INVESTIGATING THE DUAL INFLUENCES OF THEORY AND PRACTICE ON THE DESIGN AND IMPLEMENTATION OF A LEARNING PROGRAMME

A thesis submitted in partial fulfilment of the requirements for the degree of

MASTER OF EDUCATION

(Mathematics Education)

RHODES UNIVERSITY

by

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December 2011

ABSTRACT

It is widely recognized that educational research and theory should be motivated by the desire to continually improve the practice of teaching. However, bridging the divide between theoretical research outcomes and the practical constraints of classroom-based teaching has proved somewhat challenging. The involvement of teachers as the 'bridge-builders' between theory and practice could provide an effective mechanism for achieving this integration. The purpose of this study is thus to investigate whether the involvement of teachers in developing and implementing a theory-based teaching module would improve teaching practice in the classroom.

A teaching module was collaboratively developed by a group of teachers for Grade 9 linear functions using: the principles of mathematical proficiency postulated by Kilpatrick, Swafford and Findell, (2001); the teaching phases formulated by van Hiele (1986); and the cognitive classification of classroom activities developed by Stein and Smith (1998). This module was then taught to six Grade 9 classes by four teachers in one school in the Eastern Cape, South Africa over a period of 5 weeks.

The effectiveness of the module, and its application in the classroom, was assessed in terms of: (i) the extent to which theory could be used to inform the design and development of teaching materials; (ii) the efficacy of this teaching material in promoting teaching for mathematical proficiency; and (iii) the effects of extraneous influences on the usefulness of the module in teaching for mathematical proficiency.

While the theoretical framework provided a sound basis for developing the teaching module, it was found that collaboratively transforming this theory into a teaching module for practical use in the classroom is certainly possible, but it requires considerable time and effort that practising teachers do not have.

Developing the depth of understanding required for mathematical proficiency also takes time – a commodity often in short supply as teachers grapple with the demands of the curriculum. Teaching for mathematical proficiency is a layered process. It starts with thinking about an idea (like a graph) that is developed out of a related concept that then has a set of characteristic algorithms and actions which are learnt and performed in sequence. Building understanding in this way ends with a student being able to visualize and conceive the graph

as a structure that can be described as if it were an object (encapsulating all the previous concepts belonging to similar graphs in one idea). This development of understanding is important for mathematical proficiency but is not necessarily easy. When teaching with the module, it was necessary to create an extra opportunity for students to use procedural knowledge and repetition in order to provide enough examples to help them see the link between linear number patterns and linear graphs.

Extraneous influences on teaching for mathematical proficiency were grouped into two categories – endogenous and exogenous influences. Endogenous influences were teacher-related and included the attitudes, decisions and disposition of the teacher. Exogenous influences were more contextual (and in effect out of the control of the teacher) and included teaching time available, curriculum, external assessments etc. Both of these influences were seen to affect teaching for mathematical proficiency, either promoting or inhibiting it.

This research affirmed the central role that teachers play in teaching for mathematical proficiency. It is considered critical that research actively involve teachers in the evolution of mathematical theory. The development of an enabling environment (including institutional support, time, capacity, resources, skills and tools) for teachers will further enhance their capacity to teach for mathematical proficiency.

ACKNOWLEDGEMENTS

There are a number of people without whose support I would have been unable successfully to complete this project. To the following, I extend my grateful thanks.

My colleagues, whose enthusiastic support of the teaching programme we used made it possible. I was able to trust my data and seriously consider the results of teaching with the module. I need to thank them for allowing me to get a glimpse into their classrooms by videoing their lessons and for taking the time to talk to me about their teaching practice. This invaluable exercise highlighted for me the importance of teamwork, in aiming for, and achieving excellence in the teaching of mathematics.

My father, for taking the time to read through my thesis and help me with the editing of so many pages!

My mother, who gave her time and support in a way that allowed me the freedom to attend the contact sessions by taking over the care of my children when I needed a third parent!

Last but not least, my husband, James, who talked me through all the stages of the research, acted as a sounding board, gave me critical feedback on many sections that required editing, and who took the time to involve himself in an area that plays such an important role in my life. His willingness to take over parenting, to entertain children on the weekends when I was working, and to deal with so many things that I was unable to get to in the time I had available, have enabled me to pursue an academic passion. His support and interest kept me going when I wondered what I was doing. Thank you.

DECLARATION OF ORIGINALITY

I, Susan Iona Jackelman (Student number g10j5263) declare that this thesis: Investigating the dual influences of theory and practice on the design and implementation of a learning programme is my own work, written in my own words. Where I have drawn upon the words or ideas of others, these have been acknowledged using the reference practices according the Rhodes University Educational Guide to referencing.

Susan Iona Jackelman
(Signature)

30/11/2011
30 November 2011
(Date)

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LIST OF ACRONYMS

NCS – National Curriculum Statement

CAPS – Curriculum and Assessment Policy Statement

DOE – Department of Education

DBE - Department of Basic Education

Note: after 2010 the DOE was called the Department of Basic Education (DBE) and all government-sourced publications from this time (i.e. the CAPS documents) are referenced to the DBE.

HOD – Head of Department

NAPTOSA – National Professional Teachers Organisation of South Africa

UNESCO – United Nations Education, Scientific and Cultural Organisation

CHAPTER 1: INTRODUCTION AND CONTEXT

1.1 Introduction

Curiosity is a useful human characteristic because inquiry is a tool that we to can use to "enable ourselves and others to engage with key questions and issues in practice"

Jarworski (2006, p. 187).

I am a practicing teacher and have always been interested in how the way in which I teach can be used to promote understanding in my students. This curiosity led me to the realization that to learn more I needed to read about, and engage with, the ideas put forward by others in the field of mathematics education. A particular interest is one that can be considered the "art" of teaching - a process that is more complex than many people believe. This is highlighted by the work of Van Hiele (1986) in his discussion of the phases of teaching required in assisting students to become proficient in a higher level of thinking about mathematical concepts. As a mathematics teacher I am specifically interested in learning about and applying those approaches that facilitate the successful teaching of mathematics to all students, not only the able ones.

In order to examine the practice of teaching and the implications of attempting to adopt a different approach to teaching mathematics, it was necessary to first examine what is meant by good teaching practice. This has been a fascinating journey. One of the ideas that particularly struck me was the notion of "mathematical proficiency" put forward by Kilpatrick, Swafford and Findell (2001). This idea made me realise that in order to teach mathematics well, it was not only necessary to examine what 'teaching well' entailed, but also to understand what kind of mathematical understanding such a teacher might hope to promote. The need to clarify these ideas, as well as the challenges facing those teaching in the 'shifting sands' environment of current educational curricula and policy in South Africa, led me to decide what I would use as a vehicle for my research in this thesis.

The recent National Curriculum Statement (NCS) for South Africa presented a curriculum that required teachers to structure and design their own programme of learning. In the 2009 report on the implementation of the NCS, it was noted in the submissions received that there was strong resistance to the notion that teachers could be curriculum designers. One of the functions of the Curriculum and Assessment Policy Statement (CAPS) was to attempt to relieve teachers of this burden. However, as noted by Fuson, Kalchman and Bransford (2005), even when using a prepared curriculum, teachers have an important role to play in designing the experiences students have in the classroom. These occasions are examples of what they call "openings in the curriculum". Although Remillard and Geist (2002) consider these openings to consist of instances in teaching where the prescribed methods for the presentation of concepts do not work and have to be adapted or redesigned, they also refer to those opportunities that arise when using a new curriculum. At the moment, due to the rapid change of curricula that we are currently experiencing in this country (discussed in 1.2 below), publishers are unable to respond quickly enough to meet the needs of teachers for resource material and teaching guides. Without a comprehensive textbook for Grade 9 we have such an "opening in the curriculum". It is therefore necessary for us as teachers to play a more active role in examining and deciding upon these teaching approaches. This gives us an opportunity to reflect on, and attempt to develop, a teaching process that will provide students with the requisite mathematical skills.

These factors provided me with a unique opportunity to invite my colleagues to attempt a different approach to teaching one section of the Grade 9 syllabus – specifically number patterns and linear functions. The reason I used this grade and this particular topic was because (i) Grade 9 is the year in which students are required to choose between pure mathematics or what is called mathematical literacy; and (ii) this particular topic is one in which students struggle to understand the concepts involved. To consider an alternative approach, it was necessary to examine the theories underpinning mathematical proficiency and what it means to teach for mathematical proficiency. This resulted in me not only having to engage in the idea of what to teach for, but also to look at theories on how to teach for this outcome. It would be necessary to investigate the kind of activities which might promote this desired outcome and the teaching skills required to support and enable the teaching approaches I was interested in evaluating.

Reading for this degree has created an appreciation for the vast amount of effort and research that has gone into education, but I have also realised how little seems to translate into the daily teaching practices of both myself and my colleagues. This is due mainly, I believe, to a lack of either awareness of such ideas or the resources to apply such research to practice. It was this lack of understanding on my part regarding how research could be used to inform teaching practice, coupled with my interest in examining good practice in teaching mathematics that resulted in the idea of putting together a module that would help us to increase mathematical proficiency in this area of the Grade 9 syllabus. This endeavour would also provide us with an opportunity to reflect upon our own teaching practice.

1.2 Context of study

1.2.1 SA education

The context in which this study has been undertaken must be understood in order to appreciate the influence and impact it has on teaching and learning in South African classrooms. In the past twelve years the South African curriculum has undergone three curriculum changes, with another revision due for implementation in 2012. The implementation of the first post-apartheid curriculum - Curriculum 2005 - began in 1998. This curriculum and its new "reform" terminology were "intended to promote the rebuilding of a fragmented society" (Department of Education, 2003). However, it was noted that the desire for rapid change overruled the more pragmatic need for "adequate research, trialling and teacher preparation for the new ideas and learning theories" promoted by this curriculum (DOE, 2009b).

Curriculum 2005, as pointed out by Brodie (2010), leaned heavily on the constructivist learning theories developed through the work of educational researchers such as:

- Jean Piaget, whose views of how children "construct" understanding through play suggested that knowledge is built upon knowledge (Prawat, 1992);
- Ernest von Glasersfeld, who suggested that we actively construct our own knowledge, with learning taking place when we adapt our previous knowledge in order to take new knowledge into account (von Glasersfeld, 1987); and
- Lev Vygotsky, who proposed that knowledge and understanding is constructed through the use of language and interactions with others (Jaworski, 1994).

Learning, as understood in the context of these theories, implies that in the mind of the learner, new experiences are integrated into, and linked to, existing knowledge (Bodner, 1986). The developers of Curriculum 2005 therefore viewed teaching and the role of teachers in a new light. The teacher was no longer a transmitter of knowledge, but an active participant in facilitating student interaction in a "set of learning experiences designed to guide students in the development of their understanding" (Driver and Oldham, 1986, p.

112). The teachers' role was now not to provide facts and information, but rather to "guide and help the student in the conceptual organization of certain areas of experience" (von Glasersfeld, 1987, p. 16).

Unfortunately, this was not well communicated to the teachers. The curriculum emphasized general principles and "outcomes" to the detriment of the unambiguous details required by teachers in the classroom for the day-to-day teaching content in their subjects (DOE, 2009b). As Matthews (2000, p. 175) observed in his critical analysis of the impact of the theory of constructivism, particularly on mathematics and science education in New Zealand, "the catalogue of requirements for social constructivism as presented to schools has everything in it except knowing the subject matter to be taught and being able to teach it in a clear, engaging and understandable manner". The ensuing misunderstanding and misapplication of policies resulted in teachers not knowing what to teach and how to do so in order to effectively fulfill the aims of the curriculum. It undermined the development of skills - particularly, numerical skills - needed for mathematical thinking at higher levels, and prompted another revision of the curriculum.

This revision - the National Curriculum Statement (NCS) - was implemented in Grades 0 – 9 in 2004, but did not provide for a new plan for Grades 10 – 12, which led to a further revision of the curriculum in order to allow for greater continuity (National Curriculum Statement, DOE, 2008). The first Grade 12 external examinations for this curriculum were written in 2008. However, continued problems with lack of clarity regarding the aim and assessment of the curriculum, the unwieldy language used to describe the subject content and the lack of teacher training for ensuring adequate implementation of this curriculum, resulted in a *revised* National Curriculum Statement being issued in 2009 in order to provide a comprehensive view of education from Grade 1 to 12. These issues also prompted the then Minister of Education to commission a report to highlight the problems surrounding the implementation of the National Curriculum Statements and to make recommendations for the improvement of its delivery to students via their teachers. This report highlighted, among other concerns that:

> "... rather than providing teachers with a clearer specification on what to teach, the outcomes and assessment standards (language used in NCS) are distracting and suggest that teaching can be achieved through various activities rather than focusing on the teaching and application of concepts" (DOE, 2009b, p. 46).

This problem seemed to permeate many of the Grade 8 and 9 textbooks available for use in the classroom – students were exposed to a series of activities that did not develop a comprehensive understanding of the topics and their related concepts. Teachers were focusing on learning the new terminology and categories into which the content had been organised in order to report on their assessments of student learning.

The recommendations put forward by the 2009 report reviewing the implementation of the National Curriculum Statement report (DOE, 2009b) have resulted in yet another curriculum change. The Curriculum and Assessment Policy Statement (CAPS) document was published for comment in 2010 and will be implemented in Grade 10 in 2012, with the intention of the first Grade 12 exams for CAPS to be written at the end of 2014. The recommendation that a national external examination be written at the end of Grade 9 in mathematics and language was trialled in 2010. The current (2011) Grade 9 students will be the first to write the official Grade 9 and Grade 12 National Examinations for CAPS.

So, to date South African mathematics teachers have had to deal with three curriculum changes with another to follow, as the DOE attempts to clarify the purpose of its educational policies and provide teachers with a clear guide as to what they should be teaching. However, a good curriculum policy is no guarantee of improved teaching and learning. To consider the teacher as a technician who, given a curriculum to implement, will do so without playing a role in "co-constructing" the curriculum along with students offers a top down view of the curriculum and therefore of change (Kilpatrick, 2009). The Second International Mathematics Study (SIMS) considers that a curriculum has three levels: (i) *Intended curriculum* - the administrator's point of view; (ii) the *Implemented curriculum* - the teacher's point of view; and (iii) the *Attained or realized curriculum* - the student's point of view. In order to bring these into alignment we need to understand that curriculum change is not a technical matter, instead it is a blueprint for a curriculum to be realized and a personal journey for mathematics teachers (Kilpatrick, 2009, p. 119).

Constant changes to what to teach and to how mathematics is assessed have left mathematics teachers change-weary, struggling with the demands of the curriculum without time to grapple with the way in which these demands required changes in their teaching practice. As the 2009 report on the review of the implementation of the National Curriculum Statement notes: "teachers need to regain confidence in their practice and authority as subject specialists in the classroom" (DOE, 2009b). One way to do this is by actively engaging in what it means to teach for mathematical proficiency.

1.2.2 Mathematics as viewed by the South African curriculum

Any mathematical curriculum has at its heart a belief in what mathematics is and how the understanding of mathematics is developed. As the mathematics curriculum for South African schools has been revisited and revised over the past 12 years this view of mathematics has moved from one where the logic of the development of understanding was based on general constructivist principles to one where the logic was derived from the subject discipline itself. This movement is evidenced in the changing emphasis that can be found in curriculum documents published by the National Education Department from 2005 to 2011 (CAPS). In order to summarise these changes, the overview shown in *Table 1.1* below was developed.

Table 1.1: Tracking changes of the way recent SA curricula view mathematics

	Components of the way in which mathematics is viewed in the recent South African curricula.	**The curricula supporting these views of mathematics.**
1	Mathematics enables creative and logical reasoning about problems in the physical and social world and in mathematics itself.	2005 NCS 2008
2	Knowledge in mathematics is *constructed* through the establishment of descriptive, numerical and symbolic relationships.	2005 NCS 2008
3	Mathematics is *developed* and contested over time *through both language* and symbols by *social interaction* and is thus open to change.	2005
4	Mathematical ideas and concepts *build on one another* to create a coherent structure.	NCS 2009
5	Mathematics is a distinctly human activity based on observing patterns.	2005 NCS 2008 NCS 2009 CAPS
6	Mathematics involves observing representing and investigating quantitative relationships in physical and social phenomena and between mathematical objects themselves.	NCS 2009 CAPS
7	Through this process (above) new mathematical ideas and insights are developed.	NCS 2009
8	Mathematical problem solving enables us to understand the world and make use of that understanding in our daily lives.	2005 NCS 2008 CAPS
9	Rigorous logical thinking leads to theories of abstract relations.	2005 NCS 2008
10	Mathematics helps develop mental processes that enhance logical and critical thinking, accuracy and problem solving that contribute to decision making.	CAPS
11	Mathematics uses/is a specialised language that involved symbols and notations for describing numerical, geometric and graphical relationships.	NCS 2009 CAPS

Although the view of mathematics put forward by the most recent curriculum is still underpinned by constructivist learning theory - it implies that learners do engage in constructing knowledge as they come to understand certain ideas - the latest revision bases its view of mathematics upon the acceptance that these concepts are inherent in the subject (DOE, 2009b). This understanding of mathematics is closer to that suggested by Harel (2008). When describing mathematical knowledge, he uses two categories:

Category 1: *Ways of understanding*

This is the collection of structures such as axioms, definitions, theorems, proofs, problems and solutions i.e. the specifics of what is understood and has been done in mathematics. In other words, ways of describing mathematical constructs using procedures to solve problems – these procedures would form the bulk of the specific content focus in a curriculum.

Category 2: *Ways of thinking*

All the ways of thinking about mathematics which are characteristic of what Harel (2008) calls the "mental acts whose products comprise the first set". These are acts such as interpreting a mathematical representation in order to make a decision as to which procedure would be appropriate to solve a given problem or to prove a conjecture and rely on an understanding of the underlying principles involved.

The presentation, teaching and assessment of the South African curriculum focuses on the more procedural-based "*ways of understanding*" (described by Harel, 2008), which are directly content based. Yet in order to realize the intention of the curriculum, the personal journey for mathematics teachers referred to by Kilpatrick (2009), must include grappling with what it means to teach for the more conceptual "*ways of thinking*" which are the deeper mathematical principles. As Harel (2008) notes in his discussion on mathematics and the teaching of mathematics, instruction should focus on both categories so the conceptual tools necessary to construct an understanding of mathematical objects (such as those necessary to understand linear functions and their related concepts) are not neglected.

1.2.3 Grade 9 mathematics in South Africa

In Grade 9 in South African schools, the revised NCS (DOE, 2009) and CAPS (DBE, 2011) requires teachers to cover a broad spectrum of mathematical concepts. This is intended to provide students with the foundation they would require for the subject choice they make in

Grade 10. According to the curriculum documents mentioned, by the end of Grade 9 the mathematics student should (*inter alia*):

- Be able to represent numbers in a variety of ways, and move flexibly through these representations.
- Be able to describe patterns and relationships through the use of symbolic expressions, graphs and tables.
- Identify and analyse regularities and change in patterns and relationships that enable learners to make predictions and solve problems.
- Establish relationships between variables in numeric and geometric patterns.
- Express rules using algebraic language.
- Be provided with opportunities to develop mathematical thinking skills such as generalising, explaining, describing, and justifying.
- Be provided with situations in a variety of contexts so as to develop the ability to analyse them, represent them and describe them in algebraic language, formulae, expressions and graphs.
- Develop an appreciation of how algebraic manipulation is useful for solving problems (not for its own sake).
- Represent data graphically in order to interpret and make predictions about situations.

The emphasis of the syllabus on algebraic and problem solving skills that allow for representation and analysis of number relationships highlights the importance of developing an understanding of graphs.

1.2.4 Straight line graphs

One of the mathematical ideas introduced in Grade 9, and which provides a teaching challenge for teachers, is the concept of a **linear function**. The intention of this topic (linear functions) is to introduce students to ways of understanding the idea that a single relationship can be described and represented in multiple ways. This provides the groundwork for the understanding of all graphical relationships taught in mathematics in Grades 10 – 12. A high value is placed on the ability of students to work with graphical illustrations of relationships, as these representations are used to describe many situations, not only those within the field of mathematics. Gagatsis and Shiakalli (2004), suggest that this ability – the understanding of the different representations of a relationship and translating these from one representation to another – can be correlated with success in mathematics education.

As discussed, the NCS (2009) and the latest CAPS (2011) require the learner to recognise, describe and represent patterns and relationships, as well as solve problems using algebraic language and skills. The knowledge of, and ability to work with, linear functions provides a way of assessing this competence. To achieve this goal it is necessary to teach far more than just the algorithms used in working with linear functions. Gagatsis and Shiakalli (2004) suggest that instruction should encourage students to practice translating from one representation of the concept of function to another, and to use translation strategies to foster their problem-solving ability.

1.2.4.1 Challenges to teaching graphs/functions

Teaching functions and their related concepts is challenging. This is a topic that seems to show up quite fundamental divisions between those students who grasp the concepts, or at least are able to successfully navigate their way through the procedures required of them in this section, and those who don't. Some of the concerns that teachers need to address when working through this topic are highlighted below.

- Algebraic graphs are introduced for the first time in Grade 9 when we teach linear functions. This section is traditionally left to the end of the Grade 9 year by experienced mathematics teachers in the schools in which I have taught. Our experience tells us that conceptually students are not ready to deal with this mathematical idea earlier in the year. These problems seem to be mainly attributable to students' inability to link the symbolic and visual representations of functions. The teaching of algebraic graphs is as frustrating for teachers as teaching geometry. Explanations given numerous times do not seem to improve the understanding of our students; we feel out of sync, unable to perceive the gap we cannot seem to bridge.

- At the beginning of 2010, the Eastern Cape Education Department issued the work plan for the year which they intended to use when setting the Grade 9 external exam for June. Straight line graphs were expected to be taught in the first half of the year. If this is the expected order of teaching in the future, then the way in which we teach this fundamental introduction to the world of functions will need to be revised.

- One of the problems that the CAPS seeks to address is the disjuncture that still exists between the Grade 0 – 9 and grade 10 – 12 band of formal schooling. There is currently greater subject knowledge required for the beginning of grade 10 than is presently provided for at the end of Grade 9. Each band fell under different directorates within the national Department of Education and transition between these bands is not smooth. As mathematics teachers **we** need to grapple with how to teach grade 9 graphs to provide a solid foundation for Grade 10. It is also necessary to consider how to teach this section in order to realize the content specified in the CAPS documents.

- The time allocated in the year for teaching mathematics encompasses the teaching of many concepts. The South African mathematics syllabus, similar to the American syllabus discussed by Kilpatrick et al. (2001), is very broad. This makes teaching for depth difficult. As teachers we need to design our teaching approach in order to attempt do both.

These concerns, together with my interest in the way in which educational theory could be used to inform and influence teaching have therefore directed my research project. The objectives of this thesis, motivated by this curiosity, are outlined in section 1.3.

1.3 Objectives of the Research

1.3.1 Research purpose and goals

The main purpose of this research is to develop, implement and evaluate the efficacy of a theory-based teaching module, developed for Grade 9 linear functions, within the context and constraints of the South African education system.

The underlying goals of my research are to explore the following issues:

- To what extent can theory be used to inform the design and development of teaching material?

- Can such teaching material be used to promote the teaching for mathematical proficiency?
- What influences in teaching practice can be identified as affecting the teaching for mathematical proficiency?

While these questions are unlikely to be fully addressed by research of this scale, they have influenced the design approach taken in the study.

1.3.2 Research participants and site

The research involved four Grade 9 teachers at a high school in the Eastern Cape, South Africa. I am currently a practicing teacher at this school. This is advantageous, in that I had the active support and interest of my colleagues in implementing the module. A possible limitation in terms of objectivity must be noted due to my investment in what I will be teaching (Adler, Ball, Krainer, Lin and Novotna, 2005).

The teaching of this module took place during the second term of the 2011 school year – i.e. April – June. The concepts taught were examined in the June examinations of that year.

Permission was sought from the Headmaster and Governing Body of the school to conduct the research in the school and to use the module to teach the Grade 9 second term syllabus. Consent was also obtained from all the teachers involved in the implementation of the module and for the interviews required for the collection of data. (*Appendix* 8)

1.4 The structure of this thesis

The remaining chapters of the thesis are presented as follows:

Chapter 2, the literature review, discusses the educational theories informing the theoretical framework and design of the research undertaken. Different ideas that can inform and supported teaching strategies that are effective in promoting mathematical proficiency are explored.

Chapter 3 discusses the process undertaken in order to develop the teaching module using the educational theories discussed in the literature review. Here the outline and structure of the teaching module is presented.

Chapter 4 reviews the accessibility of the theories used to inform the design and development of the teaching module in terms of whether they allowed me to relate conceptual information to teaching practice in a procedural, specific and pragmatic way. The influence of these theories on the design of the module is discussed.

Chapter 5 is the methodology chapter. The focus of this chapter is on the methods used to collect information on the implementation of the teaching module in the classroom, and the experiences of the teachers.

Chapter 6 collates, presents and analysis the results obtained from the methods described in the methodology chapter. These results are analysed by looking at the influence of teaching practice on the implementation of the teaching module.

Chapter 7 discusses this analysis in terms of the research purpose and goals and presents the findings that came out of this research.

Chapter 8 concludes this thesis, giving the most important findings, exploring some of the possible implications of these findings and making some recommendations based on these.

CHAPTER 2: LITERATURE REVIEW

ELEMENTS GUIDING THE DEVELOPMENT OF THE RESEARCH

2.1 Introduction

The importance of the link between theory and practice in the field of mathematics education is widely attested to in the literature. The significance of this interrelationship, which is emphasized, *inter alia*, by Groth and Bergner (2005), lies at the heart of the methodology informing the current research project.

Elliot (2009) insisted that research on education is a way of finding out about teaching and learning. Quality instruction depends on teachers, and their preparation and continuing professional development is central to good teaching practice (Adler et al., 2005). Mathematics education research can help teachers develop and improve upon aspects of the knowledge base they need for teaching. Conversely, teachers can provide insights into teaching practice capable of providing useful directions for academic investigation.

In this literature review I present the various ideas that have informed and supported my interest in developing teaching strategies for promoting mathematical proficiency in students. I believe that these theories share a common perspective, the idea that educational research should be motivated by the desire to examine and improve teaching practice. To the extent that their underlying principles and goals are similar, the various research ideas canvassed below are mutually compatible (Radford, 2008).

2.2 The need for and discussion of proactive engagement in change:

Proactive engagement in change that supports the adaptation of teaching methods to promote the development of mathematical proficiency is, I believe, much like developing skill in teaching – a continuous learning curve. The art of teaching mathematics is complex, involving continual balancing of the knowledge of the mathematics to be taught, the students to whom it needs to be taught and the best way in which to engage these students in the learning of mathematics.

Many educational theories provide frameworks within which to position ourselves. While these theories are helpful in providing lenses through which teaching practice can be critically analyzed (Lerman, 2001), they can be unclear as to their use in providing teachers with practical advice (Jaworski, 2006; Brodie, 2010) – a translation we are often left to grapple with ourselves. This "missing part" in educational research – how to make it work in practice - is highlighted by Lester and Wiliam (2002). Their contention is that knowledge developed around teaching needs to be almost literally "taken into the classroom", so that it can be integrated into the ways in which mathematics is taught. Research needs to allow for the reality that students and teachers are not static, but respond differently to different contexts and activities (Lerman, 2001). This thought is echoed by Davis and Simmt (2006) who note that "mathematics for teaching is dynamic, always occurring in contexts that involve others". By considering the influences of theory and practice upon the teaching of mathematics it is possible to both gain insight into teaching and to promote the development of theory (Lerman, 1990; Jarworski, 2006).

An overview of research considering teacher professional learning and development compiled by The International Academy of Education (Timperley, 2008), identified studies that had a positive impact on student outcomes. This led to their conclusion that:

- Research based programs are more effective when their approaches are context-specific, attempting to translate research principles into applications directly applicable to what is being taught in the classroom.
- Practical application of theory is more effective than just teaching the theory without also considering how to translate these into practice. This promotes more meaningful and sustainable development of teaching skills which in turn promotes effective teaching.
- Participating in an investigation with other colleagues into the applicability of research to practice fosters greater integration of knowledge into practice.
- Successful programs understood, and acknowledged that:
    - Student learning is strongly influenced by what and how teachers teach.
    - Teaching is a complex activity, with the decisions teachers make both in planning and executing lessons being influences by a number of factors intrinsic to the teacher and the context within which they teach.

Stein, Remillard and Smith (2007) noted how curriculum influences student learning, that teachers influence how the written curriculum (i.e. policy documents and material resources) changes into a form they (the teachers) believe will be workable in the classroom (as illustrated in *Figure 2.1*).

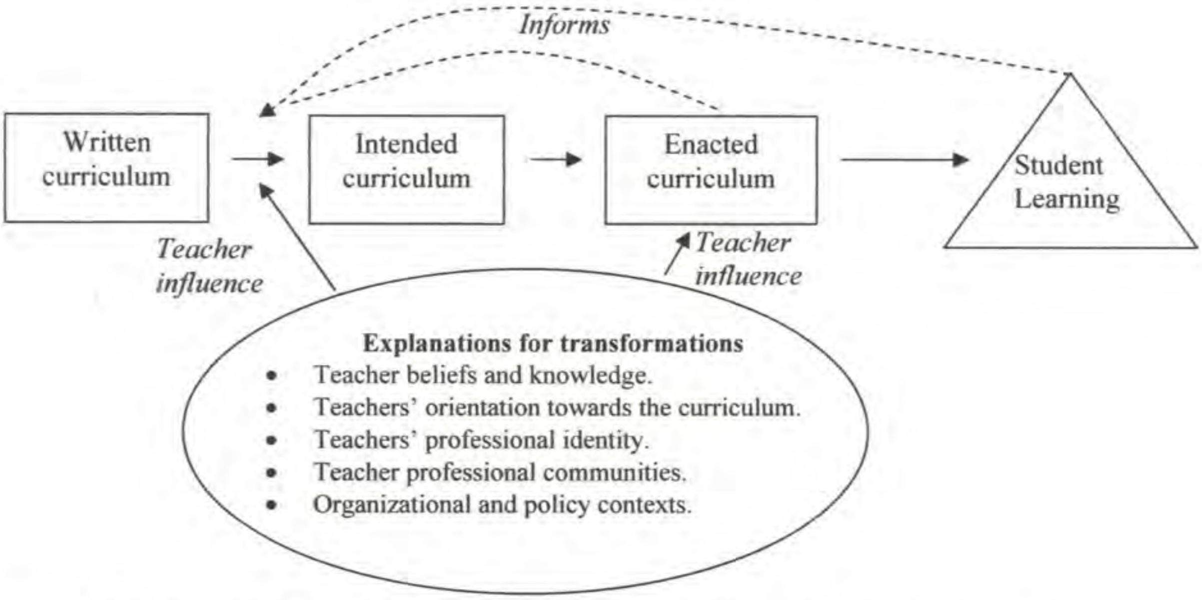


Figure 2.1: Phases of curriculum showing influences affecting student learning.
(Stein, Remillard and Smith, 2007).

Teachers who reviewed "big mathematical ideas" (Stein and Kaufman, 2010) in the curriculum, especially if they were supported in this endeavor, were found to implement lessons that were more closely aligned with curriculum ideals.

Some controversy seems to centre on what is meant by *research* – and the use of *investigation* or *inquiry* is often substituted to attempt to get around this problem (Breen, 2003; Jarworski, 2006). Perhaps this is because the studies undertaken by teachers as researchers **within the classroom** have been criticised for not adding enough theory to the body of knowledge in mathematics education (Breen, 2003). However, mathematics research, being an applied field must inform and affect teaching positively; it is not an end in itself (Sfard, 2005; Amit and Fried, 2008). There is a need for educational research to be a resource for practice rather than become a commentary on what practice should be (Kemmis, 2008). The current needs of mathematics education are very strongly classroom and teacher specific (Ball, 2001; Hill, Ball and Schilling, 2008; Ball and Forzani, 2009; Adler et al., 2005; Adler and Jarwoski, 2009; Groth and Bergner, 2005; Garfield and Ben-Zvi, 2008) and cannot wait upon semantic debate or be dismissed because such research does not seem to add to a field sometimes unfortunately regarded as separate from the practice of teaching.

2.2.1 What is proactive engagement?

To be proactively engaged in looking at how research can be applied and used in the classroom, involves more than just "experiential learning"; which can be seen as the way in which *practical, informal* knowledge may be acquired (Smith, 2001). The need to place educational research in the classroom requires informed, committed action (Smith, 2001a). This action – the process of putting theoretical knowledge into practice and reflecting and acting upon the outcome is termed *praxis*. Kemmis (2008) considers that praxis not only relates to the *action* undertaken by researchers, but to the *right conduct,* i.e. aiming towards the good of the students and necessitating reflection before, after and during the process.

Using this kind of school-based, teacher-centered inquiry investigation, Gallimore, Ermeling, Saunders and Goldenberg (2009) found that what they called "moving the learning of teaching closer to practice' led to:

- Significantly increased student achievement
- Promoted the gaining of key teaching skills and
- Allowed for identification of possible factors necessary for improving teaching and increasing student achievement.

Although this was a 5-year study, investigations of shorter duration have echoed the suggestion that, despite the challenges, this proactive approach to research proves very useful in transforming educational theory. Teachers developed a fuller understanding of the mathematics they teach and how it could be taught in order to enhance student understanding of, and engagement with mathematics (Silver, Ghousseini, Gosen, Charalambous and Strawhun, 2005). Lampart, Beasley, Ghousseini, Kazemi and Franke (2010) considered that using structured activities, based upon theory, in the classroom supports knowledge building for teachers. In their work, educating novice teachers, they have found that enacting these structured activities links coursework tightly with field work and promotes the development of the pedagogical skills necessary for effective teaching. This need to provide teachers with "opportunities that empower them both personally and professionally" is presented by Gade

(2011, p. 42) in her discussion of her experience of the use of praxis to create teaching-learning environments within her own teaching practice. She underlines the importance of the teacher's voice in educational research and contends that the opportunity to play such a role enabled her to become an effective and reflective teacher.

In attempting to put into practice the theories espoused by mathematics education research it is important to remember that they all take place within a certain context – circumstances, or what Kemmis (2008) calls practice architectures that determine what kind of praxis will be possible. Changing teaching practice is often short-lived and limited to few teachers (Gallimore et al., 2009) unless it is possible to change these circumstances. These circumstances, or practice architectures, could include a mathematics syllabus that places a high demand on the number of concepts that teachers should cover; national assessments that narrowly define what skills ensure success in mathematics; approaches to teaching that are prescribed in a way that obliges teachers to follow set methods without considering first and foremost the students in their classes. These can limit teachers to working on schooling rather than educating their students (Kemmis, 2008). Asking teachers to use the syllabus as a source of guidance and inspiration rather than a list of topics to be covered can be risky – moving out of a "comfort zone" brings an element of uncertainty to teaching and often requires teaching that goes against prescribed methods and content order (Breen, 2003).

Research conducted by Silver et al. (2005) highlighted some of the factors that would enable teachers, using research, to make effective use of student work in classroom-based tasks. These knowledge and strategic proficiency factors they labeled affective, cognitive and pedagogical. The teacher is required to be able to use their own mathematical knowledge as a resource - which, as noted by Hill and Ball (2004, p. 331), requires an "appreciation of mathematical reasoning, understanding of mathematical ideas and procedures, and knowing how ideas and procedures connect"; flexible knowledge of mathematical content and student thinking. This echoes the ideas put forward by the conceptual framework within which this research is positioned. This framework provides both a scaffold within which data collection for the research strategies can be carried out, and the tool for analysing/ making sense of the data (Leshen and Trafford, 2007). I have chosen to use Kilpatrick et al.'s model of teaching for mathematical proficiency as the theoretical framework for my research. It provides a logical and coherent framework for conceptualising what teaching for mathematical proficiency entails.

2.2.2 Illustrations of the need for integrating theory and practice

Two possible illustrations (pertinent to South African mathematics education) of the need for research ideas to permeate the classroom are the recent desire by educational authorities to use mathematics education as a vehicle for social change; and the subsequent iterations the mathematics curriculum is undergoing in order to attempt to achieve this end.

Critical mathematics education is a current topic in mathematics education research (Skovsmose, 2000; Ernest, 2001; Bishop, 2008; Tutak, Bondy and Adams, 2011), and is concerned with the way in which mathematics can play a part in creating democratic communities, by addressing issues which currently marginalize people within society. The goal of critical mathematics education is to engage with inequity in society by providing all students with the knowledge, skills and dispositions needed to be able to use mathematics effectively to promote social change. However, this requires that educational research moves beyond descriptions of projects or classroom activities (Tutak et al., 2011) and acknowledges that involving teachers in achieving change in mathematics classrooms is essential. Therefore, looking at how research can be used to influence classroom-based teaching is a worthy goal of educational research.

Curriculum change in South Africa is a continual source of frustration for teachers of mathematics. Sensitivity to past injustices and the need to find a balance that will deliver a more inclusive access to mathematics have resulted in a cycle of curriculum changes over the last 12 years (discussed in Chapter 1). These transformations have placed many demands for changes within classrooms (Tirosh and Graeber, 2003; Brodie, 2010). Curriculum change for teachers is not merely a "technical matter, but a personal journey" Kilpatrick (2009, p. 119). Therefore classroom based research into the way in which these curricula are enacted in the classroom is useful. It would enable researchers to look at ways of supporting and sustaining teacher's attempts to find new ways of teaching and learn new mathematical concepts in order to support their students needs (Olarte, Nieto, Jaramillo, Galindo and Mesa, 2004).

2.2.3 Bridging the theory-practice gap – is it possible?

Experienced and effective teachers have an understanding of how mathematics can be taught so students learn successfully. Engaging with these teachers can provide "exciting opportunities for tapping into the wisdom of those responsible for classroom practice" (Breen, 2003).

Stein and her colleagues, in their work with teachers and the activities they use in the classroom (Stein, Grover and Henningsen, 1996; Henningsen and Stein,1997; Stein et al., 2007; Stein, Smith, Henningsen and Silver, 2009 ; Stein and Kaufman, 2010) view teaching as an object of study that requires the engagement of teachers in studying and improving teaching, Heibert (in Stein et al., 2009) noted that the work done with colleagues was an "effort to bridge the divide that often exists between professional development and the improvement of practice" (p. 133). The professional development model they use has three features:

- All professional development is practice-based;
- Practice is always linked to "big ideas" in mathematics education through their Mathematical task Framework; and
- Teachers are provided with support in applying these ideas in the classroom.

Lester and Wiliam (2002) suggest that research into mathematics education will only influence teaching practice if teachers are actively involved in the creation of research questions and investigations. This participation gives teachers new perspectives to bring back to their classrooms (Magidson, 2005). Brodie (2010) noted that finding ways to talk to teachers about what they found useful and usable was a more positive step to engaging teachers in the difficult task of changing their teaching practice. Ginns, Heirdsfield, Atweh and Watters (2001) found that involving beginner teachers in action research projects (despite the need for constant guidance) helped them become more critical in their teaching practices. They also note that they found the needs of these teachers to be more technical and practical than theoretical; however the need for practical methods of successfully teaching mathematical concepts is ubiquitous to most teachers.

Reorienting research towards working with and for teachers is productive, not only so teachers gain insight into the value of practical research, but also for researchers gaining insight into the teacher-student interface – an immediate influence on the teaching and learning taking place. The interactions are characterized by the decisions made by teachers. Therefore, as Malara and Zan (2008, p. 537) point out, looking at teachers decisions that influence students' learning (and the nature of this influence) and finding out what factors influence these decisions is important (though complex) in research on teaching and can only be considered by taking research into the classroom, and involving teachers in the process.

The shared conclusion of all these writers is that the site for changing teacher practice lies in the classroom with the teachers proactively engaged in research and improving their teaching

2.3 Mathematical Proficiency: examining effective mathematics teaching

One of the theoretical frameworks which provide teachers and researchers with a useful way in which to examine both the learning and teaching of mathematics (as discussed in 2.1) is that outlining the idea of *mathematical proficiency*. Analysing literature on, and research into the learning of mathematics in primary and secondary schools led Kilpatrick et al. (2001) to conclude that it would be valuable to adopt a comprehensive view about what it means to successfully learn mathematics. These authors felt that without a focussed view of what, as mathematics educators, we are attempting to achieve, teaching would remain disparate and possibly unsuccessful.

Kilpatrick et al.'s view of mathematical proficiency is the development of skills in five areas or "*strands*" (2001, pp. 5, 117, 380). These strands (conceptual understanding, procedural fluency, strategic competence, adaptive reasoning and productive disposition) are discussed individually, but in practice they are very closely linked – "*interwoven*" – and development of ability in one area / strand influences the development of skill in another. These five strands are presented on the following pages, and their developmental characteristics are then summarised in *table 2.1*.

2.3.1 The five strands that are used to describe mathematical proficiency

2.3.1.1 Conceptual understanding

Conceptual understanding can be considered to be the "integrated and functional grasp of mathematical ideas" (Kilpatrick et al., 2001, p. 118).

Essentially, conceptual understanding of mathematics requires more than isolated facts, sections and methods of solving problems, it requires also that the student understands why a mathematical idea/concept is important and where it is useful. For example: a procedure such as factorizing needs to be seen as a useful tool that can lead to the successful solving of mathematical equations, which in turn can be seen as a way of representing a relationship between one or more variables or concepts, which in turn can be represented graphically and so on. Conceptual understanding is therefore the ability to see the big picture, and to be able to connect new ideas to those already known (Heibert and Lefevre, 1986, p. 4). This understanding of mathematical relationships supports the ability of students to remember concepts and procedures. It allows forgotten concepts to be '*reconstructed*' (Kilpatrick et al, 2001, p. 118, Heibert and Lefevre, 1986, p. 11), through their relationship to the new idea/concept. Conceptual understanding does not have to be verbalized; it is more often a 'mind map' of relationships that allows navigation through the many procedures that mathematics uses to solve problems. This does however, make it difficult to teach, as it is necessary to help students acquire '*knowledge clusters*' (Kilpatrick et al., 2001, p. 120) that enable a student to group a set of processes according to how to perform them and when and where to use them.

Students acquire conceptual understanding by learning to represent the same idea in different ways. For example: number relationships explored in Grade 9 and Grade10 mathematics such as linear number patterns are represented in tabular form, but can also be shown as a set of ordered pairs or coordinates which in turn are equally valid as represented by a straight line graph. This ability to use multiple representations (Adler et al, 2005) of a concept allows for discussions of the similarities and differences of the various representations and their advantages/disadvantages in certain situations. It also helps to develop a deeper understanding of the concept and how to use it to derive solutions in different contexts (Kilpatrick et al., 2001).

It is easier for students to develop this deeper understanding if they can perform a routine process so it is often useful for students to learn certain processes by rote, so discussion can be focused on the procedure not the process. An example of this necessity is the solving of simultaneous equations which is taught in Grade 9 and 10. The process needs to be well understood, so that when discussing the use of the procedure in e.g. the point of interception of two graphs and why the procedure is valid, the lack of knowledge of the process does not undermine the development of a new understanding of the concept. Knowledge needs to be provided within a frame of reference (Heibert and Lefevre, 1986). This allows the student to access this knowledge when it is required to solve a seemingly different problem.

Learning to represent an idea in different ways emphasizes the need for students to develop the ability to connect ideas and represent them as mathematical symbols or diagrams. This need was noted in the Kilpatrick et al. (2001) study and emphasized by Heibert and Lefevre (1986), in their discussion of the idea of how mathematical proficiency is best promoted. The conclusion was that: "Students who learn with conceptual understanding can see connections between procedures and concepts and are able to understand how some facts follow on from others and can be used to generate new ideas" (Kilpatrick et al., 2001, p. 120; Heibert and Lefevre, 1986, p. 15). Discussion of the uses of multiple representations of a concept or relationship help develop confidence in the use of mathematical language, and the thought processes that are needed as a foundation for further understanding (Kilpatrick et al., 2001). This level of understanding of mathematics also helps students avoid errors in solving problems (Kilpatrick et al., 2001, Rittle-Johnson and Alibali, 1999) by ensuring that they are able to identify which operations and procedures are suitable for a given situation. For example, in Grade 12 mathematics comprehension of arithmetic (linear) and geometric (exponential) relationships are assessed by asking very similar questions for each type of relationship. Students with good conceptual understanding will be able to see both the similarity of the relationships, the need for similar approaches to solving the problems, but the necessity of performing different processes for each type in order to answer the question.

2.3.1.2 Procedural fluency

Kilpatrick et al. (2001) refer to procedural fluency as: "The knowledge of procedures; knowledge of when and how to use them appropriately and skill in performing them flexibly, accurately and efficiently" (p. 121).

One can already see the overlap between procedural fluency and conceptual understanding. The difference between the two lies in the role procedural fluency plays in supporting and helping to develop the conceptual understanding needed to progress to another level of a topic (Kilpatrick et al., 2001). For example, in Grade 4 when learning to multiply numbers consisting of two or more digits, the procedural fluency gained in the multiplication of single digit numbers will allow for a discussion on methods of solving these types of problems. If students are not fluent in the computational skills and the understanding of when to use them, for single digits, the discussion flounders and progress is hampered as they cannot perform routine procedures. Similarly, in the example discussed earlier regarding simultaneous equations learnt in Grade 10, apart from understanding the concept that makes this process valid, and what the answers gained actually represent (conceptual understanding) the ability to perform this routine well, and to know **when it is appropriate to use**, allows other mathematical concepts (e.g. matrices and their uses) to be developed. Essentially conceptual understanding and procedural fluency should be taught together, as development of one supports the other (Rittle-Johnson and Alibali, 1999). The Kilpatrick report (p. 121-122) noted that the research they examined supported the notion that learning procedures with no understanding results in lower achievement by students. What is needed then is careful practice of skills, properly contextualized, to develop mathematical proficiency. Learning procedures **with understanding** allows students to master them more effectively when practicing these procedures to develop fluency in the processes.

In the Grade 9/10 curriculum students need to be able to factorize algebraic expressions, in particular trinomials. This important procedural skill has applications in many areas of high school mathematics, and yet is often badly mastered either as a process, or in its application. The question is why? The conceptual understanding of the number relationships that make it possible to factorize some trinomials and not others are often not explored and used to promote mastery of this skill. Lack of knowledge about where to use this procedure is often rooted in misunderstandings regarding simplification of fractions, or the meaning of the equals sign when solving equations.

This example reveals another reason why mathematical proficiency is undermined by the way mathematics is learnt at school. Often methods of working with numbers in primary schools are not "transportable beyond the classroom" (Kilpatrick et al., 2001 p. 319). By the time students reach high school where the mathematics they learn is not necessarily directly or immediately applicable to "real-life" problems, students have learnt to see mathematics as a subject and not as a skill. They view procedures as independent of the problem to be solved, especially in situations outside the classroom.

2.3.1.3 Strategic competence

Kilpatrick et al. (2001, p. 124) refer to strategic competence as "the ability to come up with, represent using the appropriate construct (algebra, graphs, geometric diagrams etc.), and solve mathematical problems".

In essence strategic competence can be understood to be a skill that allows students to approach situations in which they first have to work out what the problem to be solved is before they can work out how to resolve the question presented by the situation. Here a student needs to develop skill in dealing with what Kilpatrick et al. (2001) refers to as "non-routine" problems – problems where the solution method is not obvious. This is high level thinking and can be likened to the cognitive requirements demanded of problems classified by Stein and Smith (1998) as "doing mathematics". These are discussed in more detail later in the chapter. It is clear that this competence cannot be developed without a student developing both procedural fluency and conceptual understanding as flexibility of thinking is a fundamental requirement for success with these problems.

In the Grade 9 syllabus, this skill is not emphasized, as much of the year is devoted to developing fluency with mathematical routines, thus emphasizing procedural fluency. The areas where this skill can be developed (for example) are those of:

- probability - where it is often necessary to reinterpret a presented situation by identifying it as being the same as solving another, easier to compute problem;
- statistics – analyzing and interpreting data once it has been represented e.g. graphically and therefore
- graphs where solving problems that seem to require algebra but do not look as if they have enough information are best solved graphically.

One of the common traits of these types of problems is that they are usually descriptive – presented as "word problems".

2.3.1.4 Adaptive reasoning

Adaptive reasoning is defined as "the capacity to think logically about the relationships among concepts and situations" (Kilpatrick et al., 2001, p. 129).

Essentially this is the facility to think logically through a problem, to reflect on the method chosen to solve a problem and correct the approach if necessary. This includes the ability to effectively use language to explain and justify any such method used, the key competence for this strand of proficiency. On the surface there does not seem to be as much to learn in order to master this skill. This is not true, as justification of a chosen method or approach to a problem can only be done if the other strands of mathematical proficiency are developed simultaneously.

In the syllabus taught in South African schools the need to develop this ability is implied through the curriculum documents rather than made explicit in the content to be covered. The development of this ability is therefore the responsibility of the teacher who needs to be sufficiently skilled in this area to promote the development of this strand of mathematical proficiency.

2.3.1.5 Productive disposition

Kilpatrick et al. (2001, p. 131) refer to productive disposition as "the tendency to see sense in mathematics, to perceive it as both useful and worthwhile, to believe that steady effort in learning mathematics pays off, and to see oneself as an effective learner and doer of mathematics".

Viewing mathematics as useful; the acquisition of mathematical skills as a worthwhile pursuit and being willing to work towards this goal is important for the development of the other strands of mathematical proficiency. The way students view mathematics is important in determining their success in the subject (Kilpatrick et al., 2001). Students who do not view mathematics as a useful tool that provides ways of thinking about problems, but rather as a list of facts and processes to be remembered and produced at the right moment in an assessment will struggle to understand mathematical concepts (Fuson et al., 2005). They will avoid the effort required to engage with more challenging mathematical concepts.

Supporting the development of a positive disposition towards mathematics is in the hands of the teacher. If a teacher views mathematics as a list of concepts to be covered, "boxes to tick" then it is unlikely that students will enjoy trying difficult problems. Other factors that influence the development of a positive view are the curriculum (Kilpatrick et al., 2001) and the way in which it is assessed (Stiggens, 2002; Black, Harrison, Lee, Marshall and Wiliam, 2004). If the curriculum is very full, or assessments can be fulfilled through rote learning, teachers are less likely to teach for all the strands of mathematical proficiency and will promote those that ensure "success" for their students. Students are more likely to learn that mathematics is about learning to calculate, about following rules; about getting the answer correct and that it is about ability only and some people cannot do mathematics (Kilpatrick et al., 2001; Fuson et al., 2005).

Table 2.1: Developmental features for the strands of mathematical proficiency. (Identified in: Kilpatrick et al., 2001)

PROCEDURAL FLUENCY	CONCEPTUAL UNDERSTANDING	STRATEGIC COMPETENCE	ADAPTIVE REASONING	PRODUCTIVE DISPOSITION
Procedural fluency is developed when: • Algorithms and other procedures used in the process of solving mathematical problems are well known, and can be performed fluently, accurately and efficiently. • The student understands when it is appropriate to use a procedure and can apply it correctly in the right situation. • Students develop the understanding that some procedures can be used to solve entire classes of problems. • Recognize the usefulness of procedures as tools that can be applied to different situations. • Show an organization of thought that demonstrates an understanding of mathematics as well structured and filled with patterns that allow for predictability in familiar situations. • Students no longer make many of the common errors that demonstrate a lack of depth of understanding. • Students can connect the mathematics learnt at school with its use/ applicability to the solving of problems outside school. • Students can modify/ adapt procedures to make them easier to use.	Conceptual understanding is developed when: • Facts and methods are not isolated, but are understood in terms of why they are important and where they are useful. • Knowledge is organized coherently so as to enable the development of "knowledge clusters". • These "knowledge clusters" enable students to learn new ideas by connecting those ideas to what they already know. • Students find retention of concepts, methods and facts easier because they understand them. • Facts and methods can be reconstructed if forgotten as they have been learned with understanding. • Mathematical situations are able to be represented in different ways. • Students know how different representations can be useful for different purposes. • The similarities and differences of the representations, the advantages of each and how they must be connected are understood and can be discussed. • Students are able to use their knowledge as a base form which they can move to another level of understanding. • Students can use this understanding to avoid or correct critical errors in solving problems – e.g. problems of magnitude. • Students find that they have less to learn as they see similarities between seemingly unrelated situations.	Strategic competence is developed in a given situation when students: • Can identify the problem that needs to be solved. • Can then put that problem into words that allow them to see the structure of the problem. • Can use this structure to identify the mathematics needed to solve the problem. • Have therefore the ability to construct a mental picture to model the problem. • Are aware of the quantities, representations and relationships. • Are flexible in their thinking.	Adaptive reasoning is developed when: • Students can justify their reasoning. This means that they are able to explain the approach they have taken to solve a problem and the mathematics they have used to do so. • Students have fluent use of procedures and can tell if these are appropriate for use.	Productive disposition is developed when: • Students take responsibility and participate actively in constructing new knowledge. • Will seek out challenges. • Believe that they are capable of doing mathematics. • Will work towards understanding the mathematics.

Using these conceptual ideas allows us to examine the implications of how students become mathematically proficient (*Figure 2.2*), and how it might be possible (in a broad sense) to teach to develop this proficiency. What is meant by each of these strands in terms of effective teaching for the proficient learning of mathematics is outlined in *Figure 2.2* below.

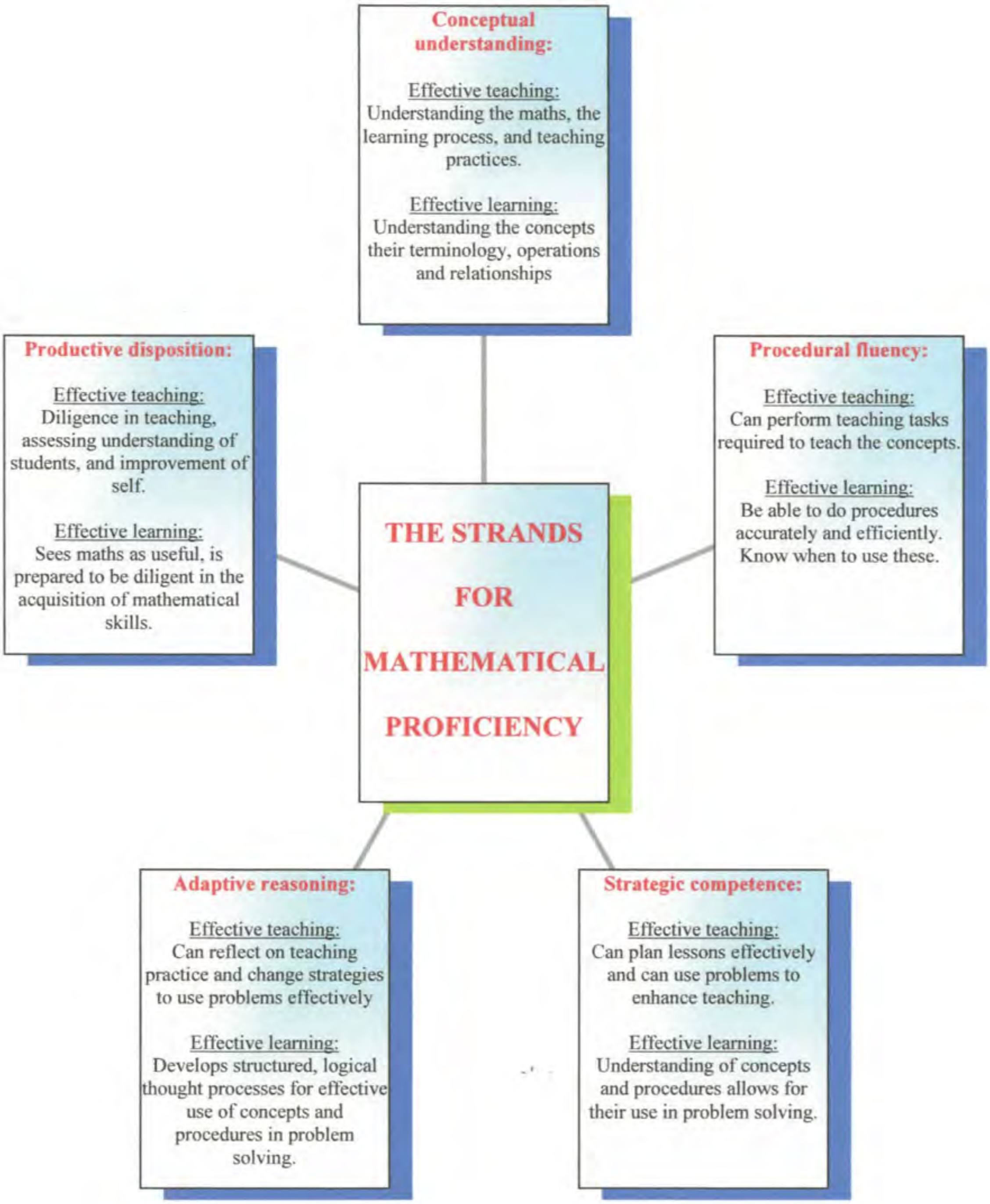


Figure 2.2: The five strands of mathematical proficiency

Kilpatrick et al. (2001) constantly emphasize that success in mathematics relies on the 'consistent integration of skills' (p. 11). To put it another way, success in learning mathematics requires that students develop the component abilities in concert, not piecemeal. In presenting the key components of mathematical proficiency in this way, Kilpatrick has also provided us with a framework to use when looking at the teaching and learning of mathematics, and the teaching and learning of the skills necessary for teaching mathematics.

2.3.2 Teaching for mathematical proficiency

If what is presented above is an outline of how mathematical proficiency can be developed in students, what then is required for teachers to develop proficiency in *teaching* mathematics? Answering this question requires the identification of three types of knowledge in which teachers need to become proficient (in terms of the five strands). These essential components of "the instructional triangle" (Kilpatrick et al. 2001, pp. 314, 371) are knowledge of mathematics, knowledge of students and how they learn, and knowledge of the art of teaching (see Silver et al., 2005; Hill and Ball, 2004, 2.1.3). The triangle (shown in *Figure 2.3*), suggests that if teachers are to provide students with the experiences that allow for the development of mathematical proficiency, the interaction of these skills is critical.

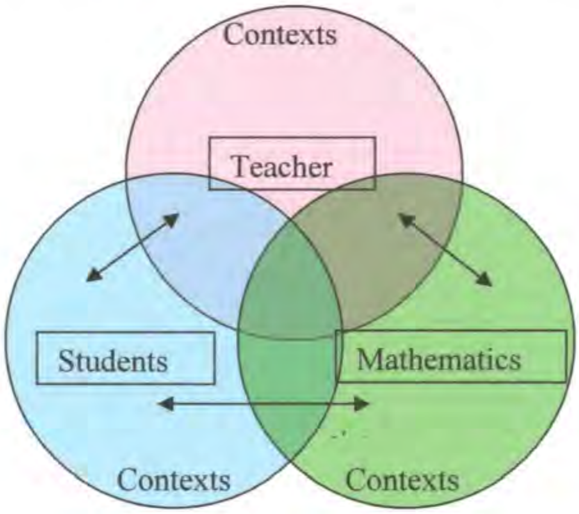


Figure 2.3: The instructional triangle showing knowledge areas and their interactions required for teaching proficiency. Adapted from Kilpatrick et al. (2001)

This is not a theory of how knowledge is acquired, but rather a model or conceptual framework enabling analysis of what knowledge and related skills are required for effective teaching for mathematical proficiency. It must be noted that teaching is seen to be about the

interaction among the mathematical material, the students and the teacher. This initial framework was refined in a later publication in order to clarify each of the three knowledge areas (Schoenfeld and Kilpatrick, 2008), and is illustrated in *Table 2.2*. These knowledge areas are discussed in more depth in section 2.3.

Knowledge of mathematics	i.e. knowing school mathematics in both depth and breadth.
Knowledge of students	• Knowing students as thinkers. • Knowing students as learners.
Knowledge of teaching	• Crafting and managing learning environments. • Developing classroom norms and supporting classroom discourse as part of "teaching for understanding. • Building relationships that support learning. • Reflecting on one's practice.

Table 2.2: A provisional framework for proficiency in teaching mathematics. Adapted from Schoenfeld and Kilpatrick (2008).

2.3.3 The benefits of teaching for mathematical proficiency

It has been suggested that to develop a student's strategic competency is a goal that, if achieved, gives a clear indication that a student is gaining mathematical proficiency and that one's teaching is having the desired effect (Lampert, Beasley, Ghousseini,, Kazemi, and Franke, 2010). But how does one get to that point? What does one focus on? Clearly, as the five dimensions of mathematical proficiency are closely linked and difficult to separate, trying to focus on each separately would present serious logistical challenges. The question is: is it possible to focus on developing two foundation skills, and by teaching towards their integration, promote the development of mathematical proficiency in students?

Tall (1995, p. 3) suggests that the cognitive growth occurring during the process of coming to know and understand mathematical concepts requires the human brain to do two things:

1. Compress knowledge appropriately (essential characteristic of mathematics) so as to manage our ability to only deal with one to two items or ideas at a time.
2. Construct linkages to other mental data to make this knowledge easy to use.

Tall argues that achievement of the first can be supported through making processes routine and using pictures to convey information; and achievement of the second, through the development of conceptual knowledge. Conceptual knowledge, in his opinion, includes what he called "concept-process links" (p. 4) which promotes success in problem solving through enabling the student to carry out appropriate mathematical procedures fluently.

Lampert et al. (2010) describe the development of students' strategic competence as requiring the teacher to get them to be willing to reason and make decisions about what procedures to use when solving problems – an intertwining of procedures with concepts. This view of procedural fluency and conceptual understanding as holding dual importance in mathematics learning is shared by Heibert and Lefevre (1986). Their contention is that "being competent in mathematics involves knowing concepts, knowing symbols and procedures, and knowing how they are related" (Heibert and Lefevre, 1986, p. 16). Davis (1986) offers a similar view when he highlights some of the shortcomings in school mathematics. His criticism is that there is an over-emphasis on rote calculations and prescribed 'rituals', and too little emphasis on understanding the problem and seeing mathematics as thinking as well as doing. When concepts and procedures are not connected, students might have an intuitive 'feel' for solving problems, but may well not understand what they are doing. Students who do not have an intuitive feel for solving problems not only will not understand what the problem is asking of them, but will not know what to do or where to start.

Heibert and Lefevre (1986) outlined some of the benefits conceptual and procedural knowledge can bring to each other when they are linked in an effort to promote the development of mathematical strategic competence. In their study on the link between procedural and conceptual knowledge, Rittle-Johnson and Alibali (1999) found that both "appear to develop iteratively, with gains in one type of knowledge leading to gains in the other", although their influence on one another did not seem to be equivalent. Their finding was that the promotion of conceptual understanding (to a greater degree) along with procedural fluency led to the acquisition of flexible problem-solving skills. The importance of the development of conceptual understanding alongside procedural knowledge was also highlighted by Schneider and Stern (2005) and Rittle-Johnson and Koedinger (2009).

2.3.3.1 Requirements and implications

The implications for teachers are two-fold: not only do they need to teach **for** conceptual understanding and procedural fluency; they must also have conceptual understanding and procedural fluency themselves with regard to the **teaching** of mathematics. These requirements and implications are discussed in section 2.4.

It is also useful to have some insight into the process that students go through in order to achieve mathematical proficiency. In a discussion of the mutually complementary relationship between procedural fluency and conceptual understanding, Kilpatrick et al. (2001) suggest that students progress from one level of a topic to another level as their understanding develops. This idea – of students moving through levels of cognitive understanding of a mathematical topic – is a common theme in mathematics education research. It appears to be based upon Jean Piaget's views of how children "construct" knowledge through play (Tall, 2004). According to the experiences that they have done, children lay down a foundation for thinking and knowledge, upon which their further understanding of new information is built. This theorization of how children learn and put knowledge together to generate an understanding of how their world works should "change the focus of learning and teaching" (Prawat, 1992, p. 357). Learning, in this constructivist perspective, happens because knowledge is built upon knowledge: new experiences are linked to, and integrated into, existing knowledge in the mind of the learner (Bodner, 1986). Varieties of constructivist theorizing include the work of Sfard (1991), who suggests a general process of understanding mathematical concepts which she terms the 'cycle of reification'. She presents an idea of how students come to understand, and are able to visualise mathematical ideas ('objectification'). Tall (2004) puts forward a possible categorisation of cognitive growth when he likens the process of coming to understand a concept as a journey through the three worlds of mathematics – the embodied (perception based), the procedural (symbolic based) and the formal (property based) worlds. Harel (2008a; 2008b; 2008c) presents a theoretical framework – what he refers to as DNR-based instruction in mathematics – which outlines three broad principles that can be used to guide mathematics teaching to enable educators to help students acquire mathematical understanding and learn to think mathematically. Finally, Van Hiele (1959; 1986) has identified different "levels" of development that students pass through in their cognitive growth.

2.4 The student/teacher interface: taking mathematical proficiency into the classroom

While these conceptual frameworks are useful as they provide teachers with models for thinking about mathematics and the teaching of mathematics, the reality of the classroom is the focus of concern for practicing mathematics teachers. Despite being informed by the research about what it is that we need to teach for in terms of mathematical proficiency, and despite acquiring some insight into what kind of process our students might go through in building understanding of a concept, we are still left with the question of how to teach for this outcome. As a teacher, one has to translate the theory into reality – no easy task. In this context, it is the interaction among the students, the teacher and the mathematics (Kilpatrick et al., 2001) that determine the learning that takes place. This interaction is influenced by the teaching process, the activities used in the classroom, and the knowledge, experience and beliefs of the teacher. If the ideas discussed above are to be considered in the context of the mathematics classroom, these are the mediating factors that must be considered.

2.4.1 Presentation of a possible teaching process towards mathematical proficiency

The frustration of seemingly being 'out of synch' with many of the students in one's class is shared by many mathematics teachers when they are teaching geometry. It was just this dissatisfaction with the teaching methods at the time that inspired the van Hiele's to investigate and to attempt to isolate, the reasons for this problem. Van Hiele (1959, 1986) identified five levels of understanding that students seem to progress through in coming to understand geometrical concepts (only four of which are applicable to school mathematics). Unless teachers are able to recognise and work with their students at the level at which these students are reasoning with respect to the topic, true communication is hampered.

The van Hieles' research focussed on the understanding of geometry, and while function relationships often involve geometric constructs of algebraic concepts, they have no clear starting point in terms of which representation comes first. But identifying different levels of understanding concepts was not the only idea put forward by van Hiele. In attempting to identify how to teach for success in mathematics, van Hiele (1959) outlined five teaching phases that a teacher could use to help students move from one level of understanding to another.

In moving through these teaching phases, the teacher will, by providing information and activities: (i) teach the basic structure of the concept under discussion; (ii) guide the student through the appropriate activities to understand the relationships within the concepts; (iii) link this experience to the appropriate symbols and language of mathematics; (iv) encourage students to develop ways to solve and present solutions; and (v) by encouraging students to reflect upon their work, guide them to consolidate the knowledge they have gained. A more detailed description of these phases in terms of students, teachers and the type of tasks required to support learning is outlined in *Appendix 1*. I developed this overview in order to better conceptualise how these phases might be used in the module. As van Hiele's (1986) description of the teaching phases is not necessarily confined to geometry, it may be adapted to promote development of the understanding of linear functions.

Teaching for mathematical proficiency requires teachers to consider the order in which they present the elements of a mathematical concept, and how best to build students' understanding of the topic. What activities they will use in the presentation of these concepts is of fundamental importance. This choice represents the way in which teachers and students interact with each other and mathematics.

2.4.2 Teaching activities and their role in teaching towards mathematical proficiency

These activities are the main way in which teachers seek to involve students in mathematics. It is therefore important that tasks set and activities employed in a classroom are examined in terms not only of how they align with the theoretical framework under consideration, but also of how they promote the teaching and learning process. Viewing the curriculum as a collection of tasks to be undertaken when mathematics is taught (i.e. classroom activities used to centre students' thinking upon a certain mathematical concept) allowed Stein, Grover and Henningsen (1996) to formulate a conceptual framework for task use (shown in *Figure 2.4*), which they later linked to the implementation of the curriculum as a whole (see Stein et al., 2007).

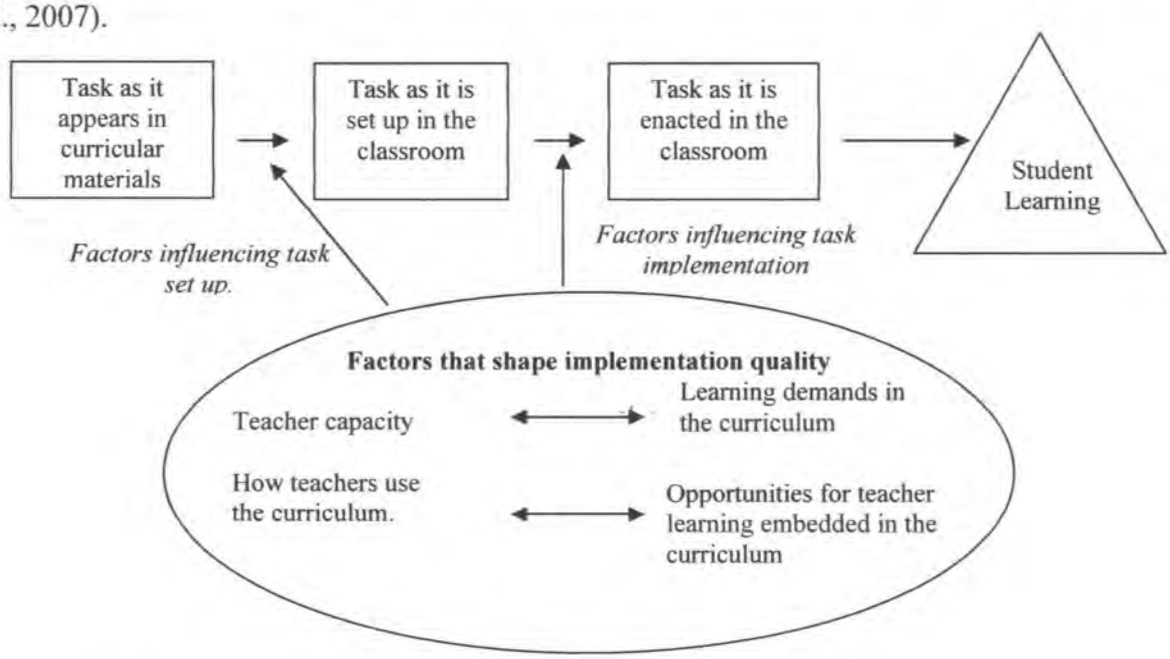


Figure 2.4: Phases of curricular task implementation and factors that shape it. (Source: Stein, Grover and Henningsen, 1996; Stein, and Kaufman, 2010)

According to this outline, a task will go through three phases: the first is the way it appears in the curriculum support materials; the second, the way the teacher decides to use or present it in the classroom; and third, the way it actually happens in the classroom. The task can be altered as it progresses through these phases. Factors that might influence the way teachers use or present tasks would include their goals for the lesson, their knowledge of the subject matter, and the knowledge of their students. Factors influencing the way the task is actually implemented in the classroom would include classroom norms, the difficulty level of the task, the teacher's way of handling student difficulties, and the way the students themselves tend to engage with tasks of varying complexity. This last set of factors was found to be so influential that Stein and Smith (1998), by categorizing tasks (*Table 2.3*) according to the cognitive demands they make, were able to investigate how factors influencing setup and implementation related to or changed the cognitive level of tasks chosen for use.

Table 2.3: The four levels of cognitive demand of mathematical tasks

Lower-level Demands	**Higher-level Demands**
1: Memorization Tasks • Involve reproducing previously learned facts, rules, formulae or definitions OR committing facts, rules, formulae or definitions to memory. • Cannot be solved using procedures because a procedure does not exist or because the timeframe in which the tasks are being completed is to short to use a procedure. • Are not ambiguous: such tasks involve exact reproduction of previously seen material, and what is to be reproduced is clearly and directly stated. • Have no connections to the concepts or meaning that underlie the facts, rules, formulae or definitions being learned or reproduced.	3: Procedures with connections Tasks • Focus student's attention on the use of procedures for the purpose of developing deeper levels of understanding of mathematical concepts and ideas. • Suggest pathways to follow (explicitly or implicitly) that are broad general procedures that have close connections to underlying conceptual ideas, as opposed to narrow algorithms that are opaque with respect to underlying concepts. • Usually represented in multiple ways (e.g. visual diagrams, manipulatives, symbols, problem situations). Making connections among multiple representations helps to develop meaning. • Require some degree of cognitive effort. Although general procedures may be followed, they cannot be followed mindlessly. Students need to engage with the conceptual ideas that underlie the procedures in order to successfully complete the task and develop understanding.
2: Procedures without connections Tasks • Are algorithmic. Use of procedure is either specifically called for, or its use is evident based on prior instruction, experience or placement of the task. • Require limited cognitive demand for successful completion. There is little ambiguity about what needs to be done and how to do it. • Have no connection to the concepts or meaning that underlies the procedure being used. • Are focused on producing the correct answers rather than developing mathematical understanding.	4: Doing mathematics Tasks • Require complex non-algorithmic thinking (i.e. there is not a predictable, well-rehearsed approach or pathway explicitly suggested by the task, task instructions or a worked-out example). • Require students to explore and understand the nature of mathematical concepts, processes, or relationships. • Demand self-monitoring or self regulation of one's own cognitive processes. • Require students to analyze the task and actively examine the task constraints that may limit possible solution strategies and solutions. • Require considerable cognitive effort and may involve some level of anxiety for the student due to the unpredictable nature of the solution process.

(Source: Stein and Smith, 1998)

The influence that activities have on mathematics learning, and therefore the need for careful consideration when choosing tasks for instruction, is emphasized by many educators in mathematics education research. For example:

- Stein, Grover and Henningsen (1996) draw attention to the fact that a demand for mathematical proficiency places more emphasis on engaging in the process of mathematical thinking. This requires exposure to meaningful and worthwhile tasks, which in turn demands from teachers an awareness of problems, of what they can be used to teach, and how this can be implemented.
- Brodie (2004a) comments that there needs to be a close link between the mathematics that teachers learn and the core activities of their practices (such as selecting tasks).
- Stein, Remillard and Smith (2007) and Stein and Kaufman (2010) note that curricula are seldom implemented as intended, and that an awareness of how teaching activities change with use is an important pedagogical skill (Stein, Grover and Henningsen, 1996, Stein, Smith, Henningsen and Silver, 2009). Although in reality, with many factors at play, it is inevitable that some change will occur, an awareness of these (illustrated in *Figure 2.4*) might allow teachers to mitigate the impact of some of these factors.
- Stein, Smith, Henningsen and Silver (2009) remark that in the work that they do, teachers are analyzers, interpreters and planners of mathematical activities, and need to be aware of the contributions their planned activities can make towards their teaching goals.
- These mathematical activities are central to students' learning (Lampert et al., 2010) and provide contexts in which they learn about mathematics (Stein, Grover and Henningsen, 1996; Henningsen and Stein, 1997). Again it is emphasized that an awareness of this importance is needed when selecting and implementing tasks in the classroom.
- Stein and Kaufman (2010) concur that teachers' use of materials has a high degree of influence over teaching and learning in their classrooms.

It is interesting to note that findings by Stein and Kaufman (2010) seem to 'debunk' the conventional wisdom that *only* high-capacity teachers teach in a high-quality way. Rather, their study suggests that "how a teacher uses a curriculum may be more important than the education, experience and knowledge she brings to the table" (pp. 688, 689). While the types of knowledge required by teachers (as discussed in 2.4.3) is not disputed, this point of view provides an interesting contrast to the emphasis placed by mathematics educators on the degree of knowledge teachers require for effective mathematics teaching. However, this might only be applicable to primary and middle (grades 1 – 9) school educators. Furner (2004) insists that research in mathematics education has shown it to be critical for secondary school mathematics teachers to have strong mathematical knowledge.

2.4.3 Areas of teacher knowledge needed for the teaching of mathematics

Developing students' strategic competence (2.2.3) requires "a kind of social management that is not necessary when students are simply expected to follow directions" … "intertwining procedures with concepts means not teaching lessons on small discrete topics, but working from different angles on big topics" Lampart et al. (2010, p. 129).

2.4.3.1 Knowledge of mathematics

The importance of mathematics knowledge specific to the teaching of mathematics has been highlighted by many educators (Kilpatrick, 2001; Tirosh and Graeber, 2003; Brodie, 2004a; Harel, 2008a,b; Adler and Pillay, 2007, Kazima, Pillay and Adler, 2008; Ball, Lebienski and Mewborn, 2001; Ball, Thames and Phelps, 2008). This specific knowledge of mathematics was seen to include: the ability to be able to break down mathematical concepts into developmental steps; to create situations for students to successfully construct concepts by working through steps; to be able to ask and respond to questions and in a way that allows these challenges to allow students to develop their mathematical knowledge and to further this understanding through the providing of opportunities for students to develop cognitive models that enhance their conceptual understanding. In essence it is the specialized mathematics teachers need to know and know how to use in their teaching (Adler and Pillay 2007; Kazima et al., 2008).

This view is confirmed by Kilpatrick. He often refers to the knowledge of mathematics required by mathematics teachers. The Kilpatrick report emphasizes that this knowledge (as studied at a tertiary level) is certainly important and necessary for proficient teaching. However, after a certain level is attained by the teacher, what has a greater impact on student performance is the ability of teachers to use and accept multiple representations of ideas and to constructively use students' mathematical solutions, explanations and questions to facilitate the learning of mathematical ideas. This idea is not new to mathematical education research – social constructivist writers highlight the importance of this as well, either directly (e.g. Ernest, 1991 p. 43, Prawat, 1992, p. 381) or by referring to the concept of scaffolding as a means of making connections between knowledge.

This means that knowing mathematics for teaching is more than just knowing the mathematics and being able to perform procedures and understand concepts. It is the capacity to use this knowledge in the teaching of the subject in ways that unpack mathematical ideas and help students construct their own unique understanding of how mathematics works.

Knowledge beyond the content of the subject – the desirable ways of understanding and thinking about mathematics (Harel, 2008a) - mentioned above - can thus be seen to allow teachers of mathematics:

- Not to view mathematics as a fixed body of facts and procedures to be transmitted by the teacher and learnt by the student. But rather as a program of activities or a network of ideas from which knowledge and skills can possibly be acquired or "constructed" (Driver and Oldham, 1986; Prawat, 1992).

- To appreciate the role of justification of ideas in mathematics which Kilpatrick views as central to developing mathematical knowledge. Ernest (1991) discusses the need for justification of mathematical knowledge in order to justify its validity. This could be done by provoking classroom discussion around the understanding of a particular concept.

- To understand and anticipate difficulties students will have with areas of mathematics and using these to inform classroom practice. (This last point highlights the fact that not only do teachers need knowledge of the mathematics, but they also need knowledge of how their students' think about and learn mathematics.)

2.4.3.2 Knowledge of students

Knowledge of students is a key area for effective teaching of mathematical proficiency (Kilpatrick et al., 2001; Tirosh and Graeber, 2003; Harel, 2008a). Accordingly, it is important for teachers to know the personal and educational background of their students in order to be aware of the mathematical skills and abilities they bring with them. Teachers need to be sensitive to the students' unique ways of learning, thinking about and doing mathematics, a thought echoed by Jaworski (1994) when she talks about the need to help students discover mathematical relationships so these can then be understood from the students' own unique perspective. Analyzing errors is a recurring theme when the individual nature of the mathematical knowledge the student brings to the classroom is discussed, and is also useful when considering the ways in which students construct knowledge and how this can be guided (see use of group discussion in section 2.4.3.3).

The emphasis Kilpatrick et al (2001, p. 344) place on the fact that "the quality of the interaction of the teacher and student around the content is critical to the success of instruction" saying that the "most successful teachers are not merely sensitive to the cultural diversity of their students but use that diversity to enrich the learning experiences provided to the class" closely mirrors Ernest's (1991, p. 43) viewpoint of mathematics as a "social" construction. It follows that the teacher is then responsible for creating a climate of learning conducive to the building of valid mathematical knowledge. In order to do this the teacher must be able to manage the class and use activities effectively, creating an environment in which ideas and methods are valued. Students then feel free to share their thinking around a problem, and mistakes are appreciated as being an appropriate step in the construction of further understanding of mathematics. This is what Kilpatrick et al. (2001) refers to as creating a "community of learners" (p. 379) when discussing the third important knowledge base for teachers.

2.4.3.3 Knowledge of instructional practices

The awareness and understanding of instructional practices (often referred to as pedagogical content knowledge) required by teachers is subject specific and is seen as a separate set of skills to those needed for the content being taught (Shulman, 1986; Krauss, Brunner, Kunter, Baumert, Blum, Neubrand and Jordan, 2008). This knowledge base covers a plethora of skills, but these can be grouped into the main areas: creating a "community of learners" referred to above; managing classroom discourse; and managing learning activities so everyone is "engaged in substantive mathematical work" (Kilpatrick, 2001, p. 379). Management of classroom discourse and the learning activities facilitate the development of an enabling classroom environment (Brodie, 2004a). Malara and Zan (2008, p. 539) qualify the need for instructional knowledge to "enable teachers to design and control the teaching-learning situation". This understanding should allow teachers to solve the practical problems they encounter and make the right decisions, and adapt their practice to their actual classroom.

The Kilpatrick et al. (2001) report stresses the fact that questioning and discussion in a mathematics classroom are required to draw out students' ways of solving mathematical problems. Argument of how the use of group and classroom discussions can support learning is widespread in (for example) constructivist literature. It is promoted by social constructivists as importance for the "two-directional flow of information between teachers and their students" (Bodner, 1986, p. 877).

In order to realize the potential of classroom discussion, the choice and management of classroom activities is vitally important. According to Kilpatrick et al. (2001), when planning a lesson, the choice of tasks should be carefully considered, so that the lesson, through the use of the activity, builds on students' previous mathematical knowledge and guides them through the process of developing greater understanding of mathematical concepts. This requires teachers to take into account what students already know. Activities must be selected for their educational value rather than their enjoyment value (the involvement of students in a classroom activity is not in itself a measure of learning).

2.4.4 Factors influencing teaching for mathematical proficiency

Teaching for mathematical proficiency requires a suite of skills on the part of the teacher. These range from mathematical knowledge and understanding to an appreciation of how different students learn and what instructional practices are needed for teaching (Kilpatrick et al., 2001; Harel, 2008a). It is difficult to separate the teaching skills needed for these two strands of the Kilpatrick model, as a skilful teacher interweaves them in providing a structured and cohesive learning environment for his or her students. Many of these skills concerned can be learnt, but not in the traditional manner, as they require practice to develop. Research has also shown that it is critical for secondary mathematics teachers to have a positive attitude towards mathematics and teaching (Furner, 2004).

2.4.4.1 Integrated teaching skills

In order to help students develop the "web of relationships" they require (Heibert, Lefevre 1986, p. 11), their teachers must themselves have developed an understanding of the links between mathematical concepts (Brodie, 2004a). However, the teacher of mathematics must also have the ability to "unpack" (Adler, 2005, p. 4-5) the mathematics that the students learn in order effectively to teach the relationships involved in the processes learnt. This in turn helps to develop understanding of the links between concepts (Kilpatrick et al., 2001). By discussing the underlying relationships in this way, the teacher helps the students develop both a mathematical vocabulary and the ability to use language when discussing methods of solving mathematical problems, and, importantly, the ability to understand the questions asked of them when their understanding is assessed.

Beyond this, teaching for mathematical proficiency requires knowledge of, and an ability to use constructively, different approaches that result in the same solution to a problem. For example, at the Grade 4 level, for multiplication of numbers with two or more digits, the traditional method is the vertical "long multiplication" method. However many children prefer a "divide and conquer" approach, which essentially applies the distributive law learnt later in high school. A teacher uncomfortable with number concept will miss out on this opportunity to develop a deeper understanding of the relationships between numbers and helping to develop the thought processes necessary for mathematical proficiency. If an opportunity arises to discuss a method that gets "remembered" arises, and the teacher is able to take into account what students know, and recognize the risks involved in using student approaches, he or she can decide on how best to use these to develop the desired mathematical results. Care needs to be taken, however, in using student approaches, as there are risks that need to be avoided. Kirscher, Sweller and Clark (2006, p. 84) note that "unguided learning activities are less effective than guided activities, especially in students without a high degree of prior knowledge". Such activities are in fact more likely to have detrimental consequences as students "acquire misconceptions, incomplete or disorganized knowledge" through learning activities that are not properly monitored by the teacher.

What is required, then, is not only an understanding of the mathematics itself, but also an understanding of where the concept being taught fits into the bigger picture of the mathematics students learn at school. This allows mathematics learnt at one level to provide a stable foundation for mathematics learnt at another. Do teachers thus need to know where their students are coming from and where they are going to in their mathematical development? Apart from having "insider" knowledge of the mathematics, mathematically proficient teachers also need to be procedurally competent in a number of other areas, including, *inter alia*:

- The ability to manage a lesson in terms of how much time to allow for solving a problem, when to provide answers, when to correct and use errors and what level of discussion to generate in a topic.
- The ability to generate an expectation that learning will be expected and will happen in their classrooms.
- The ability to recognize and choose tasks that develop the mathematics required.
- The ability to manage their classes, which change from year to year, in terms of the cultural diversity and ability of their students.
- The ability to develop a set of lessons whose sequence, together, over time, achieves the goals set.

- The ability to resolve disagreements over methodology using mathematical arguments.
- The ability to conclude a lesson and focus students' attention on the concept to be learnt, and to generate a sense of order in the learning of mathematics.

2.4.4.2 Attitudes and beliefs

According to Törner, Rolka, Rösken and Sriraman (2010), "belief" is not clearly defined in the literature. However, the suggestion that mathematical beliefs are personal philosophies, conceptions and attitudes (or disposition) regarding the nature of mathematics as well as the teaching and learning of mathematics, provides a good description of what this intangible concept might mean within the context of my research.

There is a wealth of literature outlining the effect that teachers' attitudes and beliefs towards their subject matter and their students have on their teaching of mathematics (Zollman and Mason, 1992; Brodie, 2004a; Webb and Webb, 2004, 2008; Malara and Zan, 2008; Törner et al., 2010). As Webb and Webb (2004, p. 13) observe: "what teachers consider the best way of teaching a mathematics concept is influenced by their beliefs about the nature of mathematics". It is therefore necessary to bear this in mind when considering curriculum interventions. No matter what framework or teaching approach one chooses to work within, it seems impossible to separate teachers' knowledge and beliefs, because beliefs affect the way teachers use knowledge for teaching.

Beliefs or attitudes can be categorised in terms of (i) those regarding mathematics; (ii) those regarding mathematics teaching and learning; and (iii) those regarding their students' abilities in terms of their capacity to learn (Zollman and Mason, 1992; Pehkonen and Törner, 2004; Malara and Zan, 2008).

Harel (2008b) regards knowledge (as discussed in 2.3.3) as ways not only of understanding a concept, but also of *thinking* about it. It is therefore necessary to expand our definition of what constitutes knowledge for teaching to include the way teachers *think* about mathematics and its teaching, as this could be a useful method of highlighting their beliefs or attitudes towards mathematics. These additional characteristics are described by Harel (2008b) as:

- Knowledge of mathematics: this includes ways of understanding and ways of thinking about mathematics, and it affects what and how teachers teach.
- Knowledge of students and how they learn mathematics: this includes teachers' understanding of the principles of learning, including the importance of the influence of existing knowledge on the acquisition of new knowledge, and teachers' implicit theories about the manner in which tier particular students learn. The way in which teachers understand and apply this knowledge is determined by their beliefs, an opinion promoted by Schoenfeld and Kilpatrick (2008).
- Knowledge of pedagogy: this includes how to understand and view the assessment of students' knowledge, how to use problems to promote intellectual curiosity, how to help students consolidate knowledge developed.

Harel's (2008b) opinion of current teaching practices is that teachers tend to view mathematics in terms of subject matter rather than in terms of the conceptual tools needed to understand mathematical ideas and objects. This way of approaching teaching is likely to have been experienced by mathematics teachers when they themselves were students. Brodie (2004a) and Zollman and Mason (1992) note that there is an important relationship between a teacher's beliefs and his or her style of teaching. Teachers seeing mathematics differently will teach concepts in a different order, see different tasks and activities as useful in different ways, talk differently to students about mathematics and evaluate students' mathematical understanding using different criteria. This has an influence on what the teacher does in the classroom and what decisions he or she makes. The importance of these beliefs becomes evident in the context of creating new syllabi, or exploring innovative teaching approaches: "teachers suddenly make choices that go against the spirit of the project; there is a mismatch between their stated beliefs and beliefs-in-practice" (Malara and Zan, 2008, p. 540, see Webb and Webb, 2004, p. 17).

Webb and Webb (2008), in an attempt to ascertain reasons for such disparity, found that their results in a small scale investigative study echoed those of other researchers, in that inconsistencies could be attributed variously to situations, context and culture; and that motives behind teachers' practices are possibly influenced by interactions with learners and the variety of different tasks that emerge during a lesson. These differences seem to be accentuated by stress situations such as new, innovative curricula, time constraints.

2.5 Number relationships, functions and graphs: one area that highlights the need for mathematically proficient teaching.

A concept that connects ideas in the South African high school mathematics curriculum is the graphical representation of numerical relationships, one of which is linear. The development of an understanding of algebraic functions as a mathematical construct requires students to undertake a huge conceptual journey, and requires time and care for them to fully appreciate the breadth and depth of the concept. The teaching of this section of school mathematics requires that teachers support the development of conceptual understanding and the ability to interpret the relationships among all the representations of this concept. Without the continuous underpinning of a conceptual framework, students will not develop adequate skills to recognize when and where to use certain procedures correctly (Rittle-Johnson and Alibali, 1999).

2.5.1 Students' journey when studying this section at school

Students need to move from the drawing of graphs by plotting a number of points, generated by the construction of a table of values (linked by a specific relationship between dependant and independent values), to the ability to do so from the algebraic form of a straight line graph equation by only finding the x-and y-intercepts. They also need to progress to finding the equation of the linear relationship from only two points, or from information regarding the gradient and one point, instead of a number of points given in tabular form and represented as a number pattern. In doing so, students must not lose sight of the fact that the dependant and independent values of each coordinate point are related in the same way, and that this relationship is represented by the equation of the linear function; *and* that this particular relationship is termed linear because it generates a straight line graph when these coordinates are plotted on a Cartesian plane. The fact that this graph is a representation of a relationship explored in a section called "number patterns" – often taught separately – should also be linked into the understanding of this concept.

The initial introduction to this important mathematical concept needs to establish a firm foundation for the extension of the idea of a "function", for an understanding of the different relationships that can be represented, and of how this idea can be used to solve problems – often taught in previous grades as unrelated to a potential graphical solution (for example distance-time-speed and other ratio problems that are usually only solved through algorithmic or tabular-equation methods). A high value is placed on the ability of students to work with graphical illustrations of relationships, as such representations can describe many situations, and not only those within the field of mathematics. This skill in students – the understanding of the different representations of a relationship and the ability to translate from one representation to another – has a high correlation with success in mathematics education, and can be associated with success in problem-solving (Coulombe and Berenson, 2001; Gagatsis and Shiakalli, 2004; Kieran, 2007). Effective mathematical thinking involves understanding the relationships among different representations of "the same" concept as well as their similarities and differences (Goldin and Shteingold, 2001). As Coulombe and Berenson noted (2001, p. 166): if we think of a representation not only as an *image* but also as a *process*, we can see its usefulness in the learning of mathematics. These constructs then become tools that allow students to develop algebraic thinking, make mental images of patterns and functions, and solve problems. For example, graphs and tables can be effective devices for investigating linear patterns in a variety of contexts. Friedlander and Tabach (2001) agree with this view, maintaining that using verbal, numerical, graphical and algebraic representations can make the process of learning algebra more meaningful.

There must come a point at which a function is viewed as an object in its own right, and one that can be used in the process of constructing knowledge of other, more complex ideas (Sfard, 1991). This process is often called *reification* in the literature (Sfard, 1991; Slavit, 1997; Tall, Thomas, Davis, Gray and Simpson, 2000), and can be construed (in simplified terms) as the development of a firmer understanding of an abstract entity. While it might be unrealistic to expect this understanding of functions to culminate in the complete objectification of functions at school level – certainly, this would be unlikely in the year in which they are first formalized – the process towards this understanding needs to be helped by encouraging students' understanding of the processes linked to the concept.

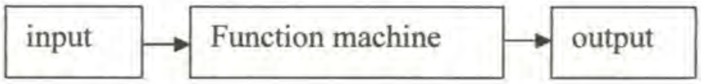
Developing the depth of understanding required for mathematical proficiency takes time – a commodity often in short supply as we grapple with the demands of the curriculum. As Sfard (1991, p. 33) says: "it is often not acknowledged that insight cannot always be expected as an immediate reward for a person's direct attempt to fathom a new idea.... It requires much effort..." and therefore time.

In order to achieve success, it is necessary to teach far more than just the algorithms used in working with linear functions. Students need to develop an understanding of the relationships involved in this concept. Instruction should encourage students to practise translating from one representation of the concept of function to another, and to use translation strategies to enhance their problem-solving ability (Gagatsis and Shiakalli, 2004).

2.5.2 Considerations of the complexity for conceptual understanding

What is a function? Slavit (1997) suggests that there are four ways in which we can think about functions:

1. In terms of actions that are undertaken to compute output values.



   This form of thinking about functions is the basic starting point of the conceptualization of the idea of a function.
2. In terms of what Slavit calls an object-orientated view, in which the action view is used to generalize a common relationship from an entire set of input-output pairs. This allows the development of the idea of correspondence or mapping. This is a traditional approach found in many textbooks.
3. A second stage in this object-orientated view involves understanding the way in which dependent and independent variables change, and analyzing, manipulating and comprehending the relationships between these changing quantities (termed a covariance view).
4. The last layer in understanding or thinking about functions is in terms of a property-orientated view: here the concept of a function is understood as a set of procedural and functional properties. Similar procedures that are performed using different notations but that do the same thing (e.g. finding intercepts by letting y=0; f(x)=0 or finding roots or zeros) are understood to be equivalent. Students are able to generalize procedures across different types and classes of functions. Functions are now understood in terms of their properties and how these are linked to their algebraic and geometric representations.

Functions and their related concepts are complex to both teach and learn. Developing this understanding of a graph on a Cartesian plane (in the case of function at school level) relies on a wider system of representations within which meanings and conventions have been established. Goldin and Shteingold (2001) define a representation as a sign or a collection of signs, characters or objects that can stand for something other than itself. They insist that understanding must happen on two levels, involving both an external and an internal system of representations. The external system is more concrete – the graph as drawn on the Cartesian plane, a function, the solutions for an algebra equation. The internal system consists of students' personal constructions of and assignments of meaning to mathematical ideas. This will have an effect on the way they perceive and work with mathematics (Keiran, 2007). It is the interaction between these systems that is so important for effective teaching and learning.

Coulombe and Berenson (2001, p. 167) presented two approaches to teaching equations with two variables – the traditional approach, and a problem-based approach, as shown in *Table 2.4*.

Traditional approach			**Problem-based approach**		
Move from	*Move to*	*Process undertaken*	*Move from*	*Move to*	*Interpretive process*
Symbols (e.g. y = 2x)	Table	Calculate values	Graph	Words	Graphical interpretation
Table	Graph	Plot points	Table	Words	Verbal pattern description
			Words	Graph	Qualitative graphical construction
			Graph	Table	Data generation
			Table	Symbols	Pattern finding

Table 2.4: Two approaches to teaching - as adapted from Janvier (1987). Source: Coulombe and Berenson (2001).

These authors noted that:

- Using a problem-based approach gives students the opportunity to interpret well-known events mathematically. This facilitates the development of a more meaningful understanding of these representations.
- The process of interpreting functions allows students to connect important mathematical ideas such as gradient to their representations.
- Interpretation and translation for assessment can be useful tools for teachers, guiding them in instructional planning by helping them to see more clearly what students understand and which mathematical ideas are still developing.

This does not mean that there is no value in traditional approaches. The judicial use of some of these approaches can prove helpful in establishing mastery of procedures that underpin the development of conceptual understanding of certain characteristics of (for example) linear functions. Greenes, Chang and Ben-Chaim (2007, p. 280) remarked upon this when they recommended that teachers devote more time to teaching and thoroughly assessing concepts of slope, y-intercept and the connection between the algebraic and graphical representation of a line in order to "enable students to build deep and meaningful understanding of the key concepts of linearity".

2.6 Conclusion

When seeking to align research and teaching practice, it is useful to examine the problems that have arisen in other such attempts. Ding and Jones (2007), identify questions that need to be clarified regarding the van Hiele teaching phases. Are these spread over the topic, or focused within a lesson? Does one mathematical idea need to be dealt with at a time when teaching using these phases, or can the whole concept be covered? In an analysis of the difficulties of reification with respect to algebra, Sfard and Linchevski (1994) noted that "making relationships between sets of input-output pairs was not an easy task" and that the problem was compounded when "students grappled with the symbolism of functions" which they dealt with in very different forms, e.g. graphs and equations. This prompted Slavit (1997) to propose developing the concept of a function by continually linking procedures to properties, so they are seen as belonging to the function as a whole, not just as a localised value with no real meaning. This requires time to be spent learning and understanding procedures that are necessary to develop mastery of the concept.

The understanding of how mathematics teaching and learning should work, as presented by the Kilpatrick model, has appealed to me both conceptually and theoretically. I have become increasingly frustrated by the "checklist" mentality of current mathematics teaching. Students have difficulty remembering how to do basic procedures e.g. apparently different methods of solving equations (grade 11). There seems to be a need to overhaul our entire approach to the concepts we teach in order to embed them in a framework which allows a cohesive and whole picture of the "puzzle" that is school mathematics.

CHAPTER 3: DEVELOPING THE TEACHING MODULE:

THE INFLUENCE OF THEORY ON THE DESIGN OF A LEARNING PROGRAMME

3.1 Review of previous teaching practice and participation in module planning

The purpose of the review and planning phase was:

- To introduce the teachers who would be teaching the module to the research process to be undertaken.
- To provide the teachers with an opportunity to become involved in the idea of trying a different teaching strategy for the section on number patterns and straight line graphs.
- To develop a teaching module for use in teaching this section.

In order to involve the participating teachers in the development of the module, I held a focus group meeting in late 2010, after teaching linear functions. I considered this to be the most appropriate method of eliciting the input from teachers that was needed for this phase of the research. I wanted the teachers' perceptions of teaching linear functions as derived from their own experience, and needed to use the interactions among the group members to stimulate ideas and ensure that a collective understanding was reached. Using a focus group meeting to obtain information of this sort is on the increase in educational research and is considered a valid data collection method in such situations (Puchta and Potter, 2004; Cohen, Manion and Morrison, 2008). By providing an opportunity to talk about the mathematics and how it should be taught, teachers were able to take ownership of, and commit themselves to the research endeavour.

My colleagues and I identified the constraints and difficulties we had experienced using traditional teaching approaches to linear functions, and outlined the manner in which we would like to proceed with teaching this section in 2011. The approach to devising an appropriate teaching strategy for 2011 provided a framework within which to develop the teaching module, and allowed space for drawing material from the literature into teaching for mathematical proficiency.

The outcome of the meeting was an agreement to approach the teaching of linear functions in Grade 9 through the use of the module to be developed by myself. A broad outline of the module and its contents was also agreed upon and summarized by the group. The focus group meeting was recorded, transcribed and analyzed for themes, in terms of the following categories.

Color	Coding for
RED	Ideas for module to teach.
BLUE	Summarizing ideas
PINK	Problems in conventional teaching
GREEN	Potential teacher's misconceptions

This analysis helped both to develop the broad structure of the module and, in tandem with the theory informing the module structure, to guide the choice of activities used in the module.

An overview of the teaching module appears below. The entire module as it was presented to the teachers can be found on the CD that accompanies this thesis.

Overview of the teaching module:

Component 1: Numeric and Geometric Number Patterns

In which number patterns, their terminology, and the ways they can be represented (including graphically) are introduced, and the understanding of the concept is developed.

Component 2: Algebraic Graphs Part 1:

An introduction to graphs - with particular reference to linear functions presented through contextual problems. The purpose of this component is to:

- Consolidate and build on Grade 8 work and number pattern concepts:
- Introduce the Cartesian plane:

Component 3: Algebraic Graphs Part 2:

Formalizing linear functions: activities designed to develop an understanding of the formal algebraic representation of straight line graphs. The purpose of this component is to:

- Introduce and revise concepts and terminology:
- Link the characteristics of a graph to its equation:
- Use observations to develop more efficient ways of drawing graphs:
- Use skills developed to promote greater understanding of relationships between graphs, gradients and equations and to consolidate concepts.

Component 4: Algebraic Graphs Part 3:

Contextualising formal algebraic linear functions: applying them to the solution of "real life" problems. The purpose of this graph is to link the formal algebra to a concrete context through:

- Graphs that tell us a story
- Simultaneous equations

3.2 Integrating the theory

The module to be developed needed to incorporate the learning goals as identified by the focus group and the South African Mathematics Curriculum. It also had to incorporate the ideas put forward by the chosen educational theories, which were:

- Using activities to relate different representations of number relationships, and promoting learning to work with these;
- Developing an approach to incorporate the use of algebraic graphs in the classroom;
- Promoting good number sense at the desired (Grade 9) level so as to support the development of 'mathematical proficiency' – with specific reference to teaching *linear functions in Grade 9.* This in turn would involve:
- Developing an understanding of the links between concepts within this topic and concepts studied in other sections of the mathematics curriculum in Grade 9.
- Developing familiarity with the language and terminology used when talking about this concept.
- Achieving this within the practical realities of the context within which we teach - one factor being the need to meet the demands of the curriculum required for Grade 9 for this topic.

3.2.1 Overview of the use of educational theories informing the module development

While many of the educational theories in the literature on mathematics education informed the development of the module, the main influences on the direction and content order of the module derived from the work of Kilpatrick et al. (2001); van Hiele (1986) and Stein et al. (1996; 1998; 2010), as depicted in *Figure 3.1.*

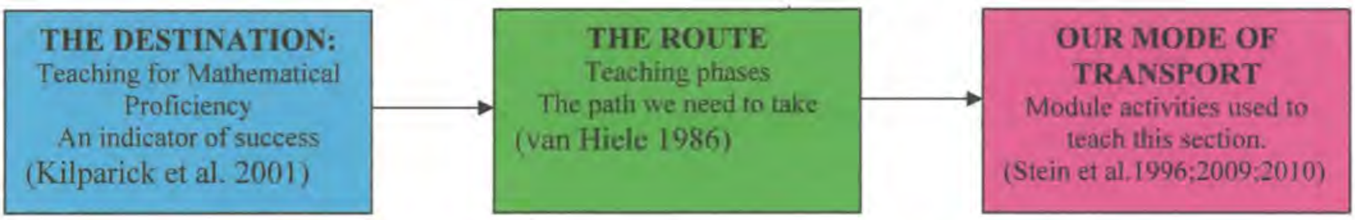


Figure 3.1: Influence of main theories on the teaching module

In putting together activities for use in teaching these topics in the Grade 9 syllabus, I was instructed by the curriculum as to *what* content had to be taught. I then endeavoured to use the literature from researchers into mathematics education to design a module to help teachers with the question of *how* to teach the topic. This resulted in a module that has far greater depth than is allowed for within the current curriculum and the time available to teach the topics. The depth is a reflection of the research. The way we are able to teach these concepts and the decisions we make in order to do so – considering the constraints within which we work – is one of the focus points of my thesis.

The notions put forward by the teaching theories that supported the development of the module, i.e. mathematical proficiency; the van Hiele teaching phases and the cognitive demands of mathematical activities, are discussed in Chapter 2. These ideas were interrogated in terms of the way in which they could be used in the development of the module.

3.2.2 Use of theory to inform the development of concepts and activities

It was necessary to understand the educational theories from a teaching viewpoint, that is, to understand how they could contribute towards effective mathematics teaching and how the activities could best be utilized to align the module with these theories. Activities used by teachers are the means by which the students engage and interact with the mathematics. Therefore, recognizing the important role that these tasks would play in ensuring the effectiveness of the module, it was necessary to put together a means of classifying teaching tasks. This then provided a guide for rating an activity according to its level of difficulty and appropriate place in the development of a concept. This was done by:

- Identifying what characteristics can be used to determine how activities can contribute towards the development of mathematical proficiency.
- Identifying the characteristics of tasks used in each of the different teaching phases as recognized by van Hiele (1986). (This provided information about the appropriate timing for the introduction of activities in a progressive teaching process.)
- Identifying what features determine a task's classification in terms of the model presented by Stein and Smith (2009), so as to assist the selection of tasks for the teaching order used in the module, and adhere to the variation in cognitive levels required by the NCS with respect to assessment tasks used in the classroom.

These classifications (shown in *Tables 3.1, 3.2* and *3.3*) were used to help choose and order the activities that populated each component of the module.

Table 3.1: Features that allow for the classification of tasks according to their promotion of the strands of mathematical proficiency

MATHEMATICAL PROFICIENCY COMPONENT	FEATURES/CHARACTERISTICS OF TASKS
Procedural knowledge	• Provide well timed practice of skills so an inability to use procedures will not prevent the development of understanding of the concept. • Are linked to problems that place emphasis on learning with understanding. • Attempts to connect the skills being practice to their use in different situations. • Students are able to succeed in task if invest reasonable effort. • Cannot be solved if common errors are made. • Provide a platform for developing understanding of the concept so as to discuss and correct mistakes. • Provide an opportunity for students to progress in learning the content in appropriate size steps. • Develops meanings for symbols used in the topic.
Conceptual understanding	• Require some scaffolding to help students apply skills and concepts learnt. • Attempts to develop relationships between pieces of information. • Focus on the meaning of the procedure and the solution it is used to find. • Requires students to transfer their knowledge of procedures to the solution of the problem. • Requires the student to choose the method or representation appropriate to the problem. • Contextualizes the use of procedures in solving problems 'beyond the classroom'. • Promotes class discussion so as to attempt to help students develop an understanding of the different approaches that could be used.
Strategic competence	• Not obvious in terms of the solution strategy. • Often presented in words that requires students to determine what is known, what must be found and to choose a way of representing the problem mathematically – this representation in not given. • Students must not be able to solve by simply focusing on the numbers in the problem. • The situation presented by the problem must be understood in order to determine the best way of representing the problem. • The way in which the problem is understood and represented must suggest the mathematical concept and procedures that might provide a solution strategy. • Requires flexibility in terms of thinking from the student – "out the box". • "Doing mathematics" (Stein et al., 2009)
Adaptive Reasoning	• These tasks or problems must require students to think about mathematical relationships. • Students must be required to justify (explain) their reasoning and conclusions. • Often a series of tasks that develop an idea requiring interlinking of various mathematical concepts. • Tasks designed to develop a situation where students ask "why" something is true, and the answer to this question must be justified using mathematics. • Tasks are specifically chosen to link conceptual and procedural knowledge by justifying the relationship.
Productive disposition	• Not necessarily single tasks, but more the result of a carefully structured **approach** to learning mathematics using a number of tasks that develop the above strands. • Tasks are graduated in difficulty, the approach is designed to provide students with belief that perseverance can lead to success. • Promotes enjoyment of challenge and mathematics, but not "just for fun", designed to develop mathematical proficiency.

(Identified in Kilpatrick et al., 2001)

Table 3.2: Characteristics that allow for the classification of problems according to the van Hiele phases of teaching and learning

PHASES		CHARACTERISTICS
1 Information		• Simple and without accidental solutions. • Not used for homework as designed for promoting class discussion under the guidance of the teacher. • Provide opportunity for the teacher to introduce the concept to be studied, its terminology and basic relationships. • Attempt to incite curiosity in students.
2 Bound orientation/ exploration		• Single solution problems. • More involved than just providing basic information. • May require guidance to help direct the student. • Introduces further relationships needed for solving problems that can be linked to this domain of mathematics. • Designed to begin to develop an understanding of the system of relationships found within the concept. • Tests progression of student.
3 Explication		• Characteristics of problems are similar to those used in the bound orientation phase. The intention of the teacher is different. • The teacher's intention is to formalize and make explicit the relationships upon which the understanding of the concept is based and to promote and consolidate the correct use of language to describe the concepts the students are studying. • Tests progression of student.
4 Free orientation		• Problems that are given must be able to be worked on by the students without the direct help of the teacher. • Consolidate and test the students understanding of the concept's relationships that have been developed so far. • Can be given as homework. • Different ways of solving these should be discussed and presented by the students. • Students who take the right direction in solving these problems should not meet unexpected difficulties. • Tests progression of student.
5 Integration	Type 1	• Consolidation and revision problems. • Often suitable as classwork, can be asked orally. • Designed to test the students' assimilation (and understanding) of the concepts introduced in the topic. • Not designed to test students' ability.
	Type 2	• Test insight. • Require application of concepts. • Require students to develop a new level of understanding of the place the topic under discussion has in the network of relationships that make up mathematical knowledge.

(Identified in van Hiele, 1986)

Table 3.3: Features that allow for the classification of tasks according to their cognitive demand

LEVEL OF COGNITIVE DEMAND	FEATURES/CHARACTERISTICS
1. Memorization tasks	• Requires the recall of previously learned information. • No understanding of the links between concepts is required. • Can be done without explanation by teacher. • Is "text-book" like – repetitive.
2. Procedures without connections tasks	• Requires the use of a well-established procedure. • There is no connection to meaning. • Any "real-world' context is window dressing and is not the focus of the procedure. • Can use a diagram, but reproduces a learned procedure, even if there are multiple steps. • Focus is on the ability to get the right answer and carry out the procedure correctly. • Used to develop a well established procedure.
3. Procedures with connections tasks	• The task focuses on the procedure, but in a meaningful context. • Real-world context can be used to provide the reason or meaning for the procedure. • Tasks provide for the use and learning of procedures, but focuses on the underlying meaning and use of the procedure.
4. Doing mathematics tasks	• No predictable pathway suggested by the task. • Focus is on looking for the underlying mathematical structure. • Requires complex thinking. • Has a "real-world" context that requires understanding of the underlying mathematics to solve. • Requires an explanation. • Uses manipulatives / diagram. • Involves multiple steps. • Is symbolic/ abstract.

(Adapted from Stein et al., 2009)

In order to help maintain the progressive development of the teaching phases suggested by van Hiele (1986), each component in the module was structured into three parts:

i. An introduction;
ii. A consolidation of basic concepts with some extension work requiring scaffolding; and finally (if appropriate to the component),
iii. Problems that required an approach not immediately obvious at the outset. This was easiest to include in component 1.

In an effort to prevent teaching for "boxes that need to be ticked off", concepts were presented as a linked extension of a previous idea and not as separate sections. It must be noted that in order to work with the concepts and ideas needed for this topic of the Grade 9 curriculum, there were certain algebraic and numeric skills that would have to be developed "inadvertently" during the teaching of the module if students did not already possess these skills.

3.2.3 Other considerations for the development of the module

Apart from the educational theories discussed above, it was necessary to take into account the following more pragmatic influences on the module that was to act as a resource for the teaching of the linear function section of the Grade 9 syllabus:

- The requirements of the curriculum as outlined in the National Curriculum Statement (NCS) and the Curriculum and Assessment Policy Statement (CAPS), the replacement curriculum document to be implemented from 2012.
- The seeming disparity of approaches in the documents in terms of the way they presented the knowledge that needed to be developed through teaching. On the one hand, the conceptual understanding and skills expected of students by the end of this topic (outlined in chapter 1) are aligned with the idea of mathematical proficiency; but on the other, the curriculum still presents the topic details to teachers as a list of concepts to be covered (the effect of which is discussed further in 6.4.2.2.).
- That the Grade 9 year in South Africa culminates in an externally set "end of Phase" examination which all students are required to write. Students of the teachers involved in this research are in addition required to write a provincial paper in June that is also externally set. The content of this paper is determined by the pace-setter provided by the Department.

It was necessary to work within these constraints where possible without undermining the purpose of the module, which was to put together a programme of work to facilitate and promote teaching for mathematical proficiency within this section of the Grade 9 syllabus.

3.3 Developing the module

The development of the module started in December 2010 and was completed by March 2011. In order to compile the resource material for the module it was necessary (as mentioned above) to review both the NCS (which provided information on the curriculum that was current in 2011) and the new CAPS document (which provided information regarding the curriculum to be implemented in 2012 – important for the life of this module beyond 2011). Activities and problems were collected from a number of sources including textbooks and worksheets developed in a range of South African high schools. These were adapted and modified for use in the module in order to attempt to align the structure of the module with ideas promoted in the literature concerning effective teaching for mathematical proficiency.

3.3.1 Module Structure

3.3.1.1 Component 1

The ability to (i) represent numbers in a variety of ways; (ii) move flexibly through these representations; and (iii) be able to describe patterns and relationships through the use of symbolic expressions, graphs and tables: these are all important proficiency skills emphasised by both the curriculum documents consulted and experts in the field of mathematics education such as Coulombe and Berenson (2001), Kilpatrick et al. (2001), Gagatsis and Shiakalli (2004) and Kieran (2007). Exploring number patterns therefore seemed to be the logical starting point for developing an understanding of the relationships underlying the linear function graph taught in the Grade 9 year. This concept would allow the teacher to explore with the student the different representations that could be used to characterize a relationship between two sets of values, and to develop the terminology used to describe this relationship. Starting with number patterns would provide teachers with a way of teaching these skills in a manner that would afford opportunities for class discussion, for student-driven "discovery" of relationships and how these are linked to their representations (including graphical representations). Working with "puzzles" nurtures students' natural curiosity and provides a way to promote their confidence before focusing on one specific representation – the graph.

3.3.1.2 Component 2

In order to move to working with the straight line graph as a representation of a linear relationship between numbers, it was necessary to consolidate and build on the understanding of the Cartesian plane and graphs as developed in Grade 8. This provided an opportunity to contextualise the use and meaning of graphs in "real life" settings while revising:

- Working with positive valued co-ordinates.
- Familiarising students with plotting of points and drawing of graphs.
- Linking representations – equations, tables of values, co-ordinates, graphs.
- Familiarising students with the meaning of the terminology to be used – gradient, dependent, independent, variable, axes.

It was not necessary to link gradient to the **equation** at this point, but rather to develop an understanding of what the "steepness" of a line means and how it is measured. This concept seems to be a difficult one for students, and so introducing the terminology and understanding early was felt to be important.

Apart from the brief re-orientation to graphs, this component was designed to extend student's knowledge of the Cartesian plane by:

- Familiarising them with all four quadrants and related terminology (x-axis, y-axis, order of naming quadrants and characteristics of points in those quadrants);
- Practicing plotting and reading off points in all four quadrants;
- Allowing teaching to move easily to the next section of algebraic graphs.

3.3.1.3 Component 3

The formalizing of the graphic representation of the linear relationship concept developed in component 1 was the main focus of this component and the most difficult to put together. Because of the complex nature of the type of understanding required in order to become proficient with graphs and their algebraic representations, this component was of necessity the largest of the four. The conceptual understanding needed for the development of procedural fluency and the other strands of mathematical proficiency does not seem to come easily. To be proficient with graphs requires algebraic and numeric skills such as solving equations; substituting and solving; understanding equating; understanding co-ordinates and their relationships; developing an understanding of concepts like gradient and intercepts and how each can be found; how all these can relate to a graph specifically and all graphs generally; and how the equation of the graph governs the relationship between the points that make up any specific graph and so determines both the uniqueness and the general characteristics of a linear function graph. In my experience, this is conceptually quite a big mental jump for students at this level of mathematical development, and full understanding does not come to all students, even by Grade 12 level. In order to encourage the eventual development of the required understanding for this topic it was necessary to promote teaching that continually attempted to incorporate new information and prevent the compartmentalization of procedures (Kilpatrick et al., 2001).

Yet this is a section in which many students develop a fragmented understanding, viewing the topic as a list of procedures to master. Students find it difficult to understand that a co-ordinate is a numerical representation of a relationship that can be written in algebraic form. They also struggle with linking the very different forms of linear functions, e.g. graphs and equations. This difficulty, with which even experienced teachers grapple, is highlighted by Sfard and Linchevski (1994). Slavit (1997) suggests that developing the concept of a function by continually linking procedures to properties, so they are seen as belonging to the function as a whole rather than as separate values that have no real meaning, might help clarify this area for students. Mudaly and Rampersad (2010) recommend the use of visualisation skills in the teaching and learning of functions, commenting that in their opinion this is essential for conceptual understanding. In order to present the mathematical ideas and link them in a structured manner, this component consisted of four sections. Each increased the number of linked ideas in an attempt to build the complexity of the conceptual understanding of the relationships between the symbolic and graphical representations of linear functions. A brief overview is given below:

Section 1: Introduction to and revision of concepts and terminology:

- Given points, plot the points, find the pattern/formula the points adhere to and recognize that points belonging to a unique formula all lay on the same straight line.

- Introduce the y-intercept.
- Given a graph, find the points that lie on that graph and the equation of the graph based on the relationship seen in the co-ordinates of the points.
- Find the y-intercept of graphs after drawing/from given graphs.
- Revision of gradient of a line – meaning and finding it from the graph.

Section 2: Linking the characteristics of a graph to its equation:

- Given the pattern/formula, find a set of points; plot them to draw a straight line. Use this to link the y-intercept and gradient to the equation of the straight line.
- Finding the y-intercept and gradient given the equation.
- Finding the equation given the gradient and y-intercept
- Finding the equation by plotting points to draw the lines and reading the y-intercept and gradient off the graph.
- Finding the equation of the graph algebraically. (without plotting first)

Section 3: Using observation to develop more efficient ways of drawing graphs:

- Gradient – intercept method.
- Dual intercept method.

Section 4: Using skills developed to promote greater understanding of relationships among graphs, gradients and equations, and to consolidate concepts.

- Revise the 3 ways of drawing a straight line (brief).
- Investigate the relationship between parallel and perpendicular graphs and their gradients by drawing graphs.
- Find equations of lines from information given either diagrammatically or descriptively. The gradient and y-intercept are not necessarily given and will need to be worked out using other information supplied.
- Determine whether or not a point lies on a line.

3.3.1.4 Component 4:

In order to facilitate the development of the strategic competence and adaptive management strands of mathematical proficiency it was necessary to add a fourth component to the module. Component 3, necessarily, focused mainly on attempting to promote procedural fluency and conceptual understanding. But in order for students to become mathematically proficient in this topic it is necessary to provide them with activities that show how the ideas developed in component 4 can be applied to solving problems. Therefore the focus of this component was the contextualising of formal algebraic linear functions by applying them to the solution of "real life" problems. In order to attempt this, activities in this component were presented in the following sections:

Section 1: Graphs that tell us a story

- What does the gradient and the y-intercept mean when a graph is used to model a real situation?
- Activities involved: - conversion graphs
  - comparison graphs

Section 2: Simultaneous equations.

- Why do we use them? – An economic situation that introduces this useful mathematical tool in a "real world" context.
- Finding the solution graphically.
- Finding the solution algebraically.
- Mathematical modelling using linear functions.

The completed module was then packaged for each of the teachers, and included a summary of the educational theories as a brief introduction to these theories.

CHAPTER 4: DISCUSSION: THE INFLUENCE OF THEORY ON THE DEVELOPMENT OF THE MODULE:

4.1 What influences the impact of research on teaching practice?

Groth and Bergner (2005) suggest four areas of concern that might restrict the impact that education research could have on teaching practice:

1. Is the research **persuasive**?
    - Would one be able to be referred to the research when seeking guidance from an authoritative voice regarding a matter of teaching practice?
2. Is it **relevant** to practice?
    - Can a teacher see this research as applicable to a concrete situation?
    - Does it relate conceptual information to teaching practice in a procedural, specific and pragmatic way?
    - Is it presented in a way that allows a teacher to link it to practice in this way?
3. Is it **accessible**?
    - Will teachers be able to find and use this information in literature that is readily accessible to them?
4. Does the **education system or the nature of the school** support the use of research to change teaching practice?
    - Is the use of a textbook too ingrained or is it seen as one resource among many?
    - Do classroom management and teamwork in schools support the use of research to look at ways to improve teaching practice?
    - What is the time allowed for face-to-face teaching in the school timetable?
    - What is the time allowed for teaching the topics and related concepts demanded by the curriculum in the school year?
    - Is the system or school inherently unstable and susceptible to teaching fads?
    - Is the system unable to promote change as it is intractable?

These four areas of concern have proved useful in determining (i) whether the theory can be used to inform the design and development of teaching material; (ii) whether such teaching material can be used to promote teaching for mathematical proficiency; and (iii) what influences upon teaching practice can be identified as affecting teaching for mathematical proficiency. Addressing these concerns allowed me to evaluate the effectiveness of the research ideas used, the efficacy of the teaching module, and the influence of teaching environment upon teaching with the module.

Research question	**Concern to be addressed to answer the question**
Can theory be used to inform the design and development of teaching material?	Was the chosen **research** persuasive, relevant and accessible? (Discussed in Chapter 4)
Can such teaching material be used to promote teaching for mathematical proficiency?	Was the **module** persuasive, relevant and accessible, thus promoting teaching for mathematical proficiency? (Discussed in Chapter 7)
What influences upon teaching practice can be identified as affecting teaching for mathematical proficiency?	This is a function of the enabling environment: what factors were present in this teaching environment to promote or inhibit teaching for mathematical proficiency? (Discussed in Chapter 7)

4.2 Using theory to inform the design and development of teaching material

4.2.1 The idea of Mathematical Proficiency

"Instruction that develops mathematical proficiency is neither simple, common, nor is it well understood. It comes in many forms and can follow a variety of paths."

(Kilpatrick, 2001, p. 359).

I found that mathematical proficiency provided an excellent way of structuring the module and was invaluable in identifying the thinking skills I needed to promote. It was not, however, very easy to convert into practice.

In sum, the theoretical framework offered a sound basis for developing the teaching module. The intention was that, through the judicious use of classroom and homework activities we would promote students' development of all five strands. The conclusion I came to was that the theory provided a necessary overarching goal but was difficult to put into practice. As Kilpatrick et al.'s (2001) report states, "[t]he goal of mathematical proficiency is an extremely ambitious one" (p. 14). This difficulty notwithstanding, thinking about how to teach for mathematical proficiency in terms of the development of the five strands, certainly proved a constructive method for reflecting upon teaching practice. It was especially helpful when considering what approaches might help develop students' understanding of the mathematics.

I concluded that in any one lesson or series of lessons what was attainable was at most some movement in a direction that would promote mathematical proficient thinking. At no point was it possible to establish that the students had become 'mathematically proficient'. When I examined the demands made by the various strands of mathematical proficiency upon class activities (see *Table 3.1*), I had more questions than answers regarding the practicalities of implementing the module. For example:

- At what point does one provide for practice of a procedure – can this be pre-empted or is it a function of one's class and therefore only ascertainable during the teaching of a concept?
- How does one characterize and recognize a problem that places emphasis on learning with understanding?
- What type of problem provides a good platform for discussion?
- What is 'good discussion' and how can it be managed?
- Does contextualizing a problem always have to mean "real life"?
- Is it possible to find a problem that develops strategic competence when one is only at the point of discovering basic linear function relationships and concepts?
- Is it possible to use activities that encourage the enjoyment of success without losing the essential mathematics?

Many of these questions arose because at the beginning of the research process I had thought that the strands to focus upon were obviously procedural fluency and conceptual understanding. I was led to this conclusion both by the way in which the South African curriculum is structured and by emphases encountered in the literature (e.g. Hiebert and Lefevre, 1986; Rittle-Johnson and Alibali, 1999; Schneider and Stern, 2005; Rittle-Johnson and Koedinger, 2009, and Lampert et al., 2010). Focussing upon these two strands made it difficult for me to envision a way in which to structure a module that might attempt to promote a more rounded view of the topic to be taught. It was also very easy to fall back into traditional 'drill and practice' examples as the main feature of a lesson. To address this problem, it was necessary to revisit the entire concept of mathematical proficiency, and this led me to conclude that procedural fluency and conceptual understanding should not be the exclusive focus of any module, even if many of the activities in such a module might be promoting these skills.

I came to the realization that exploring methods of developing adaptive reasoning was the key to promoting mathematical proficiency. I found that it was not possible to develop this particular strand in isolation. This conclusion is not unique: Kilpatrick et al. (2001, p. 129) consider adaptive reasoning to be "the glue that holds everything together", a view endorsed by Brodie (2010. p. 11), who argues that mathematical reasoning is "a key element of mathematics and thus central to learning mathematics at school". The development of adaptive reasoning skills requires procedural and conceptual understanding to be developed concurrently. Thinking about mathematical relationships, justifying answers and conclusions and using activities to promote the interlinking of skills and knowledge are all characteristics of tasks that support adaptive reasoning. Yet tasks of this type will require other strands to be developed in order to promote understanding and encourage a productive disposition towards mathematics. Considering adaptive reasoning as the "pivot" provided me with a more manageable way in which to approach the development of the module. By examining the features or characteristics of tasks that might be used to develop such a strand I was able to arrive at an understanding of mathematical proficiency as a **process** of development. It was then possible to view a series of tasks as being used progressively and collectively to promote understanding. The idea of understanding as a process is promoted by Sfard (1991; 1994), who calls this way of coming to know mathematical ideas the "cycle of reification".

Although mathematical proficiency theory supported the step-by-step idea for teaching and learning the linear function topic, a practical approach to support teaching towards mathematical proficiency is suggested in an example focusing on a primary school concept. In order to put the ideas espoused by this ideal into practice for a Grade 9 topic I needed an approach that was couched in more general terms, allowing for easier adaptation. I therefore used the work done by van Hiele (1986) on teaching phases to investigate the possibility of an approach that might support an understanding of these mathematical ideas and their related concepts.

4.2.2 Looking at teaching phases to facilitate mathematical proficiency teaching

Of the theoretical ideas influencing the development of the teaching module, the notion of "teaching phases" proposed by van Hiele (1986) was particularly useful in helping me to work out the "big picture" guiding the structuring of the module.

Van Hiele's work (as discussed in chapter 2) is mainly focussed upon the 'stages' of learning geometry. These were not helpful in developing the module as there does not seem to be an obvious starting point for teaching linear functions. However, the teaching phases that he identified as corresponding to these learning 'stages' promised to be applicable, once it was understood how they might work within the context of the linear function module. Ding and Jones (2007) developed an "operational model" of the van Hiele phases for use in classroom observations to identify whether the teaching phases were present by using the type of activities used by the teacher as identifiers of each stage. They took each stage and attempted to identify what the students should be doing in each phase and what kind of activities would typically be able to be used in order to achieve this end. While the model was rather conservative in terms of detail for most of the phases, it was interesting to see the detail for the bound orientation phase. The breakdown into activities of this particular phase serves to demonstrate the layers that need to be provided for in the activities chosen by the teacher. Ding and Jones were unable to identify the fourth and fifth phase in the teaching of this concept, and struggled to relate these phases to the type of teaching material needed at this level. The issues that arose from this research were more helpful than the actual model that was developed, as they identified questions that needed to be answered and considered when adapting the teaching phases for a concept other than geometry. I found that examining the phases of teaching – as discussed by van Hiele (1986) – and identifying the characteristics of activities that would support teaching a progressive understanding of geometry (see *Appendix 1* and *Table 3.2*) provided a very useful, generalised understanding of these phases. In this way practical information was obtained on how to introduce and develop the understanding of an idea; what types of activities were appropriate to use at which moments; and what these activities should be doing for the students. By identifying these characteristics **before** putting together any teaching material and developing a more general understanding of teaching phases, I attempted deliberately to structure the teaching model to encompass all the stages. The design that was formed once these teaching phases had been generalized, by means of classifying them according to use of activity, suited the topic of Grade 9 linear functions. This confirmed that van Hiele's idea of progressively deepening a student's understanding of a mathematical theme could be applied to another mathematical topic.

Ding and Jones (2007) also felt it was unclear how the teaching phases related to the subject matter: whether the phases should necessarily be followed in a linear fashion, and if so, whether this occurs within a topic or within a lesson; and whether one mathematical concept needs to be dealt with at a time when teaching using these phases. I found that in putting together the teaching material for the linear function module, it was logical to allow the phases to develop in a linear fashion.

The only section of the module in which an attempt was made to promote teaching using all 5 phases in one component, was the number pattern component (#1). This was because this topic could be developed to a Grade 9 problem-solving level without needing to link as many concepts together as were required for formalizing linear functions. When I designed components #2-4, I started the phases again. The teaching of this section confirmed that starting the teaching phases again when attempting to develop new ways of thinking about mathematical concepts was necessary. I also found that attempting to squash all the teaching phases into one lesson was impractical, as different activities promoted the objectives of different teaching phases. Dealing with the understanding of concepts one at a time was practical in terms of lesson structures, but the module was designed to promote the teaching of these concepts by linking them together. This way of constructing knowledge is important for supporting the development of mathematical proficiency. The idea of concepts as building blocks in the development of the understanding of a mathematical topic is compatible with the teaching phases, as this kind of linking is the very rationale for the teaching approach. I did find that the actual identification of activities for, particularly, teaching phases 4 and 5 to be difficult (as noted by Ding and Jones, 2007). This was because information regarding activities for these phases gave more insight into the use than the innate characteristics of such activities. Although the types of tasks for the geometry levels of understanding are discussed, the cognitive demands made by mathematical tasks for use during the **teaching phases** seem to be implicit rather than explicit in the literature. This is why I felt it necessary to consult research focussed specifically on tasks and their use in the classroom.

4.2.3 Identifying activities suitable for each phase according to their cognitive demand

The revised NCS and the new CAPS curricula indicate that a wider range of thinking skills needs to be included in the teaching of mathematics than that promoted by the previous curriculum. A teaching approach based upon the van Hiele (1986) teaching phases required a more specific link between tasks and their cognitive demand, in order to facilitate the progression of thinking that I was attempting to promote in the teaching module. In a way, the populating of the module with activities was my biggest concern, as these would determine the way in which teachers and students experienced working with the module. Both the mathematical proficiency framework and the teaching phases as suggested by van Hiele were invaluable in terms of the overall design and framework for the module, but the activities that filled this structure would, I felt, play the most critical role. My feeling that activities are essential is echoed by Brodie (2010) in her discussion on teaching for mathematical reasoning. One of the points she makes when talking about teachers' attempts to teach for mathematical proficiency is that teaching in this way requires teachers to be aware of the types of tasks that they choose and how those tasks should be used to promote the desired mathematical thinking. The work done by Stein and her colleagues (1996; 1998; 2007; 2009; 2010) provided a useful way of thinking about mathematical tasks and their uses, that is, in terms of the cognitive demands they make of students. The use of this research is particularly pertinent as Stein et al. (1996) emphasise the need for mathematical proficiency, stating that an interconnected understanding of mathematics is a vital skill. Stein et al. (1996) define a mathematical task as an activity used in the classroom by the teacher with the intention of focusing the students' attention towards a particular outcome. This aim of the lesson, and the type of mathematical knowledge and thinking that the teacher therefore intends to develop, should determine the activities used. Looking at mathematical activities according to cognitive demand (as discussed in 2.4.2 and outlined in *Table 3.3*) fitted in well with the theories of mathematical proficiency as presented by Kilpatrick et al. (2001) and of teaching using the van Hiele teaching phases.

Although I found that the four-level characterization of tasks was a little broad from a high school perspective (a conclusion also drawn by Sanni, 2009 in his work on tasks and teachers' use of tasks), it was not too problematic, and the cognitive categorization of tasks proved extremely useful in helping me to decide what types of activities might be used at each stage of the development of an idea. I did find, however, that a feature used to classify a task according to the hierarchy presented by Stein et al. (1998; 2009; 2010) was not invariably agreed upon by my colleagues. This means that tasks I have used in various stages of the module are not necessarily perceived in the same way by others. The subjectivity I found influencing my decisions made it difficult to find the "absolute" cognitive value of a task. I had to rely on my teaching experience when deciding on the cognitive demand a task might require of a Grade 9 student. It was also necessary to use very basic requirements (in terms of mathematical demand) for memorization tasks as I could not assume that all students were starting Grade 9 at the same level of mathematical ability. I also had difficulty deciding what activities would represent "doing mathematics" for high functioning students and yet be within the expected capabilities of a Grade 9 student. I felt that I had managed this well for the number pattern component, but had perhaps provided too much scaffolding for activities intended to provide a similar level of challenge for the graph components.

When using this categorization of tasks, I found that I was matching the cognitive levels of tasks to the graduated development of understanding suggested by the van Hiele levels. The characteristics that indicated what type of problem was suitable for a certain teaching phase (as summarised in *Table 3.2*) corresponded well with the cognitive hierarchy of tasks presented by Stein et al. (1998; 2009; 2010). In the module I attempted to use tasks or activities that required students to work out a number pattern or draw a graph using simple procedures that they would already know. The students would then be asked to use the number patterns they had generated or the graphs they had drawn to make observations. These observations could then be used to develop a hypothesis concerning how linear number patterns or graphs worked in general. Once this had been done, the idea would be summarized and consolidated by applying the knowledge to a new problem. In order to do this, I made the assumption that what might have been a 'procedures with connections' task for Grade 8 might very well now become a more straightforward task (e.g. a memorization task) for Grade 9. I did find that the line between a straightforward memorization task and a procedure without connections task blurred in component #3, and often a starting activity had elements of both.

4.2.4 Some concluding remarks regarding these educational theories

The theories derived from educational research that I made use of in this project were chosen because they were developed to provide support for practicing teachers. Mathematical proficiency remains an authoritative concept in the context of education today because it points to the kind of teaching practice that all mathematics teachers should aim for. But in my experience it is intractable, and simply not as accessible as it needs to become if it is to spread through mathematics classrooms.

Conversely, I found the idea of teaching phases and the work on tasks that I used eminently accessible, in a practical sense, even though it took quite a lot of effort to blend these into my design and my own teaching practice. This kind of work would be more effective if done collectively with teachers functioning as a team, an opinion shared by Gallimore, Ermeling, Saunders and Goldenberg (2009), and it was my intention to develop the teaching module in collaboration with my colleagues, but this did not happen. Consequently I took sole responsibility for the development of the module. My experience indicates that without the time needed for reflective practice allocated within the working day, encouraging teachers to develop into reflective practitioners, research required to produce new teaching materials will remain beyond the capacity of most individuals.

Transforming theory into a teaching module for practical use in the classroom is certainly possible, but it requires time and effort that practising teachers do not necessarily have.

CHAPTER 5: METHODOLOGY: IMPLEMENTATION OF THE MODULE

5.1 Introduction – research process and methodology

I carried out this research predominantly during 2011. As the thesis had a dual purpose – investigating the influence of both theory and practice on a teaching programme – the study was conducted in two stages. The first priority was to develop the teaching module for Grade 9 number patterns and linear functions. The second stage involved 4 teachers and evaluated the implementation of the teaching module which was designed to support teaching for mathematical proficiency. The efficacy of the implementation of the module was examined by looking for factors that might promote or inhibit effective mathematics teaching using the mathematical proficiency model described by Kilpatrick et al., (2001), as discussed in Chapter 2.

I used an adaptive management approach to provide a framework for this investigative process. In his discussion on what he termed "the work of teaching", Smith (2001b) asserts that teaching consists of certain critical activities which together form a cycle of instruction (Shown in *Figure 5.1*). The implementation of these activities is affected by certain factors that determine the way in which teachers engage in these successive phases.

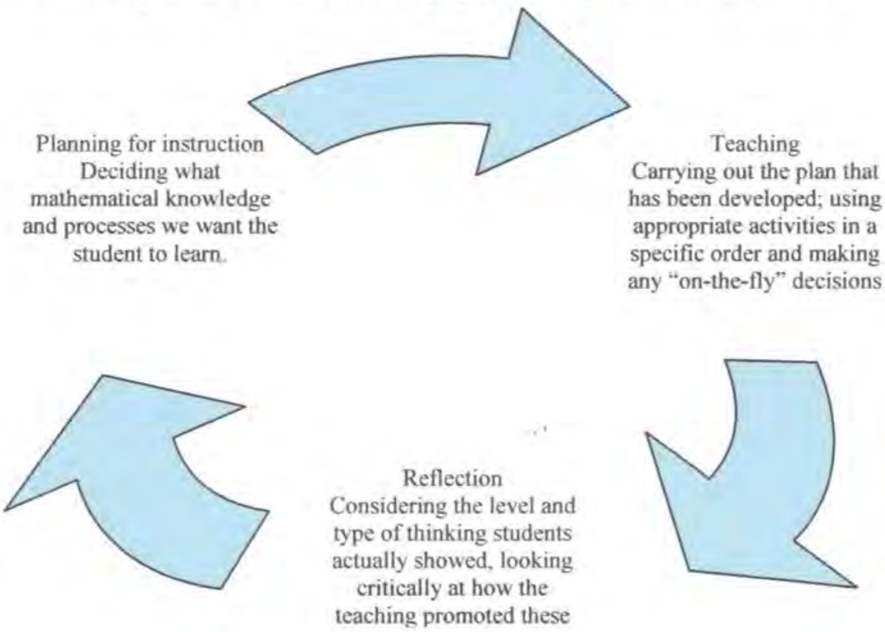


Figure 5.1: The teaching cycle (Smith, 2001b)

The principle underlying this way of working, used quite extensively in the Japanese lesson-plan approach to improving teaching practice (Fernandez, Cannon and Chokshi, 2003; Lewis

COMMENT:

The researcher as an observer and participant?

Having a researcher take on the roles of both an observer and a participant (Teacher 4) is both an advantage and a disadvantage for this kind of research. Involvement in this research process is one of the strengths of this approach, as the researcher is a part of the community and is familiar with the system within which the teaching of the module will take place. It is therefore possible to hold regular conversations with the teachers and gain greater insight into the process as the researcher's experience will mirror that of the teachers. Care must be taken regarding objectivity, a limitation of such research methodologies noted by Cohen et al. (2008). In order to mitigate the objectivity problem this loss of distance might cause, any questionnaires answered by the researcher were administered by a party uninvolved in the research. Observations used to establish themes and results came from the views and opinions put forward by the other three teachers, with the researcher's field notes being used to supplement the discussion around these themes.

Perry and Hurd, 2009), is not unique to teaching. Commonly used in business management and environmental management practices this adaptive management approach involves: (i) developing a strategy or intervention programme (bearing in mind the assumptions behind how that strategy will work); (ii) implementing the strategy and collecting data to test the assumptions and (iii) providing feedback on the efficacy of the chosen strategy. Adaptation means adjusting the strategy and/or changing the assumptions in response to the information obtained. This process is then documented to facilitate the learning process and so enable practitioners to manage projects better (Salafsky, Margoluis and Redford, 2001).

This 'learning by doing' cycle is the process that provides the framework within which this research is conducted and the study follows a small-scale cycle of planning, acting, observing and reflecting. The researcher and the teachers are involved in the design and teaching of a module that is designed to replace a traditional teaching method by a discovery method. Therefore a participatory action research (PAR) approach can provide a valid inquiry process for this project (Babbie and Mouton, 2001; Cohen, Manion and Morrison, 2008). Cohen et al. (2008) consider action research as "critical praxis" – action informed by reflection. This research is a process of putting theoretical knowledge into practice and reflecting upon the outcome. The use of observation, interviews and a research diary are recognised methods supporting PAR (McKernan, 2008).

The methodological approach described in this chapter allows my research to be divided into the phases outlined by the adaptive management approach (and illustrated in *Figure 5.2*).

Within this structure each phase is characterised by the activities that fall into that particular stage of the cycle.

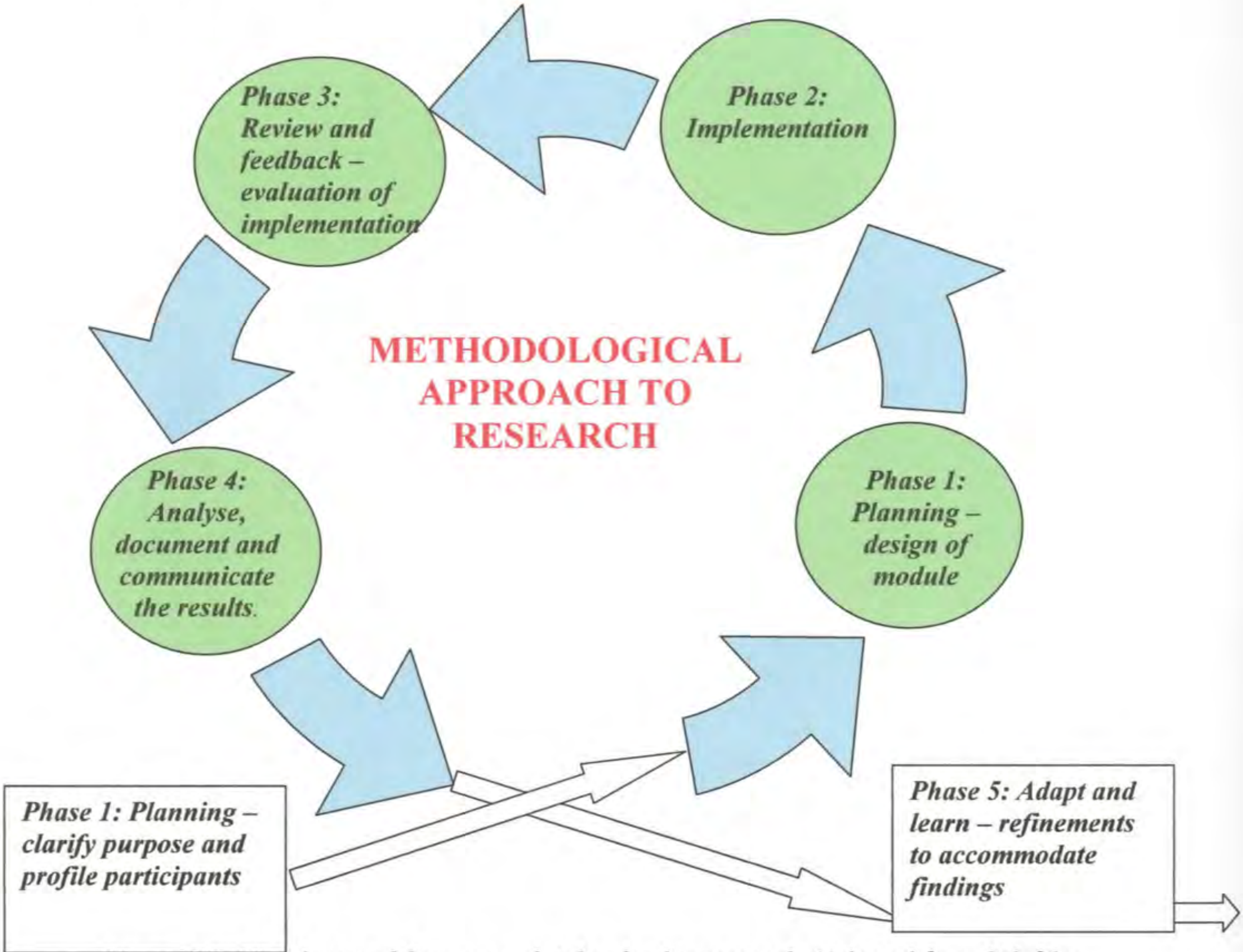


Figure 5.2: The phases of the approach taken by this research. (Adapted from Salafsky, Margoluis and Redford, 2001)

Using this approach to investigate the influences of theory and teaching practice on the implementation of a teaching module or strategy, allowed me to identify the following phases for my research:

- Develop a teaching module in the form of a resource pack based on the National Curriculum requirements for Grade 9 number patterns and linear functions which was aligned with educational theories promoting mathematical proficiency. (Phase 1 – presented in Chapter 3 and discussed in Chapter 4)

- Teach with the module – in effect test-run the module with teachers of different approaches, capabilities and students of varying mathematical ability. (Phase 2 – the focus of the research)
- Review and collect information regarding the efficacy of the module and the validity of the assumptions underpinning the development of the module. (Phase 3 – the focus of the research, presented in Chapter 6)
- Analyse, document and communicate the results of the study. (Phase 4, discussed in Chapter 7)
- Change the module and possibly the approach to teaching this section based upon what was learned, and look at the possibility of rolling out this module or teaching strategy beyond the life of the thesis. (Phase 5 which is not within the scope of this thesis)

The methods used to collect data for phases 2 and 3, which then inform phase 4, are discussed in this chapter.

Despite the study being a participatory action research approach, there was not time to finish the research cycle. Therefore this study is closer to a case study conducted within a PAR framework. The research is a single study focussed on implementation, within a single school, within a real-life context, providing an ordered description of events in which the researcher is personally involved (also see comment box above). All of these characteristics confirm that the methodology that fits such research is considered a 'case study' (Bassey, 1999; Cohen et al., 2008). That this approach is appropriate is confirmed by Baxter and Jack (2008), as the research will base conclusions on information gained though observation and experiment. These authors consider a case study to be an empirical inquiry that investigates a phenomenon within its real-life context – which is a characteristic of the research undertaken in this study. A case study method is typically used in situations where a researcher questions **how** (e.g. how the selected theoretical ideas can be integrated into teaching practice) (Yin, 2008). Also this research questions **how** a teaching module for linear functions can be developed (using theory) to promote teaching for mathematical proficiency.

When undertaking a case study approach to research it is necessary to be aware that a lack of generalization is often a shortcoming of such research (Cohen et al., 2008). Such an overview will not be possible in any case, as the boundaries within which this research is undertaken do not support generalised conclusions.

5.2 Phase 1: Planning – designing and developing the module

The module needed to incorporate the learning goals as identified by the South African Mathematics Curriculum and ideas put forward by the chosen educational theories. A focus group workshop was held in November 2010 with the grade 9 teachers involved. This allowed us to identify problems encountered teaching the section in 2010 and structure a possible teaching approach for the module. The chosen educational ideas were then interrogated in terms of the way in which they could be used in the development of the module. The main influences on the direction and content of the module were derived from the work of Kilpatrick et al. (2001); van Hiele (1986) and Stein et al. (1996; 1998; 2010). The design and development of the teaching module is presented and discussed in greater depth in Chapters 3 and 4.

5.3 Phase 2: Implementation

5.3.1 Profiling of teachers

Literature in mathematics education identifies a number of factors that influence the manner in which mathematics is taught. According to Kilpatrick et al., 2001) three of these are key influences with respect to teaching effectively for mathematical proficiency. These have been discussed in Chapter two in terms of what these authors call the "instructional triangle", and are briefly summarized below.

- *Knowledge of mathematics:* This includes the capacity to use this knowledge in the teaching of the subject in order to allow students to develop ways of understanding that enhance their conceptual understanding. (Kilpatrick et al., 2001; Ball et al., 2001, 2008; Adler and Pillay, 2007; Miles Grant et al., 2009).
- *Knowledge of students:* This refers to a teachers' general knowledge in terms of how students think about mathematics and the way they build their understanding of mathematics.
- *Pedagogical knowledge:* This knowledge base covers a range of skills that are needed to elicit and build on students' mathematical thinking (Miles Grant et al., 2009) - a clear example of how all three knowledge areas work together.

The opinions put forward by these educators together with my personal experience guided me in drawing up a list of possible factors to take into consideration when examining the small section of teaching practice under consideration in this research. I need to give these factors due consideration as they either underpin the assumptions that I made in drawing up the teaching module or influence the manner in which the module is used by the teachers involved in this project. These factors and the methods used attempt to identify them are listed in *Table 5.1*

Possible factors influencing implementation. (Based upon personal experience and exposure to ideas put forward in mathematics education literature)	**Method of attempting to identify these factors which will be used in the research.**
Disposition of teacher regarding the nature of mathematics.	Questionnaire profiling possible beliefs. Discussions arising in interview
Disposition of teacher towards the teaching of mathematics.	Questionnaire profiling possible beliefs. Discussions arising in interview.
Disposition of teacher towards methods by which mathematics is best learnt by the student.	• Questionnaire profiling possible beliefs. • Questions given in interviews.
Knowledge of mathematics.	• Pre-module questionnaire regarding qualifications and experience. • Questions given in interviews
How teachers view/understand their students	• Pre-module questionnaire regarding qualifications and experience. • Questions given in interviews.
Pedagogical knowledge.	• Pre-module questionnaire regarding qualifications and experience. • Questions given in interviews
External enabling factors: • School based: 1. Contact time. 2. Teamwork – support from colleagues. • Departmental based: 1. Number of concepts required by the curriculum. 2. External assessments for subject. 3. Professional duties taken out of the hands of teachers – effectiveness and affect on teaching. 4. Other?	• Education Department pace-setter given for Grade 9. • Curriculum set by the Education Department outlining content to be covered. • Assessments given by the Education Department and the timing of these. • School timetable – teaching time provided for. • School based assessments. • Questions in interview determining how teachers' are affected by factors. • Interview with the school mathematics Head of Department • Analysis of videos of lessons

Table 5.1: Trying to identify possible factors influencing teachers' implementation of the Grade 9 mathematics number pattern and linear function module.

(Drawn from Kilpatrick et al., 2001; Heibert and Lefevre, 1986; Ball et al., 2001, 2008, 2009; Brodie, 2004a, 2010; Prawat, 1992; Webb and Webb, 2004, 2008; Shulman, 1986; Schoenfeld and Kilpatrick, 2008; Adler and Pillay, 2007)

5.3.1.1 Data collection methods for profiling teachers

Before the start of the second school term in April 2011 the teachers concerned completed a questionnaire aimed at examining their disposition towards mathematics (adapted from the Standards Beliefs Instrument by Zollman and Mason, 1992 – *Appendix 2a*), and a questionnaire developed to examine their disposition towards the teaching and learning of mathematics (adapted from Pehkonen and Törner, 2004 – *Appendix 3a)*. The teachers were also asked to complete a questionnaire in order to obtain background information regarding their qualifications, experience and their views on the type of students that characterised the classes they were teaching. This is presented in *Appendix 4*.

I have chosen this method of data collection in order to gain insight into the opinions teachers have towards mathematics and its teaching and learning. A questionnaire can be considered to be a form of structured interview (Hofste, 2009) where all the participants are asked exactly the same questions and the structure ensures that the answers cannot digress from the concept being interrogated. A Likert rating scale was used for both questionnaires. These rating scales are widely used in research as they provide for flexibility in terms of responses, and yet can be used for statistical analysis (Cohen et al., 2008). The questionnaire on teaching and learning mathematics has 5 options. I felt this was necessary, as not all the statements here are as strong as those in the beliefs questionnaire,

As the intention behind using these questionnaires was to elicit an idea of the teachers' disposition towards mathematics and the teaching and learning of mathematics in terms of mathematical proficiency (as put forward by Kilpatrick et al., 2001); each question was designed to either present a positive or negative valence with respect to ideas espoused by mathematical proficiency. These ideas or themes and their linked questions are outlined in *Table 5.2* and *Table 5.3*.

Table 5.2: The Mathematics Questionnaire, Valence and Mathematical Proficiency basis.

Theme	Questions relating to theme	Valence	Question basis with respect to Mathematical Proficiency
Mathematics is about problem solving.	**Q5.** Mathematics is practical, creative and problem driven. **Q7.** Problem solving is a separate, distinct part of mathematics.	+ -	Problem solving is central to school mathematics and should be the site in which all of the strands of mathematics proficiency converge (pp. 421) and develops flexibility in students (pp. 126).
Mathematics as abstract or concrete.	**Q2.** Mathematics is abstract and often unrelated to the real world. **Q3.** Mathematics must be meaningful if students are to learn to apply concepts productively.	- +	Links among mathematical expressions, concrete problem settings and students' solution methods should be continually made during instruction.(pp. 426)
Mathematics as fixed or dynamic.	**Q1.**Mathematics changes all the time, it is dynamic **Q6.** Mathematics is a stable set of rules, facts and skills that need to be remembered.	+ -	Teachers frequently regard mathematics as a fixed body of facts and procedures to be learned by memorisation, a view that carries over to their teaching, (pp. 373) Techniques learned by rote may provide connections that make it easier to perform mathematical operations, but they may not lead to understanding. These connections do not best promote the acquisition of mathematical proficiency (pp.119)
Justification as central to the development of maths knowledge.	**Q12.** Mathematical knowledge is developed through justification of: ideas; and /or approaches adopted when solving problems.	+	Understanding the nature and role of justification is mathematics as a notion central to developing mathematical knowledge (pp. 373) particularly important for teaching (pp. 371).
Understanding of concepts is of fundamental importance	**Q4.** Mathematics mainly involves the manipulation of expressions and the use of set processes in solving problems. **Q15.** Understanding mathematical concepts is the most important aspect of knowing mathematics.	- +	Teachers must understand the conceptual basis for mathematical knowledge to explain and unpack ideas (pp.371; 375). Emphasising learning with understanding can lead to higher levels of skill, promoting efficient learning (pp.123) and connections between concepts.
Mathematics is about the process not the product.	**Q8.** A demonstration of good reasoning should be regarded as even more important than a students' ability to find correct answers. **Q9.** Doing mathematics is all about getting the right answer.	+ -	Giving a rule simplifies learning and can make getting the right answer the point often with little attention paid to the underlying concepts. (pp.319). Discourse should not be confined to answers only but also include discussion of connections to other problems; representations and solution methods, justification and argumentation, etc (pp. 426)
Communication and language are important for understanding maths.	**Q10.** Skillful use of a natural language is essential when doing mathematics. **Q13.** Mathematics can be thought of as a language that is used to communicate and develop a common understanding.	+ +	A factor in maintaining high levels of student engagement is emphasis on explanation and the development of meaning. The point of classroom discourse is to develop students' understanding of key ideas. Questioning and discussion should elicit students' thinking and solution strategies and build on them, leading to greater clarity and precision. (pp. 336; 345; 426)
Ideas and topics in mathematics are interconnected.	**Q11.** The study of mathematics should include opportunities of using mathematics in other curriculum areas. **Q14.** Mathematics consists of several discrete concepts such as geometry, algebra, measurement which can be best taught in isolation.	+ –	Further, the strands are interwoven across domains of mathematics in such a way that conceptual understanding in one domain, say geometry, supports conceptual understanding in another, say number. (pp. 142, 87)

Table 5.3: ***The Teaching and Learning Mathematics Questionnaire, Valence and Mathematical Proficiency basis:***

Theme	Questions relating to theme	Valence	Question basis with respect to Mathematical Proficiency
Promotion of thinking strategies is important when teaching for maths proficiency	**Q13.** Mathematics is best learnt by using justification and discussions of approaches and concepts used in solving problems in order to develop understanding. **Q21.** Teaching mathematics provides an excellent opportunity to promote the development of students' thinking.	+ +	When given instruction that emphasizes thinking strategies, children are able to develop the strands of proficiency in a unified manner. Questioning and discussion should elicit students' thinking and solution strategies and should build on them, leading to greater clarity and precision. (pp. 7; 426)
Learning strategies	**Q3.** Learners learn best when the teacher lectures to them. **Q10.** Mathematics is best learnt by doing things on your own.	- -	Instruction should not be based on extreme positions that students learn, on one hand, solely by internalizing what a teacher or book says or, on the other hand, solely by inventing mathematics on their own. (pp. 11)
Learning with understanding is the key to developing mathematical proficiency.	**Q1.** Development of concepts is the best teaching strategy to promote understanding in mathematics. **Q2.** Learning mathematics mostly involves memorising procedures and methods in order to master the required skills. **Q8.** While doing mathematics, understanding the topic is a very important idea. **Q17.** Mathematics is best taught by taking students through a process that progressively develops understanding of the concept. **Q20.** Learning mathematics is a process in which students absorb information, storing it in easily retrievable fragments as a result of repeated practice and reinforcement.	+ – + + –	But mathematics learning has often been more a matter of memorizing than of understanding. Today it is vital that young people understand the mathematics they are learning. People need to move fluently back and forth between graphs, tables of data, and formulas. The overriding premise of our work is that students should learn to think mathematically. (pp. 16) Competence in an area of inquiry depends upon knowledge that is not merely stored but represented mentally and organized (connected and structured) in ways that facilitate appropriate retrieval and application. Thus, learning with understanding is more powerful than simply memorizing because the organization improves retention, promotes fluency, and facilitates learning related material. (pp.118)
Teaching for understanding is therefore one of the key roles of the teacher.	**Q7.** Definitions of concepts are the best starting points when learners are introduced to new topics. **Q9.** A mathematical topic is best taught by working though the list of concepts to be covered in the topic. **Q12.** A vital role of the mathematics teacher is to help students develop an understanding of the language and terminology used in mathematics.	- + +	Rather than simply listing problems and exercises, teachers should plan for instruction by focusing on the learning goals for their students, keeping in mind how the goals for each lesson fit with those of past and future lessons. (pp. 425) To represent a problem accurately, students must first understand the situation, including its key features. (pp. 124).Conceptual understanding, therefore, is a wise investment that pays off for students in many ways. (pp. 118-126). Much of class time should be spent in developing mathematical ideas and methods rather than only practicing skills. Students should be given opportunities to use different representations to carry out operations and to understand and explain these operations. Instructional materials should include visual and linguistic support to help students develop this representational ability. (pp. 413)

Development of skills is important in supporting teaching for understanding.	**Q4.** In teaching mathematics, it is important to use varied exercises and applications to ensure sufficient practice of skills.	+	Procedural fluency refers to knowledge of procedures, knowledge of when and how to use them appropriately, and skill in performing them flexibly, accurately, and efficiently. Both accuracy and efficiency can be improved with practice, which can also help students maintain fluency. Practice should be used with feedback to support all strands of mathematical proficiency and not just procedural fluency. In particular, practice on computational procedures should be designed to build on and extend understanding. (pp.121)
Problem solving is a key competency for understanding mathematics.	**Q18.** Competency in mathematics is best tested through the ability to solve problems. **Q19.** Mathematics is best taught by using problem solving to promote understanding.	- +	Problem solving should be the site in which all of the strands of mathematics proficiency converge. It should provide opportunities for students to weave together the strands of proficiency and for teachers to assess students' performance on all of the strands. (pp.421-425) Studies in almost every domain of mathematics have demonstrated that problem solving provides an important context in which students can learn about number and other mathematical topics.(pp.420) Problem solving also provides opportunities for teachers to assess students' performance on all of the strands. (pp.421)
Successful teachers have high expectations of all students.	**Q11.** Mathematics is a subject that is best taught to talented pupils. **Q16.** A major goal of mathematics instruction is to help children develop the belief that they can be successful in using mathematics.	- +	Becoming mathematically proficient is necessary and appropriate for all students. Surprisingly, it is students who have historically been less successful in school who have the most potential to benefit from instruction designed to achieve proficiency (pp. 142). Students' disposition toward mathematics is a major factor in determining their educational success.(pp. 131).Successful teachers not only expect their students to succeed but also see themselves as capable of motivating and instructing students effectively.(pp. 338)
Teaching for proficiency requires use of varied resources and approaches	**Q15.** The way the textbook presents a topic provides the best option for teaching the concepts making up the topic. **Q6.** Following logic is promoted in teaching mathematics, whereas creativity and originality are not stressed.	-	Teachers need to muster and deploy a wide range of resources and approaches to support the acquisition of mathematical proficiency (pp. 370)
Knowledge of students	**Q14.** Knowing your students and how they think is very important for teaching.	+	Professional development programmes focusing on helping teachers understand both the mathematics of specific content domains and students' mathematical thinking in that domain have consistently been found to contribute to major changes in teachers' instructional practices that have resulted in significant gains in students' achievement (pp. 391)
Linking ideas is important.	**Q5.** Mathematics **in school** necessarily requires a concrete dimension; abstract mathematics alone is not enough.	+ -	Links among written and oral mathematical expressions, concrete problem settings, and students' solution methods should be continually and explicitly made during school mathematics instruction.(pp. 426)

There were two main problems to be considered when using and interpreting these questionnaires:

- Responses might be framed so as to "tell me what I want to hear", not what they actually think.
- As the questionnaire on teaching and learning mathematics has 5 options there is a tendency for people to answer questions by giving the easy 'middle" answer and to avoid committing themselves.

In order to avoid this problem, I administered all the profiling questionnaires myself. This provided me with the opportunity to clarify any questions if necessary.

Due to the internal organisation of the school, the classes in Grade 9 were streamed according to English language ability and (according to school based assessments) we found that language streaming generally reflected mathematical ability. We therefore ended up with classes grouped according to mathematical ability. This was the main characteristic that teachers used when profiling their classes. The difference in the classes taught provided another interesting way of analysing different teaching approaches.

5.3.2 Teaching with the module

Despite the time and attention that was (necessarily) devoted to the development of the teaching material in the module on Grade 9 number patterns and linear functions, the implementation of the teaching module is the main focus of this research. How does an intervention such as this influence teaching practice, and what teaching practice factors in turn influence the way in which implementation occurs? I was interested to know how teachers found teaching with the module in terms of their enjoyment of teaching this topic; how they felt their students responded to the teaching module; how their understanding compared to that of the previous year; whether our effort to link mathematical concepts was successful and what factors influenced the teaching of this section and how.

We started teaching the module a week after the beginning of the second term in April 2011 and spent the rest of the term until the end of May working with the concepts covered by the module. The following activities were planned in order to collect data on the implementation of the module during the teaching phase:

5.3.2.1 Data collection methods

- I made time for "informal' discussions with all the Grade 9 teachers during the term to track the progress of the execution of the module. Notes were kept, and when it was possible to tape these discussions they were transcribed for analysis.
- To provide information on the activities chosen and whether they worked and why they were chosen, I asked each teacher to collect copies of these in a flip file with notes explaining their use and efficacy. I also asked them to provide copies of any extra worksheets they felt were needed with an explanation as to why they felt they were important and useful (to be used for the review and feedback phase)
- In order to get a sample record of the lessons, I organised to video at least eight Grade 9 lessons during the duration of teaching the module. These lessons were analysed for teaching for mathematical proficiency using the observation schedule given in *Appendix 5*.
- The researcher kept field notes regarding the experience of designing the module, teaching using this material, and for recording other issues that came out of the informal discussions with teachers.

The use of participant observation, interviews and a research diary (which is a form of field notes) are recognised methods supporting PAR (McKernan, 2008). Observations are very useful for gaining insight into situations, but care needs to be taken as their data can be subjective (Cohen et al., 2008). The authors warn that it is necessary to maintain consistency to ensure reliability, and standardising a method of eliciting data from the observations as well as writing up notes as soon as possible is helpful. The same observation schedule was used for all the lessons and was structured in order to prevent variation in interpretation. All interviews at this stage were informal conversations to allow the teacher's thoughts and opinions to be expressed as they arose in the course of a discussion. This enabled the interview to be matched to the circumstances and the individual (Cohen et al., 2008). The field notes used to record ideas and impressions arising from these conversations were either descriptive or quick notes with key words. Using a research journal or field notes helps increase the validity of judgements made as ideas and impressions are recorded soon after the experience that engendered them (McCrindle and Christensen, 1995).

All of the Grade 9 students were required to write an externally set examination mid-year. This was scheduled for the 27th of May. In addition to this examination, the concepts covered while teaching the module were evaluated in an internal assessment written during the school examination period.

5.4 Phase 3: Review and feedback – evaluation of implementation

This phase proved to elicit the most important information– i.e. data that would provide a means of evaluating this project in terms of the following questions:

- Does the module promote teaching towards mathematical proficiency in the concepts it covered?
- What were the teachers' views regarding teaching with the module, and approaching the teaching of concepts in this way?
- Could this approach to teaching be used again, and what would be necessary in order to do so?
- Were the assumptions made valid or not and why?

5.4.1 Collection of data to support evaluation

In order to fully explore these questions it was necessary to collect additional information and to provide a review of the experience of the teaching after the work and assessment period for the term was complete. The following methods were used to provide additional support material for the investigation of these issues.

- Individual "post-mortem" semi-structured interviews were scheduled with each of the Grade 9 teachers who took part in this investigation. The questions, as outlined in *Appendix 6*, were used as a basis for directing this discussion.
- The personal experience of the researcher was analysed using the same questions that were used to direct the interviews.
- Collation of documentation such as the Eastern Cape Education Department pace-setter; the curricula (current and future) for Grade 9; external and internal assessments and the school teaching timetable.
- A semi-structured interview was scheduled with the Head of Department of mathematics who had not taught the module or Grade 9 in the current year and was therefore independent from the investigation. These questions can be found in *Appendix 7*.

Cohen et al., (2008, p. 353) consider semi-structured interviews to be a "guided approach" to interviewing where topics and issues to be covered are decided upon in advance and used to develop an outline for the interview that is used for each person. Often the interviewer decides upon the sequencing to allow for interesting issues to be explored. As these authors note, care needs to be taken in this type of interview not to miss or omit important topics and to keep the wording of the questions as standard as possible to ensure comparability.

5.4.2 How this data will be used

5.4.2.1 Use of data to examine promotion of teaching towards mathematical proficiency and the applicability of the approach to teaching in other sections

The idea of the post-interviews with the teachers concerned was to establish their views on their teaching experience and their opinion of the module and whether they felt this approach would be useful in teaching other sections of mathematics. The questions in these interviews were planned to provide a framework in which to structure the interviews, and draw out information on the key aspects of the research.

5.4.2.2 Use of data to test assumptions

Some of the assumptions made in the development and the teaching phases of the module which are further discussed in section 5.7, are briefly highlighted here in terms of how the data collected will be used to examine their validity.

(i) Sufficient hours of teaching time are available:

In order to look at the available teaching time for this module, I used the school timetable, the term calendar of teaching days available and analysed the time made available to teach this section. I also made videos of the teaching lessons and analysed them in terms of time on task and time management of the lessons. This enabled me to get an idea of how much time is actually available to the teaching and learning of concepts once students and teachers are in the classroom. It also allowed me to decide whether the module was too dense for the time available and to factor in that the classes and the teachers had differing ability levels and skills.

(i) Practicality of the approach:

In order to answer this question I used the informal discussions to look at how the activities in the module were able to be used. Additional informal interviews with the teachers were necessary in order to clarify points of interest. The personal analysis of the researcher was used to provide further depth to this discussion.

(ii) Sufficient coverage of the concept but also allowing for extension.

The post-interviews were used to determine whether the module allowed for teaching towards mathematical proficiency with students with a range of ability.

(iii) The NCS was the best basis for the foundation of the module:

The structure of the curriculum, the types of skills it sets out as desirable regarding both its objectives and the detail of the mathematical topics required to be taught, were compared and examined in terms of the way they align with the idea of teaching for mathematical proficiency.

(iv) Teachers were sufficiently skilled and interested:

Pre-module questionnaires and post-module interviews were used to probe for interest in this approach to teaching number patterns and linear functions.

(v) Enabling environment

In order to create a picture of the enabling environment, the pace-setter was analysed in terms of its help in promoting teaching in terms of mathematical proficiency.

The interview with the HOD formed the main data set for examining assumptions (iii); (iv) and (v).

5.5 Phase 4: Analyse the data, document and communicate the results.

In this phase, the data that was collected needed to be translated into useful information. The analysis and presentation of the results is presented in Chapter 6 and discussed in Chapter 7 of this thesis.

5.6 Phase 5: Adapt and learn – refinements to accommodate findings

This phase was the culmination of the work done in the intervention strategy. In terms of the sustainability of a model it was also the most important segment. The results were used to provide information from which lessons can be learnt and adaptations made to the strategy implemented. This allowed for the systematic use of information to make decisions regarding actions that will improve the project (Salafsky, Margoluis andRedford, 2001). It was difficult to do as it entailed embracing new strategies and methods. However, if this teaching strategy is to be adopted beyond the life of the thesis, it is vital to ascertain whether it is possible to implement adaptive management with the teachers involved in this project. I posed questions to this effect in the post-interviews, and the answers were analysed in terms of a willingness to do so, and possible actions that might be taken. Thereafter, the module will be adapted and these and other management changes will be documented for use next time this section needs to be taught. However, these recommended changes and the implementation of this phase will not be reported upon as this phase falls outside the scope of this thesis.

5.7 Assumptions and limitations

5.7.1 Assumptions

In putting together the teaching module for Grade 9 which needed to: (i) cover the concepts as outlined in the focus group meeting in November 2010 and by the NCS and (ii) to attempt structuring the module to encourage the promotion of the development of mathematical proficiency when teaching; I needed to make a number of assumptions. Some of these have been discussed (5.4.2.2) in terms of how they will be assessed. They are briefly reiterated as:

- That there would be sufficient hours of teaching time available to us in order to cover the concept in a way that promoted teaching for mathematical proficiency.
- The practicality of the approach taken by the module in terms of the way the activities were able to be used effectively in the classroom.
- That extent to which the module explored the concept of number patterns and linear functions and the links between them and other topics of mathematics was sufficiently clear but also allowed for the extension of higher ability students.

- That the outcome required by the NCS was the best basis for the concepts the module needed to cover.
- That each teacher was sufficiently skilled and interested to teach these concepts using this approach.
- That this module would be taught in a situation that was characterised by an effective enabling environment both at the school and the Departmental level.

5.7.2 Limitations

In a research endeavour such as this, limitation inevitably occur which inhibit generalisation of the findings. Some of these were;

- Research only took place in one school with four teachers.
- One of the teachers was the researcher – vested interest.
- The module could have covered and linked in more topics
- Having noted in the assumptions characterising the research, that the NCS curriculum was a good basis for the foundation of the module, the curriculum also proved to be a limiting factor on the development of the module.
    - Procedural skills are the main focus of any external assessment. In order for teachers to buy into teaching using the module, it was necessary to ensure that, although mathematical proficiency was the aim when developing the concepts in the module, the teachers would also be confident that the module would allow them to teach the skills needed for this assessment.
    - The breadth of the curriculum meant that it was not possible to construct the module so that it explored the topic to the extent that many educational theories into teaching mathematical concepts promoted. For example, the point of understanding conceptualised by Sfard's (2001) term reification, required a depth of understanding that we were unlikely to reach in the time available as many other topics still needed to be covered.

- The timeframe of the research meant that while improvements and adjustments to the module were suggested and developed for use in the following year, I was unable to report on them in this study.

CHAPTER 6: PRESENTATION AND ANALYSIS OF RESULTS: THE INFLUENCE OF TEACHING PRACTICE

6.1 Introduction

The results are documented, communicated and analysed in terms of the Mathematical Proficiency theory as presented in the work by Kilpatrick et al. (2001). The outcome of this analysis is then discussed in Chapter 7. These results are presented according to the phases in which each action providing this data occurred. This chapter represents the fourth phase in the adaptive management cycle forming the structure of this research project. These phases were the planning phase, the implementation phase and the review and feedback phase which are briefly summarised below:

6.1.1 *Phase 1: Planning phase*

This phase was the introductory phase in which previous teaching practice (with respect to linear functions in Grade 9) was reviewed and a structure for the proposed teaching strategy was worked out. Subsequently the teaching module was developed. This phase was presented and discussed in Chapters 4 and 5.

6.1.2 *Phase 2: Implementation phase*

Before the teaching module was implemented, the teachers who would be involved in the research were profiled to develop a picture of their views on the teaching and learning of mathematics; their academic and professional backgrounds and the classes they were teaching. Once this had been compiled; the teachers taught the linear function topic using the resources and structure provided in the module. The purpose of this phase was to attempt to ascertain how such an intervention might influence teaching practice and what factors in turn would influence the way in which such an implementation occurred.

6.1.3 *Phase 3: Review and feedback phase*

This phase looked at the teaching of this topic from the teachers' point of view regarding teaching with the module, and approaching the teaching of concepts in this way. Their responses were interrogated in terms of whether the module promoted teaching for mathematical proficiency in the concepts it covered. It was also used to determine whether this approach to teaching could be used again and examined the validity of the assumptions made when developing the module. This outcome is important to consider when making the modifications necessary to ensure the continued use of the module as a teaching tool.

6.1.4 *Phase 4: Analyse the data, document and communicate the results*

This chapter represents phase 4.

6.1.5 *Phase 5: Adapt and learn – refinements to accommodate findings*

This phase is beyond the scope of this thesis, but is scheduled for November/ December when, as a mathematics department we will meet to plan for the next teaching year. Potential changes to the module were already touched upon in the post-module interviews and these, together with points raised at the end of the year will be taken into consideration when refining the module. It is the intention of the teachers involved in this research to use the module again.

6.2 Phase 2 Implementation: Introducing the teachers

6.2.1 Qualifications, academic background and classes

Because the views a teacher might have of mathematics and how to best teach mathematics are considered, to be influenced by both their qualifications and experience (as discussed in Chapter 2) this information (summarized in *Table 6.1*) will need to be taken into consideration. The table also includes information on the classes the teachers currently teach in Grade 9. This information is valuable, as the third factor influencing teaching (apart from knowledge of mathematics and teaching strategies) is knowledge of students (Kilpatrick, 2001). This is more than just general knowledge regarding the nature of learning. It includes specific knowledge of the students in their classrooms. It is hoped that this information together with the questionnaire results and the brief glimpse into teaching practice afforded by the videoed lessons will provide insight that will facilitate analysis of this teaching practice in terms of the mathematical proficiency theory.

Table 6.1: Background information: Qualifications, experience, Grade 9 classes:

Teacher	Qualifications	Teaching experience	View of Grade 9 classes (all girls)
1 2 classes	4 year teaching degree -mathematics II -mathematics as a teaching subject.	20 years (up to Grade 12 pure maths)	**1**: Intelligent, enquiring and challenging. Don't accept facts easily, like to prove these. Question until fully confident with a concept. Need extension work with more abstract and difficult examples. Achieve 80% on average. **2**: Diverse in terms of ability and character. 10%-70% result range. Have to be schooled through concepts and then drilled until confident. Panic when don't understand. Frightened of thinking out the box. Need memory "hooks".
2 1 class	MSc; PGCSE - Mathematics III -mathematics as teaching subject	7 years (Up to Grade 10 pure maths; 11 and 12 Maths literacy)	Mixed ability class, 52% - 96% result range. Able to do extension work, generally get through work well and understand concepts. Their class teacher as well – good connection.
3 2 classes	4 year teaching diploma -mathematics as a teaching subject.	13 years (up to Grade 12 pure maths, only Standard Grade before 2008)	**1:** Stronger class; more "accepting" of teaching – easier to teach. Can extend, get focused and try different approaches with. **2**: mixed class, 19%-80% result range. More difficult to teach, cannot get through as much work, don't focus as well, more conservative in approach, like work in "blocks".
4	BSc, BComm (in progress); HDE; BEd. Mathematics II Maths as teaching subject.	20 years (pure maths up to Grade 12)	"Bottom set" according to English language stream which has coincided with the same for mathematics. 16%-60% results range, at least 5/22 < 40% Poor basic maths skills, experience difficulty with new concepts, struggle to integrate new knowledge, to use 'old' skills in 'new' concepts. 1 student diagnosed "dyscalculia" another possible.

Notes:

- Teacher 1 has two classes whom she considers to have very different mathematical abilities, and one would expect that her approach to teaching mathematics might change due to the way she views her students.
- Teacher 2 has the highest academic qualification in terms of pure mathematics. With only one Grade 9 class, with whom she considers she has a good connection and who are generally strong mathematically, one would expect that mathematical concepts could be explored in depth.

- Teacher 3 has two mixed ability grouped classes, but the one has less of a range than the other. This teacher is the only one to have a diploma qualification where mathematics was merely a teaching subject. One would expect that she would use the most traditional teaching methods.
- Teacher 4 has one class whom she considers to be the weakest of all the six mathematics classes in this grade. This expectation of her students' ability should influence her teaching methods for this module.

6.2.2 Disposition towards mathematics and the teaching and learning thereof

Both the questionnaires (given in *Appendix 2a and 3a*) investigating these views held by the teachers used a Likert scale similar to that employed by Zollman and Mason (1992) and Pehkonen and Törner (2004). The mathematics questionnaire made use of a four point scale and the teaching and learning mathematics questionnaire a 5 point scale. Because the data from such a scale is clearly ordinal (Kislenko and Grevholm, 2008; Keller, 2005) it is unsuitable for any statistical calculations but those involving ranked order or frequency. Also, due to the size of the sample (4 teachers) measurement descriptions obtained from such a statistical analysis would not necessarily be helpful. I therefore have chosen to analyze each teacher individually (questionnaires responses given in *Appendix 2b* and *3b*) using the following classification.

Strongly aligns with mathematical proficiency principles	
Aligns with mathematical proficiency principles.	
Does not necessarily align with mathematical proficiency principles	
Disagrees with mathematical proficiency principles.	
Not considered by teachers in terms of value for effective teaching	

This categorization of the responses allowed me to gain a quick overall view of how teacher's opinions regarding mathematics and the teaching of mathematics aligned themselves with the principles put forward by the idea of mathematical proficiency. This analysis is presented in *Tables 6.2* and *6.3*. Generally all the teachers' views seemed to indicate that they espoused many of these principles. In terms of mathematics there were only a few points of disagreement indicating a possible conflict of opinion concerning this theme. However, it is interesting to note that beliefs regarding the teaching of mathematics reflected more points of conflict than shown by the interrogation into their beliefs of mathematics as a discipline.

Table 6.2: Results of mathematics questionnaire presented in terms of teachers' alignment with mathematical proficiency principles.

Theme	Related Question		Teacher 1	Teacher 2	Teacher 3	Teacher 4
Mathematics is about problem solving.	Q5.	Mathematics is practical, creative and problem driven				
	Q7.	Problem solving is a separate, distinct part of mathematics.				
Mathematics as abstract or concrete.	Q2.	Mathematics is abstract and often unrelated to the real world.				
	Q3.	Mathematics must be meaningful if students are to learn to apply concepts productively				
Mathematics as fixed or dynamic.	Q1.	Mathematics changes all the time, it is dynamic				
	Q6.	Mathematics is a stable set of rules, facts and skills that need to be remembered.				
Justification as central to the development of maths knowledge.	Q12.	Mathematical knowledge is developed through justification of: ideas; and /or approaches adopted when solving problems.				
Understanding of concepts is of fundamental importance	Q4.	Mathematics mainly involves the manipulation of expressions and the use of set processes in solving problems.				
	Q15.	Understanding mathematical concepts is the most important aspect of knowing mathematics.				
Mathematics is about the process not the product.	Q8.	A demonstration of good reasoning should be regarded as even more important than a students' ability to find correct answers.				
	Q9.	Doing mathematics is all about getting the right answer.				
Communication and language are important for understanding maths.	Q10.	Skilful use of a natural language is essential when doing mathematics.				
	Q13.	Mathematics can be thought of as a language that is used to communicate and develop a common understanding				
Ideas and topics in mathematics are interconnected.	Q11.	The study of mathematics should include opportunities of using mathematics in other curriculum areas.				
	Q14.	Mathematics consists of several discrete concepts such as geometry, algebra, measurement which can be best taught in isolation				

Table 6.3: Results of mathematics teaching and learning questionnaire in terms of teachers' alignment with mathematical proficiency principles.

Theme	Related Question		Teacher 1	Teacher 2	Teacher 3	Teacher 4
Promotion of thinking strategies is important when teaching for maths proficiency	Q13.	Mathematics is best learnt by using justification and discussions of approaches and concepts used in solving problems in order to develop understanding.				
	Q21.	Teaching mathematics provides an excellent opportunity to promote the development of students' thinking.				
Learning strategies	Q3.	Learners learn best when the teacher lectures to them.				
	Q10.	Mathematics is best learnt by doing things on your own.				
Learning with understanding is the key to developing mathematical proficiency.	Q1.	Development of concepts is the best teaching strategy to promote understanding in mathematics				
	Q2.	Learning mathematics mostly involves memorising procedures and methods in order to master the required skills.				
	Q8.	While doing mathematics, understanding the topic is a very important idea.				
	Q17.	Mathematics is best taught by taking students through a process that progressively develops understanding of the concept.				
	Q20.	Learning mathematics is a process in which students absorb information, storing it in easily retrievable fragments as a result of repeated practice and reinforcement.				
Teaching for understanding is therefore one of the key roles of the teacher.	Q7.	Definitions of concepts are the best starting points when learners are introduced to new topics.				
	Q9.	A mathematical topic is best taught by working though the list of concepts to be covered in the topic.				
	Q12	A vital role of the mathematics teacher is to help students develop an understanding of the language and terminology used in mathematics.				
Development of skills is important in supporting teaching for understanding.	Q4.	In teaching mathematics, it is important to use varied exercises and applications to ensure sufficient practice of skills.				
Problem solving is a key competency for understanding	Q18.	Competency in mathematics is best tested through the ability to solve problems.				

mathematics.	**Q19.**	Mathematics is best taught by using problem solving to promote understanding.				
Successful teachers have high expectations of all students.	**Q11.**	Mathematics is a subject that is best taught to talented pupils.				
	Q16.	A major goal of mathematics instruction is to help children develop the belief that they can be successful in using mathematics				
Teaching for proficiency requires use of varied resources and approaches	**Q15.**	The way the textbook presents a topic provides the best option for teaching the concepts making up the topic.				
	Q6.	Following logic is promoted in teaching mathematics, whereas creativity and originality are not stressed.				
Knowledge of students	**Q14.**	Knowing your students and how they think is very important for teaching.				
Linking ideas is important.	**Q5.**	Mathematics **in school** necessarily requires a concrete dimension; abstract mathematics alone is not enough.				

That mathematics is rather abstract and unrelated to the real world is a common theme that seems to run through all the teachers' responses and can easily be linked to their experiences of school mathematics and the way it has been taught and the way they have learnt to teach it. This view was possibly shaped by the way in which they were taught, and does not seem to have been changed by their training.

Generally, the teachers' views of teaching mathematics seem to mirror many of those espoused by the idea of mathematical proficiency. This indicates that their teaching practice could be expected to support these ideals. However, in the areas discussed below, these opinions do not consistently endorse the ideals of mathematical proficiency.

Teacher 1 does not have a very fluid notion of mathematics, considering it to be a subject populated by discrete concepts that each has their own processes and rules and facts that need to be remembered. This view influences her views on teaching and learning mathematics as she considers that one teaches concepts by starting with the definition and then practices the processes until they are remembered and understood. Although she considers problem-solving to be useful in promoting understanding, I question (in the light of her responses) whether what she considers problems are closer to routine procedures and set questions that are typical of the assessment questions in school examinations. Since she thinks that mathematics consists of several discrete concepts involving the manipulation of expressions and use of set processes, one would expect that problem-solving would be viewed as another method to practice applying skills learnt. She also considers that when learning mathematics it is most important to first remember the rules, facts and skills that govern what she surprisingly considers to be a dynamic discipline. Which view influences this teacher's teaching the most, and for which class? The view that learning mathematics is a result of repeated practice and reinforcement corresponds to the teacher's understanding of mathematics - as indicated by her responses in the questionnaires.

Teacher 2 indicated that mathematical concepts are interconnected. Nevertheless, her view of these mathematical concepts as abstract, often unrelated to the real world and consisting mainly of expression manipulation, would influence her views on how mathematics should be taught and learnt more than her consideration of mathematics as a holistic concept. Definitions of concepts are viewed as the best starting points to developing understanding, a view that concurs with her opinion that mathematics is more abstract than concrete. This, as well as her feeling that mathematics is best taught by following a "to do" list that needs to be learnt through practice, would not promote five strand teaching. Yet she does indicate that if students are to learn to apply concepts productively the problems posed must be meaningful. The responses to the teaching and learning of mathematics questionnaire seem to indicate that mathematics is abstract and this view has the most influence on the teaching and learning of mathematics. Teachers believe mathematics is best taught by ensuring that all the concepts listed have been covered, practiced and tested, - a very traditional approach.

Teacher 3 is not really sure whether she can characterize mathematics in terms of either one view or another. Her experience of mathematics is that it changes all the time and yet once an understanding has been clarified it becomes a stable collection of facts and set processes that need to be remembered. That this might influence her teaching practice can be seen by her consideration that teaching is best done by working through a list of concepts given in the form of a lecture. She believes that this is how students learn best – absorbing and practicing information given to them by the teacher. Teacher 3 therefore presents the most traditional view of mathematics and teaching, as she considers that mathematics facts can be learnt best through judicious practice. Although understanding concepts is considered very important and should be developed progressively, the view of "concept" held by this teacher is as a process that is used in set methods to solve problems. Students need as much support as possible, but learn best when they are lectured to by the teacher.

Teacher 4's views on teaching and learning mathematics seem to be very strongly aligned with those of mathematical proficiency with no areas of conflict. However, the fact that she considers mathematics as an abstract and stable set of rules, facts and skills to be remembered seem to contradict this and may be influential when it comes to actual teaching practice. Memory requires practice and this is what may have affected her views on teaching and learning. Is it possible that the "right" answers were chosen not her actual opinions? She also indicates that mathematics consists of a stable set of rules, facts and skills that need to be remembered and practiced so they can be used fluently when required. As her students are the weakest class, one would expect that her teaching to be more traditional and that the students receive help with cognitively difficult problems. She is unlikely to consider that they have the ability to focus for long periods when they struggle with a concept.

6.3 Phase 2 Implementation: Teaching with the module

6.3.1 A glimpse into the classrooms of the four teachers - analysis of videoed lessons

6.3.1.1 Teacher 1

One of the notable features of the teacher's lessons was the willingness of her students to apply themselves to their work in order to understand the mathematics. The students were comfortable asking many questions and were used to being asked questions in turn by the teacher. Both classes were good at raising issues that needed clarification, but the top class asked questions that explored the depth of the topic. The teacher mostly directed questions to the group and not to specific students. Although the teacher was open to discussing student questions, which is an indication that the teacher is comfortable with the subject, she did not use them to generate whole group discussion. This forms a one on one question and answer discussion. An illustration of this is the way in which a student question regarding gradient was handled. The concept of gradient had started out as $\frac{rise}{run}$. When reading off the gradient, the class was taught to "always go up first". This was challenged by a student who steered the conversation into why there was automatically a negative value from the $\frac{\Delta y}{\Delta x}$ method using two points, but when you used the graph you had to put your own sign in by looking at the direction. In answering the question, the teacher focussed on the student in question and did not engage the whole class to generate the explanation, which would have been a constructive direction for a discussion.
However, concepts were discussed and queried in the lessons, although in the stronger class she would sometimes wait for the answer, but in the weaker class she mostly provided the answer to the questions she asked.

Another feature of the way in which this teacher used questions was the manner in which they were directed to a specific outcome. The discussions generally were either initiated or driven by the teacher with a specific goal in mind, or were initiated by the students in order to clarify their understanding. Questions were tightly controlled during the lesson – the teacher never lost sight of the goal of the lesson. This in part was due to time constraints imposed by the demands of the curriculum and external exam. She did miss a few opportunities to divert the lesson in a way that would not have gone off topic but might have provided greater insight for the students about the concepts.

Many formal notes were given at intervals during the lesson. These resulted from observations drawn from students via the questions that had been presented by the teacher. The teacher viewed mathematics as a stable set of rules, facts and skills to be remembered which may have influenced the number of notes that were given in the lessons. Worksheets and activities in the module were used as tools to promote the "discovery" of concepts and promote understanding. But many notes were used to provide a safety net for students, and homework was always given as she believes learning mathematics is a result of repeated practice and reinforcement.

It was interesting to note that this teacher was not consistent in the use of mathematics as the authority for explaining a concept. For example, in one of the lessons, the aim was to learn to draw using gradient and intercept. The first graphs used as examples were positive gradient and positive x-and y-intercepts or negative gradient and positive y-intercept but negative x-intercept and students were starting to see the gradient as related to the intercepts in terms of multiplication of integers. $+ \times - = -$ and $+ \times + = +$. The teacher promptly gave a graph with negative gradient and negative intercept to prevent this hypothesis from developing. However, when students queried the reason for the gradients of horizontal and vertical lines being zero or undefined respectively, the teacher used an example of a person walking on a flat surface or trying to walk on a vertical surface to indicate that it "just was" the answer.

The teacher concerned changed her teaching strategies depending on her expectations of her classes and the way in which the students fed back their understanding. She moved very quickly through concepts with the stronger class. The linking of ideas to each other was less obvious with the stronger class and if the class themselves had not pursued ideas that linked concepts together; they would not necessarily have been actively linked by the teacher. It was also observed that the material that was provided by the module created an order of teaching that promoted the linking of ideas. All observed lessons (either actively using material in the lesson or based upon material used as a homework activity) were structured to flow so that information was not discrete. Although the links were not always actively highlighted by the teacher, she always presented a lesson structure that promoted this linking. However, a great deal more support was provided for the weaker class in linking concepts together and the teacher made more use of the module in the way it was intended to be used for this class. Sometimes notes were given on "how to do", or "what is" before students had to grapple with a concept – so material used from the module was made cognitively easier a bit too soon. The teacher felt that the weaker class needed to be instructed through concepts and then drilled until they felt confident to deal with the concept. Her experience was that they panicked when they lacked understanding and they were frightened of thinking out the box. This class in her opinion needed memory "hooks".

The final lessons were tightly packed and provided little time for consolidation. The teacher felt under huge time pressure due to loss of teaching time during this term. She spent some time specifically preparing students for the exam on work that was not scheduled for the term but was nonetheless expected to be in the exam. Her teaching approach changed from student centred to teacher driven and opportunities to teach related concepts were missed. For example, she missed an opportunity to discuss a solving equation problem requiring algebraic manipulation. When the equation is not in standard form the students wanted to use the constant as the y-intercept. An example was given by the teacher but not worked with – "*it won't be in exam we will leave it till later*" was the comment.

Other observations from this classroom

- The teacher did not always use the module material in the lesson, although it was always used for homework in order to provide a foundation for the lesson ahead. This was (in the last lessons) due to time pressure and the need to cover the concepts for the exam – here the teaching was more traditional. Show and tell and use of notes was the main strategy. It was noted that the lesson in which the most use of the module material was observed was also the lesson that generated the most questions by the students.
- Confusion over gradient as a fraction, especially when written as a whole number – not seeing it as a movement between two points - another example was given and the steps repeated. The teacher used the word movement and got students to move and count to help this understanding. But the link to number pattern differences was not made.
- The teacher did not write x-and y-intercepts as co-ordinates (as per module). She taught the dual intercept method without the module due to time and exam constraints. This could set up a potential problem as a common error concerns students combining these into one point and using this point and the origin to draw the graph.

- Although no clearly identifiable introduction to the lessons were observed, this could be due to the fact that none of the observed lessons introduced a new concept or idea, but were continuations of a previous lesson. The module itself was designed to flow - linking concepts rather than presenting them as discrete sections. Therefore the fact that the homework was used as a starting point was not an indication of lack of clarity for the lesson. The teachers' understanding of the mathematics could rather be judged from the way in which the lesson progressed.

Table 6.4: Teacher 1: Observations – teaching for mathematical proficiency

Teaching activity	**Observation in terms of Maths Proficiency**	**Notes from observation of lessons.**
Presentation of concepts to be dealt with in the lesson	There is a clear introduction at the start of the lesson.	There was generally no separate introduction in the observed lessons – most of these used the previous day's homework activity on which to base the current lesson and this was then the introduction for the lesson.
	This introduction shows the teacher understands the mathematical concept that is the focus of the lesson.	
	The teacher finds out what the students already understand about the concept before sharing their own understandings of those concepts.	This understanding was taken for granted when teaching the "top set", homework was checked and the teacher moved on. However, this was rigorously done for the weaker set taught by this teacher.
	The teacher takes this understanding shown by the students into account, using it as a foundation for the further teaching of the concept.	
	The lesson *structure* encourages students to link new information and previously taught concepts to each other	Yes, all the observed lessons were structured to be able to link new information to previous ideas.
	The teacher is happy to use and discuss different ways of doing procedures and solving problems.	No. In general in the observed lesson one method was taught and adhered to and no others discussed.
	Makes references to other sections of mathematics where appropriate to promote connections between topics.	Not in the observed lessons although the mathematics provided opportunities to do so.
	Nurtures students' natural curiosity.	This was not actively promoted by the teacher's questions, but the use of the activities was designed to stimulate this and the classes were very comfortable and encouraged to ask many questions – which were answered.
	The teacher balances the use of activities that either encourage independent knowledge construction or encourage group construction of knowledge.	All the lessons observed used individual (generally silent) and whole group activities. Many notes were given on the board to copy down.
	Uses activities or problems that force students to question their methods or the "rules" they have developed to solve problems so as to promote a deeper understanding of the mathematics.	This was generally observed with the top set, but often opportunities to do so were missed. Student questions and responses indicated a greater need for this tactic.
	Lesson content and tasks are relevant and realistic to the topic.	Yes, although in two of the lessons the module activities were not used.
	Facilitates scaffolding to help students perform	This was apparently not needed for the top

	beyond the limits of their ability.	set, but was provided for the weaker student group, although the teacher showed a tendency to do the work for the students.
	Emphasises problem solving, higher-order thinking and works towards developing deeper understanding of concepts.	Yes, the teacher wanted to promote this. However, time constraints prevented full development.
Management of class discussion.	Encourages and accepts student autonomy and initiative.	This was allowed within a very tight focus. Students could only direct lesson within close boundaries – purpose of lesson most NB
	Allows student responses to drive lessons, shift instructions strategies and alter content.	
	Responds to student contributions in a way that encourages students to engage in dialogue, both with each other and the teacher (whole group discussion)	No. There was no group discussion in which mathematical knowledge was generated and ideas talked about. Conversations were teacher – student or student-teacher directed.
	Allows time for social interaction and knowledge sharing in a way that promotes the purpose of the lesson (small group discussion)	This was only observed in the weaker student group. The stronger set generally only worked in silence. Class set-up did not encourage.
Use and handling of questions	Avoids *too many* closed or narrow factual questions.	Most questions were very focused and directed to a specific answer.
	Encourages student enquiry by asking thoughtful, open-ended questions.	Very few exploratory type questions were observed in these lessons.
	Encourages students to ask questions of each other.	No.
	Asks students to elaborate on initial responses.	Yes, students were asked to do so.
	Avoids answering the questions for the students.	No. Most of the time the teacher answered the question. Most questions were whole-group directed.
	Allows for "wait time" after posing questions.	The teacher does do so, but not often enough.
	Questioning strategies encourage reasoning and justification by students.	Generally, when the student does contribute to the lesson (without being prompted) they are asked to justify their thinking.
Use of teaching strategies that encourage the development of a community of learners.	Provides opportunities for peer-review and self-evaluation.	Not evident in these lessons.
	Moves through the room interacting with students.	Yes. Manages to get to every student in majority of lessons.
	Students' errors are taken into consideration and are used to gain insight into students' previous knowledge constructions.	No, little time is spent finding the underlying problems. Errors are corrected and the right method reiterated. Time constraint noted.
	Emphasises knowledge construction as opposed to reproduction.	The teacher has planned to do so, and tries to do so, but is hindered by time constraints.
	Uses manipulative, interactive and physical materials appropriate to the lesson.	Yes. Physically drawing graphs, measuring gradients, plotting points.
	Uses and encourages the use of language and terminology appropriate to the concept being taught.	Yes. Does not water down the language. Insists on the correct terminology and presentation.
Finishing off of lesson.	Pulls concepts together to conclude lesson.	Most lessons ended in a rush and so no conclusions were observed.
	Homework set consolidates and practices skills taught during lesson.	Yes, until last lesson before exam period – new work still being taught, but no time to consolidate.

6.3.1.2 Teacher 2

The module activities were used differently by this teacher, in that the concepts were taught first, notes made and the worksheets used as homework to consolidate the teaching done. This teacher does believe that mathematics is often best taught by ensuring that all the concepts listed are covered, practiced and tested, a very traditional approach. The order of the module was used by the teacher to structure the teaching of this topic. Worksheets were often used as homework to consolidate previously taught concepts but it was not always clear if a worksheet had a preplanned use. In one instance this was disappointing to note, as the activity in question was designed to lead into a discussion of the concept of gradient. This was to familiarize students with the idea before it was applied to the algebraic and more formal linear function concept.

The most striking observation was the way the teacher stayed in the front of the class, she did not move among the students to check work, chat, or answer questions, even when individual work was being done. There was also little class discussion initiated or sustained. Even when basic questions directed to the teacher were answered, and they prompted further questions from students, this never resulted in class discussion. Answers to the worksheet given for homework were read out by the teacher while standing behind the desk.

Questions were asked both by the teacher and the students; however, teacher questions intended to check that students remembered previously taught facts and answers were often answered in a chorus by students with the teacher often answering with class.
Students asked questions to check their understanding or to confirm information. Some answers were given without referring to or linking to the graph in the exercise supporting the finding of these answers. A worksheet given as class work was answered on the board by the teacher while the students answered the same worksheet in their books.

Student errors and misconceptions were seldom uncovered during observed lessons, although the teacher showed awareness of these in informal discussions. One observed misunderstanding that arose from a student interpreting the equation of the line was dealt with by using a real life payment situation – this did clarify the concept for the student.

Other observations from this classroom

- The lessons using the worksheet on interpreting graphs and introducing equations of graphs as a continuation of number patterns introduced a new concept. Therefore an introduction to the lesson should have been observed.
- The teacher talked a lot, as did the others, but seemed less interactive.
- The answers that had been done by the students for homework were marked in a way that cost time in the lesson and kept them disparate. (Could have put graph on overhead and students could check answers quickly - class mostly correct - so that more time could have been devoted to discussion of concept linking into new work).
- Homework seemed to be either for consolidation or to finish off what was not done in class – students then finished the concept on their own.
- Lessons from the final week of the term were not observed, but this teacher noted in informal discussions that she had managed to cover all the concepts she felt were important for the external assessment. "*I liked it* (the teaching module) *and felt that it worked very well for me. I managed to finish everything that we decided to do and my marks for June were high*".

Table 6.5: Teacher 2 –Observations: Teaching for mathematical proficiency

Teaching activity	Observation in terms of Maths Proficiency	Notes from observation of lessons.
Presentation of concepts to be dealt with in the lesson	There is a clear introduction at the start of the lesson.	No clear introduction. There is no reference to the mathematics to be worked with in the lesson. The students ignore this as irrelevant.
	This introduction shows the teacher understands the mathematical concept that is the focus of the lesson.	
	The teacher finds out what the students already understand about the concept before sharing their own understandings of those concepts.	To a certain extent – the teacher reminds more than elicits information about understanding of concept.
	The teacher takes this understanding shown by the students into account, using it as a foundation for the further teaching of the concept.	No, this was not the purpose of the discussion about the Cartesian plane.
	The lesson *structure* encourages students to link new information and previously taught concepts to each other	No. the lesson does not show a structure that serves a particular purpose.
	The teacher is happy to use and discuss different ways of doing procedures and solving problems.	No evidence seen to support this.
	Makes references to other sections of mathematics where appropriate to promote connections between topics.	Yes, but the connections are not pursued.
	Nurtures students' natural curiosity.	Not in observed lessons.
	The teacher balances the use of activities that either encourage independent knowledge construction or encourage group construction of knowledge.	Yes, activities were either individual work on worksheets or teacher "telling". The activities were not used to generate discussion.
	Uses activities or problems that force students to question their methods or the "rules" they have developed to solve problems so as to promote a deeper understanding of the mathematics.	No evidence shown.
	Lesson content and tasks are relevant and realistic.	Yes, all tasks are chosen to support lesson focus.
	Facilitates scaffolding to help students perform beyond the limits of their ability.	No evidence shown.
	Emphasises problem solving, higher-order thinking and works towards developing deeper understanding of concepts.	No. Teaching to complete what seems to be a personal list of tasks.
Management of class discussion.	Encourages and accepts student autonomy and initiative.	No evidence of this.
	Allows student responses to drive lessons, shift instructions strategies and alter content.	Very little student input. Responses only elicited for giving answers to focussed questions.
	Responds to student contributions in a way that encourages students to engage in dialogue, both with each other and the teacher (whole group discussion)	No sustained dialogue or discussion to explore concepts noted in the observed lessons.
	Allows time for social interaction and knowledge sharing in a way that promotes the purpose of the lesson (small group discussion)	Students work individually.

Use and handling of questions	Avoids *too many* closed or narrow factual questions.	Only this type evidenced in lessons.
	Encourages student enquiry by asking thoughtful, open-ended questions.	No evidence of this.
	Encourages students to ask questions of each other.	No class discussion seen.
	Asks students to elaborate on initial responses.	No.
	Avoids answering the questions for the students.	Answers "with student" – a chorus.
	Allows for "wait time" after posing questions.	No.
	Questioning strategies encourage reasoning and justification by students.	No.
Use of teaching strategies that encourage the development of a community of learners.	Provides opportunities for peer-review and self-evaluation.	No.
	Moves through the room interacting with students.	No.
	Students' errors are taken into consideration and are used to gain insight into students' previous knowledge constructions.	Errors of this type are not uncovered in the lesson.
	Emphasises knowledge construction as opposed to reproduction.	Information given in top down format.
	Uses manipulative, interactive and physical materials.	Yes, but not as effectively as possible.
	Uses and encourages the use of language and terminology appropriate to the concept being taught.	Yes, always uses and expects correct terminology and language.
Finishing off of lesson.	Pulls concepts together to conclude lesson.	No.
	Homework set consolidates and practices skills taught during lesson.	Homework used to finish off work not completed in lesson. Students learn from homework tasks.

6.3.1.3 Teacher 3

This teacher was the most nervous about teaching "out of the box". (She held the most traditional view of mathematics and teaching). She put a lot of effort into using the module activities in the way we had discussed in the initial focus group meeting. I found these enjoyable lessons to observe.

A notable feature of this classroom was the whole class discussion and involvement in the concepts being dealt with. This teacher, despite being nervous of what she felt was "loose teaching" allowed her students to take ownership of their understanding in a way not observed in other classrooms. Notes were given, but were not "silent", rather group generated. She attributes the success of the discussion in her class to the type of class she is teaching – "*I have mixed ability classes that are not too spread out with respect to ability. This means there is always someone who wants or needs to ask questions, and always someone who will help to answer the question asked or who will have seen the connection I am looking for so that I don't have to say it all the time*". Yet both Teacher 2 and 3 had similar mixed ability groupings.

The teacher used the homework marking to encourage points of discovery, discussion and to generate moments to promote opportunities to teach new information. She asks many questions during marking of homework tasks. These are used for a number of reasons:

1. To refer to concepts taught before and used to remind, repeat or to encourage students to provide their understanding of these mathematical ideas. Here the teacher provided herself with an opportunity to uncover possible misconceptions and to attempt to correct these. Often if this is the case, the marking of homework can take the whole period, but was mainly used as a vehicle for teaching and consolidation of understanding.
2. To ask students to think about a new idea. This was often followed by examples (in a short lesson) and worksheets from the module for homework to promote further discovery of the new idea that will be discussed again; or by worksheets from the module that are done in class and then discussed as sections are completed (a longer lesson).

The teacher managed to get the class to "dictate" what information should be part of any notes they made to aid their learning. Classes were involved in generating this knowledge. Many of the worksheets in the module are designed to use previously developed knowledge to generate a solution to an activity that can be used to talk about new ways of working with graphs and this is the way this teacher used these worksheets. When using the module activities to teach a concept the teacher employed them either for homework, which was then followed through in the next lesson, or in a longer lesson, used them in class and while the students worked she walked around checking work, correcting mistakes, and helping students.

I witnessed the teacher using underlying mathematical principles to explain concepts. In an observed lesson, the class was looking at the gradient of lines that are either horizontal or perpendicular. The concept of a zero or undefined gradient was queried. The teacher used the mathematics and the understanding of fractions to help students understand these answers. This was repeated in a later lesson specifically dealing with these lines.

Other observations from this classroom included:

- The teacher spent a lot of time moving through the classroom looking at work, correcting mistakes helping students with work, asking questions and ensuring students were working. The whole classroom is her territory and she used the front of the classroom when she wanted the class to focus on a joint task – it is the position used for specific teaching activities.
- The class was regrouped about 20 minutes before the end of a long lesson to initiate discussion and to teach a new concept that the worksheet answers developed.
- The lessons preceding the external exam were not observed, but in discussions, this teacher indicated her frustration at having to rush through the dual-intercept method of drawing graphs. She mentioned this in a final interview – "*I must tell you – look this exam literally jumped on us and we were rather rushed at the end and just crammed it in so that we were ready for them – the external exam. But I definitely need to go and put it into exam context*".

Table 6.6: Teacher 3 – Observations: teaching for mathematical proficiency

Teaching activity	Observation in terms of Maths Proficiency	Notes from observation of lessons.
Presentation of concepts to be dealt with in the lesson	There is a clear introduction at the start of the lesson.	Yes. Often uses the homework as a point of departure and the lead in to introduce the new work. The teacher seems confident and in control of the work to be covered.
	This introduction shows the teacher understands the mathematical concept that is the focus of the lesson.	
	The teacher finds out what the students already understand about the concept before sharing their own understandings of those concepts.	This is always done, even if it on an individual basis or is a check of understanding needed from a previous lesson.
	The teacher takes this understanding shown by the students into account, using it as a foundation for the further teaching of the concept.	Always. Although often on an individual basis.
	The lesson *structure* encourages students to link new information and previously taught concepts to each other	In the majority of lessons, the structure seemed well thought out and purposeful.
	The teacher is happy to use and discuss different ways of doing procedures and solving problems.	Where appropriate this is always evidenced.
	Makes references to other sections of mathematics where appropriate to promote connections between topics.	Where appropriate does this and talks about connections, although misses a few opportunities.
	Nurtures students' natural curiosity.	Most lessons are quite focused, but one lesson was used in this way.
	The teacher balances the use of activities that either encourage independent knowledge construction or encourage group construction of knowledge.	There is evidence of this in lessons where this promotes the achievement of the objective.
	Uses activities or problems that force students to question their methods or the "rules" they have developed to solve problems so as to promote a deeper understanding of the mathematics.	This technique is used on occasion by this teacher when there is time in the lesson to "divert" the discussion.
	Lesson content and tasks are relevant and realistic.	Yes.
	Facilitates scaffolding to help students perform beyond the limits of their ability.	Yes. Although the teacher generally deals with students individually in this way.
	Emphasises problem solving, higher-order thinking and works towards developing deeper understanding of concepts.	At least two of the lessons showed this – and perhaps lent themselves to this.
Management of class discussion.	Encourages and accepts student autonomy and initiative.	Yes. The students feel that they are active participants in the mathematical endeavours.
	Allows student responses to drive lessons, shift instructions strategies and alter content.	This teacher always uses student responses to determine the lesson direction, and yet still achieves objectives.
	Responds to student contributions in a way that encourages students to engage in dialogue, both with each other and the teacher (whole group discussion)	This was observed in one of the lessons where there was time to discuss the concept properly.
	Allows time for social interaction and knowledge sharing in a way that promotes the purpose of the lesson (small group discussion)	When appropriate to lesson, although individual work is more the norm.

Use and handling of questions	Avoids *too many* closed or narrow factual questions.	Asks quite a few of these, but not in every lesson.
	Encourages student enquiry by asking thoughtful, open-ended questions.	There was no evidence of this in the observed lessons.
	Encourages students to ask questions of each other.	There was no evidence of this in the observed lessons.
	Asks students to elaborate on initial responses.	In general this teacher always asks for clarification.
	Avoids answering the questions for the students.	In lessons where questioning was a strategy, answers were all given by students.
	Allows for "wait time" after posing questions.	Yes.
	Questioning strategies encourage reasoning and justification by students.	In lessons where questioning was a strategy students were expected to justify their answers.
Use of teaching strategies that encourage the development of a community of learners.	Provides opportunities for peer-review and self-evaluation.	Not evidenced in any of the observed lessons.
	Moves through the room interacting with students.	Always gets to all the students.
	Students' errors are taken into consideration and are used to gain insight into students' previous knowledge constructions.	Student errors are extensively used in order to refine teaching approach in lesson.
	Emphasises knowledge construction as opposed to reproduction.	Yes.
	Uses manipulative, interactive and physical materials.	Yes.
	Uses and encourages the use of language and terminology appropriate to the concept being taught.	Yes.
Finishing off of lesson.	Pulls concepts together to conclude lesson.	Most of the time the lesson is concluded well.
	Homework set consolidates and practices skills taught during lesson or preceding lessons.	Yes, and this is always intended to also provide a bridge to the next lesson.

6.3.1.4 Teacher 4

A notable feature observed in this classroom was the teacher's continual use of questions to maintain focus, keep students thinking and to drive the teaching of concepts. This class was not able to structure many insightful questions, and had difficulty remembering past concepts. The teacher therefore spent much of the lesson teaching by asking questions of the class, to keep them alert – not always addressing the students whose hands were up to answer questions, using few generally directed questions and choosing students to answer from all over the classroom. Most of the questions were purposeful and directed to achieving a specific outcome such as eliciting all the steps that were needed for any procedure that used previously learnt processes. Generally the concepts were taught by asking for observations, facilitating answers, telling students and showing students the links between information using their answers as a starting point where possible. Despite the number of questions used by the teacher, class discussion in the sense that students constructively and jointly developed mathematical understanding was not obvious. But when students asked questions the teacher did bounce some of these back to other students, particularly when an answer was wrong. Questions from the students were mainly around procedural concepts.

Generally these students managed to stay focused for most of a single period (30 minutes) if their attention was engaged, and were keen to involve themselves in the lesson. To maintain this involvement, only one or maybe two concepts were ever dealt with at a time, and consolidation always followed a new idea. The teacher was very aware that she was teaching students who struggled with mathematics as many had had borderline grades in Grade 8. She did not move fast through ideas, speaking clearly and allowing wait time for students to catch up.

Many of the students had misconceptions, especially in algebra that resulted in these conceptual errors being repeated. As a consequence, the teacher spent a portion of every lesson redeveloping an understanding of certain Grade 8 algebraic concepts and correcting errors undermining accuracy of procedures. Where time allowed, no step was written down without a student explaining how to do the step. Often incorrect answers were written down without comment. Silence was to give stronger students a chance to protest or the teacher a chance to ask them to identify the problem so that the error could be discussed. This meant that this class worked more slowly that the others and that fewer worksheets were used.

This classroom, unlike the others was set up as a lecture theater which made it difficult for the teacher to move through the room making contact with all the students. To counteract this, the back students rotated every week – slowly moving forward to the front so a different group was near the teacher every week. She spent a fair amount of time in the front of the class unless an individual activity allowed time to move through the rows. There was no distance maintained that isolated her from the students.

The last few lessons were stressful and very full. The teacher felt pressure to finish off and consolidate some straight line concepts that she felt the class were not ready for. In these lessons the teaching followed a more traditional show and tell and practice routine.

Other observations from this classroom

- Middle of the range worksheets were the extension work for this class.
- Every activity had to count. There was not as much time for practice as something new had to be learnt every day.
- Did try not to develop too many methods for solving problems – pushed students to learn to apply and use one or two methods.
- Did allow students reactions to change direction or alter teaching strategy (e.g. drawing line $2x + 3y = 6$ using x-and y-intercepts only instead of plotting points. Students were very unhappy about not putting the equation into standard form first – always wanting to do so. The teacher allowed this, and built on it by using this way of working to draw graphs using x-and y-intercept and to discuss equations and working with fractions as well.
- This teacher made more use of the calculator as a tool for generating answers (e.g. a set of points for plotting graph) than the other teachers.

Table 6.7: Teacher 4 – Observations: Teaching for mathematical proficiency

Teaching activity	Observation in terms of Maths Proficiency	Notes from observation of lessons.
Presentation of concepts to be dealt with in the lesson	There is a clear introduction at the start of the lesson.	Yes. Often uses the homework as a point of departure and the lead in to introduce the new work. The teacher seems confident and in control of the work to be covered.
	This introduction shows the teacher understands the mathematical concept that is the focus of the lesson.	
	The teacher finds out what the students already understand about the concept before sharing their own understandings of those concepts.	This is only evidenced as actively done (i.e. separate from use of homework) when new ideas are introduced that rely on pre-knowledge.
	The teacher takes this understanding shown by the students into account, using it as a foundation for the further teaching of the concept.	Yes. Sometimes the lesson changes focus.
	The lesson *structure* encourages students to link new information and previously taught concepts to each other	There is always a plan that is behind the lesson to attempt to promote this.
	The teacher is happy to use and discuss different ways of doing procedures and solving problems.	Yes, as long as is can be mathematically justified.
	Makes references to other sections of mathematics where appropriate to promote connections between topics.	Not always. This depends on the lesson and its focus. But used when seems appropriate.
	Nurtures students' natural curiosity.	Only able to do so in certain topics.
	The teacher balances the use of activities that either encourage independent knowledge construction or encourage group construction of knowledge.	Tries to mix activities to prevent boredom. But not always possible due to speed of class (slow).
	Uses activities or problems that force students to question their methods or the "rules" they have developed to solve problems so as to promote a deeper understanding of the mathematics.	Yes, when feels has time. But sometimes missed opportunities to do so.
	Lesson content and tasks are relevant and realistic.	Yes.
	Facilitates scaffolding to help students perform beyond the limits of their ability.	Yes, although sometimes makes the tasks easier.
	Emphasises problem solving, higher-order thinking and works towards developing deeper understanding of concepts.	Not too evident, but in some cases does so – seemingly easy problems when does, but class does so see them as so.
Management of class discussion.	Encourages and accepts student autonomy and initiative.	Yes. Allows students ownership of mathematics, but within boundaries.
	Allows student responses to drive lessons, shift instructions strategies and alter content.	Yes. Lesson focus shifts on occasion.
	Responds to student contributions in a way that encourages students to engage in dialogue, both with each other and the teacher (whole group discussion)	Encourages students to partake in the lesson by asking many questions that demand interaction and thinking of the mathematic.

	Allows time for social interaction and knowledge sharing in a way that promotes the purpose of the lesson (small group discussion)	Not very often, or for long. Maintains tight control of this.
Use and handling of questions	Avoids *too many* closed or narrow factual questions.	Asks quite a few of these, but for a specific learning purpose.
	Encourages student enquiry by asking thoughtful, open-ended questions.	Not generally observed. Weaker class - avoids some teaching strategies that might lead to potential loss of focus.
	Encourages students to ask questions of each other.	Not very often.
	Asks students to elaborate on initial responses.	In general this teacher always asks for clarification.
	Avoids answering the questions for the students.	Answers were generally given by students, but teacher sometimes answers questions that are group directed.
	Allows for "wait time" after posing questions.	Yes, but if feels it is taking too long chooses a student to answer and then elicits help from others.
	Questioning strategies encourage reasoning and justification by students.	In lessons where questioning was a strategy students were expected to justify their answers.
Use of teaching strategies that encourage the development of a community of learners.	Provides opportunities for peer-review and self-evaluation.	Not evidenced in any of the observed lessons.
	Moves through the room interacting with students.	Always gets to all the students, but over the course of a few lessons.
	Students' errors are taken into consideration and are used to gain insight into students' previous knowledge constructions.	Student errors are extensively used in order to refine teaching approach in lesson.
	Emphasises knowledge construction as opposed to reproduction.	Yes.
	Uses manipulative, interactive and physical materials.	Yes.
	Uses and encourages the use of language and terminology appropriate to the concept being taught.	Yes.
Finishing off of lesson.	Pulls concepts together to conclude lesson.	Most of the time the lesson is concluded fairly well.
	Homework set consolidates and practices skills taught during lesson.	Yes, and this is always intended to also provide a bridge to the next lesson.

6.3.2 Teachers opinions of the approach

These opinions proved useful in assessing the assumption made regarding the practicality of this approach in providing support for teaching for mathematical proficiency - i.e. that it provided a sufficiently layered approach to the topic and concepts to be taught. Teachers' opinions were largely drawn from the many informal discussions that were held during the teaching of the module. These were used to identify in what way the use of such a module facilitated or constrained teaching for mathematical proficiency. I was looking for evidence of a process of positive change indicating a movement towards the goal of mathematically proficient teaching. In order to facilitate the analysis of the view that the module facilitated or inhibited a movement towards the goal of mathematical proficiency, I coded each response or viewpoint recorded. These were codes for the teacher; the month and then the comment, so a code **3Ap3** would refer to a remark made by teacher 3 in **Ap**ril recorded as comment **3**. This coding was used to refer to which particular teacher/s comments were chosen to support the statements made below.

6.3.2.1 Facilitating factors

The investigations involving problem solving (at the end of the number pattern component) promoted class discussions in the top set as they were pushed into thinking about solving the problems which had no immediately obvious solution. The teacher for this set noted that:
"Generally the class worked on their own or quietly in pairs and there was little class discussion around problems until they got to the investigation worksheets with more open-ended problems. This frustrated them to the point where they got vocal and loudly participated in discussions regarding the solution of the problem". (1Ap3)
There was therefore enough depth in the components to challenge the stronger students, but also graduated in difficulty so that the weaker students were challenged but not overwhelmed, especially in the number pattern component.

Formal teaching was now a means of consolidating concepts already understood, formalising setting out of work or a form of scaffolding for weaker students. In conversation, teachers commented that what they termed *"retrospective teaching"* was more for reinforcement, formalization of setting out of answers and to help those one or two students who had asked for a more structured approach to recognizing patterns. (1Ap5; 3Ap2and11; 2M2). It was felt

that with weaker sets they *"...needed to teach the concepts in a more traditional manner. Number sequences were put on the board, the class was asked to find the next term and they looked at the differences, found the formula and correlated the two"*(1Ap8).
This intervention (scaffolding) by the teacher then allowed them to work through the second worksheet but they still required support. Generally teaching in this way was felt to be quite effective.

Using the module worksheets allowed many students misconceptions (especially algebraic and integer misunderstandings) to be uncovered early. When these misconceptions were problem areas for most of the class: *"...it was sometimes necessary to work with less depth in order to consolidate and develop a good basic understanding of the topic."* (1Ap12)

Teachers were able to use familiar worksheets to link number patterns to linear functions for weaker students which allowed them to use teaching time effectively when establishing a new understanding of relationships (1Ap6; 3Ap3). But they were also able to use new worksheets to do the same for stronger students giving them the opportunity to practice old concepts and to establish new knowledge. One teacher noted that:
"I used worksheets already discussed and problems already solved to plot points and draw graphs rather than (as with the top set) using the graphs and number patterns linking worksheet which presented new number patterns. I felt that using the new and unfamiliar worksheet would result in me not being able to use teaching time effectively as this would undermine the development of new ideas".(1Ap13)

Students enjoyed working with the mathematics presented this way. One comment was that:
"This class was very excited when they picked up the link between the number pattern and the "picture" it gave when drawing". (1Ap14)

Other skills and procedures were developed or reinforced as they were used for a purpose, not only for themselves. *"This activity also allowed the class to improve upon and understand what they were doing when substituting into the formula to find, for example, a y-value for the co-ordinate"*.(1Ap15)

Teachers found that the module promoted class discussions and helped them develop their own understanding of concepts. *"I was glad to have worksheets to help lead discussions regarding dependant vs independent variable"* (3Ap3) was a comment made by Teacher 3.

This teacher felt that this had always been an area in which she was unsure of the classification. Worksheets helped her in her teaching as much as the students.

Teachers felt that by talking about and exploring concepts in number patterns it would make it easier to use these ideas in the formal linear function component.
"...the discussion of independent vs. dependant values will help facilitate the plotting required when use the patterns to move onto graphs". (3Ap4)

The way the teachers were involved in actively trying to make this method of teaching work and their enjoyment of teaching using this module (despite feeling time pressure). All the teachers commented on this in the final interviews

6.3.2.2 Constraining factors and ways in which these (if possible) were mitigated

The module did not provide enough practice activities linking the numeric and algebraic representations of number patterns to their graphical counterparts. Teachers felt this was quite a significant conceptual jump and felt it was necessary to insert an extra activity to help provide a clearer picture of this relationship (1Ap10; 3Ap5). This was suggested and provided by Teacher 3. She decided to make another worksheet as she was not confident that her classes had fully grasped the concept. She therefore inserted a second worksheet for consolidation. She did this because as she said:
"I found it very helpful to use this as a worksheet linking graphs and number patterns, because I could present new number patterns to find the relationship for and then to plot points and draw graphs representing these relationships". (3Ap5)

These extra worksheets were considered to be necessary by the teachers of the weaker classes in order to provide a clearer structure for these classes, as they were unable to perceive patterns without some help. As Teacher 1 commented:
"*They needed to see many patterns together with the same structure in order to see the links between the differences and the type and structure of the number pattern*". (1Ap10)
This worksheet was eventually used with success by all the teachers who had weaker sets to practice number patterns and link them to the idea of Cartesian plane graphs. In the end this contributed to discussions that allowed for the promotion of the idea of presenting linear

patterns graphically, of co-ordinates, of the idea of gradient (as illustrated by teaching notes from Teacher 4).

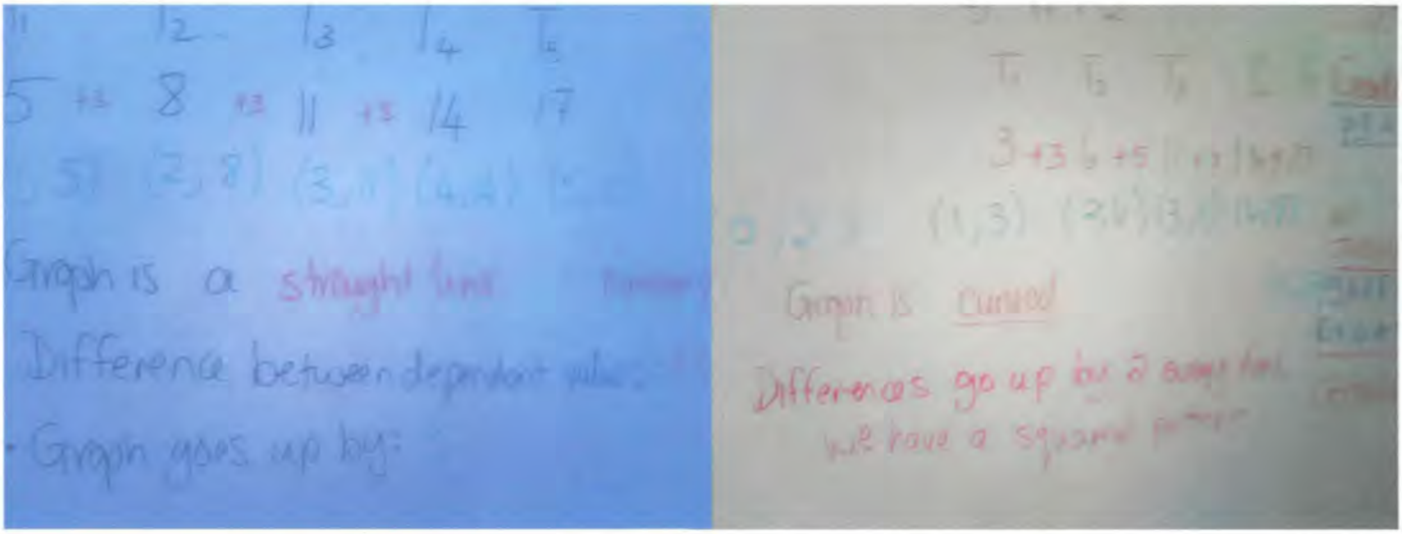


Figure 6.1: Teaching notes summarizing a class discussion on number patterns and graphs

Time constraints were a major area of concern for teachers. One comment was that "*…as you start running out of time, you pop over to old methods of teaching. I must say, the one worksheet we did find by experimenting and I used the worksheet to go into first difference, second difference – I did use it but after that, after that I said right, if you have first difference you do that with your constant difference, and if you have second difference you do that and say oh that's what you use and do, but the weaker ones I did end up teaching the recipe even if I didn't want to".* (3Ap11)

This time pressure meant that only the stronger classes got to the more difficult problem solving sections and that the module had to be "streamlined" to ensure that enough concepts got covered before the examinations.

The teachers of weaker sets of students felt they had to "tell" the class how to do things to get them moving in the right direction within the timeframe allocated to the section. *"I started with worksheet #1 and they managed the first two pages with a little help, although it took a whole period for them to do so. They got given the third page for homework, but were unable to complete it on their own. I needed to teach the concepts in a more traditional manner first and then go back."*(1Ap7)

6.4 Phase 3: Review and feedback – evaluation of implementation

6.4.1 Promotion of teaching towards mathematical proficiency: teachers' views and the applicability of approach to teaching in other sections

Interviews at the end of the second term and after we had consolidated and finished teaching the linear function topic in the third term were used to identify in what way the use of such an approach was of assistance in promoting teaching for mathematical proficiency. Again, I was looking for evidence that suggested either a process of positive change showing a movement towards the goal of mathematical proficient teaching; or a factor that might inhibit this goal.

In order to identify the elements providing evidence for promoting or inhibiting the teaching for mathematical proficiency the teacher interviews were examined; transcribed with the answers numbered A1; A2 … etc. and then coded for evidence of these themes. Evidence of areas where the approach could be refined for reuse and for where teachers felt such an approach might be applied to other areas of the mathematics curriculum were also identified. The following coding system was used for the interview transcripts:

Code	Theme (these were numbered e.g. 1P; 3H etc.)
P	Promotion of mathematical proficiency teaching
H	Inhibits teaching for mathematical proficiency
AL	Areas of module to adapt: learning from implementation
AP	Other areas in which approach could be applied.

The outcome of this analysis is summarised in *Table 6.8* and then considered in more detail. When discussing these themes, it was useful to use the teachers' responses to highlight a point. When these were quoted, they were referenced according to which teacher made the comment; which interview the comment came from (June (J) or August (A)) and which answer was quoted. So therefore a reference 2JA5 would refer to a comment by teacher **2** in the **J**une interview found in answer **5**.

Table 6.8: Summary of factors promoting or inhibiting the goal of mathematical proficient teaching

Factors promoting teaching for mathematical proficiency	1	Teacher confidence with foundation laid by component #1
	2	Teachers' experience of using module was both enjoyable and educational
	3	Teachers felt a better understanding of functions was developed this year.
	4	Teachers came to understand how topics previously seen as separate could be presented as linked concepts.
	5	These links were seen to promote students understanding.
	6	Module allowed for adaptation to teaching styles and classes.
	7	Weaker students benefited from the approach used in component 1.
	8	Better understanding of other mathematical topics was promoted.
	9	Module structure ensured promotion of basic and more advanced understanding of links between concepts.
	10	At least two teachers were encouraged to try to teach without "boundaries"
	11	Module method promoted discussions of "best options' for solving problems.
	12	Students were actively encouraged to take responsibility for their own learning.
	13	If approach were more commonly used it would develop greater confidence in the weaker students.
Factors inhibiting teaching for mathematical proficiency	1	Students' ability to transfer knowledge to other contexts was problematic.
	2	Time
	3	Basic algebra skills that were not fully mastered, especially weaker students.
	4	Need to formalise algebraic understanding of graphs too quickly.
	5	Difficulty in dealing with mixed ability classes.
	6	Students discomfort when dealing with problems having unclear solution paths.
	7	Continual linking of concepts difficult to teach.
	8	Traditional teaching methods sometimes "encouraged" by students.
	9	External examination
	10	Teachers' expectations with respect to weaker students.
	11	Question structure and language in assessments.

6.4.1.1 Evidence indicating positive movement towards teaching for mathematical proficiency

1. All the teachers felt confident that component 1 (number patterns) had provided them with a solid foundation for developing linear relationships from number patterns to functions. Most of them commented that they felt that

   "*…number patterns were definitely better and the kids had more knowledge this year and they could answer the questions in the exam*". (3JA10)

   The teachers enjoyed teaching this section and felt that even the weaker students benefited and felt they were able to do the mathematics.

   "*We will never have to re-teach patterns again. In Grade 10, 11 and 12 their understanding of patterns should be fantastic. And I can say that because my bottom class, they love it and they all got full marks for that question in the exam – all of them, the bottom class and the top class*". (1JA1)

2. Teachers' experience of using the module was both enjoyable and educational.

   "*From my own perspective, I've never really been a lover of number patterns, and I've never really liked teaching the section. But with this module that's changed, and I was much more comfortable and happy with them*". (1JA13; 1AA5)

   They felt that their knowledge of the mathematics and how to teach the mathematics had been enhanced through learning new ideas and approaches.

   "*I developed myself, which came across to the kids. So I feel very happy – I enjoyed it. I enjoyed seeing the difference by doing the same thing with the top class and then changing my approach, learning from my mistakes and what wouldn't work there and then going to teach my bottom class*". (1JA15; 1AA13)

   This was particularly true for the way in which component 1 had been linked to graphs. Teachers' comments ranged from:

   "*I liked it and felt that it worked very well for me. I managed to finish everything that we decided to do and my marks for June were high*". (2JA1, A20) To:

   "*I enjoyed teaching – it was different to "boxes method" last year. Especially with the number patterns setup, the worksheets saved time. Also, looking at dependant vs. independent, the first time we worked with relationships – not calling it just x and y, I have not done it this way before and it is important. I liked the idea of understanding this first. I also liked the pattern – graph link*". (3JA28)

One of the teachers (1JA43) went on to mention that:

"I used quite a few of your patterns with my Grade 10 class because I have the lowest Grade 10 class and they loved it. And it also made me feel more comfortable and I knew how to handle them, they were the lowest class and my Grade 9's from last year and they didn't remember doing this as well last year – we didn't cover it as well last year".

3. Teachers felt a better understanding of functions was developed this year. One commented:

   "*I am so confident that they know how to draw a graph by any method*". (1AA1)

   Although we were unable to finish the module before the June examinations due to time constraints, this feeling was confirmed when we consolidated and completed algebraic graphs in July/August.

   "*I thought, but I didn't teach them this and they are catching on, so something must have fallen in place over the term as we worked with the straight line graph. So I think their understanding, and the top children's understanding of the straight line graph will be better. Obviously your bottom children – the ones that will battle will battle no matter what. But your higher children – the straight line is not just an equation given to them and off you go. There is more depth...* ". (3AA2; AA11)

   We used their knowledge of graphs to move onto simultaneous equations (although without enough time to use much of the module activities) but teachers found that they were able to link the ideas together. "*Any simultaneous equations*" commented one.

   "*My classes loved solving it algebraically, and then I rubbed it out and then I did the same two lines graphically and I said ooh – look where they meet, do you remember what happened on this side what did we get. And they were like oh, oh, so you can do it both ways. I found the understanding of the simultaneous equation was like that (clicking fingers) quick, quick*". (1AA3)

   Generally the feeling was that:

   "*if we could spend more time doing something like that as opposed to redoing something each year the foundation would be so solid*". (1AA10)

4. During the interviews, conversation regarding the linking of concepts brought to light that the teachers had come to understand how topics previously seen as separate could be presented as linked concepts.

   "*Seeing that there is a relationship between two things – an independent and a dependant situation, and what is it that that relationship ultimately leads to – that it leads to a pattern and that leads to a picture that will usually be a straight line graph* (in this case). (1JA8; JA21)

   "*We are just used to teaching a straight line graph as "that thing" as opposed to a relationship. When I stopped using the "n" and put and "x" there it didn't frighten them, and then when I made it y = mx + c or y = something x + something it didn't frighten them. They recognized the pattern and it didn't matter about the variables*". (1JA22).

   The "*linking things together worked quite nicely. The number patterns worked quite well for me – especially with the tables for the graphs and seeing the patterns with x and y*". (2JA11)

   There was general agreement that this attempt to link concepts was what made the difference this time.

   "*I think from that point of view they see everything is one idea – all linked to graphs – it is in any case. I think they are seeing a table and a number pattern and a straight line graph and I think a link is being made. And from that point of view, yes brilliant*". (3JA45).

   In retrospect I think that it was the amount of time that we spent on patterns, cementing the relationships between the independent variable, the dependant variable and then a rule and how to derive the one from the other to the other and then drawing it. "*I'm convinced that's what did it*" (1AA6) was the consensus that was reached.

5. These links were seen to promote students understanding. Some teachers noticed that students were able to attempt to solve a problem when they were unable to "see" a solution path.

   "*When they came to the difficult question in the exam where many of them didn't know what to do – they took their own set of independent variables – which I drummed into them – that they could select at random – and they tried to develop their own pattern using them*". (1JA9)

This attempt to teach concepts as linked was also felt to help teachers move students on to function concepts more easily.

"*But the kids seem to grasp it because of where we came from. You know, the number patterns and everything else – the one lead into the other, so it's a great way, but it's just the time*". (3JA12)

6. The module allowed for adaptation to teaching styles and classes.

   "*I realized that I also had to cater according to the classes and I learnt about patterns and intelligence as well. Once you get it, if you are very bright, three worksheets can literally take 10 minutes. But the bottom class takes ages puzzling and thinking about it, testing, it took them time. I stuck more to the module with the lower class and developed the concept slowly and surely but jumped more with the top class*", (1JA17; JA19) was one of the comments. Again this freedom to match teaching the students plays an important role in being able to teach for mathematical proficiency, and was appreciated by the teachers.

   "*I think both, the module and toolkit is a divine structure for the teacher to use her own personality and judge her own classes and adapt it accordingly. I think it is a good module – not to be rigid – to allow the teacher's individual personality and flair to come into it and base it on class dynamics*". (1AA29)

   This lack of rigidity within the framework of the module allowed teachers to deal with situations that arose when the class just couldn't move past a point. As one teacher put it:

   "*I sometimes felt that I hit a block with my classes with what we were doing. Then I had to stop and go back and I had to change what I planned to do and teach it in a different order*". (2JA14)

   "*I like this idea of having a whole lot of worksheets and you deciding what you want to use for your class because not everybody has to have the same level of worksheets and if you want to have one with lots of práctice there is one for that need. But you want to carry on – this worksheet is better for your class. I like that idea*". (3AA29)

7. Teachers of students who were not strong mathematically felt that these students benefited from this teaching approach.

   "*It was the bottom class who were willing to try* (to be creative in solving problems). *Because they knew that if they tried stuff out of their heads they might be able to get to the bottom of this. I found that a lot of the weaker kids had to think about it and*

they did a lot better than the girls who learn by rote. Because they had nothing to loose and they thought about it whereas the very bright girls were thrown". (1JA35)
"And I'll tell you another thing that I've picked up with the weak kids – I've had a lot of them come to extra maths because they are too shy to ask in class, and then they come and I consolidate something, and I find that they put so much effort and care into their sketching of graphs – beautifully done because they feel so confident and happy that they can actually do something difficult well!". (1AA22)

8. Another positive outcome was that teachers were able to **use** the teaching of concepts related to linear functions such as finding the intercepts with the x-and y-axis algebraically, to tell if a point is on a line etc. to consolidate and teach other procedures.
   "Substitution, equations, I feel great after this section. My classes, if you ask them something they will be able to answer you". (1AA21)
   This allowed teachers to give meaning to some of these procedures.
   "I kept on telling them, can you see why you need to do equations in Grade 8? Because: if you don't do equations you can't do this. And kept on referring to, as you say substitution, because they need to hear that its not just this, just that but there is reason why you do integers, the reason why you do everything in Grade 8 is because you are going to use it in Grade 9 in other areas. I don't think they actually thought about it like that before, because they don't think of maths as the vocabulary and doing substitution, it's just a process that happens". (3AA9; AA10)

9. The module structure ensured promotion of basic and more advanced understanding of links between concepts. As one of the teacher interviewed commented:
   "It is the back bone of the section and what you put into it is the cherry on the top. I think a lazy teacher would just use it like that and there would be no fault in that – you would at least know the kids are being taught!". (1AA29)
   It did ensure that even if more traditional methods were generally employed, the basic understanding that we were looking for came through.
   "I found that if I made a small teaching lesson for constant/changing difference in number patterns without using the names linear and quadratic, and then after that used the work sheets to consolidate and explore further it worked very well, actually better. That's why they did not have a problem with it – even the quadratic one in

the exam", was the way one of the teachers (2JA6) explained her use of the module. She went on to say,

"I felt better if I taught a formal lesson first on gradients and graphs and then used the worksheets to back up what I had taught them. It worked very well in class. How you can identify a graph by its own gradient which we worked out with the two points method; that the gradient was a function of the line. Then I went and used the worksheets to back up what I had taught them. The class worked well and interacted with each other to work, and I was happy that they (the worksheets) helped the students to see what and understand what I had taught them when I tested them in class". (2JA9)

But on the other hand, the teacher who had the top set finished with a worksheet that challenged her class with Grade 10 level questions. As she said,

"It was very challenging, and most of them got it quite easily. The ones who didn't - as long as I said you know one of the co-ordinates– if you know the x-value or the y-value and you know the straight line, you can find anything – and then they could see what to do. And I felt that was the perfect end because it was really challenging and they really handled it well and I feel really confident". (1AA2)

10. Teacher 3 was encouraged to try to teach without the "boundaries" created by listing separate concepts under headings.

"In the end I did all three methods – but not with separate headings" she said. *"The table method we did all the time with out calling it "table method". The gradient-intercept method we also basically did but didn't actually call it that (not the dual intercept method I actually called it that). But up until then I had been mentioning "if I stand on the y-or x-axis what do I notice; what is this called, so when we actually got to that it wasn't really new to them – it was just a fancy name. But my major theme was gradient and being able to draw the graphs".* (3JA17)

She also made an effort to try to let the mathematics flow from the activities and the questions.

" And also this year I would give an exercise not knowing if it was going to work or not. Instead of giving as example - heading, example, do the exercise, I did the exercise and taught from the exercise. So I would take one and explain it and then when someone struggled with the next one – this was right at the end after doing straight line graphs – I just gave them the exercise – these last two weeks. I thought I'm going to just throw this at them and see what they do. And some of them just ran

through the whole thing, no problem. But then, as problems came up I would explain and mark. I literally did the whole thing on the board in the end. But explaining and talking about it. I've never approached it like that before. So they go home, they look, they can't do come back, I teach from the worksheet. And hopefully by looking at it before the time, trying it then teaching from it maybe it sinks in better". (3AA14; AA18)

11. Using the activities presented in the module allowed teachers to promote discussions of "best options' for solving problems.
"*I went on about how you cannot just blindly learn a method, you have to be open to all these methods because different method are sometimes better than your favorite one. This worksheet on parallel and perpendicular graphs lent itself to different methods. But when I drew it on the board I showed them where one method was better and where another method was better. And I did find in the exam that there were some who changed methods depending on what you got*". (3JA20)

12. Students were actively encouraged to take responsibility for their own learning because not all teachers used headings and formal notes. One particular teacher found this made her nervous, but she persisted.
"*I would keep on repeating things in class, but haven't got them to write it down. But I told them to make notes as I speak. It is very dangerous to do that, I was thinking to myself it will be very interesting to see who actually listens when you speak, when you mention things in class, who takes notes, because the work is there, the exercise is there, so everything will come out from the exercise if you do it. But I didn't say, right: write the heading, 1.1, 1.2, 1.3…"*. (3AA8)

A point that was mentioned both in the semi-structured interviews and in informal discussions was that if this approach were more commonly used, and we had time to develop such modules, it would develop greater confidence in the weaker students.

6.4.1.2 Factors inhibiting teaching towards mathematical proficiency

1. Students' ability to transfer knowledge to other contexts was problematic. This was particularly true when working with gradient. Teachers found that

"*The difference with the lower class was that if you changed it from a 1-1 scale when you were looking for the gradient they still counted blocks – they didn't check the scale. They couldn't' transfer their knowledge correctly*" (1JA3) and that students "*…made silly mistakes and they didn't interpret a real life situation as well as I would have liked to see, even the top class*". (1JA34)

This problem was highlighted in a discussion with one of the teachers in the interview.

"*It doesn't help that you know that's minutes and seconds or whatever, when you don't know how to bring it back to algebra (and vice versa)*" she commented. "*I think we concentrated (although it didn't show it in the paper), we concentrated on the real life set up. But you know it is typical maths, they really cannot cross borders, so unfortunately you have to show real life, teach them that, get happy with that, show them this (the algebra) then teach it and get them happy with that and then go back and put them together again*". (3JA1; JA5)

In the module the given Cartesian plane grids in component 3 worked in blocks of 1 unit and going back to a "real life" activity would only have been re-examined if the teacher either deliberately or fortuitously chose to use an activity out of component 4 which we had no time to explore.

2. Time proved to be one of the most constraining factors. We were not able, in the time that we had, to get students to the point of seeing the graph as a single object where each of its features was a characteristic of that line; specific to the relationship within the co-ordinate points. And the link between number patterns and functions – we were almost at the point, but we couldn't quite get there before running out of time at the start of the exams. Many students got the point of recognising the y-intercept and the gradient, but the understanding of what that really is telling you was not always there. We were pushing to get to this point of understanding the intercepts and the gradient and the equation being the relationship between the two co-ordinates and that the graph was the representation of the algebraic version of the relationship, and I wondered if we are asked to try and do too much in Grade 9, whether there are too many concepts that we are trying to pack in. Another comment emphasised that: "*…this exam literally jumped on us and we were rather rushed at the end and just crammed it in so that we were ready for them – the external exam*". (3JA3)

"*I did rush the double intercept method for the exam more than for understanding*", acknowledged one of the teachers. (3JA28). An opinion echoed by teacher 1 (1JA37)

"*I think we haven't spent enough time with substitution of points and teaching them the equation of the straight line and substitution to find something and equations*". (1JA30)

This problem gave rise to the feeling, expressed in one of the interviews, that

"*in the end, with graphs I think the time constraints there need us to redesign the structure and then have this as a resource to supplement maybe. It's a great method, but the time that we gave to straight line graphs – even if it is important for Grade 10 – the amount that we have to cover for Grade 9, its maybe too much time*". (3AA1)

3. Basic algebra skills that were not fully mastered, especially by weaker students hindered progression of new concepts.

   "*I had to go back and refresh their minds about equations and how to solve them. They couldn't understand why you do that and why you take it across and why it became negative*". (3AA25)

   This was particularly noted by teachers as being a problem when doing the dual-intercept method and needing to use equations.

4. There is a need to formalise algebraic understanding of graphs too quickly. This problem was raised by one of the teachers during the interviews. "*I definitely need to go and put it into exam context – where we are working towards Grade 10. Just a normal graph question, give the graph –read of the point of intersection, give the graph read off the gradient of the graph, find the x and y-intercepts. Like we do in Grade 10, not necessarily linked to real life, more like what we do in pure maths. Just to go a little further to see if they know what they are doing. Yes, the real life set up is great and it must be done as well, but also to see are they going to cope with what we are going to do in Grade 10*". (3JA3)

5. Difficulty in dealing with mixed ability classes. This mixed ability grouping influenced teaching methods in different ways. As one of the teachers observed,

   "*I think it is also just your class set up, because you have 10 out of the 26 girls who can cross and grasp it straight away. But then the others can't, and you are so busy* *trying to get them up to the right level so that you can move on – and it takes time. So you can't do exciting things – stick in more real life things because you are so pushed for time with these people at the back who you have to bring up to standard and then you can only go on*", (3JA6) when discussing her frustration with working with students who were always a little "out of sync".

However, this teacher managed to turn this frustration into a positive learning experience by actively encouraging and using discussion of the mathematics to involve all the students in the activities. Another approach can be seen from a second teacher who noted that

"*It was a mixed class so I needed to adapt my teaching strategy and work in groups to see things. I taught and used the worksheets for backup, not used the worksheet to discover and teaching as backup*". (2JA13)

6. Students discomfort when dealing with problems having unclear solution paths was a frustration for teachers.

   "*It's like they wanted a recipe – beforehand. Instead of just drawing the graph and looking at everything that you have in front of you – what is the same, what isn't the same/ why is it the same? Some kids can't do it – they just can't. They can't even look at a list and see that is the same, that the same, that is the same – they just can't do it. And it's fascinating because if you tell them they say oh yes, now I see it. But they can't do it from their own side they just can't. I don't know how to solve it*". (3JA13)

   They found that:

   "*…students who were comfortable without "recipes" were better problem solvers. Those that needed to learn methods struggled with the investigation*". (3JA23)

7. Continual linking of concepts difficult to teach and when stressed for time or unsure if something is working we would slip back into known methods. As one of the teachers commented:

   "*I think for myself - I just get in a panic state because I haven't taught it under headings - and it's scary. It's far scarier teaching it like this. I went back to the traditional teaching methods of finding the equation and most of my time was spent on that. The simultaneous equation was not a problem at all – because it was going* *back to algebra – they loved it. But I had to go and look at the equation of the straight line graph the traditional way again*". (3AA1; AA2)

8. Students have difficulty linking concepts and seeing procedures as able to be used in different settings.

   "*You know, they definitely work in compartments, so when doing the straight line graph, they don't think of doing equations*". (1JA29)

   This often results in traditional teaching methods being "encouraged" by students who were uncomfortable without headings or boundaries.

   "*Students close up and develop a "this is difficult" attitude. Also links between things and transferring knowledge - students separate things*" (3JA29), was a comment in one of the interviews. Some teachers subsequently felt that:

   "*at the end of the day I would like to write a heading – "Finding the equation of the straight line graph" and still do steps. Maybe for my sake more than for their sake – but then at least I know that I can tick, tick that I've done it. For the bottom children I still think that sometimes they need to know 'if I see this then I do this' I think that's how they learn*'. (3AA3)

   But despite this there was the general recognition that "*you see, that's the danger of teaching according to headings, because if the heading is not in the test, you don't know what to do with it. But if you teach it in a 'mumble jumble', if the students get something not obvious, they are not scared because it is not in boxes. But I don't know if it is going to work, if my pupils can get used to not having it structured by me*". (3AA19)

9. The external examination. The timing and content of the mid-year examination was a source of frustration to the Grade 9 teachers. Observations regarding the exam ranged from:

   "*I felt that the external exam did not test for understanding this section in the way that students need to understand this section. Especially since we have to develop the knowledge required for Grade 10. You could just "crash course" this section for the exam*" (3JA30) and

   "*I have a problem with the exam forcing them into one particular method*" (3JA32), to a comment on its effect on teaching methods.

   "*I think it (the module) helped me teach better. Everything could be taught like this if only we had the time. But we need in the end to teach to the exam*". (3AA19)

10. Teachers' were not really convinced that (despite saying their students had been better grounded in the topic) weaker students were really capable of learning only through this approach.

    "*I also think your weaker children will probably do that whole exercise, come to a conclusion and still not remember it - whether they discovered it or its given right there*". (3AA22)

    Their experience with these students has convinced them that weaker students are unable to see a topic as a set of interlinked concepts,

    "*The strong ones manage and seem to link skills and things, but the bottom ones because they are so insecure they don't*" (3AA19), and it is sometimes best to teach these students how to answer the exam.

    "*I know for myself, if I didn't have a heading in my book – by matric (because I was a standard grade pupil) if I didn't understand the question or know what to do I started learning the questions. I learnt the words, if I see this word, that's the process I follow. And if they change the words I am in trouble*!". (3AA20)

    This teacher, having been taught and having learnt in this way feels that the weaker students need this safety net to help them through.

11. Question structure and language in assessments. This was felt to obscure the problem. Teachers remarked that:

    "*There were too many words*" – or regarding the route to the answer – "*too much information in one table*". (3JA10)

    "*I felt that it was not that they couldn't understand the work but that they had read the question and didn't know what to do*". (2JA20)

When identifying these factors and the effect they had on teaching for mathematical proficiency, I noted that in general the factors that promoted a positive movement towards this goal stemmed from the module that had been developed, from the teachers' willingness to try new teaching approaches and their skill in working with the mathematics and the students in their classes. The factors inhibiting mathematical proficient teaching were generally either systemic, or related to beliefs that teachers held regarding students and how they learn mathematics.

6.4.2 Using data to test assumptions:

6.4.2.1 Sufficient hours of teaching time are available:

In order to test this assumption I took into consideration that there were three possible factors that would influence the time available for teaching. That is: the number of teaching days that were available in the term (April – June); the number of teaching hours that were provided for in the timetable and the manner in which the teachers used this time.

I first analysed the external time factors – the teaching days and the timetable provisions - by calculating the total number of school days available in the April-June term and the number of days used for each of the major school activities identified. The results (shown in *Figure 6.2* clearly indicate that only 52% of the term was available to teach concepts. This amounted to approximately 24 days – 5 weeks. Since in the timetable, we are provided with 4,5 hours per week (as per NCS) this results in a total of 22,5 hours of teaching time. However, as this is spread out over 4 days a week, it is only feasible to cover and consolidate a few concepts within a topic per week.

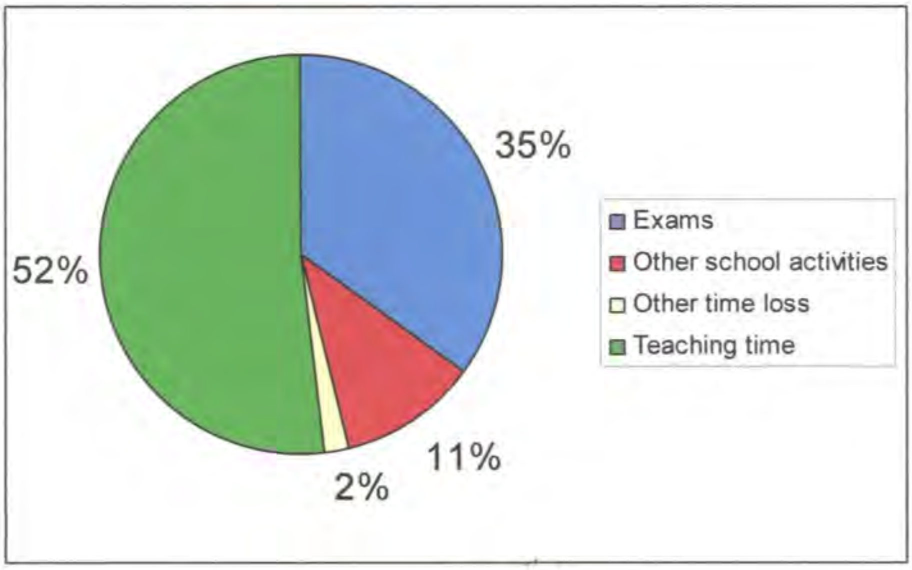


Figure 6.2 Percentage of term used by teaching and other activities.

The way the teachers made use of their teaching time was analysed by using the video material available. Eleven different class based activities were identified from this video material and the amount of time spent on these activities was calculated for each teacher and then added up. The total teaching time of all the video footage was then analysed for what proportion was spent on each activity. When considering what percentage of time is used in class to teach mathematics we see that 91% (*Figure 6.3*) of class time is devoted to teaching activities directly related to the topic being taught.

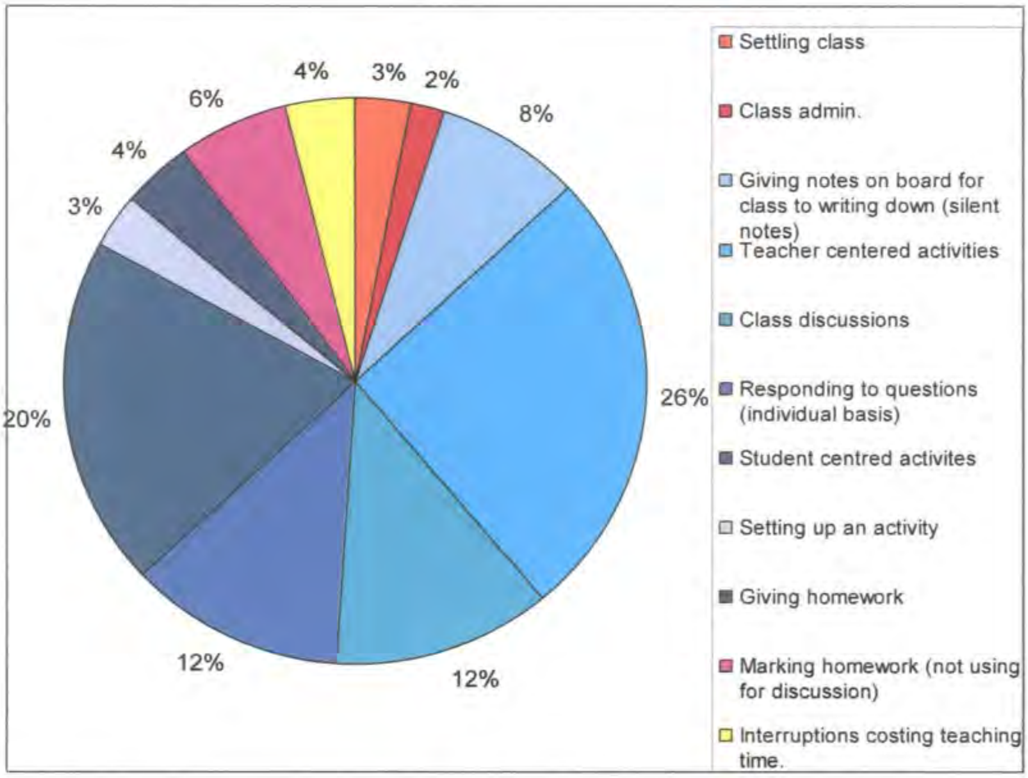


Figure 6.3: Average percentage of class time allocated to different activities.

As highlighted by the interview material, there was not sufficient teaching time available for this module despite the teachers teaching for the topic, not towards the exam and ensuring a high percentage of their available time was spent focussing on tasks used to teach these concepts. . This meant that not all the work was covered in this term. We did not finish the third component of the module satisfactorily, and made time available in the following term to complete algebraic graphs and to look at simultaneous equations. The problem solving section which was designed to allow us to apply linear function concepts to solving straight line concepts to "real life" situations has not been used at all. This is due to both the density of concepts explored in the module and the density of the curriculum for Grade 9 which necessitates the teaching of many other concepts requiring a similar depth of understanding. The time constraints put the onus on the teacher to be able to effectively choose and design a lesson series that best suited their classes as it was not possible to start with the first worksheet and go through them all.

It must be noted that many of the teachers used the module activities with their Grade 10 classes, finding them very helpful, especially with the weaker sets, to establish a firmer knowledge base than they had brought to Grade 10 with from the previous year.

6.4.2.2 External factors providing the context within which the module was taught

Again, the factors comprising the environment in which this module was taught were considered in terms of the way they facilitated or constrained teaching for mathematical proficiency. Aspects external to the school – the curriculum, the departmental pace-setter, teaching time allowed by these documents and the external June examination were analysed, and these and other factors identified as influencing teaching were discussed during an interview with the Head of Department (HOD) of Mathematics for the school. This interview was transcribed and each answer numbered A1; A2… etc. for referencing purposes. The result of this interview is presented in terms of which factors were identified as promoting or inhibiting teaching for mathematical proficiency.

Facilitating factors

- The NCS provided a useful general foundation from which to work in terms of the broad outline of skills and the level of concepts that needed to be taught.

- The timetable of the school was designed to allow for the maximum teaching time that was required by the NCS. If possible (as in the previous year) the Headmaster would allocate extra time for mathematics.

- There are enough teachers at this school to ensure that class sizes are manageable.

- These teachers are generally very competent and knowledgeable in terms of the mathematics and pedagogical knowledge.

- The teachers in the Mathematics Department in this school work together as a team, discussing teaching approaches, learning from and supporting each other. The HOD commented in the interview that the teachers were the reason teaching for mathematical proficiency might work.

  "We have an excellent, cohesive department. If it wasn't for the staff we have I don't think it would work as well as it does work. It is the staff that allows it to work, that allows this to work - us to overcome - I can see how it does not work in other schools who do not have the staff that we have". (HOD, A30)

- The educational background of the students is not dissimilar, making it easier to move from primary school mathematics to high school mathematics.

- The parent body is generally extremely supportive of the work done in the school.

- The absenteeism rate is very low, and students generally are good at catching up on work missed.

Constraining factors and ways in which these (if possible) were mitigated

- The NCS in terms of the number of concepts that were expected to be covered in the Grade 9 syllabus, many of which required a depth of understanding that needed time for development. The NCS was also sometimes vague in terms of what to do and how exactly to assess the concepts that were to be taught. (In order to alleviate this I used the CAPS draft document to clarify many of the areas in which the NCS was a little vague).

- The end of year National Exam. This assessment did affect the teaching of concepts, in that all teachers reached a point where they taught towards this examination.

- Time to teach concepts that require depth – we end up formalising concepts too quickly.

- Not enough algebraic thinking developed in Grades 7 and 8. (This problem was mentioned by all the teachers, and will be looked at as part of the review and refine step of the management cycle).

- Not enough linking of concepts both in primary school and high school. This is perpetuated by the way in which mathematics is assessed in external examinations in Grade 9.

- There is no local subject advisor for Grade 9 mathematics at the moment, and very little (if any) positive support from the Provincial department for mathematics at this level.

- Teaching time gets eroded by other activities and externally determined examination timetables (see *Figure 6.2*). As we discussed in the HOD interview:
  "..it is frustrating when you've got a section planned that has to take a specific amount of time and then you find that this time has been eroded and you then have to try and catch up and you start getting behind". (HOD A28)

- Due to a lack of subject specialisation in South African primary school teachers (are expected to teach 9 learning areas with equal expertise) there are many areas in mathematics that have weak conceptual foundations.

- The externally set provincial based June examination for Grade 9 is set according to the work the pace-setter indicates should be taught by the end of May.

- The Departmental pace-setter arrived at the end of February. This document outlines the concepts that should be taught during the year and how much time should be taken for each concept. This pace-setter determines which concepts will be examined in June. It was noted in the interview that:
  "...the work doesn't follow sequentially, the time allocation for everything is not well thought out. In the 3rd and 4th term, everything is just repeated. You just can't teach to that pace-setter. If you think about these pace-setters, why do they repeat the work? I think it is because they think that if you repeat the work and the students see it again then they will understand. But that is not necessarily true, if you are not teaching towards understanding, rushing through the work and teaching it quick, quick; then repeating it is not going to help develop that understanding". (HOD, A14)

Teachers were unable to use this document in developing a plan for teaching for understanding as it did not lend itself towards this goal. It did however, have an impact on the way teaching of the straight line concept became more traditional in the last week before exams as teachers started feeling the pressure of needing to teach towards the exam.

In order to put the topics and concepts covered by the module into context they need to be considered in relation to the entire syllabus required for Grade 9 by the NCS. The module covers only a small part of that syllabus. It took 7 weeks to teach these concepts, and we did

not complete the module. For comparison purposes the pace-setter's interpretation of the NCS, allocation of teaching time and sequencing of topics and their concepts was analysed showing that more than 60% of the Grade 9 topics were expected to have been covered by the end of May. It is necessary to understand that this pace-setter influences teaching practices directly if followed, as it encourages teaching mathematics as a "list of topics", and indirectly if used as only a guideline, as it is also the document used for setting the June exam. Teachers interviewed felt that it simplified concepts and showed no awareness of the depth or time that was required to teach such a huge section of work. This frustration with the Grade 9 curriculum is echoed by the HOD in the interview:
"*I think the syllabus is too full – more of the basics need to be taught and less of the cream on the top. And then maybe just the top class could extend the syllabus a bit more*". (HOD; A28)

Because we could not follow the suggested teaching order outlined in the document, we had to set our own assessments to gain a better understanding of our students than was possible from the June external exam. This allowed us to identify concepts that our students were struggling to understand – a necessity in teaching for mathematical proficiency. This information was invaluable in determining how to approach teaching the consolidation phase we decided was essential at the start of the third term.

6.4.3 Last but not least - other interesting insights from the interview data

1. Change needs someone to drive it.
   "*Yes, there was quite a lot in there that I would not have thought of using, and I can see in retrospect that it helped me as a teacher to expand as well, and it did – you forced that on us*". (1AA32)
   Someone will need to take responsibility for ensuring that this attempt at changing an approach to teaching linear functions is continued and improved next year.

2. In order to find a compromise that allows us to work towards the ideal of teaching for mathematical proficiency but within the systemic constraints that we have little influence over, we might need to consider a "mix and match" approach to teaching this and other sections.

3. One of the suggestions that would allow us to maybe create space for the teaching approaches that encourage mathematics proficiency was to reconsider how we structure the Grade 8 and 9 year. This comment provided an interesting compromise that might be worth looking at.

   "*I think if you look at what you teach in Grade 8 and look at what you teach in Grade 9 and spread it over the two years. It might save you time in the long run. Grade 9 is a cut off because you have that exam at the end of Grade 9, that's where we have to get to*". (3AA28)

CHAPTER 7: DISCUSSION:

THE INFLUENCE OF TEACHING PRACTICE ON THE IMPLEMENTATION OF A LEARNING PROGRAMME

Embarking upon a project that investigated the dual influences of research and practice on the implementation of a teaching module was an ambitious undertaking. The complexity of putting into practice the idea of teaching for mathematical proficiency can only be appreciated when one attempts to do so. Implementing the teaching module has emphasized the critical role played by the teacher in determining the manner in which students experience and come to view mathematics.

7.1 Using the teaching module to promote teaching for mathematical proficiency

Did this teaching module support and promote the teaching for mathematical proficiency as was intended?

In order to answer this question, I looked for evidence demonstrating that the module fostered a process of positive change (i.e. evidence that mathematical proficiency was being taught). I have presented an analysis of the module in the results chapter. The following processes of positive change were found:

(i) The modules activities, and the order in which they presented concepts, were adapted and added to by teachers to help students see how ideas previously perceived as separate topics were related. This was noted in conversations and interviews held with both Teacher 1 and 3 regarding their use of the activities that linked number patterns and graphs. This ease of use meant that the module was not so rigid that teachers were unable to adapt it and use activities from the module in a way that they felt best suited their particular class. All teachers in the interviews indicated that they adapted the module as they felt would best suit their classes and still achieve the understanding they were looking for.

(ii) The structure of the module required teachers to present concepts in the order indicated, facilitating the identification of relationships between concepts. This was easier to see in classes where teachers used the module activities in class to promote teaching of concepts and/or gave homework assignments to try to generate discussion in the next lesson. Observed lessons for Teachers 1; 3 and 4 indicated that by choosing to use these activities in this manner, conversation could be generated to promote relationships between ideas. Teacher 2 often taught first and then used the module for homework activities. Even so, the order of the module, although to a lesser degree, encouraged students to perceive the concepts as linked together.

(iii) In most classes, students were encouraged to participate by presenting their understanding of the concept. This was in part due to the way the teachers chose to present the topic, but also in part due to the structure of the module. The activities in the module often required questions regarding the nature of the concepts to be answered as part of the activity. In this way established links between pieces of information were generated by requiring students to think about relationships and teachers to discuss the answers to these questions. Teacher 3 was able in this way to clarify for herself and her students an area she had struggled to work with previously.

(iv) The first component of the module (number patterns) was very successful in promoting understanding of the mathematics and how the various concepts linked together to form a bigger picture. All teachers noted in both informal discussions and the final interviews that they felt confident that their students had understood this section. The number pattern component was enjoyed by the teachers, providing them with new ways of looking at the topic and seeing connections between concepts within this topic that they had not taught for before. This was particularly commented on by teacher 1 who had not enjoyed previous experiences teaching number patterns, but had felt that this approach had helped her to understand and become more comfortable with this concept.

(v) Students in the "weaker sets" were noted by teachers (particularly Teachers 3 and 4) to have developed a more positive attitude towards their mathematics for the number pattern component. Students in these classes were making conjectures and contributing to class conversations in a way that allowed them to develop an understanding of these concepts. A lesson identifying the relationship between the sequence of numbers and its algebraic formula was particularly successful in allowing students to take ownership of their understanding. In Teacher 4's class a method for finding the constant term in a linear relationship $T_n = an + q$ was called "Jamie's" method after the student who "discovered" it. This teacher believes that ownership of mathematical understanding is important in promoting a positive disposition towards mathematics.

(vi) An area that was indicative of a teacher's promoting mathematically proficient thinking was the way in which student errors were managed. Teachers (particularly 1, 3, and 4) explored student mistakes and considered this teaching activity important. They were generally observed to participate more frequently in discussions about mathematical concepts. Teachers 3 and 4 who asked students to explain their reasoning both for incorrect and correct answers were observed to engage more students in the lesson than those who only explored errors. The importance of following up on student thinking for both correct and incorrect answers is promoted by Brodie (2010), who emphasizes that this is a teaching technique that helps students to deepen their understanding of the mathematics. This importance was underlined by observations made of teaching practices that did not encourage classroom dialogue. In the class where no attempt at new ways of teaching was observed (Teacher 2), the module was simply slotted into established practices. The module did ensure that the order of development of understanding of concepts was maintained, but this was the only factor that could be identified as promoting proficiency. Unfortunately this was not supported by the teaching practice. In this case the emphasis was very much on procedural fluency, with understanding and the linking of concepts left up to students with that capability.

Although not all teachers considered that their basic teaching practices had changed, they all noted that they were able to use the module to think about how to present concepts in a way that linked them and the "different" topics together. This was especially true of the link between number patterns and graphs. The teachers commented on how they had enjoyed looking at teaching the two topics in this way.

It was very difficult to incorporate all the elements of mathematically proficient teaching into one lesson or even a series of lessons when one is under constraint of time. I found that at the beginning of teaching the module, activities were used in the way they were intended, and informal discussions with teachers and lesson observations indicated that the mathematics was explored in a cohesive (in terms of the 5 strands) manner. However, in the last two weeks before the scheduled examinations, teachers were stressed and anxious about finishing enough content for the external examination. The focus at this time changed to more procedural knowledge with some conceptual understanding, with little time made available for class discussion and reasoning. This, in part, explains why teachers enjoyed the number pattern component and even the introductory lessons for the algebraic component of linear functions, more so than the rest of the concepts covered in component 3, when more traditional teaching methods were used.

I found that true classroom discussion is difficult to instigate and manage. Teachers do not seem really to understand what this is and how to manage a class to ensure that it happens.

I found it difficult to gauge the "absolute" cognitive value of a task. Such assessment depended on many subjective measures, including students' ability, a teacher's perception of students' ability, and a teacher's perception of what such a task should be achieving. What a task expects from one grade should change for another grade, but this did not always seem to be the case. We also found that tasks do not hold the same cognitive level for all students, a finding with which Stein and Kaufman (2010) concur. In this case teachers often had to help students over a conceptual hurdle. This is difficult to manage without reducing the cognitive level of a task, and is done in most cases to expedite coverage of the required curricular content and was particularly noticed in the "weaker" classes of Teacher 3 and Teacher 4.

Students sometimes attempted to force teaching back into comfortable routines (for instance by wanting lots of notes for reassurance –Teacher 2's classes). Explaining their thinking and constructing their own knowledge was uncomfortable for them. I think that it is possibly more to do with a lack of practice than a lack of ability: the teachers of even the "weaker" groups (Teachers 1 and 4) found that when pushed to do so or when engaging upon a section they enjoyed – number patterns – these students were able to justify their reasoning. Their thinking was not always sophisticated, but they were able to explain their work. The notion that more practice would support skill in reasoning is supported by the teaching scenarios presented by Brodie (2010).

There seems to be constant pressure to make the mathematics we teach "concrete". As teachers we are constantly asked the question "where will I use this in real life", and we are expected to contextualise mathematics all the time. This is difficult to do: not all the concepts we deal with are directly applicable on their own, and it takes time to build up a level of understanding relating together enough concepts to be used in a "real life" setting.

This module attempted to do just that – to start and end with such activities. We did not get to re-establish a connection to a real life context as we simply ran out of time. This is a constant problem in mathematics classrooms. Despite this, the teachers using the module felt that the way of thinking about mathematics we had promoted was in itself a satisfactory outcome. As Sfard (2002) concluded, real-life usefulness is not the only good reason for learning mathematics. Mathematics can also be seen as a challenge to be appreciated for its own sake. To add to what students can already do with mathematics (although not necessarily directly applicable outside the classroom) can ultimately help to develop skills for solving problems never seen before.

Observations and discussions with teachers reiterate the fact that it takes time to teach for mathematical proficiency, and that getting to a level of understanding in, for example, linear functions, is a layered process. This process is described by Sfard (1991; 1994) as "the cycle of reification". It starts with thinking about an idea (like a graph) that is developed out of a related concept that then has a set of characteristic algorithms and actions which are learnt and performed in sequence. The cycle of understanding ends with a student being able to visualize and conceive the graph as a structure that can be described as if it were an object (encapsulating all the previous concepts belonging to similar graphs in one idea). This development of understanding is important for mathematical proficiency but is not necessarily easy. Sfard (1991) says: "it is often not acknowledged that insight cannot always be expected as an immediate reward for a person's direct attempt to fathom a new idea…. It requires much effort..." (p. 33). When teaching with the module, we found it necessary to create an extra opportunity for students to use procedural knowledge and repetition in order to provide enough examples to help them see the link between linear number patterns and linear graphs, but were unable to provide time for a similar exercise on linear functions. Developing the depth of understanding required for mathematical proficiency takes time – a commodity often in short supply as we grapple with the demands of the curriculum.

7.2 Influences upon teaching practice that affect teaching for mathematical proficiency

Influences identified in the research that played a role in determining teaching practice could be roughly grouped into two categories – endogenous and exogenous influences. Endogenous influences were teacher-related and included the attitudes, decisions and disposition of the teacher. Exogenous influences were more contextual and in effect out of the control of the teacher and included teaching time available, curriculum, external assessments etc. Both of these influences were seen to affect teaching for mathematical proficiency, either promoting or inhibiting this goal.

7.2.1 Endogenous influences

Teaching is a complex activity, with the decisions teachers make both in planning and executing lessons being influenced by a number of factors intrinsic to the teacher and the context within which they teach (Timperley, 2008). Observations from lessons and interviews with teachers led me to consider that there might be a hierarchy in terms of which factors influence teaching practice. It seems that teachers' beliefs, views and disposition towards mathematics affect their decisions and teaching practice in the following ways: (presented in order of increasing influence)

i. Disposition towards mathematics. This is related to a teacher's knowledge of mathematics and influences the way s/he approaches the teaching of mathematics (i.e. with confidence or trepidation), and to a certain degree, how s/he believes mathematics should be taught.
ii. The way teachers believe mathematics is best taught strongly influences the way they choose to teach the mathematics.
iii. Teachers' views, understanding and expectations of the students in their classes is a strong influence on the decisions they make regarding the way they teach mathematics. (Teacher 1 approached her two classes with different expectations, and provided different support for each class).
iv. The importance teachers place upon fulfilling the requirements of the subject content as set out by the curriculum and assessed in the external exam. (This is an additional opinion/view that was not tested for, but that was observed as influencing teaching in the video material.)

From these observations, it can be inferred that the mathematical knowledge base and the opinions, beliefs or views held by teachers regarding the teaching and learning of mathematics and the students they teach are the common threads that influence teaching practice. These can be placed on a spectrum that ranges from viewing mathematics as a fixed body of facts and procedures to be transmitted by the teacher and learnt by the student, to seeing it as a program of activities or a network of ideas both dynamic and problem driven and from which knowledge and skills can possibly be acquired or "constructed" (Driver and Oldham, 1986; Ernest, 1989; Olarte et al., 2004; Prawat, 1992; Webb and Webb, 2004).

Knowledge base for teaching:

Research has shown that it is critical for secondary mathematics teachers to have strong mathematical knowledge and a positive attitude toward mathematics and teaching, as well as an alignment with accepted pedagogical principles (Furner, 2004; Kilpatrick, 2001; Shulman, 1986; Krauss et al., 2008). The importance of knowledge relating to practice cannot be understated, as theory changes teachers' knowledge and therefore the base from which they make decisions (Malara and Zan, 2008). It is not necessary for all such knowledge to be theoretical in nature. It is possible for a dedicated teacher to learn, through experience, from students, colleagues and reflective practice. In my experience this is how much of my knowledge base was built. Stein and Kaufman (2010) insist that a teacher's academic or professional qualifications did not necessarily comprise an absolute measure of his or her skills as a practitioner. These authors found a strong relationship between teachers who taught for a "big" picture, trying to link concepts together, and the high-level implementation of lessons. How a teacher used a curriculum might be more important than the education, knowledge and experience of that teacher. Therefore another possible way of viewing teacher capacity is to look at it from the perspective of how the teacher is able to seek out and productively use resources. That this is worth considering was borne out by the research carried out for this thesis. One of the teachers whose lessons were the most supportive of the mathematical proficiency goal (Teacher 3), had a four year teaching diploma and no formal academic mathematics background. Yet she, despite her trepidation at attempting to teach "out of the box", was the teacher who applied herself to actively changing her teaching practice and hers were the most productive lessons observed. The students took ownership of the mathematics and were involved in discussion concerning concepts. The dialogue was promoted and well managed by Teacher 3.

It was also noticed that changing approaches did not mean that Teacher 3 abandoned previous practice. Rather she took what she knew had really worked previously and used this opportunity to change her approach to teaching concepts that had not been well understood by her previous Grade 9 students. This blending of teaching approaches has been observed by Törner, Rolka, Rösken and Sriraman (2010) and Pehkonen and Törner (1999), who, when conducting research into teaching practice, found that although a teacher might be open to new approaches, well-established beliefs, knowledge, routines and scripts were not simply replaced, but altered with the addition and assimilation of new experiences. This suggests that when a teacher has certain views regarding the teaching of mathematics and experiences success teaching in a way commensurate with these views, new methods will not displace **successful** routines; rather they will be incorporated into the teacher's repertoire of teaching approaches that can be used to promote understanding. It also suggests that a teacher needs to be open to learning and improving his or her teaching practice.

Disposition/beliefs

Beliefs in this context are personal philosophies or conceptions about the nature of mathematics, as well as about teaching and learning mathematics (Törner, Rolka, Rösken and Sriraman, 2010). That these beliefs about mathematical content, the nature of mathematics as well as its teaching and learning influence the decisions teachers make and what happens in the classroom is an accepted reality in educational literature (Törner, Rolka, Rösken and Sriraman, 2010; Kilpatrick et al., 2001; Malara and Zan, 2008; Zollman and Mason, 1992; Brodie, 2004; Webb and Webb, 2004, 2008). Malara and Zan (2008) and Törner, Rolka, Rösken and Sriraman (2010) feel that it is impossible to separate teachers' knowledge and beliefs, and I too found it difficult to do so. What I did find was that it was sometimes difficult to establish what teachers' opinions actually were. The teacher's situation was observed to strongly influence teachers' beliefs. Malara and Zan (2008) found that the professed views of teachers could change, depending on whether a teacher was objectively thinking about teaching or actually practising. The situation that appears to reflect beliefs the best is the way a teacher teaches in the classroom. This comment is based upon the great discrepancy between the responses given by one of the teachers (Teacher 2) in the questionnaire and that teacher's lessons as they were observed in the study. Opinions given in the questionnaire seemed to be at odds with the teaching practice observed. Admittedly, few lessons could be recorded, and so a teaching style might be attributed to consciousness of being "on display". The notion that beliefs are possibly best determined through observing teaching practice was also borne out by the differences (not as obvious) seen with another teacher (Teacher 1).[1]

From personal experience, observations and conversations with teachers, the way a teacher believes mathematics can successfully be taught to students seems to be highly influential on his or her teaching practice. In particular, the way teachers viewed their classes and the capabilities of their students was very relevant to the teaching approaches adopted. Attitudes

[1] It must be noted that Teacher 3's teaching was consistent with her responses in the questionnaire.

and expectations that teachers have regarding pupils influence their perception and interpretation of student behaviour. This is evidenced by the different teaching styles observed for Teacher 1, who taught two classes – a "stronger" and a "weaker" class. A great deal of support was provided for the weaker class in linking concepts together. Notes were often given before students had had a chance to grapple with the concept. The teacher helps the students over "hurdles", to the extent that material used from the module is sometimes made cognitively easier a bit too soon. This teacher was of the opinion that the weaker class needed to be schooled through concepts and then drilled until confident. She felt that they struggled to understand and were frightened of thinking "out the box", and therefore needed memory "hooks". On the other hand, the same teacher moved very quickly through concepts with the stronger class. Here she would sometimes wait for answers to questions that with the weaker class she had provided herself. Teacher 4 clearly indicated in the background questionnaire that she felt her class was the weakest class in Grade 9 mathematically. She moved relatively slowly through ideas, speaking clearly and allowing time for students to catch up. The lessons were taught through the continual asking of questions to keep the class alert. These questions were purposeful and directed to achieving a specific outcome. This teacher is careful not to develop too many methods for solving problems and encourages the students to learn and use one or two methods. The relevance of teachers' views in terms of both the decisions they make and students' performance is described by authors such as Kilpatrick et al. (2001); Malara and Zan (2008); Webb and Webb (2004, 2008); Brodie (2004a), and Zollman and Mason (1992).

It must be acknowledged that using teaching practice alone to attempt to determine a teacher's views or beliefs can be misleading. Exogenous factors are influential and can be important constraints in determining the decisions teachers make when teaching mathematics.

7. 2.2 Exogenous influences – enabling environment

As well as personal factors, there are external influencing factors: e.g. time constraints, curriculum demands, resources, students' mathematical background etc. (Brodie, 2010; Kilpatrick et al., 2001; Olarte et al, 2004) that have an impact on the effective teaching of mathematics. Work environment constraints such as time, the school environment, curriculum demands and external assessment requirements have a strong influence (both personal and practical) on teachers' decisions (Malara and Zan, 2008). The contextual constraint that was found to have the greatest influence on the teachers in this study was that of time available to teach the concepts concerned. According high importance to this factor, Kilpatrick et al. (2001) noted that "Mathematical proficiency as we have defined it cannot be developed unless regular time is allocated to, and used for, mathematics instruction" (p. 422).

Time is therefore a key requirement for teaching for mathematical proficiency. As Brodie (2010) remarks, teachers contributing to her work on teaching for mathematical reasoning (an important strand of mathematical proficiency) noted that it was very time consuming and meant that less content was covered. My data supports the comments made in the literature regarding the need for time and the effect time constraints have on teachers and teaching. As I noted in chapter 6, teachers who felt under pressure to complete a set list of concepts within a limited timeframe reverted to "old methods of teaching". The closer we got to the examination period the less the module was used to direct teaching practice, and the more teachers made the decision to teach by using more traditional methods of presentation. When teachers came under stress, they were more likely to draw on well-practised procedures (Törner, Rolka, Rösken and Sriraman, 2010), because the content to be covered becomes more important-seeming than how best to teach it. Time influenced teachers' decisions not only through imposing a practical constraint (in terms of the teaching approach chosen), but also by arousing anxiety about external curriculum and assessment requirements (Malara and Zan, 2008). All the teachers were under duress to finish the number of topics required for the Grade 9 June examination, and all chose to present topics so as to expedite the completion of the work required.

This illustrates that directly linked to time as a constraining factor, is pressure pertaining to the requirements of the curriculum. Kilpatrick et al. (2001) stated that in their opinion the American curriculum for mathematics was "a mile long and an inch deep". This is a statement which could equally be applied to that presented in the South African curriculum documents. The allocations of time suggested by these documents for teaching concepts do not reflect the stated objectives of the curricula. The NCS curriculum document states that mathematics is considered to be a subject that "helps develop mental processes that enhance logical and critical thinking, accuracy and problem solving that contribute to decision making". This statement seems to indicate that teaching for mathematical proficiency is an aim supported by the curriculum. But the number of topics required to be covered in any one grade does not support this aim by allowing sufficient time for the development of true understanding in terms of adaptive reasoning. The demand for quantity promotes teaching that turns the teacher into a technician who, given a curriculum to implement, will do so without playing a role in "co-constructing" the curriculum along with students (Kilpatrick, 2009). Without institutional support in this regard, even well-qualified, good teachers in well-resourced schools will not be able to afford the time to teach towards mathematical proficiency in all topics. My experience and conversations with the teachers involved in this research indicate that they choose a few topics deemed likely to require this approach the most, and try to make time for teaching these in more depth. The rest of the topics get taught with a focus that is largely procedural.

7.3 Reflections on this approach to teaching linear functions in Grade 9:

The opportunity to reflect on this approach to teaching linear relationship helped teachers develop and hone teaching strategies. All of the teachers indicated in their final interviews that they had thought about the way they approached teaching this section. Many helpful suggestions for improving our approach and refining the module were put forward in the final interviews that will positively influence teaching this section next time.

One of the areas of difficulty that stimulated reflection upon our approach to teaching this section is that "people seem to assume implicitly that instruction acts on students and that an opportunity to learn is actually moments of learning" (Kilpatrick et al. 2001, pp. 35). This assumption certainly was brought home to us when teaching linear functions. The concept of gradient is one with which many students seemed to struggle. It was found that while students were happy with the idea of slope and how to measure it, they found it difficult to relate this concept to its "real life" meaning and then link it to the algebraic equation of a graph once we moved into more abstract thinking. This was despite original success with the number pattern – graph link we made. This difficulty was seen to be related to the jump in thinking that was required when moving from finding the ratio difference between points that were one unit apart with respect to the x-coordinate, or plotted on graph paper of equal unit blocks, to finding the ratio difference between points more than one unit apart or plotted on graph paper of unequal units for the horizontal and vertical axes. The meaning and reason for the position of the gradient in the equation became less obvious and the link was difficult to make. This is a place where we will need to create more opportunities for connecting ideas.

This study illustrated that effective teaching required constant revision of one's approach, an opinion shared by Kilpatrick et al. (2001), who remark that learning to teach well is not a 'once learnt always known' skill but rather a career-long challenge.

Opinions from teachers contributing to this study were that it was helpful to have teaching materials and methods presented in a way that helped them think about the links between concepts. Teachers need help with materials that both provide this and allow for content to be covered in a more integrated way because of the time constraint (Brodie; 2010). This module promoted the learning not only of linear function concepts but also other mathematical procedures involved in this topic, e.g. substitution into a formula and solving equations.

Changing teaching practice is difficult, even for skilled teachers. Gallimore et al, (2009) and Timperly (2008) all note that the reality in teaching practice is that despite teachers preferring innovative teaching methods, these have limited impact on their teaching approaches if not followed through.

Teaching for mathematical proficiency requires collaborative effort from the teacher and the class. Brodie 2010 (p. 720) concludes that teaching for mathematical reasoning is only achievable through a "collaborative learning environment with effective whole-class discussions".

CHAPTER 8: FINAL OBSERVATIONS AND CONCLUDING REMARKS

"The touchstone of knowledge is the ability to teach" (Aristotle).

This comment, found on the wall of an English teacher's classroom, neatly captures one of the issues I felt this research highlighted so strongly. In fact, two interesting observations that were not anticipated in the original questions came out of this work: (i) the affirmation of a recurring theme in educational literature – that the teacher plays a critical role in determining the manner in which students experience and come to view mathematics; and (ii) the importance of focusing on teaching for adaptive reasoning.

In this chapter I consolidate the conclusions my research prompted with respect to my original questions: (i) – how can theory inform the design of teaching material; (ii) – can teaching material promote mathematical proficiency and (iii) – what influences affect the teaching of mathematical proficiency? I also examine the two observations that surprised me with their persistence in making themselves felt, despite the study not being intended to consider either of them.

8.1 What the study highlighted regarding the research questions:

8.1.1 <u>How can theory inform the design of teaching material?</u>

I found that my chosen theoretical framework provided a solid foundation that informed the direction and structure for the development of the teaching module. There were three main conclusions I drew from this experience:

- Collaborative development of teaching material was not practical due to the time required for this that the other practicing teachers did not have.
- Developing an appreciation of what is required in order to teach towards mathematical proficiency requires a depth of understanding of both teaching methods and the mathematics taught at school. This is necessary to facilitate the translation of theory into practice.

- Mathematical proficiency, while important as an overarching goal, is not "user friendly" and is unclear as to exactly what one must do in terms of the daily activities of teaching.
- The van Hiele teaching phases provided an extremely helpful idea of how to structure the module, and these, together with the work done by Stein and her colleagues on tasks answered many of the questions I had regarding what to do in the lesson material I compiled.

8.1.2 Can teaching material promote mathematical proficiency?

Regarding the efficacy of the teaching material in promoting mathematical proficiency through its use in the classroom I noticed that:

- Teaching without engaging in the process of understanding of what the desired learning outcomes are that teaching for mathematical proficiency aims towards detracts from the success of the teaching process.
- However, carefully developed teaching material can certainly help guide teachers towards this outcome.
- Teaching for mathematical proficiency relies on reflective practice and a willingness to "step out of the box" when teaching concepts. This is difficult to do and it will be interesting to see whether these teaching approaches are sustained.
- Developing the required understanding of mathematical concepts in students is a layered process that takes time to develop. This time is not always available.

8.1.3 What influences affect the teaching of mathematical proficiency?

Extraneous influences were definitely found to affect teaching towards mathematical proficiency and these could be grouped into two categories;

- Endogenous influences (teacher related) such as attitudes, disposition and the decisions teachers make. These influences, while affecting a teacher's openness to adopting a different teaching approach, also determine their belief regarding what it means to be mathematical proficient.
- Exogenous influences are contextual and not within the control of the teacher. Of these the time available for teaching, the demands of the curriculum and the external assessment were seen to have the greatest impact on teaching for mathematical

proficiency. In this study these factors were found to undermine our teaching for mathematical proficiency.

8.2 A closer look at other observations arising from the study:

8.2.1 The role of the teacher

The implementation phase of the research illustrated the critical role played by teachers. Kilpatrick et al., (p.12) affirm the teacher's influence in determining the direction taken by students with respect to mathematical proficiency, stating that: "Improving students' learning depends on the capabilities of classroom teachers. Most of the mathematics children know is learned in school and depends on those who teach it to them". Timperley (2008), reporting on research undertaken in the area of teacher professional learning and development, observes that all the evidence indicates that student learning is very much influenced by what and how teachers teach (Stein, Remillard and Smith, 2007; Stein et al., 2009; Stein and Kaufman, 2010; Kilpatrick et al., 2001). The teaching triangle – the relationship between the teacher, the students and the mathematics that is directed and managed by the teacher – should therefore be regarded as the heart of any investigation centred on teaching practice.

Lesson observations and informal interviews with teachers suggested that those teachers who provided the best support for helping students on the journey towards mathematical proficiency:

- were able to use discussion to develop group understanding of the concepts;
- used questioning to promote interaction among the students, themselves and the mathematics;
- were teachers with whom the students were able to interact and perceive as a person there to help them understand and make sense of mathematics;
- were prepared to use mathematics as the authority for justifying an idea (as teachers 3 and 4 used mathematics to justify the gradients of parallel and perpendicular lines).

This was evident in the manner in which Teacher 3 and her class worked as a collective to develop understanding of the concepts. The class participated, discussed and "developed" theories to explain concepts. They came to an understanding of the work through collaboration. They were very active in generating their own knowledge, and "gave" the teacher the information that was to appear in the study notes they were preparing. The importance of this "collaborative" development of understanding of concepts is highlighted by Brodie (2010), and is confirmed by findings presented in this study. An important result of teaching for what Brodie (2010) calls mathematical reasoning, is that students come to own their ideas, which promotes the understanding of concepts. A positive disposition towards mathematics is developed when students are afforded opportunities to explain their thinking (Frempong, 2010), another factor underlining the importance of encouraging this type of instructional practice.

A teaching technique that was observed to have an impact on the way students were able to think about and interact with the mathematics was the way in which teachers used questions and answers in their lessons. Questions asked that were "open ended" – e.g. students being asked for explanations of "why do you think ..." – were more effective in developing reasoning skills. Questions that involved asking "how do you do…" (some mathematical process) were more effective in promoting procedural fluency, and questions on why a procedure was the best option in a situation were the most effective in promoting conceptual understanding. Questions that were "closed" (e.g. students being asked "what is the answer", "what am I doing", or "clue" questions that moved activities past sticking points quickly) were not seen to promote discussion or at least encourage students to think about the mathematics rather than the answer. Closely linked to this was the way in which the teachers listened and used the students' answers in their teaching. Often a student answer was used or picked up in a way that suggested that the teacher was looking for a specific word or phase that could be used to direct the conversation or the lesson. This happened most often in lessons where the teacher was very focused on attaining the goal set for that lesson in terms of content coverage or specific concepts algebra (e.g. finding x-and y-intercepts). The more the teacher was pushed for time the more this happened. In longer (double period) lessons, teachers were sometimes more open to using answers that elicited discussion on the way a student was understanding the work. Lessons where closed questions were the most prevalent promoted little or no discussion.

That this influences the way teaching happens and students experience the mathematics is reflected in educational literature. Davis (1997), distinguished between **listening to** and **listening for** responses given by students. Listening to students meant the teacher used student answers to promote discussion focusing on the students' understanding of the mathematics. Listening for was when teachers used only student responses that provided a way of keeping a lesson focused on a particular aspect. He emphasized that questioning and listening go hand in hand. They are important factors that determine the degree to which specific-strand or whole-strand proficiency is promoted in the lesson. Brodie (2004b; 2010) talks about "teacher moves" that enable participation and communication, or focus on stimulating mathematical reasoning; and notes that the questioning and listening practices of teachers are key support activities for these.

Teaching is a creative and complex art, and I have therefore come to believe that good teachers who understand and teach key thinking skills are one of the cornerstones of successful education in general and effective mathematics education in particular. Dr Tang of The United Nations Education, Scientific and Cultural Organisation (UNESCO), stated at the 2011 Education International conference, UNESCO has made the issue of teachers a priority, as it maintains that teachers are central to quality learning (National Professional Teachers' Organisation of South Africa, Naptosa, 2011). Deputy President Kgalema Motlanthe, in an address given at the Education International 6th World Congress held in Cape Town in 2011, reiterated that "well-trained and motivated teachers are the key to the delivery of quality education" (Naptosa, 2011).

Apart from endorsing the role played by the teacher in promoting mathematical proficiency, this research highlighted the need to focus on mathematical reasoning more than just on content when developing teaching material.

8.2.2 The importance of adaptive reasoning

In developing the module I focused mainly on promoting the strands of procedural fluency, conceptual understanding and strategic competence. Grade 9 is the year in which students are introduced to functions, and it was important to develop a good base understanding of the relationships and the processes involved in working with graphs. I therefore felt that these strands would be the more useful in a module that needed to focus on building this foundation. The need for adaptive reasoning forced itself into a study that was not designed to use it - a critical finding.

I noted, when discussing the process of transforming theory into a teaching module for practical use in the classroom, that adaptive reasoning was the key to promoting mathematical proficiency. Using this strand as the focus was far more helpful when designing teaching material to support mathematical proficient teaching. The theory of mathematical proficiency, which was difficult to directly translate into practice, was made more accessible. I also found that teaching towards mathematical proficiency was more manageable if lessons were built around developing adaptive reasoning. This strand demanded the concurrent development of the other strands in "supporting roles". In this way procedural skills were given a context and a purpose. It is however essential to recognise the importance of the use of discussion when teaching for reasoning, and the complexity of initiating this in the classroom.

8.3 The role of educational theory: further recommendations

Carefully chosen educational theories can support the development of teaching material and help guide teaching practice. These theories however, need to be relevant and accessible and must be supported by the teachers' enabling environment. But theory will not have a great impact on established teaching practice unless it is easy to use and does not involve a great deal of work and take up time that many teachers do not have. Teachers need to be brought into the process, to be part of the process and support the work done. Change in any form requires that much of the responsibility is taken on by people prepared to drive it and to do most of the background work to support it. Stein and Kaufman (2010) note that curricula that supported teachers in understanding the linking of mathematical ideas and provided focused, clear objectives and support materials, encouraged high-quality teaching. I have come to believe that in order for the goal of mathematical proficiency to be pursued in more than just individual classrooms there needs to be greater institutional and research based support in terms of the following:

Institutional support

- Timetabling structured time for teachers to use reflective practice to engage in discussions about what they do.
- Teachers coming into contact with research that can directly benefit them in practical ways in the classroom.
- A curriculum that promotes cohesion and allows space for teaching for mathematical proficiency.

- Assessments that focus on what we want teachers to teach towards.

Research-based support

- Developing in teachers a better understanding of what it means to discuss a mathematical concept with a class, and how to do so successfully within the constraints within which we work.
- Identifying and promoting those management tools and techniques that we, as teachers, need to master to ensure that "moments of instruction are actually moments of learning".

As researchers it is important that we structure tasks that are meaningful and appropriate (Davis and Smit, 2006) so as to allow participants the opportunity to become involved and feel that their ideas and input are valued. Adler and Jaworski (2009), reporting on a survey undertaken of research in mathematics teacher education from 1999-2003 commented "we know less than we should about teachers' learning from experience, what they learn, whether they learn and what supports learning from experience", and "we need to know a great deal more about the kinds of mathematical learning opportunities afforded in both formal and informal sites of teacher education, particularly in relation to what and how mathematics is selected, taught and assessed". In order to investigate and attempt to come to know more about these ideas it is important that teachers' knowledge is acknowledged and that research involves teachers within the classroom. The implications of this concern for research are two-fold. First it is necessary to focus on the knowledge that is actually needed in the classroom. New teachers need practical, applicable knowledge of their subject, their students and teaching skills. Developing teachers as professionals, who are reflective practitioners involved in their own learning, needs to start the day they start studying to become teachers and should be seen as a career-long endeavour.

As Deputy President Kgalema Motlanthe went on to say, "A vibrant teaching profession is one in which teachers themselves take responsibility for their own professional development" (Naptosa, 2011).

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APPENDICES

1. An overview of the Van Hiele teaching (and learning) phases (I developed this table in order to better conceptualise how these phases might be used in the module)

2. a) Mathematics Questionnaire
   b) Summary of teachers views on mathematics.

3. a) Teaching and Learning Mathematics Questionnaire
   b) Summary of teachers views on teaching and learning mathematics

4. Background information questionnaire

5. The observation schedule I developed based on my understanding of the key areas that indicated teaching for mathematical proficiency.

6. Semi-structured interview questions for teachers (June/August)

7. Semi-structured interview questions for Head of Department (July)

8. Permission letter and consent form.

APPENDIX 1: THE VAN HIELE TEACHING (AND LEARNING) PHASES

PHASES	DESCRIPTION	INTENTION FOR STUDENT	GUIDANCE BY TEACHER	TYPE OF PROBLEM	PURPOSE OF PROBLEM
1 Information	Guided "conversation" in which teacher introduces the language and symbols with which the teacher wants the students to work.	To acquaint the student with the context of the field of study involved. Providing understandable information for the student.	Provides the material clarifying the context.	Simple and yet incites curiosity – cannot just be given as homework. Requires the guidance of the teacher.	To provide information, and help with the discovery of the field of knowledge to be taught. E.g. pictures leading to a number pattern.
2 Bound orientation/ exploration	The teacher guides the students towards the thinking that needs to be developed. The foundation for developing this higher level thinking is developed.	Student comes into contact with the main connections of the network of relations formed (e.g. $3 + 3 = 2 \times 3 = 3 \times 2$ etc). intention is for the student to be able to complete the task	Provides activities that help the student to learn the principal connections. The teacher's aim in usually invisible to the students.	One that has just a single solution path and uses one skill. If not, it will require guidance from the teacher. Requires teacher introduction, class discussion and teacher direction.	To uncover the links, relationships and systems that will be worked with in the field of study. These relationships are "discovered" by the student – implicit.
3 Expliciation	The "discoveries" from phase 2 are made explicit – they are talked about and collected into knowledge clusters. Class discussions help promote the use of technical language.	The relationships introduced in phase 2 are discussed so as to promote the learning of a technical language.	Provides activities with joint/ class solving which are discussed and used to develop language skills.	Problems are similar to those used in phase 2.	The problem is used now to formalize (i.e. make explicit) the knowledge and relationships introduced in phase 2. These phases are likely to blur.
4 Free orientation	Students are now familiar with the subject and the relationships developed through working with concrete situations. They now need to feel at home. Tasks need to be used so as to ask them to "find their way" to solutions. The skill to choose an approach from various possibilities is developed so as to promote the formation of a "network" of relationships.	Students learn to find their way through the network of relationships with the help of connections they have made – i.e. what can be done with a skill and where to use it.	Activities are provided that have instructions that encourage choice and direct students towards linking the skills they have learnt to the problems these skills can be used to solve.	Possibly more than one solution. The exact method or skill is not stated, but is not obscure. Independent work encouraged. Problems highlight relationships developed and help to consolidate the understanding of these.	Designed to encourage student to make the decision on what way to approach the solution, but the answer should be unobstructed. Problems that can be used to check progression of student. Not complicated, but highlight gaps. Encourage the development of "ready" knowledge important for the progress of the learning process.
5 Integration	"Memorisation" of methods occurs, providing student with the ability to "survey" various thinking paths. Has the student developed the desired understanding of the concept?	Students are encouraged to make their own decisions on how to solve a problem.	Provides activities that become more open ended, with <u>less</u> guidance as to which procedures provide a path to the solution.	Type 1: Tests the students understanding of the concept – this is integration. Type 2: Tests the ability of the student to apply this understanding.	Type 1: integration checking – simple, not intellectual ability tests, vision of topic tests. Type 2: problems that test insight. "What if..." These take the student beyond integration and into application.

APPENDIX 2a: MATHEMATICS QUESTIONNAIRE:

Directions: Circle the option that bests describes your feeling about the following statements on the grid provided. Use the following code:
1=strongly disagree **2**=disagree **3**=agree **4**=strongly agree

1	Mathematics changes all the time, it is dynamic.	1	2	3	4
2	Mathematics is abstract and often unrelated to the real world.	1	2	3	4
3	Mathematics must be meaningful if students are to learn to apply concepts productively.	1	2	3	4
4	Mathematics mainly involves the manipulation of expressions and the use of set processes in solving problems.	1	2	3	4
5	Mathematics is practical, creative and problem driven.	1	2	3	4
6	Mathematics is a stable set of rules, facts and skills that need to be remembered.	1	2	3	4
7	Problem solving is a separate, distinct part of mathematics.	1	2	3	4
8	A demonstration of good reasoning should be regarded as even more important than a students' ability to find correct answers.	1	2	3	4
9	Doing mathematics is all about getting the right answer.	1	2	3	4
10	Skillful use of a natural language is essential when doing mathematics.	1	2	3	4
11	The study of mathematics should include opportunities of using mathematics in other curriculum areas.	1	2	3	4
12	Mathematical knowledge is developed through justification of: ideas; and /or approaches adopted when solving problems.	1	2	3	4
13	Mathematics can be thought of as a language that is used to communicate and develop a common understanding.	1	2	3	4
14	Mathematics consists of several discrete concepts such as geometry, algebra, measurement which can be best taught in isolation.	1	2	3	4
15	Understanding mathematical concepts is the most important aspect of knowing mathematics.	1	2	3	4

(Adapted from the Standards Beliefs Instrument by Zollman and Mason, 1992. Other influences: Kilpatrick et al, 2001, Webb and Webb, 2004)

APPENDIX 2b: SUMMARY OF TEACHERS VIEWS ON MATHEMATICS:

ANSWERS TO MATHEMATICS QUESTIONNAIRE					
Question Number	**Question**	**Result: Teacher 1**	**Result: Teacher 2**	**Result: Teacher 3**	**Result: Teacher 4**
1	Mathematics changes all the time, it is dynamic.	4	3	2	3
2	Mathematics is abstract and often unrelated to the real world.	2	3	2	2
3	Mathematics must be meaningful if students are to learn to apply concepts productively.	3	4	3	3
4	Mathematics mainly involves the manipulation of expressions and the use of set processes in solving problems.	4	3	3	2
5	Mathematics is practical, creative and problem driven.	4	4	3	4
6	Mathematics is a stable set of rules, facts and skills that need to be remembered.	4	2	3	3
7	Problem solving is a separate, distinct part of mathematics.	2	2	2	1
8	A demonstration of good reasoning should be regarded as even more important than a students' ability to find correct answers	3	4	3	4
9	Doing mathematics is all about getting the right answer.	2	2	2	2
10	Skillful use of a natural language is essential when doing mathematics.	4	4	3	3
11	The study of mathematics should include opportunities of using mathematics in other curriculum areas.	4	4	3	4
12	Mathematical knowledge is developed through justification of: ideas; and /or approaches adopted when solving problems.	4	3	3	4
13	Mathematics can be thought of as a language that is used to communicate and develop a common understanding.	4	3	3	4
14	Mathematics consists of several discrete concepts such as geometry, algebra, measurement which can be best taught in isolation.	4	1	2	1
15	Understanding mathematical concepts is the most important aspect of knowing mathematics.	3	4	3	4

Code	**1**	**2**	**3**	**4**
Meaning 1	Strongly disagree	Disagree	Agree	Strongly agree

APPENDIX 3a: TEACHING AND LEARNING MATHEMATICS

Through the following questionnaire, I would like to get a profile of your ideas and conceptions concerning the teaching and learning of mathematics. Circle the option which best describes your opinion for each of these statements.

1=fully agree **2**=agree **3**=ambivalent **4**=disagree **5**=fully disagree

1	1	2	3	4	5	Development of concepts is the best teaching strategy to promote understanding in mathematics.
2	1	2	3	4	5	Learning mathematics mostly involves memorising procedures and methods in order to master the required skills.
3	1	2	3	4	5	Learners learn best when the teacher lectures to them.
4	1	2	3	4	5	In teaching mathematics, it is important to use varied exercises and applications to ensure sufficient practice of skills.
5	1	2	3	4	5	Mathematics **in school** necessarily requires a concrete dimension; abstract mathematics alone is not enough.
6	1	2	3	4	5	Following logic is promoted in teaching mathematics, whereas creativity and originality are not stressed.
7	1	2	3	4	5	Definitions of concepts are the best starting points when learners are introduced to new topics.
8	1	2	3	4	5	While doing mathematics, understanding the topic is a very important idea.
9	1	2	3	4	5	A mathematical topic is best taught by working though the list of concepts to be covered in the topic.
10	1	2	3	4	5	Mathematics is best learnt by doing things on your own.
11	1	2	3	4	5	Mathematics is a subject that is best taught to talented pupils.
12	1	2	3	4	5	A vital role of the mathematics teacher is to help students develop an understanding of the language and terminology used in mathematics.
13	1	2	3	4	5	Mathematics is best learnt by using justification and discussions of approaches and concepts used in solving problems in order to develop understanding.
14	1	2	3	4	5	Knowing your students and how they think is very important for teaching.
15	1	2	3	4	5	The way the textbook presents a topic provides the best option for teaching the concepts making up the topic.
16	1	2	3	4	5	A major goal of mathematics instruction is to help children develop the belief that they can be successful in using mathematics.
17	1	2	3	4	5	Mathematics is best taught by taking students through a process that progressively develops understanding of the concept
18	1	2	3	4	5	Competency in mathematics is best tested through the ability to solve problems.
19	1	2	3	4	5	Mathematics is best taught by using problem solving to promote understanding.
20	1	2	3	4	5	Learning mathematics is a process in which students absorb information, storing it in easily retrievable fragments as a result of repeated practice and reinforcement.
21	1	2	3	4	5	Teaching mathematics provides an excellent opportunity to promote the development of students' thinking.

(Adapted from: Pehkonen and Törner, 2004. Other influences: Van Hiele, 1986, Webb and Webb, 2004)

APPENDIX 3b: SUMMARY OF TEACHERS VIEWS ON TEACHING AND LEARNING MATHEMATICS:

ANSWERS TO TEACHING AND LEARNING MATHEMATICS QUESTIONNAIRE					
Question Number	**Question**	**Result: Teacher 1**	**Result: Teacher 2**	**Result: Teacher 3**	**Result: Teacher 4**
1	Development of concepts is the best teaching strategy to promote understanding in mathematics.	1	1	3	1
2	Learning mathematics mostly involves memorising procedures and methods in order to master the required skills.	4	4	3	4
3	Learners learn best when the teacher lectures to them.	3	3	2	4
4	In teaching mathematics, it is important to use varied exercises and applications to ensure sufficient practice of skills.	1	2	1	2
5	Mathematics in school necessarily requires a concrete dimension; abstract mathematics alone is not enough.	1	2	1	1
6	Following logic is promoted in teaching mathematics, whereas creativity and originality are not stressed.	5	3	2	2
7	Definitions of concepts are the best starting points when learners are introduced to new topics.	1	2	4	5
8	While doing mathematics, understanding the topic is a very important idea.	1	2	1	1
9	A mathematical topic is best taught by working though the list of concepts to be covered in the topic.	4	2	1	5
10	Mathematics is best learnt by doing things on your own.	4	4	4	4
11	Mathematics is a subject that is best taught to talented pupils.	4	3	5	4
12	A vital role of the mathematics teacher is to help students develop an understanding of the language and terminology used in mathematics.	1	2	1	1
13	Mathematics is best learnt by using justification and discussions of approaches and concepts used in solving problems in order to develop understanding.	1	2	1	1
14	Knowing your students and how they think, is very important for teaching.	1	2	1	1
15	The way the textbook presents a topic provides the best option for teaching the concepts making up the topic.	5	5	5	4
16	A major goal of mathematics instruction is to help children develop the belief that they can be successful in using mathematics.	1	2	1	2

17	Mathematics is best taught by taking students through a process that progressively develops understanding of the concept	1	1	2	1
18	Competency in mathematics is best tested through the ability to solve problems.	3	2	3	1
19	Mathematics is best taught by using problem solving to promote understanding.	4	2	2	2
20	Learning mathematics is a process in which students absorb information, storing it in easily retrievable fragments as a result of repeated practice and reinforcement.	2	2	1	4
21	Teaching mathematics provides an excellent opportunity to promote the development of students' thinking.	1	1	1	1

Code	**Meaning**
1	Fully agree
2	Agree
3	Ambivalent
4	Disagree
5	Strongly disagree

APPENDIX 4: QUESTIONNAIRE COLLECTING INFORMATION REGARDING QUALIFICATIONS, EXPERIENCE AND CLASSES TAUGHT IN 2011.

BACKGROUND INFORMATION

Could you please help me get a picture of your background in mathematics and mathematics teaching, as well as profile the grade 9 classes you are teaching this year by answering the following questions.

1. What are your qualifications?

   1.1) Academic: ______________________________

      1.1.1) Number of years of mathematics in this qualification:__________

   1.2) Professional:______________________________

      1.2.1) Did your course(s) include mathematics as a **teaching** subject?____

      1.2.2) If so, was the teaching of mathematics the **focus** of any of these courses? Which ones?

      ____

2. 2.1) For how many years have you been teaching **mathematics**?__________

   2.2) Do you teach any grades higher than grade 9?__________

3. Could you please describe the classes you are teaching – I am looking for the profile that determines how you feel about these students as mathematics students. (This would therefore determine the approach and strategies you would choose to use when teaching them.)

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APPENDIX 5: OBSERVATION SCHEDULE: TEACHING FOR MATHEMATICAL PROFICIENCY

Teaching activity	Observation in terms of Maths Proficiency	Notes from observation of lessons.
Presentation of concepts to be dealt with in the lesson	There is a clear introduction at the start of the lesson.	
	This introduction shows the teacher understands the mathematical concept that is the focus of the lesson.	
	The teacher finds out what the students' already understand about the concept before sharing their own understandings of those concepts.	
	The teacher takes this understanding shown by the students into account, using it as a foundation for the further teaching of the concept.	
	The lesson *structure* encourages students to link new information and previously taught concepts to each other	
	The teacher is happy to use and discuss different ways of doing procedures and solving problems.	
	Makes references to other sections of mathematics where appropriate to promote connections between topics.	
	Nurtures students' natural curiosity.	
	The teacher balances the use of activities that either encourage independent knowledge construction or encourage group construction of knowledge.	
	Uses activities or problems that force students to question their methods or the "rules" they have developed to solve problems so as to promote a deeper understanding of the mathematics.	
	Lesson content and tasks are relevant and realistic to what is being taught.	
	Facilitates scaffolding to help students perform beyond the limits of their ability.	
	Emphasises problem solving, higher-order thinking and works towards developing deeper understanding of concepts.	
Management of class discussion.	Encourages and accepts student autonomy and initiative.	
	Allows student responses to drive lessons, shift instructions strategies and alter content.	
	Responds to student contributions in a way that encourages students to engage in dialogue, both with each other and the teacher (whole group discussion)	
	Allows time for social interaction and knowledge sharing in a way that promotes the purpose of the lesson (small group discussion)	

Use and handling of questions	Avoids *too many* closed or narrow factual questions.	
	Encourages student enquiry by asking thoughtful, open-ended questions.	
	Encourages students to ask questions of each other.	
	Asks students to elaborate on initial responses.	
	Avoids answering the questions for the students.	
	Allows for "wait time" after posing questions.	
	Questioning strategies encourage reasoning and justification by students.	
Use of teaching strategies that encourage the development of a community of learners.	Provides opportunities for peer-review and self-evaluation.	
	Moves through the room interacting with students.	
	Students' errors are taken into consideration and are used to gain insight into students' previous knowledge constructions.	
	Emphasises knowledge construction as opposed to reproduction.	
	Uses manipulative, interactive and physical materials appropriate to the lesson.	
	Uses and encourages the use of language and terminology appropriate to the concept being taught.	
Finishing off of lesson.	Pulls concepts together to conclude lesson.	
	Homework set consolidates and practices skills taught during lesson or preceding lessons.	

APPENDIX 6: SEMI-STRUCTURED INTERVIEW QUESTIONS FOR TEACHERS

<u>Post module semi-structured interview questions</u>
(After teaching using the module research pack and testing the concept in the June exam assessment programme)

Now that we have been through the exam period I would like your feedback and opinions regarding the teaching of the grade 9 module we worked with this term.

1. How do you feel about the way we decided to structure the approach taken by the teaching module towards developing the straight line concept?

2. Do you feel that starting with number patterns before teaching graphs was a good idea? Why? What worked? What did not work?

3. Did this prove a useful way of connecting graphs to number relationships?

4. Were you happy with the grade 9 number pattern section – the teaching of and the development of understanding in by your students while you were teaching this section?

5. How did you feel about the way this section (number patterns) was examined in the exams? Did you feel that the way in which it was asked justified the thinking we tried to develop in our students?

6. Your students, did they show the understanding that you felt they had developed while you were teaching the section (as discussed in Q4.)?

7. How did you plan to approach the teaching of this concept and what were the mathematical ideas you planned to develop (if any) in teaching this topic?

8. Were you able to use the resources in the module to help you achieve your aims?

9. Do you feel you were able to successfully explore the interlinking of the different concepts that make up an understanding of straight line graphs?
    - Contextual relationships
    - Algebraic approach
    - Number pattern link
    - Students' recognition and understanding of the concepts gradient, y-intercepts, equation and their link to the graphical representation of the line.

11. When you were teaching this topic, did you stay with the order you had decided upon. What changes did you need to make? (Why did you need to make changes?)

12. How did your classes respond to the way in which we approached the teaching of this section this year? Did you need to change things differently for different classes?

13. What hampered/hindered/helped them? Where you able to sort out these problems?

14. Did you find yourself having to repeat the teaching of an idea because it just did not work the first time? Why do you think it did not work?

15. What were the mathematical ideas that students seemed to grapple with the most? Why do you think this was? What did you do to try and help the students?

16. Did you find that you ended up doing a lot of the talking and teaching in this section? Why do you think this was?

17. Were your classes able to participate in the discussions of the concept and in the presenting of solutions and ways of solving the problems?

18. Did you enjoy teaching this section? Why/why not? What frustrated you?

19. Talking about the exams, both internal and external,

    - Did you feel that you were trying to teach for the exam?
    - What did the way in which students answered the questions tell you about their understanding of the concepts?
    - Why do you think this is so?
    - Were you happy with the way in which this concept was asked? (internal and external)

20. What do you feel needs to be finished off next term in order to consolidate the students understanding?

21. What would you change next time you taught this section? Why – how would these changes help your students?

22. What frustrated you while you were teaching this section?

23. Is there anything else you would like to mention that we have not discussed?

APPENDIX 7: HOD SEMI-STRUCTURED INTERVIEW QUESTIONS

HOD semi-structured interview questions: The context within which the teaching takes place – getting a picture of the enabling environment outside the classroom.

1. Regarding the National Education Department: what constrains/ facilitates the teaching of mathematics in our classrooms.

- What are your feelings about the support given to us in terms of the mathematics **curriculum** for **grade 9** as it is presented by the National Education Department? (Discuss this in terms of :)

    1. The underlying theoretical concepts.
    2. This particular section for grade 9 – is it adequately prepared for in earlier grades;
    3. The practicality of the presentation of this concept in the curriculum
        - does it slowly build understanding;
        - does it cover all concepts; does it adequately prepare for grade 10; (reification)
        - Is there enough depth of understanding allowed for/required etc?

- The end of year **national assessment** for grade 9
    - Does it properly assess the mathematical proficiency of students in this area?
    - Does it encourage teachers to teach towards assessment or
    - Does it integrate concepts adequately and therefore encourage an integrated teaching approach.

2. <u>The Provincial Education Department: What constrains/ facilitates the teaching of mathematics in our classrooms.</u>

- What are your feelings about the support given to us by the Provincial Education Department in terms of:

  1. The way the National Curriculum is interpreted through the:

     - Pacesetter – its strengths and weaknesses.

     - The midyear provincial assessments – strengths and weaknesses.

  2. Support locally from curriculum advisers for Grade 9:
     - provided;
     - help/hindrance?

  3. What other benefits/constraints/ hurdles have not been mentioned?

  4. What about the provision for the professional development of teachers?
     - is it provided and by whom?
     - is it useful and pitched at the right level?

3. <u>School based factors:</u>

- Adequate time in the timetable – who determines this teaching time?

- In the existing timetable is the teaching time interrupted by many other activities? If so, does this happen too often or is it manageable?

- Of the necessary post available for mathematics teachers, are all the departmental posts filled; does the governing body provide for enough posts for the number of students that need to be accommodated?

- Are these posts filled by competent teachers?

- Are you satisfied with the professional skills and competencies of the teachers who are teaching grade 9 – that they have to ability to cover all the aspects required?

- Are there any inherited problems with respect to the grounding of the students in primary school that could affect teaching of the grade 9 syllabus?

- What about the socio-economic/ cultural diversity of the school – in terms of enhancing/ constraining/ limiting effective teaching? (e.g. student attendance, language etc)

- Interference factors – if any?

APPENDIX 8: PERMISSION LETTER AND CONSENT FORM

31 January 2011

The Board of Governors and Headmaster

I am registered as a part time student at Rhodes University, Grahamstown, and am currently teaching mathematics and accounting. I have been studying for a Master's degree in mathematics education since February 2010. I would be most grateful if you would allow me to involve the Grade 9 mathematics teachers and Head of Mathematics in contributing to my research.

The aim of my research is to collect data pertaining to participative development of a teaching module for Grade 9 linear functions. This module will be developed by myself and the other teachers currently involved in teaching Grade 9 mathematics. The information I anticipate using in my research is expected to emerge from interviews regarding the development process and teaching of the envisaged module. I will ask the teachers concerned for permission to audiotape record these interviews and the school can remain anonymous if this is required. The teachers concerned will also be assured of anonymity in the final report. The final draft will be made available to both yourselves and the teachers involved to ensure that the data collected is not misinterpreted and the thesis will be available for use by the school.

Yours truly

Sue Jackelman (Mrs)

CONSTENT FORM

I hereby agree to assist Sue Jackelman in her research. I understand that she will interview me and tape the interview for later transcription and use in the research report.

……………………………………… ………………………

TEACHERS SIGNATURE DATE