A study of mathematics teacher identity as shaped through participation in a mathematics teacher professional development programme

A thesis submitted in fulfilment of the requirements

for the degree of

Doctor of Philosophy

(Mathematics Education)

of

Rhodes University

by

Nyameka Kangela

December 2016

DECLARATION

I declare that this thesis is my own, unaided work. It is submitted for the degree of Doctor of Philosophy (PhD) at Rhodes University, in Grahamstown. It has not been submitted before for any degree or examination in any other University.

Nyameka Kangela

20th December 2016

ABSTRACT

There is an abundance of evidence suggesting that all is not well in mathematics education in South Africa. It is also common cause that the role of mathematics teachers is central to finding sustainable solutions to what is commonly referred to as a mathematics crisis. The purpose of this study is to explore the process of change in selected mathematics teachers' identities as they participated in a mathematics teacher Professional Development Programme (PDP) at Rhodes University. The core of the PDP was a teacher enrichment programme called the Mathematics Teacher Enrichment Programme (MTEP), under the aegis of the First Rand Foundation (FRF) Mathematics Education Chair at Rhodes University. MTEP foregrounded and emphasized the teaching of mathematics for conceptual understanding.

The research approach was qualitative, and it used elements of the methods associated with educational ethnography. The data was collected from five teachers from five different schools that participated in the FRF Maths Chair project. I used Wenger's (1998) three modes of belonging to analyse the identities of the five participants. This was achieved through analysing the teachers' practice with a particular focus on teaching for conceptual understanding. I used Sfard & Prusak's (2005) framework to analyse the participants' journey from an actual to a designated identity through their participation in MTEP. The participants' changing sense of belonging to MTEP was a key element in transforming their practice to teaching for conceptual understanding. I assumed the role of a participant observer during MTEP sessions, and of an outside observer as a researcher.

The study found that the selected teachers' participation in the MTEP community of practice strongly encouraged them to accumulate shared histories of learning and teaching. The study found that as participating teachers adopted and grew into their designated identity they partially embraced and implemented a conceptual teaching approach. The gap between their actual and their designated identity was partly

closed as they sought to align their teaching with MTEP’s goal of conceptual teaching.

DEDICATION

In memory of my parents Sthonga and Nomfundo Nazo, who inspired me with love for development and growth,

and

To my husband and children who have walked this journey with me.

ACKNOWLEDGEMENTS

I give thanks to God Almighty for giving me the strength, encouragement and wisdom throughout my PhD journey.

I would like to thank my supervisor, Prof Marc Schafer. I have grown academically through my interactions with him. I thank Prof for demonstrating professionalism and academic excellence. His insightful and tireless support is highly appreciated. I also appreciate his patience and understanding.

A word of gratitude to the FRF Maths Education Chair Researchers: Dr Duncan Samson and Dr Mike Mhlolo, who played a valuable role in my research journey. I also would like to acknowledge the support of Helen Averbuch, Steve Ndafengongo, Dr Gervasius Stephanus, Dr Clemence Chikiwa, Dr Clyde Felix, and Dr Ajay Narayanan. I also acknowledge and appreciate the support of my colleagues: Fortunate Gunzo, Lise Westaway and Xoliswa Magxala.

I would like to recognise the academic guidance and support of Prof Jill Adler, Chair of the Wits Maths Connect Project. Together with a special word of gratitude to all my colleagues and researchers of the Wits Maths Connect Chair.

This research study would not have been possible without my participating teachers. I appreciate their willingness to contribute to this study and for opening their classrooms for research purposes.

I would like to acknowledge the Rhodes Education Faculty lecturers for organizing PhD weeks. These research weeks have created a space for my academic growth and development.

Submission of this thesis would not be possible without the financial support of the National Research Foundation (NRF). I acknowledge and appreciate their support.

My sisters and brothers in law Sis Nolundi, Sis Prudence, Nelisa & Monde Mtyoko and Phiwe & Vuyo Mkonwana. They encouraged me and always believed in me.

To my daughter Thubelihle and sons Kulunga and Ntsukuzovuyo. A very long and tiring journey, but we have arrived now. Thank you so much for your love, support, sacrifice and encouragement.

Lastly and most importantly, to my husband: Luvuyo Kangela. I appreciate your love patience, and emotional support.

TABLE OF CONTENTS

3. RESEARCH METHODS

4. PHASE 1: PRE-OBSERVATION INTERVIEWS

LIST OF FIGURES

List of Tables

CHAPTER 1

INTRODUCTION

1.1 Mathematics teaching and learning in South Africa

Mathematics teaching and learning faces many challenges not only in South Africa, but internationally (Ogunniyi, 1996; Taylor, 1999; Taylor & Vinjevold, 1999; Ball, Lubienski, & Mewborn, 2001; Welch & Gultig, 2002; Borko, 2004; Lasky, 2005). Although South Africa, like many other countries, has embarked on a sustained effort to address these challenges, low performance in mathematics continues to prevail (Taylor, 1999; Taylor & Vinjevold, 1999; Reddy, Kanjee, Diedericks, & Winnaar, 2006). The performance of South African learners in recent international surveys has placed them in the bottom five of all participating countries (Reddy, et al., 2006).

Besides an abundance of local research data, there are international comparative studies that show that the majority of South African children are achieving performance levels well below the rest of the world. According to the report commissioned by Centre for Development & Enterprise (CDE) in 2013, there was a noticeable improvement in the grade 9 mathematics and science performance of South African learners between the years 1995 and 2011. But this improvement is still not sufficient, since South African grade 9 learners were still performing two to three grades below the average grade 8 learner from other middle income countries (Spaull, 2013).

1.1.1 Mathematics teaching & learning challenges

Research shows that if learners are to achieve excellent results in mathematics, they will need teachers to help them do so (Cuban, 1990; Adler, 1994; Borko, 2004). Teachers are necessarily at the centre of facilitating a learning environment that is conducive for learners to achieve. They are the “primary agents in education”, as Adler (1994, p. 101) puts it. If teachers are the primary agents, then efforts to improve their performance must be intensified. Teacher growth and development requires a great deal of learning on the part of participating teachers, and this will be difficult to achieve without support and guidance (Putnam & Borko, 2000; Borko, 2004).

The weak results in mathematics are an unacknowledged indicator of poor mathematics teaching (McCarthy & Oliphant, 2013). McCarthy and Oliphant also revealed that a poor attitude was one of the challenges that mathematics teaching was faced with. A poor attitude leads to resistance to interventions.

South Africa has embarked on many school support initiatives over the years, but research shows that many of these initiatives, such as help with the implementation of Curriculum 2005 (C2005), have not been sufficient. They have had little impact on the performance of most pupils in South Africa (Smit, 2001; Carrim, 2003).

Carrim (2003, p. 316) observes that "whilst C2005 was intended to professionalise South African teachers and reinforce their professional autonomy, in practice it has only managed to reinforce teachers' need for support and reinforced their lack of self-belief in their own professional competencies". Carrim (2003) also argues that the level of understanding of C2005 and the subsequent National Curriculum Statement (NCS) has generally been weak, and that there is still a wide gap between what teachers say they know and what they actually do.

Other studies reveal that C2005 was highly prescriptive in terms of policy and pedagogy, and vague in the extreme in the area of content (Taylor, 1999; Taylor & Vinjevold, 1999). Taylor (1999) concludes that C2005 unfortunately seemed to promote superficial conceptual understanding rather than systematic and grounded conceptual development. Improving the conceptual knowledge of teachers would have given them the confidence and resources to engage children at more challenging levels and undertake more adventurous tasks (Taylor & Vinjevold, 1999).

Further, within South Africa, the Eastern Cape province, where this study is located geographically, is the **second lowest** performing of the nine provinces. Alarmingly, in the Eastern Cape only 20% of grade 2 pupils from the 2001 cohort went on to pass the NSC exam in 2011, compared to 60% in Gauteng and 50% in the Western Cape (Spaull, 2013).

1.1.2 Mathematics teacher development initiatives in South Africa

Mathematics teacher improvement initiatives stemming from the Department of Education and Training mainly focus on improving mathematics content knowledge and assisting teachers with understanding the curriculum. There are few mathematics teacher development initiatives which look at how teacher identity influences mathematics teacher performance. It is thus important, if we wish to improve teaching and learning in South Africa, that we understand how mathematics and teaching combine in teachers' development of professional identities (Adler, Ball, Krainer, Lin, & Novotna, 2005).

1.2 EMPIRICAL FIELD OF THE STUDY

1.2.1 Defining MTEP

The empirical field of this study is the Mathematics Teacher Enrichment Programme (MTEP) of the First Rand Foundation (FRF) Mathematics Education Chair at Rhodes University. The First Rand Foundation (FRF) Mathematics Education Chair is one of several initiatives aimed at improving the quality of mathematics teaching, particularly in less privileged communities, by explicitly taking cognisance of individual teacher needs. The specific objectives of the Chair are a) to research sustainable and practical solutions to mathematics challenges in South Africa, b) to improve mathematics learner performance (pass rate and quality of passes) in these public schools, c) to provide leadership in mathematics education and d) to increase the dialogue around solutions to the South African mathematics education crisis. The MTEP, which is at the heart of the FRF Mathematics Education Chair, is the empirical field for this study. The FRF Chair strategy to improve mathematics teaching and learning is through a programme in which research and development are equal partners. The research agenda of the FRF Mathematics Education Chair was framed by the following research questions:

- What appropriate teaching strategies could assist in building confidence and proficiency in mathematics teachers?
- How can a positive professional identity be grown?

- What is the nature of the mathematics backlog of our pupils?
- How can effective and innovative learning and teaching resources be developed?
- What is the impact of the work of the Chair on the participating teachers?

The MTEP consists of regular contact sessions at Rhodes University for 13 participating schools, and is supported by regular in-school visits (Schäfer, 2011). The MTEP contact sessions are framed by a concept-driven model of teacher development, an approach that allows for a broad philosophy of styles and approaches, and a platform for constantly expanding and enriching mathematical ideas (Schäfer, 2011). Although the curriculum is not the departure point of MTEP, the contact sessions are nevertheless aligned with both the curriculum and classroom practice, and specifically take into account sections of the curriculum that are problematic for learners.

The MTEP sessions continuously foreground conceptual teaching as advocated by Kilpatrick, Swafford and Findell (2001). The MTEP's non-traditional approach to professional development emphasises the need to understand the contexts of teachers (Little, 1993; Joubert & Sutherland, 2008). Further, teachers who participate in MTEP willingly open up their practice to learning opportunities and participate in trying to understand problems facing mathematics teaching and learning. Effective professional development programmes change passive teachers to teachers as learners, active in shaping their professional growth (Clarke & Hollingsworth, 2002). The in-school teacher support programme which aims to link MTEP sessions with classroom practice is important in situating the professional development of teachers in realistic contexts (Clarke & Hollingsworth, 2002).

One of the five key research questions of the FRF Mathematics Education Chair relates to how a positive professional identity can be grown. This is a key question in the search for solutions to the South African mathematics crisis as it speaks directly to instilling a positive disposition in mathematics teachers (Schäfer, 2011). Kilpatrick et al. (2001) characterize a positive disposition as a productive disposition. A productive disposition is one that sees sense in Mathematics, perceives it as both useful and worthwhile, and sees the teacher as an effective learner and doer of mathematics (Kilpatrick et al., 2001).

An integral part of MTEP is the in-school support programme. The intention of the in-school support programme is to offer support in applying in the classroom what was learnt during MTEP sessions, so as to ensure a meaningful link between new knowledge and classroom practice. The in-school support programme in turn provides key insights and inputs for the planning of MTEP sessions.

Kilpatrick et al. (2001) argue that teachers should not just acquire knowledge. Teacher preparation and professional development programmes must challenge teachers and help them to analyze new knowledge so as to apply it aptly in the context of their own classrooms (Kilpatrick et al., 2001). Opportunities for teachers to discuss mathematical ideas and their representation, as well as to discuss student learning, are important for teachers learning how to teach mathematics effectively (Reed & Schaefer, 2005). MTEP therefore presents opportunities for teachers to enhance their professional development and transform their identities as teachers in order to enrich their own teaching practice.

1.2.2 Participating teachers and inviting participation

Thirteen schools in the Grahamstown Education District participate in the FRF Mathematics Education Chair project. They were selected at the start of the project in 2010 in close collaboration with the Department of Education. These schools lie within a radius of 100km from Grahamstown. Mathematics teachers at the thirteen schools participate in the project on a voluntary basis. For this study I initially selected four teachers because of their consistent participation in MTEP. Two of the teachers joined at the beginning of the project in January 2010, and the other two in March 2012. I ended up with five teachers, for reasons explained in section 3.3.2 of Chapter 3. As I was seeking stories about their teaching practice and how MTEP has contributed to their conceptual teaching in particular, I thought it would be interesting to engage with teachers who had joined at different times.

1.3 PURPOSE OF THE STUDY

The purpose of this study is to explore the process of change in mathematics teachers' professional identities through their participation in MTEP.

1.3.1 Research questions

How are mathematics teachers' identities shaped through their participation in a Mathematics Teacher Enrichment Programme and its in-school support programme?

This is the overarching research question. The three sub-questions below are more specific questions that will be used to answer the main research question. They align with Wenger's (1998) three modes of belonging, which are engagement, imagination and alignment, as discussed in the theoretical framework.

1.3.1.1 How does teacher participation in MTEP encourage or discourage teachers to accumulate shared histories of learning with respect to conceptual teaching?

1.3.1.2 How do teachers see themselves with respect to conceptual teaching through their participation in MTEP?

1.3.1.3 To what extent do teachers' styles and discourses align with the broader vision of the MTEP activities?

1.4 STRUCTURE OF THE THESIS

In Chapter 1, I discuss the rationale for the study, mathematics teaching and learning challenges in South Africa, the empirical field of the study, the purpose of the study and its research questions. In Chapter 2, I discuss the theoretical framework of the study and offer a review of relevant literature. Chapter 3 presents research methods, the data collection process, and questions validity, reliability and ethics. In Chapters 4, 5, 6 and 7, I discuss the data analysis process. In Chapters 4 and 6, I used Wenger's (1998) three modes of belonging to analyse teacher identity from first- and second-phase data gathered through a pre-observation interview schedule. In Chapters 5 and 7, I used my 'indicators of conceptual teaching' tool to analyse practice from first- and second-phase classroom observation and the post-observation interview schedule. In the final chapter, Chapter 8, I discuss the results of the study, the implications and limitations of the study, and avenues for further research.

CHAPTER 2

THEORETICAL FRAMEWORK

2.1 INTRODUCTION

This chapter outlines the theoretical framework of the study and reviews literature relevant to it. It is laid out in nine sections. In section 2.1, I discuss the ontology of identity. Identity is first defined from a psychological and then from a socio-cultural perspective. Identity from a psychological perspective emphasizes the self, whereas from a socio-cultural perspective, it is constituted through participation in a community of practice. Section 2.2 discusses the community of practice, as it is defined by Wenger (1998, 2000) and Lave & Wenger (1991). In section 2.3 I analyse Wenger's (1998) three modes of belonging (Engagement, Imagination and Alignment). In section 2.4 and 2.5, Sfard and Prusak's (2005) notion of identity is introduced, and I explain why I have combined Wenger's (1998) and Sfard & Prusak's (2005) work to explore the participating MTEP teachers' identity formation process. In section 2.6 I talk about mathematics teachers' professional identity. Traditional and emerging perspectives on teacher development programmes are introduced in section 2.7. In section 2.8, I discuss mathematics teacher development programmes (MTDP) and introduce MTEP as a MTDP. In the last section I discuss my indicators of conceptual teaching as this is a key element promoted in MTEP.

2.1.1 Definition of identity and its origins

Work on identity is an emerging aspect of the research agenda in mathematics education in South Africa (Adler, 1998; Graven, 2003) and elsewhere in the world (Lave & Wenger, 1991; Wenger, 1998; Gee, 2001; Sfard & Prusak, 2005). Early research defined identity from a psychological point of view as 'one's self concept' or 'the self' (Mead, 1934; Erikson, 1968), whereas more recent research tends to construe identity from a socio-cultural perspective (Lave & Wenger, 1991; Lave, 1996; Wenger, 1998).

2.1.1.1 Defining identity from a psychological perspective

In defining identity from this perspective, I will also define learning and teacher identity. The reason for this is to make the difference and relationship between these concepts explicit in both perspectives. Psychological perspectives on **identity** typically focus on the individual and characterize identity as something that one acquires at some point in one’s life. Its formation is internalised and is not necessarily related to participation within a group. In this perspective, identity refers to all the beliefs, ideals, and values that help shape and guide a person's behaviour. Its formation is something that begins in childhood and becomes particularly important during adolescence, but it is a process that continues throughout life. Our identity gives each of us an integrated and cohesive sense of self that endures and continues to grow as we age (Erickson, 1968).

> “Erikson therefore sees adolescence as a critical period of identity formation, in which individuals overcome uncertainty, become more self-aware of their strengths and weaknesses, and become more confident in their own unique qualities. In order to move on, adolescents must undergo a ‘crisis’ in which they address key questions about their values and ideals, their future occupation or career, and their sexual identity. Through this process of self-reflection and self-definition, adolescents arrive at their integrated, coherent sense of their identity as something that persists over time” (Buckingham, 2008, p. 2).

Hauge (2007) defines identity as the distinguishing character or personality of an individual that guides how they relate to the world around them. In this perspective, identity is internal and independent of context (Markus & Kitayama, 1991).

Learning or knowing from a psychological perspective is defined as the manipulation of symbols inside the mind of the individual. Learning is typically described in terms of an individual’s acquisition of knowledge, change in his or her knowledge structures, or growth in conceptual understanding (Peressin,i et al., 2004). Learning from a psychological perspective is seen as occurring in one or two ways: through transmission of knowledge from experts or through individual acquisition (Stein & Brown, 1997). Learning is seen as something that happens inside the brain, separate from the experience and the context of the learning situation (Hansman, 2001).

From a psychological perspective, studying **teacher identity** change emphasizes the development of new knowledge structures and cognitive skills, entities presumed to reside within the minds of teachers (Stein & Brown, 1997). Learning and identity formation is understood to involve the development in the mind of webs of chunks of knowledge and experiences and their relations to one another. These mental mappings are developed and configured over time as people develop their identity and gain a deeper understanding of areas of knowledge and their relationships with other knowledge (Hiebert, et al., 1992). Learning and identity formation in the psychological perspective can be defined independently of social participation. What is learned is independent of the context in which it is learned (Anderson, 1997). Learning and identity are thus two separate concepts.

2.1.1.2 Defining identity from a socio-cultural perspective

In contrast to psychological approaches, socio-cultural approaches focus on the groups or communities in which individuals participate. Identity from a socio-cultural perspective is formed by participating in a community of practice (CoP) (Lave & Wenger, 1991; Lave, 1996; Wenger, 1998). A CoP is defined as follows:

> A community of practice is a set of relations among persons, activity, and world, over time and in relation with other tangential and overlapping communities of practice. A community of practice is an intrinsic condition for the existence of knowledge, not least because it provides the interpretive support necessary for making sense of its heritage. Thus, participation in the cultural practice in which any knowledge exists is an epistemological principle of learning. (Lave & Wenger, 1991, p. 98)
>
> Communities of practice are the basic building blocks of a social learning system because they are the social “containers” of the competences that make up such a system. By participating in these communities, we define with each other what constitutes competence in a given context: being a reliable doctor, a gifted photographer, a popular student, or an astute poker player. (Wenger, 2000, p. 229)

Identity is an on-going and dynamic process which entails making sense of and (re)interpreting one's values and experiences (Flores & Day, 2005). Identities are malleable and dynamic, an on-going construction of who we are as a result of our participation with others in the experiences of life (Wenger, 1998). Juzwik (2006) argues that identity changes across time and space, and thus is always in motion. These changes depend, at least in part, on social and contextual interactions and not on inner or individual processes alone.

Both identity and learning involve a process of participation in a social practice (Lave & Wenger, 1991; Wenger,1998; Sfard & Prusak, 2005). Learning mathematics, for example, involves the development of each student's identity as a member of the mathematics classroom community (Anderson, 2007). Learning to teach mathematics involves the development of a mathematics teacher identity as a member of a mathematics teaching community. Learning and participation are inseparable as they involve both the 'interpersonal and informational aspects of an activity' (Greeno & Gresalfi, 2008, p. 171). "[W]e have argued that, from the perspective we have developed here, learning and a sense of identity are inseparable: They are aspects of the same phenomenon" (Lave & Wenger, 1991, p. 115).

"Learning is an integral part of a generative social practice in the lived-in-world" (Lave & Wenger, 1991, p. 35). Wenger (1998) argues that participation has broad implications for what it takes to understand and support leaning. Mathematical knowledge or learning is viewed as dynamic, constructed and reconstructed through an ongoing process of sense-making by a learner (Cochran-Smith & Lytle, 2000). "It is that learning – whatever form it takes – changes who we are by changing our ability to participate, to belong, to negotiate meaning" (Wenger, 1998, p. 226). According to Handley, et al. (2006), learning is not simply about developing one's knowledge and practice, it also involves a process of understanding who we are and in which communities of practice we belong and are accepted.

Learning should be thought of as emergent, involving opportunities to participate in the practices of the community as well as development of an identity which provides a sense of belonging and commitment (Handley, et al., 2006).

Wenger (1998) defines **learning** in terms of three modes of belonging. He argues that these modes enable one to study the formation of identities. "Students need: places of engagement, materials and experiences with which to build an image of the world and themselves and ways of having an effect on the world and making their actions matter" (Wenger, 1998, p. 271). In this perspective, learning, involves more than the acquisition of knowledge: it means becoming a certain kind of a person in the world (Horn, 2008). Learning and identity formation are thus inextricably linked. The literature on socio-cultural perspectives on learning indicates that the link is forged by the fact that they both involve a process that takes place within a framework of participation. This is in line with Adler's (2000) definition of teacher learning as a process of increasing participation in the practice of teaching, and through participation, a process of becoming knowledgeable in and about teaching.

2.2 COMMUNITY OF PRACTICE AS DEFINED BY WENGER (1998)

Wenger (1998) characterizes a Community of Practice (CoP) as comprising ***mutual engagement, joint enterprise and shared repertoire.*** He sees practice as a source of coherence within a community.

Wenger (1998) argues that associating practice with community does two things:

a) It yields a more tractable characterization of the concept of practice – in particular, by distinguishing it from less tractable terms like culture, activity and structure.

b) It defines a special type of community – a community of practice.

Within a community of practice, group members jointly share and develop practices, learn from their interactions with group members, and gain opportunities to develop personally, professionally, and/or intellectually (Lave & Wenger, 1991).

2.2.1 Mutual engagement

According to Wenger (1998), the first characteristic of a practice as a source of coherence within a community is the mutual engagement of participants.

> Practice does not exist in abstract. It exists because people are engaged in actions whose meanings they negotiate with one another. Practice resides in a community of people and the relations of mutual engagement by which they can do whatever they do. Membership in a community of practice is therefore a matter of mutual engagement. (Wenger, 1998, p. 73)

More generally, each participant in a CoP finds a unique place and gains a unique identity, which is both integrated and defined in the course of engagement in the practice. These identities become interlocked and articulate with one another through mutual engagement, but they do not fuse (Wenger, 1998). In the context of MTEP, participating teachers build relationships that enable them meaningfully to connect with the contributions and knowledge of others (facilitators and other teachers). Facilitators and participating teachers attend MTEP sessions to engage in actions aimed at improving the teaching and learning of mathematics. Wenger (1998) argues that mutual engagement involves not only our own competence, but also the competence of others. It draws on what we do and what we know, as well as on our ability to connect meaningfully to what we do not do and what we do not know. The fact that these teachers attend these sessions voluntarily is an indication of mutual engagement because it demonstrates that they are willing to share their knowledge and connect meaningfully with the contributions and knowledge of other members of the MTEP community.

> Mutual engagement does not entail homogeneity, but it does create relationships among people. When it is sustained, it connects participants in ways that can become deeper than more abstract similarities in terms of personal features or social categories. In this sense, community of practice can become a very tight node of interpersonal relationships. (Wenger, 1998, p. 76)

Participating MTEP teachers were requested to offer their lessons for video recording. The purpose of the recording was to provide an opportunity for them to examine their practice and be open to constructive criticism, with the aim of improving their teaching. This demonstrated that the teachers felt comfortable with addressing maths teaching challenges together with other teachers. According to

Wenger (2000), members must trust in each other, not just personally, but also in their ability to contribute to the enterprise of the community. This enables them to feel comfortable when addressing real problems together and speaking truthfully. In opening their practice to research aimed at providing solutions to mathematics education challenges in South Africa, they demonstrate that they are taking responsibility for their own professional development.

Mutual engagement suggests that members jointly engage in discussion, dialogue, and exchange by virtue of their co-presence and goal orientation. This engagement could include problem solving, requests for information, the discussion of developments, information seeking and coordination planning, or the negotiation of meaning (Mills, 2011). Some MTEP teachers requested support with designing projects for their learners. The Curriculum and Assessment Policy Statement (CAPS) document FET Mathematics stipulates that at most one project or assignment should be set in a year. The assessment criteria need to be clearly indicated in the project specification. The CAPS document emphasizes that the focus in the project should be on mathematics, and not involve simply duplicated pictures or the regurgitation of facts from reference material. Teachers should design rubrics which are specific to the project they set.

Through MTEP sessions and the in-school support programme (ISSP) teachers were encouraged to develop an assessment rubric specific to the project.

2.2.2 Shared repertoire

The repertoire of a CoP includes routines, words, tools, ways of doing things, stories, gestures, symbols, genres, actions or concepts that the community has produced or adopted in the course of its existence, and which have become part of its practice. It includes the discourse by which members create meaningful statements about the world, as well as the styles by which they express their forms of membership and their identities as members (Wenger, 1998).

“Shared repertoire refers to the common resources for creating meaning that result from engagement in a joint enterprise” (Clarke, 2008, p. 31). The shared repertoire in this context comprises a variety of teaching methods and learning materials which constitute a resource for teachers and their respective schools. These include participating teacher files, which document activities performed at the MTEP sessions. These materials provide teachers with a resource that promotes conceptual understanding. Teachers have also learned to develop manipulatives, which help them to promote conceptual teaching.

More generally, a shared repertoire refers to the often unspoken norms and practices by which the discipline operates (Herzig, 2010); it could include shared narratives, artifacts, discourse, and experiences (Mills, 2011), or describe common experiences shared by community members (Glazer, et al., 2005). MTEP teacher stories produced by this community in the course of its existence were presented as papers in the “How I Teach” section at the AMESA conference. (AMESA is a professional association for mathematics education in South Africa.) The papers were published in the book of abstracts of AMESA conferences held in 2012 and 2013.

2.2.3 Joint enterprise

Wenger (1998) makes three points about the enterprise that keeps a community of practice together. The first is that it is the result of a collective process of negotiation that reflects the full complexity of mutual engagement. The second is that it is defined by participants in the very process of pursuing it, and the third is that it is not just a stated goal, but creates among participants relations of mutual accountability that become an integral part of practice. Members are bound together by their collectively developed understanding of what their community is about and they hold each other accountable to this sense of joint enterprise (Wenger, 2000). The joint enterprise for MTEP is learning to teach mathematics in a manner that foregrounds conceptual understanding. This joint enterprise involves common principles to which a community adheres, and common goals toward which it strives. In teaching communities, joint enterprise reflects the mutual investment of teachers in various initiatives, such as adopting instructional resources, developing a shared curriculum, and learning new strategies and skills for the benefit of all teachers (Glazer, et al., 2005). The energies of MTEP are directed towards a common goal of teaching mathematics in a conceptually meaningful and proficient way. MTEP thus consists of a group of mathematics teachers who are making a conscious and concerted effort to collect and document a knowledge base of good practice in terms of teaching mathematics for conceptual understanding. In pursuance of that objective the MTEP community holds both facilitators and participating teachers accountable to working towards achieving this goal. It is like a thread that runs all its activities (Kangela, Mhlolo, & Schäfer, 2013).

The MTEP community exists because of mathematics researchers and educators who engage in activities aimed at improving the teaching and learning of mathematics in a way that foregrounds conceptual understanding.

Mills (2011) lists the implications for learning outlined by Mayes & de Freitas (2007), including the acquisition of habits, attitudes, values and skills in contexts and the development of identities and learning relationships. Implications for teaching in a CoP include support for identity development, facilitation of learning dialogues and relationships, and the creation of safe environments for participation and authentic opportunities for learning.

Wenger's (1998) three characteristics of a CoP, with definitions specific to MTEP, are summarized in the figure below:

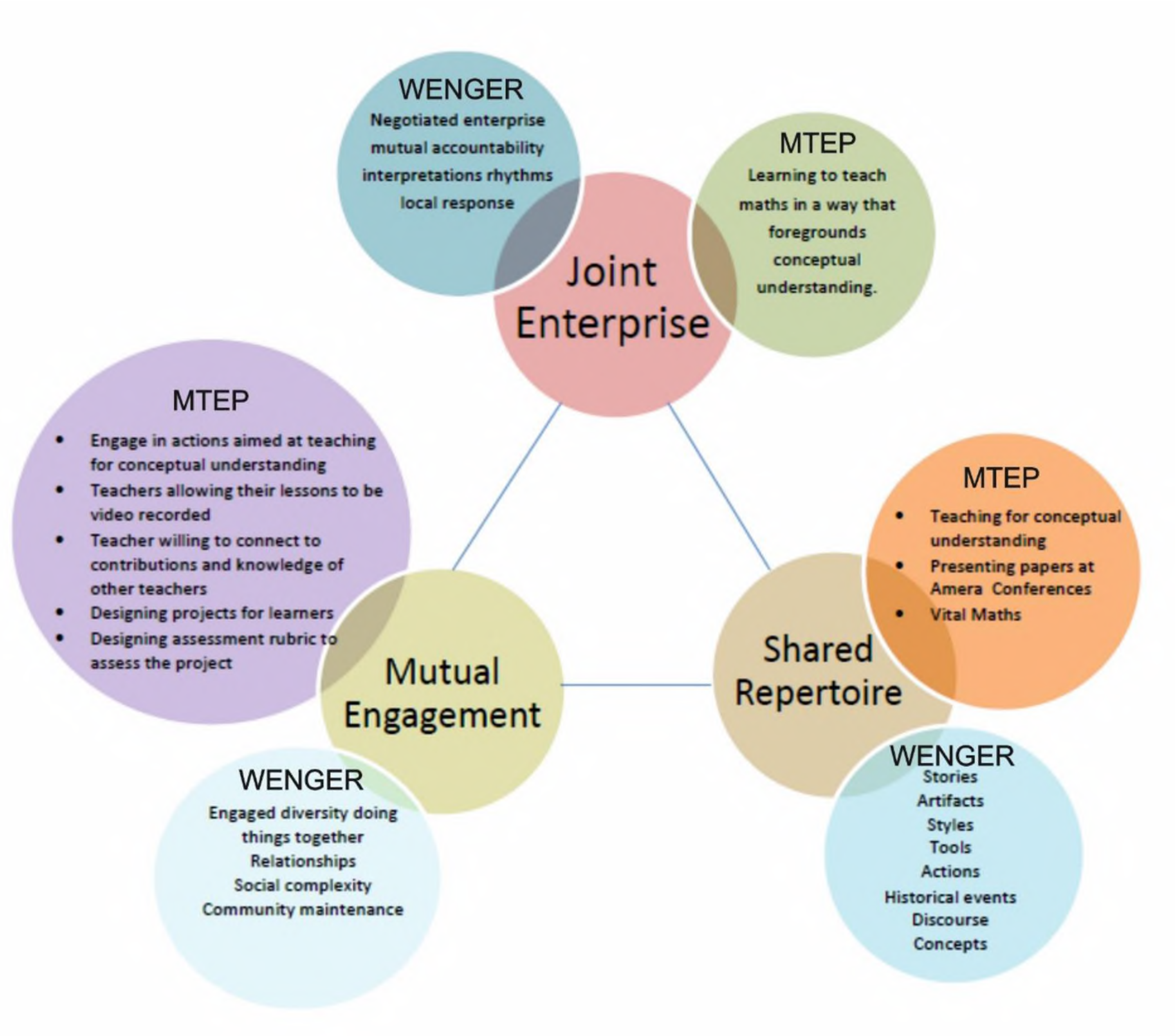

Figure 2.1 Wenger’s (1998) Three Characteristics of the Community of Practice

2.3 WENGER’S THREE MODES OF BELONGING

Wenger (1998) defines identity in terms of three modes of belonging to a CoP: engagement, imagination and alignment. According to Wenger (1998), in order to make sense of the process of identity formation and learning, it is necessary to distinguish these three modes of belonging. Figure 2.2, on page 22 provides a summary of the three modes of belonging.

2.3.1 Engagement

Engagement is defined as active involvement in mutual processes of negotiation of meaning. Engagement shapes experiences of who we are and how we are seen by others (Wenger, 1998; Sfard & Prusak, 2005; Grootenboer, Smith, & Lowrie, 2006; Anderson, 2007). Engagement means doing things together, such as talking, producing artifacts, helping a colleague with a problem. The ways in which we engage with each other and with the world profoundly shape our experience of who we are. We learn what we can do and how the world responds to our actions (Wenger, 2000). In the context of learning, engagement is not just a matter of activity, but of processes building community (Wenger, 1998). Brosnan and Burgess (2003) argue that engagement requires relationships founded on trust which allow participants to take risks and explore new ways of negotiating meaning in a supportive environment. The research questions addressed by this study are aligned with Wenger's (1998) three modes of belonging. The first question, which aligns with engagement, asks whether teachers' participation in MTEP encourages or discourages them in accumulating shared histories of learning with respect to conceptual teaching. Through this question I seek to understand if MTEP creates a safe platform for teachers to take risks in exploring new ways of teaching mathematics. I also wish to explore their unfolding histories of practice as a result of their participation in MTEP.

> According to Goodnough (2010, p. 169), "Engagement involves communities of individuals engaged in actions that are mutually negotiated. It moves beyond the notion of groups, teams or networks and involves relations that are complex, thus allowing for the establishment and sustainment of ongoing activities. Through ongoing negotiation, a joint enterprise develops over time, resulting in a shared repertoire (e.g. concepts, gestures, stories, routines and ways of acting) that guides the community and provides the impetus for continued learning. Through participation in a community or several communities, shared trajectories or pathways of learning are formed."

I seek to ascertain if there are shared trajectories formed through teachers' engagement in MTEP. According to Wenger (1998), engagement is a threefold process conjoining the ongoing negotiation of meaning, the formation of trajectories, and unfolding histories of practice.

In a community of practice, engagement in the negotiation of meanings involves the production and adoption of meanings: the two must go together. Members whose meanings are consistently rejected and whose experiences are considered irrelevant, and hence not accepted as a form of competence, will develop an identity of marginality (Tsui, 2007).

The key characteristics that emerge from the definition of engagement include mutual negotiation of activities and participation, provision of a supportive environment allowing participants to take risks, the establishment of relationships founded on trust, the establishment of sustained and ongoing activities, and a shared repertoire that guides the community and allows participants to share their histories (Kangela, et al., 2013).

2.3.2 Imagination

Imagination involves a process of producing new images and of generating new relations through time and space that become constitutive of the self (Wenger, 1998; Anderson, 2007). “Imagination involves removing ourselves from direct engagement with practice and examining action as part of the historical patterns established by a community (e. g. teachers), as well as the possibilities for future changes and developments. It focuses on self-awareness and reflection in relation to understanding others and their actions, connecting to new trajectories and locating ourselves in broader systems, creating new artifacts and processes, exploring new ways of doing things, and seeing new identities for ourselves. For teachers, this may entail envisioning how to change classroom practices in advance and during classroom action (e.g. implementing a new teaching or assessment strategy or using a new technology to support student learning)” (Goodnough, 2010, p. 169). Imagination involves exploring other ways of doing what we are doing, seeing ourselves in new ways, seeing new identities for ourselves. My second research question aligns with the concept of imagination in asking how teachers see themselves with respect to conceptual teaching through their participation in MTEP.

According to Brosnan and Burgess (2003), imagination is defined as processes involving the sharing of stories, explanations and descriptions, and the generation of scenarios (exploring other ways of doing what we are doing, other possible worlds, and other identities). Does MTEP provide a safe space for teachers to take risks, and explore other ways of teaching mathematics in order to create new images of self?

Imagination refers to an open-minded disposition evincing a willingness to explore, take risks, and make connections in order to create new images of the world and ourselves (Kangela, et al., 2013). Imagination thus helps teachers to view themselves and their profession in new ways. Teachers critically examine their practice and the teachers they would like to become. Flexibility and creativity are needed to reinvent their practices and create opportunities for novel learning (Goodnough, 2010).

2.3.3 Alignment

Alignment is revealed when members of the CoP align their activities to fit within the broader structures of the CoP and contribute to their general professional enterprise (Wenger, 1998; Anderson, 2007). Alignment involves adhering to the global practices of a community. Aligning with the practices and discourse of a broader community promotes the coordination of effort in encouraging members to direct their energies towards common goals. It involves establishing common ground and defining broad visions (Goodnough, 2010). Goodnough (2010) also warns that alignment can involve control and result in individuals being disempowered if they adhere to practices without meaningful engagement in those practices. My third research question aligns with alignment and asks to what extent teachers' styles and discourses align with the broader vision of MTEP activities. Through this question I seek to understand if teachers' styles and discourses align with the immediate and overall focus of the FRF Mathematics Chair project. A key concern of the FRF Mathematics Chair is teaching for conceptual understanding as a result of the national and global concern with the lack of skills for effectively teaching mathematics and poor learner performance in mathematics. The concept of alignment as used here does not connote a one-way process of submitting to external authority, but a mutual process of coordinating perspectives, interpretations, and actions so that they can realize higher goals (Wenger, 2000). Alignment is a process in which participants in a community become connected by bringing their actions and practices in line with a broader enterprise. It is through alignment that the identity of a large group such as an institution becomes the identity of its participants. Alignment allows us to see the effectiveness of our actions beyond our own engagement. It involves power and, as such, it is often achieved through a complex interplay of compliance and allegiance (Tsui, 2007). Alignment is therefore not just compliance but involves the co-ordination of efforts and actions for the realization of a common goal. In MTEP teachers are not required simply to comply, but are encouraged to direct their efforts towards realizing the goal of teaching mathematics for understanding.

Research (Taylor, 1999; Taylor & Vinjevold, 1999; McCarthy & Oliphant, 2013; Spaull, 2013) and policy demonstrates the necessity for mathematics teachers to be well versed in mathematics for the effective teaching and learning of mathematics. One of the most important factors in the improvement of the quality of mathematics education is excellent knowledge of mathematics content on the part of teachers.

The CoP work of Wenger (1998) provides a useful theoretical lens through which to track how teacher identities are transformed as the teachers engage, imagine and align their activities in the MTEP and broader structures of their mathematics education practice. Wenger (1998) provides the tools to work with the process of identity formation. Figure 2.2 shows Wenger’s (1998) three modes of belonging:

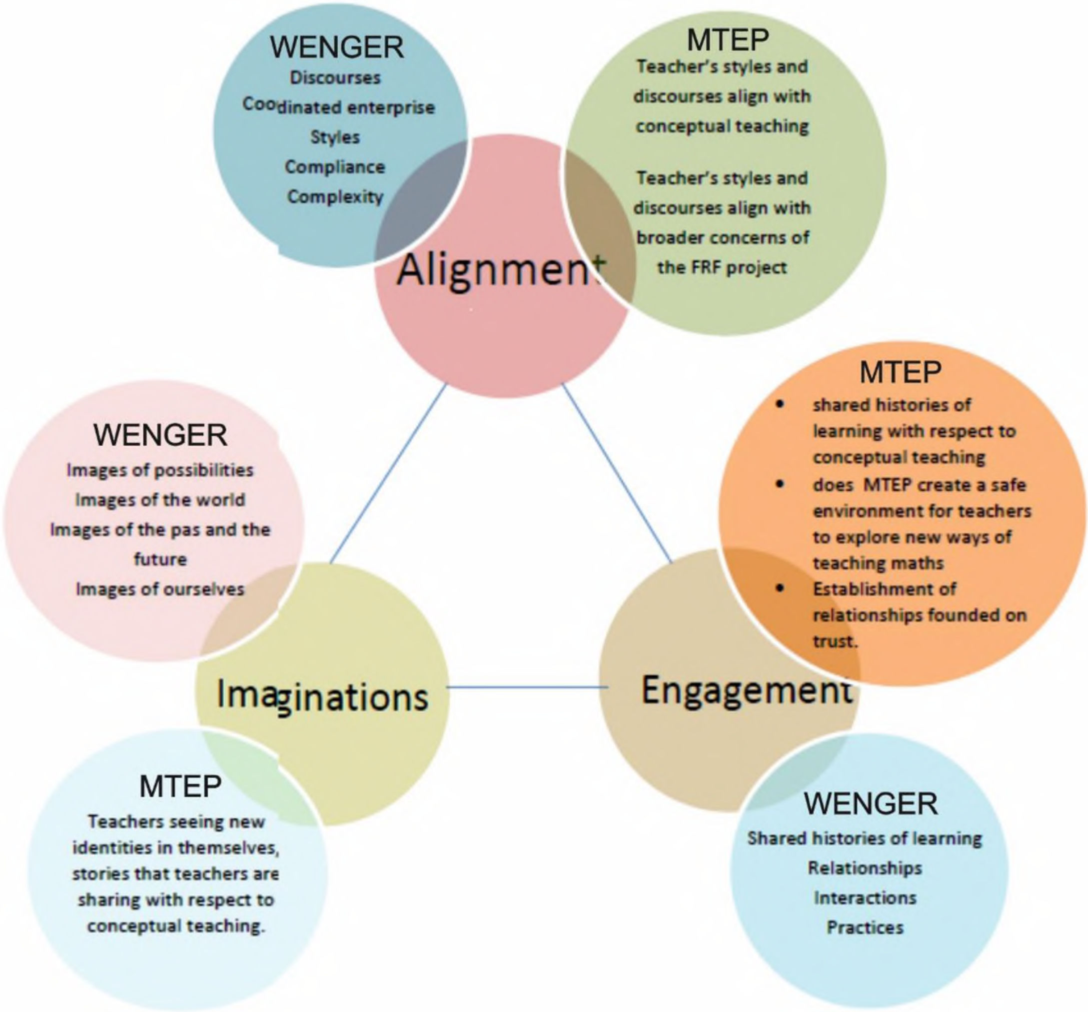

Figure 2.2 Wenger's (1998) Three Modes of Belonging

2.4 SFARD & PRUSAK'S (2005) DEFINITION OF IDENTITY

Sfard and Prusak (2005) equate identities with the stories that we agree to tell about people, including ourselves:

> Identity is a collection of stories about persons, or more specifically, as those narratives about individuals that are reifying, endorsable and significant. The reifying quality comes with the use of verbs such as be, have or can rather than do, and with the adverbs such as always, never usually, and so forth, that stress repetitiveness of actions. A story is endorsable if the identity builder, when asked, would say that it

> faithfully reflects the state of affairs in the world. A narrative is regarded as significant if any change in it is likely to affect the storyteller's feelings about the person. The most significant stories are often those that imply one's membership in, or exclusions from various communities. (Sfard & Prusak, 2005, p. 16)

They identify two sub-categories of stories, namely *actual identities*, which consist of stories about the actual state of affairs, and *designated identities*, which consist of narratives of what is expected to be in the future. Actual identities are usually told in present tense and formulated as factual assertions. Designated identities are stories believed to have the potential to become a part of one's actual identity. They can be recognized by their use of the future tense or words that express wish, commitment, obligation, or necessity, such as, have to, should, ought, must, want and so forth. According to Sfard and Prusak (2005), learning may be thought of as closing the gap between current and designated identities. Identities are likely to play a critical role in determining whether the process of learning will end with what counts as success or with what is regarded as failure (Sfard & Prusak, 2005). The work of Wenger (1998), complemented by the work of Sfard and Prusak (2005), will be used in this study as theoretical and conceptual lenses to track how participating teachers' identities grow (if at all) as they participate in the MTEP.

The MTEP 2010 teacher evaluation feedback revealed that teachers wanted more explicit support on how to teach mathematics effectively and conceptually. As the MTEP emphasized that conceptual teaching is the cornerstone of effective mathematics teaching, the content of the MTEP session therefore foregrounded the conceptual understanding of mathematical ideas and concepts. MTEP's goal was to inspire and motivate teachers to foreground conceptual understanding in their teaching. It would thus be fair to say that the designated identity of the teachers as envisaged by MTEP is that of a mathematics teacher proficient in teaching mathematical concepts that are underpinned by a solid conceptual understanding.

Although these teachers were open to learning to teach for conceptual understanding, they had their own personal designated identities; that is, they had their own stories to tell about what their identity might be at the end of the five-year project period. These stories might be different from the ones they would tell about teaching mathematics for conceptual understanding.

The MTEP and the in-school support programme provides a participation space to enable teachers to work from their current identity towards their designated identity (Sfard & Prusak, 2005). This study aims to research this journey to a designated identity and is illustrated by Figure 2.3, below. The conceptual teaching role provides the context within which to elicit participating teachers' stories and how MTEP is shaping their identities. Through conceptual teaching MTEP teachers are provided with a language which they can identify with, to talk about their identities and their practice. Research shows that there is a tendency to overemphasize procedural processes in teaching at the expense of conceptual understanding (Stein & Lane, 1996). This, according to Stein and Lane (1996), denies students opportunities to appreciate connections between mathematical ideas and to understand the mathematics behind these.

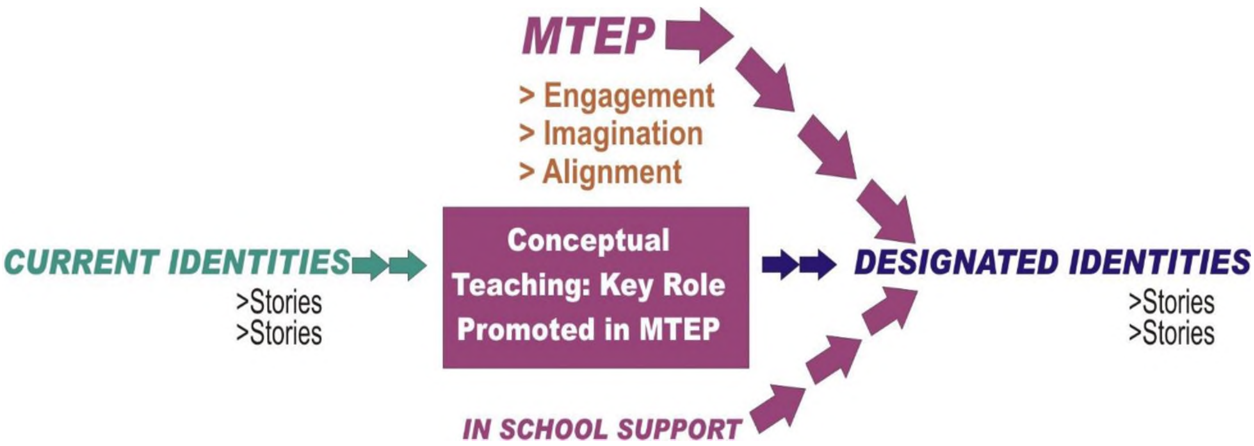

Figure 2.3: Theoretical Framing of the Study

2.5 COMBINING THE WORK OF WENGER (1998) AND SFARD & PRUSK (2005)

Although I found Wenger's (1998) modes of belonging useful in exploring the process of change in mathematics teachers, I found it important to complement them with Sfard & Prusak's (2005) notion of identity. Holland, Lachicotte, Skinner, and Cain (1998) argue that identity is not static, but fluid and constantly changing. Persons' identities develop in and through social practices. They evolve in response to the demands of social and cultural circumstances, as well as personal experiences and histories. Although one cannot deny the fact that teacher identity might be influenced by other factors outside their participation in MTEP, Sfard and Prusak's notion of identity (actual and designated identities) provides me with the tools to work specifically with stories directly linked to teachers' participation in MTEP. Identity is formed in relationships with others, extending from the past and stretching into the future. Identity is malleable and dynamic, an ongoing construction of who we are as a result of our participation with others in the experiences of life (Wenger, 1998). I combine the insights of Wenger and Sfard and Prusak to help sustain my focus on identity as influenced by the teachers' participation in MTEP.

2.6 MATHEMATICS TEACHER PROFESSIONAL IDENTITY (MTPI)

Beijaard, Verloop, and Vermunt (2000) define professional identity for the teacher in terms of the roles of subject matter expert, pedagogical expert and didactical expert. For mathematics teachers, establishing a positive professional identity involves positioning themselves within discourses of education in general and mathematics teaching in particular, in ways that allow them to be seen by others and by themselves as 'good' teachers of mathematics (Morgan, 2005). When exploring mathematics teachers' professional identities I wish to focus specifically on how their practice is informed by conceptual teaching, as this is a central focus of the MTEP and in-school support programme.

> One can design roles, but one cannot design identities that will be constructed through these roles. One can design visions, but one cannot design the allegiance necessary to align energies behind those visions. One can produce affordances for negotiation of meaning, but not meaning itself. One can design work processes but not work practices; one can design a curriculum but not learning. (Wenger, 1998, p. 229)

Identities are not roles, they are not assigned to people: they are developed and fashioned through a process of negotiating meaning in a CoP. In MTEP, participating teachers are not assigned the role of being conceptual teaching proficient mathematics teachers; but they are provided with a space in which to become proficient mathematics teachers. Learning and identity formation cannot be designed or assigned.

2.7 TEACHER PROFESSIONAL DEVELOPMENT PROGRAMMES

Below I discuss both traditional and emerging perspectives on teacher professional development programmes.

2.7.1 The traditional perspective on professional development programmes

Many South African teacher professional development programmes feature a traditional approach to teacher development, aimed at the certification of unqualified teachers, specific upgrading, and the preparation of teachers within the context of a new curriculum (Villegas-Reimers, 2003). But research shows that this traditional approach to professional development is unlikely to result in the improvement of teaching (Darling-Hammond & McLaughlin, 1995; Ball & Cohen, 1999; Shumar & Sarmiento, 2008) because as Reed and Schaefer (2005) argue, the approach often ignores the critical importance of the context within which teachers work. A traditional approach to professional development has also been criticized for failing to ensure serious and sustained learning about the curriculum, students and teaching (Ball, Lubienski, & Mewborn, 2001). (Ball, et al., 2001). Ball, et al. (2001) argue that traditional approaches to professional development lack a curriculum for teacher learning, a curriculum that considers the practices which teachers are being asked (and expected) to enact, and the mathematical knowledge that such practices entail. Hibbert (2008) defines traditional approaches to professional development as those that cast teachers as “knowledge receivers” as opposed to being “knowledge producers”. Bobis, et al. (2005) add that traditional perspectives on professional development emphasize the transmission of knowledge at one-shot workshops, as opposed to long-term classroom-based learning.

2.7.2 The emerging perspective on professional development programmes

An example of a successful professional development programme is the Lesson Study Project in Japan (Ono & Ferreira, 2010). A “lesson study” is a professional development process that Japanese teachers engage in by systematically examining their own practice. The goal of a lesson study is to improve the effectiveness of the experiences that the teachers provide to their students. The core activity in a lesson study is for teachers to reflect collaboratively on a small number of "study lessons", so called because they are used to examine the teachers' practices (Jita, Maree, & Ndlalane, 2007). A lesson study is typically characterized as classroom-situated, context-based, learner-focused, improvement-oriented and teacher-owned (Ono & Ferreira, 2010). These features have all informed the establishment of the First Rand Foundation (FRF) Mathematics Education Chair Project at Rhodes University.

In discussing the emerging perspective on teacher professional development I will focus specifically on some of the features mentioned above. These are characteristics of teacher development programmes that:

- Move beyond the boundaries of improving content only
- Are improvement oriented and context relevant
- Encourage collaborative participation
- Are teacher owned

2.7.2.1 Move beyond the boundaries of improving content only

Clarke and Hollingsworth (2002) propose an interconnected model for professional development in which change occurs through the mediating process of "reflection" and "enactment" in four distinct domains which encompass the teacher world: the personal domain (teacher knowledge, beliefs and attitudes), the domain of practice (professional experimentation), the domain of consequence (salient outcomes), and the external domain (sources of information, stimulus or support).

Campbell, Kyriakides, Muijs, and Robinson (2004) argue that professional development activities that have powerful effects on learning and help to change classroom practice, focus on content knowledge, provide opportunities for active learning, and are consonant with other learning activities. According to Desimone, et al. (2002), there are six key features that define an effective professional development programme: content focus, active learning, coherence, form, duration and participation. The first three are called core features and the last three structural.

The results of the study by Desimone, et al. (2002) showed that the six key factors influenced teaching practice positively, with respect to an increase in teachers' self-reported knowledge and skills, and changes in teaching practice. The core features worked through the structural features. Mathematical knowledge by itself is not sufficient to guarantee that teachers envision the complexity of ideas with which students must grapple (Wilson, Cooney, & Stinson, 2005). Professional development programmes should not be limited to improving content only if they are to have a positive impact on teaching practice.

2.7.2.2 Improvement-oriented and context-based

Professional development programmes are often criticized for their fragmented, episodic, and superficial nature. Teachers are thought to need "updating" rather than opportunities for serious and sustained learning about the curriculum, students and teaching (Ball, Lubienski, & Mewborn, 2001). Many of the in-service or staff development activities that teachers are offered are formal in nature, unattached to classroom life, and often a melange of abstract ideas with little attention paid to ongoing support for continuous learning and changed practices. The change from teaching to learning is significant since it implies that teacher-development opportunities must become integral to the restructuring of schools. This will of necessity involve strategies and mechanisms that are more long-range, more concerned with the interactions of groups and individual teachers, and often unique to the particular contexts in which they are invented (Lieberman, 1995).

Professional development initiatives which are not attached to the classroom and ignore context are unlikely to have a positive impact by improving practice. According to Darling-Hammond and Richardson (2009), PDPs that recognize the importance of context can help to close the gap between what teachers learn in a professional development programme and what they can actually implement in their classrooms.

Professional development opportunities should not focus on mastery of a specific teaching skill without checking whether the use of that skill has the desired effect on students (Timperley, 2008). Professional development that focuses on student learning and helps teachers develop the pedagogical skills to teach specific kinds of content has strong effects on practice (Darling-Hammond & Richardson, 2009). The MTEP and ISSP approach to mathematics teacher development is improvement-oriented and context-based. In terms of Wenger's (1998) three characteristics of a CoP (mutual engagement, joint enterprise and shared repertoire), MTEP community members share a common concern and direct their energies towards improving the teaching of mathematics.

2.7.2.3 Collective participation

The professional learning community is a new professional development model which is perceived to change classroom practice. In this model, teachers work together and engage in a continual dialogue to examine their practice and student performance to develop and implement more effective instructional practices. In ongoing opportunities for collegial work, teachers learn about, try out, and reflect on new practices in their specific context, sharing their individual knowledge and expertise (Darling-Hammond & Richardson, 2009). Birman, et al. (2000) argue that collective participation enables teachers to discuss concepts and problems that arise during the staff development programme, gives them an opportunity to integrate what they learn with other aspects of their instructional content, and contributes to a shared professional culture.

Collective participation provides a space for teachers to learn from each other. It also provides a supportive atmosphere for teachers to voice problems arising from their practice and engage in deliberations aimed at their solution. Teachers serve as support groups for one another in improving practice. Collective work within trusting environments provides a basis for enquiry and reflection, allowing teachers to raise issues, take risks, and address dilemmas in their own practice (Darling-Hammond & Richardson, 2009). Unlike many professionals, teachers generally have insufficient occasions to deliberate about or reflect upon their practice with informed colleagues. Opportunities to make their practical and often "tacit" knowledge explicit within a supportive community of educators can enable teachers to engage in issues in ways that improve their practice (Hibbert, 2008).

Clarke and Hollingsworth (2002) argue that if we are to facilitate effective professional development for teachers, we must understand the process by which teachers grow professionally and the conditions that support and promote that growth. The two authors define professional growth as an inevitable and continuing process of learning. Through collective participation, teachers learn together, grow professionally and develop new identities. In the process, teachers take ownership of problems in their practice as well as of attempting to solve them. The MTEP CoP design aligns with the three elements (domain, community and practice) of a COP as defined by Wenger (2006). A CoP is defined by a shared domain of interest, in which members build relationships to enable them to learn from each other (Kangela, et al., 2013).

2.7.2.4 Teacher owned

Learning communities in mathematics (LCM) are communities of inquiry involving teachers in developing the teaching and enhancing the learning of mathematics. In LCM, teacher groups design classroom activities that encourage pupils to get involved in inquiry in mathematics (Jaworski, 2004). Teachers are not passive receivers of knowledge in LCM, but are designers of knowledge aimed at improving practice. Teacher learning communities typically exhibit features of professional development programs, such as the establishment and maintenance of communication norms and trust, as well as the collaborative interactions that occur when groups of teachers work together to examine and improve their practice. The key components of community formation include the development of a group identity and norms for interaction, encouraging a sense of communal responsibility for the regulation of norms and behaviour, and a willingness on the part of community members to assume responsibility for colleagues' growth and development (Grossman, Wineburg, & Woolworth, 2001).

The Count Me In Too (CMIT) is another type of teacher-owned PDP initiated by the Department of Education and Training in New South Wales. In the CMIT teacher development programme, teachers participate in interviews aimed at researching the understanding of their children, as individuals and in groups. The process proved to have a powerful influence on the teachers' own professional development. They increased their knowledge of how children learn mathematics in general, they acquired a much clearer picture of their own children's understanding, and they assembled a repertoire of teaching approaches to enhance this understanding. The research team also noted teachers' willingness to engage in complex ideas over extended periods (Bobis, et al., 2005).

According to Graven (2004), teacher learning is a life-long process that teachers must themselves direct as an ongoing aspect of their professionalism. Norms of collegiality and joint problem solving encourage teachers to see that both practice and theory can be informative in the resolution of educational problems. It can, as well, lead teachers to an 'owning' of educational dilemmas and the generation of solutions specific to a context rather than relying on external authorities for solutions to classroom dilemmas (Harrington & Hathway, 1994). In MTEP teachers signal a need for support with a particular mathematics topic, and MTEP responds by providing an ISSP workshop at their schools. Teacher-owned PDPs have a positive influence on teachers' attitudes and professional growth.

2.8 MATHEMATICS TEACHER PROFESIONAL DEVELOPMENT PROGRAMMES

2.8.1 Mathematics teacher development programmes in South Africa

South Africa has embarked on a number of mathematics development programmes over the years. Some were initiated by the Department of Education, while others have been run by Universities, NGOs and the private sector. These initiatives have responded to the mathematics teaching and learning challenges that the country is facing. Research has demonstrated the need to improve mathematics teacher content knowledge in South Africa (Spaull, 2013; Taylor, 1999; Taylor & Vinjevold, 1999; Reddy, et al., 2006).

Through the work of subject advisors the Department of Education has embarked on initiatives to support and develop mathematics teachers. The role of subject advisors is to monitor and support the implementation of the curriculum; to provide and/or source relevant teaching and learning material to improve learners' performance in mathematics; to support teachers in strengthening their mathematics knowledge; and to moderate school-based assessment, including the Annual National

Assessment (www.education.gov.za). Another initiative is the Dinaledi programme, which is aimed at improving performance and increasing participation in Mathematics, Life Sciences and Physical Sciences. The purpose of Dinaledi is to promote mathematics and science teaching and learning, and improve teachers' content knowledge of mathematics and physical science (www.thutong.doe.gov.za).

Although mathematics teacher professional development programmes which are run by universities mainly focus on mathematics content knowledge and pedagogy, some have a research component attached to them, as is the case with the FRF Chair project. Research is crucial in responding to the challenges faced by mathematics teaching and learning because it explores these challenges in depth. MTEP conducts benchmark tests of grades 10 to 12 and learner performance data is collected at the beginning and end of each academic year. Each test is based on the content of the previous academic year. The benchmark tests are designed to monitor conceptual development over the course of the year. They are analysed in term of overall performance per school and in terms of specific content domains. The analysis of learner performance feeds directly into structuring the Catch-Up project as well as guiding MTEP content and in-school support. The Catch-Up project focuses on catching up pupils' mathematical backlog. It employs a cohort of qualified mathematics teachers on an ad-hoc or part-time basis, and in so doing provides an opportunity for these teachers to engage in community service. Teachers who participate in a professional development programme run at universities are exposed to research and given the opportunity to grow professionally by themselves developing as researchers.

2.8.2 MTEP as a MTDP

In Chapter 1, I described MTEP and the purpose of its existence. In this section I talk about four main themes that make MTEP different from other mathematics teacher development programmes. These are:

- In-school support programme (ISSP)
- Providing and supporting teachers to design maths teaching and learning support material

- Promoting teaching for conceptual understanding
- Highlighting the role of teacher identity in productive mathematics teaching.

2.8.2.1 In-school support programme (ISSP)

MTEP has an in-school support programme aimed at supporting teachers to link what they learned in MTEP with classroom life. A second aim of ISSP was to find out specific teacher needs for support. The projects' response to these specific needs and requests for in-school support was to run on-site workshops at schools. These workshops were aimed at equipping participating teachers with appropriate knowledge directly linked with the curriculum and their classrooms. The ISSP workshops were run in between the regular MTEP sessions.

One of the support needs raised by teachers, for example, was training in how to type mathematics question papers. Teachers were consequently trained to use Equations Editor in MS Word, as well as Geo-Gebra to teach graphs and functions. This training equipped teachers with a deeper understanding of graphs and functions and geometry, and of how to integrate these topics in their teaching.

Another specific request from teachers was for support with setting projects for their learners. The project responded by designing a grade 11 Maths Olympics project which was given to grade 11 learners. The context of the project was the Olympic Games held in London in 2012. Learners were required to design an original logo for a fictitious mathematics Olympics by using graphs to define the shapes within the logo. The project required learners to integrate GET mathematics such as shapes, symmetry and orientation with FET mathematics such as graphs, functions and algebra. It also encouraged them to investigate and explore mathematical concepts in their effort to integrate various topics in the curriculum.

Below is the logo which was given to the learners:

Figure 2.4 Olympics logo sign

This challenge presented a good opportunity for learners to learn how to use the Cartesian coordinate system to derive and apply equations in the form of straight lines, parabolas and other graphs. Designing an appropriate assessment rubric compatible with the requirements of the project emerged as another need for teacher support when they were busy with the project. Through the MTEP session and the ISSP visits teachers learned how to develop an assessment rubric for the project.

A second highlight emerging from the ISSP was the preparation and presentation of a paper for the AMESA Congress in Potchefstroom in 2012 on how to teach gradient. Gradient is a critical concept in high school mathematics. According to the requirements of the curriculum, gradient is introduced in grade 9 (GET level) and learners are expected to know and apply it throughout their FET mathematics. Grade 11 learners are expected to know how to define a gradient or slope, and how to apply the formula for calculating the gradient of a line. They are also expected to know how to sketch the graph of a linear function, given a gradient, know how to find the equation of a line, given points on that straight line. The ISSP visits revealed that some grade 11 learners lacked conceptual knowledge of gradients. Learners were provided with activities that encouraged them to look at gradients in a way that foregrounds conceptual understanding rather than a procedural protocol. This different approach was then presented at the AMESA Congress in Potchefstroom (2012).

According to CAPS FET Mathematics, teaching should not be limited to “how”, but rather feature the “when” and “why” of problem types. Learning procedures and proofs without a good understanding of why they are important will leave learners ill-equipped to use their knowledge in later life. Learners’ new conceptual knowledge of a gradient was used as a foundation to introduce and then teach analytical geometry. Through regular in-school support visits, teachers were helped to prepare presentations of the projects at the AMESA Congresses in Potchefstroom (2012) and in Cape Town (2013).

2.8.2.2 Providing and supporting teachers in the design of maths teaching and learning support materials

Part of the in-school support programme was the “ideal classroom” project which focused specifically on (re)establishing classroom spaces that are conducive to teaching and learning mathematics. This involved the equipping of classrooms with appropriate and exciting teaching aids such as posters, teaching tools such as rulers and protractors for the chalk boards, software where computers were available, and other materials developed by the FRF Mathematics Chair (Schäfer, 2011). The availability of teacher and learner support materials contributes positively to the teaching and learning of mathematics, particularly when teachers use them productively. In MTEP, participating teachers were exposed to a variety of teacher and learner support materials. These included Geo-Boards (teachers had an opportunity to design and make geo-boards), Geo-Genius, VITALmaths, wooden cubes and other algebra and geometry teaching and learning support materials.

A Geo-Board is a mathematical manipulative used to explore basic concepts in geometry such as perimeter, area and the characteristics of triangles and other polygons. It consists of a wooden board with a certain number of nails half driven in, around which rubber bands can be wrapped to create geometric shapes. Geo-Genius is a mathematics kit that gives learners the opportunity to learn the properties of shapes and objects while at play. It can also be used to teach area, perimeter, surface area and the volume of three-dimensional shapes.

VITAL maths is a multilingual collaborative research and development project between Rhodes University and University of Applied Sciences in north-western Switzerland (FHNW). The VITAL maths project produces video clips specifically designed for the autonomous learning of mathematics. These intellectually and visually appealing video clips are short (typically 1-3 minutes long) and make use of natural materials to animate and impart a variety of mathematical concepts and processes. Teachers can use these video clips for lesson preparation, personal conceptualization of mathematical concepts, and as motivational, exploratory and explanatory tools. The video clips can be freely downloaded in two formats. The MP4 format is suitable for PCs, iPods and iPhones, while the 3G2 format is designed specifically for use on older mobile phones that do not support MP4. Participating teachers have been exposed to these materials to demonstrate their significance in promoting conceptual teaching (http://www.vitalmaths.com).

Wooden cubes were used to explore the concept of an arithmetic and geometric sequence. These cubes were used to introduce grade 12 learners to Gauss's method, for example. Gauss's method is a fast and efficient way of determining the sum of an arithmetic series (http://www.pbslearningmedia.org/resource/mgbh-math-oa-gauss/ gauss-arithmetic-sequence-strategy/). The method illustrated using the sum of the first 100 natural numbers: it involves writing down the sequence to be summed *twice*, once *forwards* and once *backwards*, carefully aligning the terms as follows

$$1 + 2 + 3 + \ldots + 98 + 99 + 100$$

$$100 + 99 + 98 + \ldots + 3 + 2 + 1$$

The two terms in each column are then added, giving a sum of 101 for each of the 100 columns. This gives a total tally of 10100. However, this is double the required answer, so the sum of the original sequence is 5050. Learners were given an opportunity to use the wooden cubes to investigate Gauss's method in order to develop a visual appreciation for why the method works.

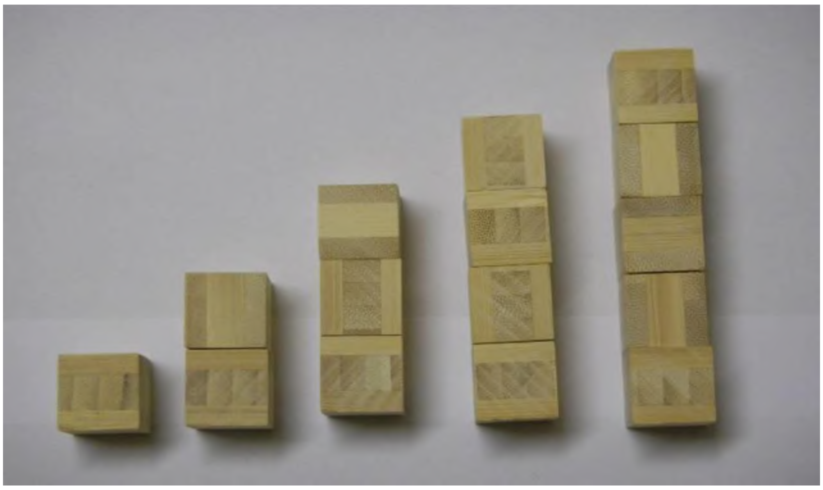

Figure 2.5 Wooden cubes

Although learners struggled to arrange the cubes they discovered that Gauss's method does not work for a Geometric series. Learners investigated, made deductions for themselves and were engaged in mathematical thinking and reasoning through this activity. The investigation provided a space for learners to develop conceptual understanding of the sum concept and taught them not to depend on formula only. This exploration led to a paper being presented by one of the MTEP teachers at the AMESA congress in Cape Town in 2013. All these teacher and learner support materials are used to equip teachers with strategies for promoting conceptual understanding, and therefore understanding of mathematics in general.

2.8.2.3 Promoting teaching for conceptual understanding

It is evident from research (Shulman, 1986; Ball, et al., 2001; Hill, Rowan, & Ball, 2005) that mathematics content knowledge is necessary, but not sufficient to guarantee success in student learning. In learning to teach for conceptual understanding, teachers do not only gain knowledge of mathematics concepts, but also enhance their pedagogical competence. Conceptual understanding is one of Kilpatrick et al.'s five interwoven and interdependent strands that define the proficient learning of mathematics. Conceptual understanding is defined as comprehension of mathematical concepts, operations and relations (Kilpatrick, et al., 2001). Teaching for conceptual understanding does not imply that the knowledge of procedures and other strands are not significant, but MTEP foregrounds conceptual understanding.

In Section 2.9.1, below, I elaborate on teaching that promotes conceptual understanding.

2.8.2.4 Identity as an important component of mathematics teaching

Graven (2003) argues that teacher development is far more complex than the simple retraining of teachers. In our context of change, teacher development needs to be supported by developing new identities for teachers (Graven, 2005). A study conducted by Graven (2005) reveals that a strengthened mathematical identity increases teacher investment in the mathematics education profession. It is thus important, if we wish to improve teaching and learning in South Africa, that we understand how mathematics and teaching combine in teachers' development and identities (Adler, et al., 2005). Identity has been used in mathematics education research as a unifying concept because it draws together and potentially connects a range of interrelated elements that are integral to our understanding of mathematics teaching and learning (Grootenboer, et al., 2006). For mathematics teachers, establishing a positive professional identity involves positioning themselves within discourses of education in general and mathematics teaching in particular, in ways that allow them to be seen by others and by themselves as 'good' teachers of mathematics (Morgan, 2005).

MTEP provides a platform for participating mathematics teachers to exchange ideas and information useful in addressing mathematics teaching challenges. In this process, these teachers try out, contribute, and learn new ways of teaching mathematics and this, according to Wenger (1998, 2000), is identity formation. In other words, the teacher learning process enhances the development of situated teacher identities. The instrumental working practices in schools enable the development of expert teachers with instrumental teacher identities, that is, teachers who adopt instrumental stances in their working lives (Kelly, 2006). Professional development initiatives that neglect the significance of transforming this default identity are thus unlikely to be successful in improving the teaching and learning of mathematics.

2.9 INDICATORS OF CONCEPTUAL TEACHING

Because this study is aimed at exploring mathematics teacher professional identity using conceptual teaching as a means to elicit teachers' stories, it is important to discuss some indicators of conceptual teaching as used in MTEP. My data collection process started two years after joining MTEP. After two years of observing MTEP sessions, participating in MTEP, being exposed to presentations and forums that defined MTEP, I came to an understanding of what MTEP was about. Literature on these conceptual teaching concepts helped define the observable indicators. Exposure and attendance of SAARMSTE conferences, research schools, and Rhodes University PhD weeks gave me insight in designing an analysis tool. The indicators for my analysis consist of the extent to which teachers:

- Promote conceptual understanding
- Promote productive maths talk
- Promote the effective use of manipulatives
- Promote and use visualization
- Show positive self-efficacy

2.9.1 Teaching that promotes conceptual understanding

Conceptual teaching is teaching that aims at conceptual understanding. Conceptual understanding describes an integrated and functional grasp of mathematical ideas. It refers to knowledge of the underlying structure of mathematics – the relationships and interconnections of ideas that explain and give meaning to mathematical procedures (Kilpatrick, et al., 2001). Rittle-Johnson and Alibali (1999) define conceptual understanding as explicit or implicit understanding of the principles that govern a domain and of the interrelations between pieces of knowledge in a domain. Researchers (Hiebert & Wearne, 1986; Rittle-Johnson & Alibali, 1999) argue that for students to have a good intuitive feel for mathematics, conceptual and procedural knowledge should be connected. This also helps to promote understanding. According to Kilpatrick, et al., (2001) a significant indicator of conceptual understanding is being able to represent mathematical situations in different ways and knowing how different representations can be useful for different purposes. Hiebert and Lefreve (1986) add that conceptual understanding acts as a screening agent to reject inappropriate procedures.

The ability to engage with learners in productive mathematical conversations about multiple ways of problem solving thus indicates a proficient conceptual understanding of mathematics (Taylor & Vinjevold, 1999; Kilpatrick, et al., 2001; Wilson, et al., 2005). Kilpatrick, et al. (2001) argue that teachers with limited conceptual understanding are ill-equipped to engage with students in productive and challenging mathematical conversations. Results of research conducted by Kazemi and Stipek (2001) shows that when conceptual understanding is promoted in classrooms, students are empowered to participate in an intellectual climate characterized by argument and justification. They argue that classroom practices that are marked by the intentional encouragement of conceptual understanding allow mathematics to drive students' engagement in activities. Establishing classroom norms that support children's development of a conceptual understanding of mathematics requires the teacher to know about both mathematics teaching and children's mathematical thinking. A sound knowledge of mathematical concepts helps the teacher to develop mathematical confidence and the freedom to move away from textbooks and a rigid curriculum (Wilson, et al., 2005). The degree of teachers' conceptual understanding relates to the richness and extent of the connections they make, and this too makes a positive contribution to their confidence in their teaching (Kilpatrick, et al., 2001). MTEP sessions are designed and presented in a manner that promotes learning and teaching for conceptual understanding. Facilitators, from both within the institution and without, present their sessions in such a way that they not only sharpen teachers' content and pedagogical knowledge but also foreground conceptual understanding. To cite an example: there is a session on statistics and probability. Teachers are grouped in pairs and asked to describe without the use of words, using gestures only, concepts such as mean, mode and median. This provides an excellent opportunity for the teachers to think deeply about these concepts and how they would present them to their learners for meaningful application of formulae.

2.9.2 Teaching that promotes productive mathematical talk

To engage learners in productive mathematical talk means taking their ideas seriously so as to support their learning (Wood, Cobb, & Yackel, 1991). Productive classroom discourse requires that teachers engage all students in discourse by monitoring their participation in discussions and deciding when and how to encourage each student to participate. By actively listening to students' ideas and suggestions, teachers demonstrate the value they place on students' contributions to the thinking of the class (White, 2003).

Teachers who promote productive mathematical talk, according to White (2000), do not stop at asking challenging questions and listening to students' answers, but go beyond that to interpret students' responses as indicators of their levels of understanding, and to adjust their teaching strategy accordingly.

According to Stein, Engle, Smith, & Hughes (2008), there are five practices that help teachers support students' engage productively with mathematics: anticipating likely student responses to cognitively demanding mathematical tasks; monitoring students' responses to the tasks during the explore phase; selecting particular students to present their mathematical responses during the discuss and summarize phase; purposefully sequencing the student responses that will be displayed, and helping the class making connections between different students' responses and between students' responses and the key ideas. Productive mathematical talk is about allowing students to access maths concepts, asking them to justify their reasoning, following up on a student answer, and questioning with encouragement to think more deeply (Morrone, Harkness, D'Ambrosio, & Caulfield, 2004).

Research by Stein, et al. (2008) shows that productive mathematical talk is a key to effective mathematics teaching. Productive mathematics talk supports students' learning of mathematics, and makes their thinking public. Productive maths talk is possible when teachers value students' ideas, explore students' answers, incorporate students' background knowledge and encourage student-to-student communication (White, 2003). Teachers must create opportunities to encourage the participation of learners (Cobb, et al. 1993), not only to answer questions but to explain their thinking (Walshaw & Anthony, 2008). They should then use these responses or explanations to stimulate a productive discussion which will eventually lead to mathematical understanding. However, some researchers (Doyle & Carter, 1984; Mercer, 1995; McClain & Cobb, 2001) warn that keeping a discussion going in a mathematics classroom does not necessarily advance student thinking. There is a difference between students' cooperating and students engaging in a productive maths talk. Fraivillig, Murphy, & Fuson (1999) argue that in order to promote productive mathematical talk teachers should intervene in advance. I therefore argue that the teacher's selection of questions and questioning style is key to promoting productive mathematical talk in a maths classroom. Productive maths talk is promoted in MTEP sessions because these sessions are interactive. Teachers are encouraged to share their ideas on how they would approach a particular mathematics concept in their teaching. Some teachers' lessons were video-recorded and the recordings were used during MTEP sessions to provide a space for teachers to engage with their own teaching and participate in discussion that sharpens their maths and pedagogy. They also presented their learner's projects in MTEP, and this helped to engender a mathematical talk. They thus experienced the value of mathematical talk in their own learning.

2.9.3 Promoting effective use of manipulatives in the teaching of mathematics

Manipulative materials are objects designed to represent explicitly and concretely mathematical ideas that are abstract. Representations commonly used in school mathematics include physical or concrete representations (manipulatives and geometric models), visual or pictorial representations (pictures, graphs, and diagrams), symbolic or abstract representations (letters, operation signs, and numerals), and dynamic electronic representations that merge characteristics of all of these modalities (dynamic geometry software and virtual manipulatives (Moyer-Packenham, Salkind, & Bolyard, 2008). Concrete manipulatives are physical objects that students can grasp with their hands (Sarama & Clements, 2009).

Kennedy and Tipps (1994) argue that manipulatives make even the most difficult mathematical concepts easier to understand by enabling students to connect abstract concepts to real objects. But Uttal, Scudder, and Deloache (1997) warn that manipulatives must be chosen and used carefully to facilitate learners' perception of the relation between the manipulative and what the teacher hopes learners will learn. An important condition for the successful use of manipulatives is that instruction and manipulative use are linked from the outset (Uttal, et al., 1997). One of the most challenging aspects of teaching using manipulatives is to facilitate students' ability to transfer what they do with the manipulatives to their conceptual and procedural understanding (Suh, 2007).

Although Ball (1992) argues that the use of manipulatives can enhance perception and thinking, she also warns that they do not carry meaning or insight: "understanding does not travel through the fingertips", as she puts it. A concrete manipulative may be interesting to young children, but may not be sufficient to advance their knowledge of mathematical concepts (Uttal, Scudder, & Deloache, 1997). Manipulatives cannot substitute for instruction, and teachers play a vital role in helping learners use manipulatives to support their understanding of mathematical concepts (Clements, 1999). Teachers should link their instruction with learners' nascent conceptions of what manipulatives represent. Learners need to be helped to see the relevant aspects and to link them to appropriate symbolism and mathematical concepts and operations (Kilpatrick, et al., 2001). Manipulatives should always be seen as a means and not an end in themselves. They require careful use over sufficient time to allow students to build meaning and make connections (Kilpatrick, et al., 2001).

Manipulatives are used in almost all of the MTEP sessions, and used purposefully. MTEP teachers experience the value of using manipulatives in their own learning. During the MTEP sessions they are asked to discuss the features of the manipulatives and share how they would use them in their teaching. Teachers are also encouraged to design a manipulative both during MTEP and in their classrooms. Among the manipulatives used in MTEP sessions are wooden cubes, which can be used to teach number patterns, sequences and series, the area and volume of shapes; and geo-boards, which the teachers designed for use in teaching geometry, perimeter and the area of shapes. The teachers are also provided with an opportunity to explore the use of technology in teaching of mathematics. Geo-Gebra, for example, is a mathematics software programme for learning and teaching mathematics suitable for mathematics at high school and university level. It deals with algebra, geometry, trigonometry and aspects of linear programming. According to the Curriculum and Assessment Policy Statement (CAPS) document, for example, teachers are expected to revise the effects of parameters a, q and p on graphs of parabolas, hyperbolas and exponential functions. The Geo-Gebra lessons have helped them to meet that expectation. Some teachers from the project have through these sessions and in-school support visits presented "How I Teach" lessons at AMESA. These lessons did not only highlight the importance of conceptual understanding, but encouraged the importance of the use of manipulatives in the teaching of mathematics.

2.9.4 Teaching that promotes visualization

There is evidence from research (Rivera, Knott, & Evitts, 2007; Makina, 2010) that promoting visualization in the teaching of mathematics can enhance learners' understanding of mathematical concepts. According to Rivera, et al. (2007), visualization in algebra enables learners to justify a formula and its parts, and offers an alternative way to understand structures and relationships through the use of variables. Mathematical visualization is the process of forming images (mentally, or with a pencil and paper, or with the aid of technology), and using such images effectively for mathematical discovery and understanding (Zimmermann & Cunningham, 1991). Presmeg (1997) defines visualization to include processes of constructing and transforming both visual mental imagery and all the inscriptions of a spatial nature that may be implicated in doing mathematics. According to Makina (2010), "Visualization incorporates those mental processes that make use of, or are characterized by, visual imagery, visual memory, visual processing, visual relationships, visual attention and visual imagination." The intuition which mathematical visualization seeks to engage is not a vague kind of intuition, a superficial substitute for understanding, but the kind of intuition that penetrates to the heart of an idea. It gives depth and meaning to understanding, serves as a reliable guide to problem solving, and inspires creative discoveries (Zimmermann & Cunningham, 1991).

Diagrams and other visual representations are essential components of the mathematics curriculum, for they convey insight and knowledge. Duval (2006) warns that there is a strong discrepancy between the common way to see figures, generally an iconic way, and the mathematical way in which they are expected to be looked at. He argues that there are many ways of “seeing”. Success in promoting visualization rests upon teachers having a clear understanding of the role of visualization, by asking carefully selected questions to help learners connect the visuals with the mathematical concept.

One of the activities practised in MTEP to promote visualization is to ask teachers to provide proofs of theorems without using the traditional procedural methods. While such activities do not imply that procedural knowledge is not important, teachers are encouraged to use their visual intuition so they can recognize its power in their own learning. In the process, teachers became familiar with the kind of questions to ask when promoting visualization in their teaching. Teachers were also provided with on-line references, such as the Visual Technology for the Autonomous Learning of Mathematics (VITAL) website, which provides teachers with suggestions on the use of manipulatives and the promotion of visualization in the teaching of maths.

2.9.5 Positive Self-Efficacy

Research shows that self-efficacy is an essential ingredient in effective teaching (Matoti, 2011; Flores & Day, 2005; Beijaard, et al., 2000). Self-efficacy refers to beliefs that individuals hold about their abilities to organize and execute the courses of action required to produce given attainments (Bandura, 1997). It is said to have a measure of control over an individual’s thoughts, feelings and actions (Matoti, 2011). The task of creating learning environments conducive to the development of cognitive competencies rests heavily on the talents and self-efficacy of teachers (Bandura, 1997).

A productive disposition is the habitual inclination to see mathematics as sensible, useful and worthwhile, coupled with a belief in diligence and one's own efficacy (Kilpatrick, et al., 2001). The more mathematical concepts that teachers understand, the more sensible mathematics becomes, and the more positive their attitudes and beliefs about themselves as learners and doers of mathematics become (Kilpatrick, et al., 2001). A belief in their own self-efficacy measures teachers' evaluation of their ability to bring about positive student change (Gibson & Dembo, 1994). Developing a productive disposition requires frequent opportunities to make sense of mathematics, to recognize the benefits of perseverance, and to experience the rewards of sense making in mathematics (Kilpatrick, et al., 2001).

The teacher of mathematics plays a critical role in encouraging students to maintain a positive attitude towards mathematics. How teachers view mathematics and its learning affects their teaching practice, which ultimately affects not only what students learn but how they view themselves as mathematics learners (Kilpatrick et al., 2001). Key characteristics common to both self-efficacy and a productive disposition are a belief in diligence, a belief in one's ability to bring about positive change, and seeing mathematics or whatever task one has to perform as sensible and worthwhile.

2.10 CONCLUSION

In this chapter, I have outlined the theory and provided a literature review relevant to the study. Identity has been defined using the arguments of Wenger (1998) and Sfard and Prusak (2005). I explain why I combined these two notions of identity to explore the process of teachers' identity formation. Mathematics teacher development programmes were discussed, with a particular focus on MTEP. Lastly, I described various indicators of conceptual teaching, since this is MTEP's approach to mathematics teacher development. In the next chapter, I discuss the research methods used to answer my research questions.

CHAPTER 3

RESEARCH METHODS

3.1 INTRODUCTION

This study is qualitative. Qualitative research presumes that people and situations are dynamic systems. A picture of an individual at a single moment is often misleading, and that individual (or group) can only be understood by observing the changes that occur over a period of time. A researcher should therefore observe over time and look for changes and processes that occur (Best & Kahn, 2006) over that period. I observed changes in the identity of teachers as they participated in MTEP over a five-year period.

In this chapter I describe the research orientation of the study. This is followed by discussion of the sampling and data collection processes. Since the study is aimed at exploring how the five teachers' identity has influenced their practice, my analysis has two parts, devoted to identity and to practice, respectively. I conclude the chapter by considering issues of validity, reliability and ethics.

3.2 RESEARCH ORIENTATION

The research orientation of this study is interpretive. An interpretive research orientation takes everyday experience and ordinary life as its subject matter, and asks how meaning is constructed and social interaction negotiated in social practices (Scott & Usher, 1999: p. 25). An interpretive approach focuses on action (Cohen, Manion, & Morrison, 2007), for according to interpretive theorists human action is inseparable from meaning. Experiences are classified and ordered through interpretive frames (Bassey, 1999; Scott & Usher, 1999). The central thrust of research within the interpretive paradigm is to understand the subjective world of human experience (Cohen, et al., 2007). This research approach therefore provides me with an appropriate platform to explore participating mathematics teachers' identities because it allows me to make sense of their world and their stories from their own perspective. Moreover, having been with the teachers for a period of five years[1] observing, questioning and trying to understand them (Bell, 2005), I was in a good position to explore how their identities were shaped though their participation in MTEP and the associated in-school support programme.

3.3 EDUCATIONAL ETHNOGRAPHY

Ethnography, sometimes known as cultural anthropology, has its roots in the late nineteenth and early twentieth century (Best & Kahn, 2006). It is defined as an approach to social research based on the first-hand experience of social action within a discrete location, in which the objective is to collect data which will convey the subjective reality of the lived experience of those who inhabit that location (Pole & Morrison, 2003). Ethnography is aligned with an interpretive paradigm, making use of participant observation, in which the observer is not a 'fly on the wall' but becomes a participant in the activity which she or he is studying (Bassey, 1999). In traditional or cultural ethnography, researchers live in the same environment and circumstances as the subjects for lengthy periods (Bell, 2005 ; Best & Kahn, 2006). Ethnography can contribute some promising techniques for the study of behaviour in an educational situation (Best & Kahn, 2006).

This study uses an educational ethnographic approach which shares some but not all the features of traditional ethnography. The primary objective of educational ethnography is to collect data that conveys the subjective reality of the lived experience of those who inhabit educational spaces (Pole & Morrison, 2003; Best & Kahn, 2006). The strength of this approach is that it recognizes the complexity of social truths (Pole & Morrison, 2003; Best & Kahn, 2006) within an educational context. In this study, the lived experiences of being and becoming a teacher of mathematics in different contexts are described and captured as stories. Being with the teachers for a five-year period has provided me with an excellent opportunity to understand their teaching context and their teaching lives. I collected data over a two-and-a-half-year period, from February 2013 to October 2015. I have captured their stories by listening to them, observing them teaching and interacting with them about their practice, all in the general context teaching for conceptual understanding.

[1] The five year period includes two years of data collection which I talk about in 3.5

My observations and interactions with teachers provided a space for me to delve deeply into aspects of their teaching stories. Through this I obtained valuable insights into their lives as mathematics teachers in general, and how their identities have been shaped by participating in MTEP in particular.

3.4 SAMPLING

3.4.1 Selection of participating schools

Thirteen schools in the Grahamstown Education District participated in the FRF Mathematics Education Chair project. They were selected at the start of the project in 2010 in close collaboration with the Department of Education. Criterion-based selection or purposive sampling was used to select the schools. Criterion-based selection requires the researcher to establish in advance a set of criteria or a list of attributes that the participants must possess. The investigator then searches for exemplars that match the specified array of characteristics (Le Compte & Preissle, 1993). Purposive sampling (McMillan & Schumacher, 2001; Cohen et al., 2007) allows the researcher to select a sample that meets his or her specific needs. Purposive sampling is often used in qualitative research studies. In case studies and ethnographies, the participants are often selected because they seem typical or are particularly interesting. The schools were selected for the project based on the criteria that they lie within a radius of 100km from Grahamstown, offer GET and FET mathematics as a subject, and have the potential for growth and development in the teaching and learning of mathematics. Mathematics teachers from the thirteen schools participated in the project on a voluntary basis.

3.4.2 Selection of participants

Initially I purposefully selected four teachers, two because of their consistent participation in MTEP since its inception, and two who joined two years later. As I explored stories about their teaching practice and how MTEP has contributed to their conceptual teaching in particular, I thought it might be interesting to engage with teachers who joined at different times.

One of the teachers who joined the project two years in withdrew from the study. The reason was that I encountered problems in securing appointments with the participant. Secondly, my interviews with the participant did not provide useful and productive data for my study. I then approached two more teachers in case of losing another participant. This is why I finally ended up with five teachers: four who joined at the beginning of the project and one who joined two years later.

Purposive sampling allows the researcher to select those participants who will provide the richest information, those who are the most interesting, and those who manifest the characteristics of most interest to the researcher (Best & Kahn, 2006). There is little benefit in seeking a random sample when most of the random sample may be largely ignorant of particular issues and unable to comment on matters of interest to the researcher. Though the participants may not be representative and their comments may not be generalizable, this is not the primary concern in such sampling. The purpose is rather to acquire in-depth information from those who are in a position to give it (Cohen et al., 2007).

The five teachers signed a participation consent form which provided them with details about their rights to participate or withdraw, issues of confidentiality, privacy and anonymity (Cohen et al., 2007). I explained to the teachers at the start of the research that I would not use their names so as to ensure anonymity. Since the teachers agreed to participate in this study, pseudonyms were used. I called them Teacher 1, Teacher 2, Teacher 3, Teacher 4 and Teacher 5. Details of the teachers' qualifications, their experience and the times when they joined the project are given in Table 3.3(a), below:

Table 3.3(a) Teacher qualification table

Teacher	Qualifications	Teaching Experience	Time when joined the project
Teacher 1	Higher Educational Diploma B Com (General) BED (In service) Master in Mathematics Education	16 years	Beginning of the project
Teacher 2	Diploma in Education BSc (Mathematics)	12 years	Beginning of the project
Teacher 3	BSc Mathematics	12 years	Beginning of the project
Teacher 4	Secondary Teacher's Diploma (STD) Further Diploma in Education (FDE) BED (Inservice) BED (Honours)	14 years	Beginning of the project
Teacher 5	Secondary Teachers Diploma Advanced Certificate in Education	11 years	A year and quarter later than the four teachers

3.5 THE RESEARCHER

A researcher should be part of, rather than detached from the research act in an educational ethnographic study (Walford, 2001). In qualitative research, the researcher is a primary instrument of data collection. I collected, partly transcribed and analysed data in this study. I am a South African woman from Duncan Village in East London. I obtained a Bachelor of Science degree in Applied Mathematics and Statistics, and taught mathematics for five years without a professional teaching certificate. I then registered for an HDE and B.ED (Mathematics and Educational Psychology), and taught for two-and-a-half years at a teacher training college. Thereafter I left teaching to join a prestigious programme at Eskom that was recruiting female science graduates to pursue a career in the management of an engineering business. Through this programme I registered for a Master of Science in Engineering Business Management degree. Although I was employed as a Regional Pricing Advisor in Eskom, the greater part of my work was to conduct pricing-related training.

In 2010, I registered at Wits University and was involved at the Wits Maths Connect Chair. While I was registered for a PhD in mathematics education I also studied theory at Masters' level. This exposure, together with my continued PhD journey at Rhodes University, has sharpened my understanding of mathematics and its teaching. My knowledge of teacher identity has also been enriched. The Rhodes Mathematics Education Chair provided me with rich, in-depth knowledge of mathematics education, particularly conceptual teaching, a key strategy promoted by the FRF Maths Chair.

3.5.1 Researcher role in the FRF Project

During the MTEP sessions that the participating teachers attended, I took a participant-observer role. In the in-school support programme, however, I took a far more pro-active role as coordinator of the programme. Both the MTEP and the in-school support context provided me with the empirical field for this study. My key role in the in-school support programme was to support teachers in linking the contents of the MTEP with classroom practice. The relationships established by the researcher with the individuals being studied can be complex and varied, ranging from outside observer to participant observer (Best & Kahn, 2006). The relationships I established with teachers during my research period were changeable. My role as a researcher in this study was complex in the sense that I was a participant observer during MTEP sessions and the in-school support and an outside observer when I conducted research with the teachers.

As a participant observer I was provided with an ideal opportunity to observe and engage with the teachers. Observation has the distinct advantage of allowing the researcher direct access to the activities they observe (Scott & Usher, 1999; Cohen et al., 2007). The weakness of participant observation is that it is prone to bias (Cohen et al., 2007), and the participant observer is likely to reproduce existing perspectives on phenomena (Yin, 1984). In order to overcome this potential shortcoming, I wrote comprehensive descriptive notes with direct teacher quotations (Best & Kahn, 2006). Participant feedback was also used to maintain interpretive validity of the research (Johnson, 1997). I shared my interpretation of teachers' stories with them so that my personal views and perspectives were eliminated.

3.6 DATA COLLECTION

My data collection process included pre-observation interviews, classroom observations, and post-observations interviews. Table 3.4(a), below, provides an overview of the data collection process.

Phases	**Data gathering tool**
1	Pre-observation Interview Classroom observation Post-observation Interview
2	Pre-observation Interview Classroom observation Post-observation Interview

Table 3.4(a) Data collection process table

3.6.1 Pre-Observation interviews

The pre-observation interviews were conducted in phases one and two, always before observing the participants in class. The main objective of these interviews was to elicit stories about the teachers' actual and designated identities, as discussed in the theoretical framework. The pre-observation interview was aimed at exploring teachers' experiences as mathematics teachers in relation to their conceptual teaching. Through the pre-observation interview I explored stories about the image of self that teachers had before they joined MTEP, and how they saw themselves with respect to conceptual teaching as a result of their participation in MTEP. The teachers were asked the same questions in both phases one and two.

I used Wenger's (1998) three modes of belonging in order to make sense of each teacher's identity formation process. The kinds of questions I asked the teachers in the pre-observation interviews relate to how MTEP and the in-school support have created a productive environment for them to accumulate a history of shared experiences, as well as their experiences of involvement in MTEP and its in-school support. These questions helped me make sense of their "engagement" (Wenger, 1998) in MTEP. In order to make sense of their "imagination" (Wenger, 1998), I explored how they saw and positioned themselves as mathematics teachers in relation to conceptual teaching. It is through these interviews that I investigated whether teachers saw themselves in new ways as a result of their involvement in MTEP. In researching their "alignment" (Wenger, 1998) with MTEP as well as the national mathematics agenda, I asked them to reflect on how they saw their activities in relation to the national curriculum and the broader mathematics education agenda. I sought to find out how the teachers saw themselves in terms of their contribution to solving the national mathematics education crisis. Through the pre-observation interview I asked:

- What are your experiences of involvement in MTEP and its in-school support?
- Has MTEP created an environment which makes it easy or difficult for you to talk about mathematics and mathematics teaching? Support your answer.

- How does your interaction with members of the MTEP community shape the way you see mathematics?
- How has your mathematics content been influenced/impacted through your participation in MTEP?
- In what ways as a result of your participation in MTEP has your teaching changed with respect to the following: a) confidence in teaching mathematics; b) how you engage with learners; c) your attitude and belief about mathematics as being sensible and worthwhile; d) your approach to teaching or any other aspect you would like to share.
- How would you like to see yourself as a mathematics teacher in future as a result of your participation in MTEP?
- Do you feel that your involvement in MTEP has created a new image of yourself as a professional mathematics educator? Please give reasons for the answer provided.
- What are your views about teaching for conceptual understanding?
- If you were to encourage other teachers to pursue conceptual teaching, would you do it, and why?
- Please tell me about an unforgettable experience during your involvement in MTEP and its in-school support. Why was it memorable?

3.6.2 Classroom observations

The pre-observation interview was followed by classroom observation of an FET mathematics curriculum lesson taught by the participating teachers. The teachers' lessons were recorded and transcribed (Bassey, 1999; Bell, 2005; Best & Kahn, 2006). The purpose of the video recording was to observe their teaching and not to make value judgements about it. The focus was on observing how their identity grew in their conceptual teaching by eliciting their stories. As articulated earlier, conceptual teaching was the key focus area promoted in MTEP, and conceptual discourse was what I used to talk to teachers about their practice and how they see themselves. The lesson formed the basis for the post-observation interview. The teachers taught the same lesson in both phases.

I initially observed the video-recorded lessons alone, and from this developed questions which helped me engage with the teachers on the subject of their conceptual teaching. The advantage of video recording is that it captures the lesson content and almost all the events that take place in the classroom, including visual and verbal content, and limits the chances of information being distorted (Opie, 2004). Video data is more versatile than any other form of data because it opens an opportunity for multiple viewers to make discoveries, and can also be kept as a valuable educational resource provided ethical compliance processes are followed (Opie, 2004).

The classroom observation and post-observation interviews helped me elicit stories of how teachers saw or positioned themselves in relation to teaching that promotes conceptual understanding, engaging learners in productive mathematics talk, using manipulatives, and visualization, and demonstrating positive self-efficacy. The video recorded lesson was also used to stimulate the teachers to talk about their practice in their real-world context.

I video recorded a total of ten lessons – five lessons in Phase 1 (at the start of the data collection period) and five lessons in Phase 2 (towards the end of the project's five-year period). Given that the teachers joined the project at different times, their identities may not necessarily have been shaped in the same way.

3.6.3 Post-Observation interviews

The post-observation interviews were conducted after each video recorded lesson. The five indicators of conceptual teaching previously articulated served as a framework for analysing the teachers' practice. It is in the post-observation interviews that I listened to the participants' stories in direct relation to the lessons that were recorded. These interviews provided an ideal space for the teachers to talk about their teaching and their journeys from their current to their designated identities in both phases. "Another advantage of interviewing is that the interviewer can explain more explicitly the investigation's purpose and just what information he or she wants" (Best & Kahn, 2006; p. 335). Both pre- and post-observation interviews were open-ended in nature, so that I could ask for the teachers' own insights and use that as a basis for further inquiry (Yin, 1984; Pole & Morrison, 2003). The interviews were conducted at the participants' schools during school hours. The interviews were audio-recorded and transcribed for complete and objective analysis (Bassey, 1999; Bell, 2005; Best & Kahn, 2006).

3.7 OBSERVATION AND INTERACTIONS WITH THE TEACHERS

Initially I intended to use the journal entries of the participating teachers. I gave each teacher a journal and asked them to record their experiences of participating in MTEP and the in-school support programme. In particular, they were asked to reflect on how they used aspects of what was learnt during MTEP in their own teaching. My in-school support interactions provided the framework and structure for the teachers to write in their journals. The teachers were asked to make notes of whether or not they incorporated concepts learnt from MTEP in their teaching. I requested the teachers to note areas of change in their teaching (if there were any) as a result of their participation in MTEP activities.

However, the teachers indicated that they did not have sufficient time to write up their experiences in a journal. I was thus obliged to abandon this intention, and rely only on the interview and observation data. This was a pity, as I thought the journal data would provide personal records of situations and events (Bell, 2005; Robinson & Reed, 1998) and valuable information (Bell 2005; Walliman, 2006).

3.7.1 Teachers' journey from actual to designated identities

Through phases one and two of the data collection process I wanted to elicit stories about the teachers' journey from their actual to their personal and MTEP's designated identity, as discussed in the theoretical framework. MTEP's designated identity involves teaching that demonstrates the five indicators of conceptual teaching. The second phase took place towards the end of the study, when I focused on each teacher's views on their journey from Phase 1 to Phase 2. I listened to teachers' stories in relation to the five indicators of conceptual teaching, and tried to understand if they were similar to or different from those told in Phase 1. This formed the basis for tracking and exploring the process of change among teachers, from their current to their individual and MTEP's designated identities through their participation in MTEP. Sfard & Prusak (2005) helped me to talk about identity in relation to practice as influenced by participation in MTEP.

3.8 DATA TRANSCRIPTION

All the data collected through pre-observation interviews, classroom observation and post-observation interviews was transcribed. Initially I transcribed the first three pre-observation interviews and this provided a beautiful space for me to deepen my understanding of the data. The nature and the design of my study required that I focus comprehensively on data collection, spending time watching the video-recorded lessons over and over so that I could design a post-observation interview schedule for each teacher. I employed a Rhodes University education department student to help transcribe my data.

There are commercially available software packages for coding qualitative data, and these can be employed in video data analysis. But software presses the researcher to make qualitative coding judgements too early in the process of reviewing the field notes or video tapes (Green, Camilli, & Elmore, 2006). I did not use the analysis software because it did not align perfectly with my coding, since I coded specifically using Wenger (1998)'s three modes of belonging and my Conceptual Teaching tool [see Table 3.5 (a)].

3.9 DATA ANALYSIS

My study was aimed at exploring how teacher identity grew from actual to designated with respect to conceptual teaching. I analysed identity using Wenger (1998)'s three modes of belonging and my Indicators of Conceptual Teaching Tool. Sfard & Prusak (2005) helped me combine identity with practice to understand how teachers' identities grew in the period of their participation in MTEP.

Data analysis, according to Bernard and Ryan (2010), starts even before one begins a research project. It is defined as a process through which we make sense of raw data and communicate the essence of what it reveals (Patton, 2002; Best & Kahn, 2006; Bernard & Ryan, 2010). The process involves organizing, accounting for, and explaining the data in terms of the participants' definitions of the situations, noting patterns, themes, categories and regularities (Cohen et al., 2007). According to Cohen et al. (2007), there are five ways of organizing and presenting the data analysis. One of them is organizing it by the posed research question.

"This a very useful way of organizing data, as it draws together all relevant data for the exact issue of concern to the researcher, and preserves the coherence of the material. It returns the reader to the driving concerns of the research, thereby 'closing the loop' on the research questions that typically were raised in the early part of an enquiry. In this approach all the relevant data from various data streams are collated to provide a collective answer to a research question" (Cohen et al., 2007: p. 468). Data analysis in this study was organized according to the research questions.

I used elements of a deductive analytical approach in this study. According to Best and Kahn (2006) and Thomas (2006), deductive data analysis allows the researcher to reach an understanding of the data by framing the research process around a preconceived theoretical perspective. As I had a well-defined theoretical foundation and design for this study, my analysis was shaped by the logic of this design and thus proceeded in a deductive manner. In order to make sense of data gathered from pre-observation interviews, my observations and interactions with the teachers, I coded and categorized the data according to Wenger's (1998) themes of engagement, imagination and alignment. Below I discuss in detail how the teachers' identity analysis was conducted.

3.9.1 Identity analysis using Wenger (1998)'s three modes of belonging

As I indicated in Chapter Two, my research questions are framed according to Wenger (1998)'s three modes of belonging, and my analysis is also guided by these three modes of belonging. I combined Wenger (1998)'s definitions with other researchers' definitions mentioned in Chapter Two in order to code my data. This framework helped analyse teacher's experiences of involvement in MTEP and the in-school support programme. The analytical framework is as follows:

3.9.1.1 Engagement

The first research question aligns with engagement and asks: How does teacher participation in MTEP encourage or discourage teachers to accumulate shared histories of learning with respect to conceptual teaching? The following questions guided my coding:

- Have the teachers produced or adopted the meaning of teaching mathematics for conceptual understanding?
- Has the MTEP environment provided a supportive environment that allowed the teachers to take risks and establish relationships founded on trust?
- Is there a shared repertoire that guides the community and allows participants to share their histories?
- Is there a new pathway of learning that has been formed through teacher participation in MTEP?

3.9.1.2 Imagination

The second research question, which aligns with imagination, asks: How do teachers see themselves in regard to conceptual teaching through their participation in MTEP?

- Do the teachers view themselves differently with regard to conceptual teaching through their participation in MTEP?
- Have teachers generated scenarios, explored other ways of teaching mathematics, other possible worlds, other possible identities?

3.9.1.3 Alignment

The third research question focuses on alignment and asks: To what extent do teachers' styles and discourses align with the broader vision of the MTEP activities?

- Do teachers' styles align with MTEP's principal goal of teaching mathematics for conceptual understanding?
- How do teachers see their participation in MTEP contributing to mathematics teaching and learning in general?

The third question will be answered by analysing if teacher practice aligns with MTEP's designated identity.

3.9.2 Analysis of practice using the conceptual teaching analysis tool

I analysed teacher practice using data gathered from classroom observations and post-observation interviews. I carefully looked at the classroom observation transcript and identified evidence of teacher utterances described in the Indicators of Conceptual Teaching Tool. The post-observation interview helped to deepen my analysis because I understood why the teachers made these utterances.

Table 3.5(a) below shows the Indicators of Conceptual Teaching Tool I used to analyse teacher practice.

Criterion	Code	Definition	Observable indicator
Questioning that promotes conceptual understanding.	QCU	Conceptual understanding refers to the knowledge of the underlying structure of mathematics – the relationships and interconnections of ideas that explain and give meaning to mathematical procedures (Kilpatrick et al., 2001). Both procedural and conceptual knowledge are considered necessary aspects of mathematical understanding. Thus to teach for mathematical understanding the teacher must include both procedural and conceptual understanding (Wearne & Hiebert, 1988b).	QCU1: The teacher asks probing questions that require mathematical reasoning. QCU2: The teacher asks questions that probe prior knowledge of mathematical concepts. QCU3: The teacher asks questions that are purposefully aimed at connecting maths ideas and concepts in order to give meaning to mathematical procedures. QCU4: The teacher asks learners to describe their solution methods. QCU5: The teacher asks learners to explain what other learners/ peers have said. QCU6: The teacher asks questions that require students to make connections with real life contexts. Non examples: The teacher mainly focuses on teaching procedures and does not encourage or link procedures with conceptual understanding.
Promoting productive mathematical talk	PMT	To engage learners in productive mathematical talk means that teachers take student's ideas seriously in their attempt to support students understanding (Wood et al., 1991) Teacher encourages students to describe their solution methods fully and uses probing or challenging follow-up questions to promote elaboration.	PMT1: The teacher invites questions from learners, give adequate time for learners to ask questions. PMT2: The teacher welcomes errors and uses them to build mathematically correct concepts. PMT3: The teacher uses learner responses and asks probing questions to promote elaboration. PMT4: The teacher asks questions which push learners to go beyond one solution method. The teacher provides a space for learners to discuss these solutions through questioning. PMT5: The teacher encourages learners to support their answers by asking the 'why' questions and probe further. PMT6: The teacher encourages mathematical reflection. Non examples: The teacher asks questions which do not challenge learners to support or justify their reasoning. In other words, the teacher only asks the 'what" questions and does not ask the "why" and the "how" questions.
Use of manipulatives	UOM	Manipulative materials are objects designed to represent explicitly mathematical ideas that are abstract. They have both visual and tactile appeal and can be manipulated by learners through hands-on experiences (Moyer, 2001). Teachers play a vital role in helping learners use manipulatives to support understanding of mathematical	UOM1: The teacher draws the attention of the learners to the manipulatives. In other words, the teacher does not just bring manipulatives to classroom, he/she uses them in teaching. UOM2: The teacher uses the manipulatives for the purposes of the lesson. UOM3: The teacher asks questions that require learners to verbalise mathematics ideas that they notice from the manipulatives.

		concepts (Clements, 1999).	UOM4: The teacher assists learners to apply maths ideas gained from using manipulatives to support the aim of the lesson. UOM5: The teacher provides learners with more than one type of manipulative to allow learners to explore rich mathematics ideas. UOM6: The teacher asks the learners to circulate these different types of manipulatives to provide a space for learners gain more maths concepts in relation to the aim of the lesson. Non- examples: The teacher brings manipulatives in the classroom and does not use them for teaching.
Promoting visualization	PVZ	Mathematical visualization is the process of forming images (mentally, or with a pencil and paper, or with aid of technology) and using such images effectively for mathematical discovery and understanding (Zimmermann & Cunningham, 1991).	PVZ1: The teacher asks questions that purposefully push learners to verbalize their mental image of a maths concept. PVZ2: The teacher asks the learners to express this/her mental image using pencil and paper. PVZ3: The teacher asks questions which require learners to share their mental image with classmates. PVZ4: The teacher assists the learners through questioning or other ways to relate their mental image with the lesson aim. PVZ5: The teacher uses manipulatives and pushes learners through probing questions to verbalize their mental images. Non- examples: The teacher fails to provide opportunities that brings learner's mental images to the fore and does not link these images with the lesson aims.
Self-efficacy	SEF	Self-efficacy refers to beliefs in one's capabilities to organize, and execute the courses of action required to produce given attainments (Bandura, 1997). Productive disposition is the habitual inclination to see mathematics as sensible, useful and worthwhile, coupled with a belief in diligence and one's own efficacy (Kilpatrick et al., 2001). I find a relationship between self-efficacy and productive disposition because they both talk about teacher's view of his or her capabilities to positively influence student learning.	SEF1: The teacher encourages and expects all learners to make a contribution in the classroom. SEF2: The teacher motivates learners to ask questions to make sure that learner's questions are answered through productive maths talk or any other way. SEF3: The teacher creates an atmosphere that is conducive for learners to express their thinking freely without fear. SEF4: The teacher uses manipulatives in a manner that motivates learners to enjoy and learn mathematics. SEF5: The teacher encourages and affirms learners. SEF6: The teacher monitors learner's work to make sure that their notes are neat and mathematically correct. Non-examples: The teacher displays a negative attitude to learner's questions and their enthusiasm to learn, does not motivate learners to be involved in the lesson, does not bother to look at learner's work and does not encourage learners to work diligently.

Table 3.5(a) Indicators of Conceptual Teaching Tool

The post-observation interview schedule had questions which were the same for all five teachers, together with other questions that were lesson-specific. Through the latter I wanted to understand why teachers did what they did in their respective lessons. The post-observation interviews helped me to obtain and explore rich data for promoting conceptual understanding.

Below are the questions of the Post-Observation Interview Schedule:

- What was the goal of the lesson?
- Do you think you have achieved what you wanted to achieve in this lesson? Why?
- What mathematics concepts do you think learners have learned from this lesson?
- Is there any part of this lesson that you would attribute to your involvement or participation in MTEP?
- If you were to share this with other mathematics educators, what aspects of this lesson would you like to highlight to them?
- What manipulatives did you use in your lesson? Why?
- Has the use of these manipulatives in your opinion supported learning probability?

The three modes of belonging, together with the conceptual teaching indicators, were therefore the central themes that I used to analyse my data. I conducted my analysis by examining how the teachers responded to these constructs during pre-observation interviews, classroom observation and post-observation interviews. This analysis design aligns directly with my research questions. Sfard and Prusak (2005) argue that as stories, identities are human-made, they have authors and recipients, and they are collectively shaped, even if individually told. People to whom our stories are told, as well as those who tell stories about us, may be tacit co-authors of our own designated identities (Sfard & Prusak, 2005, p. 18). I will thus be narrating my story of the teachers' stories.

3.10 VALIDITY AND RELIABILITY OF DATA

Engaging in multiple methods of data collection leads to a more valid, reliable and diverse construction of realities (Yin, 1984; Best & Kahn, 2006; Cohen et al., 2007). A multi-method approach contributes positively to validity and reliability because it provides a broader and richer view of a particular reality being investigated (Cohen et al., 2007). The method essentially provides multiple measures of the same phenomenon (Yin, 1984). This study involved a prolonged and often repetitive process of observing events, which helped to establish data reliability (McMillan & Schumacher, 2001; Cohen et al., 2007). "Dependability involves member checks (respondent validation), debriefing by peers, triangulation, prolonged engagement in the field, persistent observations in the field, reflexive journals" (Cohen et al., 2007; p. 149). A synthesized report drawn up from data gathered through pre-observation interviews, classroom observation and post-observation interviews was given to the teachers for accuracy and comment to ensure the dependability and reliability of the data. Coherence in this study was preserved since data was organized according to the research questions.

3.11 ETHICAL CONSIDERATIONS

This research was conducted in an ethical manner and in such a way as to guarantee integrity. The researcher requested permission for access and acceptance from the Grahamstown Education District Committees and school principals (Cohen et al., 2007; Bell, 2005; Bassey, 1999). Participants were provided with a brief outline of my study including a definition of procedures, the type of information to be gathered, and reasons for the research (Opie, 2004). The participants were informed about what would be done with the raw data; they were also informed of their rights insofar as their participation in the research was concerned, and that they were free to withdraw from the process at any point without prejudice. The research proposal was approved by the Rhodes University Higher

Degrees Committee as in compliance with the University's research codes and practices. The researcher requested written consent from the participants and all relevant authorities (Cohen et al., 2007). The researcher thus complied with the ethical issues of confidentiality, anonymity and privacy, by ensuring that participants' names were not used in the thesis (Opie, 2004; Cohen et al., 2007).

3.12 CONCLUSION

In this chapter I discussed the research orientation and methodology for the study. I described the data collection methods and data analysis processes. The sample and the sampling strategy were also discussed. To conclude, I addressed the issues of validity, reliability and ethics. In the next chapter I present the phase one pre-observation interview analysis.

CHAPTER 4

DATA ANALYSIS

PHASE 1

PRE-OBSERVATION INTERVIEWS

4.1 INTRODUCTION

This chapter details how the selected teachers' participation in the MTEP encouraged or discouraged them to accumulate shared histories of practice with respect to conceptual teaching, and how they see themselves with respect to conceptual teaching. It also explores the extent to which their teaching styles and discourses align with MTEP and the broader concerns of the FRF project. I analysed data gathered from the phase one pre-observation interviews so that I could explore the processes of identity formation using Wenger's (1998) theorizing. Wenger (1998) argues that in order to make sense of the processes of identity formation and learning, it is important to consider three modes of belonging that he defines as follows:

Engagement is defined as active involvement in mutual processes of negotiation of meaning (Wenger, 1998).

In the context of MTEP, through exploring this engagement I was interested in ascertaining the extent to which participating teachers found MTEP activities meaningful and supportive of their learning. Engagement helped me to understand if there were new trajectories or pathways of learning (with respect to conceptual teaching) being formed as these teachers interacted with other members of the MTEP community.

Imagination is defined as being creative and seeing connections through time and space by extrapolating from our own experience (Wenger, 1998).

Analysing imagination helped me to understand if participating teachers viewed themselves and their profession differently through their participation in MTEP. According to Wenger (1998, p. 185) "In terms of participation, imagination requires an opening. It needs the willingness, freedom, energy, and time to expose ourselves to the exotic, move around, try new identities, and explore new relations". I would like to know whether my participating teachers explored other ways of teaching mathematics or not, and if there were new identities formed through their involvement in MTEP.

Alignment is defined as coordinating our energy and activities in order to fit within broader structures and contribute to broader enterprises (Wenger, 1998).

Analysing alignment helped me to examine the extent to which the teachers coordinated their energies and activities towards the common goal of teaching mathematics for conceptual understanding. I wanted to hear teachers' stories that reflected alignment or non-alignment with the broader aims of the MTEP project, which arose from concerns about the underperformance of learners in mathematics and a need to improve mathematics teacher proficiency for effective mathematics teaching.

The pre-observation interview comprised of fifteen questions aligned with Wenger's three modes of belonging. The first six pre-observation questions were aimed at eliciting teachers' stories of engagement; the second three questions examined imagination stories, and the last six questions were aimed at exploring the extent to which teachers' styles and discourses aligned with MTEP and its broader vision.

The pre-observation interview was followed by a lesson observation. Teachers were asked to select and teach a lesson of their choice in the FET mathematics curriculum. The observed lesson was then followed by a post-observation interview. The purpose of the post-observation interview was to explore how teachers' practice had been influenced by what they learned in MTEP. I analysed data from the classroom observations and post-observation interviews using indicators of conceptual teaching. These indicators are:

- Promoting conceptual understanding,
- Productive maths talk,
- Using manipulatives meaningfully,
- Promoting visualization and

- Self-efficacy.

The indicators of conceptual teaching are defined in Table 3.5(a) on page 62. In Chapter 6 I demonstrate the relationship between mathematics teacher identity and mathematics teaching, and show the teachers' journeys from their actual to their designated identities. According to Sfard & Prusak (2005), learning is often the only hope for those who wish to close a critical gap between their actual and designated identities.

4.2 TEACHER 1

Teacher 1 had sixteen years' teaching experience at the time of the study and was one of the teachers who started with MTEP at the beginning of the project. She was registered for an Honours degree at the beginning of the project. In the course of the project she registered for a Master's degree in mathematics education. Below I describe Teacher 1's identity formation process using Wenger's (1998) three modes of belonging as an analytical framework.

4.2.1 Engagement

In response to the question I asked Teacher 1 about the mathematics she had learned in MTEP, she said:

There is a ***bit of maths that I learned from MTEP*** *but basically* ***what I have learned at MTEP is the different approaches to teaching*** *the maths that I knew and the maths they are exposing ourselves to, like they emphasize the conceptual understanding of mathematics or teaching of mathematics that is in my mind and that is what they over-emphasized to me that is what I am aiming at on a daily basis that I must make my maths, I must conceptualise my maths teaching and it has gone even to the other subjects that I am teaching,* ***basically is the approach not the maths as a content that I have learned.***

In her view, through her engagement, Teacher 1 has been transformed to become a better teacher with respect to teaching approaches. She has, through negotiation of meaning within the MTEP community, learned and embraced different ways of maths teaching. A significant aspect of MTEP and the in-school support was to develop a community of mathematics teachers who shared the common purpose of teaching mathematics for conceptual understanding. According to Wenger (1998), engagement transforms communities, practices, persons, and artifacts through interaction with each other. This was evident from Teacher 1's response.

Teacher 1's perspective on teaching mathematics and other subjects was positively influenced by her participation in MTEP. This is in line with Clarke's (2008) view that engagement provides a means for community members to define, maintain, and negotiate their activities and practices, while creating a space for creating and recreating identities. In the context of learning, engagement is not just a matter of activity, but community building, inventiveness, social energy and emergent knowledge ability (Wenger, 1998). There is evidence in Teacher 1's response that her participation in MTEP has positively impacted on her conceptual understanding of mathematics and her teaching.

When I asked Teacher 1 whether MTEP had created an environment which made it easy or difficult for her to talk about mathematics and mathematics teaching she responded:

Oh yes, yes, I feel ***very confident****, they encouraged me to* ***be confident*** *about what I know and the approaches that I am using because almost in all of the sessions we have they emphasized,* ***they opened up my scope of understanding and approaching lessons*** *in a classroom and by also* ***giving us opportunities to present*** *and criticize or advise us on those presentations so that makes me to* ***be confident*** *because I take those critics or those advices positive*.

Through interacting with other members of the MTEP community Teacher 1 said she had developed confidence in her mathematics teaching. Teacher 1 engaged in shared activities such as being prepared to take risks in doing presentations, and allowing members of MTEP to advise and criticize her so as to improve her mathematics teaching. Teacher 1 was open to new learning and that has made her become a better mathematics educator. Brosnan & Burgess (2003) argue that engagement requires relationships founded on trust, which permits participants to take risks and explore new ways of negotiating meaning in a supportive environment. It is evident from Teacher 1's response that involvement in the MTEP has provided a space for her to expand her scope of understanding and her approach to mathematics lessons and teaching. Learning experiences within communities influence who we are and who we will become (Goodnough, 2010). The work of engagement entails such processes as the development of interpersonal relationships (Wenger, 1998).

I requested Teacher 1 to describe how her interaction with members of the MTEP community shaped the way she sees mathematics. This is how she responded:

Being at MTEP ***made me to see mathematics, of course it is challenging, something that is doable*** *and it has made me to, because mathematics our learners they fail mathematics I take because I tend to blame myself for not knowing or I don't teach it very well,* ***because I am interacting with these people,*** *the facilitators and other teachers we share ideas that no it's not your own problem it is a general problem,* ***that motivates me that OK, of course there is problem, it is not myself only who is experiencing this, so it is not your problem, it is a general problem, so it is encouraging you go on teaching mathematics*** *and to find other ways of making it simpler, not simpler, but understandable to learners*.

The question above was followed by the question, "Is MTEP a safe place for such interactions?", to which she responded:

so I think it is a safe place, because ***you can ask whatever that pertaining mathematics****, teaching, understanding and the content and at any time those people are available to help us, especially the facilitators and I also know other people who have strong points in teaching mathematics, that* ***I can call so and so to come and help me in this topic.***

Interactions within the MTEP community motivated Teacher 1 to look at mathematics teaching differently, that is, to find ways of making learners understand better. MTEP is a safe place, a place of motivation and inspiration to continue with mathematics teaching, which encouraged her to explore new ways of negotiating meaning in a supportive environment. Her interaction with members of the MTEP community has resulted in a positive change in the way she sees mathematics and its teaching. It has motivated her to go on with mathematics teaching. There are shared concepts that the MTEP community has produced, such as, finding ways of making mathematics understandable to learners through conceptual teaching.

There was evidence that Teacher 1 developed a strong sense of belonging within the MTEP community as she said that she could call on any member of the MTEP community for help regarding her own mathematics teaching. Effective engagement requires one to take part in meaningful activities and interactions, including participation in the production of sharable artifacts, in community-building conversations, and in the negotiation of new situations. It implies a sustained intensity and relations of mutuality (Wenger, 1998). According to Goodnough (2010, p. 168) "Engagement moves beyond the notion of groups, teams or networks and involves relations that are complex, thus allowing for the establishment and sustainment of ongoing activities. Through ongoing negotiation, a joint enterprise develops over time, resulting in a shared repertoire that guides the community and provides the impetus for continued learning". Teacher 1's response also showed the working together of the three dimensions: mutual engagement, joint enterprise and shared repertoire. "Without the learning energy of those who take initiative, the community becomes stagnant. Without strong relationships of belonging, it is torn apart. And, without the ability to reflect, it becomes hostage to its own history" (Wenger, 2000, p. 230).

I asked Teacher 1 what she learned about conceptual teaching and she responded:

before I used to refer to the textbook, my preparation was based on the textbook and I would use the textbook's approach and I would follow the procedures that they are using, but now when I joined, ***exposed myself to these workshops,*** *I learned that* ***my preparation must not depend on the textbook only****, of course we do use textbooks and now* ***I use various textbooks*** *and prepare my lesson, I take my lesson from those textbooks,* ***I contextualise my teaching*** *because the context are different****,*** *like the examples that they are using sometimes,* ***they are very different*** *from what to what we are used to with the learners.*

The extract above shows that before Teacher 1 was a member of the MTEP community, her lesson preparation was dominated by textbook use and her approach to teaching was procedural. Through her involvement and exposure to

MTEP, the scope of her approach to a lesson plan, to preparation and presentation, has been broadened as she has learned to contextualize her teaching. This aligns with Wenger's (1998) view that mutual engagement draws on what we do and what we know as well as our ability to connect meaningfully to what we don't do and what we don't know – that is, to the contributions and knowledge of others.

Teacher 1 has, through learning together and engaging with other MTEP members, shaped experiences of who she is regarding the teaching of mathematics (Wenger, 2000). Wenger (1998) argues that participation in a CoP is essential to learning and at the very core of what makes human beings capable of meaningful knowing, and this is evident in Teacher 1's actions through her participation in MTEP.

4.2.2 Imagination

When I asked Teacher 1 about her experiences of involvement in MTEP and its in-school support she responded:

*It has **made me to be more committed in my teaching**, in my everything because we need to present so I need to make sure that I prepare myself both in my class and for the presentations I need to make sure that **I organize the learners**, they do the work if say they are given a project from MTEP, I must make sure that they do it up to the expected standard **so I am more committed now** to my work.*

Teacher 1s' involvement in MTEP provided a space for her to look at herself differently as far as teaching mathematics and commitment to teaching are concerned. A positive mathematics educator identity was thus developed in her through her involvement in MTEP. MTEP provided a platform for Teacher 1 to explore other ways of teaching mathematics, other possible worlds and other identities (Wenger, 1998). Mutual engagement merely creates a shared reality in which to act and construct an identity, while imagination allows the envisioning of an alternative, superior reality (Wenger, 1998). Teacher 1 saw herself as a more committed mathematics teacher, and this, according to Anderson (2007), is identity formation. Teacher 1 has through her experiences and interaction with members of the MTEP community come to learn who she is as a mathematics educator, a more committed teacher. Flores & Day (2005) argue that a sense of professional identity will contribute to teachers' self-efficacy, motivation, commitment and job satisfaction, and is therefore a key factor in becoming and being an effective teacher.

Responding to the question I asked Teacher 1 on how her teaching has been changed as a result of her participation in MTEP, she said:

*I am **now more confident,** I have got confidence, **I can run workshops** because of my exposure at MTEP.*

Teacher 1 views herself as more confident, to the extent that she can run workshops, and this she attributes to her involvement and participation in MTEP. Participation in MTEP has created a new image of self in terms of her confidence and this relates to what she said, above, about being confident about what she knows and about her teaching methods. MTEP has provided Teacher 1 with a platform which has allowed her to explore alternative identities and ways of doing things. Sfard and Prusak (2005) define identity as a collection of stories about persons or, more specifically, as those narratives about individuals that are reifying, endorsable and significant. Teacher 1 is telling a reifying story about her involvement in MTEP, which is: "I am now a more confident mathematics educator".

I asked Teacher 1 if she felt that her involvement in MTEP has created a new image of herself as a professional maths educator and she responded:

*I **can teach maths anywhere now, I have presented at Amesa***

The data above demonstrates that Teacher 1 views herself and her profession in a new way: she looks at herself as an educator who can teach mathematics anywhere. Bejaard, et al. (2004) argue that teachers derive their professional identity from the ways they see themselves as subject matter experts, pedagogical experts and didactical experts. Teacher 1 sees herself as a competent mathematics educator who sees no limitations in her mathematics teaching.

This is supported by Goodnough (2010), who argues that imagination helps teachers to view themselves in new ways and their profession in new ways: teachers critically examine their practices and the teachers they would like to become. The work of imagination entails such processes as defining a trajectory that connects what we are doing to an extended identity, seeing ourselves in new ways. The extract also demonstrates that MTEP has provided a space for Teacher 1 to see herself in a new way with respect to the teaching of mathematics, and she has therefore learned.

4.2.3 Alignment

In Teacher 1's engagement discussed above, Teacher 1 responded:

"that is what I am aiming at on a daily basis that I must make my maths, I must conceptualise my maths teaching"

Teacher 1's old style of teaching procedurally did not align well with the style of teaching that MTEP promoted, that is, teaching for conceptual understanding. Teacher 1 said that she now prefers to direct her energy towards the common goal of teaching mathematics for conceptual understanding. This aligns with Wenger's (1998, 2000) view that alignment is a mutual process of coordinating perspectives and actions in order to direct and redirect energies to a common purpose. Teacher 1's actions are also aligned with the broader concerns of the FRF project, since she aims in her teaching for conceptual understanding in other subjects as well (in her case, science). The broader concerns of the FRF project are to improve the proficiency of mathematics teachers, and her actions align well with these broader concerns.

According to Brosnan and Burgess (2003), the focus of alignment is on linking the activities of a particular learning community to external and broader issues which lie beyond the boundaries of that community. The broader styles and discourses include codes of practice that govern norms and standards for educators. The norms and standards for educators of South Africa state that teachers must be subject specialists and well-grounded in the knowledge, skills, values, principles, methods, and procedures relevant to the discipline, subject area, learning area, phase of study, or professional or occupational practice. The teacher will know about different approaches to teaching and learning (and where appropriate, research and management), and how these may be used in ways that are appropriate to the learners and the context. The teacher will have a well-developed understanding of the knowledge appropriate to the specialism. In practising conceptual teaching, Teacher 1's knowledge of mathematics and its teaching has grown, which aligns with the requirements of the norms and standards of teachers nationally.

Teacher 1 absorbed and internalized the identity that MTEP promotes. The identity of MTEP has become an identity of Teacher 1. She also aligned herself with the broader concerns of the FRF project, since she has aimed for and has implemented teaching for conceptual understanding in both maths and science. According to Goodnough (2010), there is a risk in alignment of adhering or submitting to practices without meaningful engagement in those practices. But Teacher 1 engages meaningfully with mathematics teaching practices because she has assimilated teaching for conceptual understanding.

In response to the question I asked Teacher 1 about the impact of what she learned at MTEP on her professional development, she answered:

*"it is **what I am doing and I am a aiming on my everyday preparation**", "they don't just understand how to do whatever we are doing **but they must, they need to, they need meaning behind what we do".***

Teacher 1 said that her involvement in MTEP lesson preparation had had a positive impact on her school lesson preparation. She suggested that it is not enough to simply understand – the learners must attach meaning to their understanding. This is what is encouraged in the MTEP and corroborates Wenger (1998), who said that the work of alignment entails such processes as walking boundaries, creating boundary practices, and reconciling diverging perspectives. Teacher 1's and MTEP's perspectives on teaching are reconciled by the conceptual teaching concept.

When Teacher 1 was asked about her interaction with members of MTEP community and how it shaped the way she saw mathematics, she responded:

"***so it is encouraging you go on teaching mathematics** and to find other ways to find other ways of making it simpler, not simpler, but understandable to learners*".

It is clear from what Teacher 1 said that she is committed to finding different ways to teach for understanding. She goes on to say that:

*"**Use things that learners see**, so that they understand."*

This demonstrates that she is mindful of teaching for understanding and finds the means to teach for conceptual understanding. The extract above demonstrates that she goes out of her way to ensure that learners understand what she is teaching, by using things that learners can see and identify with. The MTEP environment formed a platform of encouragement and inspiration for Teacher 1.

In the context of Teacher 1's engagement, discussed above, I pointed out Teacher 1's emergent knowledgeability about lesson preparation.

"***I learned that my preparation must not depend on the textbook only,*** *of course we do use textbooks and now I use various textbooks and prepare my lesson",*

Teacher 1 goes on to say *"I must contextualize my teaching"*, which shows that she acknowledges the importance of making or finding the means – both textual and manipulative – to make sure that learners grasp the mathematics she is teaching.

In response to the question I asked Teacher 1, regarding whether she would encourage other teachers to teach for conceptual understanding, she said:

With maths it is not easy to conceptualize only, they also need to know the procedure, but you must use these two approaches together, *so that when they are using this procedure, they understand what they are doing, so as for me I can't say they must use conceptual approach alone, from my experience I have seen that it is not easy to conceptualize factorization, they need to…*

Teacher 1 aligns by bringing her actions and practices in line with MTEP's vision of Conceptual Teaching. Wenger (2000) argues that alignment does not connote a one-way process of submitting to external authority, but a mutual process of coordinating perspectives. According to Teacher 1, the teaching of mathematics that promotes conceptual understanding enables learners to gain a better understanding of procedures, so she argues for the use of both approaches. This aligns her practice with that recommended by Hiebert and Lefreve (1987), who argue that using the two approaches together contributes to the development of a sound mathematics knowledge base. Tsui (2007) argues that it is through alignment that the identity of a large group becomes an identity of participants. MTEP's identity has become

Teacher 1's identity, since she identifies and aligns with belonging to a community of mathematics educators external to MTEP.

4.3 TEACHER 2

Teacher 2 had accumulated 12 years of teaching experience at the time of the study and is one of the teachers who started at the beginning of the project. He registered for an honours degree in Mathematics Education in the third year of the project. Teacher 2's identity formation process is discussed below.

4.3.1 Engagement

I asked Teacher 2 about the impact of what he had learned in MTEP on his professional development and he responded:

My approach to some of the topics has quite improved*, especially the topics that were challenging, because sometimes you would find you know the topic as teacher but the approach and also the effective way of teaching where* ***now I am no longer focusing on the procedure and the routine, I am now more on the conceptual way of teaching.***

Teacher 2 noted an improvement in his approach to some mathematics topics through his involvement in MTEP. Participation and involvement in MTEP had had a positive impact on Teacher 2's mathematics teaching, as he indicated that he had learned to teach for conceptual understanding. It is evident from Teacher 2's response that a new pathway of learning has been formed because of his involvement in MTEP, that is, an improved approach to mathematics teaching through teaching for conceptual understanding. Teacher 2's response reflects Wenger's (1998) definition of engagement – that it requires participation in meaningful activities and interactions, in the production of sharable artifacts, in community building conversations, and in the negotiation of new situations.

Responding to the question I asked Teacher 2 about his experiences of involvement in MTEP and its in-school support, he said:

I have gained quite a lot, my confidence as well has been boosted** and also having a better way or understanding of the software now , especially the GeoGebra, and **as I said previously that my approach has improved very well.

Teacher 2 has through interacting with members of the MTEP community gained confidence and knowledge of the use of technology in teaching mathematics. This relates to what he said in the extract above about having learned different approaches to mathematics teaching in MTEP.

I requested Teacher 2 to share an unforgettable experience of his involvement of MTEP and in-school support, and he responded by saying that:

*Yes there is quite a number of them, but the one that first come to mind is when is when **we were doing a research on how to teach a parabola**, that was very exciting, **the way learners were responding and the way we could identify the mistakes that learners do**, when they are doing this parabola.*

Teacher 2 was involved in a project to present a paper in the “How I Teach” section of the AMESA conference. In this paper he planned and prepared a presentation on his teaching of a parabola, in which he used technology to explore this subject in depth. Although he couldn’t present the paper at the conference due to study commitments, his mathematics teaching was enriched through that experience as he said that he learned more about how learners learn. Teacher 2 welcomed the invitation to present at the AMESA conference and this demonstrates that he was prepared to take a risk not only in MTEP but in a national mathematics teacher conference. Wenger (1998) argues that engagement transforms communities, practices, persons and artifacts through interaction with each other. Teacher 2 has through engaging with the MTEP community been transformed into a better mathematics educator, especially in respect of teaching approaches.

I asked Teacher 2 whether MTEP had created an environment which made it easy or difficult for him to talk about mathematics and mathematics teaching. This is how he responded:

*MTEP created an environment which makes it easy for me to talk about mathematics **because during MTEP sessions we have slots where we have to present, it***

makes one more comfortable in talking about the maths *and the challenges behind and then mh, as their doors are always* ***open for us to ask questions to ask them to come and workshop us on a particular session.***

The extract above shows that the MTEP community provided Teacher 2 with an opportunity to establish relationships and to begin to bond socially. There is a level of trust which has been developed between Teacher 2 and other MTEP members as he says that the MTEP environment has made him more comfortable with asking questions and talking about mathematics. This is in line with Brosnan and Burgess's (2003) contention that engagement requires relationships founded on trust to enable participants to take risks and explore new ways of negotiating meaning in a supportive environment. Teacher 2's response demonstrates that he attended in-school support workshops. These workshops were aimed at supporting participating teachers' mathematics teaching. Teachers who attended these workshops showed commitment to their professional growth since the workshops were run after school hours.

I asked Teacher 2 about his interaction with members of the MTEP community and how it shaped the way he saw mathematics. His response was:

The interaction has been quite good *because,* ***because you can even ask the fellow teachers*** *that are there, the supervisors as well****, you can always ask them****, so the interaction for me is pretty much good.*

The above question was followed by the question, "Is MTEP a safe place for such interactions?" and he said:

Yes to me it is, because we come from different backgrounds form different schools with different challenges that we have from our respective schools and ***then if you share them you learn from the other person the way he or she does things in his or her school.***

Teacher 2 has developed a strong feeling of belonging as he says that the environment allowed him to ask questions and share maths ideas with other members of the MTEP community. The interaction and sharing with other members has provided a good basis for Teacher 2's learning, as reflected in Wenger's (1998) contention that the work of engagement entails the development of interpersonal relationships. Through doing things together with other MTEP community members, the way in which Teacher 2 has engaged with other members has shaped his experience of who he is, a professionally developed mathematics educator (Wenger, 2000).

Responding to the question I asked about how his teaching changed as a result of his participation in MTEP he responded:

Well as for confidence as I said, ***now my confidence has been boosted*** *and when I go to class,* ***even if I know this is difficult topic for the learners, now I have got much confidence in my approach from what I have learned from the MTEP sessions*** *and as for the ways of teaching maths as I have said, during the MTEP session* ***we learn different ways of approaching, sometimes you know a way of approaching a topic****, you learn during these workshops that this is a much better way of doing this especially the demonstrations where you demonstrate a particular topic conceptually and then , so my ways of teaching mathematics has changed quite a lot now.*

The way I always was thinking about maths from the learners' point of view, learners always think that maths is difficult*, I have been always thinking that maths is not that difficult and now that I joined MTEP sessions,* ***that was assured to me that I was thinking on a right direction now so as I have thinking about maths, it is not a difficult, but it is a subject that needs a proper way of teaching and a proper way of approaching the sections, my attitude, I love maths and I have always been having a positive attitude towards maths and that made me to notice a positive attitude in learners, my learners, they enjoy mathematics most of the time.***

(R) So would you attribute the change in the attitude of learners to your interaction with the MTEP community?

Yes in a way I would do, ***I would attribute because from MTEP I came with these small games and ways of explaining some challenging topics which motivated my learners quite a lot*** *and then the practical demonstrations as well.*

(R) Can you think of a practical demonstration that you did in class?

Yes. For ***an example the projects that they did on data handling, that was to them, going out collecting the data, analysing the data which I think they enjoyed quite a lot*** *and with the grade 10's I started introducing the geo boards, I explained how they will be using the geo boards and everyone is excited to have her own geo board now.*

It is evident from the data above that Teacher 2 has through his participation in MTEP learned different ways to approach mathematics lessons. This according to Teacher 2, has boosted his confidence both in himself and his teaching of mathematics. The extract above also shows that Teacher 2 has implemented what he learned from MTEP in his classroom which has resulted in positive learner attitudes towards mathematics. Teacher 2, through being a member of the MTEP community has learned new ideas about his teaching of mathematics and his perceptions about mathematics have been positively influenced. Teacher 4 was open to learning about the use of games to explain challenging maths topics. "Through varying degrees of engagement with mathematics, their teachers and their peers, each student sees her or himself, and is seen by others as one who has or has not learned mathematics. Engaging in a particular mathematics learning environment aids students in their development of an identity, as capable mathematics learners." (Anderson, 2007, p. 8). Engaging in the MTEP learning community aided in development of Teacher 2's identity, that of a professionally grown and confident maths teacher.

I asked Teacher 2 about how he felt about being a member of the MTEP community and he responded:

*Well I feel very honoured, because one the workshops are free****, you participate and then you gain, I feel also that I am very happy with the information that we are getting there****, with the activities that they are giving to us,* ***some of the activities that are even not in the textbooks which is helping us to think*** *in that sense, ja I would say I am very excited to be one of the educators that are participating in the workshops.*

Through belonging to the MTEP community Teacher 2 has developed an attitude which made him open to learning new methods of mathematics teaching. Teacher 2's scope of approaching a lesson has been broadened as he says that he has been challenged to approach a lesson differently from the 'textbook only' route to a more diverse approach with MTEP activities. The MTEP environment has been a testing place for Teacher 2, as he has been encouraged to think laterally, in an attempt to make learners understand what he teaches. Participation in MTEP has led to the formation of trajectories in Teacher 2 as he has gained knowledge which helped him to teach maths better than when he was not an MTEP member.

4.3.2 Imagination

I asked Teacher 2, if he feels his involvement in MTEP has created a new image of self as a professional maths educator and he responded:

Yes in a way because most of the things that I gained in MTEP workshops, I have found that educators that don't attend these workshops, they don't know some of these things and as they stick to the old way of teaching, teaching the procedure so personally ***when I look at myself I see myself as someone who has grown in that regard.***

The extract above shows that Teacher 2 views himself differently through his involvement in MTEP. In his view, he has grown in his teaching from focusing on mathematical procedure to teaching mathematics for conceptual understanding. Teacher 2 has become a certain kind of mathematics educator with respect to the practices of the MTEP community (Anderson, 2007). As discussed in the engagement section above, Teacher 2 has implemented the teaching for conceptual understanding in his classroom and this reflects Wenger's (1998) definition of imagination that it entails such processes as generating scenarios, exploring other ways of doing what we are doing, other possible worlds, and other identities.

Imagination, then, helps teachers view themselves and their profession in new ways. Teachers critically examine their practices and teachers they would like to become. It requires flexibility and creativity to reinvent practices of communities and to create opportunities for novel learning (Goodnough, 2010). MTEP community practices created opportunities for novel learning for Teacher 2.

4.3.3 Alignment

Responding to the question I asked Teacher 2 about what he learnt about conceptual teaching he said:

Well before I became a member of MTEP, I think in most cases I would say I was basically teaching a procedure way of doing things *eh eh, because really I wouldn't go to asking why such a thing is happening , and then* ***after joining MTEP it is then that I started to realise that there is more than to a procedure, because these learners would understand it better if they understand the concept,*** *if I were to make an example , the question of factorising the trinomials and so on, learners would know how to factorise, but they wouldn't know the concept of what they are doing so* ***after I joined MTEP workshops then those problems were addressed and I began to grow in understanding the effects of conceptual teaching.***

Teacher 2 says that his approach before he joined MTEP was procedural. He did not emphasize the 'why' questions. Teacher 2's response demonstrates that he directs his energy towards a common goal of teaching mathematics for conceptual understanding and therefore identifies with the common purpose of MTEP. Teacher 2's actions do not only align with MTEP's vision but also with external and broader issues which lie beyond the MTEP boundaries (Brosnan & Burgess, 2003). In teaching for conceptual understanding Teacher 2 also teaches for understanding which is one of the ways of addressing challenges facing mathematics teaching in South Africa. Mathematics teaching should help learners understand the maths concepts and apply that knowledge in problem solving. Teacher 2 is mindful of teaching for this understanding.

I asked Teacher 1 if he sees himself as a conceptual teacher and if he would encourage other teachers for conceptual teaching and he responded:

As for now, I am still growing in that part, ehm I would say, partly I am more procedure and ***my other part now is beginning to grow in to a conceptual way of doing things.***

*Yes I would do, and whenever I do that I always like to make examples, like as I have indicated the other one on factorisation, there are a quite number of them and I always say to them****, that these learners understand it better when they do it conceptually and then when you bring your procedure, you bring it on something that they really understand the concept.***

In his view, he is growing into teaching for conceptual understanding and would encourage conceptual teaching. Teacher 2 argues that promoting understanding of concepts in teaching helps the learners to better understand mathematics than just giving them a procedure. MTEP's identity is that participating teachers must promote understanding of mathematics concepts and go out of their way to make learners understand what they teach. Participating teachers can choose not to own or implement conceptual teaching for varying reasons however it was not the case for Teacher 2 as he adopted and embraced MTEP's identity.

4.4 TEACHER 3

Teacher 3 had gained 12 years teaching experience at the start of the project. She, like Teacher 1 and Teacher 2, started at the beginning of the project. Below I describe Teacher 3's identity formation process:

4.4.1 Engagement

Responding to the question I asked Teacher 3 about the mathematics she has learned in the MTEP she said:

Well sure I have, ***there are things that I like when they integrate with curriculum****, I enjoyed statistics and probability and the way it was explained you know, you would be having problems with terminology and this and that* ***but as we discuss with maths teachers from there you become clear, you know***

and I also enjoyed the completing of the square issue because what really happens there it helps in the sense that they are helping us to reach out to the learners in a *way that they* ***conceptualise all what they learn in class so that they are able to relate to everyday life so that we don't just teach for exam***

*Like I said before**, being together as maths teachers it helps and when we discuss there, you get to realise you can use any method as long as it help learners to conceptualise,** you can teach any method that really relieved me.*

The extract above demonstrates a strong sense of belonging in Teacher 3, as she said that there are some mathematics concepts that became clear, that she has learned and has grown from being with the MTEP community. In her view, the discussions during MTEP sessions were fruitful and they led to the learning of new methods of teaching mathematics. These discussions also encouraged her to be purposeful in her teaching, as she said that she did not just teach for examinations, but taught for learner understanding. Teacher 3 has, through her involvement in MTEP, broadened her scope of understanding maths concepts. According to Anderson (2007) much of what students know about learning mathematics comes from their engagement in mathematics classrooms. The teaching approaches that Teacher 3 learned come from her engagement in the MTEP learning community.

"Engagement becomes a mode of belonging and a source of identity. Engagement involves communities of individuals engaged in actions that are mutually negotiated. It moves beyond the notion of groups, teams or networks and involves relations that are complex, thus allowing for the establishment and sustainment of on-going activities" (Goodnough, 2010, p. 168).

I asked Teacher 3 about the impact of what she had learned in MTEP on her professional development and she responded:

Well again the other thing I have learned at MTEP is the way they present their sessions, to ***the background of whatever you want to teach****, I think they have taught me that I should* ***have that to have a background of the origins of this, how did this start, who came up with this,*** *why did they decide to do this particular thing, you know, the history of maths I can say It won't be relevant to the curriculum but it enriches you as a person so I kind of like that. I* ***have now developed an inquisitive mind when it comes to mathematics****, if I want to teach* ***something I***

have to research about it, *know a lot about it so that I deliver to the learners* ***I am wide and broad*** *so that they can also understand what I will be talking about.*

Teacher 3's participation in MTEP had a positive influence on her approach to teaching of mathematics. Teacher 3's lesson preparation has been developed through her interaction with members of the MTEP community. Through her negotiation of meaning in MTEP, Teacher 3 has learned to broaden her knowledge through research and learning to understand the background before she delivers a lesson to learners. This experience, in her view, has taught her to be 'wide and broad' in her teaching. Through engagement with the MTEP community and the mathematics in MTEP she sees herself an educator who has learned and developed an inquisitive mind in mathematics. In order to support learning, engagement requires authentic access to both participative and the reificative aspects of practice in concert (Wenger, 1998). The MTEP learning community did not deny Teacher 3 access to participation and reification hence she learned various ideas for mathematics teaching.

I requested Teacher 3 to share the experiences of her involvement in MTEP and its in-school support and this is how she responded.

what ***I really liked most about this whole thing is the gathering of maths teachers, it really made a difference, you know when you know each other around the district it really helps because you are sharing experiences you know what other teachers are experiencing****, that really helps a lot, they also helped me to attend AMESA for the first time, I didn't know about it.*

Teacher 3's response demonstrates development of interpersonal relationships between her and the MTEP community. MTEP's supportive and safe environment has been a good foundation for Teacher 3's exploration and risk taking which had led to learning and therefore identity formation. Social relationships are fundamental to an effective learning and engagement relationships founded on trust that allow participants to take risks and explore new ways of negotiation of meaning in a supportive environment (Brosnan & Burgess, 2003) and this is evident in the abstract above.

I asked Teacher 3 whether MTEP has created an environment which makes it easy or difficult for her to talk about mathematics and mathematics teaching and she responded:

I would say they have because like when you are together when you are together, the ***other thing that has helped also is this thing of relating with classroom practice, that we get projects that we can all implement in our schools*** *we do in our schools, and there discuss about it, and we have that freedom I believe that there is that room to say that this part I didn't understand I couldn't understand,* ***I think there is that environment, that allows as to grow as maths teachers that we are not there to criticize each other, but we are there to learn from each other, so much that if you have a problem with a certain topic, you are able to bring it out to the other, and you do get genuine help from them. It helps a lot.***

According to Teacher 3, the MTEP environment has been a safe and conducive platform for her to learn and grow as a maths educator. The MTEP space also allowed for learning and gaining of knowledge on approaching certain maths topics for Teacher 3. The MTEP community helped Teacher 3 with practical solutions to maths teaching questions she had, as she said she implemented projects from MTEP in her classroom. Wenger (1998) argues that, engagement transforms communities, practices, persons, and artifacts through each other and this is reflected in Teacher 3's response.

4.4.2 Imagination

I asked Teacher 3 in what ways as a result of her participation in MTEP has her teaching changed and she responded:

Well I would say when ***it comes to confidence I will attribute to what I have learned as I said before I have learned to broaden my knowledge in mathematics when I have wide knowledge of whatever I want to teach whether it is topic or a concept I want to teach I feel confident to deliver it to the next person, so that is the part the really helped me a lot and the teaching methods.***

Well on attitude I would not say it really changed much because I have always loved maths *because I have always loved maths I have always had a passion it is only that I did not have that opportunity to develop it but I have always believed that maths relates to real life.*

It is evident from the extract above that Teacher 3 has developed confidence through her involvement in MTEP. In her view, her confidence is derived from what she learned in MTEP, which is, broadening her knowledge of mathematics through research. Imagination focuses on self-awareness and reflection in relation to understanding others and their actions, connecting to new trajectories and locating ourselves in broader systems, creating new artifacts and processes, exploring new ways of doing things, and seeing new identities for ourselves (Goodnough, 2010). Interaction with members of the MTEP community has supported Teacher 3's learning, she explored new ways of maths teaching and saw a new identity in herself.

I asked Teacher 3 how she feels that her involvement in MTEP has created a new image of self as a professional mathematics educator and she responded:

Yes it has made me to feel and it has made me to feel like a maths teacher, just being recognised that you are a maths person, and ***they want to help you develop as a maths educator it gives me the drive to want to do more in maths.***

Teacher 3 has a new identity through mutual engagement in MTEP activities, that of a mathematics educator who has a drive to want to do more in mathematics. Belonging to the MTEP community has opened new ways of seeing herself (Wenger, 1998) in relation to broader mathematics teaching and this contributes positively to her identity as a maths educator (Anderson, 2007).

4.4.3 Alignment

Responding to the question I asked Teacher 3 if she sees herself as a conceptual teacher, she said:

I have always considered myself as a conceptual teacher but I realised that sometimes you might think you have delivered the thing, ***when you share with other people and you see how they have approached it, you really see that no,***

if I could have done this way the learners could have understood it better, I think conceptual teaching is what will make learners understand better.

The question above was followed by this question “If you were to encourage other teachers for conceptual teaching, how would you do that?”

I have found that learners would learn better if they derive for themselves, they will understand the formula better ***they will be in a better position to implement it and they will know when to use it,*** *so much that if you take a formula you use it such if you are doing financial maths, if you want future value, you use this formula sometimes you will discover that they just look at the formula booklet and they pick up a wrong formula because they do not understand exactly what they are doing.*

Although it is not evident from Teacher 3’s response that she has implemented conceptual teaching in her classroom, however she is supportive of the idea and would encourage it. In her view, implementing conceptual teaching will help learners apply formulae with understanding. This aligns with Rittle-Johnson and Alibali (1999) that conceptual and procedural understanding do not develop independently, conceptual understanding influences the procedure they use.

4.5 TEACHER 4

Teacher 4 started at the beginning of the project and had 14 years teaching experience at that time. Teacher 4’s identity formation process is discussed below:

4.5.1 Engagement

Responding to the question I asked Teacher 4 about the mathematics she has leaned in MTEP she said:

Yes, ok, at MTEP we are taught to teach learners with understanding, ***they must understand concepts, you must not focus on an answer, learners must learn to do it,*** *like most of the activities are hands on activities and those hands on activities are helping them to get a deeper understanding of what they are learning.*

(R) Can you think of a topic relating to what you have just said?

Like for instance I can say about, they have done Pythagoras theorem, whereby learners Pythagoras, ***they were not just told $r^2 = x^2 + y^2$, they discover for themselves*** *using ii cell phones and they were interested, we have learnt completing of a square where learners, they have to discover a formula by doing drawings doing those squares.*

Teacher 4 has, through her engagement with the MTEP community, learned teaching that foregrounds conceptual understanding. In MTEP she learned teaching that provides learners with hands on activities which help them to explore mathematics learning at a greater depth, process focused rather than answer focused teaching and discovery supportive teaching. It is evident from the data that Teacher 4 has implemented teaching for conceptual understanding in her classroom and learners' experienced Pythagoras theorem discovery as opposed to being given the theorem to apply in problem solving. Wenger (1998) defines engagement as a threefold process which includes the conjunction of on-going negotiation of meaning, the formation of trajectories and the unfolding histories of practice. A new pathway of learning has been formed through Teacher 4's on-going negotiation of meaning in MTEP.

I asked Teacher 4 to share some experiences of her involvement in MTEP and its in-school support and she responded:

Ok, first of all ***now I am working as a team with other teachers in Grahamstown*** *because at first I didn't know, I was working alone****, I am working now as a team with other teachers****, also my learners are now interested because at MTEP they give us resources like calculators, at first my learners were suffering when we were doing calculations, but now at least...*

MTEP community membership had a positive influence on Teacher 4's perceptions about mathematics and its teaching. Wenger (1998) argues that the work of engagement entails such processes as the development of interpersonal relationships and this is reflected in Teacher 4's response above.

Responding to the question I asked Teacher 4 about whether MTEP has created an environment which makes it easy or difficult for her to talk about mathematics and mathematics teaching and she said:

They create an environment which makes it easier for me to teach mathematics. ***The reason why is that at MTEP we learned to present in classes so that makes me to be confident to talk in public*** *and* ***also they developed us in mathematics*** *so when you are confident in teaching your subject you have no fears in talking publicly to other teachers.*

(R) How have they developed you?

Like for instance there ***were strategies that I didn't even know, now I do those strategies in class***

MTEP's environment supported Teacher 4's growth professionally as she said that she was presented with an opportunity to present during MTEP sessions and at an AMESA conference. Teacher 4 presented the topic: 'Using wooden cubes to help learners investigate number patterns and to understand sum of terms of an arithmetic series' at the AMESA conference. The MTEP space made Teacher 4 feel encouraged and to be more open to contribute to the MTEP community and mathematics teacher COP nationally. Belonging to the MTEP community has developed Teacher 4's confidence to talk in public.

It is evident from the extract that Teacher 4's mathematics knowledge has been developed through her interaction with the members of the MTEP community. Engaging in the MTEP learning environment helped Teacher 4 in development of an identity of a confident and a mathematically developed educator (Anderson, 2007). Teacher 4 has also gained teaching strategies through her participation in MTEP. Community describes social configurations in which individuals engage in actions that are mutually negotiated, and identity refers to how learning can change who we are and who we become in the context of the communities to which we belong (Goodnough, 2010).

I requested Teacher 4 to share how interaction with members of the MTEP community shaped the way she sees mathematics. This is how she responded:

At first I saw mathematics as a subject that is difficult to some of the learners, ***now that I have a relationship with the MTEP community now I have developed that there is nothing difficult with mathematics if you just integrate with real life situations*** *when you teach, it is easy.*

The question above was followed by the question, "Is MTEP a safe place for such interactions?" and she said:

Yes I think so because, ***whenever you go to them and ask for help, they are willing and I feel safe, they are always willing to help****.*

Relationship with members of the MTEP community has caused Teacher 4 to develop a changed perception and attitude towards mathematics, that it is not a difficult subject, it is easy if you integrate with real life situations. Through relationships and experiences with members of the MTEP community, Teacher 4 has come to know who she is relative to mathematics and its teaching (Anderson, 2007). The extract also shows strong relationships of belonging in Teacher 4 as she says that she is free to ask for help, and the MTEP environment is conducive to building relationships of trust. This aligns with Brosnan and Burgess (2003) that engagement requires relationships founded on trust that allow participants to take risks and explore new ways of negotiating meaning in a supportive environment.

4.5.2 Imagination

I asked Teacher 4 about the impact of what she has learned in MTEP on her professional development and she responded:

Ok I can say it has an impact on me because ***now I have developed love in the subject, even my teaching has improved****, the strategies that I have learned* ***there they help my learners to get more understanding and become more interested in the subject.***

Teacher 4 saw herself differently through her participation in MTEP, as an educator who has developed a love for mathematics. According to Teacher 4, her teaching has improved because of the strategies she gained in MTEP. The extract also shows that Teacher 4 has implemented strategies learned in MTEP in her classroom and that has supported learner understanding and interest in mathematics. The work of imagination entails such processes as: defining a trajectory that connects what we are doing to an extended identity, seeing ourselves in new ways (Wenger, 1998)

I asked Teacher 4 in what ways as a result of her participation in MTEP has her teaching changed and she responded:

I have confidence, I have more confidence now in teaching maths ***as I told you that I have gained more teaching strategies, and also many ways, I have got many ways to solve problem,*** *at times I just ask learners to come up with their solutions, so I am confident now because I can even welcome the solutions from the learners,* ***I engage learners in discussions in class, so I think...***

(R) Would you attribute engaging learners in discussions to your involvement or interaction with MTEP community?

Yes I can say it is as a result of my involvement in MTEP because also at MTEP we are encouraged to discuss a topic and then we solve as a group, so that is why I also encourage learners, I open discussion in class so that those who are not clear can become clear.

Ways of thinking about maths I can say, yes because now I know that everything around me involves mathematics, you know, ***but at first when I am referring to mathematics I was just to the syllabus only,*** *you know, but now through MTEP, I know that even if I am teaching a particular topic neh, like for instance if I am teaching about shapes, you know whereas I am teaching shapes, some other topics can crop up, like for instance equation can crop up, so I must not only focus on shapes, if there is a problem, and I can see that my learners are having a problem, I must solve that problem right now and tackle it right now but at times I was thinking that man, I am doing that topic now, I am focusing on that topic.*

Teacher 4's mathematics and its teaching has been broadened through her participation in MTEP. Through her involvement in MTEP, Teacher 4 has gained more teaching strategies and this has put her at a better position to engage learners in discussions. Although data does not provide sufficient evidence of productive maths talk taking place in the classroom, however it illustrates that Teacher 4 is aware of the importance of promoting discussion and engaging learners in talk in a maths class. Imagination, then, helps teachers view themselves in new ways and their profession in new ways, teachers critically examine their practices and the teachers they would like to become (Goodnough, 2010). Teacher 4 views herself and her profession in a new way, an educator who has grown from referring to the syllabus only to an educator who links and integrates maths concepts in her teaching.

I asked Teacher 4 if her involvement in MTEP has created a new image of self as a professional mathematics educator and she responded:

Yes because now I have learned to be responsible to take care of my learner, make sure that they understand what they are doing. ***If learners have a problem, I must go out and seek information from other teachers,*** *that is why I am saying that now I am responsible, not just teaching them and just go on with the syllabus, Also now* ***I am responsible because every Tuesday I know that I must go develop myself.***

Teacher 4 views herself differently, as a responsible educator who goes out of her way to seek information that will help learners understand what she teaches. She also views herself as a learner as she said that she goes out of her way to look for information and that she is open to new ways of teaching mathematics. MTEP space provided Teacher 4 with a platform which allowed her to explore alternative identities and other ways of doing what she has been doing (Wenger, 1998).

4.5.3 Alignment

I requested Teacher 4 to share what she has learnt about conceptual teaching and she responded:

Before I joined MTEP, I was not concentrating on the concepts*, like the learners before they do a topic, they must know what the topic is meant, like to learn conceptual, learners must do projects, like hands on activities* ***before I come to MTEP, I saw those activities as just wasting of time, you know, but now I know, though they are wasting time, they last longer****, in fact they last for life because learners don't forget when they learn conceptually, you know.*

The question above was followed by the question, “Do you see yourself as a conceptual teacher?”. This is how she responded:

Yes I can see myself as a conceptual teacher because when I introduce a lesson, I always make sure that my examples are real, even the activities I give them, *they involve their attention like, meaning the activities that they do now are not the same as the ones they did in the past like solve for x and the learners do all the steps of solution process,* ***now they are involved, they must know the concepts, what they mean by doing this...***

(R) If you were to encourage other teachers for conceptual teaching, how would you do that?

Yes I will because I have seen that it helps a lot. Conceptual teaching helps learners to gain deeper understanding of what they are doing, at times when you discover something on your own, *because sometimes I give them that chance to discover a formula on their own, so when you discover on your own, it is very difficult to forget, so that is why I would encourage other teachers to use conceptual teaching.*

Teacher 4’s teaching style before she joined MTEP did not promote conceptual understanding, and she saw teaching that promotes use of manipulatives as time wasting. In her lesson introduction, she used examples that are real and this helped learners to focus and be attentive in their learning. Teacher 4’s perspective of maths teaching has positively influenced through her participation in MTEP as she said that she sees herself as a conceptual teacher. This demonstrates that Teacher 4 directs her energies towards a common goal of conceptual teaching, as she has implemented MTEP’s new ideas in her teaching. Teacher 4 therefore identifies with the MTEP vision as she encourages learners to discover things on their own. According to Teacher 4, conceptual teaching helps learners to gain a deeper understanding of what they are learning.

I asked Teacher 4 how she feels about being a member of the MTEP community and she responded:

I feel so privileged because some of the teachers did not get this opportunity that I got from MTEP ***as I told you that I have learned some new teaching strategies, I have learned to work with other teachers,*** *also I have learned how to be a learner because at MTEP I am the learner, at times when I am thinking it comes up to mind that the learners think the way I am thinking when I am teaching them. You know, those are the experiences so that is how to understand the learners in class when they don't understand, what must I do?*

Teacher 4 said in the imagination discussion above she has developed an identity of being a learner which relates to what she said in the extract above. This, according to her has changed her perceptions and approach to mathematics and its teaching. The focus in alignment is on linking the activities within a particular learning community to external and wider issues which lie beyond the boundaries of that community (Brosnan & Burgess, 2003). One of the seven roles of norms and standards for educators is to be a scholar, a researcher and a lifelong learner. The educator will achieve ongoing personal, academic, occupational and professional growth through pursuing reflective study and research in their learning area, in broader professional and educational matters, and in other related fields. Teacher 4's actions aligned with issues beyond MTEP boundaries.

4.6 TEACHER 5

Teacher 5 joined the project two years after the project started. She had 11 years of experience at the time and her identity formation process is discussed below:

4.6.1 Engagement

I asked Teacher 5 to share experiences of her involvement in MTEP and its in-school support and she responded:

Firstly in my class I had no charts but I received charts from MTEP in my class as you can see that my class is beautifully decorated with some charts that I received from MTEP, having all different types of topics and chapters that I want to teach, and also MTEP has also helped me a lot in computers. ***MTEP has also conducted training where we were trained how to type question papers using equations*** *and also how to draw different shapes,* ***I didn't know that before***

Through ongoing negotiation of meaning within the MTEP community Teacher 5 has developed into a mathematics educator who learned how to use computers in her teaching. Engaging in a particular mathematics learning environment aids students in their development of an identity, as capable mathematics learners (Anderson, 2007). Belonging to the MTEP community has developed a new identity for Teacher 5, an educator who is able to use computers to type her question papers.

I asked Teacher 5 whether MTEP created an environment which makes it easy or difficult for her to talk about mathematics and mathematics teaching and she responded:

Yes they have done that, because MTEP they are always involved in our schools. ***It is not just us who go to the workshops, they usually make a follow up, they usually come and assist us*** *in schools. It is conducive, as I have said before.*

Responding to the question I asked Teacher 5 of how her interaction with members of the MTEP community shape the way she sees mathematics she said:

Ok, as in maths you cannot work alone, you have to work with certain group of teachers, maths teachers, it shapes us in a way that you don't work in a environment where you work alone, you meet with other teachers, ***now if you encounter a problem, it is easy to meet with other teachers and discuss it.*** *That is, it* ***encourages more of the team work****.*

The above question was followed by the question "Is MTEP a safe place for such interactions? Why" and this is what she said:

Yes I would say it is a safe place because when we are there, the ***way they conduct workshops, it makes everybody to feel free in that workshop as I have said that we are working as a team there.***

(R) How does it make it a safe place for you?

You do not feel inferior to answer the questions, you are free in class. *There is more cooperation in those workshops.*

The MTEP environment encouraged Teacher 5 to be open to contribute and to the contributions of other members of MTEP. The MTEP environment, according to

Teacher 5 is conducive and supportive of learning as she said that she is free to ask questions of the members of the MTEP community. The work of engagement is basically the work of forming communities of practice. As such, it requires the ability to take part in meaningful activities and interactions, in the production of sharable artifacts, in community building conversations, and in the negotiation of new situations (Wenger, 1998). The extract above also demonstrates a level of trust and risk taking that has developed between the MTEP community and Teacher 5, as she said she found it easy to approach others when she has a problem.

I asked Teacher 5 to tell me how she felt about being a member of the MTEP community and she said:

It is good to be a member of MTEP.

(R) Why do you say it is good?

As I have said you work with other teachers, *you don't work alone you are not in the world of your own, you work with other members of MTEP you also gain a lot****, you also gain different approaches****.*

The data shows that Teacher 5 has gained different teaching approaches through participation and involvement in MTEP. As a context for learning, engagement is not just a matter of activity, but of community building, inventiveness, social energy, and emergent knowledgeability (Wenger, 1998) and this is evident in Teacher 5's response. Doing things together with other MTEP members (Wenger, 2000), has resulted in Teacher 5's learning and growth professionally.

4.6.2 Imagination

I asked Teacher 5 to share how her teaching has changed as a result of her participation in MTEP and she responded:

I will start with that of confidence in teaching maths. ***I am more confident than I used to be. There is no chapter that I would say I am not confident to teach it, or in approaching it****. Then the second one what was the second one* ***ways of teaching maths I would say it has improved, it is no more a procedural approach now, it is a conceptual approach.***

The attitude is positive, it is no longer negative as I said that I go to class now with confidence, there is no certain chapter that I am afraid of … I mean when it is not easy to approach a certain chapter, there is a group of teachers that are in MTEP, ***it is easier for me to go and approach them and find help from them and find easier ways to approach a lesson.***

Teacher 5's mathematics and mathematics teaching has been broadened through her involvement in MTEP, she looks at herself as an educator who has no limitation in her maths teaching. Interactions with MTEP members has developed Teacher 5's confidence, as she said that she is more confident that she used to be. Teacher 5 explored other ways of mathematics teaching in MTEP and has grown from being a procedural to being a conceptual teacher, and thus saw a new identity in herself (Wenger, 1998; Goodnough, 2010). The MTEP space allowed Teacher 5 to seek information which helped her to improve her teaching.

Responding to the question I asked Teacher 5 about how she feels that her involvement in MTEP has created a new image of self as a professional mathematics educator she said:

Yes I have grown, I have gained more knowledge in MTEP and certain methods that I did not understand*, I didn't know,* ***I have gained that as well, different styles of approaching maths.***

Involvement in MTEP has created a pedagogical developed maths educator in Teacher 5. Teacher 5 has learned different styles of approaching mathematics in MTEP. Exploration as one of the elements of imagination includes, opportunities and tools for trying things out; envisioning possible futures and possible trajectories, creating alternative scenarios, pushing boundaries, prototypes, play and simulations (Wenger, 1998) and this is evident in the extract above. Teacher 5 explored and tried other ways of teaching mathematics in MTEP.

4.6.3 Alignment

I requested Teacher 5 to share the impact of what she has learned in MTEP on her professional development and she responded:

Ok Professionally it has developed me in a way how I see things now**, it is different to what I used to do we used to teach and I am no longer doing **that I am practicing conceptual method

(R) Would you attribute that to your involvement in MTEP?

***I would** and I would also encourage other teachers to use this conceptual method because **it is a way of how to easily engage your learners in a classroom activity**, the learners become more active and it helps learners to be more actively involved in what you are doing.*

(R) Can you think of an example of where you tried teaching conceptually?

*I will talk about exponents, when teaching exponents before I used to **usually start with the laws of exponents where I will say I simply give the learners the rule,** now I no longer do that **I start from the numbers I let them understand how you change a number to an exponential form** then from there introducing the laws, not starting straight from the variable, starting from what they know.*

Teacher 5's teaching approach is different from how she used to teach before she joined MTEP, she foregrounds the understanding of concepts in her teaching. It is evident from the data above that Teacher 5 has implemented conceptual teaching in teaching of exponents. Teacher 5 was intentional about getting learners to understand the concept first before she introduced laws of exponents. According to Teacher 5, conceptual teaching causes learners to focus on their learning activity and it makes it easy to engage learners in the classroom. Alignment requires the ability to coordinate perspectives and actions in order to direct energies to a common purpose (Wenger, 1998) and this is evident in the extract above.

I asked Teacher 5 about what she had learnt about conceptual teaching and she responded:

*As I have said, I was introducing a procedure in class from the textbook, **now it is no more of that, I have started to do it conceptually now that is building up from what they know** to what they did not know.*

The above question was followed by the question: “Do you see yourself as a conceptual teacher?” and this is how she responded:

Yes I have started doing that in my teaching and I would encourage other teachers to do it.

(R) Why would you do that?

Because it encourages the learners to think positively in maths class. It improves their thinking, they think creatively and critically.

As discussed above, Teacher 5’s approach to maths teaching has changed to the one which aligns with MTEP’s common purpose. Teacher 5 owns and identifies with MTEP’s vision because she experienced the results of implementing conceptual teaching that it encourages learners to think creatively and critically. This does not only align with MTEP but with issues which lie beyond MTEP boundaries, that is, teaching that promotes learner understanding.

4.7 PHASE 1 PRE-OBSERVATION ANALYSIS TABLE

Table 4.1 below provides a summary of the phase 1 pre-observation analysis results:

Table 4.1

Wenger (1998)'s three modes of belonging	TEACHER 1	TEACHER 2	TEACHER 3	TEACHER 4	TEACHER 5
ENGAGEMENT	New teaching approaches. Teaching for conceptual understanding. Confidence in maths teaching. Doing presentation.s Improved approach to lesson planning, motivation and continue with maths teaching. Strong relationships of belonging (free to tell & ask for help).	New Teaching approach. Confidence & knowledge of use of computers in maths teaching. Professional growth. Confidence boosted.	New methods of teaching maths. Learned to teach for understanding. Growth in lesson preparation	Learned teaching that promotes conceptual understanding. Maths knowledge has grown in confidence	Learned use of computers in maths teaching (helped her to type question papers) Environment made it easy to ask questions. Teaching approaches.

IMAGINATION	More committed. More confidence. Professionally grown maths teacher (teach maths anywhere).	Grown to teach maths for CU. More confident.	Learned to broaden her knowledge through research and learning to understand the background. Developed an inquisitive mind in maths. Always had a positive attitude towards maths.	Changed perception and attitude toward maths (easy if you integrate with real life). Developed love for maths. Gained more teaching strategies. Engage learners in discussions. Responsible educator (goes out of her way to look for information to help learners).	Confidence in maths teaching. No limitations in maths teaching see herself as a conception teacher. Gained different approaches.
ALIGNMENT	Teaching for conceptual understanding. Implemented teaching for conceptual understanding in maths and science. Teaching for understanding.	Teaching for conceptual understanding. Teaching for understanding.	Implemented teaching for conceptual understanding.	She saw use of manipulatives as waste of time, she teaches for conceptual understanding.	Foregrounds conceptual understanding in her teaching. Teaching that promotes learner understanding.

4.8 CONCLUSION

In this chapter, I have used Wenger's (1998) three modes of belonging as an analytical tool to discuss the selected teacher's identity formation process.

4.8.1 Engagement stories

The analysis demonstrates that learning different approaches to teaching of mathematics and teaching for conceptual understanding are common engagement stories in all the participating teachers. They have, through negotiation of meaning within the MTEP community, learned and embraced different ways of mathematics teaching. The analysis also shows that the MTEP provided a safe platform for learning for the teachers.

4.8.2 Imagination stories

The analysis shows that all the participating teachers saw themselves as more confident in their teaching through their participation in MTEP. The teachers now view themselves and their profession differently, as educators who have grown professionally in various ways. Teacher 3, for example has learned to be inquisitive, to broaden her knowledge through research and to understand the background before delivering a lesson. Teacher 4 viewed herself as a teacher who has developed a love for mathematics through her involvement in MTEP.

4.8.3 Alignment stories

The analysis shows that all teachers own and identify with MTEP's vision of teaching for conceptual understanding. In teaching for conceptual understanding the teachers also teach for understanding which is one of the ways of addressing the many challenges facing mathematics teaching in South Africa. Two out of five teachers claim to have implemented conceptual teaching.

In the next chapter, I analyse data gathered from phase one classroom observation and post-observation interviews. This data analysis show how the teachers' practices have been influenced by their participation in MTEP.

CHAPTER 5

DATA ANALYSIS

PHASE 1

CLASS OBSERVATIONS AND POST-OBSERVATION INTERVIEWS

5.1 INTRODUCTION

In this chapter I analysed data gathered from phase one classroom observation and post-observation interviews. The five participating teachers selected a lesson of their choice in the FET mathematics curriculum. The purpose of the classroom observation was to observe how the teacher's classroom practices had been informed by their participation in the Mathematics Teacher Enrichment Programme (MTEP). Each of the observed lessons was then followed by a post-observation interview. The post-observation interview was aimed at gathering teacher's stories in relation to the observed lesson. Through the post-observation interview teachers had an opportunity to talk about their practice in relation to conceptual teaching. Indicators of my conceptual teaching analysis tool were used to analyse data gathered from the classroom observation and post-observation interviews. These indicators are:

Teaching that:

- Promotes conceptual understanding
- Promotes productive maths talk
- Promotes effective use of manipulatives
- Promotes visualization
- Demonstrates teacher positive self-efficacy.

See my indicators of conceptual teaching analysis tool table 3.5 of chapter 3.

The above framework of indicators determines the structure of each teacher's lesson narrative.

5.2 TEACHER 1'S LESSON

Teacher 1 taught the Area rule to grade 11 learners: It is a trigonometry topic according to the Curriculum and Assessment Policy Statement (CAPS) document. Teacher 1 introduced her lesson using a question-and-answer teaching strategy. Through this strategy learners were encouraged to demonstrate their knowledge of trigonometric ratios sine, cosine and the tangent. Teacher 1 also revised properties of triangles and the Pythagoras theorem with the learners. Teacher 1 drew a triangle ABC on the chalkboard and a perpendicular height which divided the triangle into two right angled triangles as shown in figure 5.1 below:

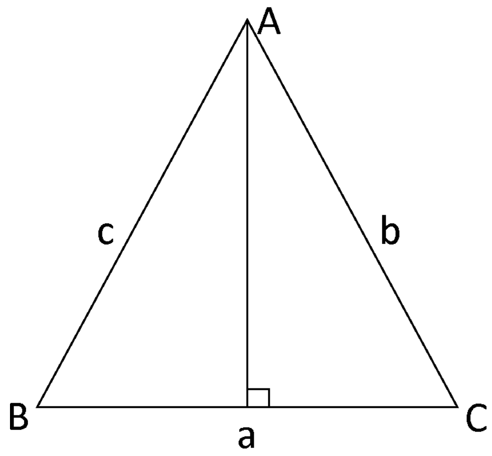

Figure 5.1

Teacher 1 then asked learners to give a formula for calculating the area of the triangle ABC. Learners were taken through the area rule proof process using their knowledge of the area of the triangle, the sine rule and changing the subject of the formula. Teacher 1 engaged learners in mathematical talk and probed for prior knowledge of trigonometric ratios, area of a triangle formula and changing the subject of the formula. Through this talk Teacher 1 encouraged learners to observe that the area of a triangle is half times the product of two sides and sine of the included angle. The area rule proof is given below:

Area of ABC = ½ base * h

Sin B = h/c

h = c Sin B

Therefore: Area of ABC = ½ a c Sin B

Area of ABC = ½ base * h

Sin C = h/b

h = b Sin C

Therefore: Area of ABC = ½ a b Sin C

By drawing a perpendicular height from B to AC, it can be shown that:

Area of ABC = ½ b c Sin A

Teacher 1 gave learners an opportunity to prove the area rule using triangles that were named differently from the one she used to prove the theorem. This activity together with Teacher 1's questioning strategy inspired the learners to observe that the area of a triangle is half the product of two adjacent sides and a sine of the included angle. Teacher 1 concluded her lesson by giving learners activities which required them to apply the knowledge of an area rule to calculate the area of a triangle, solve for sides and angles of a triangle.

5.2.1 Teacher 1 lesson and post-observation interview analysis

5.2.1.1Teaching that promotes conceptual understanding

The extract below shows Teacher 1's video recorded lesson introduction:

(T) If we can remember in grade 10 you did, you were introduced to trigonometric ratios and they would say to you, solve the following triangle, they would give you a right angled triangle, we will name the triangle ABC. Given triangle ABC, AC =4cm and BC = 2cm, they will ask you to solve that triangle using trigonometric ratios. Now remember those angles were right angled triangles, you use trig ratios or?

(Ls) Pythagoras theorem

(T) How would you solve for this side AB, which is unknown? Let AB = y, BC = x, AC = r, then Pythagoras says that $r^2 = x^2 + y^2$*, that's one method you can use. Now using trig ratios which trig ratio to find this side? We have got three basic trig ratios, which trig ratio would you use to solve this side. What is sin θ?*

(Ls) $\frac{y}{r}$

(T) Cos θ?

(Ls) $\frac{x}{r}$

(T) Tan θ?

In the post-observation interview I asked Teacher 1 to explain why she decided to introduce the lesson using the trig ratios and she responded:

(T) I felt it was important for me to start there, to remind them or to, ja to remind them about the trigonometric ratios and ***we were going to use the trigonometric ratios****. That's what I thought they knew then, I'm not sure if I'm answering your question.* ***So I was like moving from what I know they know to offering what I was going to teach them****. For continuation, I mean they must know that what they are doing in grade 10 it's not just for grade 10. They must know that it's important even for the next coming grade. So* ***these things are related, so you grow from this to this and from this simple thing to a bit complex something****. So if they don't know what they were doing in grade 10, it would be impossible for them to understand what we will be doing in grade 11. So that's why I said to myself ok, I must move back to grade 10 just to remind them. So that I can move on and* ***introduce something else to them or to relate what they know*** *to what I was going to present to them.*

The extract above shows that Teacher 1 asked questions which required learners to demonstrate their prior knowledge of trig ratios. Teacher 1's questions were aimed at first reminding learners of these trigonometric ratios and then encouraging them to use that knowledge as a foundation for the area rule that she taught to the learners. Learners made their thinking visible to Teacher 1 and this according to Walshaw and Anthony (2008) is a resource for teachers, informing them of what students already know and what they need to learn. It is evident from Teacher 1's response that her questioning style encouraged learners to relate prior knowledge to the new knowledge. This aligns with Fraivillig, et al., (1999) who argue that ascertaining what students know and how they think about mathematical concepts is a critical element for advancing learner thinking.

Below is the extract from Teacher 1's video recorded lesson:

(T) so for you to calculate the area of whatever triangle you are given, you must know these 2 sides and this angle these 2 sides and this angle. Now what will happen if now we were to talk about angle B, what would be the formula? Listen up, what you will notice here, this is an angle A, this is side b and c. Angle C, side b and a, what do you notice? If we are talking about angle C, do we have side c?

(Ls) No

(T) If we are talking about angle A, do we have side a?

(Ls) No

(T) Now let's develop the third one. We've spoken about angle C, we've spoken about angle A; now let's talk about angle B. What will happen here?

(Ls) Side a and c

(T) Can you see, side a and c; ***so as you can see, there's no need for you to memorize these formulas, you need to understand them****. That if you are talking about angle B, the sides that you are going to work with are the other sides that do not include side b. Do you understand?*

(Ls) Yes

(T) if you are talking about angle A, your sides must be the other sides that do not include A, do you understand?

(Ls) Yes

(T) Now let's do an exercise, these are the 3 formulas that you will use to calculate the area of a give triangle. Are you with me guys, they won't say to you prove these never, but you must be able to apply and use them and remember how to develop them. Calculate the, somebody read for us

(L) calculate the area of each of the given triangles

(T) yes, in a) we've been given a triangle, remember you draw a sketch for yourself before you can do anything. We've been given a diagram, is it a right angled triangle?

(Ls) No

(T) no remember I said these rules that we are going to learn they do not deal with right angled triangles but with any other shapes that do not include the right angled triangle. Now we are given, let's give a name to our triangle, it's ABC. Can you see that the triangle has not been labelled?

(L) Yes

(T) You can do that for yourself so that it's easier, we don't always have to use ABC, let's label our triangle differently; DEF, don't tell that the triangle will always be ABC, it can be XYZ, OPQ, SUV and so on. What will this side b?

(Ls) Small letter d

Responding to the question I asked Teacher 1 if she thought the lesson goal was achieved she said:

(T) *I think I achieved the goal because after the lesson I gave the learners some exercises to work on so they were able to solve the triangle using the area rule. That is side and angles of a triangle. I think I achieved that but I didn't make them prove the area rule. Err because of time frames and I don't think it was necessary. When I looked at the question papers the previous years, it was never in the question papers that they have to prove the area rule. It was just for them to be able to use it in solving the triangles.* ***But I wanted to make them aware that this, I hated to give them the formulas, but I wanted them to see how these formulas came about****. So that's why I started by proving the theorems with them even though I felt from the question papers, it was not important for them to know. And I was working for, in case they say in the question papers, prove the area rule, at least they've got an idea. It was not just for, I was not just preparing them for exams only****; I was preparing them for future use. If they go to grade 12****, like it's CAPS now, I don't know if during these exams they are still going to ask the same way as before we did, I mean during the time when we were doing NCS. Now I think I was preparing them for that*

The above question was followed by the question which asked Teacher 1 to say why she thinks it is important to prepare for the future and she responded:

*(T) For me I think knowledge**; I mean to know something is very very powerful**. Because I don't see them only doing grade 12, I think after I, what I see from there is after grade 12 they will go somewhere, they will meet these things they will have to know and remember these things from their high school education. So I wasn't preparing or I'm not preparing my learners just for passing grade 12.*

Although Teacher 1 knew that the area rule proof was not required for exam purposes, she decided to take learners through its proof process so that they understood the steps. The extract shows that Teacher 1 did not just present the rule to the learners, the proof process led to a deduction which was owned by the learners. Teacher 1's approach created an environment for learners to link conceptual understanding with procedural understanding. This aligns with Rittle-Johnson and Alibali (1999), who argue that the two types of knowledge do not develop independently, it is likely that learner's conceptual understanding influences the procedure they use. The area rule proof process together with Teacher 1's questioning strategy caused learners to observe that the area of a triangle is half times the product of two adjacent sides and the sine of an included angle.

5.2.1.2 Teaching that promotes productive maths talk

I asked Teacher 1 if there was any part of this lesson that she would attribute to her involvement in MTEP. She responded:

*(T) Yes **the conceptual part of learning it's what was over emphasized from the MTEP.** So that's why I was trying to make my learners not to memorize the formula. Because if they memorize they formula they will be wrong because there are so many things that involved in one formula. **I wanted to get the concept to them not to memorize.***

I asked Teacher 1 to share the maths concepts that she thinks learners learned from her lesson and she responded:

(T) ***I reminded them of things that they need to know like the different types of triangles and when do you apply the area rule, not only in your right angled triangle****. And the naming of triangles and the naming of the different sides;* ***not sticking to one way of naming a triangle and not memorizing the given rules that we did, even if they name the different triangles in a different way,*** *they must be able to come up with a formula. Basically that was my focus in that lesson otherwise the other things like formula I thought it's something they knew already.* ***The area of a triangle; how to calculate the area of a triangle, the different triangles and their characteristics. Basically I wanted them to know how to prove and use the area rule and not to memorize the area rule, to get to understand.*** *As I would say to them, if you're talking about angle A then on the other side of the equation, the side A will not appear. So those are the things, I mean I was strategizing or I was teaching them the strategy of how to get to understand the formula for the area rule.*

Teacher 1 has in the process of teaching the area rule proof used probing questions that required learner mathematical reasoning. Teacher 1 did this by giving learners differently named triangles, and this provided an opportunity for learners to develop an understanding of the area rule. Although there is no evidence that demonstrates that learners understood the rule, her approach encouraged learners not to merely memorize the rule without understanding. Teacher 1 monitored learner participation in the rule proof process discussions and decided when and how to encourage each learner to participate (White, 2003). Listening to learners' ideas and suggestions is one of the ways of promoting productive maths talk according to (White, 2003; Stein, et al., 2008) and this is evident in Teacher 1s' response.

Responding to the question I asked Teacher 1 aspects of this lesson that she would highlight to other educators and she responded:

(T) The first thing that I would highlight and emphasize to the teachers is that it's important for us as teachers to let our learners know how to find the formulas or the equations. We mustn't just give them the formulas that do not have meaning. ***So we need to take them through how we got the formula, so me that is very important. And then the memorization part of the formula,*** *it's not that important*

because when you memorize something if you forget one thing, then you've just lost it. So those are the two main things that I would emphasize.

5.2.1.3Teaching that promotes effective use of manipulatives

Teacher 1 did not use manipulatives in this lesson but her thoughts regarding the use of manipulatives are detailed below:

(T) ***It's a good thing to do but it's time consuming at the same time because we've got so many classes in grades that were are teaching****. And* ***in some areas it's not easy for you to use manipulatives****. Like I'm thinking of using manipulatives in developing the area rule, of course I can do of using triangles. But I'm thinking, is it going to help, it's not like doing the area where you fit this to this triangle. You are looking at different sides and for this in particular, this lesson, I don't know I didn't think of using a manipulative but ... Yes I mean if you can give a thought, a deep thought about it, what to do and how to do it, you can still use it (manipulatives).* ***So it's time consuming to use the manipulatives and you need to finish the syllabus.***

The above extract shows that Teacher 1 supports the use of manipulatives in principle, however she argues using manipulatives is time consuming which may result in not finishing the syllabus.

5.2.1.4. Teaching that promotes visualization

Teaching that promotes visualization was not evident in Teacher 1's lesson.

5.2.1.5. Teaching that demonstrates positive self-efficacy

Below is an extract from Teacher 1's video recorded lesson:

Determine Sin angle A for me, we just found Sin there to be the opposite over the hypotenuse side or y over r. ***Now help me determine Sin A in this triangle.***

(L) Height over small letter c

(T) is equal to height and there is the opposite side, over?

(L) Over small letter c

The above extract shows that Teacher 1 summarized learner understanding of the concept of sine of an angle. The manner in which Teacher 1 phrased the question which says “now help me understand” demonstrates that she expected all learners to make a contribution. Although it is not explicit that she had confidence in the learners, it would be fair to say that she had confidence in the learners’ responses. Teacher 1 encouraged the learners and this is important to create a conducive atmosphere for learning. Martino and Maher (1999) cited in White (2003) argue that by asking more open-ended questions teachers can stimulate learners’ growth of mathematical knowledge. The questions like “what do you notice” and “now help me understand” were aimed at advancing learner thinking and this demonstrates positive self-efficacy by Teacher 1. Teacher 1 wanted to make sure that all learners understand and are able to apply the area rule in problem solving in any given triangle and this demonstrates positive self-efficacy.

5.3 TEACHER 2’S LESSON

Teacher 2 taught the concept of probability to grade 11 learners. In his introduction Teacher 2 asked learners for their definition of probability as understood from their prior knowledge. Teacher 2 said, “*We are used to hear people say, probably, what does that mean*?“. Teacher 2 then summarized learners’ responses and defined probability as the ‘Chance of something happening’. One learner responded and said “Maybe”. Teacher 2 encouraged the learners to see the real life application so that learners could understand its meaning. Probability is the chance that something might happen. Teacher 2 introduced learners to 0 to 1 scale as a way of making learners understand that the probability of an event is always a number between 0 and 1. Teacher 2 further explained the meaning of zero probability and the probability of 1. Teacher 2 also reminded learners of the concepts of sample space, event, possible outcomes using an example of a classroom of 23 learners with 10 girls and 13 boys. Teacher 2 explained the concept of probability by giving real life applications of probability, to help the learners understand its application, like weather prediction. Teacher 2 brought two dice, a fair and an unfair dice to the classroom. Through a question and answer teaching method he encouraged the learners to note the difference between the two. Learners were given wooden cubes to design fair dice to help them deepen their understanding of a fair dice. Teacher 2 then introduced a formula for calculating probability:

Theoretical probability = n (E) / (n) Sample space

Teacher 2, then used a deck of cards which he brought to the classroom to assess learner's understanding of the concept of probability. The lesson was concluded by giving learners activities from the textbook. One of the activities required learners to apply the knowledge of probability in problem solving. Probability is the chance that something will happen – how likely it is that some event will happen.

5.3.1 Teacher 2 lesson and post-observation interview analysis

5.3.1.1Teaching that promotes conceptual understanding

Below is an extract from Teacher 2's video recorded lesson:

(T) Now let's do it this way, I want the probability not the sample space. I want the probability of getting an even number. I want the probability of getting an even number, so look at the sample space, how many even numbers do you have there? And then give me the probability; check your sample space, how many even numbers do you have there and then what is the probability of getting an even number when you throw a dice? Now what's the probability of getting an even number? Have I counted my sample space, how many even numbers do I have there?

(Ls) 3

(T) 3, which are they?

(Ls) 2, 4, 6

(T) 2, 4, 6; so which means we've got 3 events out of 6. So 3/6 or ½ isn't that so?

(Ls) Yes

(T) or 0.5, can you see that?

(Ls) Yes

In the post-observation interview I requested Teacher 2 to share why he asked the following questions: ‘Now what is the probability of getting an even number?’, ‘What is the probability of getting even numbers?’ ‘Which are they?’ and he responded:

(T) Firstly ***I wanted to make sure that what they understand about even numbers, can they tell me what are even numbers in a sample space.*** *So I was in a way reaching to those that are my slow learners so that they could see that in this sample space here are even numbers and they are 3 out of the total.*

I asked a follow up question of why he thought it is important to make sure that slow learners grasp the maths concepts and he responded:

*(T) Well it’s always my task that I should go down to even the slowest of my learners that whenever I do the topic, that I reach that particular learner****. So I make it a point that I know that learners are different in their levels in class, some of them I would say something once and they would quickly get it****.* ***But I need to think about that one that needs me to explain it again so that they can understand.***

Teacher 2’s response above demonstrates that he asked questions which required learner knowledge of even numbers. The extract also shows that Teacher 2 went on to encourage learners to demonstrate their understanding of even numbers verbally. It is evident from Teacher 2’s response that he questioned with an intention to ensure that all learners understand what he teaches. This aligns with Wood, et al., (1991) who argue that teachers should take students’ ideas seriously in their attempt to support understanding. Teacher 2’s response demonstrates knowledge about both mathematics teaching and learners’ mathematical thinking and this according to Fraivillig, et al. (1999) supports learner development of conceptual understanding.

The extract also shows that Teacher 2 asked learners to list the even numbers. This demonstrates that he used learners’ responses and asked probing questions to promote elaboration. When Teacher 2 asked a probability of an even number, he asked a question which was aimed at linking the concepts of sample space and probability with a formula of finding probability. Thus teaching for mathematical understanding must include both procedural and conceptual understanding (Wearne & Hiebert, 1988) and this is evident from Teacher 2’s response. Conceptual and procedural knowledge do not develop independently, it is likely that learner conceptual understanding influences the procedures they use (Rittle-Johnson & Alibali, 1999).

5.3.1.2 Teaching that promotes productive maths talk

There is no evidence of teaching that promotes productive mathematics talk in Teacher 2's lesson.

5.3.1.3 Teaching that promotes effective use of manipulatives

Below is an extract from Teacher 2's video recorded lesson:

(T) Ok, now if I have a dice I throw a dice and I want a 6, now I will throw a dice and I want a 6 can you see?

(Ls) Yes

(T) Do you see that, I want a 6 and I'll throw it, it's 6 can you see that?

(Ls) Yes

(T) I want a 6 so I'll throw it again and it's a 6, ***is there something wrong with my dice?***

(Ls) Yes

(T) What is it, it's all 6, can you see?

(Ls) Yes

(T) ***Now what is wrong with this one? Is it a fair dice or is it not****?*

(Ls) Not fair

(T) A dice or a coin must be fair, so how is a fair dice? It looks like this one, ok so something special about the dice; if it's just one we call it a die, if it's two we call it dice, so it's not a fair dice, do you agree?

(Ls) Yes

(T) ***Ok now look at this one, I want you to tell me something about this one. Can you see anything special about it, what do you notice? Pass it around***

(L) It starts from 1 to 6

(T) It starts from 1 to 6, ***what else do you notice? Look at the numbers, the way they are arranged there, what else do you notice?*** *I understand some of us bayaqala ubona i-die (it's the first time you've ever seen a dice before).* ***What else can you notice from there? Anything? There's something interesting there****, Vuyo? If you look at mine, it's got 6's but that one, something is different. The numbers go up to 6, but there is something interesting about this one and I want you to tell me. Check the arrangement of the numbers, what can you tell me about it? Ok you are going to tell me, if I put it that way, facing up; what is the number at the bottom? If the one facing up is what? If it's 3, then the bottom is what? 4, Can you see?*

(Ls) Yes

(T) if it's 5 then ngapantsi ngubani (what is underneath)?

(Ls) 2

(T) So what can you say about the numbers, if you add what's on top and what's at the bottom, the answer is what?

(Ls) 7

(T) If I add this to that one I get what?

(Ls) 7

(T) That's interesting, I understand for some of us it's the first time we've seen a die. Now you understand what is a dice akunjalo (right)?

(L) Yes

*(T****) Now, I would like us to have a feel,*** *pass it around, we are going to make fair dice. So make your own die, remember when you add the opposite numbers you should get a 7.*

(Learners passing dice around)

(T) Now make sure your die is a fair die, pass it around. Is it a fair die?

(L) Yes

(T) Let me see

(T) Good, good,

In the post-observation interview I asked Teacher 2 about manipulative he used in his lesson and he responded:

(T) I used the cubes, the wooden cubes and I also used a deck of cards

The above question was followed by "Has the use of these manipulative in your opinion supported their learning of probability and why?" and Teacher 2 responded:

(T) Yes. As I have said before, ***allowing them to build their own dice helps them to understand what we mean by rolling the dice,*** *picking a card from the deck of 52 so which I think made them to understand what is happening,* ***what we mean by picking something from a certain sample space.***

I then asked Teacher 2 to share his opinion on the importance of using manipulatives in teaching and this is how he responded:

(T) In my opinion it is very important even though at times it is time consuming. But if you use for say uhm 20 minutes, 30 minutes, that makes it easy for them to understand the concept now that they have seen. *Because remember the learners that we are dealing with, they understand based on visuals. So if they see something, then it's much easier for them to grasp.*

The classroom observation extract shows that Teacher 2 brought two dice into the classroom, a fair and an unfair dice, and drew the attention of the learners to these manipulatives. It is evident from the classroom observation extract that Teacher 2 gave learners an opportunity to design a fair dice using wooden cubes that he brought to the classroom. The extract also demonstrates how Teacher 2 engaged learners in mathematical talk which supported their understanding of a fair and an unfair dice. Teacher 2 purposefully guided the learners to observe the difference between a fair and an unfair dice. Teacher 2 asked questions which required learners to verbalise mathematical ideas they noticed from the dice, like the numbers on the dice, how the numbers are arranged and how to apply that knowledge to calculate the probability of an even number, for example.

The extract also shows that Teacher 2 assisted learners to apply the mathematical ideas they observed from the dice to achieve the aim of the lesson. This demonstrates that these manipulatives were used for the purposes of the lesson. Although Teacher 2 acknowledges that the use of manipulatives is time consuming, he argues that it supports learner understanding. Teacher 2 played a vital role in supporting learner understanding of probability through the use of manipulatives (Clements, 1999).

Although there is no evidence of a positive learner attitude from Teacher 2's response, Sowell (1989) argues that when students are instructed with concrete materials by teachers knowledgeable about their use, it improves learners' attitude towards mathematics. According to Teacher 2, meaningful use of manipulatives deepens learner understanding of mathematics concepts. Manipulatives make even the most difficult mathematical concepts easier to understand. They enable learners to connect abstract mathematical concepts to real objects (Kennedy & Tipps, 1994). According to (Meira, 1998) the meaning attached to the manipulatives is not necessarily transparent to both teachers and learners. Teacher 2 had, through his questioning style, supported learners' understanding of mathematical meaning through the use of manipulatives.

5.3.1.4Teaching that promotes visualization

I asked Teacher 2 to explain his statements and asked the questions 'Look at the numbers, the way they are arranged, what else do you notice?' and he responded:

T: Oh I see, uhm after I've noticed that some of them it was their first time to see a dice, then I wanted to familiarize them with the dice so that ***they could see that if you add the numbers on the opposite sides, on the outside they add up to 7. I wanted them to see that and that is what is interesting that I think about the normal six-sided dice****. And also uhm another thing, but I didn't go through that* *much about that; uhm if you're doing sequences, at a certain sequence in the arrangement of the numbers as well.: Ja I wanted them to notice that, the outsides, that is the sides that are, if a 1 is facing up, the bottom side will be a 6 and I wanted them to see that if 3 is facing up, the bottom number will be 4. So the numbers on the opposite sides they add up to 7*

The question above was followed by: "Why do you want learners to notice that particularly?" and Teacher 2 responded:

(T) Uhm in fact it was for their interest and that ***when they would look at a dice they would see Maths from a piece of a dice*** *and also for them as I said before, some of them it was their first time to see a dice itself.*

It is evident from the extract above that Teacher 2 asked questions which encouraged learners to visualise the number on the opposite face of the dice. It also demonstrates that Teacher 2 asked questions that purposefully pushed learners to verbalize their mental image of the number facing down in a dice. Teacher 2 supported learners in a process of forming mental images of the unseen numbers of a dice and using those images effectively for mathematical discovery and understanding (Zimmerman & Cunningham, 1991).

Teacher 2 also assisted learners through his questioning to relate their mental image to the lesson aim. Learner's mental image of the dice helped them to answer the probability questions that were asked. Duval (2006) argues that when we focus on visualization, we are facing a strong discrepancy between the common way to see the figures, generally in an iconic way, and the mathematical way that they are expected to be looked at. Teacher 2's questioning style supported learners to move from a common way of seeing to a mathematical way of seeing a dice. The mathematics concepts coming out of what learners see depends on the questioning style of the teacher. The extract shows that Teacher 2 used questions to promote visualization, to convey insight as well as understanding of the concept of probability.

5.3.1.5 Teaching that demonstrates positive self-efficacy

In response to the question I asked Teacher 2 about the following statements and questions: 'Now I'm throwing a dice, and I get a 6, what is the probability of getting a 6?' he replied:

(T) Well now after they understood what we meant by sample space, and then knowing that what the elements of the sample space. ***Then I wanted them to tell me,*** *if I throw a dice, will I get a 6, what are the chances for me of getting a 6 from that one throw. And then* ***uhm looking at the sample space they would be able to tell me*** *that there is only one 6 in your dice so you'll be able to get one 6 out of these six numbers.* ***So I wanted to bring that to their understanding as well.***

I asked Teacher 2 why he thinks it is important to ask questions in a mathematics classroom and he responded:

(T) Yes it's very important for them to ask questions because as I said some of them they would understand things differently from the way you want them to see. And then you have to allow them to ask questions at any moment of the period. And ***as I also believe that we learn by mistakes, allow them to make mistakes, allow them to tell you what they think. Because that's what is important, that's how teaching and learning goes, you learn from what they think and then you sort of build on their mistakes by trying to show the right way of doing things, from their mistakes.*** *Personally I always encourage my learners to ask question even if a person thinks that, that it is not that important. So I believe that learners should ask questions in a Maths period.*

The above question was followed by: "How do you allow them to ask those questions, so when you say 'allow' what do you mean?" and this is how Teacher 2 responded:

(T) ***You sort of let them, you open a room for them and then sometimes I will ask questions like, 'is there any question?' If they don't ask questions, you pose questions to them. What questions you wanted them to ask you, if they don't ask those questions then ask them****. If something like this happened, then*

what do you think about it. Because you would find at times some of them are shy to ask questions. So you end up creating a way to ask them those questions.

It is evident from Teacher 2's response above that he expects learners to ask questions and make a contribution in the classroom. The extract also demonstrates that Teacher 2 displays a positive attitude to learners' questions and their enthusiasm to learn. According to Teacher 2, if learners do not ask questions, he motivates them to ask questions by asking them a question. This shows that Teacher 2 created an environment that is conducive for all learners to develop cognitively. Teacher 2 was deliberate in making learners understand what he teaches. This aligns with Bandura (1997) who argues the task of creating learning environments conducive to development of cognitive competencies rests heavily on the talents and self-efficacy of teachers.

The extract shows Teacher 2's positive self-efficacy because he asked questions with an appreciation that learners are at different levels of understanding, therefore he adjusted his teaching style accordingly. This shows that Teacher 2 has a positive attitude towards learner mistakes and uses them as opportunities for learning, and to clarify and facilitate proper understanding of mathematics concepts.

5.4 TEACHER 3'S LESSON

Teacher 3 taught a lesson on factorising the difference of two squares to grade 11 learners. This was not a new topic to the grade 11 learners since they were introduced to the topic in the previous grades. Teacher 3 reminded learners of the procedural way of solving difference of two squares which is:

$$x^2 - y^2 = (x - y)(x + y)$$

Teacher 3's approach to teaching the difference of two squares was different from the usual way, that is, teaching the learners the procedure of how to factorise as it is above. Teacher 3 introduced her lesson by writing the following numbers on the chalkboard 4, 9, 16, 25, 49 and then asked learners to say what these numbers are. Learners responded and said that they are squares of numbers. Teacher 3 then asked a follow up question which required learners to define a square. Through a question and answer strategy Teacher 3 encouraged learners to focus on the literal meaning of difference between two squares using the main concepts, difference and a square.

Teacher 3 then gave learners A4 size paper which had a small square drawn inside a big one and a pair of scissors per group. Learners worked in groups of four and were instructed to label the big square x by x and the small one y by y respectively. Learners were provided with an opportunity to explore difference of two squares using the two diagrams given below:

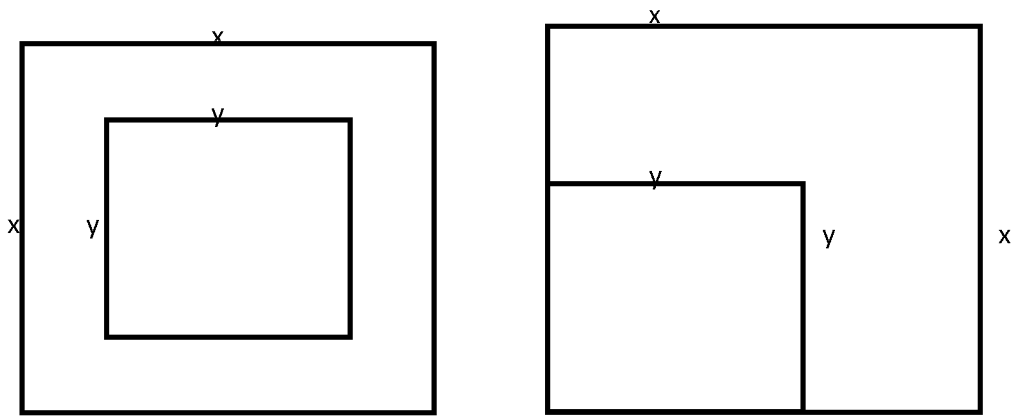

Figure 5.2 Teacher 3's Squares

(a) (b)

Teacher 3 then asked the learners to cut and remove the small square from the bigger square. Although Teacher 3 guided the learners in the cutting out process, she gave learners an opportunity to decide how they took the small squares away from the big ones.

Teacher 3 took learners through the process of taking away a small square from the bigger square and to name what is left in terms of x and y. Teacher 3 encouraged learners to write the dimensions of what is left when the smaller square is cut out.

Teacher 3 repeated the same activity using different types of squares. Through this process, learners deduced for themselves that factorising difference of two squares $x^2 - y^2$ give the product (x – y) (x + y).

5.4.1 Teacher 2 lesson and post-observation interview analysis

5.4.1.1Teaching that promotes conceptual understanding

Below is an extract from Teacher 3's video recorded lesson:

(*T) Right, what we are going to be doing today, **we are going to look at the (4; 9; 16; 25; 49) if you are looking at these numbers on the chalkboard, what are they?***

(Ls) They are squares

(T) They are squares? They are squares of they are square numbers?

(Ls) They are square numbers

(T) Right, you say they are squares, these ones particularly these one they're not just squares, they are perfect squares, right. I want to understand what it is that you mean by squares, what do you mean this number is a square?

(Ls) You can times it by itself

(T) What, you can times the number by itself to get it?

(Ls) It means that there's a number that you square to get these numbers

*(T) So when you are looking at these numbers and you are saying it's a square, **what comes into your mind is another number that you can square?***

(Ls) yes ma'am

(T) Right, now I want you to demonstrate there in your papers that you have in front of you. Can I see how you get a square from this number? You say this number is a square?

(Ls) yes

(T) How do you get a square from this number?

(Ls) It's 3 squared ma 'am

(T) Ok it's 3 squared, you get 9

(Ls) Yes

(T) So when you're talking about these numbers, ***what comes into your mind is another number that you can square to get that number?***

(Ls) Yes

(T) It never occurs to you that we can talk about this as a square as in a shape? ***What is a square, when you say square, what is it?***

(Ls) It's a shape

(T) It's a shape isn't it, so when you see these numbers you think of a shape, ***you never think of shape isn't it, you think of a number not a shape?***

(Ls) Yes

(T) But what we mean exactly is that this is a shape, a four-sided shape?

(Ls) Yes

In the post-observation interview I requested Teacher 3 to tell me what the aim of the lesson was and she responded:

(T) Ok, the aim of my lesson was to make my learners understand, ***get the concept of two squares because usually, we've done algebraically just by calculation****. And the learners they don't really associate that with the actual squares that you can manipulate.* ***So that's what I wanted the learners to see, that the actual algebraic expression that we do usually it is that, dealing with the actual squares as in shapes.*** *Because when we had done that, we had done it algebraically before and they were showing expressions like 'ok this is what we are doing, ok this is what is happening.'* ***To show that we are doing reconciling what they had learnt with what they were actually seeing.***

It is evident from the extract above that Teacher 3 asked questions that required learners to demonstrate their understanding of perfect squares and square shapes. Teacher 3 asked questions that probed prior knowledge and required mathematical reasoning from the learners. Teacher 3 asked learners if they had ever thought of a square in the difference of two squares as shape demonstrates that she was intentional in making learners focus on the literal meaning of difference of two squares. The kind of question that Teacher 3 asked were “what comes in to your mind” are open ended which required learner reasoning and not a no or a yes answer. Teacher 3 asked learners questions that required learners to demonstrate their understanding of perfect squares, a square shape and the concept of a difference. The extract above also shows that Teacher 3 asked questions that were purposefully aimed at linking procedure with concepts. Teacher 3 requested learners to demonstrate how they got a square from a perfect square number. This shows that learners were given an opportunity to explain their solution methods to the teacher and to other learners. Both procedural and conceptual knowledge are considered necessary aspects of mathematical understanding. Thus to teach for mathematical understanding the teacher must include both procedural and conceptual understanding (Wearne & Hiebert, 1988). Knowledge that has been learned with understanding provides a basis for generating new knowledge and for solving new and unfamiliar problems. When students have acquired conceptual understanding in an area of mathematics, they see the connection among concepts and procedures and can provide arguments to explain why some facts are consequences of others. They gain confidence, which then provides a base from which they can move to another level of understanding (Kilpatrick, et al., 2001). The manner in which Teacher 3 approached the lesson encouraged learners to gain conceptual understanding of factorising difference of two squares. Although there is no evidence that learners gave arguments to explain concepts, but they were encouraged to make the connection between the two as a result of Teacher 3’s questions.

The extract also demonstrates a high level of teaching for conceptual understanding. Teacher 3’s questions were purposeful and inspired learners to connect knowledge of algebra with geometry knowledge. This helps learners to think laterally and not confine mathematics knowledge into separate and unrelated topics.

5.4.1.2 Teaching that promotes productive maths talk

I asked Teacher 3 to say why she wrote the square numbers on the chalkboard and she responded:

(T) The reason why I asked that question was because I believe I had written a number, some numbers on the board. And when they see those numbers they call them square numbers ***but I wanted to find out if they can associate these square numbers with the four-sided shapes that we call squares****. That is what I was asking when I asked that question. They did not associate the square numbers with the actual shapes, the squares.*

Responding to the question I asked Teacher 3 about how she ensured that learners associate the numbers with square shapes she said:

(T) Well, from there, because I had actually written down the number, I drew squares on the board. I actually asked some of them to draw squares on the board. And then we worked out areas of those squares. ***And then that was when they started to relate the numbers they call squares with the shapes to see that it is actually the area of the shape of square.***

Teacher 3 asked the question "What is the difference between those two shapes?' and I asked her why she asked the question. She responded:

S: Well, firstly I wanted to establish if they knew what a square is because one had drawn a square and the other one had drawn a rectangle. So I wanted to find out if they knew that a square is a shape with four sides that are equal that was what I wanted to find out.

In the video recorded lesson extract given above, Teacher 3 asked learners questions like, I want to understand what is it that you mean, what is the meaning of a square, has it ever occurred to you to think of these numbers as a shape. In the post-observation interview, I asked Teacher 3 to share why she asked these questions and she responded:

Teacher 3 promoted productive maths talk in her classroom because she used learner's responses and asked probing questions to promote elaboration. Learners were provided with an opportunity to justify why they said the numbers are perfect squares. Through Teacher 3's questions, learners deduced for themselves that factorising difference of two squares

$x^2 - y^2$ will give the product (x + y)(x – y). Teacher 3 promoted productive and effective maths talk through her questioning.

Teacher 3 asked two learners to draw squares on the chalkboard and one learner drew a rectangle instead of a square. Teacher 3 used that opportunity productively to build a mathematically correct concept of a square. Teacher 3 purposefully selected two students' responses for public display. She asked the two students to display their answers on the chalkboard for discussion with the whole class (Stein, et al., 2008). She did this to promote productive maths talk which led to learners focusing of a square and therefore difference of two squares. The classroom discourse in the class was centred on purposeful mathematical talk with genuine student contributions and interactions (White, 2003).Teacher 3 had, through her questioning style, promoted discussions which were helpful to arrive at the deduction of factoring the difference of two squares.

5.4.1.3 Teaching that promotes effective use of manipulatives

I asked Teacher 3 about the manipulatives she used in her lesson and she responded:

(T) ***Ok, because I actually used paper and scissors****, that's what we used mostly because we had drawn squares on pieces of paper where they were supposed to cut. Remove one square and then cut out the remaining piece of the square so that they can get another shape and calculate the area for it.*

I then asked a follow up question concerning her choice of those manipulatives and she responded:

*Ok, the reason why I chose to, because like I said before, I had actually done this lesson with them before, algebraically before and they didn't seem to get it, they were always getting it wrong. So I was trying to make it more practical so that they see w****here exactly that is coming from so that they would be able to remember***

***most of it**. Because if you do something practically, most of the times you will remember it, you don't easily forget. **I think unlike just giving learners formulas and what have you, it is easy for learners when you manipulate things**. When you actually, **because when they manipulate for themselves, they actually make the deductions, they come up with the expressions, they come up with those formulas** which we usually give to them in class. So if they come up with those things themselves then they are not going to forget them.*

Teacher 3 drew the attention of the learners to the squares drawn in the paper and she used them for the purposes of the lesson. The extracts given in the section above demonstrate that Teacher 3 did not just bring manipulatives to the classroom, she used them in her teaching. Teacher 3 encouraged learners to verbalise mathematical ideas they observed from the drawn squares. Teacher 3 then asked the learners to name the remaining sides in terms of x and y.

The use of manipulatives supported the aim of the lesson because the learners made the deduction on their own. Teacher 3 played a vital role in helping learners use manipulatives to support understanding of mathematical concepts (Clements, 1999) and this was evident in Teacher 3's lesson. Teacher 3 had, through her questioning style, helped the learners to transfer what they did with the manipulatives into their conceptual and procedural understanding (Suh, 2007)

5.4.1.4 Teaching that promotes visualization

Teacher 3 provided the learners with an opportunity to explore the difference of two squares using two scenarios as indicated above. Responding to the question why she gave these two scenarios she said:

*(T) The idea for this was for **the learners to see the result of removing a square from another square because we are talking about the area of a square here**. I wanted them to see the result of removing the area of the smaller square from the bigger square. The area that remains when we remove the smaller square from the bigger square regardless of the shape or regardless of where the smaller square has been removed from, the area remains the same. So I wanted them to explore that so that they see, it doesn't matter where you cut the shape that is remaining, but the*

area is going to be the same and it's going to come out in that same format that we learnt algebraically.

The question that Teacher 3 asked the learners was had they ever thought of x squared as a shape. This required the learners to create a mental image of a square. Teacher 3 then asked learners to draw the squares on the chalkboard. Teacher 3's questioning style assisted learners to relate their mental image with the aim of the lesson. As Teacher 3 said that she wanted the learners to see the result of removing a square from another square. Teacher 3 provided learners a platform to literally and mentally see this result. Learners were presented with opportunities to have a mental image of the removed and the remaining square throughout the process of their calculation.

5.4.1.5Teaching that demonstrates positive self-efficacy

I asked Teacher 3 to say what maths concepts she thought learners learned in her lesson and she responded:

(T) ***Ok they have uhm, they did learn the need to investigate.*** *And they have learned also that the algebraic expressions or the formulas that we give to them have actually been done physically or let me say practically before.* ***So everything that, it's not just, like Mathematics is not just an abstract thing that you get formulas and, it has been done practically.*** *It has been proven, like being done practically. So they've, I think they have learned that Maths is not actually abstract it can be done practically.*

Teacher 3 created a conducive environment for learners to express their ideas, since she allowed them to express and demonstrate their thinking publicly. Teacher 3 gave them time to express an error and used it to build the correct understanding of square. Teacher 3 could have discouraged the wrong thinking but she decided to bring it to the fore which helped learners with an understanding of a square, but also with an understanding of the properties of quadrilaterals. This also demonstrates that Teacher 3 encouraged and expected all learners to make a contribution in her classroom.

5.5 TEACHER 4'S LESSON

Teacher 4 taught a theorem to grade 11 learners. The theorem states that; the angle subtended by the arc at the centre of the circle is twice the size of the angle subtended by the arc at any point on the circumference. In her introduction Teacher 4 drew a circle on the chalkboard and marked points A, B, C and D on the circumference of a circle. Teacher 4 had, through the question and answer teaching method, asked learners about their understanding of subtended, arc and the circumference. Teacher 4 used different colour chalk to allow the learners to observe radius and the concepts mentioned above.

Teacher 4 then began to formally prove the theorem by drawing a circle with centre O and two chords AC and CB meeting at C – see figure 5.3. Teacher 4 joined radii AO and OB so that angles AOB (at the centre) and ACB (at the circumference) are both subtended by the same arc AB.

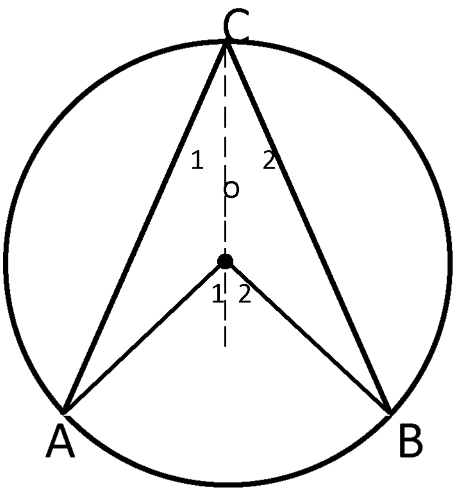

Figure 5.3 Teacher 4's theorem on the chalkboard

Theorem proof:

Given: Circle with centre O. Two chords AC and CB meeting at C to form angle $A\hat{C}B$ at the circumference.

Required to prove: $A\hat{O}B = 2\ A\hat{C}B$

Construction: Join CO, extend to D

Proof:

Angle O_1 = angle C_1 + angle CAO (An exterior angle of a triangle is equal to sum of two opposite interior angles)

Angle O_1 = angle C_1 + angle C_1 (angle CAO is equal to angle C_1 : base angles of an isosceles triangle)

Angle O_1 = 2 x angle C_1

Similarly Angle O_2 = 2 x angle C_2

Angle O_1 + Angle O_2 = 2 (angle C_1 + angle C_2)

Therefore AÔB = 2 AĈB

According to the CAPS document, this is a Grade 11 theorem, under Euclidean geometry. Learners are expected to prove the theorem and apply its results in problem solving.

5.5.1 Teacher 4 lesson and post-observation analysis

5.5.1.1Teaching that promotes conceptual understanding

Below is an extract from Teacher 4's video recorded lesson:

(T) *Today we will do theorem 3, right. Theorem three states that the angle subtended by an arc at the centre of a circle is double the angle subtended by the same arc at any point at the circumference of the circle. When you look there, there is a bold term there, which is subtended by an arc. Who can tell me the meaning of the term subtended?*

(L) Formed

(T) *Omnye , uthini? Right. Subtended by an arc means formed. Good. Formed by an arc. Do you understand? And then, an arc is defined as part of a circle. I think we know what an arc is, right. (Teacher draws a circle on the chalkboard and she asks* *the learner to finish the circle. The teacher marks four points (A, B, C and D) on the circumference of the circle. What is an arc, again?*

(L) Part of the circumference of a circle.

(T) Where is the circumference, show me the circumference? (The teacher demonstrates the circumference to the learners and ask the learners, Can you see the circumference?)

(Ls) Yes (in chorus)

(T) Show me the arc, now, where is the arc?

(Ls) Part of the circumference of a circle.

(T) In the circle on the board, where is the arc?

(L) Arc AB, BC and CD.

In the lesson Teacher 4 asked the questions ‘who can tell me the meaning of the terms subtended?’, ‘show me the arc’, ‘show me the circumference’ ‘which angle is at the centre?’. I asked her the reason for asking the questions and she responded:

(T) As I told you that, it's important first before you teach learners, to explain concepts. So as I explained the concepts before I taught the lesson, so I wanted to remind them about what we have learnt you know. ***I wanted to know if they still remembered what is the meaning of subtended, what is the circumference, they know the arc,*** *I wanted to know whether they understood. Ok I wanted to know whether they had knowledge of these terms you know so that we can explain more. Sometimes* ***they can have knowledge that the circumference is this but maybe their knowledge is incorrect*****.** ***I wanted prior knowledge, it was a prior, previous knowledge you know.*** *Because when you are introducing a lesson to learners you must first know what do they know and then take it from there*

Teacher 4’s questions were aimed at reminding learners of prior knowledge of mathematics concepts such as subtended, circumference and an arc. The knowledge of these concepts was important for Teacher 4 since she wanted to introduce new knowledge, that is, the theorem that she taught**.** Fraivillig, et al.,

(1999) argue that ascertaining what students know and how they think about mathematics is a critical element for advancing their thinking. It is evident from Teacher 4's response that she wanted to help learners understand the basic concepts before advancing their thinking. Teacher 4's questions were purposefully aimed at encouraging awareness of maths concepts. Teacher 4's questions required learners to define these concepts however the questions did not require in-depth reasoning.

Knowledge that has been learned with understanding provides the basis for generating new knowledge and for solving new and unfamiliar problems (Kilpatrick, et al., 2001). When students are encouraged to think for themselves they often incorporate their background knowledge (White, 2000). Teacher 4 encouraged the learners to think through her questioning.

5.5.1.2 Teaching that promotes productive mathematics talk

*(T) We haven't formally proven our theorem yet, but before we prove it formally, **let us identify all the properties that we already know about it, t**hat will help us in proving the theorem. Right, again, so what is O1?*

/silence/

(L) I think it will be **90°**

(T) It will be 90°, how do you know it will be 90°? do you know the size of this angle? We don't know right? Can you see?

(Ls) Yes

(T) Or are you saying this is a right angle?

(L) No

(T) Are we told that this is a right angle?

(L) No just by inspection ma'am

(T) **No, no we have to reason in a Geometry class, you don't just say anything**. Ok, right let us write it formally; we've been given, what have we been given?

The learner in the above extract gave an incorrect answer about a right angle and I asked Teacher 4 to take me through that part of the lesson**:**

(T) Ok like in geometry mos we reason, we don't just say the angle is 90^0, there must be something that supports that.

Teacher 4 asked learners to share what they knew about OA and OC. This was followed by the question that requested learners to support their answer.

(T) *Ok as I think, O was the centre of the circle,* ***so I wanted to see whether they had observed that err there are radii there****, from the centre to the circumference, from the centre to the circumference if they are radii.* ***So I wanted to make sure that, they understand the radii****, because I'm sure after that there was information that was going to be needed from the radii. Ok, I wanted them to know what is a radii you know, they must know. My aim err was for them to understand what is a radii you know. They must not say this is a radii, why is it a radii, you know, they must know that it's a chord from the centre to the circumference of the circle. You know* ***sometimes learners tend to confuse a diameter and a radius you know****. So I* ***wanted them to know that a diameter is a line that passes through the centre, it's a chord that passes through the centre. And then a radius is half of the diameter****, so it starts from the centre to the circumference. So I wanted them to understand what a radii is*

I asked her to elaborate on why she asked learners to explain why they said angle ... is subtended by AB and she responded:

*M: Ok****, first of all I wanted them to identify arcs, whenever I teach them, I ask them, look for an arc and then look for an angle that is subtended by that arc.*** *Then of the two angles are on the circumference and they are subtended by the same arc, then those two angles are equal. But if an angle is at the centre and the other angle is at the circumference, now the difference; the first two both were at the circumference but subtended by the same arc and so those angles are equal. Now the difference is that; the other one is at the centre and the other one is at the circumference but they are subtended by the same arc. So I wanted them to understand that, if it's a central angle that central angle is double the angle at* *circumference or that inscribed angle. So I think that's why **I asked them which angle was subtended by the same arc AB you know, I wanted them to identify, name the two angles that are equal or else identify the angle at centre is twice the angle at the circumference***

Teacher 4 asked the size of an angle from the learners and one of them gave an incorrect answer. Teacher 4 then used the incorrect answer and asked probing questions to promote elaboration. The learner responded and said the angle is 90^0 degrees and the reason was that he saw it by inspection. Although Teacher 4 welcomed the error, she could have asked further questions to engage learners in a productive dialogue to help them build a mathematically correct concept. Teacher 4's response was that we reason in geometry and the learner demonstrated his reasoning by giving an incorrect answer. Teacher 4 could have probed further to lead the discussion so that the learner and all other learners understand the properties of triangles and fully understand a right angled triangle (White, 2003).

Teacher 4 asked an open ended question that requested learners to tell her about what they know about radii. Teacher 4 used learners' responses productively and encouraged them to focus on the properties of an isosceles triangle. Teacher 4 then used this information from classroom discussions to show that an exterior angle of a triangle is equal to the sum of two opposite interior angles. Productive classroom discourse requires that students' ideas are encouraged, valued, and used to shape instruction (White, 2003).

5.5.1.3 Teaching that promotes meaningful use of manipulatives

Is there anything new that you have incorporated from what you have learned in MTEP?

*(T) No I didn't, because at MTEP most of the time at MTEP we are using hands-on activities. But in my class I didn't present it as a hands-on activity, it was just an investigation to them. I've drawn angles there and then identify each arc for each angle and then they noticed that, ok for example angle A and angle B are subtended by the same arc and the other one is at the centre and the other one is at the circumference so it's twice. **Otherwise I didn't use the strategy that we used in***

MTEP because to me, it's time consuming. *I've allocated time to do certain topics you know,* ***so I must work according to my schedule, so that's why***

Although Teacher 4 was exposed to the use of manipulatives during MTEP sessions, she did not think of using them in her lesson as she views their use as time consuming. The demands and the expectations of the work schedule discouraged Teacher 4 from incorporating manipulatives in her teaching. In the identity analysis chapter Teacher 4 said that the use of manipulatives promotes understanding, however she did not use them in her lesson.

5.5.1.4 Teaching that promotes visualization

There is no evidence of teaching that promotes visualization in Teacher 4's lesson.

5.5.1.5 Teaching that demonstrates positive self-efficacy

In the post-observation interview I asked Teacher 4 why she asked the questions 'What do you know about an isosceles triangle?' 'Which angles are equal there?' and she responded:

(T) Uhm whenever I teach learners, when we are talking about the shape, I want them to understand the properties of that shape. ***If we are talking about isosceles triangle, they must tell me all the properties of an isosceles triangle. Because the reason why I am saying it is an isosceles triangle, it is because err the reasoning that we are going to make, it will be based on the properties of a triangle we are talking about.*** *So that's why I asked them to tell me everything about the isosceles*

(T) Ok, I fully agree with this statement because err I think those how's are the link, we must not ask leading questions to learners, we ***must ask questions that will require them to think, to have a deep thinking about what they have been asked****. Because our aim is to give them good quality, for instance if you ask them; why angle 1 is* 90^0*, so meaning that that learner has no other solution but he must think before he answers the question. If you say why, the learner immediately will think and immediately, he will think about the properties of the shape that he or she*

is talking about, the theorems that are also involved. SO by asking those questions we are asking them err, we don't ask, these are not the leading questions you know.

Teacher 4 asked learners what they knew about an isosceles triangle, and that was followed by a question which required learners to show equal angles. Teacher 4's response in the post-observation interview demonstrates that her questions were purposeful and required learners to give properties of an isosceles triangle. The extract above shows that Teacher 4 was focused on teaching for understanding and expected all learners to make a contribution in her classroom. The kind of questions that Teacher 4 asked demonstrated confidence in her ability to influence student learning and this according to (Gibson & Dembo, 1984) shows positive self-efficacy.

5.6 TEACHER 5'S LESSON

Teacher 5 taught measurement to a grade 11 class. The purpose of the lesson was for learners to understand the concept of volume and total surface area, their formulas and applying their knowledge in problem solving. This lesson was a revision lesson for grade 11 learners since these concepts were introduced to them in the GET phase. According to the CAPS document, measurement in grade 11 is revision of grade work and should take one week of the work schedule time.

Teacher 5 introduced her lesson by asking learners to construct square and rectangular prisms using squares, rectangles and rubber bands taken from a geo-genius kit. Geo-Genius construction kit is a geometry teaching support kit. It consists of bright, precision manufactured, sturdy cardboard shapes and elastic bands. Learners can use these cardboard shapes to construct three dimensional shapes as happened in Teacher 5's class. Learners were divided into groups of five. Teacher 5 walked around the classroom observing the learners as they were constructing their prisms. Teacher 5 focused the attention of the learners on the number of edges and faces through her questioning. Teacher 5 then asked the learners to measure the length, breadth and the height of the prisms. This information was then used by Teacher 5 to revise the concept of Total Surface Area. Teacher 5 elicited prior knowledge of the area of a square and a rectangle. Learners were then introduced to the concept of a total surface area and then taken through the process of calculating it using the square and rectangular prisms they constructed. Teacher 5 defined volume as area of the base multiplied by height. Learners worked in groups to calculate the volume of the prisms they constructed. She also gave learners an opportunity to calculate volume by first calculating the volume of the prisms they designed.

5.6.1 Teacher 5 lesson and post-observation interview analysis

5.6.1.1Teaching that promotes conceptual understanding

Below is the extract from Teacher 5's video recorded lesson:

Yonzi is saying now the base times height is the formula for the volume, but I am looking for the total surface area for this shape. How can we get it? What kind of a shape is this, a rectangle or a square?

(Ls) It's a rectangle

(T) It's a rectangle, the area for a rectangle is also length times breadth yes?

(Ls) Yes

(T) ok, for the rectangle the area is length times breadth, now what is the length of the rectangle in that shape, and what is the length of the breadth? Give me the answer, what is the length of the rectangle?

(L) 7

(T) The breadth is 7, the length is?

(Ls) 11

(T) 11 times 7 what is the answer?

(Ls) 77

(L) the answer is 77 for one side, for one side right?

I asked Teacher 5 why she asked the questions 'what kind of a shape is this, a rectangle or a square?' and she responded:

(T) I wanted them to know, to see if they know the difference between the rectangle and the square that is why I asked it. ***I wanted to see if they know the features of the square and also the features of the rectangle*** *because others might think that a rectangle and a square are one and the same thing.* ***I wanted to make sure that they know the difference.*** *It is important because there are different features between a rectangle and a square. A square has got four sides equal a rectangle has got two opposite sides equal not all sides are equal in a rectangle. And that will help them in their geometry*

Teacher 5 asked questions that required learners to demonstrate their knowledge of the area of a square and a rectangle. Teacher 5's questions were aimed at focusing the attention of the learners on the difference between the two shapes. Teacher 5 then linked this knowledge with the knowledge of total surface area of a square and a rectangular prism. Teacher 5 wanted learners to know these concepts so she could build on the knowledge of a procedure, that is, the total surface area and the volume of the prisms. The questions that Teacher 5 asked in the extract above, were aimed at connecting the mathematics concepts in order to give meaning to mathematical procedure. According to Rittle-Johnson and Alibali (1999), instruction that includes a conceptual rationale for procedures leads to greater procedural skill than conventional, procedure oriented instruction.

(T) Alright now let's move on from the total surface area, by now we know that to get the total surface area we add the areas of all the faces of your shape. Now let's move on from the total surface area, let's find the volume of this, let us find the volume of this. ***Can anyone tell me what is the volume what do we mean when we're talking about the volume?*** *For the surface area we were talking about the outside area of the shape, what about the volume?* ***Are we still going to work with the outside, where is the volume of this?***

(Ls) Inside

(T) It's inside hey, now can you find the volume for this. How can we find it, are we going to use all the sides?

(L) No

(T) What can we do to get the volume, which sides are we going to use now, to get the volume of this if we were to calculate the volume? Yes Odwa

(L) Base

(T) The base by

(L) The height

Responding to the question I asked Teacher 5 about what they mean by volume she said:

(T) Before teaching the volume, I taught them the surface area. Now they would confuse the volume with the surface area. ***I wanted to be sure if they know what we mean by the volume. Can we get a volume for an example on a flat surface****, I wanted to see or I wanted to know if they knew exactly what we mean by the volume****. As a matter of fact they told me that a volume can't be outside, it's inside the object, and then I was sure that they know.***

Teacher 5 then probed for knowledge of volume from the learners. Although this was a revision, Teacher 5 could have created a space for learners to give a mathematically acceptable definition of volume through her questioning.

5.6.1.2 Teaching that promotes productive mathematics talk

There is no evidence of productive mathematics talk in Teacher 5's lesson.

5.6.1.3 Teaching that promotes effective use of manipulatives

Below is the extract from the video recorded lesson:

(T) Todays' lesson is going to be a little bit different from what we have done yesterday; ***we are going to construct some shapes*** *in today's lesson****. I want you to construct a rectangular and a square prism, for the square you construct a square and for a rectangle you construct a rectangle****, you can use the rubber bands to tie your shapes. I need a rectangular prism and a square based prism.*

Learners are quietly constructing the prisms.

(T) Then when you are done with your shapes, ***can you tell me how many faces are there? Count the number of faces and count the number of edges****. Let us all start with those with a square prism. Count the number of faces and also count the number of edges, and tell me how many faces and how many edges you have.*

(T) How many faces?

(Ls) 6 faces

(T) What shape is that?

(LS) Square prism

(T) Now, what I want us to do (let us finish up) everybody, you must have a square next to you, now, what is an area of a square, who can tell me?

I asked Teacher 5 about the manipulatives she used and why she used them and she responded:

(T) As I've said before, it makes it easy for the learners to remember the information you've given them when they see it in front of them, they touch it. Because we usually draw a sketch and tell them that this is a rectangle. ***Identify the faces and the edges of the rectangular prism. Without them seeing the object in front of them they won't be able to identify how many faces and how many edges in that triangle. That is why I gave them objects to do.*** *And also when I talk of the base of the object, with the drawing on the board or with the sketch on the board it won't be easy for them to identify the base of the object.* ***So when they see it in front of them, in a form of a box for example, it's easy to see the base of that box, it's easy to see how many corners and how many faces***

I asked Teacher 5 why she thought it is important for learners to see and touch the base when teaching volume and total surface area she said:

*(T) You cannot teach the volume if the learners don't know what the base of the substance is or what the base of the object is. So they won't be able to know what is the volume, they won't be able****, if they cannot see where the edge of a particular object is, and they won't be able to find the height.*** *So that is why it's important for them to visualise*

Teacher 5 drew the attention of the learners to the shapes that they constructed using Geo-Genius. These manipulatives were used for the purpose of the lesson to support understanding of volume and total surface area. Teacher 5 requested the learners to verbalise what they noticed from the manipulatives, that is, the number of faces and number of edges. Teacher 5's questions helped to support learners to apply mathematics ideas learned from manipulatives to support the aim of the lesson. Research (Fuson & Willis, 1989, cited in Uttal, et al., 1997) on the use of manipulatives warns that it cannot be assumed that learners will perceive the relationship between manipulatives and abstract mathematical symbols. Teacher 5 linked the use of manipulatives with instruction from the outset, and this according to (Uttal, et al., 1997) is successful use of manipulatives. Teacher 5 played a vital role in helping the learners use manipulatives to support understanding of volume and total surface area (Clements, 1999).

5.6.1.4Teaching that promotes visualization

I asked Teacher 5 to comment on whether there was any part the lesson that she would attribute to her involvement in MTEP and she said:

(T) OK, MTEP has helped a lot in dealing with this topic. MTEP has given us charts that help me to teach the lesson easily or to approach the lesson to learners easily. MTEP has also given ***us shapes so that the learners can visualize when you teach them. They can see exactly if we are talking about the rectangular shape****, they must know what we mean by the rectangle. And when we talk about the closed cylinder and the open cylinder; so MTEP has given us objects to build those shapes so that they can know exactly what we are talking about. I don't know whether I'm making sense here*

(T) When the learners ***visualize or when they see the object in front of them it makes it easy for them to remember the information.*** *They don't forget it easily when they've done it or* ***when they have touched and seen the object****, they remember it easily. Whereas if you're in front of them and you just tell them that this goes like that, it's easy for them to forget. They learn easily when they see something in front of them.*

Teacher 5 asked questions that required learners to their visual image of the prisms. Visualization according to Teacher 5, does not mean forming images mentally, but means seeing manipulatives and noting what learners see in these manipulatives. Teacher 5's lesson did not have evidence of teaching that promotes visualization.

5.6.1.5 Teaching that demonstrates positive self-efficacy

R: Ok, the last question you asked is important to me, you asked the learners not to use the rulers to measure but use a scale of 1:1, why did you do that?

F*: Ok, this is a grade 11 class, they can't always measure, and they've been taught to measure. Now,* ***as we are teaching there are lots and lots of learning areas and there's an integration between the learning areas****. So by teaching them to use a scale I'm also helping them for another subject there or another learning area. Learning areas like Geography where they are supposed to know the scale and learners like Physical Science where they are expected to know they scale.* ***I was also helping them because there is an integration of subjects***

I then asked her opinion about what she does if learners do not ask questions and she responded:

(T) ***Ok, when learners ask questions, it is a clear indication that they are with you****, they understand what you are teaching or what you are dealing with in that moment****. And for me, if learners start to ask questions, I become excited because I know exactly that they are with me.*** *Whereas if they are just sitting there quietly, you don't know if they understood you or no, so it's better for them to ask questions in class.*

(T) So if the learners don't ask questions, what do you think a teacher should do to make sure they understand?

F: Learners won't always ask questions, in class you are dealing with different types of learners. ***If they don't ask the question, you as a teacher it's your duty to pose the question back to them just to see if they understand or not.*** *And in some cases not all learners will tell you that they don't understand, that is why you have to pose the question back to them.*

The above extracts show that Teacher 5 expects all learners to make a contribution in her classroom. Although there is no evidence that demonstrates that Teacher 5 motivated the learners to ask questions, or asked questions from the learners, r she was supportive of the idea that teachers should pose questions to the learners. This demonstrates that Teacher 5 teaches for learner understanding, and this demonstrates positive self-efficacy.

5.7 PHASE 1 CLASSROOM OBSERVATION AND POST-OBSERVATION ANALYSIS TABLE

Table 5.1 below provides a summary of the phase 1 classroom observation and post-observation analysis results:

PHASE 1 CONCEPTUAL TEACHING ANALYSIS

Table 5.1

Observable Indicator	Teacher 1	Teacher 2	Teacher 3	Teacher 4	Teacher 5
Conceptual understanding	QCU2 & QCU3	QCU2, QCU3 & QCU 4	QCU1 QCU2, QCU3, QCU4	QCU1 & QCU3	QCU2 & QCU3
Productive mathematics talk	None	None	PMT3 &PMT4	PMT2	None
Use of manipulatives	None	U0M1, UOM2, UOM3, UOM4,UOM5 & UOM6	U0M1, UOM2, UOM3, UOM4 & UOM5	None	U0M1, UOM2, UOM3 & UOM4
Visualization	None	VZL1,VZL4 & VZL5	VZL1, VZL2 & VZL4	None	None
Positive self-efficacy	PSE1 & PSE3	PSE1, PSE2, PSE3 & PSE4	PSE1, PSE3 & PSE4	PSE1 & PSE3	PSE1, PSE3 & PSE4

5.8 CONCLUSION

In this chapter I used my indicators of my conceptual teaching tool to analyse data gathered through first phase classroom observation and post-observation interviews. The analysis demonstrates that all five teachers promoted conceptual understanding with respect to probing for prior knowledge of mathematics concepts and encouraging a connection of conceptual and procedural knowledge. The analysis does not show evidence of teaching that promotes productive mathematics talk for three out of the five teachers. The results demonstrate evidence of at least one indicator per teacher for the remaining three teachers. Three of the five teachers used manipulatives in their lessons and they were used productively for the purpose of the lesson. Two out of five teachers promoted visualization in their lessons. Teaching that demonstrates positive self-efficacy was evident in all five teachers and two dominant indicators were observed. These were that teachers encouraged and expected all learners to make a contribution and the environment was conducive for learners to express their thinking freely. In the next chapter, I present and analyse the second phase pre-observation interviews.

CHAPTER 6

DATA ANALYSIS

PHASE 2

PRE-OBSERVATION INTERVIEW

6.1 INTRODUCTION

In this chapter I present and analyse data gathered from the second phase pre-observation interviews. The participating teachers were interviewed a year after the first phase interviews and the same pre-observation interview schedule was used. Through this second phase pre-observation data analysis, I explore identity formation processes using Wenger's (1998) three modes of belonging. The intention of this chapter is to demonstrate whether the MTEP and the in-school support environment provided a participation space for the teachers to teach for conceptual understanding. The analysis will reveal the teachers' journey from their actual to their designated identities.

6.2 TEACHER 1

6.2.1 Engagement

I asked Teacher 1 to share how her interaction with members of the MTEP community shaped the way she saw mathematics:

(T) I was encouraged and motivated because being there ***made me to have more friends who I can relate to whenever I have challenges.*** *I knew now that if I have a problem with a certain topic I can to talk to this person and people are different even with their strengths. I knew I could talk to Facilitator 1 about this, I could talk to Teacher 4 about that,* ***so it created a situation where I would know people who can help me in Maths teaching.***

The pre-observation data analysis in phase one demonstrates that Teacher 1 has developed new teaching approaches, learned to teach for conceptual understanding and has improved in lesson planning through her involvement in the MTEP. The extract above demonstrates a relationship between Teacher 1's phase one and two engagement stories in MTEP. There is evidence from the extract above that Teacher 1 had a strong sense of belonging in the MTEP community. Teacher 1's perspective towards the teaching of mathematics has been positively influenced by her involvement in the MTEP. The extract also demonstrates that Teacher 1 saw herself as a learner who takes responsibility for her learning and the MTEP environment was a safe environment for her to ask for help in her teaching. It is evident from the extract above that Teacher 1 used her relationships with MTEP productively towards shaping her professional growth.

6.2.2 Imagination

Responding to the question in what ways has her teaching changed as a result of her participation in MTEP she said:

(T) The conceptual part of it that we can use whatever to teach Mathematics, that you can use the, whatever that you can use to teach I mean you don't just have to write numbers when you are teaching. ***If you are talking about volume you must bring things, learners must see that when you are talking about volume this is what you mean, when you are talking about perimeter this is what you mean.*** *You do the measurements you show them what's the difference between the area and perimeter so that they understand.* ***You don't base your teaching and learning on the numbers and formulas only.***

I asked Teacher 1 to share how her participation in MTEP shaped the way that she thought about Mathematics and she responded:

(T) *That I don't just base my teaching and learning on textbooks;* ***I can use whatever I can find information from people, from the internet, from different sources from people also.*** *So it's not about me concentrating on what is said in one textbook,* ***I mean I can go out of my box, think about how to present a certain lesson, what can I do to present this thing that will make learners understand.***

Mhh no, being involved there made me to see myself, I don't know if this is relevant; ***to see myself as not confined in teaching Mathematics specifically in a school***

or FET phase. I can go down, I can go beyond FET, I can influence even other teachers, so it gave me wings you know

I asked Teacher 1 if she thought that her involvement in MTEP has created a new image of self as a professional mathematics educator, and she responded:

(T) Yes in terms of uhm, because we are exposed to different conferences, we listen to different people saying different things that made me to develop. ***Just to mention one thing, I didn't know that preparation of exercises is very important.*** *I'm not sure if I did get the question correctly? Yes, in terms of preparing my work and my tasks and everything, the standard now is not the same because of the exposure that I received from MTEP that I need to do this in this manner.* ***I mean ja, the standard of my work generally in terms of preparation, marking, feedback and motivating even my learners.***

The first phase imagination stories showed that Teacher 1 saw herself as a more committed, more confident and a more professional mathematics educator through her participation in the MTEP. The data above shows that Teacher 1 has been encouraged to learn the importance of bringing and using manipulatives into her teaching in MTEP. According to Teacher 1 using manipulatives supports conceptual understanding, as she said that teaching should not be confined to formulas only. Involvement in MTEP has created a new image of self as a mathematics educator in Teacher 1, who sees the importance of looking for information from different sources in order to aid learners' understanding. Teacher 1 sees herself as a teacher who now possesses mathematics and mathematics teaching knowledge that inspired her to be confident to teach beyond the limits of the FET mathematics teaching. Teacher 1 has through participation in MTEP learned the significance of preparing exercises and tasks before she delivers a lesson. Teacher 1 sees herself differently as she said that the standard of her work with respect to task preparation, marking and giving feedback is not the same as it was before she became a member of MTEP.

6.2.3 Alignment

I asked Teacher 1 to share how she felt about being a member of MTEP and she responded:

(T) ***As a person I have developed professionally and even when it comes to confidence, I'm not a talkative person but they made me to talk****.* ***They made me to be involved, they made me to feel important, and they made me to feel valuable****. I mean we were not a big group that made easy for us to be noted, to be loved, to be missed, to be welcomed to be appreciated. So that's what I enjoyed there and I was new in Grahamstown so it was a ground for me to introduce myself to the Grahamstown community.*

I decided to practise what they told us, *that whenever we are teaching we must conceptualise the topics or whatever we are talking about* ***we mustn't just teach for the sake of teaching. But so that the learners can make and be able to remember even in 2 years*** *down the line what you have said to them****. Even in my physical sciences, I practise the conceptual part of teaching and learning.***

I asked Teacher 1 what she learnt about conceptual teaching and if she is a conceptual teacher. She responded:

(T) ***Even though conceptual teaching takes much of our teaching, it is very good because it makes learners to make sense of what you are talking about.*** *You don't just base your lesson strictly on numbers, as we know that on mathematics we deal with number. But you make learners to relate to what they are learning during mathematics lessons,* ***so I think I'm trying to be a conceptual teacher. I believe in it now because it makes learners to know and remember things easily because they are talking about,*** *if you are talking about a square, they don't have to scratch their heads to remember the properties of a square. They know that even in the classroom this is a square, this is a rectangle and they can see the properties without even going to the book. Even with graphs, if it's straight line* *graph they need to know that this must be the gradient, this is the y intercept, these are the x intercepts.*

The first phase data analysis show Teacher 1 aligned by implementing conceptual teaching and teaching for understanding. The above extract shows that Teacher 1 developed professionally: she now sees herself as an educator who is confident to talk and make presentations. Both first-phase and second-phase data show that Teacher 1 decided to implement what she learned from MTEP, not only in mathematics but in physical science teaching as well. This does not only show alignment with MTEP's vision but with the broader concerns of the FRF project. The work of alignment entails such processes as investing energy in a directed way and creating a focus to coordinate this investment of energy (Wenger, 1998).

6.3 TEACHER 2

6.3.1 Engagement

Responding to the question I asked Teacher 2 about the impact of what he has learned in MTEP on his professional development, she said:

(T) I would say first yes we've learnt quite a lot of mathematics in there. ***I would mention the main thing that comes to me is the approach, the conceptual way of approaching or of teaching mathematics and uhm in different topics.*** *So I have noticed err my approach has changed quite a lot in approaching, especially topics that are very difficult for learners to understand. And secondly,* ***I've also learnt how to use visualization in mathematics, programmes like geogebra; which makes it easy for learners to see and understand what is happening especially the uhm Euclidean geometry and trigonometry as well.***

I then asked a follow-up question which requested Teacher 2 to if he continued to use Geogebra, what he likes most about Geogebra and he responded:

(T) Yes, ever since I was introduced to geogebra I've been using it and I've tried and empower myself more on geogebra. So I'm still using it now and I will continue using it. Uhm when you ***prepare for your lesson it makes it very easy and uhm, you***

can also allow learners to be hands on. *Allow them to work on your laptop or the computers and* ***they can come up with the explanations and conclusion*** *as to maybe the theorems or why something happens like that. Those are the reason I'm still using geogebra thanks.*

I asked Teacher 2 to share his experiences of involvement in MTEP and its in-school support and he responded:

(T) Well what comes ***first to my mind is the fellowship we had with other teachers, sharing the experiences, sharing ideas with other teachers as well.*** *And then the other thing would be uhm to* ***be introduced in this way of doing things now not a routine way of doing mathematics.*** *So uhm and some of the material, teaching material we got there they really helped me quite a lot.*

I asked Teacher 2 if MTEP has created an environment which made it easy or difficult for him to talk about maths and maths teaching and he responded:

(T) Oh well the environment was quite good for teachers to talk about maths, about the problems we encounter in our classes. ***And MTEP also opened the door for us to call them whenever we needed support in a particular topic.*** *So likewise, it made it easy for us to talk about the challenges we're facing. So I would say uhm they helped us on that one quite a lot. Because in previous times, a teacher would sit there on an island with these problems,* ***but now you can call another colleague in some other school and ask them how to approach a certain topics****.*

Responding to the question I asked Teacher 2 about how his interaction with members of MTEP community shaped the way he saw mathematics, he said:

(T) Well uhm, as I have said, since we have been introduced to this conceptual way of teaching mathematics and then the interaction with uhm, with other members of MTEP was quite good. And MTEP also, because I met new people that I didn't know whom I can now call and ask. ***So it created a relationship and a friendship as well between myself and other members of MTEP.***

The above extracts demonstrate that Teacher 2 has learned new teaching approaches through his participation in MTEP, and this is evident in phase 1 data analysis as well. Both phase one and two data show that Teacher 2 has learned and implemented the use of Geo-Gebra in the teaching of Euclidean geometry and trigonometry. This according to Teacher 2 has allowed learners to be hands-on and involved in their learning. The MTEP environment has been good, safe and supportive for Teacher 2 to grow as a mathematics educator. MTEP has also has been a place of good fellowship, of sharing ideas and experiences, and a place that encouraged Teacher 2 to ask how to approach a mathematics lesson. Being a member of MTEP has encouraged and inspired Teacher 2 to understand the importance of use of manipulatives in promoting conceptual understanding in the teaching of mathematics.

6.3.2 Imagination

I asked Teacher 2 to say if his teaching had changed with regard to confidence, attitude towards mathematics, perception about mathematics through his involvement in MTEP, and he responded:

*(T) Well as I said, **confidence wise, MTEP has boosted me with regard to that.** As now you feel so confident in going to the classroom approaching the Euclidian geometry using geogebra. Same applies as to the ways of teaching mathematics because you no longer doing the textbook stuff now and then that's it. You can take your learners outside; you do the Pythagoras outside or using cubes. Uhm and then in the ways of thinking about maths, the way I was thinking about maths before I joined MTEP was the perception I got from the learners. **That learners are looking at maths as the difficult subject and now after I've been introduced to MTEP, having gone to workshops and having the material of explaining difficult and challenging topics. From what I see from the learners is that uhm, it has changed my perception of them seeing maths as difficult. Now I think that uhm they see maths as a fascinating subject as well, which changed my way of thinking about maths on my learners' side. And then, well my attitude about maths has always been positive because I'm a maths person and uhm, so I wouldn't say my attitude has changed but I've always had this passion about maths**. So and another thing that I would add on is uhm, MTEP has also, or being involved in MTEP session has boosted my I would say, the way I do things. Because*

most of the time I would say, I wouldn't break the class into groups. But now I'm able to break my class into groups and work with the groups and allow my learners to be hands on and they take part in the lesson and then after that we assess.

I asked a follow-up question which requested Teacher 2 to say why he thought it is important to give learners an opportunity to be hands-on and he responded:

(T) Well I believe in, the problem with the telling method is that they could forget, they might forget. The problem, the good thing about them doing it on their own is that they won't forget something that they have done and discovered themselves. ***So I think that's very important, they come up with conclusion, they come up with reasons as to why something has happened****. And then we collect all the answers and then we come up with the one thing from that particular section of work.* ***And I noticed that after we've done that, it's very rare to find them forgetting a theorem that we have covered in that manner.***

I asked Teacher 2 to say how he would like to see himself in future as a result of his involvement in MTEP and he responded:

(T) Well on that one, I think I should say as well, our school has been nominated in the province as the members of the Maths Science Technology Education (MSTE) Grant, because of the performance in mathematics as well as in science. ***As I was saying about the MSTE Grant, now the school has now been nominated to be a maths and a science resource centre for the nearby schools where learners can come in and then have some information on maths and science.*** *So in the near future I would like to see this community of maths learners coming to our school and not only the Bathurst community but even from Port Alfred and the surrounding schools****. I would like to see a community of maths learners where learners have got the passion of maths and also having lots of learners in our school taking maths.***

I asked Teacher 2 to share an unforgettable experience of his involvement in in-school support and he responded:

(T) Well my highlight even though I couldn't make it to the final presentation for AMESA, it was when we were writing a paper on a topic for AMESA on how I teach parabola graphs. So that was my highlight when we were doing the research of how we teach, ***and also when we were introduced on Geo-Gebra, that was most exciting.*** *And then there were also some sessions where Facilitator 1 would, I think the one that shines the most to me was when he was doing probability, different diagrams. The approach he used there really I liked it and uhm another would be when we had a session on number pattern with the (dance). When they combined a number pattern with the dance, so that was interesting and of course the Assegaai outing, the Addo Elephant park, that was my first time going there and then doing maths on the veld there.*

The first-phase data analysis and the extract above demonstrates that Teacher 2's involvement in MTEP had a positive influence on his teaching. Teacher 2 sees himself as a teacher who is better positioned to explain difficult and challenging topics through his involvement in MTEP. It is evident from the extract above that Teacher 2's lesson preparation and presentation has been enriched through his involvement in MTEP. Teacher 2 does not focus on the textbook only for his preparation, but uses other sources like Geo-Gebra. The MTEP environment has challenged Teacher 2 to allow learners to work in groups to provide a space for productive learning. The extract also demonstrates that Teacher 2 has always had a positive attitude towards mathematics. Although it is not evident from the extract that Teacher 2's school has been nominated for the MSTE grant due to his involvement in MTEP, it can be assumed that Teacher 2 associates this nomination with his involvement in MTEP. The first- and second-phase data show that Teacher 2 saw himself as a more confident and professionally grown mathematics teacher through his participation in MTEP.

6.3.3 Alignment

Responding to the question I asked Teacher 2 about what he learned about conceptual teaching and if he sees himself as a conceptual teacher he said:

So what I've learned about conceptual teaching is uhm, ***it's more effective. It's very effective it makes learners to understand what is happening*** *and also not* *to forget because they do have the concept of what was done.* ***Well looking at myself, I think I'm a bit of both conceptual teacher and a routine. Because sometimes there are some problems where you just have to stick to the procedure you see.***

The first-phase data analysis and the extract above demonstrates that Teacher 2 directs his energy towards MTEP's common goal of teaching mathematics for conceptual understanding. Wenger (2000) argues that alignment does not connote a one-way process of submitting to external authority, but a mutual process of coordinating perspectives, and this is evident in Teacher 2's response.

6.4 TEACHER 3

6.4.1 Engagement

I asked Teacher 3 to share the impact of what she has learned in MTEP on her professional development and she responded:

(T) No MTEP, generally; MTEP has, I have learnt to widen my knowledge of Maths. ***For example I have learnt to research, before I do a topic; I research on the history of that particular thing*** *and I try to get the concepts, I mean the basic, what can I say, like what lead to that formula or the formulas that we do. What lead to the formulation or to that generalisation, so from MTEP I've learnt to do that research before I can teach learners any concepts.*

Responding to the question I asked Teacher 3 about her experiences of involvement in MTEP she said:

(T) Ok, the thing that I would ***really like to say I enjoyed or that I admired with MTEP is bringing the Maths teachers together, having to ideas as Maths educators, having to share our classroom experiences and the difficulties that we face; you know you kind of brought*** *Maths teachers together so that if you have got a problem, you know who to approach. So it gave me that kind of an environment to relate to other educators in relation to Maths learning.*

I asked Teacher 3 if MTEP has created an environment which makes it easy or difficult for her to talk about maths and maths teaching and she responded:

(T) Like I have said previously, ***I think it has created an environment where it makes it easier for us to discuss about maths.*** *And it makes things easier to approach certain Maths concepts; to introduce certain Maths concepts to learners, and also to give maybe extension work to learners so that they can also broaden their understanding of Maths and also to remediate some learners where Maths is concerned.*

I asked Teacher 4 how her interaction with members of MTEP community shaped the way she saw mathematics and she responded:

(T) Well, I always feel encouraged because that is ***the group of learners who are all determined to make the maths results better or to help learners understand the so-called difficult subject.*** *So it is encouraging actually to be associated with them, because you really feel, as we are working together towards achieving certain goals, we feel encouraged, you don't despair. Even as we share experiences, you get to learn what other teachers are experiencing and you get to know that you are not on your own in the difficulties or problems that you face in the classroom.*

Teacher 3's engagement stories that emerged strongly in phase one are learning to be broad and research a topic before she delivers it, and the coming together of mathematics teachers. This is also evident in the phase 2 extracts, above. Teacher 3's involvement in MTEP has motivated and inspired her to cultivate a productive learning environment in which learners make deductions and generalizations. The MTEP environment for Teacher 3 has been a place of sharing experiences and difficulties in mathematics teaching. Belonging in MTEP has also helped Teacher 3 with approaching certain mathematics topics. The MTEP environment has been a place of learning and encouragement for Teacher 3.

6.4.2 Imagination

I asked Teacher 3 in what ways as a result of participation in MTEP her teaching has changed with regard to confidence in teaching maths and she responded:

(T) Like I did mention earlier that, MTEP has taught me to research before I teach a concept, ***so that makes me more confident because I get to be even more, I get***

the actual facts concerning that particular concept that I want to teach. *So if you've got all your facts right, it's easy to deliver the lesson to the learners, so I'm quite confident.*

I then asked a follow-up question: would she attribute that to her participation in MTEP and she responded:

(T) Yes I would because I will talk about that again, MTEP has, did you know, encourage me to research before I teach the topic that is what makes the difference.

Responding to the question, I asked Teacher 3 if she feels that her involvement with MTEP has created a new image of self as a professional mathematics educator; she replied:

(T) Yes I think it has; it has and that, MTEP I will talk mostly the actual things that MTEP ***changed in my perspective I mean perception of maths. Conceptual teaching of maths, involvement of learners using manipulatives when you are teaching learners and giving learners a chance to investigate and find answers for themselves. I think that has actually changed the way I do things, the way I see things.***

The extract above demonstrates that Teacher 3 is more confident than she was before she became a member of MTEP. Teacher 3's confidence is derived from what she learnt from MTEP: to research and approach the topic from a broad perspective before she delivers a lesson to the learners. There is evidence from the extracts above that Teacher 3's perception of maths has also been positively influenced by her involvement in MTEP. Both first- and second-phase data show that Teacher 3 sees herself as a conceptual teacher who has learned to implement manipulatives in her teaching. This, according to Teacher 3, encouraged learners to investigate and find solutions for themselves.

6.4.3 Alignment

I asked Teacher 3 to share what she has learned in MTEP and she responded:

(T) Ok, what I learnt mostly from MTEP is ***the use of manipulatives in the teaching of Maths*** *concepts. That has really made a difference that makes a*

difference in the classroom when you are teaching learners. ***And trying to like, to make pupils understand the actual concepts, that is conceptual understanding in Maths****. I think that is what I have learnt mostly in MTEP.*

The above question was followed by a one requesting Teacher 3 to say if she tried to use manipulatives in her teaching and she responded:

(T) Yes, quite a number of those I have implemented. Quite a number of them; I have used geoge…what was that, I don't remember what is it. I've also used Geo-Gebra and there are those shapes

(R) Geo-Genius

(T) Geogenius yes, ***I have also used that for surface area and err calculations of all those things. And I've also used the cubes; I've also used the cutting and folding to teach different mathematical concepts, that that we have learnt at MTEP. So I have used quite a number of them in class.*** *Ok, basically err MTEP actually introduced us to conceptual teaching of Mathematics.* ***Not teaching from a formula but trying to derive a formula so that learners can understand where everything is coming from and relate with it. So I think the method of my teaching, I no longer teach from a formula but I teach from the background which brings out the formula to the learners.***

I asked Teacher 3 if there were any other aspects that have been influenced by her involvement in MTEP and she responded:

(T) Ok, children are more involved in the teaching and learning process. It has made Maths to be more practical so that, that means learners get involved. ***Their actual involvement makes it easier for them to understand because they don't see it as an abstract subject anymore, that only deals with x and y and things like that. But they can actually relate with it practically.***

I asked Teacher 3 what she had learned about conceptual teaching and if she saw herself as a conceptual teacher, and she responded:

(T) Well I have learnt that conceptual teaching really helps learners to understand what it is they are doing and not just move numbers from A to point B. It helps them

to understand a concept right. ***I do see myself as a conceptual teacher although I have not actualised that as a result of our curriculum****. It is so packed and you really have to rush through it, and* ***sometime you find that you don't really get time to give learners that chance to investigate or to use manipulatives because we are rushing through the things that we have to cover, through the syllabus.***

Both first- and second-phase pre-observation data demonstrates that Teacher 3 directs her energy towards a common goal of teaching for conceptual understanding. The extracts above also show that Teacher 3 implemented teaching using manipulatives and discovered its importance in promoting conceptual understanding. Teacher 3 used Geo-Genius to support learner understanding of total surface area and Geo-Gebra as well. Although Teacher 3 has aligned by teaching using manipulatives, she raised the problem of being unable to cover the syllabus on time.

6.5 TEACHER 4

6.5.1 Engagement

I asked Teacher 4 the impact of what she had learned in MTEP on her professional development and she responded:

(T) Yes I can say so because err when, I've, ***the way I used to teach is different now because at first I used to give my learners formulas, just provide them with formulas, give them information;*** *I didn't give them opportunity to investigate on their own. But now we were taught that it's important to give learners a chance to come up with a solution on their own at least. And also to focus on concepts when teaching*

I asked Teacher 4 to share experiences of her involvement with MTEP and its in-school support and she said:

(T) It's teambuilding uyabona (you see) because now I've developed a friendship with other teachers that are *there, that are involved with MTEP. And then we don't end that relationship at MTEP; we go to schools****, we work together as a team you know.*** *Also, even around the school, whatever I learn from MTEP I go* *back and give other teachers those strategies and also my experience of what I have learnt at MTEP.*

I then asked Teacher 4 whether MTEP created an environment which made it easy or difficult for her to talk about Maths and Maths teaching, and she said:

(T) Oh it created an environment which makes it easy to talk about Maths and even m teaching now has improved you know. Because err the reason why I'm saying it makes it easy to talk about Maths, it's because now my learners are free to talk in class you know. Of which if they are free to talk in class, I can easily see if they understand or if they don't understand you know. ***I give them a chance to report as we used to do at MTEP, because at MTEP we work as a group and then after a group we report back what e have learnt in our groups.*** *So for me, it makes it easy to talk about Maths.*

I asked a follow-up question about whether she would attribute that to her involvement in MTEP, and she responded:

(T) ***Yes, yes I can say so because when I'm in class, I give them an opportunity to talk in class.*** *And also, they know that whenever they are going even if it's outside, they know that Maths is involved. Even if they are playing in the field, they know that there is Maths involved you know. Err they are willing to come back and report to the class you know, and they share their problems; if someone has difficulty in class, there are learners who are willing to help.* ***That's the environment that I have created because even there at MTEP, if you don't understand, there are teachers who are willing to help.***

I asked Teacher 4 if she feels that her involvement in MTEP has created a new image of self as a professional Mathematics educator and she responded:

*(T) Yes, mhh first of all; now through MTEP I used to give my learners tests****, I used to write the tests and then make copies. But now through MTEP I know that my question paper must be typed, I've learned that at MTEP.*** *And also, my teaching has developed because I can see through my results that at least now my learner, the pass rate is increasing.*

I asked Teacher 4 how she felt about being a member of MTEP and she responded:

(T) Well I feel so grateful, I'm proud of the project because uhm ***my teaching has developed a lot as I told you that err now I have confidence, more confidence that I can teach with a meaning, my learners are free to talk in class.*** *And also I can even encourage, it's a pity that it's coming to an end, otherwise if the project was going on, I would encourage other teachers to attend too.*

I asked Teacher 4 to share an unforgettable experience in MTEP and in-school support, and she responded:

*(T) Yes, an experience of me presenting at AMESA conference, even this morning I was looking at that book, page ntoni ntoni (****what what) seeing my name there written (Ms Teacher 4) is going to present this topic you know****. But unforgettable experience you know, I even boast about that to my colleagues, other Maths teachers, even encourage my learners* ***so that they know that I'm more confident I don't just teach them, I go an present to other teachers.***

New teaching strategies and confidence are two indicators that stand out in Teacher 4's involvement in MTEP, both in the first- and the second-phase pre-observation data. The extract above demonstrates a strong sense of belonging to MTEP for Teacher 4 as she said that she enjoyed being in the community of mathematics teachers. She has learned to give learners an opportunity to talk and share their thinking publicly in her teaching. Teacher 4 has developed shared histories of practice with respect to the use of technology in mathematics teaching, as she has learned to type mathematics question papers. The extract also demonstrates good experiences of involvement in MTEP for her, especially her presentation of paper at the AMESA Conference. Lastly, first- and second-phase data demonstrates that MTEP was a safe learning environment for Teacher 4.

6.5.2 Imagination

I asked Teacher 4 in what ways as a result of her participation in MTEP her teaching had changed with respect to her confidence in teaching maths and she responded:

*(T) Ok, even before I attended I attended MTEP I had confidence in teaching /**ok**/ you know. Even now, maybe I can say I am more confident now you know, because of the exposure that we were given there. **Ok, as I have told you earlier, now I have changed because at first I was concentrating only on the formula you know**. Because I knew, before I went to MTEP that formulas are not that much important the learner must investigate on their own. But I had a problem of time consuming, I would say no no if I give them a chance to do work time factors etc. But I've told myself that even if it takes time, but it will last for a longer period. It will stay, the information will not just vanish so quickly you know. **So now I give them a chance to do investigations, to investigate on their own.***

Responding to the question I asked Teacher 4 how her participation in MTEP had shaped her ways of thinking about mathematics and her attitude towards mathematics, and she said:

*(T) As I told you again that there are those real life situations, but I didn't think that they involved mathematics. Now the way I see mathematics, I see that it's all around the world you know, wherever you go there is mathematics**.** Ok, err no because I like mathematics nhe (right) I had a positive attitude towards maths. Even now I still love for my learners, I make sure that they love this mathematics in order for them to succeed and get better results, they must have a positive attitude, they must have that err positive influence towards mathematics what else?*

The first- and second-phase data demonstrate that Teacher 4 has grown from a procedure-focused teacher to one who foregrounds conceptual understanding in her teaching. The extract above also shows that Teacher 4 has grown to be a more confident mathematics teacher. It is evident from the data above that Teacher 4 has learned to give learners an opportunity to investigate and discover solutions for themselves. Teacher 4 had always had a positive attitude towards mathematics.

6.5.3 Alignment

Responding to the question I asked Teacher 4 about what she had learnt about conceptual teaching ,she said:

(T) Ok, I've learnt that whenever you teach, ***you must teach with a meaning because learners they must err have in-depth understanding of what you are teaching you know.*** *Don't just teach the learners information that they will forget soon. So if they know the meaning of what they are doing, it will stay forever.*

I then asked a follow-up question of why she related conceptual teaching to in-depth understanding and she responded:

(T) Ok, conceptual teaching err you teach with a meaning nhe (right) /ok/ the concepts, they must know concepts and if they know the concepts nhe (right), it means they will know the meaning of what you are teaching. And then in-depth teaching, they will have deep understanding of what you are doing through conceptual teaching that's why I'm combining the two

I asked Teacher 4 if she sees herself as a conceptual teacher, and to give an example of implementing conceptual teaching. She responded:

(*T) Yes I am a conceptual teacher because whenever I teach, I make sure that they understand what we are doing***.** *Mhh like for instance, ok if I'm teaching, let me talk about finance. If I'm teaching them about finance; simple interest, simple decay, compound decay, and compound interest you know. First of all, they don't know decay, they don't know compound.* ***Before I come to the topic, they must know what is simple decay, why there is a decay there and then they must also be able to tell me what is a decay.*** *For instance they will tell me, if you buy a car Miss, in three years time the price of that car will depreciate. I don't just talk about decay and then I don't relate. So I think that's conceptual teaching,* ***whenever I teach I relate to real life situations you know.*** *And then we come straight to the problems.*

The first phase and the extract above demonstrate that Teacher 4 identifies with MTEP's vision of teaching for conceptual understanding. Teacher 4 is mindful and intentional about promoting conceptual understanding and has grown from simply dispensing formulae in her teaching. According to Teacher 4, conceptual teaching helps learners to derive meaning from, and acquire in-depth understanding of, what they learn.

6.6 TEACHER 5

6.6.1 Engagement

I asked Teacher 5 to share the impact of what she had learnt in MTEP on her professional development and she said:

(T) Professionally it helped me in a way that uhm the ***way you deal with learners in class, it sort of been improved with MTEP.*** *I mean the attitude you have when you're going to class it changed. I mean it changed from,* ***I mean you tend to become a person who is more positive in dealing with learners.***

I asked Teacher 5 to share experiences of her involvement in MTEP and the in-school support, and she responded:

(T) They give you easier methods to approach the lesson, there are also skills that are used, I mean I got from MTEP. Sort of, there is ***the one of using computers, I didn't know anything in Geo-Gebra, they taught us Geo-Gebra, how to use computers,*** *how to work with computers teaching mathematics, that is also what I learnt in MTEP.*

I then asked a follow-up question: has she continued to use Geo-gebra in her teaching now, and she responded:

(T) Yes I'm still using Geo-Gebra, it also makes it easy for me to set even the papers that the kids are going to write, when I'm setting the test I used Geo-Gebra.

Responding to the question I asked Teacher 5 about whether MTEP had created an environment which made it easy or difficult for her to talk about mathematics, she said:

(T) Ok, it also helps in that, you can easily take your learners in the, from one school to another school for competing with other learners. Then, even other teachers are willing for the learners to compete with other learners from my school. ***So the attitude amongst us as the members who were there in MTEP is more positive.***

I asked if MTEP was a safe place for such interactions and she responded:

(T) Yes MTEP is a safe place because at MTEP you are more encouraged to work with others, that's what they promote there, that you must work with other members. You mustn't be in an isolated, in isolation and be there in your school only. ***You are welcome to go and interact with other members of the MTEP community.***

Teacher 5's stories of engagement in both phase-1 data analysis and the above extracts show that she learned to use computers in her teaching at MTEP. Teacher 5 has through her participation in MTEP learned different teaching approaches and has grown to be positive when dealing with learners. Teacher 5 also found the MTEP environment safe and conducive to learning.

6.6.2 Imagination

I asked if MTEP has created an environment which makes it easy for you to talk about maths and maths teaching, and she responded:

(T) Yes it is, MTEP has made it easy for me to talk about mathematics. Uhm when you meet with other teachers, ***you have that confidence to stand up and present a lesson that's what we get from MTEP. Because we used to present lessons there, so it makes it easy for me to be bold*** *and stand infront of other teachers and present a lesson.* ***Even in class I am more confident.***

I asked how Teacher 5 sees herself as a result of her participation in MTEP and she responded:

*(T) I see myself helping teaching other learners from the surrounding schools****, I have that confidence to stand there and teach the learners from the neighbouring schools around, I have that confidence.***

(R) Would you attribute that confidence to your involvement and participation in MTEP?

(T) I would say yes

(R) In other words the question is, in what ways as a result of your participation in MTEP have these been influenced, how has your confidence in teaching maths been influenced, how have your ways of teaching maths been influenced by your participation in MTEP?

(T) ***Ok the confidence, my confidence in teaching mathematics has improved;*** *I would say it has improved drastically. How has it been improved? The results in mathematics in my school have improved due to the confidence that I have in teaching maths. Uhm the ways of teaching mathematics, err I teach mathematics in such a way that even that learner who was a slow learner, even that one who was average in class now sees maths a subject which is not, maths in the first place is perceived as a subject which is most difficult. Now the learners find mathematics as subject that can easily compare to another subject which is no longer difficult as it used to be.* ***Then the attitude of learners and my attitude in mathematics; learners have a positive attitude, learners are always willing to learn each and every chapter that I'm teaching them they are always willing.*** *They are willing to work on their own, they are able now to help others in the classroom situation.* ***I mean I find it interesting for me to teach mathematics as a subject.***

I requested Teacher 5 to please tell me about an unforgettable experience of her involvement in MTEP and in-school support and she responded:

(T) My highlight is that of going to AMESA congress where we met mathematics teachers from all over South Africa that was my highlight*. And that of using a computer to teach a maths lesson****, I never thought I would be able to teach mathematics using a computer but MTEP has introduced that to me.*** *And now I find it easy to teach maths in computer, now I find it easy to use a computer, of which before I couldn't even use a computer. But due to MTEP now I can use a computer.*

The above extracts show that Teacher 5 has been inspired to be bold and has grown to be confident in presenting to other teachers through her participation in MTEP. The first- and second-phase data demonstrate that Teacher 5 has also grown to be more confident in mathematics teaching. Teacher 5's interactions with other MTEP members had a positive influence on her attitude as well.

6.6.3 Alignment

Responding to the question I asked Teacher 5 about what she had learnt about conceptual teaching and whether she saw herself as a conceptual teacher, she said:

(T) Conceptual teaching helped in such a way that in class, ***when you teach you don't just teach them to memorise, to know something step by step, you teach them that they should know it and have a deeper understanding of concepts in mathematics****, that's what has happened in my class. Learners now are no longer memorising steps, doing a problem working step by step, they no longer memorise, and they work conceptually. They have a deeper understanding of what they are dealing with now.*

I asked Teacher 5 if she would encourage other teachers toward conceptual teaching and she responded:

(T) Yes I would say conceptual teaching is good instead of promoting the rote learning, for the learners to memorise everything. Because when they memorise, they memorise and forget after that. But when they do it in a conceptual method, they have a deeper understanding for their learning.

The extract above demonstrates that MTEP's identity has become Teacher 5's, as she said that she has implemented conceptual teaching in her classroom. Conceptual teaching, according to Teacher 5, help learners to gain a deeper understanding of mathematics.

6.7 PHASE 2 PRE-OBSERVATION ANALYSIS TABLE

Table 6.1 below provides a summary of the phase 2 pre-observation analysis results:

Table 6.1 Pre-observation analysis table

Wenger (1998)'s three modes of belonging	TEACHER 1	TEACHER 2	TEACHER 3	TEACHER 4	TEACHER 5
ENGAGEMENT	Teaching approaches. Lesson planning, Strong relationships of belonging Takes responsibility of her work Improved standard of work :task preparation marking and giving feedback	Teaching approach. Use of computers in teaching (Geo-Gebra) Understand the importance of use of manipulatives in teaching	Learned to broad and research a topic before delivering a lesson Facilitator of productive learning environment MTEP safe environment	Teaching approaches Confidence Strong sense of belonging Presented a paper at AMESA conference Learned to use technology in teaching: typing question papers	Learned use of computers in maths teaching(helped her to type question papers) Environment free to ask questions Positive when dealing with learners

IMAGINATION	More committed. More confident. Learned the importance of using manipulatives to support conceptual understanding Look for information from other sources. Professional growth: to teach beyond the limits of FET	Better positioned to explain difficult topics Allow learners to work in groups Always had a positive attitude towards mathematics More confident	Confidence derived from being broad and research Sees herself as a conceptual teacher	Grown from focusing on procedure only to conceptual teaching More confident Always had a positive attitude towards mathematics	Confident Positive attitude towards mathematics
ALIGNMENT	Teaching for conceptual understanding.	Teaching for conceptual understanding.	Teaching for conceptual understanding.	Teaching for conceptual understanding.	Teaching for conceptual understanding.

6.8 CONCLUSION

6.8.1 Engagement

The first- and second-phase analysis demonstrates that gaining more teaching approaches by participating in MTEP was significant in terms of teacher growth for all the teachers. The MTEP environment has been a safe place for the teachers to grow and develop professionally in both phases one and two. A story of engagement shared by Teacher 4 and Teacher 5 was presenting papers at the AMESA conference. The analysis also reveals that Teacher 2 has come to understand the importance of the use of manipulatives in teaching through his involvement in MTEP. Teacher 2, Teacher 3 and Teacher 4 have grown to use computers in their teaching through their participation in MTEP.

6.8.2 Imagination

The analysis results show that all five teachers sustained their growth with respect to confidence in and commitment to their teaching. The teachers attribute this to their involvement in MTEP. Although Teacher 4 attributes her positive attitude towards mathematics to her experiences of involvement in MTEP, this is not the case for Teacher 2 and Teacher 3, who claim always to have had a positive attitude towards mathematics. The analysis also shows that Teacher 1 and Teacher 2 saw themselves differently with respect to lesson planning and presentation through their involvement in MTEP.

6.8.3 Alignment The first- and second-phase analysis demonstrates that all five teachers identify with MTEP's identity of teaching for conceptual understanding. In the next chapter I present and analyse data gathered through second-phase classroom observation and post-observation interviews.

CHAPTER 7

DATA ANALYSIS

PHASE 2

CLASSROOM OBSERVATION AND POST-OBSERVATION INTERVIEWS

7.1 INTRODUCTION

In this chapter I present and analyse data gathered through second-phase classroom observation and post-observation interviews. Each of the five participating teachers taught the same lesson they taught in the first phase to the same grade but with different learners. The second-phase lessons were taught a year after the first-phase ones. The lessons were followed by post-observation interviews as was the case in the first phase. Through these I analyse and discuss teachers' practice with respect to conceptual teaching. Indicators of the conceptual teaching tool were used to analyse classroom observation and post-observation interviews.

7.2 TEACHER 1'S LESSON

Teacher 1 taught the Area Rule to new group of grade 11 learners. Teacher 1 introduced her lesson by drawing triangle ABC on the chalkboard (see Chapter 5, section 5.2) as in the first phase. Through a question and answer teaching strategy Teacher 1 took learners through the process of deriving the Area Rule. One observation that was different from phase one is that Teacher 1 combined the Area Rule with the Sine Rule. The Sine Rule states that:

$$\frac{a}{SinA} = \frac{b}{SinB} = \frac{c}{SinC}$$

Teacher 1 concluded the lesson by giving learners the Area Rule application worksheets downloaded from www.everything maths.co.za.

7.2.1 Teacher 1 lesson and post-observation interview analysis

7.2.1.1 Teaching that promotes conceptual understanding

The extract below shows Teacher 1's lesson introduction:

(T) Ok, do you still remember the three basic trig ratios?

(L) Yes

(T) It's … can you mention?

(L) Sine, Cosine and Tan

(T)Yes, in this particular case we are going to use Sine. Now in this triangle, in this specific, in triangle ABD niyambona nhe (can you see?)

(L)Yes

*(T**) Ok, let's define Sine, let's define Sine.***

*(L**) Opposite over hypotenuse***

(T) Opposite over hypotenuse, so Sine A = opposite side over hypotenuse side. So our opposite side ithini igama layo (what is its name)?

(L) It's h

(T) It's h right, Yes so it's h/c, now what do we want thina (we/us) we want the value of h. Let's make h the subject of the formula from that, remember this is Sine A, let's make h the subject of the formula

In the post-observation interview I asked Teacher 1 to say why she introduced her lesson this way and she responded:

(T) In the first question I thought, it seemed to me like I was leading them to use Sine α, the trigonometric ratio, to find height, ***I wanted them to relate the trig ratio with the height that I wanted them to find. But they didn't understand that. Then I, we went back, I just said to myself, ok, let us first define the trigonometric ratios.*** *Then after that they must relate the side that we want to talk about, like the height and the adjacent side and the hypotenuse side. Which one will fit in this situation because we want the height which is the opposite side in this angle and we know our hypotenuse side? So from the three trigonometric ratios that we have defined, which one will suit this situation of ours?*

Teacher 1's lesson introduction both in phase-one and the extract above was aimed at reminding learners of their prior knowledge of trigonometric ratios. Teacher 1 had, through her questioning, required learners to demonstrate their understanding of these ratios. It is evident from the extract above that Teacher 1 was purposeful and intentional in the way she asked questions since she wanted to ascertain the learners' knowledge of ratios so she could build on this knowledge of the sine rule.

Below is an extract from Teacher 1's video recorded lesson:

(T) Area of triangle ABC = ½ b c Sine A right

(L)Yes

(T)This one ithini (what does it say) ½ b a Sine C, ***what do you notice?*** *Remember the small letters are sides and the capital letters are angles, ucinguba kengoku eyesithathu izawuthini (what do you think the third one will be) area of triangle ABC is going to be*

(L) ½ b

(T) Is it going to be b again, masithi umhlambi ngoku besiyijikile ngoku sathi ngu-B apha, ***yangu-C phaya yangu-A (let's say we changed it around and have A here, B over there and C. This side would be small letter /c/ andithi (right) /yes/ this side would be small letter a, this side would be small letter b am I confusing you /no/ niyayibona (can you see it) /yes/. Now ayizubangu small letter bani ngoku kwelicala b, ucinga ukuba ke le iformula izawuthi (we won't have small letter b this side, what do you think this formula will be)***

(L) Small letter c

The manner in which the question "what do you notice" was phrased shows that Teacher 1 required learners to reason. Teacher 1 required learners to notice the relationship between the names of angles and sides opposite those angles. However the mathematical dialogue was not as productive as it was in the first phase. In the first phase learners deduced for themselves the relationship between sides and an included angle to prove the sine rule. Teacher 1's question was aimed at making a link between a mathematics concept of a trigonometric ratio, the relationship between angle and an opposite side and the sine rule.

7.2.1.2 Teaching that promotes productive maths talk

There was no evidence of teaching that promoted productive mathematics talk in Teacher 1's lesson

7.2.1.3 Teaching that promotes effective use of manipulatives

Teacher 1 did not use manipulatives in both phase one and two.

7.2.1.4 Teaching that promotes visualization

Teaching that promotes visualization was not evident in both phases one and two.

7.2.1.5 Teaching that demonstrates positive self-efficacy

Below is an extract from Teacher 1's video recorded lesson:

(T)Masiye ke sizawuthini (what are we going to do next) **Learner Z**, *izawuthini iformula yethu ye-area rule* (what will our area rule formula be) $^1/_2$ r p Sine Q *uthi **u Learner D** kanye kulandawo mandiyicacise ndingayiyeki, no **Learner M** ingathi uxakene nento* **(Learner D** wants us to explain that step, it also seems like **Learner M** is a bit confused)

(L) *Aba r p* (those r p)

(T) *Yheke aba r p bavelaphi* (where does r p come from)?

(L) *Ingathi mos sinikwe eza ivalues zika small letter r nezika small letter p, so kengoku kuthwa masifune i-area. Kuba sinikwe i-values uba kengoku uzi-twistile u-r umbeke ngapha u-p umbeke ngapha (I think because we have been given the values of small letter r and the values of small letter p, since we have been given these values, what if you twist them and have r that side and p this side)*

In the post-observation interview I asked why she allowed the learners in the class to offer explanations and she responded:

(T) Sometimes it's easier for learners to understand each other when they are teaching each other. ***They use a friendly language an understandable language to them. Doing that one it helped me to see that at least a few individuals understood what I was teaching*** *and they can also tell other learners about what we were doing.* ***So to me it's so, I find it interesting when learners speak or talk about what we were doing in class because it is reinforcing their knowledge, the ones who are teaching others.*** *So it is helping me to see, ok, she understood what I was teaching and then it is also helping other learners to understand that ok, in our own language, in simple language as learners this is done like this*.

Sometimes I let them speak audibly when they are asking a question so that everybody hears when they are asking a question. *One because the other learner might have the same question, two one from the group might be able to answer the question. Usually I always say to them if a learner asks a question, that's not my question, that's not my question only it's not only me who is going to answer now. We are all responsible to answer that question because I have taught you this now, so at least you must try. So that is why for me it is important that for all of us to hear and understand the question. Sometimes a learner may ask a question, I don't get to understand what he or she is asking. So when everybody has heard what Learner Z, is asking a question, he will have to clarify it, 'no ma'am she is trying to say' then I will understand.*

The extract above demonstrates that Teacher 1 noted that there was a learner who did not understand the concept so she allowed other learners to explain it in their mother tongue in order for the learner to grasp it. This demonstrates that Teacher 1 expected and gave learners an opportunity to make a contribution in her lesson. It is also evident from the extract above that Teacher 1 encouraged an atmosphere conducive to discussion and exploration of mathematic concepts.

7.3 TEACHER 2 LESSON

Teacher 2 taught probability to grade 11 learners (see section 5.3). Teacher 2 used real life application of probability to help learners understand the meaning of probability. The two real life application examples that Teacher 2 used in her phase-two lesson are given in section 7.3.1 below. The lesson approach was the same as in phase one except that Teacher 2 used fewer manipulatives compared to phase one. Teacher 2 did not bring an unfair dice into his second-phase lesson. Another observation that was different from phase one was that Teacher 2 taught sampling with and without replacement. Teacher 2 concluded his lesson by giving learners a worksheet which helped learners to apply the knowledge they had gained in problem solving.

7.3.1 Classroom and post-observation interview analysis

7.3.1.1 Teaching that promotes conceptual understanding

Below are two examples that Teacher 2 used in his lesson:

Example 1

Now if we say chances are that it will rain tomorrow is 80%, u-80% wethu uzobandawoni kwi-scale ***(where will our 80% be on the scale)? Will it be here or here, 80%? Closer to 100 isn't it. So if we say that 0.8, 80% is 0.8 because it's 80/100 isn't it /yes/ so 80% is here.*** *So they say there is 80% chance that it will rain, is it going to rain or is it more likely to rain?*

(L) More likely to rain

(T) ***Ok, are we sure that it's going to rain 100%, no, we will only be sure that it's going to rain if the chances are 100%. So the chances that it will rain tomorrow are 100%, then we can say for sure it's going to rain****. 80% it's more likely, that is thetha ukuthi, ubukhulu becala izawunetha. If they say chances of a rain are 10%, is it going to rain? /no/ Chances are very low that it will rain because we are closer to 0, it might not rain can you see.* ***So the probability is between 0 and 1, or 0 and 100%.*** *Now what is the probability, we use P to represent probability. So the probability of an event, now what is an event?*

Example 2

(T)So let's say for an example I come here and say here we are 30 and then there are 18 girls, 12 boys, I'm just making an example. Then what is the probability that at random I come here and choose a girl? So lento ndiyifunayo yintoni (what I want is) to choose a girl. So we call that an event are we together /yes/ so it's what you want

to do. ***And then the probability of an event, the desired outcome that choosing of a girl ok, how many girls do we have here? So the probability is the number of the desired outcome. Favourable outcome some will say, lento siyifunayo (what we want).*** *So I want a girl out of how many girls, so we've got 18 girls here, are we together /yes/. So what is the total number of learners in this class? /30/ so the probability is what, is the number of the desired outcome divided by the total number ok, out of which we are choosing from. Now these that we are choosing from we call it a sample space. So it's the desired outcome/the total number in the sample space ok. So this is the definition of probability, it's what I want over the possible total that I have are we together /yes/.*

In the post-observation interview I asked Teacher 2 to say why he used the examples given above in his lesson and he responded:

Example 1 explanation

(T) The reason that I used the example of the rain; it's something that they often see or hear during the weather reports. They could see that in Port Alfred rain is 80% and there are some misconceptions there. Where learners will think that the rain will be 80% without clearly thinking that, the chances that it will rain is going to be 80%. ***Then using the scale, I wanted them to tell me where 80% will lie on the probability scale. So that they can understand when we speak of 80%, it's something that is beyond 50% close to 100% that's why I used the probability and the example of the rain.***

Example 2 explanation

(T) Well on that one, ***I wanted to explain the understanding of the sample space, what we mean about the sample space so that if we have 18 boys and 12 girls, so the sample space there will be 30.*** *And also, my favourable outcome, the one that I want, if I want to choose a girl, that's my favourable outcome, if I want to choose a boy that's my favourable outcome. So I wanted them to understand what we mean by a favourable outcome and sample space.*

In example 1, Teacher 2's questions were aimed at reminding learners of the meaning of probability, the meaning of a 0.8 probability and meaning of probability between zero and one. Although Teacher 2's questions were closed in example 2, he used the example to help learners understand favourable outcomes and sample space. In both examples Teacher 2 asked questions that encouraged learners to connect knowledge of probability with real life contexts. Teacher 2 asked questions that were purposefully aimed at linking the concept of probability with the formula for calculating probability.

7.3.1.2 Teaching that promotes productive mathematics talk

Below is an extract from Teacher 2's video recorded lesson:

Ok, now let's see, we roll a dice once mhh interesting, let's roll a dice once, what's the ***probability of rolling a dice once and getting a prime number?*** */3/ 3, how many prime numbers do we have there?* ***Is 1 a prime number?*** */no/ 1 is not a prime number ok /yes/ what is a prime number, anyone, yintoni (what is a) i-prime number, anyone? So i-prime number linani onoli- (is a number that you can) divide ngalo kwakunye no- (with and also) 1 but excluding 1 ngoba (because) 1 is not a prime number. So therefore, how many prime numbers do we have there? So kufuneka ujonge amanani onokwazi ukudivida ngalo ok no-1 (so you must look at a number you can divide by and also 1) ok* L: 3, 5

(T) 3, 5 is that all?

(L) And 2

(T) And 2 can you see, so how many prime numbers here? /3/ out of /6/ so which is 0.5 are we together /yes/.

The extracts above show that Teacher 2 asked a question which required learner understanding of a prime number and then used their response to push learner thinking further.

7.3.1.3Teaching that promotes effective use of manipulatives

In the post-observation interview I asked Teacher 2 to say what manipulatives he used in his lesson and why he used them and he said:

(T) ***I used uhm a dice and a deck of cards and then the reason was, using a dice makes it easy to explain to learners the outcomes, the sample space because you've got a few numbers in the sample space.*** *And then uhm, when you roll a dice, they can take a guess, so then they can understand what you mean by a probability of a number coming up. And the deck of cards, I used it to make them understand the difference first, between the cards we have and the total number of each type are there in the* ***deck of cards*** *And also to understand the probabilities of selecting a card in* ***a deck of cards*** *or a particular card of a certain type in a deck. So those were the reasons why I chose those manipulatives.*

Teacher 2 used a dice and a deck of cards in his lesson and he focused learner attention on them. Teacher 2 used these manipulatives for the purpose of the lesson since he used them to explain the concept of sample space and favourable outcomes. Teacher 2 had, through his questioning, assisted learners to apply maths ideas gained from using manipulatives to support the aim of the lesson. Learners were provided with more than one type manipulatives to allow learners to explore the concept of probability and its application in problem solving.

7.3.1.4 Teaching that promotes visualization

There was no evidence of teaching that promotes visualization in Teacher 2's lesson.

7.3.1.5 Teaching that demonstrates positive self-efficacy

I requested Teacher 2 to share the highlights of his involvement in MTEP and he responded:

(T) Well my highlight was when I prepared how I teach a parabola with a facilitator M. When we did the research with the learners, ***that was my highlight, it was my first time to do a research like that****. Something err, where learners respond actively and then you jot down all the responses and then come up with the conclusions as to why they think that way. Then secondly,* ***my other highlight would be when we were first taught how to use geogebra and how to use some***

of the manipulatives that were given to us in explaining some of the very challenging topics, those were very helpful.

I asked Teacher 2 to share whether anything he learnt in MTEP was incorporated in the teaching of this lesson and he responded:

(T) ***Yes, because the demonstration side of the topic, the practical demonstration because that's what I got from MTEP.*** *Because the previous times, in the previous years; you would go to class and teach the probabilities without even bringing a coin, without bringing a dice. You're just explaining things to learners that they don't even understand what you're talking about. And after that lesson, we had to do the venn diagram, that's where I also used what I got from one of the sessions at MTEP where we were using the papers to explain the intersections and the unions on the venn diagrams.*

Both the first-phase data and the above extract demonstrate that Teacher 2 maintained a positive attitude towards mathematics and its learning. Teacher 2 used made full use of the opportunity of being an MTEP member, he learnt to use technology in the teaching of mathematics. Although a positive attitude towards learning is not one of my conceptual teaching indicators of positive self-efficacy, Teacher 2 had a positive attitude towards new learning and he implemented what he learned from MTEP. The mathematics teacher plays a critical role in encouraging students to maintain positive attitudes towards mathematics. How a teacher views mathematics and its learning affects their teaching practice, which ultimately rolls down to how the students learn and how they view themselves as mathematics learners (Kilpatrick, et al., 2001).

7.4 TEACHER 3'S LESSON

Teacher 3's lesson was on factorisation of difference of two squares (see section 5.4). The lesson introduction was almost the same as in phase one, Teacher 3 reminded learners of the procedural way of solving difference of two squares. The teaching approach was also the same as in phase one. In conclusion, learners deduced that factorising the difference of two squares $x^2 - y^2$ gives the product (x + y) (x – y).

7.4.1 Teacher 3 lesson and post-observation interview analysis

7.4.1.1 Teaching that promotes conceptual understanding

Below is an extract from Teacher 3' video recorded lesson:

(L) Rectangle

(R) Ok, so you know the difference between a square and a rectangle /yes/ right. ***Now you say this is a square, how do you know that this is a square?***

(L) Both sides are equal

(T) Both sides are equal, ok right, so now when you work out the area of the square, let's say for instance the square that you have on the board. ***What is the area of the square that we have on the board?***

(L) Side by side

(T) Side by side, that's ok, but now I'm talking about this particular one on the board

(L) (x)(x)

(T) It's going to be (x)(x) which gives us x^2 ***now can you relate this to that? The numbers that we call squares are actually an area of a shape of a square.*** *Right, so that's what I wanted you to realise first before we got to our first question.* ***The number that we call the square is actually an area of a shape.***

So now let's come to the difference of squares, what do we mean when we say difference, what do you understand by difference? Without the difference, calculate the difference, what is supposed to be happening there? ***I've got*** $x^2 - y^2$***, we say these are squares, so if I say I've got a statement like that, what do you understand by that statement, what is happening here?***

(L) Calculate the difference

(T) Yes we are calculating the difference, ***but when we say calculate the difference, what are we supposed to do, what operation do you do,*** *mathematical operation do you do?*

(L) Subtract

***(T) Subtract, so what it actually means is, we are taking a square from this square and the answer is what remains when we remove this square from this square that is the answer that remains**. So that is what we are going to do, you are going to work in groups; I am going to give you squares ok. You are going to work in groups. You have got a bigger square, or a smaller square inside a bigger square. Label those sides, the bigger square is, you can label it (x)(x) or (y)(y) but label the sides of the square that you're working with. You are going to need a pair of scissors; doesn't anyone have a pair of scissors?*

In the post-observation interview I asked Teacher 3 to explain why he asked the question,' now can you relate this to that?' and she responded:

*(T) Alright, we had drawn a square, I had a square on the board and I discussed with them how they identify a square and they told me that a square has got equal sides. So I labelled those sides x and x so because I had put into their mind that, squares, no I wanted to **bring them to realise that the terms that we were talking about is actually area of a square.** So we multiplied the sides of that square, to get x^2 which was x by x. So I said x times x is going to give us x^2. **So I wanted them to see that, x^2 is similar to the terms that we had talked about earlier which is square numbers.***

In the same classroom observation extract above, Teacher 3 asked for learner understanding of $x^2 - y^2$. In the post-observation interview I asked why she asked for that and she said:

*(T) Right, in this case I had actually drawn two squares on the board; one square was x times x and the other one was y times y. Ok, now what I wanted to get from my learners when I asked that statement, I wanted them to be able to tell me what subtraction is about. **I wanted them to be able to figure out that when you are subtracting you are actually taking away something** from a whole or from a given item. So I wanted them to tell me, to think along those lines that we are taking this thing, this statement means we are removing this part from this other part.*

The extract above shows that Teacher 3 asked questions which probed for prior knowledge of the concept of a square shape, a perfect square number and meaning of difference. Teacher 3 was purposeful in her questioning and asked learners to relate the square shape with a square number and then the difference of two squares. Through her questioning Teacher 3 also inspired learners to connect the meaning of a square shape, perfect square and difference with a formula for factorising the difference of two squares. It is evident from the classroom observation extract above that Teacher 1's questions required mathematical reasoning from the learners. Although the questions in the extract above were phrased differently from the phase- one lesson, however Teacher 1's questions promoted conceptual understanding of the difference of two squares.

7.4.1.2 Teaching that promotes productive maths talk

The extract in the section above demonstrates that Teacher 3 engaged learners in productive mathematics talk which supported their understanding of the difference of two squares. Teacher 3 used learner response to promote elaboration by focusing on the area of the square they saw on the board.

7.4.1.3Teaching that promotes effective use of manipulatives

Below is an extract from Teacher 3's video recorded lesson:

(T) Right, from here to here is going to be how many, what is the length of that, x, right from here to there what is the length of that, in terms of x and y?

(L) x

(T) x, from here to here?

(T) y

(T) No, y is from here to here, x is from there to there, sorry, y is from here to here, x is from there to there. What is the length from here to here?

(T) xy

(T) xy? I heard someone say something that sounds correct, what is the length from here to here, x – y, because we know that this is y and we know that this whole thing

is x, so this one will be x – y. I wanted you to be able to figure it out before you cut your squares because it's going to be easier when you have the whole thing. Now let's cut our squares, ok, when you cut your squares you are going to stick them onto these pieces. The area that you want to calculate, you will stick them on these pieces. ***And keep those measurements intact because you are going to need them.***

In the post-observation interview I asked Teacher 3 to say which manipulatives she used in her lesson:

(S) Ok, I had squares drawn on pieces of paper, so I had pieces of paper with squares drawn on them which they were supposed to cut out as they do their subtracting and they were using pairs of scissors and they were going to stick the remaining piece that they were going to calculate on another paper. That is what we were doing during the lesson.

I then asked a follow-up question to probe whether the use of manipulatives supported the learning of difference between two squares and she said:

(S) Yes, a lot because now they have seen, the learners have visualised that ***they actually manipulated and did investigating themselves and came with the difference of two squares which we usually teach abstractly.*** *Now it's very easy for them to remember, they don't forget the difference of squares, so I think it has.*

I asked Teacher 3 to say why she emphasized that the learners to keep the lengths in mind and label them with an open mind and she responded:

(T) Ok, I believe at this stage, they had actually done the cutting out and that, no they were now looking at the shapes, let me say the pieces of the remaining area which they were supposed to calculate. So the length of the sides is very important in the calculation of area because we want to come up with this result that we want, so it is important to get our measurements right. ***And because they had done some cutting, there were changes in the length of what was happening.*** *So I wanted them to be very careful, that as they are cutting, they must notice the changes in the measurements in the remaining area so that they can use the correct measurements for calculating.*

It is evident from both first- and second-phase classroom observation data that Teacher 3 used manipulatives in her lesson. Teacher 3 focused the attention of the learners on the squares drawn, by asking them to keep the measurement intact because they were going to use them later. Teacher 3 used manipulatives for the purposes of the lesson, and assisted learners through her questioning to apply the mathematics ideas gained from using the manipulatives to support learner understanding of difference of two squares.

7.4.1.4 Teaching that promotes visualization

The extract in the above section shows that Teacher 3 was mindful of promoting visualization in her lesson. Although she did not ask questions which required learners to share their mental image with classmates but she had, through her questioning, assisted learners to relate the mental image area of a square, pieces of the remaining area with factoring difference of two squares. Teacher 3 asked learners to notice or keep the changes in measurement in their minds, and this shows that Teacher 3 inspired learners to see the changes literally and to have a mental picture which helped them to express the remaining side in terms of x and y.

7.4.1.5 Teaching that demonstrates positive self-efficacy

The extracts in the sections above demonstrate that Teacher 3's questions were phrased such that they accommodated and welcomed contributions from all the learners. Teacher 3 inspired learners to think along the lines of an area of a square shape and a square number, and connect that knowledge to solving difference of two squares.

7.5 TEACHER 4'S LESSON

Teacher 4 taught a theorem to grade 11 learners. The theorem states that an angle subtended by the arc at the centre is twice the size of the angle subtended by the same arc at any point at the circumference (see section 5.5). Teacher 4 used the same question and answer teaching approach that she used in phase one and took learners through the theorem proof process.

7.5.1 Teacher 4 lesson and post-observation interview analysis

7.5.1.1 Teaching that promotes conceptual understanding

Below is an extract from Teacher 4's video recorded lesson:

*(T) Right masijonge (let's look) can you tell me, vala i-text book yakho (**close your textbook please**) undixelele yonke into oyaziyo phayana (**tell me everything you know there).** O is the centre of a circle, O is the centre of a circle phayana ee (there yes)*
(L) OA uyalingana no- (is equal to) OB zi- (they are) radii

(T) Radii, OA and OB zi (they are) radii ee omnye (yes, someone else)

(L) Sithethe ngale line u-OC ma'am naye uyi-radii (and also that line ma'am, OC is also radii)

(T) Naye uyi- (it is also) radi, so sino (we have) OA, OB, OC abazi radii zethu nhe (that are radii right)

(L) Yes ma'am

(T) Now funeka sijonge i-triangle eyi-1 at a time (we need to focus on one triangle at a time right). Now let us talk about triangle AOC in triangle AOC mhh /silence/ in triangle AOC? /Silence/ Sithe ubani (what did we say) AO = OC reason?

(L) Radii

*(T) **Radii and then what kind of triangle is this**?*

(L) Isosceles

*(T**) Isosceles, what you know about isosceles triangle?***

(L) Two sides are equal

(T) Two sides are equal yes and, isosceles triangle, siyayazi mos (we all know the) i-isosceles triangle from grade 7 primary school, intsha into ye-isosceles (is this new)?

(L) No ma'am

(T) Asiqali ukuthetha ngayo mos, yeyiphi enyinto (This is not the first time we've spoken about it, what else) two sides are equal and?

(L) Opposite angles

(T) Opposite angles, ***can you tell me which opposite angles, zitheni (what about them)?***

(L) Zi- (they are) equal

(T) Two sides and two angles are equal, which angles are equal?

In the process of her lesson, Teacher 4 focused learner attention to triangle AOC and asked learners to talk about that triangle. In the post-observation interview I asked Teacher 4 to explain why she asked the question and she said:

(T) ***Ok, I wanted them to see if they can be able to identify the radius, radii.*** *Because in that isosceles triangle there was OC = OA which means two equals to be equal and also be able to identify which two angles are equal in those, in that isosceles triangle, which is the angles opposite equal sides are equal.*

The above extract demonstrates that Teacher 4 asked learners to demonstrate their understanding of radii and say why they call them radii. Teacher 4 wanted to ascertain what students know and how they think in order to advance their thinking to prove the theorem. Teacher 4's question was aimed at promoting conceptual understanding of a circle, radius, and isosceles triangle.

7.5.1.2 Teaching that promotes productive maths talk

The extract in the section above demonstrates that Teacher 4 used learner responses and asked probing questions to promote elaboration of the knowledge of an isosceles triangle.

7.5.1.3 Teaching that promotes effective use of manipulatives

I asked Teacher 4 if she thought of using manipulatives in her lesson and she responded:

(T) No we didn't use those manipulatives nhe I'm sure it's because of time, we were running out of time. Err but I've tried to explain to them the triangles because we make use of triangles you know. But I didn't use them.

Teacher 4 did not use manipulatives in both first- and second-phase lessons.

7.5.1.4 Teaching that promotes visualization

There was no evidence of teaching that promotes visualization in Teacher 4's lesson.

7.5.1.5 Teaching that demonstrates positive self-efficacy

The extract in teaching that promotes conceptual understanding in the section above, demonstrates that Teacher 4 welcomed contributions from all learners in her classroom. Teacher 4's questioning style created an enabling space for learners to express their thinking.

7.6 TEACHER 5's LESSON

The aim of Teacher 5's lesson was to teach volume and total surface area of three dimensional shapes to grade 11 learners (see section 5.6). Teacher 5 brought the Geo-Genius kit to class and asked learners to construct rectangular and square prisms. In her second-phase lesson, Teacher 5 used cylinders to support learner understanding of total surface area and volume. Teacher 5 concluded the lesson by giving learners activities which required them to calculate volume and total surface area.

7.6.1 Teacher 5 lesson and post-observation interview

7.6.1.1Teaching that promotes conceptual understanding

The extract below is from Teacher 5's video recorded lesson:

Now let's try and check, how canyou find, if you are asked to find the volume of a certain prism. ***When you talk of the volume uthetha ngantoni kanene, what do you mean when you talk of the volume?*** *Are there science learners or are you all commerce?*

(L) No there are science learners

(T) There are science learners here, ***what is the volume? Are we going to focus on the outside or?***

(L) The inside

*(T) We are going to focus on the inside****, are we going to focus on all the faces and the edges when we want to calculate the volume?***

(L) No

(T) I think you're also a science learner

(L) Yes he is ma'am

(T) Can you just help us please, when you talk of the volume, are we focusing on the outer part of it?

(L) No the inner part

(T) Inner part, now looking at this picture, we have the breadth, the length, the height and all this stuff. Now when we want to calculate the volume, ***what are we going to use? What do you think we're going to use? We want to find the volume, no one.*** *Now if you want to find the volume,* ***let me let me help you there, we are going to focus on the bottom part that is the base of your particular prism or shape. Now what shape is the shape of this prism?***

(L) Square

(T) ***Now your volume will be the area of a square times height,*** *we don't care about the other sides, we don't care about the edges, we only focus on the base of it and the height, are we together*

(Ls)Yes

Teacher 5's closed question helped to remind learners of the concept of volume, however with a limited opportunity for a mathematically rich definition of volume. Teacher 5's question assisted learners to connect the knowledge of volume with a formula for finding the volume of a square base prism.

Below is an extract from Teacher 5's video recorded lesson:

(T) Ok now, let's focus on this one

(L) Cylinder

(T) ***Yes this is a cylinder thank you, it's a cylinder. How many shapes do we have here, name them***

(L) Circle

(T) Circle, there are two; circle and circle. Now this particular shape of mine or this particular cylinder of mine is open on top, are we together */yes/. Now to find the volume of this cup, we focus on the base part of it, what is the base?*

(L) Circle

(T) It's a circle and find the area of a circle, what is the area of a circle?

L: (inaudible)

T: Someone is telling me another story and someone is telling me another story, give me one story please, what's the area of a circle? Let's help one another, yes Martin?

L: $\prod r^2$

Teacher 5 asked question that encouraged learners to give their understanding of the cylinder by asking them to identify the shapes of a cylinder. Although the third shape, that is, a rectangle was omitted in the definition, Teacher 5 assisted learners to provide a formula for the area of the base of a cylinder, which is a circle.

7.6.1.2 Teaching that promotes productive maths talk

There is no evidence of teaching that promotes visualisation in Teacher 5's lesson.

7.6.1.3 Teaching that promotes effective use of manipulatives

Below is an extract from Teacher 5's lesson:

Extract A

(T) Learner C are you creating a rectangle, you are creating a rectangle right, Learner N, a square. What are the features of a rectangle, what are the characteristics of a rectangle? Learner S, ***what are the characteristics of a rectangle?***

(T) ***A pair of opposite sides is equal in length***

In the post-observation interview I asked Teacher 5 why she asked learners to describe the characteristics of a rectangle and she responded:

(T) It was, ***I wanted to lead them to the prism, to the rectangular prism*** *where there are faces, where there are edges.* ***In a rectangle that is drawn as a 2-D (2 dimensional shape) on the board, there are no edges, there are no faces. So I wanted to lead them to the faces and the edges of a particular prism so that they will be able to differentiate between a simple rectangle shape and a prism.***

Below is an extract from Teacher 5's video recorded lesson:

Extract B

(T) Now I've given you the shapes in front of you, now I want you to tell me what type of shape do we have, just the shape in front of you, what type of shape do we have? *And tell me how many faces are there in your shape and how many edges, how many edges are there in your shape?* ***Count the number of edges you have in your shape****, Learner M, how many edges to we have there? Edge, do you know the edge guys? /yes/ Can you tell us Learner J****, how many edges do we have there?***

L: 4 edges

(T) There are 4 edges, now I want you to create a prism, I want you to create something, if you have a square, create a square prism if you have a rectangle create a rectangular prism. Who can tell me what is the area of a square?

In the post-observation interview I asked Teacher 5 to describe which manipulatives he gave to the learners and why:

(T) Giving them different shapes to restructure and to ***form other shapes from the material that I gave them****. I think it helped the learners to easily understand the lesson. For the learners to rebuild, when you talk of a square, not for them to just draw the square on the paper or on the board.* ***For them to use the Geo-Genius material that was given to them to make the prism, when you talk of the rectangular prism, yes they have the prior knowledge, they have information****, they have the idea of what a rectangle is. But you have to make them understand what is a prism, what is the difference between a prism and just a rectangle. So they know a prism that it is a 3-D shape and the one that you draw on the board is a 2 dimensional shape.*

It is evident from both extract A and B that Teacher 5 drew the attention of the learners to the Geo-Genius kit that she brought to the classroom. Teacher 5 asked learners to give the characteristics of a rectangle and this helped learners to verbalise the mathematical ideas they gained from the manipulatives. The manipulatives were used for the purpose of the lesson since Teacher 5 asked questions that promoted learner understanding of two and three dimensional shapes, total surface area and volume of three dimensional shapes.

7.6.1.4 Teaching that promotes visualization

There is no evidence of teaching that promotes visualization in Teacher 5's lesson in either the first or the second phase.

7.6.1.5 Teaching that demonstrates self-efficacy

In the post-observation interview I asked Teacher 5 to share highlights of her involvement in the MTEP programme and she responded:

(T) Iyi-1 kaloku (there's only one though) it's computers, ***you are able to do anything, I'm able to do anything now in computers regarding maths.*** *Any mathematics that needs to be done, computerised, I'm able to do it****. I can teach using Geo-Gebra of which I didn't know anything about Geo-Gebra****, I didn't even know that there was a thing called Geo-Gebra, I can now teach using Geo-Gebra. Those are the things, I can now type my own question papers, it's not so easy to type question papers for maths,* ***I can type the question papers using the***

equations now because of that programme. Icomputer bendingayazi yona nyhani, bendikwazi u-Word qha (I really didn't know much about the computer, I only knew how to use Microsoft Word) now I know everything.

The extracts in the sections above show that Teacher 5 expected all learners to contribute in her lesson. It is evident from the extract above that Teacher 5 had a positive attitude towards learning new ideas about mathematics teaching. Teacher 5 has gained knowledge on the use of technology in the teaching of mathematics. As Kilpatrick et al. (2001) argue, the more mathematical concepts one understands, the more sensible mathematics becomes. Teacher 5 sees herself as an effective learner and practitioner of mathematics, and this demonstrates positive self-efficacy.

7.7 PHASE 2 CLASSROOM OBSERVATION AND POST-OBSERVATION ANALYSIS TABLE

Table 7.1, below, provides a summary of the phase 2 practice analysis results.

Table 7.1 Conceptual teaching analysis table

Observable Indicator	Teacher 1	Teacher 2	Teacher 3	Teacher 4	Teacher 5
Conceptual understanding	QCU2 & QCU3	QCU1, QCU3 & QCU6	QCU1, QCU2 & QCU3	QCU2	QCU2 & QCU3
Productive mathematics talk	None	PMT3 & PMT4	PMT3	PMT3	None
Use of manipulatives	None	UOM1, UOM2, UOM4 & UOM5	UOM1, UOM2, UOM3 & UOM4	None	UOM1, UOM2, UOM3 & UOM5
Visualization	None	None	VZL4	None	None
Positive self-efficacy	PSE1 & PSE3	PSE1, PSE3 & PSE4	PSE1 & PSE4	PSE1	PSE1

7.8 CONCLUSION

The analysis in this chapter has revealed two dominant observable indicators for teaching that promotes conceptual understanding. These are probing for prior knowledge of mathematics concepts, and questions aimed at connecting concepts with procedures. This was observed in phase one as well. According to the analysis above, there was at least one common observable indicator for promoting productive mathematics talk for three teachers, but no evidence for the other two teachers. The second-phase results do not show significant difference in the use of manipulatives from the phase one analysis results. The analysis shows that Teacher 3, with one observable indicator, promoted visualization in her teaching. Teaching that demonstrates positive self-efficacy was evident in all five teachers, but with only one common observable indicator.

In the next chapter, I will discuss the results of the analysis. This discussion will help me demonstrate how the five teachers' participation and involvement in MTEP and in-school has influenced them in the direction of conceptual teaching. In other words, it will help me show the teachers' journey from their actual to their designated identities with respect to teaching for conceptual understanding.

CHAPTER 8

DISCUSSION OF RESULTS AND CONCLUSION

8.1 Introduction

In this chapter, I present a summary of how the five teachers' participation in MTEP and the in-school support programme shaped their teaching for conceptual understanding. The purpose of the chapter is to consolidate the study by answering the research questions. The research questions are:

1. How does teacher participation in MTEP encourage or discourage teachers to accumulate shared histories of learning with respect to conceptual teaching?
2. How do teachers see themselves with respect to conceptual teaching through their participation in MTEP?
3. To what extent do teachers' styles and discourses align with the broader vision of the MTEP activities?

The research questions focus on mathematics teacher identity and mathematics teaching practice with respect to teaching for conceptual understanding. Wenger's (1998) three modes of belonging were used to discuss teacher identity and my indicators of conceptual teaching were used to discuss their practice. Sfard and Prusak's (2005) definition of actual and designated identity helped me to demonstrate the teacher's journey with regard to conceptual teaching. In Chapters 4, 5, 6 and 7 the teachers' stories of engagement, imagination and alignment in MTEP in relation to conceptual teaching were analysed.

The teachers' actual identities are the stories they told before they became members of MTEP. The stories that they told at the end of the five-year project are their designated identities. MTEP's designated identity was determined with the aid of the Indicators of Conceptual Teaching Tool. "It is now not unreasonable to conjecture that identities are crucial to learning. Identities are likely to play a critical role in determining whether the process of learning will end with what counts as success or with what is regarded as failure. Learning is often the only hope for those who wish to close the gap between their actual and designated identities" (Sfard & Prusak, 2005, p.19). One of the aims of the study was to explore how the gap between the teachers' actual and MTEP-designated identities had been closed.

I answer my first research question by sharing teachers' engagement stories of their journey from their actual to their designated identities. The stories reveal that the teachers' participation in MTEP encouraged them to accumulate shared histories of practice with respect to conceptual teaching. The stories also showed that their interactions and relationships in and with MTEP supported them to take risks, learn and grow as mathematics educators.

The second research question is answered by sharing teachers' imagination stories of their journey from their actual to their designated identities. These stories showed that the participating teachers viewed themselves as teachers who had grown to be confident and committed through their involvement in the MTEP project.

The third research question is answered by sharing teachers' alignment stories of their journey from actual to their personal designated identities. This question is answered using the indicators of my conceptual teaching tool. These stories demonstrated that teachers were partly aligned with MTEP's goal of conceptual teaching.

In section 8.2 I discuss the teachers' actual to designated identity engagement stories. In the discussion I highlight the most significant engagement stories for all the teachers. In the next section, 8.3, I discuss teachers' actual to designated identity imagination stories with respect to conceptual teaching. In section 8.4 I consider whether the gap between teachers' actual and designated alignment closed with respect to their practice. In my discussion I also talk about teacher alignment with a broader vision external to the FRF project regarding the teaching of mathematics.

In section 8.5 I write about MTEP as a Community of Practice and in 8.6 I look at MTEP as a Professional Development Programme. In section 8.7 I discuss the implications of the study; in 8.8, the limitations of the study, and in 8.9, avenues for further research. In section 8.10 I share my own journey from an actual to a designated identity.

8.2 ACTUAL TO DESIGNATED ENGAGEMENT STORIES

Below I discuss how the MTEP environment has encouraged or discouraged the teachers to accumulate shared histories of practice with respect to conceptual teaching.

8.2.1 Accumulated shared histories of practice

8.2.1.1 Gained more teaching strategies

The analysis from both phases one and two showed that all five participating teachers identified gaps in their teaching approaches and worked together with other members of MTEP to address these gaps (Wenger, 2000). The teachers' stories told from both phases show differences in their identity in terms of their teaching approaches before and after they became members of MTEP. The formation of new pathways of learning with respect to teaching approaches differed from one teacher to another. Growth in teaching approaches for Teacher 1 and Teacher 3 included improved lesson planning. For Teacher 3 it meant facilitating productive learning environments and learning to be broadly inquisitive through research. Gaining more teaching strategies for Teacher 4 meant learning the importance of engaging learners in discussion in her teaching. Although Teacher 4 learned the importance of promoting productive mathematics talk, the analysis in Chapters 5 and 7 demonstrated that she did not fully subscribe to MTEP's designated identity in terms of promoting productive mathematics talk in her teaching.

Teaching does not necessarily result in learning. What ends up being learned may or may not be what was taught, or more generally, what the institutional organization of instruction intended. Learning is an emergent, ongoing process which may use teaching as one of its many structuring resources (Wenger, 1998). MTEP's vision was clear: to promote teaching for conceptual understanding. But what Teacher 3 learned was to broaden her scope of understanding through research. Teacher 3 was inspired and motivated to love learning and research, and this positive attitude contributed to her self-efficacy.

Although all the teachers produced meaning through their teaching for conceptual understanding, Teacher 1's story was that her participation influenced her physical science teaching positively.

Kilpatrick et al. (2001) argue that teachers' knowledge of mathematics is linked to how the teacher teaches. Teachers are unlikely to provide an adequate explanation of concepts they do not understand, and they can hardly engage their students in productive conversations about multiple ways to solve a problem if they themselves can only solve it in a single way. The participating teachers discovered more than one way of teaching mathematics through learning to teach for conceptual understanding. In the process, their mathematics content knowledge was also enriched. All five teachers learned to teach for conceptual understanding, although some aligned only partially with promoting conceptual understanding in their lessons.

8.2.1.2 Productive interactions and relationships

The interactions and relationships inherent in MTEP and the in-school support programme provided a space for the teachers to discuss difficulties they were experiencing in their mathematics teaching. MTEP responded to these with more in-school support workshops run at the schools. Teachers felt free to ask for help and the MTEP space encouraged these teachers to grow. As a result, three of the five participating teachers presented papers at the AMESA conference, and the analysis shows that they were very happy to interact and relate with other mathematics teachers nationally. These interactions opened further opportunities for the teachers to learn and contribute through their presentations to solutions to mathematics teaching and learning challenges in South Africa.

In Chapter 6, the analysis included Teacher 2's response when he was asked to say how he would like to see himself in future as result of his involvement in MTEP. Teacher 2 associated the nomination of his school for the MSTE grant with his involvement in MTEP. The relationships and interaction in MTEP provided a platform for Teacher 2 to value the importance of these in his professional growth and development.

Darling-Hammond & Richardson (2009) argue that collective participation provides a space for teachers to learn from each other. It also provides a supportive atmosphere for teachers to voice problems encountered in their practice and to engage in deliberations aimed at finding solutions to these. Teachers can serve as support groups for one another in improving practice. Collective work in trusting environments provides a basis for enquiry and reflection, allowing teachers to raise issues, take risks, and address dilemmas in their practice (Darling-Hammond & Richardson, 2009).

My findings showed that the MTEP environment has been a supportive environment, encouraging teachers to raise issues, take risks and address the dilemmas arising from their mathematics teaching. The MTEP space created a context for teacher collaboration, and provided a focus for the collaboration and a common frame for interacting with other MTEP members around common mathematics teaching problems.

8.3 ACTUAL TO DESIGNATED IDENTITY IMAGINATION STORIES

8.3.1 More confident

The analysis from both phases demonstrates that the teachers' level of confidence was increased through their involvement in MTEP. Although this was a general effect of their involvement in MTEP, they attributed it to various lessons learned from MTEP. Teacher 2 attributes it to new technology knowledge gained from MTEP, Teacher 3 to her being inquisitive and doing research before she delivers a lesson. Teacher 1, Teacher 4 and Teacher 5 attributed their confidence to the MTEP environment, which they found conducive to growth and development. Belonging to the MTEP community inspired these teachers to be confident. They even gave presentations on their teaching to other teachers.

8.3.2 More committed

My evidence showed that three of the five teachers' commitment was positively influenced by their participation in MTEP. These teachers viewed themselves and their profession in new ways with respect to commitment. They regard themselves as different from how they were before they became MTEP members. Although the teachers did not explicitly say that their designated identities were to be motivated and committed mathematics teachers, it is implicit in the data that they wanted to see themselves as committed and motivated mathematics teachers. Commitment and a positive attitude towards mathematics influence one another, although this influence may not be equal. The results of the study showed that the commitment for Teacher1 meant being prepared to continue with teaching and develop a love for the subject. Teacher 1 and Teacher 4 learned to take responsibility for their learning and growth, with their perspective on their profession positively influenced through their participation in MTEP. Commitment is important in mathematics teaching because it helps teachers to realise the importance of their professional growth, and to take ownership of that growth. To understand that one needs to grow, and then to act upon that understanding, is essential to effective mathematics teaching. Professional development programs that promote in their participants a sense that they are in control of their own learning help teachers to develop a productive disposition towards learning about mathematics, student mathematical thinking and teaching practice (Kilpatrick, 2001). Confidence and commitment make a positive contribution to improvement of mathematics teaching.

The gap between their actual and designated identities was closed for two teachers regarding their attitude towards mathematics. The MTEP space encouraged these teachers to become more positive mathematics teachers in terms of teaching strategies, confidence and commitment to their profession.

8.4 ACTUAL TO DESIGNATED IDENTITY STORIES OF ALIGNMENT

Below I discuss the five teachers' stories of alignment with MTEP and with a broader vision external to the FRF project with reference to conceptual teaching. In my discussion I demonstrate how these teachers have articulated a shared purpose and an understanding of MTEP, whether or not they are fully aligned with MTEPs' vision of conceptual teaching.

8.4.1Teaching that promotes conceptual understanding

The results of the study show two dominant indicators of teaching for conceptual understanding in all five teachers. These were asking questions about learners' prior understanding of mathematics concepts, and linking concepts with procedures. Research by Rittle-Johnson & Alibali (1999) shows that gaining conceptual understanding leads to the generation of appropriate procedures. All five teachers intertwined both conceptual and procedural understanding in their teaching.

The results did not show evidence of questions that requested learners to provide a broad and rich explanation of their solution methods. There was also no evidence of questions that required learners to explain what other learners had said in the lessons. The scarcity of these questions limited opportunities of providing learners with material for reflection and the promotion of deeper understanding of mathematics concepts. The discussion did not necessarily support and challenge learner mathematical reasoning. The results of the analysis show no evidence of questions that required mathematical reasoning from learners for four out of five teachers. Kazemi & Stipek (2001) argue that when conceptual understanding is promoted in classrooms, learners enjoy an intellectual climate characterized by argument and justification, and this was not observed in the lessons.

According to the analysis, the results showed that only one of the five teachers asked questions that required learners to make connections between mathematics concepts and real life contexts. The results did not reveal much difference between the two phases. The five teachers were partly aligned in promoting teaching for conceptual understanding, according to MTEP's designated identity.

8.4.2 Promoting productive mathematics talk

The results of the analysis showed that Teacher 2 and Teacher 4 welcomed errors from the learners, but only Teacher 2 used the errors productively to build on mathematically correct mathematics concepts. Teacher 2 requested learners to justify their reasoning, followed up on students' answers, and encouraged them to think more deeply. It is evident from results of the analysis that all five teachers required learners to demonstrate their basic understating of the mathematics concepts they taught, and used their responses to ask probing questions. However, these discussions did not promote productive mathematics talk entirely, since elaboration was not really encouraged.

The analysis results showed that the participating teachers encouraged learners to support their answers by asking 'why' questions. But these responses were not really used to stimulate productive mathematics discussion, leading to deeper understanding of maths concepts. The exchanges were not used to challenge learners to expand the boundaries of their exploration. Although many teachers find it easy to pose questions and ask students to describe their strategies, it is more challenging pedagogically to engage learners in genuine mathematical inquiry and push them to go beyond what might come easily to them (Kazemi & Stipek, 2001). White (2000) argues that teachers do not only need to ask challenging questions and listen to learners' answers to promote productive mathematics talk, they must interpret learners' responses as indicators of their levels of understanding and adjust their teaching strategy accordingly.

An effective practice which helps support productive mathematics talk is to select particular students to present their mathematical responses during discussion (Stein et al., 2008). The mathematics talk I observed did not really help learners to make connections between their different responses and the key ideas of the lessons. The analysis results for all five teachers did not provide rich evidence of questions that encouraged learners to ask questions in turn. The analysis also showed that reflection was seldom encouraged by the teachers. The gap between MTEP's actual and the designated identity was thus only partly closed with respect to promoting productive mathematics talk, since only two teachers partly aligned.

8.4.3 Teaching that promotes effective use of manipulatives

The results of the study showed that three out of five teachers used manipulatives carefully to facilitate productive learning. The manipulatives were not just brought into the classroom, but were used appropriately for the purposes of the lesson. The analysis also showed that the three teachers who introduced manipulatives provided learners with more than one type of manipulative. The teachers played a vital role in helping learners understand what they were teaching (Clements, 1999). Yet their teaching approaches were different: Teacher 2 and Teacher 3 made use of general questions to create an environment that allowed learners to explore rich mathematical ideas on the basis of the manipulatives. Teacher 5's questions encouraged learners to concentrate on features of the manipulatives for the purposes of understanding of mathematical concepts.

The manipulatives did not substitute for instruction: the teachers helped learners with relevant aspects and assisted them to link the manipulatives to relevant mathematical concepts (Kilpatrick, 2001). All three teachers facilitated an environment in which learners were able to coordinate what they did with manipulatives with their conceptual and procedural knowledge (Suh, 2007; Kilpatrick et al., 2001). The teachers asked questions which allowed learners to build meaning and make connections through the use of the manipulatives (Kilpatrick et al., 2001).

The analysis results showed that only Teacher 2 asked learners to circulate the different types of manipulatives in the class. Although the other two teachers did not use manipulatives, the results showed that their teaching also promoted understanding. The results did not show much difference between the phases. The results of the study indicated broad alignment with teaching that promotes the effective use of manipulatives.

8.4.4 Teaching that promotes visualization

The analysis results did not show broad alignment to MTEP's vision with respect to promoting visualization for the teachers. It is evident from the results that Teacher 2 and Teacher 3 asked questions that purposefully pushed learners to verbalize their mental images, to say what they were imagining in their minds. These teachers also assisted learners through questioning to relate their mental image to the lesson aim. The two teachers' questions supported learners in forming mental images and using them for their mathematical engagement (Zimmermann & Cunningham, 1991). The teachers promoted visualization in the first phase only.

The teachers concerned promoted visualization by asking carefully selected questions which assisted learners to acquire a surer understanding of mathematics concepts by connecting the visuals with mathematical ideas. The results did not show strong alignment with teaching that promotes visualization in the second phase for all five teachers.

8.4.5 Teaching that demonstrates positive self-efficacy

There is evidence from the analysis results that all the teachers encouraged and expected learners to make a contribution in the lessons they taught. Some of the teachers did not always ask questions that encouraged substantial productive mathematics talk in the classroom. The task of creating a rich environment safe for learning rests heavily on teachers (Bandura, 1997). In order for teachers to create a positive environment for learning and development, they need to have an appetite for learning, and be able to experience its value in their own learning. Teachers will not be able to present opportunities to make sense of mathematics, to recognise the benefits of perseverance and to experience the rewards of sense-making, if they themselves have not tasted and experienced these benefits (Kilpatrick, 2001).

Although the mathematical discussions did not fully align with MTEPs' vision, all five teachers created a safe environment for learners to express their thinking freely. A positive attitude towards gaining new knowledge is crucial to developing positive self-efficacy. The analysis results showed that all five teachers had a positive attitude towards gaining new knowledge. The gap between actual and the MTEP designated identity was almost closed with respect to teaching that demonstrate positive self-efficacy. All five teachers demonstrated positive self-efficacy with at least two dominant indicators in phase 1 and one dominant indicator in phase 2.

8.5 MTEP AS A COMMUNITY OF PRACTICE

The results of the study show that MTEP generated and created a shared repertoire of ideas, commitments and memories. To a large extent, the teachers assimilated the accumulated knowledge of the MTEP. This suggests that the MTEP was thus a functioning CoP (Lave & Wenger, 1991).

The MTEP teachers jointly shared and developed practices, learned from interactions with other MTEP members and gained opportunities to develop personally, professionally and intellectually (Lave & Wenger, 1991). The MTEP teachers were engaged in activities aimed at improving the teaching and learning of mathematics in a manner that foregrounded conceptual understanding.

8.6 MTEP AS A PROFESSIONAL DEVELOPMENT PROGRAMME

MTEP aimed to be improvement-oriented and context-based since it was attached closely to current classroom life. This was achieved through the in-school support and Catch-Up programmes. The MTEP CoP provided a space for the participants to become better mathematics teachers by promoting and foregrounding conceptual teaching. Collective participation was evident in the MTEP environment, since teachers learned and adopted a shared understanding of conceptual teaching. The MTEP space was safe and supportive and allowed teachers to learn about, try out, and reflect on new practices in their specific context, sharing their individual knowledge and expertise (Darling-Hammond, 2009). Although the five teachers did not fully align with MTEP's vision with respect to promoting productive mathematics talk and visualization, they grew professionally. Teacher 5's professional growth did not demonstrate that much of a difference compared to the other four teachers since similarities were observed. Clarke and Hollingsworth (2002) define professional growth as an inevitable and continuing process of learning, and this is evident in the stories told by the teachers. Alignment does not mean compliance with and submission to MTEP authority, but rather signals the coordination of efforts towards a common goal of conceptual teaching. The MTEP teachers took ownership of their own growth and development, since they were willing to learn to teach for conceptual understanding.

8.7 IMPLICATION OF THE STUDY

My research adds to knowledge that explores the relevance of teacher identity in teaching practice. The study has revealed that learning through belonging to MTEP has provided a space for five teachers to grow as mathematics education professionals. This study argues that identity as defined from a socio-cultural perspective is an important component of teacher learning and growth. Improvement of teaching practice requires acknowledging identity as a key significant factor. The results of the study have shown that embracing the notion of identity in mathematics teacher learning and teaching had a positive impact on the participants. I argue that the concept of learning as belonging rather than acquiring should be adopted for other pre-service and in-service mathematics teacher training programmes.

MTEP worked with the participating teachers for five years and produced satisfying results. Teacher development programmes need to acknowledge and recognise that teacher change is a long process and cannot be achieved by one-off mathematics teacher interventions.

MTEP has also made a positive contribution towards teaching and learning with respect to mathematics content knowledge. The MTEP teachers' content knowledge was enriched in their process of growing to be conceptual teachers. Their own conceptual understanding of mathematical content was enhanced as they developed strategies to enrich learners' conceptual understanding.

Identities are not roles, they are not assigned to people. Developing a professional identity in the context of this study was a process and a journey of negotiating meaning within a CoP. It is thus important for teacher development programmes to facilitate such negotiation by creating an environment that is safe and fosters a sense of belonging, providing a space where teachers can grow their own stories.

8.8 LIMITATION OF THE STUDY

In this study I explored the identity formation process of five teachers, and how it influenced their practice. I worked with a small sample of five teachers, which does not necessarily represent the general population of FET mathematics educators. My findings can thus not be generalized.

Secondly, I observed two lessons per teacher within the data collection period. Although there is evidence from the analysis that all five teachers have grown to become better mathematics teachers, this number of lessons is not sufficient to make a generalization. Post - doctoral studies would help to follow up and explore this identity formation process at a greater depth.

The third limitation of this study is that the stories were only told by teachers. These stories might be different from those told by their colleagues, principals, HODs and subject advisors, and even their learners.

8.9 AVENUES FOR FURTHER RESEARCH

The findings of this study indicate a close relationship between mathematics teacher identity and teaching practice. Learning as belonging to a CoP did not only create a supportive learning environment, it also inspired teachers to love and be committed to their profession. My recommendations for further research are thus:

A study that would introduce, monitor and inform South African education policy on the implementation of Mathematics Teacher Professional Development Programmes that embrace the concept of combining teacher learning and identity.

A study aimed at exploring mathematics teacher learning as belonging to a CoP in Initial Mathematics Teacher Training.

A follow-up study aimed at exploring the multiple identities of mathematics teachers belonging to a CoP.

A study that looks at how teacher identities are perceived by learners.

8.10 FINAL WORD

I joined the FRF Mathematics Education Chair in 2011 from Wits Maths Connect in Wits University. Both projects were aimed at improving mathematics teaching and learning, but their approaches were different. In MTEP, I grew to be a better educator with respect to planning and presenting lessons that promoted conceptual understanding. As an ISSP coordinator I played a key role in helping teachers to implement what they learned during the MTEP contact sessions. I worked with these teachers in a mathematics teaching environment that in addition to promoting the conceptual understanding of mathematics, foregrounded a commitment to quality planning, to producing appropriate work schedules and to producing excellent results. I would like to think that as an ISSP co-ordinator, my interactions with these teachers inspired them to learn to teach for conceptual understanding.

At the beginning of my PhD journey, I struggled with the academic writing style and paying attention to every detail of my writing. I found it difficult to remain consistent within the theoretical assumptions that I committed to at the beginning of the study. I found it quite difficult to sustain the theoretical language that underpinned my analysis. I also found it difficult, in the midst of all my references and quotes, to make my own voice heard. Through the insightful and tireless support of my supervisor, academic support workshops organized by the Chair and the PhD weeks organized by the Rhodes Department of Education, I have grown academically. My supervisor accommodated my learning style throughout our interactions, and provided feedback in a manner that suited this style. Technologically I have grown. Through my participation in MTEP and the FRF project I have assimilated the use of technology in my own teaching. I am not the same person I was before I joined MTEP. My presentation skills have been positively influenced by my participation in the FRF maths Chair and my mathematics content and pedagogical knowledge has also been enriched. My own identity as mathematics educator has grown to a designated identity that I am content with.

REFERENCES

Adler, J. (1994). Mathematics teachers in South African transition. *Mathematics Education Research Journal, 6*(2), 102-112.

Adler, J. (1998). *Lights and limits: Recontextualising Lave and Wenger to theorise knowledge of teaching and of learning school mathematics*. Johannesburg.

Adler, J., Ball, D., Krainer, K., Lin, F. L., & Novotna, J. (2005). Reflection on an emerging field: Researching mathematics teacher education. *Educational Studies in Mathematics, 60*, 359-381.

Adler, J. (2000). Conceptualising resources as a theme for teacher education. *Journal of Mathematics Teacher Education, 3*(3), 205-224.

Anderson, R. (2007). Being a mathematics learner: Four faces of identity. *The Mathematics Educator, 17*(1), 7-14.

Ball, D. L. (1992). Magical hopes: Manipulatives and reform of math education. *American Educator, 16*, 14-18.

Ball, D., & Cohen D. (1999). Developing practice, developing practitioners: Toward a practice based theory of professional education. In L. Darling-Hammond & G. Styles (Eds.), Teaching as a learning profession: Handbook of policy and practice (pp. 3-32). San Francisco: Jossey-Bass.

Ball, D., Lubienski, S., & Mewborn, D. (2001). Research on teaching mathematics: The unsolved problem of teachers' mathematical knowledge. Handbook of Research on Teaching (4th ed.) (pp. 433-456). New York: Macmillan.

Bandura, A. (1977) Self-efficacy: Toward a unifying theory of behavioral change. *Psychological View, 84*, 191-215.

Bassey, M. (1999). *Case study research in educational settings*. Philadelphia: Open University Press.

Beijaard, D., Verloop, N., & Vermunt, J. D. (2000). Teachers' perception of professional identity: An exploratory study from a personal knowledge perspective. *Teaching and Teacher Education, 16*, 749-764.

Beijaard, D., Meijer, P. C., & Verloop, N. (2004). Reconsidering research on teachers' professional identity. *Teaching and Teacher Education, 20*(2), 107-128.

Bell, J. (2005). *Doing your research project* (4th ed.). Philadelphia: Open University Press.

Bernard, R., & Ryan, G. (2010). *Analyzing qualitative data: Systematic approaches*. Thousand Oaks, CA: Sage Publications.

Best, J., & Kahn, J. (2006). *Research in educati*on (10th ed.). Boston: Pearson.

Bobis, J., Clarke, B., Clarke, T., Gill, T., Wright, R., & Young-Loveridge, J. (2005). Supporting teachers in the development of young children's mathematical thinking: Three large scale cases. *Mathematics Education Research Journal, 16*(3), 27-57.

Borko, H. (2004). Professional development and teacher learning: Mapping the terrain. *Educational Researcher, 33*(8), 3-15.

Brosnan, K. , & Burgess, R. C. (2003). Web based continuing professional development – a learning architecture approach. *Journal of Workplace Learning, 15*(1), 24-33.

Buckingham, D. (Ed.). (2008). "*Introducing identity". Youth, identity, and digital media*. Cambridge, MA: The MIT Press.

Campbell, R. J., Kyriakides, L., Muijs, R. D., & Robinson, W. (2004). Effective teaching and values: Some implications for research and teacher appraisal. *Oxford Review of Education, 30*(4), 451-465.

Carrim, N. (2003). Teacher identity: Tensions between roles. In K. Lewin, M. Samuel & Y. Sayes (Eds.), *Changing patterns of teacher education in South Africa* (pp. 306-322). Cape Town: Heinemann.

Clarke, (2008). *Language teacher identities: Co-constructing discourse and community*. Toronto, Canada: Multilingual Matters.

Clarke, D., & Hollingsworth, H. (2002). Elaborating a model of teacher professional growth. *Teaching and Teacher Education, 18*, 947-967.

Clements, D. H. (1999) "Concrete" manipulatives, concrete ideas. *Contemporary Issues in Early Childhood, 1*, 45-60.

Cobb, P., Wood, T., & Yackel, E. (1993). Discourse, mathematical thinking, and classroom practice. In E. Forman, N. Minick & C. Stone (Eds.), *Contexts for learning: Sociocultural dynamics in children's development* (pp 91-119). Oxford, UK: Oxford University Press.

Cochran-Smith, M., & Lytle, S. (2000). *The practitioner inquiry series* (pp. 1-15). Michigan State University, New York.

Cohen, L., Manion, L., & Morrison, K. (2007). Research methods in education (6th ed.). New York: Routledge Taylor & Francis Group.

Cuban, L. (1990). Reforming again, again and again. *Educational Researcher, 19*(1), 3-13.

Darling-Hammond, L., & Mc-Laughlin, M. W. (1995). Policies that support teacher development in an era of reform. *Phi Delta Kappan, 76*(8), 597-604.

Darling-Hammond, L., & Richardson, N. (2009).Teacher learning: What matters?, *How Teachers Learn, 66*(5), 46-53.

Desimone, L., Porter, A. C., Garet, M. S., Yoon, K. S., & Birman, B. F. (2002). Effects of professional development on teachers' instruction: Results from a three-year longitudinal study. *American Educational Research Journal, 24*(2), 81-112.

Doyle, W., & Carter, K. (1984) Academic tasks in classrooms. *Curriculum Inquiry, 14*, 129-149.

Duval, R. (2006). A cognitive analysis of problems of comprehension in a learning of mathematics. *Educational Studies in Mathematics, 61*, 103-131.

Erikson, E. (1968). *Identity, youth and crisis*. New York: W.W.Norton & Company.

Flores, M., & Day, C. (2005). *Contexts which shape and reshape new teachers' identities: A multi-perspective study*, 22(2), 219-232.

Fraivillig, J. L., Murphy, L. A., & Fuson, K. C. (1999). Advancing children's mathematical thinking in everyday mathematics classrooms. *Journal for Research in Mathematics Education, 30*, 148-170.

Fuson, K. C., & Willis, G. B. (1989). Second graders' use of schematic drawings in solving addition and subtraction word problems. *Journal of Educational Psychology, 81*(4), 514.

Gee, J. (2001). Identity as an analytic lens for research in education. *American Educational Research Journal, 25*(1), 99-125.

Gibson, S., & Dembo, M. H. (1984) Teacher efficacy: A construct validation. *Journal of Educational Psychology, 76*, 569-582.

Glazer, E., Hannafin, M. J., Song, L. (2005). Promoting technology integration through collaborative apprenticeship. *ETR&D, 53*(4), 57-67.

Goodnough, K. (2010). The role of action research in transforming teacher identity: Modes of belonging and ecological perspectives. *Educational Action Research, 18*(2), 167-182.

Graven, M. (2003). Teacher learning as changing meaning, practice, community, identity and confidence: The story of Ivan. *For the Learning of Mathematics, 23*(2), 25-33.

Graven, M. (2004). Investigating mathematics teacher learning within an In-Service community of practice: The centrality of confidence. *Educational Studies in Mathematics, 57*(2), 177-211.

Graven, M. (2005). Mathematics teacher retention and role of identity: Sam's story. *Pythagoras, 61*, 2-10.

Green, J. L., Camilli, G., & Elmore, P. B. (2006). *Handbook of complementary methods in education research*. London: Routledge.

Greeno, J., & Gresalfi, M. (2008). Opportunities to learn in practice and identity. In P. Moss, D. Pullin, J Gee,. E. Haertel & L. Young (Eds.), *Assessment, equity and opportunity* (pp. 170-199). Cambridge: Cambridge University Press.

Grootenboer, P., Smith, T., & Lowrie, T. (2006). Researching Identity in mathematics education: The lay of the land. In P. Grootenboer, R. Zevenbergen & M. Chinnappan (Eds.), *Proceedings of the 29th Annual Conference of the Mathematics Education Research Group of Australasia* (pp. 612-622). Australia: MERGA.

Grossman, P., Wineburg, S., & Woolworth, S. (2001). Toward a theory of teacher community. *The Teachers College Record, 103*, 942-1012.

Handley, K., & Sturdy, A., Fincham, R., & Clark, T. (2006). Within and beyond communities of practice: Making sense of learning through participation, identity and practice. *Journal of Management Studies, 43*(3), May.

Hansman, A. (2001). *New Directions for Adult and Continuing Education, 89*, Spring.

Harrington, H., & Hathway, R. (1994). Computer conferencing, critical reflection, and teacher development. Teaching & Teacher Education, 10(5), 543-554.

Harris, A., & Jones, M. (2010). Professional learning communities and system improvement. *Improving Schools, 13*(2), 172-181.

Hauge, A. L. (2007). Identity and place: A critical comparison of three identity theories. *Architectural Science Review, 50*(1), 55-51.

Herzig, H. (2010). Women belonging in the social world of graduate mathematics. University at Albany, State University of New York. *TMME, 7*(2&3), 177.

Hiebert, J., & Lefreve, P. (1986). Conceptual and procedural knowledge: An introductory analysis. In J. Hiebert (Ed.), *Conceptual and procedural* *knowledge: The case of mathematics* (pp. 1-27). New York: Lawrence Erlbaum Associates.

Hiebert, J., & Wearne, D. (1986). Procedures over concepts: The acquisition of decimal number knowledge. In J. Hiebert (Ed.), *Conceptual and procedural knowledge: The case of mathematics* (pp. 199-223). New York: Lawrence Erlbaum Associates.

Hiebert, J., Carpenter, T. P., Grouws, D. A. (Eds). (1992). Handbook of research on mathematics teaching and learning: A project of the National Council of Teachers of Mathematics (pp. 65-97). New York, NY: Macmillan Publishing Co.

Hibbert, K. (2008). “Virtual Communities of Practice: A Vehicle for Meaningful Professional Development.” *Communities of Practice,* (2),127-148.

Hill, H., Rowan, B., & Ball, D. L. (2005). Effect of teachers' mathematical knowledge for teaching on student achievement, University of Michigan. *American Educational Research Journal*, 42(2), 371-406.

Holland, D., Lachicotte, W. S., Skinner, D., & Cain, C. (1998). *Identity and agency in cultural worlds*. Cambridge, MA: Harvard University Press.

Horn, I. S. (2008). Turnaround students in high school mathematics: Constructing identities of competence through mathematical worlds. *Mathematical Thinking and Learning, 10*(3), 201-239.

Jaworski, B. (2004). Grappling with complexity: Co-Learning in inquiry communities mathematics teaching development. *Proceedings of the 28th Conference of the International Group for the Psychology of Mathematics Education*, Vol I, pp 17-36,

Jita, L., Maree, J., & Ndlalane, T. (2007). . "Lesson study (Jyugyo Kenkyu) from Japan to South Africa: A science and mathematics intervention program for secondary school teachers." *Internationalisation and globalisation in mathematics and science education*. Springer Netherlands, 465-486.

Johnson, R.B. (1997). Examining the validity structure of a qualitative research. *Education, 118*(2), 282-292.

Joubert, M., & Sutherland, R. (2008). "A perspective on the literature: CPD for teachers of mathematics." *University of Bristol: National Centre of Excellence in the Teaching of Mathematics*.

Juzwik, M. (2006). Situating narrative-minded research: A commentary on Anna Prusak's "Telling identities". *Educational Researcher, 35*, 13.

Kangela, N., Mhlolo K. M., & Schafer, M. (2013). Operationalising Wenger's three modes of belonging in the context of a mathematics teacher enrichment programme. In S. Kwofie, M. B. Ogunniyi, O. Amosun, K. R. Langenhoven & S. Dinnie (Eds.), SAARMSTE 2013, *Annual Meeting of the Southern African Association ror Research In Mathematics, Science And Technology Education, 21,* University of the Western Cape (UWC), Bellville, Cape Town, South Africa.

Kazemi, E., & Stipek, D. (2001). Promoting conceptual thinking in four upper-elementary mathematics classrooms. *The Elementary School Journal, 102*, 59-80.

Kelly, P. (2006). What is teacher learning? A socio-cultural perspective*. Oxford Review of Education, 32*(4), 505-519.

Kennedy, L. M., & Tipps, S. (1994). (7th ed.). Belmont, CA: *Guiding children's learning of mathematics* Wadsworth.

Kilpatrick, J., Swafford J., & Findell, B. (Eds.). (2001). *Adding it up: Helping children learn mathematics*. Mathematics Learning Study Committee. Washington, DC: National Academy Press.

Lasky, S. (2005). A sociocultural approach to understanding teacher identity, agency and professional vulnerability in a context of secondary school reform. *Teaching and Teacher Education, 21*, 899-916.

Lave, J. (1996). Mind, culture and activity. *Teaching as Learning in Practice, 3*(3),149-164.

Lave, J., & Wenger, E. (1991). Situated learning: Legitimate peripheral participation. New York: Cambridge University Press.

Le Compte, M. D., & Preissle, J. (1993). *Ethnography and qualitative design in educational research.*

Little, J. (1993). Teachers' professional development in a climate of educational reform. *Educational Evaluation and Policy Analysis, 15*(2), 129-151.

Lieberman, A. (1995). "Practices that support teacher development." *Phi delta kappan,* 76(8), 591.

Makina, A. (2010). The role of visualization in developing critical thinking in mathematics. *Perspectives in Education, 28*, 24-32.

Martino, A. M., & Maher. A. C. (1999). Teacher questioning to promote justification and generalization in mathematics: What research practice has taught us. *The Journal of Mathematical Behavior, 18*(1), 53-78.

Markus, H., & Kitayama, S. (1991). Culture and the self: Implications for cognition, emotion, and motivation. *Psychological Review, 98*(2), 224-253.

Matoti, S. (2011). Assessing the teaching efficacy beliefs of teacher trainees. In L. Paditz & A. Rogerson (Eds.), *Proceedings of the 11th International Conference of The Mathematics Education into the 21st Century Project* (pp. 223-228). Rhodes University, Grahamstown, South Africa.

McCarthy, J., & Oliphant, R. (2013). Mathematics outcomes in South African schools. What are the facts? What should be done*? The Centre for Development and Enterprise (CDE)*, Johannesburg.

Mc Clain, K., & Cobb, P. (2001). An analysis of a development of socio-mathematical norms in one first grade classroom. *Journal for Research in Mathematics Education, 32*, 236-266.

McMillan, J., & Schumacher, S. (2001). *Research in education: A conceptual introduction* (5th ed.). Longman: Routledge.

Mead, G. (1934). *Mind, self and society*. Chicago.

Mercer, N. (1995). *The guided construction of knowledge: Talk amongst teachers and learners*. Clevedon, UK: Multilingual Matters.

Mills, N. (2011). Situated learning through social networking communities: The development of joint enterprise, mutual engagement, and a shared repertoire. *Calico Journal, 28*(2), 345-368.

Morgan, C. (2005). Developing professionalism: Knowledge, ethics, identity. In C. Kainer (Ed.*), Making sense of curriculum innovation and mathematics teacher identity* (pp. 107-122). New York: Springer.

Morrone, A., Harkness, S., D'Ambrosio, B., & Caulfield, R. (2004). Patterns of instructional discourse that promote perception of mastery goals in a social constructivist mathematics course. *Educational Studies in Mathematics, 56*, 19-38.

Moyer-Packenham, P.S., Salkind, G., & Bolyard, J.J. (2008). Virtual Manipulatives Used By K-8 Teachers for Mathematics Instruction: Considering Mathematical, Cognitive, and Pedagogical Fidelity. Contemporary Issues in Technology and Teacher Education, 8(3).

Ogunniyi, M. (1996). "Science, technology and mathematics: The problem of developing critical human capital in Africa". *International Journal of Science Education, 18*(3), 267-284.

Ono, Y., & Ferreira, J. (2010). A case study of continuing teacher professional development through lesson study in South Africa. *South African Journal of Education, 30*, 59-74.

Opie, C. (2004). *Doing educational research*. London: Sage Publications.

Patton, M. (2002). Qualitative research and evaluation methods (3rd ed.). Thousand Oaks, CA: Sage.

Peressini, D., Borko, H., Romagnano, L., Knuth, E., & Willis, C. (2004). A conceptual framework for learning to teach secondary mathematics: A situative perspective. *Educational Studies in Mathematics, 56*(7), pp 67-96.

Pole, C., & Morrison, M. (2003). *Ethnography for education*. Glasgow: Open University Press.

Presmeg, N. C. (1997). Generalization using imagery in mathematics. In L. D. English (Ed.), *Mathematical reasoning: Analogies, metaphors and images* (pp. 267-279). Mahwah, NJ: Lawrence Erlbaum.

Putnam, R., & Borko, H. (2000). What do new views of knowledge and thinking have to say about research on teacher learning. Educational Researcher, 29(1), 4-15.

Reddy, V., Kanjee, A., Diedericks, G., & Winnaar, L. (2006). *Mathematics and science achievement at South African schools in TIMMS 2003*. Cape Town: HSRC Press.

Reed, M., & Schaefer, N. (2005). Teachers - The key to successful mathematics programs. *Prompt Intervention in Mathematics education*, (pp. 197-220).

Rittle-Johnson, B. & Alibali, M.W. Conceptual and Procedural knowledge of mathematics: Does one lead to the other? Journal of Educational psychology, 91, 175-189.

Rivera, F., Knott, L., & Evitts, A.T. (2007). Visualazing as a mathematical way of knowing. *The Mathematics Teacher, 101*, 69-75.

Robinson, D., & Reed, V. (1998). *A-Z of social research jargon*. Brookfield, USA: Ashgate.

Samuel, M. (2008). Accountability to whom? For what? Teacher identity and the force field model of teacher development. Perspectives in Education, 26(2), 3-16.

Saunders, M. (1998). Organisational culture: Electronic support for occupational learning. *Journal of Computer Assisted Learning, 14*(1), 170-182.

Samara, J., & Clements, D. (2009). “Concrete” Computer Manipulatives in Mathematics Education. *Child Development Perspectives, 3(*3), 145-150.

Schäfer, M. (2011). A discussion document reporting on growth and development of the Chair’s Programme covering the period November 2010-August 2011. Grahamstown.

Scott, D., & Usher, R. (1999). Researching education: Data, methods and theory in educational enquiry. New York: Continuum.

Sfard, A., & Prusak, A. (2005). Telling identities: In search for an analytical tool for Investigating learning as a culturally shaped Activity*. Educational Researcher, 34*(5), 14-22.

Shulman, L. (1986). Those who understand: Knowledge and growth teaching. *Educational Researcher, 1*5(2), 4-14.

Shumar, W., & Sarmiento, J. (2008). Communities of Practice at the Math Forum: Supporting teachers as professionals. In C. Kimble, P. Hildreth & I. Bourdon (Eds.), *Communities of practice: Creating learning environments for educators* (pp. 223-239). United States of America: Information Age Publishing.

Smit, B. (2001). How primary school teachers experience educational policy change in South Africa. *Perspectives in Education, 19*(3), 67-84.

Spaull, N. (2013). South Africa’s education crisis: The quality of education in South Africa 1994-2011. Report Commissioned by: *The Centre for Development and Enterprise (CDE)*, 1-65.

Stein, M. K., & Brown, C. A. (1997). Teacher learning in a social context: Integrating collaborative and institutional processes with the study of teacher change. Mathematics teachers in transition, 155-191.

Stein, M., & Lane, S. (1996). Instructional Tasks and Development of Student Capacity to Think and Reason: An Analysis of the Relationship between Teaching and Learning. An International Journal on Theory and Practice, 2(1), 50-80.

Stein, M. S., Engle, R .A., Smith, M. S., & Hughes E. K. (2008) Orchestrating productive mathematical discussions: Five practices for helping teachers move beyond show and tell. *Mathematical Thinking and Learning, 10*, 313-340.

Suh, J. M. (2007). It all together. *Teaching Children Mathematics,14*(3),163-169.

Taylor, N. (1999). Curriculum 2005: Finding balance between school and everyday knowledges. In N. Taylor & P. Vinjevold (Eds.), *Getting learning right: Report of the President's Education Initiative Research Project* (pp. 105-130). Johannesburg: The Joint Education Trust.

Taylor, N., & Vinjevold, P. (1999). Teaching and learning in South African schools. In N. Taylor & P. Vinjevold (Eds.), *Getting learning Right: Report of the President's Education Initiative Research Project* (pp. 131-162). Johannesburg: The Joint Education Trust.

Thomas, D. (2006). A general inductive approach for analyzing qualitative evaluation data. *American Journal Evaluation, 27*(2), 237-246.

Timperley, H. (2008). *Teacher professional learning and development*. International Bureau of Education. United Nations Educational, Scientific and Cultural Organization (UNESCO), (pp. 61-74).

Tsui, B. (2007) Complexities of identity formation: A narrative inquiry of an EFL teacher author(s): Amy Source. *TESOL Quarterly, 41*(4), 657-680.

Uttal, D. H., Scudder, K. V., & Deloache, J. S. (1997). Manipulatives as symbols: A new perspective on the use of concrete objects to teach mathematics. *Journal of Applied Developmental Psychology, 18*, 37-54.

Villegas-Reimers, E. (2003). Teacher professional development: an international review of the literature*. Paris: International Institute for Educational Planning*.

Walliman, N. (2006). Social Research Methods. Washington, DC: Sage Publications

Walshaw, M. & Anthony, G. (2008). The Teacher's role in Classroom Discourse: A Review of Recent Research Into Mathematics Classrooms. Review of Educational Research, 78, 516-551.

Welch, T., & Gultig, J. (2002). "Becoming competent: Initiatives for the improvement of teacher education in South Africa 1995-2002." Pan-Commonwealth Conference, Durban.

Wenger, E. (1998). *Communities of practice: Learning, meaning and identity*. New York: Cambridge University Press.

Wenger, E. (2000). Communities of practice and social learning systems. *Organization, 7*(2), 225-245.

Wenger, E. (2006). *Communities of practice and social learning systems*. Available at: www.ewenger.com/theory/communities_of_practice_intro.htm

White, D. (2000). Reaching all students mathematically through questioning. In M. Strutchens, M .L. Johnson & W. F. Tate (Eds.), *Changing the faces of mathematics perspectives on African Americans* (pp. 21-32). Reston, VA: NCTM.

White, D. (2003) Promoting productive mathematical classroom discourse. *Journal of Mathematical Behavior, 22*, 37-53.

Wilson, P., Cooney, T., & Stinson, D. (2005). What constitutes good mathematics teaching and how it develops: Nine high school teachers' perspectives. *Journal of Mathematics Teacher Education, 8*, 83-111.

Wood, T., Cobb, P., & Yackel, E. (1991). Change in teaching mathematics: A case study. *American Educational Research Journal, 28*, 587-616.

Yin, R. (1984). *Case study research: Design and methods*. London: Sage Publications.

Zimmermann, W., & Cunningham, S. (1991) Editors' introduction: What is mathematical visualization? In W. Zimmermann & S. Cunningham (Eds.), Visualization in teaching and learning of mathematics (pp. 1-7). Washington, DC: Mathematical Association of America.

www.education.gov.za

www.thutong.doe.gov.za

http://www.vitalmaths.com

http://www.polity.org.za/html/govdocs/notices/2000/not0082.html: Norms and Standards for educators