SOCIAL CONSTRUCTIVIST MATHEMATICS EDUCATION IN A CISKEIAN SECONDARY SCHOOL CLASSROOM

by

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ABSTRACT

The researcher's experience as a high school mathematics teacher in several African countries convinced her that a good number of mathematics learners exhibit serious difficulties in conceptualising mathematics properly. Her experience in teaching the subject in the Ciskei since 1990 reinforced this conviction. The researcher's natural curiosity to probe into the causes of the poor state of mathematics education in the region served as the springboard into her investigation.

Her thoughts developed in line with the emerging educational theories of social constructivism. This provided a conceptual framework for the solution of the problem, the feasibility of which was put to test practically in a Ciskeian classroom. She explicates that the difficulties experienced by the pupils in conceptualising mathematics are philosophically deep rooted and latent in the present system of mathematics education itself, which, in Ciskei, impedes the learners' conceptualisation owing to numerous problems related to their linguistic and cultural situatedness. In the analysis of the present system of mathematics education in the Ciskei she reviews a few recently published mathematics text books in the context of the topics chosen for her research study.

The results of her classroom investigation establish that a possible solution to the problem lies in the social constructivist teaching approaches.

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CHAPTER 1

RELATED THEORIES

1.1 INTRODUCTION

Experience in mathematics teaching over many years has convinced the researcher that historical and philosophical considerations that underlie the nature of mathematics deeply influence the teaching and learning of the subject. Traditional philosophers in mathematics (justificationists) promoted objectivism in mathematics education. Educationists who conceded to justificationism through axiomatic deductive approaches provided learners with objective knowledge in mathematics (the product approach) without exposing them to its constructive processes. The extent to which the learners' proper mathematical understanding is impeded by such an approach was the theme of various research projects and publications in the West for many decades towards the latter part of the nineteenth century (Freudenthal, 1972; Kline, 1973; Ernest, 1989). The republic of Ciskei follows the Western educational system in content and methods in mathematics which, among other things, might have contributed to its school pupils' poor achievement in mathematics. Accordingly, the findings of the Western educational researchers and writers could apply to the system of mathematics education in Ciskeian high schools. Social constructivism, a newly formulated philosophical perspective, emphasises a process approach in the teaching of mathematics. It explicates both the construction and objectification process of mathematical knowledge following the

'logic of mathematical discovery' (Lakatos, 1976). The present research is an attempt at testing the efficacy of the social constructivist approach in teaching mathematics in a Ciskeian high school classroom.

1.2 HOW PHILOSOPHY INFLUENCES MATHEMATICS EDUCATION

The philosophy of mathematics determines the nature of mathematics. Its fundamental epistemological questions are: what is the source of knowledge?, what is the nature and criterion of truth? and how individual knowledge construction is possible? The history of mathematics should form part of its philosophy to enable it to answer the above questions. Since the main consideration of mathematics education is the construction of subjective knowledge, its philosophy and history are very important concerns for educationists. They can gather valuable information from the developmental history of mathematics in terms of its conceptual growth to be implemented in practice.

The traditional philosophical schools of mathematics, namely logicism, intuitionism and formalism, had attempted only the foundational questions in mathematics (Wilder, 1972; Heyting, 1983; Luchins and Luchins, 1965), but failed (Ernest, 1991). Their main concern was to justify the knowledge; that is, once the knowledge is constructed they would prove it logically to legitimatise it. Such a trend was introduced into mathematics by Greek mathematicians, Euclid being the pioneer. They developed axioms as fundamentally true logical statements of mathematics to

enable them to provide logical proofs for mathematical concepts. Paying any heed to the process of construction was not in their interest. The danger thrust upon mathematics education through justificationism is that the construction and justification of knowledge are not intrinsically connected, but are grounded on different bases; construction on the reasoning of mathematical practices and justification on formal and symbolic knowledge. For the logical proofs in mathematics to be problem-free (devoid of circularity and contradictions), basic mathematical terms used should be prescriptive, undefined or implicit (Galos, 1968; Eves, 1972). But the construction of knowledge demands descriptive, explicit and well-defined basic terms.

When the pedagogicians in mathematics in the USA adopted justificationism as the guiding methodology of 'new mathematics curriculum', they were unwittingly depriving millions of mathematics learners the world over, including South Africans and Ciskeians (cf. sec. 1.1), of the benefits of the process of knowledge construction. They assumed that a prescriptive axiomatic base would enable the pupils to understand mathematics in a better manner and it took years for them to realise their mistake (Kline, 1973). But whether its negative effects on learners have been truly nullified to date is doubtful.

1.3 MORE PHILOSOPHICAL CONCERNS OVER 'NEW MATHEMATICS'

Prominent mathematicians reacted to the philosophical flaws of the 'new mathematics curriculum' implemented in the United States

towards the middle of the nineteenth century. According to Kline:

> Hence a very artificial and complicated structure of axioms and theorems was erected. The purpose of this structure was to satisfy the needs of professional mathematicians who insist on deductive structure, but it was never intended as a pedagogical approach. Yet it is this logical foundation that the new mathematics employs to cultivate understanding.
>
> (Kline, 1973, 41-42)

> It becomes quite apparent through continued analysis that the entire modern mathematics revolution has been in content, not in pedagogy.
>
> (Scott, 1972, 22)

The new mathematics curriculum introduced deductive approaches in teaching and learning of mathematics. In arithmetic the four operations were taught using the axioms; commutative (a + b = b + a); associative (a + b) + c = a + (b +c) and distributive [a x (b + c) = a x b + a x c]. The rationale was that if each step was justified using the relevant axiom, learners would refrain from rote learning. But from the learners' perspective, introduction of axioms as a 'product' made the learning of mathematics somewhat meaningless and rather difficult. They had to bypass their own intuition, imagination and free mathematical thinking to concede to the philosophers' mode of thinking.

> Moreover, the students will have to master these difficulties in about the same way that the mathematicians did, by gradually accustomising themselves to the new concept, by working with them and by taking advantage of all the intuitive support that the teacher can muster.
>
> (Kline, 1973, 40)

Pupils faced serious learning difficulties in mathematics which were reflected in their achievement:

> The report of the International Project for the Evaluation of Education Achievement gives further cause for concern. Students from the United States (the country that has purportedly led the way in school mathematics reform) were rated near the bottom in a comparison of mathematics achievement involving students from twelve industrialised countries.
>
> (Scott, 1972, 23)

1.4 THE EFFECT OF 'NEW MATHEMATICS' ON MATHEMATICS EDUCATION IN SOUTH AFRICA

Although 'new mathematics' was not the success that had been anticipated, similar curricula were followed by many developing countries because of their reliance on and confidence in the Western system of mathematics education. South Africa was not an exception. For the Blacks, the mathematics curriculum was moulded, drafted and revised according to the interest of the 'White' departments (Adler, 1991, 153) and to suit the Western standard.

Among Black South Africans, a major problem in mathematics learning has been identified as rote learning. According to van den Berg, educationists' misunderstanding about the nature of mathematics and the consequent problems in their pedagogic approaches can develop the habit of rote learning in pupils both Blacks and Europeans. He adds that this problem is more serious among the Black pupils than the European.

> ... it would seem that an over-emphasis on the axiomatic-deductive approach both in the way mathematics is taught as well as in the content matter offered, can lead to "over formalised" teaching and "dampen the creative aspect of mathematics" with a corresponding dependence on parrot-learning.... The concern of

> educationist with the misunderstanding about the nature of mathematics ... reflects the same phenomenon (parrot-learning) among the European pupils, but it does seem to be true to say that it is a problem that probably manifests itself to a much more serious extent among the Blacks.
>
> (van den Berg, 1978, 39; researcher's parentheses)

A large number of publications reacting to 'New mathematics' suggested, among other things, a 'process-approach' instead of the 'product-approach' in mathematics education. A "fallibilist" nature for mathematics as opposed to the traditional "absolute" was recommended by many of them. The absolute and fallibilist views in mathematics are explicated subsequently.

"The absolute view of mathematical knowledge is that it consists of certain and unchallengeable truths" (Ernest,1991,7). According to this view mathematical knowledge represents the unique realm of certain knowledge and this knowledge once justified and established is not questionable. This view enforced an authoritarian outlook in the teaching approaches of mathematics. The fallibilist view of mathematics is that "mathematical truth is fallible and corrigible, and can never be regarded as beyond revision and correction" (Ernest, 1991, 17). Although there are varied views of fallibilism a moderate view is that complete certainty of knowledge in mathematics is not possible and mathematics has an empirical basis.

William E. Lamon in his introductory note on Hans Freudenthal states:

> Instead of this systematic approach to new math wherein mathematics is presented as a ready-made subject, entirely structured and complete, professor Freudenthal feels that it should be presented as a subject to be discovered and learned in the same order and manner it was created; axiomatisation and formalisation of mathematics should be performed by the students, not presented to them.
>
> (Lamon, 1972, 12)

Paul Ernest points out how the professionals in the USA and UK took the challenges of the new approaches.

> Teachers of mathematics in developed countries have been bombarded with official reports and recommendations stressing the importance of the process aspects of mathematics. Thus, for example, in the U.S.A, the National Council of Teachers of mathematics in its influential 'Agenda for Action'... stated first and foremost that problem-solving should be the focus of school mathematics in the 1980s. Similarly, in the U.K., the Cockroft Report... recommended that all children should experience problem-solving and investigation as part of their mathematics education.
>
> (Ernest, 1989, 556)

Social constructivism expounds a process-oriented pedagogical approach in mathematics education. The researcher now considers the potential of social constructivism for alleviating the present pedagogical problems in mathematics eduction with special reference to the Ciskei.

1.5 SOCIAL CONSTRUCTIVISM: A PHILOSOPHY OF MATHEMATICS

1.5.1 Introduction

Social constructivism is a philosophical perspective in mathematics proposed by Paul Ernest. It does not characterise a new philosophy of mathematics, rather a consolidation of two already existing philosophies; Lakatos's quasi-empiricism and

Wittgenstein's conventionalism (Ernest, 1991,42). As a philosophy, it is descriptive and fallibilist as opposed to prescriptive and absolute. Its main thesis is that mathematical knowledge is a social construction.

Social constructivism represents a reconceptualisation in the philosophy of mathematics. Ernest contends in this regard:

> Thus much more should fall within the scope of the philosophy of mathematics than merely the justification of mathematical knowledge, provided through its reconstruction by a foundationist programme... The philosophy of mathematics must account for this complexity, and we also need to ask the questions. What is the purpose of mathematics ? What is the role of human beings in mathematics? How does the subjective knowledge of individuals become objective knowledge of mathematics? How has mathematical knowledge evolved? How does its history illuminate the philosophy of mathematics?....
>
> (Ernest, 1991, 25)

An attraction of social constructivism which makes it more promising to mathematics education is that it deals with subjective and objective knowledge simultaneously. It illustrates how subjective knowledge develops into objective knowledge through social processes. However, the way in which Paul Ernest illustrates the construction of subjective knowledge does not seem to be problem-free especially when it comes to the learning of pupils from an underprivileged society like that of Ciskei. This argument will be substantiated further subsequently. Presently, quasi-empiricism and conventionalism are examined in the social constructivist perspective.

1.5.2 Quasi-empiricism

Lakatos's quasi-empiricism represents, among other things, his challenges and reactions to the traditional dogmatic schools in mathematics, particularly to formalism. His aim was:

> to elaborate the point that informal, quasi-empirical mathematics doesn't grow through a monotonous increase of the number of indubitably established theorems but through the incessant improvement of guesses by speculation and criticism, by the logic of proofs and refutations.
>
> (Lakatos, 1976, 5)

Lakatos's quasi-empiricism deals with both the genesis and growth of mathematics. He explains the process of both the construction and objectification of mathematics knowledge following the 'logic of mathematical discovery'. He employs a 'heuristic' approach to discover and develop mathematical knowledge. His 'heuristic' approach following the 'logic of discovery' is illustrated in his book 'Proofs and Refutations' (1976) through an imaginary case study.

1.5.3 Lakatos's 'logic of discovery'

Lakatos's 'logic of discovery' explains the social process by which private mathematical constructions are transformed into accepted public knowledge. This process follows different heuristic steps as illustrated in the case study referred to above. A group of pupils and their teacher face a problem in their classroom: is there any relationship between the number of

vertices V, the number of edges E, and the number of faces F, of a regular polyhedron? After much trial and error the learners observe a relationship among them; V - E + F = 2. Now somebody among the group assumes that it is true for any polyhedron. This is termed by Lakatos a 'primitive conjecture' or theorem. A 'proof' in the traditional manner which justifies the conjecture is set up which Lakatos calls 'a thought experiment' or 'quasi-experiment'. Three steps are followed in the 'proof' or 'thought experiment':

> Step 1. A polyhedron is assumed hollow, with a surface made of rubber and if one face is cut off the remaining surface can be stretched on a blackboard. For the resulting network V and E are still the same, so that V - E + F = 1. (One face was already removed)
>
> Step 2. The network is triangulated, by drawing diagonals in those polygons which are not already triangles. By doing this both E and F are increased by one, so that the total V - E + F will not be altered.
>
> Step 3. From the triangulated network we now remove the triangles one by one. In the end it is shown that if V - E + F = 1, before the triangle is removed, it remains the same after the triangle is removed (Lakatos, 1976, 7-8).

These three steps are three sub-conjectures of the main conjecture. Testing the sub-conjectures may lead to counter-

examples. For example, 'Step 3' (third sub-conjecture) is false for cubes and for all polyhedra except the tetrahedron. These are the counter-examples which can refute this sub-conjecture. However, step-3 is further improved through arguments and criticism among the teacher and the group members which results in an improved proof while the original proof disappears. So the proof or the thought experiment leads to "decomposition of the original conjecture into sub-conjectures, instead of using it in the sense of a 'guarantee of certain truth'..." (Lakatos, 1976, 13-14). Lakatos refers to these sub-conjectures as 'lemmas'. The truth of a conjecture is tested and established through a series of refutations and improvement processes, in which all possible counter-examples against the sub-conjecture are considered. The superiority of Lakatos's quasi-empiricism over the traditional philosophy is that it finds an "intrinsic unity between the logic of discovery' and the 'logic of justification'" (Lakatos, 1976, 37). That is, the discovery and the justification are worked out on the same basis, the basis of mathematical rules and practices.

1.5.4 History is central

For Lakatos, history is a central concern of mathematics. He paraphrases Kant in this respect:

> ...the history of mathematics, lacking the guidance of philosophy, has become blind, while philosophy of mathematics, turning its back on the most intriguing phenomena in the history of mathematics, has become empty.
>
> (Lakatos, 1976, 2)

According to Paul Ernest the history and the philosophy of mathematics are intrinsically linked.

> (T)he epistemological task of the philosophy of mathematics is not simply to answer the question 'how is (any) mathematical knowledge possible?' but to account for the actual mathematical knowledge that exists. Thus, the philosophy of mathematics is indissolubly linked with the history of mathematics, since the latter is the history of the evolution of mathematical knowledge.
>
> (Ernest, 1991, 35-36)

Lakatos's quasi-empiricism does not expound mathematical practices which are entirely new. The pattern of 'thought experiment' was practised by the pre-Euclidean Greek mathematicians. A mathematical practice of conjecturing the theorem prior to developing the proof in the heuristic order was commonly followed by ancient mathematicians. "The heuristic precedence of the result over the argument, of the theorem over the proof, has deep roots in mathematical folklore" (footnote, Lakatos, 1976, 9).

But the traditional 'justificationists' replaced the culture of heurism with that of justificationism. "They want to improve their conjectures without refutations; never by reducing falsehood but by the monotonous increase of truth; thus they purge the growth of knowledge from the horror of counter-examples" (Lakatos, 1976, 37)

1.5.5 The fallibilist view

Under Lakatos's quasi-empiricism, mathematics is not a body of absolute truth. It always faces the challenges of counter-

examples and proof testing. This illustrates the fallibilist view of quasi-empiricism (Cf. section. 1.4).

1.5.6 Conventionalism

Conventionalism holds the view that "mathematical knowledge and truth are based on linguistic conventions". Both the construction and the objectification processes depend on the socially accepted rules and conventions of language. A modern proponent of conventionalism is Wittgenstein (Ernest, 1991, 31).

1.5.7 Construction of subjective knowledge according to social constructivism

The construction of subjective knowledge in mathematics is an important concern of this research. For Lakatos, construction of knowledge starts in the human mind as primitive conjectures. But how is this subjective construction possible?.

According to Paul Ernest:

> ... minds of individuals are active, conjecturing and predicting patterns in the flow of experience, and thus building theories of the nature of the world, although these may be unconsciously made theories. These conjectures and theories serve as guides for action, and when they prove inadequate, as inevitably they do, they are elaborated or replaced by new theories that overcome the inadequacy or failing of the previous theory. Thus our subjective knowledge of the external world consists of conjectures, which are continually used, tested and replaced when falsified.
>
> (Ernest, 1991, 69-70)

In order to illustrate the construction of subjective knowledge, Paul Ernest resorts to radical constructivism (Ernest, 1991, 70). An exposition of the epistemological and pedagogical implications of radical constructivism is subsequently given.

1.6 RADICAL CONSTRUCTIVISM

According to von Glasersfeld, the basic principles of radical constructivism developed mainly from the Piagetian theories are:

> 1 Knowledge is not passively received either through the senses or by way of communication. Knowledge is actively built by the cognising subject.
> 2.a The function of cognition is adaptive, in the biological sense of the term, tending towards fit or validity.
> b Cognition serves the subject's organisation of the experiential world, not the discovery of an objective ontological reality.
>
> (von Glasersfeld, 1990, 22-23)

Therefore, knowledge according to radical constructivism is that which is actively and systematically constructed. An individual or an organism cognises through organising his or her experiential world through a process of adaptation or accommodation. Accommodation or the need to 'fit' is based on the organism's previous constructions (assimilation). Also, it is the result of the individual's awareness that a certain 'scheme' is not producing the 'expected result' (von Glasersfeld, 1990, 24). For example, when a pupil simplifies the expression $7x - x$ as '7' (where $x - x$ gets cancelled according to him or her), it is a constructed answer. Such answers are very common in the Ciskeian classrooms. The pupil has systematically accommodated and developed his/her existing 'scheme' based on his/her experience.

For a radical constructivist, adaptation or accommodation whether it is physical or conceptual is strictly individual or based on the **individual's experience**. This argument is supported by the following quotation: "Conversely, when my actions fail and I am compelled to make a physical or conceptual accommodation, this does not warrant an assumption that my failure reveals something that "exists" beyond my experience" (von Glasersfeld, 1990, 24). So even though there exists a transformation or developing mechanism (reflection) within the individual, it is incapable of producing the 'expected result' acceptable to the mathematical community. But this result is acceptable to the pupil. That is why he/she does not hesitate to present any absurd answer.

Many experienced mathematics teachers would agree with the view that it is difficult for pupils to consciously understand their mistakes themselves. That is why many pupils are in the habit of making consistent mistakes in mathematics classrooms. An example is the mathematical construction of Erlwanger's Benny (1973).

> Benny had a system for converting his answers to the ones on the answer sheet provided by the curriculum. His method was systematic, and he could explain it. Converting 3/2 to .5, for example, involved adding 2 and 3 and prefixing a decimal point. That this rule also made it possible to convert 2/3 to .5 did not seem to bother Benny.
>
> (Cited by Noddings, 1990, 13)

To the radical constructivists, Benny is systematically constructing knowledge and he needs to be given credit for that. Benny's case is not an isolated one. A good number of students in the Ciskei, both in the primary and high schools, make similar or

worse constructions. If radical constructivist theories are applied in these classrooms, all those constructions are knowledge and the body of 'mathematics' would grow absurdly. According to Noddings, Benny's difficulty is that although his (Benny's) process was really constructive (according to the radical constructivists' definition) "the environment fails to press Benny to correct his misconceptions. ... the teacher's main function is to establish a mathematical environment" (Noddings, 1990, 13). Noddings further suggests that the non-constructivist instructional practices are better for pupils like Benny.

1.6.1 The role of the teacher

Radical constructivists apparently do not have a unified view about the role of the teacher in the mathematics classroom. Some do see the teacher as a facilitator of group discussion (Olivier, Murray, and Human. 1990), some as orchestrator of whole-class discussion (Wood, Cobb, and Yackel, 1990). Some maintain the view that the "teachers' main function is to establish a mathematical environment" (Noddings, 1990, 13) and some see teachers as personalities bringing out pupils' systematic construction using the power of 'discourse' (Ball, 1991, 44). The NCTM's 'Curriculum and Evaluation Standards for School Mathematics' provides only a vision about the role of teachers. Teachers in the classroom are to follow this constructivist vision according to their attitude and interest. Prawat (1992) explicates the difficulties teachers face in adopting a constructivist view in the teaching of mathematics. All the more, although the ongoing constructivist

wave makes meaningful impact, it does not seemingly prepare the mathematics teachers to develop pupils' mathematical knowledge especially in a region like Ciskei.

1.6.2 Social interaction

Present radical constructivists unanimously emphasise social interaction in learning. Their writings concentrate on two psychological traditions; Vygotskian and Piagetian. The present researcher believes that the Vygotskian tradition is more compatible with Lakatos's quasi-empiricism on strong philosophical and psychological grounds. She substantiates this view as follows: in the Vygotskian tradition, adult mediation through social interaction brings about pupils' mental development. Vygotsky specialised both in theory and practice on the interaction between spontaneous and scientific concepts in relation to instruction. Spontaneous concepts represent the child's ideas of reality developed through his/her own mental effort, and scientific concepts are those that are decisively influenced by the adult. These types of concept are developed in two different conditions; the former 'inside' and the latter 'outside'. For Vygotsky, these two interact in a mutually enhancing manner and develop the pupil mentally. He saw the bond uniting them into a total system (Vygotsky, 1934/87, 153).

Piaget's theory that supports radical constructivism views social interaction differently. He regarded spontaneous and scientific concepts as emerging from two different types of thinking: a

child's spontaneous ways of thinking and the non-spontaneous that replace the spontaneous at the age of about eleven to twelve (Vygotsky, 1934/87). According to Vygotsky, Piaget considered spontaneous and non-spontaneous thinking as separate and divided and so an interaction between the two is impossible. "Piaget attempts to present spontaneous and non-spontaneous concepts as firmly divided and self-contained entities whose interaction is impossible" (Vygotsky, 1934/87, 154).

In other words, Piaget did not envisage the type of interaction which develops the 'inside' from 'outside'. His socialisation helps the child to transform natural ways of thinking (spontaneous) to adult ways (non-spontaneous). This socialisation is characterised by antagonism and conflict. On the contrary, Vygotsky's social interaction characterised by adult mediation implies pupils' mathematical development through cooperation, wherein lies Vygotsky's strength over Piaget in mathematics instruction.

Vygotsky doubted the applicability of Piagetian theories in teaching.

> ... the results of the psychological study of children's concepts can be applied to the problem of teaching in a manner very different from that envisioned by Piaget (Vygotsky, 1934/87, 157)

Another concern is that Piaget considered language as a means to check the constructiveness of an understanding. "To become conscious of a mental operation means to transfer it from the

plane of action to that of language, i.e. to re-create it in the imagination so that it can be expressed in words" (Vygotsky, 1934/62, 88). For Vygotsky, language is a cultural tool that promotes pupils' complex thinking which is essential to solve mathematical problems.

Lakatos's view on knowledge construction agrees with the Vygotskian view because both see that learning in mathematics is possible through social interaction.

1.6.3 The process-approach

Noddings (1990) questions the constructivist denial that 'rote responses represents knowledge'. For them, a child's answer, 6 X 7 = 42 , from memory is not constructivist. People who are adamant about such a view conceive the process approach incorrectly. The social constructivist process approach in mathematics does not mean that the product is not necessary. Through the process approach, pupils are helped to axiomatise or objectify the knowledge meaningfully (Freudenthal, 1972). That means a child, after understanding the multiplication process, can objectify that knowledge himself or herself. For example, a pupil who understands the process, six 3-group pebbles make 18, gradually objectifies the knowledge as 6 x 3 = 18. Once he/she reaches that level, his or her mind had developed. Afterwards he/she simply shows the knowledge (product), and not its processes. In this respect it does not represent rote knowledge.

1.6.4 The fallibilist view leading to difficulties

The social constructivist fallibilist view does not deny mathematical truth. It means that there is no truth in mathematics, which may not be questioned one day (Cf. section 1.4). This act of questioning is a highly intelligent venture which results in the further development of knowledge (Lakatos, 1976). But some of the constructivist teachers and writers, seemingly take fallibilism to mean there is no right answer in mathematics. For example, Debora Lowenberg Ball (1991), writes that it is inappropriate for the teacher to feel that she/he has the 'right' answer. She illustrates a case where she got insight into the systematic thinking of a child's absurd answer using the strength of discourse. But she does not illustrate how she contributed as a teacher in developing the mathematical understanding of the child. She writes: "the traditional norms of classrooms, competitiveness, an emphasis on right answers, the assumption that teachers have the right answers, rejection of nonstandard ways of working or of thinking..." etc, should be replaced by constructivist norms.

1.7 SUBJECTIVE KNOWLEDGE AND OBJECTIVE KNOWLEDGE

Subjective knowledge, progressing to objective knowledge is also an important concern of social constructivism. An individual's personal construction, based on his/her personal knowledge and experiences is purely subjective. How this subjective construction in mathematics develops into objective knowledge

following Lakatos's heurism is explained in section 1.5.3. This public objective knowledge is reconstructed by individuals through their "interaction with teachers and other persons, and by interpreting texts and other inanimate sources" (Ernest, 1991, 81). In the process of reconstruction this knowledge again becomes subjective, is again improved and becomes objective. So the subjective and objective knowledge in mathematics are mutually dependent (Ernest, 1991).

Expounding the above views (cf. sec. 1.6.1 - 1.6.4), the researcher tries to explicate that radical constructivist views on teaching and instructional practices are not quite compatible with Lakatos's quasi-empiricism. Quasi-empiricism deals with the development and growth of mathematical knowledge and not with the accumulation of systematic (absurd) constructions. Because of these and other fundamental problems in radical constructivism, the researcher, for her present research, dissociates from the social constructivist formula prosed by Paul Ernest and concedes to Vygotskian tradition. Vygotsky adopted a holistic view towards educational practices. Like Vygotsky, Lakatos explains how 'outer' thought develops 'inner' thought using social processes. More reasons for the researcher's new proposal are illustrated subsequently. Hereafter, the researcher uses social constructivism to mean her newly proposed vision; that is a consolidation of conventionalism, quasi-empiricism and Vygotsky's socio-historical psychology. Terms like radical constructivism or constructivism are used to mean other types of constructivism.

CHAPTER 2

DEVELOPMENT OF THE RESEARCH THEME IN LINE WITH THE PRESENT STATE OF CISKEIAN MATHEMATICS EDUCATION

2.1 SOCIAL CONSTRUCTIVISM IN THE RESEARCHER'S VIEW

In the previous chapter the researcher has highlighted the need to reconsider the social constructivist formula proposed by Paul Ernest and concedes to the Vygotskian tradition for her research purpose. Hereafter, she uses social constructivism only in this reconceptualised form. However, the reason for such an envisagement is not only pedagogical but also philosophical and technical.

2.1.1 Philosophical aspects

Vygotsky's unprecedented holistic outlook not only on psychology and education but also on the human consciousness makes his instructional approach promising and productive. Vygotsky's philosophical intuition sees the learner in his/her socio-historical situatedness. His theories and practices seem to be more suitable for the Ciskeian mathematics education system than the atomistic radical constructivism (cf. Ch. 1). His cognition theories are grounded in society and culture. His transcendent educational theories were apparently capable of restyling even the Western dogma of individualism (the individual is a solo-learner) and the pedagogy of mathematics. His composite vision of

mathematics education does not simply differentiate between the 'weak' and the 'strong' in mathematics but suggests methods to improve the 'weak' through instructional practices. Hence instruction precedes development. The educational implications of his theories and practices are compatible with that of Lakatos's quasi-empiricism.

2.1.2 Technical aspects

Although constructivism, radical constructivism and social constructivism are frequently mentioned theories in the recent educational literature and researches, it is seemingly difficult for a reader to conceptualise the meaning of and the differences between the philosophical inclinations underlying them. Many authors, with whom the researcher has come across, use them in loose personal ways or mix up their psychological and philosophical premises indiscriminately without defining them properly. This concern is reflected in the following quotation:

> These controversies (over the genre of constructivist researches) are reflected in some of the debates between advocates and critics of constructivism (e.g. Brophy, 1980; Cobb, 1988; Confry, 1986; Kilpatrick, 1987).
> [Leder and Gunstone, 1990, 111 - 112; initial parenthesis researcher's]

The researcher believes that the premises underlying a philosophy should be made explicit before being tested. These premises also need to have some implications or connotations for the society in which the research is carried out. Considering these factors, the

researcher has to adopt a new consolidation for social constructivism for her present research.

2.2 CHANGES IN THE GLOBAL PERSPECTIVE

The problems of mathematics education are seemingly of global concern and educationists from different countries are busy with various reform attempts to alleviate these problems. So a wider consideration of the present state of mathematics education in these countries would serve in the interest of the Ciskeian mathematics education system. Reforms and research in mathematics education of Britain and the USA have always influenced similar attempts in South Africa and Ciskei (Cf. section. 1.4).

Because these two countries are among the most developed in the world, a general perception created among the developing countries is that the system of mathematics education followed in those countries must be emulated in every respect for real progress. These two countries have made substantial reforms following upon research in the field of mathematics education during recent decades (Mathematics Counts, 1982; NCTM's Agenda For Action 1980; NCTM's curriculum and evaluation standards, 1991). However, the state of mathematics education in the USA for the past few decades has been far from perfect as indicated by the following quotation.

> By now nearly everyone has probably read, or at least heard of, the recent spate of reports showing that students in the United States are not doing very well in mathematics (Dossey, Mullis, Lindquist & Chambers, 1988; Mcknight et al., 1987). Although some gains in student performance have been made in recent years, most of them have been in lower-order skills. "Most students, even at

> age 17, do not possess the breadth and depth of mathematics proficiency needed for advanced study in secondary school mathematics" (Dossey et al., 1988, p.10). The NAEP authors (Dossey et al.) note that only six per cent of 17-year-olds can solve multi-step problems involving simple algebra. Such results are deeply disappointing at the close of NCTM's "decade of problem solving.
> (Introduction to the Journal of Mathematics Education, by National Council of Teachers of Mathematics, 1990, 1).

Mcknight (1990) and Prawat (1992) have also made similar observations.

The gist of all these reform attempts mentioned above is that there is an urgent need for a shift in the educational approaches in mathematics from the traditional objectivism and cognitivism to social constructivism. Much of the literature in the field during recent times bears testimony to this argument. Paul Cobb (1990) suggests the possibility of a rapprochement in which the strong research program in cognitive science, being characterised by an emphasis on the development of runnable computer programs, is supplanted by a form of social constructivism. Gilah C. Leder and Richard F. Gunstone (1990) give research directions for considering a constructivist view of learning and assessment. Paul Ernest (1989) illustrates the emergence of a new philosophy in mathematics (social constructivism) which is relevant to the present mathematics education system. More recently, a major task of constructivism in mathematics education was undertaken by the National Council of Teachers of Mathematics (NCTM) in the USA. NCTM's vision on constructivist mathematics education is demonstrated in its **Curriculum and Evaluation Standards for School Mathematics (1991)**. These standards provide only a vision and not

a curriculum prescription for school mathematics (Crosswhite, Dossey and Frye, 1990). In other words, it is the job of the teacher to adopt appropriate instructional practices in his/her mathematics classroom to meet this vision. James Hiebert (1992) argues that these educational visions regarding teaching and learning are grounded in two psychological traditions of cognitive psychology: the Piagetian tradition, with its emphasis on individual cognition through mental transformation (reflection), and the Vygotskian tradition, which emphasises individual mental development through social interaction (communication). He also argues that the theoretical support for 'reflection and communication as understanding processes' should be established through more investigations. Thus the researchers and the teachers are entrusted with a major task in this field. This means that mathematics teachers should become classroom researchers. That is why teachers are considered as the major role players who should in the main adopt a constructivist attitude and belief (Secada, 1992). This is unfortunately not an easy task (Prawat, 1992). Although constructivism does not advocate definite teaching practices and methods, its main emphasis is the shift from traditional practices to 'something better'. In this respect the present researcher considers her investigation as part of her professional duty as a mathematics teacher, especially in the Ciskei, where to the researcher's knowledge, no such research attempts have been undertaken.

2.3 APPLICABILITY OF VYGOTSKIAN THEORIES

The researcher considers the potential of Vygotskian theory and practices in their application to mathematics education. Vygotsky's research on concept formation was centred on children. But his priorities were mathematics and language. The similarities between and the overlapping of the complex thinking processes in language and mathematics are recently being considered to be of common interest to both mathematicians and linguists. According to Vygotsky, both mathematics and language as learning subjects involve vast 'complexes of psychic functions' (Vygotsky, 1934/87). Moreover, Vygotsky's theories were tested by many Western educationists and mathematics researchers in the context of high school mathematics education (Wertsch, 1985).

2.4 CONCEPT FORMATION IN THE VYGOTSKIAN PERSPECTIVE

Vygotsky's theory of knowledge construction or concept formation is simple and realistic. He maintains that cognition is mediated through cultural tools and concept formation is the function of thought and language. The beginning of speech marks the beginning of complex thought formation in children. As they grow up, the structural and functional nature of their speech changes and their complex thinking develops into concepts. He showed experimentally that

> ... a concept is formed, not through the interplay of associations, but through an intellectual operation in which all the elementary mental functions participate in a specific combination. This operation is guided by the use of words as the means of actively centering

> attention, of abstracting certain traits, synthesizing them, and symbolising them by a sign.
> (Vygotsky, 1934/62, 81)

How language and other cultural tools function as the mediating agents in concept formation is the key to Vygotsky's learning theories.

2.5 MORE ON VYGOTSKY AND PEDAGOGY IN THE CISKEIAN CONTEXT

2.5.1 Language and thought

The learners' socio-cultural and linguistic situatedness was not regarded as a pertinent factor in mathematics education until recently. Language was considered simply as a means to expressing thought. But for Vygotsky, it serves as the very determinant of the thought process. According to him mathematical thinking and development are functions of the words the child grasps and uses from its early childhood onwards.

Communication plays a very critical role in internalising the knowledge and in the subsequent rapid mental development.

> Thought development is determined by language,i.e., by the linguistic tools of thought and the sociocultural experience of the child... The child's intellectual growth is contingent on his mastering the social means of thought, that is, language... (In other words) **The nature of development itself changes**, from biological to sociohistorical.
> (Vygotsky, 1934/62, 51; researcher's parentheses)

2.5.2 Development of word meaning and concept formation in mathematics

It is through the meaning of words that learners in mathematics, both in primary and secondary schools, construct mathematical meaning and concepts. For the young children the semantic and vocal planes are not separate and they use verbal forms and meanings without knowing their differences. At this stage they develop a sort of 'complex thinking' which later on develops into concepts (Vygotsky, 1934/62, 129). In other words, concept formation at a later stage is not a new beginning but only a development of the thought pupils have already acquired both formally and informally. About the structural and functional forms of word meaning he wrote;

> When we compare the structural and functional relations at the earliest, middle and advanced stages of development, we find the following genetic regularity: In the beginning, only the nominative function exists; and semantically, only the objective reference; signification independent of naming, and meaning independent of reference, appear later and develop along the paths we have attempted to trace and describe.
>
> Only when this development is completed does the child become fully able to formulate his own thought and understand the speech of others. Until then, his usage of words coincides with that of adults in its objective reference but not in its meaning.
>
> (Vygotsky,1934/62, 130)

When the learners can understand the signification of words independently of reference they can develop further thought using those words. Such a notion is extremely useful when one deals with pupils who learn mathematics in a second language as in

Ciskei. Lack of technical words in their mother tongue is a serious problem for the Ciskeian mathematics learners eventhough they learn mathematics in English. For example, for a pupil who learns about 'parallel' lines, understanding is easy if there is an equivalent Xhosa word for 'parallel'. In the absence of such a word it cannot be understood simply through its objective reference. So the mathematics classroom should create a mathematical context in which the learners construct both the linguistic and the mathematical meaning of the concept in the medium of instruction. Only afterwards could they understand objectively the meaning of concepts independently of reference. At that stage the mere mentioning of a concept advances a mathematical idea in the pupil's mind without any material reference. So mathematics teachers in the Ciskeian high school classrooms should not only be mathematics teachers but also language teachers in the medium of instruction.

2.5.3 Expression of thoughts through oral and written speech

An important appendage of mathematics learning is the expression of one's understanding either through writing or speech. It is through this expression that pupils' mathematical understanding is made known to others to enable evaluation. According to Vygotsky, speech and thought have distinct genetic roots and structures. Formation of a concept in mathematics, explained in the previous section as an important function of the 'verbal thought', is an inward development of language and meaning. But the development

of verbal thought into speech or written speech is an opposite process.

> ... progress in speech to the differentiated whole of a sentence helps the child's thoughts to progress from a homogeneous whole to well- defined parts. Thought and word are not cut from one pattern. In a sense, there are more differences than likenesses between them. The structure of speech does not simply mirror the structure of thought; that is why words cannot be put on by thought like a ready made garment. Thought undergoes many changes as it turns into speech. It does not merely find expression in speech; it finds its reality and form.
>
> (Vygotsky, 1934/62, 126)

The process of turning inner verbal speech into outer speech is again a function of word meaning; the thought has to be shaped "first in inner speech, then in meanings of words, and finally in words" (Vygotsky, 1934/62, 152). The learner should have enough words in his vocabulary, the meanings of which are clear, so that his thoughts are shaped in appropriate words. In written form these words have to follow yet other rules: the rules of writing. Learners who learn mathematics in their second language face difficulty when trying to grasp mathematical concepts basically due to the limitation of their mother tongue and due to their lack of second language vocabulary. No educational research, without taking into consideration such difficulties, can address the crisis in the present system of mathematics education in Ciskei.

> While learning a foreign language, we use word meanings that are already well developed in the native language, and only translate them; the advanced knowledge of one's own language also plays an important role in the study of a foreign one, as well as those inner and outer relations that are characteristic only in the study of a foreign language.
>
> (Vygotsky, 1934/87, 159)

2.6 INSTRUCTIONAL PRACTICES IN THE SOCIAL CONSTRUCTIVIST PERSPECTIVE

Based on the above argument, the researcher now attempts to envisage the instructional practices of social constructivism. Lakatos's quasi-empiricism recommends heuristic practices in mathematics instruction (cf. sec. 1.5.3). The discovery or rediscovery of mathematical concepts following the heuristic steps; identifying a problem, developing conjectures, proof generation, posing counter-example, refutation of proof-developing theorems and proof generation, are based on social interaction and criticism. Vygotsky gives strong psychological support to the heuristic instruction in mathematics through establishing the inter-evolution of spontaneous and scientific concepts which evolve under entirely different inner and outer conditions. Scientific concepts (non-spontaneous) are systematic in nature and are imparted to the learners through instruction which plays a decisive role in the mental development or in changing '**the functional composition of consciousness**' (Vygotsky, 1934/87). Spontaneous concepts represent one's natural ways of thinking.

For Vygotsky:

> The strength of scientific concepts lies in their conscious and deliberate character. Spontaneous concepts, on the contrary, are strong in what concerns the situational, empirical, and practical. These two conceptual systems, developing "from above" and "from below", reveal their real nature in the interrelations between actual development and the zone of proximal development.

> Spontaneous concepts that confront a deficit of conscious and volitional control find this control in the zone of proximal development, in the cooperation of the child with adults.
>
> (Vygotsky 1934/87, 194)

In other words, when mathematics learners face problems, they form conjectures based on their experience, which may be unconscious constructions (spontaneous). These constructions can be improved and developed through conscious cooperation with others (teacher or group members), who have a more developed mathematical understanding. That means pupils' spontaneous thinking is enhanced and developed through interactive process. It is in this context that Vygotsky has developed the concept of the 'zone of proximal development'(ZPD). He defined this as "...the distance between the actual development level as determined by independent problem solving and the level of potential development through problem solving under adult guidance or in collaboration with more capable peers" (Vygotsky, 1978, 76; cited by Stoker, 1992,137). This implies that there is a difference between a pupil's attained level of mathematical development and that pupil's potential level of such development depending on the type of instruction; problem solving through social interaction can develop his/her potential optimally which may be different from the actual development through independent problem solving. So mathematics instruction following the knowledge of the ZPD can maximise pupils achievement.

2.6.1 Instruction should utilise the zone of proximal development (ZPD)

The essence of the ZPD concept is that mathematics instruction through social interaction can help pupils to realise their potential optimally. The instructor should give pupils problems they cannot handle themselves (that is, problems that are reasonably above their natural ability). Through appropriate questions or clues, a teacher can provide the decisive step that facilitates an understanding which a pupil may not develop independently. Teaching in this respect helps to develop the child's potential optimally. In the problem to simplify $7x - x$, cited in section 1.6, if the teacher asks questions like 'what is the coefficient of x?, what is $7x + x$?, etc.,' he/she can improve pupils' ability to solve the problem correctly. Vygotsky's investigations supported this argument.

> Thus our investigation shows that the development of the psychological foundations of instruction in basic subjects does not precede instruction, but unfolds in a continuous interaction with the contributions of instruction.
>
> (Vygotsky, 1934/87, 184)

He has also recognised the need for experience, learning materials and the 'tool-mediated action' essential for learning.

2.7 THE PRESENT STATUS OF MATHEMATICS EDUCATION IN CISKEIAN HIGH SCHOOLS

Although Ciskei is a homeland region with a separate Department of Education, the state of mathematics education cannot be seen in

isolation from that of South Africa. With regard to the policy matters in mathematics education, it is similar to the Department of Education and Training (D.E.T.) in South Africa; both depend on their White 'core' department. An exposition of the present state of mathematics education in Ciskeian high schools cannot be complete without a careful consideration of a multitude of factors and concepts underlying it. However, it being beyond the scope of the present research, the researcher dwells upon a major critical factor: high school pupils' concept formation and the factors affecting it such as curriculum, text books, teacher, and teacher training.

2.7.1 Problem of mathematics education in the Ciskeian high schools

Inadequate conceptualisation and the inability to apply the concepts learned, have been identified as the major factors which impede pupils' realisation of their mathematical potential in Ciskeian schools. Lindeque (1988) states in this regard as follows;

> From discussions with the Department of Education in the Republic of Ciskei, discussion with practising teachers in Ciskei secondary schools, and the researcher's personal observations of pupils during Teaching Practice sessions, it became evident that inadequate conceptualisation, especially in Mathematics, caused serious difficulties not only to the teachers, but also to the pupils.
>
> (Lindeque, 1988, 2)

Lindeque (1988) in his study tested the effect of video-programmes on improving pupils' conceptualisation in mathematics. His

investigation was carried out in a rural Ciskeian high school the results of which supported his hypothesis.

van den Berg (1978) in his report 'A pedagogical study of the Black man's mathematical ability' gives the indication of the following problems of Black pupils: "... tendency to learn in a parrot-like fashion without insight or understanding and consequently the inability to apply their knowledge in a practical situation..." (van den Berg, 1978, 45).

2.7.2 Problems arising from the social and cultural situatedness

Lindeque (1988), in the context of the above mentioned study make mention of some factors as the causes of Black pupils' inadequate conceptualisation in mathematics. They are mainly; inadequate informal knowledge based on limited home experiences both in terms of linguistic and material factors, socio-cultural alienation from the Western-oriented education system, lack of teaching-learning materials, inadequate mastery of the medium of instruction both by students and the teachers, lack of necessary knowledge in basic concepts (previous knowledge) among the pupils, over crowded classrooms, cultural digression between home and school, ineffectiveness of the didactic techniques and authoritarian teacher-centred instructional approaches. He also wrote about the professional status of mathematics teachers in the Ciskei; "secondary school teachers in the Republic of Ciskei are unqualified to such an extent that now a mere 14 % constitutes

graduated personnel" (1988, 2). Some of his observations agree with the report by van den Berg (1978) stated above.

Cultural, linguistic and material disparity between home and the school is apparently the consequence of diverse factors, like community's cultural, social, economical and educational status.

About the literacy and numeracy of the Black South African population (including Ciskei), Marsh wrote:

> Other sad realities regarding the state of our nation are that 40 % of the adult Black population are illiterate in their first language, and 70 % of the adult Black population are illiterate in the dominant official language of the area in which they live (that is, at present, English or Afrikaans). For example, in the Pietermaritzburg area, 50 % of Black adults are literate in Zulu and 30 % of Black adults are literate in English (Van heerden, 1991).
>
> The view of Moulder (1989), as reported by du Preez (1990), is that more than 50 % of South Africa's adults are both illiterate and innumerate, and that 70 % of the Black population fall into this category.
>
> It seems reasonable to assume that people who cannot read and write language are also not able to read and write numerical processes, and can thus be classified as innumerate or mathematically illiterate. It might even be that many of those classified as literate are mathematically illiterate or, in the term used by Cockroft (1982), functionally innumerate, that is, unable to cope with the mathematics needed in everyday life.
>
> (Marsh, 1991, 137)

The majority of the people living in the rural areas, apparently fall either into the illiterate or the innumerate group or both.

The ANC in its recent publication "Discussion paper for the ANC on Education Policy" lists, among other things, the following symptoms of (causes for) the crisis in education in South Africa:

- "the high failure rates of our matriculants and our university students";
- "unqualified and poorly trained teachers";
- "the low levels of literacy and numeracy in South Africa";
- "the absence [or severe shortage] of adult education facilities" (cited by Marsh, 1991, 138; author's insertion)

Pupils' formal mathematics education is an extension of their informal consciousness which they acquire from their milieu (mainly from home). Due to the facts stated above, Black children from the rural areas in the Ciskei, may not possess adequate informal knowledge (spontaneous knowledge), which is essential for the development of their mathematical thinking and concept formation (cf. sec. 2.5.2). If pupils from these rural areas are not exposed to a proper school system planned to provide for the realisation of their intellectual potential taking into account their special problems, their mathematical conceptual system becomes increasingly inadequate and ineffective as they are exposed to more scientific concepts over the years to come. This seems to be a major problem facing the majority of the pupils learning mathematics in the Ciskeian rural high schools. Educationists term this a crisis situation in mathematics education in South Africa.

> At this early stage in the decade of the 1990's, as we approach the turn of the century and look forward to the somehow mystical promise of the year 2000, mathematics education in South Africa is in a crisis situation. This is highlighted by the appalling fact that of every ten thousand Black children who enter the school system

> only one emerges eventually with matriculation exemption in mathematics and [physical] science (Spira, 1990) (author's insertion).
>
> (Marsh, 1991, 137)

2.7.3 Concept formation: more concerns

In addition to the above concerns, there are more factors which precipitate problems in the pupils' mathematical conceptual system. These items according to the researcher's knowledge have received only limited attention in South Africa. These intricacies lying within the present mathematics education system have stemmed mainly from its philosophical and professional categories. Philosophy controls major aspects in mathematics education (cf. sec. 1.2). It (philosophy) decides, among other things, the nature of the curriculum and syllabus, text books, teachers' and learners' belief systems, the nature of professional training, etc. Each of these categories, which plays its contributive role in shaping the present status of mathematics education in the Ciskei, is now explicated.

2.7.3.1 1992 Syllabus reformation in mathematics education in Ciskei

According to the existing education system in South Africa, the curriculum matters in mathematics are under the control of the Curriculum Committee of the Department of Education and Culture (Administration, House of Assembly), the White administrative body, (now being disbanded). This committee usually prepares a core syllabus in mathematics with the professional assistance of

the experts from the four Provincial Education Departments (du Plessis, 1991). The drafting and revision of the 'core' syllabus was also effectively controlled by these four provincial 'white' education departments, subject to the interest and policy of central government. Other departments (such as that of Ciskei) adopted this syllabus either fully or with marginal adjustments to accommodate regional and 'cultural' difference (Methula, 1990 cited by Adler, 1991). However in the wake of the new political changes in South Africa positive changes are taking place in matters concerning mathematics education.

"The Department of Education and Culture (Administration: House of Assembly) has recently published new syllabuses for school mathematics", which are to be implemented chronologically in the Junior Secondary level in the following order: Standard 5 in 1991; Standard 6 in 1992; Standard 7 in 1992 (Marsh, 1991, 151). However in the Ciskei it was implemented in the following order. Standard 5 in 1991; Standard 6 in 1992; and Standard 7 in 1993. The new standard 7 mathematics syllabus recommended for the Ciskeian schools is based on the "formative educational goals" (Standard 7 Mathematics Core Syllabus, Department of Education and Culture, Republic of Ciskei, 1993). The formative curriculum goals are "based on the psychological structures of personal development (e.g. Piaget's theory)" (Ernest, 1991, 218). Social interaction is also accommodated in the standard 7 syllabus. An important underlying principle stated is:

> Pupils think and learn in different ways, so that teaching **strategies** should be varied to include direct

> teaching, activity based learning, **discussions**, application, consolidation and practice.
> (Standard 7 core syllabus , 1993, ii)

2.7.3.2 South African text books

An important concern at present is the standard of the so called 'new syllabus' mathematics text books that have recently reached the South African market and schools. Many of them are still following the traditional axiomatic approach, the residue of 'new mathematics', in content presentation (cf. sec. 1.3). Some of them, however, demonstrate a new style in the structuring through the addition of items like 'for classroom discussion', 'problem-solving', etc., but offer little innovation to teachers and pupils as stipulated in the syllabus. A subsequent review, by the researcher, of a few of the recently published standard 6 and 7 mathematics text books (new syllabus) supports her above argument. This review is based mainly on 'algebra' and a few related topics because the researcher's classroom investigation was centred on them. Although the syllabus shows the need for "recognising a real- world situation as amenable to mathematical representation" (Standard 7 Ordinary Grade Core Syllabus, 1993), some problems in these text books are shrouded in situations alien to both students and teachers. For example, in 'Just Mathematics' standard 7 (Fitten, de Jager, and Blake, 1992), the topic of graphs is introduced through a graph of 'the London smog of December 1952' (p, 42); statistics through 'research in cryptography' (p, 172). The researcher is not trying to establish that they are isolated from a 'real-world situation'; the fact of the matter is that

those situations are hardly familiar to the rural pupils and teachers in the Ciskei or South Africa. An objective approach to the introduction of new terms and concepts is another concern. For instance, 'Just Mathematics' defines graph as follows: "A graph shows in a **diagrammatic way** a **relationship** between two things called variables..." (p, 43). Terms like 'axes', 'horizontal and vertical axes', 'dependent and independent variables', etc., are simply presented as such. (Remember, graph is introduced for the first time in standard 7). Considering the linguistic limitation of the Ciskeian learners, familiar mathematical context should be presented to them so that they can construct by themselves both the word meaning and the mathematical meaning of the newly introduced mathematical concepts (cf. sec. 2.6). (Such a model will be presented in the next chapter as part of the investigation). A similar approach is adopted by the text book, 'Modern Basic Mathematics' (1992 Syllabus, Malan, Naude, van der Westhuizen), although it has adopted a simple and effective presentation of 'Probability'. Algebra is a topic very badly handled by many text books. In fact, many Ciskeian pupils find algebra a very difficult topic apparently because of their difficulty to switch from arithmetic to algebra and to grasp its abstraction. However, this is not only the problem of the Ciskeian learners but of mathematics learners the world over. A research project 'Strategies and Errors in Secondary Mathematics (SESM)' was conducted in Britain from 1980 to 1984 to identify some errors in mathematics including elementary algebra (Booth, 1984).

In this context she writes:

> The purpose of algebraic simplification is to present these statements in the most precise and economical way. But until the relationships and their algebraic representations are appreciated, and the need for more concise expression is perceived, the study of symbol manipulation itself will have little meaning.
>
> (Booth, 1984, 3)

Based on the researcher's personal experience as a high school teacher in the Ciskei region and the knowledge she has gathered through informal discussions with the subject adviser and some teachers across the region, she feels it appropriate to state that pupils in the region find it very difficult in algebra to relate letters or variables to numbers and quantities. To minimise this difficulty pupils should be introduced to proper mathematical contexts that create the necessity for the use of variables for numbers. That will help them develop the understanding by themselves of the meaning and the use of the variables in algebra. The pedagogic problems in algebra lie seemingly in the teaching methods and the text books. 'Mathematics in Focus' (Strauss, du Toit and van Tonder (new syllabus, 1992), introduces algebra in standard 6 simply by stating that 'l' and 'b' are the sides of a rectangle, and defines a variable in the most prescriptive manner (p,19). Another text book in standard 6, 'Mathematics in Action', (Gildenhuys, Paulsen and Silver, new syllabus, 1991) makes the introduction of the variable through 'replacement set' creating the potential for further difficulty in understanding amongst pupils (p, 26). Many of these text books, in fact, show the authors' knowledge about 'variable' and assume that it would serve the teaching purpose.

It is worth examining how the topic 'solution' is introduced in standard 6. Invariably all the available (standard 6) text books adopt the axiomatic or mechanical methods in the solution of problems (cf. Ch. 1). 'Mathematics in Action standard 6' introduces solution through a mechanical input- output game removing the entire algebraic meaning from the scene. Certain axioms are then demonstrated: adding, subtracting, multiplying or dividing by the same number on both sides of the equation does not make any change in the solution. These axioms are then used to solve the equation algebraically, laying more emphasis on the axiomatic approaches: 'to get x alone on the left hand side (LHS), we subtract 3 from both sides', etc. (p,119-120). A similar approach is adopted by 'Mathematics in Focus'. Although it makes mention of the 'heuristic steps' (p, 91) in solving equations, in practice it gives only prescriptive methods (inverse operation). When the same topic is dealt with in standard 7 most of the available text books extend the axiomatic methods initiated in the previous year (standard 6). It is interesting to note how 'solution' in a standard 7 book (Just Mathematics) is worked out in the 'axiomatic style'.

Eg. "Solve $5x - 3 = 2x + 7$".

The solution is argued out as follows:

You want only 'x' on the left so get rid of -3 by adding 3 on both sides. You want only numbers on the right, so get rid of the 2x by subtracting 2x from both sides. The equation now becomes; $5x - 2x = 7 + 3$. so, $3x = 10$; So, $x = 10/3$.

"Leave the answer as 10/3. You may make a mistake, and further, 10/3 is a perfectly respectable number".

The researcher doubts whether the different steps of solution and their reasoning given above could lead to a proper understanding of the solution of linear equations, apart from restricting pupils' free ways of thinking. She is sceptical about whether any mathematical thinking could be developed through the steps 'we want only x on the left side and numbers on the right'.

Such text books and practices are definitely delimiting the learners' mathematical thinking, because they simply emphasise the traditional 'axiomatisation processes'. However, the learners are definitely supposed to axiomatise by themselves the solution skills and techniques, but can do it only after mastering the construction process of 'solution'. But, when they receive these skills from others as finished products they may not find any meaning in them. The social constructivist objective of teaching 'solution' in algebra is to enable pupils to understand the mathematical meanings and mathematical processes involved in it and later on to axiomatise this knowledge. As long as the mathematics education system does not provide a systematic instruction through the use of proper materials including text books, learners would only develop algebraic misconceptions instead of proper conceptions. Numerous cases of such misconceptions can be identified in the Ciskeian classrooms. However, these misconceptions are not the exclusive trademark of the Ciskeian mathematics classrooms. A speaker at the MASA,

Katberg Conference (1992) presented different cases of mathematical misconceptions pupils exhibit in the White schools. Out of the twenty cases he highlighted, twelve were from algebra. These mistakes are commonly found in the Ciskeian classrooms also.

A very pertinent question here is: what is new in many of the 'new syllabus' mathematics text books? Implementation of any reform steps recommended in the mathematics curriculum can be undermined by improper text books. Kline (1973, 105-106) illustrates a similar situation in the United States where 'new mathematics' text books had undermined the innovations aimed at by the very program.

> One might be inclined to believe that these traditional texts "souped up" with a bit of modern mathematics could not be in wide use and so could not affect the evaluation of modern mathematics. Actually, such texts are the most popular ones because they cater to the traditionally oriented teacher who wishes or is obliged to claim that he is teaching the new mathematics.
>
> (Kline, 1972, 108)

2.7.3.3 Teachers

A counter argument against the researcher's above exposition on the 'new syllabus' text books is that text books serve merely as teaching models to the teachers who, in turn, are required to use their discretion in the classroom according to their philosophy and practices. Such an argument is acceptable for the sake of argument, but, in the Black education sector, considering the teachers' professional status in terms of subject knowledge and professional training, the argument is not a valid one.

2.8 PROBLEMS OF INSTRUCTION AND ASSESSMENT IN THE CONSTRUCTIVIST FASHION

Since radical constructivism anchors in the notion that an individual has direct access only to his/her "world of experience" (Goldin, 1990, 42), it has problems in developing instructional practices.

According to Cobb (1988, p.87):

> Although constructivist theory is attractive when the issue of learning is considered, deep-rooted problems arise when attempts are made to apply it to instruction.
>
> (Cited by Prawat, 1992, 360-361)

Jonassen illustrates this state further,

> Constructivists warn that the "knowledge" that is transmitted may not be the knowledge that is constructed by the learner. They maintain that, rather than prescribe learning outcomes, instruction should focus on providing tools and environments for helping learners interpret the multiple perspectives of the world in creating their own world view.
>
> (Jonassen, 1991, 12)

So the teachers are supposed to provide the right context and real-world problems using realistic approaches instead of predetermined instructional sequences (Lerman, 1989). Since the individual's constructions may be 'individualistic', instruction has to cater for 'cognitive flexibility', that is, multiple perspectives; various presentations are possible for the same mathematical reality or various versions of the answer are

possible for the same problem. Such a concept leads to new dimensions in assessment and evaluation.

> **Evaluation of learning would become less criterion-referenced.** If you believe, as radical constructivists do, that no objective reality is uniformly interpretable by all learners, then assessing the acquisition of such a reality is not possible. A less radical view suggests that learners will interpret perspectives differently, so evaluation processes should accommodate a wider variety of response options. Evaluation of learning, according to constructivists, should become more goal-free (Jonassen, 1991b; Scriven, 1983). Evaluation would become less of a reinforcement or control tool and more of a self-analysis tool.
>
> (Jonassen, 1991, 12)

Numerous assessment models are propagated by constructivist advocates, some are based on the NCTM's curriculum and evaluation standards. However, basically these models emphasise one constructivist principle; an individual's way of mathematical thinking is unique and even though it may not represent any mathematical reality truly, it is constructivist, if it presents a thought process systematically. A social dimension is added to instruction and assessment apparently to bring about these systematic ways of thinking through discourse. The following quotation reveals a common constructivist instructional pattern through social interaction.

> Students work in pairs to develop their own solution methods to complete the instructional activities. Following work in pairs, the teacher orchestrates whole-class discussions in which the children explain and justify their interpretations and solution to their peers.
>
> (Wood; Cobb; Yackel, 1990, 499)

Deborah Lowenberg Ball (1991), suggests an assessment model based on the 'Professional Standards for Teaching Mathematics' (NCTM 1991) in which she illustrates how she got insight into a pupil's absurd way of thinking through 'discourse'. However, she does not mention anything about her contribution as a teacher to correct the pupil's thinking. Her emphasis is that the teacher could get at the pupil's systematic way of thinking through discourse. Judith Collision (1992) gives a vision about performance assessment which is an assessment model to evaluate pupils' 'multiple dimensions' of progress; that is, to evaluate pupils' personal and mathematical disposition during problem solving. A good number of constructivist research attempts are made on 'group problem solving', 'self-evaluation', 'cooperative problem solving', etc., but whether these atomistic approaches would lead to pupils' developed mathematical thinking is something which time has to prove.

2.9 ASSESSMENT IN THE SOCIAL CONSTRUCTIVIST PERSPECTIVE

Social constructivism proposes a holistic approach to instruction and assessment as opposed to the atomistic constructivist practices. Its assessment practices take into account factors like the flaws of the traditional educational system, variety in pupils' natural disposition during problem-solving, variety in their ways of understanding and above all the entire objective of mathematics education. The most fundamental aim of any mathematics education system should be to make the pupil mathematically knowledgeable. But what is this knowledge?, Who is

capable of identifying this knowledge in the classroom? and how do the learners get the feel of this right knowledge (if the teacher should not show that he or she has the right knowledge)? Not all disputes in mathematics classrooms can be solved on the basis of consensus. If the majority forms a foolish mathematical concept then individuals with the correct concept have to accept it. By obtaining acceptability for any systematic way of thinking (including absurd ways) what mathematics knowledge are the learners achieving? Any assessment system, without appropriately considering the answers to these questions, is seemingly leading the entire mathematics education system to chaos and confusion and eventually the learners become the losers.

An important aim of social constructivism as explained so far, is to effect inner development from 'outside' through instruction (cf. sec. 1.6.2). But the time and the process of attaining this inner development are not predictable. For some pupils it comes as a sudden intuition, for some after listening to a decisive step or clue, for a few others during reflection and for some during their meditative thinking. Also, once the learners have attained the knowledge fully, they themselves process it to its objective form in the mind and, in a further problem they would naturally apply it in this objective form. For instance, a pupil who knows the identity would expand the product $(x + 6)(x - 8)$ as $x - 2x - 48$ (without showing any step). Once one has developed knowledge, the mind finds it unnecessary to show the process, but simply shows the knowledge. Krutetskii (1968/76) illustrates cases where pupils solve extremely complicated problems correctly showing

limited steps. Some of these steps are difficult for others, sometimes even for the teachers. Some pupils who solve the problems correctly are unable to explain the processes correctly. It is not due to the lack of a proper thinking pattern. These are not instances of rote learning but the result of highly developed mental thinking and construction. From the above explanation it is evident that the process of knowing is so complex and unpredictable that even the modes of and the time for thinking are not systematic. A child's ability to solve a problem may not be fully assessed and evaluated in the classroom; one may solve a problem wrongly in the class, but may get it correct at a later stage in his thinking. Hence the importance of examinations should not be underestimated. However less normative, less threatening examinations should be considered. In addition, practical examinations aimed at assessing the development can be administered. The researcher does not believe in evaluating the pupils' disposition because when children face problems they do not display their normal disposition. Pupils, especially those who are mathematically gifted, behave in unusual manners when they concentrate on mathematical problems; they would prefer to sit quietly, move around or assume unnatural postures. Krutetskii's case studies of mathematical problem solving by extraordinary pupils are the best demonstration. He illustrates cases where pupils with extraordinary mathematical abilities show abnormal physical movement and behaviour patterns before and during problem-solving. Assessment of pupils' disposition may lead to results irrelevant to their mathematical understanding.

CHAPTER 3

DESCRIPTION OF THE RESEARCH APPROACH AND THE DIFFICULTIES ENCOUNTERED IN ITS IMPLEMENTATION

3.1 RESEARCH PLAN

The research was planned to cover a period of 7 weeks (24-3-1993 to 11-5-1993) and the pilot study of 3 weeks (24-2-1993 to 17-3-1993). But due to factors beyond the researcher's control the studies could only be implemented late. Breakdown of the studies as planned and actually implemented is shown in the following table.

Breakdown of the study periods as planned	Breakdown of the study periods as implemented
Pilot study: 24-2-1993 to 17-3-1993 (3 weeks)	Pilot study: 18-3-1993 to 27-4-1993 (5 weeks)
Research study: 24-3-1993 to 11-5-1993 (7 weeks)	Research study: 28-4-1993 to 3-9-1993 (22 weeks)

(The reasons for the discrepancies in the planning and its actualization are discussed later in section 3.6).

3.2 ENTRY INTO RESEARCH SETTING

The research was intended for a sample of standard 7 pupils at Ilanga High School, Healdtown (to be known as Healdtown High School from 1994 onwards), where the researcher taught at the time of the research. Research objectives and the methods of the research were explained in detail to the Principal well in advance. He was very enthusiastic about and favourably disposed to the study. A formal application for permission to conduct the research was then forwarded to the Department of Education Ciskei, showing the summary of the research plan and the procedures. It was approved by the Department of Education timeously. The researcher made use of her own standard 7 classes for the research study. A pilot study was carried out in standard 8(b). (There were two classes in standard 8.) The concerned 8(b) mathematics teacher had been approached well in advance so as to decide upon a 'topic' for the pilot study and a mutually suitable time for its implementation.

3.3 EXPLANATION OF THE RESEARCH SAMPLE

Ilanga High School, an ordinary rural Black high school situated in Alice Circuit in the Ciskei, was taken to be representative of the population of standard 7 pupils of the region. The school consists of about 300 pupils with 10 streams covering standards 6 to 10. There are 18 members on the teaching staff, including the principal, deputy principal and the heads of division. Mathematics is a compulsory subject at the junior secondary level.

That means all pupils in standards 6, 7 and 8 are required to learn mathematics. During 1993, the year in which the research was conducted, there were two equivalent standard 7 classes with a strength of 25 pupils each initially. (Due to the problems explained in section 3.6, the effective strength of each class went down. More explanation on this aspect is given in section 3.8.3.2.1).

3.4 RESEARCH DESIGN

The research was designed in the form of an action research 'having an experimental slant' (Cohen and Manion, 1989). Of the two standard 7 classes, 7(a) was used as the experimental group and 7(b) as the control group. The pilot study was conducted in standard 8(b), it being the weaker of the two standard 8 classes at that level.

3.5 RESEARCH GOAL AND EXPLANATION OF THE TERMS

The research was aimed at evaluating the effect of a social constructivist process approach in content presentation and teaching on the mathematical development of the pupils in a rural Ciskeian classroom at the junior secondary level (standard 7). Taking into consideration the inadequate linguistic ability and the low level of understanding of the pupils, simpler terms like 'new problem-solving approaches', 'group working' and 'new teaching methods' were used instead of 'social constructivist process approach'.

3.6 DIFFICULTIES ENCOUNTERED

During her study period the researcher encountered numerous problems arising from the ongoing violence and political uprising in South Africa. The year 1993, marked by peoples' struggle on an unprecedented scale for the establishment of a democratic South Africa, which led to organised stayaways, violence and unrest in the Black townships, was not a conducive period for a classroom research. The motivation for the researcher to undertake the research under such trying circumstances was her commitment to herself and to her profession. The progress of her research was intermittently hampered by the boycott of classes by the pupils and the teachers' strike.

3.7 DETAILS OF THE COURSE ADMINISTRATION AND EVALUATION

3.7.1 Course administration

Both the pilot and the research studies were administered in the same manner.

3.7.1.1 The concept of the group

Groups were designed as envisaged in Chapter 2, to enable discussion and communication. Each group (for both the pilot and the research study) consisted of three pupils, the selection being based on their own choice. During discussions interaction took place at intra-group and inter-group levels. When the teacher's

'mediation' was needed the whole class was taken as a single group.

3.7.1.2 Pilot study

Problems like collection of fees and late arrival of students caused classes to commence late at Ilanga high school in 1993 as usual. Consequently, the pilot study could be started only on 18 March 1993 and not on 24 February 1993 as planned previously. The pre-test was administered on 18 March. The topic 'factorisation' was chosen for the pilot study in accordance with the scheme of work of the teacher concerned. First, the construction process of concepts and then their objectification were presented to follow the social constructivist process approach. The researcher herself had to design the introduction, problems and their sequence in order to teach 'factorisation' in the social constructivist fashion explained in section 2.6, because no available standard 8 text books, the researcher had come across, followed such a process approach. Instead, they followed an objective style incompatible with social constructivism.

3.7.1.2.1 On topic presentation

The social constructivist process approach was followed to introduce the topic 'factorisation'. The word meaning and mathematical meaning of 'factorisation' (cf. sec. 2.5.2) were developed initially through introductory problems. The introductory problem, "If the area of a soccer field is 6000

sq.m., what are its possible lengths and breadths" (Worksheet 1; cf. Appendix 6), gave pupils a chance to write the relationship between area, length and breadth (area = length X breadth) in numerous ways. The next task was to ensure that this knowledge was developed into the knowledge of factors; an algebraic expression can be written as the product of its factors, ('length' and 'breadth'), and vice versa (Worksheets, 2,3,4,5, cf. Appendix 7-10). By then the pupils were able to develop the mathematical meaning of 'factors' and 'factorisation'. Only then was the concept of 'factorisation' presented to them (Worksheet 6, cf. Appendix 11). Different types of factorisation such as taking the common factor out, taking the highest common factor out and splitting the middle term, were then presented to them through problems using the same approach (Worksheets 8, and 9, cf. Appendix 13 and 14).

3.7.1.2.2 Teaching approaches

Teaching was to follow social constructivist 'heuristic' approaches. A classroom innovation made in this regard was that pupil groups were formed to enable problem-solving through group interaction. However, it was found that the pupils could not develop any imaginative thinking individually or in groups to solve even the simple introductory problems (Worksheet 1). To improve the situation the teacher had to pose additional questions like (cf. sec. 2.6.1), "okay, if the length of the soccer field is 300 m., what would be its breadth?". The researcher became convinced that teacher's mediation is an integral part of the

group-learning approach. In addition to that the structure of the group had to be varied according to the level of understanding of the pupils as well as the level of difficulty of the problem. Sometimes, in the course of developing the understanding of a concept, the whole class including the teacher became a group and later small groups of pupils solved similar or even more difficult problems depending on their problem-solving abilities. During all the class discussions, the researcher/teacher presented problems and posed questions to the pupils depending on their level of understanding and the nature of the problem at hand (much flexibility and variations were accommodated in questioning). Solutions were reached mainly through classroom discourse. Pupils were encouraged to discuss, in groups, the solution process of problems by continuously asking questions like 'why?', 'what?', and 'how?'. Such discussions facilitated communication and interaction that contributed towards improving the pupils' mathematical knowledge.

3.7.1.2.3 Teaching materials

The researcher used worksheets, designed by her, which contained pre-set problems and stipulated working space. Less use than normal was made of the blackboard.

3.7.1.3 Evaluation of the study

The following enquiry methods and processes were intented to evaluate the pilot study. The same methods were used in the case of the research study also.

3.7.1.3.1 Enquiry methods

The following methods of enquiry were employed by the researcher.

a) A teacher's diary

b) Non-participant observation

c) Pupils' diaries

d) Tests

e) Interviews

* For the pilot study, item (c) could not be employed.

3.7.1.3.2 Selection of non-participant observers

There were two non-participant observers; observer (a) as the main observer and observer (b) to give a second opinion.

3.7.1.3.2.1 Observer (a)

Observer (a) was incidentally the acting Deputy Principal of the school. He is a qualified and experienced mathematics/physical science teacher who has been working in the same school at least for the past 10 years. Before becoming the Deputy Principal, he was the head of the science division. His considerable knowledge of the South African mathematics education system and his understanding of the pupils and the society from which they come have been of great value to the present research.

3.7.1.3.2.2 Observer (b)

Observer (b) is also highly qualified (M.Sc in mathematics). He has over 15 years of experience in teaching mathematics to Black children in three African countries. He is now teaching mathematics in the Ciskei and has recently completed a Higher Education Diploma course with the University of South Africa.

3.7.1.3.2.3 Familiarising observers with the problem

Both the observers were introduced to the research problem, enquiry methods and enquiry processes well in advance of the pilot study. This was accomplished through discussions and by providing them with relevant literature. Each observer had his own individual method of observation based on his conceptual framework.

3.7.1.3.3 The enquiry process

Enquiry processes employed for the pilot and the research studies were as follows:

(i) The researcher made notes in her diary on pupils' reaction to and their appreciation of the new teaching methods, the group approaches and their problem-solving abilities based on her observation and on their mathematical understanding based on their classwork.

(ii) The observers visited the classroom to observe pupils' appreciation of and reaction to problem-solving approaches in groups and the teaching approaches of the teacher in an open-ended manner.

(iii) The researcher and the observers met formally and informally to discuss their views and observations. Ideas generated through these discussions were implemented subsequently (cf. sec. 3.8.3.2.3).

(iv) Questionnaires were used to gather pupils' responses which in turn were used to evaluate the study.

(Details explained so far in this chapter were applied both in the pilot and the research studies).

3.7.1.3.3.1 The problem of time in the evaluation context

Reference needs to be made to the problem of time in the evaluation of the pilot study. Although the pilot study extended from 18 March 1993 to 27 April 1993, actual lessons could be conducted for seven days only on account of severe absenteeism caused by the problems shown in section 3.6. This made the evaluation of the pilot study partially invalid.

3.7.1.3.3.2 Testing and examination

The period of the pilot study having been very short in effect, the researcher could not employ all the enquiry methods she had envisaged in section 3.7.1.3.1. However, a pre-test, a post test and two questionnaires were given during the pilot study.

3.7.2 Summary of the pilot study

This summary is based on the enquiry process shown under section 3.7.1.3.3. However due to problems experienced during the pilot study period (see sec. 3.7.1.3.3.1 and 3.7.1.3.3.2) information gathered through these processes could not provide valuable conclusions about the pilot study. What could be suggested meaningfully about the pilot study is based on the researcher's classroom experience and observation in 8(b) together with the non-participant observers' comments on this study. However, the following summary is provided for the sake of information.

3.7.2.1 Statistical aspects

The percentage pass (standard grade) for the pre-test was 16.2 (6 out of 37) and that for the post-test was 22.2 (2 out of 9). The standard grade pass mark in standard 8 mathematics is 35%. However, the average percentage mark for the pre-test was 18.8 and for the post-test 18.7. The statistics shown here cannot be taken to reflect truly the outcome of the pilot study for the following reasons.

Of the 37 pupils who wrote the pre-test, only 9 appeared for the post-test. Among those who did the post test, 1 pupil attended six lessons, 3 five lessons, 2 four lessons, 2 three lessons and 1 two lessons of a total of seven lessons given.

3.7.2.2 Observational aspects

This part of the summary is based on the researcher's and the non-participant observers' experience during the period of the pilot study.

(i) Pupils in 8(b) are invariably incapable of developing their own thinking strategies or 'conjectures' to solve even simple problems. They lack creative thinking skills, but can acquire imitative thinking when a problem is solved through the teacher's mediation.

(ii) The whole group or the whole class would meet with an impasse when faced with new problems. This is mainly due to the serious inadequacies and misconceptions in their basic knowledge.

(iii) Pupils in 8(b) showed appreciation of the new teaching approaches in groups.

3.7.3 Conclusion of the pilot study

It has already been stated under section 3.7.2 that the summary of the pilot study cannot provide a valid conclusion about the pilot study. Now what can be suggested in conclusion about the pilot study based on section 3.7.2.2 are the following points.

Pupils in 8(b) showed appreciation of social constructivist teaching approaches in mathematics. Teacher's appropriate mediation may improve pupils' creative thinking in mathematics. The pilot study provided some support for the researcher's view about the nature of 'group' and 'group discussion' (sec. 3.7.1.2.2).

3.8 RESEARCH STUDY

The research study was conducted in standard 7, following the research design in section 3.4. The reason for choosing 7(a) as the experimental group (E) and 7(b) as the control group (C) was that 7(b) did much better in mathematics than 7(a) before the research study. This conclusion was made on the basis of the two similar monthly tests administered in both the classes before the study. The study was aimed at improving E, the weaker group, against C, the stronger group.

3.8.1 Goal of the research study

The goal of the research study was to evaluate the effect of the social constructivist process approach both in content presentation and teaching (sec. 3.8.3.1.2 to 3.8.3.1.5) on the mathematical development of the 7(a) class [E]. The study also aimed at evaluating the effect of these teaching approaches on these pupils' attitude and outlook towards mathematics and the 'new teaching approaches'. Mathematical achievement of E against C was compared using tests. Other data regarding E were gauged using different enquiry methods explained in 3.7.1.3.1 (a), (b), (c) and (e). In 7(b), non-social constructivist teaching approaches were adopted both in content presentation and teaching (cf. sec. 3.8). This means, each pupil worked out problems himself or herself and no group discussion was encouraged among the pupils. The teacher asked questions to individual pupils and never initiated class discussion. The presentation of the teaching topic (solution of linear equations) and the style of teaching in 7(b) were based on the text book approaches (Just Mathematics standard 7, 1992 syllabus) [see sec. 2.7.3.2].

3.8.2 The problem of time

The problem of time explained in section 3.6 was experienced during the research study too. However, being in the same school, both E and C were affected equally by it. A delayed conclusion of the pilot study led to the research study having to commence late, (28-4-1993 to 3-9-1993).

3.8.3 Details of course administration and evaluation

3.8.3.1 Course administration

It has already been stated that similar styles of course administration were adopted for both the research and the pilot studies. However, a few changes had to be effected based on the outcome of the pilot study to which reference will be made in due course. The research study commenced on 28-4-1993, the day both E and C wrote the pre-test (Cf. Appendix 1). E was taught following the social constructivist teaching approaches and C in the non-social constructivist approaches (see sec. 3.8.1). Identical pre-test, class tests and post-test were given to both E and C during the research. Results of these tests were used to compare the mathematical achievement of E against that of C. Details of these tests and their results will be discussed later in section 3.8.3.2.1.

Questionnaires were used to gather group E's responses with regard to their rating of the level of difficulty of mathematics, their outlook on its importance, whether they liked it or not and their appreciation of new teaching approaches at different times during the course of the research study. The aim was to gauge the effect of social constructivist teaching approaches on E's attitude to and appreciation of mathematics. The summary of pupils' responses obtained through these questionnaires is given in section 3.8.3.2.2.

3.8.3.1.1 Motivation of the research group

The research group was motivated for the social constructivist teaching methods using appropriate literature [see Appendix 15(a) and 15(b)], explanations and demonstrations. This was a major task for the researcher because the teaching approaches that they were introduced to were novel and the pupils were lacking mastery of the language of instruction. Hence, the task of motivating them in this respect was not a day's labour; rather it permeated the whole research period. A day was devoted at the beginning to 'talking' in English in order to initiate the pupils into a culture of group discussion and discourse which were to characterise the new classroom practices. At times the researcher had to allow group discussions to be conducted in their mother tongue (Xhosa), although the medium of instruction was English, but they had to definitely use English while communicating with the teacher. Their difficulties in communicating in the medium of instruction were understood with sympathy and were encouraged to express themselves using their limited vocabulary though they often failed to speak meaningfully on account of their inhibition and poor grasp of grammar.

3.8.3.1.2 The concept of group

Results of the pilot study in section 3.7.3 justified the researcher's concept of the group in section 3.7.1.1. Therefore the same group structure was adopted for the research study.

3.8.3.1.3 On topic presentation

The topic was presented to the experimental group E following the social constructivist instructional approach as explained in section 2.6. The researcher herself had to design the topic presentation, because she did not come across any text book in standard 7 appropriate in this respect. The design was based on a social constructivist 'zig-zag' (top-down/ down-top) approach. Thus 'solution of linear equations', the topic chosen for the research teaching, was introduced through a function and its graph. (Pupils had already been taught graphical representation of functions). This was accomplished as follows.

(i) The meaning and the use of 'solution' were introduced to group E before introducing the word/concept 'solution'. The graph of a linear function was obtained by plotting many true x and y values of the function (see the table below).

Table for the function, $Y = 3x-1$.

x	-2	-1	0	1	2	3
y	-7	-4	-1	2	5	8

On analogous observation of the graph and the table (cf. Appendix 17), numerous equations were constructed, for example; $2 = 3x - 1$, $-7 = 3x-1$, $8 = 3x-1$ etc,. The relationship between the function and the equation in terms of the values of x and y was illustrated in detail

using the graph and the table in an analogous manner. At this stage the pupils could understand the interplay between the letters and numbers in the context of an equation. They themselves could construct numerous equations and could grasp as well the meaning of solution of an equation. At this stage the word (concept) 'solution' was introduced.

(ii) Solution of linear equations

The method of solution was initially developed at the contextual level (through inspection);
For example, solve the equation, $5 = 3x - 1$. The solution was arrived at as follows;
5 = 3 times a number -1.
Therefore, 6, [which is (5+1)] = 3 times a number (The point stressed was the meaning of the equation) .
So the number is 2. Hence $x = 2$. [The point stressed was that the letters or variables in algebraic equations represent numbers]. Pupils were given more problems for solving.

(iii) Teaching was directed to developing the constructive process into the objective level. A variety of linear equations were given to be solved (Eg. $-16 + 5x = 4$; $39 = 3x + 2$; $-9 = 8y - 1$). These problems were designed to provide pupils with different mathematical situations which would help them to identify the mathematical

meanings of those constructions. Once they identify them (meanings of these constructions) they abstract the construction processes or their minds attain a higher level of mathematical development. Many pupils cannot reach this higher level themselves. The teacher should help them to attain this level. The social constructivist method to help the pupil in this respect is to make him or her confront a variety of questions and mathematical situations. Of the many questions one confronts, a particular one may be the decisive one to attain this level (Vygotsky, 1934/1987). So the success of a teacher in this context depends on the teaching methods and the type of questions and discussions he or she makes the pupils face. It is practically difficult for the researcher to explain in words the discussion that took place in each classroom. However a specimen sample of the social constructivist classroom discussion (Appendix 5) would highlight the type of discussion the researcher had followed in her classroom. [But on many occasions, especially towards the beginning of the study, the researcher could not carry out the discussions as explained in the above example because of the pupils' inability to respond meaningfully]. Once a pupil objectifies the knowing process himself or herself, he/she knows the meaning and the signification of taking a number or a letter from one side to the other side of the equal sign.

Now the problem, solve $7x + 1 = 10x - 8$ could be solved

as, $1 + 8 = 10x - 7x$; $9 = 3x$; $x = 3$. Or

$7x - 10x = -8 - 1$; $-3x = -9$; $x = 3$.

[The point is that pupils arrange the equation in any way they prefer using the knowing process they had objectified (without mechanically following an axiom), and solve the equation].

(iv) More problems including real world problems were introduced afterwards (for examples see Appendix 18, questions 1-4).

(v) Letters (variables) together with numbers were used in the real world situation (for examples see Appendix 18, questions 5-7).

3.8.3.1.4 Teaching approaches

Teaching approaches employed in the research study were similar to those of the pilot study (sec. 3.7.1.2.2). 'An imaginary classroom discussion' (cf. Appendix. 5) highlights the social constructivist classroom discussion the researcher intended to follow in her classroom during the investigation.

3.8.3.1.5 Teaching materials

For the research study worksheets were replaced by ordinary note books. A copy of printed problems was given to each pupil. Also more use of the blackboard was made in the research study compared

to the pilot study. These changes were made during the research study to effect more 'flexibility and variation' in the discussion process explained in 3.7.1.2.2.

3.8.3.1.6 Implementation of the enquiry processes

(i) Researcher's diary

The researcher made daily notes of the pupils' appreciation of and their reaction to the new teaching methods, their ability to make use of the group facilities and their difficulties or improvements in mathematical understanding based on classroom observation and classwork.

(ii) Non-participant observation

The non-participant observers made observation of matters like the researcher's teaching approaches, pupils' reaction to and appreciation of the new methods, their ability to participate in group discussion and their understanding. They discussed their observations with the researcher and made summaries of their evaluations. The researcher held formal meetings (sometimes weekly or fortnightly depending on the need) with the non-participant observers. In addition to these, informal meetings and discussions with them took place very often.

(iii) Pupils' diaries

Each pupil kept a diary to record his/her observation of and views on the new teaching approaches and classroom discussions.

(iv) Tests

During the research study two tests were given simultaneously to both E and C. At the end of the study a post-test was given also simultaneously to both E and C. Each time the same test was used for both E and C.

(v) Questionnaires

Pupils' responses at different times were gathered through questionnaires.

3.8.3.2 Summary of the research study

Before codifying the summary of the research study special mention needs to be made of the problem of time. Although the research period covered 22 weeks (28-4-93 to 3-9-1993), the researcher had only 20 teaching days in each class (E and C). This was due to the numerous interruptions caused during this period by the June/July holidays, stayaways by pupils and the strikes embarked on by the SADTU (the South African Democratic Teachers' Union).

These caused pupils' erratic attendance during the research study. A summary of the results of the research follows.

3.8.3.2.1 Tests and examinations

An evaluation of the mathematical achievement of E against that of C was accomplished through the test results recorded in Tables (a) and (b).

Experimental group 7(a), (E)

	monthly test before study	pre-test	class test-(1)	class test (2)	post test
Average % score	26.5	13,9	31.8	40,5	27.3
Pass rate (%)	32,5	5,4	42,9	61,1	36.8
Failure rate (%)	67,5	94,6	57,1	38.9	63,2
No. of candidates who wrote the test	18	18	21	18	19

Table (a)

[Table (b) appears on the next page]

Control group 7(b), (C)

	monthly test before study	pre-test	class test-(1)	class test-(2)	post-test
Average % score	31,5	16,7	45,5	37,9	15,2
Pass rate (%)	48,9	11,8	63,6	57,1	18,2
Failure rate (%)	51,1	88,2	36,4	42,9	81,8
No. of candidates who wrote the test	17	17	11	14	11

Table (b)

Data given in the above tables indicate that C performed (pass rate 48,9%) better than E in the monthly tests (pass rate 32,5%) before the research study. For the pre-test and class test (1) this tendency continued. For class test (2), E (pass rate 61,15%) showed an improvement over C's performance (pass rate 57,3%). For post-test, E (pass rate 36,8%) excelled over C (pass rate 18,2%). Also E recorded an improvement over C in terms of the average percentage scores for these two tests (class test (2) and the post-test). However, an interpretation of the test results needs to consider the following factors. Although, the year started with 25 pupils in both E and C, even before the commencing of the research study these numbers dropped to less than 20 in both the classes (cf. sec. 3.6 & 3.8.2). [The average numbers of pupils

who wrote the monthly test before the research study in E and C were 18 and 17 respectively (cf. Tables (a) and (b)]. Out of these a consistent group of pupils attended the classes during the research study and wrote the tests in E [Table (a)]. But in C, there was a variation in the number of pupils who wrote the different tests. But all the students who wrote the class tests and the post-test, with the exception of one, wrote the pre-test. 17 pupils wrote the pre-test, 11 class test (1), 14 class test (2), and 11 the post-test. Hence the pupils who wrote the test each time were taken as a convenient sample of the control group (Cohen and Manion, 1989). So the test results are regarded as representing the mathematical achievement of the pupils in C. Details of these tests and the summarisation of their results follow.

Before the research study, both E and C had done two class tests. The same question papers and the same timings were used for administering both these tests. At the commencing of the research study both E and C wrote the pre-test (cf. Appendix 1) This test was aimed at measuring the pupils' (from both E and C) basic understanding of the 'solution of linear equations'. The test was administered concurrently for both E and C at the beginning of the research study. Class test (1) (cf. Appendix 2) was a normal test administered concurrently for both E and C to measure their mathematical development. Class test (2) (cf. Appendix 3) was mainly to evaluate the effectiveness of 'group interaction' on mathematical understanding. A special approach was adopted here. E and C were simultaneously provided with printed worked-out

problems from a topic they had not yet learned, i.e, statistics. Group C was allowed to study the problems individually, while (E) was asked to discuss and understand them in groups. After everybody had completed this stage, a worksheet containing similar problems was given to each pupil in both the groups, to be solved individually. The result of this test is assumed to indicate the effectiveness of the 'group interaction' on mathematical understanding.

The researcher had planned to give the post-test (cf. Appendix 4), on 18-8-1993. But since the teachers' (SADTU) strike started on 16-8-1993 and continued for some time, she could administer it only on 3-9-1993 which, in effect, was equivalent to a 'delayed post-test' or created an examination situation (Pupils normally write mathematics examinations a few days after the end of lessons). Both E and C wrote the same post-test simultaneously.

Although the experimental group (E) learned through group interaction, for all the tests under discussion both E and C were tested in the conventional manner (see sec. 2.9). That is, pupils were tested singly and not in groups.

The average score percentage, pass rate%, failure rate% and the attendance of E and C in these tests are given respectively in Tables (a) and (b).

3.8.3.2.2 Questionnaires

Three questionnaires were given to the pupils in E during the investigation period [cf. Appendix 16(a), 16(b) and 16(c)], on 29-4-1993, 27-5-1993 and 12-8-1993 respectively. The aim was to gather group E's responses with regard to their rating of the level of difficulty of mathematics, their outlook on its importance as a learning subject and whether they liked it or not (cf.sec. 3.8.3.1). Simple and familiar terms were used in the questionnaires after having considered the inadequate language accomplishment of the pupils. As far as possible, questionnaires were open-ended in order to obtain a variety of views and responses. The following Tables (c), (d), (e) and (f) give a summary of the responses. The data in each category is recorded as a percentage of the total number of respondents from E. E has recorded a consistency in the number of respondents to the questionnaires. Consequently the interpretations based on the data given in Tables (c) to (f) are taken to be valid.

[Table (c) appears on the next page]

Difficulty level in mathematics.

	Questionnaire (1)	Questionnaire (2)	Questionnaire (3)
very easy	5.3	8.3	8.3
not very easy	10,5	33,3	33,3
manageable	15.8	8,3	41,7
difficult	68,4	41,7	16,7
not sure	0	8.4	0

Table (c)

Extent to which mathematics is liked

	Questionnaire (1)	Questionnaire (2)	Questionnaire (3)
very much	21,1	9,1	16,7
not very much	26,3	18,2	41,7
like it	21,1	72,7	33,3
do not like it	21,1	0	8,3
not sure	10,4	0	0

Table (d)

Importance of mathematics

	Questionnaire (1)	Questionnaire (2)	Questionnaire (3)
very important	57,9	66,7	45,5
not very important	0	0	0
important	31,6	33,3	54,5
not important	0	0	0
not sure	10,5	0	0

Table (e)

Their responses towards group work were also gauged.

	Questionnaire (1)	Questionnaire (3)
like group working	78,9	84,6
do not like group working	21,1	15,4

Table (f)

Data given in Table (c) indicate that pupils in group E found mathematics easier during the course of the research study; only 16,7% of the respondents indicated that it (mathematics) was

difficult in the end of the study against 68,4% towards the beginning. For 41,7% it was manageable towards the end against 15,8% at the beginning. Table (d) shows that there was an improvement in pupils' (E) attitude towards mathematics in terms of their liking the subject. 33,3% of the respondents liked it towards the end of the study against 21,1% at the beginning. Only 8,3% did not like it at the end against 21,1% at the beginning. However 41,7% indicated that they did not like it very much in the end against 26,3% in the beginning. Data in Table (e) indicate that many pupils in E were aware of the importance of the subject. Table(f) indicates pupils' improved appreciation of the group work; 84,6% preferred it (group work) towards the end as against 78,9% towards the beginning.

Many pupils expressed their wish to continue learning in the new ways. The reasons for appreciating working in groups were chosen as "you will get a chance to talk to your friend about your thinking and understanding so that you can together try new ways of solving the problems", by 70.8% of the pupils and "you can ask your friend to help you with solutions if you find the problem difficult", by 29,2 % of them (Questionnaire 1). This had agreed with the findings based on their personal diaries; 73,3% of the pupils initially expressed the view that the "new teaching methods" were helping them to gain a better understanding of mathematics, and 26,7 % responded to the contrary. Towards the end this changed to 80% and 20% respectively.

Reasons given by them for appreciating group learning were: "because when I learning in group I find more education", "because

in group learning I find maths very easy", "because it is improves my mathematical understanding", etc, (pupils' diaries).

3.8.3.2.3 Discussion between the non-participant observers and the researcher

The following ideas evolved during the discussion between the researcher and the non-participant observers.

(i) Initially pupils had difficulties in solving mathematical problems through group discussion. They were inclined to copy the answers or the steps from their friends without trying to get an explanation for them. Also, at that time very little talking (discourse) was taking place among the group members. At this stage the researcher illustrated in the classroom an imaginary mathematical discussion (see Appendix 5). This illustration also elaborates the researcher's social constructivist teaching methods she followed in her experimental classroom (E). The researcher had to try hard to make the pupils talk in the classroom. However, changes occurred very gradually.

(ii) Towards the end of the investigation, pupils' ability to talk had developed, although not appreciably. They gained confidence in talking to the teacher even in imperfect English. This was reflected in the following

comments made by one of the observers: "The researcher is definitely putting across what she intends and she tries very much to be well-disposed towards the pupils...There is a sign of improvement in solving problems where the value of x as a subject of the formula should be identified" [Observer(a)]. Observer (b) also expressed similar views.

3.8.3.3 Conclusion of the research study

The conclusion of the research study in standard 7 was based on its goal (cf. 3.8.1) and the summary given in sections 3.8.3.2.1 and 3.8.3.2.2.

(i) The experimental group 7(a), achieved progress in their mathematical understanding towards the end of the research study which had become evident from their improvement in terms of average score rate and pass rate over the control group 7(b) which had performed better than 7(a) before and at the beginning of the research study. This shows that the social constructivist teaching approaches envisaged and employed by the researcher had a positive effect on the mathematical achievement of the pupils in 7(a). Also exposure to the social constructivist teaching approaches developed a positive attitude and outlook towards mathematics in 7(a).

(ii) The majority of the pupils in 7(a) appreciated the 'new teaching approaches' and the groupwork in mathematics.

CHAPTER 4

DISCUSSION OF THE RESULTS

4.1 INTRODUCTION

The pilot study gave some support to the researcher's convictions about the social constructivist instructional approaches in a mathematics classroom (cf. sec. 3.7.3). The research study showed that a social constructivist process approach in mathematics teaching had a positive effect on the mathematical understanding of the pupils in standard 7(a). The major question the researcher now addresses is how far these results could be taken as true in relation to the research procedures followed (cf. Ch. 3).

4.2 DISCUSSION OF THE PILOT STUDY

The pilot study and its results are now analyzed and evaluated in the perspective explained above. The study was aimed at evaluating the effect of the social constructivist process approaches in both content presentation and teaching on pupils' mathematical understanding and development. Through subsequent analysis the researcher substantiated her presentation of the topic 'factorisation' in 8(b) in the social constructivist fashion.

4.2.3 How a process approach was effected in content presentation

The researcher has explained in sections 3.7.1.2.1 to 3.7.1.2.3, how she implemented the social constructivist process approach both in content presentation and teaching approaches for her pilot study. Through these approaches her aim was to make sure that the pupils were not given the concepts as finished products. On the other hand, they were given opportunities to construct knowledge working on appropriate problems through social interaction and teacher's mediation. They were encouraged to develop conjectures, carry out thought experiment, discuss, communicate, justify and refute following the social constructivist instructional principles (cf. sec. 1.5.3). Special care was taken, based on their rural background, in providing pupils with the right mathematical context and mediation so that they could develop the word meaning and the mathematical meaning contextually.

4.2.3.1 How the topic 'factorisation' was presented as a process

Factorisation of an algebraic expression implies that it is possible to write an expression as the product of factors. The researcher introduced the term 'factors' by capitalising on the pupils' knowledge of the area of a rectangle (area = length X breadth). Through the introductory problems (Worksheets 1 and 2, cf. Appendix 6 and 7) the pupils developed the knowledge that a number could be written as the product of two numbers and vice-versa in numerous ways. This knowledge was then used in

introducing the knowledge about algebraic factors. If the length and the width of a rectangle are (2x + 3) and 6x respectively its area is $12x^2 + 18x$. So it followed that $12x^2 + 18x = (2x + 3)\ 6x$ and $(2x + 3)\ 6x = 12x^2 + 18x$ (Worksheet 3, cf. Appendix 8). Through emphasising the forward and backward relationship between the area (expression) and the products (factors), the idea of equivalency between an expression and its factors was developed (Worksheets 3 & 4, cf. Appendix 9). In Worksheet 5 (cf. Appendix 10), pupils were asked to express algebraic expressions, given as areas of rectangles or squares, as the product of factors (length and breadth). In Worksheet 6 (cf. Appendix 11) the concept 'factorisation' was defined thus: "writing an expression as the product of factors is meant by factorisation". The concept 'factorisation' was introduced (named) only after the learners had understood its meaning and the process. Different types of factorisation were then dealt with. Factorisation by 'taking the highest common factor out' was introduced through problems in which pupils were asked to factorise an expression in as many ways as possible and then look for the case in which the highest factor appears (Worksheets 6 & 7, cf. Appendix 12). Once they understood these methods they could develop by themselves the objective methods of factorisation. Factorisation by 'splitting the middle terms' (Worksheet 9, cf. Appendix 14) was introduced through 'factorisation in groups' (Worksheet 8 cf. Appendix 13). Each piece of mathematical knowledge was formalised only after developing its process. The above explanation substantiates the researcher's attempt to present the topic of 'factorisation' following the social constructivist theory she envisaged.

4.2.3.2 The process approach in teaching

The crucial factor of a social constructivist process approach in teaching is that of exploitation of social interaction to develop mathematical understanding in the pupils. This was accomplished through the formation of groups within E (the experimental group) as envisaged by the researcher in section 3.8.3.1.2. The process approach that was adopted in teaching 'factorisation' was in line with the process approach in content presentation. Pupils were initially asked to do the introductory problem in 'factorisation' in their own groups without the teacher's help. The teacher/researcher assumed for the time being that the pupils would be able to handle the simple problem themselves. But she very soon realised that they were unable to form their own conjectures to develop their own line of thinking and argumentation in order to solve the problem reasonably. They developed difficulties in changing their mind-set and exhibited conceptual lagging and serious mathematical misconceptions. They also showed difficulties in changing their mind-set that had been attuned to the prevailing 'traditional' teaching/learning set-up. At this point the researcher was convinced of her research assumption: pupils' group interaction and the teacher's facilitation are not sufficient for adequate mathematical conceptualisation in the Ciskeian classroom (cf. sect. 1.6, 1.6.1). The researcher then started teaching the pupils through social interaction and mediation as envisaged by social constructivism (section 2.6). The researcher consequently effected some changes in her teaching methods: she appropriated

the teaching methods to make use of the ZPD. Depending on the nature and the level of difficulties of the problem being dealt with, she changed the structure of the group. During the introductory stage of developing the process of concept formation, the class was taken as a single group and discussion took place between the pupils and the teacher. At this time the researcher tried to encourage them to get involved in classroom discussion. The researcher gave hints or suggestions which could develop into a sudden fusion of understanding (Vygotsky, 1934/87). While factorising an algebraic expression by splitting the middle term a hint like "factorise in groups" (Worksheet 8) could effectively fuse a new knowledge of factorisation. After developing the process, pupils were given problems to be solved in their small groups through discussion and argumentation. But there was no guarantee that by that time each group would be able to solve problems successfully through group discussion. Many groups had to be helped out by the teacher.

4.2.3.3 On observations

The researcher and the observers had noted the pupils' initial inability to handle problems by themselves in groups. The idea to effect changes in the structure of the group evolved through discussion between the researcher and the observers.

4.2.3.4 Pupils' responses

Questionnaires were used to obtain pupils' responses to the new problem-solving approaches in the mathematics classroom. These responses supported the change effected in the group approaches mentioned in section 4.2.3.2

4.2.3.4.1 On aspects tested through questionnaires

The major aspects tested through questionnaires during the pilot study were whether pupils had difficulty in learning mathematics, to what extent they liked or disliked it, their outlook on the importance of the subject, whether they liked or disliked the new problem-solving approaches in the mathematics classroom, the reasons for their like/dislike, and whether the new problem-solving approaches helped them acquire better mathematical understanding. Responses to the first three aspects are taken to be indicative of pupils' mathematical development during the pilot study and others of their appreciation of the process-approach in mathematics instruction employed in the study. On the basis of the above responses the researcher tried to measure the pupils' development in mathematics and their appreciation of the new process approaches.

4.2.3.5 Evaluation of pupils' development in mathematical thinking

Direct evaluation of the pupils' development in mathematical thinking was not an easy task. So the researcher had to adopt

'proxy variables' (Cohen and Manion 1989, 204) for the purpose. That means that the test results and the pupils' responses were accepted as 'proxy variables' indicative of developed mathematical understanding. However, the results of the post-test or the pre-test of the pilot study could not substantiate pupils' development in mathematical thinking on account of problems already explicated in section 3.7.2.1.

4.2.3.6 Results of the pilot study

The researcher has so far substantiated in this section that she had carried out the pilot study in line with the social constructivist theories she had envisaged. That is, she had effected the social constructivist process approach in the presentation and teaching of 'factorisation' in standard 8(b) class which she chose for the pilot study, and evaluated its effect on 8(b) pupils' mathematical development, although validity of the results could not be claimed (cf. sec. 3.7.2.1).

4.3 ON THE RESEARCH STUDY

The aim of the research study was not different from that of the pilot study (cf. section 3.5). In this section the researcher evaluates the research approaches in line with the research goals.

4.3.1 About the process approach to teaching the topic 'solution of linear equations'

The social constructivist process approach in teaching the topic 'solution of linear equations' in standard 7(a) (E group) has been

discussed at length in sections 3.8.3.1.3 to 3.8.3.1.5. The approach adopted was similar to that of the pilot study. Hence, the topic 'solution of linear equations' was introduced to the pupils after they had mastered its mathematical and linguistic meaning. This implies that pupils should know the interrelation or the interplay between the variables and the numbers before they learn the concept of 'solution'. They should also know that a letter is a symbol standing for a number or numbers. This understanding was ensured through adopting a zig-zag approach as explained in section 3.8.3.1.3 in the presentation of the content. This provided them with an opportunity to understand the relationship between the letter and the number in the context of an equation. Once pupils had understood this relation (process of 'solution') the word (concept) 'solution' was introduced. Afterwards, different ways of solution were introduced through problems. Again, each method was put to them not as a product formalised by other people (the product approach explained in section 1.1), but they (pupils) were helped to develop first the process of solution and subsequently to objectify that process. At this stage, after solving different equations like $5 = 3x - 1$, pupils could develop the knowledge that subtraction of the number '1' on one side of the equation is equivalent to adding '1' on the other side. Later on this understanding was extended up to the level of variables. The researcher had also introduced complex problems and real world problems in which they could translate geometrical situations into algebraic relations, enabling them to solve them algebraically.

4.3.2 On teaching approaches to 'solution of linear equations'

Teaching approaches employed by the researcher with group E to teach 'solution' were in line with the social constructivist process approach (cf. sec. 3.8.3.1.4). In fact, in principle the researcher was following the approaches she had followed in her pilot study with 8(b), after effecting the initial changes (section 4.2.3.2). The teacher introduced the 'solution of equation' and different methods of solution after developing their processes and meanings (word meanings and mathematical meanings) (cf.sec. 3.8.3.1.3). During the development of the process the whole class was considered as a group and the teacher motivated the pupils to talk as much as possible. Only after developing the process of the concepts were they named. Subsequently, the objectification of these processes was developed after which pupils solved problems in individual groups through discussions. Some changes were effected to achieve more process approaches in teaching in the research settings; much use of the blackboard and the replacement of the worksheets by note books. Copies of printed problems and questions were given to each pupil. This allowed adequate flexibility and variability in classroom discussion depending on the pupils' level of understanding and difficulty experienced.

4.3.3 On observation

The main enquiry technique the researcher employed in the study was observation. Details of the enquiry processes are given in

section 3.7.1.3.3. At the beginning, the observers and the researcher had agreed on a time-table to effect class visits in E. Observer(a) was the main observer and observer(b)'s views were sought only when a third opinion was needed. Since both the observers were working in the same school this exercise could be implemented smoothly. After the class visit there were formal and informal discussions among the researcher and the observers. The researcher's diary was a strong research instrument in the process; it was a reflection of her classroom experiences. It indicated mainly her observations about pupils' appreciation of and reactions to the new problem-solving approaches and comments about their mathematical abilities based on her classroom observations and the classwork. The non-participant observers observed the researcher's teaching approaches and the pupils' observable reactions and abilities. Their main concerns were:

-is the researcher/teacher, putting into practice what she really intends?

-is the researcher achieving what she really intends?

-the pupils' approaches, appreciation of and reactions to group working; are they capable of following the social constructivist pattern of group discussions?

-views on pupils' development or regression while following the problem-solving approaches in groups.

-any limitations in the investigation processes.

-remedies or alternative suggestions for future actions.

Initially the non-participant observers and the researcher had agreed that the pupils in E had problems in following the social

constructivist pattern of group discussion and argumentation. The researcher had effected action immediately to take care of this difficulty as explained in section 3.8.3.2.3. As the research study progressed, observer (a) commented that the researcher was putting across in the 7(a) classroom what she really wanted. He also indicated about the positive effect of the new teaching approaches on pupils' mathematical understanding and their strong inclination to the group problem-solving approaches. Pupils' responses to these observations were then sought. All these observations and the responses were subjected to the evaluation process.

4.3.4 Evaluation of pupils' development in mathematical understanding

A crucial part of the investigation was to evaluate pupils' mathematical development. Understanding or development in understanding cannot be directly measured. So the researcher had looked for "proxy" methods (Cohen and Manion, 1989, 204). This means that pupils' test results and responses were accepted as "proxy" in measuring mathematical development. A pre-test was given to both E and C groups. Subsequently both the groups were taught by the same teacher the same topic, in E following the social constructivist process approaches as envisaged, planned and designed by the researcher (sufficiently explained by now), and in 7(b), that is C, the current 'traditional' approach as explained in the text book, "Just Mathematics" (1992 syllabus). This book was used because this was the only standard 7 mathematics text

book in 'new syllabus' available at the school. The researcher believed that the mental orientation of the pupils to mathematics at standard 7 level could be evaluated through estimating aspects like their difficulty level in mathematics, the extent to which they liked it as a learning subject and their outlook on its importance. These aspects were estimated thrice using questionnaires; at the beginning (29-4-1993), during the course (27-5-1993) and towards the closing of the investigation (12-8-1993). The researcher considered class tests as a powerful method for estimating the pupils' mathematical achievement, a measure of their mathematical development. A pre-test, two class tests (during the course of the study) and a post-test after the study were given to E and C. Each time the tests for the two groups were identical and were always administered simultaneously to both the groups.

4.3.5 On pupils' responses

Pupils' responses in group E were gathered using questionnaires. Similar aspects were estimated using questionnaires both in the pilot study and research study which were explained in section 3.8.3.2.2. An additional enquiry method used for the research study was pupils' diaries. In their diaries pupils were asked to make remarks and comments on all the aspects they had covered in the questionnaires. They were also asked to express their opinion in selecting the new problem-solving approach as a future classroom method. However, some of the responses expressed by the pupils were found to be invalid. A few responses are quoted from

their diaries: "so less teach more than take care and I think all children we must learn", "No I think to the teacher to better understanding intome in the teacher" . They were responding to the item "Do I appreciate the new problem-solving approaches in the maths classroom?". To the item "Through group discussion, are my friends helping me to have a better understanding of mathematics?", one responds as "no, I help to the teacher to better understanding get into me in the teacher discussion as groups, not better understanding for me". Incidentally, these responses strongly substantiate the researcher's explication of the difficulty of the pupils from the rural area of Ciskei in expressing themselves in coherent English.

4.3.6 On the course of the research

In this section the researcher focuses on the course of the research in order to substantiate the style she followed in the study (cf. sec. 3.4). She finds it necessary to elaborate further on pupils' erratic attendance in this context. In section 3.8.2 the researcher has explained the problem of time for her research. In the summary of the research study (cf. sec. 3.8.3.2) she further elucidated the causes for the pupils' erratic attendance. The research study started on 28-4-1993 and progressed smoothly with few hitches to the term ending (20-6-1993). But when the school reopened on 21-7-1993 no student turned up for school. This situation prevailed for a week and the researcher could only continue with her study starting on 27-7-1993. At this time the researcher had to spend some time in

revising the topics and getting the pupils back to their original mood at the time of closing the school for holidays. During this time of the year pupils attended the school very irregularly and the standard 7 pupils were not an exception. However, on whichever days the researcher taught E, she taught C also; such was her teaching time table. Absconding classes and holding meetings as they pleased were the order of the day among the pupils. Towards the second week of August the researcher, in consultation with her supervisor, prepared the post-test with the intention of administering it on 18- 8-1993 while the SADTU's threat of strike was hanging in the air. And strike they did, so that she could not give the post-test as previously planned; it was conducted after the strike was over on 3-9-1993.

Time constraints within which the researcher carried out her investigation had affected the smooth execution of her plans and actions. Consequently meetings with the observers had to be rescheduled. They met weekly or fortnightly depending on the number of teaching periods the researcher obtained per week.

4.3.7 On the summary of the research study

In this section the researcher aims at analyzing the research result in section 3.8.3.3 in the context of its proposed objectives.

4.3.7.1 On tests and examinations

In section 3.8.3.2.1 the researcher has explained how far the results of tests written by the pupils in E and C can be taken to

represent their mathematical achievements. At the beginning of the year, the average percentage score of group C in the monthly test of mathematics (31.5%) was higher than that of E (26,5%). At this time both the groups were being taught mathematics using the existing methods following the text book "Just Mathematics" (1992 syllabus). C bettered E for the pre-test: C scored 16,7% and E 13,9 %. For test (1); C 45,5% and E 31,8%. For class test(2), which was also intended to test the effect of group interaction on mathematical understanding, this trend suddenly changed: C scored 37,9% and E 40,5 %. This trend prevailed for the post-test also: C scored 15,2% and E 27,3%. In terms of the pass rate also E bettered C for both these tests [class test (2) and the post-test]. The upward trend of E compared to C in the post-test has important pedagogical implications in this situation. The researcher has already explained in section 3.8.3.2.1 about the delay caused in administering the post-test to both E and C. This enabled the researcher to give them a delayed post-test or a test in the examination situation. A trend particularly observed in the Ciskeian schools is that pupils perform poorly in the examination compared to their monthly test. This trend is seemingly caused by the existing teaching approaches in mathematics. Teachers following the available textbooks, train pupils to master concepts without explaining their processes. This simply promotes memorisation of the concepts without understanding their meanings. Concepts which are simply memorised cannot be applied meaningfully to problems and fade away from the mind. So pupils perform poorly in examinations or delayed tests. This argument is seemingly capable of supporting the other side of

the reality: E group performed better in the delayed post-test because the teaching system to which they were exposed enabled them to process the construction and the objectification of concepts. This resulted in their mental development which enabled them to solve the problem meaningfully. This development is once again shown in the November/December examination results in mathematics (E, 21,1% and C, 14,3%). All these developments support the effectiveness of social constructivist teaching approaches in E.

4.3.7.2. On Questionnaires

The researcher now turns to the interpretation of the data given in Tables (c), (d), and (e) in section 3.8.3.2.2. After the research study a good number of pupils from E found mathematics easier to learn; 68,4% regarded mathematics as difficult before the study but only 16,7% maintained this stand towards the end. Only 15,8% of the pupils saw mathematics as manageable at the beginning, but 41,7% at the end. 5,3% saw it very easy before the study, but 8,3% towards the end. 21,1% did not like mathematics before the study, but only 8,3% towards the end. Also 33,3% pupils liked it towards the end against 21,1% at the beginning.

The majority of the pupils in E had been aware of the importance of mathematics as a learning subject throughout the study [Table (e)]. This implies their attitude towards mathematics as an achiever subject. They saw the importance of the subject as the key to higher studies. The researcher's opinion about her

teaching approaches was strongly supported by group E. 78,9% of the pupils showed at the beginning, through their questionnaires, that they liked the new teaching approaches (group work), and 84,6% towards the end. Similar responses were obtained from their diaries: 80% of the pupils expressed the view that the "new teaching methods" were helping them gain a better understanding of mathematics in the end as against 73,3 % at the beginning. All the pupils (100%) would like to continue their future learning in this way.

4.3.8 Other questions of internal validity

4.3.8.1 History

The researcher had already made mention of the political events and other disturbances which had affected negatively the progress of her classroom investigation (cf .sec. 3.6). But since both E and C belonged to the same school and the same location, this effect was taken to be equal for both the groups.

4.3.8.2 Testing

Care was taken not to sensitise the pupils during the pre-test period. The pre-test and the whole research exercise were introduced to the pupils as part of a research in connection with Rhodes University. Special care was taken to inform the pupils both in E and C that the test results would not affect their 'year mark' or promotions in any way.

4.3.8.3 The question of rigour

The question of rigour is now handled within the limit of the research design. The research methods were mainly observational. The only mechanical method was the use of an audio tape to interview group E. But since the researcher could not finish interviewing all the students due to problems explained in section 3.6, the information obtained in this manner was not used in any evaluation.

4.3.8.3.1 Respondent validation

The ideas generated through discussions among the researcher and the observers were put across to the pupils to elicit their responses as explained in section 4.3.3. These responses were compared with their responses in their diaries.

4.3.8.3.2 Triangulation

The researcher chose two non-participant observers to overcome any bias in her observation. All actions during the course of the pilot study and the research were implemented based on the ideas generated through her discussions with them. Pupils' responses to these actions were obtained using questionnaires, which were then compared with their responses in their diaries.

4.3.8.3.3 Sophisticated rigour

The researcher has shown all relevant data about which mention has been made in this report "thus opening up the possibility of replication or the generation of alternative interpretations of the data" (Janse van Rensburg, 1992, pers. comm).

4.4 ON EXTERNAL VALIDITY

4.4.1 On sampling

Standard 7 pupils at Ilanga High School, Healdtown, taken as the sample group for this research, are assumed to be representative of the Ciskei rural standard 7 population because of the following reasons: Healdtown, a Black township in which Ilanga High School is situated, can be assumed to be similar to any other rural community in the Ciskei. The school, as in any other rural part of the Ciskei, admitted all the pupils in the community irrespective of their age and academic background. In this context, the history of the school is briefly mentioned.

The school originally known as Healdtown High School was one of the most prestigious Black educational institutions in South Africa until it was burned down apparently as a result of pupils' riot actions in 1976. Ilanga High School was set into function in the following year to meet the educational needs of the pupils in the neighbouring location under the Ciskei Department of Education. However towards 1993 as a result of a very strong move

to revive the old 'Healdtown', Ilanga High School was phased out and it would be again known as Healdtown High School from 1994 onwards.

Ilanga High School represents the 'dark period' in the history of Healdtown High School. The glory and the academic excellence the Healdtown High School enjoyed before 1976 and which it awaits from 1994 have no parallel during this period. The argument is that during the period in which the researcher conducted her research, the school was like any other rural Black school in the Ciskei.

Pupils' admission to the school during this period was not controlled by any admission policy. When the school opens, pupils (including the repeaters) are registered after paying the school fees. Later on, in standard 7, they are divided into different classes following the order of registration. So each standard 7 class represented a randomised group.

The standard 8(b) class was chosen for the pilot study for the following reasons:

(i) There was not a third standard 7 class, in addition to the E and C groups in the school.

(ii) The standard 8(b) class, being the non-science class was taken to be not much different from any standard 7 class because the pilot study was conducted towards the beginning of the school year. It is on the basis of this argument that the researcher has used the result of

the pilot study for her research with standard 7 wherever applicable .

4.4.2 On generalisation

Based on the above arguments the researcher believes that the research result is applicable to all Ciskeian rural standard 7 classrooms.

4.5 PRE-TEST AND POST-TEST MEASUREMENT

The researcher now analyses the strength of her tests in relation to the conclusions and implications based on their results.

4.5.1 Pre-test

Questions covered the fundamental concepts in the 'solution of linear equations' the pupils had learned in standards 6 and 7 (covered by that time). The aim of the test was to get an idea of their level of understanding in these concepts.

The knowledge tested was as follows:
In question 1, the use of letters in a general context in four operations; in question 2, the equivalency of variables; in questions 3, 4 and 6, to see variables as representing quantities in real life situations; questions 5 and 7, use of variables as particulars; question 7, generalisation of a relation (see pre-test questions in Appendix 1).

Percentage scores for both E and C are shown in Tables (a) and (b) (cf. sec. 3.8.3.2.1). Pre-test results indicated that pupils (both in E and C) had very limited knowledge of the basic concepts of solution.

4.5.2 Class test (1)

Test (1) was aimed at testing the pupils' mathematical development on the following aspects.

- knowledge of procedures for manipulating geometrical situations into algebraic equations and solving them (questions 1 and 2);
- knowledge to solve problems involving mathematical situations to be manipulated algebraically (questions 3-5) (see Appendix 2).

4.5.3 Class test (2)

Class test (2) was intended to test the effect of group interaction on mathematical understanding. The knowledge tested was from statistics; knowledge on 'arithmetic mean' (they hadn't learned this topic before). Problem 1 explained how to calculate the arithmetic mean of cricket scores. Problems 2, 3 and 4 were partially done for them (enough hints were given to complete the solution). Group E members were allowed to understand the solution of these problems through group discussion whereas group C members worked on them individually. When everybody had completed understanding the problems, each pupil from both E and

C was given separate worksheets having three problems similar to the ones they had solved. The aspect tested was mainly the effect of group discussion in developing pupils' mathematical knowledge; pupils' ability to calculate the arithmetic mean of the numbers of students in various classes in a school (question 1) and to apply this knowledge in calculating the total price and the individual score (questions 2 and 3) (see Appendix 3).

4.5.4 Post-test

The post-test tested the pupils' ability to:

- manipulate a purely mathematical context into an algebraic equation and to solve it (question 1);
- develop geometric situations into algebraic equations and solve them (questions 2 and 5);
- manipulate real life problems involving money transactions algebraically and solve them (questions 3 and 4);
- develop real life problems involving pupils' ages and number of cattle into algebraic form and solve them (questions 6 and 7);
- solve problems handling numerical quantities and variables simultaneously (questions 8-13). (Post-test question paper given in Appendix 4).

4.6 ON CONCLUSIONS OF THE RESEARCH STUDY

The summary of the research study shown in section 3.8.3.3 led to the following conclusions:

Social constructivist process approaches in the content presentation and teaching of mathematics can produce a positive effect on the mathematical development of pupils in a Ciskeian classroom. Pupils in this region should be taught mathematics through social interaction as envisaged by the researcher for proper understanding. Since language is a decisive factor in the understanding of mathematics, special care should be taken by the teacher to develop both the word meaning and the mathematical meaning before presenting a concept in the medium of instruction.

CHAPTER 5

SUMMARY AND RECOMMENDATIONS

5.1 IDENTIFICATION OF THE PROBLEM

The researcher has been serving as a mathematics teacher in different parts of Africa for the past 15 years. Her most recent posting has been in Ciskei. Her classroom experiences as a mathematics teacher in these countries are similar; pupils exhibit conceptual difficulties contributed by their inadequate conceptualisation of the related basic topics, the blame for which could be mainly attributed to the teaching/learning approaches dictated by the text books and the teachers' beliefs about knowledge construction. In the Ciskei, this difficulty is reflected in the following record of matric pass rates in mathematics;

Year	**pass rate%**
1988	36.1%
1989	28,3%
1990	12,6%

(Records held by Hlazia In-service Teacher Training Centre)
However the problem of mathematics education is not Africa's exclusive problem.

5.2 DEVELOPMENT OF THE THEORETICAL FRAMEWORK

The researcher, after much thought and reading, felt that a solution to the above problem may lie in an alternative philosophical paradigm of mathematics eduction. Her envisagement of social constructivism, as an alternative philosophy of mathematics education, is explicated in the initial chapters. The results of the present investigation indicate support in favour of the social constructivist approach in Ciskeian classrooms. In the context of the present research and the results thereof (cf. sec. 4.6) the researcher would like to propose the following suggestions and recommendations.

5.3 ABOUT THE RESEARCH TREND

In recent times research in mathematics education has been lacking in philosophical guidance. According to Kilpatrick;

> From the outset, research in mathematics education has also been shaped by forces within the larger arenas of educational research, which roughly a century ago abandoned philosophical speculation in favour of a more scientific approach
>
> (Kilpatrick, 1992, 3)

Consequently philosophical speculation in mathematics education was replaced by scientific consideration of educational psychology, the underlying causes of which are deeply philosophical. The Western philosophers like Plato, Kant and Descartes developed a kind of 'dualism' regarding knowledge construction which raised numerous unanswerable questions in the

minds of common people. As a result educationists failed to see a proper connection between the subjective and the objective levels of knowledge. This apparently resulted in a breaking off from objective considerations or philosophical speculations of knowledge constructions in mathematics eduction and a consequent confinement to subjective constructions. An example is radical constructivism (see its principles in section 1.6) based on Piaget's constructivism which permeates many ensuing educational researches. In this respect radical constructivism can only be considered as a philosophy of subjective knowledge construction although not entirely; Noddings (1990) has termed it as a 'post epistemology'. Social constructivism can be seen as a properly developed philosophy of mathematics and mathematics education because it deals with the construction of subjective knowledge and its development into objective knowledge.

Research based on wrong premises cannot lead to valid results. So the researcher humbly suggests a replacement of the radical constructivist research attempts by social constructivist research in order to obtain valid solutions to problems pertaining to mathematics education in South Africa.

5.4 ON EXAMINATIONS

Mathematics examinations the world over are planned in such a way that the pupils have to master certain examination techniques and skills. Many criticisms levelled against examinations are based on this fact; examinations test mechanically the skills and

techniques and not the knowledge construction. However this criticism needs to be analyzed in the light of the underlying philosophy of mathematics teaching. In the present system, although pupils learn these techniques and skills from the teacher and may pass the examination, they do not fully grasp the basis of mathematics. This may be responsible for the high rate of university dropouts. Pupils who become familiarised with the construction process of knowledge and its transformation into the objective level, following the social constructivist theories (cf. Ch.3), can develop for themselves examination techniques and skills. They can also manipulate the intricacies of the new problem situation in the examinations. In such a system examination techniques and skills are meaningful and productive, and do not represent mechanical, rigid or rote-learning mental tracks.

The Education Advisor, Natal Region, who is incidentally the chief examiner of DET mathematics examination HG paper-1, writes;

> Effective revision is, in many ways, the antithesis of normal teaching. Teaching aims at raising all pupils to an acceptable standard, whereas revision aims at pupils' attaining different levels, according to each ones' effort and talent.
>
> (Armstrong 1993 pers. comm)

The above quotation shows the lack of a proper teaching/learning system. Construction of knowledge and its development into 'different levels' (objective levels) does not take place simultaneously or concurrently with the learning. Pupils are

helped to develop their knowledge to the objective level only during the revision time.

Teaching and learning based on social constructivist approaches gives adequate importance, from its onset, to the transfer of knowledge from the subjective level to the objective level. Through appropriate problem-setting following this approach, mathematics learning can be made easier and more meaningful for the pupils and they can become capable of developing examination techniques and skills themselves.

The innovations to develop a social constructivist mind-set should be initiated and developed at the primary level. It is difficult for a teacher to introduce such a mind-set at a later (high school) stage.

The researcher does not mean that social constructivist teaching approaches can make every mathematics learner a mathematician, but what she contends is that, according to individual potential the pupils' mathematical thinking can develop optimally.

5.5 TO CURRICULUM DESIGNERS

The present syllabus followed by Ciskeian schools has been discussed at length in Chapter 2. This syllabus is apparently not planned in the social constructivist fashion, but follows mainly radical constructivist principles. The fundamental problems in following radical constructivism as the philosophy of mathematics

education are discussed in Chapter 2. The syllabus and the text books are important guiding documents which the teachers often follow blindly. The syllabus, if intended to follow a social constructivist outlook, should be more descriptive; also it should be developed after much thought and discussion in the social constructivist camp.

5.6 ABOUT TEACHERS

The present researcher feels that the professional status of mathematics teachers in the Ciskei is a matter of grave concern. A mathematically strong teaching force, capable of showing innovation and flexibility in the mathematics classrooms, is a primary concern of social constructivism. Teachers' belief systems (philosophies), determined by their own learning methods, the type of professional training to which they are exposed and their professional qualifications play a prominent role in the innovation. Teachers who are less confident in the subject are normally seen as less flexible and react negatively towards changes (Goldin, 1990). Consequently teacher training in the social constructivist approaches should take into consideration not only the social constructivist methodologies but also the development of content knowledge.

However to change the mind-set of thousands of present mathematics teachers in order to accept a social constructivist frame-work is not an easy task. Prawat (1992) explains the difficulties mathematics teachers encounter in adopting a constructivist

outlook. Teachers' organisations and publications which can make substantial contributions in this regard should be encouraged.

5.7 TO TEXT BOOK WRITERS

The researcher has shown in section 2.7.3.2 that many of the mathematics text books available for standards 6 and 7 in South Africa, which are used in Ciskei, simply follow the traditional axiomatic approaches, the residue of 'new' mathematics. Most of the teachers follow the text-book approaches as such in the classrooms. The writing and publishing system of these text books has to undergo drastic changes if it is to follow the social constructivist approaches. Ventures in text book writing based truly on social constructivist principles and methodologies should be encouraged by the concerned departments and publishing agents, ignoring the established monopoly. Education departments should take control of the selection of the required mathematics text books through professional advice, rather than the present system of leaving the selection at the discretion of the teachers in the field who are prone to making ad hoc selection under the persuasive influence of visiting publishers.

5.8 TO PROFESSIONAL ORGANISATIONS

Professional associations can assume a very significant role in promoting the social constructivist innovations and changes. The way in which these associations function at the moment is not very helpful for the rural teachers who really need professional help

and guidance. These associations should do a lot more in instilling professional awareness and consciousness among the rural teachers in South Africa. Conferences and seminars should be made cheaper and more frequent, taking into consideration many more interested teachers. Local associations should make efforts to maximise the teachers' enthusiasm for changes and innovations. Local and national news papers and other popular publications should reserve columns for educational publications which may help to motivate the rural teachers. Presently educational news is obtainable only through educational publications which are read apparently only by teachers who are already professionals. These publications also should rise to meet the present needs of mathematics education.

5.9 TO FUTURE INVESTIGATORS

The present investigator has made only a simple attempt to find a solution to the crisis situation of mathematics education in the Ciskei. A lot more similar attempts should be undertaken by future researchers in order to establish social constructivist teaching approaches as an effective alternative. The researchers should give special attention to research at the high school level, at which very few efforts have been made so far in South Africa. A special and meaningful suggestion in this respect is that any future social constructivist researcher should develop a thorough understanding of the underlying philosophy before implementing it.

A sad reality one should consider while accepting the triune consolidation of social constructivism (quasi-empiricism, socio-historical psychology and conventionalism) is that, the three men who propounded these philosophical views, Imre Lakatos, Lev Vygotsky and Wittgenstein each died prematurely at the time they were seriously engaged in their intellectual pursuit. So unfortunately they could not fulfil their intellectual ambitions as they might have expected, and so one may see imperfections in their theories. Taking into consideration the superiority and genius of their intellectual work it is the duty of the incoming mathematicians and researchers to develop their work to the benefit of the future society.

Classroom teachers can also become researchers or innovators in teaching the topics of their choice in any classroom following the social constructivist teaching approaches. The researcher tested the efficacy of social constructivist approaches to two topics 'factorisation' (pilot study) and 'solution of equations' (research study) in algebra. There is a wide range of other topics and aspects of mathematics which require similar exploration.

REFERENCES

Adler, J. (1991) : Vision and Constraint: Politics and mathematics national curricula in a changing South Africa. In D. Pimm and E.Love (Eds.), Teaching and Learning School Mathematics (pp. 153-170). London: Hodder and Stoughton.

Armstrong, J.P.B. (1993, 24 September) : Education Advisor: Mathematics (Natal Region).

Ball, D.L. (1991) : Implementing the Professional Standards For Teaching Mathematics. Arithmetic Teacher, 39 (3), 44-48.

Booth, L.R. (1984) : Algebra: Children's Strategies and Errors. England: NFER - Nelson.

Cobb, P. (1990) : A constructivist Perspective on Information-Processing Theories of Mathematical Activity. International Journal of Educational Research, 14 (1),67-88.

Cockcroft, W.H. (1982) : Mathematics Counts. London: His Majesty's Stationary Office.

Cohen, L and Manion, L (1989) : Research Methods in Education. Croom Helm Ltd., Beckenham.

Collision, J.(1992) : Performance Assessment to Determine Mathematical Despositions. Arithmetic Teacher, 39(6), 40-47.

Crosswhite, F.J; Dossey, J.A and Frye, S.M. (1990) : NCTM Standards for School Mathematics: Visions forImplementation. In D.C.J Wessels (Ed.), The Role of Language in the Teaching of Mathematics (pp. 4-11). Pretoria: University of South Africa.

Davis, R.B; Maher, C.A and Noddings, N (1990) : Introduction. Journal for Research in Mathematics Education, (4) 1-6.

du Plessis, N.M. (1991): The Mathematics Syllabuses at the Secondary Level in South Africa And France: A comparison. In.(Ed.), Proceedings of the 14th National Convention on Mathematics and Natural Science Education (pp. 61 -63). University of Cape Town: Macmillan Boleswa.

Ernest, P. (a) (1989) : Philosophy, Mathematics and Education. International Journal of Mathematics Education, Science and Technology, 20 (4), 555 - 559.

(b) 1991 : Philosophy of Mathematics Education. London: The Falmer Press.

Eves, H. (1972) : A survey of Geometry. Boston: Allyn and Bacon, Inc.

Fittens, S; de Jager, C and Blake, P (1992) : Just Mathematics Standard 7. Cape Town: Maskew Miller Longman.

Freudenthal, H. (1972) : Recent tendencies in the learning of Mathematics. In W.E Lamon (Ed.), Learning and the nature of Mathematics (pp 11 - 18). Chicago: Science Research Associates, Inc.

Galos, E.B. (1968) : Foundations of Euclidean and Non-Euclidean Geometry. New York: Holt, Rinehart and Winston Inc.

Gildenhuys, D.G; Paulsen R and Silver, E (1991) : New Mathematics in Action Standard 6. Pretoria: De Jager-HAUM Publishers.

Goldin, A.G. (1990) : Epistemology, Constructivism and Discovery Learning in Mathematics. In R.B. Davis, C.A. Maher and N. Noddings (Eds.), Journal for Research in Mathematics Education, (pp. 31-47). Virginia: The National Council of Teachers of Mathematics Inc.

Heyting, A. (1983) : Studies of logic. Amsterdam: North-Holland Publishing Company.

Hiebert, J. (1992) : Reflection and Communication: Cognitive Considerations in School Mathematics Reform. International Journal of Educational Research, 17(5), 439 - 454.

Hlazia In service Training Centre. (1992): Std.10 pass rates for the Ciskei Department of Education, Bisho.

Janse van Rensburg, E. (1992) : Lecture Notes. M.Ed. Research Methodology Course, Rhodes University, Grahamstowm. Personal communication.

Jonassen, D.H. (1991) : Objectivism Versus Constructivism: Do we need a Philosophical Paradigm?. Educational Technology Research and Development. 39 (3). 5 - 13.

Kline, M. (1973) : Why Johny can't add: The failure of the new math. New York, St. Martin's Press.

Krutetskii, V.A (1976) (1976) : The Psychology of Mathematical Abilities in School Children (J.Teller Trans., J Kilpatrick and I Wirszup Eds.). Chicago: University of Chicago Press. (Original work published 1968)

Lakatos, I. (1976) : Proofs and Refutations: The logic of Mathematical Discovery (J Worrall and E Zahar Eds.). New Tork: Cambridge University Press.

Lamon, W.E. (Ed.).
(1972) : Learning and the Nature of Mathematics. (pp. 11 - 12). Chicago: Science Research Associates.Inc.

Leder, G.C and
Gunstone, R.F.
(1990) : Perspectives on Mathematics Learning. International Journal of Educational Research, 14 (1), 105-116.

Lerman, S. (1989) : Constructivism, Mathematics and Mathematics Education. Educational Studies in Mathematics. 20, 211 - 223.

Lindeque, B.R.G. (1988): Conceptualization and Mathematics in the Black Secondary School. Fort Hare Papers. November (1988) vol.9, 1-13.

Luchins, A.S and
Luchins, E.H. (1965) : Logical foundations of Mathematics for behavioural sciences. New York: Holt, Rinehart Winston.

Malan; Naude and
van der Westhuizen,
(1992) Modern Basic Mathematics Standard 7. Cape Town: Maskew Miller Longman.

Marsh, T.A. (1991) : Notices Mededelings. South African Mathematical Society. 23 (3), 136 - 162.

Mcknight, C.C. (1990) : The organisation of curriculum in mathematics. International Journal of Educational Research, 14 (2), 121-135.

Noddings, N. (1990) : Constructivism in Mathematics Education. Journal For Research in Mathematics Education, (4), 7 - 16.

Olivier, A. Murray, H.
Human, P. (1990) : Alternative Approach to Early Mathematics. National Subject Didactics Symposium, (pp.326-365).

Prawat, R.S. (1992) : Teachers' Belief about Teaching and learning: A constructivist Perspective. American Journal of Education, 100 (3), 354 - 395.

Republic of Ciskei
Department of Education,
(1993) : Core Syllabus for Mathematics Ordinary Grade Standard 7. Date of Implementation: Standard 7, 1993.

Scott, L. (1972) : Increasing Mathematics Learning through Instructional Organisation. In W.E Lamon (Ed.), Learning & the nature of Mathematics (pp. 19 - 32). Chiocago: Science Research Associates, Inc.

Secada, W.G. (1992) : Introduction. International Journal of Educational Research, 17 (5), 403 - 406.

Stoker, J. (1992) : Teachers as Learners - Problem Solving and Cultural Psychology. Proceedings of the 15th National Convention on Mathematics and science Education

Strauss, J.P; du Toit, G.F and van Tonder, S.P. (1992) : Mathematics in Focus Standard 6. Pretoria: Acacia Publishers.

Tudge, J. (1990) : Vygotsky, the zone of proximal development, and peer collaboration: Implications for classroom practice. In L.C Moll (Ed.), Vygotsky and Education: Instructional Implications and Applications of Sociohistorical Psychology. (pp. 155 - 174). Cambridge, New York: Cambridge University Press.

van den Berg, D.J. (1978) : A pedagogical study of the Blackman's Mathematical ability. Pretoria:Institute of Educational Research.

von Glasersfeld, E. (1990): : An Exposition of Constructivism: Why Some Like it Radical. Journal For Research in Mathematics Education, (4), 19 - 29.

Vygotsky, L.S

(a) (1962) : Thought Language (E, Hanfmann and G Vakar, Eds. and Trans.). Massachusetts: The MIT Press. (Original work Published 1934)

(b) (1987) : Thought and Language (rev. ed.) (A. Kozulin, d. and Trans.). Massachusetts: MIT. (Original work published 1934)

Wertsch, J.V. (Ed.) (1985) : Culture, Communication and Cognition: Vygotskian Perspectives. New York: Cambridge University Press.

Wilder, R.L. (1972) : The nature of modern mathematics. In Lamon, W.E.(Ed.), Learning & the Nature of Mathematics, (pp. 35 - 48).Chicago: Science Research Associates.

Wood, T; Cobb, P and Yackel, E. (1990) : The Contextual Nature of Teaching: Mathematics and Reading Instruction in One Second-Grade Classroom. The Elementary School Journal, 90 (5), 497 - 513.

APPENDIX I

PRE-TEST QUESTIONS

Standard 7

QUESTION 1

(a) If $y/x = 3$, give different values of y and x.

x					
y					

(2 marks)

(b) If $ab = 24$, give different values of a and b.

a					
b					

(2 marks)

(c) $p + q = 19$, give different values of p and q.

p					
q					

(2 marks)

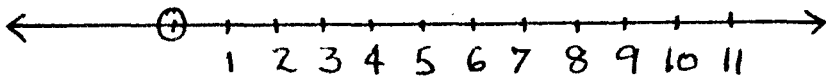
(d) $r - s = 1$, give different values of r and s.

r					
s					

(2 marks)

QUESTION 2

Using the number line answer the questions



(a) x, y, a and b are numbers between 1 and 10.

Will (x + y) be ever equal to (a + b). (3 marks)

(b) Explain your answer in 2(a). (2 marks)

QUESTION 3

In a group of 5 people John is the shortest. His height is 2x. If suko is taller than John by 10 m, how tall is Suko ?

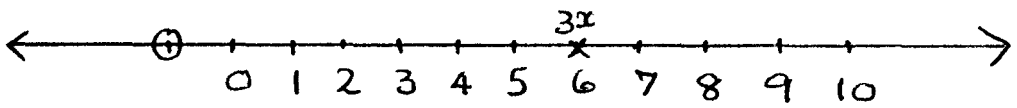
(3 marks)

QUESTION 4

The price of a radio 'Y' is calculated using the formula Y = 2x + 8. If x = 5, what is the price of the radio ?

(4 marks)

QUESTION 5



On the number line 6 is marked as 3x.

How much is x ?. How much is 10x. (4 marks)

[Continues in the next page]

QUESTION 6

From the table given form the function y = ...

x	1	2	3	4
y	2	4	6	8

(marks 4)

QUESTION 7

(a) If a + 4 = 10, how much is a ?

(b) If 4x = 24, how much is x ?

(c) If p/2 = 3, how much is p ?

(d) If x - 3 = 9, how much is x ? (12 marks)

(TOTAL MARKS: 40)

APPENDIX 2

CLASS TEST (1) QUESTIONS

Standard.7

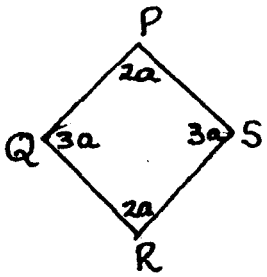
Date: 02-06-1993.

INSTRUCTIONS

Solve the problems individually

QUESTION 1

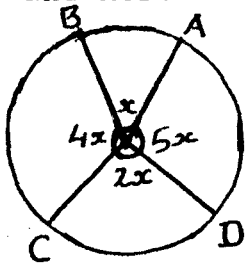
Sum of the angles in a rhombus is 360°. Calculate $\hat{P}$, $\hat{Q}$, $\hat{R}$ and $\hat{S}$? (5 marks)



QUESTION 2

Sum of the angles in a circle around the point O (centre) is 360°. Calculate $B\hat{O}A$, $B\hat{O}C$, $C\hat{O}D$, and $A\hat{O}D$?

(5 marks)



QUESTION 3

Mary is 3 times older than her son Sam. The sum of their ages is 100. How old is Mary?. How old is her son?

(5 marks)

QUESTION 4

2 times a number when subtracted from 112 gives 6 times the same number. Calculate the number. (5 marks)

(QUESTION 5)

Divide 125 into two numbers such that one number is 4 times the other number. Calculate the numbers. (5 marks)

(TOTAL MARKS 25)

APPENDIX 3

CLASS TEST (2) QUESTIONS

Standard 7

Date: 05-8-1993

PROBLEM 1

A cricketer obtains the following scores:

43, 17, 78, 105 and 61 in five innings.

The arithmetic mean of his scores is obtained as,

$$\frac{43 + 17 + 78 + 105 + 61}{5} = \frac{304}{} = 60,8$$

That is, the arithmetic mean of his scores = $\frac{\text{Total scores}}{\text{Number of scores}}$

PROBLEM 2

Johny's score rates for four subjects in his examination are given as,

English	43
Xhosa	48
Mathematics	35
Biology	39

Calculate the arithmetic mean of his scores?

Arithmetic mean =

PROBLEM 3

Jean, Sebe and Nabe are classmates. The arithmetic mean of their age is 12,5. Calculate their total age.

Arithmetic mean of their age = $\frac{\text{Total age}}{\text{Number of pupils}}$

$$12,5 = \frac{\text{Total age}}{3}$$

Total age = 12,5 x 3 =.....

PROBLEM 4

In standard 7 there are three divisions (7A, B and C). The arithmetic mean of the number of students in standard 7 is 39. The number of students in 7A and 7B are 37 and 42 respectively. How many students are there in 7C?

Total number of students in standard 7 = 39 x 3 = 117

37 + 42 + students in 7C = 117

Students in 7c =.....

CLASS TEST (2) WORKSHEET

PROBLEM 1

In a school the number of children in different standards are as follows.

Std.6	51
Std.7	62
Std.8	65
Std.9	78
Std.10	69

Calculate the arithmetic mean of the number of students in the school.

PROBLEM 2

The arithmetic mean of the price of 5 sheep is R75. Calculate the total price.

PROBLEM 3

Simon wrote 3 tests in mathematics. The arithmetic mean of his test scores is 35. For the first two tests he scored 30 and 60 marks. How much did he score for the third test?

APPENDIX 4

POST-TEST QUESTIONS

Date : 03-09-1993

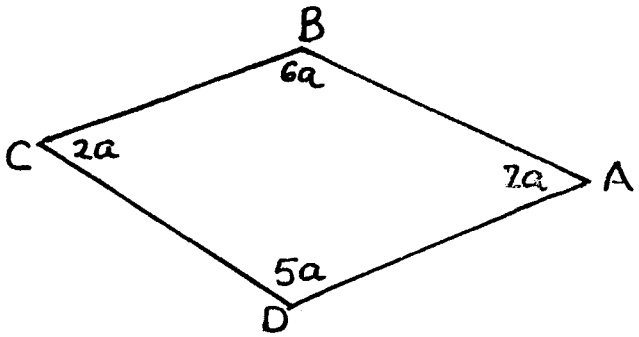
Standard 7

PROBLEM 1

2 times a number when subtracted from 189 gives 7 times the same number. What is that number? (4 marks)

PROBLEM 2

Sum of the angles in the quadrilateral ABCD is 360°. Calculate $\hat{A}$, $\hat{B}$, $\hat{C}$, and $\hat{D}$. (5 marks)



PROBLEM 3

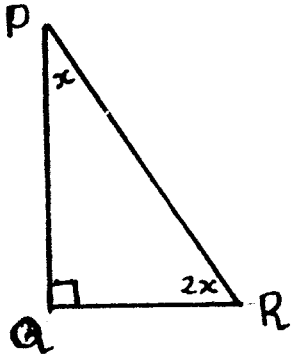
Price of a cricket ball is 4 times than that of a cricket ball. Vicky bought a cricket bat and a ball paying R80. Calculate, (i) the price of the bat (ii) the price of the ball. (5 marks)

PROBLEM 4

R900 is divided between two brothers Sam and Somi. If Sam gets twice that of Somi how much money would each one get? (5 marks)

PROBLEM 5

Sum of the angles in triangle PQR is 180°. Also angle Q is a right angle. Calculate the angles P and R.



(5 marks)

PROBLEM 6

Farmer Bengu keeps sheep and cows. The number of sheep he has is 5 times more than the number of cows. If he has altogether 234 sheep and cows, calculate (i) the number of sheep and (ii) the number of cows, he has got? (5 marks)

PROBLEM 7

Nabe is 6 times older than his daughter. Sum of their ages is equal to 126. How old is Nabe?. How old is his daughter?

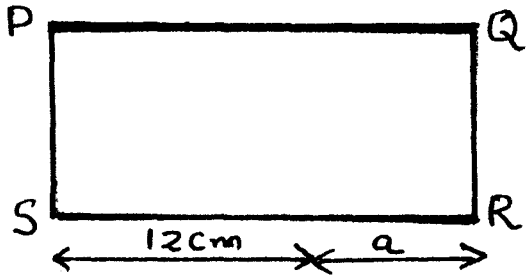
(4 marks)

PROBLEM 8

Price of a ball is R5. If the price is increased by Ra, what is the new price? (2 marks)

PROBLEM 9

Study the following diagram carefully and express the length SR of the rectangle PQRS?



(2 marks)

PROBLEM 10

Price of a book is Ra and that of a pen is Rb. What is the price of 5 books and 3 pens? What is their total price? (5 marks)

PROBLEM 11

Johny is 160 cm high. If his sister is shorter than him by p cm express, his sister's height using p. (2 marks)

(TOTAL MARKS: 44)

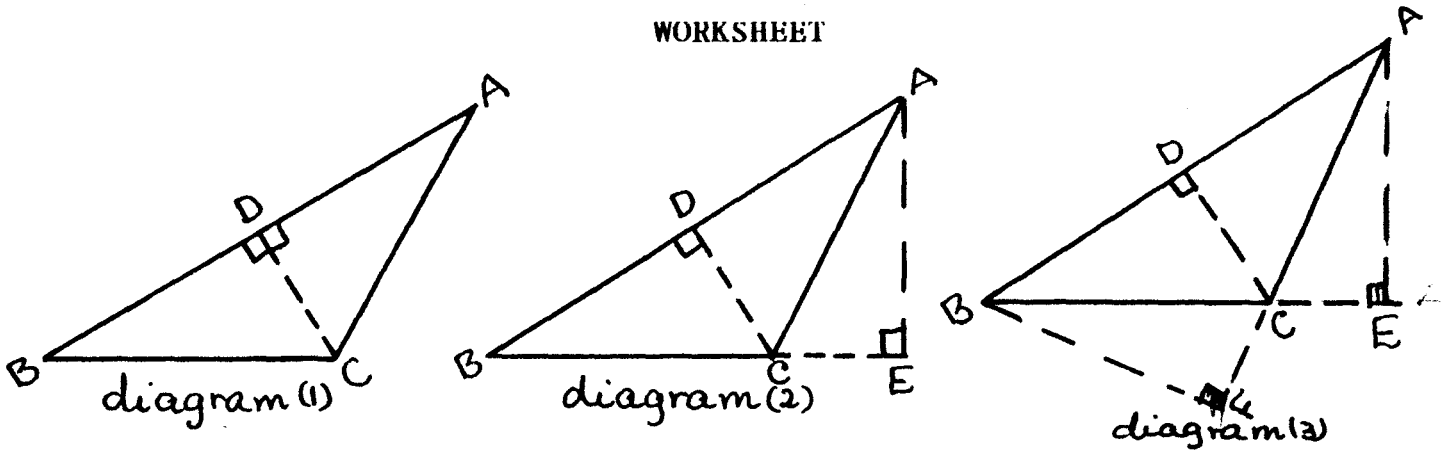
APPENDIX 5

AN IMAGINARY CLASSROOM DISCUSSION

This is the description of an imaginary group discussion taking place in a standard 7 mathematics classroom. The group consists of 3 members; Tom, Willy, and Thandi. The teacher also participates in the discussion whenever it is needed.

{The teacher hands in Worksheets to each member in the classroom which reads as follows}

WORKSHEET



You have learned in std.6 that the area of a triangle is calculated using the formula, area = $\frac{1}{2}$base x height. So the area of the triangle ABC = $\frac{1}{2}$AB x CD or $\frac{1}{2}$BC x AE or $\frac{1}{2}$AC BF. How can you arrive at these three different formulas through group discussion?.

[Tom's group starts discussion.]

Tom: Let us see the formula, area = $\frac{1}{2}$base x height.

Thandi: But do you know what is meant by the base of a triangle?

Willy: I don't know.

Tom: I don't know exactly, but let us try to study the

first formula and the diagram....It seems AB is the base and CD is the height.

Thandi: Why?

Tom:: Because the area is $\frac{1}{2}$ base x height. See the diagram (1).

Willy: Is there any difference between the side AB and the base AB.

Tom: I'm not sure. Let us ask the teacher.

(The teacher is being called. Tom explains the difficulty to the teacher)

Teacher: Good question. Yes it is the side which becomes the base. (The teacher explains using a cardboard triangle that it can stand on each side. So each side becomes the base in turn).

Tom: (To the group) Let us see the second formula $\frac{1}{2}$ BC x AE.

Tandi and Willy together:It is too difficult.

Tom: Let us try.

Willy: What is this AE?

Tnandi: How do you get than CE?. It was not in the original triangle.

Tom: I have no idea. But let us ask the teacher

(The teacher is called. Teacher asks Willy to explain the problem to her)

Teacher: BC is extended and AE is drawn perpendicular to it.

Tom: But why?. This increasing of BC is confusing us. Why can't you say that the area of the triangle is $\frac{1}{2}$ BC x AC?. That is easy.

Willy and Thandi:That is right.

Teacher: But there is a problem. AC is not the perpendicular

height of the triangle when it stands on the base BC.

Willy: Perpendicular height. What does it mean?

Teacher: In the diagram, how much is angle ADC?

Thandi: I have no idea.

Tom: It is 90 degree.

Willy: How do you know that?

Tom: (showing the diagram), see the perpendicular mark. It means 90 degree.

Willy: Oh I see.

Teacher: Do you get the point now?

Thandi: Yes, now I know that CD is perpendicular to AB.

Teacher: Is AC perpendicular to BC.

Willy: No, I don't see that perpendicular mark.

Teacher: So a perpendicular to BC is drawn which is AE.

Tom: So we should now change the original formula as, area of a triangle = $\frac{1}{2}$base x perpendicular height.

Teacher: Exactly. (compliments Tom for his ingenious observation)

Willy: But why is the area not $\frac{1}{2}$BE x AE?

Tom: Is BE the side of the triangle?. BC is the base of the triangle, not BE. BE is only a construction.

Teacher: Now carry on.

Thandi: Now I know why BF is drawn.

Willy: Me too.

Tom: When AC is the base BF is the perpendicular height.

Willy: So area is $\frac{1}{2}$AC x BF.

Tom: So the same formula can be used in three different ways to calculate the area of the triangle.

Willy: Now I follow the formula very well.

APPENDIX 6

WORKSHEET 1

Solve the following problems.

PROBLEM 1

If the area of a soccer field is 6000 sq.m., what are its possible lengths and breadths. (Hint: area = length x breadth)

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PROBLEM 2

If the area of a Rugby field is $7x^2 + 18x + 11$, what are its possible lengths and breadths ?.

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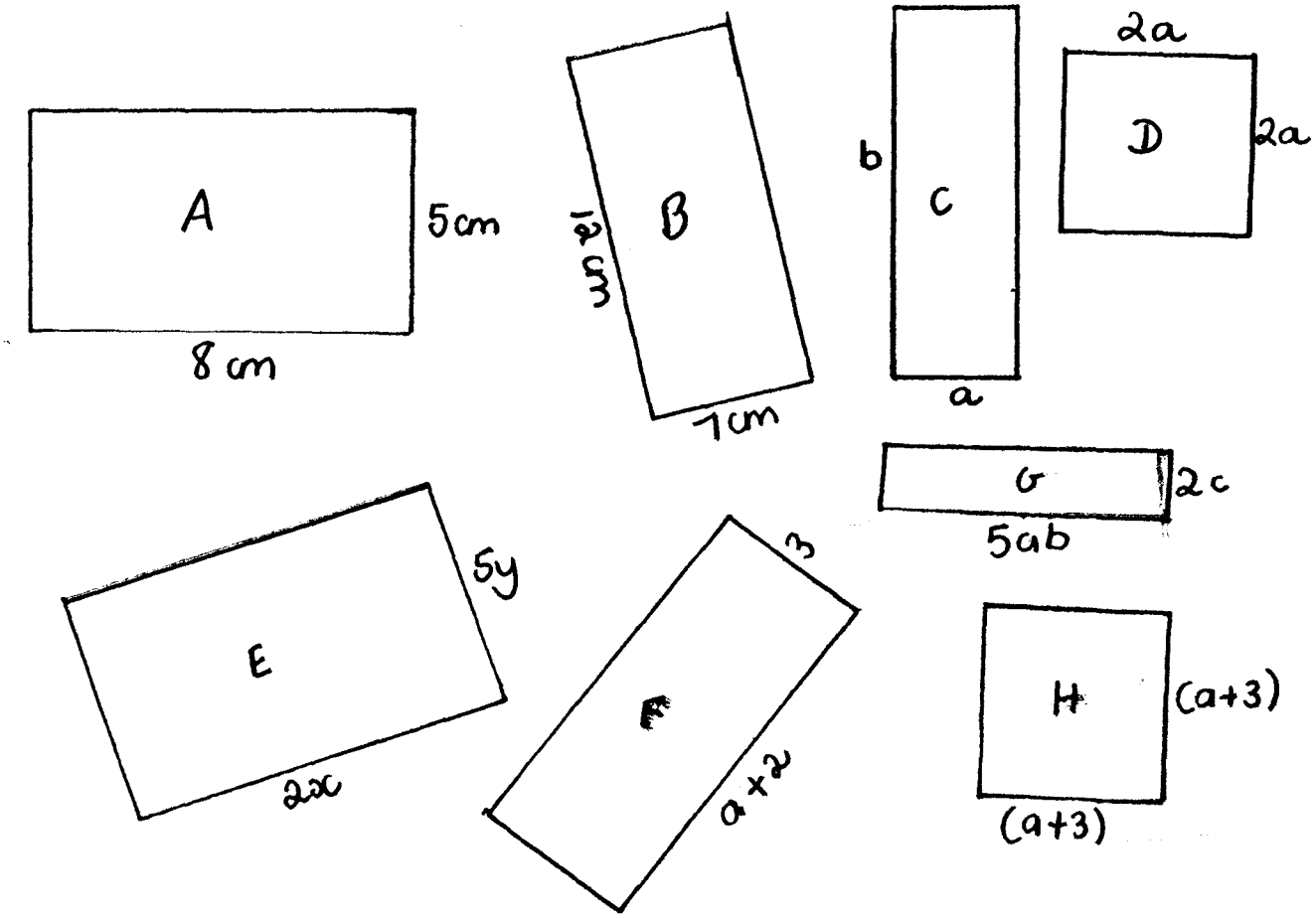
(If you cannot answer this question move to the next worksheet)

APPENDIX 7

WORKSHEET 2

Different shapes (rectangles and squares) are given with their sides as shown in the diagram. For some, sides are given in numbers and for others in algebraic terms or expressions.



For each rectangle, area = length x breadth. Consequently, length x breadth = area. That is, for rectangle A, area = 8 cm x 5 cm = 40 cm. Therefore, 40 cm = 8cm x 5 cm.

Now write the relationships between the area and the sides of all the shapes given in the diagram.

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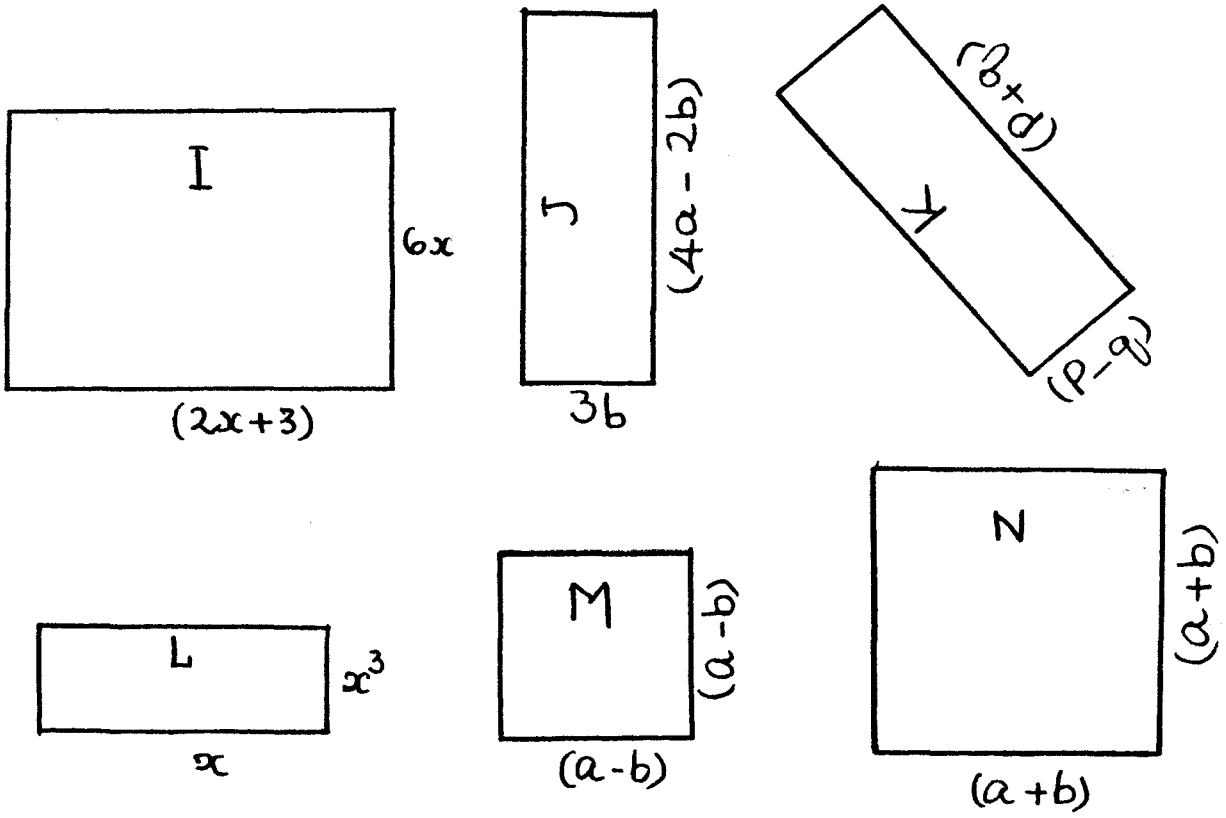
APPENDIX 8

WORKSHEET 3

Through solving problems in worksheet 2, you have seen that the areas of rectangles and squares can be written using numbers as well as algebraic terms or expressions. In worksheet 3 more algebraic terms and expressions are given as sides of rectangles and squares. Now relate these expressions or terms (sides) to the area of each rectangle.

Example; for rectangle I, area = $6x(2x + 3) = 12x + 18x$.

It follows that, $12x + 18x = 6x(2x + 3)$.



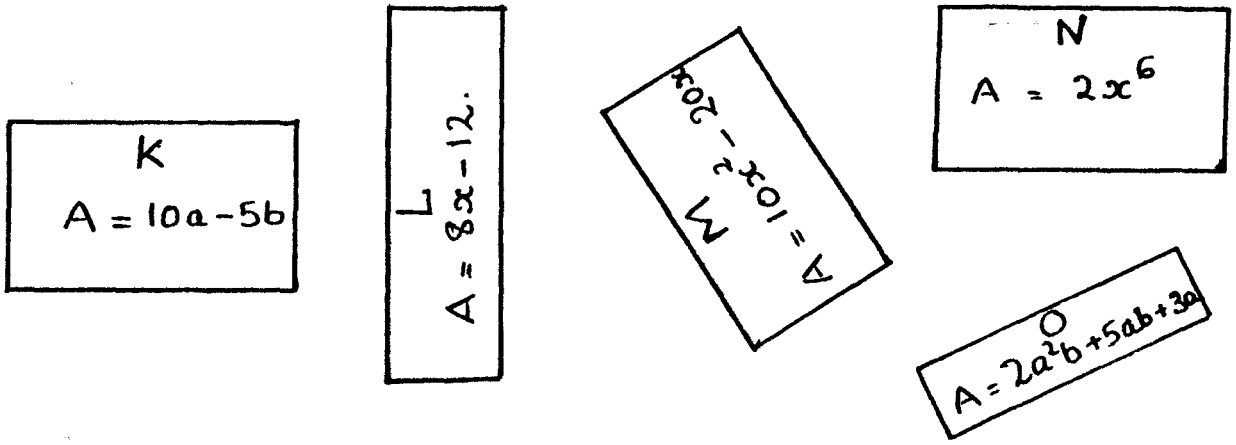
PROBLEM 2

Draw two rectangles showing sides in algebraic expressions. Relate the sides of each rectangles to its area.

APPENDIX 9

WORKSHEET 4

Different rectangles are given in the diagram with areas in algebraic expressions.



PROBLEM 1

For rectangle K, the area is 10a - 5b. If its breadth is 5 cm, find its length?.

Area = 10a - 5b = 5(2a + b). So the length is (2a + b). It follows that 5 when multiplied by (2a + b), the product is 10a - 5b. So, 5 and (2a + b) can be called the .....of (10a - 5b).

PROBLEM 2

Area of rectangle L, is 8x - 12. Write the possible values of lengths of breadths?.

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PROBLEM 3

Write all the possible values of lengths and breadths for rectangles M, N and O?.

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APPENDIX 10

WORKSHEET 5

PROBLEM

Given expressions represent the areas of rectangles or squares. Find out all possible sets of sides for each shape?. From your answer write the factors of each expression.

(1) $10a - 15b$

(2) x^4

(3) $6xy - 12xz$

(4) $(12p + 24q)^2$

(5) $(2ab^2c^3 - 10abc)$

(6) $4wa - 15wb + 16wc + 18wd$

APPENDIX II

WORKSHEET 6

FACTORISATION

What is meant by the factorisation of an expression?

So far you have seen that algebraic expressions may represent the areas of rectangles or squares. You have also seen that these expressions can be written as the products of the lengths and breadths or the sides of these rectangles and squares. These lengths and breadths or sides are called the **factors** of the expression. So when you wrote an algebraic expression (representing the area of a rectangle) you were actually factorising that expression. **Writing and expression as the product of factors or `length' and `breadth' is called factorisation.**

PROBLEMS

Write the following expressions as product of factors in as many ways as possible. Of these different ways choose the form in which the highest factor appears.

(1) $70ab - 14bc + 35cd - 21d$

(2) $15x^3y - 55x^2y + 20xy.$

(3) $30x^6y^2 - 75y^2z^3$

(4) $32g^4h - 42g^2h^2$

APPENDIX 12

WORKSHEET 7

PROBLEMS

Factorise the following expressions taking the highest common factor out.

(1) $30x^6y^2 - 55x^2y + 20xy$

(2) $32g^4h - 4g^2h^2$

(3) $5abc + 13a^4b^7c^4$

APPENDIX 13

WORKSHEET 8

PROBLEMS

Factorise the following expressions (Hint: factorise in groups)

(1) $2x + 4 + 5x + 10$

(2) $ax - ay + bx - by$

(3) $6y - 12x + 12y - 6x$

(4) $2(p - q) - s(p - q)$

(5) $x^2 + 5x + 6$

APPENDIX 14

WORKSHEET 9

PROBLEMS

Facrtorise the following expressions

(1) $x^2 + 4x + 3$

(2) $7x^2 + 18x + 11$

(3) $3x^2 + 3x - 36$

(4) $8p^2 + 35pq - 78q^2$

APPENDIX

NEW PROBLEM-SOLVING APPROACHES IN MATHEMATICS LEARNING

IMPORTANT:

Mathematical problems can be solved using so many methods. Problem-solving in groups is one of such methods.

IMPORTANT POINTS TO BE REMEMBERED ABOUT GROUP DISCUSSION IN MATHEMATICS CLASSROOM:

1. Through group discussion you get a chance to discuss your mathematical problems with your friends or colleagues.
2. It gives every pupil a chance to become familiar to others mathematical thinking and in the light of which to evaluate his/her own thinking through discussions, arguments and criticism. Perhaps such discussions might lead to you better understanding of mathematical problems.
2. Discussion in mathematical classroom does not imply that one simply agrees with or accepts others' ways of thinking without any mathematical reasoning. In group discussion each member has the right to agree or disagree with the ideas generated by others, on mathematical ground, and to ask questions like 'why?' 'what?' and 'how?'.
4. While discussing in group one should listen to others' ideas very carefully.
5. If the group members find it hard to reach a solution to a problem it should be reported to the teacher.

A SUGGESTED PROCEDURE TO HANDLE PROBLEMS IN GROUPS

STEP 1

The problem is first read and understood thoroughly through individual attempt or group attempt.

STEP 2

A solution or a solution process is developed either individually or in group. This should be mathematically reasonable to every individual member.

STEP 3

If everybody in a group is satisfied with a solution present it to the teacher to comment on it. If no agreement is reached among the group members regarding a solution it should also be reported to the teacher.

APPENDIX 15(b)

HOW DO YOU SOLVE A PROBLEM?

It is difficult to explain exactly how do solve a problem. But we all know that it involves thinking, assuming, imagination, speculation, enquiry, discovery, checking and so on.

An example for a problem is given below. Let us together find the solution to this problem.

> There is a frog which climbs everyday two steps up and then one step down. How many days does it takes <u>just</u> to reach on the 30th step?

QUESTIONNAIRE 1

Date..29.-4.-1993.....

1 How do you find mathematics as a learning subject?

very easy	not very easy	it is manageable	difficult	I don't know

2 To what extent do you like mathematics?

very much	like it	not very much	do not like it	I don't know

3 What do you think about the importance of mathematics as a learning subject?

very important	important	not very important	useless	I don't know

4 To what extent do you like group work in learning?

very much	like it	not very much	do not like it	I don't know

5 Do you practice group working in subjects other than in mathematics

yes	no

(continues in the next page)

6 If you practise group work in other subjects what do you like the best in it?

Copying answers from other group members because you yourself cannot reach the answer
Copying others' answers because it is easier
Discussing the different ways of answering the questions with your friends so that you have a better understanding
simply agreeing to others opinions and answers
Any other reasons
I don't know

7 Do you wish to have group working in mathematics?

yes	no

8 If your answer for no `7' is `no' explain your answer.

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9 If your answer for no `7' is `yes', is it because;

you can copy solutions from your friends
you can ask your friends to help you with solutions if you find the problem difficult
you will get a chance to talk to your friends about your thinking and understanding so that you can try together new ways of solution.
any other reasons
You don't know

10 While working in group, in mathematics do you think that one should always agree to what others say?

yes	no

11 If your answer for no.10 is 'no' explain your answer.

.....

12 If your answer for no. 10 is 'yes', why do you agree to what others say?

Because it is said by your best friend
Because it is said by somebody who knows maths very well
Because you do not know how to disagree
Because you do not know like to disagree
Any other reasons
You don't know

APPENDIX 16(b)

QUESTIONNAIRE 2

Date. 27-05-1993.....

1 How do you find mathematics now?

very easy	not very easy	it is manageable	difficult	I don't know

2 To what extent do you like mathematics now?

very much	like it	not very much	do not like it	I don't know

3 What do you think about the importance of mathematics as a learning subject?

very important	important	not very important	useless	I don't know

3 Do you like group working in mathematics now? yes/no Why?.....

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4 Do you think working in group improves your mathematical understanding? yes/no. Why?

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.....

5 Do you like to continue learning mathematics in group? yes/no. Why?

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APPENDIX 16(C)

QUESTIONNAIRE 3

Date. 12.-.08.-.1993.....

1 How do you find mathematics now?

very easy	not very easy	it is manageable	difficult	I don't know

2 To what extent do you like mathematics now?

very much	like it	not very much	do not like it	I don't know

3 What do you think about the importance of mathematics as a learning subject?

very important	important	not very important	useless	I don't know

3 Do you like group working in mathematics now? yes/no Why?.....

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4 Do you think working in group improves your mathematical understanding? yes/no. Why?

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5 Do you like to continue learning mathematics in group? yes/no. Why?

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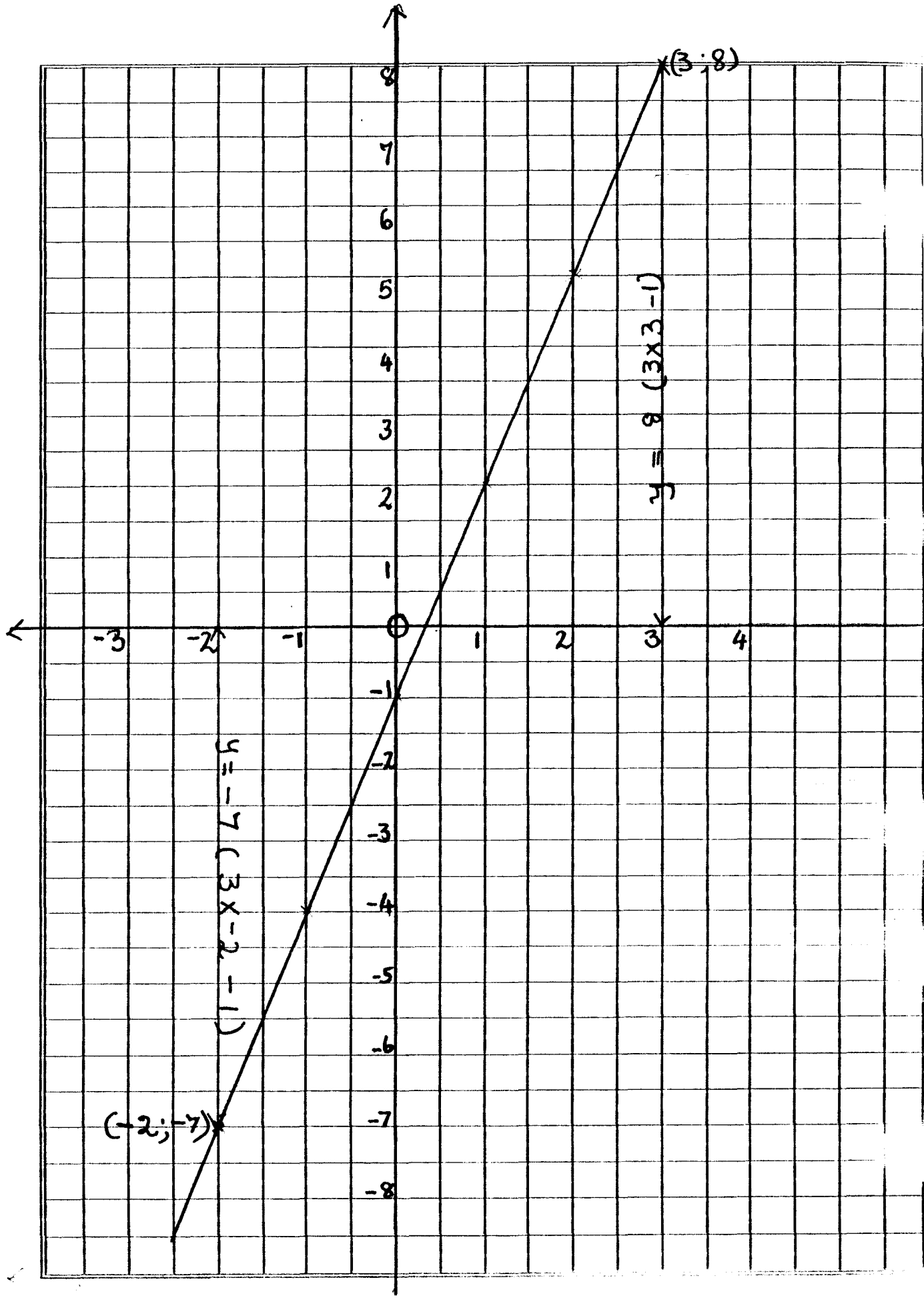
APPENDIX 17

GRAPH OF $Y = 3x - 1$.



APPENDIX 18

PROBLEMS

(1) When 8 is subtracted from thrice a number you get 81. What is that number.

(2) In an English test, David's score was only half of Johny"s. If Johny got 84 marks, how much marks did David get?

(3) Divide 175 in to two numbers such that one number is six times the other number. Calculate the numbers.

(4) A father gave R1000 to his two sons Jabu and Jebu. He told them to give R300 to their mother and divide the rest in between them such that Jabu gets three times that of Jebu. Calculate the amount they get.

(5) Price of an apple is pc (cents) and that of a banana is qc. Zebe bought 10 apples and 5 bananas. Calculate the total cost.

(6) Price of one litre petrol is 76c. If the price is increased by yc, calculate the new price.

(7) Price of a radio is R573. If the price is decreased by y, the new price is R379. Calculate the value of y.