THE SPIRAL CURRICULUM, INTEGRATED TEACHING AND STRUCTURED LEARNING OF MATHEMATICS AT THE SECONDARY LEVEL

An investigation of the effectiveness of integrated teaching in promoting structured learning of mathematics in accordance with the principles of the spiral curriculum by means of a standard 9 classroom exercise, taking two specific topics, viz., Pythagoras' theorem and absolute value.

by

Joseph Michael Alummoottil

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J.M.ALUMMOOTTIL

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ABSTRACT

The investigator's experience of teaching mathematics at a college of education since 1983 has reinforced his conviction that trainee students come to college with significant gaps, weaknesses and faults in their (mathematical) conceptual structures, probably as a result of shortcomings in the mathematics teaching to which they have been exposed.

The theme of this investigation is thus a natural choice that appeared to be of immediate relevance to secondary school mathematics teaching.

The analysis of the issue leads to a unified perspective: the problem is placed in a "theoretical framework" where Bruner [spiral curriculum], Ausubel [structured learning] and Skemp [relational understanding] are brought together.

How the curriculum, textbooks and examination influence school mathematics teaching is examined in some depth and the consequences investigated. Two specific topics, viz., the generalised Pythagorean relation and absolute value are investigated in relation to published work, curriculum and textbooks, and each (topic) is presented as a unifying theme in secondary mathematics to standard 9 pupils. The classroom exercise is assessed to test the hypothesis that structured, integrated presentation around a spiral curriculum promotes "relational understanding".

Analysis of results supports the hypothesis.

CHAPTER 1

INTRODUCING THE THEME OF THE INVESTIGATION

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1.1 INTRODUCTION

Over the years the present investigator has become convinced of the thesis that learning occurs out of the growth of a well-defined conceptual map which is a very personal acquisition of the learner. Time and again this conviction has been reinforced during teaching encounters with students showing serious gaps in their conceptual maps. Remedial teaching on such occasions has always involved retracing the path to the point of firm ground. Such first-hand experiences of teachers provide support to the curriculum model put forward by Bruner [1966].

1.2 THE "SPIRAL CURRICULUM" AS A CONCEPT RELEVANT TO MATHEMATICS TEACHING AND LEARNING

If learning is viewed as an orderly growth of the conceptual map of the learner, then mathematical education, indeed all education, should aim at unifying (integrating) the conceptual map of the learner. It is this stance which makes Bruner's view of the process of education a perspective crucial to this investigation.

> Good teaching that emphasizes **the structure** of a subject is probably **even more valuable for the less able student** than for the gifted one, for it is the former rather than the latter who is more easily thrown off the track by poor teaching.
>
> [Bruner, p.9; investigator's emphasis]

The above is a considered educational "opinion", the substance of which is justified partly by (psychological) learning theories and partly by research studies. The relevance of the statement emphasized in the quote above to secondary mathematics education in the Ciskei inspires this piece of research.

> ...giving students an understanding of the fundamental structure of whatever subjects we choose to teach. This is a minimum requirement for using knowledge, for bringing it to bear on problems and events one...encounters. The teaching and learning of structure, rather than simply the mastery of facts and techniques is, at the centre of the classic problem of transfer...
>
> [Bruner, pp 11-12]

Mere instruction/teaching does not guarantee understanding of the fundamental structure of a subject. Understanding deepens in stages and its process is intimately related to readiness for learning (of the child). The foundation of learning (leading to understanding) is the intuitive grasp of (basic) ideas and learning to use these ideas.[Cf. Bruner, p.13]. So

> A curriculum as it develops should revisit these basic ideas repeatedly, building upon them until the student has grasped the full formal apparatus that goes with them...[i.e.] the "spiral curriculum"...turns back on itself at higher levels.
>
> [Bruner, p.13; investigator's parenthesis]

The above quotes from Bruner underline the supreme importance of ordering classroom mathematical presentations into broadly structured content. This investigation examines two specific themes and how a broad structure could be built around them, and how in that process the learner is helped to climb up and down the "spiral". The themes are: "Pythagoras' theorem" [hereafter referred to simply as Pythagoras where the meaning is clear], and the role of symmetry in the teaching of "absolute value".

1.3 ON PYTHAGORAS

1.3.1 Pythagoras viewed in a theoretical framework

> To this day, the theorem of Pythagoras remains the most important single theorem in the whole of mathematics. That seems a bold and extraordinary thing to say, yet it is not extravagant; because what Pythagoras established is a fundamental characterisation of the space in which we move,...the theorem of Pythagoras in the form in which I have proved it is an elucidation of the symmetry of plane space;...Symmetry is not a descriptive nicety; like other thoughts in Pythagoras, it penetrates to the harmony in nature.
>
> [Bronowski (1977), pp 160-161]

Pythagoras is one of the most frequently found names in mathematics teachers' published notes, letters and articles. The proof itself of this theorem is still a topic of considerable interest among teachers. Hirschy [1964] gives a concise, but comprehensive account of the history and proofs of Pythagoras[*]. Most of the published material in recent years which has come to the investigator's notice is on the generation and properties of Pythagorean triples, while some is concerned with the generalization of the theorem to include the law of cosines. Occasional articles dwell on other aspects which fired the imagination of some teacher or other: for example, Ewbank [1973] plays with Pythagoras on a geoboard providing learning with insight in many directions; Didomenico [1977] is not apparently

* Hirschy reports more critically on historical anecdotes concerning Pythagoras (the man) and the famous theorem than does Bronowski. However the readers/audiences of the two are different.

talking about Pythagoras but the arithmetic and the inevitable transition to algebra which he presents do bring to the fore the Pythagorean relation. Gadanidis's [1988] chief concern is problem solving, but his argument is developed around the example of the Pythagorean theorem. Gadanidis's is the theme that comes closest to the role of this theorem in the present investigation. Gadanidis has read Bronowski but apparently not Hirschy.

> Such holistic [i.e., incorporating and integrating "facts and skills", "understanding" and "problem solving"] teaching and learning places students in situations where they are exposed to a wide variety of inter-relationships among the mathematical facts, skills, and concepts they possess and learn. **As Ausubel [1968] argues, the more relationships seen, and the more thoroughly they are understood, the more meaningful the learning that occurs [1]... Most textbooks now in use do not venture much farther than the objectives of the first dimension [i.e. facts and skills]. This narrowness is reflected in the mechanical nature of their exercises and the dryness of conceptual presentations [2].** Some textbooks are now starting to appear that are accompanied by elaborate teaching manuals...However, they are not presently used widely.
>
> [Gadanidis, pp 20-21; numbers [1] and [2], emphasis and parenthesis investigator's]

The two comments emphasized and numbered [1] and [2] above are the central issues of this investigation too. They are examined around classroom presentations of Pythagoras and absolute value as described later in the thesis. Gadanidis approaches the issues from the stance of a "generalized model of mathematics teaching... in a regular classroom setting", adopting the following working model:

> A basic assumption of the model is that the teaching of mathematics has three major areas of emphasis: facts and skills, understanding, and problem solving ....**teaching approaches must be of a holistic nature,** incorporating and integrating all three dimensions.
>
> [Gadanidis, p.16; investigator's emphasis]

The present investigator tends to adopt the approach of tying the role of the spiral curriculum with the learning process by enlarging and reinforcing the conceptual map of the learner, which ultimately is the source and basis of the skills Gadanidis stresses in the quote above. At the same time the procedure this investigator tries out in the classroom experiment is designed to emphasize the need and the role of a holistic presentation of the subject matter - holistic in the learner's conceptualisation rather than in the development of specific aspects of learning. But Gadanidis's and the investigator's ultimate goals are identical: to help students to form more appropriate long-term schemas on which to structure further learning [cf. Skemp, 1972, quoted by Gadanidis, p.18].

> Ideas required for understanding a particular topic turn out to be basic for understanding many other topics too...Unfortunately the benefits which might come from teaching them are often lost by teaching them as separate topics, rather than as fundamental concepts by which whole areas of mathematics can be inter-related.
>
> [Skemp (1976), p.24]

The present investigation assumes an instrumental role of testing the validity of this "theoretical opinion" in the practical classroom situation.

1.3.2 Pythagoras in the curriculum

The Cockcroft Commission [1982] mentions in passing [p.60], while commenting on a certain study, that many adults remembered Pythagoras' theorem by name, but not much else about it. This is so in spite of the fact that Pythagoras is mentioned hundreds of times in class by teachers through the junior as well as the senior secondary phase. It is the opinion of this investigator that this is a classic example of the failure of most teachers as well as textbooks to present different parts of the prescribed syllabi, emphasizing fundamental concepts in such manner as to inter-relate whole areas of mathematics [cf. Skemp's quote above]. Many teachers and most textbooks unwittingly deny pupils the guidance and opportunity of creating for themselves "appropriate long-term schemas on which to structure further learning" (to repeat part of an earlier quote).

Past matric question papers of the various departments of education were examined to see how examiners orientate teachers as well as pupils to Pythagoras. The following observations are made accordingly.

A fair number of questions test the ability to apply Pythagoras.

A typical example is question 6 [Cape SC HG II, November 1987].

6.1 A circle centre $Q(-3;2)$ has the points $M(-1;6)$ and $N(-1;y)$ on the circumference. The tangents to the circle at M and N meet at P.

6.1.1 Determine the equation of the circle. (3 marks)

6.2 Find the equation of the locus of P such that $PQ = 2PR$ where Q is the point $(-2;1)$ and R is the point $(3;2)$. (6 marks)

6.3 What kind of figure is formed by the equation of the locus of P in question 6.2 ? Justify your conclusion by referring to the equation of the locus. (6 marks)

The question is apparently to test knowledge of circle theory; though the Pythagorean basis of circle theory is not explicitly emphasized, the central concept underlying the solution is Pythagoras, and the question does test understanding of the concept fairly effectively. However, textbooks rarely emphasize the topic as illustrative of the central concept of Pythagoras; on the other hand they promote the compartmentalisation of learning: viz., that this is a theme in "analytical geometry" and the learner picks up reinforcements that tell him that "analytical geometry" is "something new", "something different", and his effort is focussed towards isolating this new knowledge from his existing conceptual structure: the antithesis of what a true learning situation should be achieving.

There are two more questions involving Pythagoras in the same paper, one appearing as a question in geometry while the other as one in trigonometry, questions 7.2 and 1.1 respectively.

7.2 Circle, centre O, has chord $BC = 30$ cm and a diameter $TOMX \perp BC$ such that $OM = 2MX$. Calculate giving reasons

7.2.1. TB (correct to one decimal).

Though this question is somewhat traditional in form, it calls for understanding to solve it and so is effective.

1.1 Prove by using the basic trigonometric ratios $\sec^2\theta = 1 + \tan^2\theta$.

This question is open to criticism. [Detailed examination of Pythagoras follows in the next section.] Questions 1.a(2) and 7 [Natal SC HG II], 2 and 10 [Orange Free State SC HG II], 5 and 6 [House of Representatives SC HG II], 8(a) [National SC HG II], 1.1 [House of Delegates SC HG II] and 1(b) [JMB SC HG II], all of November 1987 [cf. appendix 1] are similar to the ones examined above. In general, the emphasis is on recall and application; the criteria of mathematical understanding are open to interpretation, and at the same time it is a difficult task to test for understanding. Examiners often seem to take it for granted that ability to apply (something) implies understanding (of the thing). This may be so on many occasions, but one has to be skeptical about assuming its truth as universal.

The investigator places considerable emphasis on the Cockcroft Commission's opinion that the teaching of mathematics in schools is justified because mathematics provides pupils with a powerful means of communication. In the matric question papers examined above this aspect stands out prominently only in the geometrical contexts as far as Pythagoras is concerned. This does not have to be so. An integrated approach to themes would be encouraged in classroom teaching if examiners consistently called for similar responses in important examinations like the matric examination.

1.3.3 Possibilities with the teaching of Pythagoras

The historical aspects of the theorem known by Pythagoras' name are not of much interest to this investigation, except its traditional association with geometry. However, the theorem does not have to be treated as a purely geometrical concept/fact throughout the entire course of a pupil's mathematical education. Excellent opportunities exist for re-interpretation as the teaching/learning climbs the "spiral" of the curriculum.

Pythagoras appears as early as std 6 in the junior secondary phase; by then a pupil is familiar with expressions like $3^2 + 4^2$, $a^2 + b^2$, as well as equations involving squares. The generalised form $a^2 + b^2 = c^2$ is simply and basically a relationship among "SQUARE" numbers. One can and should descend the "spiral" when Pythagoras is presented for the first time. Once the pupil has come to see the relationship in Pythagoras, equations like $a^2 - b^2 = c^2$ could also be seen as the same relationship (by transposition); thus it is possible to interpret Pythagoras' theorem as the geometrical illustration of the more general relationship among three second-degree powers expressed as an equation. The concept extends to see $ax^2 + by^2 = cz^2$ transformed as $(\sqrt{a}x)^2 + (\sqrt{b}y)^2 = (\sqrt{c}z)^2$. [This is the kind of understanding that illuminates the logic behind completion of squares as a routine in the solution of quadratic equations.] More specifically: when $x^2 + y^2 = r^2$, x,y variables, r constant, $x,y,r \in \mathbb{R}$, we could recognize x^2 and y^2 varying between 0 and r^2, or alternatively, the equation defines an infinite set of Pythagorean triangles on the x-y plane. When the equation to a circle is introduced for the first time, the revolution of the hypotenuse of the Pythagorean triangle naturally becomes identified with the revolution of the radius vector on the Cartesian plane. However, Pythagoras need not be treated here as a property incidental to the circle; on the contrary it could be viewed as the central unifying feature, particularly when the concept locus is introduced. While presenting analytical geometry one can achieve an effective synthesis of the above perspectives. On the Cartesian plane are

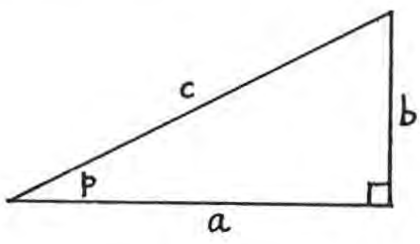
(i) sets of numbers a,b,r such that $a^2 + b^2 = r^2$;

(ii) variables x,y and constant r such that $x^2 + y^2 = r^2$;

(iii) locus of points equidistant from the origin such that the equidistance r is defined by Pythagoras;

(iv) the locus as in (iii) when r is the equidistance from any point on the plane.

Trigonometry offers another area for excellent development around Pythagoras. Interpretations of a triad of trigonometric identities as re-statements of Pythagoras should be valuable and illuminating.



With reference to the diagram:

$$\cos^2 p + \sin^2 p = a^2/c^2 + b^2/c^2$$
$$= (a^2 + b^2)/c^2$$

$$= c^2/c^2 \text{ [Pythagoras]}$$

$$= 1.$$

This is the typical textbook approach. [A number of South-African produced textbooks are reviewed in this connection in the next chapter.]

Let us look at the alternative approach:

$$a^2 + b^2 = c^2 \text{ [Pythagoras]}$$

Divide both sides by c^2:

$$a^2/c^2 + b^2/c^2 = 1 .$$

i.e. $\cos^2 p + \sin^2 p = 1$ [follows from definitions].

[There are some textbooks which adopt this approach. Detailed criticism is given in the next chapter.]

Both procedures may be considered as "derivations", but "proof" would imply proving [or having proved earlier] the theorem of Pythagoras.

The second procedure is valid for the trio of the "Pythagorean identities". It becomes immediately obvious to pupils that there are only three ways of dividing both sides in order to make one term equal to unity: hence the three forms of the same identity.

Each of the above learning experiences is a re-visiting and reinforcement of Pythagoras towards an extension of the conceptual structure of the pupil. The purely geometrical presentation of Pythagoras also reinforces diverse concepts and integrates the conceptual map of the learner leading to understanding and enhanced ability to communicate [cf. sec.1.3.2 above].

As already pointed out, one of the aims of this investigation is to see whether Pythagoras could be presented as a central theme tying together the various aspects touched on above and which appear scattered through the senior secondary [HG] mathematics curriculum. To what extent the curriculum and the textbooks [available on the market] contribute towards teaching/learning around a possible spiral curriculum in order to enhance the breadth and depth of the conceptual structure of the learner is examined in the next chapter.

1.4 ON SYMMETRY

1.4.1 Symmetry and absolute value

Symmetry could be considered as an aesthetic aspect of mathematics, but its more important role is its holistic explanatory power in complex mathematical contexts. The illustrative theme under investigation [absolute value] is comparatively simple from a symmetry point of view. However, anyone who has taught the absolute value concept and its graphing and solution of equations would know how difficult it is for most pupils in the secondary stage. The source of difficulty is of a primary nature and arises from faults and weaknesses in the "dynamic interplay between symbols and meaning" [Mick, p.244]. Mick [1987] examines the issue of meaning in mathematics (learning) from a cognitivist position and argues how interpretation as part of the learning process ties the "axiom system" with "conceptual structures". Incidentally Mick shows how his two-tier model conforms to the "surface" and "deep" structures of Skemp's two-structure model [1982]. To quote Mick:

> As people learn mathematical content and progress through its hierarchical structure, mathematics becomes more abstract and the work is done more and more within the axiom systems themselves.
>
> [Mick, p.244]

However, formalisation occurs through inductive processes after prolonged direct experiences providing "interaction between our internal structures and the external environment", particularly true of secondary mathematics. Mick goes on to illustrate "the facile movement back and forth between surface and deep structures" by means of two examples of absolute value problems.

> When mathematics is presented from within axiom systems too early, without proper support from conceptual structures, learning is characterised by mindless imitation of the teacher or text, and behavior is characterized by meaningless symbol shoving. The resulting knowledge is **characterized by isolated bits and pieces** held together by memorization and repetition; it is largely machine-like and fails to use human potential and intelligence...there can be no meaningful interpretation since there is only surface knowledge...the implications for instruction are immediate. For a particular concept, we should be aware of all the parts of its meaningful interpretation units...
>
> [Mick, pp 246-247; investigator's emphasis]

The parts emphasized in the quote above suggest how Mick's discussion closely parallels the spirit of the present investigation. The quote itself makes clear why many (if not most) secondary pupils find absolute value problems difficult: the formal symbolic definition of absolute value and its transformations [cf. textbooks] come "too **early, without proper support from conceptual structures**" [cf. quote above]. It is this support that "symmetry" can provide in the first-time learning of absolute value. Though symmetry is mentioned in textbooks in this context too little advantage is taken of its power in the classroom instructional context, with obvious disastrous consequences. To cite an example of weakness in conceptual structures: A set of cognitive ability tests developed in the United Kingdom was administered to applicants [matrics with/without exemption] for teacher training at Rubusana College of Education in Mdantsane (Ciskei) from 1984 to 1986. [The practice has been discontinued since.] The applicants' performance showed consistent development lag of three or more years compared to their age groups in the United Kingdom. Cultural as well as linguistic bias might partly account for the poor performance; but weaknesses in the applicants' conceptual structures cannot be ruled out.

1.4.2 A brief review of published work on teaching of absolute value

Mangho Ahuja [1976]: "An approach to absolute value problems"

Ahuja suggests an approach which avoids the very definition

$$|x| = x \quad \text{if } x \geqslant 0$$
$$= -x \quad \text{if } x < 0.$$

He discusses the merit of his approach which "emphasizes the metric property of the absolute value function and the fact that the real numbers form an ordered set". A number of examples are worked out by using only the primary concept that $|x - y|$ = "the <u>undirected</u> distance between A and B, where A and B are the representations of the numbers x and y, respectively, on the real line". [This approach is very similar to the symmetry approach adopted in the present investigation.]

Stephen C. Sink [1979]: "Understanding absolute value"

Sink begins by commenting that "the most haphazardly presented and least understood topic in secondary school mathematics is the solving of open sentences involving absolute value". [Sink does not cite Ahuja's article.] However, Sink takes the traditional route when he states: "It is critical that the students first understand the definition of absolute value" [cf. definitions above], but his instructional strategy is more in line with this investigator's [cf. chapter 4] when he says:

> When teaching this topic, I encourage the exclusive use of graphs instead of the rule form so that students gain a pictorial understanding of the process. After they see how different problems are solved, they begin to take shortcuts.
>
> [Sink, p.192; investigator's emphasis]

Charles Brumfiel [1980] "Teaching the absolute value function"

Brumfiel, who cites both Ahuja and Sink begins with the optimistic statement: "A unit on absolute value and inequalities can be taught in such a way that **interested, capable students will learn a great deal of useful mathematics**" [investigator's emphasis]. Unfortunately the problem in the Ciskei is how to teach the average student who is neither quite capable nor quite interested.

Brumfiel's instructional suggestions are supported by his own first-hand experience and so are likely to be successful if adopted properly. [However his experiences are with college students.] He advocates the use of five definitions of the absolute value of a real number. His definition 5 is the formal definition [cf. beginning of sec. 1.4.2].

> Students work one problem in five distinctly different ways, basing each solution on one of the definitions. They find it most instructive to observe how the choice of a definition influences their patterns of thinking...
>
> [Brumfiel, p.24]
>
> Certainly most students who have a brief exposure to absolute value concepts in high school using definition 5 don't understand the logical concepts underlying its use.
>
> [Brumfiel, p.27]
>
> The point, which I deliberately belabor, is that an

> approach to absolute value that exploits definition 5 alone is impoverished, and if not handled skillfully, will confuse students horribly.
>
> [Brumfiel, p.29]

Brumfiel's is not a symmetry approach, but he uses numberlines which provide graphic presentation extensively, and the success of his method(s) rests substantially on the symmetry apparent as well as implicit in these graphs.

Michael J. Arcidiacono [1983] "A visual approach to absolute value"

Arcidiacono cites all the three contributions briefly considered above. He recommends a strong graphic component in the presentation of this topic; he concludes:

> I have found a graphic approach to absolute value to be very helpful in the classroom. It **demonstrates a natural relationship between absolute value and the topics of graphs and linear functions and** offers students a visual way of analyzing problems by cases.
>
> [Arcidiacono, p.201; investigator's emphasis]

The emphasized part agrees with this investigator's perception that this topic also has a unifying role to play in the secondary mathematics curriculum.

Sandra C. McLaurin [1985] "A unified way to teach the solution of inequalities"

McLaurin does not cite any reference to work of a similar nature. Hers is a report on the successes she achieved by finding a unified approach to teach how to solve inequalities involving absolute values, quadratic expressions, and rational expressions. It is apparently a successful procedure, a purely algebraic routine consisting of five steps. Basic understanding of the absolute value concept is not her immediate concern. Her procedure:

Step 1: Simplify the expression.

Step 2: Determine the values of x [i.e. the unknown] for which the expression changes from true to false.

Step 3: Plot the results from step 2 on the number line.

Step 4: Select a point from each interval and test it in the original statement to see if it makes the original statement true or false.

Step 5: Write down the interval, or intervals, that make the statement true.

She concludes:

> Students I tutor in the mathematics laboratory who cannot work the problems involving inequalities by other techniques are able, with this technique, to solve inequalities involving absolute values.
> [McLaurin, p.95]

A number of responses to McLaurin's article followed.

Eleanor Dean [1985] "Letter to the editor"

Reports moderate success with McLaurin's approach.

Joseph V. Roberti [1985] "Letter to the editor"

Commends McLaurin's effort and urges "all readers to re-examine the approaches of the texts"; at the same time points out some of the deeper implications associated with the procedure.

Lynda W. Wagster [1986] "Letter to the editor"

There is no reference to other important contributions such as those considered above. She reports on the successful use of "the numberline as a tool for examining more difficult equations and inequalities involving absolute values". Hers again is an algebraic routine in which transformations eliminate the need for retaining the absolute value symbol, thus transforming the problem to a more familiar form. In concluding, she illustrates how her approach combines with McLaurin's.

Jane Mason Ballowe [1988] "Teaching difficult problems involving absolute value signs"

Ballowe refers to McLaurin's article and Wagster's response; she claims that absolute value can be taught more effectively using the definition $|x|=\sqrt{x^2}$ instead of the formal definition.

D. Sullivan [1988] [A response to Ballowe's article]

The above review shows that the problem of effectively teaching the absolute value concept is very much alive today as ever in spite of the many worthwhile contributions made by experienced teachers. It is also clear that in general textbook writers have done little to improve matters. [Textbooks are reviewed in the next chapter.]

1.4.3 Possibilities with the teaching of absolute value

Examiners too are involved in the issue, since they set standards and procedures acceptable to them, thus indirectly influencing teachers to conform to their norms and style of presentation.

A typical question taken from a past matric paper is critically examined here: OFS SC HG I November 1986, Question 3.1.1:

Solve for x:

$$|3x + 1| > 5 \qquad \text{(4 marks).}$$

(i) Solution by using rules [after typical textbook procedure]

$$|x| = x, \qquad \text{if} \qquad x \geq 0$$

$$\therefore \quad |3x + 1| = 3x + 1 \quad \text{if} \quad 3x + 1 \geq 0$$

Solution		Constraint
$3x + 1 > 5$		$3x \geq -1$
$3x > 4$		$x \geq -1/3$
$x > 4/3$	if	$x \geq -1/3.$

OR

$$|x| = -x, \qquad \text{if} \qquad x < 0$$

$$\therefore \quad |3x + 1| = -(3x + 1) \quad \text{if} \quad 3x + 1 < 0$$

Solution		Constraint
$-(3x + 1) > 5$		$3x < -1$
$-3x - 1 > 5$		$x < -1/3$
$-3x > 6$		
$-x > 2$		
$x < -2$	if	$x < -1/3.$

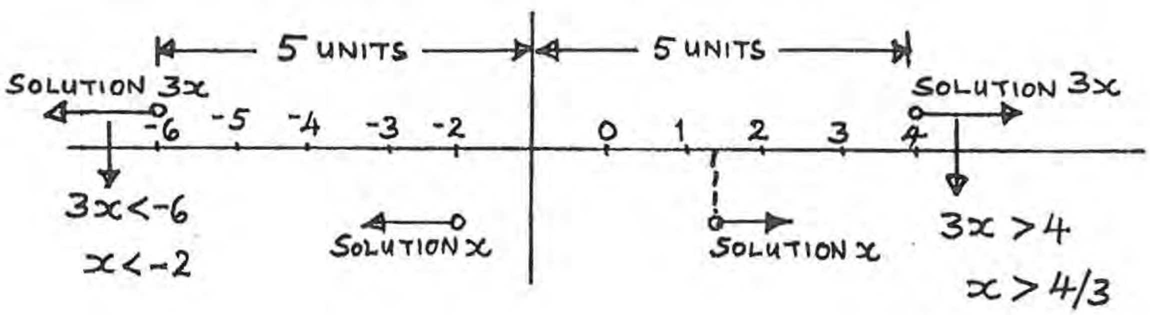
The procedure is straightforward but does not necessarily indicate pupils' understanding as argued below.

(a) The concept "absolute value" has the built-in idea of equidistance from an axis of symmetry somewhere on the numberline. This central idea is more or less lost in the above procedure. [Cf. Brumfiel's comment above on the exclusive use of the formal definition.]

(b) The correlation between the numbers 4/3 and −1/3 or between −2 and −1/3 [i.e. between the solution and the constraint] is not self-evident. To a pupil who has not understood the role of −1/3 here the statements, for example, "x < −2 if x < −1/3" and "x < −2 if x < −1,5" may not make much difference. Even a graphical representation of the solution would not resolve this difficulty of comprehension unless the axis of symmetry is indicated.

(ii) Graphical solution

The axis of symmetry for $|x| = a$, $a > 0$, is $x = 0$. It is not difficult to show that the axis of symmetry for $|ax + b| = c$, $c > 0$, corresponds to $ax + b = 0$. Thus, in the question above $3x + 1 = 0$ defines the axis of symmetry, i.e. $3x = -1$. Hence the statement $|3x + 1| > 5$ simply means that $3x + 1$ is beyond 5 units to the left or right of the axis of symmetry on the numberline.

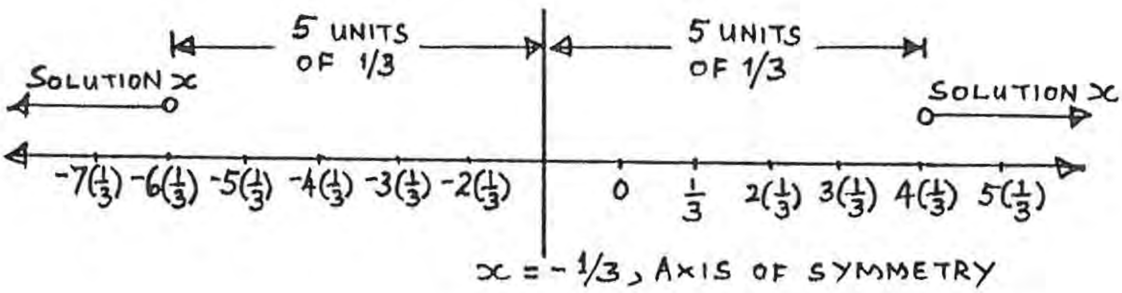


Alternatively, the axis of symmetry is defined by $x = -1/3$ in which case the variable is x on the numberline. Then $x = -1/3$ is related meaningfully to the steps in the solution as follows:

$$|3x + 1| > 5$$

$$|x + 1/3| > 5/3.$$

Axis of symmetry is defined by $x + 1/3 = 0$, i.e. $x = -1/3$. Mark the numberline in units of 1/3; then $|x + 1/3| > 5(1/3)$ means that $x + 1/3$ is beyond 5 units of 1/3 to the right or left of the axis of symmetry.



The solution is at once obvious: $x > 4(1/3)$, i.e. $x > 4/3$ or $x < -6(1/3)$, i.e. $x < -2$.

The conceptual simplicity of the procedure and its ability to reinforce the central characteristic of the concept "absolute value" are obvious.

There is no indication in the question as to whether the examiner expects this kind of approach in the solution. If not, the question does not really test the "ability to communicate" [cf. sec.1.3.2]. In fact, neither textbooks nor examiners treat the above approach as standard procedure; they probably admit it as a supplementary perspective. It is hard to justify this stance since the procedure is effective in contributing towards understanding and is capable of being used for testing understanding. However the standard procedure remains algebraic, too formal in symbolic form, with serious learning problems for pupils [cf. sec.1.4.1].

1.5 CONCLUSION

Problems in mathematics education affect both pupils and teachers; more acutely are they affected when both are of average or below average ability. Examination policies too influence them to a greater extent than the more able pupils and teachers. Therefore it is within the power of examining bodies to motivate teachers to present the topic [absolute value] meaningfully, i.e. in a form combining algebraic elegance with graphical understanding. Symmetry can play a key role in this approach.

The two topics discussed in this chapter in relation to educational principles and instructional problems will be examined in chapter 2 in relation to presentation as found in syllabi and South-African produced textbooks.

The two topics were used by the investigator as two themes around which to integrate different lines of development relevant to secondary mathematics. A classroom experiment was designed to test the hypothesis that structured, integrated presentation around a spiral curriculum promotes relational understanding. It will be seen that analysis of results supports the hypothesis.

CHAPTER 2

THE SECONDARY MATHEMATICS CURRICULUM AND THE SOUTH-AFRICAN PRODUCED TEXTBOOKS

2.1 ON THE SOUTH AFRICAN SECONDARY MATHEMATICS CURRICULUM

The core curriculum common to the various Departments of Education is the subject of examination here. Though this investigation is primarily concerned with the senior secondary phase, it is worthwhile to look at the aims of the mathematics curriculum from standard 2 upwards, since the spiral curriculum implies the conception of the mathematical development of a child from the earliest years. Further, since this investigation is concerned with secondary mathematics teaching in the Ciskei, the discussion will develop around the curriculum of the Department of Education and Training [D.E.T.].

2.1.1 Aims

Standard 2

1. To develop in pupils an insight into and understanding of mathematical principles and methods, to assist them to cope with mathematical situations they may encounter in everyday life and to solve problems based on numbers and quantities.

2. To help pupils to develop logical thinking, and systematic concise expression.

3. To develop in the pupil an interest in, an appreciation of, and a love for the unchanging nature of the laws of numbers.

4. To prepare the pupils to perform calculations which may be required in other school subjects or in later studies.

5. To develop in the pupil an ability to perform calculations accurately and quickly.

These aims are entirely compatible with the ideas of the spiral curriculum as examined in chapter 1. [Whether these aims are realized is a different matter. Indeed it is well known that achievement falls far short of aims. In fact the motivation for investigations in mathematics education comes primarily from this fact.]

Standards 3/4

The aims of the standards 3 and 4 curriculum are exactly those of standard 2.

Standard 5

The aims of standard 5 are the same as those of the lower standards except that aim number 3 is expressed slightly differently. It reads:

To develop in the pupil an interest in, a love for and an appreciation of the orderliness of numbers.

The slight change in expression might have occurred by chance or might have been intended. This investigator is inclined to think that it was intended, for on the one hand, "the unchanging nature..." does not necessarily highlight "the orderliness..." whereas the latter does imply the former, and on the other, orderliness and patterns are the essence of mathematics. Therefore it is quite appropriate to re-define the particular aim in this explicit form in the final year of the primary stage. Indeed the appreciation of the orderliness of numbers is to be seen as an expression of deeper understanding acquired by the child. Incidentally, though the aim under examination here concerns the standard 5 curriculum, it is one of the more important aims of mathematics teaching at all levels. More specifically it is of immediate relevance to the kind of order apparent in the relation described by Pythagoras as well as the order implicit in the symmetry associated with absolute values. Indeed the achievement of this aim should be seen as the ultimate testimony to the virtue of the spiral curriculum.

The syllabus for standard 5 begins with AIMS as well as GENERAL REMARKS ON SYLLABUS [cf. appendix 2]. These remarks are in line with the aspects of mathematics education discussed above. In fact some of the important points appear in the introduction to the standard 4 syllabus, e.g.:

(3) ...In teaching, the various subsections should be integrated with one another as far as possible.

(4) Situations within the experience of the pupil should, as far as possible, be used as starting points for the introduction of new concepts.

(5) New concepts should be introduced through and based on discovery and discussion. As far as possible they should be preceded by practical work and be explained by demonstrations with practical aids.

(6) All concepts taught previously that form bases for a new concept must be carefully revised before the new concept is introduced.

Comments

Number (3) is meant to promote new learning by providing structure to the content which would facilitate the assimilation of new knowledge into the existing conceptual map of the learner, as well as enlarge and modify it [assimilation and accommodation according to Piaget], and meaningful learning would occur because of the interrelations/structure [Ausubel 1968].

Number (4) reminds the teacher of the need for starting from points/locations on the existing conceptual map of the learner in order to achieve what is contemplated in (3). Number (5) stresses the need for first-hand concrete experiences.

Number (6) directly points out the procedure of the spiral curriculum.

The above Remarks on Aims are all repeated in the standard 5 syllabus in the section "General Remarks on Syllabus"; in addition, emphasis is placed on alternative procedures of solution wherever possible, (procedures which would reinforce, re-interpret and enlarge the deep structures of learning). General Remark 14 pertains to examinations and cautions on the need for testing for comprehension and insight.

Standards 6/7

The Aims and General Remarks for standards 6 and 7 are the same as those for standard 5, suggesting that the curriculum designers treat standard 5 as a point of departure towards a higher level of mathematical understanding.

Standard 8

Standard 8 Aims and General Remarks [appendix 3] indicate the next point of departure. A number of aspects of the aims up to standard 7 are combined, condensed and enhanced in scope in the first two aims of standard 8. Aim 3, viz.:

> To cultivate appreciation for the structure and the continuous theme of each section of the syllabus as well as for the underlying relation between certain sections

makes explicitly clear the emphasis on structured learning, while Aim 4, viz:

> To acquaint pupils with and train them in mathematical methods of thought and work

emphasizes the process of learning mathematics. Aims 3 and 4 are centrally related to the spiral character of the curriculum. The General Remarks are even more pointed than before. For example, Remark 2.4.2 stresses that graphs are to be treated as "unifying concept" and "should be used where possible".

Standard 9

The standard 9 Aims [appendix 4] are stated briefly in contrast to the standard 8 ones. These seem to take for granted the foundation for the senior secondary course. For example, Aim 4, viz: "To develop accuracy and mathematical insight" directly points to the development of mathematical insight, but insight cannot develop without a firm grasp of the structure of the subject.

Without further comment one can say that on the whole the aims of the South African school mathematics curriculum are well designed, on the assumption that the D.E.T. curriculum is fairly representative of the curricula of the various departments of education, and that they imply the notion of the spiral currulum.

2.1.2 Examination of syllabi

A detailed critical examination of the syllabi from standard 2 through standard 10 would be a worthwhile exercise but too lengthy as well as redundant as far as arguments are concerned. So the following examination will concentrate on the two areas of interest, viz: Pythagoras and absolute value respectively.

2.1.2.1 On Pythagoras

By "Pythagoras", in this context, is meant the general relationship between three square numbers and how it relates to other parts of the syllabus. The key preliminary concept is that of the SQUARE. The "square" as a geometrical concept is introduced as early as standard 1. Standard 2 work on "square" is intended to achieve "the particular aim of making the pupils completely familiar with the properties of a square..." [D.E.T. Manual, p. 182]. The Manual points out that the standard 2 work is to prepare the basis for the area concept to come later on. Standard 3 work is consolidation and enlargement of direct experiences. The area concept is introduced in standard 4 along with the units m^2 and cm^2. The exponential notation of the unit is an important novelty but at this stage no attempt is made to justify it. [Cf. syllabi; D.E.T. Manual.] However, the possibility of relating this notation to the exponential notation in arithmetic deserves consideration. Powers to base 10 only are introduced in standard 4 [cf. appendix 5], that too a notational device for alternative representation of large numbers. The D.E.T. Manual [pp 37-38] makes it more general and illustrates the meaning with base 2 as well. So, the pupils are ready for a proper interpretation of the notation m^2, cm^2 through "structure and spiral". Though the Manual does not explicitly suggest it, teachers might possibly be doing it. The central question is whether the "SQUARE" encountered in arithmetic is conceptually related to the "SQUARE" as a shape through the size of its area and the notation of the unit of area.

Squares and cubes of numbers are formally introduced in the standard 6 syllabus, para. 1.3, while Pythagoras is mentioned in para. 3.3 in connection with irrational numbers. [Comments appear in the next section where textbooks are reviewed.]

This is truly in line with the kind of integration which is the focus of this investigation, but surprisingly the work programme recommended by the D.E.T. for standard 6 presents irrational numbers in the 7th week while Pythagoras appears in the 23d week ! [Cf. appendix 6] The root cause can be traced to our continued adherence to a "thematic" presentation. Teachers certainly have the freedom to structure the syllabus in accordance with their views, and the D.E.T. acknowledges this freedom; however, the existence of a suggested work programme is a <u>strong deterrent</u>, in fact the existence of <u>any</u> suggested work programme, is a strong deterrent to individual teacher innovation as well as flexibility. Exchange of views and ideas at teachers' meetings and inspectors' and subject advisers' guidance could be of some value in promoting an integrated presentation of the content.

Though the syllabus relates Pythagoras to irrational numbers, it completely overlooks the possibilities with squares introduced earlier by omitting any mention of Pythagorean triples, which could in fact have prepared the ground for welcoming Pythagoras with familiarity later on [*]. The same weakness exists in algebra, and in particular in the chapter on algebraic expressions.

The standard 7 syllabus introduces the Cartesian plane, and teachers have the opportunity of making the coordinates of a point a meaningful new concept through Pythagoras. Unfortunately Pythagoras is left out of the standard 7 syllabus, which omission is a serious flaw as an opportunity for consolidation is lost. Alternatively, it breaks the "spiral" in the vertical development of the curriculum. Further, the conception of the standard 7 syllabus is weak as it does not prepare the ground for the functions $y = \pm\sqrt{r^2 - x^2}$ by making the circle familiar through the Pythagorean equation. This could easily have been achieved through the Cartesian plane, and there is no reason to think that pupils are not ready for it.

Even more serious is the omission of the circle equation in standard 8. The syllabus makers seem to have been operating under the set perception: triangles up to standard 8,

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* This investigator's personal experience as well as the experience of many teachers the world over who care to communicate through journals is that Pythagorean triples fascinate children in the mathematics classroom. In the present investigation, the standard 9 Experimental sample had not heard about Pythagorean triples, but with a little help they easily discovered one formula for generating primitive triples.

circles in standards 9 and 10, illustrating the constraining influence of the thematic approach on experts who should know better. It is this causative chain that is responsible for postponing the Pythagorean identities in trigonometry to standard 9. Had the circle been related to the Cartesian plane in standard 8, these identities could well have been introduced [at least for a first encounter] in standard 8. The semicircle functions, the Cartesian plane as a powerful graphical tool, the circle and Pythagoras could all have been integrated logically into the structures as well as learning. A more comprehensive account of this issue has been presented by Noble [1987].

In standard 9 Pythagoras is presented [cf. appendix 7] <u>without proof</u>, which was precisely what was done in standard 6. To expect pupils to retain what was done in standard 6 without reinforcement for two years is un-educational planning, to say the least. The proof of the theorem that comes in standard 10 is based on similarity, and that fact apparently justifies the placing of Pythagoras <u>with proof</u> in standard 10, but the curriculum planners missed the value of the entire story of the proofs of Pythagoras. Standards 7, 8 and 9 could have offered a variety of proofs culminating in the proof offered in standard 10. So much of value has been lost because of this omission.

If this kind of structured development of Pythagoras had been built into the curriculum, the exposition of the concept locus (of a point at constant distance from another) would have fitted naturally into the unfolding story of Pythagoras as an integral part.

2.1.2.2 On absolute value

The importance given in the syllabus to absolute value is minimal. Mirror symmetry is a key concept needed to learn this topic meaningfully. A purely algebraic treatment can lead to serious learning problems for pupils. [Cf. ch. 1, sec. 1.4.1 and 1.4.2]. The only explicit mention of symmetry in the syllabi is in para. 9 (geometry), standard 6, in the introductory comment [cf. appendix 8], but symmetry is implied and applied in standard 3 in connection with pattern work [cf. appendix 9].

It is somewhat of a contradiction on the one hand to say in the statement of intent that pupils should discover the orderliness of numbers [cf. sec. 2.1.1 above] and on the other to leave out the role of symmetry in the development of the curriculum. Even more importantly, symmetry could be given a place of value to achieve a high degree of integration of the lateral development of the curriculum. Its power was briefly indicated in chapter 1, sec. 1.4.3.

2.2 REVIEW OF SOUTH-AFRICAN PRODUCED TEXTBOOKS

The following is a brief and selective review, not quite comprehensive since not all textbooks available on the market could be collected for this purpose.

2.2.1 Primary and junior secondary levels

The following titles were examined:

Standard 3

ACTIVE MATHEMATICS: DREYER: DE JAGER-HAUM: 1981: 14TH IMPR., 1988

ROBOT: BUYS ET AL.: SHUTER & SHOOTER: 1982: 3RD IMPR., 1987

MATHEMATICS 2000: BISSESSOR ET AL.: NASOU: 1982: 5TH IMPR., 1987

PRIMARY MATHEMATICS: BOTES & VISSER: JUTA: 1980: 5TH IMPR., 1988

MATHEMATICS FOR ALL: LEVINSOHN ET AL.: JUTA-LONGMAN: 1980: 6TH IMPR.

MATHEMATICS CAN BE FUN: ENGELBRECHT ET AL.: VIA AFRIKA: 1ST ED., 1ST IMPR., (YEAR ?)

ROBOT (p.191) provides an exercise unit on mirror symmetry, [uses the term "line of symmetry"]. MATHEMATICS FOR ALL (p.138) also provides an exercise unit on mirror symmetry [uses the term "axis of symmetry"].

Standard 4

ACTIVE MATHEMATICS: 1981: 13TH IMPR., 1988

ROBOT: 1982: 2ND IMPR., 1985

MATHEMATICS 2000: 1983: 5TH IMPR., 1987

PRIMARY MATHEMATICS: 1980: 5TH IMPR., 1987

MATHEMATICS FOR ALL: 1980: 5TH IMPR., 1987

MATHEMATICS CAN BE FUN: 1988: 6TH IMPR., 1988

All except PRIMARY MATHEMATICS present exponential notation (base 10), right angles and square units, while MATHEMATICS FOR ALL and ROBOT pay attention to symmetry as well. MATHEMATICS CAN BE FUN does give a fair amount of exercise with the exponential form (base 10). [PRIMARY MATHEMATICS 3, 4 and 5 do not use the exponential notation at all.]

Standard 5

ACTIVE MATHEMATICS: 1984: 6TH IMPR. [YEAR ?]

PRIMARY MATHEMATICS: 1984: 4TH IMPR., 1988

MATHEMATICS FOR ALL: 1983: 2ND IMPR., 1986

MACMILLAN PROJECT IN MATHEMATICS: HAY & HAY: MACMILLAN-BOLESWA: 1985

CREATIVE MATHEMATICS: BARTLETT ET AL.: ACACIA: 1985: 1ST IMPR.

The area concept and the square unit for area are presented by all, but the exponential form of number is touched on (as revision) in MATHEMATICS FOR ALL only. This is a serious weakness in development of the spiral curriculum as already pointed out [sec. 2.1.2.1] in connection with the omission of Pythagoras in standards 7 and 8. The blame is not entirely on the authors of these textbooks, for they have simply followed the syllabus. Having said that, one notes that it is a more serious error on the part of the authors, for their task is not simply to write a textbook after the syllabus but to write one after critically interpreting it [*].

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* The authors of MACMILLAN PROJECT give themselves away in their very first sentence in the preface: "This textbook follows the order of the Work Programme for Standard 5 mathematics by the Department of Education and Training".

PRIMARY MATHEMATICS and MATHEMATICS FOR ALL dwell on number patterns in a logical manner and introduce SQUARE numbers which is a key concept needed to investigate and apply Pythagoras as the pupil climbs the "spiral". Between the two, MATHEMATICS FOR ALL is much more effective in exercises, particularly on square numbers. A word of commendation is due to the MATHEMATICS FOR ALL series, for frequent notes of guidance are found scattered through the texts, which are sensible as well as valuable to the teacher. Some notes concern the spiral character of the curriculum. For example:

Standard 3 [p.149]: "Please refer to Section B in Unit 13 (Length) in which your attention is drawn to the need to link this work with the work done in Unit 11 (Decimal Fractions)." Standard 4 [p.162] : "This section gives an indication of the kind of practical work which will help the pupils to comprehend the idea of symmetry. It may be supplemented by reference to symmetry in nature. The symmetrical properties of geometrical figures should also be _examined_". [Investigator's emphasis]

The emphasized aspect is very important; a topic assumes importance and acquires value in the eyes of pupils in direct proportion to its place in examinations. Educationally sound or not, it is a fact of life, and is a perception fostered by the "system".

Standard 5 [p.238]: "The work in this section, as well as the work in the following section on units of volume, is complementary to the work on area and volume in Unit 15. All this work should preferably be done together."

This is precisely the sort of approach that is shaped around Pythagoras in the investigation reported here.

Standard 6

MACMILLAN PROJECT IN SECONDARY MATHEMATICS: HAY & HAY: 1985: REPR.,1986

CREATIVE MATHEMATICS: BARTLETT ET AL.: ACACIA: 1985

CLASSROOM MATHEMATICS: LARIDON ET AL.: LEXICON: 1988

While squares and square roots are presented the geometrical square is related to the number square except in CREATIVE MATHEMATICS. CLASSROOM MATHEMATICS does this more effectively than MACMILLAN. Exponential numbers and their properties are presented in some detail in all three.

Standard 7

CREATIVE MATHEMATICS: STRAUSS ET AL.: ACACIA: 1985

CLASSROOM MATHEMATICS: LARIDON ET AL.: LEXICON: 1988

MACMILLAN PROJECT IN SECONDARY MATHEMATICS: HAY & HAY: 1985

MODERN GRADED MATHEMATICS: GONIN ET AL.: NASOU: 1985: 3RD ED., 1986

All except CREATIVE MATHEMATICS provide a certain amount of revision of exponents. MODERN GRADED MATHEMATICS does it in an applied manner, CLASSROOM MATHEMATICS does it in a fairly well organized manner, while MACMILLAN's treatment is a bare minimum. The relevance of the exponential notation and behaviour of powers in the development of algebra is apparently the motive for the revision.

Standard 8

CREATIVE MATHEMATICS: 1986: 1ST IMPR.

CLASSROOM MATHEMATICS: 1987

MACMILLAN PROJECT IN SECONDARY MATHEMATICS: 1985

MODERN GRADED MATHEMATICS: 1985: 3RD ED.: 1ST IMPR.

All except CREATIVE MATHEMATICS present the equation of the circle before graphing the functions $y = \pm\sqrt{r^2 - x^2}$. MODERN GRADED MATHEMATICS presents it briefly as "enrichment" material. MACMILLAN presents it as a necessary introduction but makes a mistake in referring back to standard 7 for Pythagoras [while it actually was in standard 6]. CLASSROOM MATHEMATICS presents a fairly comprehensive and integrated approach. It accords the circle a place of importance in a chapter, and relates it to the Cartesian plane, leads to the discovery of its equation, formalises its graph, looks at its symmetry and provides a number of useful exercises. This is truly as it should be, in spite of the weakness of the syllabus on this topic.

MODERN GRADED MATHEMATICS introduces the section on Euclidean geometry with a few worked examples,among which is one on similar triangles using the diagram which is normally used to prove Pythagoras by means of similarity. [This is prescribed material for standard 10, or for standard 9/10, taking the senior secondary as a single course. Not that there is any harm in using it in standard 8 but that the concept similarity has not been introduced in the text before, either in standard 8 or in standard 7.] Incidentally Pythagoras is applied. However, a little later they provide a short section on Pythagoras and its converse with a number of exercises which is effective in keeping the topic well revised. But they make the claim that Pythagoras (as well as its converse) was used in standards 6 and 7, while actually there is very little of Pythagoras in standard 7. One example was found in connection with the calculation of the area of a kite [p.234] and two or three exercises following it [p.236]. The note on Pythagoras tells that it is not required in all syllabuses. This "caution" is likely to put teachers off the topic; this is particularly likely considering the proof of the theorem offered by the authors. A selection of one or two historically exciting simple proofs would have served far better at this stage to keep the interest in Pythagoras alive among both teachers and pupils.

Trigonometry is introduced in standard 8: a fair amount of Pythagoras is integrated into this section in MODERN GRADED MATHEMATICS while the other titles do not compare well with MODERN GRADED MATHEMATICS in this respect.

Thus, the titles examined above give a mixed fare up to standard 8: some titles do not show any outstanding feature while some interpret the syllabus properly in parts.

2.2.2 Senior secondary level

The senior secondary phase is examined below with specific reference to Pythagoras and absolute value. The titles examined are:

ACTIVE MATHEMATICS: STANDARD 9: DREYER ET AL.: DE JAGER-HAUM: 1986

JUST MATHEMATICS 9/10 (HG): DE JAGER ET AL.: MASKEW MILLER LONGMAN: 1985: 3RD IMPR., 1986

MATHEMATICS IN ACTION: STANDARD 9: ROOS ET AL.: JUTA: 1988

SUCCESSFUL MATHEMATICS 9: COMMINS ET AL.: OXFORD UNIV. PRESS: 1987

MATHEMATICS FOR THE EIGHTIES: STANDARDS 9 & 10: ENGELBRECHT & LUBBE: VIA AFRIKA [YEAR ?]

MODERN GRADED MATHEMATICS: STANDARD 9: GONIN ET AL.: NASOU: 1986: 2ND IMPR., 1987

CREATIVE MATHEMATICS: STANDARD 9: STRAUSS & DREYER: ACACIA: 1988

CLASSROOM MATHEMATICS: STANDARD 9: LARIDON ET AL.: LEXICON: 1987

CLASSROOM MATHEMATICS: STANDARD 10: LARIDON ET AL.: LEXICON: 1987

2.2.2.1 On Pythagoras

ACTIVE MATHEMATICS [pp 213-217]

In the trigonometric context Pythagoras is taken for granted, not mentioned; and, the trigonometric form of the Pythagorean identity is first stated and "proved". A circle is shown on the coordinate plane, but neither the equation of the circle nor Pythagoras directly correlated to the situation. All illustrative exercises are worked out in trigonometric form, a parallel coordinate-algebraic form is totally absent. [This is the case with all the titles examined.]

JUST MATHEMATICS [pp 143-144]

The Pythagorean identities are obtained by a LHS-to-RHS transformation in the trigonometric form: the coordinate-Pythagorean connection provides the transformation. The two worked examples on p.144 are stated to be "harder" : but if Pythagoras is applied in the coordinate form along with the coordinate definitions of the trigonometric functions, the following alternative path suggests itself:

Simplify: $\sec^4 x - \tan^2 x\ (\sec^2 x + 1)$.

Solution:

$\sec^4 x - \tan^2 x\ (\sec^2 x + 1)$

$= \sec^4 p - \tan^2 p\ (\sec^2 p + 1)$, [the symbol x is better reserved for the coordinate]

$= r^4/x^4 - (y^2/x^2)(r^2/x^2 + 1)$

$= r^4/x^4 - r^2y^2/\ x^4 - y^2/\ x^2$

$= [r^2(r^2 - y^2)]/x^4 - y^2/x^2$

$= r^2x^2/x^4 - y^2/x^2$

$= r^2/x^2 - y^2/x^2$

$= (r^2 - y^2)/x^2$

$= x^2/x^2$

$= 1$.

The direct trigonometric solution here is much quicker, but the substitution $\sec^2 x - 1$ for $\tan^2 x$ has to become obvious to the pupil. To the extent that the main objective of such questions is to make pupils facile with direct trigonometric transformations the question merits no adverse criticism. However it is an educational virtue to let pupils know of alternative procedures which might help them out when stuck with one procedure. [A more detailed argument follows later on.]

The second example is again one of simplification.

Simplify: (tan x + cot x) sin x.

The two routes are shown side by side.

<u>As adopted in the textbook</u>	<u>Coordinate form</u>
(tan x + cot x) sin x	(tan p + cot p) sin p

$= (\sin x/\cos x + \cos x/\sin x) \sin x$

$= [(\sin^2 x + \cos^2 x) \sin x]/\cos x.\sin x$

$= \sin x/\cos x.\sin x$ [Identity : $\sin^2 x + \cos^2 x = 1$]

$= 1/\cos x$

$= \sec x$.

$= (y/x + x/y)y/r$

$= [(y^2 + x^2)/xy]y/r$

$= r^2/xr$

$= r/x$

$= \sec x$.

The value of such parallel solutions lies in the deeper insight they can provide.

<u>MATHEMATICS IN ACTION</u> [pp 204-205]

The trigonometric functions are stated in the coordinate form and the Pythagorean identities established. No mention is made of Pythagoras. It is interesting to note that the authors indicate the use of Pythagoras in the root form [$r = \sqrt{x^2 + y^2}$] which does not help to reinforce the central role of Pythagoras.

<u>SUCCESSFUL MATHEMATICS</u> [pp 104-105]

Pythagoras is properly and effectively transformed to obtain the first of the three Pythagorean identities. The value of the approach is suddenly lost sight of when the authors obtain the other two identities by means of transformation of the first.

<u>MATHEMATICS FOR THE EIGHTIES</u> [pp 432-433]

The first identity is established exactly as in MATHEMATICS IN ACTION, while the next two as in SUCCESSFUL MATHEMATICS.

MODERN GRADED MATHEMATICS [pp 304-307]

The identities are stated, not establised. Pythagoras is not mentioned, but that a right triangle is needed is mentioned. And, one of the identities is given as a worked example, wherein a right triangle is shown with Pythagoras noted in symbols but not named. Also, by using the variable x as the angle, the virtue of placing the identity on the x-y plane is lost.

CREATIVE MATHEMATICS [pp 214-217]

The presentation of the Pythagorean identities follows exactly the same direction as is favoured in this investigation. The diagram relates parts of the entire information on the coordinate plane in which Pythagoras stands out prominently, though it is not indicated. However:

(a) Pythagoras is not mentioned at all. It seems the statement $x^2 + y^2 = r^2$ is to be understood as the equation of the circle (which of course is correct). Thus the unifying role of Pythagoras is not recognized and taken advantage of in the presentation of the topic.

(b) The presentation degenerates into the "recipe" form [p. 215] when the authors list 1-to-5 items as to how to handle identities.

One example is reproduced here for comparison of the alternative routes as was done above with JUST MATHEMATICS.

Worked example 3 [p.216]:

Prove that $2\ \text{cosec}^2 A - \text{cosec}^4 A = 1 - \cot^4 A$.

Trigonometric form [Textbook]

$LHS = 2\ \text{cosec}^2 A - \text{cosec}^4 A$

$= \text{cosec}^2 A\ (2 - \text{cosec}^2 A)$

$= (1/\sin^2 A)[2 - (1/\sin^2 A)]$

$= (1/\sin^2 A)[(2\sin^2 A - 1)/\sin^2 A]$

$= (2\sin^2 A - 1)/\sin^4 A$

$RHS = 1 - \cot^4 A$

$= 1 - \cos^4 A/\sin^4 A$

$= (\sin^4 A - \cos^4 A)/\sin^4 A$

$= [\sin^4 A - (1 - \sin^2 A)^2]/\sin^4 A$

$= (\sin^4 A - 1 + 2\sin^2 A - \sin^4 A)/\sin^4 A$

$= (2\sin^2 A - 1)/\sin^4 A$

LHS = RHS.

Coordinate form

$LHS = 2(r^2/y^2) - r^4/y^4$

$= (2r^2y^2 - r^4)/y^4$

$RHS = 1 - x^4/y^4$

$= (y^4 - x^4)/y^4$

Denominators of LHS and RHS equal.

∴ Required to prove:

$2r^2y^2 - r^4 = y^4 - x^4$

$LHS = 2r^2y^2 - r^4$

$= 2(x^2 + y^2)y^2 - (x^2 + y^2)^2$

$= 2x^2y^2 + 2y^4 - x^4 - y^4 - 2x^2y^2$

$= y^4 - x^4$

= RHS.

The coordinate form deserves to be allowed alongside the trigonometric form.

CLASSROOM MATHEMATICS, STD 9 [pp 226-227]

A circle, radius r, centred at the origin of the x-y plane is made use of to obtain the three identities. However, Pythagoras is not mentioned, instead $x^2 + y^2 = r^2$ is used as the equation of the circle. All the three identities are obtained directly from this equation. The identities are repeated in standard 10 [pp 255-256], but this time the direction is reversed: the transformation from the LHS to the RHS is achieved through the squared coordinates and radius vector on the Cartesian plane as in JUST MATHEMATICS, but unlike the latter, there is no mention of Pythagoras; instead $x^2 + y^2 = r^2$ is used as a circle property.

Also, the section on "Proving Identities" merits criticism. The general tenor is akin to the "recipe" approach which was criticised above [CREATIVE MATHEMATICS]. More specifically, point number 3: "If no other approach suggests itself, express the functions in terms of sines and cosines and then simplify" misses the point of learning this topic as well as the purpose of problem solving exercises. Whether it is a classroom, homework or examination context of problem solving, the exercise is intended either to promote <u>integrated</u> understanding or to apply the integrated understanding to the solution [unravelling of the apparent mystery] of a novel situation. The "instrumentalism" hidden beneath the recipe-point number 3 is not conducive to encouraging relational understanding [cf. Skemp]. Further, the point in question is suggestive of a questionable value hidden in the instruction, viz.: to adopt this particular strategy is not "dignified", or "a mark of clever people", or "the right thing to do", etc.; briefly, there is something negative about it, and the hidden curriculum here is detrimental to pupils' long-term interests. Yet again, it would have been a far more positive suggestion, had the authors recommended adopting the coordinate form. This investigator believes that it is educationally important as well as fruitful for pupils to know that in the trigonometric form one is manipulating relations in terms of ratios of x, y and r while in the coordinate form one is handling the same situation in terms of x, y and r themselves. There are valid reasons to learn, and to learn to apply the trigonometric form, but that process will only be enriched by understanding the primary coordinate basis at all levels.

Similar comments could be made about point number 4 as well.

The general tone of the critical remarks on the textbooks reviewed above is not to be viewed in too negative a manner in relation to South-African produced textbooks. It is fair to remember what Gadanidis said as recently as 1988 about textbooks in Canada.[Cf. ch. 1, sec. 1.3.1] Briefly: Textbooks in South Africa as elsewhere generally follow the beaten track. Imaginatively written textbooks should be produced in the interests of mathematics education, even if such productions may not guarantee as much financial return as the traditional textbooks. Such innovative changes are likely to take root gradually and cause changes in teacher/pupil perceptions of quality textbooks.

The criticism that the reviewed textbooks do not use Pythagoras as a central unifying concept is to be tempered by a couple of relevant observations:

(a) What applies to Pythagoras applies to other concepts and material of value in mathematics education. To write and produce textbooks explicitly providing such networks of inter-relations in a comprehensive manner is not cost-efficient in a competitive market.

(b) From a more academic point of view, one could argue that the manner of detailed exposition of subject matter is not to be prescribed since method of presentation is a highly personalised aspect of teaching, and that the teacher is the ultimate controller in this matter, and not the textbook writer. Though logically convincing, the unfortunate fact of life is quite different: many teachers rely on a single textbook [*].

These are not new problems, they are as old as mathematics education (and printing!). This is so for the simple reason that the problems of education are far too complex to find quick and easy answers. Nevertheless academics are duty bound to worry about these and suggest answers, solutions, remedies.

Textbook business is big business: publishers, printers, authors and distributors all have a stake in the returns. Naturally competition is stiff. A variety of factors control the choice of a particular textbook at a particular school. "Instrumental Mathematics" [cf. Skemp] is far

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* There is, and always has been a minority of teachers who well research the material they present. They are the exception and the criticism here does not apply to them.

easier to produce and guarantees maximum return . To make an effective dent into this structure with established vested interests by producing textbooks which offer "relational mathematics" [cf. Skemp] is not easy, economically as well as organizationally. Substantial subsidies [from Government or Private or both sources] to promote such a venture might be a strong incentive. Production of textbooks presenting mathematics with totally new approaches to promote relational understanding in pupils, and their distribution at nominal below-cost prices should be encouraged in order to combat the persistent danger of "instrumental mathematics" in the commonly available textbooks. This is only one "front" of the battle.

Teachers themselves will be the greater obstacle in this programme.

Among the main difficulties listed and argued by Skemp [1976] on this issue is included the psychological difficulty.

> The great psychological difficulty for teachers of accommodating (re-structuring) their existing and long-standing schemas, even for the minority who know they need to, want to do so, and have time for study.
>
> [Skemp, p.24]

Provision of substantial incentives for upgrading and re-orientating one's perceptions of mathematics and the value of teaching "relational mathematics" might encourage teachers to overcome this difficulty with patience and perseverance.

Sufficiently frequent re-orientation courses and plenty of resource materials for re-orientation, and post-re-orientation support programmes are some of the many aspects of the whole issue. The scale of operation evidently calls for a national, well supported endeavour. It is not surprising that "instrumentalism" thrives in the absence of such significant interventions.

2.2.2.2 On absolute value

The problems of teaching absolute value were examined at some length in chapter 1. The virtue of teaching this topic through a symmetry-based approach was also argued. How our textbooks present this topic is now examined.

<u>ACTIVE MATHEMATICS</u> [pp 29-41]

The symmetry of the function $y = |x - a|$ about the position $x = a$ is "conceded" rather than exploited for comprehension. The presentation is fairly efficient, but caters for instrumental understanding. The algebraic procedure is centred around the formal symbolic definition of absolute value [cf. Brumfiel, definition 5, ch. 1].

<u>JUST MATHEMATICS</u> [pp 58-62]

The discussion begins properly with direct examples, and not from the formal definition. The "folding" upward of the parts of the straight lines below the x-axis is left to the teacher to explain. It would have been acceptable had its importance been indicated. This omission leads to missing out on the opportunity to relate the whole concept to symmetry. Further, though cautiously, [almost apologetically!], the formal algebraic definition is introduced as the primary means for developing the topic.

MATHEMATICS_IN_ACTION [pp 10-11; 34-35]

Totally instrumental presentation; starts from the formal definition.

SUCCESSFUL_MATHEMATICS [pp 4-5; 22-25]

Introduction based on direct experience is effective. Symmetry is implicit, also explicit in the numberline diagrams, though not stressed. The concept absolute value is linked to the square root relation, which approach contributes towards understanding. However this direction of development is not fully exploited, and each exercise is finally algebraically handled. More importantly, the illustrative diagrams [pp 4-5] leave the symmetry axis at x = 0, viewing the variable as (x - a), and do not translate the axis to x = a for the variable x. This should be treated as a serious omission. The further discussion [p. 22 onwards] is tailored more or less to "instrumental" learning.

MATHEMATICS FOR THE EIGHTIES [pp 86-87]

Very formal presentation; the authors do not seem to appreciate/ to have experienced pupils' difficulties in learning and applying this concept.

MODERN GRADED MATHEMATICS [pp 62-67]

The preliminary part of the exposition is fairly effective in forming the concept absolute value. However it turns out to be the means only for transition to formal algebraic treatment. Numberline representations are plenty but no attempt has been made to bring symmetry into the picture.

CREATIVE MATHEMATICS [pp 2-7; 46-47]

Arithmetical ground is provided only for transition to the formal algebraic form, and the approach is based on the formal definition and the examples are worked out accordingly; the numberline graphs show the inherent symmetry but its role is not exploited towards teaching for understanding. In graphs on the x-y plane the direction of discussion is towards "finding the axis of symmetry" rather than developing the graph from the fact of symmetry.

CLASSROOM MATHEMATICS [pp 152-169]

There is no pretence of anything but the formal development starting from the algebraic definition. A brief explanatory discussion comes a page later [p. 153] and thence the emphasis is on the definitive form. Plenty of examples and exercises are provided, but they are only of value to those who are able to understand the concept and its implications. To those who find the formal approach difficult the many examples and exercises are not likely to make relational comprehension easier. However, to the persistent few they are more than likely to help develop a measure of proficiency in the procedures, but relational understanding is not guaranteed. Axis of symmetry is handled as an incidental property, rather than as the central property.

2.3 CONCLUSION

(a) On the whole it seems that the authors of the reviewed textbooks are all constrained by their own "one-track" approach to the issue, viz: the development of the theme from the formal definition. What is more disappointing is the impression one gets: that these authors are either not aware of, or do not care about, the need for drastically new approaches to teaching this topic. Also, they do not seem to be abreast of the developments abroad in teaching absolute value in a more meaningful manner.

(b) The South-African produced textbooks by and large follow the traditional thematic development of school mathematics. There is hardly any indication of bold breakaway from this tradition in order to present integrated development which would contribute towards the enhancement of pupils' conceptual structures in a holistic manner.

CHAPTER 3

DESCRIPTION OF APPROACH AND IMPLEMENTATION DIFFICULTIES

3.1 INTRODUCTION

Two secondary schools, not far from the college of education where the investigator works, within about two to three km of each other, were chosen for the classroom research. Preliminary liaison was established with the principals, heads of departments and standard 9 mathematics teachers of the respective schools early in March 1989. A brief overview of the purpose and methodology of the research was presented to the mathematics teachers of the two schools and the area of investigation was identified to them. The suggested period of research, the last week of April to mid-May, was found acceptable to them.

3.2 PROBLEMS

3.2.1 Design

A two-group [E-C, Experimental-Control] design was suggested in the original proposal with a possible adoption of a three-group design, if the change was found possible. It was decided at the preliminary discussion with Professors Marsh and Irwin to give the three-group design a try. However there was only one standard 9 division in each of the two schools offering mathematics [both HG]; fortunately one class was 57-strong and so could be divided into two groups - the E [Experimental] and C1 [Control 1] while the other class (about 40-strong) could be made C2 [Control 2].

3.2.2 Administration

3.2.2.1 Timings

The investigator could not find a sufficient number of free periods at the college to carry out the investigation during regular working hours at school. Even if he had, such an arrangement was not acceptable to the E school. C2 school, however, let him do the experiment during their regular mathematics periods, (which fact turned out to be of crucial influence on results as pointed out later), and even made a few changes to accommodate him. Thus the investigation took place during regular periods at the C2 school, and during the afternoon study session at the E school.

3.2.2.2 Sample

Absenteeism in a short span of two weeks was not reckoned as a serious threat, but the investigator was totally naive in this respect. Apart from the randomness of absenteeism reducing the strength of the sample which attended <u>all</u> the lessons, the extra-curricular activities of the schools also contributed to sample depletion. At the C2 school those pupils who were members of the school choir were taken away from class during regular periods for practice for the inter-school competition, while at the E school pupils practising games were exempted from attendance at study hours in the classroom. Thus the useful sample size at the post-test was only about 20-plus.

3.2.3 Pupil motivation

The purpose of the investigation was briefly explained to the pupils at the first encounter and the pre-test [used as a diagnostic test] was administered the next day. Pupil motivation was fair at the C2 school; in contrast the scenario at the E school was near chaotic:

(i) pupils seemed not too keen to attend lessons; [it was not clear whether it was the mathematics or the timing, viz., after-hours, that contributed to lack of motivation];

(ii) discipline was lax during the study session - there occurred significant pupil movement around the campus;

(iii) absenteeism at study session did not seem to be checked effectively - teacher supervision was not strict;

(iv) one half of the class [either the E or the C1 group] had to be asked to wait in another classroom without providing a motivating programme while the investigator was busy with the other group in their own classroom. [Both groups were engaged everyday, alternating the sessions on successive days].

On the whole the E school did not offer the investigator an atmosphere conducive to education which could have ensured the success of the investigation.

3.2.4 Course administration and post-test

The absolute value concept and its application to solution of equations as well as inequalities was the topic. [This was to be followed by Pythagoras] About six hours' instruction was given to each of the three samples. C1 and C2 samples were given a formal treatment of the topic closely following ACTIVE MATHEMATICS which was the textbook used at the C2 school. [Cf. ch. 2 for a review of this textbook] The E group was offered a development around an axis of symmetry.

Identical post-tests were administered to all the three groups at the end of the instructional session on the same day. The analysis of the post-test supports the following observations:

(a) There was no significant difference in performance between the E and the C1 groups.

(b) The C2 group fared better, a negative result in relation to the contention put forward by the investigator in his proposal.

Considering the adverse factors listed above, it seemed fair to conclude that:

(a) the investigation was not likely to be of value if continued at these schools in the circumstances;

(b) the part investigation which was conducted however was valuable as a pilot study;

(c) the design of the investigation needed some revision.

3.3 REVISED PROPOSAL

In response to the investigator's suggestion Prof. Marsh accepted the proposal that the investigation be conducted at the investigator's college, taking the samples from the Course I Senior Primary teacher trainees. This was not quite the ideal alternative but appeared to be a realistic one, the advantages and disadvantages of which are examined below.

3.3.1 Advantages

(a) Easy access to samples.

(b) Sample response was likely to be better because these students were relatively more mature than the standard 9 pupils. [This "fact" was believed to contribute to awareness of the importance of such investigations and thus motivate the students. As things turned out, the investigator was naive in this respect.]

(c) Location of samples would be helpful in offering the best possible treatment in terms of instructional aids.

3.3.2 Disdvantages

(a) Most importantly, this was a proxy situation, simulating the real classrooms of schools in (urban) Mdantsane; the simulation was likely to deviate from the real.

(b) Most of the Primary Teacher trainees have standard 8 mathematics only, with relatively low grade-symbols. [The few matrics with mathematics were to be excluded from the samples without their knowledge.]

(c) The topics of investigation were not of direct interest to them.

After having examined the issue, the investigator decided to bank on the maturity of the students for the success of the investigation.

3.4 EXPERIENCE WITH THE COLLEGE SAMPLES

The same pre-test [as a diagnostic instrument] as the one administered at the schools was administered to the two classes which were chosen arbitrarily out of the five available classes doing course I. [The choice was made solely to suit the personal time table of the investigator.] A two-group design was adopted. It was found that by excluding the trainees having matric mathematics and a few other students who were very weak in mathematics, two very evenly matched samples could be drawn. There was no particular motivation to choose either as the E [Experimental] group: in the event the choice of one as the E and the other as the C [control] was arbitrary.

Lessons were presented with enthusiasm and the E group was given plenty of first-hand learning experiences on mirror symmetry, since the topic was absolute value as at the schools. Student response seemed to be encouraging. [Stationery was generously supplied also as at schools]

However, the post-test result was simply disastrous: nearly 20 out of 30 students in each group scored zero out of a possible maximum of 25. The investigator suspected lack of motivation and so administered a questionnaire to find out as much as possible about the attitudes of these trainees concerning mathematics education. The responses to the questionnaire were revealing and the investigator is convinced that lack of student motivation arose from their perceptions: <u>This exercise did not seem relevant to them.</u>

3.5 SCHOOLS AGAIN

The possibility of conducting the exercise at schools again was considered, with two changes, viz:

(a) Subtle "public relations" exercises were to be undertaken in advance to create a climate of expectation on the part of pupils;

(b) Pythagoras was to be presented instead of absolute value, the latter to be considered only if time permitted.

3.5.1 NE [New Experimental] school

It was already the last week of August, and so it was felt that the whole exercise could be "promoted" as something that would help pupils at the end-of-year examination. The standard 9 mathematics teacher of a school near the investigator's college was discreetly approached and her cooperation ensured. She agreed "to prepare" the pupils for the exercise through a series of briefs over a two-week period: the central message was that the course was going to be of <u>immediate relevance</u> to them. In the event, the investigator believes that the strategy paid off reasonably well, for when he met the pupils face to face for the first time they seemed to be positively looking forward to participation in the exercise.

School NE was chosen as the experimental school in which to present and evaluate the experimental treatment of Pythagoras without any more delay. [The search for a control school was still going on]

The choice of Pythagoras for the treatment was not the only change in the programme. The two previous experiences had highlighted problem areas: for example,

(a) time was short for such an exercise, so one had to be very economical in lesson presentation.

(b) The traditional pupil role of copying lesson material down from the chalkboard and spending time with paper and pen was likely to trigger a "psychological defence" resistance to mathematics on the part of pupils and thus make it harder for the investigator to succeed.

Some serious thought was given to these two related aspects and a solution was found in which aspects (a) and (b) made a welcome holistic union.

(i) A variety of graph papers and square nets were prepared with the necessary diagrams, definitions, hints etc. so that pupils had practically nothing to copy from the chalkboard. [Cf. appendix 16.]

(ii) Lesson issues were developed as "problems" generated with reference to the material on the hand-out sheets and pupils were immediately "hooked on" to the sheets through such activities as completing diagrams, making measurements, makimg comparisons, doing (fairly simple) numerical calculations (on the back of the hand-out sheets), etc.

(iii) Inductive discovery was adopted as the major instructional technique.

(iv) Creating opportunities for holistic structural unification was the investigator's primary role.

(v) No home work was given. [Home work seemed to be the last thing the pupils wanted: experience with the earlier samples]. Emphasis was on comprehension and a majority of pupils <u>seemed</u> to understand relationally. However, the ultimate test of understanding would be the post-test.

The post-test results did not turn out to be as successful as the investigator hoped they would; at the same time they were not mediocre either. The investigator believes that the results would have been appreciably more convincing had the pupils let themselves operate the essential reinforcement mechanism of learning at home. The investigator could not, and honestly did not expect this of them, for in his experience the school performance in mathematics of the pupils in Mdantsane schools does not indicate systematic preparation after school [*]. So the pupils were responding from their previous knowledge and understanding modified to such extent as the investigator could possibly have managed in about eight hours of

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* The investigator's first-hand experience with college trainees since 1983 is that they do not want a test or to hand in an assignment on a Monday: they do not mind a Tuesday. Morals: 1. Weekends are for activities other than serious study. 2. To them a day's preparation was sufficient to take a test or to write up an assignment.

interaction spread over two weeks during the study session.

3.5.2 Post-script on NE school

The investigator had established a positive liaison with the mathematics teacher at the NE school privately and days before he elected to approach the principal formally. He had official permission to conduct the study, granted by the Director General, Department of Education, Ciskei, well before he approached the first school. So far no principal (or the rector of the college) had really wished to verify the document granting this permission. The principal at NE did want to see the document and he questioned the investigator in detail as to what was going to be the nature of the lesson contents. [He was acting positively within his rights and responsibilities. That is not the point of interest here] When the investigator explained to him that he intended to present a certain topic in standard 9 mathematics in a particular manner and test the response of pupils in a test for comprehension at the end of the exercise, he (the principal) wondered whether it was not going <u>to confuse</u> his pupils at a time close to the end-of-year examinations. His concern was justified and the investigator did convince him (by going into some detail) that if anything, his approach was intended to integrate their knowledge in different parts of the syllabus. The moral of interest here is that there are principals (of schools) who look upon researchers with suspicion, and that a good measure of integrity and tact is essential in personal interactions of this kind. At the same time the incident also highlights the fact that some principals do not appreciate the academic worth of such exercises. In spite of this comment, this principal deserves appreciation in that he was concerned about the role of outside intervention of this kind at his school, in sharp contrast to the principals of all the other schools where the investigator was involved in this classroom research, for the latter seemed to be solely concerned about the administrative aspects of the issue such as disruption of timetable, loss of periods, etc; not one cared to probe into the educational implications. This comment unfortunately applies also to the mathematics teachers with whom the investigator came to interact in connection with this exercise. It all points to one massive truth: much of the mathematics teaching in these schools is tailored to achieve "instrumental understanding". [Cf. chapter 1.]

3.5.3 NC [New Control] school

There are a number of secondary schools in Mdantsane offering mathematics HG, so the choice of a school did not appear to the investigator to be a problem right from the beginning. When the first investigation failed on account of reasons discussed above, the choice of a school became problematic for the first time. The experience at the E school particularly suggested that a measure of order should prevail at study sessions for the exercise to succeed. So a school where the principal was a strong personality would be a good choice, if not the best in the circumstances. Just such a school was school X, only a few km away from the investigator's college. The added bonus was that the gentleman apparently seemed to value academic enrichment of teachers in general, i.e. that he was pleased to let the investigator as a teacher enjoy the benefits of academic enrichment, intrinsic as well as extrinsic. [The investigator's personal acquaintance with him since 1983 was a third consideration in favour of this school.]

However, when the investigator met the pupils to administer the pre-test he discovered to his dismay that there were only 18 pupils doing mathematics and that there was only one standard 9 class there doing mathematics and science. The sample strength was evidently not satisfactory for the investigator's purpose and he had to start the search for another school all over again.

Now, upon collecting accurate statistics about all the secondary schools in Mdantsane the investigator discovered that only about half a dozen schools are strong in mathematics HG numbers. The investigator's option was now very limited, and his next contact was made with the mathematics teacher at the NC [New Control] school with whom he literally had to make a bargain, in spite of his pleasant manners and helpful attitude, in return for his active "public relations" part with the standard-9's on the investigator's behalf: the quid pro quo was this: on the days the investigator visited the school he was also to do some tuition for the standard-10's who were shortly writing their matric examination. He gladly agreed to do this and the young teacher in return actively organized the cooperation of the pupils. The classroom research went off fairly smoothly and the same amount of time was given to the course as at the NE school. Discussion of results follows in the next chapter.

CHAPTER 4

DISCUSSION OF RESULTS

4.1 ON EXPERIMENT DESIGN

It has already been mentioned in context that the classroom exercise failed twice, and the reasons for the failure were also examined in some detail. Thus the exercise that produced some results worthy of discussion was the third and final attempt in which a two-group design was adopted.

4.1.1 Sampling

The samples were drawn from two different schools a few km apart, already referred to as NE (the New Experimental) school and NC (the New Control) school. The exercise was conducted during the study session after the regular school periods. March to August or early September is the time when study sessions are taken more or less seriously in most schools in Mdantsane. Even then absenteeism is a serious problem, notwithstanding the principal's corporal punishment the next morning. From late September the routine becomes lax.

However, compulsion was neither contemplated nor possible in the circumstances. The goodwill created by the teacher [cf. ch.3] was the best bet for obtaining a sample of reasonable size. There was dropout from the sample which took the pre-test, so the number that took the post-test at the NE school was taken as the standard for the NC school (where the course started soon after the exercise was completed at the NE school). The pre-test analysis and matching of samples was postponed until the end of the whole exercise in order to allow for sample depletion and to make sure that the finally-selected samples had attended the whole course.

4.1.1.1 Population

As indicated in the investigator's original proposal (p.12) [appendix 10] of December 1988, the classroom research was intended to draw useful and valid conclusions which would apply to standard 9 pupils doing mathematics HG in urban schools in the Ciskei. There are about a dozen senior secondary schools in Mdantsane township in which about 400 to 500 pupils do mathematics HG in standard 9. Considering the fact of vertical movement of pupils and the expectation that the mathematical learning as well as potential of this population is likely to remain constant, the population relevant to this investigation would be a few thousand-strong, looking at the flow of pupils through a five-year period, for example. In a nutshell, the population is large enough to allow the use of statistical techniques.

4.1.1.2 Samples and their representative character

The story of the choice of schools for this investigation has already been told. [Cf. ch.3.] The impracticability of random selection of samples was pointed out in that context. The next best thing to do was to examine whether the chosen samples represented the population, and if so, to what extent. Two aspects could be identified here: (i) whether the two samples, Experimental and Control respectively, belonged to the same population, and (ii) whether that population was representative of the population identified in sec. 4.1.1.1 above. These issues are discussed below.

(i) A pre-test was administered to both samples and a two-tailed **t**-test of significance was applied to settle the issue [Behr,1983]. The summary of this analysis is presented in the statistical data sheet [appendix 11].

$t_c = 0,78$; $t_t = 2,01$ [5%], 2,68 [1%] ; df = 46 .

These values show that there is no significant difference between the two samples in relation to whatever is measured by the pre-test. In other words, the samples NE and NC belong to the same population with respect to the attribute measured by the pre-test.

(ii) Whether the above population was the same as the standard 9 mathematics HG population of Mdantsane is to be established by careful analysis and argument. If the samples were randomly drawn from the Mdantsane schools, they would evidently represent this population. A random selection was however not possible due to reasons already discussed. The choice of schools, the way it actually happened, implies a dimension of "convenience sampling" [Cohen and Manion, 1984, p.100]. The NE sample was thus a "convenience sample". The NC sample was deliberately picked in order to match the former as closely as possible, and so implied "purposive sampling" [Cohen and Manion, p.100]. It is this matching that made the two samples equivalent and so represent the same population as best as possible as argued in (i) above.

Matching was achieved by a pupil-to-pupil correspondence on the basis of the pre-test scores, and in so doing 11 pupils at NC who were in excess of the 24 who took the post-test at NE were excluded. Inclusion of these 11 would make the NE and NC samples appreciably different as was seen when a t-test was carried out. Though 6 of the 11 with scores (10; 13; 12; 13; 10; 9) could be included to obtain samples which corresponded favourably at the 1% level it was decided to have excellent matching at the risk of losing a few pupils in the NC sample.

The following technique was adopted to test whether the samples were representative of the actual population which is the target of this investigation. At each school a new sample was constituted including those who took the pre-test, but dropped out as the course progressed: i.e., the actual sample became a subset of the whole sample that took the pre-test. A two-tailed t-test was carried out as before to see whether the subset and the set, i.e., the post-test sample and the whole pre-test sample, belonged to the same population [cf. appendix 11].

NE : $t_c = 1{,}58$; $t_t = 2{,}02$ [5%], 2,72 [1%] ; df = 60 .

NC : $t_c = 2{,}56$; $t_t = 2{,}02$ [5%], 2,70 [1%] ; df = 65 .

It is clear that within reasonable limits the post-test sample is representative of the entire sample that took the pre-test at NC while it is even more strongly so at NE.

Whether the larger samples at NC and NE represent the Mdantsane standard 9 mathematics HG population is another matter. That is somewhat difficult to establish within the constraints of the experiment. Nevertheless it is possible to compare all the four schools involved in the investigation by means of t-tests. Alternatively, ANOVA could be employed, but since data for t-tests had already been prepared periodically it was decided to compare two schools at a time by means of t-tests.

Findings: Of the four schools, E, C, NE and NC, E is the only exception; C, NE and NC belong to same population, [cf. appendix 11]. The E school was the subject of critical assessment in chapter 3. Those comments are justified by the observation here that it does not belong to the same population as the other three schools. Thus it is fair enough to assume that the conclusions of this investigation could be generalized to a sufficiently large population in Mdantsane secondary schools. Incidentally, the above discussion has taken care of an important aspect of external validity, viz., the representativeness of the available and target populations [Cohen and Manion, p.198].

The issue of representativeness of the sample target populations is argued above in terms of t-tests on numerical data. It is worth supplementing it by looking at the NC and NE schools in their context. Both are public (i.e., state/ community supported) schools as are all the schools in Ciskei except one in Bisho (viz., All Saints Senior College) which does not fall under the control of the Ciskei Education Department.

Mdantsane is a sprawling black township of a population about 200 000 strong having a number of primary schools, 12 senior secondary and 2 junior secondary schools. The investigator is familiar with all these secondary schools through teaching practice exercises. Admission of pupils to these schools is mostly controlled by principals who apply criteria which may vary from school to school. Half of these are large schools accommodating up to 800 or more pupils while the other half are comparatively smaller with enrolment below 500. The principal's admission criteria and the capacity of the school are the primary constraints on admission of pupils to any particular school, with the result pupils come from all over Mdantsane to almost every secondary school. The investigator's informal interviews with some of the (mathematics) teachers at both NC and NE schools have confirmed this fact.

A number of aspects pertaining to secondary schools in general and mathematics teaching in particular were touched on in the interviews. The emphasis was on collecting factual information rather than opinions, even though the latter were not ruled out. A summary is made out below for easy comparison.

Parameter	School NE	School NC
Strength:		
Pupils	850	700
Teachers	30	20
Mathematics teachers	5	4
Average number of pupils per math. class	60	50
Location	Within the central area of the town.	Just off the central area of the town.
Pupil distribution	From all over Mdantsane: at both schools.	
General facilities	No significant difference between the two.	
Teaching aids for mathematics	Chalkboard instruments only: at both schools. (Note: Pupil activity leading to inductive discovery not a routine at either school.)	
Attendance:		
Teachers	Regular at both schools.	
Pupils	Regular at both schools.	
Teacher-pupil relationship	Generally good at both schools.	
Homework	Large scale copying existed at the NE school according to the teachers interviewed.	The existence of copying denied by both teachers interviewed. They claimed that pupils were called at random to do homework on the chalkboard and those who failed to do a question after having shown the correct workout in the homework book were "taken to task".
Subject policy	Both schools follow guidance from the Ciskei Education Department.	
Syllabi and classwork	At both schools prescribed syllabi covered by engaging extra classes during afternoon study as well as on Saturdays.	

Parameter	School NE	School NC
Parent involvement in academic matters	Nil at either school.	
Parent feedback on academic report on pupils	Nil at either school.	
Individual parent visit to school	Only when called by the principal: at both schools.	
The top 5 secondary schools in Mdantsane in the opinion of the interviewed teachers	Teachers at both schools gave the same list, viz., NE, NC, C, E and school X. (Note: The order of merit was not requested.)	
Reputation of the respective schools in the opinion of the interviewed teachers and their reasons	Top school in Mdantsane until the unrest situation in 1983/84. Since 1984 there is no reason to claim the top position. Still, parents probably consider it as the best in Mdantsane.	It is a fact that school NC stood at the top (in Ciskei) in the Science Olympiad (which has a strong mathematics content) thrice in recent years: 1989, 1988 and 1985. The interviewed teachers believe that parents all over Mdantsane are aware of this fact.
Socioeconomic background of pupils	Mostly from working class families: at both schools. (Note: Children of professionals and other economically well-off families occasionally go to schools outside Mdantsane, but the vast majority of the Mdantsane population belong to the working class and are employed in various industries and businesses in and around East London.)	

The target population of this investigation was assumed to be the senior secondary schools in Mdantsane offering HG mathematics. The investigator has...

numerical data (cf. appendix 11) which establish that schools NE, NC and C belong to the same population. The descriptive analysis given above supports the assumption that on the whole NE and NC schools are similar. Whether they are typical of secondary schools in Mdantsane is still arguable. The investigator is inclined to agree with the opinion of the interviewed teachers that at least five or six secondary schools in Mdantsane are sufficiently similar (though the investigator's own experience with school E would suggest some caution in making this judgement). Hence a large enough target population exists and this investigation becomes statistically meaningful and relevant.

4.2 QUESTIONS OF EXTERNAL VALIDITY

The other questions [Cohen and Manion, pp 196-197] of external validity are briefly considered now.

4.2.1 The independent variable

The independent variable is the content and method of presentation of two specific topics in the standard 9 HG mathematics. The topics themselves were discussed at length in chapter 2. The actual content and method are presented in sec. 4.3.8 below, so that replication of the experiment is ensured for any future investigator who might wish to examine the issues involved here.

4.2.2 The dependent variable

The dependent variable is structured, relational understanding of the mathematics in the experiment. Evidently "understanding" cannot be measured directly, numerically; so, as is fair and possible in such cases the variable is measured by means of a "proxy" that can be measured numerically, viz., scores in tests. The instrument of measurement is the post-test, a test administered immediately after concluding the classroom exercise. This test was the same for both NE and NC samples. The validity of the test is examined shortly.

4.2.3 The pre-test's role in sensitizing samples to experimental conditions

The underlying threat here is that the subject of the experiment might be influenced by the pre-test, and thus "anticipate" the post-test with the result he/she might do the latter "differently" so that the investigator would not be justified in attributing the observed change entirely to the experimental treatment. This situation does not apply to the present investigation as argued below.

(a) The "pre-test" was a diagnostic test and was not meant to measure what the post-test was meant to measure. [The validity of the pre-test itself is examined shortly.]

(b) The pre-test was administered at the E and C schools in April, whereas the same [the section on Pythagoras only] was administered at the NE and NC schools in September/October, i.e., towards the end of the academic year. The scores obtained by pupils at all the four schools are comparable [except at E where pupils scored consistently lower marks compared to the other three schools]. This fact incidentally takes care of an aspect of internal validity too, viz., whether the fact that the experiment was not conducted at the different schools at the same time of the year has had any effect on the outcome of the exercise: obviously not.

(c) The pre-test measured the basic knowledge and understanding necessary for standard 9 pupils constituting the samples to participate meaningfully in the experiment. If this test could condition pupils, then all class tests and examinations as well could do the same. So, there is nothing new in the test itself which would condition the samples. The novelty of the situation, viz., an "outsider" presenting some mathematics is a different matter. There is a chance that this would condition the pupils, but it is a factor over which one has little control.

4.2.4 Hawthorne effect

The subjects of the treatment at all four schools and the college were told of the purpose of the exercise, so they were aware of the fact that the exercise would not affect the record of their regular work and performance. Yet, if knowledge of the nature and role of the exercise affected pupil response to the post-test it would probably have affected all the samples equally. The clear difference in performance in the post-test between the NE and the NC samples would however support the assumption that the Hawthorne effect is not a serious threat in this investigation. Further the negative result obtained at the E and C schools as well as at the college strengthens this view.

4.2.5 Interaction effects [Cf. Cohen and Manion, p.197]

The above assessment indicates that if interaction effects of any significance existed, these would have arisen in the context of questions of internal validity. So this aspect is considered below in that context. [Cf. sec. 4.3.9.]

4.3 QUESTIONS OF INTERNAL VALIDITY

The various aspects of this issue [Cohen and Manion, pp 194-

1953 are critically examined now.

4.3.1 History of events affecting the exercise as well as the samples

The history of this investigation has been described at length above in this chapter as well as in chapter 3. The exercise lasted only two weeks at each school, and nothing unusual came to the notice of the investigator in these short periods at either school, NE and NC. [The distractions which plagued the exercise at the E and C schools were pointed out in chapter 3.]

4.3.2 Maturation

This factor is of no consequence here because of the short duration of the exercise.

4.3.3 Statistical regression

This is a concept which could be misapplied. It is argued here that it does not apply to the scores in question here. But, first a look at the comparative analysis of the pre-test and post-test scores of some of the members of the NE sample as made out in appendix 12. A number of instances are seen where the score has moved away from the mean. The point is that there is no consistent pattern. This is not surprising as argued below.

The pre-test and the post-test were of entirely different formats [cf. sec. 4.2.3 and 4.4.2.2] and measured knowledge and understanding at different levels and of different concepts except for a single question on geometry [post-test question 9]. The concept of regression is meaningful when the same test (or at least the same type of test) is repeated. If it were not so the records of pupils on the extremes of a score continuum in the course of an acdemic year would move steadily towards the mean. This is not the case in spite of the fact that tests in a subject are of the same type generally. Thus regression effects are not of any consequence in this investigation.

4.3.4 Testing

The question whether the samples were influenced by the purpose of the experiment, and if so, in what manner and to what extent merits examination. The purpose of the experiment was briefly explained to the samples at the beginning of the experiment. Neither sample was told of the existence of the other sample: in other words, the samples were not aware of the possibility of their being compared with a similar group. [The schools C and E were about 3 km apart while NC and NE were about 5 km apart. Still the possibility of transfer of information from pupils of one school to the other due to a variety of social interactions cannot be ruled out. The only certainty is that no pupil at the NC school (where the experiment began after completion of the exercise at NE) appeared to be aware of a similar exercise at the other school.]

Both samples were given precisely the same message:

(i) This exercise is part of a programme of teaching certain topics in standard 9 mathematics for evaluation of teaching and pupil response, and has a Rhodes University connection.

(ii) A test would be given at the beginning to test their [the pupils'] understanding of certain mathematical concepts and facts. [The terms "pre-test" and "post-test" were never used; "post-test" appeared only once and that was on the test paper itself.]

(iii) That there would be a final test to test how much the pupils understood from the presentation was not mentioned at the start. This message was deliberately delayed to the mid-stage of the course. There was a response of having been taken by surprise among both samples when the message was delivered. By that time the samples and the investigator had become quite well acquainted with each other, and the latter could reassure the pupils that the results of the test would in no way be recorded by the school or used to affect them in any way. Still, the message could, in principle, have generated motivation to do well in the post-test. If so, this should have affected both samples equally, which possibility would not invalidate the difference in post-test scores observed between the experimental and control samples.

Besides, the lesson presentation was in the form of worksheets which were used in the class. There was no other material by which practice outside the classroom could have contributed towards improved test performance. However: (i) Individual pupils using the worksheets themselves for revision cannot be ruled out, but no extrinsic motivation for that can be identified. (ii) The emphasis was on comprehension, not on practice. This was not a weakness in the presentation since the topics themselves were familiar to the pupils. The whole exercise was orientated towards generating relational understanding rather than teaching something unfamiliar to the pupils.

Thus there is not sufficient ground to suspect the influence of the pre-test and hidden messages affecting the validity of the results of this investigation.

4.3.5 Sample selection and bias

Sampling was critically examined above at length. There is not sufficient ground to suspect sampling bias.

4.3.6 Sample depletion

The experience at the E and C schools highlighted this problem at the very outset of the investigation. The solution to the problem was found by choosing the samples from the available group of pupils at the end of the exercise so that the samples at both schools included only those pupils who attended the whole course and took the two tests. Steadily increasing dropout was the major source of sample depletion at both NE and NC; random absence turned out to be a minor factor [a fact which could possibly be attributed to the "season", i.e., the pre-examination season] in contrast to the state of affairs at the E and C schools [cf. ch.3].

4.3.7 Pre-test

Finally, the relatively more important question of instrumentation, viz., tests and the course itself, merits examination. The pre-test question paper and memorandum are reproduced as appendix 13.

(a) History

The same test consisting of two sections was administered at all the four schools and at the college, except that section A which concerned absolute value was omitted at schools NE and NC. The whole test took about 30 to 35 minutes to answer, while section B only (which concerned Pythagoras) took about 20 to 25 minutes to answer.

At schools E and C tests were administered on the same day with only a short interval between the two events. As questions were answered on the question paper itself there was no possibility of the test material becoming "public" knowledge. This condition was even more stringent at the college as the investigator moved from one class to the other immediately after administering the test to the first class. This was not so at NE and NC. Due to practical constraints mentioned in chapter 3 the exercise was carried out at different times at the respective schools. There is no guarantee that the contents of the test instrument were not transmitted by one or more pupils of NE to one or more pupils of NC during the course of the exercise at NE: now, that is in principle - in practice, the evidence for such loss of confidentiality should come from the behaviour and general response of the NC sample to the exercise. The investigator did not find anything unusual to suspect that the NC sample had access to the test instrument prior to the exercise.

(b) Diagnostic value: internal validity - whether the test measured what it was intended to measure.

Section A consisted of 3 questions on numbers which were intended to test whether the sample had correctly internalised concepts relevant to the presentation of absolute value. The property of additive inverses [cf. question 1] is an essential concept since it is the basis that generates the symmetry axis at zero for directed numbers. It was shocking to find that of the 71 students who took the test at the college [Course I Primary trainees] only 12 answered the question in such a way as to indicate understanding of the concept. At the E and C schools only 7 out of 48 and 10 out of 37 respectively indicated understanding.

Question 2 was intended to test understanding of directed numbers. Some of the items in this question would show whether the pupil had <u>integrated</u> the very basic concepts of addition and subtraction with symmetry as well as the concept of directed numbers. Answers such as [$^-5 - ^-5 = 0$ & $^-5 - ^+5 = 0$] & [$^-5 - ^-5 = ^-10$ & $^-5 - ^+5 = ^-10$] betrayed a total lack of concept of subtraction as well as additive inversion. At the college 51 out of 71 had no understanding at all about the given combinations of $^-5$ and $^+5$. At C, 23 out of 37 and at E, 25 out of 48 showed similar conceptual problems.

The third and final question was a further test of understanding of directed numbers: testees were asked to mark pairs of points on a numberline equidistant from given locations. This kind of understanding helps to integrate the new concept of absolute value into the existing conceptual structure of the pupil. Only a fully correct answer would indicate relational understanding. The following statistics tell the story:
At E, 5 got it right ; at C, 7 ; at the college 12.

The above analysis shows convincingly that the pre-test instrument did act as a powerful indicator of the conceptual background the investigator was looking for in the samples.

Section B consisted of 4 questions which were designed to assess the background knowledge relevant to the Pythagoras theme. Recognition of a number as the square of another number was a primary ability tested by the first question. The second question provided a further test of ability to recognize squares of numbers. The third question had two parts: the first part was intended to test ability to recognize number patterns basically related to the pattern implicit in Pythagoras; the second part tested ability to apply the relationships which were implicit in the previous part. The maximum failure rate at C, E and the college occurred on this part. The next and final question was a direct geometrical application of Pythagoras.

On the whole section B was fairly well answered by the samples at NE and NC. Again conceptual difficulties were mostly associated with the same questions as named above. The lower rate of successful response at C and E, (particularly at C which has been established to belong to the same population as NE and NC) could possibly be a consequence of the time factor, since the test was administered at NE and NC towards the end of the year while it was administered at C at the beginning of the year. However, this difference has no bearing on the difference in performance between NE and NC at the post-test since these samples returned similar responses to the pre-test questions under review here.

The general response to the pre-test highlights two conclusions:

(i) Pupils at the standard 9 level are familiar with Pythagoras. (ii) Pupils' difficulties with absolute value arise from deeper conceptual problems with directed numbers, basic operations such as subtraction, and lack of understanding of symmetry.

How comment (i) above affects the results reported here deserves attention. The core of the exercise was integration of various topics around Pythagoras as central theme. This in no way is invalidated by pupils' knowledge of Pythagoras itself. On the contrary, the exercise becomes more meaningful to the subjects of the experiment and the difference between the NE and NC samples as indicated by the post-test scores, if significant, would be a strong validation of the arguments presented here.

4.3.8 Administration of the classroom exercise

4.3.8.1 Absolute value

Absolute value was presented at schools E and C as well as at the college. At the schools the chalkboard was extensively used for development of the theme. The textbook presentation was closely followed at C while symmetry was clearly emphasized at E. The experiment failed however as was mentioned earlier, but the experience was exploited as a pilot study and the lesson materials and techniques were drastically altered before presentation at the college. The experimental and control treatments administered at the college differed substantially after a few common lessons. Several vertically mounted large-enough plane mirror strips were improvised to give first-hand learning experiences on mirror symmetry to the experimental group. Students worked in small groups and discussion was encouraged. The control group was directed closely along textbook lines; students in this group showed considerable difficulty in understanding the procedures as well as the second alternative in the formal definition itself, viz., $|x| = -x$ if $x < 0$. Stephen Sink [cf. ch.1] reports the same experience [Sink, p. 191]. In particular, a number of students could not believe/accept that $-x$ could be positive: their sole criterion was the negative sign before x. Sink reports identical experiences. On the whole the control group showed great difficulty in comprehension, whereas the experimental group "seemed" to understand the consequences of symmetry, even if not the implications. To conclude, the experiment at the college was also a failure.

4.3.8.2 Pythagoras

The failures at schools E and C and also at the college were critically examined in order to revise the methodolgy of classroom interaction. All lessons were developed as pupil activity on prepared worksheets. The first few minutes only were used for general introduction. The chalkboard was used only to enlarge diagrams etc., which were already made out on the worksheets. The investigator's major role was to go round and intervene where help was needed. Pupils were encouraged to form small groups and work on the sheets together. Emphasis was on comprehension and the atmosphere was made as informal as possible. Those who persisted through the course to the post-test seemed to enjoy it both at the NE and at the NC schools, even though the latter sample did not perform well at the post-test.

The highlights of the presentation to the NE group included:

(i) Pythagoras' theorem in its geometrical form.

(ii) Its translation to the number set, both numerical and algebraic.

(iii) Irrational numbers in relation to Pythagoras.

(iv) Pythagoras on the Cartesian plane, and the distance formula.

(v) The circle on the Cartesian plane, its equation and Pythagoras.

(vi) The "locus" concept related to Pythagoras.

(vii) The Pythagorean identities in trigonometry.

(viii) Based on (iv) to (vii) a detour to the effective use of the coordinate form of solution of trigonometric equations as well as simplification of trigonometric expressions.

(ix) Straight geometrical applications of Pythagoras.

All this was carefully presented, interwoven around Pythagoras and the essential unity of the various aspects deliberately pointed out.

In contrast, the control group was offered the same menu,the same classroom exercises using the same worksheets as those used by the experimental group, but according to textbook style as bits and pieces of various chapters in their course. Pythagoras was applied where necessary, but it was given no stress as a unifying theme. Otherwise the classroom strategies and procedures were identical to those adopted for the experimental group. The course was offered during study hours as already mentioned. The same number of days and the same number of hours were devoted to both groups.

4.3.9 Interaction effects

It was mentioned earlier [cf. sec. 4.2.5] that this question would be fully argued only in the context of the discussion on internal validity. Now that it has been done, it is noted that no serious threat to internal validity has been identified, so the question of interaction does not arise. However, if factors which caused problems of validity existed but were not identified in the above discussion, then interaction effects would exist too. No further comment on this issue is possible here.

4.4 POST-TEST

The same post-test was administered to both NE and NC samples. It was designed for an average pupil to answer in about an hour; in the event the last script was returned in about 1 hour 20 minutes at both schools.

4.4.1 Questions of validity

As the internal validity of the results of the investigation depends on the validity of this test to a great extent it deserves critical assessment.

The format of the question paper was designed in line with the worksheets and all the 11 questions were presented along with the necessary coordinate planes, square nets and other diagrams as work-material so that pupils could do the work on these aids. Only a minimal amount of construction or drawing of any kind was required of the pupils. The simple numerical work, where necessary, could be done without the use of a hand calculator, however no restriction was placed on the use of the calculator.

Answers were to be made out on the question paper itself and returned as script. The confidentiality of the test paper in relation to its repetition at the control school where the experiment was undertaken two weeks after the completion of the same at the experimental school could be questioned. However, as argued at some length in connection with the administration of the pre-test, there was no indication of any interaction between the two samples. The test result itself is the best evidence to support the view that there was no such interaction between the two groups.

The scripts were marked against a prepared memorandum to a maximum of 60 marks [appendix 14].

The important question of validity, viz., whether the test instrument measured what it was intended to measure is examined in the next section. At this point it is only claimed that the test was designed "to measure" relational understanding of Pythagoras.

4.4.2 Discussion of post-test and results

4.4.2.1 Absolute value

The failure of the experiment at schools E and C has already been attributed to several factors in various places in this report. The failure at the college was unexpected and so it was decided to explore the reasons thereof and accordingly a questionnaire [appendix 15] was administered soon after the post-test. Analysis of the response showed that a majority of students were poorly motivated to learn mathematics either as part of their course work or as a teaching subject. This finding is further supported by the fact that 11 out of 70 respondents said that mathematics should <u>not</u> be compulsory in teacher training (at the primary level) and that 14 thought that standard 5 level of mathematics was sufficient for senior primary teachers.

4.4.2.2. Pythagoras

Certain aspects of validity of the post-test were considered earlier.

Now the questions set in the test are critically examined.

<u>QUESTION 1</u>

(a) <u>Construction</u> of a right triangle on a Cartesian square net on which readings can simply be counted.

(b) <u>Measurement</u> of the length of the hypotenuse. [Measurement of distances not parallel to the vertical and horizontal axes was practised during the course.]

(c) <u>Calculation</u> of the length of the hypotenuse.

<u>OBJECTIVES</u>

(i) To test knowledge of Pythagoras' theorem.

(ii) To test skill of measurement on square nets in an approximate, simple way.

(iii) To test ability to apply the knowledge in (i) above.

(iv) To check if the answers to (b) and (c) above are the same or different.

Objective (iv) is considered to be the most important. If there is no indication in a script that the pupil noticed the difference (if the answers were different) and returned to the answer in (b) either by cancelling it or by re-doing it (whether obtaining the correct answer or not) it could be interpreted as:

1. lack of relational understanding;

2. improper as well as incorrect approach to work material/reading material where the immediate connection between closely related parts is entirely missed; or,

3. an inappropriate value system in which the pupil is not excited/upset/threatened by discrepancies in results; or, a combination of these.

RESPONSE

[The 11 pupils who were not included in the NC sample but who attended the course and took the post-test are included in this analysis. Thus the NE sample contains 24 pupils and the NC 35.]

SAMPLE NC	SAMPLE NE
12 showed serious conceptual problems.	Only 2 showed similar problems.

QUESTION 2

(a) To use the circle on the coordinate plane with convenient units of distances already marked on it to help solve an algebraic equation in the form of Pythagoras.

OBJECTIVES

(i) To test relational understanding of Pythagoras as the basis of the algebraic description of a circle. A pupil having such understanding would know/"see" the unknown "p" directly as a distance on the plane.

(ii) To test ability to recognize approximate rational values of irrational numbers: in particular to test the recognition that $\sqrt{24} \simeq 5$.

(b) Recognition of a Pythagorean statement and solution.

OBJECTIVE

Mainly to test ability to correlate the geometrical picture in (a) above with the algebraic statement in (b), a question of integration through Pythagoras.

RESPONSE

SAMPLE NC

27 failed to see the correlation between (a) and (b).

SAMPLE NE

Only 6 so failed. Further, 4 correctly translated the idea into quadrants other than the first, and in different ways - real indication of relational understanding.

QUESTION 3

Equation of a circle, centre other than the origin of the coordinate axes. The equation is to be written down by simply looking at the given diagram. Only those who understood the basis of the "distance formula" on the Cartesian plane as Pythagoras, and at the same time integrated the circle with Pythagoras would be able to answer this question correctly. To those who had such understanding the equation would be obvious since the distances involved are all simple numbers of units.

RESPONSE

SAMPLE NC

Correct: none.

Idea of right triangle: 2.

SAMPLE NE

The idea of the right triangle at the correct location, i.e., with a vertex at the centre of the circle was missed by 2 pupils, while 13 missed the fact that the radius was known, and a few made mistakes in counting the number of units of the radius.

QUESTION 4

To find the approximate value of $\sqrt{5}$, an irrational number, by using Pythagoras. The question is not in the square form. A value correct to one decimal place is possible with the given square net, though unit numbers were deliberately omitted.

OBJECTIVES

(i) Recognition of the Pythagorean pattern in any statement involving three numbers.

(ii) Skill in translation of such recognition on to the graph.

RESPONSE

SAMPLE NC

Only 1 got the diagram as well as measurement.

4 got the right diagram, but not the answer.

13 did not get the idea of the diagram at all.

Different types of conceptual problems could be identified: the majority among these (7) made right triangles with legs 1 and 4 respectively; others did not fit into specific categories.

SAMPLE NE

2 got them both.

4 got the right diagram, not the answer.

6 did not get the idea.

11 constructed the same type of triangle.

QUESTION 5

(a) To find the coordinates of a marked point P, where P is one vertex of a right triangle, with another at the origin of the coordinate axes.

(b) Pythagoras and the trigonometric connection.

OBJECTIVES

(i) The Cartesian connection of Pythagoras: identifying the leg PA of the right triangle as the y-coordinate of point P.

(ii) Skill in application and knowledge of problem solving procedure in such cases.

RESPONSE

SAMPLE NC	SAMPLE NE
All correct: 9	All correct: 10
Part (a) only correct: 9	Part (a) only correct: 5
All wrong: 14	All wrong: 1
Partly correct: 3	Partly correct: 8

QUESTION 6

To prove a trigonometric identity.

OBJECTIVES

(i) To test flexibility of approach between trigonometric form and coordinate form in such situations.

(ii) More importantly, to test the understanding that coordinate description is the basis of trigonometry.

(iii) To test ability to apply Pythagoras in context in either trigonometric form or coordinate form.

[The identity was comparatively difficult.]

RESPONSE

SAMPLE NC	SAMPLE NE
Correct: 1	Correct: 1
Partly correct: 1	Nearly correct: 3

	Partly correct: 13
Did not attempt: 4	Did not attempt: 2
All wrong: 29	All wrong: 5

QUESTION 7

A real-life application of Pythagoras. [The idea of the sketch of the ladder against a house was borrowed from a West-African textbook. The idea of its superimposition on a square net is the investigator's.] The given hint that a sketch may be drawn on the square net to solve the problem was made optional in order to allow bright pupils to do it directly without the aid of a sketch, if they so desired.

RESPONSE

SAMPLE NC	SAMPLE NE
Only 1 got both diagram and answer correct.	6 got them both.
4 got solutions with wrong diagram or no diagram.	11 got similar results.
30 went wrong completely or did not attempt the question.	Only 3 went wrong completely.
	5 got part-correct answers.

QUESTION 8

A right triangle inscribed in a circle. To prove that one of the legs is equal to a given length - constraints specified.

OBJECTIVES

(i) Recognition of right triangle and hence the role of Pythagoras.

(ii) Ability to apply Pythagoras. [In problems like this such ability is taken as a mark of relational understanding.]

RESPONSE

SAMPLE NC

Correct: 2

Did not try: 12

Wrong: 21

SAMPLE NE

Correct: 1

Comprehension of idea evident: 4

Did not try: 5

Wrong: 14

QUESTION 9

Calculation applying Pythagoras, with respect to a geometrical figure.

OBJECTIVES

(i) Recognition of Pythagoras.

(ii) Computational skill applying Pythagoras.

RESPONSE

SAMPLE NC

Correct: 11

SAMPLE NE

All correct.

QUESTION 10

Application of Pythagoras in a more complex geometrical diagram. Response demanded in the form of a proof.

OBJECTIVES

(i) Recognition of the applicability of Pythagoras.

(ii) Computational skill applying Pythagoras.

RESPONSE

SAMPLE NC

Correct: 2

Did not try: 17

SAMPLE NE

Correct: 6

Did not try: 5

QUESTION 11

Locus of a point at constant distance from a given point. Equation required by inspection and use of a ruler only.

OBJECTIVES

(i) Understanding of the nature of a circle.

(ii) Knowledge of the equation of a circle, centred away from the origin of the coordinate axes.

(iii) Ability to identify the radius of the circle by careful observation of the diagram.

RESPONSE

SAMPLE_NC	SAMPLE_NE
Comprehension evident, but answer incorrect: 1	Correct: 3
Did not try: 9	Did not try: 2
Wrong: 25	Wrong or part correct: 19

The above detailed analysis of the post-test questions and the responses was undertaken primarily to establish the following two points:

(a) The test does measure relational understanding of the theme of the classroom experiment, hence its validity remains strong.

(b) The performances of the experimental and control groups are distinctly different, the former doing much better than the latter. A one-tailed t-test was done only to confirm the obvious.

4.5 HYPOTHESIS

The null hypothesis reads:

$H_0: \mu_E = \mu_C$: There is no difference in the mean mathematics test scores of the experimental and control groups, where the experimental group is taught mathematics in a structured, integrated form, while the control group is taught the same mathematics topic by topic without providing a holistic structure.

The alternative hypothesis reads:

H_1: $\mu_E > \mu_C$: The mean test score of the experimental group is higher than that of the control group.

4.5.1 Statistical evidence

The relevant details have all been reported at various stages of the argument and discussion above. To summarise:

$t_c = 4{,}90$; $t_t = 1{,}68$ [5%] , 2,41 [1%] .

Since $t_c > t_t$ the null hypothesis is rejected.

4.6 CONCLUSION

Thus the investigation supports the view that mathematical understanding is substantially promoted by structuring topics to form an integrated whole wherever possible, and that in the process of achieving this the instructional strategy should be so designed that the pupils descend the "spiral" of the curriculum and reinforce the assimilation of the new concepts and knowledge. In the specific theme investigated here the lower levels of the spiral curriculum include Pythagoras' theorem, its equivalent form in numerical as well as algebraic form, the order that exists between numbers and their squares, the concept of rational versus irrational numbers, etc. The higher levels of the spiral include the introduction to trigonometric concepts and routines, applications of Pythagoras to complex problems, circle theory, locus concept, etc. This approach fits into the theoretical model based on the work of Bruner, Ausubel and Skemp as described in chapter 1.

CHAPTER 5

RECOMMENDATIONS

5.1 PRACTICAL CONTEXT OF THIS INVESTIGATION

Mathematics education faces unique problems the world over: Southern Africa is no exception. The present investigator has been involved in mathematics teacher education since 1983 in the Ciskei, and has first-hand experience of the complexity of the problems faced by mathematics teachers and pupils alike, both at the primary and at the secondary levels. Time and again remedial teaching sessions have reinforced the investigator's belief that serious gaps exist in the conceptual structures of many pupils, laterally as well as vertically. Experience has consistently shown that lasting results are obtained only when the pupil's "deep structures" [Skemp, 1972] are properly organized. It appeared that a short investigation to test this experience as a formal hypothesis would probably contribute something towards improvement of mathematics education in the Ciskei. In short, the investigation seemed to be immediately relevant.

Two topics were accordingly chosen - both for presentation at the standard 9 level.

5.2 THEORETICAL FRAMEWORK

The investigator found it fit to see the problem in a cognitivist perspective. Conceptual structures, their growth, weaknesses and gaps and the influence of the latter on failures in mathematics learning were all neatly organized into a single theoretical model when the ideas of Bruner [1966], Ausubel [1968] and Skemp were brought together to elucidate the problem. This analysis and synthesis at once pinpointed the significant roles played by curriculum designers, examiners, textbook writers and teachers in particular. Hence the investigator thought it fit to make a critical assessment of the South African school mathematics curriculum and textbooks which interpret/fail to interpret this curriculum. Chapters 1 and 2 of this report examine the various aspects of these issues and make specific comments which imply many messages directed towards people who determine and control education. The following sections are thus a re-interpretation of chapters 1 and 2 in the light of the results reported in chapter 4.

5.3 TO CURRICULUM DESIGNERS

The aims of the mathematics curriculum spanning the years from standard 2 to standard 10 are well designed and intended to promote new learning by providing structure to the content which would facilitate the assimilation of the new knowledge into the existing conceptual map of the learner, and on the whole the South African school mathematics curriculum agrees with the principle of the spiral curriculum. Having said that, one has to point out the flaws which missed the curriculum makers' eyes. In the context of this investigation comments which pertain to the two topics of interest, viz., absolute value and Pythagoras only are made.

5.3.1 Pythagoras

(a) Relating square units such as m^2 and cm^2 to squares of numbers by means of the exponential notation will contribute towards integrated and structured learning. The educational issue is the need and virtue of conceptually relating the SQUARE encountered in arithmetic to the SQUARE as a shape through the size of its area and the notation of the unit of area.

It is not suggested here that the curriculum precludes the relationship between the algebraic and geometric concepts of square, but only that a clear and positive suggestion built into the curriculum will carry far more weight in this respect than individual teacher's "articles", "letters to the editor", etc.

(b) Serious omissions have occurred in the curriculum as far as Pythagoras is concerned. The theorem is introduced in standard 6 for the first time, and then re-visited (almost in the same form !) in standard 9. The principle of the spiral curriculum has become the casualty in this omission. Many valuable developments also suffer.

Again, the opportunities for providing historically exciting proofs of the theorem progressively through standards 7 and 8 have also been lost, as also the possibility of integrating arithmetic and geometry through Pythagorean triples.

The secondary curriculum needs to be revised to correct such weaknesses. Also the view that the content should always be examinable deserves re-examination. Every opportunity for making mathematics learning appealing to pupils should be taken advantage of. In short curriculum makers carry the heavy responsibility of moulding teacher perceptions in this regard.

(c) The weaknesses in the Pythagoras theme as found in the curriculum are partly responsible for weaknesses in the analytical geometry theme particularly affecting circle theory and the locus concept. [Cf. ch. 2 for detailed analysis.]

5.3.2 Absolute value

Though the importance given in the curriculum to absolute value is minimal, the key concept needed to develop the theme, viz., mirror symmetry is a concept which has a powerful role to play in meaningfully integrating various themes in school mathematics. It is somewhat of a contradiction on the one hand to say, in the statement of intent, that pupils should discover the orderliness of numbers, and on the other to leave out the role of symmetry in the development of the curriculum. Its importance in "Transformation Geometry", a form of geometry which might in course of time replace Euclidean geometry in South African schools to some extent, is also a factor which favours according a prominent place for symmetry in the curriculum. It is to be hoped that the next change of curriculum will be influenced by such comments as these.

5.4 TEXTBOOKS

Textbooks in Southern Africa as elsewhere generally follow the beaten track: almost without exception authors stick to the thematic development with the result that chapters "hang together" in a topic-by-topic discussion. Imaginatively written textbooks need to be produced in the interests of mathematics education, even if such productions may not guarantee fair returns in the highly competitive textbook business where publishers, printers, authors and distributors are the first beneficiaries. Teachers and pupils often have little real choice. On sale are books often promoting "instrumental mathematics" which are easy to produce as well as sell, while mathematics education is in need of books promoting "relational mathematics". It is not easy for the latter type to make an effective dent into the structures of the vested interests of the former. Substantial subsidies from Government or the Private Sector or both sources are therefore needed to promote new ventures producing books for "relational mathematics". Social awareness of the gigantic loss of human potential as well as State resources because of large scale failure in mathematics has to be developed with a crusade-like dedication. Only such drastic changes in perceptions will set the kind of changes into motion as eventually will help in the production of good but cheaply-priced textbooks.

5.5 TO TEACHERS

Books are only one "front" of the battle. To win the war teachers have to fight "instrumentalism" put on sale by textbook writers and often condoned by examining authorities. Teachers are the ultimate controllers of the process of education. Such control operates in many directions, of which three major ones concern syllabi, textbooks and examinations respectively.

5.5.1 Syllabi

The assessment in chapter 1 has shown that on the whole the aims of the South African school mathematics curriculum are well designed and that the curriculum itself unfolds according to the principle of a spiral, envisaging growth laterally as well as vertically emphasizing structure and integration of parts, though there exist specific weaknesses in the selection and organization of the content. The latter aspect, i.e., structured , integrated development of the curriculum, is an implied principle more often than explicit expression in the syllabi. This fact coupled with the recent phenomenon of "suggested work programmes" for every standard [e.g., cf. the D.E.T. material, see appendix 6] is likely to influence the majority of teachers to stick to the traditional emphasis on a "thematic" development of the curriculum at the expense of structured, integrated presentation unless the "work programmes" themselves adopt the latter strategies. The central message of the present investigation is that the cause of structured, integrated teaching around the notion of a spiral curriculum will be substantially supported and boosted to the ultimate benefit of the pupil and mathematics education if teachers innovate integrated presentation around specific themes of their choice: <u>the Pythagoras theme only illustrates such possibilities</u>. So the existence of any suggested work programme is likely to be a strong deterrent to individual teacher innovation and flexibility.

Teachers should exercise their right to interpret syllabi at every stage of development in conjunction with the stated aims of the curriculum. Only then will the curriculum come alive and meaningful teaching take place, instead of passive, meaningless "covering" of a prescribed syllabus. The message here is that vibrant, lively mathematics classrooms can be the creations of individual teachers.

5.5.2 Textbooks

The first step in "the right direction" by the teacher is discussed and pointed out above. The most important tool in the hands of the teacher is the textbook. The teacher who has accepted the philosophy implied in sec. 5.5.1 is bound to teach "relational mathematics" and not "instrumental mathematics". It is at this stage of teacher development that individual teacher's eyes open to the truth that most textbooks better suit "instrumentalism" than "relationalism", and that there exists no such thing as the ideal textbook. The danger of depending on a single textbook for teaching will now become obvious and the need and value of comparative, critical assessment of a variety of textbooks will become a meaningful routine to the teacher. The lack of the ideal textbook can be compensated by the growth of the "ideal teacher". One of the most important qualities of the latter is the theme of this paragraph.

5.5.3 Examinations

Examinations are intended primarily to test pupils' learning of concepts, skills, etc. However it is in the nature of examinations to set norms and standards and in that process they sometimes distort the curriculum in practice. Also, there are pressures on the teacher to achieve "results" which will establish personal as well as institutional reputation. Factors like these might affect teacher values and thus promote instrumental teaching. Teachers have to be constantly vigilant about this possibility and guard against succumbing to pressures which will cause their teaching to degenerate to the instrumental level. Relational understanding of mathematics is a lasting acquisition for the pupil on the one hand and it is capable of meeting the demands of any examination on the other. Teachers should continually remind themselves of this fact in order not to let their teaching become instrumental.

5.6 TO RESEARCH INVESTIGATORS

The present investigator believes that the exercise reported here is a humble contribution to the cause of school mathematics education. The Pythagoras theme as unfolded and developed in this report illustrates his belief and contention. It is not suggested that teachers should adopt this teaching strategy as far as Pythagoras is concerned. On the contrary, teachers are simply invited to see the power of the philosophy underlying the structured, integrated approach to teaching mathematics. Future investigators could make contributions of a similar kind where the power and value of innovation are illustrated.

5.7 POST-SCRIPT

The investigator's two children were doing the senior secondary course with HG mathematics at the time of this investigation. Both of them declined his offer of teaching them how to learn absolute value "relationally" through the central use of symmetry: their reason - simple: "My teacher taught it "this" way [the formal algebraic way]". Their response underscores the tremendous control of teachers on pupils' values and perceptions. So, teachers! be imaginative and innovative!

APPENDIX 1 — Selected matric questions

NATAL SENIOR CERTIFICATE

NOVEMBER 1987 HG II

Answer all the questions. All working details must be clearly shown. Please note that the diagrams have not been drawn to scale. Unless otherwise stated an approved electronic calculator may be used.

QUESTION 1

(a) Simplify as far as possible:

NATALSE SENIOR SERTIFIKAAT

NOVEMBER 1987 HG II

Beantwoord al die vrae. Besonderhede van alle bewerkings moet duidelik aangetoon word. Let asseblief daarop dat die figure nie volgens skaal geteken is nie. 'n Goedgekeurde sakrekenaar mag gebruik word, tensy die vraag anders vermeld.

VRAAG 1

(a) Vereenvoudig so ver as moontlik:

(1) $\dfrac{\tan(180° + \theta)}{\cos(90° + \theta)\ .\ \sec(-\theta)}$ (5)

(2) $\sqrt{1 - \cos^2\theta}$ (2)

NATAL

Nov. 1987 HG II

QUESTION 7

(a) Two circles $x^2 + y^2 - 16x - 8y + 35 = 0$ and $x^2 + y^2 = 5$ are given.

(1) Determine the centres of the circles.

(2) Prove that the circles touch each other.

(b) Prove that the straight line $y = x + 3$ is a tangent to the circle $x^2 + y^2 - 6x - 4y + 5 = 0$.

(c) Determine the equation of the set of points which are twice as far from the straight line $x = -1$ as from the point (4; -1).

VRAAG 7

(a) Beskou die twee sirkels $x^2 + y^2 - 16x - 8y + 35 = 0$ en $x^2 + y^2 = 5$.

(1) Bepaal die middelpunte van die sirkels. (4)

(2) Bewys dat die twee sirkels mekaar raak. (8)

(b) Bewys dat die reguit lyn $y = x + 3$ 'n raaklyn is aan die sirkel $x^2 + y^2 - 6x - 4y + 5 = 0$. (8)

(c) Bepaal die vergelyking van die versameling punte wat tweekeer sover van die reguit lyn $x = -1$ is as vanaf die punt (4; -1). (9)

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O.F.S. SENIOR CERTIFICATE

NOVEMBER 1987 HG II

All questions must be answered. Show all the necessary calculations. Marks will be deducted for slovenly work and for faulty statements. Only non programmable pocket calculators must be used.

O.V.S. SENIOR SERTIFIKAAT

NOVEMBER 1987 HG II

Alle vrae moet beantwoord word. Toon al die nodige bewerkings. Punte word afgetrek vir slordige werk en foutiewe bewerings. Slegs nie-programmeerbare sakrekenaars moet gebruik word.

QUESTION 2

2.1 B(2; 1) is the centre of a circle which passes through A(7; 5). Determine the equation of this circle.

2.2 Determine the equation of the tangent to the circle $x^2 - 2x + y^2 + 4y = 5$ at the point A(-2; -1).

2.3 Determine the equation of the locus of a point which is always equidistant from the line $y = 1$ and the point A(1; 2).

VRAAG 2

2.1 B(2; 1) is die middelpunt van 'n sirkel wat deur A(7; 5) gaan. Bepaal die vergelyking van hierdie sirkel. (5)

2.2 Bepaal die vergelyking van die raaklyn aan die sirkel $x^2 - 2x + y^2 + 4y = 5$ in die punt A(-2; -1). (10)

2.3 Bepaal die vergelyking van die lokus van 'n punt wat altyd ewe ver vanaf die lyn $y = 1$ en die punt A(1; 2) is. (10)

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continued...

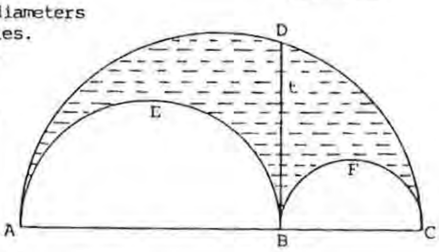
APPENDIX 1 continued

Nov. 1987 HG II — O.F.S./O.V.S.

QUESTION 10

AB, BC and AC are diameters of three semi-circles. DB ⊥ AC and DB = t. Find a formula in terms of t, which will enable you to find the area of the shaded part.

VRAAG 10



AB, BC en AC is middellyne van drie semisirkels. DB ⊥ AC en DB = t. Vind 'n formule in terme van t wat jou in staat sal stel om die oppervlakte van die ge-arseerde gedeelte te vind.

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Nov. 1987 HG II — Hse. of Reps./Huis van Vert.

QUESTION 5

5.1 A(3; -5) and B(1; 3) are two points in the Cartesian plane. Determine:

(a) the length of AB (leave your answer in surd form)

(b) the equation of the circle with AB as diameter

(c) the equation of the tangent to the circle at A.

5.2 A(-2; -1), B(2; -3) and C(3; 4) are the vertices of a triangle in the Cartesian plane.

(a) Find the equation of BF, where BF is an altitude of the triangle.

(b) Find the equation of AC and hence prove that F is the point (-1; 0).

(c) Hence, determine the area of triangle ABC.

5.3 (a) Write the following equation in the form $(x+a)^2 + (y+b)^2 = k$: $x^2 + y^2 + 4x - 6y - 3 = 0$.

(b) Hence, give an accurate description of the curve.

VRAAG 5

5.1 A(3; -5) en B(1; 3) is twee punte in die Cartesiese vlak. Bepaal:

(a) die lengte van AB (laat u antwoord in wortelvorm) (3)

(b) die vergelyking van die sirkel met AB as middellyn (6)

(c) die vergelyking van die raaklyn aan die sirkel by A. (7)

5.2 A(-2; -1), B(2; -3) en C(3; 4) is die drie hoekpunte van 'n driehoek in die Cartesiese vlak.

(a) Vind die vergelyking van BF, 'n hoogtelyn van die driehoek. (5)

(b) Vind die vergelyking van AC en bewys dan dat F die punt (-1; 0) is. (6)

(c) Bepaal nou die oppervlakte van driehoek ABC. (6)

5.3 (a) Skryf die volgende vergelyking in die vorm $(x+a)^2 + (y+b)^2 = k$: $x^2 + y^2 + 4x - 6y - 3 = 0$. (3)

(b) Gee nou 'n noukeurige beskrywing van die kromme. (3)

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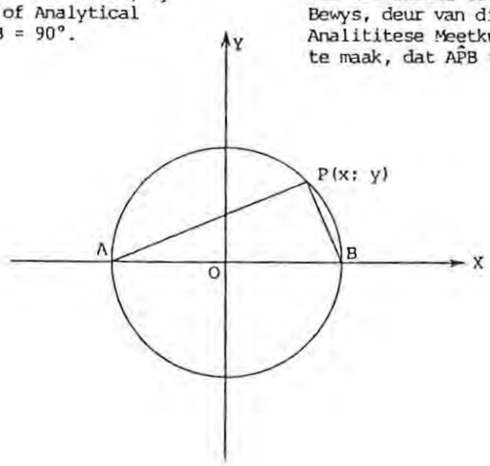
QUESTION 6

6.1 In the figure P is any point on the given circle (excluding the points A and B). The equation of the circle is $x^2 + y^2 = r^2$. Prove, by using the methods of Analytical Geometry, that $A\hat{P}B = 90°$.

VRAAG 6

6.1 In die figuur is P enige punt op die gegewe sirkel (behalwe die punte A en B). Die vergelyking van die sirkel is $x^2 + y^2 = r^2$. Bewys, deur van die metodes van Analititese Meetkunde gebruik te maak, dat $A\hat{P}B = 90°$.



(5)

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continued...

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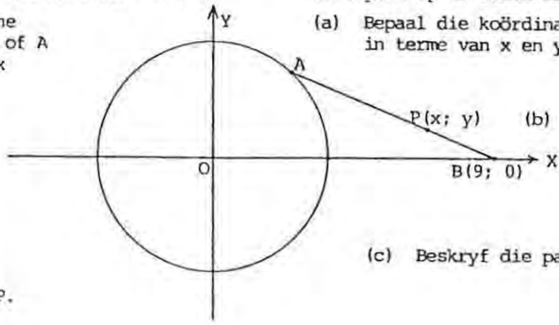
QUESTION 6 (Contd.) / VRAAG 6 (Vervolg)

6.2 In the figure, A is any point on the given circle. The equation of the locus of A is $x^2 + y^2 = 4$. B(9; 0) is a fixed point. P(x; y) is a point on AB such that AP : PB = 2 : 1.

(a) Determine, the co-ordinates of A in terms of x and y.

(b) Determine the equation of the locus of P.

(c) Describe the path followed by P.



6.2 In die figuur is A enige punt op die gegewe sirkel. Die vergelyking van die lokus van A is $x^2 + y^2 = 4$. B(9; 0) is 'n vaste punt. P(x; y) is 'n punt op AB sodat AP : PB = 2 : 1

(a) Bepaal die koördinate van A in terme van x en y. (4)

(b) Bepaal die vergelyking van die lokus van P. (4)

(c) Beskryf die pad van P. (2)

Nov. 1987 HG II — N.S.C./N.S.S.

QUESTION 8

(a) Prove that: $\sin^2\theta + \cos^2\theta = 1$, by using the definitions of the trigonometric functions.

VRAAG 8

(a) Bewys dat: $\sin^2\theta + \cos^2\theta = 1$, deur van die definisies van die trigonometriese funksies gebruik te maak. (3)

HOUSE OF DELEGATES SENIOR CERTIFICATE

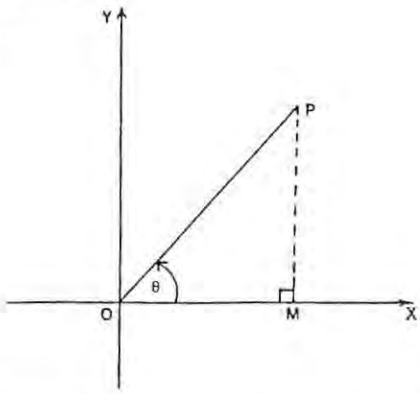
DECEMBER 1987 — HG — PAPER TWO

Answer ALL questions. All working details must be clearly shown. Where CALCULATORS/TABLES are used, answers must be rounded off to TWO DECIMAL PLACES. Marks will be deducted for incorrect statements and slovenly work. The following method of naming angles should be used where possible: $\hat{A}_1$, $\hat{C}_1$, etc. Note that diagrams are not drawn to scale. Unless otherwise stated answers must NOT be obtained by construction and measurement.

QUESTION ONE

1.1 In the accompanying figure $X\hat{O}P = \theta$ where $\theta \in [0°; 360°]$. The length of OP is r units and the co-ordinates of P are (x; y).

Use the figure to prove that $\sin^2\theta + \cos^2\theta = 1$.



(4)

J.M.B. SENIOR CERTIFICATE

NOVEMBER 1987 HG II

Answer all questions. All the necessary working detail must be shown. Neatness and clear presentation will count in the candidate's favour. Pocket calculators may be used.

1 (b) Use the formula for cos(A − B) to show that $\cos^2A + \sin^2A = 1$.

G.M.R. SENIOR SERTIFIKAAT

NOVEMBER 1987 HG II

Beantwoord al die vrae. Alle nodige bewerkings moet duidelik getoon word. Netheid en duidelike aanbieding sal in die kandidaat se guns tel. Sakrekenaars mag gebruik word.

(b) Gebruik die formule vir cos(A − B) om aan te toon dat $\cos^2A + \sin^2A = 1$. (3)

SYLLABUS FOR MATHEMATICS STANDARD 5

A. AIMS WITH SYLLABUS

1. To develop in the pupils an insight into and an understanding of mathematical principles and procedures and by so doing to help them deal with mathematical situations which they will encounter in their daily lives, and to solve problems related to quantity and number.
2. To help the pupils develop logical patterns of thought and to express their ideas systematically and concisely.
3. To develop in pupils an interest in, a love for and an appreciation of the orderliness of numbers.
4. To prepare the pupils to do calculations which they may need in their study of other school subjects or in their own further studies.
5. To develop in pupils the ability to calculate speedily and accurately.

B. GENERAL REMARKS ON SYLLABUS

1. The mathematics work programme contains the recommended order in which the contents of the syllabus should be treated.
2. The implicit relationship between subsections of the syllabus should be emphasized, making use also of integrated exercises.
3. New concepts should be introduced by and be based on discovery and discussion and should, as far as is practicable be preceded by practical work and explanations, with demonstrations involving practical teaching aids.
4. As far as is practicable situations within the pupils' experience should be used as points of departure for the teaching of new concepts.
5. In teaching a new concept all the preceding principles which form the basis of the new concept should be thoroughly revised. Where applicable, each new concept should be thoroughly consolidated by means of adequate written work.
6. Written work should not be attempted before sufficient oral work has been done. The setting out of written work by pupils should be brief and accurate. All necessary working should be shown.
7. Independent work and alternative methods of calculation should be an integral part of the pattern of work.
8. Formal drill should be meaningful and should not be undertaken merely for purposes of memorising.
9. Numeracy must be improved.
10. Pupils should be taught habits of :

continued...

10.1 determining by inspection whether their answers are within the bounds of possibility ; and

10.2 testing answers, where applicable, by working backwards.

11. Differentiation should be applied to take account of pupils' capabilities.

12. Wherever possible number work should be linked with written problems.

13. Graphical representations should, where possible, be used in order to increase meaningful comprehension.

14. Examination technique :

14.1 Examination questions should i.a. require solutions that depend on comprehension and insight. Long, cumulative calculations resulting in a shift of emphasis away from insight and comprehension should be avoided.

14.2 Questions should test all aspects of the syllabus, in proportion approximately to the time spent on the relevant section. Stereotyped questions should be avoided.

15. Period allocation :

The approximate number of periods to be devoted to any section is indicated in brackets opposite the title of that section.

C. EXPOSITION OF SYLLABUS CONTENT

Consolidation of and additional exercises on the work done in previous standards. (16)

1. Sets :

1.1 The concepts: equal sets, equivalent sets, subsets, universal sets, the complement of a set, the empty set, intersection of sets and union of sets.

1.2 The symbols $\subset$, $\supset$, $\mathcal{U}$, A', $\emptyset$, $\cup$, $\cap$, $\in$, $\notin$, $A = B$, $A \approx B$

1.3 Application of the above-mentioned concepts to known sets

1.4 Venn diagrams to illustrate the above-mentioned concepts

2. Natural numbers and whole numbers : (18)

2.1 Basic knowledge

2.1.1 Ready knowledge of addition and subtraction combinations and multiplication and division tables up to 10 x 10

2.1.2 Exercises of the type (a x b) + c and (a ÷ b) = c remainder d

2.1.3 The four basic operations.

MATHEMATICS

STANDARD 8

A AIMS WITH SYLLABUS

1. To acquaint the pupils with the part played by Mathematics in the modern world in which man is constantly required to handle the quantitative and spatial aspects of situations.

2. To contribute to the education of the pupils with special emphasis on development of logical thought and habits of systematic, accurate and neat methods of working.

3. To cultivate appreciation for the structure and the continuous theme of each section of the syllabus as well as for the underlying relation between certain section.

4. To acquaint pupils with and train them in mathematical methods of thought and work.

5. To give pupils a clear insight into, and a thorough knowledge and understanding of those basic mathematical principles which will prepare and equip them for daily life and for further study in Mathematics, the pure sciences and some of the applied sciences.

B. GENERAL REMARKS ON SYLLABUS

1. General Remarks

1.1 The syllabus content which is prescribed for Standard 8 is taken as pre-knowledge for the Standard 9 and 10 syllabuses. This knowledge is again used either directly or indirectly, in the syllabuses for the last two school years and thus also in the Matriculation examinations.

2. Remarks

2.1 In all sections of the subject pupils must be guided to tackle and solve each problem or theorem systematically by:

2.1.1 giving close attention to the data and what is required;

2.1.2 taking account of all facts and theorems that can possibly serve as key to the solution of the section of Mathematics in which they appear;

2.1.3 starting with the most obvios method and then considering other possibilities;

continued...

2.1.4 comparing the different methods of solution and making a choice;

2.1.5 giving close attention to necessary and sufficient requirements with respect to formulation and reasoning when writing down the solution.

2.2 Pupils must be trained in the correct use of notation and the making of valid deductions.

2.3 Half of the time must be spent on Algebra and the half on Geometry and Trigomometry. These two sections of the syllabus must be treated more or less simultaneously.

2.4 NB:

2.4.1 Concrete Quantities

The S.I. as used in everyday practice, must be used where applicable.

2.4.2 Graphs

Graphical representations must be treated as unifying concept and should be used where possible.

2.4.3 Non-programmable pocket calculators may be used in class where necessary and applicable to develop mathematical concepts and for calculations. Basic instruction in the practical use of pocket calculators must be given.

2.5 The allocation of periods is approximate.

3. Examination Practice

3.1 Mechanical reproduction in examinations of memorised definitions, formulae and techniques should be avoided as far as possible.

3.2 Examination questions should call for solutions based on understanding and insight. A candidate should be able to apply his knowledge and to make dedutions. Long and cumulative calculations, in which the emphasis has moved from insight and understanding must be avoided.

3.3 Seeing that final examination papers have such a great influence on the tuition, papers should be set in such a manner that they promote correct teaching methods. It should pay the teacher to concentrate on understanding and insight rather than on excessive drill. Stereotype papers should be avoided.

APPENDIX 4

SYLLABUS FOR MATHEMATICS HIGHER GRADE

STANDARD 9

A. AIMS OF SYLLABUS

1. To develop a love for, an interest in and a positive attitude towards mathematics, by presenting the subject meaningfully.

2. To enable pupils to gain mathematical knowledge and proficiency.

3. To develop clarity of thought and the ability to make logical deductions.

4. To develop accuracy and mathematical insight.

5. To instil in pupils the habit to estimate answers where applicable and where possible to verify answers.

6. To develop the ability of the pupils to use mathematical knowledge and methods in other subjects and in their daily life.

7. To provide basic training for future study and careers.

B. GENERAL REMARKS ON SYLLABUS

1. The arrangement of the content of the syllabus and its subdivision is not necessarily an indication of the sequence in which the work must be handled.

2. Non-programmable pocket calculators may be used where necessary and applicable to develop mathematical concepts and for calculation. Basic instruction in the practical use of pocket calculators must be given.

(ii) Extension to seven digits by:

(1) The use of multiples and groups of tens, hundreds, thousands, tens of thousands, hundreds of thousands, the writing of 100 as 10^2, 1 000 as 10^3, etc.

(2) Writing in figures of dictated numbers and oral practice in supplying the number names.

(3) Diagrammatic representation of numbers up to seven digits, with the aid of notation columns.

(4) Appropriate use of the number line to illustrate the relative size of numbers.

(5) The writing of numbers if the constituent digits are given and the inverse process.

(6) The analysis of numbers to show the decimal grouping; grouping in powers of 10, e.g.
$7\ 358 = (7 \times 10^3) + (3 \times 10^2) + (5 \times 10^1) + (8 \times 1)$

(iii) Rounding off of numbers to the nearest 10, 100 and 1 000. (25 periods)

(c) THE FOUR BASIC OPERATIONS APPLIED TO NATURAL NUMBERS AND 0 AND TO CONCRETE QUANTITIES (0 - 1 000 000)

(i) The four basic operations are applied to objects, quan= tities and actual situations within the experience and field of interest of the pupils.

(1) Addition and subtraction

(aa) Consolidation of these operations and their application to larger numbers and to word problems.

(bb) Estimating and checking answers.

(2) Multiplication

(aa) Consolidation of this operation, its terms and its application to larger numbers and to word problems (Multiplier limited to at most three digits).

APPENDIX 6

WORK PROGRAMME : MATHEMATICS

Standard 6

(This programme makes provision for 32 weeks of teaching time. The remaining time is available for revision and examinations.)

WEEK	CONTENT	DATE WHEN COMPLETED	PAGE NO. IN DAILY WORK BOOK FOR TEACHERS
	Natural numbers and Whole numbers		
1.	Consolidation of previous work and promotion of basic skills.		
2.	Factors and multiples prime numbers; composite numbers; prime factors.		
3.	Squares and Cubes; square roots and cube roots by factorisation.		
	Test and remedial work.		
	Integers		
4.	Extension of the number concept to the set of integers; order; the additive inverse, properties of 0.		
5.	Addition and subtraction of integers.		
6.	Multiplication and division of integers.		
	Test and remedial work.		
	Rational numbers		
7.	Extension of the number concept of the set of rational numbers. Consolidation of previous work. Equivalent fractions; order; conversion of common fractions to decimal fractions and vice versa; recurring decimal fractions. Irrational numbers.		
8.	The four main operations with common fractions; order of operations.		

continued...

WEEK	CONTENT	DATE WHEN COMPLETED	PAGE NO IN DAILY WORK BOOK FOR TEACHERS
	coefficient, term, like and unlike terms.		
16.	Addition and subtraction of algebraic terms and expressions.		
17.	Multiplication of a polynomial by a monomial; use of brackets and the distributive property; raising monomials to a power.		
18.	Division of a polynomial by a monomial.		
	Test and remedial work.		
	<u>Linear equations in one unknown</u>		
19.	Consolidation of previous work: Closed and open number sentences; solution of number sentences by inspection.		
20.	Algebraic solution of equations in which co-efficients are integers.		
21.	Problems leading to linear equations.		
	Test and remedial work.		
	<u>Geometry</u>		
22.	TRIANGLES: Consolidation of previous work: Classification of triangles. Sum of the angles; relation between exterior angle and interior angles.		
23.	TRIANGLES: Relation between length of sides and magnitude of angles. The theorem of Pythagoras. Test and remedial work.		
24.	QUADRILATERALS: Properties of a parallelogram, rectangle, square, rhombus, trapezium and kite.		
25.	TRIANGLES AND QUADRILATERALS : Calculation of perimeters and areas.		

(a) triangles;

(b) problems in two and three dimensions.

3. EUCLIDEAN GEOMETRY

(i) The following must be treated within the framework of a mathematical system. Hence only axioms in logic and definitions, axioms and theorems that occur in this list or in the lists for standards 7 and 8 may be used as reasons for statements in solving riders.

(ii) Although all theorems must be proved only proofs of theorems denoted with an asterisk (and their converses where mentioned) in the following list will be required for examination purposes.

(iii) Applications of any axiom or theorem in this list or in the lists for standard 7 and standard 8 may be set. (No constructions for examination purposes.)

(iv) No more than three tenths of the marks for geometry will be given for bookwork in the examination.

(v) A logical order of the following should be adhered to.

3.1 The theorem of Pythagoras (without proof).

3.2 The line segment joining the centre of a circle to the mid-point of a chord is perpendicular to the chord, and conversely, the perpendicular drawn from the centre of a circle to a chord bisects the chord. (Theorem)

3.2.1 Corollary: The perpendicular bisector of a chord passes through the centre of a circle.

3.2.2 A unique circle can be drawn through any three points not in a line.

3.3 The angle which an arc of a circle subtends at the centre is double the angle it subtends at any point on the circumference. (Theorem)

The concepts: variable, constant and universal set of variables.

5.3 Operations

5.3.1 Addition and subtraction, limited to three terms in an expression. The concepts: coefficient, term, like and unlike terms

5.3.2 Multiplication of a polynomial by a monomial; use of brackets. (Use of the distributive property, both left and right)

5.3.3 Raising monomials to a power

5.3.4 Division of a polynomial by a monomial

6. Linear equations in one unknown (18)

6.1 Determining the roots of simple equations by inspection

6.2 Algebraic solution of equations in which coefficients are integers.

6.3 Problems leading to linear equations

7. Ratio (14)

7.1 The concept of ratio

7.2 Division of a quantity in a given ratio

7.3 The concept of rate

7.4 Percentage increase and decrease

7.5 Profit per cent and loss per cent on cost price

8. Statistics (12)

8.1 Methods of representing statistical data, including different types of graphs

8.2 Critical discussion of statistics appearing in newspaper and periodicals

8.3 Determining the arithmetic mean, median and mode of ungrouped data

9. Geometry (56)

(Knowledge of the following should be acquired through calculations and intuitively by experimental methods, for example, by plane filling and symmetry)

9.1 Pairs of angles

Complementary angles, supplementary angles, adjacent angles

9.2 Discovery of facts in connection with (aa) Use of decimal notation, e.g. 1,500 ℓ.

(bb) Basic operations involving at most two consecutive units (ℓ, kℓ) or (ℓ, mℓ)

(cc) Conversion on sight.

(4) Time

(aa) Practical acquaintance with the concepts of seconds. Oral practice in the comparative lengths of the different periods of time and the relationship between the time units in daily use, e.g. year, month, week, day, hour, minute and second.

(bb) Simple operations involving at most two units at a time, restricted to addition and subtraction.

(45 periods)

7. SIZE AND SHAPE

(a) Revision of practical work with the square, rectangle and triangle.

(b) The circle

(i) The drawing of circles with the aid of string, tins, etc.

(ii) Paper-folding: The folding of discs.

(c) Patternwork including symmetric patterns, using the geometric shapes that the pupils already know.

(25 periods)
(Grand total 240)

D. EVALUATION

Paper 1	½ hour	40 marks
Paper 2	1½ hour	110 marks

Paper 1 must test basic facts and manipulations of numbers at speed.
Paper 2 must have questions spread proportionately over the whole syllabus.
The following distribution of marks is recommended:

APPENDIX 10

Page 12 of the original research proposal

be limited to about 30 in a group. A pre-test will be administered to two or three groups of Std-9 pupils , two groups in School A and one in School B for diagnostic purposes. Two-tailed t-tests will be employed to check whether the samples could be considered to belong to the same population. (Cf. "Statistical Analysis" below.) If the results turn out to be inconclusive a "purposive sampling" will have to be made. The final choice of samples might involve aspects of "convenience sampling" as well as of "purposive sampling". However a normal distribution of (diagnostic) test scores will be emphasized and so to that extent the purposes of "random sampling" will be achieved. The whole procedure is intended to justify reasonable generalization of the research findings to the urban population, i.e., to the Std-9 pupils of the Ciskei in the urban areas.

The problem of sample erosion is unlikely to occur as the pupils form a captive audience available for the whole year while they are needed for a few weeks only for the investigation.

Nature and design of the study

The discussion above of the sampling procedure shows that the investigation proposes a quasi-experimental design though it is not altogether the "Non-equivalent Control Group Design". (Cf. Cohen & Manion, p.193).

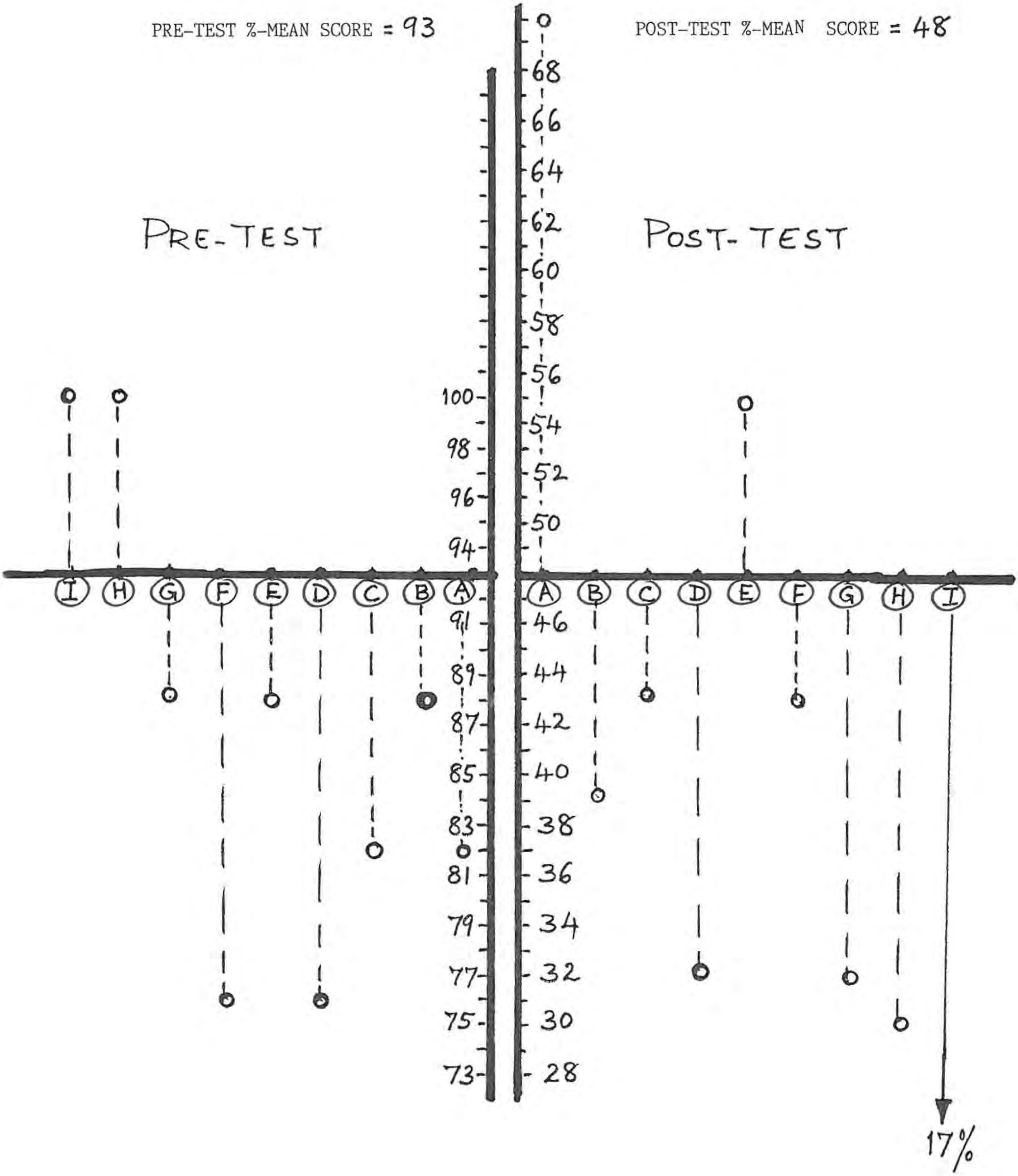
Experimental (School A)	O_1	X	O_2
Control (School B)	O_3		O_4

The possible deviation is that there may be an extra control group within the school of the Experimental group. This is intended to check on the effect of interaction between the control group and the experimental group when both groups are drawn from the same school. A further possible deviation is to include another control group in School A or B to compensate the effect of pre-test sensitization. (Cf. comment below.)

APPENDIX 11: STATISTICAL DATA

COMPARE →	NE vs NC	NE vs NC	PRE-TEST: NE vs THE WHOLE CLASS	PRE-TEST: NC vs THE WHOLE CLASS	NE, NC, C AND E AS REPRESENTATIVE SAMPLES OF MDANTSANE SCHOOLS					
TEST	POST-	PRE-	PRE-	PRE-	PRE-	PRE-	PRE-	PRE-	PRE-	PRE-
SAMPLES	1: NE; 2: NC	1: NE; 2: NC	1: NE; 2: NE-ALL	1: NC; 2: NC-ALL	1: NE-ALL 2: NC-ALL	1: C-ALL 2: NC-ALL	1: C-ALL 2: NE-ALL	1: E-ALL 2: NC-ALL	1: E; 2: C	1: E; 2: NE
N_1	24	24	24	24	38	37	37	49	49	49
$\overline{x}_1$	28,67	15,8	15,8	15,5	14,74	13,84	13,84	12,53	12,53	12,53
Σx_1^2	21290	6029	6029	5804	8604	7262	7262	7976	7976	7976
$\frac{(\Sigma x_1)^2}{N_1}$	19723	5985	5985	5766	8253	7085	7085	7694	7694	7694
N_2	24	24	38	43	43	43	38	43	37	38
$\overline{x}_2$	15,13	15,5	14,74	14,23	14,23	14,23	14,74	14,23	13,84	14,74
Σx_2^2	8139	5804	8604	8918	8918	8918	8604	8918	7262	8604
$\frac{(\Sigma x_2)^2}{N_2}$	5490	5766	8253	8710	8710	8710	8253	8710	7085	8253
$\lvert\overline{x}_1-\overline{x}_2\rvert$	13,54	0,3	1,06	1,27	0,51	0,39	0,90	1,70	1,31	2,21
df	46	46	60	65	79	78	73	90	84	85
t_c	4,90	0,7	1,58	2,56	0,86	0,80	1,45	3,49	2,57	3,75
t_t 5%	1,68	2,01	2,02	2,02	2,00	1,99	1,99	1,99	1,99	1,99
t_t 1%	2,41	2,68	2,72	2,70	2,65	2,65	2,65	2,63	2,60	2,60
t-test TYPE	ONE-TAILED	TWO-TAILED	TWO-TAILED	TWO-TAILED	TWO-TAILED	TWO-TAILED	TWO-TAILED	TWO-TAILED	TWO-TAILED	TWO-TAILED
H_0	REJECTED	ACCEPTED	ACCEPTED	ACCEPTABLE AT 1% LEVEL	ACCEPTED	ACCEPTED	ACCEPTED	REJECTED	REJECTED	REJECTED

APPENDIX 12 REGRESSION ANALYSIS: SAMPLE NE: INDIVIDUALS A TO I



APPENDIX 13

DIAGNOSTIC TEST ADMINISTERED AS PRE-TEST

CLASS:

PERSONAL DETAILS

1. SURNAME:
2. NAME:
3. AGE (in years)
4. MALE/ FEMALE (Circle the appropriate one.)
5. YOUR SYMBOL IN STD-8 MATHEMATICS EXAMINATION:

The purpose of this test is to find out about your understanding of certain concepts and relationships in mathematics.

It has nothing to do with your examinations/tests.

Please co-operate and answer the questions inthe space provided on the question paper.

Turn over the page and start answering when you are told to do so.

(TURN OVER FOR MEMORANDUM PREPARED ON A SPECIMEN PAPER.)

APPENDIX 13

PRE-TEST MEMORANDUM

QUESTION 1

Make as many ordered pairs as you can by taking one number from set A and one from Set B at a time, so that the second number is the square of the first.

$A = \{1;2;3;4;5;6;7;9;10\}$

$B = \{4;5;9;16;23;25;49;64;81\}$

Example: (2;4). Here 4 is the square of 2. NOW ANSWER IN THE SPACE BELOW.

(3;9); (4;16); (5;25); (7;49); (9;81). (5 marks)

QUESTION 2

Make any four ordered pairs by taking one number from set P and one from set Q so that both numbers are squares of natural numbers. Use a number once only.

$P = \{3; 4; 16; 25; 36; 64; 75; 90\}$

$Q = \{1; 9; 15; 49; 81; 100; 101; 125\}$

Example: (36;9). Note that $36 = 6^2$, and $9 = 3^2$. Now answer in the space below.

E.g.: (4;9); (16;49); (25;81); (64;100). (4 marks)

QUESTION 3(a)

Make FIVE statements by taking one number at a time from sets D, E and F so that;

number from D + number from E = number from F.

$D = \{1; 2; 9; 10; 11; 25\}$

$E = \{4; 7; 16; 81; 112; 144; 150\}$

$F = \{5; 9; 18; 25; 100; 115; 169; 175\}$

Example: 1 + 4 = 5. Note that 1 is from set D, 4 is from set E and 5 is from set F.

Now answer in the space below.

E.g.: 2 + 7 = 9; 9 + 16 = 25; 10 + 90 = 100; 25 + 90 = 115; 25 + 144 = 169.

(5 marks)

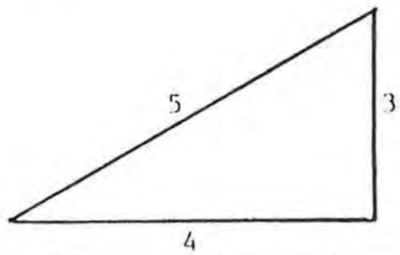
TURN OVER FOR QUESTION 3(b)

QUESTION 3(b)

One possible statement in the answer to question 3(a) above is: 9 + 16 = 25. This can be re-written as

$$3^2 + 4^2 = 5^2 .$$

This number statement can also be illustrated by a diagram as shown on the right. Make a similar statement from question 3(a) and write it here:

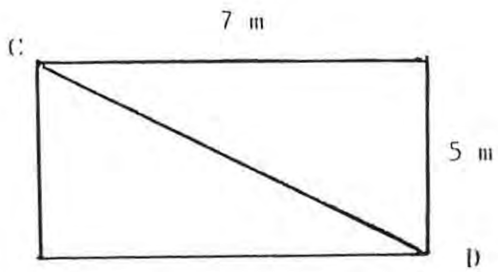


25 + 144 = 169

$5^2 + 12^2 = 13^2$. (2 marks)

QUESTION 4

A rectangle is given below. Find the length of the diagonal CD. Leave your answer in root form.



Answer here: CD = $\sqrt{74}$ m. (1 mark)

TOTAL : 17 marks.

END OF QUESTION PAPER

APPENDIX 14

NOTE: CONSTRUCTIONS ARE NOT SHOWN.

POST-TEST

SURNAME: MEMORANDUM CLASS:

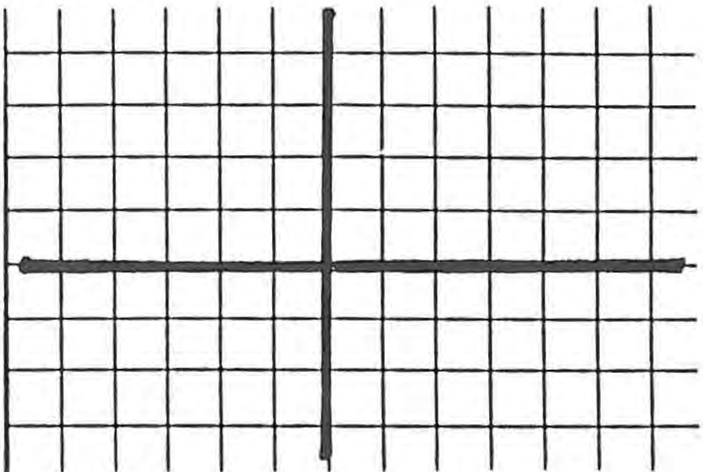
NAME :

INSTRUCTIONS: Answer all questions. Answer on the question paper itself in the space provided.

MARKS

QUESTION 1

(a) A Cartesian plane is given on the right. Construct Δ OAB with OA = 3 units and AB = 4 units. (2)



(b) (a) Measure OB and write down the length: OB = ..5.. units. (2)

(c) Calculate OB by using any relation between the sides of Δ OAB. Show the calculation in the space below:

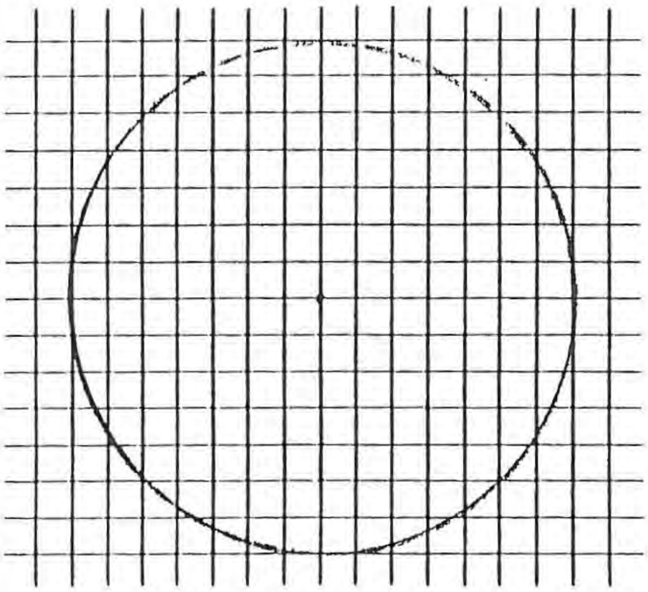
$OB^2 = 3^2 + 4^2$
(ETC.)
OB = 5 UNITS

(2)

QUESTION 2

(a) A circle is shown here on a coordinate plane. Use it to find p by geometrical construction and measurement, if (2)

$$5^2 + p^2 = 7^2.$$



Write down your answer below:

p = ..~5.. units. (2)

(b) Calculate p algebraically, if $5^2 + p^2 = 7^2$. Show your calculation below.

$p^2 = 7^2 - 5^2$
(ETC.)
$p = \sqrt{24}$ OR $2\sqrt{6}$

(2)

continued...page 2.

MARKS

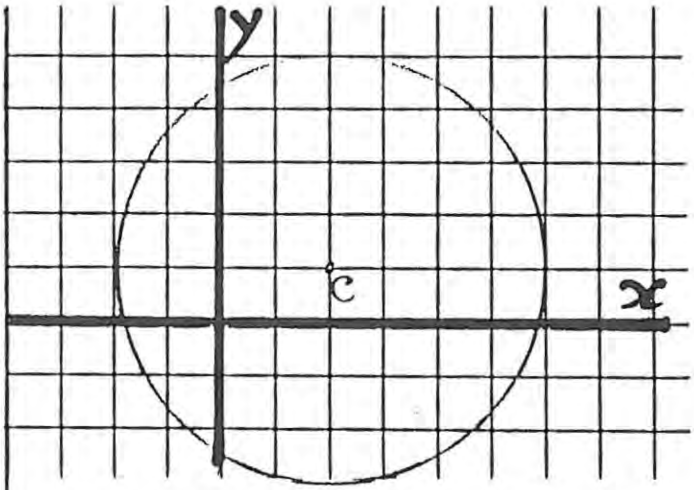
QUESTION 3

Looking at the diagram on the right write down the equation of the circle, centre C. Take 1 division on the square net as unit length.

$(x-2)^2 + (y-1)^2 = 4^2$

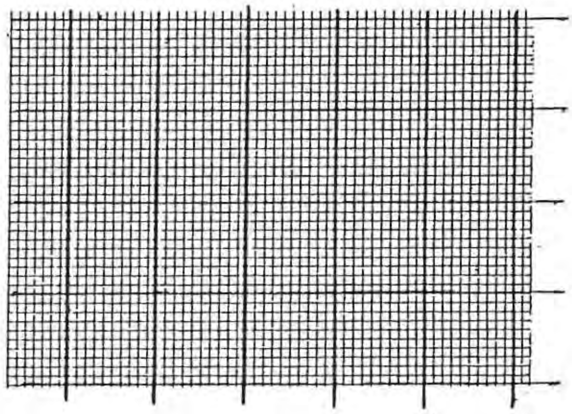
OR THE EXPANDED FORM.

(3)



QUESTION 4

Use the relation 1 + 4 = 5 to find an approximate value of $\sqrt{5}$. You may draw any suitable diagram on the square net given on the right. (2)



Write down your answer below:

$\sqrt{5}$ = 2,2 OR 2,3 (2)

QUESTION 5

P is a point marked on a Cartesian plane.

(a) Find the coordinates of P. Write down your answer below:

$13^2 = 12^2 + y^2$

$y = 5$ (ETC.)

(2)

Answer: P(12 ; 5)

(b) Show by calculation that the identity

$\sec^2\theta = 1 + \tan^2\theta$

is true for ΔOAP in the figure. Show your calculation below:

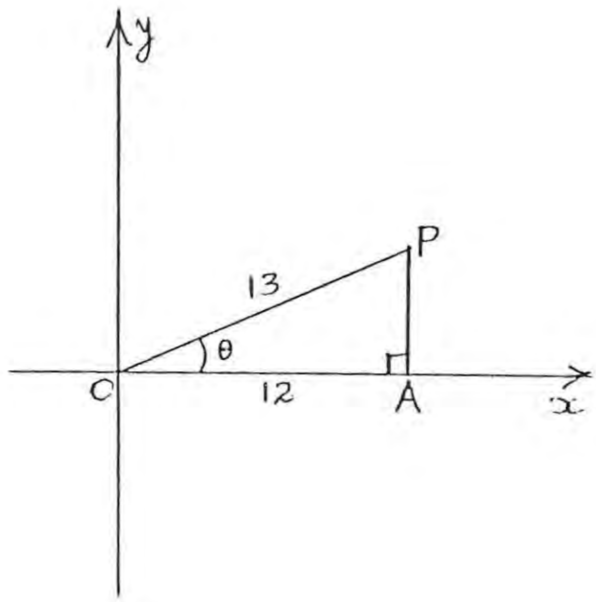
$\sec\theta = \frac{13}{12}$ ✓; $\tan\theta = \frac{5}{12}$ ✓

LHS $= \frac{13^2}{12^2}$ ✓ $= \frac{169}{144} = 1 + \frac{25}{144}$ ✓

$= 1 + \frac{5^2}{12^2}$ ✓

$= 1 + \tan^2\theta = \text{RHS}$ ✓.

(6)



continued...page 3.

QUESTION 6

Prove: $\dfrac{\sin\theta}{1+\cos\theta} + \cot\theta = \dfrac{1}{\tan\theta.\cos\theta}$.

Ignore restrictions on θ.

Show your workout below:

TRIG. FORM: $RHS = \dfrac{\cos\theta}{\sin\theta\cdot\cos\theta} = \dfrac{1}{\sin\theta}$ ✓

$LHS = \dfrac{\sin\theta(1-\cos\theta)}{(1+\cos\theta)(1-\cos\theta)}$ ✓ $+ \dfrac{\cos\theta\cdot\sin\theta}{\sin\theta\cdot\sin\theta}$ ✓

$= \dfrac{\sin\theta - \sin\theta\cdot\cos\theta + \cos\theta\cdot\sin\theta}{\sin^2\theta}$ ✓✓

$[\because 1-\cos^2\theta = \sin^2\theta]$ ✓

$= \dfrac{\sin\theta}{\sin^2\theta}$ ✓

$= \dfrac{1}{\sin\theta}$ ✓ $=$ RHS.

MARKS (8)

COORDINATE FORM:

$RHS = \dfrac{1}{\frac{y}{x}\cdot\frac{x}{r}} = \dfrac{r}{y}$ ✓

$LHS = \dfrac{\frac{y}{r}}{1+\frac{x}{r}}$ ✓ $+ \dfrac{x}{y}$

$= \dfrac{y}{r+x}$ ✓ $+ \dfrac{x}{y}$

$= \dfrac{y^2 + x(r+x)}{(r+x)y}$ ✓

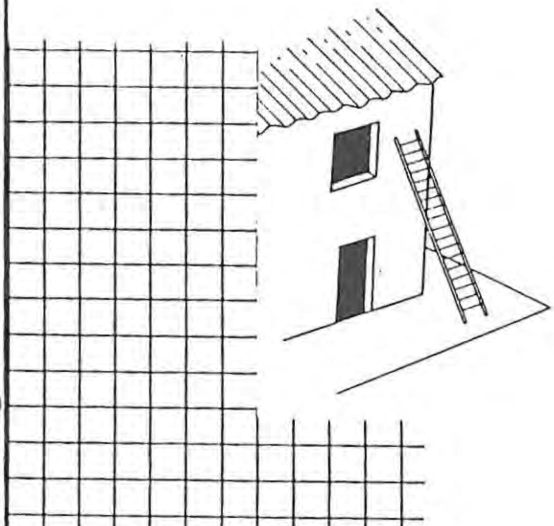
$= \dfrac{y^2+x^2+xr}{(r+x)y}$ ✓

$= \dfrac{r^2+xr}{(r+x)y}$ ✓

$= \dfrac{r(r+x)}{(r+x)y}$ ✓ $= \dfrac{r}{y}$ ✓ $=$ RHS.

QUESTION 7

A 6m-ladder rests against a vertical wall. If the foot of the ladder is 2m away from the wall on horizontal ground, at what height does the ladder touch the wall ? (You may use the square net given on the right to draw rough sketch if necessary. Leave your answer in surd form.) (2)



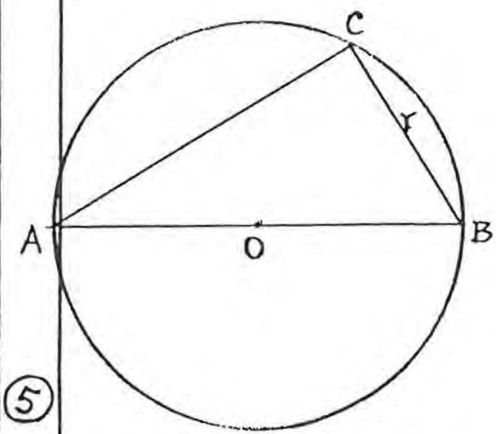
$6^2 = 2^2 + h^2$

$h = \sqrt{6^2 - 2^2}$

$= \sqrt{32}$ m or $4\sqrt{2}$ m (3)

QUESTION 8

In the figure on the right O is the centre of a circle of radius r and BC = r. Prove that $AC = \sqrt{3}\,r$.



Write down the proof below:

$AC^2 = (2r)^2 - r^2$

$= 4r^2 - r^2$

$= 3r^2$

$AC = \sqrt{3}\,r$

(5)

continued...page 4

MARKS

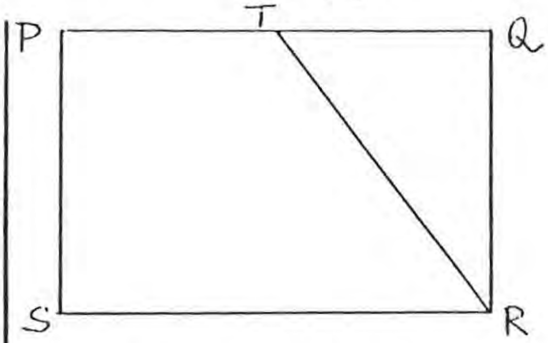
QUESTION 9

In the figure given on the right PQRS is a rectangle with sides SR = 6 units and PS = 4 units. Line segment TR bisects PQ at T. Calculate the length RT. Show your calculation below:

$RT^2 = 3^2 + 4^2$

$RT = 5$ UNITS

(4)



(You may mark lengths on the figure.)

QUESTION 10

In the figure on the right PQR is a triangle with an obtuse angle at R. Prove that

$r^2 = p^2 + q^2 + 2px$.

Write down the proof below:

IN Δ PQS:

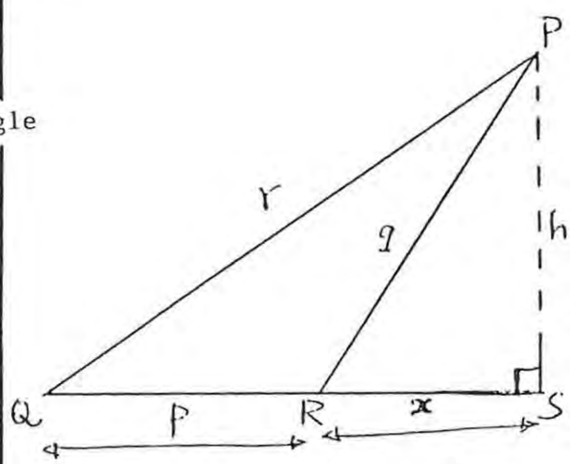
$r^2 = (p+x)^2 + h^2$ ✓✓

$= p^2 + 2px + x^2 + h^2$ ✓

$= p^2 + 2px + q^2$ ✓

$[\because q^2 = x^2 + h^2$ ✓✓
in Δ PRS$]$.

(6)



QUESTION 11

P is <u>any</u> point at a constant distance from point C on the given coordinate plane. Write down the equation of the locus of P if 1 unit on the plane is equal to unit length. YOU MAY USE A RULER ONLY FOR SOLUTION.

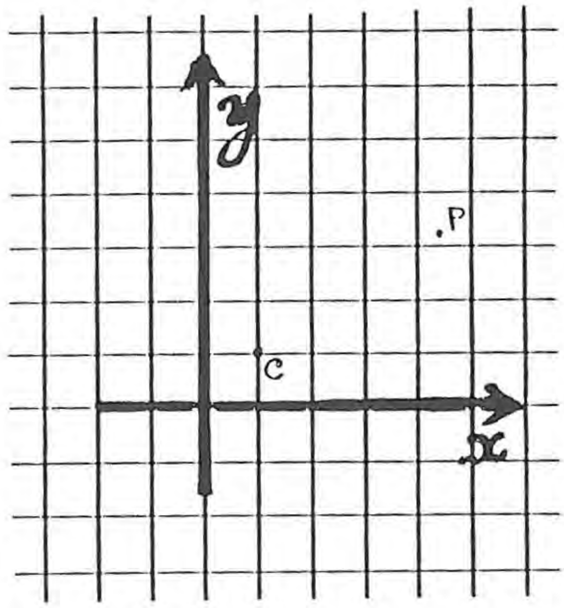
Write down the equation below.

CP = 4 ✓ UNITS (BY RULER CHECK)

C (1;1) ✓

$(x-1)^2 + (y-1)^2 = 4^2$ ✓

(5)



END OF QUESTION PAPER.

TOTAL: 60 marks.

APPENDIX 15

QUESTIONNAIRE

SURNAME:.................... CLASS:..............

NAME:...................... DATE:...............

You are shortly going to complete one year of teacher training, and you have learned about many things which were unknown to you before you started this course. Please answer the following questions as best as you can.

1. Do you like the Primary Teacher's Course ? Tick your answer in the box below:

like very much	like moderately	do not know	dislike somewhat	dislike very much

2. There are many different subjects offered in your course. Write down in the box below the name of the subject you like best.

3. When you become a teacher after completing your training, which THREE subjects would you best like to teach ? Name the three subjects in order of liking in the box below.

best	
next best	
last	

4. Do you think Mathematics must be compulsory for teacher training? Tick Yes or No in the box below.

YES	
NO	

5. Do you think ALL pupils should be taught Mathematics in the Primary School ? Tick your answer Yes or No in the box below.

YES	
NO	

6. Do you think that ALL Primary teachers should teach Mathematics ? Tick YES or NO in the box below.

YES	
NO	

continued...2/-

7. What level of knowledge in Mathematics is essential for Primary Teachers? Tick your answer in the box below.

Standard 5	Standard 8	Standard 10	Name any other if you think of such

Questions about the special course we conducted with you

8. Do you think it was relevant to you ? Tick YES or NO in the box below.

YES	
NO	

9. Did you understand the lessons ? Tick YES Or NO in the box below.

YES	
NO	

10. What score do you expect in the test you wrote at the end of the course? Tick your answer in the box below.

POOR	
AVERAGE	
GOOD	

11. If you have any other comment about the course write down what you think about it here below.

Thank you for your co-operation.

APPENDIX 16

Two specimen worksheets used in the course administered on the Pythagoras theme.

(a) Inductive discovery of the equation of a circle centred at $C(x_c;y_c)$, radius r.

A few points on the circle, scattered in the different quadrants, are marked, labelled and their distances from the centre C related to their coordinates (x;y) through Pythagoras. It is then established that in all cases the horizontal leg (of the relevant right triangle) is $(x - x_c)$ and the vertical leg $(y - y_c)$; the radius r is simply measured by counting the squares on the grid, either in the x-direction or in the y-direction. Hence the equation to the circle follows as

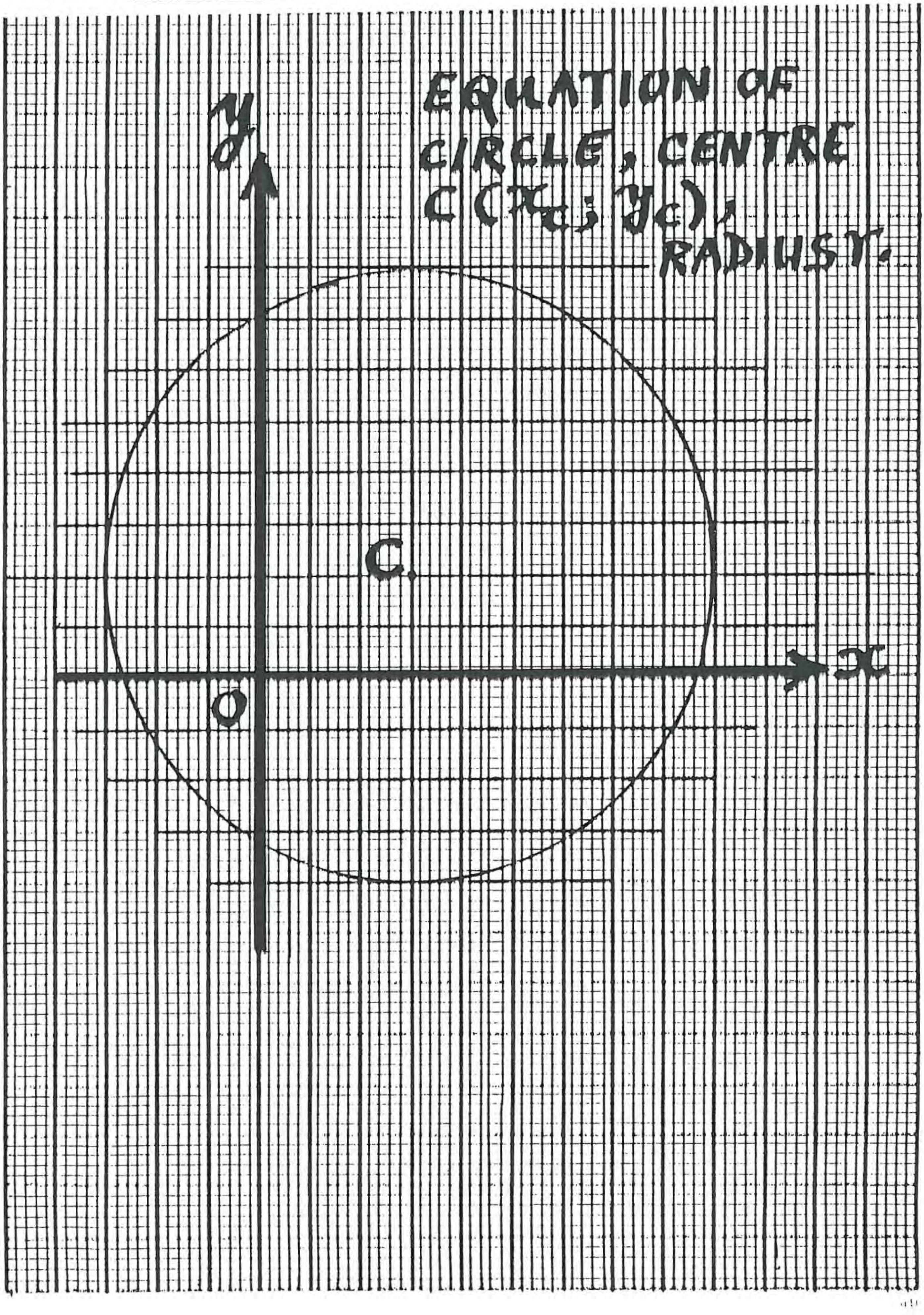
$$(x - x_c)^2 + (y - y_c)^2 = r^2.$$ (Cf. appendix 16(a).)

(b) Word problems involving Pythagoras theorem can be successfully translated on to square grids, (cf. appendix 16(b)). The legs of the relevant right triangle can be drawn as line segments horizontally and vertically while the hypotenuse will be a length in some other direction. Two procedures are available, both of which were adopted in solving each problem.

(i) The hypotenuse is measured by means of dividers and read off by placing the points of the dividers along the horizontal or the vertical squares of the grid.

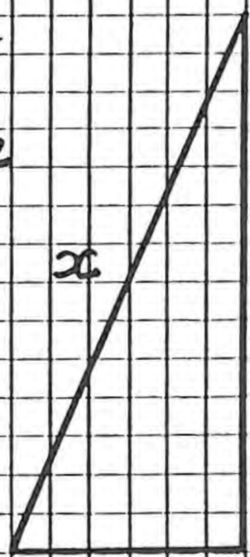
(ii) By calculation applying Pythagoras, with two lengths measured as in (i) above.

APPENDIX 16(a)



APPENDIX 16(b)

Word problems applying Pythagoras are easily transferred to square grids like these.



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