

AN INVESTIGATION ON HOW LEARNERS MAY
USE MULTIPLE REPRESENTATIONS IN A
SOCIAL INTERACTION TO PROMOTE
LEARNING OF PERCENTAGES AND
FRACTIONS: A CASE STUDY

A thesis submitted in partial fulfillment of the requirements for the
degree of Master of Mathematics (Mathematics Education) of:

RHODES UNIVERSITY

BY

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December 2012

DECLARATION OF ORIGINALITY

I, *NGOLA-KAZUMBA MARIA* (Student number: 04N5524) declare that this thesis **An investigation on how learners may use multiple representations to develop proficiency in percentages and fractions: a case study** is my own work written in my own words. Where I have drawn on the words or ideas of others, these have been acknowledged using the reference practices according to the Rhodes University Education Department Guide to Referencing.

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15 December 2012

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ABSTRACT

The study examined the use of multiple representations such as the real world, written symbols, spoken symbols, diagrams and manipulatives by learners to promote the learning of percentages and fractions through social interaction. This investigation was carried out through a teaching and learning programme which was developed and implemented by me, the researcher. The effect of the implemented programme was the main focus of the research. The qualitative study was oriented in the interpretive paradigm – a paradigm that seeks to understand the meaning attached to human actions.

Twenty learners participated in the implementation of the programme and 9 learners were selected for focus group interviews. The purpose of the interviews was to explore learners' understanding and feelings about the use of multiple representations in the learning of percentages and fractions through social interactions. The other tools employed in this study were pre-and-post diagnostic tests, observations, learners' work and a journal. The pre-test was used to determine learners' prior knowledge for the program design and implementation, while the post-test and learners' work were used to analyze the effect of the programme. Observations were used to investigate how multiple representations promoted or did not promote the learning of percentages and fractions. The teacher's journal was to record and reflect on any relevant information gathered on each lesson observed.

The data shows that the effective use of multiple representations helped learners learn the concept of percentages and fractions better. Learners were able to look at representations in useful ways; multiple representations made some aspects of the concept clear; and multiple representations enabled learners to correct errors. Through the interaction between the teacher and learners, the following was found: all the learners changed words to change focus; learners made links between multiple representations; the learners deepened their concepts of percentages and fractions; learners could convert between fractions using multiple representations; learners could work out percentages of a quantity; and learners could express one quantity as a percentage of another. Furthermore, through the interaction between learners and learners all learners could identify more equivalent fractions of an initial fraction which was given to them; and they could increase and decrease a quantity by a given percentage.

On the basis of this research, it can be concluded that the programme promoted the learning of percentages and fractions through three effective methodologies. The first methodology consisted of the effective use of multiple representations; the second methodology concerned the interaction between the teacher and learner during the learning process and the last methodology related to the interaction between the learners - interactions that were not strongly mediated by the teacher. I would recommend that teachers use these three effective approaches when teaching percentages and fractions to promote the learning of the concepts.

DEDICATION

I dedicate this thesis to my three daughters (Melinda, Unify and Fernanda) and particularly to my husband (Kazumba Paulus Dihscor) for the love, support and care that he has shown throughout my study. His caring and understanding of all the hardships I went through during my study inspired me more to progress.

ACKNOWLEDGMENT

First and foremost I would like to thank GOD for protecting and being there for me during all the challenges, distractions and difficulties I faced throughout this study. I thank GOD for giving me strength when I was weak, for encouraging me when I was discouraged, for providing time when time was limited, and for giving me his everlasting love and care throughout my study.

I would like to acknowledge my supervisor BRUCE BROWN for supervising, guiding, advising and assisting me with the entire process of the study. I gained insight on the context, content and concept of research in mathematics education, especially with regard to percentages and fractions. I'm grateful for all the support that he has provided. Thanks for believing in me and having high hopes in my study. Without his mentoring, this thesis would never have been realized.

I would like to extend my gratitude to Robert Kraft and Marc Schäfer, who helped me through working on the academic administration of this study. My thanks goes to Marc for helping me work according to the timeframe of this course by encouraging and informing me on which chapters to tackle and the due dates.

I am very grateful to my participants who provided me with the necessary data that I needed for this study. They were such a wonderful and exciting group to work with. Their participation in the study contributed hugely to the completion and success of the study.

I would like to acknowledge all my colleagues at the school where the research was conducted but more especially Mr. Chindongo, Mr. Sipeta, Ms. Irengya and Mr. Hamutenya for assisting me with my teaching and administration during the one month period I went away to GRAHAMSTOWN for my study.

I would like to acknowledge my husband, my children, my parents and my siblings for their moral support. I would like to thank my sisters and Priscilla for typing all the video transcripts for this study. I would like to thank Ryan for editing my work.

Lastly I would like to acknowledge the Ministry of Education for the financial assistance they have provided to me through the duration of this course.

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ACRONYMS

1. JSC – Junior Secondary Certificate
2. MKO – More Knowledgeable Others
3. ZPD – Zone of Proximal Development
4. VAT – Value Added Tax

CHAPTER ONE

INTRODUCTION

1.1 CONTEXT/BACKGROUND

Since gaining independence in 1990, Namibia has had the dream of becoming a country that can stand alongside other leading countries with respect to technology, education, and standard of living. Education is the most important aspect of a country's development, for without a steady supply of workers for all necessary fields of work, a country cannot grow. In Namibia, according to national reports from 2011, 48.5% of learners failed grade 10, thus halting their education (Smit, 2011). Furthermore, 53.3% of these grade 10 learners failed mathematics (Smit, 2011). In trying to find the root of this problem, we see that 59% of upper primary learners are failing mathematics as well (Sasman, 2011). This implies that learners are not attaining a proper grasp of the foundations of this subject from an early age. The topic of percentages and fractions is one of the building blocks of this foundation. The Report of Southern and Eastern Consortium for Monitoring Educational Quality project (SACMEQ 2000 and 2007) shows that learners in Namibia do not perform well in this topic. The Report on the National Examination for the grade 10 candidates for 2011 states that many learners got correct answers in topics such as algebra, geometry and money and finance, but topics such as percentages, fractions, ratio and proportions and probability seems challenging to the majority of the learners (Junior Secondary Certificate (JSC) Examiners Report 2011). Percentages form an important part of the Namibian grade 8 syllabus. Learners need to acquire greater understanding of percentages and fractions so that when they reach grade 10, percentages and fractions provide the foundation for topics like proportion, ratio and probability. For this reason, this research is focused on grade 8 teaching of percentages and fractions.

In my teaching experiences, I have observed that learners lack conceptual understanding of percentages and fractions - they only develop some procedural skills in this domain. One cause of the lack of conceptual understanding in learners on this topic may be because teachers use instructional approaches that do not encourage social interaction. Vygotsky's theories stress the fundamental role of social interaction in the development of cognition; he strongly believed that community plays a central role in the process of "making meaning" (Vygotsky, 1978). He further

states that learning is strongly influenced by social interaction, which takes place in meaningful contexts (Van Der Stuyf., 2002). Children's social interaction with more knowledgeable or capable others and their environment significantly impact on their ways of thinking and interpreting situations. A child develops his/her intellect through internalizing concepts based on his/her own interpretation of an activity that occurs in a social setting. The communication that occurs in this setting with more knowledgeable or capable others (MKO) helps the child to construct an understanding of the concept (Bransford, Brown & Cocking, 2002). Moreover a learning activity that occurs in a social setting should fall within the zone of proximal development (ZPD) of the child in order to promote development (Vygotsky, 1986). Vygotsky further states that a person's potential for learning lies in their ZPD. For this reason, this study will involve research into activities that encourage interaction between the learning child and MKO (teacher and other capable learners), and creates a zone of proximal development.

Kilpatrick, Swafford and Findell (2001), argue that interaction in an instructional triangle among the teacher, the students, and the mathematics develops proficiency in mathematics. Haas (1998) also reported that the reason for learners' lack of conceptual understanding in fractions is because instruction on fractions is delivered neither appropriately nor adequately so that the connections between manipulative representations and symbolic representations are built upon. With these concerns, I therefore thought of using two instructional orientations to develop mathematical proficiency in learners.

The first orientation was to use multiple representations in the learning and teaching of fractions and percentages. The use of multiple representations may yield learning and teaching interactions that promote mathematical proficiency. Many researchers have strongly recommended the use of multiple representations in order to help students understand mathematical concepts in depth (Ball, 1988; Edgardo, 2001; Kaput, 1989). I therefore used multiple representations to teach percentages and fractions to my grade 8 learners.

However the use of multiple representations in isolation does not promote mathematical proficiency, as multiple representations only become representations when someone gives them meaning by interpreting them (Van Someren, Reimann, Boshuizen & De Jong, 1998). They become useful when they are used as mediating tools within the learning interaction process (Vygotsky, 1978). Tools (conceptual and manual) are seen as central in mediating between the

person and the world (subject and object) and for the development of a person's competencies (Hedegaard, 2001). That is why this study looked at meaningful learning as a product of interaction between the teacher as a mediator, other learners as influences and artifacts or tools as mediation or manipulation.

The second orientation is to move from traditional theory (of transmission whereby the teacher transmits knowledge and the student learns by listening or copying exactly what the teacher has to say), to participation in social communities and developing competence (Hedegaard, 1999). In this approach, the child is seen as a participant in learning, and she/he plays an important role in the interaction among the teacher as a mediator, other learners' influence and multiple representations or artifacts as mediation or manipulation.

1.2 THE PURPOSE OF THE STUDY

The aim of this study was to investigate the extent to which multiple representations in a social interaction may be used to enhance the learning of percentages and fractions by grade 8 learners in a Namibian school. This investigation was carried out through a teaching and learning programme which was developed and implemented by the researcher. The effect of the implemented programme was the main focus of the research. This research focused on the following four research questions:

1.3 RESEARCH QUESTIONS

The overall question

- How did the teaching and learning programme promote, or not promote the learning of percentages and fractions?

This question was addressed by means of the following sub-questions:

1. How did the use and manipulation of multiple representations as tools by the learners influence the learning process?
2. How did the teacher influence the dynamic learning interaction process in ways that promoted or did not promote learning?
3. What learners' interactions that were not strongly mediated by the teacher, promoted or did not promote the learning of percentages and fractions?

The teaching and learning processes generally related to all three of the above sub-questions. These three questions are strongly inter-related but for the purpose of analysis they show different and complementary perspectives to be taken in the teaching and learning process.

1.4 DEFINITION OF KEY TERMS

The following operational definitions are defined as they are used in this study:

Multiple Representations: “Multiple representations are external mathematical embodiments of ideas and concepts to provide the same information in more than one form (Ozgun-Koca, 1998) such as pictures, diagrams, sketches, tables, written symbols, oral representation and the use of manipulates” (Edgardo, 2001).

Manipulatives: “are concrete objects used to model corresponding mathematical ideas by providing hands-on experiences” (Cramer & Henry, 2002). In this study, the manipulatives used were the 11 boxes.

Social Interactions: “interactions that provoke learning, learning that occur through participation in social or culturally embedded experiences” (Raymond, 2000).

Zone of Proximal Development: “is the gap between the levels of the child’s actual development, determined with the aid of tasks that he can solve independently, and the level of his possible development” (Vygotsky, 1978).

More Knowledgeable Others: refers to “anyone who has a better understanding or a higher ability level than the learner with respect to a particular task, process, or concept” (Vygotsky, 1978).

Interpsychic: “is when the child and the adult interact through their actions (feelings, thoughts). What is born at the moment of this meeting between the child and adult that neither of the participants had before this meeting, due to the child’s thoughts and actions being guided by the teacher this process is called interpsychic” (Zuckerman, 2007).

Intrapsychic: “is the process whereby the child develops a higher understanding of the concept individually after acquiring it first in a social, interpsychic setting” (Zuckerman, 2007).

Scaffolding: “a structure of “supporting point” for performing an action” (Obukhoval and Korepanova, 2009).

Tool mediation: “the key aspect of learning that influences and changes human’s relation to the world and the human mind” (Vygotsky, 1997).

Internalization: “starts on the intersubjective plane where the child interacts with another person. On the subjective plane, the child still takes part in a kind of interaction but this time the child takes all the roles in the interaction” (Hedegaard, 1990).

Fractions: a rational number represented as a “part of a whole”.

Percentages: “Percentages are numbers on a fixed scale that runs from 0 to 100” (Van Galen, et al., 2008).

Network of relationship: is defined as “the knowledge that children develop about the relations between different types of fractions” (Van Galen et al., 2008).

1.5 METHODOLOGY

This study is a qualitative investigation and it was conducted within the interpretive paradigm. A paradigm that seeks to understand the meaning attached to human actions (learning through using multiple representations when presenting percentages and fractions). Using this paradigm helped me to engage deeply with the data on how the participants interacted, interpreted and related to the work with multiple representations in presenting percentages and fractions. I used five research tools to collect my data namely; pre and post diagnostic tests, observations, focus group interviews, learners’ work and the teacher’s journal.

The purpose of the pre- diagnostic test was used to determine learners’ prior knowledge for the design and implementation of the intervention programme, while the post-diagnostic test and learners’ work were used to analyze the effect of the intervention programme. The interviews were mainly to explore learners’ understanding and feelings about the use of multiple representations in the learning of percentages and fractions through social interactions. The observations were used to investigate how multiple representations promoted or did not promote the learning of percentages and fractions. The teacher’s journal was to record and reflect on any relevant information gathered in each lesson observed.

1.6 FINDINGS

I present my findings in detail in chapter four. However I will highlight some of the main findings here. The study reveals that the effective use of multiple representations helped learners to learn the concepts of percentages and fractions better, learners were able to look at representations in useful ways; multiple representations made some aspects of the concept clear; and multiple representations enabled learners to correct their errors. Again, through the interaction between the teacher and learners, all the learners changed words to change focus;

learners made links between multiple representations; learners deepened the content and concepts of percentages and fractions as they could convert between percentages and fractions using multiple representations, they could work out percentages of a quantity and they could express one quantity as a percentages of another one. Furthermore, through interactions with each other (through exploratory talk), all learners could identify more equivalent fractions of the initial fractions given to them and they could increase and decrease a quantity by a given percentage.

1.7 SIGNIFICANCE

This study is significance because firstly, it encourages teachers to use the three effective approaches when teaching percentages and fractions to promote the learning of the concepts. Secondly, it gives a reflection on how the programme enhances the learning of percentages and fractions, how learners become fully engaged in the lessons and how they are motivated because of the use of multiple representations. Thirdly it shows how teacher and learners' interactions are essential during the process of learning about percentages and fractions.

1.8 OVERVIEW OF THE THESIS

This thesis is divided into six chapters. Chapter One gives a brief introduction to the thesis. Chapter Two reviews literature relevant to the study and is divided into four main sections. The first section focuses on literature that discusses the use of multiple representations in mathematics. This section is further subcategorized into three discussions: descriptions of multiple representations, the effect of using multiple representations in learning mathematics and the effective framework for multiple representations in percentages and fractions. The second section of Chapter Two discusses learning as a product of interaction between the social process and artifact mediation. This section is further categorized in two topics of discussion. Firstly, it discusses the interaction between the learning child and the MKO (teacher and other capable learners) that creates a zone of proximal development. Secondly it discusses how tool or artifact mediation develops meaningful learning through social interaction. The third section of Chapter Two discusses the instructional triangle as an approach that develops proficiency in mathematics. The fourth section discusses the learning of percentages and fractions.

Chapter Three is about the methodology. This chapter gives a detailed description of the research process, including the research context, the research tools used to collect data, how the tools were used and why they were used.

Chapter Four discusses the presentation of the data and data analysis. An overview is provided through comparing the pre-and-post diagnostic tests. An analysis of how learners' usage of the real world objects, manipulatives, diagrams, written mathematical symbols and spoken symbolic representation of percentages and fractions was conducted through the learners' written activities, through extracts from video transcriptions, through themes emerging from the teacher's field notes in the journal and through extracts from transcriptions of the interview. Chapter Five presents the discussion of findings.

A summary of the findings, recommendations, limitations and challenges encountered in the research and suggestions for further research are some of the aspects that are discussed in the Chapter Six.

CHAPTER TWO

LITERATURE REVIEW

2.1 INTRODUCTION

The aim of this research was to investigate the extent to which multiple representations may be used to enhance the learning of percentages and fractions by grade 8 learners in a Namibian school. This chapter reviews literature relevant to this study and presents a theoretical framework for the research. The chapter is divided into four main sections. The first section focuses on research those talks about the use of multiple representations in learning and teaching mathematics. This section is subcategorized into three topics: a description of multiple representations, the effect of using multiple representations in learning mathematics and the framework for multiple representations in fractions and percentages. The second section covers research that discusses how meaningful learning takes place as the product of interaction between the social process and artifact mediation. This section is subcategorized into two discussion topics: firstly, the interaction between the learning child and MKO (teacher and other capable learners) that creates a zone of proximal development; secondly, how tool/artifact mediation develops meaningful learning through social interaction. The third section talks about the instructional triangle as an approach that develops proficiency in mathematics. The last section focuses on previous research on the learning of percentages and fractions.

2.2 THE USE OF MULTIPLE REPRESENTATIONS IN MATHEMATICS

2.2.1 Description of multiple representations

Until recently, researchers have understood the need for using multiple representations in learning and teaching. Therefore it is important to look at how researchers describe multiple representations. Edgardo (2001) states that multiple representations include pictures, diagrams, sketches, tables, written symbols, oral representation and the use of manipulatives. A further description by Schultz & Waters (2000) said that multiple representations include graphs and diagrams, tables and grids, formulas, symbols words, gestures, software code, videos, concrete models, physical and virtual manipulates pictures and sounds. Ainsworth, Billy and Wood (2002) confirm that multiple representations in technology include calculators, computers, graphing, games and simulation. They can all be useful in helping students grasp essential concepts and supporting computational skills development. In addition to this Ozgun-Koca

(1998) defines multiple representations as “external mathematical embodiments of ideas and concepts to provide the same information in more than one form” (p.1). He classified representations as external and internal. Each of them possesses a considerable amount of sub-themes exposed to more and deeper research linked with other fields. Furthermore, Kaput (1989) stated that multiple representations are ways to symbolize, describe and refer to the same mathematical entity. They are used to understand and to communicate different mathematical features of the same object or operation, as well as connections between different properties. Multiple representations are embedded in the learning theory of cognitive flexibility theory. According to Spiro, Feltovitch and Coulson (1988), in cognitive flexibility theory, learners grasp the nature of complexity more readily by being presented with multiple representations of the same information in different contexts. This theory encourages the use of multiple representations in the process of learning and teaching.

2.2.2 The effect of using multiple representations

The use of multiple representations has been strongly recommended by a number of researchers, because they help students understand mathematical concepts in depth (Edgardo, 2001; Kaput, 1989). Edgardo says that multiple representation provides opportunities for learners to make connections and realize the relationship between what they are learning and their experiences or prior knowledge. In addition, Defour-Janvier, Bednarz, and Belanger (1987) said that multiple representations can be used to mitigate certain difficulties through multiple concretizations (p. 110-111). Similarly, Rau, Aleven and Rummel (2009) point out that multiple representations of learning content can enhance student learning in complex domains, compared to learning with only single representations. Rau et al. (2009) have shown that students learned more with multiple graphical representations of fractions than with a single representation, only however when prompted to self-explain how the graphics related to the symbolic fractions representation. They said that concrete models can help students represent numbers and develop number sense; they can also help bring meaning to students’ use of written symbols and can be useful in building place-value concepts. Moreover, Kilpatrick et al. (2001) said that multiple representations such as manipulates can enhance students’ understanding and help students correct their own errors.

Ainsworth et al. (2002) say some representations, such as pictures, videos and manipulates can motivate because of their richness, possibilities of play, technology involved, or connection with

interesting areas of life. Edgardo (2001) said one of the main advantages when using technology is the capability to represent information in multiple representations. Furthermore Edgardo (2001) points out that the use of technology can increase accuracy and speed of data collection and allow real-time visualization and experimentation. They said that spreadsheet programs provide dynamic links among formulas, grids and several types of graphs.

Furthermore Kaput (1989) argues that multiple representations may also remove gender bias that exists in mathematics classes. Each student can choose a presentation suitable for him/herself. Edgardo (2001) adds that students can pass from one representation to another, knowing the possibilities, the limits and effectiveness of each one. Students can develop an understanding of multiple methods of solutions and forms of answers. Multiple representations such as visual representation, manipulates, gestures, and to some degree grids promote qualitative reasoning in the learning of mathematics (Edgardo, 2001). However Rau et al. (2009) argue that learners must perform a number of cognitive tasks in order to benefit from multiple representations. Learners acquire a deep understanding only if they are able to link multiple representations of the same concept and to coordinate between them. If students fail to link multiple representations their learning may be jeopardized.

Besides the essence of using multiple representations, some research studies point out the weaknesses of multiple representations. Defour-Janvier et al. (1987) state that the use of representations is sometimes abstract to students, especially if heavy emphasis is placed on symbols and their manipulation. This could provoke a lack of meaning of the mathematical concepts. They affirm that the inappropriate contextual use of representations, as well as the prematurity of their use results in negative consequences to students. Edgardo (2001) agrees that “the use of multiple representations encourages a play on symbols, puts the emphasis on the syntactical manipulations of symbols without reference to the meaning. The significance is absent! Mathematics is reduced to a formal language” (p. 17). Van Someren et al. (1998), further argue that when information is presented to students in varied forms, without interpreting them or teaching students the connections or relations between representations, it becomes difficult for students to construct the relations themselves. Representations become representations when someone gives them meaning by interpreting them. Thus it is important for teachers to interpret,

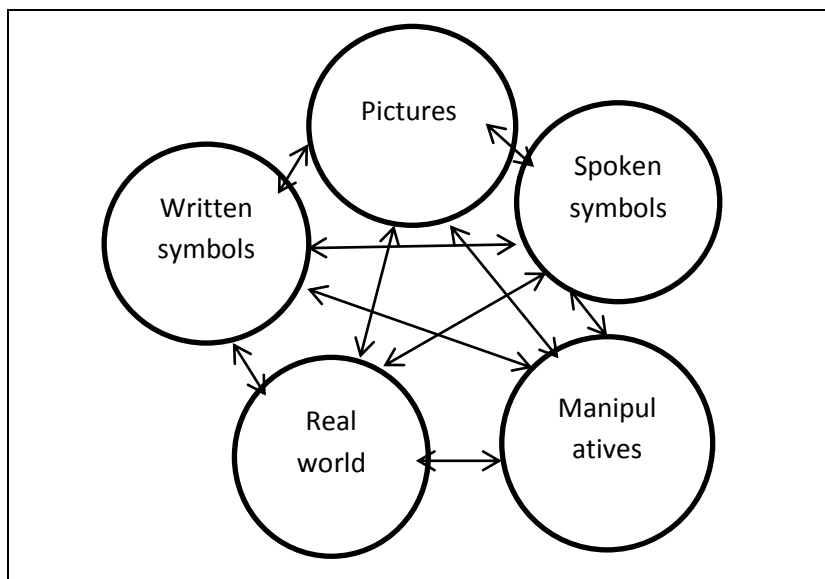
connect and give the relationship between representations, and most of all to let learners use them as manipulations.

2.2.3 The framework for multiple representations in mathematics

Lesh's model

Lesh (1979) proposed five multiple representations which are interwoven and essential for enhancing conceptual understanding and procedural fluency in mathematics concepts. The five multiple representations are: pictures, spoken symbols, written symbols, manipulative and real-world. I therefore designed an intervention programme, which includes multiple representations appropriate to use in the learning of percentages and fractions and adopted Lesh's model. The five representations below are included in Lesh's model (1979).

Fig 2.1: Lesh's five multiple representations model



For the purpose of my study I have chosen to adopt and use the four multiple representations that correspond with those of Lesh, which are: spoken symbols, written mathematical symbols, manipulatives and real world objects. However I replaced the fifth representation on Lesh's model - pictures with diagrams. I proposed to use diagrams instead of pictures because diagram representations are thought to be the best models for reasoning with fractions and percentages (Van Galen, Feijs, Figueiredo, Gravemeijer, Van Herpen & Keijzer, 2008; Sun, 2005). I will elaborate more on this at a later stage. The five-stage model of the multiple representations which were included in this programme were all as follows:

- **Spoken symbols** – i.e. the teacher saying the number “fifteen percent” is different from the teacher writing it on the chalkboard for learners to see.
- **Written mathematical symbols** – these can include numbers, mathematical expressions, and others, e.g. 25% of 300, $\frac{1}{2} = 50\%$ and etc.
- **Diagrams replacing Lesh’s ‘pictures’**– for example using pie-charts, tables, sketches, figures and grids are examples of diagrams.
- **Manipulatives** – like bottle tops, pieces of pipe, and pieces of boxes, counting pins, pieces of plastic bottles, bottle necks, erasers, chalk, pens, rulers, paper sticks, and pairs of scissors.
- **Real world objects**- like small stones, almond seeds, money, cash slips and calculators.

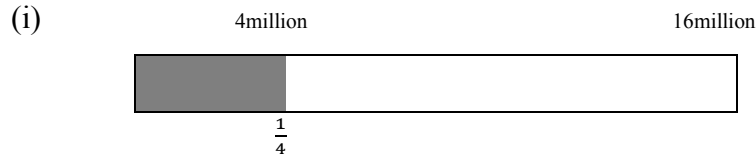
According to various researchers, each of these five multiple representations used during this study have had positive effects in the teaching of percentages and fractions (Cramer & Henry, 2002; Van Galen et al., 2008; Lampert, 2003; Sun, 2005; Zhang, 1997). Different forms of representation such as real world objects, manipulatives, pictures, spoken symbols and written symbols contributed differently to conceptual understandings (Sun, 2005). Applying real world representations could motivate learning and make learning meaningful; it also serves as an intuitive foundation on which later learning can be built (Campbell, 1996; Sun, 2005).

Manipulatives were also reported as benefiting students’ learning by providing hands-on experiences to make symbolic representations more concrete (Cramer & Henry, 2002). Written symbol representations were also reported to be one important factor (Lampert, 2003). Written symbolic representations stimulated reflective thinking (Lampert, 2003), which served as the bases of “logical, analytic, rational, and scientific” thoughts (Sun, 2005). Therefore, it served not only as a result but also as a process of thinking (Zhang, 1997).

Zhang (1997), states that mathematics concepts, for example rational numbers, usually involve at least four sub-constructs. So a single representation cannot address all of these sub-constructs significantly. He used diagrammatical representations such as number lines to demonstrate the part-whole sub-construct. Here number lines and fraction strips were used to demonstrate fractions. Van Galen et al. (2008), argue that bars and number lines are central models for reasoning with fractions and percentages: “In a context problem, a fraction is a certain part of

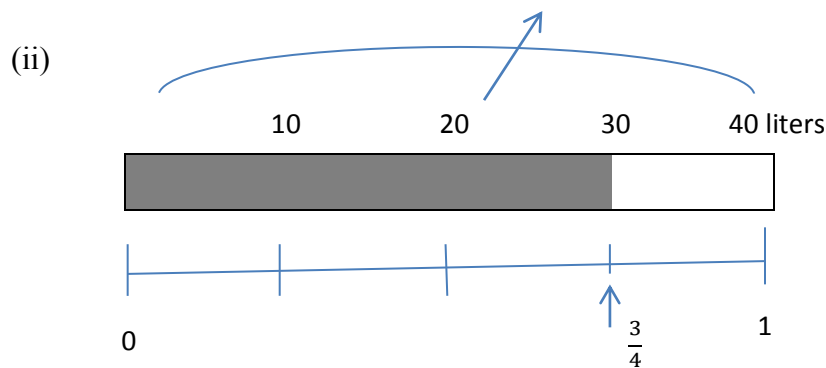
something, and that “something” is often a quantity. When we refer to “one-fourth of the Dutch population”, this is approximately equal to one-fourth of 16 million people. We are therefore talking about two relationships at the same time: the relationship between one-fourth and a whole, and the relation between 4 million and 16 million. A bar can represent double relationships very accurately as shown in the figure below.” (p. 75).

Fig 2.2: Model for reasoning with fractions (Van Galen et al., p. 75)



Van Galen et al. (2008) further argue that double meaning appears on both a bar and a double number line. For instance if we take an example of the fuel gauge below, everyone sees that the tank is three-fourths full, but if you know that the tank holds 40 liters, you can also understand that there are 30 liters full and 10 liters empty.

Fig 2.2: Models for reasoning with fractions (Van Galen et al., p. 75)



When it comes to percentages, the most valuable model is still a bar. The percentages are written above the bar and the corresponding numbers below the bar, or the other way around (Van Galen et al., 2008). According to Van Galen et al. (2008), the advantage of the bar is that it has “body” – area. For children, this makes it easier to talk in terms of “the whole” and “the so-much part” of the whole. They further state that circles play a role in teaching fractions, as one advantage of the circle is that the orientation of the lines give the parts their own character. However, Van Galen et al. (2008) pointed out that although a circle may be easier to interpret as an illustration, it can be more difficult to use as a conceptual model. Mack (1990) criticizes the use of circle or pie graphs for their limitations in demonstrating fractions.

2.3 LEARNING AS THE PRODUCT OF INTERACTION BETWEEN SOCIAL PROCESS AND ARTIFACT MEDIATION

2.3.1 The interaction between the learning child and MKO (teacher and other capable learners) that creates a zone of proximal development

According to Vygotsky, learning occurs through participation in social or culturally embedded experiences (Raymond, 2000 p. 176). He further explains that in Vygotsky's view, learning is strongly influenced by social interactions which take place in meaningful contexts. Children's social interactions with more knowledgeable or capable others and their environment significantly impact their ways of thinking and interpreting situations. A child develops his/her intellect through internalizing concepts based on his/her own interpretation of an activity that occurs in a social setting. The communication that occurs in this setting with more knowledgeable or capable others (parents, teachers, and peers) helps the child construct and understand the concept (Bransford, Brown & Cocking, 2002).

According to Hedegaard (1990), children not only learn through their participation in the social world, but also become involved in a reciprocal process in which their motives and personalities play a part in the interaction with the other persons (MKO) in the classroom – the teacher and their classmates, and thereby contribute to their own learning conditions. Vygotsky again points out that children learn by taking part in an activity that they are not able to perform on their own, but they are able to perform with the help (mediation) of others. The adult or another child offering the help are already capable of performing the activity and so are called more knowledgeable others (MKO) (Vygotsky, 1986). Vygotsky said that only activities that fall within the zone of proximal development of the child, promote development. He continued stating that if a child can already competently perform the activity on their own, little learning is involved when performing it. On the other hand, if the activity is too far beyond their capability, they will not be able to perform it even with help, or if they are helped to perform it, they will not do enough of the work themselves to learn the concept. The child's zone of proximal development, or ZPD, consists of those activities for which the child needs help to perform competently, and is also able to contribute significantly (Vygotsky, 1986).

According to Obukhoval and Korepanova (2009), the ZPD concept is seen as scaffolding, a structure of “support point” for performing an action. Wood, Bruner and Ross (1976) suggest that an activity that leads to learning has to be within a social relation where it is possible to scaffold the child’s action. Wood et al. (1976), define scaffolding as carrying “those elements of the task that are initially beyond the learner’s capacity, thus permitting him to concentrate upon and complete only those elements that are within his range of competence. They further explain that scaffolding is the way the adult guides the child’s learning via focused questions and positive interactions.

The use of scaffolding is another factor that helps to maintain student engagement at a high level (Hiebert & Wearne, 1993; Stein, Grover, & Henningsen, 1996) by offering a subtle hint, posing a similar problem, or asking for ideas from other students. They further state that teachers must scaffold learning; they need to continually probe students’ understanding, especially by allowing them to explore new ideas on their own. Wood & Middleton (1975) explain that scaffolding is effective when the support is matched to the needs of the learner. They then mention certain processes that aid effective scaffolding such as:

- Gaining and maintaining the learner’s interest in the task.
- Making the task simple
- Emphasizing certain aspects that will help with the solution.
- Controlling the child’s level of frustration.
- Demonstrating the task.

“The child’s zone of proximal development (ZPD) is the gap between the levels of his actual development, determined with the aid of tasks that he can solve independently, and the level of his possible development, determined with the aid of tasks that he can solve under the guidance of an adult or in collaboration with more capable companions” (Vygotsky, 1978). Zuckerman (2007) adds that the ZPD is described not in the language of the content of tasks but in that of the kinds of help that to a greater or lesser degree aid the child in solving a task. Bruner (1982) on the other hand, describes the zone of proximal development as “the child’s ability to recognize the value of hinges and props even before he is conscious of their full significance”.

Moreover, in the ZPD the teacher doesn't teach what is well within the child's own capacity. The child comes to school not knowing how to do certain things, and with the teacher's help he/she becomes increasingly independent. Finally he is able to act without outside assistance and needs no further guidance, instructions or collaborations. The help that the adult provides should change the learner's actions under the adult's guidance (Zuckerman, 2007). Vygotsky (1986), believed that when a student is at the ZPD for a particular task, providing the appropriate assistance will give the student enough of a "boost" to achieve the task. Moreover the child's performance can thus be extended, developed and improved by taking on and internalizing the performance of the helping MKO when carrying out the activity. Therefore learning always originates in some form of participation that is internalized and consequently changes our mental functioning.

"Any function in the cultural development of the child appears on the scene twice, on two planes - first on the social and then on the psychological plane, first between people, as an interpsychic category, and then within the child, as an intrapsychic category (Vygotsky, 1986, p. 145).

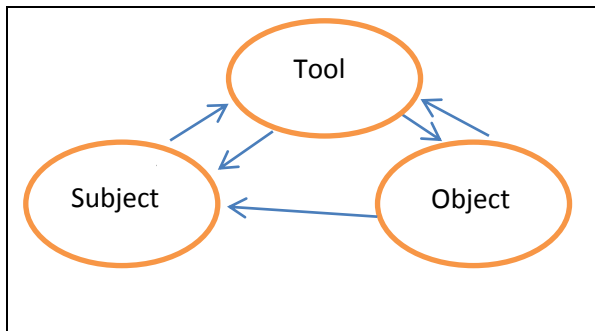
Zuckerman (2007), states that the child and the adult interact through their actions (feelings, thoughts). What is born at the moment of this meeting between the child and adult that neither of the participants had before this meeting is called the interpsychic process, due to the child's thoughts and actions being guided by the teacher. The intrapsychic on the other hand is the process whereby the child develops a higher understanding of the concept individually after acquiring it first in a social, interpsychic setting.

Not all interactions, in which the child is involved, automatically create the interpsychic form of a higher mental function. It only occurs in interaction where the intentions of the child and of the adult intersect in the body of a sign, symbol or tool (Zuckerman, 2007). I will therefore discuss research on tool mediation that develops meaningful learning.

2.3.2 Tool mediation

Both Vygotsky (1986), and Hedegaard (1990) state that learning and development take place through social interaction in institutional practice. Hedegaard (1990) elaborates that learning has to be related to tool use, guided by teaching and related to the institutional activities in which the children participate. The model of the mental act by Vygotsky explains the social interaction (see fig 2.3 below) (Vygotsky, 1997, p. 67).

Fig 2.3: The model of the mental act



Vygotsky focused on tool use as the key characteristic of human mental activity and characterized the process that combines persons, tools and world as the mental act. His theoretical focus was primarily on humans’ “psychological functioning with mental tools (i.e. oral and written language, models, blueprints, number systems)”. It is the mediation of these mental tools that especially influences humans’ psychological relation to the world (Vygotsky, 1997).

Vygotsky’s theory explicates tool mediation as the key aspect of learning. This aspect influences and changes human’s relation to the world and the human mind. Tool mediation is related to cultural practice and tradition in different types of institutions. However to carry out a mediated action well, we need to learn to use the mediating tool effectively. We need to learn to act in a way that fits the structure of the tool such as:

- ❖ We need to perceive the important properties of the tool that are related to the function it affords. e.g. length, weight.
- ❖ We set up neural paths to control that action of „person and tool“.
- ❖ We set up neural paths to allow us to predict responses and so react quickly and efficiently to improve the action.

Learning to use a tool changes the way we think about the action. We think in terms of ourselves and also the tool as a single element. Learning to use a tool also changes the way we view objects and actions. Use of the tool and the potential it offers, structures the way we see actions and potential actions (Vygotsky, 1986).

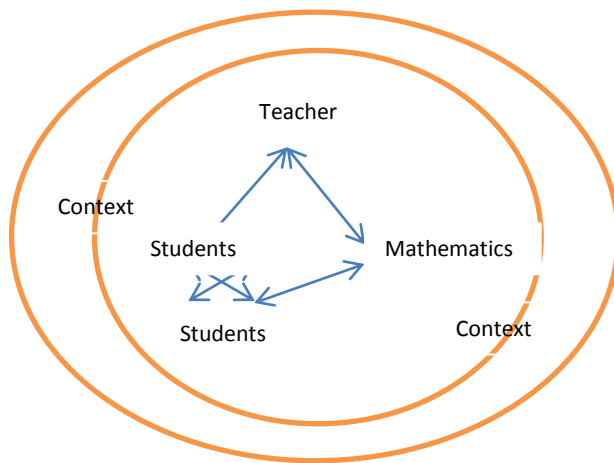
According to Hedegaard (1990), speech and communication are tool activities that are central in Vygotsky’s theory on understanding how the human psyche is created. She says that Vygotsky

did not mean copying, but transforming the external interaction to a new form of interaction that guides the child's actions (p. 10). Hedegaard (1990) supports the view of Vygotsky that internalization does not directly mirror the external social relations, but is a transformed reflection. Internalization starts on the intersubjective plane where the child interacts with other persons (ideally a child and an adult). On the subjective plane, the child still takes part in a kind of interaction but this time the child takes all the roles in the interaction (i.e. the regulating as well as the action role). Therefore this means that the person's relation to the world changes because his or her possibilities for mental interaction changes too (Luria, 1961; Vygotsky, 1997).

2.4 THE INSTRUCTIONAL TRIANGLE AS AN APPROACH THAT DEVELOPS PROFICIENCY IN MATHEMATICS

Kilpatrick et al. (2001) view the teaching and learning of mathematics as the product of interactions among the teacher, the students, and the mathematics in an instructional triangle (p. 313).

Fig 2.4 Kilpatrick's Instructional triangle



SOURCE: Adapted from Cohen and Ball, 1999, 2000, in press

They discuss the impact on student learning of how teachers select and use content (the teacher-content side of the triangle), how teacher and students interact (the teacher - student side), and how students interact with content (the student-content side). Although they discuss each side of the instructional triangle separately, they emphasize that instruction is not about one side alone but the quality of instruction is about the trilateral interaction among teacher, students and

content. Kilpatrick et al. (2001) point out features in a quality instructional triangle, and I have chosen to discuss the few features that talk directly to my research.

Kilpatrick et al. (2001) say that teachers should create opportunities for learners to learn, so that in each lesson the teacher allows students to engage in and spend time on academic tasks such as working on problems, exploring situations and gathering data, listening to explanations, reading texts, or conjecturing and justifying. Teachers should step back in order to give learners opportunities to think during the interaction among them and the learners. In addition, Sun (2005) states that students should actively engage in maths and teachers should carefully guide this work while being cautious about simply telling students the “answers”. Teachers should ask students to explain how they know and allow them to share multiple ways to solve problems. Essentially students need opportunities to work collaboratively, share ideas, and present ideas. Their understandings should be challenged and students should be allowed to build and construct knowledge for themselves.

Moreover, according to Kilpatrick et al. (2001), in a quality instructional triangle the use of manipulatives allows students to build meaning and make connections. When used well, manipulatives can enhance student understanding. Additionally, manipulatives can, for example, enable teachers and students to have a conversation that is grounded in a common referential medium, and they can provide material on which students can act productively provided they reflect on their actions in relation to the mathematics being taught (Thompson & Lambdin, 1994). Moreover manipulatives also help students correct their own errors. Again, manipulatives can provide valuable support for student learning when teachers interact over time with the students (Ball, 1992) to help them build links between the object, the symbol, and the mathematical idea being represented. However, simply putting manipulatives/ concrete materials on desks or suggesting to students that they might use manipulatives is not enough to guarantee that students will learn appropriate mathematics from them. Fuson & Briars (1990), state that when students use manipulatives, they need to be helped to see their relevant aspects and to link those aspects to appropriate symbolism and mathematical concepts and operations. In addition, Ball (1992) points out that if students do not see the connections among objects, symbols, language, and ideas, using a manipulative becomes just one more thing to learn rather than a process leading to a larger mathematical learning goal.

In a quality instructional triangle, the use of calculator by grade 8 is recommended by many researchers (Groves, 1993; Shuard, 1992). Again Kilpatrick et al. (2001), stress that the use of calculators can enhance conceptual understanding, strategic competence, and dispositions toward mathematics. They emphasize that the use of calculators helps students carry out their problem solving. Students using calculators in mathematics spend relatively less time on algorithms and more on problem solving. Students using calculators are found to possess a better attitude toward mathematics and a better self-concept in mathematics. Students who use calculators often during mathematics lessons do not become reliant on calculators at the expense of other methods of calculations. Groves (1993) has found that calculator use is not detrimental to competency in mathematics. The question therefore is not whether not we should use calculators, but rather how calculators should they be used. Kilpatrick has also found out that instructional emphasis in a calculator-inclusive environment can shift from computational procedures to problem solving and mental arithmetic. Van Galen et al. (2008) have similar ideas about the use of calculators that they can be a means of allowing students to think about the relationships between fractions, percentages and decimals.

Furthermore, Kilpatrick et al. (2001) stress that teachers should manage discourse in their classroom. They must make judgments about when to tell, when to question, and when to correct. Teachers should decide when to guide with prompting and when to let students grapple with a mathematical issue. The point of classroom discourse is to develop students' understanding of key ideas. It also provides opportunities to emphasize and model mathematical reasoning and problem solving and to enhance students' dispositions toward mathematics.

Moreover, Kilpatrick et al. (2001) suggest that teachers should create cooperative groups during the lessons. According to them, cooperative groups are usually groups of three, four, or five students who have been given a task to work on together, with some effort by the teacher to specify the role each child is to play in the group's work (p. 348). Kilpatrick et al. (2001) stress the benefit of creating cooperative groups during mathematics lessons. Cooperative grouping arrangements promote friendships and positive social interactions among students who differ in achievement, gender, race, or ethnicity.

In addition, Mercer, Wegerif and Dawes (1999) pointed out that during this social interaction, children's talk promotes the development of reasoning. They termed this process exploratory talk. They define exploratory talk as

“That in which partners engage critically but constructively with each other's ideas. Statements and suggestions are sought and offered for joint consideration. These may be challenged and counter-challenged, but challenges are justified and alternative hypotheses are offered. In exploratory talk, knowledge is made publicly accountable and reasoning is visible in the talk”, (p. 97).

They mention features of exploratory talk, such as: all partners in the interaction offer their opinions and give reasons to support them, they seek each other's views, all relevant information is shared, all the children are actively involved, their reasoning is often made explicit in the talk, and they come to agreement before taking joint action. Mercer et al. (1999) say that using exploratory talk helps children reason together more effectively. Using exploratory talk for joint reasoning helps children develop better ways of using language as a tool for reasoning individually. Furthermore, Kilpatrick et al. (2001) advise teachers of the need to teach students how to work in the cooperative mode or in social settings. Skills for working cooperatively have to be taught directly, and students need to be prepared for both the social and the cognitive demands of such work.

2.5 LEARNING OF PERCENTAGES AND FRACTIONS

Most research papers dealing with percentages view percentages as one representation of rational numbers (Kilpatrick et al., 2001; Payne & Rathmell, 1975). It follows from this that the skills important for learning percentages and fractions generally relate directly to rational number understanding. Thus it is important for students to know the relations or connections between percentages, fractions and rational numbers. Van Galen et al. (2008) similarly explain that it is essential that students learn to see how fractions, percentages, decimals and proportions are related to each other. Besides the relationship between these forms, it is also important that students understand their difference. It is important that students learn to acknowledge the differences and to see the value of these differences. Students must also know why in one situation you use one descriptive form and in a different situation another descriptive form.

Students should understand that numbers can be represented in various ways; again they should see that $\frac{1}{4}$, 25% and 0.25 are all different names of the same number. Percentages, which can be

thought about in ways that combine aspects of both fractions and decimals, offer students another useful form of rational number. Students often seem to develop fractured, isolated ideas about rational numbers. They seem to believe that percentages are different to fractions, and that the concepts applicable to common fractions, decimals and percentages are all separate and have to be learned one at a time (Kilpatrick et al., 2001), however percentages, decimals and fractions are the same number written in different representation (Kilpatrick et al., 2001). It is therefore important for teachers as mediators to use multiple representations when presenting percentages and fractions, so that students should learn to perceive all rational numbers as being very similar. Van Galen et al. (2008) discuss about the core insight into fractions and percentages.

2.5.1 Core insight into fractions

There are four main features in developing core insight into fractions: 1. learning about fractions as a foundation, 2. developing a network of relationships, 3. models for reasoning with fractions, 4. doing arithmetic with fractions (Van Galen et al., 2008).

1. Learning about fractions as a foundation: According to research, fractions are the foundation of learning about rational numbers (Kilpatrick et al., 2001; Sun, 2005; Van Galen et al., 2008). Van Galen et al. (2008), state that there are two major reasons why fractions should be taught: firstly, students often figures and think in terms of fractions even when fractions are not explicitly involved. For example, if one has to estimate how much 72% of 600 is, one probably figures that 72% is about three-quarters. Then one figures that one-half of 600 is 300 and one-quarter of 600 is 150, so the answer is about 450 (p. 63). Fractions give meaning to percentages and decimals, and they play an important role in mental arithmetic when percentages and decimals are involved (Van Galen et al., 2008).

Secondly a didactic reason is that if one understands fractions; then she/he has a good foundation for percentage, proportion and decimal. Modern mathematics teaching uses situations in daily life of a child that lead naturally to fractions. Example of situations can be where children want to indicate how big something is compared to something else (Van Galen et al, 2008). Moreover, children become familiar with situations involving splitting into fractions when they are very young. Fractions are created when a certain number of things are to be shared out and this number is not equal to the number of people who want to receive a share or to a multiple of the number of people, thus sharing often leads to situations involving fractions (Van Galen et al,

2008). In addition to that, Empson (1999) states that students' informal notions of sharing provide a starting point for developing the concept of rational number. Kilpatrick and others stress that young children appreciate the idea of "fair shares," and they can use that understanding to partition quantities into equal parts. Their experience in sharing equal amounts can provide an entrance into the study of rational numbers, p. 232.

Furthermore Pothier & Sawada (1983), argue that many children when they are very young, are concerned more that each person gets an equal number of things than with the size of each thing. But as they move through the early grades of school, they become more sensitive to the size of the parts as well. Kilpatrick et al. (2001) state that soon after entering school, many students can partition quantities into equal shares corresponding to halves, fourths, and eighths. These fractions can be generated by successively partitioning by half (Pothier & Sawada, 1989), which is an especially fruitful procedure since one half can play a useful role in learning about other fractions. Van Galen et al. (2008) further state that by the time children are seven or eight, most of them already know a number of simple fractions, even if these were not addressed in the arithmetic lessons. Kilpatrick et al. (2001) continue saying that an informal understanding of rational number, which is built mostly on the notion of sharing, is a good starting point for instruction. Confrey (1994) agrees that sharing quantities and comparing sizes of shares provides a foundation in students' concepts of fractions. Equal shares, for example, open the concept of equivalent fractions e.g. if there are 6 children sharing 4 pizzas, how many pizzas would be needed for 12 children to receive the same amount? (Behr, Harel, Post & Lesh, 1994).

Apart from equal shares, measuring also opens the concept of fractions (Sun, 2005). Van Galen et al. (2008), agree that the difference between measuring situations and sharing situations is not absolute, because the result of sharing can also be regarded as a measurement. For instance, when eight people share six pizzas, it can be said that everybody gets " $\frac{3}{4}$ pizza". They are not concerned with exactly how the pizzas are cut; but they use "pizza" as unit of measurement. Fractions should be taught first because understanding fractions is the foundation for arithmetic using percentages and decimals. The concept of fractions links up closely with the way young children think. Thus fractions should be taught first, then percentage and then decimal (Kilpatrick et al., 2001)

2. Developing a network of relationships: Van Galen et al. (2008), state that the knowledge that children develop about the relations between different types of fractions is called a “network of relationships”. Children do not develop a network of relationships around fractions simply by practicing a lot. If the emphasis is on purely numerical fraction problems (without context), there is a high probability that the children will begin using calculation rules that they do not understand. Thus it is essential to develop the network of relationships from contextual situations – situations with labeled fractions.

Van Galen et al. (2008), further point out that, children should work with labeled fractions problems as much as possible at first, which means that they should write down explicitly what every fraction refers to e.g. in the case of a problem about pizzas, one should write “ $\frac{3}{4}$ pizza = $\frac{1}{2}$ pizza + $\frac{1}{4}$ pizza” rather than just writing “ $\frac{3}{4} = \frac{1}{2} + \frac{1}{4}$ ”. Students have to eventually understand that $\frac{1}{2} + \frac{1}{4}$ is $\frac{3}{4}$ whether it concerns pizzas, litres or numbers of children in order to avoid misconception (Post & Cramer, 1987). They must understand, without any context that:

“ $\frac{3}{4}$ is larger than $\frac{2}{3}$. They must reason that $\frac{4}{5}$ is smaller than $\frac{5}{6}$, for example by looking at what remains relative to the whole: $\frac{1}{5}$ and $\frac{1}{6}$. Children should be able to imagine situations themselves that „prove” that an answer is correct, and they should be able to support this reasoning with a drawing for example of a bar or a circle. They should also be able to give examples of what such a bar or circle represents. When children develop a network of relationships they reason as follows: “ $\frac{3}{4}$ is less than $\frac{4}{5}$, because with $\frac{3}{4}$, you have $\frac{1}{4}$ left over, and with $\frac{4}{5}$, you have $\frac{1}{5}$ left over. So you have less left over with $\frac{4}{5}$ ” (Van Galen et al., 2008, p. 81).

Furthermore with fractions, the issue is always to share or divide in a very specific way. The pieces or parts all have to be equal in size. Students should develop skills of “fair sharing”. When students investigate the geometry of fair sharing, they also investigate the relationships between fractions. For example, one can divide a strip into six pieces by first finding “half” and then dividing each half into three pieces. In terms of fractions, this means; $\frac{1}{2} = \frac{1}{6} + \frac{1}{6} + \frac{1}{6}$ (Van Galen et al., 2008).

3. Models for reasoning with fractions: As pointed out in section 2.2.3 of this chapter, Zhang (1997) and Van Galen et al. (2008) argue that bars and number lines are very effective models for reasoning with fractions. The number line is the most difficult yet important model in terms of the measure sub-construct (Behr & Post, 1992; Kilpatrick et al., 2001). Figures 2.2 (i) and (ii)

show and describe a bar and number line that represent double relationship very accurately. On the other hand circles also play a role in teaching fractions. The advantage of a circle is that the orientation of the lines gives the parts their own character.

4. Doing arithmetic with fractions: With regard to doing arithmetic with fractions, according to Van Galen et al. (2008), the first step in doing arithmetic with fractions is comparing fractions. Children must be able to compare simple fractions and say which one is bigger. Similarly, Lesh, Post and Behr (1987) say that children should have to be able to work out why two fractions are “equally big”. This process explicitly involves reasoning, not memorization.

The second step in doing arithmetic with fractions is by adding, subtracting, multiplying and dividing with fractions. Adding and subtracting can also be limited largely to reasoning in a tangible context. To go a step further, addition and subtraction are based primarily on finding the least common denominator; thirds and fourths, for example, can only be added to and subtracted from each other by making them into twelfths. When students can compare fractions by finding the common denominator, it is only a small step to adding and subtracting those fractions. Using a “sub-unit” can be an aid towards routine addition and subtraction with fractions. For example: when students have to calculate the difference between $\frac{3}{4}$ and $\frac{2}{3}$ of a candy bar, it would be convenient if this bar happened to consist of 12 pieces. Using these “pieces” as the unit of measurement, the approach changes to the subtraction problem (9-8), after which the sub-unit can be converted back to the original unit (candy bar): “nine pieces minus eight pieces is one piece, so the answer is $\frac{1}{12}$ candy bar”. In some cases, students can devise a suitable sub-unit themselves (Van Galen et al., 2008).

Multiplying with fractions: as long as multiplication with fractions can be understood as repeated addition, students have little trouble: “four people eat $\frac{3}{4}$ pizza each” ($4 \times \frac{3}{4} = \frac{12}{4}$, which is 3 pizzas). This is a situation in which a whole number is multiplied by a fraction. Situations in which the fractions are the multiplier are not recognized by the children as a multiplication problem: “Apples cost €2.00 per kilo. How much would you pay for $\frac{3}{4}$ of a kilo?” In a contextual situation, students can probably handle this problem, but if the numbers become so difficult that the children have to use a calculator, things can go wrong. For example this question of apples involves multiplying a fraction with a whole number.

Most students understand that $\frac{3}{4}$ is equal to 0.75, but the step towards “0.75 x something” is quite a big one. Multiplying with fractions is especially difficult because students associate multiplying with making something larger. It seems strange to them that the result of $\frac{3}{4} \times 2$ is actually smaller than 2. Van Galen et al. (2008) state “it is difficult for the children to interpret “a certain fraction of something” as multiplication” (p 83). They also have difficulty with purely numerical fraction problems. For instance, “ $\frac{3}{4} \times 2$ ” is a difficult problem because children cannot give a meaning to “ $\frac{3}{4}$ times something”. This is also caused by the fact that they understand multiplication almost exclusively from situations involving repeated addition, and “ $\frac{3}{4}$ times something” cannot be interpreted as repeated addition (Van Galen et al., 2008).

There are two ways to give meaning to problems like $\frac{3}{4} \times 2$. First, it is a good idea to have the students look at multiplication problems on a more formal level, in this case by discussing the commutativity principle. In multiplication, you can switch the terms around without affecting the result. The students are more familiar with a notation such as $2 \times \frac{3}{4}$, but then they understand that this is the same as $\frac{3}{4} \times 2$. The fact that these multiplication problems have the same outcome becomes natural to children, and they do not see any difference between the two multiplication problems. Children have to be made aware that something weird is indeed happening: two different situations can be described with the same multiplication problem (Van Galen et al., 2008).

Another way to give multiplying with a fraction a meaning, is to make sure that students understand multiplication as more than just repeated addition. A multiplication problem can also be a factor problem. Activities involving scale and enlargement are suitable for broadening the concept of multiplication. The important thing is the proportion, not repeated addition (Van Galen et al., 2008).

According to Behr and Post (1992), dividing – similar to multiplying – a fraction divided by a whole number has much more meaning for students than a whole number divided by a fraction. An important reason for this is that students associate dividing with “making smaller”. This association applies correctly to the case of $\frac{1}{2} : 2$, because the fraction you get is smaller than the original one. In the case of $2 : \frac{1}{2}$, this association does not apply, because you end up with a larger number than the 2 you started out with. As in multiplication, students should not associate

division only with repeated subtraction, but also with describing a proportion. In addition, Van Galen et al. (2008) continue saying that there is a good reason why the division sign ($:$) is used - to indicate the scale of a drawing or map. A map with a scale of 1: 50 000 means that every centimeter is 50 000 cm in reality, or half a kilometer, and that every distance on the map is $\frac{1}{5000}$ part of the real distance. In this case, you do not divide with a fraction, but you use a fraction to define a ratio.

The third step in doing arithmetic with fractions is developing language about fractions. Language about fractions therefore must be carefully developed. Sharing offers a good opportunity for this, because sharing situations lead to all kinds of fractions. An important step in developing this language is the transformation of a “certain part of” into a measurement unit. For example, at first we refer to “one-sixth of a pastry bar” and later on to “ $\frac{1}{6}$ pastry bar”. This is followed by the expansion to fractions with numerators that are not equal to 1, where $\frac{5}{6}$ stands for five times $\frac{1}{6}$ pastry bar. It is important that students wean themselves from making and counting parts as soon as possible. They must start to see fractions as descriptions of part-whole proportions, in which $\frac{2}{5}$ refers to the proportion “2 out of 5”. Based on that proportion, the students can understand that $\frac{2}{5}$ is smaller than $\frac{1}{2}$ (Van Galen et al., 2008).

2.5.2 Core insight into percentage

Percentages are numbers on a fixed scale that runs from 0 to 100. It is not coincidental that the scale goes to 100, instead 75 for example. This is because in this way it is possible to place percentages neatly into our number system so that we can easily convert them into decimals. Moreover, a scale from 0 to 100 is one with which we are very familiar. When we see the figure “64%”, most people have a clear image of how much that is with respect to the whole; it is more than half, but not that much more. The scale from 0 to 100 is precise enough for most situations (Van Galen et al., 2008).

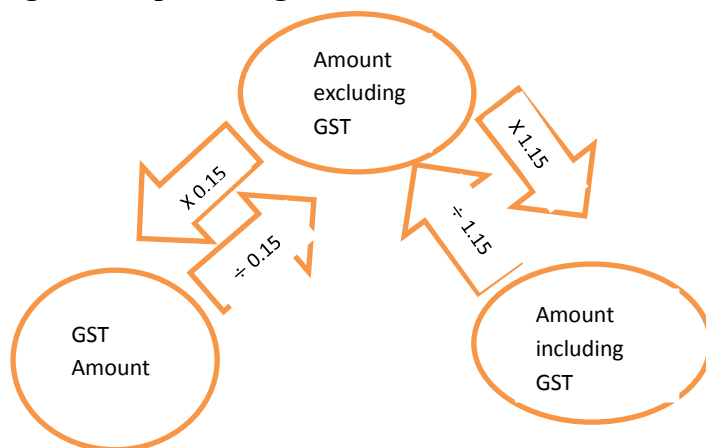
According to Van Galen et al. (2008), there are three features in core insight into percentage, the first one is the two approaches to introducing percentage, the second one is models for percentage and the third is calculating with percentages. The introduction of percentages often takes place using the example of discount percentages. A justification for this is that the children are already familiar with the context of sales and discounts and also that this means beginning

with rounded-off percentages such as 10%, 20% and 25%, which makes it easy to establish a direct relationship with fractions. However percentages derive their right to exist from the limitations of regular fractions: fractions are difficult to compare with each other, and the scale that they provide is rather unrefined.

The first feature in core insight into percentage is the two approaches to be used when introducing percentage. The first approach is to introduce percentages as a new way of writing down part-whole relationships, in addition to the fractional notation that the students already know. During this process, the usefulness of percentage notation is not addressed directly. The second approach is to introduce percentages as an invention, or the answer to a problem. Teachers should allow the students to make the leap to percentages themselves.

The second feature in core insight into percentage is the models for percentages. According to Van Galen et al. (2008), the most important model for percentages is the bar. As stated in section 2.2.3 of this chapter, the advantage of the bar is that it has “body” – area. For children, this makes it easier to talk in terms of “the whole” and “the so-much part” of the whole. The percentage bar supports flexible mental arithmetic with percentages. Besides the bar, the ratio table can also be used as an arithmetic aid, but it is only effective when working together with the bar. If you want to choose only one of the two for a standard aid, then you should choose the bar. Moreover, Nic (2012) states that a well-thought-out diagram helps students in understanding and retaining new knowledge. She has designed a diagram that is very useful and effective for answering questions about increasing and decreasing by a given percentage. Nic’s diagram is as follows: (posted on 1 March, 2012; A picture is worth).

Fig 2.5 Diagram for percentages increase and decrease



The third feature in core insight into percentage is calculating with percentages. And the first point discussed about calculating with percentages is about conversion. It is important that students convert percentages into fractions and the reverse. Students must convert between the two forms because percentages and fractions are often used interchangeably in daily life; instead of “72%”, someone might say “nearly three-fourths”. It is also important because the process of converting the percentage into a regular fraction makes students familiar with the magnitude of percentages and therefore contributes to their numerical insight. Conversion should not only involve rounded percentages such as 25%, 75%, 20% etc., but it should involve also the interpretation of percentages such as “64%” as “more than half, but less than $\frac{2}{3}$ etc.”. Moreover the relationship with fractions must be established by means of reasoning.

The second point discussed in calculating with percentages is exercises that involve calculations such as 23% of 674. Exercises that involve rounded percentages such as 20%, 25%, 60% and 75 and not 23% or 79% in one respect they negate the essence of what percentages have to offer, because percentages actually help us to understand proportions with large or difficult numbers. It is important to use both types of exercises - those that involve rounded percentages and also those exercises that involve percentages that aren't nicely rounded. This develops estimation skills in students. In daily life, estimation plays a more important role than exact arithmetic. Moreover, children develop a feeling for numbers through estimation. Nevertheless, it is also important that students at a certain point learn to solve percentage problems with the calculator. Many calculators have a percentage button, but the way this button works is not always

immediately obvious. If you only occasionally have to calculate a percentage, it is easier to use the standard buttons for dividing and multiplying. Doing arithmetic on the calculator therefore amounts to applying standard procedures. It is important that students use these procedures insightfully.

In addition, three ways on how percentage can be asked are firstly, by calculating a part, secondly is by calculating a percentage and thirdly is by calculating the whole. If a question is for example, how much is 12% of €750? Such a question can be answered by first figuring out how much 1% of this amount is and that is 7.5 ($750 \div 100$). Then that amount is multiplied by the percentage and the answer is 90 (7.5×12). This means that the following steps are carried out sequentially. Another way by figuring a part is using the table ratio as follows (Van Galen et al., 2008, p. 93).

Fig 2.6 Ratio table

100%	1%	12%
€750.00	€7.50	€90.00

When students understand that a percentage can be understood as a factor, the two steps can be combined into a single operation: multiplying by the decimal equivalent of the percentage, i.e. the whole multiplied by the factor and you get the part (Leder, 1992).

The types of questions that require figuring the percentage are for example: “In a school, 60 of the 150 parents are opposed to the plans of the school administration. What percentage of the parents is opposed?” One can use the following reasoning: there are 150 parents, 1% of this number is 1.5. In order to know how much 60 of 150 is in percent, one must know how often 1.5 goes into 60. And if that part is divided by 1% of the whole, the answer is the percentage. The common way is to divide the given amount by the total and multiply the result by 100. The ratio table can also be used.

The type of questions that require figuring the whole is such as: “during a sale, you get a 15% discount on an MP3 player which originally cost €82. How much will you have to pay?” one can also answer this question by first calculating how much 1% is, i.e. €0.82, then 15% is 15×0.82 ; then you will pay €82,00 - €12,30 is €69.70. The other way is by converting the percentage to a decimal, so one can calculate 15% of 82 by multiplying 82 by 0.15, then one can calculate the price of the MP3 player with 15% discount as 0.85×82 . The fact that this method of calculation provides the same result is not difficult to understand: $85 \times €0.82$ compared to $0.85 \times €82$, but conceptually there is a major difference. In the first case, we make the step via 1%, where the multiplication – so many times 1% - can be interpreted as repeated addition. While in the second case, one views the percentage as a multiplication factor. The latter assumes a relatively abstract, formal way of looking at percentages. For most children in primary school, this is asking too much, but the better mathematics students can make this step (Van Galen et al., 2008).

2.6 CONCLUSION

The literature in this chapter confirms that the use of multiple representations in mathematics can promote learning of percentages and fractions. A definition of multiple representations is given; and the effect of using multiple representations when teaching mathematics is also stated. The chapter further explains that the appropriate multiple representations in the teaching of percentage and fractions are real world objects, manipulatives, spoken symbols, written symbols and diagrams.

Moreover, literature in this chapter explains learning as the product of interaction between social processes and artifact mediation. It elaborates on how the interaction between the learning child and MKO (teacher and other capable learner) creates a zone of proximal development. Tool mediation also stimulates learners to learn mathematics. Furthermore, the literature explains the instructional triangle as an approach that develops proficiency in mathematics. The chapter concludes with literature that specifically explains in detail the learning of percentages and fractions by pointing out features of core insight into fractions and percentages.

CHAPTER THREE

METHODOLOGY

3.1. INTRODUCTION

This chapter restates the research goal of this study. The study is a qualitative investigation that is embedded within the interpretive paradigm, the aim being to investigate how multiple representations may be used to enhance the learning of percentages and fractions through social interaction by the grade 8 learners in a Namibian school. It is a case study of 9 learners who were involved in the implementation of a learning programme. This learning programme was designed by me the researcher, and the design process is discussed in detail in this chapter. The learning programme was designed in such a way that the study focused on a specific issue in mathematics i.e. learning through using multiple representations when presenting percentages and fractions in a social interaction.

In this chapter, I have mentioned and discussed the techniques/tools used to collect data. They were as follows: focus group interviews, observations, learners' work, journal (notebook) and tests (pre-and-post diagnostic tests). It further elaborates the sampling and research site of the study. This chapter gives a brief description on how the data were analyzed. Furthermore, in this chapter I explain how I overcame some ethical issues and I also discuss some threats to validity and reliability and how I overcome them. I indicate some challenges that compromised the research process. I also explain what I would do differently if I had to do this research over again.

3.2. RESEARCH ORIENTATION AND THE CHOSEN PARADIGM

3.2.1 Qualitative investigation within the interpretive paradigm

This study is a qualitative investigation and it was conducted within the interpretive paradigm. Cohen, Manion, and Morrison (2007), discuss qualitative research as that which draws the researcher into the phenomenological complexity of participants' worlds: here situations unfold, and connections, causes and correlations can be observed as they occur over time (p. 397). Moreover, the interpretive paradigm gives the researcher an opportunity to understand and interpret the world in terms of its actors (Cohen et al., 2000 p. 180). In addition, the researcher searches modes of explanation from the data themselves, be they descriptive, analytical or conceptual (Cohen & Manion 1980 p. 28). Using this paradigm helped me to engage deeply with

the data on how the participants interacted, interpreted and related to the work with multiple representations in presenting percentages and fractions.

3.3. RESEARCH METHODOLOGY

3.3.1 Case study

According to Bell (1993), a case study gives an opportunity for one aspect of a problem to be studied in some depth within a limited time scale. My experience was similar to what Yin (2006) have states: “the strength of the case study method is its ability to examine, in-depth, a “case” within its “real-life” context”, (p. 111). This is a case study involving nine grade 8 learners who were involved in the implementation of the learning programme. Bassey (1998) adds that a case study allows the researcher to extract meanings and to understand ideas more clearly. So this study focused on a specific issue in mathematics, i.e. learning through using multiple representations when presenting percentages and fractions in a social interaction.

3.4. SAMPLE AND RESEARCH SITE

The research was carried out at one school in the Kavango region of Namibia. The school currently has 1234 learners from grade 5 to grade 10. Each grade 8 class has 44 learners. I purposefully selected one grade 8 class, of mixed gender and having with various learning abilities. Purposeful sampling is a strategy to choose small groups or individuals likely to be knowledgeable and informative about the phenomenon of interest. It further refers to a selection of cases without needing or desiring to generalize to all such cases (McMillan & Schumacher, 2001).

I chose the sample from those to whom I had easy access. I purposefully selected this class because I am their subject teacher and the topics on the intervention programme are in their syllabus. It was therefore possible for me to observe and interact with familiar participants for a longer period. The other classes were not taught using the intervention programme because of time constraints. If the study shows that the programme promotes the learning of percentage and fractions, then this will be a stepping stone to implement it to the whole grade. The whole class participated on the implementation of the intervention programme, but only nine learners were purposefully selected for the focus group interviews. I obtained informed consent from these 9 participants and allowed any of these learners approached to choose not to participate in the research interviews.

3.5 PROFILES OF PARTICIPANTS

For the purpose of anonymity I have decided to call my participants as letters. Each learner was given a letter as their names, so I have learners A - I. Learners A, C and D are girls and learners B, E, F, G, H and I are boys. As I have mentioned, my sample was a purposeful sample. I did not select these learners at random but because of the circumstances already described, I used them as my participants.

Learner A is the youngest participant in the group. Her favourite subject is English. She said she likes mathematics but the problem is that the subject is difficult. She does not like to raise her hand even if she knows the answer she is a little bit shy. She obtained a D in mathematics last year, which is an average symbol.

Learner B is a boy, a top learner in terms of performance among the other learners. He performed very well last year in mathematics, his symbol being an A. He loves mathematics and he is active in the class. He likes participating during the lessons and most of the time he has the correct answer.

Learner C is a girl who likes participating during the mathematics lessons. Her favourite subject is Natural Science. She got a D symbol last year in mathematics. She likes to raise her hand when she has the correct answer and she also likes to go and write her explanations on the chalkboard.

Learner D is a girl who is an outspoken and active learner among the participants. She likes mathematics, she likes participating in the class, and she likes explaining her answers when giving feedback. Last year she obtained a B symbol in mathematics.

Learner E is a boy who is very quiet in the class. Even though he is a quiet learner among others, he does not often concentrate in the class. He is the lowest learner among the participants in terms of performance. He doesn't like mathematics at all. To him mathematics is the most difficult subject he has ever faced. He doesn't have a favourite subject, because he does not understand them all. He obtained a D symbol in mathematics last year.

Learner F is an active boy. His favourite subject is Arts and Social Science. And he also likes mathematics. He participates during mathematics lessons. He likes asking questions and likes to explain his work. Last year he obtained a C symbol in mathematics.

Learner G is a quiet boy. His favourite subject is English. He does not like to talk, especially explaining his answers to the others. He prefers not to work with others. He performs well in mathematics, with his symbol in mathematics last year being B.

Learner H is an active boy, he likes to participate in class, asking questions and giving his ideas. His favourite subject is Natural Science and Arts. He obtained a C symbol in mathematics last year.

Learner I is a humorous boy, who likes to be funny sometimes in the class, but at the same time he participates by discussing in groups, asking questions and answering questions. His favourite subject is mathematics even though he says it is difficult. He obtained a D symbol in mathematics last year.

3.6 RESEARCH DESIGN

3.6.1 Research plan and Research Journey

I designed the following plans and programs for the study: (a) Permission letters; (b) Pre-and-post diagnostic tests; (c) Intervention programme; (d) Learners' activities; (e) Journal (note book) and (f) Interview questions.

At the end of last year (2011), I consulted with the intended participants and at the beginning of this year I had two letters to obtain permission from participants' parents and the school's principal. The first letter was given to the principal of the school to allow the study to be conducted at the school (Appendix G); the second letter was given to the participants' parents for their permission (Appendix H). Early this year I developed, (a) pre-and-post diagnostic tests, (b) the intervention programme, (c) learners' activities, and (d) interview tasks, and piloted them.

The pre-and-post tests were developed by extracting questions from two Namibian Mathematics textbooks, which are: "Y = mx + c Grade 8-10 (D'Emiljo, 2007); Math for life Grade 7 (Silver, 2007)". These questions were based on the topic of fractions, percentages and decimal numbers. Both the pre-and-posttests consisted of 13 similar questions. All these questions required learners to use multiple representations appropriate to the learning of percentages and fractions. Moreover the 13 questions were designed to assess five dimensions of percentages and fractions. From question 1 to question 4 the questions consisted of converting between percentages and fractions; while questions 5 and 7 were about expressing one quantity as a percentage of another.

These quantities were in the following units: N\$ (Namibian dollars), cents and grams. Questions 6 and 8 required working out the percentage of a quantity; and questions 9, 10, and 11 were on decreasing and/ increasing a quantity by a given percentage. The last questions - 12 and 13 were on working out the VAT of a quantity and include or exclude it from the original price.

I designed an intervention programme, as I have already stated in chapter two which includes multiple representations appropriate to use in the learning of percentages and fractions. I used Lesh's model of representations to help me design my intervention programme. The entire modification of Lesh's model is elaborated and explained in more detail in chapter two, section 2.2.3. The intervention programme was spread over a two week period that consisted of eight lessons of one hour each. For each lesson, there were learner's activities to do, which related to the learning objectives of each lesson (See Appendix B). Learners were given feedback after each activity was marked. Throughout the lessons I used a journal (notebook) to note any action of any learner related to the study. The programme began on the 26th April 2012 and continued until 05 May 2012 but a week before this date, the participants wrote the pre-diagnostic test, while the post test was written a day after the programme.

Alongside the intervention programme, I developed 11 boxes. In each of these boxes, there were 100 objects (manipulatives and real world objects). These various little objects served as the unit segment of each box. Learners were expected to work in pairs and used these boxes to make meaning of what a fraction or percentage is, by taking out a portion of the objects inside the box and making a fraction from it. When they had made a fraction from the objects, learners were asked to tell a story on how they made their fractions, write their fractions on the chalkboard and read their fractions. The boxes were as follows: one contained 100 stones, two contained 100 bottle tops, another contained 100 N\$1 coins, another had 100 almond seeds, another had pieces of pipe, another had pieces of plastic bottles, another had necks of plastic bottles, another contained pieces of boxes, and the last two held pins. Learners made use of the 11 boxes on three lessons out of the eight of the learning programme. Learners were expected to count out the number of objects in their boxes, to explain in detail what they were doing, and to write their fractions of objects removed from the box.

I designed two focus group interviews, the first of which was held before the programme. The purpose of this interview was to gather data of learners' prior knowledge on the topic of

percentages and fractions. There were five pre-determined questions in this interview but I left space to add questions in response to answers that I would receive. The first two questions were included for the purpose of writing the profile of my participants. The five questions were as follows: What is your favorite subject and why? Do you like to study mathematics? If yes why? If no why? Percentages and fractions are one of the topics in mathematics, define percentages and fractions and give an example of your definition? What did you learn last year on the topic of percentage and fractions? Give examples. Do you have a problem/or not in the topic of percentages and fractions? If yes, what is/are the problems you have in the topic of percentages and fractions?

The second focus interview I conducted after the programme. The main reason for the second focus group interview was to gather data on the experiences learners learn/or did not learn through the programme. This interview had five pre-determined questions as well, but I left space to add questions during the interview process. The questions were as follows: How was the programme? What did you enjoy/-not enjoy during the programme? Define percentages and fractions and give examples: What did you learn/or not learn from the programme? For those who had a problem before the programme, did the programme help you/or not with the problem you had? I found it helpful to conduct the focus group interview of nine participants immediately one day after the programme.

3.6.2 Data collecting techniques/tools

Lankshear & Knobel (2004) state that when collecting data through qualitative investigation it is essential to use multiple sources of evidence. Multiple sources of evidence can be used to explain complex phenomena, because “the more researchers examine their data from different viewpoints, the more they may reveal – or construct – participants’ complexity”. For the purpose of „triangulation“ (Cohen et al., 2000), I used multiple methods to collect the data. The data was collected using the following techniques and tools:

1. Pre-and post-diagnostic tests
2. Observations
3. Journals
4. Learners work
5. Interviews

(a). Pre and post diagnostic tests: the pre and post diagnostic tests were developed to measure participants' percentage and fractions skills and concepts. The pre diagnostic test was mainly designed to reveal learners' prior knowledge on the topics of percentage and fractions. These results provided the baseline information that made it possible to identify what the children learned through their work in the programme. The post diagnostic test was designed to analyze the effects of the use of multiple representations in the learning of percentage and fractions. The fact that these two tests didn't have the same questions, made it challenging for me to compare them for the purpose of analysis.

(b). Observation: Observations provide an opportunity for the researcher to "gather live data from live situations" (Cohen et al., 2000, p. 305). Throughout the study I used observations to help me collect quality data. The observation data were generated as the participants engaged in the activities of the programme. All the lessons were videotaped and through that it was possible for me to see how multiple representations developed proficiency in learners on the topic of percentages and fractions. Cohen et al. (2007) adds that observational data enables researchers to understand the context of programmes, to be open-ended and inductive, to see things that might otherwise be unconsciously missed, to discover things that interview situations, to move beyond perception-based data (e.g. opinions in interviews) and to access personal knowledge. I had someone who walked around with the video, focusing it on the interactions between the teacher and the learners. Even though at first the video distracted learners, when they got used to it they started to concentrate.

(c). Journal: Throughout the implementation of the intervention programme of this study, I made use of a personal journal to make field notes on each and every lesson in the programme. According to Lankshear & Knobel (2004), collecting data as qualitative investigation can be through observation and can include written records of direct observations, e.g. field notes made in the "heat of the moment" as things are happening (p.175). Burton, Brundrett, & Jones (2008) mentioned that field notes may be on pre-determined themes or in response to events as they unfold (p. 100). Thus in this journal, the moment an action from the participants was observed I immediately recorded the information found related to the research questions. I managed to record the notes mostly when learners were either discussing in groups, or using multiple representations, or and when they were doing their calculations on the calculators or in their

books. The paramount concern in field notes is that important events or occurrences are captured and that the chronology is captured so that the evolution of patterns can be determined (Burton et al., 2008).

(d). Analysis of learners' work: The learning programme included a number of learning and teaching activities on the topic of percentages and fractions using multiple representations, with which learners were engaged. This documentary data was analyzed to provide further insight into the learners' experiences of the programme.

(e). Focus group interviews: I conducted two focus group interviews with the selected learners involved in the programme. The first of the interviews I conducted before the programme, while the second was conducted after the programme. According to Ashworth (2001), a research interview should be regarded as a conversational partnership in which the interviewer assists a process of reflection. Therefore, the main reason for the first interview was to find out learners' prior experience on the topic of percentages and fractions, while the reason for the second interview was to gather deep and rich data by asking the participants about their experiences regarding the use of multiple representations in learning about percentages and fractions.

3.6.3 Summary of design and tools

Phase	Techniques/tools	Purpose	Data and analysis
I	Diagnostic tests	Identifying understandings, misconceptions and misunderstandings that learners might have on the topic of percentages and fractions.	Qualitative results. The data was used to determine learners' prior knowledge for the programme design and implementation.
II	Classroom observations	Video recording of the classroom process.	„Thick description“ on how multiple representations promotes or did not promote the learning of percentages and fractions. Data obtained from observation schedule.
III	Journals	To record and reflect on any information gathered on each lesson observed.	Ongoing reflective qualitative data. Coding in themes.
IV	Analysis of learners' work	Engaging the learners in using multiple representations during the learning of percentages and fractions.	Qualitative results in the form of analyzing the learners' work.
V	Focus group interview	To explore how learners feel about the use of multiple representations in the learning of percentage and fractions.	Qualitative results in the form of interview transcripts.

3.7 DATA ANALYSIS PROCESS

Data were analyzed through comparing the two pre-and-post diagnostic tests. A bar graph of the two tests was used to summarize the data findings. An analysis was conducted into learners' usage of real world objects, manipulatives, diagrams, written mathematical symbols and spoken symbolic representations of percentages and fractions through social interaction and learners' written activities, through extracts from video transcriptions, through themes emerging on the teacher's field notes in the journal and through extracts from transcription of the interviews.

Within the five multiple representations mentioned above, an analysis of the learners' use of various strategies to calculate equivalent fractions, convert between percentages and fractions, calculate the percentage of a quantity, express one quantity as a percentage of another, and increase and decrease a quantity by a given percentages were observed and coded.

3.8 VALIDITY IN DATA COLLECTION AND ANALYSIS

According to Cohen et al. (2007) when using qualitative data, validity is essential and should be addressed through the honesty, depth, richness and scope of the data achieved, the participants approached, the extent of triangulation and the disinterestedness or objectivity of the researcher. Furthermore, Cohen et al. (2007) defines triangulation as the use of two or more methods of data collection in the study of some aspect of human behaviour. Newby (2010) suggests that triangulation offers a way of ensuring validity in interpretive research. By comparing various independent sources of data, it is possible to triangulate the findings and thus ensure that the study is valid. Moreover, Anderson (2000) adds that triangulation helps eliminate bias and can detect errors or anomalies in the researcher's findings. To validate the research findings of this study, I used various methods such as interviews, tests and observations to collect and analyze the data. I therefore discuss how I ensured that the techniques or tools used to collect data for this study were valid.

Focus group interviews were one of the tools I used. Interviews use a set of standard questions but allow the researcher to add questions in response to answers that have been given (Elton-Chalcraft, Hansen, & Twiselt, 2008). In terms of validity, this has the advantage of not assuming that the researchers know all the right questions to ask before she/he starts. In fact it happen that, from the pre-determined questions of the focus group interviews of this research, I came up with re-phrased questions, supportive questions, and clarifying questions that led me to collect in-

depth experience data from participants. Again the relationship that I had with the participants influenced the answers I got. Since I made the participants aware of my studies, participants were free to express their feelings out and also to be honest.

The other technique used was observation. Cohen et al. (2007) affirms that there are several threats to validity and reliability in observation-based research such as: the researcher, in exploring the present, may be unaware of important antecedent events, and for this reason I chose to use a journal during all the lessons to note down all events that were related to my study. It is very helpful to gather data from various techniques rather than from just one technique; the presence of the observer might bring about different behaviours, the researcher might “go native”, becoming too attached to the group to see it sufficiently dispassionately. To address this, Newby (2010) suggests triangulation of data sources and methodologies, which I did. I analyzed by extracting data from all the techniques I used, and this made it possible for me to obtain in-depth findings.

The other technique used to collect data was through pre-and-post diagnostic tests. Taking into consideration the range of issues which might affect the reliability of the test, there were still threats to reliability using this technique. These threats are much the same as the one identified by Cohen (Cohen et al., 2007). The first threat is: individuals - some individuals might be forgetful compared to others. In the first test (assessing prior knowledge), an individual was assumed not to know the concept when he/she could have merely forgotten the ideas. Nevertheless the tests consisted questions that required learners to remember the learned concepts. On top of that some questions in the test were also asked during the first focus group interviews to help me gather data that is valid, so even if the child forgot during the test, during the interview she/he had a further opportunity to remember. The second threat is that participants may vary from one question to another - a participant may have performed better with a different set of questions which tested the same matters. Cohen et al (2007) argues that two questions which, to the expert, may seem to be asking the same thing but in different ways, to the students might well be seen as completely different questions. Since my study focus on multiple representations, I deliberately set questions about the same concept in different ways, the main reason of which was to validate the data of my study. The most essential way to validate data is through triangulation which I then did.

3.9 ETHICAL CONSIDERATIONS

According to Stake (2000), “the value of the best research is not likely to outweigh injury to a person exposed; the researcher’s manners should be good and ethically strict”. Elton-Chalcraft et al. (2008) encourage the researcher to follow some ethical framework produced by a professional body, university or college. Nevertheless they have five important ethical issues which are essential for the researcher to consider and they call it the 5 Cs, which are as follows:

1. *Conduct* before, during and after the research has been undertaken;
2. *Confidentiality* of responses and in terms of identifying participants within the research;
3. *Consent* and permissions to undertake the research possibly from a governing body;
4. *Choosing* a methodology that is fit for the purpose;
5. *Contextualizing*.

I therefore took the following ethical issues into account but in another manner. The first ethical issue was to get permission. Since this study involved grade 8 learners, I wrote a letter to the school principal, and to participants’ parents asking for the consent to carry on with this research on the participants mentioned above. I asked permission from the learners to carry interviews on them and to observe them.

The second ethical issue was about participants’ rights. To ensure that the participants participated freely without fear, the participants were told that parents may withdraw their children from taking part any time they wish. Again during the analyses of the data it was the participants’ right to choose what was not to be written.

The third ethical issue was about confidentiality; I tape recorded all the interviews and transcribed them. I safely stored the disc as well as the transcripts for reference. Yet again I videotaped all the lessons and stored the video discs for reference.

The last ethical issue was about anonymity, participants were made aware that during the analyses of the data no names were to be mentioned. I used false names to identify my participants.

3.10 CHALLENGES ENCOUNTERED

One of the challenges I faced was the period of implementing the programme - the intervention programme was conducted during the school holiday, and during this period parents expect their children to help them with household chores. Each and every day participants did not come on time, and sometimes some did not come at all. It was difficult to analyze some of the participants' lesson activity because of their attendance, and for validity purposes I needed to have data of each learner in each technique I used. Thus this situation forced me to take only nine learners' lesson activities and focus group interviews because they were the one who attended the entire implementation of the programme.

The other challenge was that the focus of my research was only on percentages and fractions and not on decimal numbers. The fact that these three representations are forms that represent the same number means that one cannot teach them separately. Even though important themes on decimal numbers arose during my analysis, it was difficult for me to discuss about it, because I didn't pay much attention to it during the implementation of the programme, learners activities and the pre-and-post diagnostic tests.

Another challenge was that questions on the pre-and-post diagnostic tests were not the same, they were just similar. During the analyses of the two tests, it was not easy to conclude that learners did better in one test than in another test because of the fact that the questions were not the same. Therefore I could not make conclusions using only the tests, but conclusions resulted only after I compared data from all techniques that I used.

If I had to do this research over again, there are three major aspects that I would take into consideration. The first one is that the period of the implementation of the programme should not be during the school holidays. It should be a period where learners are not affected because of household chores. The second aspect would be to expand the programme by including decimal numbers in the design of the programme, the learners' activities and also in the pre-and-post diagnostic tests, because percentages, fractions and decimals cannot be taught separately. The last aspect is that questions on the pre-and-post diagnostic test should exactly be the same, so that it will be easier to analyze learners' performance.

3.11 CONCLUSION

This chapter included the discussion of the methodology and the procedures followed in this research study. The chapter focused on the research orientation in which I described the research paradigm used in this study i.e. interpretive paradigm. The chapter briefly explained that the research is a case study. This was followed by the sample and research site, I also included a brief profile for my participants.

Furthermore, the chapter also included a detailed description of the research tools used for data collection. These are pre-and-post diagnostic tests, observations, learners' activities, a journal and focus group interviews. Thereafter, a description of the data analysis process was given. In this section, I explained how I analyzed my data. The chapter also includes a discussion on validity in data collection and analysis. It includes a description of ethical issues pertaining to this study. This chapter was concluded with reflections on the challenges that I encounter during the research process. Despite the challenges encountered during the research process, it was worth continuing. I gained a lot of experience which can be used to refine future research procedures in this field.

CHAPTER FOUR

DATA PRESENTATION

4.1 INTRODUCTION

This chapter presents the data analysis of this study and addresses the main research question posed in this study – whether a teaching and learning programme using multiple representations in various social settings that place emphasis on student centered learning, promotes, or does not promote the learning of percentages and fractions. This main question has three other sub-questions. Data from both the pre-and-post diagnostic tests is presented on the bar-chart to show the learners’ results from the pre-and-post diagnostic tests, as a summary of findings. However, presenting data on a bar-chart only does not show sufficient validity for my research results. Therefore a more comprehensive response will then be discussed based on data from all the research techniques I used. Data from observation, interviews, learners’ work, the journal and again the diagnostic tests will provide answers to the three sub-questions. In this data presentation, extracts from focus group lessons and interviews; extracts from the learners’ work; and an analysis of each participant’s test results will be provided. The data show how the programme promoted the learning of percentages and fractions. The following sections will answer each one of the sub questions separately.

- (a) How did the use and manipulation of multiple representations as tools by the learners influence the learning process?
- (b) How did the teacher influence the dynamic learning interaction process in ways that promoted or did not promote learning?
- (c) What are the learners’ interactions that were not strongly mediated by the teacher that promoted or did not promote the learning of percentages and fractions?

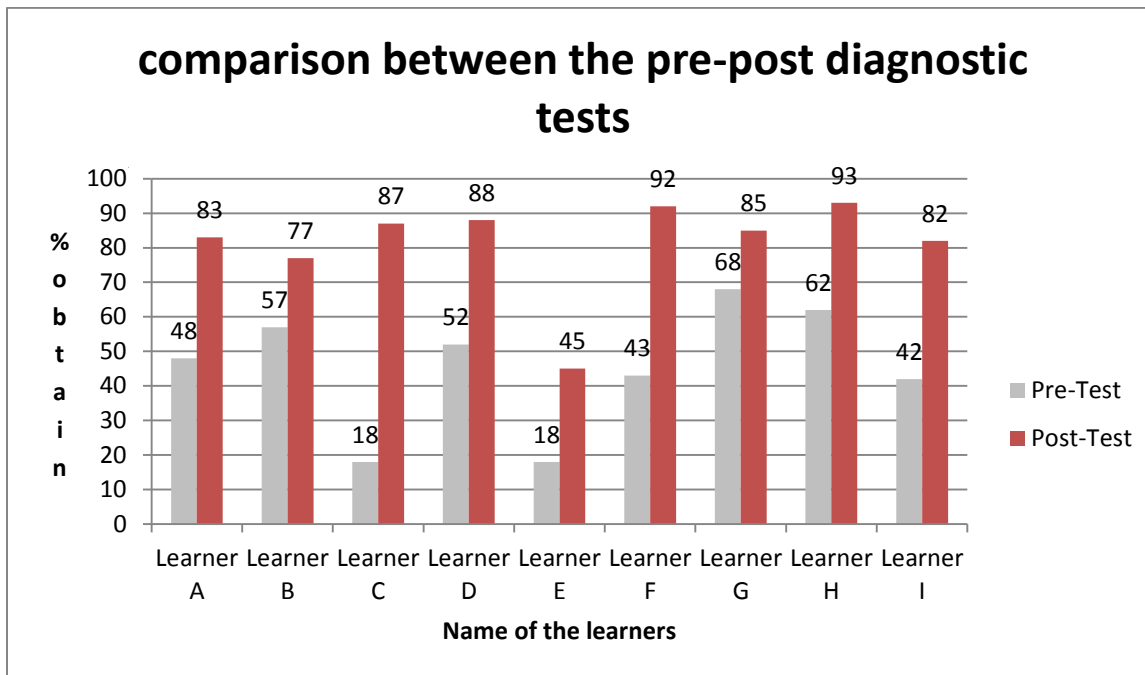
First, a summary of the formally assessed learning will be given, based on the results of the pre-and post-diagnostic tests.

4.2 SUMMARY OF ASSESSED LEARNING OF PERCENTAGES AND FRACTIONS

In order to quantify the hypothesis that learners would perform better in mathematics and gain a higher understanding of the various concepts required of them in the curriculum, the understanding of percentages and fractions acquired from traditional teaching methods was compared to the understanding acquired from teaching using multiple representations and learner centered social settings in a selected group of learners. The learners were first taught the material based on the traditional teaching methods of using a textbook, and with the teacher presenting the material to learners without an emphasis on group learning. After this, the pre-test was administered. These same learners were then taught the material with the use of multiple representations in special social settings to promote learner-centered interaction. A post-test was then given. A comparison was made between the performances on each test in order to obtain a rough global measure of whether this programme did indeed promote the learning of percentages and fractions.

The data shows that the use of multiple representations by grade 8 learners in a setting that encouraged social interaction did indeed promote the learning of percentages and fractions. The bar graph in figure 4.1 displays how the learners performed in the two diagnostic tests. It is evident from this graph that the programme promoted the learning of percentages and fractions.

Fig 4.1 The comparison of learners' results in the pre-and post-diagnostic tests



The analysis of each sub-question below will present data that shows how the programme promoted the learning of percentages and fractions.

4.3 HOW DID THE USE AND MANIPULATION OF MULTIPLE REPRESENTATIONS AS TOOLS BY THE LEARNERS INFLUENCE THE LEARNING PROCESS?

The use and manipulation of multiple representations as tools by the learners positively influenced the learning process as follows:

- (a) Learners were able to look at representations in „useful“ ways
- (b) Multiple representations could make some aspects of a concept clear
- (c) Multiple representations enabled learners to correct errors
- (d) The use of calculators contributed to learners' conceptual understanding of percentages and fractions

4.3.1 Learners were able to look at representations in ‘useful’ ways

The multiple representations used during the programme were real world objects; written representations; spoken representations; manipulatives and diagrams. During the programme, learners looked at these representations in useful ways and this contributed to their learning.

Real world objects and manipulatives (the 11 boxes) helped bring meaning to the learners’ use of both oral and written symbols of percentages and fractions. Each of the 11 boxes contained 100 real objects (see the description of the 11 boxes in chapter three, section 3.5.1), and learners were asked to remove a portion of the 100 objects in each box. By doing so they were able to describe the number of objects remaining as a fraction of the full box. They were able to reason why a fraction is part of a whole. The 100 real objects in the boxes are objects that learners know and play with, but the fact that these learners were removing a portion of their 100 objects in a box, helped them to look at these representations in useful ways.

Before the beginning of the learning programme, seven learners showed that they were able to colour in identified parts out of a whole and write a correct fraction to describe this (see learners’ answers on table 4.1(i) below); these learners were also able to write down the correct fraction for the shaded part/s on a diagram (see learners’ answers on table 4.1(ii) below); and they were able to find the percentage of each of the various diagrams which were shaded (see Appendix C, question 2(a)-(e)). Even though these learners showed the correct usage of oral and written symbols of fractions and percentage, two of the same learners could not present an accurate description of what a percentage is. The extract from lesson one shows some learners’ responses when they were asked what percentage is: **Extract 1 Lesson1**. *Learner C: Percentage is hundred. Learner G: A fractions out of hundred. Learner A: Anything out of hundred. Learner E: Any number out of hundred. Learner I: Percentage is over hundred (line 2).*

Again, in the first focus group interview, three other learners from the above mentioned seven showed that they were not able to explain what a fraction and percentage is. Learners were asked to define percentage in their own words and give examples. Three of the learners’ responses were as follows:

Extract 2 interview1. *Teacher: In your own opinion, what do you understand by the word percentage? Learner D: Percentage is one hundred number only. Teacher: One hundred number only, what do you mean? Learner D: Percentage is.....ah is hundred. Teacher: This*

hundred is what? Learner D: This hundred is when I use that unit ma, madam, this unit that look like this, (learner D, demonstrating this unit (%) in the air). Teacher: Give me an example. Learner D: 55%. Teacher: mmm. OK, who else has another idea on what percentage is? Learner B: Percentage is number which is over hundred. Teacher: What do you mean by the word over? Learner B: It is like, ah, ah, you have one number up and one down. Teacher: What are those numbers? How are they related? Learner B: It is any number which can be one, a small on t, ah up and one down. Teacher: Give me an example. Learner B: Thirteen over hundred. Teacher: Oho, ok. Ah, who wants to add in his ideas? Learner F: A fractions out of hundred. Teacher: And that is? Learner F: Percentage. Teacher: You said a fraction is out of hundred, give me an example. Learner F: $\frac{25}{100}$. Teacher: Is that a fractions or percentage? Learner F: Is fraction. Teacher: What do 25 and 100 represent in that fraction? Learner F: A numerator and a denominator. Teacher: What is a numerator or a denominator? Learner F: (smiling) Ah, am, am, numerator is the number up and denominator is the number down (Line 5).

From the extracts above of different techniques I used, the data shows that before the learning programme, learners B, C, D and I lacked an understanding of percentages or fractions, even though they had a good understanding of written and oral symbols for percentages and fractions, and were able to give good examples of these. But the fact is that they could not explain what the written percentages or oral symbols of percentages and fractions meant to them. The real world objects and manipulatives (the 11 boxes) helped bring meaning to their use of these oral and written symbols.

Data from the second focus group interview shows how learners had improved their definitions of percentages and fractions and they were also able to explain what a percentage and fractions is because of the help of multiple representations.

Extract 3 Interview 2. *Teacher: That's great; yes what about you, what did you learn? Learner I: I enjoyed a lot so perfect, actually, first of all I was told, how understanding, how to, how to say out of a number. I was using the word over and then the teacher told us that we must not use the word over, because over does not mean anything. Teacher: Did it really, did I really mean something? Learner I: Yes! When I was, when i say 3 over 5, I didn't understand the meaning much? Teacher: What do you mean you didn't understand? Learner I: I don't know what over something means. Teacher: Now how do you understand the meaning if it is 3 over 5? Learner*

I: I will say 3 out of 5. Teacher: What does that mean? What do 3 out of 5 mean to you? Learner I: mm mm... three divide by 5. Teacher: What is that 5? Learner I: Five is my total things in one box, like when we used 100 bottle tops in one box. Teacher: OK. So it's a group of how many things? Learner I: Five! Teacher: Then, if there are 5 things, make it five bottle tops then what did you do with the five bottle tops? Learner I: I subtract three, mmm, I took three bottle tops out of five now I can write it as: 3 that line (showing the line (-) in the air) 5. (Learner I actually meant $\frac{3}{5}$). If I work out three out of five in the calculator, I will press three divide by five (line 4).

Teacher: OK, learner B, what did you learn? Learner B: I have learnt a lot, I now know how to convert between fractions, especially to convert percentage to fractions, especially if the total things in a box is out of hundred. Teacher: Give an example. Learner B: If I want to write fifty two percent as a fractions, then I read it as fifty two out of hundred and also write it correctly, with that line, madam, th..., that line, that line means you remove parts of things from the total things, that is what a fractions is, Teacher: What is a fractions, you said? Learner B: A fractions is part of the total things one has in a box, for example if I have 20 pins in a box and want to make a fractions out of my box, let's say fifteen pins to remove from the box, my fractions will be fifteen out of twenty. I converted twenty percent to fractions like that. Teacher: Wow, that's great (line 5).

Teacher: OK, what about you learner D, what did you learn? Learner D: I learned how to work out a quantity of a percentage, like if you have twenty percent of hundred Namibian dollars is twenty Namibian dollars. Teacher: How did you work it out? Learner D: I took twenty out of hundred, because twenty percent is twenty out of hundred as fractions, then I multiply my answer by hundred and got twenty Namibian dollars. Teacher: In your understanding what is twenty percent? Learner D: I have learnt that percent is a number out of the total group, if the total is out of hundred bottle tops, then I remove twenty from the hundred bottle tops in a box, I made a fractions of twenty percent. Teacher: Oho. That's very good (Line 6).

Evidence shows that relating symbolic representations to real world objects and manipulative representations (11 boxes) enabled Learners B, D and learner I, not only to know how to give a correct example of a fraction but to understand the meaning of that specific percentage or fraction they said or wrote.

TABLE 4.1 (i) Translating fractions diagrams into written symbols










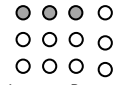
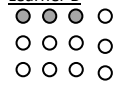
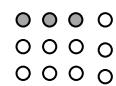
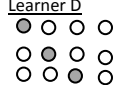
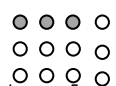
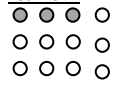
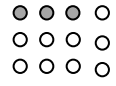
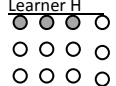
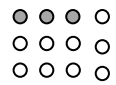
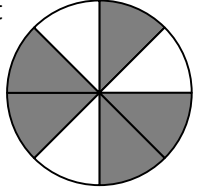
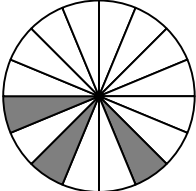
Pre – test (9 learners response)	Post – Test (9 learner’s response)
<p>(a) Color in two out of eight circles below and write your answer as a fraction.</p> <p><u>Learner A</u></p>  <p>Ans: $\frac{250}{1000}$</p> <p><u>Learner B</u></p>  <p>Ans: $\frac{2}{8}$</p> <p><u>Learner C</u></p>  <p>Ans: $\frac{2}{8}$</p> <p><u>Learner D</u></p>  <p>Ans: $\frac{2}{8}$</p> <p><u>Learner E</u></p>  <p>Ans: $\frac{2}{8}$</p> <p><u>Learner F</u></p>  <p>Ans: $\frac{2}{8}$</p> <p><u>Learner G</u></p>  <p>Ans: $\frac{2}{8}$</p> <p><u>Learner H</u></p>  <p>Ans: $\frac{2}{8}$</p> <p><u>Learner I</u></p>  <p>Ans: $\frac{2}{8}$</p>	<p>(a) Color in three out of twelve circles below, and write your answer as a fraction.</p> <p><u>Learner A</u></p>  <p>Ans: $\frac{3}{12}$</p> <p><u>Learner B</u></p>  <p>Ans: $\frac{3}{12}$</p> <p><u>Learner C</u></p>  <p>Ans: $\frac{3}{12}$</p> <p><u>Learner D</u></p>  <p>Ans: $\frac{3}{12}$</p> <p><u>Learner E</u></p>  <p>Ans: $\frac{3}{12}$</p> <p><u>Learner F</u></p>  <p>Ans: $\frac{3}{12}$</p> <p><u>Learner G</u></p>  <p>Ans: $\frac{3}{12}$</p> <p><u>Learner H</u></p>  <p>Ans: $\frac{3}{12}$</p> <p><u>Learner I</u></p>  <p>Ans: $\frac{3}{12}$</p>

TABLE 4.1 (ii) Translating fractions diagrams into written symbols

Write down the fractions of the shaded part/s

<p>pre-test</p> 	<p>post test</p> 
<p><u>Answers</u></p> <p>Learner A $\frac{60}{100}$ or $\frac{3}{5}$</p> <p>Learner B $\frac{5}{8}$</p> <p>Learner C $\frac{5}{3}$</p> <p>Learner D $\frac{5}{8}$</p> <p>Learner E $\frac{5}{8}$</p> <p>Learner F $\frac{5}{8}$</p> <p>Learner G $\frac{5}{8}$</p> <p>Learner H $\frac{5}{8}$</p> <p>Learner I $\frac{5}{8}$</p>	<p><u>Answers</u></p> <p>Learner A $\frac{3}{16}$</p> <p>Learner B $\frac{3}{15}$</p> <p>Learner C $\frac{3}{16}$</p> <p>Learner D $\frac{3}{16}$</p> <p>Learner E $\frac{3}{16}$</p> <p>Learner F $\frac{3}{16}$</p> <p>Learner G $\frac{3}{16}$</p> <p>Learner H $\frac{3}{16}$</p> <p>Learner I $\frac{3}{16}$</p>

4.3.2 Multiple representations can make some aspects of a concept clear

During the third lesson, learners worked in groups and they were asked to identify equivalent fractions of the initial fraction given to them by their teacher. Learners used multiple representations to generate equivalent fractions; they again used those representations to justify why the generated fractions were equivalent to the original fractions.

There were five groups of learners in the class, and in each group there were three learners. Group five with learners I, J and N used diagrams to generate five more equivalent fractions of their initial fractions. Their initial fractions were $\frac{4}{6}$ and their identified equivalent fractions were: $\frac{6}{9}$; $\frac{8}{12}$; $\frac{10}{15}$; $\frac{2}{3}$; $\frac{12}{18}$. Learner I, J and N draw the same size of bars to identify the five extra equivalent fractions as follows:

Extract 4 Lesson3.



These learners explained that using bars was the most effective method, because one can see clearly how their identified fractions fitted exactly to their original fractions. Diagrams made the idea of equivalence easier to understand by looking at the same size in the five bars.

On the other hand, groups one and three used manipulatives (11 boxes) to generate equivalent fractions of their initial one. Learners A, C and D were in group one and they used pins (11 boxes) to make repeated patterns that generated equivalent fractions. For example, their initial fractions was $\frac{1}{3}$ and the five fractions which they generated were: $\frac{2}{6}, \frac{3}{9}, \frac{4}{12}, \frac{5}{15}, \frac{6}{18}$.

This is how they generated their equivalent fractions, placing one pin above and three pins below and started repeating the same patterns.



Learners G and K were in group three and they also used manipulatives (11 boxes) to identify their five equivalent fractions of the initial one. They used bottle tops to make repeated patterns that generated equivalent fractions just as group one did. Their initial fractions was $\frac{2}{8}$ they then identified the following five equivalent fractions; $\frac{4}{16}, \frac{6}{24}, \frac{8}{32}, \frac{10}{40}, \frac{12}{48}$.

Group four had learners B and H, and these learners identified the five equivalent fractions of their initial one by repeated patterns but they didn't use manipulatives (11 boxes). Rather, they multiplied both the numerator and denominator of their initial fractions ($\frac{15}{20}$) by a factor (2). Each time they got the new fractions, they multiplied the numerator and denominator of the new fractions by the same factor. E.g. $\frac{15 \times 2}{20 \times 2} = \frac{30 \times 2}{40 \times 2} = \frac{60 \times 2}{80 \times 2} = \frac{120 \times 2}{160 \times 2} = \frac{240 \times 2}{320 \times 2} = \frac{480}{640}$. When learners were asked to explain why they carried out this multiplication by two, they couldn't explain why (Teacher's journal).

Groups one and three on the other hand could explain why they used concrete models (11 boxes). When they were asked to explain their work, these were their responses:

Extract 5 lesson3. Teacher: *What did you do?* **Learner C:** *Our original fractions were one third; we used pins to make other equivalent fractions of the original. Teacher:* *Explain how you worked with pins.* **Learner C:** *We arranged the pins as follows (pointing at the pins, which were arranged as shown above), we grouped the first pins, then repeat the same pattern, we counted the top pins separate from the bottom pins and count the new pins, so we then got a new equivalent fractions, we repeated that for five times to get five equivalent fractions (line11).* **Teacher:** *OK, what about you group three, what did you do?* **Learner G:** *We took our original fractions and make two groups according to our original fractions, then we count all the bottle tops together and get our first equivalent fractions. We then add again another group of our original fractions to the new group and count all together; we repeat this until we generate five equivalent fractions (line 12).*

The way groups one and three generated their equivalent fractions shows that they looked at the relationship of top pins or bottle tops to that of the bottom pins or bottle tops to generate their equivalent fractions.

Group two had learners F and E, and these two learners used calculators to find out all the fractions that have the same percentage as their initial fractions that is, their equivalent fractions. Their initial fractions were $\frac{1}{8}$ and they converted one eighth to a percentage which is 12.5%. Then they looked for all the fractions that have 12.5%. The equivalent fractions which were identified were: $\frac{2}{16}$; $\frac{4}{32}$; $\frac{125}{100}$; $\frac{5}{40}$ and $\frac{8}{64}$. During this activity of identifying equivalent fractions, learners showed that they used representations that made the task easier and clearer to them.

4.3.3 Multiple representations enable learners to correct their errors

Three incidents showed that multiple representations enabled learners to correct their errors. The first incident was during lesson two, when learners were asked to remove a portion of the objects out of the box. All learners were expected to have a total of 100 objects in each box. Learner G didn't count his bottle tops correctly, and had only 84 bottle tops. He removed 62 from the 84 bottle tops, whilst he thought he had a total of hundred. When he was asked to put aside the fraction of bottle tops which were removed from the box, there were only 22 bottle tops left. Learner G insisted that his work was correct. The teacher helped learner G to realize the error by re-counting the bottle tops together and this recounting of bottle tops helped learner G detect his error. Learner G took a piece of paper and calculated $\frac{62}{100} + \frac{22}{100} = \frac{84}{100}$, and that was when he admitted that the total of his bottle tops were not enough. So he added 16 extra bottles tops, removed 62, and was left with 38 in the box. He then tested again his work $\frac{62}{100} + \frac{38}{100} = \frac{100}{100}$. The extract below shows the dialogue between the teacher and the learners about this incident.

Extract 6 Lesson2.

*Teacher: What did you tell your partner to do? (Teacher asked learner O). **Learner O:** I told him to remove sixty two bottle tops from the box. **Teacher:** OK, learner G, what did you do?*

***Learner G:** I took sixty two out of hundred bottle tops. **Teacher:** Where are your sixty two bottle tops? **Learner G:** It is here (pushing aside 6 ten groups of bottle tops together) while holding two bottle tops. (After learner G pushed groups of bottle tops together, there were only twenty two bottle tops left. Both the teacher and the learners discovered this). **Teacher:** When you counted you bottle tops, where they hundred all together? **Learner G:** Yes (teacher asked learner G if he is sure).*

Teacher: *There is something wrong with your bottle tops, is either the sixty two you took out from the box or the twenty two remaining. (Teacher told learner G to re-count again). Before counting he insisted that he had 100 bottle tops. When he counted, there were actually 62 bottle tops and there were 22 left in the box. Learner G then took a piece of paper and calculated $\frac{62}{100} + \frac{22}{100} = \frac{84}{100}$. He finally admitted that he only had 84 bottle tops instead on 100. He then said: “That”s why I’m not getting hundred; I made a mistake when counting the entire bottle tops together (line 3).*

The second incident was during Lesson three, when learners worked in groups identifying equivalent fractions of the initial one given to them by their teacher. Group two decided to look for fractions with the same percentage as their initial fractions, i.e. $\frac{1}{8}$ is equal to 12.5%. Learners from this group had given one another task; each one had to look for two equivalent fractions of the same percentage as one eighth. Afterwards each of these learners came together and discussed their identified equivalent fractions. Learner E insisted that $\frac{125}{100}$ is one of the correct equivalent fractions of $\frac{1}{8}$ because they have the same percentage. Others complained that $\frac{125}{100}$ does not have the same percentage as one eighth. Learner E explained that 125 divided by 100 is 12.5%, without using a calculator. Then the others with a calculator gave learner E a calculator to work out his fractions, he got a wrong answer. That was when he admitted that his fraction was not equivalent to $\frac{1}{8}$ (Teacher”s journal).

The third incident was during Lesson eight, when learners were calculating VAT inclusive and VAT exclusive of the original price. For example learners were asked to fill in the table below showing VAT inclusive and exclusive of the original price.

Extract 7 Lesson 8.

4.2 Calculating prices including and excluding VAT

<i>Items</i>	<i>Original price</i>	<i>VAT of 15%</i>	<i>Price with VAT of 15%</i>
<i>Can beans</i>	<i>N\$4.00</i>	<i>N\$</i>	<i>N\$</i>
<i>Oros (Caribbean)</i>	<i>N\$8.00</i>	<i>N\$</i>	<i>N\$</i>
<i>Chicken (braai pack)</i>	<i>N\$</i>	<i>N\$</i>	<i>N\$30.00</i>
<i>Soup</i>	<i>N\$</i>	<i>N\$</i>	<i>N\$5.00</i>

All the learners used the same method to calculate the prices for all the four items, be it VAT inclusive prices or VAT exclusive prices. For the first two items, learners were expected to calculate the VAT of fifteen percent and then to calculate the price of the items with VAT included. For example the original price for the first item was N\$4.00 VAT excluded. All the learners first worked out 15% of the original price ($\frac{15}{100} \times 4 = N\0.60) and the answer was N\$0.60 VAT, then they added the value of VAT to the original price to get the price of a can of beans including VAT and they got N\$4.60. The same with the second item, where the learners worked out the VAT first and then added the answer to the original price.

This method just worked with the first two items. For the last two items, learners were expected to work out the VAT and also the original price without VAT. Learners first worked out the problems as in the first two calculations. They took 15% of the original price of the items and subtracted their answers from the price of an item with VAT. e.g. for the calculation of the chicken, learners worked out 15% of N\$30.00 and got N\$4.50, they thought this answer was for the VAT amount, subtracted N\$4.50 from N\$30.00 and got N\$25.50 for the original price.

But then five of these learners (learners A, B, D, G and H) tested their answers by working out 15% of N\$25.50 and they didn't get the VAT of N\$4.50. They realized that their last two calculations were wrong, and they were eager to find out their mistake. After I introduced the diagram for percentage increase and decrease (see fig 2.5 on chapter two) learners A, B, D, G and H were able to rectify their error by using the diagram. The learners took the price of chicken with VAT which was N\$30.00, divided it by $\frac{115}{100}$, got an answer of N\$26.09 (the original

price), and calculated the VAT amount and got N\$3.91, fifteen percent of N\$26.09. When these five learners tested their answer, there was correlation between the three prices, as this time their calculations were correct. This diagram was very useful to the learners, and using it enabled them to detect their errors.

From there, many learners used this diagram to work out VAT inclusive and VAT exclusive of the original price. During the learners' class activity 8(a) all the learners got 100% (See Appendix B, Question 8 (a)). Again, in the class activity 8(b) Question 1, learners also showed that using the diagram for percentage increase and decrease helped them work out the correct answer. The question was as follows:

Extract 8 Learners' Activity 8. Work out the prices needed to complete the table below: either the original price, the price with VAT of 15% or the VAT price of 15%.

Items	Original price	VAT price of 15%	Price of items with VAT included
Cellphone			N\$230.00
Packet of fish	N\$84.15		
2litre of coke			N\$15.00
2litre cooking oil	N\$34.00		

Learners A, B, C, D, G, H and I got all the answers correct, and their responses were as follows; the cellphone was N\$200 for the original price and N\$30 for the VAT; the packet of fish was N\$12.62 for the VAT and N\$96.77 for the price with VAT included; the 2 liters of coke was N\$13.04 for the original price and N\$1.96 for the VAT; the 2l cooking oil was N\$5.10 for the VAT and N\$39.10 for the price with VAT included.

Learner E got one mistake in his work; he got the original price of the 2 litres of coke correctly as N\$13.1, but he got the VAT price wrong as N\$11.3 instead of N\$1.96. Learner F got two mistakes in his work - he worked out the original price of the cellphone and got N\$195.50 with the VAT of N\$34.50 instead of N\$200 for the original price and N\$30 for the VAT. Learner F worked out 15% of N\$230 and got N\$34.50 as his VAT, and later subtracted N\$34.50 from N\$230 and got N\$195.50 as his original price of the cellphone. One can see that Learner F didn't use the diagram. Instead he used the method described above, which others did before they were introduced to the diagram of percentage increase and percentage decrease.

4.3.4 The use of a calculator contributed to learners conceptual understanding of percentages and fractions

The data presented below shows how learners used calculators in a way that helped them carry out and complete their problem solving. In many cases, using the calculator enabled learners to perform the calculations required to complete their solutions correctly. Without the calculators, learners either calculated incorrectly, or did not calculate at all and left their solutions incomplete.

In the pre-test learners A, B, C, H and I showed that solving problems in percentage without a calculator did not help them to work out their questions fully and correctly. Question 5(b) of the pre-test required learners to work out the percentage of “20 cents of 150 cents”. Learner A answered as follows: $\frac{20}{150} \times 100 = \frac{20 \times 10}{15} = \frac{200}{15}$ (see Appendix C, Question 5(b)). Learner A knew how to solve this problem, but because of not having a calculator, she did not complete her calculations. It was easy for her to go to the second step ($\frac{20 \times 10}{15}$) without a calculator because she divided by ten, and also it was easy for her to get $\frac{200}{15}$ because she multiplied 20 by 10, but it was difficult for her to divide 200 by 15 mentally without a calculator. Whereas in the post- test she solved a similar question and it was correctly and completely done, because of the help of a calculator. This question was to work out the percentage of “N\$54 of N\$112” and this is how she answered: $\frac{54}{112} \times 100 = 48, 21\%$.

Question 5(a) of the pre-test required learners to work out the percentage of “40 cents of 200 cents”. Learner I answered as follows: $\frac{40}{200} \times \frac{100}{1} = 40\%$ (see Appendix C, Question 5(a)). In the same way as described in the previous paragraph, Learner I, knew how to solve this problem, but because of not having a calculator, he got an incorrect answer. However in the post-test, Learner I could solve a similar question correctly because of the use of a calculator. Learner I’s calculation on what the percentage of N\$54 of N\$112 is was as follows: $\frac{54}{112} \times 100 = 48, 2\%$ (see Appendix C, Question 5(a)).

In Question 7 of the pre-test, learners A and I again showed that without a calculator they could not solve percentage problems correctly. The question was as follows: “you have read 71 pages of the mathematics text book that has 300 pages. What percentage of the pages have you read so far?” Learners A’s and I’s response to this question was as follows: $\frac{71}{300} \times 100 = 71$, but in the same question learner B’s answer was: $\frac{71}{300} \times 100 = \frac{71}{3} = 5$. All these three learners used the correct

method of working out the answer, but because of not having a calculator, they didn't get the correct answer.

Throughout the programme, I encouraged learners to use a calculator; I helped learners working out calculation on percentage and fractions by using a calculator. I explained that "out of" means "divide by": for example $\frac{50}{100}$ means fifty divided by a hundred (see Appendix D, Lesson 2 line 7). I also explained that learners should use the multiplication sign if the question has the word "of" as in 20% of 400 cents. Thus it would be $20 \div 100 \times 400$. During the lessons, learners learned how to use a calculator, and from there learners were eager and interested to use calculators.

Many of their calculations in the post-test showed that the use of a calculator helped them solve percentages and fractions problems fully and correctly.

In other cases, learners used calculators as efficient tools to explore and check their answers, or to compare two different answers in order to identify mistakes. By doing this, the use of the calculator helped the learners develop and improve their conceptual understanding. For example, during Lesson 3, learners worked in groups to identify equivalent fractions of the initial one given to them by their teacher. There were five groups, and four of these groups used manipulatives (11 boxes) to identify their equivalent fractions while one group used calculators to find out all the fractions that have the same percentage as their initial fractions. Their initial fraction was $\frac{1}{8}$ so their percentage was 12.5%. Without using a calculator, learner E insisted that $\frac{125}{100}$ is one of the correct equivalent fractions of $\frac{1}{8}$, because the two fractions have the same percentage. After learner E discussed with Learner F and explored how to calculate the percentage of $\frac{125}{100}$ using a calculator, learner E realized that he had made a mistake. Learner E and Learner F pressed as follows on their calculators: $125 \div 100 \times 100 = 125\%$, (Teachers journal). Learner E compared his answer to that of their initial fractions and admitted that $\frac{125}{100}$ was not one of their equivalent fractions because it is a different percentage from one eighth.

Moreover, learners C and B also used calculators as a tool in their discussion with the teacher, in ways that helped improve their understanding. For example, in lesson 3, learner C was tasked to convert 0.125 to a fraction and percentage. Learner C wrote $\frac{125}{100}$ and said "as percentage it will be one hundred and twenty five percent". I asked learner C how to convert a number from a decimal to a percentage. She then answered "you take zero point one two five and multiply by one hundred". I asked for learner C's answer and she used a calculator, pressing 0.125×100 and

got 12.5. I praised her and requested that she correct her previous mistakes. She erased 125% and wrote 12.5%. Again she changed her fraction of $\frac{125}{100}$ and wrote $\frac{12.5}{100}$. Other learners raised their hands, thinking that learner C wrote the incorrect fraction. Learner B suggested $\frac{125}{1000}$ as the correct fractions and he then said “*because there is three numbers after the comma, so 125 will be out of 1000*”. I instructed learner C to convert $\frac{12.5}{100}$ to a decimal, and also learner B to convert $\frac{125}{1000}$ to a decimal, then asked both of them to tell me their answers. After working on the calculators, both of them got 0.125. Learner C pressed on the calculator as follows; $12.5 \div 100 = 0.125$ while learner B pressed $125 \div 1000 = 0.125$, (teacher’s journal). With the help of the calculators, learners could understand the concept of converting decimals to percentages and fractions better.

4.4 HOW DID THE TEACHER INFLUENCE THE DYNAMIC LEARNING INTERACTION PROCESS IN WAYS THAT PROMOTED OR DID NOT PROMOTE LEARNING?

Three particular interaction types would be seen to positively influence the process dynamics. They are as follows:

- (a) All the learners changed words to change focus
- (b) Learners made links between multiple representations
- (c) Learners deepened their content knowledge and concepts of percentage and fractions;
 - (i) Learners were helped to convert between percentages and fractions using multiple representations
 - (ii) Learners were helped to work out percentages of a quantities
 - (iii) Learners were helped to express one quantity as a percentage of another

4.4.1 All the learners changed words to change focus

In the beginning of the programme, learners showed that they were accustomed to using the word “OVER” referring to the line in fractions. e.g. $\frac{7}{100}$ will be read as seven OVER one hundred. The focus of the word “over” was only on the line or the written symbol and not on the meaning of the line or the written symbol. In the following example it is shown how the interaction between the teacher and the learners using real world objects and manipulatives (the 11 small boxes) helped bring meaning to the learners’ use of both oral and written symbols of percentage and fractions.

Once again learners made use of the 11 boxes (see the description of the 11 boxes in Chapter 3, Section 3.5.1). Learners were asked to remove a portion of the 100 objects in each box. These boxes were designed for the learners to use, so that they made, said, wrote down and read each fraction which was removed from their boxes.

During lesson 1, learners worked in pairs and each pair was expected to make, say, write and read each fraction which was removed from their boxes. I instructed each pair to remove a certain portion of their objects inside their boxes, then to explain and write the portion of their objects as a fraction on the chalkboard. I drew the following table on the chalkboard.

Table 4.3 Converting percentages to fractions using the 11 boxes

Objects	Number of objects out of the box	Write it as a fractions
1. Stones	20	
2. N\$1 coins	34	
3. Bottle tops (1)	50	
4. Bottle tops (2)	5	
5. Almond seeds	70	
6. Piece of pipes	43	
7. Piece of plastic bottles	96	
8. Cola of plastic bottles	66	
9. Piece of boxes	82	
10. Pins (1)	14	
11. Pins (2)	100	

After learners counted out the number of objects in their boxes, I started pointing out each pair to say the fractions they had written and read it. These are the learners' responses:

Extract 9 Lesson 1. Learners I and K: (Learner I was talking about the groups of ten objects that they counted). We took ten, ten, ten, ten and three pieces of pipes from the box, so it is forty three over hundred. **Teacher:** Say it as a fraction. **Learner I:** It is forty three over hundred. (He then wrote forty three over hundred (in words) on the table which was on the chalkboard next to his number). That's their fractions. **Learners A and D:** (they were in a pair but learner D is the one who spoke). We subtracted twenty from hundred which makes it twenty over hundred and the stones which are left in the box are 80 (Learner A re-counted the stones inside the box, and she started grouping the stones in tens). **Teacher:** Writes as a fraction. **Learner D:** Writes $\frac{20}{100}$ and read it as twenty over a hundred. **Learners J and E:** (They were in a pair but learner J is the one who spoke) We count fifty bottle tops out, and that is fifty percent. **Teacher:** Write and read it. **Learner J:** Writes it as $\frac{50}{100}$ and read it as fifty percent over hundred. (While reading, learner J writes next to 50 in $(\frac{50}{100})$ this unit % and it looks as follows $\frac{50\%}{100}$). **Learner L:** (working alone) I subtracted seventy from hundred and that is seventy over hundred, he writes $\frac{70}{100}$ and read it as

seventy over hundred. **Learners M and N:** (learner M was the one talking) We subtracted eighty two from one hundred which is eighty two over hundred. **Learner M:** Writes $\frac{82}{100}$. **Learner B:** (working alone) I took thirty four coins out of hundred coins. He wrote his fraction as $\frac{34}{100}$ and read it as thirty four over hundred.

Learner G: (Working alone) I took sixty six out of hundred pieces then I got sixty six percent. He then wrote $\frac{66}{100}$. **Learner O:** (working alone) I took fourteen of hundred pins, it will be fourteen percent and as a fractions is fourteen over hundred. She wrote her fraction as $\frac{14\%}{100}$. **Learners F and C:** (Learner F was the one talking) We took ninety six out of hundred and it is ninety six over hundred as a fractions and ninety six percent. (Teacher asked them to write their answer on the chalkboard) Learner F wrote $\frac{96}{100}$ and 96%. **Learner H:** (working alone) I took hundred from hundred pins, as a fraction is hundred from hundred, he then write it as $\frac{100}{100}$ (line 9).

It is clear that all the learners were accustomed to using the word “over” instead of “out of”. Many learners were able to write the correct fractions, but could not read it correctly. Learners used the word “over” to read the line in their fractions. Although learners used the word “over” when reading their fractions, when they were explaining the fractions they took from the box, they used words such as “out of” and “take from”. I then asked learners to explain why all of them were using the word “over” to read their fractions. Many learners explained that they meant to remove a portion from a whole, and they admitted that the right word to use would be “out of” rather than “over”. This extract provides the evidence:

Extract 10, Lesson 1. Learner F: “We were taking out our numbers from the box. **Teacher:** From your statement you didn’t use the word over, but rather what did you use? **Learner F:** I used taking out. **Learner D:** “We learnt over when we were in grade 7, it means reading any number as a fractions. e.g. $\frac{7}{10}$ we will read it as seven over ten”. **Teacher:** OK, what do you think you mean, when you say, when you say, seven over ten? I mean, I mean taking few things from the total (laugh a little). **Teacher:** Then why can’t you just use the word “taking from” than “over”? It’s true ma.... (Laugh aloud). **Learner I:** “Yes we are used to say over and even when you asked us at first what percentage is I said percentage is over hundred but it looks better if I will use the word out of, because fractions means to remove few same things of the total things. **Learner H:** Yes out of is better than over. Over cannot explain... cannot mean anything, especially in the fractions. Out of means you are taking few things out of the 100 things (Line 10).

I then encouraged them to use the word “out of” than “over”.

Extract 11, Lesson 1. *Teacher: “Alright, from today on, let’s avoid saying that $\frac{34}{100}$ is thirty four over hundred, because we don’t understand what over means, like you said it yourselves. We should rather say thirty four N\$1 coins out of 100 N\$1 coins. It makes sense when we use the word “out of or from”, as you can see that all of you were taking a portion of your real world things out of 100 real things.” Do we all agree? (All learners shouted) yes, (line 11).* I then wrote a few percentages on the chalkboard and asked the learners to read them as fractions. The percentages were as follows: 34%; 27%; 18%; and 100%. All the learners which I pointed to could read as thirty four out of a hundred; twenty seven out of a hundred; eighteen out of hundred and a hundred out of a hundred. I then asked the learners to remove a different portion of objects from their boxes for the second time. Most of the learners, who were quiet during the first responses, were the ones talking this time. Learners were reading their percentages and fractions correctly and they also wrote their fractions and percentages accurately. Learners reminded themselves to say “out of” the moment they were about to say “over”.

Before the programme, learners A and C could not write the correct fractions of the shaded part/s of a diagram. Even though these two learners showed that they knew what a fraction looks like ($\frac{a}{b}$), they could not interpret from the diagram which values would be attached to “a” and “b”. In question 1(b) of the pre-test, the learners were expected to write fractions of the shaded part/s of a diagram. The diagram was a pie-chart with eight equal parts and five of the parts were shaded. The correct fraction was supposed to be $\frac{5}{8}$, but learner A’s answer was $\frac{60}{100}$ or $\frac{3}{5}$, while learner C’s answer was $\frac{5}{3}$ (see Table 4.1 (ii) above).

Letting learners engage deeply in the activity of removing a portion of objects from a box made them change the customary word “over” to “out of”, and this change made many learners change focus. Learners A and C improved in the way they wrote their fractions. Furthermore, after the programme learners B, D, F, H and I could explain what a fraction and percentage is, using examples (Interview 2). Besides this, all learners could answer questions on fractions correctly, related to the activity of removing a portion from a box. The extract shows how learners performed in the class activity 1:

Extract 12 Learners’ Activity 1. The first question was for learners to fill in 62 of the 100 seeds drawn on the paper (Appendix B, Learners’ Activity 1), and write their answer as a fraction. All the learners filled in 62 seeds and wrote $\frac{62}{100}$ as its correct fraction. The second question was for the learners to color in 4 out of 16 bottle tops already drawn on the paper and write their answer

as a fraction. All the learners coloured in four bottle tops out of sixteen and wrote the correct fraction as $\frac{4}{16}$.

The third question was a grid of 100 equal squares with 65 of the squares shaded, and learners were expected to write down the fraction of squares which are not shaded. Many learners got it right as $\frac{35}{100}$. Learners E and I got a wrong answer, Learner I probably because he thought the fraction should be for the parts which were shaded ($\frac{65}{100}$), while learner E's answer was $\frac{100}{100}$. The fourth question was a pie-chart cut into six equal parts and learners were asked to write down the fraction of the shaded parts. Apart from learner F who wrote $\frac{3}{3}$ as his answer to the fourth question, all learners wrote $\frac{3}{6}$ as the correct fraction. The last question was for learners to circle in 2 out of 10 N\$1 coins already drawn and write the correct fraction. Many learners got it right: they circled two N\$1 coins and wrote $\frac{2}{10}$ as its fraction. Learner E was the only one who did not circle, and wrote an incorrect fraction as $\frac{2}{100}$ instead of $\frac{2}{10}$.

4.4.2 Learners made links between multiple representations

After two lessons of learners using manipulatives (11 boxes), they started making links between multiple representations. At first, learners changed words to change focus: they changed from reading a fraction as “a number over another number” to “a number out of another number”, and gained insight into what a fraction means. From there learners understood that apart from reading a fraction as “out of”, the phrase “out of” means the same as “divide by”. The extract from Lesson 3 shows how learners linked their experiences of using the word “out of” to “divide by”.

I started the lesson by asking the learners to work out their homework activity from the previous lesson on the chalkboard, and allowed each learner to explain their work. The homework activity was to convert between fractions, percentages and decimals using the relationship table. Each learner was asked to fill in the table at the appropriate place.

Table 4.4 The conversion table between fractions, decimals and percentages

Fractions	Fractions	Decimal	Decimal	Percentage	Percentage
$\frac{1}{3}$					
			Zero point four		
				85%	
		0.125			
					One hundred and four percent
$\frac{2}{4}$					
				52%	

Learners were given different coloured chalks to complete the table. The extract below shows learners’ work and explanations.

Extract 13 Lesson 3. Learner H worked out the answers for the first row. **Learner H** wrote $\frac{1}{3}$ as a word as follows; one third, as decimal; zero point three, he then explained that, I divided, one divide by three and get zero point three, three, three, and three..... he then wrote 0.33 as a percentage (33, 3333%), and he wrote 33.33%. He then explained, I divided one divide by three and the answer I multiply it by hundred and I got 33% as in words is, thirty three percent. (Teacher praised learner H (line 2).

Learner E answered the second row, the row of zero point four. **Learner E:** “I calculated zero point four the answer multiplied by hundred and then get 40%. As a fraction is four out of ten $\frac{4}{10}$ or four tenth”. **Teacher:** “How did you convert zero point four to fractions?” **Learner E:** “I just take zero point four and count the number after the comma is one, so four will be out of ten”. **Teacher:** “From there how can you write zero point four as another decimal number?” (**Learner E**, pointing at 0.40 on the chalkboard). **Teacher:** “How did you work it out?” **Learner E:** “I took forty and divide it by hundred and get 0.40”. **Teacher:** Very good (line 3).

Learner G: answered the row showing 85%. He started saying that “85% is read as eighty five percent”. He said, “As a fractions is $\frac{85}{100}$, or eight five hundred”. (Teacher asked someone to read the fractions properly) **Learner O:** read as “eighty five hundredth”. **Learner G** continued explaining his work that “85% as decimal is 085”. **Teacher:** You should put the comma, there’s no comma, where should the comma be in 085? **Learner G:** Between zero and eight. **Teacher:**

OK, how did you get 0.85? **Learner G:** I divided. Eighty five out of hundred means to divide eighty five by hundred. **Teacher:** OK, that's good (line 4).

The next row was to convert 0.125 into the three forms. **Learner C:** Zero point one two five will be $\frac{125}{100}$, and in words is one hundred and twenty five out of hundredth, as percentage it will be one hundred and twenty five percent. (Teacher asked learner C how to convert a number from decimal to percentage) **Learner C:** You take zero point one two five and multiply by one hundred. **Teacher:** And what is your answer? Learner C used a calculator, pressed 0.125 x 100 and got twelve point five. (The teacher praised learner C and requested her to correct her previous mistakes). She erased 125% and wrote 12.5%. She changed her fraction of $\frac{125}{100}$ and wrote $\frac{12.5}{100}$. (Other learners raised their hands thinking that learner C wrote an incorrect fraction). **Learner B:** It should be $\frac{125}{1000}$ because there is three numbers after the comma, so 125 will be out of 1000. **Teacher:** I want learner C and some of you sitting to convert $\frac{12.5}{100}$ to decimal and also learner B to convert $\frac{125}{1000}$ to decimal too and both of you tell me your answers. (Both of them got 0.125). **Teacher:** "None of you got this answer wrong; all of you got the correct answer. **Teacher:** How did you work your answer out? **Learner B:** I divide one hundred and twenty five by hundred because any number out of another number you divide by that number. I got zero point one two five. **Teacher:** Good, what about you learner C? **Learner C:** I took twelve point five and divide it by hundred and get zero point one two five. **Teacher:** Good (line 5).

Learner F wrote one hundred and four percent as $\frac{104}{100}$ in a fraction and in words he wrote it as "1 hundred and for hundredth". **Teacher:** corrected learner F's mistake, you should write all the numbers in words e.g. "1 hundred and for hundredth, will be one hundred and four hundredth. 1 in word is „one' and the 4 is not written as "for" but the correct way to write 4 in word is "four". **Learner F** continued explaining $\frac{104}{100}$ as decimal is one point zero four (1.04). I divide one hundred and four by hundred and the answer is one point zero four. As percentage I press one point zero four in the calculator and multiplied by one hundred and get 104%. (Teacher praised learner F), (line 6).

Learner D could convert between percentages, fractions and decimals correctly. Her fraction was $\frac{2}{4}$, and she could read it as two fourths or two quarters. She explained how she converted $\frac{2}{4}$ to a decimal as: **Learner D:** I divided two by four and get zero point five, I then convert 0.5 to percentage by multiplying the decimal number by hundred and get 50%. **Teacher:** Why did you

divide two by four? **Learner D:** Because two out of four is the same thing as two divide by four. (Teacher praised learner D), (line 7).

Learner B could also convert between percentages, fractions and decimals correctly. His initial number was 52%. **Learner B** explained as follows: *I converted the 52% to fractions as fifty two out of hundred $\frac{52}{100}$, and he read the fraction as fifty two hundredths. He continued: To convert 52% to decimal I pressed fifty two divide by hundred because fifty two out of hundred means you divide fifty two by hundred if you are working it out in the calculator. I got an answer of zero point five two. **Teacher:** Very good, learner B (line 8).*

Furthermore, during the second focus group interview I conducted, two learners admitted that they learnt that “out of” can be also mean “dividing by”, especially if you are working on the calculator. The extract below shows learners’ responses to “out of” to mean “divide by”.

Extract 14 Interview 2. Teacher: What does that mean? What do 3 out of 5 mean to you? **Learner I:** Mm... three divide by 5. **Teacher:** What is that 5? **Learner I:** Five is my total things in one box, like when we used 100 bottle tops in one box. **Teacher:** OK, so it’s a group of how many things? **Learner I:** Five! **Teacher:** Then, if there are 5 things, make it five bottle tops then what did you do with the five bottle tops? **Learner I:** I subtract three, mmm, I took three bottle tops out of five now I can write it as: 3 that line (showing the line (-) in the air) 5. (**Learner I** actually meant $\frac{3}{5}$) If I work out three out of five in the calculator, I will press three divide by five. (Line 4). Learner C, on the other hand, when she was asked what she learnt from the programme, answered thus: **Learner C:** I learnt that, when you are saying “out of” with fractions, you press divide in the calculator. **Teacher:** Give an example. **Learner C:** For example fifty out of hundred, means fifty divide by hundred if you are working on the calculator, (Line 7).

4.4.3 (c) Learners deepened their content knowledge and concepts of percentage and fraction

(i) *Learners were helped to convert between percentages and fractions using multiple representations*

One of the aspects that showed learners’ deepened content knowledge and concepts of percentage and fractions was that they could convert between percentages and fractions using multiple representations. Before the programme, learners showed that they could not convert between percentages and fractions using multiple representations. I therefore present evidence that shows how learners could not convert between percentages and fractions. In Question 2(a)-(e) of the two diagnostic tests, learners were required to convert fractions to percentages using five diagrams. They were asked to write the percentage of each part/s of the five diagrams which was shaded. The diagrams used were: pie-chart, a grid, a number line and figures (see Appendix C, Question 2(a)-(e)). The two tables below show the scores of each of the nine learners on both pre-post-tests for this question.

Table 4.5 Scores of the nine learners on both pre and post-tests for Question 2 (a)-(e).

Pre-test

Scores	0	1	2	3	4	5
Number of learners	3	0	0	2	3	1

Post-test

Scores	0	1	2	3	4	5
Number of learners	1	0	0	0	1	6

Out of the three learners who scored zero on question 2(a)-(e) of the pre-test, two scored five on the post-test but one still scored zero on the post-test. The former two learners are Learners C and Learner I. On the pre-diagnostic test learner C and learner I showed that they were not able to use diagrams well enough to convert to percentages. From the five diagrams used, which were a pie-chart, a grid, a number line and figures, learner C and learner I could count the shaded part/s only, and could not count the un-shaded parts or all the parts.

They also couldn't compare the shade part/s within the un-shaded part of the whole. However in the post-test learner C and learner I showed a huge improvement on how to use diagrams to convert to percentage. They gain the know how to use the diagrams. They were able to count the shaded parts and also the un-shaded parts or all parts and compare the two parts and convert the fractions correctly to percentage. (See Appendix C, Question 2(a)-(e)).

On the other hand, the third learner (learner E) who also scored zero in this question of the pre-diagnostic test still scored zero in the post-diagnostic test. Learner E showed confused ideas in his solution when using diagrams to convert to percentages. e.g. the first diagram on the pre-test was a pie-chart equally divided into four parts, of which one of the parts was shaded. The question was to write the percentage of the shaded part. Learner E's answer was 400%; he worked out his answer as $\frac{40}{10} \times 100$. In the post-diagnostic test the pie-chart was divided into eight equal parts where three of the parts were shaded, and learner E wrote $\frac{30}{10} \times 100 = 30\%$. From the two responses, one can see that he has the correct idea of multiplying a fraction by a hundred. However his fractions are not correctly written. In the second and third diagram on the pre-test learner E showed that he has the correct idea of multiplying fraction by a hundred, but this time he reversed his fraction upside down. The identified parts were supposed to be out of all parts, but in his case the identified parts become his subsection (denominator). On the post-test learner E showed that he counted his shaded parts but he did not compare them to all the parts. After he counted the parts he just added zero to the total number of parts (See Appendix C, Question 2(a)-(e)).

On the same question, learners A and learner H scored three in the pre-test and five in the post-test (see Table 4.2 above). Learners A and H were able to convert to a rounded percentage using diagrams effectively, but when the convention did not give a rounded percentage for an answer, they could not find the correct answer. Moreover, the answer to the last diagram was not a rounded percentage. It was a diagram with three rows of five circles in each row with four of the circles shaded and learners were expected to write the percentage of the shaded parts in the diagram. Learners B and F could not give the correct answer to the last diagram. Nevertheless, learners A, B and F showed in the post-test that they gained ideas on how to use diagrams to convert to percentage, even when the answer was not a rounded percentage. They answered correctly and even showed their work to the rest of the class. (see Appendix C, Question 2(a)-(e)).

Furthermore, in the pre-test earners H and G struggled to convert to percentages using the shaded points on a number line. The fourth diagram was a number line of ten points with five of the points shaded. Learner H's answer was 0% while learner G's answer was 4%. Their answers showed that they did not understand how to use a number line to convert to a percentage. Conversely on the post-test, apart from learner E, all the other learners showed that they gained ideas on converting to percentages using diagrams (see Appendix C, Question 2(a)-(e)).

During the second lesson, I used the same five diagrams from the pre-test for the class activity. After I marked the activity, I discussed it together with the learners. I chose learners who got the correct answer to explain to others how they got their answers right. The extract shows learners' explanations of their answers.

Extract 15 Lesson 2. The first diagram was a pie-chart divided into four equal parts with one part shaded. **Learner F:** *It is one out of four, and it is twenty five percent. I took one divide by four and multiply by one hundred.* **Teacher:** *Very good learner F, you should try to read " $\frac{1}{4}$ " as one quarter than one out of four, (line 15).* The second diagram was a grid of 35 parts, seven by five grids with 4 parts shaded. **Learner L:** *Wrote $\frac{4}{35}$.* **Teacher:** *Write it as percentage.* **Learner M:** *It is eleven point four two eight five....there is a lot of numbers after the comma.* **Teacher:** *Round off to the nearest digit, if there is a lot of number after the comma?* **Learner B:** *Then you round off and the answer is 11.4%.*

Teacher: *Very good to all of you, many of you in the pre-test wrote a correct fraction $\frac{4}{35}$ but couldn't convert it right to percentage. What was the main problem? I needed to find out the learners' problems in this question, so we worked out the problem together and learned from one another.* **Learner M:** *I thought percentage doesn't have a lot of number after the comma, only decimal.* **Teacher:** *It does, if you get an answer that has a lot of digits after the comma, we round off the number. That's why we learned already how to round off numbers to the nearest unit.* **Learner A:** *I couldn't convert fractions that gives me an ugly answer, I like converting fractions that gives me an exact answer.* **Teacher:** *Do you now know how to convert this fraction $\frac{4}{35}$ to percentage?* **Learner A:** *No, only the first example $\frac{1}{4}$.* **Teacher:** *How do you convert $\frac{1}{4}$ to percentage?* **Learner A:** *I divide one by four and times hundred.* **Teacher:** *Exactly, the same way you convert one quarter to percentage, the same way you should convert $\frac{4}{35}$, try now.* **Learner A:** *Four divide by thirty five, times hundred.* **Teacher:** *Equals to what?* **Learner A:** (pressing the

calculator) *It is 11.4285. Teacher: From there what should one do? Learner A: Convert. Teacher: Great, (line 16).*

The third diagram was another grid of 10 parts, five by two grids with 2 parts shaded. **Learner K:** *Is read as two out of ten and is written as $\frac{2}{10}$. Learner F: wrote 20% and explained - I divided two by ten then the answer I multiply it by hundred and got 20%. Teacher: Very good, (line 17).*

The fourth diagram was a number line from one to ten, and five of the number points were shaded. **Learner A:** *wrote $\frac{5}{10}$ and read it as five out of ten, she wrote 50%, and read it as fifty percent. Teacher: How did you convert from the fraction to a percentage? Learner A: I multiply hundred by five divide by ten. Teacher: That's good, (line 18).*

The last diagram was fifteen small circles in five rows of three in each row. Four of the fifteen small circles were shaded. **Learner D:** *As a fraction is four out of fifteen and as percentage is twenty six percent. Teacher: Is the answer an exact one? Learner D: No, it is 26, 66666667. Teacher: Round off to the nearest one decimal place. Learner I: (shouted) 26.7%. Teacher: Yes, if to the nearest whole number what will it be? Learner E: It is 26. Teacher: Is 26 correct to the nearest whole number? Learners: No it is wrong, learner H: It is 27%. I then reminded learners how to round off again, (line 19).*

The data shows that the interaction between the teacher and the learners and also the learners' interactions amongst each other made it possible for them to gain knowledge on how to convert between percentages and fractions. I gave the learners examples of decimal numbers with only one digit after the comma to be converted to fractions and percentages e.g. 0.6; 0.9; 0.2; 0.5. The learners' answers were as follows:

Learners D, A, C and I all had the same answers: (a) $0.6 = \frac{6}{10} = 6\%$; (b) $0.9 = \frac{9}{10} = 9\%$; (c) $0.2 = \frac{2}{10} = 2\%$; and (d) $0.5 = \frac{5}{10} = 5\%$ and Learner B's response on the other hand was as follows: (a) $0.6 = \frac{6}{10} = 60\%$; (b) $0.9 = \frac{9}{10} = 90\%$; (c) $0.2 = \frac{2}{10} = 20\%$; and (d) $0.5 = \frac{50}{100} = 50\%$. Learner B got his answers correct, while all the other learners got it wrong. I asked learner D to explain how she converted decimals to fractions and then to percentages. I wanted learner D to realize by herself that her answer was not correct, without me telling her the correct answers straightaway. The following extract shows the discussion between me and learner D.

Extract 16 Lesson 2. Learner D: *Zero point six to fractions is six out of ten, because there is only one number after the comma, so the six will be out of ten. If there was two numbers after the*

point, then it will be out of hundred. **Teacher:** Very good, what about converting zero point six to percentage? **Learner D:** It will be 6%. **Teacher:** OK, let's remember what we have learnt. (I wrote $6\% = \frac{6}{100}$). **Learner D** responded $6\% = \frac{6}{100}$, because this (%) unit represent out of hundred. **Teacher:** That's correct, your first answer was $\frac{6}{10} = 6\%$, this is different from how you are explaining now. **Learner D:** Admitted that her answer was wrong. **Teacher:** Why are you saying that the first answer is wrong? **Learner D:** Because six percent is six hundredth and not six tenth. **Teacher:** so, six tenth is what percent? **Learner D:** Umm, I am not sure, (line 12).

After the discussion with learner D, I still did not give the answers straight away, but gave the learners a chance to give their views on how to convert $\frac{6}{10}$ to percentage, and I created an opportunity by giving them a chance to explain or give their views.

Extract 17 Lesson 2. Teacher: Nice try, OK who has a different answer from learner D? **Learner B** raised his hand and explained that "Six out of ten is equal to sixty percent because in ($\frac{6}{10}$), one should change ten to become hundred because we learn that a fractions to percentage should only be out of hundred not any number. Therefore six times ten is sixty and $\frac{60}{100}$ is 60%. **Teacher:** Very good, who still has a different idea on how to get 60%? **Learner G:** I divide six by ten, then I multiply by one hundred and my answer is sixty percent. **Teacher:** Very good, both learner B and learner G have the correct way to convert decimal to fractions and percentage. **Learner B** explained that 0.6 converted to a fraction is $\frac{6}{10}$, and he then needed to make "out of ten" to be "out of a hundred". He said that percentage is out of a hundred, and that was why he multiplied his fraction by ten - so that he got an equivalent fraction out of a hundred. When the fraction is out of a hundred it is easier to convert to a percentage. **Learner G**, on the other hand, converted $\frac{6}{10}$ to a decimal i.e. by taking six and dividing it by ten and got the answer 0.6. Then he multiplied the decimal number by a hundred and got sixty percent. **Teacher:** Now I want you all to convert the other decimal numbers 0.9; 0.2; and 0.5 using learner B or G's methods or even yours, as long as it works, (line 12). I moved around and checked how the learners were doing the work. I observed them correctly converting the decimal numbers to percentages. Many of them used learner G's method.

I again taught learners how to convert between percentages and fractions using the table below. At first I drew table, and thereafter I explained to learners that there are two ways for each of the three forms to be written. I filled in the first row of the table to explain in detail how each of the

three forms is written. I then asked learners to complete row two and three together with me. I then asked learners to explain their work.

Table 4.4 The conversion table between fractions, decimal and percentage

Fractions	Fractions	Decimal	Decimal	Percentage	Percentage
$\frac{3}{10}$	Three tenth	0.3	Zero point three	30%	Thirty percent
		0.5			
				112%	

The extract below shows learners' explanations.

Extract 18 Lesson 2. *Learner P:* wrote $\frac{5}{10}$ and said five out of ten. *Learner N:* Wrote, five tenths as another way of writing fractions. *Learner E:* wrote 0.5 in decimals and in words zero point five. *Learner L:* wrote 50% and explained I multiply five out of ten by hundred. *Learner A:* wrote fifty percent in words. I asked the learners to complete row three. *Learner F:* 112% will be one hundred and twelve out of hundred - and he then wrote $\frac{112}{100}$. *Learner B:* In another fraction is one hundred twelve of hundred. Of hundred (teacher repeats after the answer) *Learner B:* Of hundredth and he then wrote "one hundred and twelf hundredth". I then corrected him on the spelling of 12, not "twelf" but "twelve". *Learner O:* wrote 1.12 and read it as "one point twelve". I corrected learner O, to read 1.12 as "one point one two" and not "one point twelve". I again asked learner O how she got the answer. Learner O's response was that I divide one hundred and twelve divide by hundred. I asked learner M to write 112% as a decimal in words, and - *Learner M:* wrote one point one two. *Learner D* on the other hand wrote 112% as a percentage in words as "one hundred and twelve percent" (line 14).

(ii) Learners were helped to work out percentages of quantities

The other aspects that showed that learners deepened the content knowledge and conceptual understanding of percentages and fractions were that learners could work out percentages of quantities. Before the programme, three learners showed that they were not able to work out the percentage of a quantity. Question 6 of the pre-test was for the learners to: (a) work out 30% of

N\$300 and (b) 17% of N\$20. Learners C, D and E's responses showed how these learners were not able to work out the percentage of a quantity. Their responses were as follows:

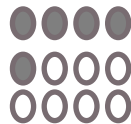
Extract 19 pre-test. Learner C: (a) $30 \times 300 = N\$ 9000$, (b) $17 \times 20 = N\$340$. **Learner D:** (a) N\$10, (b) N\$85. **Learner E:** (a) N\$ 9000, (b) N\$1,420. These learners had no idea on how to work out the percentage of a quantity. In particular learner E he did not know that percentage is always a number out of a hundred, and he also did not understand how to work out the percentage of a quantity.

The interaction between the teacher and learner E helped learner E to understand the concept of working out the percentage of a quantity. I mediated the learning of learner E by first giving him a chance to try out an activity around this concept. The extract below shows how it happened.

The question on the board was to shade in 25% of this diagram.



Learner E's answer was as follows:

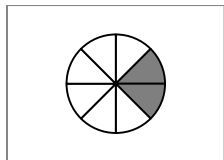


He explained his work in writing as follows: $\frac{25}{12} \times 100 = 208.33\%$. I asked learner E if he thought his answer was correct, and he said it was not correct because he got a larger answer. I then asked him what the right answer was and learner E said it was five. I then asked learner E to explain how he got five. Learner E could not explain. When I realized that learner E did not understand the concept I made use of the other learners to help learner E. I asked learner D to come and work out the answer in front of others while learner E watched and listened. Learner D shaded in three of the circles and this is how she explained her answer:

Extract 20 Lesson 5. Learner D: *Three out of twelve is twenty five percent and twenty five out of hundred multiplied by twelve is three. She writes her work as follows: $\frac{25}{100} \times 12 = 3$, the percentage given is twenty five percent, so twenty five percent will be twenty five out of hundred, that's where I got that fractions, And therefore $\frac{3}{12} \times 100 = 25\%$, (Line 2).* I then explained to learner E that percentage is always out of a hundred by reminding him of the work they did with the 11 boxes.

I asked learner C to shade in 25% of a pie that was equally divided into eight parts. This is how she worked out her answer $\frac{25}{100} \times 8 = 2$, she then explained her working as follows: *Percent is always out of hundred, I took the percentage and multiplied it by the equal number of parts to*

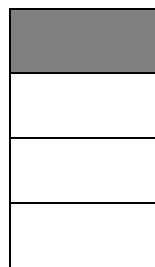
know the parts which I will shade, and that's why I shaded two parts here (pointing at her answer) (line 3).



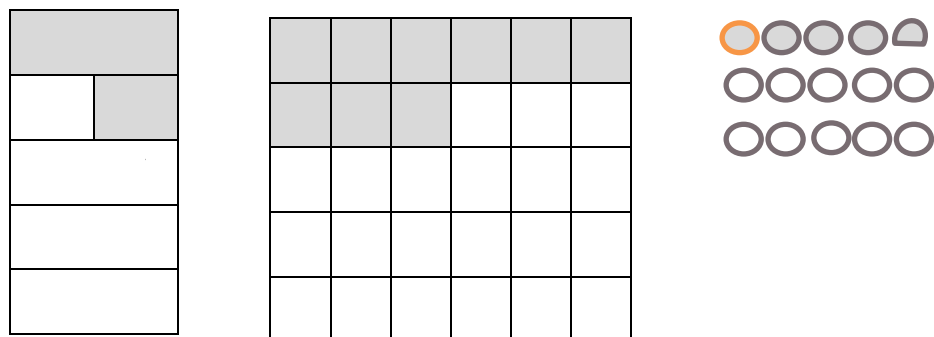
The third thing I did to give learner E a second chance was to work out 25% of a grid that had four equal rectangles. Throughout this exercise, I asked a lot of questions on the concept, to help him. The extract shows how it happened.

Extract 21 Lesson 5. *Learner E:* wrote $\frac{25}{100} \times 100 = 25$. *Teacher:* OK, I can see that you converted 25% to fractions correctly, very well, but where did you get the hundred you used? Learner E immediately realized that he had used an incorrect quantity and erased the “100” and wrote “4”, so his second answer looked as follows; $\frac{25}{100} \times 4 = 16$. *Teacher:* How did you get 16? (*Learner E* pressed 25×100 on the calculator). *Teacher:* How do we read this $\frac{25}{100}$? *Learner E:* Twenty five out of hundred? *Teacher:* What does that mean, if you want to work it out in the calculator? *Learner E:* Twenty divide hundred. *Teacher:* OK, that's correct, now work out your work, (Line4). **Learner E** pressed twenty five out of a hundred as twenty five divide by hundred, then he multiplied his work by four and got one as his answer. He then shaded in one of the diagrams given.

This was his answer:



This class activity evidence shows that learner E improved on how to work out a percentage of a quantity correctly. Question 2 of the activity was for the learners to shade in 30% of the diagrams below and learner E's responses were as follows:

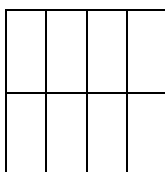


As I pointed out before, learners C, D and E had problems on how to work out the percentage of a given quantity. However, in lesson 5 learners C and D helped learner E a lot (see Extract 21 above), after they had gained the knowledge of how to work out a percentage of a given quantity the previous day in Lesson 4. I introduced lesson 4, by drawing the diagrams below and asking the learners to work out the percentages of the quantities, using the diagrams.

(a) Colour in 50% of the 18 circles below;



(b) Shade in 40% of the figure below



(c) Colour in 25% of the pie-chart below.



Extract 22 Lesson 4. Learner D worked out the first diagram. She shaded in 9 circles. **Learner D:** *I shaded nine circles.* **Teacher:** *How did you get the answer?* **Learner D:** *I put nine out of eighteen and multiply it by one hundred and I got 50%.* She showed her answer as $\frac{9}{18} \times 100 = \frac{900}{18}$

= 50%. **Teacher:** How did you decide to use nine from all the numbers? **Learner D:** I looked for a fraction out of 18 that can give me 50%. **Teacher:** How did you look for that fraction? **Learner D:** Fifty percent is the same thing as fifty out of hundred and fifty out of hundred is equivalent to nine out of eighteen because they all have the same percentage. (line 2). Learner D showed that she gained the knowledge of converting percentages to fractions ($50\% = \frac{50}{100}$), thereafter she deepened this knowledge by thinking of an equivalent fraction of 50% which is $\frac{9}{18}$. Furthermore, she explained why fifty out of a hundred is equivalent to nine out of eighteen.

I gave a chance to the other learners who used different methods to explain their methods. In this way, learners could understand that there are always different methods to tackle a question. After I asked if there was someone with another method, learner H raised his hand and explained. **Learner H:** I did this: 50% is equal to $\frac{50}{100}$, and multiplied that fractions by eighteen out of one. **Teacher:** What is your main idea? **Learner H:** My main idea is to find out the half of that number, because 50% is always half of the quantity. (line 2).

Learner I worked out the second graph, and he explained as follows: **Learner I:** I took ten, ten no, four, four out of ten multiplied by hundred i.e. $\frac{4}{10} \times 100 = 40\%$. **Teacher:** How did you get four? **Learner I:** I got the answer four by trying numbers that will work. I first tried number three it didn't work, but then when I tried number four it worked. Learner B on the other hand explained the same question as follows: I took forty divide it by hundred then multiply it by ten, then I got four, (line3). Learner A work out the last diagram as follows: **Learner A:** wrote $\frac{1}{4} \times 100 = 25\%$. **Teacher:** How do you know that a quarter is equal to 25%? **Learner A:** $\frac{25 \div 25}{100 \div 25} = \frac{1}{4}$, (line 4). I then praised all the learners for their very good explanations.

I then gave more representations on working out the percentage of a quantity to the learners. This time the representations used were without graphs. Examples: (a) 20% of N\$300, (b) 25% of N\$60 and (c) 40% of N\$5. Learners worked out their answers and explained their workings when I pointed to them. The extract below shows the learners' explanations.

Extract 23 Lesson 4. **Learner H:** wrote $\frac{20}{100} \times 300 = N\60 . **Learner G:** wrote $\frac{25}{100} \times 60 = N\15.00 . **Learner C:** wrote $\frac{40}{100} \times 5 = N\2.00 (line 5). Each of them talked about first converting their percentage value to a fraction by making their fractions out of a hundred, and then multiplied their answer by the given quantity.

The other representations I used were grids. I gave the learners three grids: 1. 25 equal rectangles, 2. 100 equal rectangles and 3. 200 equal rectangles. I wanted learners to paste into all their grids small rectangles of different colours. I gave the learners the number of small rectangles needed for each colour. The numbers of small rectangles were percentages. Example 1: I asked each learner to paste on his/her 100 grid 40% small yellow rectangles, 30% small red rectangles, 25% small blue rectangles and 5% small white rectangles. All the learners covered their 100 grids correctly. For the learners to know how many small rectangles they needed to paste into their grid for each colour, they needed to work out the percentage of the hundred grid. (line 6).

After learners finished pasting onto their hundred grids, I told them to also paste onto their 200 rectangle grid, using the same percentages as for the hundred grid. Some learners worked out the percentage with calculators, and pasted the correct number of small rectangles on the 200 grid. Learners calculated as follows: $40\% = \frac{40}{100} \times 200 = 80$ small yellow rectangles; $30\% = \frac{30}{100} \times 200 = 60$ small red rectangles; $25\% = \frac{25}{100} \times 200 = 50$ small blue rectangles and $5\% = \frac{5}{100} \times 200 = 10$ small white rectangles. A few learners, such as learners D, E and K did not paste the correct number of small coloured rectangles. They worked out their answers as follows: 40% of small yellow rectangles as $\frac{40}{200} \times 100$ and got 20 yellow rectangles, $\frac{30}{200} \times 100$ and got 15 red rectangles, $\frac{25}{200} \times 100 = 12.5$ blue rectangles, $\frac{5}{200} \times 100$ and got 2.5 white rectangles (line 7).

These learners did not convert percentages to fractions correctly. Instead of writing the fractions out of hundred (because percentages are out of a hundred), they instead wrote their fractions out of two hundred. I wanted the learners to notice themselves that their fractions were incorrectly written, rather than me telling them about it. I asked learner D to explain how she got her first answer, this is how she answered: **Learner D:** *I converted 40% to fractions.* **Teacher:** *I can see that you have converted percentage to fractions, let's do it together.* I wrote $40\% = \frac{\quad}{\quad}$ on the chalkboard and asked learners D, E and K to complete the answer. **Learner D:** *wrote $40\% = \frac{40}{100}$.* **Teacher:** *OK, can you see where you gone wrong? (line 8).*

Learner D admitted that she wrote forty out of two hundred instead of forty out of one hundred. I then encouraged the three learners to always write percentages out of a hundred. Learners D, E and K later pasted the correct number of small coloured rectangles onto the 200 grid.

When the learners completed the 25 grid in the same way, all pasted correctly. This time I changed the percentages of the small coloured rectangles to 40% yellow, 20% blue, 30% white and 10% red. The learners worked out the answers as follows: $40 \div 100 \times 25 = 10$ yellow rectangles, $20 \div 100 \times 25 = 5$ blue rectangles, $30 \div 100 \times 25 = 7.5$ white rectangles and $10 \div 100 \times 25 = 2.5$ red rectangles. When the learners pasted on the white and red rectangles, some of them cut one of the rectangles in half and other learners folded one of the rectangles in half. To show the answer for the small white rectangles as 7.5, they took 8 of them, pasted 7 and some learners cut the remaining rectangle in half, and pasted the half, while other learners folded the remaining rectangle and pasted the folded part. I was impressed with learners' initiative cutting and folding the small rectangles in half.

After using multiple representations to explain how to work out the percentage of a given quantity I gave the learners an algorithm they should use if necessary for this concept. The name of the algorithm is for figuring a part (Van Galen, 2008), and that is the same as working out the percentage of a quantity. I introduces the algorithm almost at the end of Lesson 4, with the following example: 20% of N\$300 (Lesson 4, line 10) **Teacher:** *From all the correct explanations you gave me on this concept of working out percentage of a quantity, this is what you did first, figuring out how much 1% of the amount, in our example it will be 1% of N\$300 ($300 \div 100$). Then that amount is multiplied by the percentage and the answer is N\$60 (3×20), (line 10).*

All the learners used this algorithm during the post-test in a way showed their deepened understanding of the concept of working out the percentage of a quantity. This was because I did not introduce the algorithm at the beginning of the lesson. Firstly I found out what the learners' experiences in this concept were; secondly, I used their experiences as examples when I explained the concept in detail, and thirdly I mediated by discussing and communicating together their new experiences. Only after all the mediation, I introduced the algorithm to them. The methods that they used were appreciated and valued.

All the learners performed very well during the Learners' Activity 5 (See Appendix B, Question 5). In Question 1, learners were expected to work out the following (a) 64% of 500 cows; (b) 17% of 2m; (c) 29% of N\$36; (d) 7.5% of N\$16.40; (e) $8\frac{1}{2}\%$ of 2400g. Learners got their work correct, apart from learners A and E who got their last answer wrong and this was because the percentage was mixed number. In the second question where learners were expected to shade in

30% of the three different diagrams (see Appendix B, Question 5) all the learners including learner E shaded in all three diagrams correctly.

Learners even work out the correct answers to questions which were more abstract, but in the pre-test all learners showed that they could not answer word problems on the percentage of a given quantity (See Appendix C, Question 5). In Questions 3 and 5 of the Learners' Activity 5, learners were expected to solve word problems working out the percentage of a quantity, and they got the correct answer. Question 3 was: "*Your uncle bought for you school shoes of N\$150.00. It did not fit on you and now you are selling it at a profit of 30%. How much is your profit?*" All the learners' answers were: $\frac{30}{100} \times 150 = \text{N}\45.00 , but few of them forgot to add the unit "N\$". Question 5 was: "*You have read 40% of the Rukwangali story book (Kotokeni zaHaitana). The book has 240 pages. How many pages did you read so far?*" All the learners' answers were: $\frac{40}{100} \times 240 = 96$ pages.

Even after the programme all the learners showed that they were able to work out the percentage of a quantity correctly, specifically learners C, D and E who had problems before the implementation of the programme. Question 6 of the post-test was for the learners to: (a) work out 28% of N\$600 and (b) 70% of N5.00. Learners C, D and E's responses were as follows: **Extract 24. Learner C:** (a) $\frac{28}{100} \times 600 = \text{N}\168 , (b) $\frac{70}{100} \times 5 = \text{N}\3.50 . **Learner D:** (a) $\frac{28}{100} \times 600 = \text{N}\168 , (b) $\frac{70}{100} \times 5 = \text{N}\3.50 . **Learner E:** (a) $\frac{28}{100} \times 600 = \text{N}\168 , (b) $\frac{70}{100} \times 5 = \text{N}\3.50 . These learners showed that they deepened their content knowledge and conceptual understanding of working out the percentage of a quantity. The interaction between the teacher and learners during the lessons helped learners to deepen their understanding.

(iii) Learners could express one quantity as a percentage of another one

The third aspect that showed how learners deepened their content knowledge and conceptual understanding of percentages and fractions was that learners could express one quantity as a percentage of another quantity. In Question 7 of the pre-test, learners were expected to express one quantity as a percentage of another one. Learners A, C, E, F and I showed that they were not able to answer the question correctly (see Appendix C, Question 7).

The interaction between the teacher and the learners helped them to understand the concept of expressing one quantity as a percentage of another one. I used multiple representations to explain the concept. I used various written symbols and also graphs to present the concept. Further, I also used learners' responses to explain the concept in detail. I asked learners to justify their answers,

and when they were not able to justify them, I helped or asked other learners to help. Near the end of the lesson I used learners' useful explanations to explain and introduce an algorithm to use for this concept. The extract below shows the interaction between the teacher and the learners.

Extract 25 Lesson 6. The first representations used were written (word problems). They were as follows: (a) What percentage is twenty marks out of twenty five marks? (b) There are thirty learners in a class. Five are boys. What percentage of the learners are boys? I first asked any learner to read the question and to take out the main information from the question. Learner K read the first question and said that the main information in the question was that twenty five marks is the total number of marks and twenty marks were obtained. After that, Learner I worked out the first example as follows: **Learner I:** $\frac{20}{25} \times 100 = 80\%$. While Learner I was working on the chalkboard, I observed the other learners' answers and found that learner E worked as follows: **Learner E:** $\frac{20}{100} \times 25 = 5\%$ (Teacher's Journal). Back in front of the class I asked, *OK, so what did you do?* **Learner I:** *I took twenty out of twenty five and multiplied it by hundred, and got 80%.* **Teacher:** *Very good, alright, why are you multiplying the whole by that fractions (teacher pointing at $\frac{20}{25}$?* **Learner I:** *I don't know why, but I just know that questions about percentage is either you multiply by hundred or divide by hundred. This question automatically is multiply by hundred because the twenty is not percentage, if it could be percentage then it will be twenty divide by hundred.* **Teacher:** *OK, mm, OK, very good, (Line 3).* Learner I could answer the question but not understand why he divided twenty by twenty five.

When I gave learners the algorithm of this concept, I explained more about why Learner I divided twenty by twenty five (see extract below). After learner I's response, I asked learner E for his work. He did not answer me, so I used learner I's response to elaborate more on when to divide by hundred and when to multiply by hundred. I explained that if the question was changed to: 20% of 25 marks, that's when one should write twenty out of hundred times twenty five. I reminded learner E to remember that "out of" is the same as "divide by". I then asked any learner to read the question and take out the main information.

Learner D read the question and said: *The main information is the total number of learners which is thirty and the number of boys in the class.* **Teacher:** *Very good, what percentage are boys?* **Learner A:** *writes $\frac{5}{30} \times 100 = 16.666 = 17$ boys.* When learner A was working out her answer on the chalkboard, I observed that all learners worked as learner A did, apart from learner

E who worked it out as follows: $\frac{30}{5} \times 100 = 600\%$, (line 5). I then asked learner A for her explanation as follows: **Teacher:** *How did you get that?* **Learner A:** *Five divided by thirty, times hundred it gives me sixteen point six, six, six then I rounded off and my answer is 17boys.* **Teacher:** *Is it seventeen boys?* **Learner A:** *Seventeen percentages.* **Teacher:** *Why did you multiply?* **Learner A:** *I want to work out the answer.* **Teacher:** *OK, I realized that the learners could answer, but could not justify their working.*

I then explained to the learners using the first example: What percentage are twenty marks out of twenty five marks? One needs to take 1% of the total marks-which is 0.25. In order to know how much 20 is of 25 in percent, one must know how often 0.25 goes into 20. If one divides by 1% of the whole, the answer will be 80%.

The learners understood the work better after my explanations. I then gave them other representations, this time simple symbolic calculations: e.g. Convert to a percentage (a) $\frac{21}{25}$; (b) $2\frac{1}{2}$; (c) $\frac{5}{8}$; (d) 0.38; (e) 1.05. The following extract is the learners' responses: **Learner G:** *wrote $\frac{21}{25} \times 100 = 84\%$.* **Teacher:** *What did you do?* **Learner G:** *I took twenty one divided by twenty five and multiplied by hundred then I got 84%.* **Teacher:** *Why are you dividing by twenty five?* **Learner G:** *I want to find out one percent of my total amount.* **Teacher:** *That's great (line 7).* Learner G has showed that he was able to express one quantity as a percentage of another and even justified his working correctly.

Learner B: *wrote $2\frac{1}{2} = \frac{5}{2} \times 100 = 250 = 2.5\%$.* **Teacher:** *Yes, explain your work.* **Learner B:** *I took two and a half than convert it to improper fractions, than I multiply by hundred than I got two hundred and fifty.* **Teacher:** *Where did you get two point five percent?* **Learner B:** *I divide my answer by hundred.* **Teacher:** *Why?* **Learner B:** *Because percentage is out of hundred.* **Teacher:** *Yes, percentage is out of hundred, but where the percentage is?* **Learner B:** *OK, I, wanted, wanted, to write my final answer as percentage.* **Teacher:** *You took $2\frac{1}{2}$ to percentage, that's why you multiplied by hundred, and get 250. This supposed to be your final answer.* **Learner B:** *How can I get a percentage answer more than hundred, if percentage stop only at hundred.* **Teacher:** *No, learner B, percentage doesn't stop at hundred. Percentage can be any number out of hundred even 250.* **Learner B:** *OK, (line 8).*

Learner B used prior knowledge of converting the mixed fractions to improper fractions and multiplied his fractions by a hundred. He could not accept a number greater than a hundred to be his answer, so he decided to divide his final answer by a hundred again, because he believed that

percentages stop at one hundred. The discussion between me and him enabled him to realize his mistake and agree to change. During the second focus group interview he mentioned that that was one thing he had learned.

Extract 26 Interview 2. Learner B: *I learned that percentage doesn't stop only at hundred. I was confused with the meaning between percentage to be out of hundred and percentage to be hundred. I thought percentage should be only number below hundred, it cannot be more than hundred. Now I realized that I can get even five hundred percent and it means five hundred out of hundred (line 5).*

The last representations I used to explain the concept of expressing one quantity as a percentage of another one was graphs. I drew two types of graphs on the chalkboard for learners to express the percentages of the shaded parts in the graph. The first graph had six faces; and two of the six faces were shaded as follows:



I asked learner I to work out the percentage of the shaded faces. This was his answer:

Extract 27 Lesson 6. Learner I: *wrote $\frac{2}{6} \times 100 = 33.33$. Teacher: What did you do? Learner I:* *I took two of the face which were shaded out of the total faces, and that is six faces and multiply my fractions by hundred and got thirty three, three,. The reason of my work is first to work out one percent of my total amount, after getting that answer I then wanted to find out how many times does my one percent goes in two of the shaded parts, to know how much 2 of 6 in percent is. Teacher: Very good, that's great. (line 13).* I then asked learner F to work out the second diagram. It was a diagram of twelve "Z" letters, with three rows of four "Z" in each. Among the twelve, eight of the Z; were shaded. **Learner F's response was as follows:** $\frac{8}{12} \times 100 = 66.66\%$. **Teacher:** *What did you do? Learner F:* *I took eight divided by twelve multiply hundred and I got sixty six point six, six percent. Teacher: Can you give reason to your work. Learner F:* *A question like this one should first find the one percent of the total then divide the parts which are shaded by one percent because you want to know how many eight are there in twelve as percentage (line 14).* Through learners I and F's justifications of their working, they showed a deep understanding of how to express one quantity as a percentage of another.

The results of the Learners' Activity 6 showed that all learners were able to express one quantity as a percentage of another quantity. This activity had five different ways of questioning their

understanding of this concept. The first question was for the learners to solve their work on diagrams using this concept. There were three diagrams with shaded parts and learners were expected to express their fractions of the shaded parts as percentages. In Question 2, learners were expected to convert the following to percentages; (a) $\frac{5}{20}$; (b) $\frac{17}{50}$; (c) $2\frac{1}{2}$; (d) 0.125 and (e) 1.02. In Question 3 learners were asked to work out the percentages of: (a) 20 cents of 150 cents and (b) 26 grams of 100 grams. In Question 4 learners were expected to fill in a table with fractions down one side for the learners to fill in the correct percentages of each specific fraction. In Question 5 and 6 learners were expected to solve word problems on expressing one quantity as a percentage of another one. (See Appendix B). Apart from learner E, who only scored 10%, the other learners scored in the range of 95% to 100%. Learner E's problem was that he was calculating all his work using the algorithm of figuring out a part. The rest of the learners showed that they were able to express one quantity as a percentage of another correctly and even showed their working effectively. This result reflected the same in the post test: in Question 7 learners were expected to use this concept, and they were able to calculate their work correctly apart from learner E who could not.

4.4 WHAT LEARNERS' INTERACTIONS THAT WERE NOT STRONGLY MEDIATED BY THE TEACHER PROMOTED OR NOT DID PROMOTE THE LEARNING OF PERCENTAGES AND FRACTIONS?

Through interaction amongst the learners, the following learning took place:

- (a) All the learners could identify more equivalent fractions of the initial fractions which they were given
- (b) Learners could increase and decrease a quantity by a given percentage

4.4.1 All the learners could identify more equivalent fractions of the initial fractions which were given to them

Learners worked in groups during Lesson 3, and through their discussions during their interactions among each other, all the learners could identify more equivalent fractions of the initial fractions which were given to them. These interactions were not strongly mediated by the teacher, and they promoted the learning of percentages and fractions.

There were five groups of learners in the class, and in each group there were three learners. Group 5 with learners I, J and N used diagrams to generate five more equivalent fractions of their initial fractions. Their initial fraction was $\frac{4}{6}$, and their identified equivalent fractions were: $\frac{6}{9}$; $\frac{8}{12}$; $\frac{10}{15}$; $\frac{2}{3}$; $\frac{12}{18}$. These three learners negotiated the best representations to use, to identify more equivalent fractions of their initial one. During their discussions, they pointed out reasons why

they wanted to use a particular representation and not another one. They listened to each other; they disagreed with each other's reasons and also agreed to each other's reasons. Through this, the learners learned what an equivalent fraction is. The extract below shows learners' interactions through their discussions.

Extract 28 Lesson 3. Learner J: *Let's work out like this - you take $\frac{4}{6}$ and multiply it by two, or three, or any number. Learner I: Why should we multiply by two of three or any number? Learner J: Because the fractions we will get will be equivalent to this first one. Learner I: But how do we explain so that it is clear? Learner J: Let me show you (took a piece of paper and worked out) $\frac{4 \times 2}{6 \times 2} = \frac{8}{12}$; this new fractions is the same as the first fractions. Learner N: That is true, our problem is that how do we tell that the two fractions are equal. Learner I: What if we work out their sizes. Learner N: How? Learner J: By working out all to be decimal, like four divide by six is (pressing in the calculator) zero point six, six, six and also eight divide by twelve is zero point six, six, six. Learner I: That's true. Learner N: Yes. Learner I: Let's try to draw our decimals. Learner N: It will be difficult to use decimal maybe fractions. Learner J: Yes fractions. Learner I: OK (started drawing fractions). Learner N: Make sure the line should be of the same size even if the parts will be different. Learner I: Yes (line 10).* This group agreed to use representations to bring meaning to their symbolic representation. They agreed that using bars were the most effective representation because one can clearly see how their generated equivalent fractions fitted exactly to their original fractions.

Groups 1 and 3 on the other hand, used manipulatives (11 boxes) to generate equivalent fractions of their initial one. Learners A, C and D were in Group 1, while learners G and K were in Group 3. Group 1 used pins (11 boxes) and Group 3 used bottle tops (11 boxes) to make repeated patterns that generated equivalent fractions. Group 1's initial fraction was $\frac{1}{3}$ and the five fractions which they generated were: $\frac{2}{6}$; $\frac{3}{9}$; $\frac{4}{12}$; $\frac{5}{15}$; $\frac{6}{18}$, while Group 3's initial fraction was $\frac{2}{8}$ and the five fractions which they generated were: $\frac{4}{16}$; $\frac{6}{24}$; $\frac{8}{32}$; $\frac{10}{40}$; $\frac{12}{48}$. These two groups generated their equivalent fractions by making a group of their first initial fractions with the manipulatives and then repeating the patterns to generate new equivalent fractions. Groups 1 and 3 discussed and convinced one another, why they should use concrete models (11 boxes). Through this, learners were able to learn equivalent fractions.

Extract 29 Teacher's Journal Lesson 3. Learner D: *Now how are we going to do this? Learner A and C: Quite. Learner D: Yes what if we use one of the boxes. Learner A: How?*

Learner D: We can do it this way (she took the box, counted out pins and placed one pin on top and three pins at the bottom, and then arranged a second group of the pins but this time using two). **Learner C:** OK, you mean this new group is one of the equivalent fractions? **Learner D:** Yes. **Learner C:** Let me count (counted the group) and wrote $\frac{2}{6}$. **Learner A:** Oho, the more the pins increase up, the more it increases down. **Learner D:** Yes it is increasing and we are adding the same group every time. **Learner C:** OK, let's write the rest of our equivalent fractions.

Learners G and K also discussed, and then later came to an agreement about the representations to use. This was their discussion: **Learner K:** Let us look for our equivalent fractions. **Learner G:** We can start counting bottle tops and put them in groups. **Learner K:** What do you mean? You know what: equivalent fractions are fractions that are the same. **Learner G:** Yes. Now we can show it and explain to the teacher how they are the same fractions, we can put them in groups like this (taking out bottle tops and start grouping them), some bottle tops should be up and some down.

Then it will be like the same thing that we are grouping all over again, each time we add a group into the new fractions. **Learner K:** (helped to group the bottle tops by following the patterns). **Learner G:** started counting and learner K wrote the five equivalent fractions. The way Group 1 and 3 generated their equivalent fractions shows that they looked at the relationship of top pins or bottle tops to that of the bottom pins or bottle tops and generated their equivalent fractions.

Group 4 had learners B and H, and these learners identified the five equivalent fractions by repeated patterns but instead of using manipulatives (11 boxes) they multiplied their initial fraction ($\frac{15}{20}$) by a factor (2), and each time they got the new fraction they multiplied the new fraction by the same factor. e.g. $\frac{15 \times 2}{20 \times 2} = \frac{30 \times 2}{40 \times 2} = \frac{60 \times 2}{80 \times 2} = \frac{120 \times 2}{160 \times 2} = \frac{240 \times 2}{320 \times 2} = \frac{480}{640}$. When learners were asked to explain why they multiplied by two, they could not explain why (Teacher's Journal, line 16).

Group 2 had learners F and E, who used calculators to find out all the fractions that have the same percentage as their initial fraction, i.e. equivalent fractions. Their initial fraction was $\frac{1}{8}$ which they converted to a percentage 12.5%. Then they looked for all the fractions that show 12.5%. The equivalent fractions which were identified were: $\frac{2}{16}$; $\frac{4}{32}$; $\frac{125}{100}$; $\frac{5}{40}$ and $\frac{8}{64}$. During the activity involving identifying equivalent fractions, these two learners discussed which of the equivalent fractions has the same percentage as the initial one. Through this discussion there were disagreements and agreements about their identified equivalent fractions. Learner E learned

to identify correct equivalent fractions through interacting with learner F. The extract below shows the interaction between learner E and F.

Extract 30 Teacher's Journal, Lesson 3. *Learner F: You know what, we should use our calculators, so that we look for fractions that have the same percentage as one eighth have, listen you look for three equivalent fractions and I will look for three also. Learner E: How is that? Learner F: Look, I want you to work out the percentage of one eighth. Learner E: (pressed one divide by eight, multiplied by one hundred, got twelve point five and said) I got twelve point five. Learner F: Yes, that's the correct percentage, now we should look for any fractions with the same percentage. Learner E: Start trying on the calculator. After a few minutes, these learners came together and discussed their identified equivalent fractions. One of the fractions that Learner E identified was $\frac{125}{100}$, which he insisted was one of the correct equivalent fractions of $\frac{1}{8}$ because they have the same percentage. This is how learner F answered:*

Learner F: No, learner E, this fraction cannot be one of them because it doesn't have the same percentage as one eighth. Learner E: But one hundred and twenty five divided by hundred is 12.5% (Learner E did not use a calculator, then learner F gave learner E a calculator to work out his calculations, and learner E got a different answer). Learner E: OK, I got a wrong answer, I thought it will give me twelve point five; it gave me one point two five. Learner E: OK, let's write all our correct fractions in one paper (line 14).

4.4.2 Learners were helped to increase and decrease a quantity by a given percentage

Before the programme, all the learners apart from learner F have showed that they were not able to increase a quantity by a given percentage (See Appendix C, Questions 5 and 7). Interactions amongst learners helped most learners to learn how to increase and decrease a quantity by a given percentage.

In Lesson 7, the learners were arranged in groups of three. Learners in Group 1, 2 and 3 used grids to increase or decrease a quantity by a certain percentage. Group 1 was given a grid of 24 rectangles to reduce by 25%. Learners A, B, and F were in Group 1. The extract below shows the learners' discussion of this activity:

Extract 31 Teacher's Journal Lesson 7. *Learner B: Reduce means we should subtract. Learner A: Yes, we should subtract 25% from 24 rectangles. Learner F: We need to work out twenty five percent of twenty four rectangles. Learner B: It's true (he then press $25 \div 100 \times 24 = 6$ and said) The answer is six. Learner F: No, the answer is not only six, six we work out*

percentage of a quantity only, we need to continue. **Learner A:** Yes twenty five of twenty four rectangles are six rectangles. **Learner B:** OK, you mean we need to minus six. **Learner F:** Yes, we minus six from twenty four rectangles and that's eighteen. **Learner F:** Yes eighteen is our answer. **Learner A:** What about this grid which the teacher gave us. **Learner B:** We need to reduce by the answer we got. **Learner F:** How do we do it? **Learner B:** We need to subtract six rectangles of this grid. **Learner F:** Yes, you are right. **Learner A:** (gave scissors to learner B). **Learner B:** Cut six rectangles from the grid and count the new grid. **Learner F:** What did you get? **Learner B:** Eighteen rectangles. **Learner F:** That's nice. It is visible that our answer eighteen is clearly shown on the paper (referring to the grid) (line 2).

Group 2 had only learners E and G. These two learners were given a grid of 50 rectangles to reduce by twenty five percent. The extract below shows the learners' discussion of the activity. **Learner G:** Count the rectangles if they are fifty. **Learner E:** (counted the rectangles on their grid, then later said) Yes, it is fifty. **Learner E:** Now is what? **Learner G:** We should calculate the percentage of our grid. **Learner E:** (pressed $25 \div 100 \times 50 = 12.5$). **Learner G:** What are you doing? **Learner E:** I calculate twenty five percent of fifty rectangles and I got twelve point five. **Learner G:** Now this answer we will minus it **Learner E:** (worked out the answer on the calculator as follows $50 - 12.5 = 37.5$). **Learner G:** Let me see the answer. **Learner E:** It is thirty seven point five. What should we do now? **Learner G:** We should minus, we should minus, and we should minus thirty seven point five from our grid. **Learner E:** How, but I minus already, unless this thirty seven point five should be our final answer. **Learner G:** Yes, ok. Now, we cut our grid of fifty rectangles. **Learner E:** (cut twelve rectangles and asked) What about point five. **Learner G:** (folded the thirteenth rectangle and said) It should be like this, showing half of the thirteenth rectangle. **Learner G:** You know what, I think machines in shops that reduce photo sizes, use this method to calculate it (line 3). The two extract above showed that through interactions, learners offered opinions to one another; they gave reasons to support their opinions; they were sought each other's view points and agreed with others' opinions, so through their communications they learned from one another.

In Lesson 8 the learners worked in groups and discussed how to calculate VAT inclusive and VAT exclusive price. At the beginning of the lesson all the learners used the same method to calculate the prices of items with VAT inclusive and prices of items with VAT exclusive. The method they used was only appropriate to calculate the prices of items excluding VAT. It could not be used to calculate prices including the VAT. e.g. The original price for the first item (can

of beans) was N\$4.00 VAT excluded. All the learners first worked out 15% of the original price ($\frac{15}{100} \times 4 = N\0.60) so the answer for VAT was N\$0.60. They then added this amount to the original price to get the price of a can of beans including VAT and they got N\$4.60. Likewise with the second item - learners worked out the VAT first and they added the answer to the original price.

This method was only appropriate to the first two examples and not to the last two examples. For the last two items, learners were expected to work out the VAT and also the original price without VAT. Learners worked out their work as in the first two calculations. They took 15% of the original price of the items and subtracted their answers from the price of an item with VAT. e.g. For the calculation of the chicken, learners worked out 15% of N\$30.00 and got N\$4.50, and they thought this answer was for the VAT amount. Learners later on subtracted N\$4.50 from N\$30.00 and got N\$25.50 for the original price.

But then when five of these learners (learners A, B, D, G and H) tested their answers by working out 15% of N\$25.50, they did not get the VAT of N\$4.50. They realized that their last two calculations were wrong, and they were eager to find out their mistake.

After I introduced the diagram for percentage increase and decrease (see Fig 2.5 in Chapter Two) to the learners, learners A, B, D, G and H were able to rectify their error by using the diagram. Learners were in groups and discussed how to work out the last two examples. The extract below shows how the interactions among the learners helped other learners to rectify their error.

Extract 32 Teacher's Journal, Lesson 8. Learners A, G and H were in Group 1. **Learner H:** *This answer that we got N\$25.50 as the original price, is not giving us the N\$30.00, the price with VAT included. Unless we try the sketch, the teacher had shown us.* **Learner A:** *We should take N\$30.00 then divide by $\frac{115}{100}$. What is the answer?* **Learner G:** *It is N\$26.09.* **Learner A:** *Now let us try to subtract our answer from N\$30.00. What are we getting?* **Learner H:** *It is N\$3.91.* **Learner G:** *Our answers this time are correct because all the three prices are going hand in hand (line 5).* Learners B and D got the correct answer also after discussing how to use the sketch that I introduced to them. During their interactions, these five learners shared opinions on how to calculate; they tested their answers and agreed on the final answer.

During the learners' Class Activity 8, on how to work out the price of an item with VAT inclusive and exclusive, all the learners got 100% (see Appendix B - the sample of the activities). This chapter has shown how in Class Activity 8(b) Question 1, learners showed that using the diagram for percentage increase and decrease helped them work out the correct answer.

CHAPTER FIVE

DISCUSSION

5.1 INTRODUCTION

This study investigated how learners may use multiple representations in a social interaction to promote the learning of percentages and fractions. First, this study examined how the use and manipulation of multiple representations as tools by the learners influenced the learning process. Second, it investigated how the teacher influences the dynamic learning interaction process in ways that promote learning. Lastly, the interactions between the learners that were not strongly mediated by the teacher were also investigated. The participants included nine, Grade 8 learners. This chapter discusses the major findings from the results, while addressing the three sub-research questions of this paper, and compares these findings with the findings of others who have conducted similar research on this topic.

5.2 HOW DID THE USE AND MANIPULATION OF MULTIPLE REPRESENTATIONS AS TOOLS BY THE LEARNERS, INFLUENCE THE LEARNING PROCESS?

5.2.1 Learners were able to look at representations in “useful” ways

The results of the study showed that the use and manipulation of multiple representations as tools by the learners influences the learning process. Learners were able to look at representations in “useful” ways. After the learners worked with the (11 boxes) tool, they were able to reason why a fraction is part of a whole. Learners could understand the meaning of percentages and fractions in depth - not only their written and spoken symbols, but their explanation in terms of actions. Therefore this study has provided empirical data to support the research findings reported by Edgardo (2001) and Kaput (1989), that multiple representations help students to understand mathematical concepts in depth, and develop their number sense (Rau et al., 2009).

Research in the literature indicated that learning to use a tool changes the way we view objects and actions. The use of the tool and the potential it offers, structures the way we see actions and potential actions (Vygotsky, 1986). This study supported the idea that the use of these tools lets learners gain knowledge of what percentage and fractions means. Before the programme, learners could just write and say a correct fraction without knowing the meaning of that written or spoken symbol. After learners used manipulatives (11 boxes) as tools for learning, suggested also by Hedegaard (1990) they developed a meaningful understanding of what a percentage or a fraction is. This supports the research findings reported by Cramer & Henry (2002) that

manipulatives benefit students" learning by providing hands-on experiences to make symbolic representations more concrete (Cramer & Henry, 2002).

5.2.2 Multiple representations may make some aspects of a concept clear

Rau et al. (2009), state that students learn more with multiple graphical representations of fractions than with a single representation, however only when prompted to self-explain how the graphics relate to the symbolic fraction representation. In this study, evidence shows that learners prompted to self-explain how the graphics relate to symbolic fraction representation, by drawing five bar graphs with various symbolic fractions with of the same size. The five bar graphs showed clearly how various symbolic fractions are equivalent, because their sizes fit exactly to one another. Therefore the bar graph is a central model for reasoning with fractions and percentages because the advantage of it is that it has "body" area. For children, this makes it easier to talk in term of "the whole" and "the so-much part" of the whole (Van Galen et al., 2008).

Apart from some learners who used bar graphs, others used concrete models (11 boxes) to identify equivalent fractions of the initial one given to them. Edgardo (2001) encouraged students to use various representations suitable for them, so that they can pass from one representation to another, knowing the possibilities, the limits and effectiveness of each one. In this study learners used different representations, and this helped them to understand and accept multiple methods of solutions and forms of answer. Group 4, in particular could not explain why they were "multiplying" their initial fractions "by 2" - $\frac{15 \times 2}{20 \times 2} = \frac{30 \times 2}{40 \times 2} = \frac{60 \times 2}{80 \times 2} = \frac{120 \times 2}{160 \times 2} = \frac{240 \times 2}{320 \times 2} = \frac{480}{640}$ to generate a new one. After they learned from Group 1 and 3 who used concrete models (11 boxes) to generate equivalent fractions through repeated patterns of their concrete models, they accepted and understood their work better than before.

5.2.3 Multiple representations enable learners to correct their error

Kilpatrick et al. (2001) say that multiple representations such as manipulates can enhance students" understanding and help students correct their own errors. In this study evidence show that manipulatives (11 boxes) helped learners detect their errors. Evidence shows that when Learner G translated his work from removing a portion out of a box into written symbols, these representations helped him detect his mistake faster. Learner G removed sixty two bottle tops out of a hundred bottle tops and he was only left with twenty two bottle tops in the box. He then calculated his work on a piece of paper as follows: $\frac{62}{100} + \frac{22}{100} = \frac{84}{100}$. Learner G was expecting to get the answer $\frac{100}{100}$, and he could see clearly that something went wrong. The teacher helped learner G

to realize the error by re-counting the bottle tops together and this recounting helped learner G to detect his error. That was when he admitted that the total of his bottle tops were not enough. So he added 16 extra bottle tops, removed 62 and was left with 38 in the box. He then tested his work again: $\frac{62}{100} + \frac{38}{100} = \frac{100}{100}$.

Moreover, the use of the diagram for percentage increase and decrease (see Fig. 2.5) helped learners to detect their errors. Evidence has shown that before the diagram for percentage increase and decrease was introduced to the learners, learners A, B, D, G and H did not know how to calculate prices excluding VAT, only prices which included VAT. These learners used the same method of calculating prices including VAT, as for prices excluding VAT. e.g. The question was for learners to work out the price of the chicken without VAT if the VAT is 15% and the amount including VAT is N\$30.00. Learners worked out 15% of N\$30.00 and got N\$4.50, they then thought this answer was for the VAT amount. Learners later on subtracted N\$4.50 from N\$30.00 and got N\$25.50 for the original price. But then these learners (learners A, B, D, G and H) tested their answers by working out 15% of N\$25.50 and they did not get the answer VAT of N\$4.50. They realized that their last two calculations were wrong, and they were eager to find out their mistake. After I introduced the diagram for percentage increase and decrease (see Fig. 2.5 in Chapter Two) to them, learners A, B, D, G and H were able to rectify their error by using the diagram. Learners took the price of chicken with VAT which was N\$30.00 and divided it by $\frac{115}{100}$. They got an answer of N\$26.09 - the original price. They then calculated the VAT amount and got N\$3.91 - 15% of N\$26.09. When these five learners tested their answer, there was a correlation between the three prices, so this time their calculations were correct. This diagram was very useful to the learners, and using it enabled them to detect their errors.

5.2.4 The use of calculator contributed to learners conceptual understanding of percentages and fractions

There is evidence in this study that the use of calculators helped learners to carry out their problem solving more effectively. Therefore this study provides observed data that supports the research findings reported by Kilpatrick et al. (2001) that the use of the calculator helps students carry out their problem solving. Students using calculators in mathematics spent relatively less time on algorithms and more on problem solving. In this study specifically, results have shown that the use of calculators allowed learners to think about the relationships between percentages and fractions. e.g. learners knew that the line on a fraction means “out of” and on the calculator it

means “divide by”, and because of this learners A, B and I worked out their word problems accurately, which they could not do before the programme.


Research in the literature showed that the use of calculators enhances learners’ understanding of the concept (Van Galen et al., 2008). This study supports the idea that the use of calculators helped the learners develop and improve their conceptual understanding such as: using calculators as an efficient tool to explore and check their answers, or to compare two different answers in order to identify mistakes.

5.3 HOW DID THE TEACHER INFLUENCE THE DYNAMIC LEARNING INTERACTION PROCESS IN WAYS THAT PROMOTED OR DID NOT PROMOTE LEARNING?

5.3.1 Changing the word to change the focus

Results of this study showed that learners moved into their ZPD, after they changed from using the word “over” to “out of”. Vygotsky (1986) believed that when a student is at the ZPD for a particular task, providing the appropriate assistance will give the student enough of a “boost” to achieve the task. Zuckerman (2007) said the help from the teacher changes the students’ actions. In this study, evidence shows that learners came to school not knowing the meaning of fractions and percentages. After my help in letting them change the word “over” to “out of” learners became increasingly independent and finally they were able to understand the meaning of fractions and percentages.

At the beginning of the programme, learners showed that they were accustomed to using the word “OVER” referring to the line of any fraction. e.g. $\frac{7}{100}$ would be read as seven OVER a hundred. The action of the learners was determined by the focal-action of the word “OVER” on (something OVER something). This is misleading because it only describes the writing of the fraction. To the learners, the word OVER related to the experience of writing the symbols. It seems to be a closed loop of meaning between two symbols: the word “OVER” and the FRACTION SYMBOL.

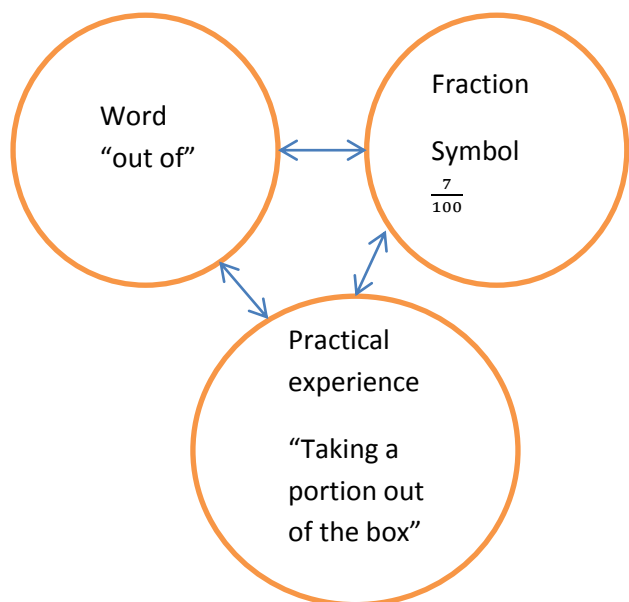
(Word “OVER”)  (Fraction symbol $\frac{7}{100}$). Nothing steps out of this loop to relate to the rest of the learners’ other experiences and certainly not to any meaningful action beyond the symbols.

The interaction between the teacher and the learners using real world objects and manipulatives (the 11 small boxes) helped bring meaning to learners’ use of both oral and written symbols of percentages and fractions. During the process of the programme learners changed their focus;

they started using the word “OUT OF” rather than “OVER”. Thus the focal action of the learners responded to the meaning of the word “OUT OF” and they removed a portion of objects from a box. Therefore, to the learners, the word “OUT OF” related to both the experience of writing the symbols and the practical experience.

This type of relationship deepens the understanding of the concept. Therefore this study provided realistic data that supports the research findings reported by Bransford, Brown & Cocking (2002), that communication that occurs in social settings with MKO helps the child construct an understanding of the concept. The figure below shows how the change of word “OVER” changed the focus and also where the focus pointed to after learners had changed the word “OVER” to “OUT OF”.

Fig 5.1 Changing the word to change focus.



5.3.2 Learners made links between the concepts learned.

Research in the literature indicates that the teacher’s appropriate guidance on how to use manipulatives helps students to build links between the object, the symbol, and the mathematical idea being presented (Ball, 1992). Students need to be helped to see relevant aspects (e.g. manipulatives) and to link those aspects to appropriate symbolism and mathematical concepts and operations (Fuson & Briars, 1990). Thus in this study, evidence shows that the teacher’s role was that of a mediator, guiding learners how to use manipulatives (the 11 boxes), and this helped learners to build links between the concept learned when using manipulations, to appropriate written and oral symbols.

After learners changed the word to change focus, i.e. from using the word “over” to “out of” they gained insight into what a fraction means. Learners further linked their understanding of this concept “out of” to “dividing by”, especially if one is expressing a fraction on a calculator. This study provides evidence and supports the research finding reported by Rau et al. (2009), that learners acquire a deep understanding only if they are able to link multiple representations of the same concept and to coordinate between them. Therefore in this study learners could link the concept of “out of” to “divide by” and evidence shows that they acquire a deep understanding through this linkage.

5.3.3 Learners deepened their content knowledge and conceptual understanding of percentages and fractions

This study has shown evidence that learners deepened their content knowledge and conceptual understanding of percentages and fractions through the three aspects: 1. learners could convert between percentages and fractions using multiple representations; 2. learners could work out the percentage of a quantity, and 3. learners could express one quantity as a percentage of another.

Many learners who could not convert to percentages before the programme showed that they were able to do it after the programme through - using multiple representations. In this study, evidence shows that learning took place through interactions between the teacher and the learners. As Vygotsky (1986) pointed out, learning is strongly influenced by social interactions which take place in meaningful contexts (Raymond, 2000). In this study the teacher as a mediator in social interactions, helped the learners to gain knowledge on how to convert by creating an environment that motivated and promoted learning.

When learners were presented with the concept of converting between fractions and percentages using multiple representations, I scaffolded their learning by first letting all the learners who could already convert between fractions and percentages, explain in front of the others how to do it; second, by asking learners to justify their explanations; third, when learners made mistakes I help them by continually probing their understanding, enabling them to notice their own mistakes without me just telling them the answer. Through all this scaffolding, learners learned the concept of converting between percentages and fractions. This type of scaffolding provides realistic data to support the research findings reported by Obukhoval & Korepanova (2009) that scaffolding is seen as a „structure point“ for performing an action.

Furthermore, evidence in this study shows that learners deepened their concept of working out percentages of a quantity. This is because learners took part in activities that they were not able to perform on their own, but did so with the teacher's help. Research in the literature indicates that children's social interactions with more knowledgeable or capable others and their environment significantly impacts their ways of thinking and interpreting situations (Raymond, 2000). It is the role of the teacher to make the social interaction effective. Evidence in this study shows that the teacher (MKO) helped learners develop understanding of the concept. I made sure that learners were actively involved in the interactions, and also motivated, just as Hedegaard (1990) pointed-out that children not only learn through their participation in the social world, but also become involved in a reciprocal process in which their motives and personalities play a part in the interaction with the other persons (MKO) in the classroom (in this case - the teacher), and thereby contribute to their own learning.

During the two lessons - Lessons 4 and 5 - that required learners to work out the percentage of a quantity, I created opportunities for the learners just as Kilpatrick et al. (2001) suggest. Teachers should allow students to engage in and spend time on academic tasks such as working on problems, exploring situations and gathering data, listening to explanations, reading texts, or conjecturing and justifying. Evidence in this study show that I used some of these features, creating opportunities for learners to learn, and these are as follows: first, ask learners about their prior knowledge on the concept, and then I used this prior knowledge to elaborate and explain the concept in detail. Second, when a learner is stuck, not knowing how to work out a question, I used other learners to help explain the concept in detail. Again I gave more examples on the same concept for learners to tackle and I also asked learners to give more methods to tackle the same question. Third, when learners made mistakes, I directed them to the right answer by asking them questions that led them to think of the answers. Fourth, I asked learners to justify their answers, and all the learners' answers and justifications I summarized when I gave them the algorithm by Van Galen et al. (2008) to work out percentages of a quantity. Lastly, I used multiple representations to explain the same concept. This approach helped learners gain knowledge on how to work out percentages of a quantity.

Moreover, evidence of this study has shown that learners deepened the content knowledge and concept of expressing one quantity as a percentage of another. The interaction between the teacher and the learners helped learners deepen this concept. I used multiple representations to explain the concept, and the two representations I used were written symbols and graphs. After I

taught this concept using only written symbol and realized that learners had gain the idea, I again taught the same concept using graphs. I also used learners' responses to explain the concept in detail. Again I asked learners to justify their answers. If they were not able to justify them, I helped or used other learners to help. Near the end of the lesson I used learners' justifications and explanations and introduced another algorithm to use for this concept by Van Galen et al. (2008), but reassured them that their methods were also a hundred percent accurate. This approach helped deepen learners' concepts of expressing one quantity as a percentage of another. These findings give realistic data to support the research findings by Kilpatrick et al. (2001), that teachers may manage discourse in their classroom by judging when to tell, when to question, when to correct, and also by deciding when to guide with prompting and when to let students grapple with a concept. In this way students' understanding of key ideas is developed, and opportunities are provided to emphasize and model mathematical reasoning and problem solving and to enhance students' dispositions toward the particular concept.

5.4 WHAT ARE THE LEARNERS' INTERACTIONS THAT WERE NOT STRONGLY MEDIATED BY THE TEACHER THAT PROMOTED OR DID NOT PROMOTE THE LEARNING OF PERCENTAGES AND FRACTIONS?

5.4.1 All the learners could identify more equivalent fractions of their initial fractions given to them

Research in the literature shows that during social interactions, children's talking promotes the development of reasoning, and they term this process exploratory talk (Mercer et al., 1999). Evidence in this study has shown that a number of learners' interactions that were not strongly mediated by the teacher promoted the learning of percentages and fractions. Through the interactions between learners, they gained knowledge on how to identify fractions equivalent to their initial one. During these interactions, learners discussed, and pointed out reasons why they used certain representations to identify more equivalent fractions. Learners disagreed and agreed with each other's reasons they listened to one another. These features are the ones that Mercer et al. (1999) also identify as helping children reason together more effectively.

When learners were asked to identify more equivalent fractions using multiple representations, they worked in groups. Learners I, J and N used diagrams to generate five more equivalent fractions of their initial fractions. Throughout their discussions, there were agreements and disagreements on the best representations to use. Later they came up with the final decision of using diagrams to identify equivalent fractions. Their main reason was that graphs bring meaning

to symbolic representations. They said it is easy for one to clearly see how their generated equivalent fractions fitted exactly to their original fractions. Learners A, C, D and also learners G and K used concrete models (11 boxes such as pins and bottle tops) to generate new equivalent fractions. They grouped their pins and bottle tops in such a way that they compared the top pins/bottle tops to the bottom pins/bottle tops. They repeated the patterns until they got the number of equivalent fractions needed. Their main reason for using concrete models is that it was easy for them to see how equivalent fractions are generated because of the same increasing pattern.

5.4.2 Learners were helped to increase and decrease a quantity by a given percentage

Evidence in this study shown that interaction amongst learners helped most of them learn how to increase and decrease a quantity by a given percentage. In Lesson 7 where learners were tasked to increase or decrease grids by a given percentage, they interacted by offering opinions to one another. They gave reasons to support their opinions; they sought the other's views and agreed on the final answer. These interactions helped many to acquire more knowledge on this concept. Results for the Learners Activity 7 showed that learners could increase and decrease quantities by given percentages - which they could not do before the programme.

In Lesson 8, after learners were introduced the sketch to use when increasing and decreasing a quantity by a given percentage, evidence shows that learners acquired the knowledge on how to use the sketch, and not only that but how to calculate this concept using the sketch. Through the interaction between learners, by testing their answers, discussing, sharing opinions and agreeing on the final answer, learners learned more. Evidence shows that during their talk, learners were reasoning. Therefore this study has provided empirical data to support the research findings reported by Mercer et al. (1999) that in exploratory talk, knowledge is made publicly accountable, and reasoning is visible in the talk.

CHAPTER SIX

CONCLUSION

6.1 INTRODUCTION

This chapter serves to conclude my study by providing a summary of the research findings, followed by a brief discussion of the limitations and challenges encountered in this thesis. Thereafter, the potential value of my research will be discussed and finally, I will present my recommendations and suggestions for further research.

6.2 FINDINGS

Learners attained a stronger grasp on the topics of percentages and fractions through the use of: Multiple Representations, Teacher and Learner interactions, and Learner to Learner interactions.

- (i) **Effects of Multiple Representations.** Multiple representations have shown positive effects in the learning of percentages and fractions.
 - Learners were able to look at representations in useful ways
 - A representation may make some aspects of a concept clear
 - Multiple representations make it easier for learners to correct their own errors
 - The use of calculators contributed to learners' conceptual understanding of percentage and fractions
- (ii) **Teacher and Learner Interactions.** Interactions between the teacher and learners were important in the learning process. Three particular interaction types are seen to positively influence the process dynamics. They are as follows:
 - Learners change words to change focus
 - Learners made links between the concepts learned
 - Learners deepened their content knowledge and conceptual understanding of percentages and fractions
- (iii) **Learner to Learner Interactions.** There were interactions between learners themselves that were not strongly mediated by the teacher that promoted the learning of percentages and fractions. This type of interaction was essential for learning of percentages and fractions. It was as follows:
 - Exploratory talk

1. **Multiple Representations** – multiple representations allowed learners to visualize the - concepts clearly and they learned how to use visual diagrams and real world examples to help

them reason through the setup and calculation of difficult problems. Aspects of these problems became clearer and learners were able to easily identify errors made in their calculations. Moreover, the use of calculators contributed to learners' conceptual understanding of percentages and fractions. Therefore this study provided empirical data to support the research findings reported by Edgardo (2001) and Kaput (1989) that multiple representations help students understand mathematical concepts in depth, and develop number sense (Rau et al., 2009).

2. Teacher and Learners Interactions – the teacher positively influenced the learning process, by letting learners change words to change focus. Learners were accustomed to using the word “over”, referring to the line in all fractions. The learners' actions were reliant on something over something and the focal action of the word “over” was only on the written symbol. The teacher's help changed the students' actions. Learners started using the word “out of” rather than “over”. The focal action of the learners was because the word “out of” relates to the action of removing a portion from a box. Through this activity the teacher let learners see the manipulatives (11 boxes) in useful ways. Thus this study provides data that supports the research findings reported by Vygotsky (1986) that the use of tools and the potential they offer, structures the way we see actions and potential actions. Therefore for the learners the meaning of the word “out of” relates to both the experience of writing the symbols and practical experiences. So this study supports the research findings by Bransford, Brown & Cocking (2002) that communication that occurs in social setting with MKO helps the child construct and understand of the concept.

The teacher's appropriate guidance on how to use manipulatives helped students to build links between the concepts learned. After the teacher helped learners change the word to change their focus, the learners built links between the new concepts “out of” being the same as “divide by”. Research in the literature also supports this finding (Ball, 1992) that the teacher's appropriate guidance on how to use manipulatives helps students to build links between the object, the symbol, and the mathematical idea being presented.

Furthermore, the teachers' mediation through social interactions helped the learners to deepen their content knowledge and conceptual understanding of percentages and fractions. Learners could convert between percentages and fractions using multiple representations, they could work out a percentage of a quantity and they could express one quantity as a percentage of another one. Learners deepened their concepts of percentages and fractions because learning was

strongly influenced by social interactions which took place in meaningful contexts, just as Vygotsky suggests (Raymond, 2000). In this study, it was not any interaction that the teacher mediated, but rather it was interactions that created the child's zone of proximal development. Learners were actively involved in the interactions and they were motivated. The teacher also scaffolded learning that allowed the learners to perform an action. Obukhova & Korepanova (2009) see scaffolding as a structure point for performing an action.

3. Learner to Learner Interactions – learners' interactions that were not strongly mediated by the teacher promoted the learning of percentages and fractions, in particular through learners' exploratory talks. According to Mercer et al. (1999), exploratory talk promotes the development of reasoning. Hence in this study, reasoning was promoted: through learner to learner interactions involving discussions, by pointing out reasons why they used a certain representation to identify more equivalent fractions, by disagreeing and agreeing with each other's reasons, by listening to each other, and giving each other a chance to speak. In this way, the learners identified more equivalent fractions of their given one, and they also increased and decreased the quantity of a given percentage.

6.3 LIMITATIONS AND CHALLENGES ENCOUNTERED

One of the challenges I faced was the period of implementing the programme - the intervention programme was conducted during the holiday, and during this period parents expected their children to help them with household chores. Each and every day participants did not come on time, and sometimes some did not come at all. It was difficult to analyze some of the participants' lesson activities because of their attendance, and for validity purposes I should have had data from each learner for each technique I used. Thus this situation forced me to take only nine learners' lesson activities for the focus group interviews because they were the ones who attended the entire implementation of the programme.

The other challenge was that the focus of my research was only on percentages and fractions and not on decimal numbers. The fact that these three representations are forms that represent the same number means that one cannot teach them separately. Even though important themes on decimal number arose during my analysis, it was difficult for me to discuss them, because I did not pay much attention to decimal numbers during the implementation of the programme, learners' activities and the pre-and-post diagnostic tests.

Another challenge was that questions in the pre-and-post diagnostic tests were not the same, merely similar. During the analyses of the two tests, it was not easy to conclude that learners did better in one test than in the other test because of the fact that the questions were not the same. Therefore I could not conclude using only the tests only. The conclusion came only after I compared data from all techniques that I used.

6.4 SIGNIFICANCE

This study is significant because firstly, it encourages teachers to use the three effective approaches when teaching percentages and fractions to promote the learning of the concepts. Secondly, it gives a reflection on how the programme enhances the learning of percentages and fractions, how learners become fully engaged in the lessons and how they are motivated because of the use of multiple representations. Thirdly, it shows how the teacher and the learners' interactions are extremely essential during the process of learning about percentages and fractions.

6.5 RECOMMENDATIONS AND SUGGESTIONS FOR FURTHER RESEARCH

If I had to do this research over again, there are three major aspects that I would take into consideration. The first one is to think about the period of the implementation of the programme, so that it would not take place during the school holidays. It should rather be a period where learners are not affected because of household chores. The second aspect would be to expand the programme by including decimal numbers in the design of the programme, the learners' activities and also on the pre-and-post diagnostic tests, because percentages, fractions and decimals cannot be taught separately. The last aspect is that questions on the pre-and-post diagnostic tests should exactly be the same, so that it would be easier to analyze the learners' performance. Therefore I recommend other teachers to use the three effective approaches when teaching percentages and fractions to promote the learning of the concepts.

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APPENDICES

APPENDIX A – INTERVENTION PROGRAMME

{Investigating how learners may use multiple representations to promote learning of percentage and fractions in a social interaction.}

I designed an intervention programme, which includes multiple representations appropriate to use in the learning of percentage. The intervention programme is spread over a two week period consisting of 8 lessons of one hour each. Multiple representations included in the programme are:

- **Spoken symbols** – i.e. the teacher saying the number fifteen percent is different from the teacher writing it on the chalkboard for learners to see.
- **Written mathematical symbols** – these can include numbers, mathematical expressions, and others. E.g. 25% of 300, $\frac{1}{2} = 50\%$ and etc.
- **Diagrams** – for example using pie-chart, tables, sketches, figures and grids.
- **Manipulatives** – like bottle tops, pieces of pipes, and pieces of boxes, counting pins, pieces of plastic bottles, bottle's neck, erasers, chalks, pens, rulers, paper sticks, and pairs of scissors.
- **Real world things**- like small stones, almond seeds, money, cash slips and calculators.

Lesson 1

Topic: percentage

duration: 1hour

Date: 26 April 2012

Learning objectives: develop percentage language and basic conceptual understanding.

- **Types of representations to use:** spoken symbols; written percentage symbols; diagrams such as grids, pie-charts, tables and figures; manipulatives such as like bottle tops, pieces of pipes, and pieces of boxes, counting pins, pieces of plastic bottles, bottle's neck, erasers, pencils and pens and real world such as stones, almond seeds and money.
- **How to use each of the representations:** Spoken symbols; learners will be given chase to make various fractions using the things in the boxes (*I talked about the boxes below*) given by their teachers. And they will talk about what they are doing. For instance if this learner will have a box of 100 almond seeds and I asked him/her to remove 17 seeds from the box, she should say that, I took 17 seeds out of 100. The purpose of this activity is for learners to have a concept of what $\frac{a}{b}$ (a fractions) really means. Written percentage symbols; after learners saying out fractions they will again write down on the chalkboard all the fractions they are saying. Manipulatives and real world; I have prepared **11 boxes** of 100 things in each boxes, the table below show the list of boxes of 100 things in each.

Names of things in the boxes	Number of things in the boxes
Bottle tops	100
N\$1 coin	100
Almond seeds	100
Pieces of boxes	100
Pieces of plastic bottles	100
Pieces of pipes	100
Round counting pins	100
Round counting pins	100
Bottle tops	100
Plastic bottle's necks	100
Stones	100

Learners in pairs will count all their manipulatives and or real things in the boxes to make sure that they have 100 things inside their boxes. (*I made each box to have 100 things because we were dealing with percentage*). Each pair will start making fractions using the manipulatives and or real things in their boxes. The instructions will be given to them on what fractions each pair should make. (The teacher writes a table like the one below with the names of all the things in the 11 boxes, but this time I will include the number of little things that I want each pair to remove out of their one hundred things). E.g.

Names of the things in the box	Number of things out of their boxes	Write it as fractions
Bottle tops	50	
Almond seeds	70	
Pins	100	

Each pair will be busy counting out the number of things out of their 100 things in the box, after that I will asked them what they did and again ask them to write their work as a fractions on the chalkboard next to their box's name, e.g. the pair with bottle tops will say, they took 50 bottle tops from hundred bottle tops and that is the same as $\frac{50}{100}$ they should then write their fractions in the spaces provided on the table. When learners complete the table by writing the fractions they made, then they can be told that they converted from percentage to fractions. Learners should repeat the same work for the second time with different numbers of things which should be removed out of their boxes. Diagrams; I will give learners a written activity (**see learners' activity 1**) to work out fractions using various diagrams. There are five types of diagrams in the written activity, the first one has 100 circles of 10 per row and I will ask learners to fill in 62 of 100 circles and write their answers as fractions. The second one has 16 smaller rectangles which I said they are bottle tops, I will ask learners to color in 4 out of 16 bottle tops and write their answer as fractions. The third diagram is a grid of 100 squares whereby I shaded 65 out of 100 and will ask learners to write the fractions of the squares which are not shaded. The fourth diagram is a pie-chart which is divided into 6 equal parts; I shaded 3 parts and will ask learners to write down the fractions of the shaded part. The last diagram is pictures of 10 N\$1 coins, I will ask learners to circle in 2 of the 10 coins and write down the fractions.

Lesson 2

Topic: percentage

duration: 1hour

Date: 27 April 2012

Learning objectives: realize the relationship between percentage, fractions and decimal

- **Types of representations to use:** spoken symbol; written percentage symbol; diagrams such as tables, grids, pie-chart, number line, figures; manipulatives and real world things such as calculators, pen and the 11 boxes of things.
- **How to use each of the representations:** spoken symbol; when learners will be given the activity to convert from percentage to fractions and from fractions to percentage, they will need to explain how they got their answers in front of others. Written percentage symbol; learners will show their work on the chalkboard on how they calculate percentage to decimal to common fractions and vice versa. Diagrams: this is the main representation in this lesson. The first diagram which will be used is the table that shows the relationship between percentage, decimal and fractions. e.g.

fractions	fractions	decimal	decimal	percentage	Percentage
$\frac{3}{10}$	Three tenth	0.3	Zero point three	30%	Thirty percent
		0.5			

After the teacher explains and complete the first column the learners need to calculate and explain in front of others how to do it so that they complete the table above which will be on the chalkboard. For instance if 0.5 is given, they need to write the next decimal representation of 0.5 as in words. They need to write 0.5 as a fraction $\frac{5}{10}$ and explain how, again to write it as in word i.e. five tenth. They need to change 0.5 again to percent and explain how. I have given the same table as learners' activity so that they work out the extra rows I added. (See learners' activity 3). The other diagrams which I used are grids of rectangles; I shaded some rectangles so that learners will write the fractions of the shaded rectangles and later to write it as percentage. (See learners' activity 2, b & c and learners' activity 3, c). I have also used pie-chart cut them into parts and asked learners to write down the fractions as well as the percentages of the shaded parts. (See learners' activity 2 and learners' activity 3 b). The other diagrams which learners will use is circles, I drawn circles and shade some for learners to write the fractions as well as the percentage of the shaded circles. (See learners' activity 2e and activity 3a). The last diagram which learners will use is number line, I will give learners number line of 10 dots and shaded 5 of them and ask them to write as fractions and percentage. (See learners' activity 2 d). Manipulatives and real world: at the beginning of the lesson I will ask learners to work with the *11 boxes* of manipulatives and real world things in it, to reflect on previous lesson. In this lesson learners will use the calculator more, especially when converting between percentage, decimal and common fractions and vice versa.

Lesson 3

Topic: percentage

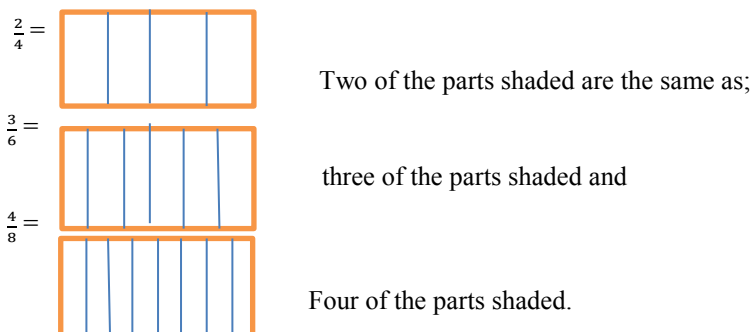
duration: 1 hour

Date: 30 April 2012

Learning objectives: realize fractions of the same percentage (equivalent fractions).

- **Types of representations to use:** spoken symbols; written percentage symbols; diagrams such as grids and tables; manipulatives and real world things such as calculators, pen, papers, the 11 boxes of things.
- **How to use each of the representations:** spoken symbols; each learner will explain how they came up with five equivalent fractions of their original one given to them by the teacher. Written percentage symbols; each learner will write up five equivalent fractions of their original one, and write down how did they come up with the five equivalent fractions. E.g. if the original fractions is half then they are expected to write $\frac{1}{2} = \frac{2}{4} = \frac{50}{100} = \frac{9}{18} = \frac{55}{110} = \frac{5}{10}$ and again expected to write down how did they came up with their fractions. After learners identify the five equivalent fractions, they continue again and look for another five extra ones, and so on.

Diagrams; learners should show equivalent fractions using diagrams; they should draw rectangles of the same sizes and cut them like the ones below, they should realize that all the parts of their fractions in the diagrams have the same size, that's why they are equivalent. E.g.



Manipulatives and real world things; learners are expected to use the calculators more in this lesson, to work out equivalent fractions of their original fractions. They will use the calculator by trying out what fractions have the same percentage as their original fractions, if the fractions have the same percentage then that is an equivalent fraction of their original fractions. When one learner will have the chance to list down all his/her equivalent fractions on the chalkboard, the other learners try to test whether all the listed fractions have the same percentage as his or her original fractions. Learners may also make use of the **11 boxes**. Each pair will try to use their manipulatives and or real world things in their boxes to find the equivalent fractions of their original fractions.

Lesson 4

Topic: percentage

duration: 1 hour

Date: May 01, 2012

Learning objectives: work out the percentage of a quantity (use the 3 A4 paper grids of rectangles)

- **Types of representations to use:** spoken symbols; written percentage symbols; diagrams such as grids and shapes; manipulatives such as pair of scissors, paper sticks, pen, papers and chalks, A4 paper of 25, 100 and 200 grids; real world things such as calculators.
- **How to use each of the representations:** Spoken symbols; this type of representation will be used throughout the lesson especially after a learner is asked to explain his or her ideas of working out percentage of a quantity. Written percentage symbols; learners will be asked to write how to calculate percentage of a quantity. E.g. 20% of N\$300, write how she/he will get the correct answer. Diagrams: I prepared 3 A4 white paper grids for each learner to use, i.e. a grid of 25 rectangles, a grid of 100 rectangles and a grid of 200 rectangles. (*I then decided to call the three A4 paper grids to be mats, see learners' activity four*). Each learner should have 3 mats; their job will be to cover the white 3 mats with different colors. They will use small rectangles which were cut into pieces that fit exactly in one of each rectangle on their mats/grids of 25 or 100 or 200. These pieces which they will use are in different colors. They will cover their mats by sticking in the correct number of different color rectangles into their mats. I will give instructions to learners that, on the grids or mats of 100 and 200 rectangles, learners should cover 40% of the mats or grids yellow, 30% red, 25% blue and 5% white. On the grid/mat of 25 rectangles, learners should cover 40% yellow, 10% red, 20% blue and 30% white. The main purpose of this job is for learners to work out percentage of quantity, examples learners in this regard are given 3 quantities; 25, 100 and 200. For them to know how many yellow rectangles need to cover their mats of 25, 100 and 200 rectangles, they should work out 40% of 25 = 10 rectangles that they should cover, for 100 it will be 40% of 100 = 40 rectangles and for 200 it will be 40% of 200 = 80 rectangles to be covered in the 200 mat, they should do that for all the colors given to them. Manipulatives and real world things; learners will use a pen and papers to show their work on working out the number of rectangles of different color to cover their 3 mats. They will use their calculators to press their work in and get the correct answer very quick.

Lesson 5

Topic: percentage

duration: 1 hour

Date: May 02, 2012

Learning objectives: calculate percentage of a quantity (more examples on written symbols).

- **Types of representations to use:** spoken symbols; written percentage symbols; diagrams such as grids and shapes; manipulatives such as pen, papers and chalks; real world things such as calculators.
- **How to use each of the representations:** Spoken symbols: during the beginning of the lesson learners will be given chance to work out percentage of diagrams with different quantity drawn on the chalkboard, they should explain to others how to get the correct answers. E.g. color in 25% of the circles (000000000000), learners should color in 3 and explain why color in 3 from the 12 circles. Each and every time if they are given work to do, they need to explain how they worked it out. Written percentage symbols: learners will write down how they are calculating percentage of a quantity, example if they are asked, (a) 5.5% of N\$6.22, (b) 80% of N\$62.50, (c) $12\frac{1}{2}\%$ of N\$88.50, (d) 20% of 60kg, (e) 40% of 360° , they should show their work in writing on the chalkboard. I gave them individual work of the same type of question. **(See learners' activity five no. 1 (a)-(e))**. The other written percentage symbols I will use in this lesson are word problems; I have come up with four word problems of calculating percentage of a quantity for learners to work out. The word problems are as follows:
 - (a) Melinda scored 82% in her mathematics test. The test was out of 50 marks. How many marks did she obtain?
 - (b) The cost price of a bicycle is N\$700. A discount of 25% is given to anyone who buys the bicycle cash. Calculate the discount price?
 - (c) A school has 950 pupils. 48% of them are boys, how many boys are at that school?
 - (d) Chilambo receives a commission of 1% for selling second-hand cars. The car cost N\$80 000. Calculate chilambo's commission. For each question I will point a learner to read it for the class, I then will give them time to calculate in their papers, later asked any of the learners to work out the answer on the chalkboard. The other three word problems each learner will solve individually are on the learners work. **(See learners' activity five no. 3, 4 and 5)**. Diagrams: I used four types of diagrams for learners to work out the percentage of each. E.g. the question is for learners to shade in 25% of each diagram below: (a) I will draw 12 circles, 4 in each row, (b) I will draw a pie of 8 parts, (c) I will draw a rectangle of 4 parts and (d) I will draw a grid of 6 parts. The other diagrams which learners will use to work out the answer are on the learners work. **(See learners' activity five no. 2 (a) – (c))**.

Lesson 6

Topic: percentage

duration: 1 hour

Date: May 03, 2012

Learning objectives: express one quantity as a percentage of another.

- **Types of representations to use:** spoken symbols; written percentage symbols; diagrams such as shapes; manipulatives such as pen, papers and chalks real world such as calculators.
- **How to use each of the representations:** spoken symbols; throughout the lesson learners will discuss with the teachers how they are calculating one quantity as a percentage of another. Each time a question will be written on the chalkboard, a volunteer learner will be asked to come and explain how to work out the answer. Written percentage symbols; this representation will be used throughout the lesson; it will be the main focus. There are three types of written representations which I will use in this lesson. One is just to ask learners to convert a fractions or decimal to percentage. E.g. (a) $\frac{21}{25}$

(b) $2\frac{1}{2}$

(c) $\frac{5}{8}$

(d) 0.38

(e) 1.05

The second example of the written symbols on this lesson is; express N\$2 as a percentage of N\$10. The third examples of the written symbols on this lessons are word problems such as (a) what percentage is twenty marks out of twenty five marks?

(b) There are thirty learners in a class. Five are boys. What percentage of the total learners are boys?

All this type of questions we will discuss it with the learners, as usual, a learner reads the question and I will asked all to work out the problem and ask one of them to explain on the chalkboard, with my help. I have repeated asking similar questions on the three types of written representations for learners to answer individually. **(See learners' activity six no. 2, 3, 4, 5 and 6).** Diagrams; I will include some diagrams during the lessons as examples to work out one quantity as a percentage of another one. For instance I will draw six faces on the chalkboard and shade two of the faces and ask learners to write the shaded faces as percentage of all the faces. Again I will draw 16 letter Z and color in 8 of the Z and ask learners to express the color Z as percentage of the 16 Z's. I again used three types of diagrams which I asked learners individually to answer on the same concept of expressing one quantity as a percentage of another. **(See learners' activity six no. 1).**

Lesson 7

Topic: percentage

duration: 1 hour

Date: May 04, 2012

Learning objectives: increase or decrease a quantity by a given percentage.

- **Types of representations to use:** spoken symbols; written percentage symbols; diagrams such as grids; manipulatives such as pair of scissors, pen, papers and chalks real world such as calculators.
- **How to use each of the representations:** spoken symbols: throughout the lesson learners will discuss with the teachers how they are calculating increase or decrease a quantity by a given percentage. Each time a question will be written on the chalkboard, a volunteer learner will be asked to come and explain how to work out the answer. Written percentage symbols; this representation will be used throughout the lesson; it will be the main focus. There are two types of written representations which I will use in this lesson. Examples of the first written representations are: increase 24 rectangles by 25% and or decrease N\$30 by 40%. The second written representations are word problems such as;
 - (a) A pair of shoes cost N\$200 and there is a sale of 25% in the shop for each item. How much will be the selling price of a pair of shoes?
 - (b) A packet of cool drinks cost N\$130, 40% is added to the packet as a profit, and how much is the selling price?

All this type of questions we will discuss it with the learners, as usual, a learner reads the question and I will asked all of them to work out the problem on the papers and ask one of them to write and explain on the chalkboard. I have repeated asking similar questions on the two types of written representations for learners to answer individually. **(See learners' activity seven no. 3,4,5,6 and 7).** Diagrams; in the beginning of the lesson I will give learners grids of different sizes and ask them to increase or decrease them by a given percentage. Example I will give each learner a grid of 24 rectangles and ask them to reduce it by 25%. Learners should cut out 6 rectangles away from the 24 rectangles; therefore the new grid will be 18 rectangles. Pairs of scissors will be available. Learners will still be given grids of other rectangles to increase it by a certain percentage and they should show their work to the teacher. I have asked learners similar questions in their activity of increasing or decreasing rectangles on certain diagrams by given percentage. **(See learners' activity seven no. 1).**

Lesson 8

Topic: percentage

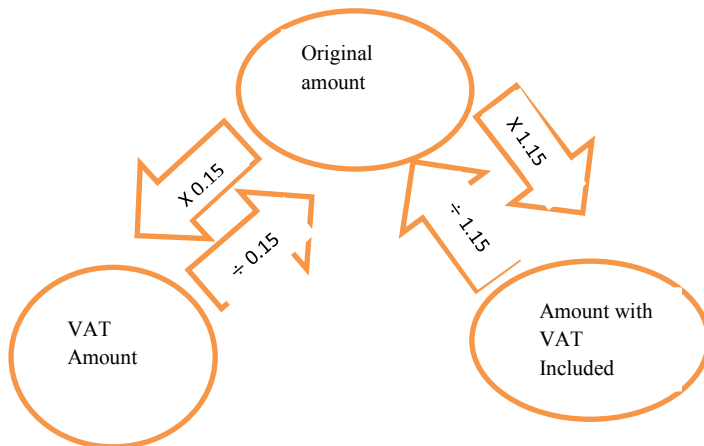
duration: 1 hour

Date: May 05, 2012

Learning objectives: calculate quantity with VAT inclusive and VAT exclusive.

- **Types of representations to use:** spoken symbols; written percentage symbols; diagrams such sketch and tables; manipulatives such as poster, pen, papers and marker pens; real world such as calculators, sales slips and money.
- **How to use each of the representations:** spoken symbols; throughout the lesson the learners need to explain themselves how are they calculating VAT inclusive and exclusive. Written percentage symbols; learners need to solve word problems on VAT inclusive and VAT exclusive. Word problems such as:
 - (a) The price of 320mt of Fanta is N\$6.99 VAT included of 15%. How much was the price without VAT?
 - (b) The price tag on a 20kg paper bag of sugar reads N\$150.00. VAT excluded at 15%. How much does the customer have to pay?

The learners need to work out their answers on the papers and if raised by the teacher he/she will explain to the rest of the class how she/he got the correct answer. Diagrams; in this lesson learners will use the sketch below to calculate VAT amount only or Amount with VAT exclusive or an Amount with VAT inclusive.



For instance if the question will be for learners to work out the original price of an item that cost N\$40, VAT of 15% is included, how should learners use the above sketch? Learners will therefore take the N\$40 divide it by 1.15 and the answer will be N\$34.78. The next diagram which I will use is in the form of a table. Examples, learners need to fill in the answers in the table below:

Items	Original price	VAT of 15%	Price with 15% VAT
1. Can beans	N\$4.00	N\$	N\$
2.Oros (Caribbean)	N\$8.00	N\$	N\$
3.Chicken (braai park)	N\$	N\$	N\$30.00
4. Soup	N\$	N\$	N\$5.00

This table is written on a poster, so learners together with the teacher discuss how to complete the table. I have also asked similar type of questions to the learners on VAT inclusive or Exclusive. **(See learners' activity number 8).** Manipulatives; I will use a poster written the table above. Real world; I will take along sales slips to the learners for them to calculate VAT of items on the sales slip. I will take along money for learners to use during the class activity which I prepared on increasing each item with 15% VAT. (See class activity 8). Example in pair learners will work out the percentage increase of 15% as VAT for each item on the table in the class activity 8. Then learners use real money to add on the prices given. In this case learners will understand the concept clearly, i.e. after calculating the amount, they need to add the VAT price to the original amount and count the whole amount together.

APPENDIX B –

LEARNERS' ACTIVITY 1

Lesson: 1

Topic: percentages and fractions

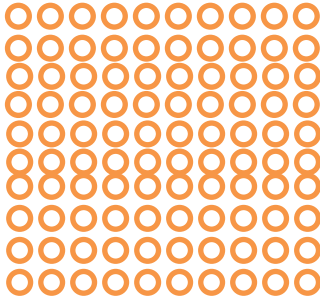
grade: 8

Date: 26 April 2012

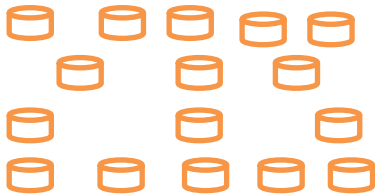
Learning objectives: develop percentage language and basic conceptual understanding.

Evaluation questions

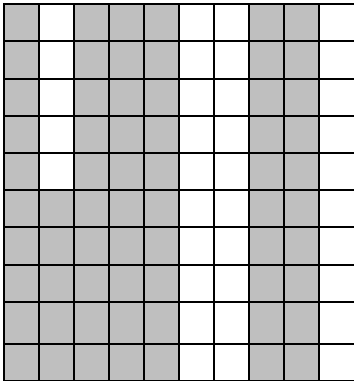
1. Fill in 62 of 100 seeds below. Write your answer as a fraction.



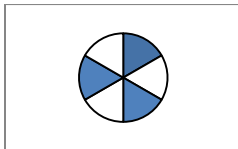
2. Color in 4 out of 16 bottle tops. Write your answer as a fraction.



3. Write down the fractions of the parts which are not shaded.



4. Write down the fractions of the shaded part.



5. Circles in 2 out of 10 N\$1 coins, and write your answer as fractions.
N\$1 N\$1 N\$1 N\$1 N\$1 N\$1 N\$1 N\$1 N\$1 N\$1

LEARNERS' ACTIVITY 2

Lesson 2

Grade: 8

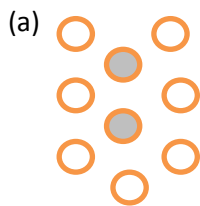
Mathematics: percentages, decimal and common fractions

Date: 27 April 2012

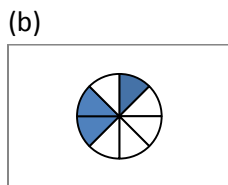
1. Complete the table below.

Fractions	fractions	decimal	decimal	percentage	Percentage
$\frac{1}{10}$	One tenth	0.1	Zero point one	10%	Ten percent
$\frac{1}{2}$					
	Three tenth				
			Zero point four		
				85%	
		0.125			
					One hundred and four percent
$\frac{3}{5}$					
	Nine tenth				
$\frac{10}{10}$					

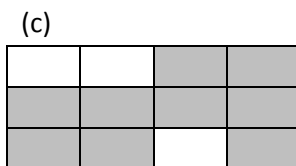
2. In each of the following diagrams write as fractions and as percentage.



= -----%



= -----%



LEARNERS' ACTIVITY 5

LESSON 5

TOPICS: PERCENTAGE AND FRACTIONS GRADE: 8

02 MAY 2012

1. Workout the following:

(a) 64% of 500 cows

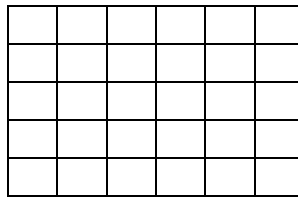
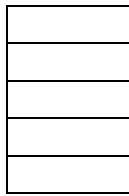
(b) 17% of 2m

(c) 29% of N\$36

(d) 7.5% of N\$16.40

(e) $8\frac{1}{2}\%$ of 2400g

2. Shade in 30% of each of the diagram below:



3. Your uncle bought for you school shoes of N\$150. It did not fit on you and now you are selling it at a profit of 30%. How much is your profit?

4. A lady work at a furniture shop earns a basic wage of N\$600 per week and 20% commission on all furniture she sell. Calculate the commission she earns on a television worth N\$5000.

5. You have read 40% of the Rukwangali story book (kotokene ZaHaitana). The book has 240 pages. How many pages did you read so far?

LEARNERS' ACTIVITY 6

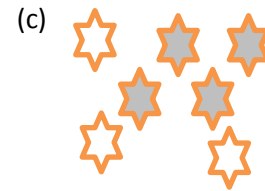
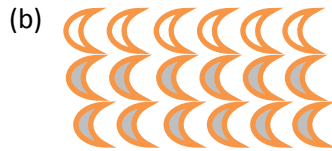
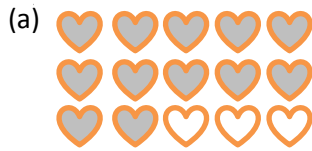
LESSON 6

TOPICS: PERCENTAGE

GRADE: 8

03 MAY 2012

1. In each of the following diagrams below, express the shaded objects as a percentage of the total number of objects.



2. Convert to percentage

(a) $\frac{5}{20}$ -----

(b) $\frac{17}{50}$ -----

(c) $2\frac{1}{2}$ -----

(d) 0.125 -----

(e) 1.02 -----

3. What percentage is

(a) 20 cents of 150 cents (2)

(b) 26 grams of 100 grams (2)

4. Fill in the table below.

Common fractions	percentage
$\frac{12}{18}$	%
$\frac{8}{12}$	%

5. You have read 120 pages of the mathematics textbooks that have 300 pages. What percentage of the pages did you read so far? (2)

6. There are 33 learners in grade 8A class. 20 are girls, what percentage of the class are girls? (2)

LEARNERS' ACTIVITY 7

Lesson: 7 topic: percentage

grade: 8

04 MAY 2012

1. Increase the diagram below by 25%. (Re-draw your final diagram). (2)

2. Decrease the diagram below by 20%. (Re-draw your final diagram). (2)

--	--	--	--	--	--	--	--	--	--

3. (a) Increase N\$300 by 40% (b) decrease 4m by 40% (4)

4. School shoes cost N\$150. If they are marked up with 20%, calculate the selling price? (3)

5. A baby was born with a mass of 2kg and now the mass has increased with 40%. Calculate the baby's new mass. (3)

6. Ok furniture shop offers 25% discount on fridges. The cash price of the fridge is N\$2500. Calculate the selling price. (3)

7. A large snake normally weighs 12kg. After swallowing a rat, the weight of the snake increases by 7%. How much does it weigh after dinner? (3)

LEARNERS' ACTIVITY 8(a)

Lesson 8

topic: percentage

grade: 8

May 5, 2012

Learners' class work

Instructions

You are the manager of a mini-market at Sauyemwa. You were asked to include VAT of 15% on each of the items in the shop. Workout the price of each of the items in the table below with VAT included.

ITEMS	ORIGINAL PRICE	THE PRICE OF VAT (15%)	PRICE OF ITEMS plus VAT OF 15%
750ml cooking oil	N\$ 10.00	N\$	N\$
1 20dollar tango	N\$ 20.00	N\$	N\$
A 3 pocket pen sac	N\$ 30.00	N\$	N\$
Biscuit (eet-sum)	N\$ 7.00	N\$	N\$
2.5kg of sugar	N\$ 12.00	N\$	N\$
Body lotion (Nivea)	N\$ 25.00	N\$	N\$
5kg of bread flour (backpro)	N\$ 40.00	N\$	N\$

LEARNERS' ACTIVITY 8(b)

Lesson: 8

Topics: percentage

Grade: 8

May 5, 2012

1. Workout the prices needed to complete the table below. Either the original price, the price with VAT of 15% or the VAT price only of 15%.

Items	Original price	VAT price of 15%	Price of items with VAT included.
Cellphone	N\$	N\$	N\$230
Packet of fish	N\$ 84. 15	N\$	N\$
2l of coke	N\$	N\$	N\$ 15.00
2l cooking oil	N\$ 34.00	N\$	N\$

(8)

2. The price on a Walkman player reads N\$160 plus VAT at 15%. How much does the customer have to pay? (3)

3. Mr. Arthur bought a digital camera for N\$1880 VAT inclusive of 15%. Calculate the cost of the digital camera without VAT. (3)

4. The original price of a guava juice is N\$ 5. 20. How much will you pay for a juice if the VAT of 16% is added to the original price? (3)

5. The price of a school bag is N\$ 80.00 VAT included. 15% VAT was added to all the items in the shop.










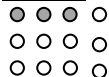
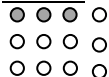
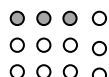
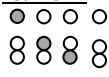
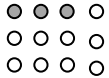
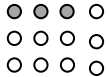
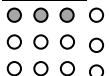
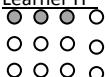
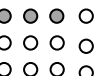
(a) How much is the VAT price? (2)

(b) What was the original price of the school bag without VAT? (1)

APPENDIX C –

COMPARISON OF LEARNERS’ ANSWERS THE PRE-AND-POST DIAGNOSTIC TESTS

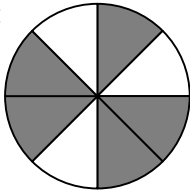
Question 1 (a)

Pre-test (9 learners response)	Post-test (9 learner’s response)
<p>Question 1</p> <p>1. (a) Color in two out of eight circles below and write your answer as a fraction.</p> <p><u>Learner A</u></p>  <p>Ans: $\frac{250}{1000}$</p> <p><u>Learner B</u></p>  <p>Ans: $\frac{2}{8}$</p> <p><u>Learner C</u></p>  <p>Ans: $\frac{2}{8}$</p> <p><u>Learner D</u></p>  <p>Ans: $\frac{2}{8}$</p> <p><u>Learner E</u></p>  <p>Ans: $\frac{2}{8}$</p> <p><u>Learner F</u></p>  <p>Ans: $\frac{2}{8}$</p> <p><u>Learner G</u></p>  <p>Ans: $\frac{2}{8}$</p> <p><u>Learner H</u></p>  <p>Ans: $\frac{2}{8}$</p> <p><u>Learner I</u></p>  <p>Ans: $\frac{2}{8}$</p>	<p>Question 1</p> <p>1. (a) Color in three out of twelve circles below, and write your answer as a fraction.</p> <p><u>Learner A</u></p>  <p>Ans: $\frac{3}{12}$</p> <p><u>Learner B</u></p>  <p>Ans: $\frac{3}{12}$</p> <p><u>Learner C</u></p>  <p>Ans: $\frac{3}{12}$</p> <p><u>Learner D</u></p>  <p>Ans: $\frac{3}{12}$</p> <p><u>Learner E</u></p>  <p>Ans: $\frac{3}{12}$</p> <p><u>Learner F</u></p>  <p>Ans: $\frac{3}{12}$</p> <p><u>Learner G</u></p>  <p>Ans: $\frac{3}{12}$</p> <p><u>Learner H</u></p>  <p>Ans: $\frac{3}{12}$</p> <p><u>Learner I</u></p>  <p>Ans: $\frac{3}{12}$</p>
<p>127 Page</p>	

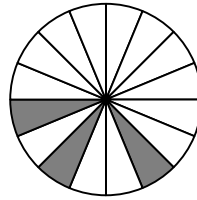
Question 1(b)

Write down the fractions of the shaded part/s

pre-test



post test



Answers

Learner A $\frac{60}{100}$ or $\frac{3}{5}$

Learner B $\frac{5}{8}$

Learner C $\frac{5}{3}$

Learner D $\frac{5}{8}$

Learner E $\frac{5}{8}$

Learner F $\frac{5}{8}$

Learner G $\frac{5}{8}$

Learner H $\frac{5}{8}$

Learner I $\frac{5}{8}$

Answers

Learner A $\frac{3}{16}$

Learner B $\frac{3}{15}$

Learner C $\frac{3}{16}$

Learner D $\frac{3}{16}$

Learner E $\frac{3}{16}$

Learner F $\frac{3}{16}$

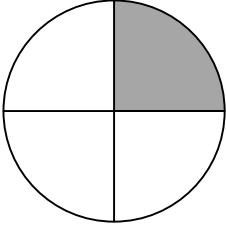
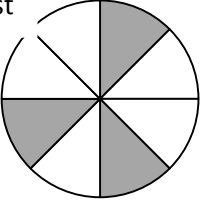
Learner G $\frac{3}{16}$

Learner H $\frac{3}{16}$

Learner I $\frac{3}{16}$

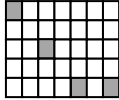
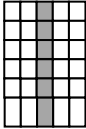
Question 2(a)

What percentage of each of the diagrams/figures below is shaded?

Pre-test	Post test
	
<u>ANSWERS</u>	<u>ANSWERS</u>
Learner A 25%	Learner A $\frac{3}{8} \times 100 = 37.5$
Learner B 25%	Learner B $\frac{3}{8} \times 100 = 37.5$
Learner C 1%	Learner C $\frac{3}{8} \times 100 = 37.5$
Learner D 25%	Learner D 37.5
Learner E 400%	Learner E $\frac{30}{10} \times 100 = 30$
Learner F 25%	Learner F $\frac{3}{8} \times 100 = 37.5$
Learner G 25%	Learner G $\frac{3}{8} \times 100 = 37.5$
Learner H 25%	Learner H $\frac{3}{8} \times 100 = 37.5$
Learner I 1%	Learner I $\frac{3}{8} \times 100 = 37.5$

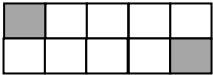

Question 2(b)

What percentage of each of the following diagram/figure is shaded?

Pre-test		post-test	
<u>ANSWERS</u>		<u>ANSWERS</u>	
Learner A	4%	Learner A	$\frac{6}{30} \times 100 = 20\%$
Learner B	11%	Learner B	20%
Learner C	4%	Learner C	$\frac{6}{30} \times 100 = 20\%$
Learner D	11%	Learner D	20%
Learner E	875%	Learner E	$\frac{30}{100} \times 6 = 1.8\%$
Learner F	11%	Learner F	20%
Learner G	11.4%	Learner G	$\frac{6}{30} \times 100 = 20\%$
Learner H	11.42857143	Learner H	$\frac{6}{30} \times 100 = 20\%$
Learner I	4%	Learner I	$\frac{6}{30} \times 100 = 16.7\%$

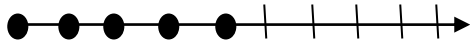
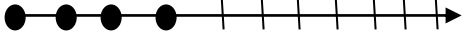
Question 2(c)

What percentage of each of the diagrams/figures is shaded?

Pre-test		Post-test	
<u>ANSWERS</u>		<u>ANSWERS</u>	
Learner A	20%	Learner A	$\frac{1}{3} \times 100 = 33.33\%$
Learner B	20%	Learner B	33%
Learner C	2%	Learner C	$\frac{1}{3} \times 100 = 33.33\%$
Learner D	20%	Learner D	33%
Learner E	500%	Learner E	10%
Learner F	20%	Learner F	33%
Learner G	20%	Learner G	$\frac{1}{3} \times 100 = 33.3\%$
Learner H	2%	Learner H	$\frac{1}{3} \times 100 = 33.3\%$
Learner I	2%	Learner I	$\frac{1}{3} \times 100 = 33.3\%$

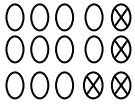

Question 2 (d)

What percentage of the diagrams/figures below is shaded?

pre-test		Post test	
<u>ANSWERS</u>		<u>ANSWERS</u>	
Learner A	50%	Learner A	$\frac{4}{11} \times 100 = 36.36\%$
Learner B	50%	Learner B	40%
Learner C	5%	Learner C	$\frac{4}{11} \times 100 = 36.36\%$
Learner D	50%	Learner D	40%
Learner E	100%	Learner E	40%
Learner F	50%	Learner F	40%
Learner G	4%	Learner G	$\frac{4}{11} \times 100 = 36.4\%$
Learner H	0%	Learner H	$\frac{4}{11} \times 100 = 36.4\%$
Learner I	5%	Learner I	$\frac{4}{11} \times 100 = 36.4\%$

Question 2(e)

What percentage of the diagrams/figures below is shaded?

Pre-test		Post-test	
<u>ANSWERS</u>		<u>ANSWERS</u>	
Learner A	4%	Learner A	$\frac{5}{12} \times 100 = 41.66\%$
Learner B	6%	Learner B	42%
Learner C	4%	Learner C	$\frac{5}{12} \times 100 = 41.66\%$
Learner D	26%	Learner D	41.66% = 42%
Learner E	275%	Learner E	50%
Learner F	7%	Learner F	41.7%
Learner G	26.6%	Learner G	$\frac{5}{12} \times 100 = 41.7\%$
Learner H	26.66666%	Learner H	$\frac{5}{12} \times 100 = 41.6666667\% = 41.7\%$
Learner I	4%	Learner I	$\frac{5}{12} \times 100 = 41.7\%$

Question 3(a)

Write as a fraction and as percentage

Pre-test

ANSWERS

LEARNER A-I



- A. $\frac{6}{12} = 6\%$
- B. $\frac{6}{12} = 50\%$
- C. $\frac{6}{6} = 1\%$
- D. $\frac{6}{12} = 25\%$
- E. $\frac{6}{12} = 200\%$
- F. $\frac{6}{12} = 50\%$
- G. $\frac{6}{12} = 50\%$
- H. $\frac{6}{12} = 50\%$
- I. $\frac{6}{12} = 6\%$

Post- test

ANSWERS

LEARNER A-I

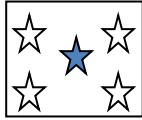


- A. $\frac{4}{6} \times 100 = 66.6$
Fractions = $\frac{4}{6}$
Percentage 66.66%
- B. $\frac{4}{6} = \frac{4}{6} \times \frac{100}{1} = 67\%$
- C. $\frac{4}{6} \times 100 = 66.66\% = 67\%$
- D. $\frac{4}{6} = 66.66\% = 67\%$
- E. $\frac{4}{6} = 0.666 \dots$
- F. $\frac{4}{6} = 66.7\%$
- G. $\frac{4}{6} = 4 \div 6 \times 100 = 66.7\%$
- H. $\frac{4}{6}$ is Fractions $\frac{4}{6} \times 100 = 66.7\%$ is percentage
- I. $\frac{4}{6} \times 100 = 66.7\%$

Question 3(b)

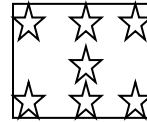
Write as a fraction and as percentage

Pre-test
ANSWERS
LEARNER A-I



- A. $\frac{20}{100} = 20\%$
- B. $\frac{1}{5} = 20\%$
- C. $\frac{1}{4} = 25\%$
- D. $\frac{1}{5} = 20\%$
- E. $\frac{1}{5} = 500\%$
- F. $\frac{1}{4} = 10\%$
- G. $\frac{1}{5} = 20\%$
- H. $\frac{1}{5} = 20\%$
- I. $\frac{1}{5} = 1\%$

Post-test
ANSWERS
LEARNER A-I



- A. $\frac{7}{7} \times 100 = 100\%$
- B. $\frac{1}{7} = \frac{1}{7} \times 100 = 14\%$
- C. $0.7 \times 100 = 70\%$
- D. $\frac{7}{7}$ Fractions
= 100%
- E. $\frac{7}{7} = 1\%$
- F. $\frac{7}{7} = 100\%$
- G. $\frac{7}{7} = 7 \div 7 \times 100 = 100\%$
- H. $\frac{7}{7} \times 100 = 100\%$
- I. $\frac{7}{7} \times 100 = 100\%$

QUESTION 4
Convert to percentage (show your work)

ANSWERS

- A. $0.86 = 86$
 $\frac{4}{7} = \frac{4}{7} = 57\%$
 $1.2 = 12\%$
 $1\frac{1}{4} = 11\%$
 $\frac{4}{5} = \frac{4 \div 20}{5 \div 20} = \frac{80 \div 20}{100 \div 20} = \frac{4}{5} = 80\%$
- B. $0.86 = \frac{86}{100} = 86\%$
 $\frac{4}{7} = 4 \div 7 = 57\%$
 $1.2 = 1 \frac{2}{100} = \frac{12}{100} = 12\%$
 $1\frac{1}{4} = 5\%$
 $\frac{4}{5} = \frac{4 \times 2}{5 \times 2} = \frac{8}{10} = 80\%$
- C. $0.86 = 86\%$
 $\frac{4}{7} = 0.571428571\%$
 $1.2 = 120\%$
 $1\frac{1}{4} = 5\%$
 $\frac{4}{5} = 0.8\%$
- D. $0.86 = \frac{86}{100} = 86\%$
 $\frac{4}{7} = \frac{4}{7} \times \frac{100}{1} = \frac{400}{7} = 57\%$
 $1.2 = 1.2 \times 100 = 60\%$
 $1\frac{1}{4} = 5 \times 100 = 25\%$
 $\frac{4}{5} = \frac{4}{5} \times \frac{100}{1} = \frac{400}{5} = 80\%$
- E. $0.86 = 0.86 \times 100 = 86\%$
 $\frac{4}{7} = \frac{4}{7} \times 100 = 174\%$
 $1.2 = 1.2 \times 100 = 1200\%$
 $1\frac{1}{4} = 1\frac{1}{4} \times 100 = 500\%$
 $\frac{4}{5} = \frac{4}{5} \times 100 = 80\%$
- F. $0.86 =$
 $\frac{4}{7} =$
 $1.2 =$
 $1\frac{1}{4} = 1.5$
 $\frac{4}{5} =$
- G. $0.86 = \frac{86}{100} = 86\%$
 $\frac{4}{7} = \frac{4}{7} \times 100 = 57\%$
 $1.2 = \frac{1}{2} \times 100 = 50\%$
 $1\frac{1}{4} = 1 \frac{25}{100} = \frac{125}{100} \times 100 = 125\%$
 $\frac{4}{5} = \frac{4}{5} \times 100 = 80\%$
- H. $0.86 = \frac{86}{100} \times 100 = \frac{86}{1} = 86\%$
 $\frac{4}{7} = \frac{4}{7} \times \frac{100}{1} = 57.14285714\%$
 $1.2 = \frac{1}{10} \times \frac{100}{1} = 1.20\%$
 $1\frac{1}{4} = \frac{5}{4} \times \frac{25}{1} = 1.25\%$
 $\frac{4}{5} = \frac{4}{5} \times \frac{100}{1} = 80\%$
- I. $0.86 = \frac{86}{100} = 86\%$
 $\frac{4}{7} = 0.57 = \frac{57}{100} = 57\%$
 $1.2 = 1 \frac{2 \times 10}{10 \times 10} = \frac{120}{100} = 1.20\%$
 $1\frac{1}{4} = 1 \frac{1 \times 25}{4 \times 25} = \frac{125}{100} = 1.25\%$
 $\frac{4}{5} = \frac{4 \times 20}{5 \times 20} = \frac{80}{100} = 80\%$

ANSWERS

- A. $0.58 = 0.58 \times 100 = 58$
 $\frac{3}{15} = \frac{3}{15} \times 100 = 20\%$
 $1.04 = 1.04 \times 100 = 104\%$
 $1\frac{1}{4} = \frac{5}{4} \times 100 = 125\%$
 $\frac{4}{5} = \frac{4}{5} \times 100 = 80\%$
- B. $0.58 = \frac{058}{100} \times \frac{100}{1} = 58\%$
 $\frac{3}{15} = \frac{2}{15} \times \frac{100}{1} = 20\%$
 $1.04 = \frac{104}{100} \times \frac{100}{1} = 104\%$
 $1\frac{1}{4} = \frac{5}{4} \times \frac{100}{1} = 125\%$
 $\frac{4}{5} = \frac{4}{5} \times \frac{100}{1} = 80\%$
- C. $0.58 \times 100 = 58\%$
 $\frac{3}{15} = x \times 100 = 20\%$
 $1.04 \times 100 = 104\%$
 $1\frac{1}{4} = \frac{5}{4} \times 100 = 125\%$
 $\frac{4}{5} \times 100 = 80\%$
- D. $0.58 = 0.58 \times 100 = 58\%$
 $\frac{3}{15} = \frac{3}{15} \times 100 = 20\%$
 $1.04 = 1.04 \times 100 = 104\%$
 $1\frac{1}{4} = \frac{5}{4} \times 100 = 50\%$
 $\frac{4}{5} = \frac{4}{5} \times 100 = 80\%$
- E. $0.58 = 0.58 \times 100 = 58\%$
 $\frac{3}{15} = \frac{3}{15} \times 100 = 20\%$
 $1.04 = 1.04 \times 100 = 104\%$
 $1\frac{1}{4} = 5 \times 100 = 500\%$
 $\frac{4}{5} = \frac{4}{5} \times 100 = 80\%$
- F. $0.58 \times 100 = 58\%$
 $\frac{3}{15} = x \times 100 = 20\%$
 $1.04 \times 100 = 104\%$
 $1\frac{1}{4} = 133\%$
 $\frac{4}{5} = 80\%$
- G. $0.58 = \frac{58}{100} = 58\%$
 $\frac{3}{15} = 3 \div 15 \times 100 = 20\%$
 $1.04 = 104 \div 100 \times 100 = 104\%$
 $1\frac{1}{4} = 1 \frac{25}{100} \times 100 = 125\%$
 $\frac{4}{5} = 4 \div 5 \times 100 = 80\%$
- H. $0.58 = 0.58 \times 100 = 58\%$
 $\frac{3}{15} = \frac{3}{15} \times 100 = 20\%$
 $1.04 = 1.04 \times 100 = 104\%$
 $1\frac{1}{4} = \frac{5}{4} \times 100 = 125\%$
 $\frac{4}{5} = \frac{4}{5} \times 100 = 80\%$
- I. $0.58 = 0.58 \times 100 = 58\%$
 $\frac{3}{15} = \frac{3}{15} \times 200 = 20\%$
 $1.04 = 1.04 \times 100 = 104\%$
 $1\frac{1}{4} = \frac{5}{4} \times 100 = 1.25\%$
 $\frac{4}{5} = \frac{4}{5} \times 100 = 80\%$

What percentage is?

5(a) 40 cents of N\$2

ANSWERS

- A. $\frac{40}{2} \times 100 = \frac{4000}{2} = 2000$
- B. $\frac{2}{40} \times \frac{5}{1} = \frac{10}{2} = 20\%$
- C. $40 \times 2 = \frac{2}{40} \times 100 = 50\%$
- D. 26%
- E. 2.000%
- F.
- G. 2000
- H. $\frac{40}{N\$2} \times \frac{100}{1} = 200 = 100 = 20\%$
- I. $\frac{2}{40} \times \frac{40}{200} \times \frac{100}{1} = 40\%$

5(a) 120g of 500g

ANSWERS

- A. $\frac{120}{500} \times 100 = 24\%$
- B. $\frac{120g}{500g} \times \frac{100}{1} = 24\%$
- C. $\frac{120}{500} \times 100 = 24\%$
- D. $\frac{120}{500} \times 100 = 24\%$
- E. $120 \times 500 = 60.000\%$
- F. $\frac{120}{500} \times 100 = 24\%$
- G. $\frac{120}{500} \times 100 = 24\%$
- H. $\frac{120}{500} \times 100 = 24\%$
- I. $\frac{120}{500} \times \frac{100}{1} = 24\%$

QUESTION 5(b) What percentage is?

5(b) 26 grams of 100 gram

ANSWERS

- A. $\frac{26}{100} \times 100 = 26\%$
- B. $\frac{26}{100} \times \frac{100}{1} = \frac{26}{1} = 26\%$
- C. $26 \times 100 = 2000$
- D. 26
- E. 2.600%
- F.
- G. 26%
- H. $\frac{26g}{100g} \times \frac{100}{1} = 26\%$
- I. $\frac{26}{100} \times \frac{100}{1} = 26\%$

5(b) N\$54 of N\$112

ANSWERS

- A. $\frac{54}{112} \times \frac{100}{1} = 48.21\%$
- B. $\frac{54}{112} \times 100 = 48\%$
- C. $\frac{54}{112} \times 100 = 48.21\%$
- D. $\frac{54}{112} \times 100 = 48.21428571\%$
 $= 48.21\%$
- E. $\frac{54}{100} \times 112 = 60.48\%$
- F. $\frac{54}{112} \times 100 = 48.21\%$
- G. $\frac{54}{112} \times 100 = 48.2\%$
- H. $\frac{54}{112} \times 100 = 48.21428571$
 $= 48.2\%$
- I. $\frac{54}{112} \times \frac{100}{1} = 48.2\%$

QUESTION 5(c)

What percentage is?

Pre-test	Post-test
5(c) 20 cents of 150 cents	5(c) 300 cents of 1000 cents
<u>ANSWERS</u>	<u>ANSWERS</u>
A. $\frac{20}{150} \times 100 = \frac{20 \times 10}{15} = \frac{200}{15}$	A. $\frac{300}{1000} \times \frac{100}{1} = 30\%$
20 x 150 = ?	
B. $\frac{150}{20} \times \frac{100}{1} = 75\%$	B. $\frac{300}{1000} \times \frac{100}{1} = 30\%$
C. 20 x 150 = 3000%	C. $\frac{300}{1000} \times 100 = 30\%$
D. 13%	D. $\frac{300}{1000} \times 100 = 30\%$
E. 13.3%	E. 300 x 1000 = 300.000
F. 10%	F. $\frac{300}{1000} \times 100 = 30\%$
G. 13.3%	G. $\frac{300}{1000} \times 100 = 30\%$
H. $\frac{20}{150} \times \frac{100}{1} = \frac{200}{15}$ = 13.33333%	H. $\frac{300}{1000} \times 100 = 30\%$
I. $\frac{20}{150} \times \frac{100}{1} = \frac{200}{15}$ = 13.33333%	I. $\frac{300}{1000} \times \frac{100}{1} = 30\%$

QUESTION 6(a)

Work out the following;

Pre-test	Post-test
<p>6(a) 30% of N\$300 <u>ANSWERS</u></p> <p>A. $\frac{30}{100} \times \text{N\\$}300 = \frac{30}{1} \times \frac{30}{1} = \frac{90}{1} = \text{N\\$}90$</p> <p>B. $\frac{30}{100} \times \frac{3}{1} = \frac{90}{1} = \text{N\\$}90$</p> <p>C. $30 \times 300 = \text{N\\$}9000$</p> <p>D. N\$10</p> <p>E. $\frac{30}{100} \times 300 = 90$</p> <p>F. $\frac{30}{100} \times 300 = 90 = \text{N\\$}90$</p> <p>G. N\$90</p> <p>H. $\frac{30}{100} \times \frac{\text{N\\$}300}{1} = 90 = \text{N\\$}90$</p> <p>I. $\frac{30\%}{100} \times \frac{300}{1} = 90$ $= \text{N\\$}90$</p>	<p>6(a) 28% of N\$600 <u>ANSWERS</u></p> <p>A. $\frac{28}{100} \times 600 = \text{N\\$}168.00$</p> <p>B. $\frac{280}{100} \times \frac{600}{1} = \text{N\\$}168.00$</p> <p>C. $\frac{28}{100} \times 600 = \text{N\\$}168.00$</p> <p>D. $\frac{28}{100} \times 600 = \text{N\\$}168.00$</p> <p>E. $\frac{28}{100} \times \text{N\\$}600 = \text{N\\$}168.00$</p> <p>F. $\frac{28}{100} \times 600 = \text{N\\$}168.00$</p> <p>G. N\$168</p> <p>H. $\frac{28}{100} \times \text{N\\$}600 = \text{N\\$}168.00$</p> <p>I. $\frac{28}{100} \times 600 = \text{N\\$}168.00$</p>

QUESTION 6(b)

Work out the following;

<p>6(b) 17% of N\$20 <u>ANSWERS</u></p> <p>A. $\frac{17\%}{100} \times \text{N\\$}20 = \frac{17 \times 2}{10 \times 1} = \frac{34}{10} = \text{N\\$}3.4$</p> <p>B. $\frac{17\%}{100} \times \frac{20}{1} = \frac{17}{5} = \text{N\\$}3.6$</p> <p>C. $17 \times 20 = \text{N\\$}340$</p> <p>D. N\$85</p> <p>E. N\$1.420</p> <p>F. 3.4</p> <p>G. N\$3.40 cents</p> <p>H. $\frac{17}{100} \times \frac{20}{1} = \text{N\\$}3.40$</p> <p>I. $\frac{17}{100} \times \frac{20}{1} = \frac{34}{10} = \text{N\\$}3.4$</p>	<p>6(b) 70% of N\$5.00 <u>ANSWERS</u></p> <p>A. $\frac{70}{100} \times \text{N\\$}5.00 = \text{N\\$}3.50$</p> <p>B. $\frac{70}{100} \times \frac{5.00}{1} = \text{N\\$}3.50$</p> <p>C. $\frac{70}{100} \times 5 = \text{N\\$}3.50$</p> <p>D. $\frac{70}{100} \times 5 = \text{N\\$}3.50$</p> <p>E. $\frac{70}{100} \times 5.00 = \text{N\\$}3.50$</p> <p>F. $\frac{70}{100} \times 5.00 = \text{N\\$}3.50$</p> <p>G. N\$3.50</p> <p>H. $\frac{70}{100} \times \text{N\\$}5.00 = \text{N\\$}3.50$</p> <p>I. $\frac{70}{100} \times 5 = \text{N\\$}3.50$</p>
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QUESTION 7

Pre-test

7. You have read 71 pages of the mathematics textbook that has 300 pages. What percentage of the pages did you read so far?

ANSWERS

A. $\frac{71}{300} \times 100 = 71\%$

B. $\frac{71}{300} \times \frac{100}{1} = \frac{71}{3} = 5\%$

C. 21 300

D. 23%

E. 21.300

F. 65%

G. 23.6%

H. $\frac{71}{300} \times \frac{100}{1} = \frac{71}{3}$
= 23.6666667%

I. $\frac{71}{300} \times \frac{100}{1} = 71\%$

Post-test

7. There are 40 learners in grade 8A class. 24 are boys. What percentage of the class are boy?

ANSWERS

A. $\frac{24}{40} \times 100 = 60\%$

B. $\frac{24}{40} \times \frac{100}{1} = 60\%$

C. $\frac{24}{40} \times 100 = 60\%$

D. $\frac{24}{40} \times 100 = 60\%$

E. $40 \times 24 = 960\%$

F. $\frac{24}{40} \times 100 = 60\%$

G. $\frac{24}{40} \times 100 = 60\%$

H. $\frac{24}{40} \times 100 = 60\%$

I. $\frac{24}{40} \times 100 = 60\%$

QUESTION 8

Pre-test ANSWERS

8. Ms. Mauve is a sales lady for Beauty Care. She earns a basic wage of N\$300 per week and 20% commission on all products she sells.
- (a) Calculate the commission she earns on. (Show your work)
- A. (i) Make up worth N\$800

$$\frac{20}{100} \times 800 = \frac{20}{1} \times \frac{8}{1} = \text{N\$}160$$
- (ii) Body lotion worth N\$250

$$\frac{20}{100} \times 250 = \frac{20 \times 25}{10 \times 1} \times \frac{500}{10} = \text{N\$}50.00$$
- B. (i) $\frac{300}{800} \times \frac{20}{100} \times \frac{600}{80000} = \text{N\$}79.400$
- (ii) $\frac{280}{300} \times \frac{20}{100} = \frac{5000}{30000} = \text{N\$}25000$
- C. (i) $\frac{20}{800} \times 300 = 7.5$
- (ii) $\frac{20}{250} \times 300 = 24$
- D. (i) $\frac{20}{800} \times \frac{100}{1} = \frac{80}{32} = \text{N\$}2.5$
- (ii) $\frac{20}{250} \times \frac{100}{1} = \frac{40}{5} = \text{N\$}8.00$
- E. (i) $800 \times 100 = \text{N\$}80\,000$
- (ii) $250 \times 100 = \text{N\$}25\,000$
- F. (i)
- (ii)
- G. (i) $30\% \div 100 \times 800 = 160$
- (ii) $20 \div 100 \times 250 = ?$
- H. (i) $\frac{20}{100} \times 800 = \text{N\$}160.00$
- (ii) $\frac{250}{300} \times \frac{800}{1} = 2.000$
- I. (i) $\frac{300}{800} \times \frac{20}{100} = \frac{60}{8} = 7.5$
- (ii) $\frac{250}{300} \times \frac{7.5}{100} = \frac{1875}{8} = 625$

Post-test ANSWERS

A lady work at furniture shop and earns a basic wage of N\$1200 per week and 15.5% commissions on all furnisher she sells?

- A. (a) Calculate the commission she earns on a sofas with N\$10 200.

$$\frac{15.5}{100} \times 10200 = \text{N\$}1581.00$$
- (b) Calculate the commission she earns on a fridge worth N\$800

$$\frac{15.5}{100} \times 5000 = \text{N\$}775$$
- B. (a) $\frac{15.5}{100} \times \frac{10200}{1} = \text{N\$}1581$
- (b) $\frac{15.5}{100} \times \frac{5000}{1} = \text{N\$}775$
- C. (a) $\frac{15.5}{100} \times \frac{10200}{1} = \text{N\$}1581.00$
- (b) $\frac{15.5}{100} \times 5000 = \text{N\$}775.00$
- D. (a) $\frac{15.5}{100} \times 10\,200 = \text{N\$}1581$
- (b) $\frac{15.5}{100} \times 5000 = \text{N\$}775$
- E. (a) $\frac{15.5}{100} \times 1200 = \text{N\$}186$
- (c) $\frac{15.5}{100} \times 5000 = \text{N\$}775$
- F. (a) $\frac{15.5}{100} \times 10200 = \text{N\$}1581.00$
- (b) $\frac{15.5}{100} \times 5000 = \text{N\$}77$
- G. (a) $\frac{15.5}{100} \times 10200 = \text{N\$}1581$
- (b) $\frac{15.5}{100} \times 5000 = \text{N\$}775$
- H. (a) $\frac{15.5}{100} \times 10200 = \text{N\$}1581$
- (b) $\frac{15.5}{100} \times 5000 = \text{N\$}775$
- I. a) $\frac{15.5}{100} \times 10200 = \text{N\$}1581.00$
- (b) $\frac{15.5}{100} \times 5000 = \text{N\$}775.00$

QUESTION 9

Pre-test

9. Melinda pays N\$240 for a carpet & sells it at a profit of 30%. Find the selling price of the carpet?

(Show your work)

ANSWERS

$$A. \frac{30}{100} \times \frac{240}{1} = \frac{30}{10} \times 24 = 720 \\ = \text{N\$}72$$

$$B. \frac{240}{1} \times \frac{30}{100} = \text{N\$}72$$

$$C. \frac{30}{100} \times 240 = 72$$

$$D. \frac{30}{240} \times \frac{100}{1} = \frac{150}{12} = 12,5 \\ = 240 + 12,5 = \text{N\$}252,5$$

$$E. 240 \times 30 \quad \frac{30}{100} \times 240 = 72 \\ = 7.200$$

$$F. \frac{30}{100} \times 240 = 72 + 240 = \text{N\$}72$$

$$G. 30 \div 100 \quad 0.3 \times \text{N\$}240 = \text{N\$}72$$

$$H. \frac{30}{100} \times \frac{240}{1} = \frac{720}{10} = 72 \text{ selling price is} \\ \text{N\$}75$$

$$I. \frac{240}{1} \times \frac{30\%}{100} = \frac{7200}{100} = 72$$

Post-test

9. School shoes costs N\$50.00 if they are marked up with 20%. Calculate the selling price?

ANSWERS

$$A. \frac{30}{100} \times 150 = \text{N\$}30 \text{ is the selling price} \\ 20 = 100 \times 150 = \text{N\$}30$$

$$B. \frac{20}{100} \times \frac{150}{1} = 30 = 150 + 30 \\ = \text{N\$}180$$

$$C. \frac{20}{100} \times \frac{150}{1} = 30 \\ = 150 + 30 = 180$$

$$D. \frac{20}{100} \times \frac{150}{1} = 30 \\ = \text{N\$}150 + \text{N\$}30 = \text{N\$}180$$

$$E. \frac{20}{100} \times \frac{150}{1} = 30$$

$$F. \frac{20}{100} \times 150 = 30 \quad 150 + 30 = \text{N\$}180 \\ \text{N\$}30 + 150 = \text{N\$}180$$

$$G. \frac{20}{100} \times 150 = \text{N\$}30 \text{ the selling price is N\$}180 \\ \text{N\$}30 + 150 = \text{N\$}180$$

$$H. \frac{20}{100} \times 150 = \text{N\$}30 \\ \text{N\$}150 + \text{N\$}30 = \text{N\$}80.00 \text{ selling price}$$

$$I. \frac{20}{100} \times 150 = 30 \\ 150 + 30 = \text{N\$}180.00$$

QUESTION 10

Pre-test

10. Best furniture offers discount on fridges. A fridge that cost N\$2500 is sold for N\$2000. Calculate the percentage discount.
(Show your work)

ANSWERS

A. $2500 - 2000 = 500$
 $\frac{500}{2500} \times 100 = \frac{500}{25} \times 1 = \frac{500}{25} =$

B. $\frac{2000}{2500} \times \frac{100}{1} = \frac{200\ 000}{2\ 500} = 80\%$

C. $2500 - 2000 = 500$

D. $\frac{2000}{25000} \times \frac{100}{1} = \frac{400}{5} = 80\%$

E. $2500 - 2000 = 500$
 $\frac{500}{100} \times 200 = 5.00$
 $= 5.000.000$

F. $2500 - 2000 = 500 = 50\%$

G. $\frac{\text{Discount}}{\text{Original price}}$

$$\frac{2500}{2000} \times 100 = 125\%$$

H. $\frac{2000}{2500} \times \frac{100}{1} = \frac{200000}{250} = 80$

I. $\frac{2000}{2500} \times \frac{100}{1} = \frac{2000}{25} = 80\%$

Post-test

10. A large snake normally weighs 12kg. After swallowing a rat, the weight of the snake increases by 12%. How much does it weigh after dinner?

ANSWERS

A. $\frac{12}{100} \times 12 = 1.44 \text{ kg}$
 $12 \div 100 \times 12 = 1.44 \text{ kg}$

B. $\frac{12}{100} \times 12 = 1.44$ = 13.44 kg

C. $\frac{12}{100} \times 12 = 1.44$
 $12 + 1.44 = 13.44 \text{ kg}$

D. $\frac{12}{100} \times 12 = 1.44 + 1.44$
 $= 13.44$

E. $\frac{12}{100} \times 12 = 1.44$

F. $\frac{12}{100} \times 12 = 1.44 + 12$
 $= 13.44 \text{ kg}$

G. $\frac{12}{100} \times 12 = 1.44$

H. $\frac{12}{100} \times 12 = 1.44 \text{ kg}$
 $1.44 \text{ kg} + 12 \text{ kg} = 13.44 \text{ kg}$

I. $\frac{12}{100} \times 12 = 1.44$
 $1.44 + 12 = 13.44 \text{ kg weigh}$

QUESTION 11

Pre-test

11. The price tag on CD player reads N\$289 plus VAT at 15%. How much does the customer have to pay? (Show your work)

ANSWERS

A. $\frac{15\%}{100} \times \frac{N\$289}{1} = \frac{4335}{100} = N\43.35

B. $\frac{289}{1} \times \frac{16}{100} = N\143.84

C. $\frac{15}{100} \times 289 = 43.35$

D. $\frac{15}{289} \times \frac{100}{1} = \frac{1650}{822} = N\5

E. $289 + 15 \times 100 \quad \frac{15}{100} \times 289 = 43.35$
 $= 1.789$

F. $\frac{15}{100} \times 289 = 43.35 + 289$
 $= 332.35$

G. $15 \div 100$
 $0.15 \times N\$289$
 $= N\$43.35$ cents that's what customer pay

H. $\frac{15}{100} \times \frac{N\$289}{1} = 43.35$
 $= 43.35$

I. $\frac{289}{1} \times \frac{15}{100} = \frac{4335}{100} = 43.35$

Post-test

11. The price tag on CD player reads N\$300, VAT excluded at 15%. How much does the customer have to pay? (Show your work)

ANSWERS

A. $300 \times 0.15 = 45$ original VAT price with VAT of 15%
 $300 \times 1.15 = N\$345$ $N\$300 + 45.00 = N\345.00

B. $1.15 \div 300$
 $= N\$296$ to pay

C. $\frac{15}{100} \times 300 = 45$ $300 + 45 = N\$345$

D. $\frac{15}{100} \times 300 = 45$ $300 + 45 = N\$255$

E. $\frac{300}{100} \times 15 = 45$

F. $\frac{15}{100} \times 300 = 45 + 300 = N\345

G. $300 \times 0.15 = 45$ the customer must pay 45
 $45 + 300 = N\$345$

H. $300 \times 0.15 = 45$
 $300 + 45 = N\$345$ selling price the customer have to pay

I. $300 \times 1.15 = N\$345.00$

QUESTION 12

Pre-test

12. In a furniture shop the price tags must be charged to make the prices VAT inclusive. If VAT is 16%, work out the new prices of the sofas below? (Show your work).

ANSWERS

A. $\frac{15\%}{100} \times 999 = 16\ 483.5$
 $= \text{N}\$164.835$

B. $\frac{899}{1} \times \frac{16}{100} = \text{N}\143.84

C. $\frac{16.5}{100} \times 999 = 164.835$

D. $\frac{16.5}{822} \times \frac{100}{1} = \frac{1\ 650}{822} = 2.00$

E. $100 \times 16.5 \frac{16.5}{100} \times 100$
 $= 1.650 = 0.16$

F. $16.5 \times 1000 = 165 + 1000 = 1165$

G. $16.5 \div 100 = 0.165 \times 999 = \text{N}\1648.35

H. $\frac{16.5}{100} \times 999 = 437.685$
New price = 47.685

I. $\frac{15\%}{100} \times \frac{16.5\%}{100} = 247.5$

Post-test

12. Mr Arthur bought a digital camera for N\$1 880 VAT inclusive of 15%. Calculate the cost of the digital camera without VAT.

ANSWERS

A. $1\ 880 \times 1.15$ original VAT of 15 price with Vat 15%
 $1\ 880 \times 0.15 \text{ N}\$1\ 880\ 282 \text{ N}\$2\ 162$

B. $1.15 \div 1880 = \text{N}\$1\ 878.88$

C. $\frac{15}{100} \times 1880 = \text{N}\282

D. $\frac{15}{100} \times 1880 = 282 \text{ N}\$1\ 880 + \text{N}282$
 $= \text{N}\$1\ 598$

E. $\frac{1880}{100} \times 15 = \text{N}\282

F. $\frac{15}{100} \times 1880 = 282$
 $= 1880 - 282$
 $= 1\ 598$

G. $1880 \div 1.15 \quad \text{N}\1880.00
 $= 1634.88 \quad - \text{N}\1634.88
 $= \text{N}\$245.12$

H. $\text{N}\$880 \div 1.15 = \text{N}\637.782609
 $= \text{N}\$1637.78$

I. $1880 \div 1.15 = \text{N}\1637

APPENDIX D

LESSON 1'S TRANSCRIPTS

Line1. After the teacher writes the learning objectives and the dates on the chalkboard, she greet the learners, and introduce the lesson by asking learners what percentage is and she wrote all learners response on the chalkboard.

Line2. Learners' response where as follows: **Learner C:** Percentage is hundred. **Learner G:** A fractions out of hundred. **Learner A:** Anything out of hundred. **Learner E:** Any number out of hundred. **Learner I:** Percentage is over hundred.

Line3. Teacher praise learners and she then explain to the learners what percentage is using some of the learner's response and rectify learners C and I. (Percentage is not a number hundred; percentage is a fractions out of hundred like what learner, F, D and E has said. Percentage is any number out of hundred parts.

Line4. Teacher asks a learner to write any number as a percentage on the chalkboard. Learner J writes 50% on the chalkboard and the teacher point at this unit (%) next to the learner J's answer and ask the meaning of %. Learners J then answer that it is percentage. Teacher gave chance to other learners. Learner D; "it is the unit of percentage." the teachers continue to ask, so what does this unit says? Learner D; "it means 50 is out of 100." After the teacher praise learner D the then explain that (%) this is the unit showing that a number is expressed as percentage. 50% means, there were 100 equal parts and 50 out of these 100 parts, is the one which is said to be 50%. 50% is also a fractions; it is a special fractions that is always out of hundred. Percentage is the same number written in different form as fractions. How can one write 50% as fractions?

Line5. Learners were all quite for some time. The teacher then asked again, anyone from the learners to write 50% as fractions. Again all of the learners were quite. The teacher than told learners that they will do it together she let all learners to be in pairs and work with the II boxes.

Line6. Teacher requested all the learners in pairs to open each box and count their things in it, and they should make sure that in each box there are 100 things. Before learners started counting, the teacher asked each pair to tell her what things were inside their boxes. Learners responses were: **Pair 1:** we have piece of boxes. **Pair 2:** nongongo (teacher said for now we will call it seeds.). **Pair 3:** pieces of plastic bottles. **Pair 4:** Pins. **Pair 5:** bottle tops. **Pair 6:** bottle tops. **Pair 7:** stones. **Pair 8:** Pieces of pipes. **Pair 9:** Pins. **Pair 10:** pins. **Pair 11:** money (N\$1 coins)

Line7. Learners started counting their things inside their boxes. Some pair where counting in tens, some in one's some in five's and some in fifty. Some pair got extra things and put aside and took their 100 things back in their boxes.

Line8. Teacher asked learners to do the following: she writes a table on the chalkboard and instruct each pair to remove some of their things inside the box, and write the percentage that they made and also the fractions that they made.

Names of things	Number of things out from the box	percentage	Fractions
Stones	20		
N\$1 coins	34		
Bottle tops	50		
Bottle tops	5		
Seeds	70		
Piece of pipes	43		
Piece of plastic bottles	96		
Cola of bottles	66		
Piece of boxes	82		
Pins	14		
Pins	100		

Each pair said what they did with their 100 things in the box, they said the percentage and fractions they made and wrote their answers on the chalkboard on the space provided of the table above. Each pair response was as follows:

Line9. Learner I and K: (learner I was talking each group of their ten real things that they counted). We took ten, ten, ten, ten and three pieces of pipes from the box, so it is forty three over hundred. **Teacher:** say it as a fraction. **Learner I:** it is forty three over hundred. He then writes forty three over hundred (in words) on the table which was on the chalkboard next to his number and say that's their fractions. **Learner A and D:** (they were in pair but learner D is the one who spoke). We subtracted twenty from hundred which makes it twenty over hundred and the stones which are left in the box are 80 (Learner A re-counted the stones inside the box, and she started grouping the stones in tens). **Teacher:** writes as a fraction. **Learner D:** writes $\frac{20}{100}$ and read it as twenty over hundred. **Learner J and E:** (they were in pair but learner J is the one who spoke) we count fifty bottle tops out, and that is fifty percent. **Teacher:** write and read it. **Learner J:** writes it as $\frac{50}{100}$ and read it as fifty percent over hundred. (While reading, learner J writes next to 50 in $(\frac{50}{100})$ this unit % and it looks as follows $\frac{50\%}{100}$).

Learner L: (Learner L worked alone) I subtracted seventy from hundred and that is seventy over hundred, he writes $\frac{70}{100}$ and read it as seventy over hundred. **Learner M and N:** (learner M was the one talking) we subtracted eighty two from one hundred which is eighty two over hundred. **Learner M** writes $\frac{82}{100}$. **Learner B:** (learner B worked alone) I took thirty four coins out of hundred coins, he wrote his fractions as $\frac{34}{100}$ and read it as thirty four over hundred. **Learner G:** (learner G worked alone) I took sixty six out of hundred pieces then I got sixty six percent. He then writes $\frac{66}{100}$. **Learner O:** (learner O worked alone) I took fourteen of hundred pins, it will be fourteen percent and as a fractions is fourteen over hundred. She writes her fractions as $\frac{14\%}{100}$. **Learner F and C:** (learner F was the one talking) we took ninety six out of hundred and it is ninety six over hundred as a fractions and ninety six percent. (Teacher asked them to write their

answer on the chalkboard) learner F writes $\frac{96}{100}$ and 96%. **Learner H:** (he worked alone) I took hundred from hundred pins, as a fraction is hundred from hundred, he then write it as $\frac{100}{100}$.

Line10. Teacher praised all the learners and informs learners that they converted percentage to Fractions using their 100 things in the box. Teacher reminds learners that she asked them to write 50% as fractions, and everybody was quiet; she then asked learners the answer. **Learners A:** writes $\frac{50}{100}$. The teacher explains why she wanted learners to use only hundred things in their boxes. She said “I didn’t use this unit (%) but rather I used what? **Learner B:** “you gave us boxes of 100 things in each box. Teacher explain further “yes that’s true, “I gave you boxes of hundred things each, like I told you, percentage is a special fractions out of hundred. Percentage is fractions. When you take out any number of things out of hundred things, that number is expressed as percentage.

Learner F: “we were taking out our numbers from the box. **Teacher:** From your statement you didn’t use the word over, but rather what did you use? **Learner F:** I used taking out. **Learner D:** “we learnt over when we were in grade 7, it means reading any number as a fractions. E.g. $\frac{7}{10}$ we will read it as seven over ten”. **Teacher:** ok what do you think you mean, when you say, when you say, seven over ten? I mean, I mean taking few things from the total (laugh a little). **Teacher:** Then why can’t you just use the word „taking from” than „over”? It’s true ma... (Laugh aloud). **Learner I:** “yes we are used to say over and even when you asked us at first what percentage is I said percentage is over hundred but it looks better if I will use the word out of, because fractions means to remove few same things of the total things. **Learner H:** yes out of is better than over. Over cannot expla... cannot mean anything, especially in the fractions. Out of means you are taking few things out of the 100 things.

Line11. Teacher: “alright, from today on, let’s avoid saying that $\frac{34}{100}$ is thirty four over hundred, because we don’t understand what over means, like you said it yourselves. We should rather say thirty four N\$1 coins out of 100 N\$1 coins. It makes sense when we use the word “out of or from”, as you can see that all of you were taking a portion of your real world things out of 100 real things.” Do we all agree? (All learners shouted) yes, (line11).

Line12. Teacher writes few percentages on the chalkboard and asks learners to read it. 34%; 50%; 100%; 2, 5%. Responses from learners were: **Learner B:** Thirty four percent, and as fractions is thirty four out of hundred. **Learner I:** Fifty percent and as a fractions is fifty out of hundred bottle tops. **Learner D:** Hundred out of hundred percent. **Learner F:** Two comma five percent and that are two comma five out of hundred things.

Line13.Teacher asked learners to remove things out of their boxes for the second time. She erases the numbers of things that learners remove at first from their boxes and created other numbers. The second time learners were reading their percentage and fractions correctly and also they were writing their fractions and percentage accurately. This time many of the learners didn’t use the work “over”, to read their fractions but rather they used the word “out of”. Learners’ responses were: **Learner I:** “we took thirty out of hundred pieces of pipes, which is thirty percent piece of pipes. **Learner K:** “we took seventy nine pins out of hundred pins, is seventy nine percent of the pins”. **Learner O:** we subtract eight five out of hundred bottle tops. Fifteen percent is not out percentage. Eight five percent of bottle tops is our percentage. (Teacher asks how many bottle tops learner O took from the box). **Learner O** responds as “eight five bottle tops, that is eight five out of hundred bottle tops and it is eight five percent. **Learner E:** “we took fifteen out of hundred bottle tops, and that is fifteen percent.” **Learner N:** we took three out

of hundred pieces of boxes and that is three percent.” **Learner L:** we took twelve seeds out of one hundred seeds and that is twelve out of hundred seeds. **Learner F:** we took six colorless pieces of bottles out of one hundred, and that is six percent of piece of plastic bottles. **Learner**

H: I took eight pins out of one hundred pins and that is eight percent of the pins. **Learner G:** I took twenty eight out of hundred; in percentage is twenty eight percent of cola of plastic bottles.

Line14. Teacher praised all the learners that they have improved on the way they are reading and saying percentage & fractions. She again encouraged learners to continuously use the word “OUT OF” than “OVER”. The teacher concluded her lesson by giving learners activities to do relate to the topic.

APPENDIX D

LESSON 2'S TRANSCRIPT

Line1. This is the second lesson; the teacher writes the date and the learning objectives on the chalkboard, she then greets the learners.

Line2. The lesson was introduced by letting learners do the previous activity of taking out numbers of things from their boxes. This time, one partner in a pair tells the other partner to remove the number of things she/he want the partner to remove and the one who is removing should tell the teacher, what type of fractions & percentage she/he made?, how to read and write them?.

Line3. When learner G counted his bottle tops together, it was not hundred, it was just eight four. He then removed sixty two out of eight four bottle tops. Throughout his counting, he is thinking that he has hundred bottle tops in his box. When the teacher asked him where his sixty two bottle tops were; he pushed aside 6 groups of ten bottle tops while holding two of the bottle tops and said “it’s her”. After he pushed groups of bottle tops together, they were only twenty two bottle tops left. **Teacher:** What did you tell your partner to do? (Teacher asked learner O).

Learner O: I told him to remove sixty two bottle tops from the box. **Teacher:** Ok, learner G, what did you do?

Learner G: I took sixty two out of hundred bottle tops. **Teacher:** Where are your sixty two bottle tops? **Learner G:** it is here (pushing aside 6 ten groups of bottle tops together) while holding two bottle tops. (After learner G pushes groups of bottle tops together, they were only twenty two bottle tops left. Both the teacher and the learners find it out). **Teacher:** When you counted you bottle tops, where they hundred all together? **Learner G:** Yes (teacher asked learner G if he is sure). **Teacher:** There is something wrong with your bottle tops, is either the sixty two you took out from the box or the twenty two remaining. (Teacher told learner G to re-count again). Before counting he insists that his bottle tops were sixty two. When he counted, the bottle tops were really sixty two and the one’s left in the box were twenty two. Learner G then took a piece of paper and calculates $\frac{62}{100} + \frac{22}{100} = \frac{84}{100}$ he then admitted that his bottle tops were not hundred together they were just eighty four. He then said: “that’s why I’m not getting hundred; I made a mistake when counting all the bottle tops together.

Line4. The other pairs managed to say their fractions and percentage correctly. Again none of them used the word “OVER” all of them used the word “OUT OF”. Moreover the first partners in each pair were telling the teacher that they told others to “subtract out” things from their boxes. They were using the word “Subtract out”. Examples: **Teacher:** What did you tell your partner to do? **Learner B:** I told him to “subtract out” fifty from hundred cola of plastic bottles.

Teacher: And what did you do? (Pointing to the other partner in that pair). **Learner I:** I took fifty out of hundred cola of plastic bottles and this make fifty percent. **Teacher:** (While moving to the next pair, she point to the pair with N\$1 coins). What did you do? Who was the first partner? **Learner D:** It’s me, I told her to subtract 30 out of hundred. **Learner A:** the answer is thirty out of hundred because I subtract hundred minus seventy. That is thirty percent.

Line5. When the teacher realized that learners were subtracting out their portion from hundred, she then used that opportunity to ask the two partners in the pair to say the fractions which they made and the other fractions which is left in the box. **Teacher:** Who was the first partner? What

did you tell your partner to do? **Learner C:** (Raised her hands) I told her to make sixty eight out of hundred. **Teacher:** That makes what fractions? **Learner K:** Sixty eight out of hundred and that is sixty eight percent. **Teacher:** Very good, learner C if your friend took sixty eight percent from the box, what fractions of piece of pipes is left in the box? **Learner C:** I subtract sixty eight out of hundred from the box. So the fraction which is left is thirty two out of hundred. They will all be out of hundred because the total of piece of pipes were hundred.

Teacher: What about this pair, what fractions did you make? **Learner E:** forty over hundred, ai... forty out of hundred and that is forty percent. **Teacher:** (while putting the 40% of plastic bottles aside she ask) if this is forty percent of plastic bottles, what is the fractions left in the box?

Learner E: sixty percent, I subtract hundred minus forty percent. (Teacher praised learner E and go to the other pair).

Teacher: Yes learner F, what's your story (the seeds of learner F were in groups of ten, so he had three groups of ten and seven seeds). **Learner F:** (he pointed at his thirty seven seeds) our fraction is thirty seven percent of seeds. **Teacher:** If this is 37 percent of seeds, what percent of seeds is left in the box? **Learner F:** I subtract hundred out of hundred seed minus thirty seven out of hundred seed and it gave me sixty three out of hundred seed. **Teacher:** very good. (She dealt with all the pairs by asking the fractions left in each box. The pair was subtracting their fractions from the whole (hundred things) to get the fractions left in the box.

Line6. The teacher praised all pairs and informed learners that, they subtracted fractions, she then asked a question based on today's lesson. "What is the relationship between percentage, decimal and fractions? **Learner D:** the relationship between the three is that they are all out of hundred. **Teacher:** (praised learner D and asked someone with another idea, but all the learners were quite). "Alright the relationship between the three forms is that the same number is represented in different form. E.g. let take 50%, in which form is this (learners shouting) "Percentage". How can I write the same number as fractions? **Learner E:** we read it as fifty out of hundred and we write it as $\frac{50}{100}$. **Teacher:** "we have seen 50% written as fractions is $\frac{50}{100}$ the same number can also be written as decimal, who has an idea?"

Line7. Learner M: "it is zero comma fifty (the teacher praised learner M and asked her to write her answer on the chalkboard and to explain how she got the answer). Learner M writes 0.50 and explains how she got the answer. "I just added zero and a comma and I wrote fifty." **Teacher:** Ok, but how's that? **Learner M:** I just know that all decimal has a comma. **Teacher:** Then? **Learner M:** Then I count how many numbers fifty has, then I wrote 0, 50. **Teacher:** Why did you count the digits fifty has? **Learner M:** so that I know how many number will be behind the comma. (Teacher praised learner M and ask other learners with different ideas). **Learner I:** Mostly, if a number is out of hundred, after the comma, there should be two number. **Teacher:** What do you mean? **Learner I:** (went on the chalkboard and explain). If you are changing from fractions (point at $\frac{50}{100}$) to decimal (point at 0. 50) after the comma must have two numbers. **Teacher:** what is the reason of having two numbers after the comma? **Learner I:** because in fractions is out of ten, then the answer is zero comma five, only one number after the comma. The teacher then praised learner M and I's answers. She used learner's answers to elaborate that if one is converting from fractions $\frac{50}{100}$ to percentage, they should divide 50 by hundred. She asked learners to try out. All the learners used calculators and they got 0. 5 and they shouted "zero comma five."

Line8. The teacher explains to learners that 0.5 and 0.50 and 0.500 are all the same number; she explained that the number five in each of the decimal number is on the same place value that is why the three decimal numbers are the same. She again explained that the decimal number 0.50 are read as zero point five zero and not zero point fifty. After the explanations from the teacher, she then gave learner examples to do, to convert from percentage to fractions then to decimal examples 25%; 42%; 75%; 60%.

Line9. All the learners converted these examples correctly from percentage to fractions and decimals. Their answers were: $25\% = \frac{25}{100} = 0.25$; $42\% = \frac{42}{100} = 0.42$; $75\% = \frac{75}{100} = 0.75$; $60\% = \frac{60}{100} = 0.6$

Line10. Teacher asked learners to convert decimal to fractions and to percentage. She first gave examples that have two digits after the comma. e.g. 0.33; 0.27; 0.55; 0.78. Learner's answers were; $0.33 = \frac{33}{100} = 33\%$; $0.27 = \frac{27}{100} = 27\%$; $0.55 = \frac{55}{100} = 55\%$; $0.78 = \frac{78}{100} = 78\%$.

Line11. Teacher gave examples of decimal number with only one digit after the comma to be converted to fractions and percentage. e.g. 0.6; 0.9; 0.2; 0.5. Learner's answers: **Learner D** (a) $0.6 = \frac{6}{10} = 6\%$; (b) $0.9 = \frac{9}{10} = 9\%$; (c) $0.2 = \frac{2}{10} = 2\%$; (d) $0.5 = \frac{5}{10} = 5\%$. **Learner B** (a) $0.6 = \frac{6}{10} = 60\%$; (b) $0.9 = \frac{9}{10} = 90\%$; (c) $0.2 = \frac{2}{10} = 20\%$; (d) $0.5 = \frac{5}{10} = 50\%$. **Learner A** (a) $0.6 = \frac{6}{10} = 6\%$; (b) $0.9 = \frac{9}{10} = 9\%$; (c) $0.2 = \frac{2}{10} = 2\%$; (d) $0.5 = \frac{5}{10} = 5\%$. **Learner C** (a) $0.6 = \frac{6}{10} = 6\%$; (b) $0.9 = \frac{9}{10} = 9\%$; (c) $0.2 = \frac{2}{10} = 2\%$; (d) $0.5 = \frac{5}{10} = 5\%$. **Learner I** (a) $0.6 = \frac{6}{10} = 6\%$; (b) $0.9 = \frac{9}{10} = 9\%$; (c) $0.2 = \frac{2}{10} = 2\%$; (d) $0.5 = \frac{5}{10} = 5\%$.

Line12. The teacher asked learner D to explain her first answer. **Learner D says:** Zero point six to fractions is six out of ten, because there is only one number after the comma, so the six will be out of ten. If there was two number after the point, then it will be out of hundred. **Teacher:** Very good. What about converting zero point six to percentage? **Learner D:** It will be 6%. **Teacher:** Ok let's remember what you have learnt (teacher write) $6\% + \underline{\quad}$? **Learner D:** $6\% = \frac{6}{100}$, because this % represent out of hundred. **Teacher:** That's correct, but you wrote that $\frac{6}{10} + 6\%$ this is different from how you are explaining. **Learner D:** (Admits that her answer is wrong). **Teacher:** why are you saying that the first answer is wrong? **Learner D:** because six percent is six hundredth and not six tenth. **Teacher:** so, six tenth is what percent? **Learner D:** umm, I am not sure. (The teacher calls any of the learners with a different answer). **Learner B:** (Raised his hands) and explain that six out of ten is equal to sixty percent. You change $(\frac{6}{10})$ ten to become hundred because we learn that a fractions to be percentage only if it is out of hundred not any number. Therefore six times ten is sixty and $\frac{60}{100}$ is 60%. **Teacher:** Praised learner B and asked learners who has a different way to get 60%. **Learner G:** "I divide six by ten, then I multiply by one hundred and my answer is sixty percent. **Teacher:** very good, both learner B and learner G have the correct way to convert decimal to fractions and percentage. Learner B explained that 0.6 to fractions is $\frac{6}{10}$, he then need to make „out of ten“ to be „out of hundred“, because he said percentage is out of hundred, that's why he multiply his fractions by ten so that he get an equivalent fractions out of hundred. When the fractions is out of hundred it is easier to convert to percentage. Learner G on the other hand convert $\frac{6}{10}$ to decimal and that is by taking six and

divide it by ten and the answer is 0.6, then you multiply your decimal number by hundred and get sixty percent. **Teacher:** now I want you all to convert the other decimal number 0.9; 0.2; and 0.5 using learner B or G's methods or even yours, as long as it works.

Line13. The teacher draw a table below that shows the relationship between percentage and fractions and decimal she then explain that there are two ways on how each of the three forms is written. For instance the teacher explain how to fill in the first row of the table as shown below, and left the other rows for learners to do it.

Fractions	Fractions	Decimal	Decimal	Percentage	Percentage
$\frac{3}{10}$	Three tenth	0.3	zero point three	30%	thirty percent
		0.5			
				112%	

Line14. Teacher explains how to convert between percentage, fractions and percentage by filling in the first row. She then asked learners to complete row two and three together with her. She called learners to complete the table and ask them to explain their work. **Learner P:** writes $\frac{5}{10}$ and say five out of ten. **Learner N:** writes, five tenth as other way for fractions. **Learner E:** writes 0.5 as in decimal and in words, zero point five. **Learner L:** writes 50% as in percentage and explains as follows, i multiply five out of ten by hundred, **learner A:** write fifty percent in words. I again asked learners to complete row three. **Learner F:** 112% will be one hundred and twelve out of hundred, he then writes $\frac{112}{100}$. **Learner B:** in another fractions is „one hundred twelve of hundred. Of hundred (teacher repeats after the answer)? **Learner B:** of hundredth, he then writes like this, „one hundred and twelf hundredth“. I then corrected him on the spelling of 12, not to write twelf but twelve. **Learner O:** writes 1.12 and read it as one point twelve. I corrected learner O, to read 1.12 as one point one two and not one point twelve. I again asked learner O, on how she got the answer. Learner O's response was that, I divide one hundred and twelve divide by hundred. I asked learner M to write 112% as decimal in words, **learner M:** writes one point one two. **Learner D** on the other hand writes 112% as percentage in words as one hundred and twelve percent. The teacher praised all the learners and informs them that they will receive a class activity of such table

Line15. The teacher worked with the learners how to convert from fractions to percentage using 5 diagrams. The question was; write the shaded part of figure /diagram as fractions then as percentage. (See learner's activity two for the diagrams). **Learner F:** it is one out of four, and it is twenty five percent. I took one divide by four and multiply by one hundred. **Teacher:** very good learner F, try to read ($\frac{1}{4}$) as one quarter than one out of four. **Teacher:** after praising learner F she asks; “one out of four has a special mathematic term as what? **Learner L:** “One quarter” Teacher emphasis more on how learners should read or say $\frac{1}{4}$.

Line16.The teacher moved to the next diagram. **Learner L:** writes $\frac{4}{35}$. **Teacher:** write it as percentage. **Learner M:** it is eleven point four two eight five....there is a lot of numbers after the

comma. **Teacher:** round off to the nearest digit, if there is a lot of number after the comma? **Learner B:** then you round off and the answer is 11.4%. **Teacher:** very good to all of you, many of you in the pre-test wrote a correct fractions $\frac{4}{35}$, but couldn't convert it right to percentage. What was the main problem? I needed to find out the learners problem in this question, so that we work out the problem together and learn from one another. **Learner M:** I though percentage doesn't have a lot of number after the comma, only decimal. **Teacher:** it does, if you get an answer that has a lot of digits after the comma, we round off the number. That's why we learned already how to round off numbers to the nearest unit. **Learner A:** I couldn't convert fractions that gives me an ugly answer, I like converting fractions that gives me an exact answer. **Teacher:** do you now know how to convert this fraction $\frac{4}{35}$ to percentage? **Learner A:** no, only the first example $\frac{1}{4}$. **Teacher:** how do you convert $\frac{1}{4}$ to percentage? **Learner A:** I divide one by four and times hundred. **Teacher:** exactly, the same way you convert one quarter to percentage, the same way you should convert $\frac{4}{35}$, try now. **Learner A:** four divide by thirty five, times hundred. **Teacher:** equals to what? **Learner A:** (pressing her work on the calculator) it is 11.4285. **Teacher:** from there what should one do? **Learner A:** convert. **Teacher:** great.

Line17. The teacher moved to the third diagram. **Learner K:** is read as two out of ten and is written as $\frac{2}{10}$. **Learner F:** writes 20% and explain that, i divided two by ten then the answer i multiply it by hundred and got 20%. **Teacher:** very good.

Line18. **Learner A:** writes $\frac{5}{10}$ and read it as five out of ten, she again writes 50% and read it as fifty percent. **Teacher:** how did you convert from fractions to percentage? **Learner A:** I multiply hundred by five divide by ten. **Teacher:** that's good.

Line19. **Teacher:** very good, what about the last diagram? **Learner D:** as a fraction is four out of fifteen and as percentage is twenty six percent. **Teacher:** is the answer an exact one? **Learner D:** no, it is 26, 66666667. **Teacher:** round off to the nearest one decimal place. **Learner I:** (shouted) 26.7%. **Teacher:** yes, if to the nearest whole number what will it be? **Learner E:** it is 26. **Teacher:** Is 26 correct to the nearest whole number? **Learners:** no it is wrong, **learner H:** it is 27%. I then reminded learners how to round off again.

APPENDIX D

LESSON 3'S TRANSCRIPT

Line1. The teacher writes the dates, and the learning objectives on the chalkboard, she then greet learners and reflect on the previous Lesson. She draws the table below on the chalkboard and informs learners that they will discuss the table before going to the learning objective for today.

Fractions	fractions	decimal	decimal	percentage	percentage
$\frac{1}{3}$					
			Zero point four		
				85%	
		0,125			
					One hundred and four percent
$\frac{2}{4}$					
				52%	

Line2. Learners were given different color chalks to complete the table. The answers from learners for the first row were: **Learner H** writes $\frac{1}{3}$ as word as follows; one third, as decimal; it will be zero point three, he then explains that; “I divided. One divide by three and get zero point three, three, three, and three..... he then writes 0.33 as percentage and it is 33, 3333%, he again writes thirty three point three, three percent. He then explain that, “I divided one divide by three and the answer I multiply it by hundred and I got 33% as in words is, thirty three percent. **Teacher:** very good. Ok next row.

Line3. Learner E: „I calculated zero point four the answer multiplied by hundred and then get 40%. As a fraction is four out of ten $\frac{4}{10}$ or four tenth. **Teacher:** how did you convert zero point four to fractions? **Learner E:** I just take zero point four and count the number after the comma is one, so four will be out of ten. **Teacher:** From there how can you write zero point four as another decimal number? (**Learner E**, pointing at 0.40 on the chalkboard). **Teacher:** how did you work it out? **Learner E:** I took forty and divide it by hundred and get 0.40. **Teacher:** very good. (The teacher praised learner E and ask the next learner who filled in the next row to come and explain.

Line4. Learner G: „85% is read as eighty five percent. He said as a fractions is $\frac{85}{100}$, or eight five hundredth“. (Teacher asked someone to read the fractions proper) **learner O:** read as „eighty five hundredth“. **Learner G** continues explaining his work that „85% as decimal is 085. **Teacher:** you should put the comma, there’s no comma, where should the comma be in 085? **Learner G:** between zero and eight. **Teacher:** ok, how did you get 0.85? **Learner G:** I divided. Eighty five out of hundred means to divide eighty five by hundred. **Teacher:** ok, that good.

Line5. Learner C: .0.125 will be one hundred and twenty five out of hundred, in word it will be one hundred and twenty hundredth, as percentage it will be one hundred and twenty five percent. **Teacher:** “How do we convert a number from decimal to percentage?” **Learner C:** you take zero point one two five and multiply by one hundred. **Teacher:** and what is your answer? **Learner C:** used a calculator and press 0.125 times hundred and got twelve point five. (The teacher praised learner C and requested her to rectify her mistakes). She erased 125% and writes 12.5% again she changed her fractions of $\frac{125}{100}$ and wrote $\frac{12.5}{100}$. (Other learners were raising their hand thinking learner C wrote a wrong fraction). **Learner B:** it should be $\frac{125}{1000}$, because there is there numbers after the comma, so 125 will be out of 1000. **Teacher:**” I want learner C and some of you sitting to convert $\frac{12.5}{100}$ to decimal and also learner B to convert $\frac{125}{1000}$ to decimal too and both of you tell me your answers”.(both of them said is 0.125). **Teacher:** “None of you got this answer wrong; all of you got the correct answer”. **Teacher:** how did you work your answer out? **Learner B:** I divide one hundred and twenty five by hundred because any number out of another number you divide by that number. I got zero point one two five. **Teacher:** good, what about you learner C? **Learner C:** I took twelve point five and divide it by hundred and get zero point one two five. **Teacher:** Very, very good. Ok let’s move to the next row.

Line6. Learner F: writes “1 hundred and for hundredth”. **Teacher:** „You should write all the numbers in words e.g. “1 hundred and for hundredth, will be, one hundred and four hundredth. 1 in word is „one“ and the 4 is not written as for with this word but the correct way to write 4 in word is four. **Learner F** continued explaining that $\frac{104}{100}$ as decimal is one point zero four (1.04). I divide one hundred and four by hundred and the answer is one point zero four. As percentage I press one point zero four in the calculator and multiplied by one hundred and get 104%. **Teacher:** Very good. Ok let’s go to the next row.

Line7. Learner D: (all her conversion between percentage, fractions and decimal was correct). She explained how she converted $\frac{2}{4}$ to decimal as, **Learner D:** I divided two by four and get zero point five, i then convert 0.5 to percentage by multiplying the decimal number by hundred and get 50%. **Teacher:** why did you divide two by four? **Learner D:** because two out of four is the same thing as two divide by four. **Teacher:** Very good.

Line8. Learner B explains as follows; „I converted the 52% to fractions as fifty two out of hundred $\frac{52}{100}$ and he reads the fractions as fifty two hundredth, he continues saying that to convert 52% to decimal I pressed fifty two divide by hundred because fifty two out of hundred means you divide fifty two by hundred if you are working it out in the calculator. I got an answer of zero point five two. **Teacher:** Very good, learner B.

Line9. From the table the teacher took $\frac{2}{4}$ as equal to 50% and asks learners to think of other fractions that gives us 50%. Learner responses were: **Learner I:** three out of six. **Teacher:** how did you get that? **Learner I:** three divide by six times one hundred. **Teacher:** how did you to think of $\frac{3}{6}$? **Learner I:**(quiet for some time), I was testing that there is a two and down is four, and I was also testing three up to six down and I got 50%. **Teacher:** very good, what do you have as your fractions learner H? **Learner H:** four out eight. **Teacher:** how did you get four out eight? **Learner H:** I took the given fractions and say $(\frac{4}{2})$, two in four, 2 times, I took $(\frac{3}{6})$, three in six, 2 times. I took $(\frac{4}{8})$ four in eight, 2 times. **Teacher:** that’s good, anyone else? **Learner A:** one half. **Teacher:** yes, how did you come to that fraction? **Learner A:** how many ones are in two, how many two’s are in four and I got the same answer. **Teacher:** very good, another one with a

different fractions? Learner N: 6 out of 12 give me 50%. Teacher: how did you come across this fraction? Learner N: I said six divide by twelve times one hundred and got 50%. Teacher: how did you get 6 out 12, how did you decide that $\frac{6}{12}$ will make 50%? Learner N: (very quiet) later she said I just know that six out of twelve is half, all the fractions which other got is half. Teacher: how did you get the half six divide by twelve is 0.5 two divide by four is 0.5 three divide by six is 0.5. Teacher: ok, very good way of thinking. Yes someone else with other fractions. Learner C: five out ten, because ten divide by five is 2. Teacher explains that equivalent fraction has the same size using diagram (see intervention, lesson 3).

Line10. Learners worked in groups and they were asked to identify equivalent fractions of the initial fractions given to them by their teacher. There were five groups of learners in the class, in each group there were three learners. Group five with learner I, J and N used diagrams to generate five more equivalent fractions of their initial fractions. Their initial fraction was $\frac{4}{6}$ and their identified equivalent fractions were: $\frac{6}{9}$; $\frac{8}{12}$; $\frac{10}{15}$; $\frac{2}{3}$; $\frac{12}{18}$. Learner I, J and N draw the same size of bars to identify the five extra equivalent fractions as follows:

- (a) $\frac{6}{9}$ (b) $\frac{8}{12}$ (c) $\frac{10}{15}$ (d) $\frac{2}{3}$ (e) $\frac{12}{18}$



Learner J: let's work out like this, you take $\frac{4}{6}$ and multiply it by two, or three, or any number.

Learner I: why should we multiply by two of three or any number? **Learner J:** because the fractions we will get will be equivalent to this first one. Learner I: but how do we explain so that

it is clear? **Learner J:** let me show you (take a piece of paper and work out) $\frac{4 \times 2}{6 \times 2} = \frac{8}{12}$; this new fractions is the same as the first fractions. **Learner N:** that is true, our problem is that how do we

tell that the two fractions are equal. **Learner I:** what if we work out their sizes. **Learner N:** how? Learner J: by working out all to be decimal, like four divide by six is (pressing in the calculator) zero point six, six, six and also eight divide by twelve is zero point six, six, six.

Learner I: that's true. **Learner N:** yes. Learner I: let's try to draw our decimals. **Learner N:** it will be difficult to use decimal maybe fractions. Learner J: yes fractions. Learner I: ok (started drawing fractions). **Learner N:** make sure the line should be of the same size even if the parts will be different. **Learner I:** yes.

Line11. Group one and three used manipulatives (11 boxes) to generate equivalent fractions of their initial one. Learner A, C and D were in group one; they used pins (11 boxes) to make repeated patterns that generated equivalent fractions. For example, their initial fraction was $\frac{1}{3}$ and the five fractions which they generated were; $\frac{2}{6}$; $\frac{3}{9}$; $\frac{4}{12}$; $\frac{5}{15}$; $\frac{6}{18}$. **Teacher:** what did you do?

Learner C: Our original fraction was one third; we used pins to make other equivalent fractions

of the original. **Teacher**: explain how you worked with pins. **Learner C**: we arranged the pins as follows (pointing at the pins, which were arranged as shown above), we grouped the first pins, then repeat the same pattern, we counted the top pins separate from the bottom pins and count the new pins, so we then got a new equivalent fractions, we repeated that for five times to get five equivalent fractions. This is how they generated their equivalent fractions, placing one pin up and three pins down and started repeating the same patterns.



Line12. Learner G and K were in group three; they also used manipulatives (11 boxes) to identify their five equivalent fractions of the initial one. They used bottle tops to make repeated pattern that generated equivalent fractions just as what group one did. Their initial fraction was $\frac{2}{8}$ they then identified the following five equivalent fractions; $\frac{4}{16}$; $\frac{6}{24}$; $\frac{8}{32}$; $\frac{10}{40}$; $\frac{12}{48}$. **Teacher**: ok, what about you group three, what did you do? **Learner G**: we took our original fractions and make two groups according to our original fractions, then we count all the bottle tops together and get our first equivalent fractions. We then add again another group of our original fractions to the new group and count all together; we repeat this until we generate five equivalent fractions.

Line13. Group four had learner B and H, these learners identified the five equivalent fractions of their initial by repeated pattern but they didn't use manipulatives (11 boxes) rather they multiplied both the numerator and denominator of their initial fractions ($\frac{15}{20}$) by a factor (2). Each time they get the new fractions they multiplied the numerator and denominator of the new fractions by the same factor. E.g. $\frac{15 \times 2}{20 \times 2} = \frac{30 \times 2}{40 \times 2} = \frac{60 \times 2}{80 \times 2} = \frac{120 \times 2}{160 \times 2} = \frac{240 \times 2}{320 \times 2} = \frac{480}{640}$.

Line14. Group two had learner F and E, these two learners used calculators to find out all the fractions that have the same percentage as their initial fractions, and that is their equivalent fractions. Their initial fraction was $\frac{1}{8}$ they converted one eighth to percentage and is 12.5%. So they looked for all the fractions that have 12.5%. The equivalent fractions which were identified were: $\frac{2}{16}$; $\frac{4}{32}$; $\frac{125}{100}$; $\frac{5}{40}$ and $\frac{8}{64}$. **Learner F**: you know what; we should use our calculators, so that we look for fractions that have the same percentage as one eighth have, listen you look for three equivalent fractions and I will look for three also. **Learner E**: how is that? **Learner F**: look, I want you to work out the percentage of one eighth. **Learner E**: (press one divide by eight and multiply by hundred and got twelve point five and say) I got twelve point five. **Learner F**: yes, that's the correct percentage, now we should look for any fractions with the same percentage. **Learner E**: start trying on the calculator. After some minutes, these learners came together and discuss their identified equivalent fractions. One of the fractions that Learner E identified was $\frac{125}{100}$ he insisted that $\frac{125}{100}$ is one of the correct equivalent fractions of $\frac{1}{8}$ because they have the same percentage. This is how learner F answered. **Learner F**: no, learner E, this fraction cannot be one of them because it doesn't have the same percentage as one eighth. **Learner E**: but one hundred and twenty five divided by hundred is 12.5% (learner E didn't use a calculator then learner F gave learner E a calculator to work out his calculations, learner E got a different answer). **Learner E**, ok, I got a wrong answer, I thought it will give me twelve point five; it gave me one point two five. **Learner E**: ok, let's write all our correct fractions in one paper.

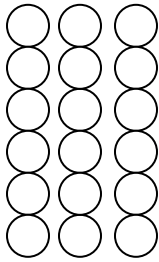
Line15: Teacher was impressed with different ways learners used to come to their answer, she praised all the groups and take out the paper from learners that they work out their equivalent fractions.

APPENDIX D LESSON 4'S TRANSCRIPT

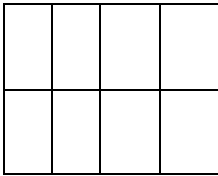
Line1. The teacher writes the learning objectives and dates on the chalkboard. She greets her learners and announces the learning objectives. After greeting the learners she reflect on the previous lesson by asking learners what was the topic of the previous day. Learners remembered. She then asked few examples for learners to work out the equivalent fractions. All the learners were getting the correct answer.

Line2. The teacher writes the following activities on the chalkboard and ask learners to answer.

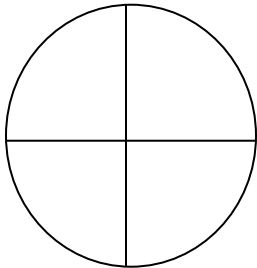
(a) Colour in 50% of the 18 circle below



(b) Shade in 40% of the figure below



(c) Colour in 25% of the pie chart below



Learner D: „Ishaded nine circles“. **Teacher:** how did you get the answer? **Learner D:** I put nine out of eighteen and multiply it by one hundred and I got 50%, this is how she have shown her answer $\frac{9}{18} \times 100 = \frac{900}{18} = 50\%$. **Teacher:** how did you decide to use nine from all the numbers?

Learner D: I looked for a fraction out of 18 that can give me 50%. **Teacher:** how did you look for that fraction? **Learner D:** fifty percent is the same thing as fifty out of hundred and fifty out of hundred is equivalent to nine out of eighteen because they all have the same percentage.

Teacher: Is there someone with another method (learner H raised his hands and explained).

Learner H: “I did this 50% is equal to $\frac{50}{100}$ times eighteen over one and is equal to nine $\frac{50}{100} \times 18 = 9$ circles. (Teacher asked learner H his main ideas) my main idea is to find out the half of eighteen because 50% is half of that number (teacher praised learner H and moved to the other activity).

Line3. Learner I: (Learner I worked out the second graph, and he explained as follows):
Learner I: I took ten, ten no, four, four out of ten multiplied by hundred i.e. $\frac{4}{10} \times 100 = 40\%$.
Teacher: how did you get four? **Learner I:** I got the answer four by trying numbers that will work. I first tried number three it didn't work, but then when I tried number four it worked. Learner B on the other hand explains the same question as follows: I took forty divide it by hundred then multiply it by ten, then I got four.

Line4. Learner A work out the last diagram as follows: **learner A:** writes $\frac{1}{4} \times 100 = 25\%$.
Teacher: How do you know that a quarter is equal to 25%? **Learner A:** $\frac{25 \div 25}{100 \div 25} = \frac{1}{4}$, **Teacher:** very good. You all had excellent explanations. Ok let's do the next thing.

Line5. Teacher then gave more representations on working out percentage of a quantity to the learner. This time the representations used were without graphs. Examples: (a) 20% of N\$300, (b) 25% of N\$60 and (c) 40% of N\$5. Learners were working out their answers and explain their work when I pointed them. The extract below shows learners explanations. **Learner H:** writes $\frac{20}{100} \times 300 = N\60 . **Learner G:** writes $\frac{25}{100} \times 60 = N\15.00 . **Learner C:** writes $\frac{40}{100} \times 5 = N\2.00 . (The teacher praised learner H, G and C and explain again how to work out percentage of a quantity).

Line6. Teacher hand in 3 grids to the learners, (a) 25 grids, (b) 100 grids and (c) 200 grids. She instructs learners to cover their grids with various colors. She first asked learners to take their 100 grids and paste in.

40% small piece of rectangle in yellow colour,

30% small piece of rectangle in Red colour,

25% small piece of rectangle in Blue colour,

5% small piece of rectangle in white colour.

Many learners cover the 100 grids correctly using the correct percentage. They got it correct because they were calculating 40% of 100 = 40 rectangle in yellow, 30% of 100 = 30 rectangles in Red, 25% of 100 = 25 rectangle in Blue and 5% of 100 = 5 rectangle in white. Therefore the grid was covered with 40 yellow rectangles, 30 red rectangles, 25 blue rectangle and 5 white rectangles, together is 100 different rectangles covered.

Line7. After learners finished pasting their hundred grids, the teacher told them to also paste their 200 equal rectangle grid using the same percentage for the hundred grids. Some learners were calculating their work using calculators and paste the correct small rectangles on the 200 grid. Learners were calculating as follows: $40\% = \frac{40}{100} \times 200 = 80$ yellow small rectangles; $30\% = \frac{30}{100} \times 200 = 60$ red small rectangles; $25\% = \frac{25}{100} \times 200 = 50$ blue small rectangles and $5\% = \frac{5}{100} \times 200 = 10$ small rectangles. Few learners, such as learner D, E and K didn't paste the correct number of the small rectangles of different colour. They worked out their answers as follows: 40% of yellow small rectangles as $\frac{40}{200} \times 100$ and got 20 yellow rectangles, $\frac{30}{200} \times 100$ and got 15 red rectangles, $\frac{25}{200} \times 100 = 12.5$ blue rectangles, $\frac{5}{200} \times 100$ and got 2.5 white rectangles.

Line8. These learners didn't convert percentage to fractions correctly. Instead of them writing the fractions out of hundred because, percentage is out of hundred, they rather wrote their fractions out of two hundred. I wanted learners to notice by themselves that their fractions were wrongly written than me telling them about it. I asked learner D to explain how she got her first answer, this is how she answered: **Learner D:** I converted 40% to fractions. **Teacher:** I can see that you have converted from percentage to fractions, let's do it together. I wrote $40\% = \frac{40}{100}$ on the

chalkboard and ask learner D, E and K to complete. **Learner D:** writes $40\% = \frac{40}{100}$. **Teacher:** *ok, can you see were you gone wrong?* Learner D admits that she wrote forty out of two hundred instead of forty out of hundred. I then encouraged the three learners to write percentage out of hundred, all the times. Learner D, E and K later pasted the correct small rectangles of different colour on the 200 grid.

Line9. Teacher instructs learners to finish converting a grid of 25 rectangles with yellow, blue white and red rectangle. They should use 40% of yellow colour, 20% of blue colour, 30% of white colour and 10% of red colour. When learners pasted the 25 grid, all pasted correctly. This time I changed the percentages of the small rectangles of different colour, it was 40% of yellow colour, 20% of blue colour, 30% of white colour and 10% of red colour. This is how learners worked out their answers; $40 \div 100 \times 25 = 10$ yellow rectangles, $20 \div 100 \times 25 = 5$ blue rectangles, $30 \div 100 \times 25 = 7.5$ white rectangles and $10 \div 100 \times 25 = 2.5$ red rectangles. When learners pasted the white and red rectangles, some of them were cutting one of the rectangles into half and other learners were folding one of the rectangles into the middle. Example the answer of white small rectangles was 7.5 they took 8 of them, pasted 7 and the remaining rectangle, some learners cut it in the middle and pasted the half, and other learners folded the remaining rectangle in the middle and pasted the folded part. I was impressed with learners' initiative of cutting and folding the small rectangles in half.

Line10. After using multiple representations to explain how to work out the percentage of a quantity given, I then gave learners an algorithm they should use if necessary on this concept. The name of the Algorithm is when figuring a part (Van Galen, 2008), and that is the same thing as working out percentage of a quantity. I introduce this algorithm almost at the end of lesson 4, with the following example; 20% of N\$300. **Teacher:** From all the correct explanations you gave me on this concept of working out percentage of a quantity, this is what you did; first, figuring out how much 1% of the amount, in our example it will be 1% of N\$300 ($300 \div 100$). Then that amount is multiplied by the percentage and the answer is N\$60 (3×20).

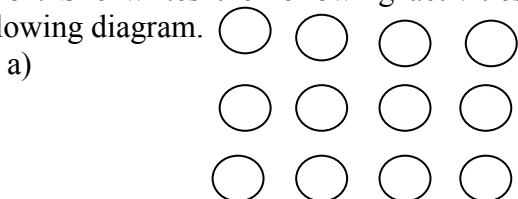
Line11. The lesson was then concluded by summarizing all the different ways of working out the percentage of a quantity. The teacher lastly collects all the three grids from each learner.

APPENDIX D

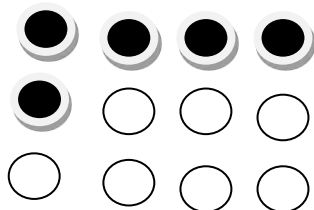
LESSON 5'S TRANSCRIPT

Line1. After the teacher wrote the learning objective and the date on the chalkboard, she then greets learners. She then said the lesson will be a continuation of the previous lesson, and that is to work out percentage of a quantity.

Line2. She writes the following activities on the chalkboard. Shade in 25% of each of the following diagram.



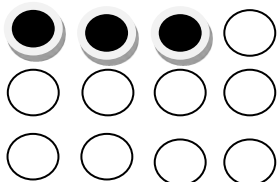
Teacher: Alright, if that is the question and you are asked to shade 25% of each of the following diagram. “What will be the answer? “Ask the learner to answer the question on the board. **Learner E:** Shades 25% of the diagram like this.



Teacher: Asks the learner E to explain on how he came to the conclusion of his answer.

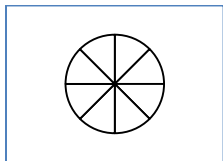
Learner E: Writes his explanation on the chalk board. ($\frac{25}{12} \times \frac{100}{1} = 208.33\%$). **Teacher:** yes, is this answer correct or not? **Learner E:** it is wrong, because I got a larger answer. **Teacher:** why do you think your answer is wrong? Explain your answer. **Learner E:** I don't know how. **Teacher:** but this is what we learned yesterday, ok, which answer is right, the five you shaded or 208.33%? **Learner E:** five. **Teacher:** how? **Learner E:** didn't answer. **Teacher:** ok, learner D come and explain, while you learner E should listen. **Learner E:** Returns to his seat. **Teacher:** Asks learner E back to the board. **Teacher:** Here you have 12 circles and you shaded 5, my question is how did you get 5? **Learner E:** I am not quite sure. **Teacher:** Who can help him? Yes, learner D come! and explain.

Learner D: Shade in as follows:

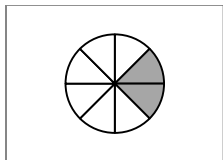


She then explains, three out of twelve is twenty five percent and twenty five out of hundred multiplied by twelve is three. She writes her work as follows: $\frac{25}{100} \times 12 = 3$, the percentage given is twenty five percent, so twenty five percent will be twenty five out of hundred, that's where I got that fractions, And therefore $\frac{3}{12} \times 100 = 25\%$. **Teacher:** very good. Ok. Percentage is always out of 100, so if you see percentage you have to know that it's out of 100. **Learner E,** you used a wrong fraction you could have used this fraction, (teacher pointing at $\frac{25}{100}$).

Line3. Teacher: Ok let's try the second diagram. Shade in 25% of this diagram.

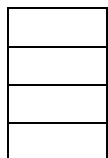


Learner C: Writes $\frac{25}{100} \times 8 = 2$, and say „percent is always out of hundred, I took the percentage and multiplied it by the equal number of parts to know the parts which I will shade, and that’s why I shaded two parts here“ (pointing at her answer).

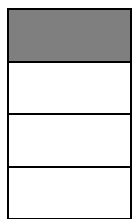


Teacher: Very good learner C.

Line4. Teacher: Ok, now learner E you should come and work out the next diagram. Color in 25% of this diagram.



Learner E: writes $\frac{25}{100} \times 100 = 25$. **Teacher:** ok, I can see that you converted 25% to fractions correctly, very well, but where did you get the hundred you used? Learner E erased the hundred and wrote four, (his second answer looked as follows); $\frac{25}{100} \times 4 = 16$. **Teacher:** how did you get sixteen? (Learner E pressed 25×100 on the calculator). **Teacher:** how do we read this $\frac{25}{100}$? **Learner E:** twenty five out of hundred. **Teacher:** what does that mean, if you want to work it out in the calculator? **Learner E:** twenty divide hundred. **Teacher:** ok, that’s correct, now work out your work. **Learner E** pressed twenty five out of hundred as twenty five divide by hundred, then he multiplied his work by four and got one as his answer. He then shade in one of the diagram given, this is how his answer looked: Teacher: very good. Good try.



Line5. Teacher: now the last diagram, what is 25% of the following diagram?



Learner K: calculate it as; $\frac{25}{100} \times 6 = 1.5$; she then shades one and a half parts of six parts. **Teacher:** explain your answer. Learner K: twenty five over 100 times 6 is equal to 1.5.

Line6. Teacher writes other representations of the same concept. e.g. (a) 5.5% of N\$6.2.

Learner F: answer as follows: $= \frac{5.5}{100} \times 6.22 = 0.34$. **Teacher:** very good learner F, next question is (b) 80% of N\$62.50. What will be the answer? **Learner A:** writes $\frac{80}{100} \times 62.50 = \text{N}\50.00 .

Teacher: very good learner A, next question is (c) 20% of 60kg. What will be the answer?
Learner G: writes $\frac{20}{100} \times 60 = 12\text{kg}$. **Teacher:** very good learner G, yes the next question is (d) $12\frac{1}{2}\%$ of N\$88.30, what will be the answer? **Learner B:** writes $\frac{12.5}{100} \times 88.50 = 11.062 = \text{N}\11.10 .
Teacher: very good, yes the number should be rounded off to the nearest decimal place, because money only has two numbers after the comma, even if your calculator gives an answer that has many number after the comma, you just have to round it off. Alright, the last question is 40% of 360. What will be the answer? **Learner I:** writes $\frac{40}{100} \times 360 = 144$. **Teacher:** very good, now $\frac{144}{360}$ is equal to what percentage. **Learner I:** 40%. **Teacher:** great, ok now we will do the other representations of working out percentage of a quantity.

Line7. Teacher: These examples are word problem questions, I will quickly write one question at a time on the chalkboard. (Teacher writes on the board); 1. Fernanda stored 80% in her mathematics test. The test was out of 150 marks. How many marks did Fernanda obtain? Alright that's the question. Now who will read for us? Learners were raising their hands. **Learner A** read the question. **Teacher:** yes, how many marks did she obtain, yes learner G? **Learner G:** 41 marks. **Teacher:** Ok! Work it out for us, how did you get 41 marks? **Learner G:** writes $\frac{82}{100} \times 50 = 41$ marks. **Teacher:** yes, so if she gets 82%, I just gave you the percentage (%) and I also gave you the total marks of what Fernanda got then now that's enough for you to know how many marks was it. Because you just said 82% out of 100 are 41 marks. Good! (Teacher writes on the chalk board the next question).

Line8. The cash price of a bicycle is N\$700; a discount of 25% is given to anyone who buys the bicycle cash. Calculate the discount price? **Teacher:** Who will read for us?
Learner M: the cash price of a bicycle is N\$700 a discount of 25% is given to anyone who buys the bicycle cash calculate the discount price? **Teacher:** mmm...learner M, were there is a full stop in the sentence, then you need to pause. Before we work out this question, does everybody know what is a discount is? What is a discount? Yes learner B, what is a discount? **Learner B:** discount is like a bank of saving, money so that you can buy. **Teacher:** yes, so discount is money subtracted from the cash price because you bought the item cash. Most of the time, discount is given only if you buy a larger item. The good example is this one about the bicycle. What will be the answer? Yes learner M? **Learner M:** writes $\frac{25}{100} \times 700 = \text{N}\175.00 . **Teacher:** Ok, what did you do? **Learner M:** Twenty five percent is out of hundred so twenty five divide by hundred times seven hundred is equals to 175. **Teacher:** very good.

Line9. **Teacher:** the third question is as follows, (teacher writes the question on the chalkboard). "A school has 950 pupil, 48% of them are boys, how many boys are there at the school?"
Learner E: writes $\frac{950}{100} \times 48 = 456$ boys. **Teacher:** very good. The fourth question is, if the percentages of the boys are 48%, then what is the percentage of the girls? **Learner K:** 29%. **Teacher:** no, try again. I need the percentage of the girls. **Learner H:** 52%. **Teacher:** how did you get that, come and work it out for us. **Learner H:** writes 4 then he erase the four, he then writes $100\% - 48\% = 52\%$. He explains that; I took the percentage of the boys minus the 100% then I got 52%. **Teacher:** ok, very good. **Teacher:** did you get the number of girls? **Learners:** yes. **Teacher:** how did you do it? **Learner K:** 950 minus 456. **Teacher:** very good. Class, apart from what learner K did, is there anyone with another method of calculating this question? **Learner D:** 450 divide by 950 multiply by 100 equals to 52%. **Teacher:** very good.

Line10. Ok, let's finish with the last question. (Teacher writes the question on the chalkboard). Chilambo receives a commission of 1% for selling second hand cars, the car cost N\$80000, calculate Chilambo's commission, yes what will the answer be? Learner A: read the question. Learner C: writes $\frac{1}{100} \times 80000 = \text{N}\800 . Teacher: so what did you do? Learner C: percentage is always out of 100 multiply this fraction by 80000. Teacher: Good, that is correct. Ok. Now I will give you activities to write and through that I will see, who understand well how to work out percentage of a quantity.

APPENDIX D

LESSON 6'S TRANSCRIPT

Line1. The teacher greets the learners, write the learning objectives and date on the chalkboard and ask learners what they learned the previous day.

Line2. Learner M: We calculate the percentage of a quantity. **Teacher:** yes, we were trying to calculate percentage of a quantity. Now today we are going to express one quantity as a percentage of one, for instance a question can be as followed:

Line3. Teacher: (writing on the chalkboard) example 1. a) What percentage is 20 marks out of 25 marks? The question can be as followed. Yes learners K, you can read it for us. Learner K: (stood up and read the question). **Teacher:** Yes, so what do you think the answer will be?

Learner I: calculating and rising up, stood up going to the chalkboard and wrote down: $\frac{20}{25} \times 100 = 80\%$. **Teacher:** Ok, so what did you do? **Learner I:** I took twenty out of twenty five and multiplied it by hundred, and got 80%. **Teacher:** very good, alright, why are you multiplying the whole by that fractions (teacher pointing at $\frac{20}{25}$? **Learner I:** I don't know why, but I just know that questions about percentage is either you multiply by hundred or divide by hundred. This question automatically is multiply by hundred because the twenty is not percentage, if it could be percentage then it will be twenty divide by hundred. **Teacher:** ok, mm, ok, very good. Yes, learner E, how did you do it? **Learner E:** was quiet. **Teacher:** if the question is changed to; 20% of 25 marks, that's when one, should write twenty out of hundred multiply by twenty five. You learner E need to remember that out of is the same as divide by if you are pressing the work in your calculator. OK.

Line4. **Teacher:** Okay very well, now class, what learner I just did is, he got the answer correct. I went through some of your calculator, some of you still did not get correct answer, and few of you got all 80%, and yesterday example the percentage was given to us in the story. Isn't? In this example they want us to work out the percentage, so what percentage is 20 marks out of 25 marks, 20 is not in percentage, but it is as marks, 25 is also marks. So it means it would be a fractions, 20 out of 25 is a fractions, so this fractions is the one we want to work out to find the percentage and what did we do, what did we learn so far. If we want to calculate percentage we are taking fractions to percentage, we need to do what we need to multiply, so many of the confusion, when I was marking your previous tests, your confusion is now you will be telling me 20 out of 100 times 25. If that supposed to be 20% of 25 marks, that's when you tell me 20 out of 100, because of that % but this was 20 marks out of 25 marks. So it will be a fraction which looks like that. So learner I is right $\frac{20}{25} \times 100 = 80\%$, so what did we do, we express the percentage of 20 marks out of 25 marks. Alright, let me give you another question.

Line5. **Teacher:** (writing the question on the chalkboard) b) There are 30 learners in a class. 5 are boys. What percentage of the learners are boys? So if the question is of that type what will be the answer? Learner D you can read it for us. **Learner D:** (stood up and read the question on the chalkboard). **Teacher:** Yes, what is the main idea on the question? **Learner D:** the main information is the total number of learners which is thirty and the number of boys in the class. **Teacher:** very good, what percentage are boys? **Learner A:** writes $\frac{5}{30} \times 100 = 16.666 = 17\text{boys}$.

When learner A was working out her answer on the chalkboard, I observed that all learners worked as learner A did, apart from learner E who work it out as follows: $\frac{30}{5} \times 100 = 600\%$, I then asked learner A for her explanations as follows: **Teacher:** how did you get that? **Learner A:** five divided by thirty, times hundred it gives me sixteen point six, six, six then I rounded off and my answer is 17boys. **Teacher:** is it seventeen boys? **Learner A:** seventeen percentages. **Teacher:** why did you multiply? **Learner A:** I want to work out the answer. **Teacher:** ok. Let me give you an example, for me to explain why you are multiplying your fractions by one hundred. What percentage are twenty marks out of twenty five marks? That is to take 1% of the total marks which is 0.25. In order to know how much 20 of 25 in percent, you must know how often 0.25 goes into 20. And that is part divide by 1% of the whole and the answer will be 80%.

Line6. **Teacher:** Now I went through some of your work on your calculator you did 30 out of 5 times 100 and you did got 600, this is not correct, the correct is this one 17 percent. why is it 5 out of 30 and not 30 out of 5, normally if I ask you to work out percentage of the boys, work out the percentage of the boys of the total learners in the class, so you need to tell like you are taking the boys out of the group. Boys and girls together, so you are taking the boys out, remember when we were doing fractions, the first lesson we were doing well were writing numbers as percentage. So what we will be doing with our things, coins and so false we were taking out of the group. So now if it is 5 means it should be 5 out of 30. Know now to write your fractions, it is very important. Ok that's how to work out if you are given question like word problem and so false, the second examples sometimes they won't give you solve word problems.

Line7. Teacher: (writing the question on the chalkboard) Example 2: convert to percentage

- a) $\frac{21}{25}$ b) $2\frac{1}{2}$ c) $\frac{5}{8}$ d) 0.38 e) 1.05

The question is for you to convert to percentage. What do they want us to do? Yes, convert to percentage, and the rest of us let's try to work it out, convert to percentage. **Learner G:** (went and write down the answer on the chalkboard). **Learner G:** writes $\frac{21}{25} \times 100 = 84\%$. **Teacher:** what did you do? **Learner G:** I took twenty one divided by twenty five and multiplied by hundred then I got 84%. **Teacher:** why are you dividing by twenty five? **Learner G:** I want to find out one percent of my total amount. **Teacher:** that's great. **Teacher:** Yes, you multiply by 100, all what you do is, this is a fractions by its own, it can be 21 marks out of 25 marks or maybe 21 Namibian dollar out of 25 Namibian dollar, so as long as its fractions, what we want you to bring it into percentage is just to multiply by 100 and that is it, so can you see that this time is not word problem the first example they were in word problems and were we doing it the same we were taking the fractions multiply it by 100. Now this time we are still doing the same, a fraction multiply by 100. Ok, so what will be the answer of the second one?

Line8. **Learners:** (rising up and were given chacks). **Learner F:** (stood up while calculating and went to the chalkboard) $\frac{5}{8} \times 100 = 62.2\%$. **Learner B:** (another learner stood and went to write the next answer). $2\frac{1}{2} = \frac{2}{2} \times 100 = 250 = 2.5\%$. **Teacher:** yes, the next one. **Learner H:** (writing down answer) $\frac{1.05}{100} \times 100 = 1.05\%$. **Learner I:** $\frac{0.38}{100} \times 100 = 0.38\%$. **Teacher:** Ok, I will start with the second one, letter B, whoever answered it come and explain. **Learner B:** writes $2\frac{1}{2} = \frac{5}{2} \times 100 = 250 = 2.5\%$. **Teacher:** yes, explain your work. **Learner B:** I took two and a half than convert it to improper fractions, than I multiply by hundred than I got two hundred and fifty. **Teacher:** where did you get two point five percent? **Learner B:** I divide my answer by hundred. **Teacher:** why? **Learner B:** because percentage is out of hundred. **Teacher:** Yes, percentage is out of hundred, but where is the percentage? **Learner B:** ok, I, wanted, wanted, to write my final

answer as percentage. **Teacher:** you took $2\frac{1}{2}$ to percentage, that's why you multiplied by hundred, and get 250. This supposed to be your final answer. **Learner B:** how can I get a percentage answer more than hundred, if percentage stop only at hundred. **Teacher:** no, learner B, percentage doesn't stop at hundred. Percentage can be any number out of hundred even 250. **Learner B:** ok. **Teacher:** Did you use a calculator? **Learner B:** Yes madam.

Line9. Teacher: try it again, just this method. Just this way (showing the learner which fractions to use) I want you to put it in your calculator. Okay. **Learner B:** I got 250. **Teacher:** yes, that's what you get on your calculator. **Learner B:** I divided it by 100. **Teacher:** why did you divide by 100? **Learner B:** because the percentage is out of 100. **Teacher:** what is this one is the original of this one. **Learner B:** Yes. **Teacher:** you are taking $2\frac{1}{2}$ to percentage so you are multiplying by 100. So you are taking $\frac{5}{2}$ multiply by 100 to give you 250. This 250 supposed to be your answer like I said, so $2\frac{1}{2}$ multiply by 100 are equal to 250 that supposed to be your last answer. You should not again divide. 2.5% supposed not to be your last answer, but 250. Okay, Good. Who has the same answer but different method for letter B, who got 250 but you did it another way. Are you raising your hand? **Learner J:** No. **Teacher:** who have the same answer but did it in another way (no one from the learner is rising his/her hands). **Teacher:** Do you know why learner B converts from $2\frac{1}{2}$ to $\frac{5}{2}$; do we know how he did it? **Learners:** No. **Teacher:** right, he changed mixed number to improper fractions. He either knows it mentally or he worked it out, actually, so let's find out from him. **Learner B** what did you do? I multiply two by two plus one is 5 then five will be out of two, that's what i did. **Teacher:** Okay, that's great. Another way is to convert $2\frac{1}{2}$ as decimal and multiply the decimal by one hundred. Ok, what will be the answer to letter C?

Line10. Learner F: (went on the chalkboard to explain her answer) I divide 5 by 8 times one hundred is equal to 62.5%. **Teacher:** Very good, so that fraction times one hundred is 62.5%. Yes the next one? Who worked it out? **Learner I:** (went to explain also his answer on the chalkboard) I took 0.38 divide by one hundred, sorry times one hundred. **Teacher:** can you explain to us why are you divide there ($0.38 \div 100$)? **Learner I:** I made a mistake. **Teacher:** Where now? **Learner I:** Here where I put a divide sign ($0.38 \div 100$) it could be a times. **Teacher:** could be times? **Learner I:** Yes. **Teacher:** Okay, so you suppose not to put a divide, you were supposed to put times. **Learner I:** Yes. **Teacher:** Okay, so what is the answer than? Work it out and tell me the answer. **Learner I:** (working out his method on a calculator) Zero point three eight (0.38). **Teacher:** zero point three eight? **Learner I:** yes. **Teacher:** Okay, now tell what are you doing here from 0.38 to $\frac{0.38}{100}$ this fraction, what did you just do? **Learner I:** I divide by 100. **Teacher:** why are you dividing zero point three eight by hundred, why? **Learner I:** because is a decimal. **Teacher:** it is? **Learner I:** it is a decimal. **Teacher:** if it is a decimal you write it over one hundred? **Learner I:** yes.

Line11. Teacher: is that true, Okay, who will help? **Learner H:** (stood up to help) writes $\frac{38}{100} \times 100 = 38\%$, and explain, I took 0.38 in decimal and change it to fractions. **Teacher:** Very good, I now see, why learner I put 0.38 out of hundred. You wanted to bring this decimal (0.38) to fractions, ok, yes learner H, why are you saying $\frac{38}{100}$. **Learner H:** because there are two numbers before the comma, your number not say 0.38, so as fractions this is how it's supposed to look ($\frac{38}{100}$). **Teacher:** Lets' continue, after you change to fractions? **Learner H:** You multiply the

fractions by 100. **Teacher:** Okay, you get the answer as 38 percent good. Now learner I, try this one quickly (0.6), I want you to write this number as percentage. **Learner I:** (working out the number on the calculator and wrote it down) $\frac{62}{100} \times 100 = 62\%$. **Teacher:** Good, that is the correct way to do it, if you want to take it from decimal to fractions that's how you do it. Who used another way, but got 38 percent, yes learner L, how did you get 38 percent, come and do it on the chalkboard. **Learner L:** (went to write another method on the chalkboard) $0.38 \times 100 = 38\%$. **Teacher:** so what did you do? **Learner L:** zero point thirty eight times one hundred the answer is 38 percent. **Teacher:** Yes so you just multiply by 100, (0.38×100) 38 out of 100, it is the same number as zero point three eight, so doing like this $(\frac{38}{100} \times 100)$ or doing 0.38×100 , you are taking the same number and multiply it by one hundred so is either you are taking $\frac{38}{100}$ multiply by one hundred, so taking this number 0.38 to $\frac{38}{100}$, you are not changing anything. It just depend on which one, are you comfortable, so you use the method you are comfortable with, all what I am saying is that this is the same thing as that one (teacher pointing at the work of learners). Ok the last answer, who did it?

Line12. Learner J: (went on the board to explain her method) I took 1.05 over one hundred and multiply by hundred, the answer I got is one point five percent. **Teacher:** one hundred and five percent, that's very good. How did you come to this fraction? **Learner J:** one point zero five, I divided it by one hundred, because it has two numbers after the comma. **Teacher:** very good, excellent, alright now class is either this way (1.05%) or mm.., is this a point (1.05%) on your answer? **Learner J:** Yes madam. **Teacher:** press the whole story in your calculator and see what you are getting. **Learner J:** pressing her work on the calculator as follows; $1.05 \div 100 \times 100 = 105$. **Teacher:** your answer is 1. where is your mistake? **Learner J:** at the comma. **Teacher:** yes, the comma suppose not to be there, alright, there is no different between someone who will press straight away 1.05×100 and someone who will convert the 1.05 to fractions first, then multiply by hundred. Ok, we are done with these examples. Now the third representations on working out percentage of another one.

Line13. Teacher: In each of the diagrams on the chalkboard, write down the percentage of the shaded part out of the total, what will be the answer? **Learner I:** Should I write as percentage? **Teacher:** yes, dear the question is what is the percentage of the shaded faces in this diagram?



Learner I: (writing on the chalkboard) $\frac{2}{6} \times 100 = 33.33$. And explain, I just wrote thirty three point three three percent. **Teacher:** Very good, now where did you get this fraction? (Teacher pointing at $\frac{2}{6}$). **Learner I:** From the diagram. If two of the diagram is shaded then two is out of the total diagram, that's where I got my fractions. **Teacher:** what did you do? **Learner I:** I took two of the faces which were shaded out of the total faces, and that is six faces and multiply my fractions by hundred and got thirty three, three.. The reason of my work is first to work out one percent of my total amount, after getting that answer I then wanted to find out how many times does my one percent goes in two of the shaded parts, to know how much 2 of 6 in percent is. **Teacher:** very good, that's great. **Teacher:** that is very good. Ok the next diagram.

Line14. Teacher: What if you were given this diagram, how will you do it? In this diagram what is the percentage of the letter Zs is color yellow?

Z Z Z Z

Z Z Z Z

Z Z Z Z

Learners: (working out using calculator and raising their hands) **Teacher:** (pointing at learner B to say the answer). **Learner F:** (writing on the chalkboard) $\frac{8}{12} \times 100 = 66.66\%$. **Teacher:** What did you do, yes? **Learner F:** I took 8 divided by 12 times 100 I got 66.66 percent. **Teacher:** Can you give reason to your work? **Learner F:** a question like this, one should first find the one percent of the total then divide the parts which are shaded by one percent because you want to know how many eight are there in twelve as percentage. **Teacher:** alright, very good. The yellow were eight (teacher pointing at Zs with yellow color) out of 12 Zs in total. If one wants to get this fraction as percentage, then you multiply by 100. Ok. Now class the last examples which can be asked, for you to express one quantity as percentage of another one is follows;

Line15. Teacher: Express N\$2 dollars as percentage of N\$10. What will be your answer? (Learners raising their hands). Who did not say anything today, should be the one to answer this question, yes I got him. Yes Learner G. **Learner G:** (going to the board). **Teacher:** (going to other learners and check the answers). **Learner G:** writes $\frac{N\$2}{N\$10} \times 100 = 20$. **Teacher:** (Asked) what did you do? **Learner N:** (explain) I I took two dollars divide by 10 dollars times hundred. **Teacher:** and your answer is? **Learner G:** 20. **Teacher:** 20 what? **Learner G:** 20 percent (while writing the unit % next to 20). **Teacher:** 20 percent, very good, so 20 percent of 10 is 2 dollar (repeat the same) while writing on the chalk board) 20% of N\$10 = ?.

Line16. Teacher: How to work on this one? Who will come and work it out? **Learner C:** (writing the following) $\frac{20}{100} \times N\$10 = N\2 . **Teacher:** Okay, what did you do? **Learner C:** I take 20 divided by 100 times N\$20 then I got 2. **Teacher:** Why divide by 100? **Learner C:** Because if you were given a (pointing at) % you divide by 100. **Teacher:** Very good, because 20% (while pointing at this unit (%)) means? **Learner C:** per hundred. **Teacher:** Very good, Okay, you can sit down.

Line17. The teacher sum-up the lesson reminding the learners about the main ideas of this lesson and then she hand out activity6 to the learners to do, regarding the concept learned.).

APPENDIX D

LESSON 7'S TRANSCRIPT

Line1: The teacher writes the learning objectives and the dates on the chalkboard and then later greets the children. She then announces that the lesson will be on how to increase or decrease a quantity by a given percentage.

Line2: the teacher let learners be in groups of three each. Each group was given work to do. Group one was given grid of 24 rectangles to reduce it by 25%. Learner A, B, and F were in group one. **Learner B:** reduce means we should subtract. **Learner A:** yes, we should subtract 25% from 24 rectangles. **Learner F:** we need to work out twenty five percent of twenty four rectangles. **Learner B:** it's true (he then press $25 \div 100 \times 24 = 6$ and say) the answer is six. **Learner F:** no, the answer is not only six, six we work out percentage of a quantity only, we need to continue. **Learner A:** yes twenty five of twenty four rectangles are six rectangles. **Learner B:** ok, you mean we need to minus six. **Learner F:** yes, we minus six from twenty four rectangles and that's eighteen. **Learner F:** yes eighteen is our answer. **Learner A:** what about this grid which the teacher gave us. **Learner B:** we need to reduce by the answer we got. **Learner F:** how do we do it? **Learner B:** we need to subtract six rectangles of this grid. **Learner F:** yes, you are right. **Learner A:** (give a scissor to learner B). **Learner B:** cut six rectangles from the grid and count the new grid. **Learner F:** what did you get? **Learner B:** eighteen rectangles. **Learner F:** that's nice it is visible that our answer eighteen is clearly shown on the paper (referring to the grid). **Teacher:** very good.

Line3. Teacher: Group two had learner E and G only. These two learners were given a grid of 50 rectangles so that they reduced it by twenty five percent. **Learner G:** count the rectangles if they are fifty. **Learner E:** (counting the rectangles on their grid, then later say) yes, it is fifty. **Learner E:** now is what? **Learner G:** we should calculate the percentage of our grid. **Learner E:** (press $25 \div 100 \times 50 = 12.5$). **Learner G:** what are you doing? **Learner E:** I calculate twenty five percent of fifty rectangles and I got twelve point five. **Learner G:** now this answer we will minus it: **learner E:** (work out the answer on the calculator as follows $50 - 12.5 = 37.5$). **Learner G:** let me see the answer. **Learner E:** it is thirty seven point five. What should we do now? **Learner G:** we should minus, we should minus, and we should minus thirty seven point five from our grid. **Learner E:** how, but I minus already, unless this thirty seven point five should be our final answer. **Learner G:** yes, ok. Now, we cut our grid of fifty rectangles. **Learner E:** (cut twelve rectangles and ask) what about point five. **Learner G:** (folded the thirteenth rectangles and say) it should be like this, showing half of the thirteenth rectangle. **Learner G:** you know what, I think machines in shops that reduce photo sizes, use this method to calculate it.

Line4. Teacher gives learner C and D a grid of 30 rectangles and instructs them to increase the grid by 25%. **Learner D:** we said 25 percent divide by one hundred times 30 then it gives us 7.5 rectangles. So we cut extra seven rectangles and a half then add it to our grid of 30 rectangle and the new rectangles become 37.5 rectangles. **Teacher:** very good.

Line5. Learner H and learner I was given a grid of 60 rectangles and instruct them to increase the grid by 25%. **Learner I:** we took twenty five percent and work out the percentage of 60 rectangles; I got the answer of 15 rectangles. **Learner H:** our answer therefore is sixty plus fifteen is equal to 75 rectangles. We take fifteen here at this extra grid and add it to our 60

rectangles, now all together is seventy five. **Teacher:** very great. Ok we will now work out similar questions but this time, we will use other representations. Not using graphs (grids) again.

Line6. The teacher then asks learners to increase N\$30 by 40%. **Learner F:** writes $\frac{40}{100} \times \text{N\$}30.00 = \text{N\$}12 + \text{N\$}30.00 = \text{N\$}42.00$. he explain that: I took forty out of hundred and multiply my answer by thirty and get twelve, then twelve plus thirty is forty two Namibian dollars. **Teacher:** very good. Where do we use percentage increase or decrease? **Learner D:** " percentage decrease is when you are given discount, the subtract money from the cash price. **Teacher:** very good.

Line7. The teacher explains further that, "sometimes you get an advertisement written like this one on the chalkboard. You will not pay the shoes with N\$200.00, but how much will you pay for the shoes? **Learner O:** "N\$150". She writes $\text{N\$}200 - \text{N\$}25 = \text{N\$}175$, she then erase and writes another calculation $\text{N\$}200 - \text{N\$}50 = \text{N\$}150.00$ she explain that $\text{N\$}200 - \text{N\$}50$ is equal to N\$150, (teacher asked learner O how she got N\$50) **learner O:** I took twenty five percent and multiply it by N\$200 is N\$50 dollars. Therefore two hundred minus fifty will be one hundred and fifty dollars. **Teacher:** very good.

Line8. Teacher: in practical situation when do we increase a quantity by a given percentage? **Learner B:** when one what to make a profit. **Teacher:** yes, learner B is right, e.g. a person with a mini-market buys a pack of cool drink at a whole sale for N\$130, he decide to make a profit of 40%. So how much will he get in one pack of cool drink? **Learner D:** writes $\frac{40}{100} \times 130 = \text{N\$}52$, $\text{N\$}130 + \text{N\$}52 = \text{N\$}182, 00$ after writing she explain her work. **Learner D:** The selling price of one pack of cool drink will be N\$182, 00 and the profit will be N\$52.00. **Teacher:** very good, learner D.

Line9. Teacher explains to the learners another way to work out the answer of the question above. We have another method of calculating percentage decrease and increase. If you want you can use it to work out the answers. It is as follows; we have three main prices that you need to remember. The first one is the original price and the second one is the profit price or markup price and the last price is the total amount includes or excludes markup or markdown. The original price will always be out of hundred percent, if it is written as percentage, then the mark-up you work out the percentage of a quantity. Then the total you either subtract it or add it to the markup or markdown, it depends on the question. E.g. let's work out the example of a pack of cool drinks which is 130 and the profit of 40%. You then add the percentage

Original price 100% or N\$130 Mark-up 40% or N\$52 Total 140% or N\$182.00.

Line10. **Teacher:** But I advise you to use methods of their choice. **Teacher** gave learners, activity seven for them to do and work out the correction together with their teacher.

APPENDIX D

LESSON 8'S TRANSCRIPT

Line1. The teacher writes the learning objectives and dates on the chalkboard. She then announces that the lesson will be on VAT inclusive and VAT exclusive.

Line2. Teacher explain what VAT is, teacher give cash slips to learners and ask them to find the percentage of VAT on each pay slip. (Learners were able to read out the correct percentage of VAT on the cash slip.

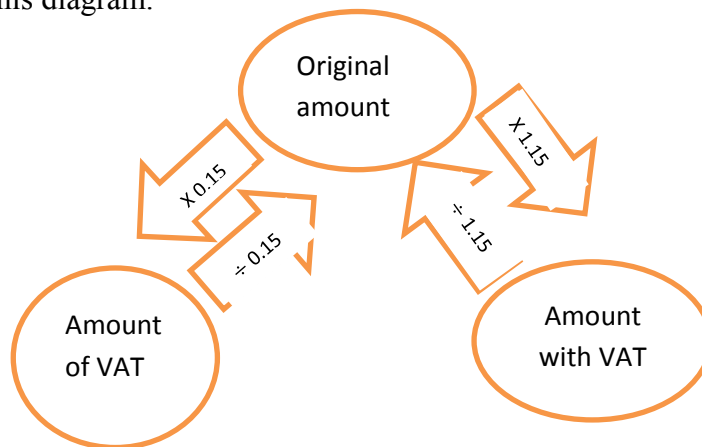
Line3. Teacher paste a poster on the chalkboard with activities on VAT inclusive and VAT exclusive. Examples; learners were calculating VAT inclusive and VAT exclusive of the original price. For example learners were asked to fill in the table below of VAT inclusive and exclusive of the original price after working in their groups.

4.2 Calculating quantity with VAT inclusive and VAT exclusive

<i>Items</i>	<i>Original price</i>	<i>VAT of 15%</i>	<i>Price with VAT of 15%</i>
<i>Can beans</i>	<i>N\$4.00</i>	<i>N\$</i>	<i>N\$</i>
<i>Oros (Caribbean)</i>	<i>N\$8.00</i>	<i>N\$</i>	<i>N\$</i>
<i>Chicken (braai pack)</i>	<i>N\$</i>	<i>N\$</i>	<i>N\$30.00</i>
<i>Soup</i>	<i>N\$</i>	<i>N\$</i>	<i>N\$5.00</i>

Teacher: what will be the answer? Yes group two? Learner B: the VAT amount is N\$0.60 and the oros is N\$1.20 and the chicken is N\$4.50 and for the soup is N\$0.75. Teacher ok, are you testing your answers. All the groups were using the same method to calculate the prices for all the four items, being it prices with VAT inclusive or prices with VAT exclusive. For the first two items, learners were expected to calculate the VAT of fifteen percent and then to calculate the price of the items with VAT included. For example the original price for the first item was N\$4.00 VAT excluded, all the learners first worked out 15% of the original price ($\frac{15}{100} \times 4 = N\0.60) and the answer was for VAT which is N\$0.60, then they added the value of VAT to the original price to get the price of beans can that includes VAT and they got N\$4.60. The same with the second item, learners worked out the VAT first and the answer they added it to the original price.

Line4. Teacher: I want you to test your answer and discuss in your groups how to work out the prices of all the four items. I will first explain how to work out VAT inclusive and VAT exclusive using this diagram.



Teacher: they will always give you one of the prices from the three and the percentage of the VAT. Therefore you need to use those arrows to know where you are going. If you are given the original price and the VAT percentage then you multiply your original amount by the decimal percentage. If you are coming from amount with VAT then you divide the amount by the decimal percentage. In your groups, I want you to talk about how to use this sketch on the chalkboard.

Line5. Learner A, G and H were in group one. **Learner H:** this answer that we got N\$25.50 as the original price, is not giving us the N\$30.00, the price with VAT included. Unless we try the sketch, the teacher had shown us. **Learner A:** we should take N\$30.00 then divide by $\frac{115}{100}$. What is the answer? **Learner G:** it is N\$26.09. **Learner A:** now let us try to subtract our answer from N\$30.00 what are we getting? **Learner H:** it is N\$3.91. **Learner G:** our answers this time are correct because all the three prices are going hand in hand.

Line6. Teacher: yes, did you finish working out your answers, what is your answer? **Learner B:** the answers which are wrong are for the chicken and the soup. I have discussed with learner D and find out that the first answers when we tested to add the VAT amount on the original amount does not give the amount with VAT. But after we use the sketch we got a different answer. **Teacher:** what is the price of the chicken? **Learner D:** it is N\$26.09, because 30 divide by 1.15 according to the sketch is N\$26.09. I rounded off the answer; there was a lot of number after the comma. **Teacher:** well done. So you can use the same sketch or your method you like to work out VAT inclusive or VAT exclusive.

Line7. Teacher: if you decide to use this sketch (pointing the sketch on the chalkboard), always remember the VAT percentages for you to convert it to decimal. Not all the time the VAT is fifteen percent. So concentrate, sometimes the VAT is sixteen and half percent. If the VAT is that what will be the decimal number in this arrow? Learner I: one point six five. Teacher: very good, if you are coming from the amount include VAT, what should you do? Learner F: you divide the number by one point six five. Teacher: very good. If you are coming from the VAT amount to the original price what to do? Learner G: you divide by zero point one six five. Teacher: excellent. Now do this class activity.

Line8. Work out the prices needed to complete the table below; either the original price, the price with VAT of 15% or the VAT price of 15%.

Items	Original price	VAT price of 15%	Price of items with VAT included
Cellphone			N\$230.00
Packet of fish	N\$84.15		
2litre of coke			N\$15.00
2litre cooking oil	N\$34.00		

Learner A, B, C, D, G, H and learner I got all the answers correct, their responses were as follows; the cellphone was N\$200 for the original price and N\$30 for the VAT; the packet of fish was N\$12.62 for the VAT and N\$96.77 for the price with VAT included; the 2ℓ of coke was N\$13.04 for the original price and N\$1.96 for the VAT; the 2l cooking oil was N\$5.10 for the VAT and N\$39.10 for the price with VAT included.

Line9. Learner E got one mistake in his work; he got the original price of the 2litre of coke correct as N\$13.1, but the VAT price he got it wrong as N\$11.3 instead of N\$1.96. Learner F got two mistakes in his work; he worked out the original price of the cellphone and got N\$195.50 with the VAT of N\$34.50 instead of N\$200 for the original price and N\$30 for the VAT. Learner F worked out 15% of N\$230 and got N\$34.50 as his VAT, and later subtracts N\$34.50 from N\$230 and got N\$195.50 as his original price of the cellphone. One can see that Learner F didn't use the diagram instead he still used the method described above, which others did at first before they were introduced with the diagram of percentage increase and percentage decrease.

Line10. Teacher calls all learners to work out their answers on the chalkboard. Many were taking about using the sketch. The teacher concluded the lesson by giving lesson activity for learners to do. She then summarizes the lesson by pointing out the main point.

APPENDIX E

TRANSCRIP FOR THE FOCUS GROUP INTERVIEW 1

Line1. Teacher: good morning learners. **Learners:** good morning madam. **Teacher:** how are you today? **Learners:** we are very well thank you, how are you madam? **Teacher:** I am also fine, thank you.

Line2. Teacher: I would like to find out your experience and knowledge in mathematics particularly in the topic of fractions and percentage. So feel free and try to be as honest as you can. Your respond will be very much helpful for my studies.

Line3. Teacher: first of all I want to find out from each of you, what your favorite subject is and why it is your favorite subject. I will start from here, yes, learner B, what is your favorite subject and why? **Learner B:** My favorite subject is Mathematics, English and Natural science; I like these subjects because I understand them well. **Teacher:** ok, next, what is your favorite subject? **Learner E:** I don't have a favorite subject because I don't understand them all. **Teacher:** ouw, that's bad, you don't understand them, and do you ask teachers for help? **Learner E:** no. **Teacher:** okay, what about you, what is your favorite subject? **Learner A:** my favorite subject is English, because the teacher who teaches us English knows how to explain very well for someone to understand. **Teacher:** that good. Ok, next one? **Learner I:** my favorite subject is mathematics, even though it is difficult, I just like it because in the future I need it, and all of us need it. **Teacher:** how do you need mathematics in the future? **Learner I:** I want to be a doctor in the hospital. **Teacher:** you mean you need mathematics for your future career? **Learner I:** yes. **Teacher:** ok, what about you learner F? **Learner F:** my favorite subject is Arts and social science; they are my favorite because I like arts, like, music, like, singing, like, drawing, all that is arts. **Teacher:** ok, can you draw well? **Learner F:** (laughing) ye..s. **Teacher:** ok, someone else, what is your favorite subject? **Learner C:** my favorite subject is Natural science, because I like nature. **Teacher:** ok, what about you learner D, what is your favorite subject? **Learner D:** I like mathematics, because it is easy and I understand it well. **Teacher:** ok, next. **Learner G:** my favorite subject is English, because I like to learn languages. **Teacher:** good, yes, next? **Learner H:** my favorite is Natural science and arts, I like Natural science because I understand it better from the all subjects, and I like arts, because since I was young I like drawing. **Teacher:** that is good. Ok.

Line4. Teacher: I am very happy to hear your favorite subjects. Now I want to find out whether you like to study mathematics, yes, or no, if yes why, and if no why? Yes, I will still start from you again. **Learner B,** like I said, I love mathematics, because I understand it more. **Teacher:** yes. **Learner E:** I don't like mathematics at all. To me mathematics is the most difficult subject I ever faced. **Teacher:** mmm, next? **Learner A:** I like mathematics, but the problem is it is a difficult subject. **Teacher:** ok, next? **Learner I:** I have said mathematics is my favorite subject because it will help me in the future, **Teacher:** yes, you said it already why math is your favorite. Ok, next? **Learner F:** I also like mathematics but is not my favorite subject. **Learner C:** I am

not sure, if it is to choose between subjects to be taught, I will not choose it. But now is compulsory, I just take it. **Teacher:** (laughing) okay, alright, next? **Learner D:** I like mathematics because I understand it well. **Learner G:** I like study math, but is a little bit difficult. **Learner H:** I like mathematics, it is important to study mathematics, because it is a subject which is needed to all the career. **Teacher:** ok, good to hear from you all.

Line5. Teacher: percentage and fractions are one of the topics in mathematics, who will define percentage and fractions in your own words, you can give examples too. In your own opinion, what do you understand by the word percentage? **Learner D:** percentage is one hundred number only. **Teacher:** one hundred number only, what do you mean? **Learner D:** percentage is.....ah is hundred. **Teacher:** this hundred is what? **Learner D:** this hundred is when I use that unit ma, madam, this unit that look like this, (learner D, demonstrating this unit (%) in the air). **Teacher:** give me an example. **Learner D:** 55%. **Teacher:** mmm. Ok, who else has another idea on what percentage is? **Learner B:** percentage is number which is over hundred. **Teacher:** what do you mean by the word over? **Learner B:** it is like, ah, ah, you have one number up and one down. **Teacher:** what are those numbers? How are they related? **Learner B:** it is any number which can be one, a small on t, ah up and one down. **Teacher:** give me an example. **Learner B:** thirteen over hundred. **Teacher:** oho, ok. Ah, who wants to add in his ideas? **Learner F:** a fractions out of hundred. **Teacher:** and that is? **Learner F:** percentage. **Teacher:** you said a fraction is out of hundred, give me an example. **Learner F:** twenty five over hundred. **Teacher:** is that a fractions or percentage? **Learner F:** is fraction. **Teacher:** what does twenty five and hundred represent in that fractions? **Learner F:** a numerator and a denominator. **Teacher:** What is a numerator or a denominator? **Learner F:** (smiling) ah, am, am, numerator is the number up and denominator is the number down. **Teacher:** ok, who want to add? **Learners:** (all quiet).

Line6. Teacher: ok, what did you learn last year on the topic of percentage and fractions give example/s? **Learner I:** I learnt about to change percentage to fractions. **Teacher:** give me an example. **Learner I:** mm, like, I forgot. **Teacher:** ok, change fifty five percent to fractions. **Learner I:** it will be fifty five over hundred. **Teacher:** correct. Who else has something to add? **Learner F:** I learn about smaller fractions and larger fractions. **Teacher:** give me an example. **Learner F:** a half and one over four, the small is one over four; the big one is a half. **Teacher:** correct. Next, someone with other examples? **Learners:** (all quiet). **Teacher:** ok, no other examples, if no, do you then have a problem or not in the topic of percentage and fractions? If yes, what are the problems you have in the topic of percentages and fractions?

Line7. Teacher: what problem do you have in the topic of percentages and fractions? **Learner D:** equivalent fractions. **Teacher:** what problem do you have in equivalent fractions? **Learner D:** I don't understand it. **Teacher:** ok, next? What was the problem? **Learner G:** also about equivalent fractions. **Teacher:** ok, what? **Learner G:** I also don't understand it. **Teacher:** ok, what about you, what problem do you have in the topic of percentages and fractions? **Learner C:** about the selling price and the cost price, I don't know how to calculate the percentages and fractions of selling price and the cost price. **Teacher:** yes, who have a problem? **Learner L:** me, I don't understand, to convert percentage to decimal. **Teacher:** ok, what problems do you have? **Learner F:** of increasing and decreasing, I always get confused. I don't know how to do it.

Teacher: ok, another one with a problem? **Learner J:** to me is all about percentage. **Teacher:** what exactly was your problem? **Learner:** converting. **Teacher:** ok,

Line8. Thank you very much for sharing with me your experience in mathematics particularly in the topic of percentages and fractions. Thank you very much. We will have lessons on percentage and fractions and I am hoping that most of the problems may be solved. Thanks again for your time. Good bye.

APPENDIX F

TRANSCRIPT FOR THE FOCUS GROUP INTERVIEW 2

Line1. Teacher: good afternoon boys and girls. **Group:** morning madam. **Teacher:** ok, ah... the interview is about the learning programme we had throughout the past days. So I will be asking you..., some of your experiences, and feelings, try to be as am mm..., as honest as you can and try to say anything you want to tell me. Alright, I want to find out from each of you how the programme was. I will start with this one (teacher pointing at the first learner sitting at her right). How was the programme?

Line2. Learner A: the pro, the programme was very good, we were studying about how to find dec, how to find percentage from fractions to decimal and we, and we were studying how, how to find the quantity of a fractions. **Teacher:** ok, alright next, how was the programme? **Learner C:** the programme was very, very good and we were learning about how to..., how to express percentage of, of one quantity. **Teacher:** ok, how was the programme? (Teacher pointing at the next person). **Learner G:** the programme was nice; we learn how to convert from percentage to decimal and from decimal to...mmm...percentage. **Teacher:** ok, how was the programme? (Teacher pointing at learner D to say something). **Learner H:** the programme was fantastic, it help me a lot, I understand more how, how to, I understand more about percentage, I hope and trust that I will never forget it.

Teacher: yes what do you say (teacher pointing at learner E to speak). **Learner B:** the programme was inclusive, because it has helped me how to include VAT in the percentage and in the money. I think I never, never forget about it. **Teacher:** ok, what do you say? (Teacher pointing at learner E to speak). **Learner F:** the programme was enjoyable, I study the increase or decrease of the quantity given and from now on I will never forget, increase is plus and decrease is minus. **Teacher:** what do you mean? Mm gives me an example. **Learner F:** example if, if they give me a fractions like N\$16 to increase 15%. **Teacher:** how will you do it? **Learner F:** 15 percent out of 100 times N\$16 dollar. **Teacher:** ok that's great. **Teacher:** yes? (Teacher looking at learner D). **Learner D:** the programme in general was incredible it really consolidated me a lot, I thank for, for having, for having such a wonderful teacher to teach us all about percentage, and I think I would, I would put myself; put my mathematical skills together so that I should be the best learner.

Line3. Teacher: What did you enjoy or not enjoy during the programme? **Learner G:** I, I enjoyed the, the programmes all of the programmes I enjoyed them. **Teacher:** Alright! What is that, that made you to enjoy the programme? **Learner G:** it help me, it help me to understand more about the percentage? **Teacher:** give me some examples. **Learner G:** like equivalent fractions. I know how to work out equivalent fractions. **Teacher:** ah...ha. What is that, that you how did you enjoy it? **Learner G:** I enjoy it like when they give me to find the equivalent fractions of two out of four at first I didn't know how to find the equivalent fractions so when we discussed, I was given two out of four I can put the similar group together and add them together,

either using bottle tops or pins. **Teacher:** and what will be the answer? **Learner G:** and it would be four out of eight. **Teacher:** that's great; yes what about you, what did you learn?

Line4. Learner I: I enjoyed a lot so perfect, actually, first of all I was told, how understanding, how to, how to say out of a number. I was using the word over and then the teacher told us that we must not use the word over, because over does not mean anything. **Teacher:** did it really, did I really mean something? **Learner I:** yes! When I was, when i say 3 over 5, I didn't understand the meaning much? **Teacher:** what do you mean you didn't understand? **Learner I:** I don't know what over something means. **Teacher:** now how do you understand the meaning if it is 3 over 5? **Learner I:** I will say 3 out of 5. **Teacher:** What does that mean? What do 3 out of 5 mean to you? **Learner I:** mm mm... three divide by 5. **Teacher:** what is that 5? **Learner I:** five is my total things in one box, like when we used 100 bottle tops in one box. **Teacher:** okay. So it's a group of how many things? **Learner I:** five! **Teacher:** then, if there are 5 things, make it five bottle tops then what did you do with the five bottle tops? **Learner I:** I subtract three, mmm, I took three bottle tops out of five now I can write it as: 3 that line (showing the line (-) in the air) 5. (**Learner I** actually meant $\frac{3}{5}$). If I work out three out of five in the calculator, I will press three divide by five. **Teacher:** Ok, learner B, what about you, and what did you learn?

Line5. Learner B: I have learnt a lot, I now know how to convert between fractions, especially to convert percentage to fractions, especially if the total things in a box is out of hundred. **Teacher:** Give an example. **Learner B:** if I want to write fifty two percent as a fractions, then I read it as fifty two out of hundred and also write it correctly, with that line, madam, th..., that line, that line means you remove parts of things from the total things, that is what a fractions is, **Teacher:** What is a fractions, you said? **Learner B:** a fractions is part of the total things one has in a box, for example if I have 20 pins in a box and want to make a fractions out of my box, let's say fifteen pins to remove from the box, my fractions will be fifteen out of twenty. I converted twenty percent to fractions like that. **Teacher:** wow, that's great. **Learner B:** I also learned that percentage doesn't stop only at hundred. I was confused with the meaning between percentage to be out of hundred and percentage to be hundred. I thought percentage should be only number below hundred, it cannot be more than hundred. Now I realized that I can get even five hundred percent and it means five hundred out of hundred.

Line6. Teacher: Ok, what about you learner D, what did you learn? **Learner D:** I learned how to work out a quantity of a percentage, like if you have twenty percent of hundred Namibian dollars is twenty Namibian dollars. **Teacher:** how did you work it out? **Learner D:** I took twenty out of hundred, because twenty percent is twenty out of hundred as fractions, then I multiply my answer by hundred and got twenty Namibian dollars. **Teacher:** In your understanding what is twenty percent? **Learner D:** I have leant that percent is a number out of the total group, if the total is out of hundred bottle tops, then I remove twenty from the hundred bottle tops in a box, I made a fractions of twenty percent. **Teacher:** Oho. That's very good.

Line7. Teacher: yes? What did you learn? **Learner C:** The programme was great full because when I started I did not know on how to convert percentage to fractions like this time I know how to convert fractions to percentage, percentage to fractions. **Teacher:** what do you do with a number if you want to convert it from percentage to fractions? **Learner C:** if it is 30 percent it will be 30 out of one hundred. **Teacher:** very good! Yes what else did you learn? **Learner C:** I normally enjoy about percentage and I understand now all about it, example, they give you one hundred and six to convert it to fractions you get one hundred and six over hundred. **Teacher:** ok, **Learner C:** I also learnt that, when you are saying „out of“ on a fractions. You press divide in the calculator. **Teacher:** give an example. **Learner C:** for example fifty out of hundred, means

fifty divide by hundred if you are working on the calculator. **Teacher:** ok, good, next. What did you learn from this program?

Line8. Learner J: About percentage. **Teacher:** yes, but what exactly? **Learner J:** to convert, how to convert, how to convert from fractions to decimal and decimal to percentage and back to fractions. **Teacher:** so if you want to change a number from fractions to percentage, what do you do? **Learner J:** so that it become percentage? **Teacher:** yes. **Learner J:** multiply by one hundred. **Teacher:** yes, for instance, give me a number? **Learner J:** like, like twenty. **Teacher:** twenty out of fifty. **Learner J:** twenty out of fifty times, twenty out of fifty times, twenty out of fifty, twenty out of hundred times fifty. **Teacher:** we are taking twenty out of fifty? That is a fractions that we are taking it to percentage, what do you do? **Learner J:** divide. **Teacher:** but you were all along right, you multiply by one hundred.

Line9. Teacher: Okay, what did you learn (teacher checking at learner K)? **Learner K:** I learnt about expressing one quantity as a percentage. Like an example; four, four out of ten to convert it to percentage then, then number 4, 4 then I convert it by 4 divide by 10 then the answer will be.... (Learner K, thinking about the answer). **Teacher:** (after a minute) no, I don't need the answer I just want you to tell me how you can come to the answer. So you were talking about taking 4 out of 10 to what? **Learner K:** to percentage. **Teacher:** so what do you do? **Learner K:** so I take 4 divide by 10 times hundred. **Teacher:** so how do you get that answer? Like now, do you use a calculator or what is that, that will help you to come to the answer? **Learner K:** by a calculator. **Teacher:** a calculator, ok. Is there someone that can take four out of ten to percentage and don't use a calculator? (Learner H, raising his hands) What do you use?

Line10. Learner H: like for instance four out of ten, four times twenty five and take that twenty five. Four times twenty five is equal to four times twenty five, one hundred. **Teacher:** yes, mmm. You are.... **Learner H:** oh! But I made a mistake. **Teacher:** I was saying....., **Learner H:** four out of ten to percentage? **Teacher:** yes, I was saying four out of ten, to bring it to percentage, is there someone who can get that answer without using a calculator or what else can you use? That's my question. Yes what did you say? (Teacher pointing at learner H). **Learner E:** (while learner H, was very quite thinking of what to answer) yes it is ah, four out of ten without using a calculator I just have to make it like this, four times ten and the denominator is ten and ten times ten, ten times ten will give me hundred and four times ten will give me 40.

Line11. Teacher: so what did you, why did you change the denominator? **Learner E:** because the denominator is, are always out of one hundred because it is a percentage. **Teacher:** ok, so you were looking for an equivalent fraction? **Learner E:** yes. **Teacher:** out of what? **Learner E:** hundred. **Teacher:** very good, ok. Yes Learner A, did you had a problem aah-aah previously on percentage.

Line12. Learner A: no I don't. **Teacher:** you did not? Oh, who still had a problem on percentage? Yes previously. Yes Learner D, how did the programme help you with the problem that you faced? **Learner D:** the pro, the programme made me understand more about percentage. **Teacher:** what problem were you having specifically? **Learner D:** Problem with equivalent fractions. **Teacher:** how did the programme help you with that? **Learner D:** it helps me now to

know, how, how can you find equivalent fractions. **Teacher**: ok: if a given fractions and how do you do that? **Learner D**: for instance if you are given mmm... mmm... **Teacher**: one out of two. **Learner D**: ya. **Teacher**: yes, how do you get the next equivalent fractions? **Learner D**: times two, times two. **Teacher**: oh, the first number times two and ah? **Learner D**: the second number times two. **Teacher**: times two it would be? One out of two it would be? **Learner D**: two out of five, au, two out of four.

Line13. **Teacher**: two out of four (repeat the learners' answer), good. Yes aah...mhh were you having a problem (pointing at learner G). **Learner G**: yes. **Teacher**: aah, what was the problem? **Learner G**: also about equivalent fractions. **Teacher**: how did the programme help you? **Learner G**: it help me a lot by, by giving ma, like aah, shortest way on how to find another equivalent fractions by grouping them and look for the same similar fractions. **Teacher**: ah... ah. Ok. So you are talking about grouping. What will you do with the grouping afterwards? **Learner G**: you group them then you add them together.

Line14. **Teacher**: ok. Yes, where you having a problem on percentage. **Learner C**: yes on how to find out the selling price and the cost price. **Teacher**: mmm. How did the programme help you? **Learner C**: it helped me how to find out the price for from a given quantity. **Teacher**: percentage of a quantity. **Learner C**: yes. **Teacher**: ok, give me an example? **Learner C**: fifteen percent m. Fernanda (the learner mentioned a person's name, Fernanda) bought a topper for four dollar and he wanted to sell it for 40 percent. **Teacher**: mmm...mmm. So what will you do then? **Learner C**: you take 40 percent over one hundred because always a percentage is out of hundred, then you, you multiply it by the mmm qua, mmm. **Teacher**: the quantity four dollars? **Learner C**: yes. **Teacher**: ok, oho. **Learners C**: then you get the answer. **Teacher**: ok, what, how will you work out the answer in your case? Will you use a calculator? **Learner C**: Yes.

Line15. **Teacher**: ok, yes anyone who had a problem? **Learner L**: yes. **Teacher**: what was the problem? **Learner L**: to convert percentage to decimal. **Teacher**: ok, I remember that, you explained that already. Who, who else had a different problem like for instance we talk about; there were some of you who had a problem of equivalent, (teacher pointing at learners and reminding them about their problems they have mentioned in the beginning of the interview). You had a problem of working out the percentage of the quantity; you had a problem of equivalent also. Aah, are these the only problems that we had? (Learners raising their hands). Yes, what problems were you having? (Teacher pointing at learner F to speak).

Line16. **Learner F**: I was having a problem between increasing and decreasing, I was confused, increasing, the, what is the meaning of increasing and the decreasing. Since we had a lesson of decreasing and increasing, now I know the different between increasing and decreasing. **Teacher**: ok. Give aah.., a practical example when do we use percentage to increase the quantity? **Learner F**: for example, if they, if they..., decrease or increase? **Teacher**: any..., **Learner F**: if they will ask me, that seventy percent, seventy dollar decreases thirty percent. **Teacher**: ow...ho. **Learner F**: and I will do it like this thirty percent out of one hundred times seventy dollars. **Teacher**: mm mm? **Learner F**: (looking for a calculator) I will then get the answer from the calculator. **Teacher**: alright, like that you have increased, is it not that...? **Learner F**: is like to decrease seventy dollar, for you to decrease thirty percent. **Teacher**: ok,

decrease seventy dollars by thirty percent? What will you do? **Learner F**: (learner F was very quiet). **Teacher**: what will you say? **Learner F**: from seventy dollars you take, you decrease thirty percent, and then it will be N\$21.00 (working out the answer from the calculator).

Line17. Teacher: ok, good, what did you say, apart from equivalent fractions, decrease and increase, who had the same problem of increase and decrease? Who had the same problem? **Learners**: (no one is raising the hands). **Teacher**: no one. Ok, is there another problem that the programme helped you with? (Teacher point at learner E). **Learner E**: yes **Teacher**: what was the problem? **Learner E**: the problem is, during lesson five, percentage of money, for me the way I used to understand it is that... when, when, you are given a percentage of money, me I used to calculate, the percentage given by the money, when I get the answer I wrote it as a percentages but it's over then out of hundred. **Teacher**: ow...ho. **Learner E**: yes **Teacher**: give me an example. **Learner E**: example is that, twenty percentages of fifty dollars. **Teacher**: mm...mm (teacher wanting to hear more from the learner). **Learner E**: so for me, I will write as a fifty out of one, times twenty percent out of hundred. So it was a little confused for me, when I will get the answer then, the number is quiet big and even more than one thousand, so I was having a little problem there. **Teacher**: ow...ho. **Learner E**: but when the teacher taught us then I was starting to get the idea. **Teacher**: yes the idea. So you are telling me that twenty percent of fifty dollars. **Learner E**: yes **Teacher**: this time after the programme, how will you do it? **Learner E**: this time I will do it like twenty percent, because the percent is meaning that is out of hundred, then twenty percent, twenty out of hundred, times fifty dollars out of one. **Teacher**: then you will work it out? **Learner E**: then the answer will be ten dollars.

Teacher: wow, you can even answer it without a calculator. **Learner E**: (few laughing). **Teacher**: ok, any other problems that you faced but maybe, maybe, aah, not only your problem. Maybe you knew something on percentage and the programme came and confuse you, you can also say about that. Is there someone that had good idea on percentage, but after this programme, you got confused, you should also talk to me about that? **Learner E**: yes madam. **Teacher**: yes **Learner E**: yes, the problem is, the little bit confusion is that, for us as learners we only knew that the percentage is only up to hundred. **Teacher**: um um. **Learner E**: but, but when you come taught us is that a percentage, can even go up to 200 or 125. **Teacher**: um um. **Learner E**: that's why it was a little bit confused there. **Teacher**: now/ what will you say? **Learner E**: now, I do understand that the percentage can go even up to over hundred. **Teacher**: oh ho. As long as that number is out of what? **Learner E**: out of hundred. **Teacher**: um. Good. **Learner E**: ya. **Teacher**: that's correct, yes? What where you saying? **Learners**: (looking at one another).

Teacher: yes, I think you want to say something? (Pointing at a learner). **Learner A**: when they give us a number like three, like one out of three to find the percentage, it is difficult to find the percentage, when they taught us then it is easily to find it, to find the answer. **Teacher**: how is the, give me an example? **Learner A**: one out of three. **Teacher**: ok, one third? **Learner**: yes **Teacher**: ok, so you want to get the percentage of one third? **Learner**: the percentage, the numbers are a lot. **Teacher**: ow ho. So how do you do that? How do you work it out now? **Learner A**: when you taught us, you must round off, if 5 up then you round off the number, if zero, like the number zero five and so on, then you round off, the five will round off then the

zero will be one. **Teacher:** oh ho. ah...But now before you get the rounded off number, before you round off, how do you, like you said is one third, how do you? You want to bring it to? **Learner A:** to percentage. **Teacher:** how do you do that? That's my question. **Learner A:** one divided by three times one hundred. **Teacher:** ok, so the answer is the one you are saying, it will be a lot of number after the comma. **Learner A:** yes.

Teacher: good, alright, what about you, what problem were you having? **Learner J:** to me is all about percentage. **Teacher:** what exactly was your problem? **Learner:** converting. **Teacher:** you did not know how? **Learner:** yes **Teacher:** do you now know how? **Learner:** yes **Teacher:** ok, what about you, what was your problem?

Learner K: equivalent fractions. **Teacher:** how did the programme help you with equivalent fractions? **Learner K:** it helps me to know the meaning, that are, that are the fractions that has the same percentage. **Teacher:** wow, ok. So you were looking at equivalent fractions as fractions that do what? **Learner K:** that has the same percentage. **Teacher:** ow ok. That's very great; okay, um, someone else with something to say.

Learners: (all quite). **Teacher:** (after a min) ok nothing. **Learner G:** Mrs., something in general. **Teacher:** yes something thing in general, no problem. **Learner G:** even not that of percentage? **Teacher:** yes. **Learner G:** ah, um. On my behalf I would like to say thank you for, for, for spending time with us to teach us more about percentage, we are great full and, and just saying thing, just talking wouldn't make sense, is rather putting it in doing and success is not the money you have but success are the people you make to become successful. I thank you. **Teacher:** alright, someone else? **Learners:** (all quiet) **Teacher:** (after some seconds) ok, thank you very much, thanks you all of you for your time and your effort of really doing and getting something on the programme. I was really impressed with you attendance especially and your effort. Otherwise thank you, good bye.

APPENDIX G

PERMISSION LETTER TO THE PRINCIPAL

P.O.BOX 2366

RUNDU

NAMIBIA

26 February 2012

TO: THE PRINCIPAL

[REDACTED]

P. O. BOX [REDACTED]

RUNDU

RE: REQUESTING TO CONDUCT MY RESEARCH AT THE SCHOOL WITH THE GRADE 8 LEARNERS.

I am a student at RHODES UNIVERSITY, doing my Master course in Mathematics Education. I would like to request the school to grant me permission so that I conduct my research at the school.

The aim of this study is to investigate the extent to which multiple representations may be used to enhance the learning of percentages by grade 8 learners at our school. To carry out this investigation, I have developed a teaching programme focusing on the learning of percentages by using multiple representations. I will implement the programme, it is an intervention programme for two weeks and it consists of 8 lessons of one hour each. The programme will start as from 25 of April 2012 up to the 03 of May 2012.

I would look forward for your respond.

Yours in Education

MRS. KAZUMBA-NGOLA MARIA

APPENDIX H

PERMISSION LETTER TO THE PARTICIPANTS' PARENTS

P.O.BOX 2366

RUNDU

NAMIBIA

26 March 2012

TO: THE PARENTS

RE: REQUESTING TO CONDUCT MY RESEARCH AT THE SCHOOL WITH THE GRADE 8 LEARNERS.

I am a student at RHODES UNIVERSITY, doing my Master course in Mathematics Education. I would like to request the parents of ----- to grant me permission so that I conduct my research with him/her.

The aim of this study is to investigate to what extents multiple representations may be used to enhance the learning of percentages by grade 8 learners at our school. To carry out this investigation, I have developed a teaching programme focusing on the learning of percentages by using multiple representations. I will implement the programme, it is an intervention programme for two weeks and it consists of 8 lessons of one hour each. The programme will start as from 25 of April 2012 up to the 03 of May 2012.

I would look forward for your respond.

Yours in Education

Mrs. NGOLA-KAZUMBA MARIA

APPENDIX I
CONSENT FORM

I....., voluntarily agree to participate in Mrs. Ngola-Kazumba Maria's research. I am aware that the data that will be collected will be reflected in her report but I am assured of the principles of confidentiality and anonymity as far as data handling is concerned. I am also aware that I can withdraw my participation at any stage of the research process.

Signature of participant

Date