and Jones, 1988) can be promoted.

CHAPTER 3 : ACTION RESEARCH

3.1 THE NATURE OF ACTION RESEARCH

Action research techniques, as described by Kemmis (1988), enable the teacher to research his own teaching methods. As researcher, he monitors a deliberate experiment with his teaching practice and uses his reflection on and his evaluation and interpretation of this "action" as a "basis for future action". The classroom thus becomes the teacher's laboratory.

The interpretations of other participants in the process under investigation are relevant in that they inform the teacher-researcher about the consequences of his practice. Only the teacher-researcher himself is, however, seen as having access to all the perspectives and intentions underlying a particular action.

Kemmis (1988) sees the method of action research as being based on a spiral of cycles of "planning, acting, observing and reflecting". He does not, however, see action research as being characterised by a particular set of research techniques. The use of a research diary to record events and of interviews with pupils to further investigate their interpretation of events in lessons is nevertheless common. So too is the analysis of these records in order to produce interpretations and recommendations for future approaches. These techniques were employed in the research described in Chapter 5.

Olson (1988) describes action research as leading to "educational theory based on practical wisdom derived from the interpretation of practice". Action research can consequently be seen as falling within the interpretive paradigm of educational research, rather than the empirical-analytic paradigm. In terms of the interpretive paradigm, education is seen as what Kemmis (1988) terms a "lived experience" and the research aims at providing practically useful insights to teachers. The empirical-analytical paradigm, on the other hand, views educational events and practices as phenomena which are open to objective investigation, measurement and analysis (eg. statistical).

3.2 THE APPROPRIATENESS OF ACTION RESEARCH

Action research appears particularly appropriate for use in curriculum development projects. The steps in the cycles of action research method described above fit in very well with those of the curriculum development cycle outlined in Chapter 2, with the "observing and reflecting" stages of the action research cycle matching the "evaluation" stage of curriculum development.

Olson (1988) argues that action research is a "way of testing curricular ideas in practice" and sees action research used in this way as a process in which teachers design their own projects, decide on approaches and resources to use, discover constraints as they go and adjust their activities accordingly. This view of action research again closely matches the classroom-based curriculum development model expounded in Chapter 2. The compatibility of action research methods with the curriculum development model served as motivation for the use of action research in the project described in Chapter 5.

Olson also argues that "change through action research" is a suitable way for teachers to work toward matching "existing procedures with new technologies". In terms of this argument innovations involving the use of microcomputers are not only amenable to action research, but should in fact result from such classroom-based research. Olson suggests that if "the computer does offer an environment in which new ways of dealing with subject matter can be explored and new forms of teacher-student relationship established, teachers will have to discover them".

Support for the use of action research methods in investigating the use of computers in mathematics education can also be found in a variety of research papers. Adams (1988) characterises projects in the above field as being free of objective tests and statistical measures, but instead following a "new paradigm" which involves observing pupils at work and interpreting small events. Fong (1989) specifically recommends that teachers carry out action research based on his model for integrating the computer into mathematics curricula. This recommendation again associates curriculum development, action research and computer-inspired innovation with each other. In the same vein, Smith and Keep (1988) advocate action research type approaches to software evaluation. They motivate this by arguing that "interaction between teacher, student and computer may lead to significant changes in the processes rather than the measurable products of education" (author's emphasis) and see information concerning these processes in "real" schools within "real" teaching as what practising teachers really need.

The argument regarding what information teachers need goes some way toward answering criticism of action research projects' lack of empirical measurements. In this regard Underwood (1988) points out that teachers "often feel that objective educational evaluation misses the point of what is happening in their classroom". She sees advantages in teachers being able to explain their intentions and their interpretations of outcomes. Furthermore it is often precisely those processes taking place during a lesson that are of interest when teachers experiment with computer related innovation. The potential influences (discussed in 1.5) of the computer on the methodological aspects of doing and teaching mathematics suggest a need for information about these "real" lesson processes and how teachers and pupils interpret their interactions with one another and the computer. Papert's view (quoted by Adams, 1988) that experiments involving computers and learning are often not amenable to scientific measurement because "something very little happened" or because "too many factors changed at once" is also relevant in answering this criticism.

Likewise criticism of action research as being too subjective must be weighed against the value of knowing about teacher intents and the teacher's interpretation of his practice. Olson (1988) suggests that much of so-called "objective" descriptive research by outsiders makes judgements about the "surface events" of observed practice and fails to discern what the practice "means" or accurately interpret the intentions which underlie the practice. The views of non-participant observers can, however, provide the teacher-researcher with different perspectives on the processes and events which form part of his lessons and in so doing assist him in the reflection and interpretation process. The variety of perspectives also make findings somewhat less subjective. Fellow teachers filled this role in the research project described in later chapters.

Action research method was chosen for this research project because the project's goals encompass the aspects for which the above arguments suggest action research is appropriate. These are: classroom-based curriculum development, innovation by incorporating microcomputer use into the learning situation and an emphasis on the process of learning rather than just on the products produced by pupils.

CHAPTER FOUR : PROJECT MOTIVATION

The goal of the action research project described in Chapter 5 is to develop a course module for standard 7 mathematics pupils which will both facilitate the development of their reasoning skills and expose them to microcomputer usage. As part of the course, pupils work in groups and interact with the microcomputer, used in tutee mode, via the PROLOG system. Previous chapters have considered the value of classroom-based curriculum development in general and the appropriateness of action research methods. In this chapter, the specific approaches and course content of the above project are justified.

4.1 WHY REASONING SKILLS OF STANDARD 7 PUPILS ?

Papert (1980) poses the question as to whether there is such a thing as a "mathematical way of thinking" and whether it can be taught or learnt. The inclusion of an aim to develop mathematical (logical) reasoning in all the collections of aims mentioned in 1.4 provides some motivation for a positive answer. The new *Junior Secondary Course Syllabus* of the Cape Education Department (1991) clearly specifies its view of the skills encompassed by such an aim. It lists these as: "the ability to think logically, to generalise, specialise, organise, abstract, draw analogies and to prove". The NCTM *Standards* document includes the ability to use inductive and deductive reasoning in its *Mathematics as Reasoning* standard, while Romberg (1990) argues that all pupils should be "taught to <u>reason</u>, to design models, to <u>create</u> and to solve problems" (author's underlining).

Satisfying the syllabus aims exemplified above provides some of the motivation for a course to develop reasoning skills. Under the present (1984) Cape Education Department syllabus, pupils' first encounter with formal mathematical language and arguments has been via the standard 7 Euclidean geometry course. In the experience of the researcher this course has not been an adequate vehicle for achieving the aim of promoting pupils' reasoning skills. There seems to be a great deal of truth in the comment Papert (1980) makes in the article *Teaching Children Thinking* when he suggests that you "can take a child to Euclid, but you can't make him think". In this and other articles, Papert argues that pupils understand neither the nature of nor the need for mathematical structures and the formal use of language. The researcher has also found this in his own practice. Significant numbers of standard 7 pupils, for example, do not see the need for proving theorems about what they regard as "obvious" and make logical errors in trying to reproduce even the proofs of short theorems (eg. vertically opposite angles are equal). The same pupils who experience difficulty with formal proofs are, however, able to apply the theorems in calculations. Analysis of this situation suggests the need for the development of a new course which attempts to overcome these difficulties in the acquisition of reasoning skills.

What appears to be important for such a course is not that pupils specify and produce proofs (often by rote learning) of various theorems, but rather a focus on the processes involved. The course should provide opportunities for pupils to use and enhance their reasoning skills in a meaningful way, so that they do become aware of situations in which formal language and reasoning are necessary. In the proposed course use is made of microcomputers and the PROLOG system to attempt to create such an environment.

Blease (1986) advocates the use of process objectives which instead of specifying new content to be learnt, specify the activities in which the pupils should be involved. Based on these objectives the teacher plans and sets up the necessary learning environment, but is also forced to recognise the pupil as an active participant in the learning process. Such an approach appears appropriate for a course which focuses on the meaningful use of reasoning skills and the construction by pupils of their own formal systems.

The other reasons for targeting the course specifically at standard 7 pupils are of a logistical nature. The standard 7 syllabus provides opportunities for curriculum compacting. In order to make time for the "new" course, the researcher cut the time allocated to formal Euclidean geometry and also condensed the statistics course. This was possible, inter alia, because the constraints of an over-full syllabus and an eventual external exam do not apply to the junior secondary course. Furthermore, standard 7 is the last standard in which all pupils take mathematics, so that by choosing to work with them, the researcher was able to ensure that the research project was not restricted to mathematically able pupils.

4.2 WHY MAKE USE OF MICROCOMPUTERS ?

The emphasis on reasoning processes and on the pupils learning in an active manner places the approaches of the proposed course in the realm of the learning theories advocated by the so-called cognitive theorists. Romberg (1990) points out that in terms of cognitive theory, learning mathematics requires the pupil to build up appropriate cognitive structures and to actively integrate new perceptions of the environment with existing structures.

The microcomputer can play a useful role in the provision of an appropriate environment for active learning. Bork (1984) highlights this ability and the computer's role in individualizing learning as reasons for the computer being a "powerful learning device". This view is also supported by Underwood (1988), who reports that in her evaluation of "computer-based information processing investigations in eighteen classrooms", she found evidence of active learning in all cases. Jones and Lamb (1985) suggest that in developing and using computer programs a collection of logical reasoning skills are developed and that rules "closely akin to geometric theorems" are internalised. Papert's views on the computer's role in active learning are particularly relevant. He argues as follows :

> "Most children never see the point of the formal use of language. They certainly never had the experience of making their own formalism adapted to a particular task. Yet anyone who works with a computer does this all the time. We find that terminology and concepts properly designed to articulate this process are avidly seized by the children who really want to make the computer do things. And soon the children have become highly sophisticated and articulate in the art of setting up models and developing formal systems".

The prospect of pupils actively developing formal systems for themselves provides strong motivation for using the microcomputer in the proposed course.

A further reason for computer usage is that in addition to developing reasoning skills, the communication aim of the mathematics curriculum is likely to be promoted as pupils use formalised language to interact with the computer. The parallel acquisition of basic computer skills is also valuable, particularly if Evan's (1986) view that such skills are "the fundamental requirements for the society of the future", is accepted.

The decision to use the microcomputer is not the end of the teacher's responsibility. Benn (1990) as well as Woodhouse and Jones (1988) point out that the success of computer use in the classroom depends on how effectively the teacher manages the situations in which it is used. These management tasks include the selection of software and the mode of computer use and facilitating appropriate use by the pupils. The decision to use PROLOG and the tutee mode therefore also needs justification.

4.3 WHY TUTEE MODE ?

Taylor (1980) sees the use of the computer as tutee as being able to shift "the focus of education in the classroom from end product to process". Such an approach fits in well with the process orientated objectives of a course to promote reasoning skills. Further motivation for use of the tutee mode flows from Chatterton's (1988) finding that many successful computer-based lessons were characterised by the knowledge-base being controlled by the pupils rather than the teacher and from Luehrmann's (1980) argument that pupils' learning is more meaningful when they "teach" the computer than when the roles are reversed.

Use of the tutee mode also fits in with active learning approaches. While "teaching" the computer, the pupil tries out various ideas and based on the feedback he receives builds new insights and develops his own understanding of concepts and of the need for formal language. In this way it might be possible for the pupil to develop his reasoning skills in an active manner by using them to communicate with the computer. Philosophically, use of the tutee mode is also in line with an approach that aims at developing in pupils a sense of what Papert (1987) calls "empowerment" - a feeling that even in the mathematics classroom they have the power to discover or create something for themselves, independent of the teacher.

Harper (1987) and Fong (1988) both cite a number of studies which suggest that the computer can be used successfully in tutor mode to enhance routine computational skills via drill and practice type software. They also, however, point out that evidence does not reveal a similar improvement in the understanding of mathematical concepts or in pupils' attitudes towards mathematics. It therefore appears necessary to use the computer in a different mode when pursuing the process type objectives of the proposed course. Hubbard (1988) observes a trend away from highly specific software packages (suitable mainly for use in tutor mode) to powerful general purpose systems (suitable for tool or tutee use) which enable the teacher and pupil to "assemble, create, and explore domains of knowledge". The PROLOG system provides for such activity and allows pupils to construct their own formal system of facts and rules. In so doing they "teach" the computer. The tutee mode therefore appears suitable for this application.

4.4 WHY LET PUPILS WORK IN GROUPS ?

The major reason for deciding that pupils should work in groups is a practical one. The researcher's school's computer laboratory contains 10 microcomputers and the standard seven class that used them comprised of 27 pupils. Pupils were therefore asked to work in groups of 3.

Working in groups can, however, also be motivated as appropriate on the basis of the literature surveyed. Pollak's summary (quoted in Crosswhite, 1990) of the mathematical abilities needed by employees in modern industries includes the "ability to work with others on problems". Crosswhite's own vision of a class in which the NCTM *Standards* have been implemented also includes "students who work together in groups", while Romberg (1990) sees value in pupils communicating with each other and so developing the ability to "reflect on, evaluate and clarify their own thinking". In the same vein Artzt and Newman (1990) advocate cooperative learning as a means of enhancing pupils' mathematical skills and attitudes.

Cooperative learning requires a small group of pupils to work together as a team, discussing problems and helping one another. In a cooperative learning classroom the teacher becomes a facilitator, rather than the major knowledge source. Pupils become "actively involved" in learning mathematics and an accompanying improvement in attitude results. This model for the mathematics classroom fits in well with the approaches chosen for the research project. Active learning is promoted and there is a focus on the processes by which group members "construct" their knowledge. It also seems possible to adapt the model for use in a computer laboratory where group members interact not only with each other, but also with the microcomputer.

There is some evidence to suggest that the use of small groups is desirable specifically in learning situations which involve the computer. Lieberman (1985) quotes a finding by Cox that the performance and interest in computer-based learning amongst adolescents is improved when they work in groups, while on the basis of his observations and research, Bork (1985) recommends this group work approach in order to promote effective learning. This view is supported by Blease (1986) who sees concept formation being promoted by such group work. Chatterton (1988) provides a useful description of the positive effect of such group work on classroom interactions. He sees the microcomputer as facilitating the sort of cooperative learning described above, where pupils "develop and test their ideas" and the teacher by listening to this process gains a better understanding of the progress of individuals in the class. Like Artzt and Newman he sees such group work situations as being less threatening to pupils. This results in pupils being prepared to discuss problems, so gaining new insights and causing "mistakes" to become "part of the learning process, rather than a place at which to stop".

In the research project, cooperative learning is encouraged and the pupils within each group of three are expected to work together on developing a joint collection of facts and rules in the PROLOG system. The effectiveness of such groups is one of the areas on which the research project focuses.

4.5 WHY USE PROLOG ?

PROLOG gives the computer tutee its "character". The logical structure provided by the PROLOG system is seen by Nichols (1986) as forcing pupils who interact with it to "make clear distinctions between different types of data, to order information consistently, to express themselves precisely and to make explicit logical connections between different events or pieces of information". These activities are compatible with the process objectives of the reasoning skills course. In order to provide a fuller justification for the use of PROLOG to enhance reasoning skills, it is necessary to investigate the nature of PROLOG.

Users can interact with the PROLOG system via three types of *clauses*. These clauses always terminate with a full stop and are made up of two parts, a head and a body. The head of a *rule* clause is a conclusion which is true if one or more conditions making up the body of the clause are satisfied. *Fact* clauses have an empty body as they are used to declare relationships which are unconditionally true. *Question* clauses again are seen as having only a body. A PROLOG database is built up by defining relations using *fact* and *rule* type clauses. The database can then be queried using *question* type clauses.

The form and nature of these elementary clauses model situations which pupils encounter in the mathematics curriculum. *Facts* declare relationships in a manner akin to functions which are a major focus of the mathematics syllabus. The fact that John plays cricket can, for example, be declared in PROLOG as

```
¦ plays(john,cricket).
```

This can also be interpreted as "john" being related to "cricket" by the function "plays". A further example is a fact like

```
¦ age(john,15).
```

As indicated above, *rules* declare conclusions that are true depending on the truth of given conditions. An example is

```
¦ member(X,team) :- age(X,15), plays(X,cricket).
```

This rule declares that the variable X is a member of the team *if* the conditions that his age is 15 *and* that he plays cricket are satisfied. The logic the rule displays is similar to that encountered by pupils in Euclidean geometry, for example : a quadrilateral is a parallelogram if two opposite sides are equal *and* parallel.

Question type clauses such as

```
?- member(john,team).
```

require the PROLOG system to look for facts and rules from which the truth of the question can be deduced. In terms of the examples given, the reply to the above question would be

```
yes
```

This process appears similar to proving geometrical assertions.

Variables can also be used in questions. The response to

```
?- plays(john,Y).
```

would, for example, be

```
Y = cricket
```

as the system is able to deduce the truth of the question when the *object*, cricket, is *instantiated* for the variable Y. This is reminiscent of the familiar notion of substituted solutions satisfying equations.

The logic of PROLOG is such that the system will answer "*no*" if the answer to a question cannot be logically deduced ("proved") from the facts and rules already in the database. If the database contains only the above rule and facts, the response to the question

```
?- plays(john,rugby).
```

would be

```
no
```

as there are no facts in the database from which the answer can be deduced.

The type of logic described above with its "closed world" assumption matches that found in deductive systems like Euclidean geometry. Bratko (1986) uses a mathematical analogy to explain how a program is interpreted in PROLOG. The facts and rules in the database are accepted as axioms by the PROLOG system and the user's question as a "conjectured theorem" which the system tries to derive from the axioms. Clocksin (1987) summarises this by stating that a "computation is seen as a deductive proof generated from a set of axioms". The manner in which answers to questions were deduced in the above examples illustrates this process.

The above discussion suggests that the PROLOG system confronts the user with logical reasoning similar to that found in Euclidean geometry. A significant difference is, however, that the PROLOG system is interactive and that by using PROLOG the user is able to test his reasoning skills and adapt his actions according to the feedback he receives from the system. Where questions, for example, do not receive anticipated answers, the user can revise the facts and rules he has used to "teach" his tutee and in so doing develop his own reasoning skills. PROLOG therefore appears to turn the computer into an appropriate tutee for use in a reasoning skills course - one that forces the pupils who "teach" it to use (and so develop) their reasoning skills in an active and precise manner.

Latham (1984) reports on the use of PROLOG for small group work in English lessons. He observes that pupil groups were actively involved in creating their own data bases and that they were able to test generalizations and conjectures against the data base. Such activity also appears apposite for the course described in Chapter 5. Blease (1986) is also a proponent of using data base systems in the classroom and sees the decision-making skills developed by interacting with them as already being sufficient motivation for their use. This view also provides some motivation for the use of PROLOG in a course aimed specifically at the development of reasoning skills. A full description of the planned and actual interaction between pupils and the PROLOG system during the course is given in Chapters 5 and 6.

CHAPTER FIVE - THE COURSE DEVELOPMENT CYCLE

Whereas previous chapters have aimed at establishing the context for the development of a course to promote reasoning skills using the microcomputer, this chapter describes the actual development cycle. As indicated in chapters 2 and 3, curriculum development cycles and action research cycles can be broken down into similar parts. These divisions are also used for the sub-sections of this chapter.

5.1 SITUATION ANALYSIS AND PLANNING

Much of the situation analysis was done as part of the course motivation in chapter 4. In order to present a full description of the development cycle, the needs which the course seeks to satisfy and the milieu within which it was implemented are also specified here.

The present (1984) standard 7 mathematics syllabus of the Cape Education Department requires that pupils be acquainted with the nature of a deductive system via Euclidean geometry. As indicated in section 4.1, the researcher has found that pupils tend to experience difficulty with formal language and formal arguments. The addition of a computer laboratory equipped with 10 IBM-compatible microcomputers, each with a hard drive, to the school facilities made it possible to consider developing a new course to replace part of the Euclidean geometry course. The researcher decided to use the microcomputer facility and the PROLOG system (Portable Prolog Release 2.1) in developing a course with the following objectives:

5.1.1 enabling pupils to use and develop their reasoning skills

5.1.2 developing in pupils an appreciation of the need for formal language use

5.1.3 enabling pupils to construct a logical system for themselves

5.1.4 developing pupils' computing skills

The rationale for approaching the course in this manner was provided in chapter 4.

The class with which the course was developed, is a mixed ability group of 27 boys. The performance of these pupils in the school's mid-year mathematics exam ranged from 26% to 90% with a mean of 60%. Only 8 members of the class had worked with computers before and in most cases this had only involved playing games. Pupils worked in groups of 3 throughout the course. They were used to sitting in pairs and helping one another during their usual mathematics lessons. They did not, however, have much experience at working in groups to produce a common product. The course plan provided for pupils to decide the composition of their groups for themselves.

The class has six 30 minute periods of mathematics per week. On one day per week, two of these periods combine to form an hour long double period. The course was planned to have a duration of 3 weeks. In planning the course, it was, however, accepted that once pupils had become familiar with the PROLOG system, their progress would be determined by their own ingenuity and reasoning skills. Because of the pupil-driven nature of the course and its focus on processes, the division into periods given in the initial course plan was necessarily only approximate.

The initial plan for the course was the following:

Week	Period	Activity
1	1 & 2	The PROLOG system is loaded prior to lessons. Pupils are shown how to enter *facts*, change between *learn* mode and *query* mode and pose *questions*. They practise this.
	3	Pupils learn to load the PROLOG system from disk and continue interacting with the system via *questions* and *facts*.
	4	Pupils learn to save their *facts* and use the editing facility.
	5 & 6	Pupils start building up a database of *facts* about a topic of their choice, eg their class or sport.
2	1 & 2	Demonstrate *rules* and their formulation to pupils. Pupils experiment with *rules*.
	3 to 6	Pupils use *rules* to extend their databases or replace collections of *facts*. They discover the difference between *and*-conditions and *or*-conditions. The teacher points out the relationship between the database pupils are constructing and a formal deductive system on a group by group basis.
3	1 to 3	Groups complete their database of *facts* and *rules* and prepare presentations. These presentations should include an explanation of what they have done, a demonstration of questions which the system can answer and a description of the role of *rules* in their database.
	4 & 5	Group presentations.
	6	Written test

In terms of the research method described in chapter 3, a variety of teachers were invited to attend periods during the course as non-participant observers. As the school has an established system of inter-class visits by teachers, the presence of an additional teacher in the class was readily accepted by the pupils as being quite normal. These teacher-observers were expected to provide the researcher with written comments on the lesson and aspects of pupil interactions. By inviting a number of fellow mathematics teachers to act as observers, the researcher planned not only to have access to different perspectives when evaluating the course, but also to influence colleagues to consider incorporating microcomputer activities in their own lessons.

5.2 IMPLEMENTATION AND OBSERVATION

A research diary was kept to record events and observations during the implementation phase. This diary is used as a source for this section which aims only at providing a record of these events and observations. Interpretation is left for chapter 6.

The first two periods of the course proceeded according to plan. A note on an overhead projector transparency was used as an aid in explaining how to enter facts, change modes and ask questions. The information given to the pupils in a formal teaching situation in the above periods was limited to discussing the examples and information contained in the following "note":

```
+-----------------------------------------------------------+
¦ONCE PROLOG IS ACTIVE IT IS IN ONE OF TWO MODES            ¦
¦?- [in QUERY mode]                                         ¦
¦¦  [in LEARN mode]                                         ¦
¦                                                           ¦
¦TO TEACH THE SYSTEM NEW FACTS                              ¦
¦?- consult(user).      [switches to LEARN mode]            ¦
¦¦ bats(ryan).                                              ¦
¦¦ bowls(andre).                                            ¦
¦¦ bats(andre).         [teaching facts]                    ¦
¦¦ ?-end.               [switch back to query mode]         ¦
¦                                                           ¦
¦TO QUERY THE EXISTING CLAUSES                              ¦
¦?- bats(X).  [your query]                                  ¦
¦  X = ryan   [system response]                             ¦
¦  X = andre  [system response after you enter a semi-colon]¦
¦  no.        [system response after you enter a semi-colon]¦
¦?- bats(ryan).  [your query]                               ¦
¦   yes.         [system response]                          ¦
¦                                                           ¦
¦TO LEAVE PROLOG                                            ¦
¦?- end.                                                    ¦
+-----------------------------------------------------------+
```

Pupils were keen to start interacting with the computer in the manner demonstrated to them. Despite the fact that the lesson was for many their first opportunity to use a microcomputer, they showed no hesitance in starting to try things. Due to a lack of precision in formulation many groups experienced problems at this stage and the teacher was kept busy answering appeals for help. These problems, however, were not an impediment to pupils' progress as they continued their efforts and learnt from their mistakes.

The course hit its first snag in the third period when there was an unexpected clash of venue and a standard 10 class was already using the laboratory for a weekly woodwork design lesson. The standard 7 class returned to a classroom and were set the task of formulating their facts on paper. The teacher used the period to check the format of pupils' facts and made arrangements to avoid clashes in future weeks.

A further problem emerged when pupils were shown how to use the PROLOG system's edit facility and save their facts on disk. A number of groups were unable to save their facts despite using the correct commands. The researcher attributed this to the inferior quality of the disks which were being used. In order to solve the problem, the researcher loaded the PROLOG system onto the hard drive of each computer. Pupils consequently were taught the appropriate DOS commands to activate the PROLOG system during the next period. By the end of the week, the course was back on track with groups building up their databases of facts and saving them on hard drive.

At the start of the second week, pupils were introduced to rules. An overhead projector was used to show pupils a few examples of correctly formulated rules. A brief explanation was also given of the role rules could play in replacing large collections of facts or as a means of extending the database. Thereafter pupils were allowed to continue experimenting with rules, while the teacher-researcher assisted with problems. Initial problems encountered by groups were caused by failing to formulate the condition part of a rule in exactly the same way as the facts which had to satisfy it and by switching the condition and conclusion parts of rules around. An example of the first error is a rule like

```
¦  cricketer(X) :- bowler(X).
```

in a database where the relevant facts are of the form

```
¦  bowls(jason).
```

In assisting groups to correct this error by formulating the rule as

```
¦  cricketer(X) :- bowls(X).
```

the teacher was able to emphasize the need for precise use of language. A further error of this type which occurred was the omission of a variable from part of a rule. An example is stating

```
¦ englishteacher(neil) :- mclass(X).
```

instead of

```
¦  englishteacher(neil,X) :- mclass(X).
```

The second type of error mentioned, is illustrated by the use of a rule like

```
¦  sevenm(X) :- mathsteacher(max,X).
```

to extend a database which contains no facts about teachers, but does already contain a collection of facts of the form

```
¦  sevenm(brandon).
```

which state the names of pupils in 'seven m'. On discovering that their formulation of the rule did not produce desired results, pupils asked the teacher for assistance. This provided a relevant setting for the teacher-researcher to discuss with pupils the nature of deductive reasoning in which the truth of a conclusion depends on a condition being satisfied.

As the week progressed, groups' competence and confidence in formulating appropriate rules increased. The manner in which they initiated interactions with the teacher-researcher changed from the somewhat tentative - "Can we make one like this?" - to a confident - "Look at our rules". The discovery that their rules worked and produced anticipated results appeared to give pupils a feeling of accomplishment. Extending the database by using rules also appeared to provide a welcome challenge for a few group members (mainly those who had previous experience of computers) who had shown signs of boredom.

The overall motivation of the class appeared to be good, except that two groups in which members did not work well together made slower progress than the others. These groups contained individuals who also struggled to apply themselves in other lessons. On the other hand, the researcher also observed that groups making good progress and formulating the most sophisticated rules did not always contain the most able mathematics pupils. One such group for example were able to formulate a rather complicated relationship like

```
¦  grandpa(X,Y) :- father(X,Z) , father(Z,Y).
```

The teacher's role at this stage was to act as a consultant and sounding board for pupils. In constructing their database of facts and rules, pupils used their imagination and topics ranged from family relationships to class activities and surfing equipment. Rules were tested and reformulated where necessary. In this way pupils were able to develop their reasoning skills through actual use. Although the success of this approach will be evaluated in chapter 6, two pupil comments are relevant here to illustrate the processes taking place:-

- "I learnt to think before typing, to be precise in what I do and finally to try and reason with the computer."
- "It showed us how maths can be used without actual numbers."

Both comments are indicative of pupils learning in an active manner, while the first comment also suggests an appreciation by the pupil of the appropriate manner of interacting with the computer as tutee - by using his reasoning skills.

The written test planned for the end of the third week actually took place at the start of the week due to the school's test timetable. Pupils were set a question on the work they had been doing with PROLOG as part of the test. This question counted 30% of the total marks. In the question pupils were given a collection of facts and rules which made up a database. These included the following examples:

```
¦  mclass(adrian).
¦  mclass(sean).
¦  mathsteacher(max,adrian).
¦  mathsteacher(max,sean).
¦  pupil(X,Y) :- mathsteacher(Y,X).
```

Pupils were then asked what the computer's response would be to a variety of questions. These included:

```
?-  mclass(adrian).
?-  pupil(adrian,T).
?-  pupil(X,max).
?-  pupil(max,X).
```

```
?-  mathsteacher(X,sean).
```

Pupils were also asked to formulate a rule which could replace the mathsteacher facts, to extend the database by formulating a rule about school membership and to specify a fact which could be added to the database to elicit a *yes* response to a given question.

Pupils' achievement in the PROLOG question did not differ markedly from that in the rest of the test. They did, however, have difficulty in properly formulating the rule which could replace the *mathsteacher* facts.

Pupils spent most of the third week in building up and refining their databases and in preparing to explain and demonstrate their work during the last two periods of the week. Most groups were so pleased with what they had done that they requested the teacher-researcher to invite a number of their teachers to attend the presentations. During these presentations each group was able to demonstrate how the computer was able to use the system they had constructed to answer questions. They were also able to explain what they had done in an effective manner and their explanations in most cases also indicated an appreciation of the functions of rules in their systems. The explanations of two groups contained particularly noteworthy aspects. In one, the spokesman who had used a computer for the first time during the course, started his presentation with the machine switched off. He then went through every step needed to activate the PROLOG system, starting with switching the machine on and ending with an explanation of changing between the different modes of the PROLOG system. In so doing, he demonstrated confidence with the computing skills he had acquired and impressed the visiting teachers. The other explanation which the researcher found significant came from a member of a group who had constructed a system to answer questions about group members' family relationships. The pupil was able to explain that the reason the computer included him in a list of his brothers was that his rule

```
  |  brother(X,Y) :- mother(Z,X) , mother(Z,Y).
```

was not strong enough.

The course was concluded after these presentation periods. During the first period of the next week, pupils were asked to write a short description of their perceptions of the course, indicating what they had learnt, how they felt about using computers, how they felt about working in groups and how much thinking they had to do. After reading these responses, the researcher interviewed selected pupils.

5.3 EVALUATION

A detailed analysis of what was learnt from the course is given in chapter 6. This section seeks only to highlight the methods of evaluation and to outline general themes. Four sources were used in the evaluation. These were: the observations of visiting teachers, pupil comments, pupil products and the researcher's own observations.

The teacher who acted as an observer during the first week commented on the "fearless" manner in which pupils took to the computer and how quickly they were able to help themselves (once the logistical problems mentioned earlier had been sorted out). She also commented that the 30 minute periods seemed a bit too short for the lessons. Teachers who observed lessons in the second week again commented on the motivational role of the computer and the fact that interacting with the computer made for an active learning situation. They also commented that as a result of hearing about the course from friends, their classes were asking to follow a computer-assisted course as well. Teachers who attended the presentations during the last week were drawn from different subjects (history and accounting). They commented that their visits were interesting and that the pupils' apparent prowess made them wonder about computer assisted lessons in their own subjects. They also commented on a number of individual pupils. They were impressed by the confident manner in which pupils were able to explain what they had done, but also noticed three individuals who were not making a constructive contribution to their group.

Pupil comments also suggest that while most groups functioned effectively - "It was very nice working in a group because we could all share ideas" - there were problems in others - "...sometimes it was rather a hassle because certain pupils in our group wouldn't discuss what we had to do".

The value of active learning noted by the visiting teachers is also reflected in pupil comments like the following:

- "In the beginning, I didn't know what was going on, until we started working on our own. The rules weren't that hard to formulate, all you had to do was think logically."
- "At first formulating rules was difficult, but after a while of working, our group made rules with ease."

The above comments also suggest that some progress was made in achieving the objective of promoting pupils' reasoning skills, while an appreciation for the need to use language in a formal, precise manner is evident from a comment like:

- "You had to think a lot because if you had one tiny error the whole thing would not work."

The development of pupils' skill in using the computer is evident from some of the observations mentioned in 5.2 and also from comments like:

- "I learnt not to be scared to press a button in case it was the wrong one."
- "I learnt a lot about computers while we were working with them as I don't have one at home."

The positive attitude to computer usage which was noted by the observers also emerges from pupils' comments. Consider, for example, the following:

- "... it was a lot of fun and I would love to work on the computers again."
- "... it was a nice change as I usually only play games on the computer."
- "I found it rewarding when you type in questions and it gives you the answers you taught it."

The pupil perceptions illustrated above and the researcher's observations in 5.2 suggest that the course did promote the objectives stated in 5.1. This is also confirmed by the groups' products. Each database contained rules which indicated appropriate use of *and*-conditions and *or*-conditions, while the rules also provided suitable extensions of the facts. Only in two of the nine groups was the progress limited.

The researcher's findings and own evaluation of the course will be detailed in chapter 6. The following aspects were seen as being worthy of comment and analysis: group work and class organisation, interactions between pupils and the computer, the effectiveness of active learning situations, the changed role of the teacher, pupil attitudes and products and the overall success of the course in making pupils think and achieving its objectives.

CHAPTER SIX - ANALYSIS OF OBSERVATIONS AND FINDINGS

6.1 GROUP WORK AND CLASS ORGANIZATION

Pupils interactions with members of their groups, with the teacher and with the microcomputer largely proceeded in a manner which matches the findings of Chatterton (1988) described in section 4.4. In most groups, pupils were happy to experiment and progress through a process of trial, discussion and improvement, with the teacher acting as consultant from time to time. Pupil comments as well as observations by the researcher and by the teachers acting as observers also suggest that many of the potential benefits of small group work and peer teaching listed by Chen (1985) and Kyriacou and Newson (1991) did in fact accrue during the course. These include: reducing anxiety and increasing motivation and pupil involvement; enabling pupils to share, discuss and systematise ideas; allowing pupils to be responsible for what they and other members of their group learn; promoting the development of the inter-personal skills needed for working in a cooperative manner; encouraging investigative learning; promoting self-evaluation by the group; changing the role of the teacher from lecturer to that of facilitator and consultant and providing for greater individualisation in instruction.

However, all groups did not function equally well and a comparison of effective and non-effective groups can highlight specific advantages and problems of group work in the course. Effective groups were characterised by what might be termed an "us"-approach. Members of these groups shared ideas, helped one another and worked together in a constructive manner. In the less effective groups there was what might be termed a "me"-approach.

The group dynamics in these groups did not promote collaborative efforts, but rather a "my turn"/ "your turn" type of approach in which group members took turns to interact with the microcomputer and did not really work together or assist one another in achieving a common group goal.

Pupil comments about different aspects of their work clearly reveal these distinct types of groups. Consider firstly attitudes toward different ideas. Members of "us"-groups saw these in a positive light. Their comments include the following:

- "It was very nice working in a group because we could all share ideas and work with our friends"; and
- "Working in a group was better as there were more ideas".

On the other hand, members of "me"-groups seemed to attach negative connotations to differences of opinion. Their comments for example include:

- "I think I would prefer to work on my own because there were a few things we argued about ..."; and
- "Groups were very annoying because people have different viewpoints".

Attitudes to helping other group members or being helped by them also differed markedly. The "us"-group approach is evident from the following two comments in which the concern of group members for one another is evident:

- "I found that it was better to work in groups because ... some pupils didn't know a thing about computers and they needed a little help"; and
- "Working with a group was better than working on my own since I often made mistakes which were pointed out by them".

Contrasting with the above comments are the following views where the individuals in the group lacked patience with one another:

- "If you kept on making a mistake they would shout at you ..."; and
- "...sometimes when another member made a mistake you wanted to do it for him and it became frustrating".

The two approaches are perhaps best summed up by the follow pupil opinions:

- "Working in a group was good because you could all give in ideas and if someone didn't understand something, it could be explained to them by another member in the group. Working in a group is very difficult, but this is a good way to practice it."
- "Because I was working with a group that did not know what was going on, I found it hard as I had to do a lot of thinking to work things out and make everything work ..."

The first pupil shows a great deal of insight into what the purpose of working in groups was and it is encouraging that he describes a group in which these goals were achieved. The final product produced by his group also proved to be one of the best. The second pupil describes a group in which there apparently was very little collaboration and the work was left to one member who tried to do things himself. One positive aspect of his comment is, however, that he was made to think, thus achieving one of the course aims. His group was one of the two which the observer teacher noted as not being very industrious.

It is encouraging that only two of the nine groups were identified by the researcher and observers as being of the non-effective type. Even in these groups progress was, however, evident and only in one of them was the final product produced markedly inferior to that of other groups. Furthermore the ability to work together was a skill which most groups had to work at improving and developing during the course. Pupils' appreciation of this aspect is apparent from responses such as the following to a question about what they learnt during the course:

- "We learnt to work together which I think is very important";
- "I learnt to work together and saw other people's point of view and ideas";
- "Working in a group is not an easy task, because everyone normally wants to do their own thing, but in our group everything worked out okay and everyone tried working on the computer"; and
- "I also learnt how to combine my ideas with those of others in our group".

The fact that two types of groups could be identified raises questions about the composition of groups. The researcher allowed pupils to form their own groups for the study, but did ensure that those with some experience of microcomputers were spread across the groups. In most cases this "experience" did not extend much beyond being able to use a keyboard and did not appear to be a particularly significant determinant of the progress of groups. The compatibility of group members did, however, seem to play a major role in this regard. Observations by the researcher and subsequent interviews with pupils support this contention and suggest that problems in less productive groups were largely caused by disruptive individuals. The difficulty these particular individuals had in making a positive contribution to group activity was not limited to their mathematics class groups, but was also apparent in their sports teams and in their relationships with the class as a whole.

A further problem as regards the composition of groups was the frustration expressed by some pupils with the pace at which their groups worked and with slower members of the group. This frustration was not experienced by all the brighter pupils as many enjoyed being able to take the lead in their group or were able to assist their group to progress at an acceptable rate. In two cases brighter pupils continued in this role after the completion of the computer-based course and maintained a concern for the progress of their "pupils". The fact that the material and approach of the course was new to the class also seemed to enable different pupils to emerge as "experts" and one of the groups which made the best progress did not contain any pupils who had previously been identified as top mathematics pupils.

The problems experienced in some groups do perhaps indicate that the teacher should play a somewhat more active role in allocating pupils to groups in order to provide for the needs of some academically able pupils and to attempt to minimize the adverse effect of disruptive individuals. The fact that all but two groups did settle down to function constructively, however, suggests that such teacher interference in group formation need only be of limited extent. The pupil comments about learning to work in groups imply that there is a need for group work skills to be practiced and that by using such an approach more regularly in the mathematics class, pupils' ability to work together can be enhanced.

6.2 INTERACTION WITH THE MICROCOMPUTER

Findings in this regard largely agree with those reported by Underwood (1988) in terms of growth in pupils' computer skills and the dynamic nature of the interactions between pupils and the microcomputer. Findings reported by Chen (1985) and Lieberman (1985) concerning the motivational role of the microcomputer and its ability to promote peer interaction and cooperation in groups are also supported by the researcher's observations, the feedback from the non-participant observers and the information gleaned from pupil interviews. The role of the microcomputer and the PROLOG system in facilitating interaction which promoted the process objectives of the course was also particularly striking.

The novelty effect of working in a microcomputer environment served as an initial motivating factor as illustrated by the following pupil comments:

- "I enjoyed working on computers since it is a new approach to learning ..." and
- "The best thing was working on the computer and missing working in our books".

This novelty effect wore off for a few pupils as is apparent from a comment like:

- "When we were told we were going to work on computers I was really excited. I found working on a computer interesting, but near the end I got bored."

On the other hand the expectations of the majority of pupils were met, with many commenting that working with the microcomputer was "a lot of fun". The expectations of some pupils were even exceeded as is evident from the following opinion:

- "I thought working on computers would be quite interesting and it was very interesting."

The microcomputer played a central role in group activities with pupils adapting their responses according to the feedback they received from the microcomputer. The interaction was so intense in some groups that pupils referred to the microcomputer in terms which came close to being anthropomorphic. Consider, for example, the following pupil comments:

- "I have learned ... to try and cooperate and reason with the computer";
- "I enjoyed working with the computer very much and it also helped us to think ..." and, as mentioned in section 5.3,
- "I found it very challenging and rewarding when you type in the questions and it gives you the answers you taught it".

The above comment and teacher observations suggest that the microcomputer effectively filled the role of group tutee. This is also illustrated by the following pupil opinions:

- "I ... found that it was simple to teach the computer"; or
- "I learnt that it is very difficult to teach an ignorant machine to be capable to be intelligent".

Although a tutee in the group, the microcomputer and PROLOG system imposed a discipline on group members. They had to learn how to teach the system and in the process acquired an appreciation for the reasoning skills and the precise use of language which were necessary to interact with the microcomputer via the PROLOG system. This aspect of the two-way interaction between group members and the microcomputer is illustrated by some of the above comments and also by pupil responses like:

- "I learnt how to ask the computer questions properly and I've also learnt a lot about substitution";
- "I learnt how to put facts in and then to ask questions to get an answer".

The dynamic nature of the interactions with the computer seemed to promote active learning by the pupils and the process objectives of the course. The objective of developing pupils' computer skills was also promoted. This happened in an informal manner and occurred almost as a by-product of the course. Evidence of the growth in pupils' computer skills is: the example mentioned in section 5.2 of the confidence with which the operation of the computer and the PROLOG system was explained to visiting teachers; the increasingly competent manner in which pupils were able to arrive for a lesson, activate the PROLOG system, work with the system for the period, save their work in a file and exit the system at the end of the period; the positive comments of observer teachers and pupils' own comments. Pupil responses to a question about what they had learnt ranged from:

- "I now know where the keys are and what they are for"; to
- "It also helped me to understand how to use it as I usually only play games on the computer";
- "I have learned ... to be precise in what I do..."; and the more sophisticated
- "I learnt a whole new computer language which was very different from the normal programs you find on computer".

Although not part of the original course plan, the two periods spent away from the computer near the start of the course also seemed to have a positive effect. These periods provided pupils with an opportunity to think, plan and formulate the first collection of facts for their databases. Having done this preparation, pupils had a better idea of what they wanted to "teach" the PROLOG system and on returning to the computer laboratory were ready to start entering facts about their chosen topic. This allowed interaction with the microcomputer to proceed much more smoothly when it resumed.

One most encouraging pupil response to the computer-based reasoning skills course was the following:

- "The computers showed us a way in which maths is used in real life and this made it a lot more interesting."

That interacting with the PROLOG system was able to make this pupil see the relevance of an aspect of mathematics which requires logical reasoning and formal language use, provides support for Papert's view, quoted in section 4.2, that working with computers can make children see the point of formal language. It also provides some indication that at least in that pupil's group an important aim of the course was achieved and that interaction with the microcomputer played a vital role in making it possible.

6.3 THE ROLE OF THE TEACHER

Both Chatterton (1988) and Underwood (1988) highlight a shift in the role of the teacher when pupils work in groups at microcomputers. The important ways in which the teacher's role changed during the course has already been alluded to a number of times and will be specifically analysed here.

Chatterton (1988) claims that provided pupils have control of the knowledge base needed for lessons and can formulate and test conjectures independent of the teacher, a major shift in the teacher's role can result. He reports that because of the support of the microcomputer the teacher is able to spend "less time in managing the lesson ... and more in listening to the pupils and responding to their problems on an individual/small-group basis". The researcher followed the approach described by Chatterton and by Underwood and took care not to intervene unnecessarily and be overly prescriptive. The researcher found that his role as teacher did change markedly during the course in a manner similar to that described above.

A number of different teacher roles can be identified as having operated during the course. Different roles predominated at different stages of the course and included the following:

6.3.1 The teacher as facilitator

This role started at the very practical level of loading the PROLOG system onto the various microcomputers and ensuring that all necessary preparations were made. It also included working to create a climate in which pupils were willing to try things and which promoted group interaction.

6.3.2 The teacher as manager and lecturer

This more prescriptive role was still needed, particularly at the start of the course. Pupils had to be shown how to operate the microcomputer and had to be provided with an explanation of the rudiments of working in the PROLOG system. The role of lecturer predominated for no more than 3 periods and fell away once pupils had got started on their work. The manager role remained only in terms of time usage due to the stricture of fixed period lengths.

This aspect at times had a negative effect on the progress of groups as they often had to be interrupted whilst working well. The observer-teacher also identified this aspect as a problem. The researcher was, however, unable to find an immediate solution due to the constraints of a complex school timetable.

6.3.3 The teacher as consultant

This role tended to predominate for the first two weeks of the course, while groups were still making many errors in their formulations and hence were unsure of the quality of their work. During this time the researcher-teacher interacted with pupils almost exclusively on a small group basis. Most of this contact was initiated by the various groups rather than by the teacher. The teacher tended to be called in as a consultant to give advice, to assist in solving problems identified by the group and to answer questions of the "Can we...?"-type.

6.3.4 The teacher as listener

As the pupils became more familiar with the PROLOG system, demands on the teacher to act as consultant became less and the teacher had more time to move around listening to group discussion and watching pupils try out their rules on the PROLOG system. In this way the researcher was also able to monitor progress. Discussion in groups at this stage tended to focus on making rules to produce desired results and involved pupils in "if ...then..."-type reasoning. The teacher only intervened in these groups when his presence was noted and he was asked for an opinion or used as a sounding-board. In giving an opinion care was taken not to be prescriptive.

6.3.5 The teacher as critic

During the second half of the course, groups had built up a database of facts and rules which in most cases gave desired answers to questions. Pupils were keen to show off what they had accomplished and consequently expected the teacher to be an appreciative critic. The teacher's role therefore became one of praising what had been done, but also pointing out areas in which improvements could be made or ways of testing systems which seemed to contain errors. Once again this type of interaction was largely pupil initiated, often with a "Look at our ..." - approach. On a few occasions the teacher did initiate contact in this role by commenting when noticing a particularly good rule formulation or asking appropriate questions where a group had become bogged down.

6.3.6 The teacher as evaluator

Lester and Kroll (1991) suggest that a teacher "can learn a great deal about students by circulating unobtrusively as students work in small groups and by interjecting questions to clarify their observations". They also include this approach of "observing and questioning" in their list of alternative evaluation techniques. Many of the teacher roles described above involved such observation and questioning and simultaneously provided an opportunity for the teacher-researcher to act as evaluator. In this role the researcher was able to monitor the extent to which the process objectives of the course were being achieved. The researcher tried to determine the quality of the interactions between group members and the quality of their interactions with the PROLOG system, rather than trying to quantify and assign marks to their efforts. The assessment of the quality of group products is, however, an area which also deserves attention and is discussed in section 6.5.

The informal evaluator role enabled the researcher to gain valuable insights as regards the operation of the groups and the specific problems experienced by individuals. Feedback aimed at encouraging continued improvement and refinement of products and approaches could therefore be given in a manner appropriate for a particular group or individual.

A significant feature of the various teacher roles outlined above is that only one of them is of the "teacher-talk" variety and that time spent by the researcher in this role accounted for only about 10% of the entire course. This represents a major shift away from an approach to mathematical instruction which Romberg (1990) terms "the model of teacher as teller" to a situation in which the teacher with the aid of the microcomputer allowed the learning process to be pupil driven. The positive reactions of pupils and observer teachers, the meaningful nature of the alternative roles filled by the teacher-researcher and the achievement of most course objectives suggest that this shift was beneficial.

6.4 THE ACTIVE LEARNING SITUATION

Observation of the pupil groups showed that pupils learnt to use a trial and improvement approach in building up their databases in the PROLOG system. The immediate feedback they received from the computer when asking questions of the system enabled pupils to judge the efficacy of their rules. Where necessary the pupils would then reformulate and retest their rules. In doing so the pupils learnt the need for precision in their formulations (cf. the pupil comment quoted on page 50) in an active manner. Pupils also developed their reasoning skills during this process of learning to ask appropriate questions and formulate rules which related to the facts in their database. One group for example discovered the need for an *or*-condition in the rule

```
¦ cricketer(X):- bats(X) ; bowls(X).
```

after noticing that the use of an *and*-condition did not produce anticipated answers to questions. The rule asserts that X is a cricketer if he bats or bowls. The same group were later able to use an *and*-condition correctly in the rule

```
¦ allrounder(X):- bats(X) , bowls(X).
```

which asserts that X is an allrounder if he bats and bowls.

In addition to observations such as the above, the effectiveness of the active learning situation is also indicated by pupil comments which suggest learning from experience. These are exemplified by the comments, quoted on page 54, where pupils describe progressing from not knowing what to do to confidently formulating rules and by an opinion such as:

- "The rules were at first difficult, but we soon got used to the commands and created more complicated rules".

As indicated in section 6.2, a further area in which the learning-while-doing approach proved to be effective was the acquisition of computer skills. In all the cases mentioned, pupils acquired new skills or knowledge after experiencing a need in their group. The acquisition process continued while pupils experimented with their conjectures and often led to a confident application of the new knowledge. The knowledge gained in this way was seen as meaningful by pupils and the active learning environment consequently appears to the researcher to be appropriate for pursuing the goals of the course.

6.5 PUPIL PRODUCTS

The open-ended nature of the task set for the groups resulted in a variety of databases of facts and rules being built up by the different groups. To evaluate these, the researcher chose a method based on the "holistic scoring" method described by Lester and Kroll (1991) which focuses on whether certain characteristics are present in a pupil's solution to a problem. The researcher identified characteristics which could be present in the logical systems built up by the groups. The various group products were then assessed according to the presence or absence of these characteristics. Ten such characteristics were identified. In the discussion which follows an indication is given of the number of group products displaying each characteristic.

6.5.1 Fluency

The researcher took the number of relevant facts and rules present in group products to be an indication of the group's fluency in formulating facts and rules. The number of facts formulated was found to be limited in only one of the nine groups. Two groups, however, appeared to struggle with the formulation of rules and produced significantly less than other groups.

6.5.2 Scope

This characteristic refers to the breadth of information in the databases produced by the groups. The researcher found that three groups stood out in this regard, but that all groups provided an adequate range of information in their databases.

6.5.3 Compatibility of facts and rules

All groups were able to produce rules in which the condition part was formulated in a manner compatible with the facts (or the conclusion part of other rules) which had to satisfy it. An incorrectly formulated rule was found in the database of two groups. One, for example, was the rule

```
¦ cheers(X) :- spectator(X).
```

when the related facts were of the form

```
¦ spectator(ian,tennis).
```

These erroneous rules, however, appear to be isolated cases which had not been properly checked.

6.5.4 The use of rules to replace collections of facts

This use of rules was evident because five groups neglected to remove all superfluous facts from their systems. Members of the group which successfully formulated a rule expressing the grandfather relationship, for example, neglected to remove all the facts which asserted who their grandfathers were. As previously indicated, their rule was

```
¦ grandpa(X,Y) :- father(X,Z) , father(Z,Y).
```

6.5.5 The use of rules to build onto facts

The products of all groups exhibited this characteristic in that rules were present which extended the information contained in the database without making any fact clauses redundant. An example is a rule like

```
¦ teacher(neil,X) :- mclass(X).
```

which relies on a collection of facts of the form

```
 | mclass(sam).
```

to satisfy its condition part and extends the information about members of the 'm class'.

It is possible that in some cases such rules were originally formulated to replace a collection of facts, but that the group remembered to remove the facts which became redundant from their system. In other cases groups were, however, observed to simply formulate the rule to add to existing information.

6.5.6 The use of rules as definitions

New mathematical concepts or relationships are often defined in terms of existing concepts or relationships. This process became more meaningful to pupils in that their use of definitions resulted from actual needs. Rules were used as definitions by five of the groups. Examples are the rules below in which a parent is defined as an individual who is a father or mother and a "prosurfer" is defined as an individual who uses a particular make of surfboard:

```
  | parent(X,Y) :- father(X,Y) ; mother(X,Y).
  | prosurfer(X) :- surfboard(X,bluehawaii) ;
             surfboard(X,andrewcarter).
```

6.5.7 Use of facts expressing binary relations

All groups progressed beyond declaring only simple properties of objects in unary relations like

```
  | underfourteen(jordan).
```

Their databases all contained a number of correctly formulated facts which expressed the relationships between pairs of objects.

Examples are:

```
¦ bestmove(daniel,cutback).
¦ mother(june,dean).
```

6.5.8 Use of rules with two variables

All but one group demonstrated the ability to formulate rules expressing a relationship between two variables. Examples are the parent relationship defined above and a rule like

```
¦ coaches(max,X) :- firsts(X).
```

6.5.9 Use of "if and only if"-type conditions

Two groups found it necessary also to include the converses of some rules in their systems. The group which formulated the rule in 6.5.8 also needed to use its converse,

```
¦ firsts(X) :- coaches(max,X).
```

6.5.10 Distinguishing between *and*-conditions and *or*-conditions

The databases of all but one group contained rules which used or-conditions correctly. Five groups also made appropriate use of the conjunction of conditions in their rules. These rules ranged from the rather complex expression of the grandpa relationship cited in 6.5.4 to a simpler rule like

```
¦ allyear(X) :- cricket(X) , rugby(X).
```

Although question clauses were not saved, the researcher also observed a number of the above characteristics in the questions groups used to interact with the system. One group whose database contained facts about their ages, subjects and sports made use of conjunction in their questions in order to restrict the possible answers. The researcher noted a question like

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?- art(X) , underfourteen(X).
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Questions involving binary relations like

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?- teaches(max,X).
?- teaches(X,Y).
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were also noticed. The presence of the above characteristics in pupils' databases and questions suggests that constructing the systems did involve pupils in deductive reasoning and provides evidence that they did develop some appreciation of the nature of a mathematical system. It is also indicative of groups' success in constructing logically coherent systems.

6.6 THE ACHIEVEMENT OF COURSE OBJECTIVES

The achievement of the objectives regarding the acquisition of computer skills, the development of an appreciation for formal language usage and the construction of a logical system has been dealt with in sections 6.2, 6.4 and 6.5. This section concentrates on the sometimes elusive process objective of making pupils think and hence developing their reasoning skills. In investigating the achievement of this objective, the researcher made use of a method which Lester and Kroll (1991) term "student self-reports". This approach requires pupils to write a "retrospective account" of the mathematical activity they have just completed. It also fits in with the "open-ended writing" method advocated by LeGere (1991) in terms of which pupils are asked to "write what they think they are learning". The researcher tried to focus pupil reflections by asking them to consider four questions in their reports. These pupil reports produced a great deal of useful information. Findings are, however, not based solely on the pupil reports, but are supported by the observations of the researcher and observer-teachers.

Pupil responses to the questions: "Did you learn anything ?" and "How much did you have to think to formulate rules ?" provide motivation for a finding that the course's process objectives were largely achieved. The following pupil responses provide a good overview of the thinking processes involved:

- "You had to think a great deal, firstly to get ideas and then to put (these) into words, but learning to make rules to shorten your facts can be quite a challenge."
- "The rules were not very hard but you always had to think, as if you were a person getting rules from another, to make them understandable. In this form of maths I learnt how to make rules and then apply them and ask questions."

Pupils' appreciation of the need for rules to be consistent with the rest of their systems and the consequent need for logical thought when formulating and testing rules, is also revealed in comments like:

- "When we made rules we had to take all the information into account and plan our rules accordingly. I learnt how to think more logically than before."
- "Forming rules wasn't easy because if your facts were incorrect your rules wouldn't work."
- "The rules are the most difficult to do. You have to think logically to think up rules. I learnt a little to think logically about something and also to work in a group."
- "The rules weren't that hard to formulate, all you had to do was think logically."

Further examples of responses which reflect the achievement of the objective of making pupils think, are:

- "To formulate rules I had to think carefully and sometimes it even made me a bit confused."

- "We had to think quite a bit to formulate rules and sometimes we had to ask the teacher for advice."
- "You had to think very carefully and logically."

The above pupil comments indicate that constructing the databases required a great deal of thought. They also reflect an appreciation of the logical nature of the systems that were created. Further evidence to support the finding that the process objectives of the course were achieved in most groups is provided by: the manner in which pupils were able to explain what they had done to visiting observer teachers, and the discussions noted in groups while they were building up their databases.

CHAPTER 7 - RECOMMENDATIONS

Recommendations can be divided into two groups. On the one hand recommendations are made in sections 7.1 and 7.2 for the improvement and future use of the course described in previous chapters. In later sections, recommendations are made as regards other possibilities for microcomputer-inspired adaptations to the secondary school mathematics curriculum.

7.1 ORGANISATIONAL CONSIDERATIONS

7.1.1 Group work

The benefits of group work were discussed in section 4.4 and emphasised in section 6.1. This approach is strongly recommended. As indicated in 6.1, working in groups, however, requires skills which need to be nurtured by regular use in the mathematics classroom. Regular use of group work could also enable teachers to discover the most effective way of dividing pupils into groups and perhaps eliminate the problem, encountered during the course development, of individuals who were not compatible with their groups. The emphasis on problem solving in the new Cape Education Department (1991) Junior Secondary Course syllabus provides further opportunities for experimenting with problem solving by groups.

Support for the above argument in favour of group work in mathematics is also provided by Chazan (1990). He highlights the need for developing communication skills in mathematics and the opportunities provided by microcomputer-aided investigations for developing the skills necessary to work effectively in groups. He sees a need for pupils to learn to "listen to each other, to critique constructively and gently, to accept criticism, and to accept different working styles" when working in groups. He also indicates that certain pupils require assistance in developing these skills and that teachers should insist on collaboration and a sharing of ideas within groups.

The size of groups also requires attention. Groups need to be large enough to ensure that sufficient ideas and points of view to provoke discussion are generated. They also have to be small enough to provide all group members with opportunities for contributing to the group activity. Groups of three worked well during the course development and are recommended. The only problem was that there was sometimes disagreement as regards which group member should use the keyboard. This problem disappeared once the novelty effect of using the keyboard wore off.

7.1.2 Time constraints

The length of periods is significant. Both the researcher and the observer teacher felt that 30 minute periods were too short in the early stages of the course. A mechanism to create a larger number of double periods for use in the early stages of the course needs to be found. Cross-curricular work with other subjects can provide a possible solution. Pupils could build up a PROLOG database about a topic in another subject. The periods of that subject could then possibly be combined with mathematics periods, increasing the number of double periods available for the course.

Once pupils get started and acquire some skill in using the microcomputer, 30 minute periods are acceptable. During the early stages of the course one or two periods away from the computer are also advisable. These periods allow ideas to incubate and provide pupil groups with time for planning their approach.

The three week duration of the course also appears appropriate and is recommended. It proved long enough for pupils to build up a meaningful database and acquire basic computer skills. It was also short enough to maintain pupil interest.

7.2 PLACING THE COURSE IN THE CURRICULUM

As indicated in chapter 6, the PROLOG-based course did achieve its objectives. Furthermore pupils' exam results indicate that they were not disadvantaged by the compacting of the Euclidean geometry and statistics sections of the curriculum to make place for the course. The inclusion of the PROLOG-based reasoning skills course in the curriculum is therefore recommended. The positioning of the course in the curriculum, however, needs to be reconsidered in the light of the new mathematics syllabus for standard 7 due to be implemented in 1992 and for standard 8 due to be implemented in 1993. In terms of these syllabuses, pupils' formal acquaintance with a deductive system via Euclidean geometry has been moved up to standard 8, although the standard 7 syllabus does still provide for a limited amount of deductive proofs to explain or verify results. The new standard 8 syllabus requires that pupils learn to organise geometrical results into a deductive system by building up a system of definitions and theorems about parallelograms. The PROLOG-based course can complement this process or serve as preparation for it.

Two approaches to the positioning of the PROLOG-based course appear feasible. The one is to leave it in standard 7 and use it to develop an appreciation of the need for formal language and deductive reasoning. The course would then serve as preparation for the formal introduction to a deductive system in standard 8. Having learnt to work in the PROLOG system in standard 7, it might even be possible for pupils to follow this up by using the system to build up a database about parallelograms in standard 8. The second approach is to move the PROLOG-based course to standard 8 and adapt it so that pupils build up databases of facts and rules about parallelograms.

If the first approach is adopted, time for the course can again be created by trimming the Euclidean geometry and statistics sections of the new standard 7 syllabus. The time needed for the statistics section can be cut by taking up the suggestion in the notes to the syllabus of using spreadsheets and so avoiding time-consuming pen-and-paper calculations and charts. The computer usage skills which pupils acquire during the PROLOG-based course are also likely to stand them in good stead when using the computer as a tool in the statistics section.

If the second approach is adopted, the PROLOG-based course can be presented as an alternative to the parallelogram section of the new standard 8 syllabus. The syllabus aim of providing pupils "with the experience of organising geometrical results into a deductive system" would remain. Use of the PROLOG-based alternative can, however, result in many of the advantages of active learning, discussed in chapter 6, accruing.

7.3 "EMPOWERMENT" IN MATHEMATICS

As indicated in chapter 4, the use of the microcomputer in tutee or tool mode can provide the pupil with opportunities for experimentation and for investigating his own ideas and perceptions. In this way the pupil is "empowered". The course described in previous chapters exemplifies such a pupil-driven approach. The advantages of the active learning which took place are discussed in section 6.4.

Allowing pupils to feel in control of their mathematics lessons can serve as a useful antidote for the difficulty barriers which they sometimes encounter in mathematics. As illustrated in chapter 6, it can also have a positive effect on their attitude to the subject. It therefore seems advisable to explore the possibilities for creating more opportunities for pupil-driven investigations in the mathematics class. In particular, the creation of further opportunities for using the microcomputer as investigative tool or as tutee should be pursued. Providing pupils with such learning experiences would also meet the instruction in the Draft Core Syllabus of the Department of Education and Culture (1991) that teaching approaches should include the provision of opportunities for "activity-based learning" and "open-ended investigations".

7.4 FURTHER USE OF COMPUTERS IN MATHEMATICS LESSONS

In section 1.5 the effect of microcomputers on the methodology of mathematics and mathematics education is discussed and the role of the computer as investigative tool highlighted. In section 4.3, Hubbard's (1988) comments on a trend toward the use of general purpose software packages for this purpose are reported.

Further investigation of the possibilities for the use of such packages as tools in mathematics is recommended. A few examples are given here.

A general purpose package to be supplied to some schools by the Cape Education Department is *Microsoft Works*. Wood (1990) argues that the use of the spreadsheet and charting capabilities of the *Microsoft Works* package can promote active learning in the mathematics classroom. She provides examples in which the package is used as a tool for exploring topics which also form part of the new Cape Education Department (1991) standard 7 syllabus. These topics are the gradient of straight lines and the use of charts to represent statistical data. She also indicates how the package can be used to investigate exponential functions - a topic which forms part of the new draft standard 9 algebra syllabus of the Department of Education and Culture (1991). Andrews (1990) expresses the view that "the potential for spreadsheets in mathematics classes is enormous" and that as a result of the power of the spreadsheet to do the necessary calculations the mathematics which can be explored is greatly increased. He provides an example of using a spreadsheet to investigate patterns in a number sequence and to find the limit of the sequence.

In section 1.6 reference is made to the fact that the new Department of Education and Culture (1991) draft syllabus for standard 10 Higher Grade includes amongst its optional modules a course on numerical methods. This course is likely to provide further opportunities for the use of the microcomputer as a tool in mathematics classes. Heideman (1991) gives an overview of the prescribed course and points out that computers and calculators "remove the drudgery and uncertainty of hand computation". This makes the solution of higher order polynomial equations by numerical methods far more accessible to pupils. The spreadsheet can be particularly useful as a tool when interval halving or iterative methods are used and it has the advantage that pupils can see the accuracy of solutions improve with successive calculations. They also have the advantage of making it easy for iteration steps to be set out neatly in a table.

The new draft Department of Education and Culture (1991) standard 8 syllabus also provides for trial and improvement methods of solving first or second order equations and systems of equations in two unknowns. Here too, there appears to be room for using the microcomputer as a tool in the mathematics class by employing spreadsheets.

The remarks section of the Department of Education and Culture (1991) draft core syllabus for standards 8 to 10 mathematics states:

> "The fact that problem solving is a central focus means that it must not be regarded as a separate content area, but as a method of enquiry, the process by which one learns and does mathematics, and should therefore be an integral part of all mathematical content and activity".

In section 1.5 the potential of the microcomputer as a tool in problem solving and as a means of generating problem situations is pointed out. Using a microcomputer in this context is, however, likely to be a new skill for many pupils and teachers. Furthermore, the design of courses which provide opportunities for using a microcomputer in a problem solving context is likely to be a non-trivial exercise and it is recommended that such courses flow from classroom-based research projects. Marsh (1988) describes a number of computer programs which can be used as tools in a variety of problem solving settings ranging from the summing of series to the solution of triangles. French (1991) indicates how short, simple BASIC programs can be used to create a problem solving environment for learning algebra. The programs mentioned above can act as a starting point for teachers to experiment with computer-assisted problem solving in their classes.

7.5 TRAINING IMPLICATIONS

As implied above and indicated in section 1.5, teacher training is crucial if teachers are to make informed and innovative decisions regarding the use of the microcomputer as a tool in mathematics classes. Possibilities for providing the necessary teacher training are in-service courses where teachers can learn to incorporate the new technology into their teaching methods and their "doing" of mathematics - the "mathematics in the making" of part 1.2. In the Cape Province such courses could be presented at Teachers' Centres under the auspices of mathematics study groups.

A further possibility is that such courses be included in post-graduate B.Ed., M.Ed., or Further Diploma in Education curricula. This would enable teachers to obtain the relevant training whilst upgrading their qualifications. The M.Ed. (Computers in Education) offered by Rhodes University is an example of an appropriate degree course. It does, however, seem desirable that similar courses be offered also at a lower level where they can be accessible to more teachers. Universities should also ensure that new teachers are trained to make use of microcomputers in their teaching by including appropriate courses in mathematics method curricula.

The in-service training envisaged for teachers by the Cape Education Department (1991) in terms of its Computers in Schools and Colleges (CISC) project has the potential of making a contribution to meeting the training needs outlined above. In terms of this project, in-service teacher training is divided into three phases: "basic computer training", "mastery of the computer as everyday tool" and "mastery of the computer as a teaching aid". This basic training in the first phase is based largely on the *Microsoft Works* package and will be provided for two teachers per school in the first year of the project. Whereas the courses in this initial phase of the project will be the responsibility of regional computer coordinators, based at teachers' centres, the Cape Education Department regards its resources as inadequate for providing the training for the second and third phases of the teacher training part of the project. In these phases the Cape Education Department suggests that it will be necessary for schools and subject study groups to arrange the necessary training. Mathematics study groups could also, in terms of this departmental plan, assume responsibility for ensuring that courses which equip teachers to integrate the microcomputer specifically into mathematics teaching, are presented.

A need also remains for a greater dissemination of "classroom-tried" techniques and information on available software. Articles in journals such as *Pythagoras* can play a valuable role in this regard. Teachers in the Cape Province also need to be encouraged to make more use of the services of the Education Department Library which makes available relevant articles from many foreign journals such as *School Science and Mathematics; The Australian Mathematics Teacher; Micromath; Mathematics Teacher; Mathematics Teaching* and *Mathematics in School*. Many useful ideas on teaching approaches which use the microcomputer or encourage active learning can be gleaned from these articles.

The regional computer coordinator should be responsible for collecting journal articles on the use of microcomputers in the classroom and making teachers in his region aware of them. He should also be trained to act as a consultant to teachers who undertake the development of courses incorporating microcomputer use. As suggested in chapter 2, teachers' centres can also be instrumental in arranging for teachers in their region to share their knowledge and learn from one another.

7.6 FURTHER COMPUTER-INSPIRED COURSES

Most of the recommendations made thusfar are inspired by the use of the computer in the role of tutee or investigative tool. In section 1.3 reference is also made to the computer as a user of mathematics. This role of the computer also merits consideration when planning possible further adaptations to the mathematics curriculum. A possible course inspired by the computer in this role is one in which algorithms and elementary programming are studied. The views of advocates of such a course are presented in section 1.6. It, however, seems prudent to develop such a course by using the action research methods and classroom-based development cycle employed for the PROLOG-based reasoning skills course.

The previous Cape Education Department junior secondary course syllabus (1973) did in fact contain a module on computers as an option in standard 7. In addition to presenting an overview of the nature and uses of computers, this course provided for learning about algorithms and flowcharting. It could therefore serve as a reference during the design phase of a new course on algorithms and programming. Such a new course can include specifying algorithms in the form of flowcharts to solve problems. The problems can range from specifying a method for making a cup of coffee to the processing of test marks. Further suitable problems appear in at least one of the current standard 7 textbooks, that of Laridon et al. (1984). Having mastered the techniques of specifying algorithms and flowcharting, pupils can move on to writing simple programs in order to be able to interact with the microcomputer in testing their algorithms. Based on such interaction pupils also need to learn to modify their algorithms.

If research indicates that such a course on algorithms is viable, it could be incorporated into the standard 7 curriculum as an optional module in addition to the option on transformations in the new (1991) syllabus. The duration of such a course is likely to be 2 to 3 weeks.

Further possible computer-inspired courses which can be considered as future optional modules in senior standards are courses on mathematical logic or formal languages. The feasibility of presenting such modules could also be the subject of classroom-based research.

7.7 PLANNING CONSIDERATIONS

The Cape Education Department (1991) standard 5 - 7 syllabus has adopted a useful new format by providing a "Notes" column to amplify its content statements and recommend teaching approaches. The inclusion of notes which indicate how the microcomputer can be used as a teaching aid and an investigative tool in the various syllabus sections would facilitate the integration of this tool with the mathematics curriculum. Teachers would then be able to consider a role for the microcomputer when planning their lessons and when drawing up annual schemes of work.

It is recommended that regular updates to such "notes" be published and that such notes also accompany the senior secondary syllabus. Such an approach could ensure that teachers are continually made aware of the possibilities for appropriate microcomputer usage in the mathematics curriculum.

In planning and drawing up annual schemes of work for their school, teachers should also consider building in opportunities for classroom-based research and course development similar to that described in chapter 5. In this way they can ensure that their methods and approaches are continually updated and revitalised.

CHAPTER EIGHT - CONCLUDING REMARKS

As indicated in previous chapters, the microcomputer has influenced the way in which mathematics is done, making numerical methods more accessible and facilitating trial and improvement approaches and experimentation. The acquisition of microcomputers by schools has enabled the microcomputer also to influence the way in which mathematics is taught and learnt. Microcomputer application in mathematics education has furthermore moved beyond use in tutor mode for individual drill and practice exercises aimed at improving pupils' routine computational skills. As emphasised throughout previous chapters, microcomputers used in tool or tutee mode can also facilitate group work, an investigative approach, active learning and problem solving. The creation of opportunities for learning experiences of this nature is encouraged by the new mathematics syllabuses of the Cape Education Department.

The researcher identified a need for a new approach to developing pupils' reasoning skills and promoting an appreciation for formal language use in mathematics. The microcomputer, used in tutee mode via the PROLOG-system, proved to be an effective vehicle for creating an active learning environment within which the above objectives could be pursued. The objective of making pupils think in a logical manner was achieved. At the same time, pupils' skill in using the microcomputer improved.

In developing the above course, use was made of action research techniques which blended well with the curriculum development cycle described in chapter 5. This cycle started with a situation analysis and planning stage which took cognisance of the potential of the microcomputer. It continued with an implementation stage in which extensive use was made of the microcomputer and an evaluation stage which included an assessment of the value of using microcomputers for the course. The process outlined above can serve as a method for integrating the microcomputer into the mathematics classroom. At the same time it appears appropriate for future use in doing the classroom-based research which Laridon (1990) advocates as a basis for curriculum development in mathematics.

The microcomputer can, as illustrated in chapter 7, stimulate a great deal of the above type of research and development, both in terms of the methods of teaching and doing mathematics and in terms of possible new curriculum content. If teachers can be motivated to undertake such research, then the way forward can be most exciting. The mathematics classroom can become a "laboratory" in which new approaches are continually tried as the teacher learns to incorporate microcomputer-inspired methods and content into the curriculum. The most important result can be that pupils experience mathematics as a vibrant, interesting and relevant activity.

REFERENCES

Andrews, P. (1990, Autumn). The power of spreadsheets. Micromath Vol. 6, No. 3.

Artzt, A.F. and Newman, C.M. (1990, September). Cooperative learning. Mathematics Teacher.

Atiyah, M F. (1986). Mathematics and the computer revolution. In A G Howson and J P Kahane (Eds). The influence of computers and informatics on mathematics and its teaching. Cambridge : Cambridge University Press.

Blease, D. (1986). Evaluating educational software. Croom Helm.

Borke, A. (1984). The fourth revolution - computers and learning. In Terry, C. (Ed.). Using microcomputers in schools. London : Croom Helm.

Borke, A. (1985). Children and interactive learning environments. In Chen, M. and Paisley, W. (Eds.). Children and microcomputers. London : Sage.

Bratko, I. (1986). Prolog programming for artificial intelligence. Wokingham : Addison-Wesley.

Cape Education Department. (1987). Guidelines for Curriculum Development.

Cape Education Department. (1991). Junior secondary course syllabus for mathematics ordinary grade standards 5 to 7.

Cape Education Department. (1991). The CISC project [Computers in Schools and Colleges].

Chatterton, J.L. (1988). Knowledge control: the effect of CAL in the classroom. Computers and Education, Vol. 12, No. 1.

Chazan, D. (1990, November). Students' microcomputer-aided exploration in geometry. Mathematics Teacher.

Chen, M. (1985). A macro-focus on microcomputers: Eight utilization and effects issues. In Chen, M. and Paisley, W. (Eds.). Children and microcomputers. London : Sage.

Clocksin, W. (1987,August). A Prolog Primer. Byte.

Crosswhite, F.J. (1991, October). National Standards: A new dimension in professional leadership. School Science and Mathematics, Volume 90 (6).

Department of Education. (1973). Junior secondary course syllabus for mathematics. Provincial Administration of the Cape of Good Hope.

Department of Education. (1984). Junior secondary course syllabus for mathematics. Provincial Administration of the Cape of Good Hope.

Department of Education and Culture: House of Assembly. (1989). Syllabus for mathematics : standards 5 - 7 (Third draft).

Department of Education and Culture: House of Assembly. (1991). Draft core syllabus for mathematics standards 8,9 and 10 Higher grade.

Eggleston, J. (1980). School-based curriculum development in Britain. A collection of case studies. London : Routledge & Kegan Paul.

Evans, N. (1986). The future of the microcomputer in schools. London: Macmillan.

Fong, H. (1989). Models for the integration of computing into mathematics curricula. Computers and Education, Vol. 13, No. 2.

French, D. (1991, March). Keep it short. Mathematics in School, Vol. 20, No. 2.

Gardiner, T. (1988, Spring). Will computers count?. Micromath.

Graf, K-D. (1985). Computer Science/Informatics - A challenge to mathematical education and vice versa. In Duncan, K and Harris, D (Eds). Computers in Education. North-Holland.

Halpin, M.N. (1986). Logic for problem solving. Proceedings of the Eighth National Congress on Mathematics Education. MASA

Harper, D.O. (1987). The creation and development of educational computer technology. In Thomas, R.M. and Kobayaski, V. (Eds.). Educational technology. Pergamon Press.

Hart, M. (1982). Using computers to understand mathematics. Four years on. Mathematics Teaching, 98.

Heid, K.M. (1989, September). How symbolic mathematical systems could and should affect precollege mathematics. Mathematics Teacher.

Heideman, N. (1991, July). Numerical methods at school level. Pythagoras, No. 26.

Higgo, J.R. (1984). The effect of micros on the mathematics curriculum. Mathematics Association Teaching Committee - Subcommittee on Microcomputers. Oxford

Hubbard, G. (1988). Recent history and imminent prospects for CAL in the U.K. Computers and Education, Vol. 12, No. 1.

Johnson, D.C. and Anderson, R.E. (1985). Algorithms and programming as part of computer literacy - some assessment results with implications for secondary school mathematics. Computers in Education. North Holland.

Jones, K.L. and Lamb, C.E. (1985). Programming in mathematics education : an essential component in the development of reasoning skills. Computers in Education. North-Holland.

Kemmis, S. (1988). Action Research. In Keeves, J.P. (Ed.). Educational research, methodology and measurement : an international handbook (pp.42-49). Oxford : Pergamon Press.

Kyriacou, C and Newson, G. (1991, May). Small group work in secondary school mathematics. Mathematics in School, Volume 20 no. 3.

Laridon, P.E. et al. (1984). Classroom Mathematics std. 7. Johannesburg : Mc Graw Hill.

Laridon, P.E. (1986). Computers and the mathematics curriculum. Proceedings of the Eighth National Congress on Mathematics Education. MASA

Laridon, P. (1990, April). Fundamental curriculum development issues relating to the current senior secondary syllabus proposals. Pythagoras No. 22.

Latham, J. (1984). PROLOG and English teaching. In Ramsden, E. (Ed.). Microcomputers in education 2. Chichester: Ellis Horwood Ltd.

LeGere, A. (1991, March). Collaboration and writing in the mathematics classroom. Mathematics Teacher.

Lester, F.K. and Kroll, D.L. (1991, April). Evaluation: A new vision. Mathematics Teacher.

Lieberman, D. (1985). Research on children and microcomputers. In Chen, M. and Paisley, W. (Eds.). Children and microcomputers. London : Sage.

Lynch, J.K., Fischer, P. and Green, S.F. (1989, December). Teaching in a computer-intensive algebra curriculum. Mathematics Teacher.

Marsh, T.A. (1988). Computer programs as tools for mathematical problem solving. Proceedings of the Ninth National Congress on Mathematics Education. MASA

Marsh, T.A. (1989, April). Microcomputer applications in mathematics education. Pythagoras No. 19.

Murakami, H and Hata, M. (1986). Mathematical education in the computer age. In Howson, A G and Kahane, J P (Eds). The influence of computers and informatics on mathematics and its teaching. (pp. 43-51). Cambridge : Cambridge University Press.

Murray, J.C. (1988). On the quality of learning in computer-supported learning environments. Proceedings of the Tenth National Congress on Mathematics Education. MASA.

National Council of Supervisors of Mathematics. (1989, September). Essential mathematics for the twenty-first century. Mathematics Teacher.

National Mathematics and Statistics CBL Review Panel. (1986). Will mathematics count?. AUCBE : Hatfield, England.

Nichol, J., Dean, J. and Briggs, J.H. (1985). Powerful PROLOG. Times Educational Supplement, 25 October.

Nichols, J. (1987). Why PROLOG ? . In Scanlon, E. and O'Shea, T.(Eds.). Educational Computing. Chichester: John Wiley and Sons.

Norton, D.A. (Project Leader) (1991). An investigation into the results obtained by pupils in the senior certificate mathematics examination. Summary Report. Cape Education Department.

Olson, J. (1988). Schoolworlds/microworlds: computers and the culture of the classroom. Oxford : Pergamon Press.

O'Shea, T. and Self, J. (1983). Learning and teaching with computers. Artificial intelligence in education. Brighton : The Harvester Press.

Papert, S. (1980). Computer-based microworlds as incubators for powerful ideas. In Taylor R.P. (Ed). The computer in the school: tutor, tool, tutee. (pp.177 - 196). New York : Teachers College Press.

Papert, S. (1980). Teaching children thinking. In Taylor R.P. (Ed). The computer in the school: tutor, tool, tutee. (pp.161 - 176). New York : Teachers College Press.

Papert, S. (1980). Teaching children to be mathematicians vs. teaching about mathematics. In Taylor R.P. (Ed). The computer in the school: tutor, tool, tutee. (pp.177 - 196). New York : Teachers College Press.

Papert, S. (1981). Mindstorms : Children, Computers and Powerful Ideas. Brighton : Harvester Press.

Papert, S. (1987). A critique of technocentrism in thinking about the shool of the future. In Sendov, B. and Stanchev, I. (Eds.). Children in the information age. Pergamon Press.

Pendley Manor Conference Report. (1983). Mathematics and Microcomputers.

Polya, G. (1957). How to solve it. Princeton.

Romberg, T.A. (1990, October). Evidence which supports NCTM's Curriculum and Evaluation Standards for School Mathematics. School Science and Mathematics, Volume 90 (6).

Romiszowski, A. J. (1981). Designing Instructional Systems. London : Kogan Page Ltd.

Rudduck, J. (1984). Curriculum development and teacher research. In Skilbeck, M. (Ed.). Readings in school-based curriculum development. London : Harper & Row.

Schmalz, R. (1989, December). Problem Solving - an Attitude as well as a Strategy. Mathematics Teacher.

Skemp, R R. (1971). The psychology of learning mathematics. Penguin.

Smith, D. and Keep, R. (1988). Eternal triangulation: case studies in the evaluation of educational software by classroom-based teacher groups. Computers and Education, Vol. 12, No. 1.

Steen, L A. (1986). Living with a new mathematical species. In Howson, A G and Kahane, J P (Eds). The influence of computers and informatics on mathematics and its teaching. Cambridge : Cambridge University Press.

Sweeten, C. (1982). Mathematics and microcomputers. ISMEC Discussion Paper.

Tall, D. (1983). Introducing algebra on the computer. Mathematics in Schools, Vol. 12 No. 5.

Taylor R.P. (Ed). (1980). The computer in the school: tutor, tool, tutee. New York : Teachers College Press.

Underwood, J. (1988). An investigation of teacher intents and classroom outcomes in the use of information-handling packages. Computers and Education, Vol. 12, No. 1.

Van Rooy, M.P. (1988). Die wiskunde kurrikulum : knelpunte van die van die verlede en uitdagings van die toekoms. Proceedings of the Tenth National Congress on Mathematics Education. MASA

Wood, J.B. (1990, Spring). Utilizing the spreadsheet and charting capabilities of Microsoft Works in the mathematics classroom. Journal of Computers in Mathematics and Science Teaching, Vol.9(3).

Woodhouse, D. and Jones, A. J. (1988). Integrating CAL with other instructional activities in schools. Computers and Education, Vol. 12, No. 4.

Wright, S. (1987, Summer). Micros and the aims of mathematics teaching. Micromath.